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Abstract

Public transport networks (PTN) are affected daily by different types of disturbances. In fact, between a single delay and a long service interruption, there is a range of disruptions with different impacts, dependent by their characteristics. Despite this, in literature the common definition of disruption is a link closure for a certain amount of time. Low interest is given to different types of disruptions or to the connection between delays and disruptions. In addition, in multimodal PTN a link closure is not always observable, but rather people experience delays or cancelled stops on different lines. The aim of this work is to explore the relationship between delays and disruptions, analyzing different degrees of disruptions, in terms of duration, delay, size and network characteristics. Real disturbances of the Zürich and Bern PTNs are analyzed to identify disruptions with different characteristics. Hence, the disruption impact is computed on simulated ODs, based on the sets of possible paths with and without the disruption. Finally, relationships between the disruption characteristics and the impact are analyzed to identify the main features of a disruption.

Keywords
Disruption; Public Transport network; Vulnerability; AVL Data; Features Importance

1 Introduction

Public transport networks are characterized by daily unexpected delays or cancelled runs. The impact of each of them depends by multiple factors, such as network characteristics and the entity of the disturbance. For instance, a cancelled run of a bus travelling in a city center has a different impact than a bus travelling in a rural area. In addition, combination of delays and failed trips can affect particular areas of the PTN more than single disturbances. Typically, a major exceptional event, in terms of duration and effects, is defined as a disruption. Nevertheless, there is not a clear distinction between small and big disruptions, but rather there is a continuous range of disruptions with different impact.

With this study, we aim to understand the impact of disruptions to the affected demand from their characteristics. In that way, knowing the characteristics of a disruption, public transport providers can better deal with unexpected events. To identify different disruptions, real disturbances of the Zürich PTN and Bern PTN are extracted from one year of AVL data and grouped through a clustering algorithm. Therefore, the estimated impact of each disruption is analysed on different ODs considering a range of possible paths for the passengers. Finally, the relationships between the disruption characteristics and the impact are analysed through machine learning and feature importance metrics.

2 State of the Art

In previous works, the definition of disruption in PTN is generally simple and network based. Typically, a disruption is described as a node or a link failed for a certain amount of time, without traffic admitted through it (Cats and Jenelius (2014), Rodríguez-Núñez and García-Palomares (2014)). This definition can be consistent with a railway/metro network and with long disruptions. Instead, for multimodal networks including buses, a disruption can be better defined from the operational perspective, taking into account delays or missed runs. In this regard, Sun and Guan (2016) analyse vulnerability from line operation perspective, but they consider only a metro network and a disruption as formed by cancelled trips. In the literature of transport disruption and vulnerability studies, few works examine public transport networks compared to road networks (Mattsson and Jenelius (2015)). In addition, most of them are focused only on metro (Rodríguez-Núñez and García-Palomares (2014), Lu (2018)) or railways (Van der Hurk (2015)), instead of considering a multimodal PTN. On that area, Leng *et al.* (2018) analysed the user's behaviour in a multimodal network, but they considered only railway disruptions.

Most of the previous works are focused on identifying critical links or stations and few attention is given on analyse the impact of disruptions with different characteristics. Burgholzer *et al.*

(2013) described a disruption by its duration (2 hours the smallest), its time of occurrence and the capacity reduction. Cats and Jenelius (2018) analysed the relation between the extent of capacity reduction and its consequences on PTN performance, but they did not examine other disruption characteristics. One of the few works considering different characteristics of a disruption is Jenelius (2009), even if on a road network (e.g. road density, user travel time, traffic flow). They investigate the dependence of the effects of link closure on several indicators using a regression model.

Focusing on the methodology, Mattsson and Jenelius (2015) identified two distinct traditions in disruption analysis: topological vulnerability analysis, based on the topological properties of the transport network; system-based vulnerability analysis, that represents also the demand of the transport systems. In the first group, Angeloudis and Fisk (2006) study the degree distribution of different subway networks of the world and they simulate attacks on the stations to analyse the network robustness. In the second group, the interaction between demand and supply is simulated by means of transport system models. Typically, the passengers' behaviour is modelled as the shortest travel time (Van der Hurk (2015), Rodríguez-Núñez and García-Palomares (2014), Lu (2018)) or using discrete choice models (Cats and Jenelius (2014)). Therefore, the impact of a disruption is primarily measured based on the whole traffic in the network (Cats and Jenelius (2014), Cats and Jenelius (2018), Burgholzer *et al.* (2013)). To the best of our knowledge, the impact is never analysed on single ODs or considering the entire choice-set of a user.

A key missing aspect in literature is the analysis of short disruptions (in the order of minutes), although they are the most frequent disruptions people experience in daily trips. In addition, the relationship between disruptions and operational delays is seldom analysed. Instead, it is reasonable to think that they are linked phenomena and there is not a strict boundary between them.

3 Methods

The methods used to understand the impact of different types of disruptions on a PTN, can be divided in three parts. First, the concept of disruption is defined and several disruptions are identified; second, the impact of the disruptions is computed; third, the relationships between the disruptions' characteristics and their impact are analyzed.

3.1 Disruption Identification

The definition of a disruption as a link closure is not realistic in the case of a multimodal PTN. In fact, the network traffic is characterized by delays or missed stops (i.e. a bus that did not stop at a stop), that can not be described by a link closure. Therefore, a new definition of disruption is necessary, able to include delays and failed trips, and able to represent both short and long-term disruptions. We define an event as an arrival of a public transport means at a stop (considering departures do not change the analysis significantly). Therefore, we define a disruption as a set of delayed or missed events near to each other in time and space. This definition is not strict, but it allows both to connect delays to disruptions and to determine many characteristics for disruptions, of which impact can be analysed afterwards.
To identify real cases of disruptions, AVL data are used, seeking clusters of delayed or failed events. To find the clusters, the ST-DBSCAN algorithm is used (Birant and Kut (2007)). This algorithm is a variant of the clustering algorithm DBSCAN, used to cluster spatio-temporal data. DBSCAN is a density-based algorithm that groups together points close to each other, based on a distance metric. In ST-DBSCAN both a spatial distance and a temporal distance are used, to form clusters of points (representing events) close in time and space. Given a set of delayed or failed events, ST-DBSCAN is able to detect groups of related events that satisfy our definition of disruption. The algorithm takes as input the following values:

- *P*: set of points to cluster, with two spatial and one temporal coordinates (i.e. latitude, longitude, time)
- *epsSpace*: maximum spatial distance to consider two points as near
- *epsTime*: maximum temporal distance to consider two points as near
- *minPoints*: minimum number of points within *epsSpace* and *epsTime* to form a cluster

Therefore, the output is a label assigned to each point (event), representing its cluster. A point can also be marked as noise if it is not part of any cluster. In that way, an isolated event is not considered as part of any disruption. Referring to one day of service, we define *P* as the set of all the events with a delay $\geq$ *minDelay*. Failed events are considered as delayed events with a delay equals to the time difference with the next same event (same line at the same stop). The parameter *minDelay* acts as a threshold for too small delays, since we assume they have a marginal impact compared to others. Furthermore, the $\Delta\epsilon$ parameter of the ST-DBSCAN is not used ($\Delta\epsilon = \infty$). For a detailed description of the algorithm, we refer to Birant and Kut (2007).

3.2 Disruption Impact Evaluation

Since we aim to consider also short disruptions (in our experiments we set *minDelay* = 6 minutes), we decided to evaluate the impact of a disruption only on ODs directly affected by it, without considering capacity constraints or a full OD matrix of the network. Therefore, we considered ODs starting from the center of mass of the disruption at its beginning (the planned time of the first event of the disruption). The destinations are chosen randomly among the stops of the network. For each OD two choice sets are generated to model the possible paths with and without the disruption. The first is based on the timetable, without considering disturbances in the network; the second considers as the only disturbances in the network the events of the disruption. In this way, we can evaluate the impact of a disruption comparing the two sets of alternative for a OD. Using the whole choice set, instead of a single optimal path, can better describe the disruption impact, since more possibilities for the user are taken into account.
We modelled the PTN operations as a graph $G = (N, A)$ from the AVL data. Each node in N is a triple *(A/D, tripId, stopId)*, representing the arrival or departure (*A/D*) of a public transport vehicle (*tripId*) at a stop (*stopId*). The arcs in A model the trips in the network and the possible transfers. A trip arc connects two nodes with the same *tripId*; a transfer arc connects an arrival node with a departure node with different *tripId* but with near stops.
For each OD the choice sets are based on the K-shortest paths (K-SPs) (Yen (1971)), choosing as cost function the total travel time with a transfer penalty of 5 minutes. Douglas and Jones (2013) reviewed transfer penalty estimates in literature and they showed there is not a common used value, even if most of the estimates range between 5 and 9 minutes of travel time. Regarding the choice set, the following paths are not considered: paths passing two times at the same stop; paths with the same means but different stops of paths already selected (e.g. boarding on the same bus at a different stop). The walking speed for transfers is set to 1.4 m/s. Instead, the walking times from the origin to the first stop and from the last stop to the destination are set to 0, to give more flexibility to the user's choices. For modelling and computational reasons, the following constraints are added to the model: max distance within nearby stops considered in transfers = 350 meters; max waiting time of 20 minutes; the K-SPs are limited to a cost double the first SP cost or to K = 250 paths.
We defined the impact of a disruption on a certain OD as the difference of the average travel cost of the two choice sets (Equation 1), that represents the difference of travel cost in case of disruption with the case of no disruptions. Full information on the disruption is assumed for the users. Each path is weighted by the probability to use it, computed using a multinomial logit model. The cost function used (C_j) is the same to build the choice set and the calibrated parameters are based on Montini *et al.* (2017).

$$impact(od, dis) = \frac{\sum_{j}^{j \in P(od,dis)} e^{-\beta C_j} C_j}{\sum_{j}^{j \in P(od,dis)} e^{-\beta C_j}} - \frac{\sum_{j}^{j \in P(od)} e^{-\beta C_j} C_j}{\sum_{j}^{j \in P(od)} e^{-\beta C_j}} \tag{1}$$

$P(od, dis)$ = choice-set for the given od and disruption. (2)

$P(od)$ = choice-set for the given od without any disruption. (3)

3.3 Features Analysis

Analysing the relationship between an OD and the impact of a disruption, we can determine how much the impact of the disruption on the OD depends on its characteristics and which of these are more important. First, we extracted 19 features for each OD, describing size of the disruption, duration, service frequency, network metrics and other characteristics of the disruption and the OD. The list of features is shown in Table 1. Therefore, the features importance to predict the impact is analysed computing the mutual information and using random forest regression.
The mutual information is a measure of the amount of information one random variable contains about another (Cover and Thomas (2006)). Given two random variable X and Y, the mutual information is the following:

$$MI(X, Y) = \int_X \int_Y p(x, y) log(\frac{p(x, y)}{p(x)p(y)})\, dx\, dy \tag{4}$$

This metric determines the similarity between the joint distribution *p(x,y)* and the products of the marginal distributions *p(x)* and *p(y)*. In fact, if the two variable are independent, $MI(X, Y) = 0$. Therefore, it is possible to rank the features by their mutual information with the impact. Applications of this metrics for a related task can be found in Chandrashekar and Sahin (2014). Indeed, they presents different feature selection methods, including methods based on mutual information to rank features by their importance. Nevertheless, this measure does not capture the relationships among features and it is possible that a feature has a great importance only if combined with others. In contrast, a random forest regression considers multiple features in one single model. To fit the regression model, we used 67% of dataset as training-set and cross-validation to estimate the parameters. The regression can show how much the features are able to describe the impact, and can rank them based on a metric called *Mean Decrease Impurity* (MDI) (Breiman (2002)). Considering a feature x, its MDI value is computed as follows:

$$MDI(x) = \frac{1}{|T|} \sum_{T} \sum_{n \epsilon T : v(s_n) = x} p(n) \Delta i(s_n, n) \tag{5}$$

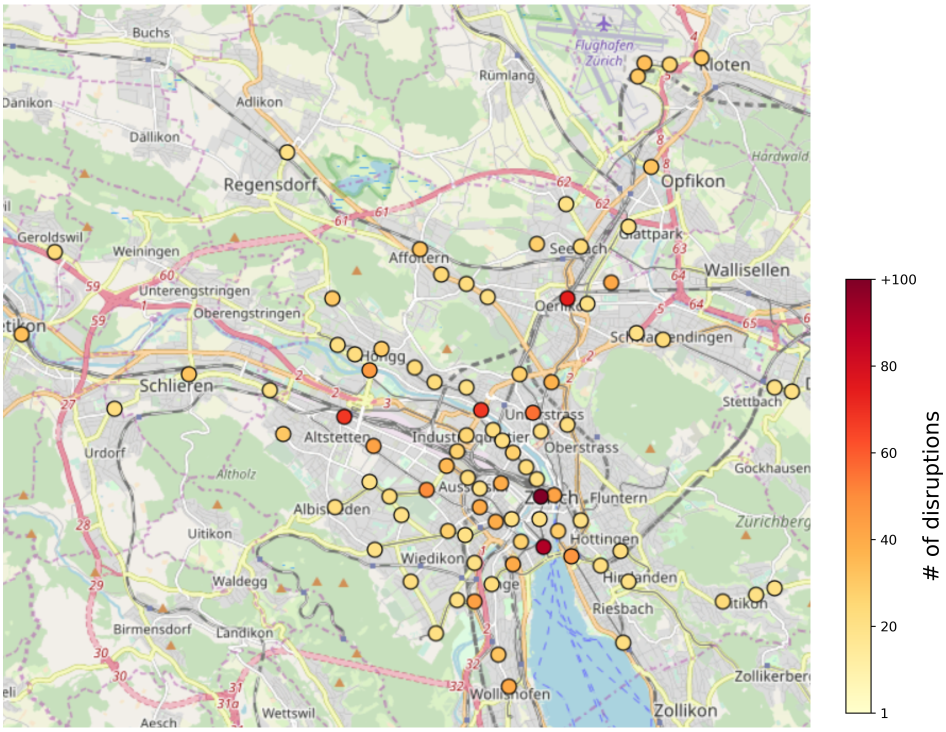
where T is the set of trees in the forest; n is a tree node s.t. the split (s_n) is made on the feature x; $p(n)$ is the proportion of samples reaching n; $\Delta i(n)$ is the decrease (difference) of the considered impurity measure after the split s_n. In case of regression tree i is the variance. For more details on feature importance in random forest, we refer to Louppe *et al.* (2013). We want to remark that particular attention must be given to correlated features, since this metric tends to distribute their importance.

4 Experiments and Results

4.1 Disruption Identification

For our experiments, we used 8 months (01-08/2018) of AVL data of the city of Zürich ($\approx$ 57 million of events) to analyse realized disruptions. In Section 5 we present similar analysis on the city of Bern. To identify disruptions with the ST-DBSCAN algorithm, for each day, all the events with a delay $\geq$ 6 minutes (*minDelay*) are selected for clustering. A threshold of maximum 3 hour of delay is also considered to filter possible errors in the data.
The following values are assigned to the ST-DBSCAN parameters: *MinPts* = 5, *epsSpace* = 250 meters, *epsTime* = 4 minutes. Given the intentional ambiguity of the definition of a disruption, a precise tuning of these parameters is not possible. Therefore, they have been selected by manual experiments by the authors to have a moderate number of disruptions per day and events per disruption. In our experiments, 2528 disruptions were detected ($\approx$ 10 per day). To avoid bias due to different timetables (e.g. during weekend), we considered in this analysis reported only disruptions with events that have happened also on a normal working day, the 01-10-2018 (1301 disruptions). The spatial distribution is shown in Figure 1. We can see that most of the disruptions are located near the city center or railway stations. Figure 2 presents the distribution of number of events per disruption. This shows that the number of events seldom become very high (>= 16+), leading to clusters formed by events close to each other. This is also shown by the average number of stops involved in a disruption, that is 2.67. Figure 3 shows the average delay among the events of each disruption. The analysed disruptions have a median of average delay of $\approx$ 10 minutes, showing that the analysis is focused principally on small disruptions. The spatial distribution of the number of events and the average delay are shown in Figure 4. There are no particular region where the number of events per disruption is higher, showing that the clustering algorithm is not sensitive to the specific area. Instead, the average delay is slightly higher in the city center, due to the higher amount of traffic. Regarding the types of mode, on average disruptions involve 97% of times buses and trams and 3% of times trains. This is explained by the fact that only 5% of events (arrival of a vehicle at a stop) are made by trains

Figure 1: Distribution of disruptions in the city of Zürich (from 01-01-2018 to 31-08-2018).



Source: map from openstreetmap.org.

Figure 2: Distribution of events per disruption.

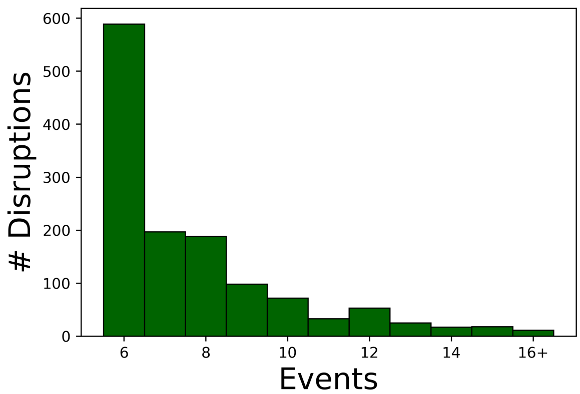
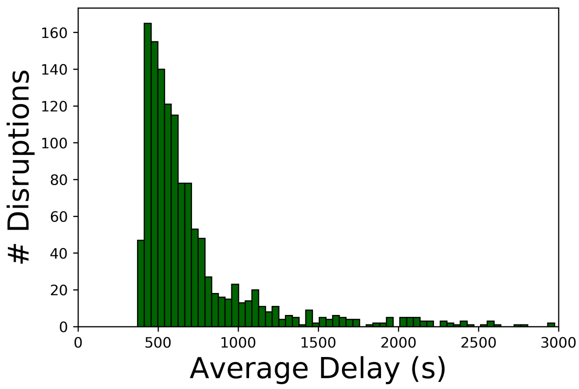


Figure 3: Distribution of the average delay among the events of each disruption.



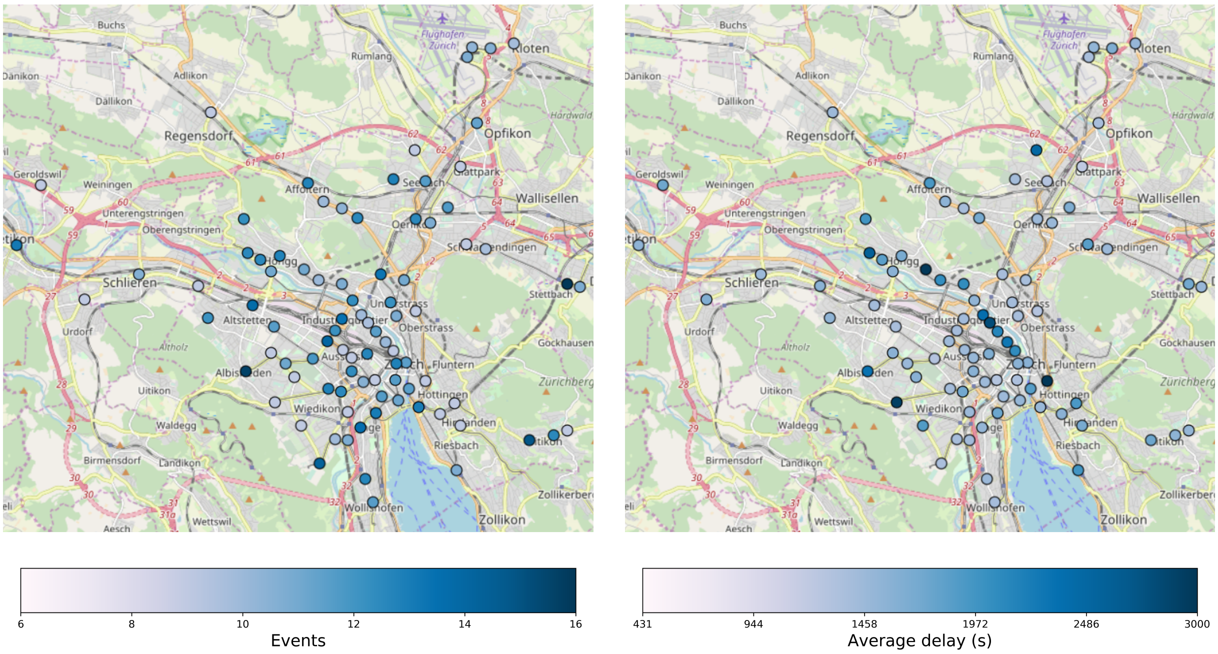
in Zürich.

4.2 Disruption Impact Analysis

To evaluate the impact of the identified disruptions, for each disruption 10 different random ODs are created and the impact on each OD is computed, as explained in Section 3.2. The ODs not

Figure 4: Disruption properties distributions: (a) number of events per disruption; (b) average delay of the events per disruption. For disruption in the same location, the average value is reported.

(a) (b)



Source: map from openstreetmap.org.

affected by the disruption (i.e. none of the disrupted means is ever used to reach the destination) are discarded (8.7%). In total, 11313 OD pairs were analysed. We want to remark that, since the impact function is based on the multinomial logit model, a disruption can also have a negative impact. For instance given an OD, if a disruption affects the worst path in its choice-set, the probability to choose a better path increases, reducing the average travel cost.

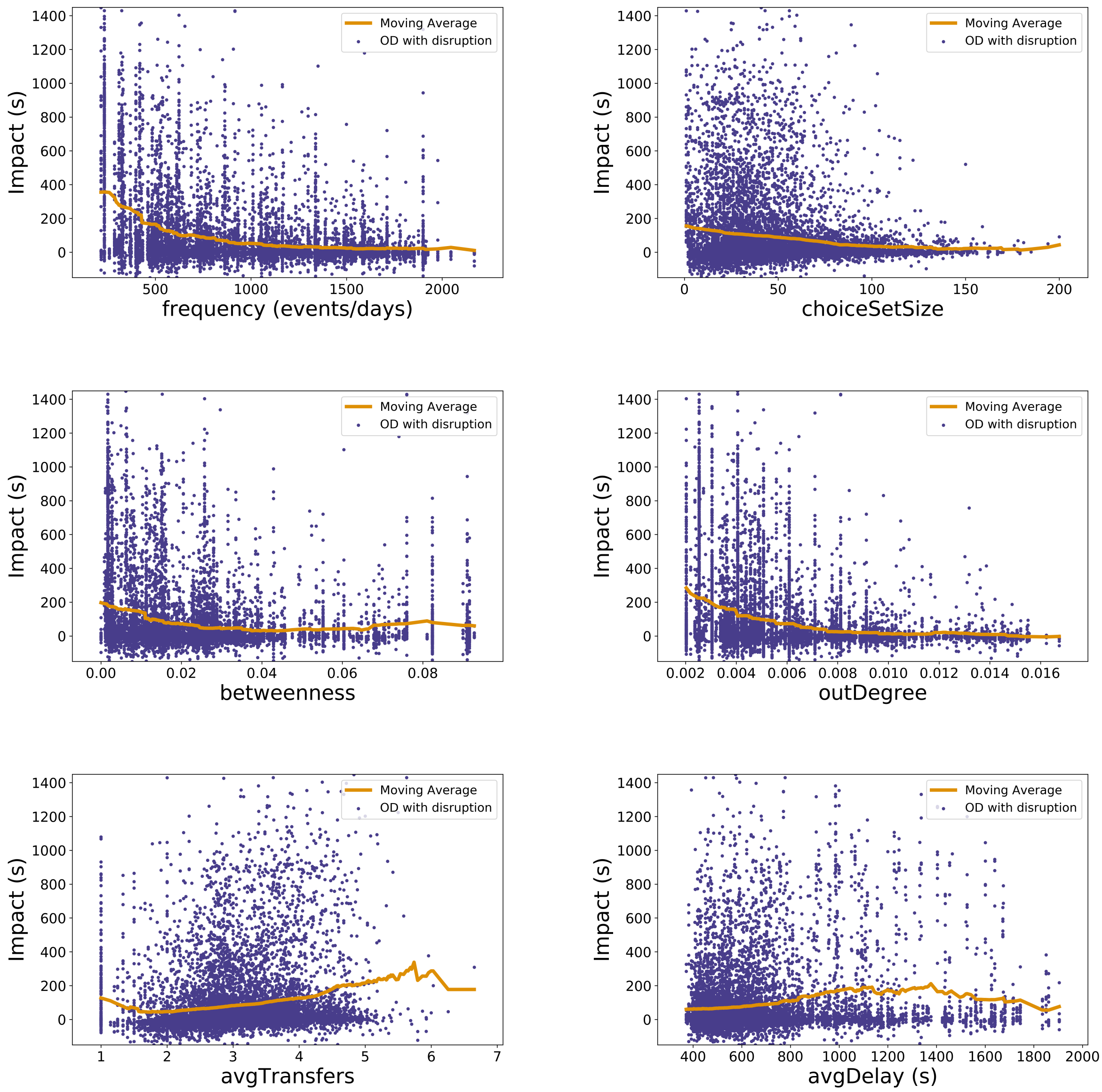
The relationship between the features of each OD and the impact are analysed as explained in Section 3.3. The random forest regressor gives an $R^2 = 0.48$ (i.e. half of the variance in disruption impact depends on the identified features), that can be considered an acceptable value, even if there are not studies with which to compare the results. This proves that it is possible to predict the impact of a disruption (as defined by the authors) from its characteristics. The results of the features importance analysis are shown in Table 1. Given the complexity of the task and the high correlation among the features, the values in Table 1 must be judged as useful to make general conclusions and not as strict rankings. The most relevant feature in both the metrics is the *frequency* (of service). This proves that a high-frequency service can contrast delays or single failures. Slightly less important, with an high MI, are three network metrics (out-degree, closeness and betweenness centrality), proving that the impact of a disruption is dependent by its location and connectivity in the network. These metrics are computed on a static network with a node for each stop and arcs weighted by the travel time. Two stops are

Table 1: Feature importance: features rankings based on mutual information (MI) and mean decrease impurity (MDI). Features are sorted by MDI. The mark (AVG) means that the feature is computed as the average among the events of the disruption.

Feature	MDI	MI	Description
frequency	0.173	0.207	Number of events per day (AVG)
betweenness	0.104	0.171	Betweenness centrality (AVG)
choiceSetSize	0.085	0.113	Size of the timetable choice set
outDegree	0.081	0.187	# of reachable stops (AVG)
avgTravelCost	0.066	0.082	Avg. travel cost in the timetable choice set
distance	0.064	0.053	OD distance
closeness	0.063	0.189	Closeness centrality (AVG)
avgTransfers	0.060	0.095	Avg. # of transfers in the timetable choice set
avgDelay	0.049	0.063	Delay (AVG)
closenessDest	0.046	0.045	Destination closeness
totalDelay	0.045	0.070	Sum of delays of the disruption events
betweennessDest	0.041	0.022	Destination betweenness
events/Perimeter	0.040	0.128	# events / disruption perimeter
duration	0.032	0.027	Disruption duration
trips	0.017	0.030	# vehicles involved
events	0.012	0.021	# events
busPercentage	0.001	0.035	% of buses involved respect to other means
tramPercentage	0.001	0.034	% of trams involved respect to other means
trainPercentage	0.001	0.018	% of trains involved respect to other means

connected if they are connected by a service or if they are distant less than 350 meters. The disruption density (*events/Perimeter*) has a moderate influence, showing that an increase of disturbances in the same area have a greater impact. Instead, features with a lower influence are the duration and the number of events of the disruption. Interesting is that network metrics of the destination have low influence on the impact, proving that it is more important to go away from the disrupted zone. Finally, the type of mode involved in the disruption is not relevant (*trainPercentage*, *tramPercentage*, *busPercentage*). The relationships between a part of the features and the impact are shown in Figure 5. We can see that the impact decreases with higher *frequency*, *choiceSetSize* and *outDegree*. This is realistic, since high values of these features corresponds to a better quality of service in the disrupted area. With the increase of *betweenness* the impact first slightly decreases, but then it increases again. This shows that a disruption has higher impact in a poorly connected area or in a hub, and less in intermediate zones. Considering the delays of the disruption events, the impact increases with *avgDelay* until a certain value ($\approx$ 20 minutes), then an increase of delay is no longer important. Finally,

Figure 5: Relationship between features and impact.



the impact also increases with *avgTransfers* (that means reaching the destination requires more transfers), even if it decreases for values between 1 and 2. This shows that if the OD is only directly connected (*avgTransfers*=1), a disruption has higher impact.

5 Bern network analysis

To strengthen our results respect to a possible bias due to the specific PTN, we repeated the same analysis for a different city of Switzerland, Bern. The major differences between the PTN

Table 2: Zürich and Bern PTNs comparison. We refers to bus/tram lines that provided AVL data in the analysed period.

	Zürich	Bern
area considered	330 km^2	64.25 km^2
pt stops	987	365
bus/tram lines	126	25
events per day (≈)	235000	73700
avg # connected stops per stop	5.34	4.53
std # connected stops per stop	3.7	3.8
avg stop distance (minutes)	1156	735

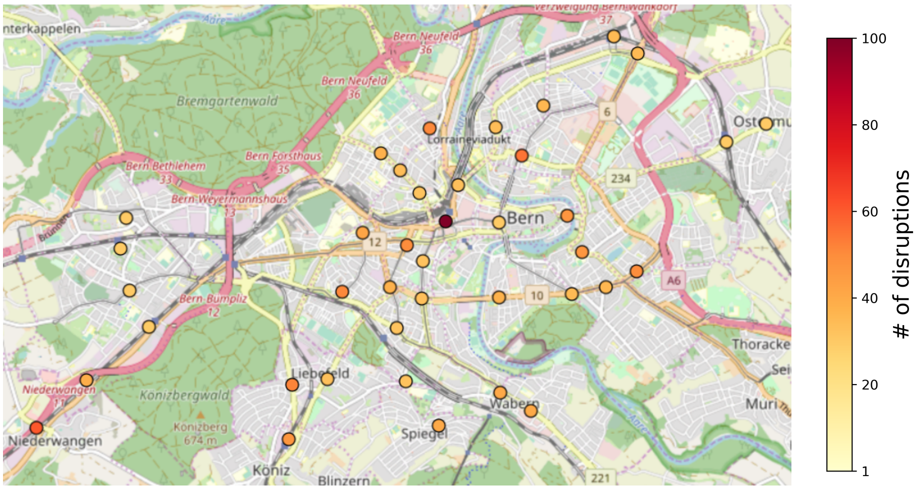
of the two cities considered are highlighted in Table 2. The network of Bern is smaller in terms of area and service and the geographical characteristics of the cities are also different, given for instance the presence of the lake in Zürich. The same disruption identification algorithm was applied in Bern, considering data of all the days of the 2018. Therefore, 463 disruptions were analysed and their distribution is shown in Figure 6. As in Zürich, the biggest number of disruptions is near the main station. For the impact analysis, we selected 20 different ODs for disruption. After discarding ODs not affected by the disruption, we analysed a total of 8060 ODs. The random forest regressor gives an $R^2 = 0.69$, that is higher the one from Zürich. This can be explained by the fact that the PTN in Bern is smaller, therefore it is easier to identify heterogeneity among different disruption location. The feature importance analysis is shown in Table 3. Comparing Table 1 and Table 3 we see that the order of the features is similar for both the tables. This demonstrates that the features of an OD can explain the disruption impact independently by the PTN. In this sense, we can confirm that frequency of service, choice-set size and network metrics play a key role on the disruption impact.

In conclusion, in this Section we showed that the same analysis can be applied to different networks, obtaining similar results. A detailed analysis of how the disruption impact varies in different networks is left for future works.

6 Conclusions

The classical definition of PTN disruption as a link closure has been overcome in this study. A new definition is given, based on the combination of delays and failed trips, in order to represent disruptions with different characteristics and link them with small disturbances in the PTN. AVL data of the cities of Zürich and Bern were used to identify real cases of disruptions. This fills the gap in the literature on short (in the order of minutes) disruptions analysis in multimodal public

Figure 6: Distribution of disruptions in the city of Bern (from 01-01-2018 to 31-12-2018).



Source: map from openstreetmap.org.

Table 3: Bern feature importance: features rankings based on mutual information (MI) and mean decrease impurity (MDI). Features are sorted by MDI.

Feature	MDI	MI	Feature	MDI	MI
closeness	0.095	0.519	outDegree	0.052	0.389
frequency	0.095	0.454	totalDelay	0.045	0.260
choiceSetSize	0.094	0.218	closenessDest	0.045	0.066
events/Perimeter	0.070	0.383	betweennessDest	0.037	0.055
betweenness	0.068	0.517	trips	0.032	0.120
avgTransfers	0.067	0.209	events	0.027	0.088
duration	0.060	0.181	busPercentage	0.021	0.155
avgTravelCost	0.060	0.148	tramPercentage	0.020	0.116
distance	0.055	0.085	trainPercentage	0.005	0.076
avgDelay	0.052	0.293			

transport. An additional aspect in our analysis that differs from the literature is the level of detail. Instead of computing the disruption impact on the whole network, we modelled it on single ODs affected by the disruption, based on two different choice sets, allowing to consider the impact in a fine-grained level and to analyse it for different types of OD. We showed for Zürich that there is a high relationship between the impact of a disruption and its characteristics and we ranked the disruption characteristics according to their influence. In particular, the service frequency, the choice set size of the considered OD and network metrics of the disruption area play a key role on the disruption impact. In contrast, destination's metrics are not so relevant. An interesting finding is that the size and the number of events in a disruption are less relevant than the characteristics of its location. Finally, we repeated the analysis on a different city, Bern, showing that our main findings can be generalized and they are not limited to the particular case study. This paper represents a first step in the analysis of different types of disruptions and several future directions are possible, such as using a different disruption identification method or a different impact function. Analysing the sensitivity of the results to the parameters defining what is a disruption can help to identify the relevant characteristics of a disruption. A further interesting analysis is to evaluate the impact of multiple disruptions at the same time. Moreover, testing the same methodology on big disruptions, that involve the whole network, can help to make a better distinction between small and big disruptions.

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