Modelling of rocking structures: Are our models good enough?

J.A. Bachmann, M.F. Vassiliou¹, M. Broccardo, B. Stojadinovic
ETH Zurich, Switzerland

M. Strand
NTNU, Tondheim, Norway

ABSTRACT

Numerical models developed to predict the seismic response of structures deforming into the inelastic range often fail the conventional validation test: they fail to reproduce the experimentally obtained response to a particular ground motion with acceptable accuracy. This paper claims that this is too strict of a test for seismic response modelling. The Earthquake Engineering design problem involves predicting the statistics of the response to an ensemble of ground motions characterizing a given seismic hazard; not to a single ground motion. This is a weaker model validation test that requires that the structural model to only to be unbiased and to introduce less uncertainty than the uncertainty of the excitation itself. In this paper, we propose a weak form of seismic response analysis model validation. As an example, we use the 1963 Housner dynamic response model of a rocking structure. We performed 600 shaking table seismic response tests using a well-defined and repeatable rocking structure as well as 600 numerical simulations of these tests, and compared both the individual test response and the statistical aggregates of these response focused on predicting limit states such as overturning or maximum tilt angle. We show that the 1963 Housner model passes the weak validation test even though it fails the strong validation test. Therefore, the model is good enough for the scope of Earthquake Engineering. In conclusion, we propose broader application of weak validation models and outline possible acceptance criteria consistent with probabilistic seismic performance objectives for design and evaluation of structures.

Keywords: rocking structures, model validation, uncertainty quantification, structural modelling

INTRODUCTION

Since the publication of Housner's seminal paper (Housner, 1963), the rocking block has drawn the attention of many researchers and engineers. Rocking behavior has been studied to address, for example: a) large structures that use uplift and rocking as a seismic response modification technique (Deng et al. 2012, Makris and Vassiliou 2013 and 2014, Vassiliou et al. 2014 and 2017a, Kokkali et al 2015, Dimitrakopoulos and Paraskeva, 2015, Dimitrakopoulos and Giouvanidis 2015, Vassiliou and Makris 2015, Acikgoz and DeJong 2017, Agalianos et al. 2017, Bachmann et al 2017a, Giouvanidis and Dimitrakopoulos 2017a, Vassiliou 2018); b) precious museum artifacts and unanchored equipment whose seismic overturning stability is important (Di Egidio and Contento 2009, Chiozzi et al. 2015, Wittich and Hutchinson 2015 and 2017, Dar et al. 2016, Ibarra et al. 2016, Sextos et al. 2017a,b, Voyagaki et al. 2018), c) ancient Greek and Roman temples (Konstantinidis and Makris 2005), and d) masonry arches and walls (Dimitrakopoulos and DeJong 2013, Tondelli et al. 2016).

¹ Corresponding Author: M.F. Vassiliou, ETH Zurich, vassiliou@ibk.baug.ethz.
As the response of the rocking block is highly non-linear because of its negative post-uplift stiffness, some researchers have treated the rocking block as a chaotic system (Lin and Yim 1996), in the sense that small perturbations of its governing parameters (be it the block properties, initial conditions, or the excitation) result in widely diverging outcomes. Experiments to validate Housner's model have shown that, given the modelling uncertainty, especially of the one related to impact, it is difficult to confidently predict the time history response of a specific rocking block to a specific ground motion. Even if the coefficient of restitution is estimated accurately, or determined experimentally, the rocking block is so sensitive that predicting the entire seismic response time history is practically impossible (Truniger et al. 2015). This has generated a discussion on Housner's approach to determine the energy losses during impact and has resulted in various different energy dissipation models (Vassiliou et al. 2015, Kalliontzis et al. 2016, Chatzis at al. 2017, Giouvanidis and Dimitrakopoulos 2017b).

An accurate model prediction of the dynamic response of a rocking block prototype to a single ground motion is a sufficient, but not a necessary condition to validate, and ultimately trust, the model. Validation is a process of determining whether a verified model accurately represents the observed behavior of the prototype, preferably obtained from controlled experiments. In the presence of uncertainties (in the ground motion excitation, in the coefficient of restitution, and in the model), an objective approach to model validation necessarily involves a statistical comparison of probability distributions of response quantities that characterize the seismic response of the rocking block prototype and its models.

In this paper, a statistical approach is employed to validate the 1963 Housner numerical model of a rigid rocking block against experimentally obtained response data. It is notable that Yim et al. (1980) pioneered the probabilistic approach to modeling the response of a rocking block. Using synthetic ground motion excitation and the 1963 Housner rocking block model, they observed order in the response of the rocking block and indicated that it could be predicted in a statistical sense.

**NUMERICAL MODEL OF THE ROCKING BLOCK**

With reference to Figure 1 and assuming that the coefficient of friction is large enough to prevent sliding, the equation of motion of a free-standing rigid block of size $R$ and slenderness $\alpha = \tan(B/H)$ measured using the tilt angle $\theta$ subjected to a horizontal ground acceleration $\ddot{u}_g(t)$ rocking in plane about pivot points O and O’, is (Dimitrakopoulos and Fung 2016 among others):

$$
\ddot{\theta}(t) = -p^2 \left( \sin[\alpha \text{sgn} \theta(t) - \theta(t)] + \frac{\ddot{u}_g}{g} \cos[\alpha \text{sgn} \theta(t) - \theta(t)] \right)
$$

(1)

where $p$ is the frequency parameter of the rigid rocking block defined as $p = \sqrt{\frac{mgR}{I_0}}$.

Housner (1963) assumed that the impact is instantaneous and that the impact forces are concentrated at the pivot points O and O’. Under these two assumptions, conservation of angular momentum an instant before the impact and immediately after the impact gives the coefficient of restitution:

$$
r = \left( 1 - \frac{2mR^2}{I_0} \sin^2 \alpha \right)^2
$$

(2)

**SHAKING TABLE TESTS**

A series of shaking table tests was conducted to measure the in-plane response of a virtually rigid rocking specimen to earthquake-like ground motion excitation (Bachmann et al. 2017b). A rocking specimen (Figure 2, left) consists of four wedge plates (Nr. 1), two linking plates (Nr. 2) and two hollow columns (Nr. 3). All specimen parts were made of aluminum. The specimen's height is 500mm and its depth 400mm. This shape of the specimen was chosen in order to keep the rocking motion in plane. The wedges used in the test series
presented in this manuscript had a total width of 82mm, but the effective rocking width (distance between the pivot points O and O') was 75mm resulting in an effective slenderness of \( \tan \alpha = 0.15 \). The eigenperiod of the specimen, corresponding to both columns bending in phase out of the specimen's plane, was measured to be 0.04s.

The model examined in this study represents structures that are designed to rock, meaning that their contact with the support surface is designed such that the pivot points are well defined and the impact forces are concentrated as close to the corner as possible to decrease the uncertainty concerning the position of the impact forces. However, even if the pivot points are well defined and the coefficient of restitution is accurately estimated using measured data, the response of a rocking specimen is so sensitive to the initial conditions and the contact surface imperfections that the outcomes of individual experiments are often not repeatable (Bachmann et al. 2016).

Free rocking motion tests were performed to establish the frequency parameter \( p \) and the coefficient of restitution \( r \) of the numerical model of the specimen. Figure 3 plots the measured free rocking response of the specimen and the response obtained using the numerical model with the frequency parameter \( p \) and the coefficient of restitution \( r \) obtained from response measurements. Notably, the two response histories compare well for more than 20 rocking cycles. The measured and the Housner restitution coefficient values differ by a relative error of less than 1%. However, as shown in a next section, even such small discrepancies between the numerical model and the experimentally tested specimen lead to large errors in predicting the response of the rocking specimen to a single ground motion excitation.

The rocking response of the specimen was induced by a dynamic excitation of its support. This was achieved by placing the specimen on the top of the ETH shaking table (Figure 2, right). The movement of the rocking specimen during the tests was measured using an NDI infrared-emitting diodes with an accuracy of 0.1mm at a sampling frequency of 500Hz. This corresponds to a tilt angle accuracy of 0.8mrad, which equals 0.53% of the rocking specimen slenderness \( \alpha \).

Figure 1. Rigid block rocking freely in plane on a rigid surface

Figure 2. Left: Rocking specimen; Middle: Close-up of wedges (units [mm]) Right: Specimen on the ETH unidirectional shaking table
SYNTHETIC GROUND MOTION ENSEMBLES

To constrain the uncertainty of the ground motion excitation, the ensemble of ground motions used in the shaking table tests and the subsequent numerical model response analyses was synthesized using a spectral version of the Rezaeian and Der Kiureghian stochastic ground motion model (Razaeian and Der Kiureghian 2008, Broccardo and Der Kiureghian 2014, Broccardo and Dabaghi 2017). We define the generated synthetic motions as “target-spectrum-consistent”. A stochastic method to generate ground motions with similar elastic response spectra was adopted as the best-available precondition for a statistical comparison of the numerical and experimental rocking block responses, even though motions with similar elastic spectra could have very different overturning potential (Vassiliou et al. 2017b).

Two recorded ground motions, the longitudinal component of the 1940 El Centro Array #9 record and the transverse component of the 2003 Lefkada record, were used as the "seed" ground motions. Two ground motion ensembles, each with 100 synthetic ground motions, were generated from the two seed ground motions. More details on the ground motions produced, are given in Bachmann et al. (2017).

Scaling of the ground motion ensembles

Changing the size (length scale $S_L$) of the rocking block affects its response the same as changing the squared frequency of the ground motion excitation in the opposite direction. To facilitate an experimental investigation of multiple prototype sizes using the same specimen, the ground motions used to excite the specimen were scaled in time by the time scale factor $T_L = S_L$, while preserving the acceleration scale.

Three different prototype rocking blocks, with heights $2H$ equal to 5m, 10m, and 20m (in the prototype scale), were tested. Given that in the specimen height $2H$ is 0.5m, the ground motions in the two synthetic ensembles were time-scaled by a factor of $\sqrt{10}$, $\sqrt{20}$, and $\sqrt{40}$, respectively (i.e. the frequency of the ground motions was increased without changing the amplitude scale).

The ETH shaking table used to conduct the rocking response tests is not perfect: the dynamics of the table further modifies the motion that is applied at the base of the rocking specimen. For consistency, the measured (as-applied) ground motions are used as excitation to compute the response of the numerical model of the rocking block.

DETERMINISTIC ROCKING RESPONSE HISTORY COMPARISON

Figure 4 compares the tilt angle time histories of rocking blocks with heights $2H$ equal to 5m, 10m, and 20m, obtained in shaking table tests, to the ones obtained using numerical models excited by the same measured (as-applied) time-scaled ground motions randomly picked from the ensembles (one for El Centro and one for Lefkada). Two numerical models, one with the 1963 Housner restitution coefficient (equation (3)) and the
Figure 4. Deterministic comparison of the tilt angle response time histories to time-scaled recorded 1940 El Centro and 2003 Lefkada target-spectrum-consistent ground motions obtained from an experiment and from 2 numerical models.

Figure 5. Comparison between the normalized maxima of the tilt angle time histories for the 1940 El Centro (top) and the 2003 Lefkada (bottom) synthetic ground motion ensemble: experimental vs numerical ($r = r_{exp}$).
other with a restitution coefficient measured in free rocking tests (Figure 3 (left)) were run. In some cases, the two models are capable of matching the measured response time history but, generally, they fail to do so. Figure 5 compares the normalized maxima of the tilt angle time histories, $\theta/\alpha$, obtained by shaking table tests, to the corresponding maxima of the numerical models (using the measured coefficient of restitution) for all ground motions and all heights $2H$. Even though there is no clear bias, it is clear that a deterministic prediction is not possible and oftentimes the error is large.

**STATISTICAL COMPARISON OF ROCKING RESPONSE QUANTITIES**

The experimental response data were obtained by conducting a total of 600 shaking table rocking block response tests. The response of $2H=5m$, $2H=10m$ and $2H=20m$ rocking blocks excited by each one of the hundred ground motions in the two target-spectrum-consistent ensembles was recorded. Using Housner’s model (i.e. Eq. 1), a numerical analysis was made for each prototype size and two restitution coefficient values, one measured and one theoretically computed (Equation (3)).

Moreover, in order to test the method proposed by Priestley et al. (1978) we also computed the response using their elastic spectrum based method, even though Makris and Konstantinidis (2003) have claimed that the method is fundamentally flawed. The reason for using this approach is to check whether the method can pass the weaker test of statistical validation, even though it fails the stronger test of deterministic comparison.

The responses of the tested and the numerically modelled rocking blocks are compared in terms of the maximum rocking body tilt angle (tilt angle) $\theta_{\text{max}}$, obtained in response to the same measured (as-applied) ground motion excitation.

![Figure 6](image-url). Cumulative distribution functions of the normalized maximum tilt angle $\theta_{\text{max}}/\alpha$ for the 1940 El Centro (top) and 2003 Lefkada (bottom) synthetic ground motion ensembles.
Explanatory Data Analysis

Following Yim et al. (1980), the maximum tilt angle $\theta_{\text{max}}$ normalized by the rocking block slenderness $\alpha$, $(\theta_{\text{max}}/\alpha)$ data from the shaking table tests and from response analyses using numerical models with the 1963 Housner model and the measured coefficient of restitution values are arranged in ascending order and plotted in the form of empirical cumulative distribution functions (CDFs) in Figure 6. This plot shows the probability that the maximum tilt angle of a rocking block is smaller or equal to a specific value of $\theta_{\text{max}}/\alpha$. In the same graph, the 90 and 95% nonparametric Confidence Intervals (CI) (Wasserman 2013) are reported for the experimental CDF. A CI of 90% for the experimental empirical CDF means that if the experiments were performed with a different ensemble of hundred simulated ground motions there is a 90% probability that the new empirical CDF would be contained in this CI. Observe that the experimental CDF discontinuities, which is the probability of the overturning event. The following conclusions are drawn from the data in Figure 6 and from Table 1:

Comparison with Housner’s model

- There is no systematic bias between the experimentally obtained empirical CDFs and numerically obtained empirical CDF. The non-exceedance probability obtained from the numerical models is larger than that obtained from prototype tests for some values of the normalized tilt angle and smaller for others (Figure 6). The same pattern emerges from the relative error data in Table 1.

- The numerical CDF curves for the $2H = 5m$ and $2H = 10m$ rocking blocks are well inside the confidence intervals (CIs) of the experimental CDFs. The exception are the numerical CDFs for the $2H = 20m$ rocking blocks where both numerical CDFs are outside the 90% and 95% CIs for small values of normalized tilt angles. In the context of earthquake engineering, these data indicate that the 1963 Housner numerical models (with the theoretical as well as with the measured restitution coefficient) can predict the statistical distribution of the maximum response of a rocking block with sufficient accuracy.

- The ground motion variability, even within a single target-spectrum-consistent ground motion ensemble, overshadows both the numerical model error and the chaotic behavior arising from the large sensitivity of rocking response. Ground motion uncertainty is expected to be even more dominant when the source and site variability is included.

- For both ground motion sets and for $2H=5m$ and $2H=10m$ rocking blocks, the maximum error in predicting the 25th, the 50th, and the 75th percentile of the tilt angle $\theta_{\text{max}}$ is 16% (Table 1, 2003 Lefkada, $2H=5m$), with most other relative error values less than 10%.

- The relative distance between two CDFs is lower when the experimentally measured coefficient of restitution is used in place of the 1963 Housner theoretical value. Distance refers to the Kolmogorov distance between the two CDFs. The distance between the two numerical models is more pronounced for the smallest ($2H=5m$) rocking block.

### Table 1. Comparison of the rocking block maximum tilt angle relative errors from the prototype tests and the numerical model with the measured coefficient of restitution.

<table>
<thead>
<tr>
<th>$2H$</th>
<th>El Centro</th>
<th></th>
<th></th>
<th>Lefkada</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_{\text{num}}$</td>
<td>$\theta_{\text{exp}}$</td>
<td>$\Delta\theta/\theta_{\text{exp}}$</td>
<td>$\theta_{\text{num}}$</td>
<td>$\theta_{\text{exp}}$</td>
<td>$\Delta\theta/\theta_{\text{exp}}$</td>
<td></td>
</tr>
<tr>
<td>5m</td>
<td>25th percentile</td>
<td>0.430</td>
<td>0.360</td>
<td>0.194</td>
<td>0.600</td>
<td>0.550</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>50th percentile</td>
<td>0.710</td>
<td>0.650</td>
<td>0.092</td>
<td>0.880</td>
<td>0.970</td>
<td>-0.093</td>
</tr>
<tr>
<td></td>
<td>75th percentile</td>
<td>OT</td>
<td>OT</td>
<td>-</td>
<td>OT</td>
<td>OT</td>
<td>-</td>
</tr>
<tr>
<td>10m</td>
<td>25th percentile</td>
<td>0.180</td>
<td>0.170</td>
<td>0.059</td>
<td>0.220</td>
<td>0.260</td>
<td>-0.154</td>
</tr>
<tr>
<td></td>
<td>50th percentile</td>
<td>0.300</td>
<td>0.280</td>
<td>0.071</td>
<td>0.370</td>
<td>0.370</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>75th percentile</td>
<td>0.430</td>
<td>0.510</td>
<td>-0.157</td>
<td>0.550</td>
<td>0.540</td>
<td>0.019</td>
</tr>
<tr>
<td>20m</td>
<td>25th percentile</td>
<td>0.005</td>
<td>0.018</td>
<td>-0.722</td>
<td>0.060</td>
<td>0.032</td>
<td>0.875</td>
</tr>
<tr>
<td></td>
<td>50th percentile</td>
<td>0.019</td>
<td>0.033</td>
<td>-0.424</td>
<td>0.117</td>
<td>0.081</td>
<td>0.444</td>
</tr>
<tr>
<td></td>
<td>75th percentile</td>
<td>0.051</td>
<td>0.095</td>
<td>-0.463</td>
<td>0.208</td>
<td>0.180</td>
<td>0.156</td>
</tr>
</tbody>
</table>
A comparison between the probabilities of overturning when the Housner coefficient of restitution and when the measured coefficient of restitution are used reconfirms the 1980 finding of Yim et al. (1980): “From a probabilistic point of view, the coefficient of restitution influences the response […] to a much lesser degree than the other system parameters.”

Comparison with the model of Priestley et al.
Even when we attempt to validate the model in the weak, statistical sense, the error is still large. It reaches 500% in terms of predicting the median response for the 2H=20m El Centro case. Moreover, it is not clear whether the method is consistently conservative or not, or whether its accuracy depends on the size of the examined block. Therefore, it is confirmed, that the conclusions of Makris and Konstantinidis (2003) hold, even when the problem is treated statistically.

CONCLUSIONS

The seismic response of a rigid rocking block is sensitive to the imperfections of the specimen and the support surfaces, and the variation in initial conditions, making it practically impossible to exactly repeat the outcome of a single experiment indicating that the seismic response motion of a rocking block is chaotic, in the sense that small perturbations lead to very different results for a single excitation.

The numerical model of a rigid rocking block is deterministic, but is sensitive to modeling assumptions, particularly those about energy dissipation during repeated impacts, to such an extent that accurate reproduction of the time history of the measured response of a rocking block prototype to a known support motion is quite difficult. Such sensitivity of rocking models makes practicing engineers hesitant to adopt rocking as an earthquake response modification strategy (Deng et al. 2012): predicting the response of a rocking block to a specific ground motion with sufficient confidence in the experiment or analysis outcome is not possible.

However, since the seismic response problem is inherently stochastic, that is, the ground motion excitation is not known a priori, modelling uncertainties should not be the reason to deter engineers from using rocking response modification solutions. The design question is to predict the response of a given system to an ensemble of ground motions. Then, the statistics of the maxima of the response become important (e.g. the median and the dispersion of the response). To illustrate this issue, we tested experimentally and analyzed numerically the response of a rigid rocking specimen to 600 ground motions. The numerical model response data were obtained using the 1963 Housner model with measured and with theoretically computed restitution coefficient. Cumulative probability distributions of the maximum tilt of the rocking block prototype and the rocking block numerical models (that differ in the restitution coefficient) were compared. The CDFs produced by the numerical models are within the 90% and the 95% of the confidence interval bounds of the experimental CDFs. These findings indicate that even though the response of a rigid rocking block to a single ground motion is chaotic, the statistics of the response to ensembles of ground motions is predictable with sufficient accuracy for seismic design and evaluation, when the 1963 Housner model is used. On the contrary, the simplified approach proposed in Priestley et al. (1978) proves inadequate.

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