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Risk Assessments of Complex Infrastructure Systems Considering Spatial and Temporal Aspects

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RISK ASSESSMENTS OF
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CONSIDERING SPATIAL AND TEMPORAL
ASPECTS

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To my parents Annemarie and Erich.
Abstract

Infrastructure systems are important for social development and economic growth. They play a fundamental role in the use and distribution of spatial services, such as transportation and communication. Recent historical events such as the 2002 European flood, hurricane Katrina (2005), or the Tohoku earthquake and tsunami (2011) have shown that the analysis and understanding of large-scale infrastructure systems are essential for research, engineering and society. Especially due to the complexity and interdependence of these systems, localised failures can cascade dramatically, leading to widespread, unforeseen and often disproportionate disruptions compared to the actual physical damage.

Infrastructure managers plan and execute interventions to guarantee the operational state of their infrastructures under various circumstances. This also applies in the aftermath of natural hazard events. As the resources available to managers to protect their infrastructures are limited, it is essential for them to be aware of the probable consequences (i.e., risk) in order to set priorities and be resource-efficient. In order to support infrastructure managers in their risk assessments, this work aims to develop a methodology and corresponding techniques to understand and quantify the risk of complex infrastructure systems affected by natural hazards, considering spatial and temporal aspects.

The first part of this work focuses on the development of a risk assessment process for infrastructure systems affected by natural hazards using computational models to simulate different hazard scenarios and estimate the associated consequences. In this part, a general risk assessment process for infrastructure systems affected by natural hazards is introduced. Based on this process a simulation engine is presented which is constructed as a computational platform to estimate risk by supporting the combination of models from different disciplines. This allows the application of the proposed process to estimate the spatio-temporal risk of a realistic road network due to the occurrence of time-varying multi-hazard events, considering physical and functional effects on network objects (e.g. bridges and road sections), the functional interrelationships of the affected objects, the resulting probable consequences, duration of network disruption, and the restoration of the network. To give better insights into the resilience of the infrastructure system to natural hazards and to help the infrastructure managers to make better decisions in such situations, a restoration model is formulated to determine optimal recovery responses in the aftermath of such events.

While the implementation of the risk assessment process in the first part is mainly based on computational models, the second part focuses on the development of innovative mathematical models from the field of network sciences. First, a network model for interdependent infrastructure systems is presented, which is based on the mathematical concept of the spatially embedded random network, and therefore, needs only a limited amount of data. Second, a complex network approach is used to investigate traffic flow dynamics on road networks, which provides reasonable estimates for traffic flow changes and significantly reduces the computing time of classical simulation models. These models can be used to support the risk assessment of complex infrastructure systems in terms of data requirements and computing power, i.e. computational models can be substitu-
ted if only limited data is available, or a decrease in the computational effort is required.

This work contributes to the field of risk assessment and its application to complex infrastructure systems, by providing a methodology and corresponding techniques to understand and quantify the risk of complex infrastructure systems, affected by natural hazards, considering spatial and temporal aspects. More precisely, this work provides a novel risk assessment process for infrastructure managers, designed to estimate the spatio-temporal risk of complex infrastructure systems due to the occurrence of time-varying multi-hazard events. In doing so, it not only extends the state-of-the-art research in this field but also helps to provide decision support for infrastructure managers.
Zusammenfassung


Infrastrukturbetreiber planen und führen Massnahmen durch, um den Betrieb ihrer Infrastrukturen unter verschiedenen Umständen zu gewährleisten. Dies gilt auch nach dem Auftreten von Naturereignissen. Da die den Managern zum Schutz ihrer Infrastrukturen zur Verfügung stehenden Ressourcen begrenzt sind, ist es für sie unerlässlich, sich der wahrscheinlichen Folgen (d.h. des Risikos) bewusst zu sein, um Prioritäten zu setzen und Ressourcen so effizient wie möglich einzusetzen. Um die Infrastrukturbetreiber bei ihren Risikobewertungen zu unterstützen, zielt diese Arbeit darauf ab, eine Methodik und entsprechende Techniken zu entwickeln, um das Risiko komplexer, von Naturgefahren betroffener Infrastruktursysteme unter Berücksichtigung räumlicher und zeitlicher Aspekte zu verstehen und zu quantifizieren.


Während die Umsetzung des Risikobewertungsprozesses im ersten Teil hauptsächlich auf rechnergestützten Modellen beruht, konzentriert sich der zweite Teil
auf die Entwicklung innovativer mathematischer Modelle aus dem Bereich der Netzwerkwissenschaften. Zunächst wird ein Netzwerkmodell für voneinander abhängige Infrastruktursysteme vorgestellt, das auf dem mathematischen Konzept des räumlich eingebetteten Zufallsnetze basiert und folglich nur eine begrenzte Datenmenge benötigt. Zweitens wird ein komplexer Netzwerkansatz zur Untersuchung der Verkehrsflussdynamik auf Strassen netzen eingesetzt, der angemessene Abschätzungen für Verkehrsflussänderungen liefert und die Rechenzeit klassischer Simulationsmodelle deutlich reduziert. Mit diesen Modellen kann die Risikobewertung komplexer Infrastruktursysteme in Bezug auf Datenbedarf und Rechenleistung unterstützt werden, d.h. die ursprünglichen rechnergestützten Modelle können ersetzt werden, wenn nur begrenzte Daten verfügbar sind oder ein gerin gerer Rechenaufwand erforderlich ist.

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Foreword

Managers of infrastructure networks are responsible for ensuring that infrastructure stakeholders obtain the maximum net-benefit from the infrastructure, i.e. the difference between the benefits and costs of the infrastructure. If there is to be no change in the expected service, this means that they have to plan interventions to ensure that there is an optimal trade-off between the costs of executing and the costs of not executing interventions. As the future costs, due to the ability of infrastructure to provide the expected service are uncertain, risk assessments are an important part of the decision making process. Mr. Hackl has concentrated his efforts on improving this part of infrastructure management.

In his thesis, Mr. Hackl has made numerous suggestions that will enable infrastructure managers to obtain a better understanding of what might go wrong with their infrastructure in the future and the resulting consequences. Mr. Hackl’s five proposals are:

1. A risk assessment process to evaluate the risk associated with infrastructure networks due to natural hazards that is more suitable for use in infrastructure management state-of-the-art processes, due to its completeness and its modular set-up, which enables it to be used with a wide range of models and different levels of computer support. It was devised to ensure that the models used in the process could be easily updated or replaced when required. The general process is given in the first article. A detailed version of the process and an example are given in the second article. The example includes the modelling of multiple time-varying hazard events, the physical and functional effects of these events on the infrastructure assets, the functional interrelationships of the affected assets, how the restoration of the network is likely to occur and estimates of the costs of restoration, traffic changes and duration of network disruption.

2. An algorithm to determine near-optimal restoration programs for transportation networks following natural hazard events using simulated annealing. This algorithm helps give improved estimates of the best ways for infrastructure assets to be restored following natural hazard events. The algorithm is useful in the estimation of risk when many simulations of how networks might be damaged due to natural events and how they might be restored are required. It could, however, also be used to identify the optimal way to restore infrastructure assets following the occurrence of natural hazard events.

3. A methodology to create random networks that conserves their spatial properties and thereby increases the usefulness of random networks in infrastructure risk assessments, in situations where the exact topology of the network is not known, either now or in the future. The methodology includes the use of a spatial non-homogeneous point process for vertex creation, which accounts for the spatial distribution of vertices, considering clustering effects within the network, and a hybrid connection model for edge creation.
4. A multi-layer spatially embedded random network to be used to analyse multiple spatially distributed interacting systems, which, with further development, could help with risk assessments involving multiple interacting networks. The model combines Markov marked point processes for vertex creation, which accounts for the spatial distribution, layer assignment, clustering effects of the vertices and a hybrid connection model for the edge creation.

5. A networks in networks approach for rapid estimation of traffic flow changes in urban areas that can considerably improve the estimation of travel time increases due to damaged infrastructure over classical diffusion processes. This approach can be used in simulation based risk assessments, where it is necessary to quickly assess the resulting traffic flows across damaged networks for many simulations.

Each of Mr. Hackl’s proposals are improvements on the state-of-the-art in their own areas. His proposals, once adopted by infrastructure managers in appropriate ways, will lead to a better understanding of the infrastructure related risks and better decisions as to how to use limited financial resources to reduce this risk. In other words, Mr. Hackl’s proposals will help ensure that infrastructure managers are intervening on our infrastructure in ways to ensure that we are obtaining the maximum net-benefit from our infrastructure.

Although Mr. Hackl, principally used road networks for his examples, his proposals have ramifications on how risk assessments are conducted on other infrastructure networks. I am convinced that researchers focused on many types of infrastructure networks will reference his work for years to come.

Through his work, Mr. Hackl has demonstrated that he has the ability to conduct work rigorously at a high academic level and the ability to use advanced mathematical tools to model the behaviour of complex systems. On behalf of the Institute for Construction and Infrastructure Management at the Swiss Federal Institute of Technology, Zürich, I thank him for his thorough and constant investment to his thesis, his personal drive and motivation, his substantial help over the years with the supervision of Master students, and for his personal contributions to team spirit in IBI.
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Prologue
1. Introduction

1.1. Motivation

Infrastructure systems are important for social development and economic growth. They play a fundamental role in the use and distribution of spatial services, including transportation and communication. Recent historical events such as the 2002 European flood, hurricane Katrina (2005), or the Tohoku earthquake and tsunami (2011) have shown that the analysis and understanding of large-scale infrastructure systems are essential for research, engineering and society. Especially due to the complexity and interdependence of these systems, localised failures can cascade dramatically, leading to widespread, unforeseen and often disproportionate disruptions compared to the actual physical damage.

Natural hazards pose a particular threat to infrastructure systems, as they are spatially distributed, which means that their rate of hazard exposure is higher than that of single-site facilities. This is particularly true of infrastructure systems where multiple objects within the system are likely to be affected simultaneously by extreme loads, i.e., exposed to geographically distributed floods, landslides, or earthquakes. Switzerland alone suffered infrastructure damages, for road, rail and power networks, of CHF 10 billion between 1972 and 2013, as a result of climate-related events such as floods and mudflows (WSL, 2014). It should be noted that the infrastructure damages provided by WSL represent only the value of reported immediate damage when the hazards occur. Hence only direct costs are included such as costs related to clean up, repairs, rehabilitation and reconstruction but no future indirect costs such as the cost of business interruption, temporal prolongation of travel or costs due to missed trips.

The primary objective of infrastructure managers is to ensure that their infrastructure system provides adequate service. This also applies in the aftermath of natural hazard events. As the resources available to managers to protect their networks are limited, it is essential for them to be aware of the probable consequences (i.e., risk) in order to set priorities and be resource-efficient. Since natural hazards can result in substantial negative economic and societal consequences, a significant portion of the probable consequences of infrastructure failures can be attributed to how infrastructure managers restore services once they have been lost.

In particular, managers, whose infrastructures are exposed to natural hazards, are in need of methodologies and techniques to understand and quantify the risk of their complex infrastructure systems, affected by natural hazards. Hence risk assessments are important and can help to identify probable hazard events and evaluate their impact on networks and users. Nonetheless, conducting such assessments can be a particularly challenging task, due to a large number of scenarios that need to be taken into account; for instance, the computational modelling of these events, the relationships among them, and the availability of data and support tools to run the models in an integrated way. In building scenarios multiple types of hazards need to be considered, along with the complex nature of networks, specifically, their large number of objects, their spatial distribution, and functional interrelationships. Moreover, it is crucial to estimate the performance of infrastructure systems during the hazard events and through the recovery to
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an adequate level of service that is driven by restoration strategies.

1.2. Background

1.2.1. Risk assessments of complex infrastructure systems

The purpose of infrastructure systems is to balance supply and demand for services that are essential for modern society. For example, the demand for transport is derived from people trying to satisfy their mobility needs (work, leisure, health, education) through activities in specific places. Transport supply is the service provided at a certain point in time. This includes the infrastructure (e.g. road network) and a set of mobile units (e.g. persons, vehicles, goods). In combination with a set of rules for operation, the movements of persons and goods can be ensured (de Dios Ortuozar and Willumsen, 2011).

In contrast to other engineering systems, most of the infrastructure systems cannot be understood by only analysing its individual parts or components. Interactions or relations of the system elements can lead to non-trivial effects, also implicitly affecting elements that were not part of the immediate interaction. Topological structures and hierarchical order between the elements can impose rules that cannot be explained based on the element level. A system exhibiting such properties can be regarded as complex and requires more than just collecting data and observations to fully understand it (Anderson, 1972; Johnson, 2009). For example, the interruption of an important traffic link might influence the traffic flow on the whole network since travellers have to take detours and thereby affect other travellers. Likewise, most infrastructure systems are rarely isolated but usually interdependent and interact with other systems (Bashan et al., 2013). These complex interactions can facilitate the propagation of failure from one system to another. For example, in 2003 a failure of a power station in Italy caused an interruption of the provision of telecommunication services, which in turn contributed to the failure of even more power stations (Rosato et al., 2008; Buldyrev et al., 2010).

A risk assessment must therefore, take a systems view of infrastructure that requires consideration of a number of crucial elements beyond just the obvious physical parts (Dawson et al., 2018). For example, an appropriate risk assessment process must consider, among other things, the scope of the methodology, the audience to which it is addressed (e.g. policy makers, decision makers, research institutes) and their level of detail and abstraction (e.g. element level, system level, system-of-systems level). These attributes are not mutually exclusive, in the sense that the level of detail defines to a certain extent the target group of the methodology. For instance, a risk assessment methodology which is applicable to system-of-systems at national or even supranational level is mostly addressed to policymakers and relevant authorities and less to operators or to infrastructure managers at a local level (Giannopoulos et al., 2012).

In a traditional (technical) context, the risk is often expressed as the likelihood of the occurrence of a scenario and its consequences for a given area and time period (Kaplan and Garrick, 1981). In order to quantify the risk of complex infrastructure systems affected by natural hazards, a methodology is required which considers spatial-temporal properties and occurrence probabilities of natural hazard events. It should also account for the exposure and vulnerability of the elements which form the infrastructure network, and relationships between natural hazard events, infrastructure systems, and potential societal and economic impacts (Pant et al., 2016b).
Due to the importance of infrastructure systems, a considerable number of risk assessment methodologies have been developed in recent decades. The choice of methodologies is vast and reaches from the classification of interdependencies (Bashan et al., 2013; Nan and Sansavini, 2017; Haimes, 2018), over their use in vulnerability (Erath et al., 2009a; Jenelius and Mattsson, 2015; Pant et al., 2016a) reliability (Li et al., 2016b; Paredes et al., 2018) and cascading failure analysis (Kiremidjian et al., 2007; Dueñas-Osorio and Vemuru, 2009; Zhao et al., 2016b), to system-of-systems analysis at national and supranational scales (Hackl et al., 2016; Hall et al., 2016b; Guo and Haimes, 2016; Thacker et al., 2017; Papathanasiou et al., 2018). An overview of several risk assessment methodologies for infrastructure systems is given by Giannopoulos et al., (2012).

Most of these risk assessment methodologies are designed for policymakers and relevant authorities to get a general overview where critical system elements are located and how prone their infrastructure systems are to natural hazards. However, there is still a gap in methodologies that support infrastructure managers. Managers of transport, water, power and other infrastructure networks are in need of methodologies that also support decisions on maintenance, repair, retrofit, renewal and other infrastructure management interventions, in order to guarantee an operational state under various circumstances, including the occurrence of natural hazard events.

1.2.2. Spatial and temporal aspects

Most infrastructure systems are spatially embedded and cover a wide area. This spatial distribution and the connectivity of the infrastructure system components make the system susceptible to natural hazards. In other words, the spatial proximity between the components of the infrastructure means that they are exposed to similar local environmental impacts. Exposure to natural hazards changes depending on the area. For example, individual components may be exposed to rockfalls, while other components of the same infrastructure may be exposed to floods at the same time.

It is usually easy to specify the possible locations of the events, hazards and infrastructure components; however, it is more difficult to determine how they are related, e.g. heavy rain causes a flood hazard. This becomes even more challenging when the location of possible consequences is to be specified. Consequences can be far away from the location of the events, hazards, and infrastructure, and may be outside the direct area of responsibility of the infrastructure manager (e.g. the collapse of a highway bridge on a trans-European highway network can have consequences on the free flow of goods in many countries).

The spatial scale of the risk assessment can vary from analysing the global picture to the identification of local-level conditions which strongly affects the scale-dependent data information sources applicability and usability (Aubrecht et al., 2013b). Geographic information and related technologies can offer the basis for estimating and mapping risks, for determining damage potentials and impacted areas, for evacuation planning and for resource distribution during recovery (Aubrecht et al., 2013a; Heitzler et al., 2016). Challenges include the data collection, analysis and representation of geographic information, as well as associated dynamics and uncertainties (Cutter and Finch, 2008).

In addition to the spatial dimension, time aspects must also be taken into account in the risk assessment of infrastructure systems. In classical risk assessments, the temporal component is often expressed by the probability of occurrence. The so-called return period describes the probability an event of a certain
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Magnitude will occur within a certain period of time. Besides estimating the likelihood of natural hazard events such as earthquakes (Tsompanakis, 2015), floods (Brink et al., 2005), or landslides (Peres and Cancelliere, 2016), the concept of return period (probability of failure) is also used when dealing with structure design expectations (JCSS, 2008).

However, since the main task of infrastructure systems is to provide services, a further time component needs to be considered for a comprehensive risk assessment. As a result of the dynamic nature between demand and supply, provided by infrastructure systems, temporal attributes such as traffic flow, water demand and electricity consumption are constantly changing. Due to the strong dynamic coupling between supply and demand and in combination with the topology and capacity limitation of the underlying networks, infrastructure systems exhibit atypical dynamic behaviours. Performance deteriorates as soon as the demand in the network exceeds a critical accumulation (Daganzo and Geroliminis, 2008; Hoogendoorn and Knoop, 2012), e.g., vehicles block each other and the flow decreases, leading to spill backs and gridlock effects. This phenomenon is amplified by the fact that even small (unexpected) failures or damage to the infrastructure can lead to significant disruptions that are disproportional to the actual physical damage itself. In order to consider such situations and to estimate the associated indirect costs, the temporal evolution of the infrastructure before, during and after a natural hazard event has to be taken into account.

In order to address this issue, risk assessment methodologies have been extended to take account of the time-varying events and consequences. As a result, the concept of resilience has developed in engineering science over the past years. Regarding the definition and quantification of resilience, they vary by discipline and application (Vugrin et al., 2010b; Ouyang et al., 2012), most concentrate on the ability of a system to withstand hazard impacts and recover in a timely fashion (Thacker, 2015). Studies of infrastructure resilience include among others water distribution systems (Diao et al., 2016; Didier et al., 2018), power networks (Ouyang et al., 2012; Ouyang and Dueñas-Osorio, 2014), transportation networks (Vugrin et al., 2014; Mattsson and Jenelius, 2015; Gómez and Baker, 2019), and consider earthquakes (Bruneau et al., 2003; Gehl et al., 2018), floods (Sayers et al., 2012; Fekete et al., 2017), and landslides (Aydin et al., 2018; Schlögl et al., 2019).

While resilience considerations focus mainly on the post-hazard period, scientists pointed out that the evolution of natural events should also be considered, especially for dealing with cascading events and multiple hazard events (Aubrecht et al., 2013a; Mignan et al., 2014; Gallina et al., 2016). Risk, related to natural hazards, is subject to temporal changes since the risk-influencing factors are variable over time. Therefore, methodologies, focused on identifying, analysing and modelling the temporal development of hazard processes and infrastructure systems at risk, are needed in order to provide information on short-term as well as long-term changes of risk (Fuchs et al., 2013).

The time frame of natural hazard events can vary from a few seconds (e.g. earthquake) to a few days (e.g. flood) to several months (e.g. drought). In order to cope with this, the temporal scale of the risk assessment must also range from high temporal resolutions of monitoring results to long-term planning and investment decisions. The importance of considering such developments has been highlighted by several scholars (Ouyang and Dueñas-Osorio, 2012; Fuchs et al., 2013; Hall et al., 2016a) and is of particular importance with respect to the different temporal dimensions in maintenance planning practices (Adey et al.,
1.2. Background

These issues pose a challenge not only from a scientific point of view but also for infrastructure managers, who are responsible for the design and maintenance of (future) infrastructure systems, as the consideration of this temporal development in risk assessments becomes necessary.

To summarise, any effort to understand and quantify the risk of complex infrastructure systems, affected by natural hazards, requires the ability to deal with the assessment of spatial and temporal dimensions. This has so far often been neglected in traditional risk assessment methodologies, which focus on either temporal or spatial variation, but not both. There is an important feedback between time and location in space, i.e., temporal variability of risk is affected by the change of spatial characteristics of risk and vice versa. Understanding the spatial and temporal aspects of natural hazard events, as well as the geographical limits of their impacts, and the development or materialisation of risk over time is crucial for the risk assessment of infrastructure systems. Particularly when combined with information on social systems affected, for both risk reduction measures prior to an event and response and recovery efforts. Increasing knowledge about one part, such as an advanced understanding of natural hazard events, infrastructure systems and their vulnerabilities, without analysing the interaction between these components and their spatial and temporal dynamics, may therefore not provide a complete picture to develop comprehensive strategies for risk mitigation (Aubrecht et al., 2013a).

1.2.3. Advances in computational modelling

To support infrastructure managers with their task to assess infrastructure-related risks, a modelling framework is required that captures not only the dynamic processes in time and location in space but also integrates different modelling tools, required for solving complex scenarios, i.e., chains of interrelated events. For example, a transportation network is exposed to rainfall, which causes multiple hazards like riverine floods and mudflows. At the same time, these events damage components of the infrastructure leading to direct costs, linked to clean-up, repair, rehabilitation and reconstruction activities, and indirect costs, associated with loss of connectivity and temporal prolongation of network user travel time, linking the modelling of these latter effects with the traffic flow dynamics of the network.

The inclusion of these needs in a comprehensive modelling environment is limited and, where it occurs, it often focuses on specific areas of interest and does not explicitly address all the important factors relevant for an exhaustive risk assessment of complex infrastructure systems (Speight et al., 2017). For example, research has been focused either on the technical aspects of hazards or those of networks. In the first case, scholars have focused on improving the understanding, the modelling or the prediction of single hazard events (Apel et al., 2004; Pritchard et al., 2015; Schlögl and Laaha, 2016; Pellicani et al., 2017) without explicitly considering the complexity and dynamics of networks. In the second case, scholars have investigated the vulnerability (Jenelius et al., 2006; Rupi et al., 2015; Shabou et al., 2017) or resilience (He and Liu, 2012; Boccini and Frangopol, 2012; Vugrin et al., 2014; Sharma et al., 2018) of networks due to disruptions, without evaluating the cause and/or the probability of such disruptions.

Some work has been conducted to consider multiple hazards and their effects...
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(i.e., multiple vulnerabilities and consequences) in a unified framework (Komendantova et al., 2014; Mignan et al., 2014; Gallina et al., 2016; Contento et al., 2019). Assessing the risk in such a way is relatively new, and until now only partially developed by experts with different backgrounds such as statistics, engineering and various fields of geosciences (Komendantova et al., 2014). Only a limited number of scenario-based and/or site-specific studies have been proposed due to the difficulty and novelty of the task (Mignan et al., 2014). Having a diverse team of experts, whose discipline-specific approaches to risk assessment may differ, presents an additional challenge: their contributions and models are not always easy to aggregate to a level that is useful for infrastructure managers.

An integrated modelling framework that considers all contributing factors and the spatial and temporal dependencies between them is therefore required, i.e. a modelling environment that can link multiple natural hazard events and their effects on infrastructure systems to social, economic, and environmental consequences is fundamental to an understanding of the risks and impacts of proposed management decisions (Hall et al., 2003; Apel et al., 2004; Guidotti et al., 2019).

1.2.4. Advances in complex networks

Network theory is often used to describe and analyse complex systems, found in nature, technology, society and human-made structures (Newman, 2010; Boccaletti et al., 2014). The ideas of mathematical graph theory are used to generate insights into the behaviour of complex systems by abstracting information into ordinary graphs (networks). In these representations, the network comprises vertices, connected by edges, where vertices represent single elements and edges indicate interactions or relationships between them. Although this approach is simple in many respects, it enables the characterisations of the complex system so that traditional graph-theoretic metrics can be used and analyses can be done. For example, such abstractions have been used to study growth mechanisms (Barabási and Albert, 1999; Barabási, 2002), processes of collective dynamics (Watts and Strogatz, 1998), and to illustrate that certain vertices play a central role in complex systems (Freeman, 1977; Wasserman and Faust, 1994).

In recent years, the modelling of complex systems as networks of connected elements has become subject to intense study in various fields, including physics, social science, and biology, with the goal of analysing the structure and dynamics of large-scale networks of all kind (Watts and Strogatz, 1998; Albert et al., 1999; Barabási, 2002; Barabási and Bonabeau, 2003; Newman, 2003). Notable contributions to understand real-world phenomena are among others, random networks (Gilbert, 1959; Erdős and Rényi, 1959), small-world networks (Molloy and Reed, 1995), and scale-free networks (Barabási and Albert, 1999).

In order to study more realistic systems, a transition from relatively simple graphs to more realistic ones has started (Kivelä et al., 2014). This enables the development of models which more realistically exhibit real-world properties; for example, an essential property of many real-world systems, such as technological (Andrews et al., 2010) and transportation infrastructure (Chan et al., 2011), or biological systems (Bullmore and Sporns, 2009; Stella et al., 2016, 2018), is that they are embedded in the physical space. Consequently, vertices and edges are subjected to spatial and physical constraints, which affect their topological properties and the dynamic processes, associated with the network (e.g., traffic flow across a road network) (Barnett et al., 2007; Barthelemy, 2011, 2018). Notably, in a case where costs are associated with the length of the edges, the topological properties are strongly correlated to spatial aspects (Schweitzer et al., 1998;
1.3. Aim and objectives

Gastner and Newman, 2006; Louf et al., 2013).

While structural properties are still important in constraining the behaviour of a system (Marr and Hütter, 2005), the focus has expanded to an understanding of the relationship between structure and dynamics that takes place in networks and the impact of this relationship on network design (Toroczkai, 2005). Most technological, biological, economic, social or infrastructural networks support a number of dynamic (transport) processes, such as the movement of information packages (Wang et al., 2006), finance and wealth (Coelho et al., 2005), rumours (Moreno et al., 2004), diseases (Newman, 2002), people or goods. Gradually, these theories have been introduced to the field of transportation. More and more scholars have conducted research on the characteristics of various transportation networks, among others those of (urban) road networks (De Montis et al., 2007; Erath et al., 2009b; Barthélémy, 2011; Lin and Ban, 2013), railway networks (Latora and Marchiori, 2002; Sen et al., 2003), and transit networks (Guo and Lu, 2016; Solé-Ribalta et al., 2016). Furthermore, current studies use complex networks to analyse traffic time series (Tang et al., 2013; Yan et al., 2017; Bao et al., 2017).

In addition to classical modelling approaches, complex network theory could offer new possibilities to gain a deeper understanding of infrastructure systems. For example, innovative mathematical models from the field of network sciences might support the risk assessment of complex infrastructure systems in terms of data requirements and computational performance.

1.3. Aim and objectives

The aim of this work is to develop a methodology and corresponding techniques to understand and quantify the risk of complex infrastructure systems, affected by natural hazards, considering spatial and temporal aspects. Contributing towards the overall aim, the work is divided into five objectives and structured in two parts:

(i) Computational risk and resilience modelling.

The first part focuses on the development of a risk assessment process for infrastructure systems, affected by natural hazards using computational models to simulate different hazard scenarios and estimate the associated consequences. The three objectives related to this part are:

(1) Develop a general process that infrastructure managers can use to assess the risks of their infrastructure systems, in order to quantify the consequences, caused by natural hazards.

With special emphasis on:

- a universal process applicable to different types of infrastructure systems and natural hazards;
- an interdisciplinary approach allowing different disciplines work together;
- a flexible design, so that different levels of detailing and abstraction can be depicted.
- ensuring that temporal and spatial correlation of events are considered;
1. Introduction

(2) Apply the proposed process to estimate the spatio-temporal risk of a realistic road network due to the occurrence of time-varying multi-hazard events.

With special emphasis on:

- quantifying the risk of a complete chain of events, from a source event to its resulting probable consequences;
- assessing socio-economic impacts, caused by performance losses of infrastructure systems;
- a simulation-based approach, supporting the inclusion of uncertainties and their propagation throughout the risk assessment;
- developing a computational platform, supporting the combination of models from different disciplines, and their modular update and replacement.

(3) Develop a restoration model for identifying optimal recovery responses in the aftermath of a natural hazard, in order to quantify the system resilience.

With special emphasis on:

- taking into consideration the direct costs of executing the physical interventions and the indirect costs incurred due to the inadequate service being provided;
- considering multiple object types, damage states and different interventions associated with each state of each object;
- accounting for time-dependent resource limitations and budget constraints;
- a computational method allowing to analyse realistically sized problems.

(ii) Complex network modelling for infrastructure systems.

The second part focuses on the development of innovative mathematical models from the field of network sciences to support the risk assessment of complex infrastructure systems in terms of data requirements and computational performance. The two objectives related to this part are:

(4) Develop a network model for interdependent infrastructure systems, using only a limited amount of data.

With special emphasis on:

- an appropriate mathematical representation for spatially embedded interdependent infrastructure systems;
- interactions with the surrounding environment such as vertex clustering and edge assignments.
- building a representative interdependent infrastructure system at a local and national scale.

(5) Develop a model for rapid estimation of traffic flow changes due to road interruptions.

With special emphasis on:
1.4. Scope

The primary objective of infrastructure managers is to ensure that their infrastructure system provides adequate service. This also applies in the aftermath of natural hazard events. As the resources available to managers to protect their networks are limited, it is essential for them to be aware of the probable consequences (i.e., risk) in order to set priorities and be resource-efficient. Due to the complexity and interdependence of infrastructure systems, however, localised failures can cascade dramatically, leading to widespread, unforeseen and often disproportionate disruptions compared to the actual physical damage. In order to understand and quantify the risk of such complex infrastructure systems, affected by natural hazards, infrastructure managers are in need of novel methodologies and tools to support them.

In the scope of this work, a novel risk assessment process is proposed with a specific focus on infrastructure managers (Objective 1). The process gives guidance on how to break down the complex problem into smaller pieces, allowing a structured analysis. The process is designed in such a way, that different decisions, levels of details, spatial and temporal boundaries, as well as an interdisciplinary approach can be considered. This is archived by using an iterative event-based approach, where each action or decision is represented as an interrelated event. To be as flexible as possible (e.g. account for different types of infrastructure systems and natural hazards), the risk assessment process is designed very generically. To give more detailed guidance on its implementation and show its applicability, several tools to support this risk assessment process are developed in the scope of this work. To show the application of the risk assessment process and demonstrate the usefulness of the supporting techniques and tools, one real-world inspired case study is used throughout this work. The area of interest is located in the Canton of Grisons, Switzerland. Although the process is not limited to any particular infrastructure system or natural hazard, the example focuses on a road network affected by flooding and landslides.

To quantify the infrastructure related risk of this region, according to the proposed process, a computational platform is created (Objective 2). The structure and setup of the computational platform follow the principles and ideas of the general risk assessment process and allows to incorporate and link different events. Thereby, each event is represented as a spatio-temporal model, which needs a specific set of input data and produces corresponding output data; other models can use. To quantify the risks, monetary values are put to the societal events, i.e. the losses society face but also the actions of persons or groups of persons to which a value can be placed. For example, from an infrastructure managers point of view, direct costs address the restoration of the infrastructure system while indirect costs are related to the reduced services provided\(^1\). After assembling the

\(^{1}\)In the scope of this work, the term indirect cost is used to address costs related to reduces
events according to the risk assessment process, the computational platform allows to perform a simulation-based risk assessment, considering the occurrence of time-varying multi-hazard events, the inclusion of uncertainties and their propagation throughout the risk assessment. In the scope of this work, the data and models selected are chosen considering the need to keep the computational time low. Nonetheless, the selection of data and models is sufficient for illustrating the use of the methodology. The proposed risk assessment process is unaffected by these limitations, and the modular computational platform supports the updating of data and models.

Knowing the locations and the severity of the damaged objects, infrastructure managers have to ensure an optimal recovery response in the aftermath of a natural hazard. Currently in practice objects are ranked using priority rules. For example, objects that cause loss of connectivity are restored first and then objects are restored based on their average traffic volume. Due to the complexity of the infrastructure system, however, the strong interdependencies between network structure and dynamics and limited resource availability, this approach does not always lead to the best solution. To address this issue, and help infrastructure managers by their decision-making process, a restoration model is developed to identify the optimal recovery process, considering direct and indirect costs as well as time-dependent resource limitations and budget constraints (Objective 3). The proposed restoration model is formulated as a bilevel optimisation problem, which provides the infrastructure manager with an optimal restoration schedule while reducing the consequences for the road users. This includes the minimisation of additional travel time and lost connectivity. Additionally, this improved restoration model is used to show the capability of updating or replacing models within the computational platform, i.e. the previously used rule-based model is replaced with the new restoration model.

Setting up a computational platform or a sophisticated restoration model requires a comprehensive amount of data and time, especially if the risk assessment is performed at a very high level of detail. In addition, simulation-based risk assessments are computationally intensive, especially when probabilistic modelling and uncertainty quantification are required. To enable infrastructure managers to cope with these situations, tools from the field of network science are presented. In the scope of this work, two approaches are introduced to support the risk assessment process. First, the network structure (topology) is investigated, allowing the infrastructure manager to gain a deeper understanding of their complex infrastructure systems in terms of spatial embedding and interdependencies with other networks. Second, the network dynamic (e.g. traffic flow) is investigated, to study the relationship between complex dynamic processes and the underlying network structures as well as changes in case of interruptions.

The modelling and simulations mentioned above is only possible when proper data is available. Conducting a risk assessment in areas where such data is not given can be challenging. In the scope of this work, a tool is presented to generate artificial infrastructure systems, which have similar network properties than the real-world system (Objective 4). Infrastructure managers can use such networks in the risk assessment process as substituted for infrastructure systems where data is not (publically) available (e.g. infrastructure systems of national importance, such as power grids or water distribution systems) or to test new services, such as prolongation of travel or loss in connectivity, other socio-economic costs are not considered.
hypotheses in a probabilistic setup where more than one network configuration is needed. Another advantage of such a mathematical model lies in its scalability, i.e, with relatively little effort infrastructure managers can move their analysis from a regional to a national level. Furthermore, considering multiple layers allows to model different hierarchies within the network and with other infrastructure systems.

Although complex networks made their appearance in transportation research through several structural measures, such as betweenness or other centrality measures, little research has been done to investigate the dynamics of transportation networks. In general-network-science for example, one widely used approach to study network dynamics is the use of diffusion processes. While this approach has a well-behaved solution in the form of a stationary distribution, it fails to capture the real world phenomena, such as traffic redistribution, because origin-destination data, which is essential for transportation networks, is not considered. In the scope of this work, a novel network model is presented, which allows, through its multi-layer structure, to encode multiple pieces of information such as the topology, used paths and origin-destination information, within one consistent graph structure (Objective 5). Infrastructure managers can use the gained insights and computational savings to improve and accelerate the risk assessment process, for example by substituting the computational intensive traffic flow model with the complex network approximation.

1.5. Outline and overview

The work is organised in two parts, eight chapters and four appendices. Chapter 1 gives an overview and scope of the work. It includes a general introduction, motivation for the research work, a brief introduction to the recent advantages in the fields of risk assessments, computational modelling and complex networks, research objectives, and the outline of the work. A separate literature review is not included since literature reviews are comprised in each of the subsequent chapters. Chapters 2-7 and appendix B represent research work, out of which six are published in peer-reviewed journals, and one is published as a book chapter. Each chapter focuses on one of the stated objectives described above. In general format, each chapter is arranged as abstract, introduction, methodology, results, discussion and conclusion sections. Chapter 8 contains the conclusions of this research and presents relevant issues for future research. Supplementary information for the main chapters is given in the appendices.

Part I The first part focuses on the development of a risk assessment process for infrastructure systems, affected by natural hazards, using computational models to simulate different hazard scenarios and estimate the associated consequences. In this part, a general risk assessment process for infrastructure systems, affected by natural hazards, is introduced. Based on this process a simulation engine is presented, which is constructed as a computational platform to estimate risk by supporting the combination of models from different disciplines. This allows the application of the proposed process to estimate the spatio-temporal risk of a realistic road network due to the occurrence of time-varying multi-hazard events, considering physical and functional effects on network objects (e.g. bridges and road sections), the functional interrelationships of the affected objects, the resulting probable consequences, duration of network disruption, and the restoration of the network. To give better insights into the resilience of the infrastructure system to natural hazards and help the infrastructure managers to make better decisions in such situations or to avoid them respectively, a restora-
1. INTRODUCTION

The first chapter of Part I introduces a general process that infrastructure managers can use to assess the risks of their infrastructure systems, in order to quantify the consequences, caused by natural hazards. The process is based on the concept that considered events are described using generic categories; i.e. as a set of interdependent spatial-temporal events, which share functional relationships with each other. For addressing infrastructure-related risk due to natural hazards five categories were identified: source, hazard, object, network and societal. Source events are ones that, at least from a modelling perspective, are considered to simply happen with a certain probability and may lead to hazard events. Each object in the network is vulnerable to these hazard events, which may cause damage and/or functional losses. These impacts can be the origin of failures in the network, resulting in a deficient level of service. These effects can impair the functioning of society during the hazard events and restoration periods. The execution of restoration interventions enables the network to provide an adequate level of service again by changing the state of damaged objects. The proposed process is meant to fit within the risk management process of any infrastructure owner. It is designed to be coupled with detailed sub-processes to achieve varying levels of detail in risk assessment. This flexibility ensures that the process is applicable for different types of infrastructure, different types of hazards, different levels of detail in the assessment, different sizes and types of regions and different levels of abstraction. It is also developed to ensure that the temporal and spatial correlation of events can be taken into account.

Chapter 3 The proposed process from Chapter 2 is applied in Chapter 3 to estimate the spatio-temporal risk of a road network located in Switzerland due to the occurrence of a time-varying rainfall that caused flood and mudflow events. To achieve this objective, a modular simulation engine is introduced to couple rainfall, runoff, flood, mudflow, damages, functional losses, traffic, and restoration models. Consequences are monetised into direct and indirect costs, considering restoration interventions, prolongation of travel time, and missed trips. Decisions of the infrastructure managers are incorporated in the restoration procedure. The modularity of the simulation engine supports the updating and swapping of models according to the needs of infrastructure managers. The application includes the modelling of (i) multiple time-varying hazard events; (ii) their physical and functional effects on network objects (i.e., bridges and road sections); (iii) the functional interrelationships of the affected objects; (iv) the resulting probable consequences in terms of expected costs of restoration, cost of traffic changes, and duration of network disruption; and (v) the restoration of the network. The simulation-based approach supports the inclusion of uncertainties and their propagation throughout the models used.

Chapter 4 While in Chapter 3 a traditional method to develop restoration programs, based on subjective ranking and priority rules, is used, in this chapter, a restoration model is formulated to determine near-optimal recovery responses in the aftermath of the natural hazard event, in order to better quantify the systems resilience. The objective function of the model is to minimise the costs, taking into consideration the direct costs of executing the physical interventions, and the indirect costs that are incurred due to the inadequate service being provided by the network. Besides the minimisation of the costs for multiple object types, damage states and different interventions associated with each state of each object, the model accounts for time-dependent resource limitations and
budget constraints as well as different traffic assignments, caused by the applied
restoration strategies. The restoration model is applicable to real-world networks
by utilising heuristic processes to solve the complex bilevel optimisation prob-
lem, as illustrated in a realistically sized case study in Switzerland. Computing
such a network shows that the provided restoration model can be of great use for
infrastructure managers planning and overseeing the restoration process of com-
plex infrastructure systems, due to disruptive events, by obtaining information
concerning the investment in recovery operations such as insights on the trade-
off between budget and resources spent, and effects on the system’s performance
gained over time.

Part II While the implementation of the risk assessment process in the first
part is mainly based on computational models, the second part focuses on the de-
velopment of innovative mathematical models from the field of network sciences.
First, a network model for interdependent infrastructure systems is presented,
which is based on the mathematical concept of the spatially embedded random
network, and therefore, needs only a limited amount of data. Second, a complex
network approach is used to investigate traffic flow dynamics on road networks,
which provides reasonable estimates for traffic flow changes and significantly re-
duces the computing time of classical simulation models. These models can be
used to support the risk assessment of complex infrastructure systems in terms
of data requirements and computing power, i.e. the initial computational models
can be substituted if only limited data is available, or a decrease in the computa-
tional effort is required.

Chapter 5 In this chapter a network model is developed to describe infras-
tructure systems using only a limited amount of data. The proposed model
combines a non-homogeneous point process for vertex creation, which accounts
for the spatial distribution of vertices (e.g. road intersections or power stations),
considering clustering effects of the network and a hybrid connection model for
the edge creation (e.g. road sections or power cables). Risk, related to the net-
work topology, is estimated by performing percolation studies, which describe
how a network transitions from a connected to a disconnected state, by removing
vertices or edges randomly or systematically. This work is a first step towards
the improved understanding of real-world spatial networks. It enables the gen-
eration of different realisations of the same spatially embedded network. These
randomly generated networks can be used to gain insight into the functioning of
the infrastructure system, taking into consideration its spatial properties. This
has potential to be useful in many types of analysis such as the risk assessment of
complex spatially distributed systems, the modelling of networks where there is
only a limited amount of data available, and the modelling how networks might
change over time.

Chapter 6 While in Chapter 5 only single infrastructure systems are con-
sidered and no interactions between different vertices, in this chapter a network
model is developed to describe multiple interdependent infrastructure systems us-
ing only a limited amount of data. This chapter extends the initial idea presented
in Chapter 5 and offers a first attempt to model complex infrastructure systems
using multi-layer spatially embedded random networks. Concepts from spatial
statistics and graph theory are applied to map complex systems with interde-
pendent subsystems to a simplified and condensed mathematical representation.
The developed model combines Markov marked point processes for vertex cre-
ation, which accounts for spatial distribution, layer assignment, and clustering
effects of the vertices; and a hybrid connection model for the edge creation. The
1. Introduction

Efficacy and utility of the investigated model are demonstrated with some examples. With only a few input parameters such as population density and terrain elevation, a wide variate of multi-layer spatially embedded infrastructure systems can be modelled, including the power grid and road network of Switzerland. It is observed that with a limited amount of data and a few simple assumptions, the randomly generated networks estimated reasonably well topological properties of the real-world infrastructure systems.

Chapter 7 In this chapter a model is presented for a rapid estimation of traffic flow changes due to road interruptions in order to reduce computational effort for the risk assessment process. To achieve this objective a networks in networks approach is presented, which closes the gap between classical simulation models and classical complex network models. The networks in networks approach, where nodes themselves represent networks, enable a computationally efficient way to study the complex dynamical processes on transportation networks. This approach encodes more information than conventional single-layer networks, and in combination with a multi-layer diffusion process, a good approximation of traffic flow changes due to topology changes can be made. The proposed approach captures both topological and spatial-temporal patterns in a simple, static representation, while classical graphic-analytical methods might lead to misrepresentations of such complex systems. In addition, the computing time can be significantly reduced, compared to conventional simulation models with only a moderate decrease in accuracy.

Chapter 8 The last chapter presents a synthesis of the results produced during the research work and their discussion. It also provides the main conclusion of the research and its significant contribution to scientific and practical knowledge. Finally, future research directions and recommendations are provided.

Appendix Finally, the main chapters are supplemented by four appendices. A detailed description of the models used in Chapter 3 is given in Appendix A. This supplement includes the inputs and outputs needed for each model, as well as the mathematical definitions, a description of the process and data used to calibrate the models, and some of the major underlying assumptions and limitations. Appendix B offers supporting material to Chapter 3 focusing on the description of the source and hazards events and their functional relationships, since the way such events are simulated has implications on (i) the overall computational cost of the entire risk assessment, which increases as the complexity of the network of interest increases, (ii) the accuracy of the individual risk estimations, as well as (iii) the quantified uncertainty of resulting risk estimations. In this section, a method is presented to develop rainfall-triggered hazard events, namely riverine flood events and mudflow events. The method enables the generation and simulation of hazard events that (i) are of a specific modeller-defined return period, enabling the characterisation of the uncertainty of risk estimation for given return periods, and (ii) changes over space and time, leading to the spatio-temporal estimation of network-related risk. The method is designed for network managers, and therefore, integrates computationally-efficient models that can be quickly coupled, and require data which is generally available or can be easily obtained or estimated, without impacting the integrity of the results. Appendix C contains the connection functions and algorithm, used for the generation and simulation of multi-layer spatially embedded random networks. Finally, the nomenclature used throughout the work is given in Appendix D.
Part I.

Computational Risk and Resilience Modelling
2. An overarching risk assessment process to evaluate the risks associated with infrastructure networks due to natural hazards

This chapter corresponds to the published article:¹


Abstract: In Europe, extreme natural hazard events are not frequent, but due to the complex interdependency of the infrastructure systems, these events can have a devastating impact in any part of Europe. Protection against the impacts of natural hazards must be guaranteed for people to work and live in a secure and resilient environment. People who manage infrastructure have to handle these risks. The proposed overarching risk assessment process is constructed in a way so that computational support can be constructed in modules. Therefore, each module interacts with other modules by receiving and delivering information. The content of the modules depends on the established context of the risk assessment process. The use of the overarching risk assessment process is demonstrated by using it to evaluate infrastructure-related risk due to natural hazards for an example region in Switzerland.

¹Please note, this is the author’s version of the manuscript published in the International Journal of Performability Engineering. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at http://www.ijpe-online.com. When citing this chapter, please refer to the original article found in the reference above.

For reasons of consistency, the text has been amended to British English.
2. Risk assessment process

2.1. Introduction

Infrastructure networks are the backbone of modern society. If they do not work as intended, which can happen due to natural hazards, there is a high probability that there will be significant consequences (Bensi et al., 2011). This can be predominantly attributed to system effects both during the event and following the event and depends greatly on how all of the objects within the affected infrastructure networks behave, and how fast and how they will be restored so that they once again provide an adequate level of service. People who manage infrastructure, herein referred to as infrastructure managers, have to handle these risks. Each infrastructure manager relies on his own risk management processes. These processes are systematic, timely and structured processes that when followed will provide the infrastructure manager with a better understanding of what may go wrong with the system in which the infrastructure is embedded, the probability of this happening and the associated consequences. This risk assessment process is particularly challenging for managers of infrastructure networks, due to the large number of scenarios that need to be analysed in order to assess the risks appropriately, the spatial and temporal correlations between these events (Vinchon et al., 2011), and the correlation between event occurrences, or so-called cascading events (Garcia-Aristizabal and Marzocchi, 2011).

In addition to the challenges in the physical world, the process is made even more complex because the risk assessment process requires that persons work together from many different disciplines whom each have their own discipline-based approaches to risk assessment that are not always harmonious with those in other disciplines. This makes it so that independent risk assessments from different persons are not always easy to aggregate to a level that is useful for the infrastructure manager.

The overarching process presented in this work is meant to be helpful to infrastructure managers who want to assess the infrastructure related risks due to natural hazards. It is to be used to help bring together people from many different disciplines so that they can provide information in a way that will be useful to an infrastructure manager. It has been specifically developed to deal with road and rail infrastructure networks, but it is believed to be generally applicable to all types of infrastructure networks. The proposed overarching process is meant to fit within the risk management process of any infrastructure owner. This process is developed so that it can be coupled with detailed sub-processes to achieve varying levels of detail in risk assessment. This flexibility ensures that the overarching process is applicable for different types of infrastructure, different types of hazards, different levels of detail in the assessment, different sizes of regions, different types of regions and different levels of abstraction. It is also developed to ensure that the temporal and spatial correlation of events can be considered. More detailed information can be found in the report Adey et al., (2014a) which was submitted as a deliverable in the INFRARISK project. The work builds on that done for the Swiss Federal Roads Authority in 2005 (Adey et al., 2009, 2010).

2.2. Overarching risk assessment process

The overarching risk assessment process is based on the ISO 31000, (2009), including different principle activities: communicating and consulting, establishing the context, and identifying, analysing, evaluating, treating, monitoring and reviewing risk. Besides the basic concepts of the ISO 31000, the proposed framework has been extended to allow explicit consideration of the spatial and temporal
2.2. Overarching risk assessment process

correlation between hazards as well as the modelling of the functional interdependencies between multiple objects in the infrastructure networks, including physical dependencies, cybernetic dependencies, geographical dependencies and the modelling of impacts. The process is described using generic definitions of sources, hazards, objects of the network and the network itself, which eases the application to different decision-making situations.

It is constructed keeping in mind that for many decision-making situations it will be desired to have the process be computer-supported, for example to model specific parts of the system. It has also been constructed keeping in mind that different decision situations will require the use of different types of models and models that will provide different levels of detail. In the following, a brief overview of the different subprocesses of the overarching risk assessment is given.

2.2.1. Problem identification

The first step is to identify the question to be answered. This step includes the generation of preliminary thoughts on the area to be investigated. It is only once this question is identified that a meaningful risk assessment can be conducted. It will affect the system definition, the requirements of the assessment in terms of both input, e.g. manpower, and output, e.g. the accuracy of the results or the number and types of scenarios to be investigated. It will also affect the scope of the assessment and the level of detail.

2.2.2. System definition

The system representation is a model of the relevant part of reality used for the evaluation and consists of all important realisations of stochastic processes within the investigated time period. It includes sufficiently good representations of the hazards, infrastructure, and consequences, as well as the interaction between them so that it can be reasonably certain that there is an appropriate understanding of the system and that the risks and the effectiveness of the strategies can be determined.

The system to be modelled includes all things required to assess risk, including the natural environment, e.g. amount of rain, amount of water in rivers, the physical infrastructure, e.g. the behaviour of a bridge when subjected to high water levels, and human behaviour, e.g. traffic patterns when a road bridge is no longer functioning. As it is necessary to model the system over time, it is necessary to also model the spatial and temporal correlation between events and activities within the investigated time period. This includes the consideration of assumptions, agreements as to how the system will react in specific situations, and drawing fixed system boundaries where it is clear that the things outside the considered system are not being modelled. It also includes the consideration of cascading events.

Boundaries

By establishing spatial boundaries, the part of the natural and man-made environment to be specifically modelled is determined. In addition to the definition of the geographical space, this includes specification of where the objects are located, where the events and hazards can occur and where the consequences could take place. It is usually easy to specify the possible locations of the events, hazards and objects. It is more difficult to, however, determine how they are related, e.g. heavy rain causes a flood hazard. This becomes even more difficult when the location of possible consequences is to be specified. Consequences can
be far away from the location of the events, hazards, and infrastructure, and may be outside the direct area of responsibility of the infrastructure manager (e.g. the collapse of a highway bridge on a trans-European highway network can have consequences on the free flow of goods in many countries).

By establishing temporal boundaries, the time period over which risk is to be assessed is fixed, as well as how this time period is to be subdivided for analysis purposes. With respect to time, the system representation can be made either: static or dynamic. In the case of a dynamic representation, the model evolves over time whereas in the case of a static representation time is not explicitly modelled.

Elements

It is proposed to group the system elements from initiating events to the events that are considered to be quantifiable and no further analysis is required. It is considered that the element types can be further grouped as either elements to which no value can be directly assigned or elements to which a value can be assigned. In the assessment of risk related to infrastructure due to natural hazards, one can label these further as “hazard elements” and “consequence elements”. Although the number of element types to be considered vary depending on the type of problem and the desired level of detail. Each element type is considered to correspond with events, which can be considered to have a probability of occurrence. Five basic element types, or event types, that should be regularly considered are:

*Source events*, or initiating events, are events, which occur regularly (rainfall, tectonic plates movements, ground movement etc.). The occurrence of such an event does not necessarily mean that a hazard will be triggered.

*Hazard events*, or loading events, are events related to an earlier event or that may lead to consequences. A hazard always has a source event. It may also trigger another one (e.g. earthquake triggers landslide). Most hazards evolve through space and time and interact with their environment. The time frame can vary from a few seconds (e.g. earthquake) to over a few days (e.g. flood) to several months (e.g. drought). The area that is affected can range from very local, to global. In defining the hazards to be considered it is important to define the intensities of the hazards to be considered. This should include consideration of the return period of the hazards to be used, e.g. 1/500 year flood or earthquake, and the loads to which the infrastructure will be subjected, e.g. the amount of water in the river during a flood, the magnitude of ground motions during an earthquake, the amount of displaced soil during a landslide.

*Infrastructure events* include all the objects and the condition states of these objects to be considered, e.g. a bridge collapse is an infrastructure event. How the infrastructure networks to be modelled are subdivided into infrastructure objects depends on the specific problem and the level of detail desired in the risk assessment. For example, a 10 km road link may be modelled as one element, although it consists of three bridges, four road sections and a tunnel, or it may be subdivided to explicitly consist of all eight of these objects. If more detail is required, then each object could be subdivided. For example, one of the bridges could be seen as being composed of columns, bearings, decks, etc. In the development of the system representation, it is important to consider which infrastructure objects are affected by which hazard and how the states of these objects may change over time. This is a difficult task as in many cases many objects could be affected, but the effect might range from very small, e.g. yielding of a reinforcement bar.
in a bridge during an earthquake, to very large, e.g. collapse of the bridge. An example of a value that could be assigned to this element type may be the cost of reconstruction of the infrastructure object if damaged. This value depends on the level of damage that might happen and how the infrastructure manager plans to intervene on the object if it is damaged. Sometimes these are referred to as direct consequences, although this terminology is not used consistently. For more in-depth analysis, one might decide to not assign values directly to infrastructure elements and to model the human activities involved in restoring the infrastructure, which would allow a substantially higher level of detail regarding the costs related to multiple objects in a network being affected simultaneously.

**Network use events** include the states of use of the infrastructure network that might occur. For example, due to a tunnel collapse, the freight corridor between Rotterdam and Genoa is closed and no vehicles can travel on it. The probabilities of these events occurring are particularly difficult to estimate as their occurrence depends on spatial and temporal correlation, and physical relationships between initiating events, hazards and infrastructure events. The latter, which can lead to cascading events. An example of a value that can be assigned to this element type is the cost of deviating traffic around a closed road. For more in-depth analysis, one might decide to not assign values directly to network elements and to model the human activities involved in redirecting traffic, which would allow a substantially higher level of detail. Another example is the value of lost travel time due to the closed link. Of course, the value assigned is highly dependent on the flow of traffic if the road is closed which in turn depends on the decisions of many persons in society.

**Societal events** include the actions of persons or groups of persons. For example, due to the freight corridor between Rotterdam and Genoa being closed 50% of goods is put onto trucks, 40% of goods are diverted over other train routes and 10% is not delivered. In order to model the actions of persons or groups of persons, it is often beneficial to group them into categories based on their general behaviour, which in turn is coupled with how their behaviour is to be modelled. Societal events may lead to other societal events. If they, however, do not then a value needs to be assigned to the event. This value then enters the risk assessment as a consequence.

As the events form the initiating event to the event upon which a value is placed forms a causal chain, it is convenient to think of them in the form of an event tree, where each chain of events is represented by a path in the event tree. To build the tree, it is necessary to determine the intensity measures to be used to define the events to be investigated, e.g. the water height above which a flood event is considered to have occurred. At each branch in the event tree, a decision is required to determine the value of the intensity measures, which allow classification of the event. The number of intensity measures used to describe the events depends on the problem being investigated and the level of detail required in the analysis. A very simple example is given in Figure 2.1.

As can be seen from this simple example, there is an infinite number of ways to represent reality. Due to this, particular care needs to be used in the development of an appropriate system representation. The necessary detail to be used depends on the specific problem and the level of detail desired. If events at any level or complete ranges of the values of intensity measures are excluded, it should be explicitly explained and documented why, because in the following risk assessment, the risk coming from those hazards cannot be taken into account.
2. Risk assessment process

Figure 2.1: Example of a simple event tree for the risk assessment of infrastructure networks.

Relationships

In order to estimate the likelihood of each subsequent event in the causal chain of events appropriate models of the relationship between them are to be developed. For example, in order to determine the amount of water coming in contact with a bridge during a flood, it is necessary to model how the water which falls as rain reaches the river, taking into consideration, for example, the amount of water that seeps into the ground or evaporates, or is held in temporary retention ponds.

The amount of effort to be invested in this depends on the exact problem and the level of detail desired. For example, in some cases, it may be sufficient to use one-dimensional vulnerability curves based on expert opinion to estimate the amount of damage that a single building might incur during an earthquake. In other cases, it may be desirable to use multidimensional vulnerability curves based on detailed finite element models to estimate the amount of damage a large dam might incur during an earthquake. In general, extra effort should be spent to achieve more detail when it is suspected that the results will add additional clarity for decision-making. If additional clarity is not provided, the extra effort is not worth it.

Although specific examples are given here, the general thoughts apply to all system elements, i.e. initiating events, hazard events, infrastructure events, network events and societal events. If possible, the availability of data to be used to model the relationships should be taken into consideration in determining the level of detail to be used.

2.2.3. Risk identification

In the previous step, an emphasis is made on identifying the correct system elements to be used in the risk assessment and how to model the relationships between these. In its most extensive form, the definition of these elements and relationships will provide all possible scenarios or risks. As it is unrealistic to attempt to quantify all of these it is necessary to identify the specific scenarios that are to be part of the risk assessment. Each branch in Figure 2.1 is a scenario which has an associated risk.

The identification of the scenarios should be made in this step without an explicit estimation of their probability of occurrence or putting a value on the consequences. The starting point for the development of this set of scenarios is all combinations of the system elements in the system representation. It is useful in the identification of scenarios to first determine for who the risk assessment is to be done and then to:

- start with the initiating events and think through how the infrastructure
2.2. Overarching risk assessment process

will be affected and then how humans will react to this,

- to start with the consequences and think through how the infrastructure would have to behave to something to cause these consequences, and

- to start with infrastructure behaviour and think in the other two directions.

Comprehensive identification of relevant scenarios is critical because scenarios excluded in this step will not be included in the further analysis and may result in an underestimation of risk. To minimise the possibility of this happening it is important that experts in each area are involved.

2.2.4. Risk analysis

The analysis of risk has to do with estimating the probability of occurrence of the scenarios and the value of the consequences of the scenario if it occurs. It is only through doing this that an infrastructure manager can decide if action needs to be taken and if multiple options are available, which one is the best. It can be done using a qualitative or a quantitative approach. In both cases, however, the goal is to gain a better understanding of the probability of occurrence of a scenario and the consequence of that scenario.

Risk analysis, as with risk identification, can be undertaken with varying degrees of detail, depending on the specific problem, the information, data and resources available. Analysis can be qualitative, semi-quantitative or quantitative, or a combination of these, depending on the circumstances. The certainty with which both the probabilities of occurrence of each of the scenarios and the consequences can be estimated, as well as the sensitivity of these values to the modelling assumptions, need to be given appropriate consideration in interpreting the results. Indicators of the sensitivity of these values are the divergence of opinion among experts, the availability of information, the quality of information, the level of knowledge of the persons conducting the risk analysis, and the limitation of the models used.

2.2.5. Risk evaluation

Risk evaluation has to do with verifying the meaning of the estimated risk to persons that may be affected, i.e. stakeholders. This is true regardless if a qualitative or a quantitative approach is used. A large part of this evaluation is the consideration of how people perceive risks and the consideration of this over- or under-valuation with respect to the analyst’s point of view used in the risk analysis step of the risk assessment. Through the risk evaluation, there is the possibility to bring into the risk assessment aspects that have not been explicitly modelled in the risk analysis step. The risk evaluation steps help to bring decision makers closer to finding a solution that is more acceptable to all stakeholders. One possible result of this step is that the risk analysis needs to be redone with more detailed system representations, improved models and different values. Another possible result is that it is decided that the risks are acceptable and no exploration of possible interventions are required (ISO 31000, 2009).

2.2.6. Modules

The proposed risk assessment process is constructed in a way so that computational support can be constructed in modules. Providing a platform in which the necessary modules can be integrated does this. A module is a self-contained set of (computational) instructions with unambiguously defined input and output interfaces. Inputs are either provided via external input (e.g. user input) or
2. Risk assessment process

via internal input (i.e. by using outputs of other modules generating compatible datasets). Therefore, each module interacts with other modules by receiving and delivering information. The type of information to be exchanged between modules is to be constant. Modules can perform a function itself or can be composed of sub-modules that each performs functions. The modular construction was chosen to allow continual updating of models as new information becomes available or better or detailed models are developed. The content of the modules depends on the established context of the risk management process. Thereby, modules can be described in terms of the functions they perform (e.g. a specific quantitative model) and the data they exchange.

In order to provide efficient and accurate risk analysis, the structure of the models and the framework in which they are embedded have to be adapted for their specific needs. For example, a damage calculation module that evaluates damage curves for streets based on inundation values may only take one inundation file for execution. Therefore, this module needs to be executed for each time step separately. Other modules may, in contrast, need a time series as input and therefore only need to be executed once. Relationships between modules are defined through the order of execution (module 2 can only be executed after the data of module 1 is present) as well as the data to be exchanged. For example, a damage calculation module needs inundation depths stored in a file of type “GeoTIFF”. This “GeoTIFF” is provided by a flood calculation module which produces this kind of data.

Additionally, there might be implicit assumptions for certain datasets. For example, when analysing geodata, typically it is adopted that the datasets use the same Coordinate Reference System and lie within a similar extent. Infrastructure managers do not necessarily create modules themselves since it can be assumed that certain tasks, existing tools can be reused and assembled. Also, one module may be reused within several configurations.

The different modules need different information for the risk assessment. The type of input and output of each module has to be specified. In some cases, this is done through the problem identification and the system definition steps of the process.

An information exchange structure has to be constructed together with the experts, stakeholders and infrastructure managers. For instance, for each module things such as the area of application, the type of model, or the kind of intensity measurement, have to be specified. Data compatibility between modules is ensured through the concepts of syntactic and semantic interoperability. According to IEEE, (1990), interoperability is defined as “the ability of two or more systems or components to exchange data and use information”. In the context of the overarching methodology, these systems or components are represented in the form of modules.

Once, the modules and data are assembled appropriately, the infrastructure manager may perform simulations based on this framework. Running a simulation when specific external inputs are provided does this. These inputs may be defined by the infrastructure manager or potentially automatically when performing multiple runs (e.g. by sampling a certain distribution using the Monte Carlo Method).

2.3. Example

In this section, the use of the overarching risk assessment process is demonstrated by using it to evaluate infrastructure-related risk due to natural hazards for an
2.3. Example

Figure 2.2.: Overview of the area of interest.

example region. For the sake of simplicity, the example is presented in a sequential manner, although the process itself is highly iterative. The results of this example should be treated with care since only very simple physical models are used to evaluate the risk.

2.3.1. Problem identification

The target area is located around the city of Chur, the local capital of the easternmost Canton of Switzerland, Graubünden. The region is home to companies of different sectors such as finance, engineering and chemistry (e.g. EMS-Chemie AG, Hamilton AG) and its road network is part of one of two major transport links for goods from Italy to Northern Switzerland. Also, the main station of Chur is an important railway junction to other regions of Graubünden. Most of these objects are located in a valley between several mountains (e.g. Calanda, Montalin) with many watercourses draining into the main river Rhine.

The addressee of this risk assessment is the city administration (city planners) being interested in damage, cost and other consequences resulting from a low probability/high impact natural hazard scenario in the Chur region consisting of a coupled flood and landslide event.

2.3.2. System definition

Boundaries

The spatial boundary of the system has been selected to be that shown in Figure 2.2. The system is spatially bordered by a bounding polygon which is aligned to the main valley of the region of interest and covers an area of approximately 150 km² in the Swiss coordinate reference system CH1903/LV03. Since the focus lies on the main watercourses, only those watercourses are taken into account. If a more detailed study is attempted, it is suggested that a thorough examination is undertaken to identify watercourses relevant for the target area.

The risk assessment is done for a flood hazard with a return period of 500 years. The occurrence of this hazard takes three days, i.e. water rises slowly and inundates the surrounding areas, and finally the flood water goes down. In order to model the temporal evolvement of the flood hazard, the period of 3 days is subdivided into 72-time steps of one hour. To compare the risk with other cities and regions, the losses resulting from this analysis are converted into an average annualised loss.
2. Risk assessment process

Elements

Source event precipitation: The model of precipitation was constructed using the precipitation data from a historical event which occurred from 07.08.2007 to 09.08.2007 and is scaled in such a way that it corresponds to a precipitation event resulting in a flood with a return period of 500 years.

Hazard event flood: The model of the amount of water on each land surface area and in the rivers was developed using a set of interrelated tools. These are the Hydrological Modelling System (HMS) and the River Analysis System (RAS), both being maintained by the Hydrologic Engineering Center (HEC), as well as their interface applications GeoHMS and GeoRAS for the Geographic Information System ArcGIS.

Hazard event landslide: In this scenario, the increase in soil saturation due to precipitation triggers one of the pre-modelled debris flows from the SilvaProtect project (Losey and Wehrli, 2013) affecting the small town of Haldenstein. These potential debris flows are modelled using the software packages MGSIM and dfwalk.

Infrastructure event residential and industrial buildings: Information on buildings on the footprint level are taken from the swissBUILDINGS3D dataset (swisstopo). The buildings are represented by polygons and are additionally enriched with information on their type of use (e.g. residential, industrial, agriculture).

Infrastructure event hospitals: In the area of interest, only one institution is present for ambulant care, the hospital of the Canton of Graubünden. This hospital consists of three separate buildings of which each is converted to a point geometry to be used as a source for network analyses.

Infrastructure event road segments: Since road geometries for the target area can have lengths up to several hundred metres, these are partitioned in such a way that spatial analysis can be undertaken on a feasible resolution. For this application, a segmentation interval of 4m was considered to give a reasonable trade-off between computational effort and accuracy. For reasons of performance, the segmentation process was limited to those regions which are affected by a hazard at any time step during the scenario. For flooding, it was considered that all roads affected by the flooding during the scenario could be selected by intersecting them with the floodplain with the greatest extent. A similar approach can be followed to consider the landslide geometry

Network events: The road network for the target area is extracted from the VECTOR25 dataset. Each road is represented by a linear geometry with assigned attributes on their type (swisstopo). Roads of subordinated types (agricultural, forest or bicycle way) are removed because they are considered to be unsuitable for most motorised vehicles.

Societal events: Societal events are how the traffic behaves on the network when it is not fully operational. It is estimated using traffic simulations to estimate how much additional time is required to travel from anywhere in the hospital catchment area to the hospital.

Relationships

The interactions between infrastructure networks, elements and components of elements at the one hand side and between hazards, infrastructure and consequences on the other side, should be represented completely. This is necessary to determine dependencies in failure scenarios and evaluate common influencing factors.

Source-Hazard-Interaction: For reasons of simplicity and efficiency only a sim-
ple hydrological model for the runoff calculation is used. In the simple model, the precipitation can fall on the watershed’s vegetation, land surface, and water bodies (streams and lakes). The runoff volume is computed by the volume of water that is intercepted, infiltrated, stored, evaporated, or transpired and subtracted it from the precipitation. Interception and surface storages are intended to represent the surface storage of water by trees or grass, local depressions in the ground surface, etc. Infiltration represents the movement of water to areas beneath the land surface. The ModClark model (Kull and Feldman, 1998) is used to estimate the discharge during the precipitation event. This model accounts for retention by using a Linear Reservoir Model (LRM) and translation by taking into account a grid-based travel-time model.

**Hazard-Infrastructure-Interaction:** To estimate damage resulting from inundation, simple damage curves are used. These take into account the inundation depth $d$, in the range of 0 to 5 m, associated with the infrastructure object and return a dimensionless damage factor $\alpha \in [0, 1]$ where 0 represents no damage and 1 represents complete failure. The damage functions associated with the different categories are listed in Deckers et al., (2009). For infrastructure affected by the landslide, the damage is assumed to be 1 for both, roads and buildings independent of their type.

**Infrastructure-Society-Interaction:** It is assumed that if the infrastructure is damaged that it would be restored to the condition it had prior to being damaged. These costs are estimated by multiplying the area of the affected object with the unit cost of constructing the object from scratch. For buildings, the area is directly derived from the geometry of the polygon. For roads, the area is calculated by multiplying the length of the linestring with the width associated with the corresponding road type. The unit values used are taken from Kutschera, (2008).

**Infrastructure-Network-Interaction:** Since this connectivity changes during the scenario due to node failure, for each time step a distinct network needs to be created. Impassable road segments due to natural hazards are excluded from the network, e.g. by deleting segments with assigned inundation depths $\geq 0.3$ m.

**Network-Society-Interaction:** The quantification of consequences related to travelling across the network resulting from the failure of infrastructure network nodes was undertaken in terms of the following non-exhaustive list of examples: travel time costs (e.g. man-hours of work time lost), vehicle operating costs (e.g. increase of fuel needed), accident costs (e.g. number and type of injuries/deaths), environmental costs (amount of additional nose/pollution) (Adey et al., 2012a). These predominately depend on the amount of additional travel time that will be incurred on the network when the network is in less than a fully operational state. In this example, this additional time was estimated by determining the shortest paths to be used when the network was in a failed state. For road networks, this measure typically is represented by the length of a road segment (shortest path) or, if additional information such as speed limits are available, by the time needed to pass a segment (fastest path). While this approach assumes an idealised behaviour of a virtual car driver, it should be sufficient to estimate the true route through the target area coarsely. After their computation, the shortest path lengths were decomposed by road class. Not only the total length of the shortest path is increasing with more and more streets becoming inaccessible, but also that the driver needs to use alternative roads of lower capacity.
2. Risk assessment process

2.3.3. Risk identification

The target area has been historically prone to the mentioned natural hazards flooding and landslides. Information on past events is stored in the database “Unwetterschadens-Datenbank” (Hilker et al., 2009) for the period ranging from 1975 to 2007. The database holds 43 natural hazard events located within the region of interest, including inundation, mudflow and mass movement events. In addition, two more recent projects, AquaProtect and SilvaProtect (Losey and Wehrli, 2013) provide model-based information on regions vulnerable to floods and landslides.

Based on the problem identification, the risk assessment was conducted on a medium scale area where buildings are taken into account on the footprint level and streets are represented by connected linear geometries.

For the sake of simplicity, only one scenario is considered. This scenario is comprised of the following events: Source event is rainfall, the hazard events are a flood, defined as being more severe as the largest volume of water expected in the main river expected in 500 years, and a landslide. The infrastructure events are derived from the buildings, road sections and hospitals being in specified damage states. The network events are derived from the different combinations of damage states of the different infrastructure objects. The societal events are derived from modelling the traffic flow results from the different network condition states.

2.3.4. Risk analysis

For the risk analysis of the considered scenario, a quantitative approach is used. This approach is based on historical information, expert knowledge as well as physical and mathematical models. Most of the analysis is performed within a GIS framework. For example, the identification of buildings and roads at risk is undertaken using standard GIS functionality by spatially relating the geometries of the hazards to those of buildings and roads. Depending on the characteristics of the objects in question different approaches are used. For the infrastructure network analyses based on graph theory is performed, e.g. to estimate the increased travel time required to reach the Chur hospital when the infrastructure network is not fully operational.

In order to aggregate risk that has been estimated based on the specific scenario, it is necessary to ensure that they are directly comparable and that they are not double counted. There is an especially high chance of this happening when cascading events are part of the scenarios.

The value associated directly to the condition of the infrastructure objects, i.e. the infrastructure events assuming that the objects will be restored to a like new condition at a later point in time, are added. It is assumed that the maximum damage predicted throughout the three-day period is the amount of damage that needs to be repaired. No consideration was made as to how the repair work would be executed or whether or not there would be a reduction in costs because multiple objects would be repaired at the same time. It is considered that the costs required to restore the objects from damaged condition states to fully operational due to either flood and landslide are additive. Based on the cost associated with the single objects for each time step, the development of the total losses for the whole region of interest can be calculated.

The costs related to the disruption of traffic on the road network are estimated by counting the number of additional hours of travel time that is required on the network while the network is not fully operational. These costs are added for each time step in the three-day period. In this case study, it is assumed that all
2.3. Example

Figure 2.3.: Example results of the main processes of the overarching methodology for the time steps 20 and 38 for the area under investigation.

road sections are restored to normal immediately following the three-day period. For those that could not travel no extra costs were estimated as it was assumed that they could postpone their trips. The estimation would be significantly more complicated if these assumptions are not made and instead the time until actual repair of the infrastructure is estimated and the travel on the network is modelled for this entire duration, as well as the effects of not being able to travel.

Figure 2.3 exemplarily illustrates the results of this process. Here, for each event a pair of maps illustrates one stage of the overarching process in top-to-bottom order. To illustrate the change of the system, the left maps represent the state of the system for time step 20 and the right maps for time step 38. The simplified legends should suffice to conceive the relevant information.

The source maps show heavy rainfall over the region of interest, which decreases towards the end of the simulation period. The hazard maps show the maximum inundation depths of the resulting flood for each surface area until the respective
time step. It becomes apparent that the maximum inundation depths increase with time, which therefore leads to increasing damages of affected infrastructure objects such as buildings and street segments. This causes rising reconstruction costs, which is shown in the element maps for the Haldenstein region. As indicated by the red rectangle in the hazard maps, this region is located in the northern part of the area of interest and is affected by flood as well as by the landslide. Because of the damage induced by these hazards, the road networks functionality is reduced as shown in the network maps. Here, red road segments indicate that they are isolated from the green main network. Impassable road segments are not shown. This reduced network state results for some regions, in particular in the northern and south-western parts, to be cut off from important infrastructure objects. For example, it is impossible for people in these areas to get to the hospital in Chur as indicated by the society maps.

2.3.5. Risk evaluation

In this work, risk evaluation is not performed. If a complete risk management process is being conducted this work would need to be done in conjunction with the city administration of Chur. The results coming from the risk analysis would support this task in order to plan further analyses, safety measures or risk treatments.

2.4. Discussion

The example demonstrates that the proposed overarching risk assessment process is useful to assess infrastructure-related risk due to natural hazards. Computer systems can highly accelerate its distinct steps so that the results can be delivered to infrastructure managers in a timely manner. However, in order to refine the results, the methodology needs to be applied to a greater number of scenarios.

The process can be used for a wide range of different problems at different levels of detail. In addition, the changes over time and interactions between different events can be modelled as shown in the example.

Although the proposed overarching risk assessment process can be used conceptionally for all kinds of different problems, its usefulness depends on the quality of available models and data. Often the physical models do not take into account interaction with their environment. For example, if a bridge collapses, the cross-section of the river will be changed, too.

In the presented example a deterministic point of view was chosen. In order to take the numerous uncertainties into account, a probabilistic approach seems more suitable, especially when dealing with natural hazards. If one associates a probability of occurrence with the occurrence of the particular precipitation then one could quantify the risk. A more sophisticated example will require not only the consideration of the probability of occurrence of different rain patterns, but also given the rainfall patterns, the probability of different water run-off events, different levels of water in different parts of the rivers, different behavior of the infrastructure objects in the network, and different behavior of the vehicles on the network. It would also require consideration of more extended periods of time, in which multiple rain events occur and perhaps even different types of source events that may result in consequences.

In the expansion of the example to do this, there are substantial hurdles with respect to the infinite number of scenarios possible, the uncertainties associated with many different models to be used to make approximations and the temporal changes in the probabilities of event occurrences.
2.5. Conclusions

This work describes a generic overarching risk assessment process as well as an example of how it can be used and how it can be implemented using a GIS framework. Even in its current form, it is believed that this process would be useful to infrastructure managers in the assessment of their infrastructure-related risks due to natural hazards. It is applicable to different types of infrastructure, different types of hazards and different types of consequences and can take into consideration both simple and complex system representations.

The overarching risk assessment process will be further improved by taking into account multiple scenarios, including multiple initiating events, multiple hazards, multiple infrastructure events, multiple network events and multiple societal events. It will also be expanded to deal properly with the spatial and temporal consideration in the estimation of the probability of occurrence of scenarios and the establishment of the scenarios. More work is required to emphasise the human interaction in conducting the risk assessment.
3. Estimating network related risks: A methodology and an application in the transport sector

This chapter corresponds to the published article:


Abstract: Networks such as transportation, water, and power are critical lifelines for society. Managers plan and execute interventions to guarantee the operational state of their networks under various circumstances, including after the occurrence of (natural) hazard events. Creating an intervention program demands to know the probable direct and indirect consequences (i.e., risk) of the various hazard events that could occur in order to be able to mitigate their effects. This work introduces a methodology to support network managers in the quantification of the risk related to their networks. The methodology is centred on the integration of the spatial and temporal attributes of the events that need to be modelled to estimate the risk. Furthermore, the methodology supports the inclusion of the uncertainty of these events and the propagation of these uncertainties throughout the risk modelling. The methodology is implemented through a modular simulation engine that supports the updating and swapping of models according to the needs of network managers. This work demonstrates the usefulness of the methodology and simulation engine through an application to estimate the potential impact of floods and mudflaws on a road network located in Switzerland. The application includes the modeling of (i) multiple time-varying hazard events; (ii) their physical and functional effects on network objects (i.e., bridges and road sections); (iii) the functional interrelationships of the affected objects; (iv) the resulting probable consequences in terms of expected costs of restoration, cost of traffic changes, and duration of network disruption; and (v) the restoration of the network.

1Please note, this is the author’s version of the manuscript published in the Journal Natural Hazards and Earth System Sciences. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at https://www.nat-hazards-earth-syst-sci.net. When citing this chapter, please, refer to the original article with DOI: 10.5194/nhess-18-2273-2018.

For reasons of consistency, the text has been amended to British English. References to supplementary files have been replaced with links to the appendix.
3. Estimating network related risks

3.1. Introduction

Managers of networks, such as transportation, water and power, have the continuous task to plan and execute interventions to guarantee the operational state of their networks. This also applies in the aftermath of (natural) hazard events. As the resources available to managers to protect their networks are limited, it is essential for managers to be aware of the probable consequences (i.e., risk) in order to set priorities and be resource-efficient (Eidsvig et al., 2017). Consequences are often expressed in monetary values, and these are distinguished between direct costs (e.g., costs related to clean up, repairs, rehabilitation and reconstruction) and indirect costs (e.g., in the transport sector, costs related to additional travel time, vehicle operation and an increase in the number of accidents). Indirect costs have a wide spatial and temporal scale (Merz et al., 2010) and are potentially more significant than direct costs (Vespignani, 2010).

Conducting a risk assessment can help identify probable hazard events, and evaluate their impact on networks and users. Nonetheless, conducting such assessments can be a particularly challenging task due to the large number of scenarios (i.e., chains of interrelated events) that need to be taken into account, the modelling of these events, the relationships among them, and the availability of support tools to run the models in an integrated way. In building scenarios, multiple types of hazards need to be considered, along with the complex nature of networks, specifically, their large number of objects, their spatial distribution, and functional interrelationships. Moreover, it is crucial to estimate the performance of networks during the hazard events and through the recovery to an adequate level of service that is driven by restoration strategies (Lam and Adey, 2016). Therefore, network managers need to think of ways to model the cascade of events, interdependencies, and the propagation of uncertainties (Hackl et al., 2015b).

As a result of these challenges, risk assessment methods for networks have been the subject of increasing research interest in recent years. Most research has been focused either on the technical aspects of hazards or those of networks. In the first case, scholars have focused on improving the understanding, the modelling or the prediction of single hazard events (Apel et al., 2004; Pritchard et al., 2015; Schlägl and Laaha, 2016; Pellicani et al., 2017) without explicitly considering the complexity and dynamics of networks. In the second case, scholars have investigated the vulnerability\(^2\) (Jenelius et al., 2006; Rupi et al., 2015; Shabou et al., 2017) or resilience (He and Liu, 2012; Bocchini and Frangopol, 2012; Vugrin et al., 2014) of networks due to disruptions, without evaluating the cause and/or the probability of such disruptions.

Some work has been conducted to consider multiple hazards and their effects (i.e., multiple vulnerabilities and consequences) in a unified framework (Komendantova et al., 2014; Mignan et al., 2014; Gallina et al., 2016). Assessing the risk in such a way is relatively new, and until now only partially developed by experts with different backgrounds such as statistics, engineering and various fields of geosciences (Komendantova et al., 2014). Only a limited number of scenario-based and/or site-specific studies have been proposed due to the difficulty and

\(^2\)Numerous definitions for network vulnerability have been proposed in the literature. In most cases, the literature has defined network vulnerability in terms of the effects of service disruption, irrespective of its probability (Berdica, 2002; Taylor and D’Este, 2007; Wang et al., 2014). In the context of the present work, vulnerability is defined in the same terms, acknowledging that it is an inherent attribute of any network, and thus, an essential element of risk assessments.
novelty of the task (Mignan et al., 2014). Having a diverse team of experts, whose discipline-specific approaches to risk assessment may differ, presents an additional challenge: their contributions are not always easy to aggregate to a level that is useful for network managers. In addition, current research also shows the need to take into account the spatial-temporal quantification of risks across different levels of scale for future sustainable risk management (Fuchs and Keiler, 2008; Fuchs et al., 2013).

Open research: In conclusion, there has been little work to bring research outputs concerning the modelling of hazard events and network vulnerability together in a way that their spatial and temporal uncertain behaviours are assessed in a unified framework that fosters multidisciplinary collaboration.

Contributions: To overcome these challenges, this work presents the application of a novel risk assessment methodology. The methodology can be used to investigate multiple scenarios, starting from the modelling of a source event (e.g., rainfall, fault rupture), and ending in the estimation of the probable consequences of related societal events (e.g., communities without access to transportation, water or power service). While the methodology is applicable to different types of hazards, networks, regions, sizes of regions, and levels of resolution, the application of the methodology is illustrated on a regional road network in Switzerland prone to floods and mudflows. Specifically, this work advances the state-of-the-art in the field of network related risks due to hazard events as follows.

- The risk of a complete chain of events, from a source event to its societal events is quantified over space and time. In the application, this means considering precipitation, runoff, flood, mudflows, (physical) damages, functional losses, traffic flow changes, and restoration interventions. The presented links between cascading hydrometeorological hazards, a road network and society will be of interest to the international research community and practitioners working in the fields of network management, urban planning, public policy, and emergency response.

- When quantified, risk can be categorised into probable direct and indirect costs, with the latter estimated throughout the hazard events and restoration periods, not just immediately after the occurrence of the hazard events. In the application, direct and indirect costs included costs of interventions, prolongation of travel time, and missed trips, allowing for the evaluation and comparison of the socio-economic impacts of the multiple hazard scenarios considered.

- The simulation-based approach supports the inclusion of uncertainties and their propagation throughout the risk model. The application includes results related to the simulation of 1,200 rainfall events, causing floods of return periods ranging from 2 to 10,000 and, depending on the rainfall intensity and duration, stochastically triggering a number of mudflows in the area. Furthermore, as suggested by Lam et al., (2018a), the approach can support the testing of additional scenarios based on uncertain relationships (e.g., fragility functions relating hazard intensities with damage state exceedance probabilities).

- A novel simulation engine was constructed as the computational platform to estimate risk, supporting the combination of models from different disciplines, and their modular update and replacement. The application demon-
strates the coupling of existing models and information from literature in geosciences, engineering, network theory, transportation, and economics.

The example application makes further research contributions on ways that road network managers can estimate the risk related to their networks due to climate-related hazard events. With a changing climate, exacerbated by an increase in urbanisation, the frequency of extreme hydrometeorological hazard events is expected to rise, impacting economic corridors, disrupting supply chain, and stressing emergency and rescue operations, among other effects (Keiler et al., 2010; Fuchs et al., 2017). As a result, particular focus is now given to these hazard events and their associated risks. In times of scarce public resources and increasing incidences and damages caused by floods (Bowering et al., 2014), network managers have the need to become increasingly aware of their causes and consequences so that they can appropriately manage their risks, for example, through the adaptation of networks (Elsawah et al., 2014) or the planning of actions following a hazard event (Taubenböck et al., 2013). A number of methods have been developed to quantify the damages and costs due to floods (Merz et al., 2010; Hammond et al., 2015). Nevertheless, most of these studies have been focused on buildings and the estimation of direct costs.

In the transport sector, some work has been done to consider the probable direct costs of floods (e.g., Scawthorn et al., 2006, Deckers et al., 2009, and Bowering et al., 2014), but these works have often neglected the spatial and temporal attributes of these networks. Only a few scholars have investigated flood risk in combination with the actual changes in traffic flow. Dawson et al., (2011) implemented an agent-based model to simulate the human response to floods considering different storm surge conditions. Suarez et al., (2005) studied the impacts of floods and climate change on the urban transport system of the Boston Metro Area, using a conventional analytical framework for simulating traffic flows under different flood scenarios, changes in land use, and demographic and climatic conditions. In both cases, the direct costs resulting from damages were not taken into account. Furthermore, only the events unfolding during the hazard events period were analysed, and hence, no network restoration was considered, which is essential for quantifying indirect costs.

While floods are the most common rainfall-triggered hazard events in mountainous areas, landslides, including mudflows, are second in place. As road networks in these areas generally have a low level of connectivity (e.g., remote villages that are only connected by few mountain roads), there is a high probability that connections between some areas within a road network can be interrupted when one or a limited number of objects fail (Rupi et al., 2015). Usually, risk estimates related to networks, including road networks, due to landslides have been obtained by overlapping hazard and consequence maps (Ferlisi et al., 2012; Pellicani et al., 2017), and thus the indirect costs for specific hazard scenarios have not been considered.

In general, according to Mattsson and Jenelius, (2015), there are two dominant types of risk assessment methods for transport networks. The first set of methods is rooted in graph theory and is focused on the study of the topological properties of the networks. This analytical approach requires network topology data and considers the importance of different edges (Jenelius et al., 2006; Rupi et al., 2015), cascading failures (Dueñas-Osorio and Vemuru, 2009; Hackl and Adey, 2017a), and interdependencies between different networks (Thacker et al., 2017). The second group of methods is focused on understanding the dynamic behaviour
3.2. Risk assessment methodology

The purpose of the risk assessment methodology is to support network managers in the quantification and subsequent management of risk. The methodology is founded on the principles of systems engineering (Adey et al., 2016), and thus the methodology is structured keeping in mind that (i) different decisions require different models, (ii) models provide different levels of detail, and (iii) this is an iterative process, requiring changes as data and model insufficiencies are discovered, and new data and models become available. Considering these principles, the risk assessment methodology is structured as follows:

1. **Set up risk assessment** – the rough outline of the planned risk assessment.
2. **Determine approach** – the agreements on how to perform the risk assessment, including the involved parties and the activities to conduct.
3. **Define system representation** – the agreements on system behaviour, system boundaries, assumptions and limitations.
   - Define boundaries – the parts of the system to be analysed, both spatially and temporally.
   - Define events – the events to be analysed.
   - Define relationships – the relationships between events.
   - Define scenarios – the scenarios combining the events and relationships.
   - Determine tools – the selection of tools, models and software to analyse the scenarios.
4. **Estimate risk** – the computed probabilities of the scenarios and their consequences.
5. **Evaluate risk** – the interpretation of the risk estimates.
6. **Update approach and/or system representation** – the parts of the system to be analysed in more detail in order to decrease the uncertainties of the result.

The considered events are described using generic categories: **source**, **hazard**, **object**, **network** and **societal**. Source events are ones that, at least from a modelling perspective, are considered to simply happen with a certain probability, and may lead to hazard events. Each object in the network is vulnerable to these...
3. Estimating network related risks

hazard events, which may cause damage and/or functional losses. These impacts can be the origin of failures in the network resulting in a deficient level of service. These effects can impair the functioning of society during the hazard events and restoration periods. The execution of restoration interventions enables the network to provide an adequate level of service again by changing the state of damaged objects.

Such a chain of events can be assembled using a novel simulation engine (Heitzler et al., 2017a) that supports the coupling of multiple heterogeneous models, where a given model encapsulates the behaviour and state of a part of the system. To support the data exchange between models, models are grouped into modules, with each module comprising distinct execution instructions and data requirements. The data consist of model parameters, static input (e.g., the location of mudflows, the extent of the road network, the distribution of a variable to sample from) and the states of models in other modules (e.g., to determine the damaged objects, the water extend from a flood model is needed). The advantage of such a modular approach is that only input and output have to be defined for each module. Consequently, different models and software solutions can be used to simulate scenarios. For example, a flood can be simulated using a custom model or off-the-shelf software such as HEC-RAS (Brunner, 2016), Basement (Vetsch et al., 2018), etc., without having to modify other models. This approach allows scientists to implement and test their own models, including their effects on risk estimations.

Once the modules and module interfaces are defined (as a reference, see Figure 3.1 for a schematic representation of the modules used in the application), the network manager can conduct the risk assessment. The risk is here expressed in terms of expected monetised consequences, calculated as a product of the probability of occurrence of a certain scenario and the associated costs of that scenario – both the direct (dc) and the indirect costs (ic) should be considered. Their cost functions \( C_{dc} \) and \( C_{ic} \) are associated with the modeled societal events \( E_{soc} \) that occur as a result of object events \( E_{obj} \) and network events \( E_{net} \), and that can be traced back to hazard events \( E_{haz} \) and source events \( E_{src} \). This representation (for a given source event, see Equation 3.1) can also be used when considering multiple hazard events. As the spatial and temporal correlation between events is considered, risk can be said to be spatially and temporally distributed (i.e., risk estimates vary in space within a defined area of study, and in time within a defined period of analysis).

\[
R|E_{src} = \frac{\text{probability}}{\text{consequences}} \times \frac{C_{dc}(E_{soc}|E_{obj}, E_{haz})}{\text{direct}} + \frac{C_{ic}(E_{soc}|E_{net}, E_{obj}, E_{haz})}{\text{indirect}}
\]

(3.1)

Furthermore, network managers may often be interested in investigating the effect of hazard loads on their critical objects. Therefore, managers need to be given the option to select hazard events based on the periodicity of the manifested site-specific hazard loads (i.e., managers may not be as concerned with selecting a hazard event based on the return period of the preceding source event). Then, Equation 3.1 can be modified by first selecting the hazard events according to the return periods related to the site-specific loads (i.e., risk is conditioned on the hazard event \( R|E_{haz} \), and thus the probability of occurrence of a scenario is defined as \( P[E_{src}|E_{haz}] \)). For example, the network manager might be interested.
3.3. Application

The application presented in this section is used to demonstrate the usefulness of the methodology considering a specific problem. The application shows the design and implementation of an assessment focused on estimating the risk related to a road network in the Canton of Grisons in Switzerland. In the study, the network was exposed to rainfall, which caused multiple hazards, specifically riverine floods and mudflows. At the same time, these events led to direct costs linked to clean-up, repair, rehabilitation and reconstruction activities, and indirect costs associated with loss of connectivity and temporal prolongation of network user travel time, linking the modelling of these latter effects with the dynamics of the network. A large number of uncertain rainfall leading to floods of multiple return periods was considered in the analysis.

The data used for this application were representative of actual entities and processes in the region or were derived from such data. Moreover, the models selected for the application were chosen considering the need to keep the computational time low. The authors of this work are fully aware that other models may be available in the literature, some of which are more sophisticated and precise, and hence, demand much more detailed data than the data that were available. Nonetheless, the selection of data and models was sufficient for illustrating the use of the methodology. The proposed risk assessment methodology is unaffected in evaluating a flood with a return period of 100 years in a specific point in the network. In this case, the simulation engine finds a suitable rainfall that causes the flood of interest.
3. Estimating network related risks

Figure 3.2.: The area of study is located in the east of Switzerland. The red areas indicate the considered catchments of the Rhine rivers. The risk assessment was performed on the road network located within the green boundary. The data to calibrate and validate the rainfall, runoff and flood model were collected from nine precipitation measurement stations (green nodes) and seven gauging stations (red nodes). Historical landslides, floods and their damages to road networks are illustrated as symbols, where their size represents the magnitude of the associated direct costs. Four hazard events with damages over CHF 5 million occurred within the last decade in the investigated area (Felsberg, Chur and Trimmis).

by these limitations, and the modular simulation engine supports the updating of data and models.

3.3.1. Area of study

The investigated road network is located in the Rhine Valley between Trin and Trimmis (Figure 3.2). This target area is located around the city of Chur, the capital of Grisons, the largest and easternmost canton of Switzerland. It is the largest city in Grisons with approximately 34,500 inhabitants and a large business centre. Chur is also an important transportation hub, linking Switzerland, Germany, Austria and Italy. The Canton of Grisons is crossed in a north-south direction by the A13 motorway. The considered road network comprises circa 121 bridges and 605 km of roads, including 51 km of national roads.

Lake Toma in Grisons is generally regarded as the source of the Rhine. Its outflow is called Rein da Tuma, and after a few kilometres, the outflow forms the Anterior Rhine. The Anterior Rhine is about 76 km and has a catchment area of 1,512 km². The Posterior Rhine is the second tributary of the Rhine, with less length, but a larger discharge than that of the Anterior Rhine. The basin of the Posterior Rhine is approximately 1,698 km². The river begins to be called the Rhine at the confluence of the Anterior Rhine and Posterior Rhine in Reichenau. For both rivers, detailed runoff and flood information are available from the stream gauging stations in Ilanz (No. 2033, 2498) and Fürstenau (No. 2387). Both towns are at the considered boundaries of the investigated area. Another stream gauging station is located in the study area at Domat/Ems (No. 2602), which was used as a reference point for the hydraulic model. Nine precipitation gauging stations with sufficient historical series are available in the basins un-

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3A more detailed overview of the canton of Grisons and its damage potential against natural hazards can be found in Fuchs and Bründl, (2005)
3.3. Application

Precipitation data since 1887 are available to determine and calibrate extreme rainfall. The investigated area is exposed to floods and landslides on an annual basis as the historical records of the Swiss Flood and Landslide Damage Database show. Figure 3.2 illustrates some of the recorded hazard events (45 in the area of interest) from 1975 to 2013 that affected networks. From these, 27 events fell into the category of floods/debris flow, 13 events fell into the category of landslides (other than debris flow), and the remaining 5 events were classified as rockfalls. The costs were in the range between CHF 10 thousand and CHF 6.85 million. According to the dataset, four events caused "high/catastrophic" damages while five and 36 caused "medium" and "minor" damages, respectively. Consequences included 12, 31 and two disruptions on the railway network, the road network and traffic, and the power supply, respectively. Over the period of 38 years, two bridges were severely damaged, and three bridges collapsed. One of the worst floods in recent history occurred in 2005. This flood was responsible for approximately CHF 695 million in damages to roads and railways in Switzerland (Bezzola and Hegg, 2007).

3.3.2. Application of the methodology

Additional specifications complemented the setup of the risk assessment described in the opening of Section 3.3. The assessment only considered rainfall that could happen within a one year period (i.e., the interest was on understanding what could occur in a given year before changes in the system representation due to network renewal works, network extension, and urban sprawl, among other factors). Moreover, it was assumed that only one rainfall leading to hazard events could happen throughout this period (i.e., a sequence of rainfall events within this period was not explored). The floods to be simulated would have return periods of 2, 5, 10, 25, 50, 100, 250, 500, 1,000, 2,500, 5,000, and 10,000 years. One hundred (100) events would be simulated for each return period, leading to a total of 1,200 simulated floods.

With respect to the quantitative approach to the risk assessment, as the simulation engine supported the integration of the spatio-temporal properties of the events, the estimation of direct and indirect costs, and the consideration of the aleatory uncertainty (due to natural and stochastic variability) and the epistemic uncertainty (due to incomplete knowledge of the system), no additional specifications were necessary. However, to represent the elements pertaining to the physical and natural phenomena within the simulation engine, an entity-based (vector) approach and a continuous field (raster) approach were used. An entity-based approach views space as a place to be populated by entities with clearly defined spatial boundaries and associated properties (e.g., an object within a road network and its type of use). A continuous field approach represents phenomena as a set of spatially varying values of some attribute, such as precipitation or elevation. Given these distinctive approaches, to estimate the impact of the hazard events on the road network, road sections had to be split into a set of 74,466 unidirectional entities (i.e., road subsections) in a way that an entity can be assigned to a single 16 m × 16 m cell of the hazard events' continuous fields. This is illustrated in Figure 3.3.

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4 WSL Unwetterschadens-Datenbank der Schweiz
5 Events that caused damage to buildings or forestry/agricultural assets are not considered here.
6 A definition of these terms can be found in Hilker et al., (2009).
3. Estimating network related risks

Figure 3.3.: The road network consists of numerous objects. The objects considered are bridges and road sections, which are mapped as edges in the network. Given spatial considerations during the risk assessment, the road sections are divided into subsections. The length of subsections is defined by a projected raster grid matching the grid of the hazard events.

With regards to determining the system representation, the analysed events (organised using the generic categories introduced in Section 3.2) are presented next. It is worth noting that all events were described in terms of their spatial and temporal characteristics, and measures of the intensities were assigned to these events to describe attributes of interest. Once these events were defined, scenarios were built considering the relationships among events as suggested by the caption of Figure 3.1.

- Source event: rainfall, runoff.
- Hazard events: floods and mudflows.
- Object events: bridge local scour, road section inundation, road section mud-blocking, speed reduction, and capacity reduction.
- Network events: network functionality.
- Societal events: restoration interventions, traffic changes.

To model the spatio-temporal behaviour of rainfall and the changes in river discharge values, a larger area than the area used for assessing the hazard events was considered for the source events. This is illustrated in Figure 3.2. The hazard and object events were bounded to the corresponding catchment area of the region. The boundaries of the network and societal events exceeded the boundary of the object events to model the effects at the network level properly. The costs were only accounted for the Chur region. National-level impacts were not considered (e.g., blockade of the transit route from northern Europe to the south). Furthermore, scenarios were divided into time steps, with the duration of each time step varying, depending on the occurred events in the scenario. Source and hazard events were observed every hour, in order to account for gradual changes in the system (e.g., the travelling paths of rainfall, the increase of river stage). Object, network, and societal events were evaluated on an hourly basis during the hazard events and on a daily basis after the hazard events as the restoration time was expressed in working days.

3.3.3. Modules

Due to the modular approach of the simulation engine, events were represented as autonomous physical models grouped in modules, where only the metadata such as needed input and output had to be defined (a module can contain one
or more models). Modules and models were developed in-house using Python to support the detailed study of (i) the behaviour of the comprising models as the source code was available, and (ii) the uncertainties and propagation as all variables and functions of the individual models were known and could be modified. As most network managers use geographic information systems (GIS), a GIS data interface was developed to facilitate the import and export of data. Furthermore, the program code was optimised for massively parallel computing to reduce the computational time of the optimisation process (i.e., each simulation ran on a designated CPU, and thus an increase in the amount of CPUs increased the number of simulations for a given time frame). Brief descriptions of the modules used are provided in the following sections. For the interested reader, mathematical definitions of each module are given in Appendix A.

Rainfall

The experimental precipitation catalogue described by Wüst et al., (2010) was used as the data source for rainfall modelling. The dataset covers all of Switzerland for the period between May 2003 and May 2010. This catalogue was derived from a combination of precipitation-gauge-based high-resolution interpolations and an hourly composite of radar measurements. The aggregation of the two data sources allowed for a catalogue of very high resolution at the spatial scale (1 km$^2$) and at the temporal scale (1 h). The initial precipitation fields were sampled from this catalogue using randomised start hours and durations. The precipitation values for each raster cell for each time step were then scaled to associate the return period of a generated river discharge value at a location of interest with that of the rainfall (Hackl et al., 2017b).  

**Inputs** $^8$: The return period desired to be investigated.

**Outputs** $^9$: A time series of precipitation fields (i.e., raster file for every time step), where the cell values represented the rainfall intensity per time step [mm/h].

Runoff

The modified Clark (ModClark) model by Kull and Feldman, (1998) was used to estimate the runoff. After parsing a watershed into a uniform grid (matching that of the rainfall), this model used a linear quasi-distributed transformation method to estimate the runoff based on the Clark conceptual unit hydrograph (Clark, 1945). The method accounted for spatial differences in precipitation and losses (Paudel et al., 2009), allowing to model runoff translation and storage. The implementation of this model was calibrated by comparing the calculated values with measured values from the stream and precipitation gauging stations in the area.

**Inputs** : A time series of precipitation fields, where the cell values represented the rainfall intensity per time step [mm/h].

**Outputs** : Hydrographs for different sections of the rivers in the area of study, which were generated using the excess of cells located at the basin outlets [m$^3$/s].

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$^7$It should be noted that by scaling the precipitation values for high return period, events could be under- or overestimated.

$^8$The term “inputs” refers to those inputs that are provided by other modules in the simulation engine, or externally by the network manager.

$^9$The term “outputs” refers to those outputs that are provided by the module and can be used by all other modules, or ultimately can be regarded to be the estimated consequences.
3. Estimating network related risks

Flood
A one-dimensional hydraulic model for gradually varied steady flow in an open channel network was implemented for the 31 km-section of the Rhine river in the area of study. This model was used to simulate the floodplain inundation by first computing the water surface profile of a given cross-section to the next, and then interpolating inundation values in between cross-sections to obtain the inundation field for the area of study. The 198 cross-sections, with an average distance of 150 m between them, were obtained from a digital elevation model (DEM) of the area. The model boundary conditions were represented by (i) the discharges at Ilanz and Fürstenau, (ii) the water level at the Rhine outlet, and (iii) the evolution of the additional discharges per cross-section derived from the computed hydrographs. The (Manning) roughness coefficients were estimated based on soil cover and land use. The hydraulic model was calibrated based on historical records of the stream gauging stations in the area.

Inputs: Hydrographs for different sections of the rivers in the area of study [m$^3$/s].
Outputs: A time series of inundation fields (i.e., raster file for every time step), where the cell values represented the floodwater depth above ground [m].

Mudflow
The mudflow model consisted of three main processes: (i) determination of potential mudflow locations, (ii) modelling the potential geometries and volumes, and (iii) estimation of the probability to trigger a mudflow. Potential locations and geometries were obtained from Losey and Wehrli, (2013). Potential locations were determined using geological data and relief parameters as described in Giamboni et al., (2008), and geometries were calculated using the random walk routine of Gamma, (2000). In total, 54 potential mudflows were considered in the target area of study. The volume of each mudflow was then estimated by calculating the runout length of the fan using the empirical relation of Rickenmann, (1999). The increase in elevation per cell was calculated by dividing the mudflow volume by the area of the fan.

The probability of occurrence was estimated by first using the empirical intensity-duration function for sub-alpine regions proposed by Zimmermann et al., (1997) to determine which mudflows could be triggered at a given time step. When intensity-duration thresholds were exceeded at the site of a potential mudflow, a probability of being triggered was assigned to the mudflow event. This probability was related to the corresponding estimated slope factor of safety (Skempton and Delory, 1952). When determined to be triggered, the mudflow process was assumed to happen within one simulation time step.

Inputs: A time series of precipitation fields, where the cell values represented the rainfall intensity per time step [mm/h]
Outputs: A time series of mudflow fields (i.e., raster file for every time step), where the cell values represented the deposited mudflow volume [m$^3$].

Object fragility
Fragility functions represented the relationships between hazard and object events. These functions related hazard intensity measures $\Xi$, such as river discharge, inundation depth, and mudflow volume, to the likelihood of meeting or exceeding a determined damage (limit) state $s$ of a bridge, road section/subsection. As fragility functions do not consider monetised consequences (as opposed to vul-
3.3. Application

Table 3.1: Damage states for bridge local scour, road section inundation and road section mud-blocking.

<table>
<thead>
<tr>
<th>State</th>
<th>Label</th>
<th>Bridge local scour</th>
<th>Road section inundation</th>
<th>Road section mud-blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>operational</td>
<td>no changes in bridge response</td>
<td>no observed damages, negligible sign of sediments</td>
<td>no observed damages</td>
</tr>
<tr>
<td>$s_1$</td>
<td>monitored</td>
<td>first noticeable changes in bridge response</td>
<td>presence of sediments and debris</td>
<td>encroachment limited to verge/hard strip blockage of hard strip and one running lane</td>
</tr>
<tr>
<td>$s_2$</td>
<td>capacity-reduced</td>
<td>significant changes in the bridge response</td>
<td>elements of the road section slightly damaged</td>
<td></td>
</tr>
<tr>
<td>$s_3$</td>
<td>closed</td>
<td>lack of pier stability to support the bridge</td>
<td>loss of subgrade layer</td>
<td>complete blockage of carriageway and/or repairable damage to surfacing</td>
</tr>
</tbody>
</table>

Fragility functions\(^{10}\), which measure monetised consequence ratios), their use allows treating damages and their consequences separately. The fragility functions developed for this work were assumed to be log-normally distributed\(^{11}\) (Equation 3.2), where $S$ is the realization of the damage state, $s$ is a possible damage state of an object, $\Xi$ is the observed intensity measure of the hazard event at the object location, $\Phi$ denotes the standard normal cumulative distribution function, and $\mu$ and $\sigma$ are the parameters of the estimated probability distribution.

$$P[S \geq s|\Xi] = \Phi\left(\frac{\ln \Xi - \mu}{\sigma}\right) \quad (3.2)$$

The estimated damage state exceedance probabilities output considered the cumulative effects of hazard events. In other words, only the maximum hazard intensity measure observed up until time step $t$ was used at that time step to determine the damage state exceedance probabilities. The following subsections provide specific details on the development of the fragility functions for bridge local scour, road section inundation and road section mud-blocking.

**Bridge local scour** Assuming a negligible contraction of the riverbed cross-sections (Gehl and D’Ayala, 2015), only local (pier) scour was considered for the application, resulting in the selection of five bridges that could fail as a result of this phenomenon. To quantify different levels of local scour, four damage states were defined as described in Table 3.1. Fragility functions were used to relate these damage states with a range of possible river discharge values for bridges with one pier and those with two piers. The method used the local scour model of Arneson et al., (2012) and a Monte Carlo scheme to generate 100,000 uncertain local scour depths. These depths were compared with depth thresholds assumed to correspond to each damage state, leading to the estimation of damage state exceedance probabilities. The derived fragility parameters are given in Table 3.2. The fragility functions are illustrated in Figure 3.4.

**Road section inundation** The determination of the probable damages of a road section, whether part of a major or minor road, due to floods is an active

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\(^{10}\)Such functions are also referred to as damage functions and loss functions.

\(^{11}\)Due to its simplicity and good alignment with the data, a log-normal distribution was used; see Appendix A.
3. Estimating network related risks

Table 3.2: Fragility function parameters for bridge local scour, road section inundation and road section mud-blocking.

<table>
<thead>
<tr>
<th>State Label</th>
<th>Bridge scour</th>
<th>Road inundation</th>
<th>Road mud-blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-pier</td>
<td>2-pier</td>
<td>major</td>
</tr>
<tr>
<td>s₀</td>
<td>operational</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>s₁</td>
<td>monitored</td>
<td>5.52 0.66</td>
<td>4.67 0.64</td>
</tr>
<tr>
<td>s₂</td>
<td>capacity-reduced</td>
<td>8.34 0.58</td>
<td>7.44 0.54</td>
</tr>
<tr>
<td>s₃</td>
<td>closed</td>
<td>10.1 0.61</td>
<td>9.10 0.54</td>
</tr>
</tbody>
</table>

Figure 3.4: Fragility functions for (a) bridge local scour, (b) road section inundation and (c) road section mud-blocking. The horizontal axis represents the intensity measure Ξ of the corresponding hazard. For (a) bridge local scour and (c) road section mud-blocking the axes are displayed in log-scale. The vertical axis represents the exceedance probabilities for the different damage states. The parameters µ and σ of these functions are given in Table 3.2.

field of research. The small amount of available data makes describing such a relationship in a numerical form a challenging task. Some works (e.g., De Bruijin, (2005), Kok et al., (2005), Koks et al., (2012), Tariq et al., (2014), and Li et al., (2016a)) have sought to numerically describe the relationship between damages and inundation depth. However, there are observed differences in the methods and results (e.g., some works bundled direct and indirect costs in their estimates), leading to the conclusion that these numerical relationships can only be applied to specific contexts and confined geographical areas.

To overcome this challenge, a different approach was taken, which involved proposing fragility functions based on data and information published by previous works seeking to qualitatively illustrate the impact of floods on road sections (e.g., ALA, (2005), ADEPT, (2011), Walsh, (2011), Vennapusa et al., (2013), and Roslan et al., (2015)). Despite the use of these studies, the proposed functions remain to be coarse illustrations that cannot be used in practice without further analysis. To qualify different levels of damage, four damage states were defined as described in Table 3.1. The fragility function parameters are given in Table 3.2. The fragility functions are displayed in Figure 3.4.

Road section mud-blocking To determine the level of road section mud-blocking, a distinction was made between high-speed (major) and local (minor) roads. The damage states defined by this work were based on the descriptions of Winter et al., (2013). These states are shown in Table 3.1. Fragility functions were estimated using expert data from the survey conducted by Winter et al., (2013), where experts were asked to relate debris flow volumes and damage state exceedance probabilities for different damage states and road categories (in doing
so, it was here assumed that the results of this survey, focused on debris flow, could be used for determining a relationship between mudflows and road sections. The fragility function parameters are given in Table 3.2. The fragility functions are shown in Figure 3.4.

**Inputs**: (i) Hydrographs for different sections of the rivers in the area of study (out of which only those hydrographs for the river sections with the bridges of interest were selected during the analysis) \([\text{m}^3/\text{s}]\), (ii) a time series of inundation fields, where the cell values represented the floodwater depth above ground \([\text{m}]\), and (iii) a time series of mudflow fields, where the cell values represented the deposited mudflow volume \([\text{m}^3]\).

**Outputs**: Time series of damage state exceedance probabilities considering cumulative damages for bridges due to local scour, road sections / subsections due to inundation, and road sections / subsections due to mud-blocking.

**Object functionality**
In terms of loss of functionality, a distinction was made between speed reduction and capacity reduction. The reduction of travel speed was used as a temporary measure during the hazard events period, indicating the drivers’ response to the changed driving conditions (i.e., on an inundated road, drivers reduced their speed). In addition to this temporary measure, it was assumed that damages in objects would result in a reduction of capacity (lane closure). In order to return to an adequate level of service, such objects had to be repaired (see Section 3.3.3 Restoration).

**Speed reduction** During the hazard events period, the relationship between inundation depths and feasible vehicle speeds on the road was derived from the data presented by Pregnolato et al., (2017). An exponential function was fitted to these data to describe the limit vehicle speed in a road as a function of inundation depth. The maximum acceptable velocity that ensures safe control of a vehicle through a specific section at a certain time when considering the inundation depth \(i\) was estimated by \(v = v_{\text{max}} \cdot e^{-0.10814 \cdot i}\). Where \(v_{\text{max}}\) describes the maximum allowed speed on a road (e.g., 120 km/h). It was assumed that vehicles were only operated until an inundation depth of 0.3 m.

**Capacity reduction** Functional loss functions (Lam and Adey, 2016) represented the relationships between the hazard events and the reduction in capacity. Expected functional losses were determined as functions of time-dependent hazard intensities \(\Xi\), damage state \(s\) probabilities derived from fragility functions’ damage state exceedance probabilities, and functional loss values \(\lambda\) associated with the investigated damage states (Equation 3.3). The expected functional loss of a specific object at a specific time in the simulation is represented by \(\langle \lambda \rangle \in [0, 1]\).

\[
\langle \lambda \rangle = \mathbb{E}(\lambda|\Xi) = \sum_s \mathbb{E}(\lambda|s) \cdot P[S = s|\Xi]
\]

(3.3)

Table 3.3 presents the estimated loss values \(\lambda\) used. The values were either directly obtained or inferred from a survey conducted by D’Ayala and Gehl, (2015). A loss value of \(\lambda = 0\) represents no reduction in capacity (i.e., all lanes are open) while a loss value of \(\lambda = 1\) indicates that no capacity is available (i.e., all lanes are closed). The functional loss functions are illustrated in Figure 3.5.
3. Estimating network related risks

Table 3.3: Functional loss estimations for bridge local scour, road section inundation and road section mud-blocking.

<table>
<thead>
<tr>
<th>State</th>
<th>Label</th>
<th>Bridge scour</th>
<th>Road inundation</th>
<th>Road mud-blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1-pier</td>
<td>2-pier</td>
<td>major</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 3.5: Functional loss functions for (a) bridge local scour, (b) road section inundation and (c) road section mud-blocking. The horizontal axis represents the intensity measure $\Xi$ of the corresponding hazard. For (a) bridge local scour and (c) road section mud-blocking the axes are displayed in log-scale. The vertical axis represents the expected functional loss.

**Inputs**: (i) A time series of inundation fields, where the cell values represented the floodwater depth above ground [m], and (ii) time series of damage state exceedance probabilities considering cumulative damages for bridges due to local scour, road sections / subsections due to inundation, and road sections / subsections due to mud-blocking.

**Outputs**: (i) A time series of speed reduction for inundated road sections / subsections, and (ii) time series of expected capacity reduction for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.

**Object restoration needs**

For each damaged bridge, road section/subsection, a restoration intervention had to be executed. Associated with each intervention were (i) the functional losses due to the execution of the intervention (i.e., functional loss during the restoration intervention), (ii) the length of time required to execute the intervention (i.e., restoration time), and (iii) the cost of the intervention (i.e., restoration cost). These values were estimated using the same convention as that of Equation 3.3, where $\lambda$ is replaced by the functional loss during the restoration intervention (in this application, these losses were assumed to be the same as those in Table 3.3), restoration times and restoration costs.

While a restoration cost was composed of a fixed part (e.g., site setup) and a variable part (e.g., CHF/m$^2$ of pavement, CHF/m$^3$ of concrete), the corresponding restoration time was approximated as a single value (i.e., no distinction between activities related to fixed and variable costs). These costs and durations are shown in Table 3.4. Cost estimates were based on Staubli and Hirt, (2005) and from a survey conducted by D’Ayala and Gehl, (2015). For each damage state, a
restoration strategy was derived, and for each strategy, a restoration cost\textsuperscript{12} and a restoration time were approximated. It was assumed that the selected restoration program did not affect intervention costs.

**Inputs**: Time series of damage state exceedance probabilities considering cumulative damages for bridges due to local scour, road sections / subsections due to inundation, and road sections / subsections due to mud-blocking.

**Outputs**: Time series of the expected capacity reduction during restoration intervention, restoration costs [CHF], and restoration times [h] for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.

**Network**

In cases where a road section had to be split into subsections, assigned functional losses, restoration costs and restoration times to subsections were aggregated to the section level. Aggregation routines of subsections’ functional losses, restoration costs and restoration times were implemented, which assigned the maximum functional loss of subsections in a given section to that section and the sum of the restoration costs and the sum of the restoration times of subsections in a given section to that section. The routine corresponding to the estimation of functional losses is schematized in Figure 3.6. As described in this figure as well, prior to this aggregation (from subsection level to section level), another aggregation had to be performed to consider the impact of the multiple time-varying hazard events. This latter estimation of functional losses, restorations costs, and times for subsections is described in Lam et al., (2018b). The algorithm presented in Lam et al., (2018b) is also applicable for bridges.

Once aggregated to the section level, functional losses, restoration costs and restoration times for road sections, along with those for bridges, were assigned as attributes to the network. This required modeling the road network as a graph composed of 1,520 vertices (i.e., 37 centroids, 1,056 junctions, and 427 changes in road geometric features) and 3,202 directed edges, where each edge represented a bridge or a road section. As the road network is located in a mountainous area, the topology of the network is such that certain areas are served by a single road, which means that, if part of this road is disrupted, there is no valid rerouting alternative and part of the demand remains unsatisfied. This results in missed trips. For this network, in particular, these edges (also referred to as cut links)

\textsuperscript{12}Costs taken from the literature were adjusted to 2017 price levels.
3. Estimating network related risks

Figure 3.6.: The hazard events (a) were represented in the form of a spatially distributed grid where the intensity measures change for each cell (b). Each road section of the network had to be split up according to this grid, which resulted in a set of subsections. This process was also previously illustrated in Figure 3.3. In this specific example, the considered section is split into four subsections (b). The intensities are then processed independently for each of these subsections. For each intensity measure, the expected functional loss (i.e., speed reduction, capacity reduction) for each subsection is determined using computed functional loss functions $f_{FL1}$ and $f_{FL2}$ (c). Schematized in panel (d) are the resulting functional losses that are encoded in the colours of the subsections. A subsection aggregator $A^{I}$ (e) is executed for each subsection and combines all functional loss values derived from different hazard events into a single value (f). This aggregator simply takes the maximum functional loss values. Finally, a section aggregator $A^{II}$ (g) combines the functional losses of the subsections into a single functional loss for the section (h). Losses at the section level are then used in the network model (i). (This figure was adapted from Heitzler et al., (2017a).)

After the hazard event period, the network was restored using a restoration model. This model supported the updating of damaged objects to restored objects (see Section 3.3.3 Restoration), leading to updated network graphs to be used in the traffic model to estimate the indirect costs.

**Inputs**: (i) A time series of speed reduction for inundated road sections / subsections, (ii) time series of expected capacity reduction for bridges with scoured piers, inundated road sections / subsections, and mud-blocked road sections / subsections, and (iii) time series of the expected capacity reduction during restoration intervention, restoration costs [CHF], and restoration times [h] for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections, and (iv) a restoration program, defining when each damaged object is to be restored.

**Outputs**: (i) A time series of routable network graphs that can be used for traffic assignment, and (ii) time series of the expected capacity reduction during restoration intervention, restoration costs [CHF], and restoration times [h] for bridges with scoured piers, inundated road sections and mud-blocked road sections.

**Traffic**

Vehicle travellers were assumed to behave according to the user equilibrium principle, which states that they choose a route from their origin to their destination that minimises their travel cost. This means that the travel cost between each origin-destination pair is uniquely defined (Jenelius et al., 2006). This cost depends on the travel costs of all edges in an origin-destination path, which change

represented about 11% of the entire edge set.
with traffic flow. A stable state is reached only when no traveller can reduce his/her own costs of travel by unilaterally changing routes (Sheffi, 1985). Travel time was estimated using the formulation proposed by the Bureau of Public Roads, (1964).

Regarding the demand model, the origin-destination matrix was estimated by a gravity model based on population density and was related to the number of vehicles on an average hourly and an average daily basis. This matrix was calibrated and updated using information of recent traffic counts along a set of edges. The study area was divided into 37 zones. Every internal centroid corresponded on average to an area of 15 km$^2$ with a population of about 1,000.

To reduce computational complexity, only changes in route choices were considered within the risk assessment (i.e., travel demand was assumed to be inelastic). This was deemed a reasonable assumption as a vast majority of the travel demand was caused by trips to work, which are made under normal circumstances. Therefore, changes in destination choices, mode choices, or trip frequencies were not considered. Moreover, it was assumed that the duration of network closures was long enough for all travellers to be aware of them so that a new user equilibrium could be reached.

**Inputs** : A time series of routable network graphs that can be used for traffic assignment.

**Outputs** : (i) A time series of traffic flow and travel time for each edge in the network, (ii) a time series of missed trips in the network, and (iii) a time series of damaged bridges and road sections that caused loss of connectivity.

**Restoration**

Once the discharge values along the river returned to normal conditions and no further damages to objects could occur, the impaired network was restored. A basic restoration model simulated this procedure (an improved version of the work of Lam and Adey, (2016)). The prioritisation of restoration activities was done by first restoring objects that caused loss of connectivity and then restoring objects based on their average traffic volume (i.e., the average daily traffic volume for each object under normal conditions). It was assumed that ten reconstruction crews could work simultaneously and each crew could at most be assigned to one object. For each day until the restoration was finished, the model updated the objects’ states and executed the traffic model described to support the estimation of indirect costs during the restoration period. The restoration process ended once the network was completely restored.

**Inputs** : (i) Time series of the expected capacity reduction during restoration intervention, and restoration times [h] for bridges with scoured piers, inundated road sections, and mud-blocked road sections, and (ii) a time series of damaged bridges and road sections that caused loss of connectivity.

**Outputs** : A restoration program, defining when each damaged object is to be restored.

**Direct and indirect costs**

The direct costs were estimated to be the sum of the direct costs for each intervention to be executed during the restoration period. These costs were actually accrued during the hazard events period as bridges and road sections were impacted by the hazard events (see Section 3.3.3 Object restoration needs). The direct costs were composed of a fixed part (e.g., side setup) and a variable path (e.g., CHF/m$^3$ of pavement). The direct costs associated with each damage state...
3. Estimating network related risks

Figure 3.7.: Spatio-temporal representation of the system. The columns represent the investigated events, while the rows illustrate different time steps of the considered scenario. Time step 1 represents the beginning of the simulation with all elements at their initial state. Time step 10 illustrates the system state close to the peak of the flood. As observed, road sections were damaged by the flood, which resulted in functional losses and a change of traffic flow. In this scenario, people living in the northern part of the area were cut off from the rest of the network due to severely inundated roads.

are given in Table 3.4. The indirect costs were comprised of costs for the prolongation of travel time and costs due to a loss of connectivity. These costs were accrued during the hazard events period and the restoration period. The travel time costs were estimated based on the increased amount of time vehicles spent travelling. The Swiss Association of Road and Transport Experts (VSS, 2009b) approximated the travel time cost per vehicle to be 23.29 CHF/h.

In addition to considering the time factor, indirect costs also accounted for the costs of vehicle operation, which were incurred as a result of fuel consumption and vehicle maintenance. Based on the estimates of VSS, (2009a), the mean fuel price was approximated with 1.88 CHF/litre with mean fuel consumption of 6.7 litres/100veh - km and the operating costs per vehicle without fuel consumption was assumed to be 14.39 CHF/100veh - km. The costs due to a loss of connectivity were estimated based on the unsatisfied demand and the resulting costs due to missed trips. For every hour of delay of a trip, costs in the amount of 83.27 CHF were charged.

**Inputs**: (i) A time series of the expected restoration costs [CHF] for bridges with scoured piers, inundated road sections and mud-blocked road sections, (ii) a time series of traffic flow and travel time for each edge in the network, and (iii) a time series of missed trips in the network.

**Outputs**: Time series of the estimated direct and indirect costs.

3.4. Results

The results section is organised in three subsections that present (i) the results of a single simulation run, (ii) the aggregation of results of multiple simulations with the same return period, and (iii) the results of multiple scenarios with multiple return periods.

3.4.1. Single scenario

Figure 3.7 shows the changes in a scenario due to a flood of 500-year return period. For this specific chain of events, the hazard events extended over a period of 18 hours while the restoration period lasted 23 days (please note that while only two time steps are shown in this figure).
3.4. Results

Figure 3.8.: Aggregation of multiple simulations for hazard events with a 500-year return period. This map visualises the uncertainties related to these events. For example, only 5% of the floods exceeded the light purple floodplain (95% percentile), while only 5% of the floods resulted in floodplains smaller than the dark purple area (5% percentile).

In this scenario, the rainfall started at hour 1 and ended at hour 10, with the maximum precipitation observed around hours 6 and 7. The precipitation moved from the northwestern part of the study area to southeastern part. At hour 6, parts of the motorway to Fürstenau were flooded, which led to a detour of the traffic through the village of Bonaduz. At hour 8, the motorway was flooded between Tamins and Domat/Ems, which interrupted the west-east connection in the valley. Traffic had to detour through Chur and drive south. Due to the lag runoff, the maximum extent of the flood was reached at hour 11. Most of the flood damage was caused in the western part of Chur, while in the northern part, only the motorway and several minor roads were flooded. Additionally, the triggering of a mudflow next to the village of Felsberg can be observed in hour 10, causing the blockage of two minor roads. This type of analysis is useful, to investigate the spatial and temporal evolution of the events. However, a single scenario, cannot capture the uncertainties of the events. To overcome this issue, multiple scenarios with the same initial conditions (i.e., the same return period) can be performed.

3.4.2. Multiple scenarios with the same return period

Figure 3.8 illustrates the aggregated simulation results for 100 scenarios with 500-year return period. On the top left, the 25, 50 and 75-percentile precipitation fields are shown, with darker areas indicating more intense rainfall. The hazard events of interest are also presented, specifically the 5, 50 and 95-percentile of possible inundation depths as well as the location of possible mudflows colour-coded according to their probability of occurrence. The expected discharge along the river is also illustrated in the graph. It can be observed that the 5, 50 and 95-percentile values were approximately 1,690 m$^3$/s at section 30. This value corresponded to the targeted river discharge value for a 500-year flood at the predefined gauging station located in that section. While the estimated direct costs are illustrated in Figure 3.8, the indirect costs are not shown because these latter costs are a property of the whole system, and hence no clear spatial positions can be assigned to them.

As illustrated in Figure 3.8, the mapped floods can be interpreted as a prob-
3. Estimating network related risks

Figure 3.9: Direct and indirect costs related to the hazard events with a 500-year return period. A heat map of expected direct costs for all scenarios (a) can be used to analyse high-risk areas. Here, five clusters were detected. The estimated direct costs per cluster and their associated uncertainties are illustrated in panel (b) using box plots. While in the second cluster, the median direct cost was around CHF 4.8 million, in the fifth cluster, a much smaller median value was estimated. However, the uncertainty of this cluster on the positive direction was comparable to that of the second cluster in the same direction. Comparing direct and indirect costs (c), their medians were similar. Nonetheless, the distribution of direct costs was positively skewed (heavy tail) while the distribution of indirect costs was close to symmetrical.

A probabilistic hazard map for events of a 500-year return period. While such maps can be used to reach some conclusions about the probable inundated areas, it is possible also to use geostatistical tools to analyse these and other information. Figure 3.9 shows the automatic cluster of damaged objects in the network using a mathematical algorithm that also supports the visualisation of the associated direct costs. Such analysis can also be used for the planning of mitigation measures as part of the management of risk.

Figure 3.9 gives additional insights into the resulting costs. Direct costs followed a heavy-tailed distribution (i.e., scattered high values in the positive direction of the distribution). Such distributions are often observed when considering extreme events, which is the case in the application that modelled random mudflows in addition to floods. For example, a mudflow was occasionally triggered in cluster five (see Figures 3.8 and 3.9). When the mudflow was not triggered, little to no damage was observed in this cluster; however, when the mudflow was triggered, severe damage occurred. Contrary to direct costs, indirect costs followed a symmetric distribution due to the network redundancy (i.e., as long the network is not completely out of service, vehicles can take detours, leading to a redistributing of the indirect costs).

3.4.3. Multiple scenarios with multiple return periods

Figure 3.10 shows the obtained risk curves (return period vs costs) with confidence intervals and annualised risk estimates (how much risk can be expected per year on average over a long period of time). Once a risk curve of a specific percentile was determined, its corresponding costs were used to estimate the matching annualized risk using Equation 3.4 (Deckers et al., 2009), where $R^a$ is the annualized risk and $C_i$ the costs associated with the annual exceedance probability $\frac{1}{T_i}$, which is estimated based on the return period $T$ of the event $i$. To use this equation, the events had to be arranged in decreasing order (e.g., $\frac{1}{2}$, $\frac{1}{5}$, $\ldots$, $\frac{1}{10000}$).
3.5. Discussion

Since the primary goal of the presented application was to illustrate the use of the methodology, models were selected based on the data available and a desire to keep computational time low to support the exploration of a large set of scenarios. This way, multiple hazards of various return periods could be considered along with changes in the traffic flow and restoration activities, leading to a more encompassing way to estimate risk. Due to the modular approach used, if desired, models that are more sophisticated may be integrated in the future. For example, the traffic model used here provided a low-complexity representation of driver behaviour (e.g., travellers had full knowledge of the traffic conditions). Moreover, the traffic model did not account for dynamic phenomena like queues, spillbacks, wave propagation, or changes in travel patterns after a disruptive event (although studies show these can be considerably different after a disruptive event; e.g., Chang and Nojima, (2001) and Kontou et al., (2017)). In general, the integration of more sophisticated models can potentially lead to improved risk estimates. A prerequisite for this integration is conducting uncertainty and sensitivity analyses to prioritise the parts of the system to be analysed in more detail.

Due to the modular approach and the universal nature of the models used, the implemented simulation engine conceptually can be used to estimate the risk related to other river systems and road networks, provided the required datasets are available. It is worth noting that the quantity and quality of data needed to be used in this study are mostly available for many locations around the world. Alternatively, single parts of the simulation engine may be applied

\[
R^a = \sum_i \frac{1}{T_i} \cdot (C_i - C_{i-1})
\]  

(3.4)

When using this equation, direct costs were found to follow a heavy-tailed distribution. The median annualised risk related to direct consequences (CHF 1.5 million) was more extensive than that of indirect consequences (CHF 1 million). If other indirect consequences had been considered in addition to prolongation of travel time and missed trips (e.g., business interruptions), the annualised risk related to indirect consequences would be expected to increase significantly.

Figure 3.10.: Risk curves and annualised risk estimations. The risk curves related to direct and indirect costs are illustrated in (a). The 50% and 90% confidence intervals are given for the sum of the costs. Costs associated with individual simulation runs are illustrated as grey dots. Equation 3.4 was used to calculate the annualised risk (b) for different cost percentiles. The median annualised risk for the sum of the costs was found to be CHF 2.4 million.
3. Estimating network related risks

independently (e.g., to investigate the probability of bridge failure due to local scour at a given location). Hence, the system may be profitably used for a number of additional purposes (e.g., as a tool for cost-benefit analysis of flood and mudflow protection measures, as a decision support system for operational flood and mudflow control). As implemented, the risk assessment can provide a mechanism for a region-wide screening of priority locations for risk reduction based on the analysis of the road network and traffic properties (e.g., Heitzler et al., (2017b)). This function can be enhanced with the use of visualisation tools, enabling network managers to dynamically see how different environmental systems (i.e., rainfall, flood and mudflow) influence each other, and how their impact on networks can affect society (Heitzler et al., 2016).

Finally, combining several models results in a significant degree of uncertainty. Whenever possible, the results should be compared with and calibrated against empirical data, when available. For example, the probability of damages obtained through simulations could be calibrated against collected data from field structural surveys when such data exist. Calibration for societal events is somewhat difficult because such data are difficult to measure and monitor. Nonetheless, in some cases, basic data are available, including the duration of a network’s loss of functionality and the estimated number of network users affected.

3.6 Conclusions

The purpose of estimating the risk related to networks is, among others, to provide an overview of the probable adverse events that may negatively affect the network, assess their societal effects (e.g., costs), and provide a basis for planning risk-reducing interventions. Assessing the risk related to networks exposed to multiple types of hazards is not trivial due to the large number of events that need to be modelled in an integrated manner, their uncertainties and the propagation of these uncertainties affecting risk estimates. This is particularly evident when dealing with complex system representations, where the costs of indirect consequences can be multiple times higher than the costs of direct consequences, with no linear relationship between these types of cost.

This work describes a risk assessment methodology for networks when there is a need to represent the system containing these networks using various degrees of complexity. The methodology is supported by a modular simulation engine that fosters multidisciplinary collaboration. Following this methodology, a state of the art application was designed and implemented, which aimed at estimating the spatio-temporal risk of a road network in Switzerland due to the occurrence of a time-varying rainfall that caused flood and mudflow events. To achieve this objective, the modular simulation engine was used to couple rainfall, runoff, flood, mudflow, damages, functional losses, traffic, and restoration modelling. Consequences were monetised into direct and indirect costs, considering restoration interventions, prolongation of travel time, and missed trips. The costs of the 1200 scenarios simulated were analysed at three different levels: (i) those of a single simulation, (ii) those of multiple simulations with the same return period, and (iii) those of multiple simulations of multiple return periods. This number of scenarios supported the analysis of cost uncertainties.

The use of the methodology is not limited to hydro-meteorological hazard events or road networks. For example, the methodology can be applied to other hazards (e.g., earthquakes, coastal floods, rockfalls) or other networks (e.g., railway, waterways, inter-modal). Such analyses would require appropriate hazard models and descriptors of the relationships between the hazard events, dam-
age and functional losses (e.g., no rail traffic due to a settlement of the rail tracks following an earthquake) as well as appropriate datasets. Additionally, other societal events, such as business interruption, rescue missions, and access to education/health services, among others could be implemented in future work. Nonetheless, depending on the complexity of the system representation, some of these applications may result in computationally intensive risk assessment designs, increasing the time required to compute risk estimates. However, for the simulation engine to be of value to other researchers, it is necessary to refactor and clean up the code base.
4. Determination of near-optimal restoration programs for transportation networks following natural hazard events using simulated annealing

This chapter corresponds to the published article:


Abstract: Disruptive events, such as earthquakes, floods, and landslides, may disrupt the service provided by transportation networks on a vast scale, as their occurrence is likely to cause multiple objects to fail simultaneously. The restoration program following a disruptive event should restore service as much, and as fast, as possible. The estimation of risk due to natural hazards must take into consideration the resilience of the network which requires estimating the restoration program as accurate as possible. In this work, a restoration model using simulated annealing is formulated to determine near-optimal restoration programs following the occurrence of hazard events. The objective function of the model is to minimise the costs, taking into consideration the direct costs of executing the physical interventions, and the indirect costs that are being incurred due to the inadequate service being provided by the network. The constraints of the model are annual and total budget constraints, annual and total resource constraints, and the specification of the number and type of interventions to be executed within a given time period. The restoration model is demonstrated by using it to determine the near-optimal restoration program for an example road network in Switzerland following the occurrence of an extreme flood event. The strengths and weaknesses of the restoration model are discussed, and an outlook for future work is given.
4. Restoration programs for transportation networks

4.1. Introduction

Infrastructure networks are essential for economic growth and development. Critical services such as power, water distribution, and transport are provided by these infrastructure networks. The damage of such a network could cause severe disruption to service, probably out of all proportion to the actual physical damage (Vespignani, 2010). With this in mind, the quick restoration of damaged infrastructure following a disruptive event is critical for the society. Managers of these infrastructure networks have the challenging task to determine optimal restoration programs, i.e. the optimal plans of how, and the order in which, the damaged infrastructure objects will be restored so that they provide adequate levels of service (LOS), taking into consideration the possible improvements in service, the costs and the limited available budget and resources (Cavdaroglu et al., 2011).

In this work, a restoration model using simulated annealing is formulated to determine the near-optimal\(^2\) restoration program to restore service on transportation networks following the occurrence of a disruptive event. The objective of the model is to minimise the costs, taking into consideration the direct and indirect costs. Direct costs are associated with the execution of interventions, such as cleaning-up, reparation, rehabilitation or reconstruction. Indirect costs are associated with the traffic flow of the network and include costs for additional travel time, vehicle operating costs or increased risk of accidents. Constraints, such as limits on available funding and resources, and limits on the type of intervention that can be executed per damage state are taken into consideration. The restoration model can be classified as a multilevel problem, where multiple problems have to be solved dependently (e.g. the minimum costs depend on the planned interventions, which influences the traffic, which in turn causes additional (indirect) costs). Under certain assumptions, (e.g. the optimisation of the costs depends on the traffic assignment that satisfies Wardrop’s user equilibrium conditions (Wardrop, 1952), which can be expressed as first-order optimality conditions for a convex program (Beckmann et al., 1959)) the problem can be expressed as a bilevel optimisation problem. However, also, in this case, classical optimisation methods cannot be used for multiple reasons, including non-linearity, non-convexity, and non-differentiability. Hence, in this work simulated annealing (SA), is used to solve the problem.

The presented restoration model can be used to determine near-optimal, restoration programs, which include the objects to be repaired and the time and type of interventions to be executed to minimise overall costs. The model is illustrated by using them to develop a restoration program on parts of the road network in the Canton of Grisons, Switzerland, comprised of circa 51 km national roads, 165 km main road, 395 km minor roads and 116 bridges. Although the structure of the network is real in the example, fictive damage states following a fictive flood event were used, as were fictive origin-destination data for the traffic assignment and fictive costs. The use of fictive but realistic data in no way diminishes the illustration of the usefulness of the model applied. The restoration model is shown to work in situations where the network topology, traffic demand, and the available resources are to be taken into consideration.

The proposed restoration model is novel in the sense that it is the first model that schedules a tactical restoration program by minimising direct and indirect

\(^2\) In the remainder of this work, a “near-optimal solution” in context of SA means an approximated optimal solution, which is a good, though not necessarily the global optimal solution.
4.2. Related work

Costs for multiple object types, damage states and different interventions associated with each state of each object. Thereby, the model accounts for time-dependent resource limitations and budget constraints as well as different traffic assignments caused by the applied restoration strategies. Furthermore, the restoration model is applicable to real-world networks by utilising heuristic processes to solve the complex bilevel optimisation problem, as illustrated in a realistically sized case study in Switzerland. Computing such a network, shows that the restoration model presented here can be of great use for infrastructure managers overseeing the reliability and resilience of critical infrastructures to disruptive events, by obtaining relevant information concerning the investment in recovery operations such as insights on the trade-off between recovery budget and quality of the resulting restoration program.

The remainder of this work is organised as followed: In Section 4.2 related work about restoration modelling for transportation networks after the occurrence of disruptive events is summarised and discussed. The restoration model is presented in Section 4.3, and the simulated annealing is presented in Section 4.4. The example is given in Section 4.5. In Section 4.6 a discussion about the advantages and disadvantages of the restoration model is given. A summary of the work and suggestions for future work in this area are given in Section 4.7.

4.2. Related work

Consequences of transportation network failure depend greatly on how all of the objects within the affected transportation network behave, and on the restoration program adapted to, again, provide an adequate LOS (Hackl et al., 2016). Insight into how such network-related consequences can be classified and estimated is given in (Hackl et al., 2015b; Adey et al., 2016; Hackl et al., 2016; Lam and Adey, 2016). In particular, Lam and Adey, (2016) discusses the interrelationship that exists between the restoration program for damaged objects and the ability of networks to provide an adequate LOS over time. Understanding the restoration process is critical for evaluating risks related to transportation networks, and for designing and operating robust transportation networks to provide an adequate LOS.

The determination of the optimal restoration program for transportation networks has been the focus of a substantial amount of research over the past few decades. These have focused on modeling the restoration program after disruptive events, including earthquakes (Çagnan and Davidson, 2004; Luna et al., 2011; Isumi et al., 1985; Chen and Tzeng, 2000; Bocchini and Frangopol, 2012), storms (Liu et al., 2007; Ramachandran et al., 2015), and floods (Lertworawanich, 2012), considering different individual infrastructure networks such as road networks (Chen and Tzeng, 1999, 2000; Chen and Miller-Hooks, 2012; Bocchini and Frangopol, 2012; Lertworawanich, 2012), power networks (Çagnan and Davidson, 2004; Liu et al., 2007), water distribution networks (Luna et al., 2011), or interdependent networks (Isumi et al., 1985; Ramachandran et al., 2015). Similar optimization models are used for maintenance and rehabilitation planning (Ouyang and Madanat, 2004; Ng et al., 2009; Kuhn, 2010; Sathaye and Madanat, 2011).

One type of model often used, and the type used in this work, is a deterministic resource constraint model (Çagnan and Davidson, 2004). In this type of model, the restoration program is modelled determined using a set of simple equations and rules. Constraints in resources can be taken into consideration, e.g. the number of work crews available, the rate of repair for different objects, or traffic flow on the network. An advantage of models of this type is that they can deter-
mine a restoration program both in time and space. A disadvantage, however, is that they are deterministic, which does not allow for effective modelling of the uncertainties related to inputs such as restoration time and cost, which does not necessarily capture reality (Luna et al., 2011).

Isumi et al., (1985) used differential equations to determine optimal restoration programs taking into consideration the number of available personnel and the efficiency of repair as a function of time, for each supply area. Chen and Tzeng, (2000) developed more detailed models using a multilevel approach that takes into consideration intervention costs and travel time costs, where the latter involved determining an equilibrium in the traffic flow on the network at different periods of time during the restoration program. In the developed models, several different objective functions have been used, including the minimization of travel time (Chen and Tzeng, 1999), the minimization of the number of impassable paths (Nolz et al., 2011), the minimization of the amount of freight that cannot be shipped (Chen and Miller-Hooks, 2012), and the minimization of the total cost of the restoration activities (Bocchini and Frangopol, 2012). In most cases, these objective functions have been constrained due to a limited budget. In a few cases, however, other constraints, such as limited resources have been considered. In the majority of these models, it is possible to take into consideration different intervention types (Vugrin et al., 2010a) and different damage states of the objects (Bocchini and Frangopol, 2012).

To solve many of the models, due to the computational complexity of the problem, the optimal restoration program has been determined either using stochastic mixed-integer programs solved by Monte Carlo simulations (Chen and Miller-Hooks, 2012) or using heuristic procedures, such as genetic algorithms (GA) (Chen and Tzeng, 1999), ant colony system (ACS) (Yan and Shih, 2012), or simulated annealing (SA) (Vugrin et al., 2014). Although many of these models can be run relatively quickly to find near-optimal restoration programs, none of them have been used in real-world situations of comprising complexity (e.g. networks larger than a dozen of edges).

Notable work in determining restoration programs by using SA has been conducted by Vugrin et al., (2010a, 2014). They used a modified resource-constrained project schedule approach (Boctor, 1996) in order to optimise the tasks within an intervention.

In addition to the research that has used Monte Carlo simulations or heuristic procedures to determine near-optimal intervention programs, some have been conducted to determine the optimal intervention programs directly, such as Hajdin and Adey, (2005, 2006), Lethanh et al., (2014), and Eicher et al., (2015). Although these models allow for very realistic representations of the world, their use becomes severely difficult, if not intractable, when intervention programs are to be determined for large networks and actual traffic flow has to be considered.

4.3. Restoration model

The mathematical model used in the work to determine the optimal restoration program for a transportation network minimises the weighted sum of direct and indirect costs that occur over the time period between the occurrence of the hazard event and the moment that the restoration of the network is complete.

Preliminaries and an introduction to the network model and the definition of damage states and functional losses are given in Sections 4.3.1 and 4.3.2. The mathematical model is explained in Section 4.3.4. The notation used is given in Appendix D.
4.3. Restoration model

4.3.1. Network design

A transportation network is denoted by $G = (V, E)$ and hereinafter referred to as the network. Formally, $G$ is a mathematical structure, which describes a set of vertices $V$ and a set of edges $E$. Both vertices and edges correspond to objects in the physical network. Vertices are well suited to represent objects that are to be seen as points in the physical network, e.g. bridges or road crossings, whereas edges are well suited to represent objects that are to be seen as objects with length, e.g. road sections and rail sections. The choice depends on the purpose of the model. Here, the notation $n \in \mathcal{N}$ refers to any object in the network $G$, where $\mathcal{N} = V \cup E$ is the set of all considered objects.

The movement of vehicles on the network is defined by the graph $H = (V^H, E^H)$ where the vertex set $V^H \subseteq V$ contains origin vertices $O \subseteq V^H$ and destination vertices $D \subseteq V^H$. Two vertices $o \in O$ and $d \in D$ are connected by an edge in $H$ if and only if there is a traffic demand with positive demand value between them, hence an edge in this graph is also called od-pair.

Graphs are assumed to be directed. An edge is, therefore, an ordered pair $e = (u, v) \in E$ indicating that $u$ and $v$ are directly connected and vehicles travel only from $u$ to $v$. A set of capacities is defined for each edge. The capacity $y_e$ of an edge $e = (u, v)$ is the upper limit on flow in a specific time interval, e.g. in vehicles per hour. A demand from $o$ to $d$ is denoted by $od$ and stands for the directed demand $(o, d)$.

Vehicle movements from origin to destination vertices, which occur along edges, are represented as paths. A path $P \in \mathcal{P}$ is considered to be a sequence of edges that ordered so that two vertices are adjacent if and only if they are consecutive. $\mathcal{P}$, therefore, denotes the set of all nonempty simple paths in $G = (V, E)$. The set of od-paths is denoted by $\mathcal{P}_{od} \subseteq \mathcal{P}$.

4.3.2. Damage states and functional losses

The occurrence of a hazard event can result in the loss of functionality of the objects within a transportation network, which in turn results in the loss of functionality of the network. These functionality losses may occur due to objects being damaged by a hazard, e.g. a bridge being knocked off its bearings due to excessive ground motions, a road section being eroded by scour or is blocked by flood waters. More detailed examples of road objects can be found in Lethanh et al., (2015).

The set of considered states is denoted $\mathcal{S}$. These states are defined taking into consideration the ability of the objects to provide the required LOS, which involves the consideration of aspects such as the capacity, i.e. the maximum number of vehicles that can travel over the object in a specified period of time. The objects in a normal state are denoted by 0 and all other states are denoted by $s \in \mathcal{S}$. For example, $n \in N^s$ describes objects with reduced functionality, where $N^s \subseteq \mathcal{N}$ denotes the set of objects in state $s \in \mathcal{S}$. Full functional loss, i.e. the state when no traffic flow over an object is possible, is denoted by $g \subseteq s$.

4.3.3. Costs

Direct costs

Direct costs are intervention costs, i.e. the costs of restoring objects to states where they function again as intended. For each object with reduced functionality $n \in N^s$, a (finite) set of possible interventions $\mathcal{I}(n|s)$ is assigned. Only one of these can be selected at a time to restore functionality of $n$. Associated with each intervention $i \in \mathcal{I}(n|s)$ are (i) the flow capacity $\Delta y_{n,i}$ following the execution of
4. Restoration programs for transportation networks

the intervention, (ii) the length of time required to execute the intervention $\tau_{n,i}$, (iii) the amount of resources required $r_{n,i,k}$ for resource $k \in K$, and (iv) the cost of the intervention $C_{n,i} \geq 0$. This cost is composed of a fixed part, a length-dependent part, and a variable part dependent on the used resources. The overall direct costs $C^{DC}$ are the sum of the direct costs for each intervention executed. It is assumed that intervention costs are not affected by the selected restoration program.

$$C^{DC} = \sum_{n \in N_s} \sum_{i \in I(n|s)} \sum_{t \in T} \delta_{n,i,t} \cdot C_{n,i}$$  \hspace{1cm} (4.1)

where $\delta_{n,i,t}$ is a binary variable, which has a value of 1 if intervention $i \in I(n|s)$ is executed on object $n$ in state $s$, initiated at period $t$ and 0 otherwise. Since at most one intervention is executed on an object at a time (i.e., at most one of the $\delta_{n,i,t}$ has to be 1 and all others have to be 0), the capacity of object $n \in N_s$ at time $t$ is

$$y_{n,t} = y_{n,0} + \sum_{i \in I(n|s)} \sum_{j \in T - \tau_{e,i}} \delta_{n,i,j} \cdot \Delta y_{n,i} \hspace{1cm} \forall n \in N$$  \hspace{1cm} (4.2)

where $y_{n,0}$ is the capacity immediately following the disruptive event.

**Indirect costs**

Indirect costs are divided into two categories: (1) those associated with temporal prolongation of travel, and (2) those associated with the loss of connectivity, as suggested by Adey et al., (2004). Both can be associated with states of the objects. In the case of prolongation of travel, indirect costs are principally caused by such things as additional travel time, additional vehicle operating costs and additional accidents. While in the case of a loss in connectivity, indirect costs are principally due to the loss of economic activity that occurs while travel is not possible. The magnitude of each depends on the network design and the traffic flow. A complete list of both types of indirect costs can be found in Adey et al., (2012b).

Indirect costs $C^{IC}$ are measured as the difference between indirect costs at $t$ and the indirect costs at $t = 0$ when the network was fully functional.

$$C^{IC} = \sum_{t \in T} \left[ \sum_{P \in \mathcal{P}^{x_e,t}_{e \in E}} \Pi(t|x_e,t) - \Pi^0(t|x_e,0) + \sum_{P \in \mathcal{P}^{\phi}_{od}} \Lambda(t) \right]$$  \hspace{1cm} (4.3)

where $\Pi$ is a cost function dependent on the link traffic flow $x_e,t$ on edge $e$ in period $t$, and $\Lambda$ is a cost function dependent on a loss of connectivity. $\Pi^0$ is associated with the costs in when the network if fully functional. $\mathcal{P}^{x_e,t}_{od}$ refers to the set of $od$-paths where at least some flow is still possible, while $\mathcal{P}^{\phi}_{od}$ refers to the set of $od$-paths where no flow is possible. The former refers to the those containing no objects with zero functionality $g$. A vertex $v^g$ that has zero functionality $g$, renders all incident edges non-functional. Thus, $\mathcal{P}^{x_e,t}_{od}$ is the set of $od$-paths where at least some flow is still possible.

$$\mathcal{P}^{x_e,t}_{od} = \{ P \in \mathcal{P}^{x_e,t}_{od} \mid e^g, \Gamma(v^g) \notin P \}$$  \hspace{1cm} (4.4)

where $\Gamma(v)$ denotes the set of incident edges of a vertex $v \in V$ in $G$. 

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4.3. Mathematical model

The objective of the mathematical model is to find a restoration program that minimises the sum of costs, considering Eq. (4.1) for the direct costs and Eq. (4.3) for the indirect costs. The mathematical model allows considerations of the functional losses of edges and vertices in the network. The objective function is written as:

\[
\min Z^R = \sum_{t \in T} \left[ \sum_{n \in N^s} \sum_{i \in I(n[s])} \delta_{n,i,t} \cdot C_{n,i} + \gamma \sum_{P \in P^{\lambda g}} \Pi(t|x_{e,t}) - \Pi^0(t|x_{e,0}) \right]
\]

subject to

\[
\sum_{n \in N^s} \sum_{i \in I(n[s])} \delta_{n,i,t} \cdot C_{n,i} \leq \Omega_t \quad \forall t \in T
\]

\[
\sum_{n \in N^s} \sum_{i \in I(n[s])} \sum_{j \in t-\tau_{n,i}+1} \delta_{n,i,j} \cdot r_{n,i,k} \leq \Psi_{k,t} \quad \forall k \in K, \forall t \in T
\]

\[
x_{e,t} \in \min Z^T = \sum_{e \in E^s} \int_0^{x_{e,t}} C^T_e(\omega) \, d\omega
\]

subject to

\[
\sum_{P \in P^{\lambda g}} f_{od}(P) = d_{od} \quad \forall od \in V^H
\]

\[
f_{od}(P) \geq 0 \quad P \in P^{\lambda g}, \forall od \in V^H
\]

The optimisation problem presented, is a bilevel optimisation problem, i.e. a problem where a subset of the variables from a part of the model is constrained to the optimal solution of another part of the model. Here the indirect costs (Eq. (4.3)), which are part of the upper-level optimization (Eq. (4.5a)) depend on the link traffic flow \(x_{e,t}\). Beckmann et al., (1959) established that the solution to the traffic assignment (network equilibrium) problem, in the case of user travel cost functions \(C^T_e\), in which the cost on a link only depends on the flow on that link and is assumed to be continuous and an increasing function of the flow, can be obtained by solving the (lower-level) optimization problem (Eq. (4.5e)) (Nagurney, 2007). In return, the travel cost function \(C^T_e\) and the feasible paths \(P \in P^{\lambda g}\) are determined by the interventions selected in the upper-level optimisation.

The accompanying constraints of Eq. (4.5a) are continuity constraints, budget constraints, and resource constraints. The continuity constraints Eq. (4.5b) force the model to select only one intervention per object, which is executed in one time interval, throughout the investigated time period. The budget constraint Eq. (4.5c) forces the model to select no more interventions than for which funding is available, i.e. the total cost of all interventions cannot exceed \(\Omega\) for the
investigated time period \( t \). The resource constraint Eq. (4.5d) forces the model to select no more resources than available in a time interval \( t \), i.e. the amount of resources \( k \) in time interval \( t \), needed for the interventions, cannot exceed \( \Psi_{k,t} \).

Other constraints for the direct costs could be added if desired; for example, time constraints, accessibility constraints, or maximum or minimum work-zone constraints (Lethanh et al., 2016).

The link traffic flow \( x_{e,t} \) is estimated, by solving the user equilibrium assignment, Eq. (4.5e) subjected to Eqs. (4.5f) and (4.5g). The costs of travel on each edge change with the flow and, therefore, the costs of travel on several of the network paths change as the edge flow changes. A stable state is reached only when no traveller can reduce his costs of travel by unilaterally changing routes (Sheffi, 1985).

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The lower-level optimisation Eq. (4.5e) for the user equilibrium assignment, corresponds to the sum of the integrals of the travel cost function \( C^T_e \) for all edges \( e \) in the network. The demand constraint Eq. (4.5f), stating that the flow on all \( od \)-pairs has to equal the demand \( d_{od} \geq 0 \) for all \( od \in \mathcal{V}^H \). The non-negativity constraints Eq. (4.5g) are required to ensure that the solution of the program will be physically meaningful. Eq. (4.5e) is formulated in terms of edge flows, whereas the constraints are formulated in terms of path flows. Eq. (4.6) expresses the relationship between edge and path flow, and incorporates the network design into the optimisation problem.

\[
x_e = \sum_{od \in \mathcal{V}^H} \sum_{P \in \mathcal{P}^{\lambda,\gamma}_{od}} x_{e}(P) \tag{4.6}
\]

An overview of traffic assignment methods can be found in Nagurney, (2007), de Dios Ortuozar and Willumsen, (2011), Hoogendoorn and Knoop, (2012), and Patriksson, (2015). A similar approach can be used for other transportation networks such as, power grids (Quelhas et al., 2007; Van den Bergh et al., 2014), utility networks (Geidl and Andersson, 2007; Leong and Ayala H., 2013; Sarbu, 2014), or communication networks (Riley et al., 2004; Casalicchio et al., 2011).

Since the quantification of indirect costs is a non-trivial task, and there are high levels of uncertainty associated with the estimated values, a weighting factor \( \gamma \) is used to allow a relative weighting between both costs; i.e. decision makers can decide to which extent indirect costs will affect the determination of the optimal restoration program.

4.4. Heuristic optimization

While there are solution strategies for solving the upper-level and lower-level optimisation separately, classical methods fail to solve this multilevel optimisation model exactly. This is due to the computational complexity, including non-linearity, non-convexity, and non-differentiability of the combined problem. Furthermore, the upper-level optimisation can be classified as a combinatorial optimisation, where the optimal restoration program comes from a finite set of restoration program, i.e. the combinations of different interventions in time are finite. In many such problems, an exhaustive search is not feasible, but heuristic procedures can provide a way of computing near-optimal solutions.

In this work, a SA based metaheuristic procedure is used to approximate an optimal solution of the upper-level optimisation. The lower-level optimisation is embedded in the SA but solved with classical methods. Other metaheuristics, including genetic algorithms (GA), ant colony system (ACS) and tabu search (TS)
might also be possible procedures but often perform, for this type of optimisation, worse than SA (Adewole et al., 2012; Antosiewicz et al., 2013; Mukhairez and Maghari, 2015; Fredrikson and Dahl, 2016).

4.4.1. Simulated annealing

SA was introduced by Kirkpatrick et al., (1983) and Černý, (1985) for solving combinatorial optimisation problems. This heuristic procedure originated from an analogy with the physical annealing process of finding low energy states of a solid in a heat bath (Metropolis et al., 1953). In the process of physical annealing, a crystalline solid is heated and then allowed to cool very slowly until it reaches its most regular possible crystal lattice configuration, i.e. its minimum lattice energy state. SA establishes the connection between this type of thermodynamic behaviour and the search for the global minimum for a discrete optimisation problem (Henderson et al., 2003).

In computational terms, the typical SA based heuristic seeks to minimise a given objective $Z$ by applying small random changes to the control variables and considering the change in the value of the objective function, $\Delta Z$. When there is an improvement of the value of the objective function, the resulting change in the solution is accepted and a further search is initiated in the neighbourhood of this point. If the resulting new solution yields no improvement, it is accepted with probability $\exp(-\Delta Z/T)$, where $T$ is a control parameter referred to as the temperature of the system. The probability of accepting a new solution is high for large temperatures and conversely small for low temperatures. This acceptance criterion is helpful in avoiding convergence to local optima. One can think, here, of temperature as representing the level of disorder in the search process. Thus, the search is initiated under a high temperature to allow as much exploration of the design space as possible and, as the temperature is then decreased to zero in some regulated fashion, the search converges onto a globally near-optimal solution (Suppapitnarm et al., 2000).

The design of a good SA based heuristic is nontrivial and problem-specific. The following elements are required: (1) A specific objective function; (2) A rule to create a neighbour solution randomly from the current solution; and (3) A cooling schedule to control the temperature during the annealing process (Eglese, 1990).

4.4.2. Choice of the objective function for SA

In general, a classical SA can handle complex objective functions but without constraints. Finding constrained global minimums is challenging since non-convex and highly nonlinear constraints make it difficult even to find a feasible solution or a feasible neighborhood. Additionally, it is possible that feasible neighbourhoods may be disjoint and the search may need to visit multiple feasible neighbourhoods before finding the global minimum (Wah and Wang, 1999).

In order to avoid infeasible solutions for Eq.(4.5a), the constraints (Eqs. (4.5b) to (4.5d)) were taken into consideration by using a dynamic penalty function, known as annealing penalty (Michalewicz and Schoenauer, 1996). In other words, infeasible solutions were penalised by adding a penalty to the solution of the objective function, such that it becomes worse than a feasible solution.

$$\text{eval}(X) = \begin{cases} Z(X) & X \in \text{feasible} \\ Z(X) + P(X) & \text{otherwise} \end{cases} \quad (4.7)$$

where the penalty function $P(X)$ is zero if no violation occurs and otherwise it is positive. In addition, the penalty function accounts for the degree of infeasible
4. Restoration programs for transportation networks

solutions by weighting the distance to a solution from a feasible neighborhood. In the case of an annealing penalty, the penalty function also depends on the temperature:

\[ P(X) = \frac{1}{2T} \sum_{j \in m} Y_j(X)^2 \]  

(4.8)

where \( Y(X) \) is a constraint of \( Z \), and \( m \geq 0 \) the amount of constraints. The initial penalty value is relatively low and gradually increases as the temperature reduces. This construct allows a wider solution space in the beginning but puts increasing pressure on infeasible solutions as the simulation goes on.

4.4.3. Choice of neighbourhood

The efficiency of SA is highly influenced by the neighbourhood function, which is used to create a neighbour solution from the current solution. The definition of the neighbourhood function is mainly a problem-specific choice. The problem of finding an optimal restoration program can be reduced to a combinatorial optimisation problem with a finite solution space. For example, for a specific object type, three different intervention types exist, i.e. in a SA process, the object can only be in three stages. Furthermore, \( \delta_{n,i,t} \) can be decomposed in a sequence of tuples \([(n_j, i_j), (n_k, i_k), \ldots]\) where the order indicates the period in which an intervention of type \( i \) is executed on object \( n \). The problem is solved by generating a number of sequences in a feasible neighbourhood and choosing the best. To generate a neighbour solution, one of the tuples in the sequence is chosen randomly and assigned to a new position; additionally, the intervention type is changed with a certain probability.

4.4.4. Choice of cooling schedule

The lowering of the system temperature through a cooling schedule is required to reduce the amount of time required to find a near-optimal solution. In literature many heuristic cooling schedules are available, e.g. for engineering applications a comprehensive review of different cooling schedules is given by Siddique and Adeli, (2016). However, the effectiveness of these schedules is highly problem-specific and can only be compared through experimentation (Henderson et al., 2003). The cooling schedule implemented is as follows. An initial temperature \( T_0 \) is set at the beginning. At every \( i \)-th iteration out of \( J \), the temperature \( T_0 \) is multiplied by \( \exp(\kappa \cdot i/J) \), where \( \kappa \) is a parameter for the decay rate and here defined as \( \kappa = -\ln(T_0/T_{\text{min}}) \). The initial temperature is estimated based on the work of Ben-Ameur, (2004).

4.5. Example

The restoration model is demonstrated by using them to determine the near-optimal restoration program for an example road network in Switzerland following the occurrence of an extreme flood event. The road network investigated is in the area around the city of Chur, the capital of Grisons the largest and easternmost canton of Switzerland. Grisons is a mountainous area and includes parts of both the Rhine and Inn river valleys. The investigated area of Grisons includes the districts of Imboden and the northern part of Plessur. This area is located next to the river Rhine and contains the city of Chur and Grisons’ industrial area, together with the most important transportation links in the canton. The considered road network is comprised of circa 51 km national roads, 165 km main roads, and 395 km minor roads. The canton is crossed in a north-south direction.
from the A13 motorway. The example network consists of 2,153 objects which include 2,011 road sections and 116 bridges, as shown in Figure 4.1.

4.5.1. Investigated network

The network investigated is shown in Figure 4.1 and was assumed to consist of only motorways, main roads, and minor roads\(^3\). The road sections were described by their direction, length, free flow speed, capacity and the parameters of the capacity restraint function. Bridges were modelled as vertices with a degree of 2, i.e. the bridge location was defined in the middle of the river where two road segments are joined. This assumption allowed to account for both, local scouring and inundation of the bridges. A sample of bridges and road sections are shown in Table 4.1 and 4.2. For reasons of brevity, the complete list of 2,153 objects is not given.

Table 4.1.: Sample of considered bridges and their assigned attributes.

<table>
<thead>
<tr>
<th>ID</th>
<th>class</th>
<th>type</th>
<th>material</th>
<th>Affected components</th>
</tr>
</thead>
<tbody>
<tr>
<td>2052</td>
<td>major</td>
<td>box girder</td>
<td>concrete</td>
<td>piers: 1, abutments: 0</td>
</tr>
<tr>
<td>2064</td>
<td>minor</td>
<td>single span</td>
<td>concrete</td>
<td>piers: 0, abutments: 1</td>
</tr>
<tr>
<td>2070</td>
<td>major</td>
<td>single span</td>
<td>concrete</td>
<td>piers: 0, abutments: 2</td>
</tr>
</tbody>
</table>

Table 4.2.: Sample of considered road sections and their assigned attributes.

<table>
<thead>
<tr>
<th>Object</th>
<th>ID</th>
<th>class</th>
<th>One way</th>
<th>Capacity</th>
<th>Speed limit</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>motorway</td>
<td>461</td>
<td>f</td>
<td>t</td>
<td>4,000</td>
<td>120</td>
<td>193</td>
<td>10.0</td>
</tr>
<tr>
<td>major</td>
<td>554</td>
<td>f</td>
<td>t</td>
<td>1,200</td>
<td>100</td>
<td>47</td>
<td>6.0</td>
</tr>
<tr>
<td>minor</td>
<td>1279</td>
<td>f</td>
<td>f</td>
<td>850</td>
<td>30</td>
<td>92</td>
<td>4.0</td>
</tr>
<tr>
<td>minor</td>
<td>1692</td>
<td>f</td>
<td>f</td>
<td>600</td>
<td>30</td>
<td>902</td>
<td>2.8</td>
</tr>
<tr>
<td>major</td>
<td>1702</td>
<td>f</td>
<td>f</td>
<td>900</td>
<td>50</td>
<td>40</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The trips in the region were considered to start and end in the 37 zones based

\(^3\)The information of the Chur road network was extracted from the VECTOR25 data-set, provided by swisstopo (JD100042). This set of data exhibits a full national coverage and describes approximately 8.5 million objects with their position, shape and its neighbourhood relations (topology), as well as the kind of object and further special attributes. The accuracy of the spatial data is in the range of 3 to 8 m and available as an ESRI Shapefile for the Swiss coordinate system CH1903/LV03 LN02 (ESPG code: 21781)
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on judicial districts as shown in Figure 4.1. All trips made from an origin to a destination during a particular time period are stored in a so-called OD matrix. Since not enough trip distribution information was available for the area of interest, a gravity distribution model (de Dios Ortuozar and Willumsen, 2011) was used to estimate the trips based on the population of zones. The obtained gravity model was calibrated based on the Swiss national traffic model (FOSD, 2015), which provides data for the motorway and main roads.

4.5.2. Determine damage states and functional losses

Functional losses due to local scour at bridge piers and inundation of road sections caused by an extreme flood event was considered. The states of the objects were derived by fragility and functional loss functions (Lam and Adey, 2016), which were used in a flood simulation with a 500 year return period. The detailed quantification and computer-supported model used to determine the damage state and functional losses are described in (Hackl et al., 2016). The relationships between the hazard event which caused physical damage, and functional losses of the objects used are given in Table 4.3 and 4.4.

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
<th>LOS</th>
<th>Loss [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>no damage - object is in a normal state.</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>minor damage - local scour at pier(s) and/or abutment(s) observed; some service can be provided</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>major damage - Pier(s) and/or abutment(s) with footing(s) exposed; structural reliability is no longer guaranteed; Object cannot provide any service</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

a $s_2 = g$

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
<th>LOS</th>
<th>Loss [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>no damage - object is in a normal state.</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>minor damage - debris lying on the road; some service can be provided</td>
<td></td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>major damage - road is washed out by the flood; passing of the road is no longer possible; Object cannot provide any service</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

a $s_2 = g$

The scenario for which the near-optimal restoration program was determined contains three bridges in state 1, two bridges in state 2, twenty road segments in state 1 and four road segments in state 2. The spatial distribution of the objects is illustrated in Figure 4.2.

4.5.3. Setup restoration model

The objective function introduced in Eq. (4.5a) was used, considering Eq. (4.1) for the direct costs and Eq. (4.3) for the indirect costs, where Eq. (4.3) depends on the traffic assignment specified by Eqs. (4.5e) to (4.5g).

Estimation of the direct costs

The intervention cost was calculated using the following equation:

$$C_{n,i} = \epsilon_{n,i} + \zeta_{n,i} + \eta_{n,i,k} \quad \forall n,i$$

The total costs for intervention $i \in I(n|s)$ executed on object $n$ in state $s$, were considered to be the summation of the fixed costs $\epsilon$ (e.g. building site
facilities), the variable costs $\zeta$ (e.g. $\text{mu}/m^2$ pavement, $\text{mu}/m^3$ concrete), and resource-related costs $\eta$ (e.g. labor costs) to execute the intervention. The terms fixed and variable costs are used for non-resource related material costs, where resource costs are used to describe labour and construction machinery cost of the restoration work crews.

These costs vary depending on the type of object and the type of intervention. For each type of object, three different intervention types are considered: level 1 interventions, level 2 interventions, and level 3 interventions. Level 1 interventions require less time than level 2 interventions, but more resources and additional costs. Level 2 interventions restore the capabilities in a default way. Level 3 interventions require less time and costs than a level 2 intervention. Both level 1 and level 2 interventions restore objects so that they once again provide the expected LOS, level 3 interventions do not.

The LOS recovery, durations, resources and costs related to interventions of each type for road sections and bridges are given in Table 4.5 and 4.6.

Table 4.5.: Intervention types and associated recovery rate, resources and costs for bridge local scour.

<table>
<thead>
<tr>
<th>State</th>
<th>Intervention</th>
<th>LOS recovery</th>
<th>Duration</th>
<th>Resources</th>
<th>Fixed costs</th>
<th>Variable costs</th>
<th>Resource costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>damage</td>
<td>type</td>
<td>$\Delta y$</td>
<td>$\tau$</td>
<td>$\rho$</td>
<td>$\epsilon$</td>
<td>$\zeta$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>1 minor</td>
<td>1 level</td>
<td>100</td>
<td>20</td>
<td>2</td>
<td>10,000</td>
<td>24,000</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>2 level</td>
<td>100</td>
<td>40</td>
<td>1</td>
<td>10,000</td>
<td>15,000</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>3 level</td>
<td>20</td>
<td>35</td>
<td>1</td>
<td>10,000</td>
<td>13,000</td>
<td>900</td>
</tr>
<tr>
<td>2 major</td>
<td>1 level</td>
<td>100</td>
<td>90</td>
<td>2</td>
<td>48,000</td>
<td>64,000</td>
<td>1,200</td>
</tr>
<tr>
<td></td>
<td>2 level</td>
<td>100</td>
<td>160</td>
<td>1</td>
<td>30,000</td>
<td>40,000</td>
<td>1,200</td>
</tr>
<tr>
<td></td>
<td>3 level</td>
<td>10</td>
<td>145</td>
<td>1</td>
<td>30,000</td>
<td>37,000</td>
<td>1,200</td>
</tr>
</tbody>
</table>

el ... affected elements of the bridge (e.g. pier(s), and/or abutment(s))

rwc ... restoration work crews

mu ... monetary unit (e.g. USD, EUR, CHF)

**Estimation of indirect costs**

The indirect costs are comprised of costs for the temporal prolongation of travel time and costs due to a loss in connectivity $\Lambda$. The costs attributed to traffic flow...
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Table 4.6: Intervention types and associated recovery rate, resources and costs for inundated road sections.

<table>
<thead>
<tr>
<th>State damage</th>
<th>Intervention type</th>
<th>LOS recovery</th>
<th>Duration $\Delta t$</th>
<th>Resources $t$</th>
<th>Fixed costs $\epsilon$</th>
<th>Variable costs $\zeta$</th>
<th>Resource costs $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>i</td>
<td>$\Delta t$ [%]</td>
<td>$t$ [h/m²]</td>
<td>$r$ [#rwc]</td>
<td>$\epsilon$ [mu]</td>
<td>$\zeta$ [mu/(rwc · h)]</td>
<td>$\eta$ [mu/(rwc · h)]</td>
</tr>
<tr>
<td>1 minor</td>
<td>level 1</td>
<td>100</td>
<td>0.001</td>
<td>2</td>
<td>5,250</td>
<td>22.00</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>level 2</td>
<td>100</td>
<td>0.003</td>
<td>1</td>
<td>3,500</td>
<td>16.50</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>level 3</td>
<td>30</td>
<td>0.003</td>
<td>1</td>
<td>3,500</td>
<td>14.50</td>
<td>500</td>
</tr>
<tr>
<td>2 major</td>
<td>level 1</td>
<td>100</td>
<td>0.006</td>
<td>2</td>
<td>14,400</td>
<td>110.00</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>level 2</td>
<td>100</td>
<td>0.012</td>
<td>1</td>
<td>9,600</td>
<td>82.50</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>level 3</td>
<td>10</td>
<td>0.010</td>
<td>1</td>
<td>9,600</td>
<td>78.50</td>
<td>700</td>
</tr>
</tbody>
</table>

Include travel time $\Phi$ and vehicle operation costs $\Upsilon$.

$$\Pi(t|x_{e,t}) = \Phi(t|x_{e,t}) + \Upsilon(t|x_{e,t}) \quad \forall t, e \in P, P \in \mathcal{P}_{od}^s$$ (4.10)

**Travel time:** Travel time costs are estimated as the increased amount of time that people spend travelling. They are linked directly to the flow on an edge, due to the cost-flow relationship $C^T_e$ where cost is handled in terms of travel time per unit distance (de Dios Ortuozar and Willumsen, 2011). The cost-flow relationship used is the function proposed by the Bureau of Public Roads (Bureau of Public Roads, 1964):

$$C^T_e := t_{e,t}(x_{e,t}) = t_0^0 e \left( 1 + \alpha_e \left( \frac{x_{e,t}}{y_{e,t}} \right)^{\beta_e} \right)$$ (4.11)

where $t_{e,t}$ is the travel time at edge $e$ in period $t$ given the traffic flow $x_{e,t}$, $t_0^0$ is the free flow travel time, $y_{e,t}$ the edge capacity, and $\alpha$ and $\beta$ are parameters for calibration, typical chosen as $\alpha = 0.15$ and $\beta = 4$, which corresponds to the design capacity (normally LOS C) of the Highway Capacity Manual. The travel time multiplied by the value of travel results in the travel time costs.

$$\Phi(t|x_{e,t}) = t_{e,t}(x_{e,t}) \cdot \sum_{w \in \mathcal{W}} \mu_{e,w} \cdot \xi_{e,w}$$ (4.12)

where $w \in \mathcal{W}$ represent different vehicle types, $\mu_{e,w}$ the proportion of vehicles of type $w$ on edge $e$, and $\xi_{e,w}$ the value of travel. Only two types of vehicles are considered, i.e. cars and trucks. Based on the work of the Swiss Association of Road and Transport Experts (VSS, 2009b), $\xi_{e,w}$ was assumed to be 23.02 mu/hour for cars and 130.96 mu/hour for trucks. There was assumed to be the same proportion of cars and trucks on all roads. Hence, the overall fraction of trucks ($\mu_{w=\text{truck}}$) was estimated as 6% of the total traffic volume (FEDRO, 2015).

**Vehicle operation:** Vehicle operation costs are incurred for fuel consumption and maintenance of vehicles. They were estimated as the sum of the operating costs for all vehicle types.

$$\Upsilon(t|x_{e,t}) = x_{e,t} \cdot \ell_e \cdot \sum_{w \in \mathcal{W}} \mu_{e,w} \cdot (\nu \cdot F_w + \rho_w)$$ (4.13)

where $\ell$ is the length of edge $e$, $\nu$ is the mean fuel price (1.88 mu/liter), $F_w$ is the mean fuel consumption, depending on the vehicle type (6.7 liter and 33 liter per...
100 km for car and truck respectively), and \( \rho_w \) are the operating costs without fuel for different vehicle types (14.39 \( \text{mu} / (100 \cdot \text{vehicle} \cdot \text{kilometer}) \) for cars and 32.54 \( \text{mu} / (100 \cdot \text{vehicle} \cdot \text{kilometer}) \) for trucks)(VSS, 2009a).

\[ \text{Loss in connectivity: } \] The costs due to a loss in connectivity are estimated based on the unsatisfied demand per time period \( t \) and the resulting costs due to a loss of labour productivity. Labour productivity is an economic indicator that measures the value of goods and services, produced in a period of time, divided by the hours of labour used to produce them (Freeman, 2008).

\[ \Lambda(t) = f_{od,t}(P) \cdot v(t) \quad \forall t, P \in P_{od} \] (4.14)

where \( f_{od,t}(P) \) is the demand loss and \( v \) is the labor productivity at period \( t \). Based on the data from the Federal Statistical Office (Reutter and Bläuer Herrmann, 2016), the labor productivity per hour worked is 83.27 \( \text{mu} / \text{hour} \).

### 4.5.4. Considered scenarios

In order to estimate the optimal restoration program, different assumptions were made, including the number of restoration work crews available (\( \#\text{rwc} = 3 \)), the considered time intervals (\( \Delta t = 4 \) hour), the working hours per day (8 hour), and the weighting factor for indirect costs (\( \gamma = 1 \)). The investigated scenarios vary in context of different levels of available resources and different levels of budget and are shown in Table 4.7.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Resources</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Psi_{k,t} = 2 ), ( t \in [0, 4] ) = 0, ( \Psi_{k,t} = 3, t \in [10, 15] ) = 0</td>
<td>unlimited</td>
</tr>
<tr>
<td>2</td>
<td>( \Psi_{k,t} = 2, t \in [0, 4] ) = 0, ( \Psi_{k,t} = 3, t \in [10, 15] ) = 0</td>
<td>3,630,000</td>
</tr>
<tr>
<td>3</td>
<td>all available</td>
<td>unlimited</td>
</tr>
</tbody>
</table>

Scenario 1 had resource constraints but no budget constraints. Here, the second restoration work crew (resource B) was not available in the first four days, while restoration work crew three (resource C) was not available between the 10\(^{th}\) and the 15\(^{th}\) day of the restoration program. Scenario 2 had the same resource constraints as 1 but additionally budget constraints of 3,630,000 \( \text{mu} \). The budget constraint was set low enough to force the use of the penalty function\(^5\) (4.8). Scenario 3 had no available resources and budget constraints. Additionally, a sensitivity analysis was conducted using scenario 1 by varying the value of the weighting factor.

Due to the complexity of the problem and the extensive solution space, it is not possible to solve this mathematical model exactly by analytics or an exhaustive search. In order to evaluate the obtained results, a comparison between the proposed model and a (standard practice) benchmark model was made.

The traditional methods to develop restoration programs are mostly heuristic and based on subjective ranking and priority rules developed by domain experts. These prioritisation rules can be based either on economic or engineering criteria, such as structure vulnerability, road class, traffic volume and various socioeconomic factors. For example Buckle et al., (2006) prioritise road object based on expected damage ranks, where the object with the highest damage or functional loss are given the highest priority for restoration. Because this ranking does not account for the importance of the object in the network, Miller, (2014) proposed

\(^5\)i.e. all interventions had to be of type 3
4. Restoration programs for transportation networks

prioritisation based on the average daily traffic volume for each object under normal conditions, as a benchmark model. Since this model does not account for disconnected parts of the network, a modified version was implemented as the benchmark:

1. sorting the objects based on their average daily traffic volume under normal conditions,
2. restoring objects that cause a loss of connectivity,
3. restoring the remaining objects;

4.5.5. Implementation

The model was programmed in python. A conjugate direct Frank-Wolfe (CFW) algorithm (Mitradjieva and Lindberg, 2013), was implemented to solve the traffic assignment problem (Eq. (4.5f)). The convergence criterion was set to a relative gap of $10^{-4}$. The SA was based on the python package simanneal\(^6\) and modified according to Section 4.4. A GIS interface was developed, allowing easy in- and export of GIS data. Furthermore, the program code was optimised for parallel computing, in order to reduce the computational time of the optimisation process.

The optimisation for the example was conducted on a 2x10 Core Intel Xenon E5-2690v2 3.0Ghz, 384GB DDR2 server running on Linux 64bit operating system (Ubuntu 14.04). A simulation of $J = 10,000$ steps took approximately 10 hours. In total ten searches per scenario were conducted. The initial temperature, estimated based on Ben-Ameur, (2004), was $T_0 = 25,323$.

4.5.6. Results

In 1, the near-optimal restoration program obtained from the model (Table 4.8), include level 1 and level 2 interventions at the beginning of the restoration period in order to first restore network connectivity. Once network connectivity is restored, level 3 interventions are included. The majority of the damaged objects are located in regions where there is little traffic flow as shown in Figure 4.2. The overall costs for the restoration is 7,935,172 mu, with 3,735,844 mu direct and 4,199,328 mu indirect costs over a period of 39.5 days. Restoring 1 with the benchmark model, the incurred restoration costs were 9,032,603 mu. This is 1,097,431 mu or 13.83% above the estimated optimum.

Table 4.8 contains the near-optimal restoration program for scenario 1, including the optimal order, the objects should be restored, the interventions to be executed with the estimated restoration duration (i.e. start and end time of the task), the assigned restoration work crews and direct costs.

Figure 4.3 shows the restoration program over time for scenario 1, showing the allocated resources per object, the development of the direct and indirect costs, and the consequences on the network flow, average travel time and change in connectivity. Objects 1 and 7 are these objects which yield the highest net improvement, i.e. increase in the provided service minus the costs of intervention. The economic costs due to lost connectivity are mainly caused by object 7, while object 1 causes the highest additional travel time.

Figure 4.4 shows the evaluation of the values of the objective function during the SA optimisation for scenario 1. All ten searches for the near-optimal restoration program show convergence to 7,935,172 mu. 9 out of 10 searches reached this value exactly.

\(^6\)https://github.com/perrygeo/simanneal
4.5. Example

Table 4.8.: Near-optimal restoration program for scenario 1, ordered according to the importancies of the objects.

<table>
<thead>
<tr>
<th>Nr</th>
<th>ID</th>
<th>Object</th>
<th>State</th>
<th>Interv.</th>
<th>Schedule</th>
<th>Res.</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>type</td>
<td>damage</td>
<td>type</td>
<td>start</td>
<td>end</td>
<td>dur.</td>
</tr>
<tr>
<td>1</td>
<td>2042</td>
<td>Bridge</td>
<td>1</td>
<td>minor</td>
<td>1</td>
<td>0.0</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>554</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>2</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>332</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>4</td>
<td>461</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>5</td>
<td>1803</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>3.0</td>
<td>4.5</td>
</tr>
<tr>
<td>6</td>
<td>1095</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>7</td>
<td>2052</td>
<td>Bridge</td>
<td>2</td>
<td>major</td>
<td>1</td>
<td>4.0</td>
<td>15.0</td>
</tr>
<tr>
<td>8</td>
<td>1802</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>4.5</td>
<td>6.0</td>
</tr>
<tr>
<td>9</td>
<td>1706</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>6.0</td>
<td>8.0</td>
</tr>
<tr>
<td>10</td>
<td>460</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>8.0</td>
<td>10.0</td>
</tr>
<tr>
<td>11</td>
<td>562</td>
<td>Road</td>
<td>2</td>
<td>major</td>
<td>3</td>
<td>15.0</td>
<td>27.5</td>
</tr>
<tr>
<td>12</td>
<td>2069</td>
<td>Bridge</td>
<td>2</td>
<td>major</td>
<td>3</td>
<td>15.0</td>
<td>33.0</td>
</tr>
<tr>
<td>13</td>
<td>1913</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>15.0</td>
<td>24.0</td>
</tr>
<tr>
<td>14</td>
<td>1703</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>24.0</td>
<td>26.0</td>
</tr>
<tr>
<td>15</td>
<td>1692</td>
<td>Road</td>
<td>2</td>
<td>major</td>
<td>3</td>
<td>26.0</td>
<td>29.0</td>
</tr>
<tr>
<td>16</td>
<td>1279</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>2</td>
<td>27.5</td>
<td>28.0</td>
</tr>
<tr>
<td>17</td>
<td>2131</td>
<td>Bridge</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>28.0</td>
<td>32.5</td>
</tr>
<tr>
<td>18</td>
<td>1907</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>29.0</td>
<td>30.0</td>
</tr>
<tr>
<td>19</td>
<td>1905</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>30.0</td>
<td>30.5</td>
</tr>
<tr>
<td>20</td>
<td>1237</td>
<td>Road</td>
<td>2</td>
<td>major</td>
<td>3</td>
<td>30.5</td>
<td>33.0</td>
</tr>
<tr>
<td>21</td>
<td>1276</td>
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<td>minor</td>
<td>3</td>
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<td>22</td>
<td>1202</td>
<td>Road</td>
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<td>minor</td>
<td>3</td>
<td>33.0</td>
<td>33.5</td>
</tr>
<tr>
<td>23</td>
<td>1233</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>33.0</td>
<td>33.5</td>
</tr>
<tr>
<td>24</td>
<td>1798</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>33.5</td>
<td>34.5</td>
</tr>
<tr>
<td>25</td>
<td>1814</td>
<td>Road</td>
<td>2</td>
<td>major</td>
<td>3</td>
<td>33.5</td>
<td>38.0</td>
</tr>
<tr>
<td>26</td>
<td>2043</td>
<td>Bridge</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>34.0</td>
<td>38.5</td>
</tr>
<tr>
<td>27</td>
<td>1498</td>
<td>Road</td>
<td>2</td>
<td>major</td>
<td>3</td>
<td>34.5</td>
<td>38.0</td>
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<tr>
<td>28</td>
<td>471</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>3</td>
<td>38.0</td>
<td>38.5</td>
</tr>
<tr>
<td>29</td>
<td>1371</td>
<td>Road</td>
<td>1</td>
<td>minor</td>
<td>2</td>
<td>38.0</td>
<td>38.5</td>
</tr>
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<tbody>
<tr>
<td>sum:</td>
<td>228,500</td>
<td>2,770,944</td>
<td>736,400</td>
<td>3,735,844</td>
<td></td>
<td></td>
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The changes in direct, indirect and total costs, for various weighting factors $\gamma$ are shown in Figure 4.5. The considered values for $\gamma$ are 0.001, 0.01, 0.5 , 1, 2.5, 5, and 10. The indirect costs were normalised by the corresponding $\gamma$ value. A weighting factor of $\gamma = 0$ means that indirect costs (Eq.(4.3)) are not considered in the optimisation while increasing weights gives more significance to the impact of flows on the total costs.

Figure 4.6 shows the evolution over time of the required resources per object and the development of the direct and indirect costs for all three scenarios. Similar to the restoration program associated with scenario 1, the restoration programs associated with scenario 2 and 3 include objects 1, 2, 3, 4 and 7 at the beginning of the restoration program. This is because the restoration of these objects has the most significant net improvement. Considering, the resource constraints in scenario 1 and 2, object 1 and 7, which yield the highest net improvement, cannot be executed in parallel, as in 3. Due to the additional budget constraints in scenario 2, only level 3 interventions are included in the near-optimal restoration program, i.e. only the minimum possible effort to restore the network is made (e.g. execution of clean-up). An overview of all costs is given in Table 4.9.
4. Restoration programs for transportation networks

Figure 4.3.: Near-optimal restoration program for scenario 1 over time. **Top:** accumulated direct and indirect costs and their cost components (fixed, variable, resources, travel time, vehicle operation, and lost connectivity). **Middle:** restoration schedule for the work crews A, B, and C over time. The observed damage states are indicated by the fill colors and the assigned intervention types by the hatchings. **Bottom:** functional losses (loss of connectivity and additional travel time) of the network over time.

4.6. Discussion

The presented restoration model using SA has been described and its use was demonstrated by determining the near-optimal restoration program on a road network around the city of Chur following an extreme flood event. It is suspected that this model enables improved estimates of the costs incurred due to the loss of service following a disruptive event and, therefore, enable the develop-
4.6. Discussion

Development of restoration programs that are better than would be determined with state-of-the-art models. Additionally, it is suspected that the use of the restoration mode enable higher order dependencies to be discovered and taken into consideration, which otherwise might not. For example, in some cases it might be more beneficial to restore an object slowly (a level 2 intervention instead of a level 1 intervention), because over the entire restoration period this brings more benefits (e.g. the execution of an intervention on one road section does not need to progress quickly if the traffic flow over the road section in hindered by the execution of an intervention on a bridge within the same road link.)

In any case, however, the methodology is useful in the estimation of risks due to natural hazards, where an estimate of the resilience of the network is considered, i.e. when the network will provide service following the natural hazard event.

It is also interesting to note that the model identifies the critical objects in the network, i.e. the objects whose failure will result in relatively large disruptions to service. For example, the restoration determined includes an intervention of level 1 on Bridge 2042 (object 1) in time interval 1, as this intervention would reduce to a large amount the additional travel time from 378 to 119 hours per half day on the network. Additionally, Bridge 2052 (object 7) is scheduled as soon as possible with an intervention of level 1, since this reduces the number of missed trips by 306 (see Figure 4.3). These substantial improvements are because of the restoration of Bridge 2052, as it is the only connection from Haldenstein to Chur (see Figures 4.1 and 4.2), reconnects the people to the main region of Chur. Without resource constraints, object 7 would be scheduled first (see Figure 4.6), because lost trips have higher costs than prolonged travel.

The restoration programs of all scenarios outperform those developed using the

Figure 4.4.: Illustration of the convergence process for all searches for the near-optimal restoration program of scenario 1. Total costs plotted against the number of simulated steps.

Figure 4.5.: Costs for different weighting factors $\gamma$ for scenario 1. Indirect costs normalized by $\gamma$ and direct costs plotted against various weighting factors.
benchmark model based on traffic volume-based importance measure. Even in the early phase of the SA process, better solutions than those found using the benchmark model could be observed (see Figure 4.4). In general, more substantial improvements could be observed with scenarios with constraints, which is because the benchmark model cannot adapt well to this. While the used benchmark model represents the current state in practice, it does not indicate how good the observed near-optimal restoration program actually is, hence the lack...
4.6. Discussion

Table 4.9.: Costs of the scenarios in comparison with the benchmarks.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Costs</th>
<th>Benchmark</th>
<th></th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>direct</td>
<td>indirect</td>
<td>total</td>
<td>costs</td>
</tr>
<tr>
<td>1</td>
<td>3,735,844</td>
<td>4,199,328</td>
<td>7,935,172</td>
<td>9,032,003</td>
</tr>
<tr>
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<td>3,628,226</td>
<td>5,547,319</td>
<td>9,175,545</td>
<td>9,841,807</td>
</tr>
<tr>
<td>3</td>
<td>3,724,125</td>
<td>3,462,792</td>
<td>7,186,918</td>
<td>7,784,687</td>
</tr>
</tbody>
</table>

of such a benchmark model makes it difficult to test the solutions found and the computational complexity truly.

The extent with which indirect costs are considered in the analysis is essential. Without considering network flows ($\gamma=0$), the model yields the restoration program with minimum direct costs; however, the impacts on the users might be out of all proportion. Increasing the importance of indirect cost leads to more efficient but also more costly restoration programs. As the value of $\gamma$ is increased, there are decreasing benefits, i.e. there is a decreasing benefit of restoring objects so that they can be used in their full capacity faster.

The assumptions with respect to the damage states and functional losses used in the example are very general. There is, however, no conceptual problem with making them more detailed. A practical problem though is that increasing detail in the descriptions of damage states and functional losses leads to increasing complexity of the computational models (e.g. computational fluid dynamics, finite elements, etc.) to be used. The appropriate level of detail needs to be determined in cooperation with appropriate stakeholders and experts, utilising existing knowledge of how the object’s function.

Accurate estimates of restoration times and costs are essential for the quality of the near-optimal restoration programs. The provision of this quality is challenging because they often depend on the extent and severity of the damage, the considered object, and on the number of interventions that are being simultaneously executed in the same area. Using historical data, public and private records, and expert knowledge-based evaluations, these estimates can be improved, and so, therefore, can the restoration programs. Unfortunately, the deterministic resource constraint models do not allow effective modelling of the uncertainties related to these estimates. One way to quantify uncertainties related to restoration time and costs is to conduct a sensitivity analysis in order to analyse how much the results are affected by changes in these parameters.

The restoration program determined, depends greatly on the underlying traffic assignment model. In this work, a static user equilibrium traffic assignment model, based on the BPR functions to emulate the traffic flow conditions, was implemented to simplify the problem to a bilevel optimisation problem. While this model is mathematical rather simple, computationally inexpensive and widely used in literature, it has some limitations when it comes to a realistic representation of traffic flow, e.g. it is assumed that travellers have full knowledge of the traffic conditions, which is clearly not the case. It also does not account for changes in the travel pattern after a disruptive event, although studies show this behaviour is considerably different than before a disruptive event (e.g. Chang and Nojima, (2001) and Kontou et al., (2017)). Although BPR functions are widely used as travel cost functions, they have several disadvantages, as Horowitz, (1991) pointed out, including the use in urban areas (e.g. traffic controlled intersections) and the inability to represent dynamic traffic phenomena like queues, spillbacks, wave propagation, capacity drops, and so forth. In order to improve this, more sophisticated traffic assignment models (e.g. dynamic traffic assignment) could
4. Restoration programs for transportation networks

be used (e.g. Nagurney and Dong, (2002), Smith, (2013), Bliemer et al., (2014), and Bliemer and Raadsen, (2017)). However, in this case, the assumption of a bilevel optimisation problem has to be relaxed.

Although it may be theoretically possible to determine a restoration program for a network of basically any size, substantial and practical difficulties exist regarding modelling, and data availability and quality. For example, as infrastructure networks are often owned, operated and maintained by different entities, data is often not accessible, of considerably different quality and in different forms. Cost data is particularly challenging to obtain. Another challenge for infrastructure managers is the determination of the correct balance between the level of detail and speed of analysis (Adey et al., 2016). Abstraction and simplifications are thus necessary for viable modelling. One needs to keep in mind here that the developed model needs to be detailed enough but not overly detailed.

In the work presented in this paper one, such trade-off was made by using a metaheuristic procedure. While these enables the solving of complex problems, where no analytic solution can be found, the results are only an approximation of the optimal solution. Consequently, it is hard, or in most cases impossible, to quantify how close the observed solutions are to the global minimum. Beyond that, it is also difficult to compare the results within different heuristics. For example, the performance of SA depends as much on the specific problem as on the heuristic itself, i.e. the choice of an appropriate neighbourhood function, of an efficient cooling schedule, and of sophisticated data structures that allow fast manipulations can substantially reduce the error as well as the running time (Aarts et al., 2005).

An additional challenge lies in the efficient and effective implementation of the restoration model. In order to use it as near-real-time decision support, the computational time has to be reduced (e.g. using compiled languages such as C++ and utilising parallel computing). Besides technical challenges, optimal disaster response and restoration planning are made even more complex, due to the contribution of different agencies and people, legal requirements and different policies.

4.7. Conclusions

This work introduces a restoration model for identifying near-optimal recovery responses that allow a quantitative analysis of the costs, resources and time needed for a disrupted network to regain full operation after a disruptive event. An optimisation problem to determine the most effective restoration program is presented, aiming to minimise the sum of the direct costs, which are related to the execution of the interventions, and the indirect costs associated with the traffic flow on the network until service is restored. This involves a bilevel optimisation problem which is heuristically solved using the SA method, in order to overcome the issue of the computational complexity.

The model is general enough to be applicable to a variety of infrastructures, including the road, power, water, wastewater, or communication infrastructures. As a realistic case study, a part of the Swiss road network, damaged by an extreme flood event, is considered. Solving the model on the realistic size network shows that the restoration model can produce solutions that are within 7 to 14% above the best possible solution observed by currently used prioritisation schemes. Furthermore, the analysis shows that the restoration model presented here can be of great use for infrastructure managers overseeing the reliability and resilience of critical infrastructures to disruptive events, by obtaining relevant information.
4.7. Conclusions

concerning the investment in recovery operations such as insights on the trade-off between recovery budget and quality of the resulting restoration program.

This type of information can be useful in several ways. Immediately following a disruptive event, the model could be run to offer early estimates of how long it is likely to take to restore service after the occurrence of the event. By running the model for a suite of disruptive events, in order to provide estimates of the range of possible restoration scenarios that might affect the system. This can be useful for utilities that have not experienced recent hazards, as well as for those that have and are tempted to assume future events will be similar to the last ones. It is also a means to compare different approaches to efficiently revise operational plans, e.g. by simulating different restoration strategies and mitigation measures (e.g. increasing the number of restoration work crews or adding redundancy to the network), and then compare the resulting output with the original output to determine their effectiveness in making the restoration faster and more efficient.

At the current state, the proposed restoration model is at a tactical level. In order to bring it to an operational level future work will be required, including the development, calibration, and validation of more accurate damage states and intervention types. Further, more sophisticated and accurate traffic assignment strategies (e.g. dynamic traffic assignment) will create a more realistic and robust traffic analysis, allowing to account for elastic demands, queues, and spillbacks, wave propagation, capacity drops, etc. Additionally, to increase the usability the computational efficiency has to be improved and a straightforward user interface has to be developed that can input local data for infrastructure managers to develop restoration strategies.

The used benchmark also requires increased sophistication in its design. Currently, there is no established benchmark procedure such that the solutions found and the computational complexity can be genuinely tested among various. Notably, in the context of resilience infrastructure, there is an urgent need to test and compare the performance of different approaches.
Part II.

Complex Network Modelling for Infrastructure Systems
5. Generation of spatially embedded random networks to model complex transportation networks

This chapter corresponds to the published article:\(^1\)


Abstract: Random networks are increasingly used to analyse complex transportation networks, such as airline routes, roads and rail networks. So far, this research has been focused on describing the properties of the networks with the help of random networks, often without considering their spatial properties. In this work, a methodology is proposed to create random networks conserving their spatial properties. The produced random networks are not intended to be an accurate model of the real-world network being investigated but are to be used to gain insight into the functioning of the network taking into consideration its spatial properties, which has potential to be useful in many types of analysis, e.g. estimating the network related risk. The proposed methodology combines a spatial non-homogeneous point process for vertex creation, which accounts for the spatial distribution of vertices, considering clustering effects of the network and a hybrid connection model for the edge creation. To illustrate the ability of the proposed methodology to be used to gain insight into a real-world network, it is used to estimate standard structural statistics for part of the Swiss road network, and these are then compared with the known values.

\(^1\)Please note, this is the author’s version of the manuscript published in the book 14th International Probabilistic Workshop. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at https://link.springer.com. When citing this chapter, please, refer to the original article with DOI: 10.1007/978-3-319-47886-9_15.
5. Spatially embedded random networks

5.1. Introduction

Transportation networks are essential for economic growth and development. Over the past decades, it has become apparent that the analysis and understanding of large-scale infrastructure networks are essential for research, engineering and society. The failure or damage of an infrastructure system could cause massive social disruption. It could be out of all proportion to the actual physical damage (Vespignani, 2010). Thus, understanding the general principles, leading to the complex structures of these networks and their ability to withstand failures, natural hazards and human-made disasters, is critical for evaluating risk related to transportation networks, and for designing robust transportation networks to keep the risk within acceptable limits (Schneider et al., 2011).

In order to estimate this risk, both the probability of different types of network failures and their associated consequences must be estimated. The estimation of the types of network failures is often estimated using percolation models in which some fractions of the total number of networks vertices or edges are removed. Using percolation models, as the number of vertices or edges are removed, the network is seen as undergoing transitions from the phase of connectivity (fully functional network) to the phase of disconnectivity (non-functional network) (Li et al., 2015).

A good estimation of the type of failure requires a good estimation of how the failure of part of the network can spread, which is something that is dependent on the network topology and requires appropriate consideration of the mutual interplay between the structural complexity and functional dynamics of the network (Kröger and Zio, 2011). The estimation of the type of network failure is usually made using the exact network topology. Although good when possible, this approach is not always possible. For example, it is increasingly difficult with increasing network size and complexity, with increasing periods of time to be investigated, and increasing detail required (Cadini et al., 2015). It would be useful to have an easier, although a less accurate way of being able to approximate the types of failures.

In this work, a model framework is proposed that has the potential to be used for this purpose. The model framework can be used to create random networks that conserve the spatial properties of the networks to be analysed, and, therefore, allows more features of the transportation network to be captured than when using random networks developed using other model frameworks. The resulting random networks are, even if they are not an accurate model of the real-world network being investigated, useful in gaining insights into the different types of network failure taking into consideration its spatial properties. The random networks developed taking into consideration their spatial properties are herein referred to as spatially embedded random networks.

5.2. Methodology

5.2.1. Definition of a spatially embedded random network

Spatially embedded random networks can be seen as a special kind of random network, where vertices are placed randomly according to a specific distribution within a metric space. Edges are assigned to each pair of vertices with a probability, taking spatial properties into account.

**Definition 1** (Spatially embedded random network). A random network $G = (V, E)$ defined on $S$ is called a spatially embedded random network if:
5.2. Methodology

(i) \( V \) is a mapping \( X \) from a probability space \((\Omega_1, \mathcal{F}_1, \mathbb{P}_1)\) into a metric space \((N_1, N_f)\). Where \( \Omega_1 \) is a sample space of all possible outcomes, \( \mathcal{F}_1 \) is a set of all considered events, and \( \mathbb{P}_1 \) is a probability measure of the events. Where \( N_f \) is defined as a space of finite subsets of \( \mathbb{R}^d \) and \( N_f \) is a Borel \( \sigma \)-algebra \( B \), such that for all bounded Borel sets \( B \subseteq S \), the mapping \( X \rightarrow N_X(B) \) is measurable.

(ii) \( M(X, g) \) is a connection model mapping from \( \Omega = \Omega_1 \times \Omega_2 \) into \( N_f \times \Omega_2 \) defined by \( (\omega_1, \omega_2) \rightarrow (X(\omega_1), \omega_2) \), where the sample space \( \Omega_2 \) is defined as \( \prod_{\{K(n_u, z_u), K(n_v, z_v)\}|[0,1]} \). The realization corresponding to \( (\omega_1, \omega_2) \) is obtained for any two vertices \( u \) and \( v \) of \( X(\omega_1) \), considering binary cubes. An edge \( \{u, v\} \) exists if and only if \( \omega_2 < g \) where \( g \) is the spatially dependent connection function.

Barnett et al., (2007) pointed out that spatially embedded networks are essentially “ensemble-of-ensembles”, with the following two levels of randomness: the placement of the vertices, and an edge assignment giving the coordinates of the vertices. Therefore, the properties of the graph \( G \) are conditional on the vertex placement \( X \). In order to create such a spatially embedded network, a stochastic model for the spatially embedded vertices has to be introduced. As well as a connection model for the assignment of the edges, depending not only on the distances between the vertices but also other (spatial) properties.

5.2.2. Vertex creation

The spatially embedded vertices are to be created using (inhomogeneous) Poisson point processes.

**Definition 2** (Poisson point process). A Poisson point process with intensity function \( \lambda \) is a point process \( X \) on \( S \), characterized by the following two properties:

(i) for any \( B \subseteq S \), the number \( N(B) \) of points in \( B \) is a Poisson random variable with parameter \( \lambda(B) \)

\[
\mathbb{P}\{N(B) = k\} = \frac{\lambda(B)^k}{k!} \exp\{-\lambda(B)\} \quad k \in \mathbb{N}_0, B \in S. \tag{5.1}
\]

(ii) \( N(B_1), \ldots, N(B_n) \) are independent random variables for each \( n \in \mathbb{N} \) and pairwise disjoint sets \( B_1, \ldots, B_n \in S \) with \( \lambda(B_i) < \infty \).

If \( \lambda \), the expected number of points per unit area, is constant, the Poisson process is called a *homogeneous Poisson process* on \( S \) with rate or intensity \( \lambda \); otherwise it is said to be an *inhomogeneous Poisson process* on \( S \). Moreover, if \( \lambda = 1 \) the process is called the standard Poisson point process or unit rate Poisson process on \( S \). If the average density of points is a function \( \lambda(u) \) defined at all spatial locations \( u \), then it is possible to fit an inhomogeneous Poisson process model to an observed covariate. Thereby, a covariate is a spatial function \( Z(u) \) defined at all spatial locations \( u \). For instance, the covariate \( Z(u) \) might be the altitude or population density at location \( u \).

5.2.3. Edge creation

The spatially embedded edges are to be created using a combination of a *deterministic* and a *random* connection model. In the first step, vertices are connected if they are close to each other (deterministic connection model). In the second
step the edges are added, removed or rewired, by applying a random connection model. A parameter is used to weight the two connection models. As $\kappa \to 0$, the connection model becomes a purely deterministic one. As $\kappa \to 1$, the connection model becomes purely a random one. More formally the connection model is:

**Definition 3** (Hybrid connection model). Let $X$ be a finite point process on $\mathbb{R}^d$, $\mathcal{M}_d(X)$ a deterministic connection model, $\mathcal{M}_r(X, g)$ a random connection model with connection function $g$, and a parameter $\kappa$ satisfying $0 \leq \kappa \leq 1$. To build the network

1. Construct a graph $G(V, E)$, where $V = X$ and $E = \mathcal{M}_d(X)$.
2. Rewire every edge $(v_i, v_j)$ for every vertex $v \in V$ with probability $\kappa$ according to $\mathcal{M}_r(X, g)$.

This model draws on the research in two major fields: *computational geometry* and *random graphs*. Computational geometry is the study of (deterministic) algorithms for the solution of geometric problems. It often makes use of graphs. Once graphs, and therefore, the connections between vertices, are established many network properties can be determined. For example, there are many constructs in computational geometry that encode important aspects of the notion of nearness, such as Voronoi regions. Voronoi regions are defined in terms of nearest neighbours, their dual, the Delaunay triangulation, minimum spanning trees, which are defined in terms of the shortest path through the vertices, and various extensions and generalisations of these ideas (Marchette, 2004).

Random graphs are generated using stochastic models to connect vertices. The “classical” theory of random graphs, has been established by Gilbert, (1959) and Erdős and Rényi, (1959, 1960, 1964). Specifically, in the *Gilbert model* $G(n, p)$ every possible edge occurs independently with a certain probability, while in the *Erdős-Rényi model* $G(n, M)$ equal probabilities are assigned to all graphs with exactly $M$ edges. Many other connection models have been developed. For example, in order to restrict the degree distribution, which is not done in the generation of a classical random graph model, so-called *configuration models* can be used, in which the degrees of vertices are fixed before the random graph is generated. This makes it possible to generate random networks with the desired degrees of vertices. Another example is the use of *preferential attachment* mechanisms to help represent how networks change over time. Something which is not possible with *static* models, such as classical random graphs or random graphs developed using configuration models. When preferential attachment mechanisms are used the model vertices are added sequentially with a fixed number of edges connected to them.

The methods used in both of these fields both have advantages and disadvantages if they are to be used to generate spatially embedded random networks. Methods used in the field of computational geometry have the advantage that they are usually able to create edges that are not only dependent on the distance between two vertices but also on the position of other vertices (Yukich, 1998). They, however, have the disadvantage that through the use of a deterministic connection algorithm, many structural characteristics, observed in real-world networks, cannot be modelled (Penrose, 2003). Methods used in the field of random graphs have the advantage that vertices can be connected with a certain probability, which depends both on the types of vertices and the distance between them (Meester and Roy, 1996). They, however, have the disadvantage that as-
5.2. Methodology

sumptions of complete independence among possible edges are largely untenable in practice (Kolaczyk, 2009).

In order to overcome the shortcomings of both, deterministic and random connection models, a hybrid connection model is introduced in this work (see Definition 3) and a generic connection function.

Let $g$ be a connection function, that is a Borel measurable function from $[0, \infty) \to [0, 1]$. In the common random connection models, two distinct vertices $u, v \in V$ are connected with probability $g(\text{dist}(u, v))$. In other words, the probability of assigning an edge to a pair of vertices depends only on their Euclidean distance to each other. However, in the context of spatially embedded random networks, the probability of connecting two edges might not only depend on the distance but also on some spatial properties. Therefore, the connection function can be generalised.

Theorem 1. Let $g : [0, \infty) \to [0, 1]$ be a connection function, $z : U \subseteq \mathbb{R}^d \to \mathbb{R}$ some scalar field, and any edge $e : [u, v] \to \mathbb{R}^d \forall u, v \in V$ a piecewise smooth curve $C \subset U$. Two distinct vertices $u, v \in V$ are connected with probability $g(I_{e_i})$, with

$$I_{e_i} = \int_{e_i} z ds := \int_u^v z(e(t))||\dot{e}(t)||dt,$$

(5.2)

where $e(t)$ is a parametrisation of curve $C$, with derivative $\dot{e}(t)$.

Proof. Assuming an edge is a straight line between two vertices, it is trivial to show that if $z \equiv 1 \Rightarrow I_{e_i} = \text{dist}(u, v)$. i.e. no additional (spatial) properties influencing the connection probability.

$$\int_{e_i} ds = \int_u^v ||\dot{e}(t)||dt = I_{e_i} = \text{dist}(u, v).$$

(5.3)

5.2.4. Network generation

The spatially embedded random network is to be generated using the following three steps.

Step 1 Select an inhomogeneous Poisson point process model with an intensity function that depends on an observed covariate. This includes the selection of scaling parameters for all covariates.

Step 2 Create the spatially embedded vertices, which include the selection of the number of simulated vertices $N$.

Step 3 Create the spatially embedded edges, which includes the selection of the values of the parameter $\kappa$.

If possible, the values of the parameters should be selected so that they correspond with real-world data. If this is not possible, e.g. there is no readily available data, the data for similar areas may be used to together with expert opinion.
5.3. Example

5.3.1. Background

To illustrate the ability of the proposed methodology to be used to gain insight into a real-world network, it is used to estimate standard structural statistics for part of the Swiss road network, and these are then compared with the known values. The part of the Swiss road network (Figure 5.1) that is investigated is located around the city of Chur, the capital of Grisons (or Graubünden in German) the largest and easternmost canton of Switzerland. Grisons is a mountainous area and includes parts of both the Rhine and Inn river valleys. Forty-one per cent of the population of Grisons lives at altitudes above 1’000 MASL. The highest mountain is the Piz Bernina at 4’049 m, and the lowest point is the border with Ticino at 260 m. The Grisons’ road network is comprised of circa 163 km national roads, 597 km main roads and 835 km minor roads. The canton is crossed in a north-south direction from the A13 motorway. The investigated area of the Grisons includes the districts of Imboden and the northern part of Plessur. This area is located next to the river Rhine and contains the city of Chur and Grisons’ industrial area, together with the most important transportation links in the canton.

![Figure 5.1.: Overview of the investigated area. (The real-world road network is shown with black lines, without distinctions between different types of roads.)](image)

5.3.2. Description of data

The two types of data used to generate the spatially embedded random network were 1) terrain elevation (altitude) and 2) population density (Figure 5.2). The population density was obtained from the Swiss census statistic. The data is of high enough quality that, it is possible to observe how the structures and the development of the population and households have changed over time. The data for the statistics of population and households (STATPOP) is geo-referenced, where each hectare represents a data record (Federal Statistical Office, 2011). The source for the terrain data is the RIMINI terrain model. This is a matrix
model with a spatial resolution of 100 m and covers all of Switzerland (Kölbl, 2006). Both databases were available as a GeoTiff for the Swiss coordinate system CH1903/LV03 LN02 (ESPG code: 21781). The road network data was taken from the VECTOR25 data-set, provided by Swisstopo, (2015). This set of data describes approximately 8.5 million objects with their position, form and its neighbourhood relations (topology), as well as the kind of object and further special attributes. VECTOR25 is composed of 9 thematic layers, one layer representing the road network. The classification of roads is based on Swisstopo, (2011) guidelines. The roads are represented as lines. If there are level crossings, then roads share the same intersection point. The VECTOR25 data-set exhibits a full national coverage in homogeneous form and quality, with an accuracy of 3 to 8 m and it is delivered as an ESRI Shapefile for the Swiss coordinate system CH1903/LV03 LN02.

![Figure 5.2: Spatial input data for the model. On the left the terrain elevation and on the right the population density.](image)

5.3.3. Generation of random networks

In this example, the road intersections are modelled as vertices, while the road itself is represented by the edges. The population density is used as a covariate for the spatially embedded vertices, and the terrain elevation is used as an input for the connection model. These are considered to be reasonable assumptions, since more road intersections occur in densely populated areas, while roads at high elevations are rather rare. The intensity function is, therefore, proportional to the population density:

\[
\lambda(u) = \beta \cdot Z(u)
\]

where \( \beta \) is a scaling parameter and \( Z(u) \) is the population density at \( u \).

The deterministic connection model used is a relative neighbourhood graph. It was proposed by Toussaint, (1980) and accounts for the relative closeness between points. The random connection model used is a classical random connection model, i.e. edges are randomly added or removed to or from the graph,
5. Spatially embedded random networks

Table 5.1.: Properties of the real world network.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vertices</td>
<td>1329</td>
</tr>
<tr>
<td>Number of edges</td>
<td>1892</td>
</tr>
<tr>
<td>Total length of the road network in km</td>
<td>711</td>
</tr>
<tr>
<td>Diameter</td>
<td>63</td>
</tr>
<tr>
<td>Average shortest path length</td>
<td>23.64</td>
</tr>
<tr>
<td>Average clustering coefficient</td>
<td>0.079</td>
</tr>
<tr>
<td>Degree assortativity</td>
<td>-0.143</td>
</tr>
</tbody>
</table>

Table 5.2.: Fitted model parameters.

<table>
<thead>
<tr>
<th>Sym. Parameter</th>
<th>Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>4.71</td>
<td>(+2.96, -2.32)</td>
</tr>
<tr>
<td>$N$</td>
<td>4059</td>
<td>(+814, -922)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.909</td>
<td>(+0.0655, -0.0517)</td>
</tr>
<tr>
<td>$p$</td>
<td>0.001</td>
<td>(+0.0429, -0.0000)</td>
</tr>
</tbody>
</table>

$^a$ Range is based on the 90% confidence level and observed from the MCMC simulation.

making the probability of the existence of each edge indirectly proportional to the accumulated altitude over the edges. Hence, as the altitude difference between two vertices increases, the probability that they are connected decreases. The values of the parameters were determined using data from the real-road network and a Markov Chain Monte Carlo simulation, which minimised the standard error of: the average clustering coefficient, the average shortest path length, the degree assortativity, the diameter, the number of edges and vertices, and the total length of the road network. The values of the real network are given in Table 5.1 and the estimated parameters are given in Table 5.2.

With the values of the parameters determined, the spatially embedded random networks were generated. One example is shown in Figure 5.3.

5.3.4. Illustration of usefulness

In order to illustrate the ability of the proposed methodology to be used to gain insight into a real-world network, the relationship between the size of the largest connected cluster, measured in terms of number of edges, and the ratio of the number of failed edges in the network and the total number of edges in the network, i.e. the failure rate, which is also sometimes taken as a measure of robustness (Schneider et al., 2011). The estimation is done by performing a percolation study, which describes how a network transitions from connected to disconnected (Li et al., 2015). The percolation study was done by removing edges of the network both randomly and systematically for both the random networks and the real network. When removed randomly, an edge was selected at random and removed, thereby the probability of selecting an edge is uniform over all edges in the network. When removed systematically they were removed in descending order of their betweenness centrality, which was calculated as

$$c_B(v) = \sum_{s,t \in V} \frac{\sigma(s, t|e)}{\sigma(s, t)}$$  \hspace{1cm} (5.5)

where $V$ is the set of vertices, $\sigma(s, t)$ is the number of shortest $(s,t)$-paths, and $\sigma(s, t|e)$ is the number of those paths passing through edge $e$. 15’000 realisations of the random network were used in both cases. The results are shown in
5.3. Example

Figure 5.3.: One example of a generated spatially embedded random network.

Figure 5.4.: Relationship between the size of the largest connected part of the network and the failure rate within a confidence interval of 2σ.

A failure rate $f = 0$ indicates that no edges have been removed, and $f = 1$ indicates that all edges have been removed. In general, the higher the $f$, the smaller the remaining parts of the network that are still connected with each other. When the network contains a cluster, then it percolates, and the point at which the percolation transition occurs is called the percolation threshold. Thus, the presence of the largest connected cluster is an indicator of a network that is at least partly performing its intended function. Whereas, the size of the largest connected cluster indicates how much of the network is working (Newman, 2010). The curves (i) on the right-hand side in Figure 5.4 represent the results of random removal of edges. The curves (ii) on the left-hand side in Figure 5.4 show the systematic removal of edges based on their betweenness centrality. The values obtained from the simulations using random removal on the
5. Spatially embedded random networks

random network overestimate those using random removal on the real network, i.e. the real network has lower robustness than the random network. The values using the real network, are, however, within the $2 - \sigma$-confidence interval of the simulation, except at the beginning of the percolation process. This might be caused by the finite amount of simulations performed. The confidence interval was generated from estimating the relationship for all 15'000 random networks. The values obtained from the simulations using systematic removal on the random network underestimates those using systematic removal on the real network until the largest connected component reaches half their initial size. Afterwards, the random approach overestimates the real behaviour, i.e. the random network has at the beginning of the percolation process lower robustness than the real network and in the end higher robustness. This is caused by the choice of the connection model. While the network is densely connected in high populated areas, only a few connections are created between those areas. The values using the real network, are, however, within the $2 - \sigma$-confidence interval of the simulation for the whole percolation process, as shown in Figure 5.4. Apparently, only one realisation for the real network was possible, because the topology is not changing. While for the random network several realisations can be simulated, resulting again in a confidence interval.

5.4. Discussion

As illustrated in the example, with only a few parameters, a random network can be created which mimics the properties of a real network relatively accurately. Here, only four parameters (Table 5.2) and two spatial maps (Figure 5.2) are necessary to generate a spatially embedded random transportation network. In other words, only very little information on the target network is needed. The analysis using random removal shows that both the real network and the random networks behave similarly. The largest connected cluster decreases continuously and at a failure rate of $22.26 \pm 3.26\%$ no cluster can be observed at the real network and a failure rate of $23.94^{+7.42}_{-6.02}\%$ at the random network. Both numbers indicate that the network is not densely connected which can also be observed in Figures 5.1 and 5.3. The analysis using systematic removal shows that both the real network and the random networks have relatively significant increases in failure rate with the removal of a small number edges. For example, removing systematically 1.93% edges of the real network or $2.41^{+0.75}_{-0.64}\%$ of the random network leads to a disappearance of the largest connected cluster (Figure 5.4). This behaviour is due to the specific spatial properties of the road network and how the edges are removed. In other words, there are only a few edges are used to connect different clusters, e.g. here cities and villages, and they are removed using betweenness centrality when removes these edges first. It might be a closer fit if edges were removed taking into consideration for the traffic flow across the network. This may also be better in the estimation of risk related to the network. The differences between the real and the random networks, although not significant, maybe even smaller if: 1) the number of simulations performed was increased, 2) connection models the better accounted for the real network topology were used, 3) the uncertainty in the parameters used was reduced, or 4) spatial input parameters in addition to the population density and the elevation model were used to capture more properties of the target road network.
5.5. Conclusions

In this work, a methodology to develop models of spatially embedded random networks is proposed. It combines a spatial non-homogeneous point process for vertex creation, which accounts for the spatial distribution of vertices, considering clustering effects of the network and a hybrid connection model for the edge creation. In contrast to many other methodologies, this methodology takes into account spatial restriction. It was shown that the proposed methodology could be used to develop models to estimate reasonably well the relationship between the largest connected part of a part of the Swiss road network and the ratio between the number of failed links and the total number of links. This was demonstrated by conducting percolation studies with both random and systematic removal of edges. In the illustrated example the input parameters were population density and a terrain model, and the random network models were calibrated using the seven parameters shown in Table 5.1. It could be shown that the behaviour of the randomly created networks is similar to that of the real network. It is suspected that even more similar behaviour is possible.

The work presented here is a useful first step towards the improved understanding of real-world spatial networks, as it enables the generation of different realisations of the same network. This is useful to take into consideration uncertainties in network properties, in situations where it is too difficult to model the network directly. Until now, uncertainties in the network topology have mainly been avoided and opens up a myriad of possibilities in network analysis. Three important overlapping areas where it is suspected that this methodology has substantial use are:

(i) to assess risk on complex spatially distributed networks,
(ii) to model networks where there is a limited amount of information available, and
(iii) to model how networks might change over time in the future.

Future work is to be focused on:

- the development of a model of multiple interdependent networks
- the use of functional and spectral based measures to characterise the networks
- the use of random networks to estimate risk taking into consideration the probabilities of occurrence of loading of the network and the consequences of the possible network states.
6. Modelling multi-layer spatially embedded random networks

This chapter corresponds to the published article:¹


**Abstract:** Most real and engineered systems, including transportation infrastructure, are embedded in space and interact with one another in a variety of ways. To study such systems, a novel multi-layer spatially embedded random network model is proposed. In the development of this model, concepts from spatial statistics and graph theory are used to map complex systems with interdependent subsystems to a simplified and condensed mathematical representation. The developed model combines Markov marked point processes for vertex creation, which accounts for spatial distribution, layer assignment, and clustering effects of the vertices, and a hybrid connection model for the edge creation. To test the capabilities of, and gain insights with respect to, a real-world network, the model was used to model a complex infrastructure system, comprised of the power grid and road network of Switzerland. It was found that, even with very simple assumptions, topological properties could be estimated reasonably well.

¹Please note, this is the author’s version of the manuscript published in the *Journal of Complex Networks*. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at https://academic.oup.com. When citing this chapter, please, refer to the original article with DOI: 10.1093/comnet/cny019.

For reasons of consistency, the citations have been amended to author-year style.
6. Multi-layer spatially embedded random networks

6.1. Introduction

Network theory is often used to describe and analyse complex systems found in nature, technology, society and human-made structures (Newman, 2010; Boccaletti et al., 2014). The ideas of mathematical graph theory are used to generate insights into the behaviour of complex systems by abstracting information into ordinary graphs (networks). In these representations, the network comprises vertices connected by edges, where vertices represent single elements and edges indicate interactions or relationships between them. Although this approach is simple in many respects, it enables the characterisations of the complex system so that traditional graph-theoretic metrics can be used and analyses can be done. For example, such abstractions have been used to study growth mechanisms (Barabási and Albert, 1999; Barabási, 2002), processes of collective dynamics (Watts and Strogatz, 1998), and to illustrate that certain vertices play a central role in the complex system (Freeman, 1977; Wasserman and Faust, 1994).

In recent years, the modelling of complex systems as networks of connected elements has become subject to intense study in various fields including physics, social science, and biology, with the goal of analysing the structure and dynamics of large-scale networks of all kind (Watts and Strogatz, 1998; Albert et al., 1999; Barabási, 2002; Barabási and Bonabeau, 2003; Newman, 2003). Notable contributions to understand real-world phenomenon are among others, random networks (Gilbert, 1959; Erdős and Rényi, 1959), small-world networks (Molloy and Reed, 1995), and scale-free networks (Barabási and Albert, 1999).

In order to study more realistic systems, a transition from relatively simple graphs to more realistic ones has started (Kivelä et al., 2014). This enables the development of models that more realistically exhibit real-world properties. For example, an essential property of many real-world systems, such as technological (Andrews et al., 2010) and transportation infrastructure (Chan et al., 2011), or biological systems (Bullmore and Sporns, 2009; Stella et al., 2016, 2018), is that they are embedded in the physical space. Consequently, vertices and edges are subjected to spatial and physical constraints, which affect their topological properties and the dynamic processes associated with the network (e.g., traffic flow across a road network) (Barnett et al., 2007; Barthelemy, 2011, 2018). Notably, in the case where costs are associated with the length of the edges, the topological properties are strongly correlated to spatial aspects (Schweitzer et al., 1998; Gastner and Newman, 2006; Louf et al., 2013).

The concept of spatially embedded networks is not new to transport engineers and geographers. Quantitative studies of transportation networks began in the 1960s (Garrison, 1960; Kansky, 1963; Haggett and Chorley, 1969), and was focused mainly on simple topological and geometric properties, due to the limitation of available data and computational power. Nevertheless, many important properties and measures were derived in this period, including size, length, diameter and connectivity of networks (Kansky, 1963).

In contrast to theoretical models of networks, spatial networks are embedded in a physical (Euclidean) space where vertices and edges occupy a precise position (Erath et al., 2009b). This, in turn, constrains the number of connections and orientates the layout of the network, with consideration of borders (Ducruet and Lugo, 2013). For example, in many real-world systems the actual costs of edges are related with the length of the connection (e.g., the cost for road construction and maintenance), thus the number of long-range edges is limited and most of the edges will connect vertices close to each other (Barrat et al., 2004). Because
the most interesting properties of spatially embedded networks lie in their spatial organization and in the relationship between space and topology, several scholars have thus explored the development of new theoretical network models to study spatial systems such as power grids (Schultz et al., 2014; Soltan and Zussman, 2016; Cadini et al., 2015), flight networks (Barrat et al., 2004; Gastner and Newman, 2006), road networks (Schweitzer et al., 1998; Barthélémy and Flammini, 2008, 2009; Hackl and Adey, 2017a), railway networks (Louf et al., 2013) and telecommunication networks (Wang et al., 2010; Dettmann and Georgiou, 2016).

A general description of spatially embedded random networks is given by Barnett et al., (2007), which demonstrated general constraints on connectivity and the existence of small-world spatial networks.

However, almost all of these models assume spatial independence between the vertices by using “classical” Poisson point processes, where the vertices are independently distributed in space. Clearly, this is not the case for real-world systems, where tendentially clusters or separations of vertices can be observed. For example, road intersections tend to be clustered in urban areas, while major airports tend to be further away from each other. Additionally, in these models, spatial features of edges are purely considered by the distance between their vertices, while interaction with their (physical) environment is neglected.

Using such models, Hayashi and Matsukubo, (2006) showed that spatial networks are less vulnerable to random failures and targeted attacks as networks without spatial properties. Others (Buldyrev et al., 2010; Vespignani, 2010; Donges et al., 2011; Bashan et al., 2013), however, have found that interdependent networks are more vulnerable than single networks. In order to study the interdependencies between networks based on their physical correspondence through spatially embedded vertices and edges, there is a need for improved modelling, which can be obtained through the use of multi-layer network models.

Most recently, efforts have been undertaken to study such interdependent systems using network theory, as it allows the modelling of complex systems comprised of multiple subsystems (Kielhauser et al., 2017). In the classical graph representations, it is assumed that edges of a single static type connect the vertices. This is not adequate for modelling interdependent networks, as they require edges of multiple types to connect the vertices, which are often organised in layers of connectivity (De Domenico et al., 2013; Cozzo et al., 2016). Such modelling is especially required for infrastructure systems, when even a small level of sophistication is required, as they consist of numerous interdependencies, e.g. the physical interdependencies between where material or people flow from one infrastructure network to another, and geographical interdependencies, e.g. the close spatial proximity of the objects, such as water distribution pipes, in one infrastructure network to the objects, such as roads, in another (Vespignani, 2010).

The use of such multi-layer descriptions make it possible to investigate the dynamic processes that produce cascading failures, where failures in one network propagate to other networks (Bashan et al., 2013), e.g. a water pipe breaks and disrupts traffic flow on the road network, something that it is necessary to take into consideration when planning interventions in urban areas (Kerwin and Adey, 2017). In fact, failure of a small percentage of vertices or edges in one network often causes widespread disruption in the whole system. For example, in 2003 a failure of a power station in Italy caused an interruption to the provision of telecommunication services, which in turn contributed to the failure of even more power stations (Rosato et al., 2008; Buldyrev et al., 2010). Thus, understanding
6. Multi-layer spatially embedded random networks

the general principles, leading to the complex structures of these networks and their ability to withstand failures, natural hazards and human-made disasters, is critical for evaluating risk and designing robust transportation infrastructures (Hackl and Adey, 2017a).

Spatially embedded transportation networks have usually been studied separately (Strano et al., 2015). This undoubtedly leads to an understanding of network behaviour, but most real-world systems are comprised of multiple subsystems, and therefore, require the modelling of multiple networks. To determine in detail the importance of the different vertices and edges in the multiple networks with respect to the interaction and communication between different networks, as well as to characterise the topology in order to study the network related risks and vulnerabilities, novel network models are necessary. As a result of these challenges, multi-layer spatially embedded network models have been the subject of increasing research interests in recent years (Vaknin et al., 2017; Korkali et al., 2017; Stella et al., 2018).

In many cases, it is possible to use and extend methods developed for single-layer networks for multi-layer networks, assuming that the topology of each network and their connections between them are known precisely. Unfortunately, creating networks based on real-world systems often involves a lack of information about the details of their structures and interdependencies (Wider, 2016). This is especially true for infrastructure systems that are of national importance, such as power grids or water distribution systems, where data is not available for public use. In such situations, network science can be used to build models that reproduce ensembles of artificial representations of real-world systems with their expected properties.

Contributions. To study multi-layer spatially embedded random networks (MLSERNs) systematically, it is useful to develop a precise mathematical representation of them. In this work, a novel general model is presented that can be used to model such complex networks. In the development of this model, concepts from spatial statistics and graph theory are used to map complex systems with interdependent subsystems to a simplified and condensed mathematical representation. Going beyond the previous studies, this work advances the state-of-the-art in the field of spatially embedded networks as follows:

- Using advanced Markov marked point processes, it allows describing spatial interactions of vertices, in which the position and layer assignment of individual vertices depends on the properties of their neighbour vertices (e.g. their positions and/or layer assignments), causing them to stay away from each other (repulsion) or come closer together (attraction). Additionally, the use of spatial fields allows modelling vertex interactions with their surrounding (physical) environment. (e.g. road intersections are more likely located in urban areas than in the countryside)
- The edge assignment is done with a novel connection model utilising existing connection functions and combining them into hybrid connection functions. A weighted distance based on spatial fields is introduced to account for the fact that the probability of connecting two vertices might not only depend on the distance but also on some spatial and layer properties.
- In the case of the interdependent power and road network of Switzerland, it is shown that the MLSERN model constitutes highly accurate approximations for the true spatially embedded network, in terms of single and multi-layer measures.
6.2. Multi-layer spatially embedded random networks

While the model presented in this work has been developed explicitly for complex infrastructure systems such as power grids, railway and road networks, the proposed model is generally applicable to all types of spatially embedded networks. As network theory, upon which the model is based, relies on the hypothesis that complex systems can be explained by relationships among their components (i.e., vertices and edges), thus the resulting random networks are artificial representations of the real-world systems, which are expected to capture many of the complex features.

The remainder of this work is organised as follows. In Section 6.2, the general formulation for the multi-layer spatially embedded random network representation is presented. Two applications of usage are given in Section 6.3. In particular, this section is divided into an overview of the data used (6.3.1), the modelling of the multi-layer power and road network of Switzerland (6.3.2), and a hierarchical model of the road network considering different road classes (6.3.3). In Section 6.4 the results and a critical discussion about the findings, advantages and shortcomings of the model are given. Concluding remarks and suggestions for future work in this area are given in Section 6.5. The notation used throughout this work is given in Appendix D.

6.2. Multi-layer spatially embedded random networks

Multi-layer networks represent complex systems formed by several networks (layers) each one representing interactions of different nature and connections. Due to the distinction between the different nature of edges, multi-layer networks encode significantly more information than conventional single-layer networks (Iacovacci and Bianconi, 2016). One subclass of multi-layer networks are networks of networks, which are formed by layers composed of different vertices. Edges which connect different networks do not necessarily indicate dependency relations (Boccaletti et al., 2014; Kivelä et al., 2014). Notable examples can be found in complex infrastructure networks, such as power grids, transportation and water supply systems, where each layer represents a separate infrastructure (Vespignani, 2010; Donges et al., 2011; Bashan et al., 2013).

Following the notation of Kivelä et al., (2014), a multi-layer network can be represented as a graph $G_M$ which is an ordered tuple $G_M = (V_M, E_M)$ considering a non-empty labelled vertex set $V_M$ and a multiset $E_M \subseteq V_M \times V_M$ of edges. A vertex $(u, \alpha) \in V_M$ is a tuple representing vertex $u$ on layer $\alpha \in \Lambda$, where $\Lambda$ is the set of layers in the network. Multi-layer networks are undirected if $\{(u, \alpha), (v, \beta)\} \in E_M \implies \{(v, \beta), (u, \alpha)\} \in E_M$ and they have no loops if $\{(u, \alpha), (u, \alpha)\} \notin E_M$. Edges can be categorized into different groups, depending if the cross layers stay within a single layer. Intra-layer edges $E_\alpha = \{(u, \alpha), (v, \beta)\} \in E_M | \alpha = \beta$ describe edges within the layer $\alpha$, inter-layer edges $E_{\alpha \beta} = E_M \setminus E_\alpha$ describe edges which connect vertices across layers, and coupling edges $E_C \subseteq E_{\alpha \beta}$, edges for which the two vertices represents the same entity in different layers. Consequently, an intra-layer graph $G_\alpha = (V_\alpha, E_\alpha)$, an inter-layer graph $G_{\alpha \beta} = (V_{\alpha \beta}, E_{\alpha \beta})$, and a coupling graph $G_C = (V_C, E_C)$ can be defined.

6.2.1. Definition

Multi-layer spatially embedded networks can be seen as a special kind of multi-layer, where vertices of one, or several layers respectively, are placed within a metric space. Many real-world networks are embedded in two or three-dimensional space, including the internet, airline networks, transportation and wireless communication networks. In order to create an artificial multi-layer spatially embed-
ded network two levels of randomness have to be considered: (i) the placement of the vertices and their assignment to a specific layer, and (ii) the assignment of the edges to pairs of vertices, taking spatial and layer properties into account. In mathematical terms, such a network can be formally described as:

A multi-layer spatially embedded random network (MLSERN) defined on $S$ is a graph $G_M(V_M, E_M)$ defined through its vertices set $V_M = Y$ and edge set $E_M = \mathcal{M}(Y, g)$. Where:

- $S = A \times L$ is a complete separable metric space, with Borel $\sigma$-algebra $\mathcal{B}$, $|A \times L| < \infty$. The network is embedded in a $d$-dimensional space $A \subset \mathbb{R}^d$ comprised of $L \subseteq \mathbb{N}$ layers.

- $V_M$ is a mapping $Y$ from a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ into a metric space $O_Y = \{y = \{(u, \alpha) : u \in A, \alpha \in L\} \subseteq A \times L : n(u_B) < \infty, \forall B \subseteq A\}$, where $n(u_B)$ is the number of realizations for each Borel set $B \subseteq S$. A realization of $Y$ is an unordered set $y = \{(u, \alpha), \ldots, (v, \beta)\}$, $u, v \in A$, $\alpha, \beta \in L$, with $u$ describing the location of the vertex and $\alpha$ the corresponding layer.

- $E_M$ is a connection model $\mathcal{M}(Y, g) = \{\{(u, \alpha), (v, \beta)\}|g_{\alpha\beta}\}$ to assign the edges, where $g$ is a matrix of connection functions $g_{\alpha\beta} : [0, \infty) \rightarrow [0, 1]$, which gives the probability of assigning an edge between two vertices assigned to layer $\alpha$ and $\beta$. The connection probability may vary depending on spatially properties and type of edge (e.g. intra-layer, inter-layer or coupling edges).

6.2.2. Model

A MLSERN can be interpreted as a multi-stage marked point process. The first stage corresponds to the placement of the vertices and their assignment to a specific layer, where the corresponding mark describes the layer. In the second stage, a set of probabilities is assigned to each vertex, which indicates how likely they are to be connected to all other vertices. Where the connection probability not only depends on the assigned layer but also on other features, e.g. the geodesic distance or spatial constraints, they are captured in the connection function $g_{\alpha\beta}$. Based on this interpretation, a stochastic model is used to assign the spatially embedded vertices within the assigned layers, and a connection model is used to assign the edges, taking into consideration the distances between the vertices and other (spatial and layer) properties.

Vertex generation

Most of the models used in literature to simulate spatially embedded vertices exhibit some kind of spatial independence between the vertices. For example, in a binomial process all vertices are independent, in a Poisson process the vertices are independently distributed in disjoint regions, and in Cox processes, they are conditionally independent given the intensity function (Lieshout, 2000). In many real-world applications, however, spatial interactions in terms of inter-point interactions play an essential role, i.e., some vertices attract or repel each other,
which is reflected in clustered patterns. For example, for economical, functional or physical reasons there are minimum distances between physical infrastructure objects such as airports, power transmission stations or road intersections. Similar patterns appear in many biological applications, but also in physics and materials sciences.

To overcome these issues and model spatially interacting vertices\(^3\), Markov marked point processes are applied. Such models are constructed based on densities for point processes with respect to Poisson processes and imposing certain relationship conditions among the points to ensure Markovian properties (Møller and Waagepetersen, 2003). This point processes analysis ensembles of vertices scattered in space, where the vertices are characterised not only by their positions but also by additional properties, such as the assigned layer. In statistical physics, these types of processes are often called canonical ensembles and used for the description of large interacting particle systems (Ruelle, 1969; Georgii, 1976).

To build the model to generate the vertices, let \(S = A \times L\) be a complete separable metric space, with \(|A \times L| < \infty\), \(A \subset \mathbb{R}^d\), and \(L \subseteq \mathbb{N}\). \(\nu(\cdot)\) is a finite, non-atomic Borel measure on \(L\), \(\ell(\cdot)\) a probability distribution on the Borel \(\sigma\)-algebra of \(L\), and \(\pi_{A \times L}\) the distribution of a Poisson process on \(A \times L\) with intensity measure \(\nu \times \ell\). Let \(Y\) be a marked point process with position in \(A\) and marks in \(L\) specified by an unnormalized density \(h\) with respect to \(\pi_{A \times L}\). Then \(Y\) is a Markov marked point process with respect to the symmetric, reflexive relation \(\sim\) on \(A \times L\) if for all \(y\) such that \(h(y) > 0\), \(h(z) > 0\), \(\forall z \subseteq y\), and \(\forall (v, \beta) \in A \times L\), \(h(y \cup \{(v, \beta)\})/h(y)\) depends only on \((v, \beta)\) through \(N_{(v, \beta)} \cap y = \{(u, \alpha) \in y : (v, \beta) \sim (u, \alpha)\}\), where \(N_{(v, \beta)} = \{(u, \alpha) \in S : (v, \beta) \sim (u, \alpha)\}\) defines the neighbourhood of \((v, \beta)\), which contains all its neighbors, where \((u, \alpha)\) and \((v, \beta)\) are neighbors if \((u, \alpha) \sim (v, \beta)\).

The class of Markov functions used to generate the vertices is characterised by the Hammersley-Clifford-Ripley-Kelly theorem (Ripley and Kelly, 1977), which provides an explicit factorisation of the probability density \(f\) of a Markov marked point process in terms of interactions between the points being modelled, which are in this case the vertices. Thus, a Markov function \(h\) defines a Markov marked point process with respect to the neighbourhood relation \(\sim\) defined on \(S\) if and only if it can be factorised as

\[
f(y) \propto h(y) = \prod_{z \subseteq y} \phi(z), \quad \forall y \in S, \tag{6.1}
\]

where \(\phi : S \to [0, \infty)\) is a interaction function (Lieb, 2000). To observe the probability density \(f\) the unnormalized density \(h\) must be normalized \(f(y) = ah(y)\). The intensity of a Markov marked point process is defined by the Papangelou conditional intensity (Papangelou, 1974), which does not depend on the normalization constant \(a\):

\[
\lambda((u, \alpha), y) = \frac{f(y \cup \{(u, \alpha)\})}{f(y)} = \frac{h(y \cup \{(u, \alpha)\})}{h(y)} \quad \text{for } (u, \alpha) \notin y \tag{6.2}
\]

Heuristically, \(\lambda((u, \alpha), y)du\) can be interpreted as the conditional probability of finding a point of the point process \(Y\) inside an infinitesimal region around the vertex \((u, \alpha)\) of area \(du\), given the position of all other vertices outside this region.

\(^3\)i.e. the position of a vertex is influenced by the locations of the other vertices
In order to construct spatial point processes that exhibit stochastic dependence between vertices, a probability density that depends on more than one vertex has to be introduced. The most common models are pairwise interaction processes. A marked pairwise interaction point process has a probability density.

\[
f(y) \propto h(y) = \prod_{(u,\alpha) \in y} \phi_1(u, \alpha) \prod_{\{(u,\alpha),(v,\beta)\} \subseteq y} \phi_2((u, \alpha), (v, \beta)),
\]

where \(\phi_1(\cdot) : S \to (0, \infty)\) and \(\phi_2(\cdot, \cdot) : S \times S \to (0, \infty)\) are interaction functions.

In using marked point processes to generate vertices, it is necessary to take into consideration that different approaches of assigning locations and layers can be used, which lead to different stochastic models and have different inferential interpretations. A statistical model for vertex location in A and layers in L can be formulated in several ways:

(i) The independent model \(Y(A, L)\) is the simplest marked point process. The positions and the layers of the vertices are assigned conditional independent by random mechanisms.

(ii) If a model \(Y(L|A)\) is conditional on location, then first the locations of the vertices in A are determined via the spatial point processes and subsequently the layers L are assigned to the vertices by a random mechanism.

(iii) If a model \(Y(A|L)\) is conditional on marks, then the layers L are first assigned to the vertices according to some random mechanism and subsequently they are placed at certain locations in A by point processes.

### Edge generation

In modelling networks, one of the most important parts is an accurate assignment of the edges to vertices, as this affects all topological features and properties of the network being modelled. In the last few decades, scientists developed various connection functions to describe and reproduce networks. Notable connection functions are among others, the Gilbert and Erdős-Rényi model (Gilbert, 1959; Erdős and Rényi, 1959, 1960, 1964) which build the foundation of random graph theory, by assigning edges at random with equal probability; configuration models (Molloy and Reed, 1995) where the degree distribution of the vertices is restricted; preferential attachment (Barabási and Albert, 1999) mechanisms which help represent how networks change over time; or rewiring models (Watts and Strogatz, 1998) where existing edges are rearranged.

In this work, a more general approach of a connection model \(M(Y, g)\) is implemented for the edge assignment. Thereby, \(g\) is a set of connection functions \(g_{\alpha\beta}\), which gives the probabilities of a direct link between two vertices \((u, \alpha)\) and \((v, \beta)\). In the common connection functions for spatially embedded networks, two distinct vertices \(u, v \in V\) are connected with probability \(g(\text{dist}(u, v))\). In other words, the probability of assigning an edge to a pair of vertices depends only on their Euclidean distance to each other. However, in the context of MLSERNs, the probability of connecting two vertices might not only depend on the distance but also on some spatial and layer properties. Therefore, a weighted distance based on spatial fields is introduced as proposed by Hackl and Adey, (2017a).

Let \(Z : U \subseteq \mathbb{R}^d \to \mathbb{R}\) some scalar field, and \(r : [u, v] \to \mathbb{R}^d \forall u, v \in V_M\) is an arbitrary bijective parametrization of a piecewise smooth curve \(C \subset U\) such that

---

4Higher-order interaction processes are possible but mathematically more demanding. For illustration purpose, pairwise interaction processes are used in the remainder of this work.
6.2. Multi-layer spatially embedded random networks

\( \mathbf{r}(u) \) and \( \mathbf{r}(v) \) give the endpoints of \( C \). The weighted distance is the line integral

\[
\int_C Z \ ds := \int_u^v Z(\mathbf{r}(t)) \cdot ||\dot{\mathbf{r}}(t)|| \ dt, \tag{6.4}
\]

where \( \dot{\mathbf{r}}(t) \) is the derivative of \( \mathbf{r} \). Assuming an edge is a straight line between two vertices, it is trivial to show that if \( Z \equiv 1 \Rightarrow w(u, v, Z) = \text{dist}(u, v) \). i.e. the weighted distance is equal to the Euclidean distance. If \( Z \) is representing a vector field, the weighted distance depends not only on the distance, but also on the direction of the edge. An overview of several connection functions is given by Dettmann and Georgiou, (2016) and Parsonage and Roughan, (2017).

In this work a distinction is made between deterministic and probabilistic connection functions. In the case of deterministic connection functions, edges are assigned based on a computational geometric algorithm, taking the distances between the vertices into account. For example, there are a number of algorithms that encode important aspects of the notion of nearness, such as nearest-neighbour graphs (NNG), Minimum spanning trees (MST), relative neighbourhood graphs (RNG), Gabriel Graphs (GG), Delaunay triangularization (DT), and random geometric graphs (RGG). In the case of probabilistic connection functions, the edges are assigned independently of each other, with probability \( g \in [0, 1] \). While “classical” models, such as Gilbert, Erdős-Rényi, configuration, preferential attachment and rewiring models, do not take spatial properties of the network into account, other models taking the distance between two vertices into account for evaluating the connection probabilities. For example the Waxman model (Waxman, 1988) links vertices with a probability

\[
g = a \cdot \exp\{-\text{dist}(u, v) / b\},
\]

where \( a \) and \( b \) are constants.

Because different connection functions have different properties, the spatially embedded edges are generated by the use of a hybrid connection function \( \bar{g} \). Such a hybrid combines two or more connection functions by assigning a certain probability \( \kappa_i \) to each considered connection function \( g_i \):

\[
\bar{g} = \sum_{i=1}^{n} \kappa_i g_i \quad \text{with} \quad g_i \in [g_1, \ldots, g_n], \quad \sum_{i=1}^{n} \kappa_i = 1 \tag{6.5}
\]

The resulting hybrids are particular useful for constructing connections which exhibit interactions at different spatial scales.

Network generation

To generate MLSERNs the following parameters are required: \(|L|\) - number of layers, \(|V|\) - number of vertices, \( Y \) - spatial point process, \( \phi_i \) - interaction functions, \( g \) - matrix of \( L \times L \) connection functions. Optional are \( Z_V, Z_E \) - spatial fields for vertex and edge generation, \( \kappa_i \) weight parameters if \( i \) hybrid connection functions are used. The network is constructed by:

1. drawing \(|V|\) vertices \( \mathbf{y} = \{(u, \alpha), \ldots, (v, \beta)\} \) from \( Y \);
2. for each pair \((u, \alpha), (v, \beta)\) of possible edges an edge is applied with probability \( g_{\alpha \beta}(u, v) \).

If spatial fields are considered the vertices are drawn from \( Y(Z_V) \) and instead of \( \text{dist}(u, v) \) the weighted distances \( w(u, v, Z_E) \) are used in for the connection functions.

To represent real-world systems, appropriate parameters have to be chosen. This is done by calibrating the model with the networks, which should be approximated. If this is not possible, e.g. there is no readily available data, then data for similar networks may be used together with expert opinion.
6.3. Application

The application presented in this section is used to demonstrate the usefulness of the proposed model considering a specific problem. The application shows the design and implementation of an MLSERN to model a complex infrastructure system, comprised of the power grid and road network of Switzerland.

Both networks are essential for modern societies. Their resilience is strategically important to the security and sustainability of the communities, which have become considerably more sophisticated and integrated than ever before. As indicated by the recent history, unfortunately, such infrastructure systems were proven to be vulnerable under natural hazards like earthquakes, storms, floods, etc., leading to negative economic and societal ramifications for the entire community (Sun, 2017). In the case of the power grids, such failures or malfunctions might lead to widespread disruption in the whole system, causing shutdowns of coupled power-dependent systems, while interrupted road networks aggravate rescue missions and restoration of other critical infrastructures, i.e., a functional road network is needed to reach and repair parts of the damaged power grid. Because recovery in the post-disaster stage is at least as critical as the disaster robustness, considerable research focus has shifted to the resilience of communities and infrastructure systems (Hackl et al., 2018a). However, studying such interdependent spatially embedded infrastructure systems raises several issues:

- Data for infrastructure systems of national importance, such as power grids or water distribution systems, are often not available for public use. Furthermore, in many cases, there is no geo-referenced data available at all.
- Transportation infrastructures are unique structures, meaning that they cannot be reproduced under the same conditions, i.e., vertices and edges are subjected to spatial and physical constraints. Therefore, it is hard to test new hypotheses if only one predefined, deterministic network structure is given.

In such situations, MLSERN can be used to build models that reproduce ensembles of artificial representations of real-world systems with their expected properties.

The remainder of this section is structured as follows: First, an overview of the investigated networks and data used is given in Section 6.3.1, followed by the implemented MLSERN model conditional on marks for the multi-layer power and road network of Switzerland, in Section 6.3.2. Thereby, this application is used to introduce a simple Markov marked point process and showed the usage of weighted distance and hybrid connection models. In Section 6.3.3 MLSERN model conditional on location is used to describe the hierarchical structure of the road network, in order to generate a better approximation of the real-world system.

6.3.1. Data

For the analysis, the road network and power grid data were taken from the VECTOR200 data-set, provided by swisstopo\textsuperscript{5}. This set of data represents a great generalisation (scale 1:200,000) of the natural and human-made features of Switzerland. The road (power grid) data-set describes approximately 98,000 (1,300) objects with their position, form and topology, as well as the kind of object. The classification of roads is based on Swiss cartography guidelines (Swisstopo, 2011). The roads are represented as lines. Roads with level crossing were

\textsuperscript{5}Federal Office of Topography
6.3. Application

Figure 6.1.: Visual representation of the investigated real-world networks. [left] The blue layer at the top represent the high voltage power grid of Switzerland, where vertices correspond to power substations. The orange layer in the middle shows the road network, while the bottom layer illustrated the topography of Switzerland. [right] In the detailed section of the map, the actual power lines, the roads classified into primary (light orange) and secondary (white) and the topography is illustrated.

Table 6.1.: Single-layer network measures of the real-world networks.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Measure</th>
<th>Power</th>
<th>Road</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>V</td>
<td>$</td>
<td>Number of vertices</td>
</tr>
<tr>
<td>$</td>
<td>E</td>
<td>$</td>
<td>Number of edges</td>
</tr>
<tr>
<td>$L$</td>
<td>Total length of the edges [km]</td>
<td>4.055e+3</td>
<td>28.698e+3</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter</td>
<td>32</td>
<td>101</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius</td>
<td>18</td>
<td>55</td>
</tr>
<tr>
<td>$\langle \ell \rangle$</td>
<td>Average shortest path length</td>
<td>12.479</td>
<td>42.621</td>
</tr>
<tr>
<td>$\langle C \rangle$</td>
<td>Average clustering coefficient</td>
<td>4.413e-2</td>
<td>7.497e-2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Degree assortativity</td>
<td>-6.824e-2</td>
<td>2.954e-3</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Transitivity</td>
<td>5.769e-2</td>
<td>8.214e-2</td>
</tr>
<tr>
<td>$\langle \ell \rangle^w$</td>
<td>Weighted average shortest path length</td>
<td>1.760e+5</td>
<td>2.177e+5</td>
</tr>
</tbody>
</table>

* the weights are the real lengths of the edges

considered to share the same intersection point. Only roads of national and regional importance were considered, i.e. the road classes primary roads (e.g. motorways and main long distance roads) and secondary roads (e.g. local connections and important roads in towns and cities). All other road classes, e.g. tertiary and residential roads, were not considered. Power lines of the high voltage network of Switzerland were considered, where vertices represent extra-high voltage substations. Due to the confidentiality of the data, only the topology of the power grid was used. The roads connecting the substations with the remaining road network were considered as inter-layer edges. As spatial fields, the population density and the land cover were used. The population density was obtained from the Swiss census statistic and aggregated per municipality. Land cover data were also taken from the VECTOR200 data-set. The VECTOR200 data-set exhibits a full national coverage in homogeneous form and quality, with an accuracy of 20 to 60 m and it is delivered as an ESRI Shapefile for the Swiss coordinate system CH1903/LV03 LN02. Figure 6.1 illustrates these real-world networks.

In a first step the data was cleaned by filtering out unused road classes, which lead to an over-representation of crossings; i.e. edges might be interrupted even if there is no crossing anymore. To eliminate this bias, the edges were dissolved, i.e. the entities become aggregated, dependent on specific attributes, such as the road classes. Common network measures of the two real-world networks
6. Multi-layer spatially embedded random networks

Figure 6.2.: Degree distribution of the unified networks. Vertices with degree two were removed by joining the edges in order to reduce the representation bias. (a) Degree distribution of the power grid. (b) Degree distribution of the road network. (c) Joint degree distribution of the networks.

are given in Table 6.1. The degree distributions of the networks are shown in Figure 6.2. Due to physical constraints of spatially embedded networks, the degree distribution has a specific form, which cannot be approximated easily by binomial or Poisson distribution. The maximum degree of intersections is limited to a finite number. The majority of the vertices have degree one, three or four and only a limited number of vertices connect five and more edges. Vertices with degree one describing dead ends or an intersection with the considered spatial boundaries. Using only a simplified set of roads, a vertex with degree one also might describe a connection to a subordinated network, which is not represented. In the scope of a flow-based topology, without considering varying attributes (e.g. road classes), vertices with degree two cannot be observed. This is based on the assumption that a vertex with degree two can be removed and the edges can be joint without changing the routing of the network\(^6\). The degree-degree distribution between both networks indicates how likely hubs in the power grid tend to be close to hubs in the road network.

6.3.2. Power grid and road network

In the first application, the MLSERN was composed of the power grid layer \(p\) and the road network layer \(r\), i.e. \(L = [p, r]\), with \(\alpha, \beta \in L\). The substations of the power grid and road intersections and were modelled as vertices \(V_p\) and \(V_r\), while power cables and road sections were represented by the edges \(E_p\) and \(E_r\). Thereby, an aggregated road network was used, i.e. no distinctions between different road classes were made. The study region \(A_{\ominus r} \subseteq A\) was defined by the national boundary of Switzerland. To minimize the sampling bias (so-called boundary or edge effects), the sampling space included a \(r = 10\) km buffer around the Swiss territory.

Vertex generation

A MLSERN model conditional on marks was implemented because the vertex locations of the two different infrastructure networks do not (strongly) depend on each other, however, the vertex placement inside a layer depends on the position of the other vertices of the same layer, i.e. the placement of a power substation is not influenced by the surrounding road intersections but depend on the positions of other power substations in the neighbourhood. The Markov marked point process used to model such a behaviour was a non-stationary Strauss process

\(^6\)In the remainder of the work such networks were analysed and denoted as unified networks.
6.3. Application

(Strauss, 1975), which is a simple, but non-trivial, pairwise interaction process, with interaction functions:

\[ \phi((u, \alpha)) = Z_{V, \alpha}(u) \cdot b_\alpha \quad \text{and} \quad \phi((u, \alpha), (v, \beta)) = \begin{cases} c_{\alpha\beta} & \text{if } \text{dist}(u, v) < r_{\alpha\beta} \\ 1 & \text{otherwise} \end{cases} \]  

(6.6)

and probability density function:

\[ f(y) = a \cdot \prod_{(u, \alpha) \in y} Z_{V, \alpha}(u) \cdot b_\alpha \prod_{\{(u, \alpha), (v, \beta)\} \subseteq y} c_{\alpha\beta}^{s_r(y)} \]  

(6.7)

where \( a \) is the normalization constant, \( b_\alpha > 0 \) is the first order trend parameter, \( Z_{V, \alpha}(u) \) is a covariate that has been measured at every location \( u \in A \), \( 0 \leq c_{\alpha\beta} \leq 1 \) is the interaction parameter, and \( s_r(y) = \sum_{u, v \subseteq y} 1_{\{\text{dist}(u, v) \leq r\}} \) is the number of unordered neighbour pairs of distinct vertices in \( y \) which lie closer than \( r \) units apart. The conditional intensity of a Strauss process at location \( u \) is:

\[ \lambda((u, \alpha); y) = Z_{V, \alpha}(u) \cdot b_\alpha \cdot \prod_{\beta \in L} c_{\alpha\beta}^{t_{\beta}(u, \alpha); y} \]  

(6.8)

where \( t_{\alpha\beta}(u, \alpha); y = s_r(y \cup \{(u, \alpha)\}) - s(y) \) (for \((u, \alpha) \notin y\)) is the number of vertices in \( y \) that lie within a distance \( r_{\alpha\beta} \) of location \( u \), such that the model is Markov at range \( r_{\alpha\beta} \). By neglecting higher order interaction terms, i.e. \( c_{\alpha\beta} = 1 \), the point process is reduced to a “classical” non-homogeneous Poisson point process with intensity \( Z_{V, \alpha}(u) \cdot b_\alpha \), where the vertex locations are independent of each other and completely random if \( Z_{V, \alpha}(u) = \text{constant} \). If \( c_{\alpha\beta} = 0 \) the model is a so-called hard core process, where vertices cannot lie closer than \( 2r \) units apart.

In order to simulate such a process the parameters \( b_\alpha, c_{\alpha\beta}, r_{\alpha\beta}, \) the number of simulated vertices \( N_\alpha \) and \( Z_{V, \alpha}(u) \) had to be defined for both layers \( \alpha, \beta \in [p, r] \). Since interaction between \((u, p)\) and \((v, r)\) were not considered, the interaction terms \( c_{pr} = c_{rp} = r_{pr} = r_{rp} = 0 \). For this example, the population density was used as covariate \( Z_V \), under the assumption that road intersections and substations occur more likely in denser populated areas (see Gastner and Newman, (2006)).

**Edge generation**

Usually, the edge assignment for infrastructure systems has to consider two essential requirements: (i) the costs of constructing and maintaining the infrastructure, which should be proportional to the total length of the edges, and (ii) the efficiency of travel, which should be proportional to the shortest path length between two vertices (Schweitzer et al., 1998). This ratio between total length and shortest path length has been studied theoretically and numerically in computational geometry. In this context, several scholars pointed out that the topology of infrastructure systems such as power grids and road network can be approximated by relative neighbourhood graphs (RNG) and Gabriel Graphs (GG) (Aldous and Shun, 2010; Osaragi and Hiraga, 2014; Zhao et al., 2016a). The RNG is a subgraph of a GG, which means that a GG has a higher efficiency and thus higher costs, i.e. the GG has more edges that are more expensive to build, while the RNG is cheaper in construction because it has fewer edges, but therefore it is not as efficient.

Hence, for both intra-layer connections, a hybrid model was chosen, combining a RNG with a GG model. Because of the deterministic nature of both connection
functions (see Section 6.2.2), a classical random connection model was additionally added to account for stochasticity in the networks, by removing randomly edges \( \langle g_{ER} \rangle = 0 \).

\[
\bar{g}_\alpha = (1 - p_\alpha) \cdot q_\alpha \cdot g^\text{RNG} + (1 - p_\alpha)(1 - q_\alpha) \cdot g^\text{GG} + p_\alpha \cdot g^\text{ER} \quad \alpha \in [p, r] \tag{6.9}
\]

The factors \( \kappa_i,\alpha \) were composed of \( p_\alpha \), the ratio between the deterministic and probabilistic contribution of the connection functions, and \( q_\alpha \), the ratio between RNGs and GGs. All connection functions used are given in Appendix C.1.

For the connection functions, a weighted distance based on a land cover spatial field was used. For simplification, the values of the spatial field were set to \( Z_{E,\alpha} = 1 \) except for impassable obstacles, such as lakes or mountains, were \( Z_{E,\alpha} = \infty \). Inter-layer edges \( E_{\alpha\beta} \) were assigned based on the nearest neighbour of a substation, i.e., these edges represent physical connections from the substations to the road network.

\[
g_{pr} = g^\text{NNG} \quad \text{and} \quad g_{rp} = 0. \tag{6.10}
\]

In order to assign the edges according to this connection model the parameters \( p_\alpha, q_\alpha \), and \( Z_{E,\alpha}(u) \) had to be defined for both layers \( \alpha \in [p, r] \).

**Network generation and calibration**

According to Section 6.2.2, the network was created in two phases; first the vertices were generated, and afterwards, the edges were assigned. To generate the vertices, the spatial point process was simulated. Contrary to Poisson point processes, which can be simulated directly, Markov marked point processes have to rely on more complicated methods, in order to account for vertex repulsion or attraction. Here, a birth-death-move process was implemented in a Markov chain Monte Carlo (MCMC) framework (Metropolis-Hastings algorithm) to simulate the spatially embedded vertices. The main idea of the Metropolis-Hastings simulation algorithm was that at every step a random proposal was made, which was accepted or not by a certain probability. The proposal contained either (i) deleting some vertices (death), (ii) generating new ones (birth) or (iii) moving existing ones (move). Thereby, the acceptance probability for the Markov marked point processes was based on the non-normalised density \( h(y) \). For each possible pair of edges, probabilities were assigned according to Eqs. (6.9) and (6.10). The algorithm used for the example and additional information on the simulation process is given in Appendix C.2.1.

Bayesian inference and MCMC methods were applied to estimate the model parameters used for the network generation. This was done by minimising the normalised mean square errors of selected network measures from the real-world networks. Because the vertices were modelled conditional on marks, i.e. the network structures of both infrastructures were modelled independent of each other, only single-layer measures were used in this example for the parameter estimation. In accordance with the edge generation, the total length \( L \), as a representative of the costs, and the average shortest path length \( \langle \ell \rangle \), as a representative of the efficiency of travel, were used as network measures to estimate the model parameters. The estimated parameters are listed in Table 6.2, more details on the parameter estimation process are given in Appendix C.2.2.
6.3. Application

Table 6.2: Estimated model parameters for the power grid and road network.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter Description</th>
<th>Power</th>
<th>Road</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
<td>std</td>
</tr>
<tr>
<td>b</td>
<td>First order trend parameter</td>
<td>0.0339</td>
<td>0.0027</td>
</tr>
<tr>
<td>c</td>
<td>Interaction parameter</td>
<td>0.825</td>
<td>0.0311</td>
</tr>
<tr>
<td>r</td>
<td>Interaction distance</td>
<td>12772</td>
<td>638</td>
</tr>
<tr>
<td>N</td>
<td>Number of simulated vertices</td>
<td>549</td>
<td>31</td>
</tr>
<tr>
<td>q</td>
<td>Ratio RNG/GG</td>
<td>0.947</td>
<td>0.0065</td>
</tr>
<tr>
<td>p</td>
<td>Ratio det./prob. connection function</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

6.3.3. Road network with hierarchies

Functional hierarchy of road network classifies individual roads into several levels by taking account of the priority for mobility, access or residential functions. This concept has been widely recognised and applied to road planning, design and operation (Goto and Nakamura, 2016), i.e. motorways and main long distance roads (primary roads) follow other design principles and provide other functionals than local connections (secondary roads) or residential roads. While in the previous section the road network was treated as one unified network, in this application a hierarchical behaviour between primary and secondary roads was modelled.

Vertex generation

A MLSERN model conditional on location was implemented because the vertex locations of the two road hierarchies depend on each other, but also on the position of the other vertices in the same layer. For example, to connect regions over long distances, access to motorways are more likely located next to secondary roads, but due to cost and functionality reasons, motorway accesses are spread apart from each other. To model such behaviour, a Strauss process as defined in Section 6.3.2 was used. Contrary to the previous application, the interaction terms $c_{\alpha\beta} \neq 0$ and $r_{\alpha\beta} \neq 0$ for $\alpha \neq \beta$ were not neglected, to account also for the inter-layer interactions of the vertices.

For the modelling of the road intersections three layers were considered: (i) layer $h_{11}$ contained intersections of primary roads with other primary roads, (ii) layer $h_{12}$ represented intersections between primary and secondary roads, and (iii) layer $h_{22}$ comprised intersections only between secondary roads. In order to simulate such a process the parameters $b_\alpha$, $c_{\alpha\beta}$, $r_{\alpha\beta}$, the number of simulated vertices $N_\alpha$ and $Z_{V,\alpha}(u)$ had to be defined for all layers $\alpha, \beta \in [h_{11}, h_{12}, h_{22}]$.

Edge generation

The same assumption about costs of constructing and maintaining the infrastructure as well as the efficiency of travel, as stated in Section 6.3.2, was assumed for this network. Hence, the same hybrid connection function as defined in Equation (6.9) was used for the edge assignment. Since vertices $V_{h_{12}}$ occur in the primary and secondary road network, no inter-layer connection model was needed. The edges $E_{h_{1}}$ for the primary roads were composed of the intra and inter-layer edges of $h_{11}$ and $h_{12}$ ($E_{h_{1}} = E_{h_{11}} \cup E_{h_{11},h_{12}} \cup E_{h_{12}}$), similar to the edges $E_{h_{1}}$ for the secondary roads ($E_{h_{2}} = E_{h_{12},h_{22}} \cup E_{h_{22}}$).

Network generation and calibration

The same birth-death-move processes as before were implemented to simulate the spatially embedded vertices (see Section 6.3.2 and Appendix C.2). After the
vertex generation, the vertices $V_{h_{11}}$ and $V_{h_{12}}$ where temporally assigned to the primary road layer $h_1$ and the vertices $V_{h_{12}}$ and $V_{h_{22}}$ where temporally assigned to the secondary road layer $h_2$, i.e. vertices from $V_{h_{12}}$ were assigned to both layers. For each possible pair of edges in these temporal layers, probabilities were assigned according to Equation (6.9).

For reasons of comparability, this model was also calibrated using the same approach and network measures as used in Section 6.3.2; therefore, the three layers were aggregated to a unified network. The estimated parameters are listed in Table C.1.

6.4. Results and discussion

The results of the previously defined applications are discussed in this section in order to help illustrate the applications of MLSERNs and map out the next steps for research. The presented model allowed the generation of random networks, which exhibit the topological properties of real-world systems relatively accurately. Novel is the use of marked point processes, to generate multi-layer structures. Additionally, the use of hybrid connection models, utilises existing connection models, in order to enable a wider variety of possible ways to assign edges. In the examples, it could be shown, that with only a few parameters (see Table 6.2), a random network could be created which mimics the topological properties of real-world networks relatively accurately. The use of Markov marked point processes allowed, in addition to the modelling of the layers, to take spatial interactions between vertices into account. This made it possible to describe clustering effects between the vertices of the network.

As mentioned in Section 6.2.2, how the marks are assigned changes the placement of the vertices. This allowed the construction of multi-layer networks in different ways and enabled a new view on layers and spatial properties. For example, hierarchies within a spatial system could be examined as shown in Section 6.3.3, where the location of primary road intersections also depended on the positions of the secondary road intersections and vice versa. Additionally, the use of weighted distance based on spatial fields made it possible to capture more realistic spatially embedded edges, which were not only depended on the distance between two vertices but also on some spatial properties in between. For example, obstacles such as lakes or mountains, which are difficult to be physically passed by roads, were taken into consideration (see Figure 6.3.a).

However, even if the model is built on well-defined mathematical expressions, in combination, the mathematical tractability becomes difficult. Thus, the illustrated examples were limited to simulations studies rather than deriving mathematical properties and analytical solutions. Further studies are necessary to do so. With the values of the model parameters determined, for each application 10,000 MLSERNs were generated for further analysis. Figures 6.3.a and 6.3.b illustrate one realisation of a MLSERN for a power grid and road network, and a hierarchical road network respectively.

The estimation of the model parameters was done by minimising the normalised mean square errors of the total length $L$ and the average shortest path length $\langle \ell \rangle$ for each infrastructure network. With this process, the average relative error for the power grid / road network and / hierarchical road network could be reduced to $-26\% / -24\% / 5\%$ and $5\% / 11\% / -2\%$ respectively, as shown in Figure 6.4 and Table C.2. However, estimating the model parameters based on common topological measures should perhaps not always be done for the following reasons. Firstly, these measures were derived from non-spatially embedded single
6.4. Results and discussion

Figure 6.3.: Realizations of a MLSERN simulation. (a) A realization of a multi-layer power grid and road network for Switzerland. Similar to Figure 6.1, the power grid is represented in the blue top layer and the road network in the orange middle layer. In the green layer below, the spatial field, representing the obstacles $Z_E$ for the edge assignment is illustrated. Dark green areas are representing impassable obstacles such as mountains or lakes. The purple bottom layer shows the population density $Z_V$ used for the vertex placement. For reasons of comparability, the same detail is shown as in Figure 6.1. In the background, the population density and the obstacles are shown. Because a unified road network is considered, no distinction between different road classes is made. (b) A realization of a hierarchical road network. Here layers represent different types of road intersections, namely $h_{11}$ intersection between primary and primary roads, $h_{12}$ intersection between primary and secondary roads, and $h_{22}$ intersection between secondary and secondary roads. Contrary to the detail in (a), the hierarchical structure of the road network is given in the detailed section of the map.
networks. In other fields, these measures provide useful insights, but without considering (actual) space, these properties are more of a benchmark instead of providing a deeper understanding of the network configuration. Additionally, these measures can lead to false conclusions since the measures are boundary affected. This is especially the case if boundaries are artificial and not inherent in the underlying system of interest (e.g., borders of countries) (Rheinwalt et al., 2012). Secondly, most real-world networks are built to fulfil a purpose. Therefore, measures of the dynamical behaviour might give better, or at least additional, insights into the structure of the network. For example, if there is a higher need for transportation between two vertices (e.g., cities), more edges (e.g., roads) and/or higher capacities will be observed.

This statement is supported by the observed low rewiring probability of \( p_p = 0.0 \) and \( p_r = 0.029 \) indicate, that edges in human-made infrastructure systems are not attached randomly, but rather defined through their neighbourhood, i.e., most of the connections from a given vertex will go to the closest neighbour. Additionally, the hybrid connection model parameters \( q_p = 0.947 \) and \( q_r = 0.896 \) inclined to the RNG model instead of the GG model which represent a more sparse network. Both observations are consistent with real-world observations, where transportation infrastructure should provide direct connections between vertices to be efficient, and costs for construction and maintenance depend on the length and density of edges (Aldous and Shun, 2010; Osaragi and Hiraga, 2014; Zhao et al., 2016a).

Average relative errors of other, single-layer network measures are also given in Figure 6.4 and Table C.2, including the number of vertices \( |V| \) and edges \( |E| \), diameter \( D \), radius \( R \), average clustering coefficient \( \langle C \rangle \), degree assortativity \( \rho \), transitivity \( \tau \), and weighted average shortest path length \( \langle \ell \rangle^w \). While in some cases the approximation of the real-world properties was good, in other cases, over and underestimations could be observed. For example, the actual length of the networks was underestimated by almost all simulations (\( L_p = -26\% \) and \( L_r = -24\% \)). This is because the actual shape of the roads and transmission lines were approximated by straight lines between the vertices. Consequently, network measures weighted by this length also did not correspond to the observed values. This shows the need for connection models, which better accounted for the real network topology, and spatial input parameters, which capture more properties of the target network.
In the case of the road network $r$, the number of vertices and edges were underestimated. This is caused by the complex structure of the real-world system, which cannot be captured by a single network approach. For example, primary roads are used to connect cities and for more important roads inside theses, while secondary roads are used to exploit rural areas. Hence, both networks have different topological properties. Using a multi-layer approach within the same network (see Section 6.3.3), led to an improvement in describing the real-world network, since more characteristics could be correctly estimated. The hierarchical modelling of the road network reduced the average relative error for the vertices ($|V_h| = -3.38\%$), edges ($|E_h| = -0.38\%$), and total length ($L_h = 5.12\%$), as shown in Figure 6.4.

Beside single-layer network measures, the degree-degree distribution and correlation of inter-layer vertices were used to give a brief insight into the multi-layer structure of the investigated infrastructure networks. Figure 6.5 illustrates the average relative errors of the degree-degree distribution of inter-layer vertices between (a) the power grid and the road network, (b) the power grid and the hierarchical road network, and (c) the primary and secondary roads in the road network. Contrary to the unified road networks, between primary and secondary roads, vertices of degree two could be observed, indicating a change of the road class within one road section, i.e. between two intersections the road classes change.

In addition to the classical intra-layer degree assortativity $\rho$ (degree-degree correlations), where vertices having a certain degree are preferentially connected to other vertices having similar (assortative correlations) or dissimilar degree (disassortative correlations), in a multi-layer network a similar concept of inter-layer degree-degree correlations can be defined (Nicosia and Latora, 2015; Arruda et al., 2016). As Nicosia and Latora, (2015) showed, a compact way to quantify the presence of inter-layer degree correlations is to make use of one of the standard correlation coefficients to measure how the degree sequences of two layers are correlated. For example, the Pearson’s linear correlation coefficient can be defined
as

\[ \rho_{\alpha\beta} = \frac{\langle k((u, \alpha)) \cdot k((v, \beta)) \rangle - \langle k((u, \alpha)) \rangle \cdot \langle k((v, \beta)) \rangle}{\sigma_{(u,\alpha)} \cdot \sigma_{(v,\beta)}} \quad \forall \{(u, \alpha), (u, \beta)\} \in E_{\alpha\beta} | \alpha \neq \beta \quad \text{(6.11)} \]

where \( k((u, \alpha)) \) denotes the degrees of vertex \( u \) respectively at layer \( \alpha \) and layer \( \beta \), and \( \sigma_{(u,\alpha)} \) the standard deviation. To avoid the bias due to the relatively small multiplexity of real-world systems, the averages are taken over all the vertices which are active on both layers. The degree-degree correlations are: for the real-world network \( \rho_{rp} = 0.2193 \), for the MLSERN of the power grid and road network \( \bar{\rho}_{rp} = 0.2087 \) (\( \sigma_{\bar{\rho}_{rp}} = 0.0619 \)), and for the power grid and the hierarchical road network \( \bar{\rho}_{rh} = 0.2352 \) (\( \sigma_{\bar{\rho}_{rh}} = 0.0738 \)). Also, here the hierarchical road network is a better approximation of the real-world system compared to a unified road network. The degree-degree correlation between the real-world primary and secondary roads shows a disassortative correlation \( \rho_{h1,h2} = -0.3465 \), i.e. high-degree vertices in one layer are preferentially connected to low-degree vertices in another layer and vice-versa. This behaviour is caused because often several secondary roads usually lead to a primary road (e.g. motorway access). The MLSERNs of the hierarchical road networks have also a disassortative correlation with \( \bar{\rho}_{h1,h2} = -0.1331 \) (\( \sigma_{\bar{\rho}_{h1,h2}} = 0.0839 \)), although not as distinct as in the real-world network. This is caused by the implemented connection functions, which have not as strict criteria as observed in reality.

Another often neglected issue, but a source of uncertainty for generating spatially embedded networks, are boundary effects as introduced in Section 6.3.2. These effects, do affect not only network measures but also have a strong influence in the simulation process. Since the point process is limited to a sampling space \( A \), vertices and edges outside this region cannot be considered. Consequently, this artificially changes the vertex degrees and the amount of longer edges in the remaining network. This effect is enhanced, in the case of MLSERNs, especially when the different layers contain different levels of scale and importance. For example, the consideration of the Swiss high voltage power grid is only reasonable at a national or international scale, but not at a municipality level, where the whole road infrastructure could be modelled. To overcome this issue, in this work a so-called border method was introduced as a simple edge correction strategy. Here the study region is reduced from the initial sample space \( A \) to a smaller region \( A_{cr} \). The main drawback of this method is that more vertices and edges have to be generated, as used in the final model.

6.5. Conclusions

A network theory approach to the study of multi-layer spatially embedded random networks is a natural consequence of the study of complex interconnected real-world systems, such as technological and transportation infrastructure, or biological systems, but it has not widely explored to date. In order to gain an understanding of such systems, new mathematical approaches and tools are needed. Indeed, this work offers the first attempt to model these systems using multi-layer spatially embedded random networks. Concepts from spatial statistics and graph theory are applied to map complex systems with interdependent subsystems to a simplified and condensed mathematical representation. The model investigated combines Markov marked point processes for vertex generation, which accounts for spatial distribution, layer assignment, and clustering effects of the vertices,
6.5. Conclusions

and a hybrid connection model for the edge generation. In contrast to many other methodologies, this model takes into account spatial restrictions.

To test the capabilities and gain insights into a real-world system, the model was applied to a complex infrastructure system, comprised of the power grid and road network of Switzerland. It could be shown that, with very few parameters, the randomly generated networks estimated reasonably well topological properties of the real-world system. Nevertheless, it could also be shown, that these simplifications led to an over and underestimation of other important network characteristics such as the number of vertices and edges, or the total length of the networks. This issue could be resolved by using the model to represent the hierarchical structure of the road network (i.e. different classes of roads) as another multi-layer spatially embedded random network.

The work presented here is a useful first step towards the improved understanding of real-world spatial networks, as it enables the generation of different realisations of the same multi-layer network. This is useful to take into consideration uncertainties in network properties, in situations where it is too difficult to model the network directly or only incomplete or no information is available. Despite the various advantages, there are still many problems, which might be interesting for research at a theoretical and applied level, including the following topics:

- **Boundary effects.** There is no literature available, which studies boundary effects in the context of simulating spatially embedded networks. In current simulations, these effects are completely neglected. However, it is an essential part of modelling and understanding real-world systems, because recent studies (Rheinwalt et al., 2012; Gil, 2017) show that network measures are strongly affected by such boundary effects.

- **Connection models.** As shown in the work, simple connection models are not able to precisely capture the properties of complex real-world systems. While they have nice mathematical properties, they lack in applicability for more applied analyses.

- **Metrics for MLSE RNs.** Graph theoretic metrics and analyses, are of fundamental importance to study networks. However, there is only a very limited amount of appropriate metrics for (ML)SE RNs, which also account for spatial properties.

- **Network dynamic.** Most real-world systems are built to fulfil a purpose. In the case of infrastructure systems, this is mainly to transport information, goods or people from one point in space to another. The network topology is selected to achieve this purpose best. In order to gain a deeper understanding of these systems, these dynamics have to be taken into account.

- **Network uncertainties.** Until now, uncertainties in the network topology have mainly been avoided in engineering, i.e. only networks with predefined topologies are used. As a result of this, uncertainties due to variations of edges and vertices are not considered in engineering analyses.

- **Network risks.** MLSE RNs opens up a myriad of possibilities in estimating risks for complex systems, taking into consideration the probabilities of occurrence of loading (e.g., earthquakes) of the network and the consequences of the possible network states.
7. Estimation of traffic flow changes using networks in networks approaches

This chapter corresponds to the accepted article:\(^1\)


Abstract: Understanding traffic flow in urban areas has great importance and implications from an economic, social and environmental point of view. For this reason, numerous disciplines are working on this topic. Although complex network theory made their appearance in transportation research through empirical measures, the relationships between dynamic traffic patterns and the underlying transportation network structures have scarcely been investigated so far. In this work, a novel Networks in Networks (NiN) approach is presented to study changes in traffic flows, caused by topological changes in the transportation network. The NiN structure is a special type of multi-layer network in which vertices are networks themselves. This embedded network structure makes it possible to encode multiple pieces of information such as topology, paths, and origin-destination information, within one consistent graph structure. Since each vertex is an independent network in itself, it is possible to implement multiple diffusion processes with different physical meanings. In this way, it is possible to estimate how the travellers’ paths will change and to determine the cascading effect in the network. Using the Sioux Falls benchmark network and a real-world road network in Switzerland, it is shown that NiN models capture both topological and spatial-temporal patterns in a simple representation, resulting in a better traffic flow approximation than single-layer network models.

\(^1\)Please note, this is the author’s version of the manuscript accepted in the Journal Applied Network Science. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at https://appliednetsci.springeropen.com/. When citing this chapter, please, refer to the original article with doi: 10.1007/s41109-019-0139-y.
7. Networks in Networks

7.1. Introduction

Mobility and accessibility are essential factors for lifestyle and prosperity. People travel to satisfy their needs, by carrying out certain activities at specific places such as work, leisure and learning. The spatial distribution of these activities often leads to a coordination problem, which can significantly affect the equilibrium between the demand for, and the supply of transportation. To determine network flow, costs and other aspects of interest, the satisfaction of a given demand for movements of persons and goods with different trip purposes, at different times, using various modes of transport, must be ensured in a transport system with a given operational capacity (de Dios Ortuozar and Willumsen, 2011). As a result of the dynamic nature between mobility demand and supply, in combination with the topology and capacity limitation of the underlying network, transportation networks exhibit atypical dynamic behaviour. Unlike many other networks, network performance deteriorates as soon as the number of vehicles in the network exceeds a critical accumulation (Daganzo and Geroliminis, 2008; Hoogendoorn and Knoop, 2012), i.e. vehicles block each other and the flow decreases, leading to spillbacks and gridlock effects. This phenomenon is amplified by the fact that even small (unexpected) failures or damage to the infrastructure (i.e. changes in topology) can lead to significant disruptions that are disproportionate to the actual physical damage itself (Vespignani, 2010). To prevent such situations, scientists and engineers are working on the implementation of resilient systems capable of withstanding failures, natural hazards and human-made disruptions. Part of the research deals with the quantification of network-related risks, including the modelling of traffic flows after multiple link failures (Erath, 2011; Hackl et al., 2018a,c).

Traffic models are needed to simulate the current and predict future traffic flows. An essential component of such models is the so-called traffic assignment process, which aims to reproduce the pattern of vehicular movements based on certain behavioural rules (Wang et al., 2018). For example, a common behavioural rule is that travellers choose paths with minimum travel time (Wardrop, 1952) or maximise their utilities (Charypar and Nagel, 2005). In order to satisfy the needs of all travellers, an equilibrium between demand and supply has to be found, i.e. no traveller wants to change his path. This complex and computationally intensive mathematical problem is still being actively researched. To make matters worse, in order to quantify network-related risks, resilience, or optimal intervention strategies, the traffic assignment problem must not only be solved once but many times with different network topologies (e.g. see (Erath, 2011; Vugrin et al., 2014; Hackl et al., 2018a,c; Schögl et al., 2019)).

While addressing such problems have led to a substantial body of work in areas such as geography, economics, and transportation research, complex network theory still plays a minor role. Although complex networks made their appearance in transportation research through empirical measures, little research has so far been done to investigate the relationship between dynamic traffic patterns and the underlying structures of the transportation networks (Barrat et al., 2008).

In this work, the application of a novel Networks in Networks (NiN) approach is presented. This approach is used to study traffic flow changes caused by topological changes in the transportation network (e.g. due to multiple link failures) from a complex network perspective. NiNs are based on a multi-layer approach where each vertex itself represents a network. This embedded network structure allows encoding multiple pieces of information such as the topology, paths
used and origin-destination information, within one consistent graph structure (i.e. using only vertices and edges). In combination with a multi-layered diffusion process, an approximation to changes in traffic flow due to topology changes can be made. Specifically, this work advances the state-of-the-art in the field of complex network science in transportation research as follows.

- Using a modified multi-layer hypergraph it is formally feasible to describe vertices that are networks themselves. Thereby the relationships in the incidence graph represent the edges connecting different layers. The edges within the different layers are given by a connection model, which allows different topologies in the different layers.

- Because each vertex is an independent network in itself, it is possible to implement multiple diffusion processes. Therefore, it is possible to assign different physical meanings to the processes. For example, one process can describe how individual travellers switch between different paths, while another process describes the propagation of disturbances through the network.

- The proposed approach allows approximating traffic flow changes due to multiple edge failures. Using the Sioux Falls benchmark network and a real-world road network in Switzerland, it is shown that NiN models capture both topological and spatial-temporal patterns in a simple representation, resulting in a better traffic flow approximation than single-layer network models.

This work is organised as follows. A brief overview of the modelling, functionality and complexity of transport systems is given in the following section. In addition, advances in complex network theory regarding transport systems and dynamic processes are discussed. A general formulation for the NiN representation is presented in the Methodology section. In addition, the application to transportation networks and the modelling of traffic flow changes are discussed on a general level. Two applications are presented, the modelling of a small benchmark network and the modelling of a real network located in Switzerland. In particular, this section is divided into an overview of the data used, the assumptions made and the implementation of the methodology. Subsequently, the results and a critical discussion about the results, advantages and disadvantages of the method are given. Finally, concluding remarks and suggestions for further work in this area are presented. The notation used in this work is listed in the appendix.

7.2. Background

7.2.1. Transportation networks (preliminaries)

The purpose of transport systems is to balance supply and demand for mobility. The demand for transport is derived from people trying to satisfy their needs (work, leisure, health, education) through activities in specific places. Transport supply is the service provided at a certain point in time. This includes the infrastructure (e.g. road network) and a set of mobile units (e.g. persons, vehicles, goods). In combination with a set of rules for operation, the movements of persons and goods can be ensured (de Dios Ortuozar and Willumsen, 2011). In order to predict how the need for mobility will manifest itself in space and time, a formal representation of the transport system is required. In a mathematical sense such
7. Networks in Networks

systems are often represented as graphics or networks, which are denoted by \( G = (V, E) \) and consist of a set of edges \( E \) and a set of vertices \( V \). In this work, the term infrastructure network is used to refer to networks where only topology and connectivity are considered (Rodrigue et al., 2009), i.e. the network comprises vertices and edges that form a connected component. If, in addition to topology, flow characteristics, such as origin-destination demands, capacity constraints, path choice and travel costs, are taken into account to represent the movement of people, vehicles or goods, the network is referred to as transportation network.

In a transportation network, the edges represent the movement between vertices, which in turn represent points in space. An edge \( e \in E \) connects two vertices \( v_i, v_j \in V \) and a vertex connects two or more edges. Edges can be either directed \( e = (v_i, v_j) \in E \), indicating that \( v_i \) and \( v_j \) are directly connected and movement is only possible from \( v_i \) to \( v_j \), or undirected \( (e = \{v_i, v_j\} \in E) \). Important properties of transportation network edges include edge length, edge cost and edge capacity. The edge length corresponds to the length of the road section connecting two vertices. The term edge cost is used to describe the disutility perceived by the network user for travelling on this edge. It is a composite measure of all factors known to be important for decision-making. Travel time and direct costs such as fuel consumption, parking fees and tolls are often taken into account for this purpose. Since transportation networks are physically constrained, it is assumed that each edge has a maximum capacity, i.e. a maximum rate at which people, vehicles or goods can travel on an edge during a given period under prevailing roadway, traffic and operation conditions (Hoogendoorn and Knoop, 2012). The movements in a transportation network correspond to flows with a distinct origin and destination. Origins and destinations can represent particular locations such as residential buildings, offices, shopping centres, or specific zones. In the context of transportation networks, origins and destinations are represented as vertices \( o, d \in V \). It should be noted that not all vertices in the network need to be an origin or a destination. Vehicle movements from origin \( o \) to destination \( d \) occurring along edges are represented as paths. A path \( p \in \mathcal{P} \) is considered as a sequence of edges that ordered so that two vertices are adjacent if and only if they are consecutive. \( \mathcal{P} \) therefore denotes the set of all simple non-empty paths in \( G = (V, E) \). The set of \( o-d \)-paths is denoted by \( \mathcal{P}_{od} \subseteq \mathcal{P} \).

To estimate the movements in a transportation network, it is necessary to find an equilibrium between demand and supply, i.e. in the equilibrium situation, the user chooses the path that he perceives to be the least costly at the time. Economic theory admits that this equilibrium may never really actually occur in practice, as the system of demand and supply levels is constantly adapting to cope with internal and external changes. However, the concept of equilibrium is still valuable to understand movement in transportation networks, assuming that the system is at least near an equilibrium situation. In order to find this equilibrium, various traffic flow models have been developed in recent decades. The most common classification in current traffic flow research is the distinction between macroscopic and microscopic traffic flow modelling approaches (Hoogendoorn and Knoop, 2012). The macroscopic perspective considers the overall or average state of traffic, while the microscopic perspective considers the behaviour of individuals interacting with surrounding vehicles. Macroscopic models were the first to be derived by scientists (Wardrop, 1952; Lighthill and Whitham, 1955) who studied vehicle flow as an analogy to the flow of continuous media such as fluids or gases. These models are based on a limited number of partial differential equations, which reduces the computational complexity. The disadvantage,
7.2. Background

however, is that dynamic features cannot be modelled as accurate as with microscopic models. Microscopic models have been developed to try to emulate human behaviour in traffic situations. To accomplish this, the models contain different driving conditions to describe typical driving reactions. As each vehicle is an autonomous entity, microscopic models become very computationally expensive with increasing system size.

In order to reduce the computational time for both modelling approaches, scholars have developed various techniques. This includes among others, the improvement of the optimisation algorithms to find an equilibrium solution (Charypar and Nagel, 2005; Mitradjieva and Lindberg, 2013; Gentile, 2014); the development of speed-up techniques for sub-problems of the traffic assignment (e.g. finding the shortest paths) (Geisberger et al., 2008; Delling et al., 2009; Buchhold et al., 2018); or the utilisation of GPU cards to parallelise (agent-based microscopic) traffic models (Song et al., 2017; Heywood et al., 2018). Another way to address these problems could be through the use of complex network approaches.

7.2.2. Traffic on complex networks

Complex networks are based on the ideas of mathematical graph theory in order to gain insights into the behaviour of complex systems by abstracting information into ordinary graphs (networks). In these representations, the network comprises vertices connected by edges, with vertices representing individual elements and edges indicating interactions or relationships between them. Although this approach is simple in many ways, it allows the characterisation of the complex system so that traditional graphic-theoretical metrics can be used and analyses performed. For example, such abstractions have been used to study growth mechanisms (Barabási and Albert, 1999; Clauset et al., 2009), processes of collective dynamics (Watts and Strogatz, 1998), and to illustrate that certain vertices play a central role in the complex system (Freeman, 1977; Wasserman and Faust, 1994).

The strength of the complex network paradigm lies in its ability to capture some of the essential structural features of interacting systems while reducing the details of both the elements and their interactions. Consequently, the early complex network literature focused almost exclusively on the structural properties of networks (Smith et al., 2011). This topology-driven analysis can reveal relevant properties of the structure of a complex system (Albert et al., 1999; Watts and Strogatz, 1998) by highlighting the role of vertices and edges (Bavelas, 1950; Freeman, 1977) or global network properties (Taaffe et al., 1973; Cliff et al., 1979). The robust mathematical framework allows the derivation of analytical solutions even for large complex systems. For example, hierarchical network representations are used to study large complex transportation systems (Gómez et al., 2013; Lim et al., 2015). Thereby, hierarchical models are obtained by successive clustering of networks, i.e. decomposition of the system into different levels of details (Ferrario et al., 2016).

While structural properties are still important in constraining the behaviour of a system (Marr and Hütt, 2005), the focus has expanded to an understanding of the relationship between structure and dynamics that takes place in networks and the impact of this relationship on network design (Toroczkai, 2005). Most technological, biological, economic, social or infrastructural networks support a number of dynamic (transport) processes, such as the movement of information packages (Wang et al., 2006), finance and wealth (Coelho et al., 2005), rumours (Moreno et al., 2004), diseases (Newman, 2002), people or goods. Gradually,
these theories have been introduced to the field of transportation. More and more scholars have conducted research on the characteristics of various transportation networks, among others those of (urban) road networks (De Montis et al., 2007; Erath et al., 2009b; Barthélémy, 2011; Lin and Ban, 2013), railway networks (Latora and Marchiori, 2002; Sen et al., 2003), and transit networks (Guo and Lu, 2016; Solé-Ribalta et al., 2016). In addition, current studies use complex networks to analyse traffic time series (Tang et al., 2013; Yan et al., 2017; Bao et al., 2017).

In the field of complex network sciences, a widely used approach to study the relationship between dynamic processes and the underlying network structures is through the use of random walks. In such a model, random walkers move in the network and visit various edges and vertices over time. An extensive overview of the use of random walks and diffusion on complex networks is given by Masuda et al., (2017). Researchers, using this technique, have been able to gain insights into topological features such as vertex centralities (Brin and Page, 1998) or community structures (Newman, 2006; Jeub et al., 2015).

In many real transportation systems, however, the assumption of such simple random walker models may not always be justified, since it is assumed that the walker moves randomly in the network without considering its origin or destination. For example, in a diffusion process on a road network, the next position of a vehicle depends only on its current position (occupied vertex) and the outgoing roads (edges), but not on one of the previously visited locations. In reality, however, travel in a network has a specific purpose: a person starts at home and navigates through the network to reach a particular destination (e.g. work), and then returns home with a high probability (Salnikov et al., 2016). Consequently, the naive application of (static) network paradigm in modelling dynamic complex systems might lead to wrong conclusions (Rosvall et al., 2014; Scholtes et al., 2014; Scholtes, 2017). One way to address this issue is through the extension to a multi-layer networks representation.

Multi-layer networks represent complex systems that are formed from several networks (layers), each of which represents interactions of different nature and connections. Due to the distinction between the different types of edges and vertices, multi-layer networks encode significantly more information than conventional single-layer networks (Iacovacci and Bianconi, 2016). In network science, the two most prominent classes of multi-layer networks are multiplex networks and networks of networks. Networks of networks are formed by layers composed of different vertices. Edges connecting different networks do not necessarily indicate dependency relationships. Examples can be found in complex infrastructure networks such as road networks, railway networks and flight networks, where each layer represents its own infrastructure. Multiplex networks, on the other hand, are formed by the same set of vertices connected by edges indicating different types of interactions. In the context of transport systems, this approach has been used, for example, for the analysis of flight networks (Cardillo et al., 2013). An overview of other types of multi-layer networks is given by Boccaletti et al., (2014), Kivelä et al., (2014), and Bianconi, (2018). In all these definitions, it is assumed that a multi-layer network can be represented as a graph $G_M$, which is an ordered tuple $G_M = (V_M, E_M)$ considering a non-empty labelled vertex set $V_M$ and a multiset $E_M \subseteq V_M \times V_M$ of edges. A vertex $v^\alpha \in V_M$ is a tuple representing vertex $v$ on the layer $\alpha \in \mathcal{L}$, where $\mathcal{L}$ is the set of layers in the network.
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7.3.1. Networks in networks

A Networks in Networks (NiN) structure is a special type of multi-layer network in which vertices themselves are networks, i.e. \( G_{\text{NiN}} = (G_{\text{NiN}}, E_{\text{NiN}}) \) is a network with a set of graphs \( G_{\text{NiN}} \) acting as vertices and a multiset \( E_{\text{NiN}} \subseteq G_{\text{NiN}} \times G_{\text{NiN}} \) of edges. A vertex \( v_{i}^{\beta} \in G_{\beta} \) is a tuple representing a graph \( v_{i}^{\beta} := (G_{\alpha}, E_{\alpha}) \), where \( \alpha < \beta; \alpha, \beta \in \mathcal{L} \) are the layers describing the order of hierarchy in the network. It should be noted that the vertices for each layer are defined recursively since they are constructed from a previous layer.

The set of vertices \( G_{\text{NiN}} \) can be interpreted as a modified multi-layer hypergraph \( H_{M} = (X_{M}, Y_{M}) \), where \( X_{M} \) is a set of basic elements and \( Y_{M} \) is a set of non-empty subsets of \( X_{M} \), here denoted as hypervertices.\(^2\) Hypervertices are arbitrary sets of vertices and can contain therefore any number of vertices. In a multi-layer structure, higher-order hypervertices are arbitrary sets of lower-order hypervertices, whereby the order indicates the position in a hierarchical layered structure.

\[
\bigcup_{j=1}^{n_{\beta}} y_{j}^{\beta} = \bigcup_{i=1}^{n_{\alpha}} y_{i}^{\alpha} \quad \text{with} \quad \alpha < \beta; \alpha, \beta \in \mathcal{L} \quad (7.1)
\]

where \( y_{i}^{\alpha} := (y_{i}^{\alpha})_{i \in I_{\alpha}} \) with index set \( I_{\alpha} \) and \( n_{\alpha} \) is the number of hypervertices in layer \( \alpha \). The same is true for layer \( \beta \). In the layer with the lowest order, the hypervertices are sets of the basic elements:

\[
\bigcup_{i=1}^{n_{\alpha}} y_{i}^{\alpha} = X \quad \text{with} \quad \alpha = \min\{\alpha, \beta, \ldots\} \in \mathcal{L} \quad (7.2)
\]

The relationships between the different types of hypervertices are described by their incidence structure. Unlike the classical incidence graph associated with the hypergraph, the incidence graph for a NiN consists of inter-layer edges, which define the relationship between hypervertices of different orders. The intra-layer edges, which describe edges within a layer, are defined by adjacency matrices for each layer. The incidence matrix between two consecutive layers \( \alpha < \beta, \alpha, \beta \in \mathcal{L} \) is given by \( I_{\alpha \beta} = i_{ij}^{\alpha \beta} \) where:

\[
i_{ij}^{\alpha \beta} = \begin{cases} 1 & \text{if } y_{i}^{\alpha} \in y_{j}^{\beta} \\ 0 & \text{otherwise} \end{cases} \quad \text{with} \quad i = 1, \ldots, n_{\alpha}; \quad j = 1, \ldots, n_{\beta}. \quad (7.3)
\]

The adjacency matrix of each layer \( \alpha \in \mathcal{L} \) is given by \( A_{\alpha} = a_{ij}^{\alpha} \) where:

\[
a_{ij}^{\alpha} = \begin{cases} 1 & \text{if } (v_{i}^{\alpha}, v_{j}^{\alpha}) \in E_{\alpha} \\ 0 & \text{otherwise} \end{cases} \quad \text{with} \quad v_{i}^{\alpha}, v_{j}^{\alpha} \in G_{\alpha}. \quad (7.4)
\]

Considering that hypervertices itself are networks, a transformation into a classical supra-adjacency matrix (Boccaletti et al., 2014; Kivelä et al., 2014; Cozzo et al., 2016), (i.e. the representation of a multi-layer network as a single-layer network), may not always be possible or intended because information is lost in the transformation process (Kivelä et al., 2014).

---

\(^2\)In a classical single-layer hypergraph \( X_{M} \) usually refers to hyperedges, which are finite sets of basic elements (vertices) representing their relationships (connections). In the scope of a multi-layer hypergraph, the vertices of each layer are defined as the hyperedges of the previous ones. To avoid confusion in terminology, such higher-order elements are referred to as hypervertices or vertices in this work.
7.3.2. Connection models

In order to consider the influence of the different lower-order hypervertices on the connection between two hypervertices, connection models are used for the intra-layer edge assignment (Meester and Roy, 1996; Hackl and Adey, 2017a, 2019b). A connection model $\mathcal{M}(G_\beta, g_\beta)$ has two characteristics. First, the set of vertices $G_\beta$ which should be connected and second the so-called connection function $g_\beta$, which gives the probability of a direct link between two vertices $v^\beta_i$ and $v^\beta_j$ on layer $\beta$ (Meester and Roy, 1996). Since the connection function can be any arbitrary functional relationship between two vertices, it allows to define the entries of the adjacency matrices for each layer depending on the network properties of the embedded vertices.

Formally, the multiset $\mathcal{E}_\beta$ can be defined as a connection model $\mathcal{E}_\beta := \mathcal{M}(G_\beta, g_\beta) = \{(v^\beta_i, v^\beta_j) \mid g(v^\beta_i, v^\beta_j); v^\beta_i, v^\beta_j \in G_\beta\}$, with the connection function $g : v^\beta_i \times v^\beta_j \rightarrow \{0, 1\}$, which is a mapping from the underlying graphs $v^\beta_i = G_\alpha$, and $v^\beta_j = G_\beta$, to 0 (not connected) or 1 (connected) with $\alpha < \beta$: $\alpha, \beta \in \mathcal{L}$. In addition, $g$ can be a mapping to $\mathbb{R}^+$ that indicates the weight of the edge $(v^\alpha_i, v^\alpha_j)$. An overview of several connection functions is given by Dettmann and Georgiou, (2016), Parsonage and Roughan, (2017), and Hackl and Adey, (2019b).

7.3.3. NiN structure for transportation networks

The following section presents considerations on how a NiN model can be used for transportation networks. An important point to consider is the concept of origin and destination. Therefore, the network representation should contain information of an origin-destination matrix as well as the path statistics, i.e. how many trips from $v_i$ to $v_j$ are taken and which paths are used therefore? To achieve this, a NiN network with four layers is presented. The first layer ($\alpha$) consists of the vertices of the infrastructure network, where the basic elements correspond to the intersections of the transport network. In the second layer ($\beta$) possible paths of the network are mapped. These are grouped in the third layer ($\gamma$) according to their origins and destinations. In the last layer ($\delta$) these origin-destination paths are combined to describe all trips in the network. A conceptual illustration of a simple transportation network is given in Figure 7.1. The individual layers are described in more detail below, with references to Figure 7.1.

Layer $\alpha$. The underlying infrastructure network comprises vertices and edges. Vertices correspond to objects or locations in the physical world such as road junctions or facilities. Edges connecting these vertices represent road sections. The basic elements of the NiN model are the vertices of the underlying infrastructure network. Based on these vertices, all other layers of the NiN are constructed. For example, the set of basic elements for the infrastructure network in Figure 7.1 is given by $\mathcal{X} = \{v^\alpha_1, v^\alpha_2, v^\alpha_3, v^\alpha_4, v^\alpha_5, v^\alpha_6, v^\alpha_7\}$ [Equation (7.2)].

Layer $\beta$. Considered paths $\mathcal{P}$ in the network are described in the second layer. Each path is represented as a hypervertex $v^\beta_i \in G_\beta$, composed of a set of connected basic elements. The hypervertex $v^\beta_i$ is a graph that describes a sequence of vertices connected by edges in one direction, so that movement from the first vertex to the last vertex is possible. Obviously, edges can only be assigned if they are also present in the underlying infrastructure network. In Figure 7.1 five paths are considered. Path $v^\beta_1$, for example, connects vertex $v^\alpha_1$ with vertex $v^\alpha_6$ via $v^\alpha_2 \rightarrow v^\alpha_3 \rightarrow v^\alpha_4 \rightarrow v^\alpha_5$ and is defined as $v^\beta_1 := \{(v^\alpha_1, v^\alpha_2), (v^\alpha_2, v^\alpha_3), (v^\alpha_3, v^\alpha_4), (v^\alpha_4, v^\alpha_5), (v^\alpha_5, v^\alpha_6)\}$. The first set represents a subset of the lower-order hypervertices [Equation (7.1)]. The relationship be-
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a) Infrastructure network  

b) Considered paths  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha$  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha$  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha \rightarrow v_5^\alpha$  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha \rightarrow v_5^\alpha$  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha \rightarrow v_5^\alpha$  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha \rightarrow v_5^\alpha$  
$v_1^\alpha \rightarrow v_2^\alpha \rightarrow v_3^\alpha \rightarrow v_4^\alpha \rightarrow v_5^\alpha$

c) Network in Network structure

Layer $\gamma$

Layer $\beta$

Layer $\alpha$

Layer $\delta$

Figure 7.1: NiN structure for transportation network example. (a) Visualisation of the underlying infrastructure network. (b) Considered paths in the network. (c) Incidence graph of the NiN model with the associated adjacency matrices of the hypervertices.

tween different orders of hypervertices (inter-layer edges) is described by an incidence matrix [Equation (7.3)] or an incidence graph, as depicted in Figure 7.1. The second set represents the connections among the lower-order hypervertices (intra-layer edges), represented in Figure 7.1 as adjacency matrices [Equation (7.4)]. Indices of the adjacency matrices are the lower-order hypervertices.

Layer $\gamma$. Paths with the same origin and destination are grouped in layer $\gamma$. In other words, each hypervertex $v_1^\gamma \in \mathcal{G}_\gamma$ comprises a set of paths $v_2^\beta \in \mathcal{G}_\beta$ with the same origin destination vertices $v_0^\alpha \rightarrow \cdots \rightarrow v_d^\alpha$, where $v_0^\alpha, v_d^\alpha \in \mathcal{G}_\alpha$. The connection between the vertices can be interpreted as the choice a traveller has, in order to switch from the initially assigned path $v_1^\beta$ to any other path available between the origin and the destination, including self-loops where he remains on his initial path. The edge weighting corresponds to the probability of changing the path, i.e. the adjacency matrix is a direct representation of the transition matrix. For example, in the situation of an equilibrium between supply and demand, no one will alter their path choice and therefore only self-loops with weight 1 are observed, as illustrated in Figure 7.1. In order to consider the influence of lower-order hypervertices and their properties (e.g. interrupted or congested paths), connection models can be used for the intra-layer edge and weight assignment. Thus hypervertex $v_1^\gamma$ can be expressed as $v_1^\gamma = \{(v_1^\beta, v_2^\beta, v_3^\beta), \{v_i^\beta, v_j^\beta \mid g(v_i^\beta, v_j^\beta)\}; v_i^\beta, v_j^\beta \in \mathcal{G}_\beta\}$, where an edge is assigned if the criteria of the connection function $g$ are fulfilled (e.g. assign an edge with
weight 1 if \( v^\beta_i = v^\beta_j \), otherwise not).

Layer \( \delta \). The top layer combines all origin-destination paths, similar to an origin-destination matrix. In contrast to the classical matrix representation, where each cell represents the number of trips from the origin (row) to the destination (column), the hypervertex \( v^\delta_i \in G^\delta \) comprises all paths \( v^\gamma_j \in G^\gamma \), grouped by their origin-destination vertices. Furthermore, this lower-order hypervertices contain detailed path information \( v^\beta_k \in G^\beta \) derived from the underlying basic elements \( v^\alpha_l \in G^\alpha \). Edges in this layer can have different meanings. In this work, it is assumed that the edges represent the common road sections between different origin-destination paths. For example in Figure 7.1, travellers from \( v^\alpha_1 \) to \( v^\alpha_5 \) share three common road segments with the travellers from \( v^\alpha_3 \) to \( v^\alpha_7 \). The edges and weights can also be assigned using a connection model (e.g. assign an edge with weight \( n(E^\beta_{v^\gamma_i} \cap E^\beta_{v^\gamma_j}) \) if paths in \( v^\gamma_i \) and \( v^\gamma_j \) share common edges, where \( E^\beta_{v^\gamma_i} \) is the set of edges in layer \( \beta \) associated with hypervertex \( v^\gamma_i \) and \( n(\cdot) \) is a cardinality operator. In Figure 7.1, for example, \( W(v^\gamma_1, v^\gamma_1) = n(\{(v^\alpha_1, v^\alpha_2), (v^\alpha_2, v^\alpha_4), (v^\alpha_4, v^\alpha_5), (v^\alpha_5, v^\alpha_6), (v^\alpha_6, v^\alpha_5), (v^\alpha_4, v^\alpha_7), (v^\alpha_7, v^\alpha_6)\}) = 7. \)

Figure 7.2.: NiN dynamics for traffic flow change example. (a) Visualisation of the underlying infrastructure network. (b) Incidence graph of the NiN model with the associated adjacency matrices of the hypervertices. (1) Change of the path topology. (2) Update of the edges and weights. (3) Random walker to determine path changes. (4) Random walker to determine cascading effects. (5) Modification and assessment of not directly influenced paths. (i), (ii), and (iii) show the network representations of the adjacency matrices, i.e. the Markov chains. (i) Travellers which original use path \( v^\gamma_i \) have to diffuse to \( v^\gamma_2 \) or \( v^\gamma_3 \), also a change between \( v^\gamma_2 \) and \( v^\gamma_3 \) is possible. The strength of the cascading effect is determined in (ii), which influences the traffic flow redistribution in (iii).
7.3. NiN dynamics for traffic flow changes

Once the NiN structure has been created, dynamic processes on the network can be studied. Among other things, it is important to understand how the system behaves when several road sections fail, i.e. how do the affected trips change and does they have an influence on a global level? To study such behaviour, diffusion processes on two layers of the NiN model are applied. On layer $\gamma$ this is used to estimate how the path choices of the travellers are changing, while on layer $\delta$ the strength of the cascading effect is determined. This process is schematically illustrated in Figure 7.2 and described in more detail below.

In the first phase, a set of interrupted edges $\bar{\mathcal{E}}_\alpha \subseteq \mathcal{E}_\alpha$ is removed from the considered paths $v^\beta_i \in \mathcal{G}_\beta$. This interrupts the sequence of vertices connected by edges, so that no movement from the first to the last vertex is possible, i.e. with the number of edges available, the path cannot be completed ($n(\bar{\mathcal{E}}_\alpha) \leq n(\mathcal{G}_\alpha) - 2$). In Figure 7.2, for example, it is assumed that the edge $(v^\alpha_4, v^\alpha_5)$ is interrupted. Hence, path $v^\beta_1$, containing this edge, can no longer be used to reach the desired destination $v^\alpha_5$.

As a first consequence, people, vehicles or goods that initially used these paths must be redistributed to other possible paths. The likelihood of changing to another path is determined by the weights of the adjacency matrix of $v^\gamma_i \in \mathcal{G}_\gamma$. Using an extension of the previously introduced connection model, the edge and weight assignment can be done based on the properties of the lower-order hyper-vertices (e.g. if the underlying path is interrupted, assign edges to other paths with equal weights). Since in Figure 7.2 path $v^\beta_1$ is interrupted, travellers have either to use path $v^\beta_2$ or $v^\beta_3$. Here, it is assumed that these travellers distribute equally on the remaining paths. Furthermore, travellers from neighbouring paths might also reconsider their path choices resulting in 20% additional path changes.

The behaviour of changing paths based on an underlying transition matrix can be described by a diffusion process and modelled with the help of random walkers. In this context, a random walker represents a traveller who randomly decides which path to take, given some preferences (e.g. edge weights). In the case of an interrupted path, the random walker remains with zero probability and visits instead another path. This stochastic process can be described as a sequence of random variables $X_0, X_1, \ldots, X_m$, where $X_m$ denotes the position of the random walker in the network at step $m$. Assuming that each movement simulated in the network is only dependent on the current position, the problem is reduced to a first-order Markov model:

$$ P(X_m | X_{m-1}, X_{m-2}, \ldots, X_0) = P(X_m | X_{m-1}) \quad (7.5) $$

Thus, all this information can be captured by the transition matrix $T$:

$$ P(X_m = v_j | X_{m-1} = v_i) = T_{ij} = t_{ij} = \frac{W(v_i, v_j)}{\sum_k W(v_i, v_k)} \quad (7.6) $$

measuring the probability that a random walker at vertex $v_i$ will go to vertex $v_j$ considering the connection strengths.\(^3\) These connection strengths indicate the preferences of the traveller to choose one path over another. If $m \to \infty$ and the Markov chain is ergodic, a unique stationary distribution $\pi^\beta$ can be observed, i.e. $\pi^\beta T = \pi^\beta$. This vector can be interpreted as a new equilibrium distribution of the selected paths given a set of interrupted paths. Multiplied

\(^3\)A division by $\sum_k W(v_i, v_k)$ is needed to normalise the values such that $\sum_j W(v_i, v_j) = 1$. 

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with the initial observed traffic flows $f^\beta_i$ on paths $v^\beta_i$, the traffic flows $\hat{f}^\beta_i$ in the interrupted network can be estimated. A simple trajectory of a random walker on $v^\gamma_1$ is illustrated in Figure 7.2. The walker starts on the interrupted path $v^\beta_1$ and moves to the other paths, where he stays with probability of 80%.

This redistribution of path flows can cause cascading effects since the road sections of the new paths are also used by others. In order to consider these interdependencies, a second diffusion process is modelled in the uppermost layer $\delta$. In this layer, edges represent the connections between different origin-destination paths, weighted with the number of common road sections. In this work, it is assumed that this weight indicates how strongly different origin-destination paths influence each other, i.e. paths which share no common road sections will not (directly) influence each other, while paths which share almost all their road sections will have a strong influence on each other. Similar to the previous layer, a diffusion process is used to determine the propagation of the cascading effect. The stationary distribution $\pi^\gamma$ indicates how strongly other vertices are affected, even if they do not contain paths with interrupted edges.

In such a case, the diffusion process on layer $\beta$ is repeated, however, the new distribution of path flows is a combination of initial distribution and the redistribution, weighted by the mapped strength of the cascading effects $h(\pi^\gamma) \rightarrow [0,1]$: \[\hat{f}^\beta_i = f^\beta_i \cdot (h(\pi^\gamma_j|v^\beta_i) \cdot (\pi^\beta_i - 1) + 1).\] (7.7)

where $\hat{f}^\beta_i$ is the estimated path flow, $f^\beta_i$ is the initial path flow, and $\pi^\beta_i$ is the flow distribution of path $v^\beta_i$. The influence of cascading effects for $v^\beta_i$ is determined by the stationary distribution $\pi^\gamma_j|v^\beta_i$. For example, in Figure 7.2 at hypervertex $v^\gamma_2$ a random walker is initialised, even if no of the underlying paths are interrupted.

7.4. Application

The application presented in this section is used to demonstrate the usefulness of the methodology in addressing a specific problem. The application shows the design and implementation of a NiN model for estimating changes in traffic flow due to edge interruptions on two road networks. The first road network is the classical benchmark network of Sioux Falls, which is used within transport research to test, demonstrate and compare methods and algorithms. The second investigated road network is a real-world example from the region around the city of Chur, the capital of Grisons, the largest and easternmost canton of Switzerland.

In both cases, the initial traffic flow assignments were performed using a macroscopic traffic flow model. The networks were modelled as NiN and calibrated with the initial assignment results. Edges were randomly removed, and the traffic flows were estimated using diffusion processes as described above. To show the influence of the multi-layer approach, the same procedure was conducted with a single-layer network (SLN) model. The results of both modelling approaches were compared with the ground truth observed from the macroscopic traffic flow model with the same network topologies.

The remainder of this section is structured as follows. First, an overview of the networks and data used is given, followed by a description of the assumptions made during the modelling process. Finally, the implementation and simulation process is described in detail.
7.4. Application

Figure 7.3.: The infrastructure networks of Sioux Falls and Chur. (a) The Sioux Falls network is located in South Dakota, USA and consists of 24 vertices and 77 edges. Each vertex is associated with an origin and destination location. The edges are directed, i.e. there is an edge (1, 2) and (2, 1). (b) The Chur network is located in the canton of Grisons, Switzerland and consists of 1,262 vertices and 3,199 edges. The area is divided into 37 zones which represent the origin and destination locations.

7.4.1. Data

Sioux Falls

Although the Sioux Falls scenario is not considered realistic, it is used in many publications. Morlok et al., (1973) first introduced the network as a traffic equilibrium network. Later, the network was adapted as a benchmark and test scenario in many publications, including (LeBlanc et al., 1975; Suwansirikul et al., 1987; Meng et al., 2001; Chakirov and Fourie, 2014). A more detailed list of use cases and the original data set is given by (Stabler et al., 2018).

The network was directed and consisted of 24 vertices and 76 edges, where each vertex also represented an origin-destination vertex, i.e. the area was divided into 24 zones. All network data including the vertex labels given in Figure 7.3a were taken from (LeBlanc et al., 1975). Except for the vertex coordinates, which were taken from (Chakirov and Fourie, 2014), since the original article does not contain this information. The origin-destination flows in the original article are given in thousands of vehicles per day with integer values up to 44, i.e. the origin-destination flows were the values from the table multiplied by 100. They thus amount to 0.1 of the original daily flows and could, therefore, be considered as approximate hourly flows. This conversion was initially carried out to compare the objective values with the articles published in the 1980s and 1990s. The units of free flow travel time are 0.01 hours, but they are often interpreted as being minutes (Stabler et al., 2018). As an edge cost function the “traditional” BPR function proposed by the Bureau of Public Roads, (1964) was used:

\[
c_e := t_e(x_e) = t_e^0 \left( 1 + a_e \left( \frac{x_e}{y_e} \right)^{b_e} \right)
\]

where \( t_e \) is the travel time at edge \( e \) given the edge traffic flow \( x_e \), \( t_e^0 \) is the free flow travel time, \( y_e \) the edge capacity, and \( a_e \) and \( b_e \) are parameters for calibration, here chosen as \( a_e = 0.15 \) and \( b_e = 4 \), \( \forall e \in \mathcal{E} \).

Chur

As a real-world example, the road network in the Rhine Valley around the city of Chur, Switzerland was investigated. An overview of the network is given in Figure 7.3b. Only national, main and secondary roads were considered for the
7. Networks in Networks

analysis. This corresponded to about 51km of national roads, 165km of main roads and 395km of secondary roads. The network was represented as a directed graph with 1,262 vertices and 3,199 edges.

The information of the Chur road network was obtained from the VECTOR25 dataset of swisstopo (JD100042). This data set shows a complete national coverage and describes 8.5 million objects with their position, shape and their neighbourhood relations (topology) as well as the type of object and other special features. The accuracy of the geodata is in the range of 3 to 8 m and is available as ESRI shapefile for the Swiss coordinate system CH1903/LV03 LN02 (ESPG-Code: 21781). The road sections were described by their direction, length, free flow speed, capacity and the parameters of the edge cost function. For consistency reasons, the same cost function as in the Sioux Falls scenario \[\text{Equation (7.8)}\] was used.

The trips in the region were performed between 37 zones, based on judicial districts, as shown in Figure 7.2b. All trips made from an origin to a destination zone in a given period were stored in an origin-destination matrix. Since there was not enough information available on the distribution of trips for the area of interest, a gravitational distribution model (de Dios Ortuozar and Willumsen, 2011) was used to estimate the trips based on census data for each zone (e.g. demographics, households, work locations). The obtained gravitational model was calibrated by the Swiss national traffic model (FOSD, 2015), which provided data for the motorway and main roads. In addition, data from traffic counting stations in the study area were used to calibrate the initial traffic assignment, by adapting the estimated origin-destination matrix.

7.4.2. Assumptions

In order to reduce complexity and apply the methodology to real problems, several assumptions were made during the modelling process. The assumptions and their justifications are listed below.

1. The objective was to model non-recurring situations such as road works or interruptions due to extreme weather events (e.g. floods, mud blockades). Although these events occur unexpectedly, it was assumed that travellers already had some information about the situation. In other words, it was not the traffic flow immediately after the event that was taken into account, but the one after a few days, so that travellers could adapt to the new situation. This also made it possible to compare the results with the results of the macroscopic traffic flow simulation, which assume an equilibrium between supply and demand in the interrupted scenario.

2. It was assumed that interrupted edges were removed entirely from the network, although it would be possible to consider capacity constraints (see Discussion section). As indicated under assumption number one, the interest was in extreme events where the effects could also be observed a few days later, so a complete road closure was chosen to express this severity.

3. It was assumed that there was no mode change, i.e. people who used a car in the initial configuration also used the car in the modified configuration, and did not change to another means of transport such as walking or cycling.

4. Due to the high complexity, not all possible paths were modelled. Instead, only the already selected paths and their \(k\) most similar paths were considered. A similarity was determined using the BPR edge cost function
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This means that the paths between origin and destination were evaluated by the travel costs observed in the baseline scenario. This approach ensured that very unlikely paths did not have to be considered.

5. Also to reduce complexity, it has been assumed that travellers can make a maximum of three decisions: (i) they can stay on the path initially chosen, (ii) they can switch to the next cheaper path or (iii) to the next more expensive one. This assumption is based on the assumption that travellers know about their paths and alternatives. In the baseline scenario, which is in equilibrium, there is no incentive to change paths, but in the interrupted scenario, travellers can speculate about using an initially better path with the risk of others using it, or an initially worse path that might be better under the new configuration. If a traveller is on an interrupted path, he has only the options (ii) or (iii). If the traveller is also on the best path in the baseline scenario, he just has the option (iii). The costs of the paths were determined using Equation (7.8).

6. In order to model the cascading effects in the network, only paths where the original traffic load changed, were used, i.e. edges between pairs of origin-destination groups where there was no change were weighted by 0 and therefore were not considered in the diffusion process.

7.4.3. Implementation

Initial traffic flow assignment

The starting point was the simulation result of a classical macroscopic traffic flow model on an intact transportation network. Path statistics were generated from this data, i.e. the path flows were simulated and evaluated. Thereby, a path flow represents the quantitative amount of movements from an origin to a destination on a particular path. In the simplest case of a static user equilibrium, the traffic assignment problem is presented as follows:

$$\min Z = \sum_e \int_0^{x_e} c_e(\omega) d\omega$$

subject to the demand and non-negativity constraints given by

$$\sum_p f_{od}^p = q_{od} \quad \forall p \in P_{od}$$

$$f_{od}^p \geq 0$$

where $x_e$ is the flow on edge $e$ comprised of the sum of flows on the paths sharing edge $e$, $c_e(\omega)$ is the cost on edge $e$ for a flow of $\omega$, $f_{od}^p$ is the flow on path $p$ connecting origin $o$ and destination $d$ and $q_{od}$ is the total traffic demand between $o$ and $d$.

Substantial research has been carried out on this problem and its extension to more practical approaches, including the representation of dynamic traffic phenomena such as queues, spillbacks, wave propagation, capacity drops and so on. In this work, this simplified approach was used due to the universality of the problem and its simple mathematical handling. In addition, the simple and commonly used Frank-Wolfe algorithm was used to solve the optimisation problem (Jayakrishnan et al., 1994; Chen et al., 2002). However, this does not affect the applicability of the proposed method, as it can be applied independently of the problem.

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7. Networks in Networks

NiN setup and failure propagation

According to the proposed methodology, the basic elements in layer $\alpha$ were formed by the vertices of the infrastructure networks. The network paths in layer $\beta$ were obtained from the initial macroscopic traffic flow simulations. Paths with the same origin and destination are grouped in layer $\gamma$.

The contained pathways were ranked by travel costs [Equation (7.8)] and connected accordingly, i.e. the path with the lowest travel costs was located at the beginning of the Markov chain, followed by the path with the second lowest costs, and so on. As described in assumption number five, only the neighbouring paths were connected. To take into account alternative routes that were not used in the initial configuration, additional paths were added to each origin-destination group. These paths were generated using the Yen’s algorithm (Yen, 1970) to calculate the $k$-shortest loop-less paths for a graph with non-negative edge costs. These additional paths have been disabled, meaning that such a path could not be visited in the initial configuration, i.e. the edge weights and the initial traffic flow were set to 0. In the top layer, $\delta$ all origin-destination groups were combined and connected based on their common road sections.

To initialise the failure propagation a set of edges were randomly removed from the infrastructure network. Care was taken to maintain the connectivity of the network such that there were no disconnected origin or destination vertices. Paths that contained these interrupted edges were disabled by setting the probability to zero that someone would visit or maintain on this path. With a certain probability, additional paths were enabled. Based on the new topology of the Markov chain and the (estimated) transition probabilities, the new stationary distributions were calculated. As a result of these changes in traffic flow, positive edge weights were assigned to edges in layer $\delta$. The second diffusion process was performed to initiate other redistribution processes at layer $\gamma$, as described in the methodology. This process continued until the traffic flow no longer changed.

Bayesian inference and MCMC methods were applied to estimate the transition probabilities for the diffusion process. This was done by minimising the normalised mean square errors of edge flows based on the results from the traffic flow simulations performed on the interrupted network topology where on edge was interrupted. For the parameter estimation (training process) 100 different network configurations were used.

Single-layer network approach

To evaluate the obtained results, a comparison between the proposed NiN model and a traditional complex network diffusion process on a single-layer network (SLN) was made. Since the traffic flows are edge properties, the initial topology of the road network has been converted into a higher-order network of order two (Rosvall et al., 2014; Scholtes, 2017; Lambiotte et al., 2018), i.e. intersections were turned into edges and road sections into vertices (Porta et al., 2006). To consider the failures of the road sections, the incoming edges of the affected higher-order vertices were removed, i.e. in a diffusion process, the traffic flow can only flow away from these vertices. On this modified higher-order network, a diffusion process was carried out where the diffusion quantity corresponded to the traffic flows of the removed road sections. In other words, the travellers move away from the interrupted road sections. Finally, the new traffic flow configuration was mapped back to the initial road network to allow a comparison with the NiN model and the results of the traffic flow simulations.
7.5. Results and discussion

7.5.1. Sioux Falls scenario

Due to the design and size of the Sioux Falls scenario, traffic flows were observed on each road section in the initial configuration. Figure 7.4a illustrates the initial flow by the varying line width, which can also be interpreted as an impotence measure, i.e. higher flows indicate more critical connections. The average relative error between the NiN model and the macroscopic traffic flow model [Equation (7.9)] is colour-coded, with positive (red) values representing overestimation and negative (blue) values representing underestimation. Figure 7.4a shows the average relative error for a scenario where a single edge was randomly removed. It can be observed that the flow near the centre (e.g. near the vertices 10, 15, 16, 17, 19) is well estimated (less than 2.5%). In addition, the estimates for edges with a high initial traffic flow are just as good (e.g. Edge (20, 18)). Edge (5, 6) shows the highest overestimation at 6.5%, while Edge (2, 1) underestimates the traffic flow by 4.1%, resulting in both edges having a low initial traffic flow. The average

Technical implementation

For reasons of rapid development and comparison, all the models were programmed in pure Python and executed on a single core. (Consequently, a considerable reduction of the computation time is to be expected by using compiled languages such as C++ and utilising parallel computing.) The simulation runs were conducted on a single Intel Core i7-4770 CPU 3.40Ghz, 16GB DDR2 PC running on Linux 64 bit operating system (Ubuntu 16.04). A conjugate direct Frank-Wolfe (CFW) algorithm (Mitradjieva and Lindberg, 2013), was implemented to solve the traffic assignment problem [Equation (7.9)]. The convergence criterion was set to a relative gap of $10^{-4}$. The network modelling for the NiN and the higher-order networks was done with the python package cnet.

Results and discussion

The results of the previously defined applications are discussed in this section in order to help illustrate the applications of the NiN model and map out the next steps for research. The presented approach allowed the estimation of the traffic flow by utilising modified multi-layer hypergraphs to model vertices themselves as networks. This allowed encoding multiple pieces of information such as the topology, paths used and origin-destination information, within one consistent graph structure.

Both the Sioux Falls scenario and the Chur scenario were both evaluated with a NiN model and an SLN model. As ground truth, the results of a macroscopic traffic flow simulation were used to determine the errors of the traffic flow estimation. In order to assess the behaviour in case of edge failure, 1 to 15 random edges were removed from the infrastructure network. With the help of diffusion processes, the traffic flow changes of both approaches were estimated and compared with the (exact) result of the traffic flow simulation. In the Sioux Falls scenario, the relative deviations of the traffic flows were compared. This was not possible in the Chur scenario, therefore, the relative differences were used for the analysis. In both scenarios, it could be shown that the NiN model provides better estimates than the SLN model.

In the remainder of this section, the two scenarios and their results will be discussed in more detail, followed by a general discussion about the observations and future improvements.

4https://github.com/hackl/cnet
total relative flow error for all scenarios in which an edge was randomly removed is 
$-2.7\% \ (\sigma = 0.064)$, and 95\% of the estimated values are in the $+13.4\%$ and 
$-15.9\%$ ranges, as shown in Figure 7.4b. Since the implemented cost function 
[Equation (7.8)] considers the estimated traffic with the performance of $b_e = 4$, 
the cost estimate shows higher uncertainties, i.e. 95\% of the cost estimates are 
in the range between $+46.7\%$ and $-56.6\%$.

If the NiN model is compared with the SLN model, there are hardly any deviations 
if only a few edges are randomly removed. However, if several edges are 
removed, the SLN model begins to underestimate the traffic flow (see Table 7.1 
and Figure 7.5). This is caused by the redistribution of the traffic flow due to 
the diffusion process. Since the traffic flow is not constrained by an origin 
and destination pair, travellers can diffuse to all edges in the network, which assigns 
traffic flows more evenly over the whole network. In contrast, the traffic flow in 
the NiN model can only be redistributed to other paths with the same origin 
and destination, so that traffic in the network cannot spread too far. However, an 
over- or underestimation can occur if too many or too few travellers are assigned 
to the new paths or if paths are taken which were not considered by the $k$-shortest 
paths. Figure 7.5 also shows that the distribution of the relative error is more 
symmetrically allocated for the NiN model than for the SLN model, which tend 
to underestimate the traffic flows.

7.5.2. Chur scenario

In contrast to the Sioux Falls scenario where traffic flows occurred on every road 
section, the Chur scenario had road sections that were not used in the initial 
traffic configuration. To compare deviations from the true value of zero, instead 
of the relative error measure, the relative difference measure was used, which 
compared the difference between two values to their average magnitude:

$$d_r(x_e, \hat{x}_e) = 2 \frac{\hat{x}_e - x_e}{|\hat{x}_e| + |x_e|} \quad (7.12)$$

This is a signed expression, positive when the estimated edge traffic flow $\hat{x}_e$ 
exceeds the observed edge traffic flow $x_e$ and negative when $x_e$ exceeds $\hat{x}_e$. Its 
value always lies between $-2$ and $2$, i.e. if a flow is assigned to an edge where
7.5. Results and discussion

Table 7.1.: Average relative errors and standard deviations of the Sioux Falls scenario when multiple edges are removed.

<table>
<thead>
<tr>
<th>No. of rem. edges</th>
<th>SLN model</th>
<th>NiN model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std.</td>
</tr>
<tr>
<td>1</td>
<td>-0.036</td>
<td>0.077</td>
</tr>
<tr>
<td>2</td>
<td>-0.054</td>
<td>0.107</td>
</tr>
<tr>
<td>3</td>
<td>-0.063</td>
<td>0.126</td>
</tr>
<tr>
<td>4</td>
<td>-0.073</td>
<td>0.148</td>
</tr>
<tr>
<td>5</td>
<td>-0.082</td>
<td>0.170</td>
</tr>
<tr>
<td>6</td>
<td>-0.092</td>
<td>0.181</td>
</tr>
<tr>
<td>7</td>
<td>-0.100</td>
<td>0.194</td>
</tr>
<tr>
<td>8</td>
<td>-0.108</td>
<td>0.207</td>
</tr>
<tr>
<td>9</td>
<td>-0.116</td>
<td>0.223</td>
</tr>
<tr>
<td>10</td>
<td>-0.118</td>
<td>0.247</td>
</tr>
<tr>
<td>11</td>
<td>-0.121</td>
<td>0.259</td>
</tr>
<tr>
<td>12</td>
<td>-0.127</td>
<td>0.276</td>
</tr>
<tr>
<td>13</td>
<td>-0.141</td>
<td>0.280</td>
</tr>
<tr>
<td>14</td>
<td>-0.141</td>
<td>0.288</td>
</tr>
<tr>
<td>15</td>
<td>-0.142</td>
<td>0.295</td>
</tr>
</tbody>
</table>

Figure 7.5.: Violin plot of the relative errors for the Sioux Falls scenario when multiple edges are removed. In red (left) the kernel density of the SLN model is shown, while in green (right) the kernel density of the NiN model is shown. The number of removed edges ranges from 1 to 15, corresponding to a removal of 1.3% to 19.7% of the edges in the network. Dotted lines within the densities represent the 25% and 75% quantiles, respectively, while the dashed lines represent the median values.

the initial flow is 0 the relative difference is 2. By using absolute values in the denominator, negative numbers are handled in a reasonable way.

Both Figure 7.6 and 7.7 clearly show that an SLN model approach to estimate traffic flow changes in a real-world network leads to misrepresentations of the actual traffic flow by assigning flows to edges which are not in use. In other words, travellers were assigned to edges, which were not part of feasible paths between the desired origin and destination locations. The NiN model showed no such behaviour, as travellers were only allowed to change their paths while retaining their initial origin and destinations.

In particular, this overestimation can be observed in the city centres of Chur and Domat-Ems, using the SLN model. Access roads and side roads are wrongly estimated. Since the original macroscopic traffic flow model is a regional model, i.e. no traffic flow is assigned to these secondary roads. This could change if a more detailed model is used (e.g. microscopic model).

Increasing the number of removed edges also led to an overestimation of traffic flow in the NiN model, as shown in Figure 7.6b. Instead of exploring and accounting new paths between the origins and destinations, the NiN model assigned more
7. Networks in Networks

Figure 7.6.: Traffic flow estimation of the Chur scenario. (a) The Chur network where the initial traffic flow defines the edge width. Relative differences between the SLN model and the NiN model respectively and the results of the traffic flow simulations are colour-coded. Overestimations of the traffic flows are coloured red, underestimations blue. Dark red edges (i.e. a value of 2) indicate a traffic flow assignment on edges that do not carry traffic. (b) Histogram for the Chur scenarios where 15 edges were randomly removed. The bin at 0 shows the correctly assigned traffic flows, while the bin at 2 shows the flow assignment to edges that should not have any traffic flow. In contrast, at −2 the bin illustrates an underestimation of traffic flow, where certain traffic flow is observed, but the model estimate is 0.

flows to the existing paths. This could be due to the model calibration, which was done on a network where only one edge was randomly removed and not several edges.

7.5.3. Discussion

In both scenarios, the NiN model performed better than the SLN model. This can be attributed to the fact that the NiN model takes into account the path statistics and the origin and destination locations in addition to the network topology. This was clearly shown in the Chur scenario where the SLN model had assigned traffic flows to roads where none should be. Consequently, if one wants to investigate dynamic processes on transport networks using a complex network approach, this should be taken into account, otherwise, it could lead to wrong conclusions.

Since the NiN model is able to encode multiple information, more data has to be provided to create such a model. While an SLN model only requires the network topology, a NiN model also requires path statistics. This is a particular problem for historical networks where such data is no longer available. However, due to increasing digitalisation (GPS tracking, telephone data, Twitter data) and advancing development in traffic models, more and more such data is nowadays available for research (e.g. Song et al., (2010), Tang et al., (2015), and Jurdak et al., (2015)).

The introduced NiN model represents the first step to represent complex dynamic processes on transport networks using complex network approaches. For example, the decision of which path to take is reduced to a simple diffusion process instead of solving a complex optimisation problem. Naturally, this approach does not provide the same accuracy as classical traffic flow models, but it allows general statements to be made with relatively simple means, e.g. showing the cascading effects of several edge failures in the network.

In this example, some assumptions were made to reduce the complexity of the problem. However, this also led to a reduction in accuracy and applicability. In future work, some of these assumptions could be relaxed. For example, it was as-
7.6. Conclusions

Understanding traffic flow in urban areas has great importance and implications from an economic, social and environmental point of view. For this reason, numerous disciplines are working on this topic. Although complex network theory made their appearance in transportation research through empirical measures, little research has so far been done to investigate the relationship between dynamic traffic patterns and the underlying structures of the transportation networks.

In this work, the application of a novel Networks in Networks (NiN) approach

Figure 7.7.: Violin plot of the relative differences for the Chur scenario when multiple edges are removed. In red (left) the kernel density of the SLN model is shown, while in green (right) the kernel density of the NiN model is shown. The number of removed edges ranges from 1 to 15. Dotted lines within the densities represent the 25% and 75% quantiles, respectively, while the dashed lines represent the median values. Due to the kernel density estimation, overestimates are presented continuously around the value of 2, while in reality, this corresponds to a point distribution, as shown in Figure 7.6b.

sumed that interrupted edges were removed entirely from the network. However, it would be possible to consider capacity reductions similarly as the cascading effects were studied, i.e. instead of removing the edges, a path diffusion process is performed, and the traffic flow corresponding to the capacity reduction is redistributed. This implementation would represent a more realistic representation of the real weld and thus significantly extend the application area of the model. In addition, it was assumed that travellers could only make three choices: (i) they could stay on the path initially chosen, (ii) they could switch to the next cheaper path or (iii) to the next more expensive one. This simplifies the estimation of the parameters but limits reality. Especially in the context of real observations, it would be interesting to see how travellers behave in a disturbance and what alternative paths they take.

If real-world observations are not available, another improvement would be the extension to an underlying microscopic traffic flow modelling approach (e.g. agent-based approach), to reproduce the real-world behaviour of the daily path of individuals in an urban environment. Thereby individual agents choose activities at different locations. Sequences of activities are generated and equilibrated based on a co-evolutionary algorithm that alters the agent’s behaviour from iteration to iteration. The objective is to find optimal routes, modes and departure times in order to maximise the total utilities of the agents’ daily activity schedules (Chakirov and Fourie, 2014; Horni et al., 2016). In contrast to the aggregated path flows from a macroscopic point of view, where each path is assigned to an individual agent. The setup of the NiN would be similar, but the underlying model would provide additional details.
is presented to study changes in traffic flow caused by topological changes in the transportation network, from a complex network perspective. The NiN structure is a special type of multi-layer network in which vertices are networks themselves. This embedded network structure makes it possible to encode multiple pieces of information such as topology, paths, and origin-destination information, within one consistent graph structure. Since each vertex is an independent network in itself, it is possible to implement multiple diffusion processes with different physical meanings. In this way, it is possible to estimate how the travellers’ paths will change and to determine the cascading effect in the network.

To test the capabilities and gain insights into real-world systems, the model was applied to two complex infrastructure systems, the Sioux Falls benchmark network and a real-world road network in Switzerland. The networks were modelled as NiN and calibrated with initial traffic flows from a macroscopic model. Edges were randomly removed, and the traffic flows were estimated using diffusion processes. To show the influence of the multi-layer approach, the same procedure was conducted with a single-layer network (SLN) model. The results of both modelling approaches were compared with the ground truth observed from the macroscopic traffic flow model. In both scenarios, the NiN model performed better than the SLN model. Especially for the real-world network, the NiN model gives better results, since in addition to the network topology also path statistics and the origin and destination information were taken into account.

The work presented here is the first step towards a better understanding of complex dynamic processes in transport networks using approaches from complex network theory. Naturally, this approach does not provide the same accuracy as classical traffic flow models, but it allows general statements to be made with relatively simple means. In addition, NiN’s uniform graph structure offers a new perspective on the problem of traffic assignment that goes beyond classical approaches. For example, while classical traffic flow models evaluate the (user) equilibrium by solving an optimisation problem, the proposed method uses a (path) diffusion process instead. This is computationally more efficient and should be explored in future research. Furthermore, the presented approach is limited to the estimation of traffic flow changes caused by multiple edge failures, given an initial traffic flow configuration. Future research should focus on further developing this approach to address relevant issues such as mode changes, temporal changes, spillbacks and gridlock effects.
Epilogue
8. Conclusions

8.1. Introduction

This chapter reviews the main elements of the work. It summarises how the aim and the associated objectives of the thesis were achieved and to what extent they were successfully accomplished. It also provides suggestions and recommendations for future extensions of this research. Detailed conclusions for the individual studies have been presented in the respective chapters. In this chapter, the general conclusions from this work are summarised, and the key findings of this research, in relation to the research objectives, are discussed.

The aim of this research was to develop a methodology and corresponding techniques to understand and quantify the risk of complex infrastructure systems, affected by natural hazards, considering spatial and temporal aspects. This aim was accomplished by focusing on five main objectives: (1) The development of a general process that infrastructure managers can use to assess the risks of their infrastructure systems, in order to quantify the consequences, caused by natural hazards; (2) The application of this process to estimate the spatio-temporal risk of a realistic road network due to the occurrence of time-varying multi-hazard events; (3) The development of a restoration model for identifying optimal recovery responses in the aftermath of a natural hazard, in order to quantify the system resilience; (4) The development of a network model for interdependent infrastructure systems using only a limited amount of data; and (5) The development of a model for rapid estimation of traffic flow changes due to road interruptions.

The first three objectives focused on the risk assessment process and its implementation into a computational model framework. The emphasis was put on an interdisciplinary approach, which allows quantifying the spatio-temporal risk of a complete chain of events, considering multi-hazard events as well as socio-economic impacts, caused by performance losses of infrastructure systems. The last two objectives were technical complements to the previous ones. The focus was on innovative mathematical models from the field of network sciences to support infrastructure managers in their risk assessments when only a limited amount of data was available or when a less computational effort was required.

The following sections summarise the results of each of the objectives stated in Section 1.3, indicating their importance and impact.

8.2. Synthesis of the objectives

8.2.1. Development of a general process that infrastructure managers can use to assess the risks of their infrastructure systems, in order to quantify the consequences caused by natural hazards

Chapter 2 introduced an overarching risk assessment process to support infrastructure managers in the quantification and subsequent management of risk. The methodology is founded on the principles of systems engineering (Adey et al., 2016), and thus the process is structured, keeping in mind that (i) different decisions require different models, (ii) models provide different levels of detail, and (iii) this is an iterative process, requiring changes as data and model insufficiencies are discovered, and new data and models become available. Considering these principles and the high-level risk management process proposed in ISO 31000,
(2009), an element based process was presented. All elements were described in space and time, and measures of the intensities of interest. In order to estimate the likelihood of each subsequent event in a causal chain of events (scenarios), models of the relationships between the elements had to be developed. In its most extensive form, the definition of these elements and relationships will provide all relevant scenarios for the risk assessment.

Specifically, this work advances the state-of-the-art in the field of risk assessment of complex infrastructure systems as follows:

- The risk assessment process is general enough to be applicable to different types of infrastructure systems, a wide range of natural hazards and various levels of abstractions. It provides the infrastructure managers with a clear and structured way to answer the questions of how to set up their risk assessment process, what are important considerations, and how it can be used and implemented.

- Using the principles of systems engineering ensures that all likely aspects of a system are considered and integrated into a whole. Assessing and managing risks are one of the interdisciplinary parts of systems engineering. By providing a holistic view of the development effort, systems engineering helps mould contribution from diverse disciplines into an unified team effort. It is forming a structured risk assessment process that proceeds from concept to analysis to evaluation and, in some cases, to planning and execution of interventions to reduce the risk to acceptable levels.

- The modular definition of the elements and a clear description of their relationships allow a flexible design of the individual system components, such that different levels of detailing and abstraction can be depicted. This allows that the risk assessment process can be used with and without computer support, and both simple and complex system representations can be considered.

- Through a decided definition of the system boundaries, both spatial and temporal conditions are delimited. As it is necessary to model the system over time, it is also necessary to model the spatial and temporal correlation between events and activities within the investigated time period. This includes the consideration of assumptions, agreements, as to how the system will react in specific situations, and the consideration of cascading events.

8.2.2. Application of the risk assessment process to estimate the spatio-temporal risk of a realistic road network due to the occurrence of time-varying multi-hazard events

Chapter 3 presented an application of the proposed process from Chapter 2. The application presented was used to demonstrate the usefulness of the process, considering a specific problem. It showed the design and implementation of an assessment focused on estimating the risk related to a road network in the Canton of Grisons in Switzerland. In the study, the network was exposed to rainfall, which caused multiple hazards, specifically riverine floods and mudflows. At the same time these events led to direct costs, linked to clean-up, repair, rehabilitation and reconstruction activities, and indirect costs, associated with loss of connectivity and temporal prolongation of network user travel time, linking the modelling of these latter effects with the dynamics of the network. A large number of
8.2 Synthesis of the objectives

uncertain rainfall leading to floods of multiple return periods was considered in the analysis. The data used for this application were representative of actual entities and processes in the region or were derived from such data.

Specifically, this work advances the state-of-the-art in the field of risk assessment of complex infrastructure systems as follows:

- The risk of a complete chain of events, from a source event to its societal events, is quantified over space and time. In the application, this means considering precipitation, runoff, flood, mudflows, (physical) damages, functional losses, traffic flow changes, and restoration interventions. The presented links between cascading hydrometeorological hazards, a road network and society will be of interest to the international research community and practitioners, working in the fields of network management, urban planning, public policy, and emergency response.

- When quantified, risk can be categorised into probable direct and indirect costs, with the latter estimated throughout the hazard events and restoration periods, not just immediately after the occurrence of the hazard events. In the application, direct and indirect costs included costs of interventions, prolongation of travel time, and missed trips, allowing for the evaluation and comparison of the socio-economic impacts of the multiple hazard scenarios considered.

- The simulation-based approach supports the inclusion of uncertainties and their propagation throughout the risk model. The application includes results related to the simulation of 1,200 rainfall events, causing floods of return periods ranging from 2 to 10,000 and, depending on the rainfall intensity and duration, stochastically triggering a number of mudflows in the area. Furthermore, as suggested by Lam et al., (2018a), the approach can support the testing of additional scenarios, based on uncertain relationships (e.g., fragility functions relating hazard intensities with damage state exceedance probabilities).

- A novel simulation engine was constructed as a computational platform to estimate risk, supporting the combination of models from different disciplines, and their modular update and replacement. The application demonstrates the coupling of existing models and information from literature in geosciences, engineering, network theory, transportation, and economics.

8.2.3 Development of a restoration model for identifying optimal recovery responses in the aftermath of a natural hazard, in order to quantify the system resilience

In the scope of system resilience, Chapter 4 introduced a restoration model for identifying near-optimal recovery responses that allow a quantitative analysis of the costs, resources and time needed for a disrupted network to regain full operation after a natural hazard event. An optimisation problem to determine the most effective restoration program was presented, aiming to minimise the sum of the direct costs, which are related to the execution of the interventions, and the indirect costs, associated with the traffic flow on the network until service is restored. Utilising heuristic processes to solve the optimisation problem allowed the analysis of realistically sized problems.

Specifically, this work advances the state-of-the-art in the field of risk assessment of complex infrastructure systems as follows:
8. Conclusions

• The restoration model determines the optimal restoration program for a transportation network by minimising the weighted sum of direct and indirect costs that occur over the time period between the occurrence of the hazard event and the moment that the restoration of the network is complete. Indirect costs are associated with the traffic flow of the network, which changes according to the chosen restoration program.

• To address real-world problems, the restoration model is designed to account for multiple object types (e.g. road sections and bridges) which might be in different damage states (e.g. minor or major damage), affecting the performance of the infrastructure system. In order to restore network functionality different intervention types can be associated with each state of each object (e.g. emergency bridge intervention due to major damage, caused by local scour).

• Using a modified resource-constrained project schedule approach, time-dependent constraints such as resource limitations and budget constraints can be considered. This is particularly important after a natural hazard, as the resources are usually not on site and must first be procured.

• A simulated annealing based metaheuristic procedure is used to approximate an optimal solution of the optimisation problem. This allows to analyse realistically sized problems and obtain better solutions than with currently used restoration strategies.

8.2.4. Development of a network model for interdependent infrastructure systems using only a limited amount of data

To overcome the issue of limited topological data available, in Chapter 5 and 6 a mathematical model of multi-layer spatially embedded random networks was presented. Concepts from spatial statistics and graph theory were applied to map complex infrastructure systems with interdependent subsystems to a simplified and condensed multi-layer network representation. The developed model combined Markov marked point processes for vertex creation, which accounts for spatial distribution, layer assignment, and clustering effects of the vertices, and a hybrid connection model for the edge creation. The multi-layer spatially embedded random network approach was used to build models that reproduce ensembles of artificial representations of real-world infrastructure systems with their expected properties, based on only a few input parameters such as population density and terrain elevation. Risk related to the network topology was estimated by performing percolation studies, which describes how a network transitions from connected to the disconnected state, by removing vertices or edges randomly or systematically.

Specifically, this work advances the state-of-the-art in the field of risk assessment of complex infrastructure systems as follows:

• Using advanced Markov marked point processes, allows describing spatial interactions of vertices, in which the position and layer assignment of individual vertices depends on the properties of their neighbour vertices (e.g. their positions and/or layer assignments), causing them to stay away from each other (repulsion) or come closer together (attraction). Additionally, the use of spatial fields allows modelling vertex interactions with their surrounding (physical) environment. (e.g. road intersections are more likely located in urban areas than in the countryside)
8.3. Contributions

- The edge assignment is done with a novel connection model, utilising existing connection functions and combining them into hybrid connection functions. A weighted distance based on spatial fields is introduced to account for the fact that the probability of connecting two vertices might not only depend on the distance but also on some spatial and layer properties.

- In the case of the interdependent power and road network of Switzerland, it is shown that the multi-layer spatially embedded random network model constitutes highly accurate approximations for the true spatially embedded networks, in terms of single and multi-layer measures.

8.2.5. Development of a model for rapid estimation of traffic flow changes due to road interruptions

In Chapter 7, complex network theory was applied to study problems, related to classical modelling approaches, such as traffic flow models, which are very computationally intensive. To close this gap and combine the advantages of both fields (physical and network models), a novel networks in networks approach was presented, which made it possible to study complex dynamical processes in traffic networks in a computationally efficient way. In contrast to classical network models, this is a multi-layer approach where each vertex itself represents a network. This embedded network structure allows encoding more information than conventional single-layer networks (e.g. sets of possible paths a traveller can use.). In combination with a multi-layered diffusion process, a good approximation to changes in traffic flow due to topology changes can be made.

Specifically, this work advances the state-of-the-art in the field of risk assessment of complex infrastructure systems as follows:

- Using a modified multi-layer hypergraph it is formally feasible to describe vertices that are networks themselves. Thereby the relationships in the incidence graph represent the edges connecting different layers. The edges within the different layers are given by a connection model, which allows different topologies in the different layers.

- Because each vertex is an independent network in itself, it is possible to implement multiple diffusion processes. Therefore, it is possible to assign different physical meanings to the processes. For example, one process can describe how individual travellers switch between different paths, while another process describes the propagation of disturbances through the network.

- The proposed approach allows approximating traffic flow changes due to multiple edge failures. Using the Sioux Falls benchmark network and a real-world road network in Switzerland, it is shown that NiN models capture both topological and spatial-temporal patterns in a simple representation, resulting in a better traffic flow approximation than single-layer network models.

8.3. Contributions

8.3.1. Scientific contributions

Summarising, this work contributes to the field of risk assessment and its application to complex infrastructure systems. Understanding natural hazards, complex infrastructure systems and deducing appropriate statements is not always
8. Conclusions

a straightforward task. In order to gain a deeper understanding of such complex systems, novel methodologies and computational models, as well as new mathematical approaches, are needed.

This thesis provides a methodology and corresponding techniques to understand and quantify the risk of complex infrastructure systems, affected by natural hazards, considering spatial and temporal aspects. More precisely, this work provides a novel risk assessment process for infrastructure managers, designed to estimate the spatio-temporal risk of complex infrastructure systems due to the occurrence of time-varying multi-hazard events. In doing so, it not only extends the state-of-the-art research in this field but also helps to provide decision support for infrastructure managers.

So far, six journal publications [1-6] five conference publications [7-11] and eleven other presentations [12-22] cover some paths of the research presented in this thesis and make the results available to the scientific community. The seven most important contributions [1-6,9] are contained as individual chapters in this work.

Journal Publications


Conference Publications


Presentations


8. Conclusions


8.3.2. Practical contributions

One of the overarching objectives of this thesis was to develop a methodology and applied methods that would be applicable and useful for infrastructure managers. The proposed risk assessment process is set up in a way that takes the infrastructure manager through all the necessary steps, from problem identification, system definition, data collection and analysis to model development and integration, required in the development of accurate risk estimation models in a consistent and systematic way. It allows managers to have a better understanding of the risks related to their infrastructures while giving them flexibility during the decision-making process, by allowing the consideration of different alternatives for interventions at a given time and location.

The methodology and all corresponding techniques have been demonstrated on realistically sized real-world-data based examples. While the primary focus of this examples lay on Swiss road infrastructure, the methods provided in this work are sufficiently general, by design, to be applied to other regions of the world and other infrastructure systems. In order to make such an implementation as simple as possible, special emphasis was put on using open data wherever possible.

The process was constructed keeping in mind that for many decision-making situations it is desired to have the process be computer-supported, for example to model specific parts of the system. It also has been constructed keeping in mind
that different decision situations will require the use of different types of models and models that will provide different levels of detail. The proposed modular structure is suitable for efficient software implementation.

Beside infrastructure managers, these methods will also aid city planners and public officials in determining the best mitigation policies so that the response of their infrastructures is improved. Results from a risk assessment provide clues about the deficiencies of the networks and about the expected magnitude of the direct and indirect consequences, caused by natural hazard events. This type of analysis can also shed some light on common network failure modes and on how to better plan and coordinate restoration efforts for increasing the infrastructure resilience against natural hazards.

8.4. Limitations

This thesis has succeeded in making relevant contributions to the field of risk assessment and its application to complex infrastructure systems, but of course, there is room for improvement. The following section points out some limitations of the current work.

Model complexity. The case studies have, by design, focused on the illustration of the proposed risk assessment process. The computational models selected for the applications were chosen considering the need to keep the computational time low. Other models may be available in the literature, some of which are more sophisticated and precise, and hence, demand much more detailed data than the data that were available. Nonetheless, the selection of data and models was sufficient for illustrating the use of the methodology. The proposed risk assessment process is unaffected by these limitations, and the modular simulation engine supports the updating of data and models. However, more sophisticated models might increase the accuracy of the estimates. A few examples:

- The current flood simulations are based on a one-dimensional hydraulic model for gradually varied steady flow in an open channel, which gives a rough estimation of the floodplain. However, for a more precise flow modelling, which also includes debris and sediment movements, two- or even three-dimensional hydraulic models are needed. Notably, in the case of estimating complex flows around the bridge piers, three-dimensional methods usually reveal considerably more information (Hackl et al., 2018b).

- The types of damage considered and the associated functional losses also offer an opportunity for refinement. In this work, the case studies have focused on bridge damage from local scour and road damages from inundation and mudblocking. This was done by design to limit the scope. Prior work; however, has shown the importance of other types of damage, such as to bridges and roads from logjams, erosions, debris and sediment removal (Hofstad et al., 1969), or tunnels and other infrastructure objects. In addition, the bridge models for local pier scour could be refined by replacing some or all of the fragility models used in this study with structural model results.

- The indirect costs depend significantly on the underlying traffic assignment model. In this work, a static user equilibrium traffic assignment model, based on the BPR functions to emulate the traffic flow conditions, was implemented to simplify the problem. While this model is mathematical rather simple, computationally inexpensive and widely used in literature,
it has some limitations when it comes to a realistic representation of traffic flow, e.g. it is assumed that travellers have full knowledge of the traffic conditions, which is clearly not the case. It also does not account for changes in the travel pattern after a disruptive event, although studies show this behaviour is considerably different than before a disruptive event (e.g. Chang and Nojima, (2001) and Kontou et al., (2017)). Although BPR functions are widely used as travel cost functions, they have several disadvantages, as Horowitz, (1991) pointed out, including the use in urban areas (e.g. traffic controlled intersections) and the inability to represent dynamic traffic phenomena like queues, spillbacks, wave propagation, capacity drops, and so forth.

**Model calibration.** Whenever possible, the results of the models should be compared with and calibrated against empirical data, when available. For example, the probability of damages obtained through simulations could be calibrated against collected data from field structural surveys when such data exist. Calibration for societal events is somewhat difficult because such data are difficult to measure and monitor. Such a calibration approach was carried out for the models used, i.e. each model was calibrated individually. A calibration of the complete chain of events, i.e. all models interacting with each other, was not done due to the lack of data.

**Computational complexity.** Using pure physical based computational models, as done in Chapters 2, 3 and 4, the computational time for a single scenario was relatively high. In the case of Chapter 3 a single scenario needed approximately 4 hours while in Chapter 4 it took approximately 10 hours. Due to this fact, only a limited number of simulations were performed. Improvements in the results and their uncertainties are expected if more simulations would be performed.

**Probabilistic modelling.** Currently, the inputs for the simulations are defined by the infrastructure managers (expert knowledge) or available datasets. From this set of information probabilistic model parameters were inferred. The parameters for each simulation run were sampled from these input distributions using a Monte Carlo approach. Sources of randomness for the examples were introduced by the spatio-temporal precipitation fields, the flood event and the mudflow events. In the considered scenarios, the initial condition that the network manager could freely choose was limited to the return period(s) of the flood. This feature was of particular importance to support scenario selection for infrastructure management applications (e.g., assess the performance of a network with respect to a given return period) and the quantification of the uncertainty of probable consequences for the desired return period. For the rest of the models used, it was assumed that the inputs were known, e.g. the roughness coefficients for every cell.

### 8.5. Outlook

Despite the contributions made in this thesis and by advances from other researchers around the world, the understanding of the risks of complex infrastructure systems remains in its infancy. The following section provides possible solutions to several of the previously identified limitations and a selection of future research directions that will help to improve this understanding and ensure that theoretical insights have the best chance of making an impact with infrastructure managers who are charged with the safe and reliable operation of a multitude of
8.5. Outlook

8.5.1. Modelling

Since a large part of quantitative risk assessment has to do with the modelling of events, a relevant direction for future research is the improvement of these. Based on the proposed risk assessment process and the provided modular simulation engine, it is easy to adapt the corresponding models. While this is a refinement of the models, another important future direction is the investigation of model uncertainties and how they cascaded through the system. Some ideas in this direction are presented in the following:

**Model complexity.** Due to the modular design of the simulation machine it is not a problem to exchange the existing models with more sophisticated ones. Only the inputs and outputs of the models have to be adapted. This approach also enables existing models from science and practice to be included in the risk assessment process. As mentioned in the limitations, this could be used; for example, to improve applications such as flood modelling, damage modelling or traffic modelling.

In particular, the traffic model could be improved by implementing a microscopic traffic flow modelling approach (i.e. an agent-based model) to reproduce the real-world behaviour of the daily path of individuals in an urban environment. Thereby individuals choose activities at different locations. Sequences of activities are generated and equilibrated based on a co-evolutionary algorithm that alters the agent's behaviour from iteration to iteration (Chakirov and Fourie, 2014; Horni et al., 2016). This approach would allow modelling the traffic flow before, during and after a hazard event, considering the individual behaviour of each agent. Such quantifications may lead to a more accurate characterisation of risks and improved assessment of the effectiveness of risk management strategies and investments (Aerts et al., 2018).

**Computational complexity.** The consideration of more sophisticated models is usually accompanied by a higher computing effort. There are basically two ways to overcome this computational complexity: the optimal use of new hardware (e.g. high-performance clusters, graphics processing units (GPU)), and the development of surrogate or metamodels.

- Nowadays one GPU card integrates thousands of computing cores and can provide a powerful computational capability (Heitzler et al., 2017b). Hence, a way forward to reduce the computational burden of large-scale flood simulations might be due to the utilisation of GPU cards to parallelise the shallow water model (Liu et al., 2018). Also in the field transportation, research started to work on simulating agent-based microscopic traffic models using GPUs (Song et al., 2017; Heywood et al., 2018). Possibly, this technology paves the way for real-time analysis and thus allows an interactive simulation and visualisation of the risks related to the infrastructure systems.

- The other approach is to construct surrogate models of the computational models or the simulation engine. Thereby, a surrogate model is constructed in order to emulate the input-output relation of the original model (Nagel et al., 2017). The main purpose of such a model is that it is cheap to evaluate and can, therefore, supersede the full model in analyses that require many model runs, e.g. uncertainty propagation, sensitivity analysis and parameter estimation. This enables the analysis of systems where brute-force-
8. Conclusions

Simulations are not possible due to the incurred computational complexity (Schöbi et al., 2017). While such approaches are more and more common in structural engineering (e.g. Abdallah et al., (2018) and Moustapha et al., (2018)), further research is needed for spatial-temporal risk assessment of complex infrastructure systems.

Natural hazards. While the case studies focused in particular on the development and investigation of flood and mudflow events, triggered by heavy rainfalls, the proposed methodology can be applied to other (natural) hazard events. Examples include catastrophic events, such as earthquakes which are often accompanied by significant infrastructure damage (Miller, 2014), storms which cause the highest economic losses (CRED, 2018), or more and more frequent forest fires (Stevens-Rumann et al., 2018). Besides considering new hazard events, future research could also focus on multi-hazard approaches, i.e. taking several simultaneous events into account. For example, heavy rain triggers a landslide, which in turn changes the topography and causes a flood event. This means that different physical hazard models have to interact with each other on a temporal and spatial level, but also that novel damage models are necessary because different impacts of different kinds jeopardise the structural integrity.

Uncertainty quantification. Considering a chain of events, where events are represented as probabilistic models, the uncertainties of the results might be very high. For example, the stochastic output of one model is at the same time the stochastic input of the next model. In order to answer the question, which model (parameter) causes the highest uncertainty, or in other words which model should be improved, further research has to focus on uncertainty quantification of such complex infrastructure systems. Possible examples might include: Bayesian techniques for calibration and validating of the models (Nagel and Sudret, 2015; Meert et al., 2018); sensitivity analysis (Sudret and Mai, 2015; Deman et al., 2016), which aims at determining the input parameters of the models whose uncertainty explains at best the system’s performance variability; or structural reliability analysis (Schöbi and Sudret, 2017; Marelli and Sudret, 2018), which aims at computing the probability of failure of a system, given uncertain input parameters.

8.5.2. Infrastructure systems

While this work has focused mainly on road infrastructures, the methodology can be easily adapted to other interdependent infrastructure systems. Some ideas, related to the investigated infrastructure systems, are presented in the following:

Other types of infrastructure. The proposed risk assessment process can be easily adapted to other infrastructure systems as Papathanasiou et al., (2018) has shown in the case of railway infrastructure systems. Due to the flexibility of the process, it is possible to extend future research to other types of infrastructures. Besides, an extension to classical transportation networks such as energy, (waste) water or communication. The method could also be applied to single infrastructure objects that are functionally related to each other. For example, healthcare facilities such as hospitals that are exposed to natural hazards. The aim of future research in this area should be to gain a basic understanding in order to mitigate the risk of disasters in hospitals and improve the continuity of health services during or after such events (Achour et al., 2014; Nia et al., 2018).

Interdependent infrastructures. In addition, future studies should highlight the interactions and dependencies between different infrastructure systems.
This is particularly important for transportation systems, as people can change means of transport if the standard option is not available. This can facilitate cascading events further as the capacity limits of the other transportation systems can be reached. Methods that enable these risks to be analysed are emerging (Hall et al., 2016b; Nan and Sansavini, 2017; Haimes, 2018) but a deep understanding of such cascading events and their associated risks is still in its infancy and requires further investigation; especially, as infrastructure becomes increasingly interdependent (Dawson et al., 2018).

**Long-term decision-making.** A significant portion of the work on the resilience of transportation infrastructure has been attributed to how infrastructure managers restore service once it is lost. Such a reactive approach of resilience is emphasising a focus on “bouncing back” to a normal state. If resilience is to be a useful concept in informing design strategies, it must ultimately instruct how to change. This includes a need to plan proactively, to adapt when necessary, with some culturally acceptable risk and loss, and with transformative capacity. Long-term sustainability necessitates a shift from reactive to proactive planning, which would allow a fundamental, fact-based discussion about long-term investments in measures protecting transportation infrastructures from natural hazards. At present; however, sound scientific tools and methods that provide the required information for supporting decisions on investments in infrastructure resilience for natural hazard risk reduction are missing.

### 8.5.3. Complex Networks

As shown in Chapters 5, 6 and 7, complex network theory can be used to gain insights into the behaviour of infrastructure systems by abstracting information into mathematical representations. While the results might not be as detailed as observed by simulations, such methods can be used to represent the bigger picture. Some ideas in this direction are presented in the following:

**Network growth.** The development of infrastructure systems is closely linked to spatial development. For example, the demand for infrastructure systems is highly dependent on population growth. In order to estimate the future performance of infrastructure systems, novel models are needed. As described in Chapters 5 and 6, an existing infrastructure network can be mathematically modelled with little information. The next step would be to extend such a complex network model to estimate temporal changes in order to give a rough overview of how future infrastructure systems might look like.

**Systemic risk.** The economics literature has used the term *systemic risk* in the context of financial systems for many years, in order to address the risk of market-wide illiquidity or chain reaction defaults. In recent years, network scientists have started to study such financial networks, deriving various different cascade models (Hurd, 2016; Battiston and Martinez-Jaramillo, 2018). Future research for the risk assessment of large-scale infrastructure systems could build on these concepts. For example, instead of connected banks, connected infrastructure objects could be assumed, where their assets and liabilities correspond to supplied and demanded utilities (e.g. road capacities). Due to external shocks, i.e. hazard events, an infrastructure object fails if the assets are insufficient to cover its liabilities. Similar to the banking case, these shocks might then build up and cause further infrastructure object defaults, and trigger new shocks, creating cascades.
Appendix
A. Computational Models

This chapter corresponds to the supplement of the published article:\(^1\)


This file provides supplementary information for Chapter 3. In this file, the modules used in the risk assessment are explained in more detail. An overview of the modules and their relationships is given in Figure A.1. The modules are described in terms of: inputs, outputs, resources, process, calibration, and assumptions and limitations.

![Figure A.1.: Schematic overview of the modules used for the application.](image)

\(^1\)Please note, this is the author’s version of the manuscript published in the Journal Natural Hazards and Earth System Sciences. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at https://www.nat-hazards-earth-syst-sci.net. When citing this chapter, please, refer to the original article with DOI: 10.5194/nhess-18-2273-2018.

For reasons of consistency, the text has been amended to British English.
A. Computational Models

Inputs: The term “inputs” refers to those inputs that are provided by other modules in the simulation engine, or externally by the network manager.

Outputs: The term “outputs” refers to those outputs that are provided by the module and can be used by all other modules, or ultimately can be regarded to be the estimated consequences.

Resources: The term “resources” refers to model specific data, which are needed by the current module. Depending on the model used for a specific event, these data might change. For example, while a simple traffic assignment model only needs the network and an origin-destination matrix, more complex models need additional (sociodemographic) data such as housing and workplace locations, population and employment statistics, among others.

Process: This section gives a brief description of how the module is applied.

Calibration: This section describes the process and data used to calibrate the models.

Assumptions and limitations: This section lists some of the major underlying assumptions and limitations.

A.1. Modules

A.1.1. Rainfall

Inputs:

$T_{\text{rain}}$ – Return period desired to be investigated [years]. This input can be chosen by the network manager or determined by the desired return period of the flood event $T_{\text{flood}}$.

Outputs:

$P_{\tau_{\text{rain}}}$ – Time series of precipitation fields (i.e., raster file for every time step) over period $\tau_{\text{rain}}$, where the raster cell values $p_{c,t} \in P_t$ with $t \in \tau_{\text{rain}}$ represented the rainfall intensity per time step [mm/hour].

Resources:

$P_{\text{rain}}$ – Precipitation catalogue of historical events, represented as a time series of precipitation fields, with a spatial resolution of 1 km $\times$ 1 km and a temporal resolution of 1 hour. The source data were taken from Wüst et al., (2010).

Process: The first part of this process was choosing the time series of precipitation fields $P_{\tau_{\text{rain}}} \in P_{\text{rain}}$ to be used in a given simulation from the precipitation catalogue of Wüst et al., (2010). This involved two steps: (a) setting the beginning of the rainfall event from this catalogue using a simple random sampling algorithm, and (b) selecting the duration of the rainfall event $\tau_{\text{rain}}$.

The latter was accomplished using a simple random sampling algorithm on a scaled Beta probability distribution representing possible duration lengths, ranging from 1 to 72 hours. Each return period of interest had an assigned Beta probability distribution, with larger durations to be observed with higher frequency when modeling events of larger return periods. To further characterize a rainfall event, a second set of actions was needed to relate that event to a given return period. The precipitation values $p_{c,t} \in P_{\tau_{\text{rain}}}$ for each raster cell $c$ at time $t \in \tau_{\text{rain}}$ were iteratively scaled as described in Hackl et al., (2017b) until the rainfall event generated a discharge value at a point of interest matching that of the desired return period. The result of this entire process was a time series of scaled precipitation fields $P_{\tau_{\text{rain}}}$. 

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Finally, in order to match the spatial resolution to be used throughout the entire analysis (set at 16 m × 16 m), the resolution of all precipitation fields $P_{\text{rain}}$ (originally set at 1 km × 1 km), was adapted using a re-gridding process.

**Calibration** : Records from precipitation measurement stations located near the study area (see Figure 3.2.) were used to calibrate the model. Extreme events that had been not recorded were extrapolated from the data using extreme value statistics.

**Assumptions and limitations** :
- This approach was limited to modifying historical events to recreate new rainfall events. In this process, values could be up or downscaled to produce the desired return period. New events could not be produced (e.g., changes in the movement of the rainclouds, the spatial coverage of the clouds).
- The rainfall catalogue was limited to the year 2007.
- Rainfall patterns could not span monthly borders.
- A rainfall event could only occur in a period between 1 and 72 hours.

### A.1.2. Runoff

**Inputs** :
- $P_{\text{rain}}$ – Time series of precipitation fields over period $\tau_{\text{rain}}$ [mm/hour].

**Outputs** :
- $Q_{r,t}$ – Hydrograph for river station $r$, which was generated using the excess of raster cells located at the basin outlets, as a function of time $t$ [m$^3$/s].

**Resources** :
- CN – Raster file with the runoff curve numbers [-] to predict direct runoff and infiltration from rainfall excess.
- DEM – Raster file containing the terrain’s elevation (digital elevation model) [m] to calculate the direction of runoff.
- $Q_{r,0}$ – Base flow for river station $r$ [m$^3$/s].
- $\varrho$ – Storage coefficient for linear reservoirs [hour].

**Process** : The precipitation excess was computed for each raster cell using the Soil Conservation Service (SCS) Curve Number (CN) model. This model estimates precipitation excess as a function of cumulative precipitation, soil cover, land use, and antecedent moisture (Feldman, 2000):

\[
pe_{c,t}^{\text{excess}} = \frac{(\text{CN}_e \cdot (p_{c,t} + 50.8) - 5080)^2}{\text{CN}_e \cdot (\text{CN}_e \cdot (p_{c,t} - 203.2) + 20320)} \tag{A.1}
\]

where $pe_{c,t}^{\text{excess}}$ is the accumulated precipitation excess for raster cell $c$ at time $t$, $p_{c,t}$ is the corresponding precipitation value, and $\text{CN}_e$ is the curve number for the raster cell $c$. Each raster cell’s excess was then lagged to the basin outlet. The translation time to the outlet was computed through a grid-based travel-time model:

\[
t_{c}^{\text{runoff}} = t^{\text{runoff}} \cdot \frac{d_{c}^{\text{runoff}}}{d_{\text{runoff}}} \tag{A.2}
\]

where $t_{c}^{\text{runoff}}$ is the lag (travel) time from raster cell $c$, $t^{\text{runoff}}$ is the time of concentration for the watershed, $d_{c}^{\text{runoff}}$ is the travel distance from raster
A. Computational Models

cell $c$ to the watershed outlet, and $d_{\text{runoff}}$ is the travel distance from the most distance raster cell to the watershed outlet.

The individual raster cell outflows $f_{c,t}^{\text{out}}$ were routed through a linear reservoir, to account for the effects of watershed storage. The routing was done using Clark’s original methodology:

$$f_{c,t}^{\text{out}} = \frac{2 \cdot \Delta t Q \cdot (f_{c,t}^{\text{in}} - f_{c,t-1}^{\text{out}})}{2g + \Delta t Q} + f_{c,t-1}^{\text{out}}$$  \hspace{1cm} (A.3)

where $f_{c,t}^{\text{in}}$ is the average inflow to the storage of raster cell $c$ at time $t$ composed of the accumulated precipitation excess $P_{c,t}^{\text{excess}}$ and the outflows of the neighbor cells at $t-1$, $g$ is a storage coefficient for linear reservoirs (defined in time units), and $\Delta t Q$ is the time interval of a hydrograph $Q$ (here set to 1 hour).

The results from each raster cell were combined to produce the final hydrographs $Q_{r,t}$ for each river station $r$ for all time steps $t$. These flows were estimated by adding the outflow values $f_{c,t}^{\text{out}}$ of the raster cells $c$ located at the watershed outlet that correspond to the river station $r$, and the base flow $Q_{r,0}$ of that station:

$$Q_{r,t} = Q_{r,0} + \sum_{c_r} f_{c_r,t}^{\text{out}}$$  \hspace{1cm} (A.4)

Calibration: The model was calibrated using records from past precipitation events and their resulting increase in river discharge, measured at the gauging stations located near the study area (see Figure 3.2).

Assumptions and limitations:

- The basins could be subdivided into grid-cells; therefore, all grid-cells within a sub-basin had the same loss-rates at the beginning of each simulation.
- The infiltration rate would approach zero during a rainfall event of long duration, rather than constant rate as expected.
- The initial abstraction did not depend upon the rainfall characteristics or timing.
- The storage behavior was simplified in terms of evaporation, infiltration and groundwater flow, the latter of which was not considered.

A.1.3. Flood

Inputs:

- $Q_{r,t}$ – Hydrograph for river station $r$ as a function of time $t$ [m$^3$/s].
- $T_{\text{flood}}$ – Return period desired to be investigated [years]. This was an alternative to specifying the return period of the rainfall event. In this case, the simulation engine produced suitable rainfall patterns, such that the resulting hydrographs led to the targeted flood event return period.

Outputs:

- $I_t$ – Time series of inundation fields (i.e., raster file for every time step), where the raster cell values $i_{c,t}$ represented the floodwater depth above ground [m].
- $Q_{r_B,t}$ – Hydrograph for river station $r$ near the bridge of interest $B$ as a function of time [m$^3$/s].
A.1. Modules

**Resources**:

DEM – Raster file containing the terrain’s elevation (digital elevation model) [m] to extract the geometries and generate the inundated areas.

$S_{friction}$ – Friction slopes between river cross-sections, which were estimated using empirical laws (e.g., the Manning formula).

**Process**: The governing equation describing the flow problem of the one-dimensional hydraulic model was derived by the energy equation for two neighboring cross-sections, enclosing a channel reach of length $L_{i,i+1}$ (index $i$ denoted an upstream cross-section, and index $i+1$ denoted a downstream cross-section):

$$z_i + h_{i,t} + \frac{\gamma_i \cdot v_{i,t}^2}{2g} = z_{i+1} + h_{i+1,t} + \frac{\gamma_{i+1} \cdot v_{i+1,t}^2}{2g} + S_{friction,i,i+1} \cdot L_{i,i+1}$$ (A.5)

where $z_i$ is the bed elevation with regard to the datum, $h_{i,t}$ is the water depth at time $t$, $\gamma_i$ is the energy correction factor, and $v_{i,t}$ is the average flow velocity at time $t$, with all of these variables for a given cross-section $i$. Moreover, $g$ is the gravitational acceleration, $S_{friction,i,i+1}$ is the average friction slope between both cross-sections. The average flow velocity $v_{i,t} = \frac{Q_{r=i,t}}{A_{i,t}}$ can be expressed as a function of the discharge $Q_{r=i,t}$ and the wetted cross-sectional area $A_{i,t}$. At the same time, for a given cross-section $i$, this area $A_{i,t} = h_{i,t} \cdot b_i$ is expressed as a function of the water depth $h_{i,t}$ and the width of the channel $b_i$. Equation (A.5) allowed to compute the water surface profiles from one cross-section to the next. For most cases, this was done numerically. Finally, the water depth $h_{i,t}$ values at each river cross-section were interpolated to obtain an inundation field $I_t$, representing a raster file for time $t$.

**Calibration**: Historic records from gauging stations along the rivers (see Figure 3.2) were used to calibrate the model. Extreme events that had not been recoded were extrapolated from the data using extreme value statistics. Simulation results were compared, additionally, with hazard maps from the region.

**Assumptions and limitations**:

- The flow was assumed to be unidirectional (i.e. parallel to the main channel flow).
- No sediment transport or debris were considered.
- Storage and recirculation areas were not considered.
- The model could not reproduce flood events with extreme non-uniformity and spatial variability of flow patterns.

A.1.4. Mudflow

**Inputs**:

$P_{\tau_{min}}$ – Time series of precipitation fields over period $\tau$ [mm/hour].

**Outputs**:

$L_t$ – Time series of mudflow fields (i.e., raster file for every time step), where the raster cell values $l_{ct,t}$ represented the additional elevation caused by the mudflows [m]$^2$.

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2The elevation data contained in the raster files were used to determine the mudflow volume [m$^3$] deposited on road sections / subsections.
A. Computational Models

Resources:
DEM – Raster file containing the terrain’s elevation (digital elevation model) [m].
$L$ – Set of potential mudflow locations and geometries, where $\ell \in L$ was a mudflow event with specific location $c_\ell$. The source data were taken from Losey and Wehrli, (2013).

Process: Potential mudflow locations $c_\ell$ were obtained from Losey and Wehrli, (2013). The probability that a mudflow could occur was estimated based on precipitation thresholds obtained by using the empirical intensity-duration function for sub-alpine regions proposed by Zimmermann et al., (1997):

$$P_{\text{mudflow}}^{c_\ell,\tau} = 32 \cdot \tau_{c_\ell,t}^{-0.72}$$ (A.6)

where $P_{\text{mudflow}}^{c_\ell,\tau}$ is the precipitation threshold in mm/hour and $\tau_{c_\ell,t}$ is the duration of the rainfall event until time $t$ at the potential mudflow location $c_\ell$. For each potential mudflow location, the respective precipitation values $p_{c_\ell,t}$ were extracted from the rainfall model and used as points of comparisons. If the threshold was exceeded ($\sum_{t \in \tau_{c_\ell,t}} p_{c_\ell,t} > p_{\text{mudflow}}^{c_\ell,\tau}$) at a given time step, a probability of being triggered was assigned to the event, based on the slope factor of safety (FS) (Skempton and Delory, 1952):

$$FS_{c_\ell,t} = \frac{(c^s + c^r) + (\gamma^s - m^t \cdot \gamma^w) \cdot z^s \cdot \cos^2 S_{\text{angle}} \cdot \tan \phi}{\gamma^s \cdot z^s \cdot \sin S_{\text{angle}} \cdot \cos S_{\text{angle}}}$$ (A.7)

where $c^s$ and $c^r$ are the cohesion of soil and roots respectively, $\gamma^s$ is the specific weight of soil, $m^t = z^w_t / z^s_t$ is the fraction between water table depth $z^w_t$ at time $t$ and the soil depth $z^s_t$, $\gamma^w$ is the specific weight of water, $S_{\text{angle}}$ is the slope angle, and $\phi$ is the angle of internal friction. The water table depth $z^w_t$ is composed of the initial water table depth $z^w_0$ and the additional depth $\sum_{t \in \tau_{c_\ell,t}} p_{c_\ell,t}$. All values can be assumed to correspond to the potential mudflow location $c_\ell$, and therefore to mudflow $\ell$. Based on probabilistic input parameters (Table A.1), a Monte Carlo scheme was used to generate $j = 100,000$ FS values. This data set was then used to derive the triggering probability ($P[\ell|t] = \frac{1}{j} \sum_j \mathbbm{1}_{FS_{c_\ell,t} < 1}$).

Table A.1.: Probabilistic inputs for mudflow triggering.

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Description</th>
<th>Distr.</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^s$</td>
<td>cohesion of soil</td>
<td>Norm</td>
<td>5.04, 2.18</td>
<td>kPa</td>
</tr>
<tr>
<td>$c^r$</td>
<td>cohesion of roots</td>
<td>Norm</td>
<td>3.41, 2.36</td>
<td>kPa</td>
</tr>
<tr>
<td>$\gamma^s$</td>
<td>specific weight of soil</td>
<td>Unif</td>
<td>18, 33</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>$\gamma^w$</td>
<td>specific weight of water</td>
<td>Det</td>
<td>9.81</td>
<td>kN/m$^3$</td>
</tr>
<tr>
<td>$z^s$</td>
<td>soil depth</td>
<td></td>
<td>0.1, 1.5</td>
<td>m</td>
</tr>
<tr>
<td>$S_{\text{angle}}$</td>
<td>slope angle</td>
<td>Unif</td>
<td>35, 65</td>
<td>Deg</td>
</tr>
<tr>
<td>$\phi$</td>
<td>angle of internal friction</td>
<td>Norm</td>
<td>30, 5</td>
<td>Deg</td>
</tr>
</tbody>
</table>

The volume $V_\ell$ of each mudflow was estimated by taking into account the runout length $R_\ell$ of the fan using an empirical relationship proposed by Rickenmann, (1999):

$$V_\ell = \left( \frac{R_\ell}{15} \right)^2$$ (A.8)

The increase in elevation per raster cell was calculated by dividing the mudflow volume by the area of the fan. The output of the model was a
time series of raster files $L_t$, whose raster cell values corresponded to the additional elevation caused by the mudflows.

**Calibration**: Historical records of rainfall intensity and triggered mudflows were used to calibrate the triggering probability of these events. The pre-calculated mudflow locations and geometries were compared with observations from different test areas and qualitatively evaluated by experts (see Losey and Wehrli, (2013)).

**Assumptions and limitations**:
- The intensity-duration function and a probabilistic infinite slope model could describe the triggering of mudflows.
- This approach was limited to the modification of predefined potential mudflow locations and geometries. New events could not be produced.

### A.1.5. Object fragility

**Bridge local scour**

**Inputs**:
- $Q_{r_B,t}$ – Hydrograph for river station $r$ near the bridge of interest $B$ as a function of time [m$^3$/s].

**Outputs**:
- $D_{scour}$ – Time series of damage state exceedance probabilities considering cumulative damages for bridges due to local scour.

**Resources**:
- DEM – Raster file containing the terrain’s elevation (digital elevation model) [m] to extract the river geometries.
- $e$ – Objects (i.e., bridges $B$) with associated properties such as type of bridge and number of piers.

**Process**: Empirical relationships from Arneson et al., (2012) were used to quantify the excavated depth $h_{scour}^{e=B,r_B,t}$ of an object $e = B$ located near river station $r_B$ due to local scour at time $t$:

$$h_{scour}^{e=B,r_B,t} = 2.0 \cdot \kappa_1 \cdot \kappa_2 \cdot \kappa_3 \cdot h_{r_B,t} \cdot \left( \frac{a_{e=B}}{h_{r_B,t}} \right)^{0.65} \left( \frac{v_{r_B,t}}{g \cdot h_{r_B,t}} \right)^{0.43}$$  \hspace{1cm} (A.9)

where the $\kappa$ parameters are corrective coefficients and $a_{e=B}$ represents the pier width. The relationship between the flow depth directly upstream of the pier $h_{r_B,t}$, mean velocity of flow directly upstream of the pier $v_{r_B,t}$ and discharge $Q_{r_B,t}$ is given in Section A.1.3. Based on probabilistic input parameters (Table A.2), a Monte Carlo scheme was implemented to generate 100,000 scour depths $h_{scour}^{e=B,r_B,t}$ and to analyze the probability of damage exceedance ($P = \frac{1}{j} \sum_j 1_{h_{scour}^{e=B,r_B,t} > h_{scour max}}$).

Table A.2.: Probabilistic inputs for bridge local scour.

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Description</th>
<th>Distr.</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$</td>
<td>factor for pier shape</td>
<td>Det</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>factor for angle of attack</td>
<td>Norm</td>
<td>1.23, 0.16</td>
<td>–</td>
</tr>
<tr>
<td>$\kappa_3$</td>
<td>factor for bed from</td>
<td>Norm</td>
<td>1.1, 0.055</td>
<td>–</td>
</tr>
<tr>
<td>$a_e$</td>
<td>pier width</td>
<td>Unif</td>
<td>0.8, 0.85</td>
<td>m</td>
</tr>
<tr>
<td>$h_{scour max}$</td>
<td>critical scour depth</td>
<td>Norm</td>
<td>5.7, 1.12</td>
<td>m</td>
</tr>
</tbody>
</table>

This dataset was then entered into a maximum likelihood estimation function to generate fragility functions for the four damage states given in Ta-
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Table 3.1 with respect to flow discharge $Q_{r,t}$. The functions followed lognormal relationships:

$$\mathbb{P}[S \geq s_i|Q_{r,t}=b,t] = \Phi \left( \frac{\ln Q_{r,t} - \mu_{s_i}}{\sigma_{s_i}} \right)$$ (A.10)

where $S$ represents a realized damage state, $s_i$ represents a possible damage state $i \in [0, 1, 2, 3]$, and $Q_{r,t}$ represents the hazard intensity measure at river station $r$ near bridge $B$ at time $t$. The derived fragility parameters, namely $\mu_{s_i}$ and $\sigma_{s_i}$, for bridge local scour damage states are given in Table 3.2.

**Assumptions and limitations**:

- Only local pier scour was assumed while scour at the embankments was neglected.
- The scour process was only determined by the discharge values (i.e. flow properties).
- Sediment transport was not (directly) considered in the scour depth calculation.

**Road section inundation**

**Inputs**:

$I_t$ – Time series of inundation fields, where each raster cell value $i_{c,t}$ represented the floodwater depth above ground [m].

**Outputs**:

$DS_{inun}$ – Time series of damage state exceedance probabilities considering cumulative damages for road sections / subsections due to inundation.

**Resources**:

$n$ – Section/subsections of the objects with associated properties such as the type of road.

**Process**: The fragility functions were constructed assuming that (a) the general width of high-speed (major) roads was 12 m and that of local (minor) roads was 6 m, (b) all pavements had a sub-base, with major roads having a sub-base twice as thick as that of minor roads, (c) major road layers were considered to always be thicker than local road layers, (d) one day of inundation could compromise the performance of the subgrade layer (Roslan et al., 2015), and (e) any amount of traffic on a road section with a compromised subgrade layer would result in reconstruction. Log-normal fragility functions were fitted based on three additional assumptions:

- The sub-base of a linear meter of major road section can store 0.35 m$^3$ of water (Walsh, 2011), leading to assume that inundation depths below 2.92 cm caused problems to major road sections with 5 % probability (the same threshold for minor road sections was set to 1.46 cm).
- An inundation depth of 30 cm is the average depth at which passenger cars start to float, which implies that objects as heavy as passenger cars can be transported throughout the road network, leading to assume the collapse of the drainage function and significant damages to various road elements in addition to making the subgrade vulnerable with 95 percent probability.
- The median inundation depth values of the fragility functions arbitrarily increase by 5 cm as the damage states increase, with median
values of major roads higher by 10 cm than those of local roads to illustrate that pavement thickness is a vulnerability factor as indicated by Zhang et al., (2008) and acknowledge that major roads undergo a more rigorous design process than local roads.

Table 3.2 shows the parameters of the fragility function when inundation depth was used as an intensity measure. Such depth was associated with the need to clean up a given road section, damages to selected elements, and the eventual loss of the subgrade.

Assumptions and limitations:
- Other modes of failure, in particular, the blockage of drainage, delamination, erosion and washed out elements can be associated with runoff flow. Although important to model, these phenomena were not included in the model, but should certainly be considered in the future.

Road section mud-blocking

Inputs:
- $L_t$ – Time series of mudflow fields, where the raster cell values $l_{ct,t}$ represented the additional elevation caused by the mudflows [m]$^3$.

Outputs:
- $DS_{\text{block}}$ – Time series of damage state exceedance probabilities considering cumulative damages for road sections / subsections due to mud-blocking.

Resources:
- $n$ – Section/subsections of the objects with associated properties such as the type of road.

Process: As part of a survey conducted by Winter et al., (2013), experts assigned damage state exceedance probabilities to debris flow volumes for specific damage states and road categories (i.e., major roads and minor roads). Volumes were understood to intersect a road section of 500 m. Experts also provided a score representing their level of expertise. This dataset was used to derive fragility functions for road section mud-blocking. For every combination of damage state and road category, four expert responses were randomly sampled from the survey dataset. This process resulted in different scenarios of relationships between debris flow volumes and damage state exceedance probabilities. These sampled relationships, along with the recorded expertise level scores, were entered into a maximum likelihood estimation function to generate the parameters of the fragility functions given in Table 3.1.

Assumptions and limitations:
- The results of the survey by Winter et al., (2013), focused on debris flow, could be used for determining a relationship between mudflows and road sections.

A.1.6. Object functionality

Capacity reduction

Inputs:
- $DS_{\text{scour}}$ – Time series of damage state exceedance probabilities considering cumulative damages for bridges due to local scour.

---

The elevation data contained in the raster files were used to determine the mudflow volume [m]$^3$] deposited on road sections / subsections.
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\( \text{DS}^\text{inun} \) – Time series of damage state exceedance probabilities considering cumulative damages for road sections / subsections due to inundation.

\( \text{DS}^\text{block} \) – Time series of damage state exceedance probabilities considering cumulative damages for road sections / subsections due to mud-blocking.

**Outputs** :

\( \langle \lambda \rangle_{n,t} \) – Time series of expected capacity reduction for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.

**Resources** :

\( n \) – Section/subsections of the objects with associated properties such the road type.

**Process** : The expected capacity reductions were determined as functions of time-dependent damage state exceedance probabilities \( \bar{D}_S_{s_i,t} \) and capacity reduction values \( \lambda_{n,s_i} \) associated with section/subsection \( n \) and the investigated damage states \( s_i \) \((i \in [0, 1, 2, 3])\) (see Table 3.3):

\[
\langle \lambda \rangle_{n,t} = \sum_{i=0}^{3} \lambda_{n,s_i} \cdot \bar{D}_S_{s_i,t}
\]

where \( \langle \lambda \rangle_{n,t} \in [0, 1] \) is the expected capacity reduction of a specific section/subsection \( n \) at a specific time \( t \) in the simulation.

**Assumptions and limitations** :

- The estimated loss values \( \lambda_{n,s_i} \) were obtained from a survey conducted by D’Ayala and Gehl, (2015). The most conservative values were selected whenever possible. For bridge local scour, the survey had a range of answers for a general bridge local scour category, which did not necessarily match with the proposed damage limit state functions of this work.

**Speed reduction**

**Inputs** :

\( I_t \) – Time series of inundation fields, where each raster cell value \( i_{c_n,t} \) represented the floodwater depth above ground \([m]\).

**Outputs** :

\( \langle \phi \rangle_{n,t} \) – Time series of speed reduction for inundated road sections / subsections.

**Resources** :

\( n \) – Section/subsections of the objects with associated properties such as maximum allowed speed.

**Process** : During the hazard event period, the relationship between inundation depths and feasible speed of vehicles on the road was derived from the data presented by Pregnolato et al., (2017). An exponential function was fitted to these data to describe the limit vehicle speed in a road as a function of inundation depth.

\[
v_{n,t}(i_{c_n,t}) = \begin{cases} v_{n}^{\text{max}} \cdot \exp\{-0.10814 \cdot i_{c_n,t}\} & \text{for } i_{c_n,t} \leq 30 \text{ cm} \\ 0 & \text{otherwise} \end{cases}
\]

The content presented in this section is applicable to bridges as well. The estimation of \( \langle \lambda \rangle_{e=B,t} \) follows the same process as the estimation of \( \langle \lambda \rangle_{n,t} \).
where \(v_{n,t}\) is the maximum acceptable velocity that ensures safe control of a vehicle through subsection \(n\) at time \(t\) when considering the inundation depth \(i_{cn,t}\), and \(v_{n}^{\text{max}}\) is the maximum allowed speed on any road. The functional loss due to speed reduction for a section \(n\) at time \(t\), was determined by:

\[
\langle \phi \rangle_{n,t} = \max(0, v_{n}^{\text{max}} - v_{n,t}(i_{cn,t}))
\]

where \(\langle \phi \rangle_{n,t} \in [0,1]\) is the expected speed reduction at a specific section/subsection \(n\) at time \(t\) in the simulation and \(v_{n}^{\text{max}}\) is the maximum allowed speed on section/subsection \(n\).

**Assumptions and limitations**:

- The maximum allowed speed \(v_{n}^{\text{max}}\) in Equation (A.12) was set to be 120 km/h.
- No distinction was made between different types of vehicles (e.g., cars, trucks, etc.).

### A.1.7. Object restoration needs

**Inputs**:

- \(\text{DS}_{\text{scour}}\) – Time series of damage state exceedance probabilities considering cumulative damages for bridges due to local scour.
- \(\text{DS}_{\text{inun}}\) – Time series of damage state exceedance probabilities considering cumulative damages for road sections / subsections due to inundation.
- \(\text{DS}_{\text{block}}\) – Time series of damage state exceedance probabilities considering cumulative damages for road sections / subsections due to mud-blocking.

**Outputs**:

- \(\langle c \rangle_{R,n,t}^R\) – Time series of the expected restoration costs [CHF] for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.
- \(\langle \lambda \rangle_{R,n,t}^R\) – Time series of the expected capacity reduction during restoration intervention for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.
- \(\langle \tau \rangle_{R,n,t}^R\) – Time series of the expected restoration times [hours] for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.

**Resources**:

- \(C_{dc}^{\text{dc}}\) – Set of direct cost parameters including fixed costs \(c_{n,s_i}^{R,\text{fix}}\) and variable costs \(c_{n,s_i}^{R,\text{var}}\) for the restoration of bridge local scour, road section inundation and road section mud-blocking (see Table 3.4.).

**Process** : For each section \(n\) in a damage state \(s_i\) \((i \in [0,1,2,3])\), a restoration intervention was assigned. Associated with each intervention were (a) the capacity losses due to the execution of the intervention \(\lambda_{n,s_i}^R\), (b) the length of time required to execute the intervention \(\tau_{n,s_i}^R \geq 0\), and (c) the cost of the intervention \(c_{n,s_i}^R \geq 0\). This cost was composed of a fixed part \(c_{n,s_i}^{R,\text{fix}}\) (e.g., site setup) and a variable part \(c_{n,s_i}^{R,\text{var}}\) (e.g., CHF/m² of pavement, CHF/m³ of concrete). Based on the derived time series of damage state exceedance probabilities \(\text{DS}_{s_i,t}\), expected capacity reduction during restoration \(\langle \lambda \rangle_{n,t}^R\), the expected restoration costs \(\langle c \rangle_{n,t}^R\) and durations \(\langle \tau \rangle_{n,t}^R\) for each section were calculated.
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\[
\langle \lambda \rangle_{n,t}^R = \sum_{i=0}^{3} \lambda_{n,s_i}^R \cdot \bar{D}S_{s_i,t} 
\]
(A.14)

\[
\langle c \rangle_{n,t}^R = \sum_{i=0}^{3} \left( c_{R,\text{fix}} + c_{R,\text{var}} \right) \cdot \bar{D}S_{s_i,t} 
\]
(A.15)

\[
\langle \tau \rangle_{n,t}^R = \sum_{i=0}^{3} \tau_{n,s_i}^R \cdot \bar{D}S_{s_i,t} 
\]
(A.16)

Assumptions and limitations:

- Although multiple restoration strategies might be possible (e.g., putting more effort into the restoration of critical objects), it was assumed that only one strategy with expected costs and restoration time was implemented.

A.1.8. Network

Inputs:

- \( \langle c \rangle_{n,t}^R \) – Time series of the expected restoration costs [CHF] for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.
- \( \langle \lambda \rangle_{n,t} \) – Time series of expected capacity reduction for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.
- \( \langle \lambda \rangle_{n,t}^R \) – Time series of the expected capacity reduction during restoration intervention for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.
- \( \langle \phi \rangle_{n,t} \) – Time series of speed reduction for inundated road sections / subsections.
- \( \langle \tau \rangle_{n,t}^R \) – Time series of the expected restoration times [hours] for bridges with scoured piers, inundated road sections / subsections and mud-blocked road sections / subsections.
- \( R_t \) – Restoration program, defining when each damaged object is to be restored.

Outputs:

- \( G \) – Time series of routable network graphs that can be used for traffic assignment.
- \( \langle c \rangle_{e,t}^R \) – Time series of the expected restoration costs [CHF] for objects \( e \).
- \( \langle \lambda \rangle_{e,t} \) – Time series of the expected capacity reduction for objects \( e \).
- \( \langle \lambda \rangle_{e,t}^R \) – Time series of the expected speed reduction for objects \( e \).
- \( \langle \phi \rangle_{e,t} \) – Time series of the expected aggregated speed reduction for object \( e \).
- \( \langle \tau \rangle_{e,t}^R \) – Time series of the expected restoration times [hours] for objects \( e \).

Process: The road network was modelled as a graph \( G = (\mathcal{V}, \mathcal{E}) \) composed of 1,520 vertices (i.e., 37 centroids, 1,056 junctions, and 427 changes in road geometric features) and 3,202 directed edges \( e \in \mathcal{E} \), also referred to as links or objects.

An aggregation routine of subsections’ functional losses was implemented, which computed the expected functional loss at an edge level by identifying the maximum expected functional loss of the subsections that are part of...
the edge. The functional loss related to road capacity reduction \( \langle \lambda \rangle_{e,t} \) for an edge \( e \) at time \( t \), was determined by\(^5\):

\[
\langle \lambda \rangle_{e,t} = \max_{n \in e} \langle \lambda \rangle_{n,t}
\]  
(A.17)

where \( \langle \lambda \rangle_{n,t} \) is the expected capacity reduction of a specific section/subsection \( n \in e \). At the same time, the functional loss due to speed reduction \( \langle \phi \rangle_{e,t} \) for an edge \( e \) at time \( t \), was determined by:

\[
\langle \phi \rangle_{e,t} = \max_{n \in e} \langle \phi \rangle_{n,t}
\]  
(A.18)

where \( \langle \phi \rangle_{e,t} \) is the expected speed reduction for a specific section/subsection \( n \in e \). The restoration cost \( \langle c \rangle_{e,t}^R \) and time \( \langle \tau \rangle_{e,t}^R \) for an edge \( e \) at time \( t \) were determined by:

\[
\langle c \rangle_{e,t}^R = \sum_{n \in e} \langle c \rangle_{n,t}^R
\]  
(A.19)

\[
\langle \tau \rangle_{e,t}^R = \sum_{n \in e} \langle \tau \rangle_{n,t}^R
\]  
(A.20)

where \( \langle c \rangle_{e,t}^R \) and \( \langle \tau \rangle_{e,t}^R \) are the expected restoration cost and time for a specific section/subsection \( n \in e \), respectively.

**Assumptions and limitations**:
- The section with the worst condition determined the condition of the corresponding object (i.e., a weakest link approach).

A.1.9. Traffic

**Inputs**:
- \( G \) – Time series of routable network graphs that can be used for traffic assignment.
- \( \langle \lambda \rangle_{e,t} \) – Time series of the expected capacity reduction for object \( e \).
- \( \langle \phi \rangle_{e,t} \) – Time series of the expected speed reduction for object \( e \).

**Outputs**:
- \( \mathcal{P}^0_{od,t} \) – Time series of \( od \)-paths where no flow is possible.
- \( t_{\text{traffic}}^{e,t} \) – Time series of travel time for each edge \( e \) in the network.
- \( x_{e,t} \) – Time series of traffic flow for each edge \( e \) in the network.

**Resources**:
- \( od \) – Origin-destination matrix of the area.

**Process**: The traffic flow \( x_{e,t} \) for edge \( e \) at time \( t \) was estimated by solving the user equilibrium assignment, Equation (A.21a) subjected to Equations. (A.21b) and (A.21c).

\[
x_{e,t} \in \min \sum_{e} \int_0^{x_{e,t}} C_{\text{traffic}}(\omega) \, d\omega
\]  
(A.21a)

subjected to

\[
\sum_{P \in \mathcal{P}^1_{od,t}} f_{od}(P) = d_{od} \quad \forall od \quad (A.21b)
\]

\[
f_{od}(P) \geq 0 \quad \forall P \in \mathcal{P}^1_{od,t}, \forall od \quad (A.21c)
\]

\(^5\)The estimation of \( \langle \lambda \rangle_{e,t}^R \) follows the same process as the estimation of \( \langle \lambda \rangle_{e,t} \).
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where

\[
x_{e,t} = \sum_{od} \sum_{P \in P^{1}_{od,t}} f_{od}(P)
\]

(A.21d)

where \(f_{od}(P)\) is the function to estimate the flow between origin \(o\) and destination \(d\) on path \(P\). While \(P^{1}_{od,t}\) refers to the set of \(od\)-paths where some flow is still possible, \(P^{0}_{od,t}\) refers to the set of \(od\)-paths where no flow is possible. The demand constraints Equation (A.21b) state that the flow on a given \(od\)-pair has to equal the demand \(d_{od} \geq 0\), for all \(od\). The non-negativity constraints Equation (A.21c) are required to ensure that the solution of the program will be physically meaningful.

In terms of the edge cost function \(C_{\text{traffic}}\), which estimates the travel time \(t_{e,t}\) through edge \(e\) at time \(t\) when using the corresponding traffic flow as an input, has been defined using the formulation proposed by the Bureau of Public Roads, (1964):

\[
C_{\text{traffic}}(x_{e,t}) := t_{e,0}^{\text{traffic}} \left( 1 + \alpha_e \left( \frac{x_{e,t}}{1 - \langle \lambda \rangle_{e,t}} \cdot y_{e,0} \right) \right) \beta_e
\]

(A.22)

where \(t_{e,0}^{\text{traffic}}\) is the initial free flow travel time, \(\langle \phi \rangle_{e,t}\) is the expected speed reduction, \(\alpha_e\) and \(\beta_e\) are parameters for calibration, with typical values \(\alpha = 0.15\) and \(\beta = 4\), \(\langle \lambda \rangle_{e,t}\) is the expected capacity reduction, and \(y_{e,0}\) is the initial edge capacity.

**Calibration**: Data from traffic count stations in the study area were used to calibrate the initial traffic assignment (i.e., before the hazard events occurred).

**Assumptions and limitations**:

- A static user equilibrium traffic assignment model, based on the Bureau of Public Roads function could be used to simulate traffic flow conditions. Although this model is mathematically simple, computationally inexpensive and widely used in literature, it has limitations when representing traffic flow (e.g., it is assumed that travelers have full knowledge of the traffic conditions, which is clearly not the case). Also, it does not account for changes in the travel pattern after a disruptive event although studies show that this behavior can be considerably different from that exhibited before a disruptive event.

A.1.10. Restoration

**Inputs**:

- \(P^{0}_{od,t}\) – Time series of \(od\)-paths where no flow is possible\(^6\).
- \(\langle \lambda \rangle_{e,t}^{R}\) – Time series of the expected capacity reduction during restoration intervention for objects \(e\).
- \(\langle \tau \rangle_{e}\) – Time series of the expected restoration times [hours] for objects \(e\).

**Outputs**:

- \(R_t\) – Restoration program, defining when \((t)\), each damaged object \((e)\) is restored and the assigned work crew.

**Resources**:

- \(x_{e,0}\) – Initial traffic flow on the network.

\(^6\)The number of missed trips can be derived from this variable.
A.1. Modules

Process:
1. All edges with functional capacity losses greater than 10% were labelled as “objects in need of restoration”.
2. All edges with a need of restoration were ranked according some prioritization criteria. First, edges which disconnect parts of the network \( (e \in \mathcal{P}_{od}) \) were restored. Afterwards, edges with high initial traffic flows \( x_{e,0} \) were prioritized.
3. The expected restoration durations \( \max_t(\langle \tau \rangle_{e,t}^R) \) for the edges were assigned.
4. Available work crews were assigned to the top ranked edges. The capacity of the edges under restoration was set to \( \max_t(\langle \lambda \rangle_{e,t}^R) \).
5. Once an edge was restored (i.e., after a corresponding period \( \max_t(\langle \tau \rangle_{e,t}^R) \)), the edge removed from the list of “objects in need of restoration”.
6. A work crew was assigned to the next object in the ranking (iterate steps 4 through 6).

Assumptions and limitations:
• Objects were restored only if the capacity loss was greater than 10%; otherwise, it was assumed that the objects were restored during their normal maintenance process. Only one work crew could repair an object (i.e., multiple work crews working on the same object was not supported).

A.1.1. Direct and indirect costs

Direct costs
Inputs:
\( \langle c \rangle_{e,t} \) – Time series of the expected restoration costs [CHF] for objects \( e \).

Outputs:
\( C_{dc} \) – Expected direct cost for restoring damaged objects.

Process:
Only restoration costs were considered as direct costs. The overall expected direct costs \( C_{dc} \) was the sum of the expected direct costs for each intervention executed. It was assumed that the selected restoration program did not affect intervention costs.

\[
C_{dc} = \sum_e \max_t(\langle c \rangle_{n,t}) \quad (A.23)
\]

Cost estimates were based on Staubli and Hirt, (2005) and from a survey conducted by D’Ayala and Gehl, (2015). For each object type and damage state, a restoration strategy was derived, and for each strategy, cost and duration values were approximated (Table 3.4).

Assumptions and limitations:
• Costs taken from the literature were adjusted to 2017 price levels.
• Variable costs were only dependent on the length of the edge.
• To avoid over interpreting the specific values that were in the example, monetary units were used instead of real currency.

Indirect costs
Inputs:
\( \mathcal{P}_{od,t}^0 \) – Time series of \( od \)-paths where no flow is possible\(^7\).

\(^7\)The number of missed trips can be derived from this variable.
A. Computational Models

\( P_{od,t}^1 \) – Time series of od-paths where flow is possible.

\( t_{traffic}^{e,t} \) – Time series of travel time for each edge \( e \) in the network.

\( x_{e,t} \) – Time series of traffic flow for each edge \( e \) in the network.

**Outputs:**

\( C_{ic} \) – Expected indirect cost due to prolongation of travel time and missed trips.

**Resources:**

\( C_{ic} \) – Set of indirect cost parameters including: the value of travel time \( (\xi) \), mean fuel consumption \( F \), mean fuel price \( (\zeta) \), operating costs excluding fuel \( (\rho) \), and the value of a missed trip \( (\epsilon) \).

\( l_e \) – The length of each edge \( e \) in the network

\( t_{traffic}^{e,0} \) – The initial travel time on the network.

\( x_{e,0} \) – The initial traffic flow on the network.

**Process:** The indirect costs were comprised of costs for the temporal prolongation of travel and costs due to a loss of connectivity. The overall indirect costs \( C_{ic} \) were measured as the difference between indirect costs at time \( t \) and the indirect costs at time 0 when the network was fully functional.

\[
C_{ic} = \sum_t \left[ \sum_{e \in P_{od,t}^1} C_{ic,pt}^{e,t}(x_{e,t}) + C_{ic,lc}^{e,t}(P_{od,t}^0) \right] \tag{A.24}
\]

where \( C_{ic,pt} \) is a cost function dependent on the edge traffic flow \( x_{e,t} \) through edge \( e \) at time \( t \), where edge \( e \) is part of the set of feasible od-paths \( P_{od,t}^1 \) at time \( t \), and \( C_{ic,lc} \) is a cost function dependent on a loss of connectivity, which is determined based on the set of unfeasible paths \( P_{od,t}^0 \) at time \( t \).

**Temporal prolongation of travel** – The cost function attributed to traffic flow included sub-functions to estimate the costs related to travel time \( C_{ic,tt} \) and vehicle operation \( C_{ic,vo} \).

\[
C_{ic,pt}^{e,t}(x_{e,t}) = C_{ic,tt}^{e,t}(x_{e,t}) + C_{ic,vo}^{e,t}(x_{e,t}) \tag{A.25}
\]

**Travel time costs** – These costs were estimated based on the increased amount of time people spent travelling, which was linked directly to the flow on an edge.

\[
C_{ic,tt}^{e,t}(x_{e,t}) = (t_{traffic}^{e,t} \cdot x_{e,t} - t_{traffic}^{e,0} \cdot x_{e,0}) \cdot \xi \tag{A.26}
\]

where \( t_{traffic}^{e,t} \) is the travel time on edge \( e \) at time \( t \) in hours and \( \xi \) is the value of travel time. Based on the work of the Swiss Association of Road and Transport Experts (VSS, 2009b), \( \xi \) was assumed to be 23.02 CHF/hour per vehicle.

**Vehicle operation costs** – These costs were incurred as a result of fuel consumption and vehicle maintenance.

\[
C_{ic,vo}^{e,t}(x_{e,t}) = (x_{e,t} - x_{e,0}) \cdot l_e \cdot (\zeta \cdot F + \rho) \tag{A.27}
\]

where \( l_e \) is the length of edge \( e \), \( \zeta \) is the mean fuel price (1.88 CHF/litres), \( F \) is the mean fuel consumption (6.7 litres per 100 km per vehicle), and \( \rho \) is the operating cost without fuel (14.39 CHF/(100 · veh − km)) (VSS, 2009a).
**Loss of connectivity** – The costs due to a loss of connectivity were estimated based on the unsatisfied demand per time $t$ and the resulting costs due to a loss caused of the missed trips.

$$C^{lic,lc}(P^0_{od,t}) = \sum_{od} \sum_{P \in P^0_{od,t}} f_{od}(P) \cdot \epsilon$$

where $f_{od}$ is a function used to estimate the demand on any given path $P \in P^0_{od,t}$ for a specific origin-destination $od$, and $\epsilon$ is the monetary loss due to missed trips (i.e., cost of lost labour productivity per hour). The missed trip cost was assumed to be 83.27 CHF for every time step of simulation during the hazards event period and 666.16 CHF (i.e., eight times 83.27 CHF) for every simulation time step during the restoration period.

**Assumptions and limitations:**

- Business interruptions and other indirect costs were not considered.
b. Development of flood and mudflow events for the spatio-temporal risk assessment of networks

This chapter corresponds to the published article:¹


Abstract: Networks such as transport, water and power are critical lifelines to society. Network managers plan and execute interventions to guarantee their operational state under various circumstances, including after the occurrence of (natural) hazard events. Creating an intervention program demands knowing the probable network-related consequences (i.e., risk) of the various stochastic hazard events that could occur. The way such events are simulated has implications on (i) the overall computational cost of the entire risk assessment, which increases as the complexity of the network of interest increases, (ii) the accuracy of the individual risk estimations, as well as (iii) the quantified uncertainty of resulting risk estimations. To support network managers in their task to assess network-related risks, a method is presented here to develop rainfall-triggered hazard events, namely riverine flood events and mudflow events. The method enables the generation and simulation of hazard events that (i) are of a specific modeller-defined return period, enabling the characterisation of the uncertainty of risk estimation for given return periods, and (ii) change over space and time, leading to the spatio-temporal estimation of network-related risk. The method is designed for network managers, and therefore, integrates computationally-efficient models that can be quickly coupled, and require data that is generally available or can be easily obtained or estimated, without impacting the integrity of the results. An example is presented to illustrate the application of the method to develop flood and mudflow events to be used in the assessment of risk for a road network in Switzerland.

¹Please note, this is the author’s version of the manuscript published in the Journal European Water. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at http://www.ewra.net. When citing this chapter, please refer to the original article found in the reference above.

For reasons of consistency, the text has been amended to British English.
B. Hazard events

B.1. Introduction
Managers of transport, water, power and other networks need methods that support decisions on maintenance, repair, retrofit, renewal and other infrastructure management interventions. Since their networks are spatially distributed and have temporal attributes (e.g., traffic flow, water demand and electricity consumption vary throughout the day), managers understand that independent and dependent disruptive events could occur in one or multiple points within the network, and manifest at different times for various lengths. As a result, each possible scenario will lead to a different set of consequences in terms of restoration costs and loss of level of service. In particular, managers, whose networks are exposed to (natural) hazards, are in need of probabilistic methods that can support the quantification of probable consequences, here referred to as risks, along with their uncertainty, by considering the various stochastic ways hazard events could manifest.

A number of research works have put forward methods, models and support systems in response to these needs. These works cover a wide area of contributions, from those solely concerned with assessing the probable costs of restoration (e.g., Padgett et al., (2010) for the transport sector due to earthquake hazard events) to assessing the probable loss of service (e.g., Hackl et al., (2015b) for the transport sector due to flood hazard events). There are also works that have investigated the effects of cascading events (e.g., Kiremidjian et al., (2007) for the transport sector with respect to liquefaction and landslide hazard events due to ground shaking) and a large set of stochastic events (i.e., over 1000; e.g., de Riesgos Naturales – América Latina, (2012) for the water sector due to earthquake hazard events). Methods that integrate these various important elements are still needed to support the management of networks. Furthermore, there is a need to expand methods related to the assessment of consequences due to spatio-temporal hazard events such as hydro-meteorological hazards as a vast majority of the literature focuses on geological hazards, specifically earthquakes, which are often simulated as sudden events with only spatial characteristics.

The probabilistic method presented here supports the numerical estimation of risk while reducing the computational cost of modelling a complex system, which comprises the continuous, non-linear interaction between a network and its surrounding environment, using peer-reviewed simplified models. As a result, the method supports the simulation of a large set of hazard events and the subsequent quantification of the uncertainty of risk results. The method is solely concerned with rainfall-triggered hazard events, specifically riverine flood events as well as mudflow events, which are modelled as events that change over space and time. The application of the method is illustrated by an example of a road network in Switzerland.

B.2. Probabilistic method
The presented probabilistic method was constructed using the iterative process described in Adey et al., (2016). The detailed quantitative and computer-supported models used to estimate network-related risk are described in Hackl et al., (2016) and Heitzler et al., (2016). In this work, the focus lies on the *hazard module* (i.e., the modelling of the different rainfall-triggered events and their interactions over space and time).

The environmental conditions, whose data is generally available (e.g., terrain, hydrological network, gauge station data), can be approximated (e.g., precipi-
B.2. Probabilistic method

Model rainfall
Model runoff and discharge
Model flood
Spatio-temporal precipitation fields
Geological features
Gauge station data
Geological features
Hydrological network
Terrain

Determine suitability of rainfall-runoff-flood event
Suitable rainfall event
Suitable discharge event
Terrain changes
Landslide geometries
Intensity-duration function
Geological features
Hydrological network
Terrain

Model mudflow
Model damage
Suitable landslide event
Suitable flood event

Figure B.1: Hazard module for rainfall-triggered events.

Rainfall event needs calibration

Most network managers are mainly interested in the additional loads impacting their critical structures, and therefore, need to have the capability to select hazard events based on the periodicity of the manifested site-specific loads, as opposed to being primarily concerned with the preceding source rainfall events leading to those loads. The presented probabilistic method accounts for this particular need by enabling the selection of the hazard events based on the return period of interest related to loads at a specific site. This selection is achieved through a calibration process (Figure B.1). In this process, a random rainfall event is produced, and the corresponding discharge values are computed. Should the obtained discharge value at a location of interest be in the 50-percentile of the discharge values calculated for that specific location using gauge data and for the desired return period, the simulation continues; otherwise, the rainfall event gets calibrated, and the evaluation process starts again.

In order to provide a computationally-efficient and accurate estimation of risk, the structures of the models and of the modelling environment in which the models are embedded have to be adapted for the specific context and needs of a network manager. For example, the hazard module illustrated in Figure B.1 describes two hazard events: riverine flood events and mudflow events. Both hazards are triggered by a rainfall event, which causes additional discharge in the rivers and an increase in surface-water flow.

Once the models and data are assembled appropriately, the modelling environment can be used to perform simulations. The inputs for the simulations may be...
B. Hazard events

defined by the network manager or potentially automated when performing multiple runs (e.g., by sampling a specific distribution using Monte Carlo methods). The result of a simulation represents the outcome of a so-called scenario, which is one possible sequence of events based on a set of initial conditions.

The use of stochastic inputs allows expressing the uncertainties related to the environmental conditions. For example, the use of a random spatio-temporal precipitation field allows considering uncertainties related to the rainfall event. These uncertainties propagate through the entire hazard module and will be passed on to succeeding models (i.e., damage model and consequence models for costs of restoration and loss of level of service), influencing the estimated risks. Instead of single risk values for a specific return period, a distribution of risk can be obtained.

B.3. Example

B.3.1. Introduction

An example is included here to illustrate the application of the method described to determine the probable environmental conditions surrounding road networks and the resulting riverine flood events and mudflow events, for the Rhine Valley area around Chur, Switzerland in the Canton of Grisons (Figure B.3.b) This area was suspected to have an unacceptable level of road network-related risk due to riverine floods as well as mudflows. Historical records and previous studies suggested that the heavy rainfalls in this area have the potential to result in the listed hazard events. The road network in the area of study, which is considered to play an essential role in the economy of the eastern part of Switzerland, consists of 32 km of high-speed roads, 559 km of local roads, and 92 bridges, with many of these objects exposed to the hazards of interest. A detailed description of the area in the context of the example can be found in Hackl et al., (2016).

B.3.2. Application of method

To describe the environmental conditions and hydro-meteorological hazard events, the model shown in Figure B.1 was utilised. This model included five defined sets of activities:

1. The rainfall events were generated based on the spatio-temporal precipitation fields of the RdisaggH-Dataset (Wiest et al., 2010). This dataset was reduced to the extent of the target area and regridded to conform to the target extent and resolution. Based on a predetermined start date and duration, the corresponding subset from the spatio-temporal precipitation catalogue was extracted. The fields of this catalogue subset were then normalised according to the maximum rainfall amount over the considered time period. Afterwards, each normalised value was multiplied by a randomised precipitation volume. Once a precipitation volume was assigned to each dataset, the datasets themselves were normalised. By multiplying the assigned precipitation volume for a given time step to that precipitation field, the actual amounts of rainfall were determined.

2. The runoff volume was computed from the volume of rainfall that was considered to be intercepted, infiltrated, stored, evaporated, or transpired. Therefore, the ModClark model (Kull and Feldman, 1998) was used to estimate the discharge during the rainfall event. This model explicitly accounts for translation and storage, where the storage was determined through a
linear reservoir model. The translation was accounted for through a grid-based travel-time model. The grid was superimposed on the watershed. For each cell of the grid representation of the watershed, the distance to the watershed outlet was specified. Translation time to the outlet was computed. The area of each cell was specified, and from this, the volume of inflow to the linear reservoir for each time interval was computed as the product of area and rainfall excess. This excess was computed for each cell using the watershed data extracted from the GIS model. Each cell’s excess was then lagged to the basin outlet according to the cell’s travel time. Next, individual lagged cell outflows were routed through a linear reservoir, with a lag time due to natural storage effects. The lagged and routed outflows were then summed, base flow was added, and the watershed’s outlet hydrograph was produced.

3. The rainfall and the runoff model were used together for determining whether the resulting discharge scenario was corresponding to the discharge value of the desired return period, which was estimated based on available gauge data. If the desired return period was not achieved, the rainfall event was calibrated (i.e., up-scale, down-scale of predetermined spatio-temporal precipitation fields), and the analysis was performed again.

4. A simple 1D model was used and justified by the relatively large size of the investigated area (approximately 20 km) and the need of a model that would require relatively little computation time to run simulations. In particular, a steady and gradually varied flow in open channels was considered. The required discharge and stage data were acquired by the hydrological model, described above, using rainfall data as an input, rather than being measured by gauge stations. The outputs of interests were the velocity, shear stress, and water depth in each cross-section of the rivers, which could be used for estimating the effects on network objects such as bridges, ramps, and dams.

5. Also due to computational constraints, the design of the mudflow model was kept simple, considering only predetermined areas of potential landslides (Losey and Wehrli, 2013). In total, 54 potential mudflows were considered for the target area. Because the duration of landslides to occur usually is in the range of seconds to minutes while the time step of the simulation was one hour, the dynamic process itself was not modelled. For each time step, an iteration was performed over all triggering locations. For each location, the respective values were extracted from the precipitation field, and used as input for the intensity-duration function (Rickenmann, 1999): $T = 32 \cdot D^{-0.72}$. Where $T$ was the threshold that caused the mudflow to be potentially triggered when exceeded, and $D$ was the duration of the rainfall event in the triggering location. The output of this model for each time step was a digital elevation map (DEM), whose elevation values were updated according to the heights of the triggered landslides at a given time step.

The primary source of randomness was introduced by the directionality of the spatio-temporal precipitation fields and the scaling of these. In the considered scenarios, the initial condition that the network manager could freely choose was limited to the return period(s) of the flood. This feature was of particular importance to support scenario selection for infrastructure management applications.
B. HAZARD EVENTS

(e.g., assess the performance of a network with respect to a given return period) and the quantification of the uncertainty of probable consequences for the desired return period.

In total, 1,180 simulations were conducted: 20 simulations for each of the return periods between 1 and including 9 years as well as 100 simulations for each of the return periods of 10, 25, 50, 100, 250, 500, 1,000, 2,500, 5,000 and 10,000 years - such selection enables the generation of a loss exceedance curves (i.e., functional representations of cost of restoration or monetized loss of level of service over return periods). Depending on the return period, rainfall patterns with different durations were calculated. For return periods smaller than 100 years, the rainfall patterns had a duration of 7 to 13 hours, for periods between 100 and 1,000 years. Patterns had a duration between 10 and 16 hours, and for patterns greater than 1,000, they had a duration between 13 and 19 hours. The durations of the resulting flood events varied greatly and were in the range of 20 to 30 hours.

B.3.3. Software and hardware

The code used in the determination of the risk was programmed in Python. Since most network managers use geographic information systems (GIS), a GIS interface was developed to facilitate the import and export of data. Furthermore, the program code was optimised for massively parallel computing in order to reduce the computational time of the optimisation process (i.e., each simulation ran on a designated CPU, and therefore, an increase in the amount of CPUs, increases the number of simulations).

The computation of the scenarios was conducted on a 4×10 Core Intel Xenon E5-2690v2 3.0Ghz, 384GB DDR2 server running on Ubuntu 14.04 64bit operating system. Overall, the real-time computational time for all hazard events amounted to approximately two days.

B.3.4. Results

Figure B.2 shows how the rainfall, the flood and the mudflow events, and the consequences on the road network change over time for one specific simulation run. While the beginning of the simulation (hour 1) corresponds to a period before the hazard events occur, hours 11 and 21 correspond to periods during and at the end of the hazard events, respectively. In addition to the evolution of the flood event over time, the triggering of different mudflow events can be observed, which can be used in a later processing step to estimate the probable consequences on the network, which in this case are changes of free flow speed on the inundated and damaged road network.

The aggregated simulation results for a scenario with a return period of 500 years are illustrated in Figure B.3. Figure B.3.A shows the 25, 50 and 75-percentile of precipitation fields, where darker areas indicate more intense rainfall.

An overview of the area is given in Figure B.3.B. In this figure, the hazard events of interest are also presented, specifically the 5, 50 and 95-percentile of possible inundation depths and the location of possible mudflows colour-coded according to their probability of occurrence. Figure B.3.C zooms into a particular area. The expected discharge along the river is illustrated in Figure B.3.D. It can be observed that the, 5, 50 and 95-percentile values are approximately 1,800 m$^3$/s at section 30 because in the chosen scenario this value corresponds to the targeted discharge value of the 500-year flood event at the predefined gauging station located in that section.
B.3. **Example**

Figure B.2.: Changes over time for a specific simulation run.

Figure B.3.: Aggregated simulation for a return period of 500 years.
The probabilistic method introduced here supports the modelling of the environmental conditions and hazard events impacting networks. As defined here, hydro-meteorological hazards such as riverine flood events and mudflow events change over space and time. With the decomposition into peer-reviewed models, this complexity was reduced, which resulted in a decrease of the overall computational cost of the entire risk assessment and allowed to quantify the uncertainty of resulting risk estimations. Therefore, the network manager rely on computationally-efficient models that can be quickly coupled and require data that is generally available or can be easily obtained or estimated, without impacting the integrity of the results. When an increased level of detail is required of any model, this could be achieved at a computational cost.

The method was implemented in the context of an example that aimed at stochastically simulating spatio-temporal rainfall events that caused flood and mudflow events. The results of this application are of significance to risk assessments for networks. For example, besides temporary inundation, flood events can cause scour to selected bridges that play a crucial role in communicating two sides of a city. Also, mudflow events can cause permanent damage to road sections that are critical due to lack of redundancy (e.g., alternative routes).

Despite the virtues of the probabilistic method presented here, there is still work to be done in the development of flood and mudflow events to support network managers in calculating network related risk. Future work may focus on enhancing the hazard module described in the example to output refined results. This may include improving the determination of the time periods for the rainfall events and the intensity-duration function that triggers the mudflow events. Furthermore, given the need to determine the uncertainties related with the results, it is necessary to further explore the adequate distribution of the inputs and initial conditions. Further work could also consider an enhancement to account for the uncertain calculated discharge values in the calibration process.
c. MLSERN Model

This chapter corresponds to the appendix of the published article:¹


c.1. Connection functions

The connection function for a relative neighbourhood graph (RNG) of $V$ is defined as

$$g_{\text{RNG}} = \begin{cases} 1 & (B(u, w(u, v)) \cap B(v, w(u, v))) \cap V = \emptyset \\ 0 & \text{otherwise} \end{cases}$$

(C.1)

where $B(u, r) = \{ v : w(u, v) < r \}$ denotes an open sphere centred at $u$ with radius $r$. The connection function for a nearest-neighbour graph (NNG) of $V$ is defined as:

$$g_{\text{NNG}} = \begin{cases} 1 & v = \arg \min_{v \in V} (\text{dist}(u, v)) \\ 0 & \text{otherwise} \end{cases}$$

(C.2)

The connection function for a Gabriel graph (GG) of $V$ is defined as:

$$g_{\text{GG}} = \begin{cases} 1 & B \left( \frac{u+v}{2}, \frac{\text{dist}(u,v)}{2} \right) \cap V = \emptyset \\ 0 & \text{otherwise} \end{cases}$$

(C.3)

The connection function for a random geometric graph (RGG) of $V$, with a hard disk model, is defined as:

$$g_{\text{RGG}} = \begin{cases} 1 & \text{dist}(u, v) \leq r_0 \\ 0 & r_0 > 0 \end{cases}$$

(C.4)

The connection function for a Erdős–Rényi (ER) random graph $G(V, p)$ with $0 \leq p \leq 1$ is defined as:

$$g_{\text{ER}} = p$$

(C.5)

¹Please note, this is the author’s version of the manuscript published in the Journal of Complex Networks. Changes resulting from the publishing process, namely editing, corrections, final formatting for printed or online publication, and other modifications resulting from quality control procedures may have been subsequently added. The final publication is available at https://academic.oup.com. When citing this chapter, please, refer to the original article with DOI: 10.1093/comnet/cny019.

For reasons of consistency, the citations have been amended to author-year style.
C.2. Algorithms

C.2.1. Network generation

Algorithms

controlled by the following simulation parameters:

ones (move), which is accepted or not, depending on chance. Proposals are deleting some vertices (death), generating new ones (birth) or moving existing step a random proposal is made for a change in the current configuration by

The idea of this birth-death-move Metropolis-Hastings algorithm is that at every point process can be found in Møller et al. (Møller and Waagepetersen, 2003).
4. $q_b(y, \cdot)$ is the proposal density function for the location of the new vertex $(u, \alpha)$ with Hastings ratio:

$$
    r_b(y, (u, \alpha)) = \frac{h(y \cup (u, \alpha))}{h(y)} \cdot \frac{(1 - p(y \cup (u, \alpha))) \cdot q_d(y \cup (u, \alpha), (u, \alpha))}{p(y) \cdot q_b(y, (u, \alpha)) / p_L(\alpha)}
$$

(C.7)

5. $q_d(y, \cdot)$ is the probability for the proposal to delete the vertex $(u, \alpha)$ from $y$ with Hastings ratio:

$$
    r_d(y, (u, \alpha)) = \frac{h(y \setminus (u, \alpha))}{h(y)} \cdot \frac{p(y \setminus (u, \alpha)) \cdot q_b(y \setminus (u, \alpha), (u, \alpha)) / p_L(\alpha)}{(1 - p(y)) \cdot q_d(y, (u, \alpha))}
$$

(C.8)

where $p_L(\alpha)$ is a discrete or continuous density on $L$.

C.2.2. Model calibration

1. set $\theta_c \leftarrow \theta_0$, $P_c \leftarrow P(D|\theta_c)$ and $C \leftarrow \emptyset$

   /* Likelihood function */

2. Function $P(D|\theta)$:

   3. generate $y \leftarrow$ getNetworkMeasures(MLSERN($\theta$))

   4. $\chi^2 \leftarrow \sum_i \frac{(D(t_i) - y(t_i))^2}{\sigma^2}$

   5. return $\exp\{-\chi^2\}$

   /* MCMC parameter estimation */

6. for $i \in [0, 1, \ldots]$ do

   7. $\theta_p \leftarrow \theta_c + \sigma \cdot \text{Normal}([1, \ldots, n])$ and $P_p \leftarrow P(D|\theta_p)$ /* new proposal */

   8. generate $U \sim \text{Uniform}(0, 1)$

   9. if $U \leq P_p / P_c$ then /* acceptance criteria */

   10. $\theta_c \leftarrow \theta_p$

   11. $P_c \leftarrow P_p$

   12. $C_i \leftarrow \theta_c$

13. return $C$

A detailed description of the implemented algorithm for the parameter estimation can be found in Gelman et al. (Gelman et al., 2004). The idea is to find the best model parameters $\theta$ that fits the dataset $D$, by using Bayesian inference and model comparison, computed by a MCMC algorithm. As error function the normalized mean square error $\chi^2$ is used, where the $\sigma$ is some estimate of the error of the data. A Gaussian likelihood function $P(D|\theta) = \exp\{-\chi^2\}$ is implemented, which is maximal when the error is minimal. A Metropolis-Hastings algorithm is used to do a maximum likelihood estimation. A new set of proposed parameters $\theta_p$ is accepted with probability $\alpha_p = \min\{1, r_p\}$, where $r_p$ is the Hastings ratio, defined as $P(D|\theta_p) / P(D|\theta_c) = \exp\{-\chi_p^2 + \chi_c^2\}$.
C. MLSERN Model

Table C.1.: Estimated model parameters for the hierarchical road network.

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Parameter</th>
<th>Hierarchical road network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean</td>
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<tr>
<td>b</td>
<td>First order trend parameter</td>
<td>0.0415</td>
</tr>
<tr>
<td>c</td>
<td>Interaction parameter</td>
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<tr>
<td></td>
<td></td>
<td>[0.0733 0.0507 0.0733]</td>
</tr>
<tr>
<td>r</td>
<td>Interaction distance</td>
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<td>10352</td>
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<td></td>
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<td>N</td>
<td>Number of simulated vertices</td>
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</tr>
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<td>q</td>
<td>Ratio RNG/GG</td>
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<tr>
<td>p</td>
<td>Ratio det./prob. connection</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>function</td>
<td></td>
</tr>
</tbody>
</table>

Table C.2.: Relative errors of single-layer network measures.

| Sym. | Power L | | Road \langle \ell \rangle | | Hierarchical road \langle \ell \rangle | |
|------|---------| |-----------------| |-----------------|
|      | mean    | std | mean    | std | mean    | std |
| L    | -0.26   | 0.032 | -0.24   | 0.045 | 0.05   | 0.262 |
| \langle \ell \rangle | 0.05   | 0.104 | 0.11    | 0.110 | -0.02  | 0.136 |
| | -0.02   | 0.076 | -0.17   | 0.084 | -0.03  | 0.138 |
| | 0.04    | 0.083 | -0.22   | 0.083 | 0.00   | 0.147 |
| | 0.08    | 0.184 | 0.14    | 0.178 | 0.02   | 0.139 |
| | 0.01    | 0.140 | 0.10    | 0.158 | -0.03  | 0.125 |
| | 0.02    | 0.112 | -0.16   | 0.144 | 0.23   | 0.153 |
| | -0.01   | 0.103 | -0.00   | 0.181 | -0.06  | 0.156 |
| | 0.04    | 0.098 | -0.03   | 0.197 | 0.26   | 0.135 |
| | -0.24   | 0.060 | -0.03   | 0.062 | 0.02   | 0.093 |
d. Nomenclature

Chapter 3

DEM digital elevation model

General

B bridge object

c raster cell

e object

g gravitational acceleration

ℓ mudflow event

n section/subsection of an object

Φ standard normal cumulative distribution function

P probability

r river station

t time step

τ duration

Subscripts

c variable or function associated with raster cell c

cℓ variable or function associated with raster cell c where mudflow ℓ occurs

cn variable or function associated with raster cell c where section/subsection n is located

cr Variable or function associated with raster cell c located at the watershed outlet that corresponds to the river station r

e variable or function associated with the object e

i,j running variable (e.g., index of a river cross-section)

ℓ variable or function associated with mudflow ℓ

n variable or function associated with the section/subsection n

od variable or function associated with origin-destination od

r variable or function associated with river station r

rB variable or function associated with river station r near bridge B

si variable or function associated with damage state i

t variable or function at time step t

Supscripts

dc direct costs

ic indirect costs

in inflow

out outflow

Rainfall

Pt precipitation field at time t

Prain time series of precipitation fields over period τrain

Prain precipitation catalogue of historical events

pc,t precipitation intensity at raster cell c at time t

Train return period

τrain duration of the rainfall event

Runoff

CN field of runoff curve numbers

ΔtQ time interval of a hydrograph Q

drunoff travel distance from the most distant raster cell to the watershed outlet

dc runoff travel distance from raster cell c to the watershed outlet

finlc average inflow into raster cell c at time t

foutlc outflow of raster cell c at time t

pexcess accumulated precipitation excess for raster cell c at time t

Qr,0 base flow for a river station r
### Nomenclature

- \( Q_{r,t} \): hydrograph for river station \( r \) as a function of time \( t \)
- \( \varrho \): storage coefficient for linear reservoirs
- \( \tau_{\text{runoff}} \): duration of the runoff event
- \( t_{\text{runoff}} \): time of concentration for the watershed
- \( t_{\text{runoff}}^c \): lag (travel) time from raster cell \( c \) to the watershed outlet

#### Flood

- \( A_{i,t} \): wetted cross-sectional area at cross-section \( i \) at time \( t \)
- \( b_i \): channel width at cross-section \( i \)
- \( \gamma_i \): energy correction factor at cross-section \( i \)
- \( h_{i,t} \): water depth at cross-section \( i \) at time \( t \)
- \( I_t \): time series of inundation fields at time \( t \)
- \( i_{c,t} \): inundation depth at raster cell \( c \) at time \( t \)
- \( L_{i,i+1} \): channel reach length between cross-sections \( i \) and \( i+1 \)
- \( Q_{r_B,t} \): hydrograph for river station \( r \) near bridge \( B \) as a function of time \( t \)
- \( Q_{r=i,t} \): hydrograph for cross-section \( i \) as a function of time \( t \)
- \( S_{\text{friction},i,i+1} \): average friction slope between cross-sections \( i \) and \( i+1 \)
- \( S_{\text{friction}} \): friction slope between river cross-sections
- \( T_{\text{flood}} \): return period of the flood event
- \( v_{i,t} \): average flow velocity at cross-section \( i \) at time \( t \)
- \( z_i \): bed elevation with regard to the datum at cross-section \( i \)

#### Mudflow

- \( c^r \): cohesion of roots
- \( c^s \): cohesion of soil
- \( c^{s+r} \): cohesion of soil and roots
- \( \text{FS}_{\ell,t} \): factor of safety for mudflow \( \ell \) at time \( t \)
- \( \gamma^s \): specific weight of soil
- \( \gamma^w \): specific weight of water
- \( L \): set of potential mudflow locations and geometries
- \( \ell \): mudflow event
- \( L_t \): mudflow elevation field at time \( t \)
- \( \ell_{c,t} \): mudflow elevation at raster cell \( c \) at time \( t \)
- \( m_t \): fraction between water table depth and the soil depth at time \( t \)
- \( p_{\text{mudflow}} \): precipitation threshold for intensity-duration function at raster cell \( c \) for a period \( \tau \)
- \( p_{c,t} \): precipitation intensity at raster cell \( c \) at time \( t \)
- \( \phi \): angle of internal friction
- \( R_{\ell} \): runout length of mudflow \( \ell \)
- \( S_{\text{angle}} \): slope angle
- \( \tau_{c,t} \): duration of the rainfall event until time \( t \) at raster cell \( c \)
- \( V_\ell \): volume of mudflow \( \ell \)
- \( z^s \): soil depth
- \( z^w_t \): water table depth at time \( t \)

#### Object

- \( a_{e=B} \): pier width for bridge \( B \)
- \( D_{S_{i,t}} \): probability for damage state \( i \) at time \( t \)
- \( \text{DS} \): time series of damage state exceedance probabilities
- \( h_{e=B,r_B,t} \): scour depth for bridge \( B \) located near river station \( r \) at time \( t \)
- \( h_{r_B,t} \): flow depth directly upstream of the pier
- \( h_{\text{scour max}} \): critical scour depth
- \( \kappa \): corrective coefficients
- \( \lambda_{n,s_i} \): capacity reduction for section/subsection \( n \) given damage state \( i \)
- \( \langle \lambda \rangle_{n,t} \): expected capacity reduction for section/subsection \( n \) at time \( t \)
\(\mu_{s_i}\) logarithmic mean for damage state \(i\)
\(\langle \phi \rangle_{n,t}\) expected speed reduction for section/subsection \(n\) at time \(t\)
\(S\) realization of a damage state
\(s_i\) damage state \(i\)
\(\sigma_{s_i}\) logarithmic standard deviation for damage state \(i\)
\(v_{n,t}\) maximum acceptable velocity that ensures safe control of a vehicle through section/subsection \(n\) at time \(t\)
\(v_{rB,t}\) mean velocity of flow directly upstream of the pier
\(v_{\text{max}}\) maximum allowed speed on section/subsection \(n\)

**Network**
\(E\) set of edges in the network graph
\(G\) routable network graph
\(\langle \lambda \rangle_{e,t}\) expected capacity reduction for object \(e\) at time \(t\)
\(\langle c \rangle_{e,t}\) expected costs of the intervention for object \(e\) at time \(t\)
\(l_e\) length of edge \(e\)
\(\langle \phi \rangle_{e,t}\) expected speed reduction for object \(e\) at time \(t\)
\(\langle \tau \rangle_{e,t}\) expected restoration time for object \(e\) at time \(t\)
\(V\) set of vertices in the network graph

**Restoration**
\(\langle c \rangle_{n,t}^R\) expected costs of the intervention for section/subsection \(n\) at time \(t\)
\(R_{\text{fix}}^{n,s_i}\) fixed intervention costs for section/subsection \(n\) given damage state \(i\)
\(R_{\text{var}}^{n,s_i}\) variable intervention costs for section/subsection \(n\) given damage state \(i\)
\(C^{dc}\) set of direct cost parameters
\(\lambda_{n,s_i}^R\) capacity reduction during restoration for section/subsection \(n\) given damage state \(i\)
\(\langle \lambda \rangle_{n,t}^R\) expected capacity reduction during restoration for section/subsection \(n\) at time \(t\)
\(R_t\) restoration program
\(\langle \tau \rangle_{n,t}\) expected restoration time for section/subsection \(n\) at time \(t\)
\(\tau_{n,s_i}^R\) restoration time for section/subsection \(n\) given damage state \(i\)

**Traffic**
\(\alpha_e, \beta_e\) calibration parameters for traffic estimation through edge \(e\)
\(C^{\text{traffic}}\) cost function to estimate travel time
\(d_{od}\) demand for a given od-path
\(f_{od}(P)\) function to estimate traffic flow on path \(P\) that connects origin-destination \(od\)
\(od\) origin-destination pair
\(P\) path
\(\mathbb{P}_{od,t}^0\) set of unfeasible od-paths at time \(t\)
\(\mathbb{P}_{od,t}^1\) set of feasible od-paths at time \(t\)
\(t_{\text{traffic}}^{e,t}\) travel time through edge \(e\) at time \(t\)
\(t_{\text{traffic}}^{e,0}\) initial free flow travel time
\(x_{e,t}\) traffic flow through edge \(e\) at time \(t\)
\(y_{e,0}\) initial edge capacity

**Costs**
\(C^{\text{dc}}\) expected direct cost
\(C^{ic}\) set of indirect cost parameters
\(C^{ic}\) expected indirect cost
\(C^{ic,lc}\) cost function for loss of connectivity
\(C^{ic,pt}\) cost function for prolongation of travel time
\(C^{ic,tt}\) cost function for travel time
\(C^{ic,vo}\) cost function for vehicle operation
\(\epsilon\) value of a missed trip
\(F\) mean fuel consumption
Nomenclature

\( \rho \) operating costs excluding fuel
\( \xi \) value of travel time
\( \zeta \) mean fuel price

Chapter 4
\( \alpha, \beta, \gamma \) parameters
\( C^{DC} \) direct costs
\( C^{IC} \) indirect costs
\( C_{n,i} \) intervention costs for object \( n \) due to intervention \( i \)
\( C_{T} \) travel cost function for edge \( e \)
\( D \) set of destination or demand vertices
\( \delta_{n,i,t} \) binary variable, which has a value of 1 if an intervention \( i \) is executed on object \( n \), initiated at period \( t \) and 0 otherwise
\( d_{od} \) flow demand between origin \( o \) and destination \( d \)
\( \Delta y_{n,i} \) restored capacity of \( n \) due to intervention \( i \)
\( \Delta Z \) change in value of the objective function
\( E \) set of edges of the network
\( \epsilon_{n,i} \) fixed costs of intervention \( i \) on object \( n \)
\( e \) edge in the network
\( f_{od}(P) \) path flow between origin \( o \) and destination \( d \), on path \( P \)
\( F_w \) mean fuel consumption depending on the vehicle type \( w \)
\( \Gamma(v) \) set of incident edges of vertex \( v \)
\( G \) transportation network graph
\( g \) state of complete damage \( g \subseteq s \)
\( \eta_{n,i,k} \) resource related costs of resource \( k \) for object \( n \) due to intervention \( i \)
\( H \) demand graph
\( i \) intervention
\( \kappa \) decay rate
\( \ell_{e} \) length of edge \( e \)
\( \Lambda \) cost function for loss of connection

\( \mu_{e,w} \) proportion of vehicles of type \( w \) on edge \( e \)
\( \mathbb{N}^s \) set of usable objects in state \( s \in S \)
\( \nu \) mean fuel price
\( n \) object
\( \mathcal{O} \) set of origin or supply vertices
\( \Omega_t \) available budget in period \( t \)
\( od \) demand from \( o \) to \( d \)
\( \mathbb{P} \) set of all non-empty paths
\( \Pi \) cost function for prolongation of travel
\( P \) specific path in the network
\( \mathbb{P}^{g}_{od} \) set of all disconnected \( od \)-paths
\( \mathbb{P}^{od}_{od} \) set of all \( od \)-paths
\( \mathbb{P}^{s,g}_{od} \) set of all usable \( od \)-paths
\( P(X) \) penalty function
\( \rho_w \) operating costs (without fuel) for a vehicle of a specific type
\( r_{n,i,k} \) resource requirement for resource \( k \) on object \( n \) due to intervention \( i \)
\( s \) state
\( \tau_{n,i} \) intervention time for intervention \( i \) on object \( n \)
\( T \) control parameter (temperature)
\( t \) time period
\( t_{e,t}^0 \) free flow travel time at edge \( e \)
\( t_{e,t} \) travel time at edge \( e \) in period \( t \)
\( \upsilon \) labor productivity
\( u, v, o, d \) vertices in the network
\( \mathcal{V} \) set of vertices of the network
\( \mathcal{W} \) set of all considered vehicle types
\( w \) vehicle type
\( \xi_{w} \) value of travel for a vehicle of type \( w \) on edge \( e \)
\( X \) state of the variables of \( Z \)
\( x_{e,t} \) link flow on edge \( e \) in period \( t \)
\( \psi_{k,t} \) available resource \( k \) in period \( t \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>functions of the constraints of $Z$</td>
</tr>
<tr>
<td>$y_e$</td>
<td>capacity of edge $e$</td>
</tr>
<tr>
<td>$y_{n,t}$</td>
<td>capacity of $n$ in period $t$</td>
</tr>
<tr>
<td>$\zeta_{n,i}$</td>
<td>variable costs of intervention $i$ on object $n$</td>
</tr>
<tr>
<td>$Z$</td>
<td>objective function</td>
</tr>
<tr>
<td>$Z^R$</td>
<td>objective function for the restoration problem</td>
</tr>
<tr>
<td>$Z^T$</td>
<td>objective function for the user equilibrium assignment</td>
</tr>
<tr>
<td>$\sim$</td>
<td>neighbourhood relation</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>layers</td>
</tr>
<tr>
<td>$A$</td>
<td>$d$-dimensional space</td>
</tr>
<tr>
<td>$A_{\subset r}$</td>
<td>study region with buffer of distance $r$</td>
</tr>
<tr>
<td>$B$</td>
<td>Borel set</td>
</tr>
<tr>
<td>$B(u, r)$</td>
<td>open sphere centred at $u$ with radius $r$</td>
</tr>
<tr>
<td>$b_\alpha$</td>
<td>first order trend parameter</td>
</tr>
<tr>
<td>$\langle C \rangle$</td>
<td>average clustering coefficient</td>
</tr>
<tr>
<td>$C$</td>
<td>piecewise smooth curve</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>error function</td>
</tr>
<tr>
<td>$c_{\alpha\beta}$</td>
<td>interaction parameter</td>
</tr>
<tr>
<td>$D$</td>
<td>data set</td>
</tr>
<tr>
<td>$\mathcal{D}$</td>
<td>network diameter</td>
</tr>
<tr>
<td>$\text{dist}(\cdot, \cdot)$</td>
<td>Euclidean distance</td>
</tr>
<tr>
<td>$E_M$</td>
<td>multiset of edges</td>
</tr>
<tr>
<td>$E_{\alpha\beta}$</td>
<td>set of inter-layer edges</td>
</tr>
<tr>
<td>$E_{\alpha}, E_{\alpha\alpha}$</td>
<td>set of intra-layer edges</td>
</tr>
<tr>
<td>$\text{ER}$</td>
<td>Erdős-Rényi (ER) random graph</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>set of all considered events</td>
</tr>
<tr>
<td>$\phi$</td>
<td>interaction function</td>
</tr>
<tr>
<td>$f$</td>
<td>probability density</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>hybrid connection function</td>
</tr>
<tr>
<td>$g$</td>
<td>matrix of connection functions</td>
</tr>
<tr>
<td>$G_\alpha$</td>
<td>intra-layer network</td>
</tr>
<tr>
<td>$G_M$</td>
<td>multi-layer network</td>
</tr>
<tr>
<td>$g_{\alpha\beta}$</td>
<td>connection function</td>
</tr>
<tr>
<td>$\text{GG}$</td>
<td>Gabriel graph</td>
</tr>
<tr>
<td>$h$</td>
<td>hierarchical road network layer</td>
</tr>
<tr>
<td>$h_1$</td>
<td>primary road layer</td>
</tr>
<tr>
<td>$h_{11}$</td>
<td>primary - primary road intersection layer</td>
</tr>
<tr>
<td>$h_{12}$</td>
<td>primary - secondary road intersection layer</td>
</tr>
<tr>
<td>$h_2$</td>
<td>secondary road layer</td>
</tr>
<tr>
<td>$h_{22}$</td>
<td>secondary - secondary road intersection layer</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>weighting probability</td>
</tr>
<tr>
<td>$\langle \ell \rangle$</td>
<td>average shortest path length</td>
</tr>
<tr>
<td>$\langle \ell \rangle^w$</td>
<td>weighted average shortest path length</td>
</tr>
<tr>
<td>$\ell(\cdot)$</td>
<td>probability distribution on the Borel $\sigma$-algebra of $L$</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>total length of the edges</td>
</tr>
<tr>
<td>$\lambda((u, \alpha), y)$</td>
<td>conditional intensity</td>
</tr>
<tr>
<td>$L$</td>
<td>set of layers</td>
</tr>
<tr>
<td>$\mathcal{M}(Y, g)$</td>
<td>connection model</td>
</tr>
<tr>
<td>$\nu(\cdot)$</td>
<td>finite, non-atomic Borel measure</td>
</tr>
<tr>
<td>$n(\cdot)$</td>
<td>number of realizations</td>
</tr>
<tr>
<td>$N_{(v, \beta)}$</td>
<td>neighbourhood of $(v, \beta)$</td>
</tr>
<tr>
<td>NNG</td>
<td>nearest-neighbour graph</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>set of all possible realizations</td>
</tr>
<tr>
<td>$O_Y$</td>
<td>metric space</td>
</tr>
<tr>
<td>$\mathbb{P}$</td>
<td>probability measure</td>
</tr>
<tr>
<td>$p$</td>
<td>power grid layer</td>
</tr>
<tr>
<td>$p(y)$</td>
<td>birth probability</td>
</tr>
<tr>
<td>$p_\alpha$</td>
<td>ratio between deterministic and probabilistic connection functions</td>
</tr>
<tr>
<td>$p_L(\alpha)$</td>
<td>probability density on $L$</td>
</tr>
<tr>
<td>$p_m$</td>
<td>move probability</td>
</tr>
<tr>
<td>$\pi$</td>
<td>acceptance probability</td>
</tr>
<tr>
<td>$\pi_{AXL}$</td>
<td>distribution of a Poisson process</td>
</tr>
<tr>
<td>$q_{\alpha}$</td>
<td>ratio between RNGs and GGs</td>
</tr>
<tr>
<td>$q_b(y, \cdot)$</td>
<td>proposal density function to create a new vertex</td>
</tr>
<tr>
<td>$q_d(y, \cdot)$</td>
<td>proposal density function to delete a vertex</td>
</tr>
</tbody>
</table>
Nomenclature

$q_m(y, \cdot)$ proposal density function to move a vertex
$\mathcal{R}$ network radius
$\rho$ degree assortativity
$\rho_{\alpha\beta}$ inter-layer degree correlation
$r$ road network layer
$r$ Hastings ratio
$r_{\alpha\beta}$ interaction radius
RGG random geometric graph
RNG relative neighbourhood graph
$\sigma$ standard deviation
$S$ complete separable metric space
$s_r(y)$ number of unordered neighbour pairs of distinct vertices in $y$ which lie closer than $r$ units apart
$\tau$ transitivity
$\theta$ model parameter
$t((u, \alpha), y)$ number of vertices in $y$ that lie within a distance $r$ of location $u$
$(u, \alpha), (v, \beta)$ multi-layer vertices
$u, v, \cdot$ vertices (positions)
$V_{\alpha}$ set of intra-layer vertices
$V_M$ multiset of vertices
$w(\cdot, \cdot, Z)$ weighted distance
$y, z$ realization of a marked point process
$Y$ marked point process
$Z$ spatial field

Chapter 7

$A$ adjacency matrix
$\alpha, \beta, \gamma, \delta$ layers of the network
$a_{ij}$ entry of the adjacency matrix
$a_e, b_e$ parameters
$c_e$ edge costs
$d$ destination vertex
$d_r(\cdot, \cdot)$ relative distance operator
$\bar{E}$ set of interrupted edges
$E$ set of edges of the network
$e$ edges in the network

$f_p$ path flow
$\mathcal{G}$ set of graphs
$G$ network graph
$g$ connection function
$I$ set of indices
$\Xi$ incidence matrix
$i_{ij}$ entry of the incidence matrix
$L$ set of layers
$\ell_e$ edge length
$\mathcal{M}$ connection model
$m$ number of steps
$n$ number of vertices
$n(\cdot)$ cardinality operator
$o$ origin vertex
$\mathbb{P}$ set of all nonempty paths
$\mathbb{P}_{od}$ set of all $od$-paths
$p$ specific path in the network
$\pi$ stationary distribution
$q_{od}$ traffic demand between $o$ and $d$
$\sigma$ standard deviation
$T$ transition matrix
$t_{ij}$ entry of the transition matrix
$t_e$ edge travel time
$\mathcal{V}$ set of vertices of the network
$v, o, d$ vertices in the network
$\omega$ incremental traffic flow
$W$ weight function
$X$ set of basic elements
$X$ random position in the network
$x_e$ edge traffic flow
$\mathcal{Y}$ set of hypervertices
$y$ hypervertex
$y_e$ edge capacity
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