A general framework for multi-modal macroscopic fundamental diagrams (MFD)

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Author(s):
Loder, Allister; Bressan, Lea; Wierbos, María J.; Becker, Henrik; Emmonds, Andy; Obee, Martin; Knoop, Victor L.; Menendez, Monica; Axhausen, Kay W.

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Allister Loder
Lea Bressan
Maria J. Wierbos
Henrik Becker
Andy Emmonds
Martin Obee
Victor L. Knoop
Monica Menendez
Kay W. Axhausen
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Allister Loder  
IVT, ETH Zurich  
aloder@ethz.ch

Lea Bressan  
IVT, ETH Zurich

Maria J. Wierbos  
Transport & Planning, Faculty of Civil Engineering and Geosciences, TU Delft

Henrik Becker  
IVT, ETH Zurich

Andy Emmonds  
Surface Transport, Network Management Directorate, Transport for London

Martin Obee  
Surface Transport, Network Management Directorate, Transport for London

Victor L. Knoop  
Transport & Planning, Faculty of Civil Engineering and Geosciences, TU Delft

Monica Menendez  
Division of Engineering, New York University Abu Dhabi

Kay W. Axhausen  
IVT, ETH Zurich

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Abstract

Car traffic streams in urban networks are rarely homogeneous and usually contain some disturbances by other transport modes of different physical characteristics, e.g. bicycles or buses. Inevitably, these interactions decrease the network's capacity and performance, but, so far, no methodology exists to assess the local interaction effects on the network level as for instance described in the macroscopic fundamental diagram (MFD). This paper proposes an analytical framework to link general microscopic disturbances to the shape of the MFD and thus to network capacity. The influence of disturbances is established by linking the two-fluid theory of urban traffic to travel times derived from the MFD. We apply the framework to the interactions of bicycles, buses and cars with an empirical calibration using data from London (UK). This framework allows to identify the maximum possible travel production of a given network and its associated modal split, as well as to identify for a given demand the optimal modal split.

Keywords

MFD, multi-modal, bus, car, bicycle, two-fluid, Bose-Einstein condensation
1 Introduction

Urban transportation is complex due to the interactions of many road users with many different transportation modes, e.g. cars, bicycles, buses, taxis, pedestrians, etc. In general, the mere presence of a road user in the network imposes negative externalities to all other users potentially leading to delayed journeys. Thus, which is the optimal allocation of time and space resources to transportation modes for a given city? This classic question has frequently been asked and answered in many ways, but, so far, no comprehensive methodology exists that quantifies the multi-modal interactions at the network level. However, the macroscopic fundamental diagram (MFD) offers an approach for understanding network-wide traffic (Daganzo, 2007; Daganzo and Geroliminis, 2008; Godfrey, 1969; Williams et al., 1987; Mahmassani et al., 2013). Some studies have already investigated interactions between cars and buses (Geroliminis et al., 2014; Loder et al., 2017; Castrillon and Laval, 2018; Dakic et al., 2019), and cars and pedestrians (Daganzo and Knoop, 2016).

In this paper, we propose a novel and general methodology to describe analytically the effects of local and microscopic disturbances in multi-modal traffic, e.g. cars, buses, bicycles, on the overall performance of urban networks. We use a recently formulated functional form for the MFD (Ambühl et al., 2018) in conjunction with the two-fluid theory of urban traffic by Herman and Prigogine (1979). The latter theory sees traffic analogous to the Bose-Einstein condensation at low temperatures (Ardekani and Herman, 1982; Dixit, 2013) and allows to bundle additional delays caused by other modes to the unimodal travel times derived from the unimodal MFD, resulting in a multi-modal MFD. In contrast to the existing 3D-MFD approach by Geroliminis et al. (2014); Loder et al. (2017), this method retains the individual mode’s speed information, which is very important for both modeling and control purposes.

We organize this paper as follows. The first Section introduces the mechanism to link the delays caused by the interaction between modes to the MFD. Thereafter, we discuss the delay functions for the case with interactions between bicycles, buses, and cars in the MFD. Then, we present the model calibration, before closing the paper with an overall discussion and concluding remarks.
2 The model

The proposed interaction model applies to the interaction of $m$ modes. Here we focus on the interactions between cars (subscript $c$), buses (subscript $b$), and bicycles (subscript $v$, for velo - french bicycle). We define $k$ as the three-dimensional state vector of the system with elements car density, $k_c$, bus density, $k_b$, and bicycle density, $k_v$. Each mode has a well-defined MFD. Generally, the MFD is an upper envelope to all possible states in the relationships between network’s average flow, $q$, and density, $k$ (Daganzo 2007; Daganzo and Geroliminis 2008). Originally defined for car traffic only, we transfer the idea to buses and bicycles as the three modes basically only differ in propulsion and some operational characteristics, e.g. speeds, passenger occupancy, vehicle size. Here, we denote this upper envelope as the upper MFD (uMFD) and assume that it is known a priori (Ambühl et al. 2018; Daganzo et al. 2018). Then, all observed traffic states will always be located below this uMFD due to traffic heterogeneity (e.g. Mazloumian et al. 2010; Geroliminis and Sun 2011; Gayah and Daganzo 2011; Daganzo et al. 2011) and network dynamics (e.g. Mariotte et al. 2017). Here, we use a functional form for the MFD proposed by Ambühl et al. (2018) that captures the gap between the a priori known uMFD and the observed MFD with just a single parameter, $\lambda^0$. This parameter can be seen as a quantification of network homogeneity or the between-vehicle interactions. Eqn. 1 shows this functional form for a trapezoidal uMFD. Such uMFD shape has been used, for example, by Daganzo et al. (2018). For the reader’s convenience we omit the subscript $m$ for the mode as it appears for every item.

$$q (k) = -\lambda^0 \ln \left( \exp \left( -\frac{v^f k}{\lambda^0} \right) + \exp \left( -\frac{Q}{\lambda^0} \right) + \exp \left( -\frac{(\kappa - k) w}{\lambda^0} \right) \right)$$

Here, $v^f$ is the free flow speed in the network, $Q$ is the network’s capacity as defined by the most constraining intersection (Daganzo and Geroliminis 2008), $\kappa$ is the jam spacing in the network, and $w$ the backward wave speed. Arguably, each mode $m$ has its characteristic values. In Figure 1a we illustrate the behavior of this MFD function for different values of $\lambda^0$ in comparison with the uMFD as defined by the minimum operator with the trapezoidal shape (Eqn. 1). Here, each of the trapezoidal shape’s three segments enters as an argument to the minimum operator. With $\lambda^0$ approaching zero, the resulting curve approaches the uMFD, in this case the trapezoidal shape. When $\lambda^0$ increases, the curve moves further down but still describes the familiar MFD shape.
We also define the pace (travel time per unit length) $T_m$ of mode $m$. It consists of two parts as given by Eqn. 2. The first term, $T_m^0 (k_m)$, denotes the undisturbed pace of mode $m$ given its current accumulation levels, $k_m$, i.e. without any interactions with other modes. The second term, $\Gamma_m (k)$, describes the additional delays caused by the interactions across modes. In other words, the undisturbed pace of $m$ is given by each modes’ own MFD, while the additional interaction delays $\Gamma_m$ jointly depend on all modes’ accumulations.

$$T_m (k) = T_m^0 (k_m) + \Gamma_m (k) \tag{2}$$

The additional interaction delays $\Gamma_m (k)$ will decrease the flow of all vehicles as interactions always negatively effect each other. This means, that an increase in $\Gamma_m (k)$ will increase $\lambda$ of Eqn. 1. In other words, we want to find $\mu (k)$ as defined in Eqn. 3 which increases $\lambda^0$ to $\lambda$ due to the interactions between modes at a given state vector $k$. Here, $\mu (k)$ is a function with $\mathbb{R}^m \rightarrow \mathbb{R}$. The model proposed in this paper allows to calculate $\mu (k)$, and
then either $\lambda$ or $\lambda^0$ if one of the latter two values is measured.

$$\lambda = \lambda^0 + \mu(k)$$

(3)

We will now establish the link between the additional interaction delays $\Gamma$ and $\lambda$ with the well-established two-fluid theory of urban traffic (Herman and Prigogine 1979). The two-fluid theory of urban traffic is analogous to the Bose-Einstein condensation of particles at low temperatures (Ardekani and Herman 1982; Dixit 2013). In this theory, traffic consists of running vehicles (subscript $r$) and stopped vehicles (subscript $s$), where the running speed of vehicles $v_r$ is related to the fraction of running vehicles $f_r$ by Eqn. 4 and by definition Eqn. 5. Here, $n$ is a network-wide constant and assumed to result from driving behavior, network topology, and signal settings, $v_f$ is the free-flow speed, and $v$ is the average space-mean speed in the network. The inverse of $v$ is equal to the pace or travel time per unit distance $T$. As there are stopping and moving vehicles, $T$ equals the sum of stopping time per unit distance $T_s$ and the running time per unit distance $T_r$, i.e. $T \equiv T_r + T_s$. We assume that the two-fluid theory of urban traffic applies to all considered modes $m$ in the same fashion. For each mode, the parameter $n$ is derived from $\lambda$ as we show later.

$$v_r = v_f (f_r)^n$$

(4)

$$v = v_r f_r$$

(5)

By definition the fraction of vehicles stopped $f_s$ and the fraction of running vehicles $f_r$ always add up to one: $f_s + f_r \equiv 1$. Then, the space-mean speed in the network $v$ results from Eqn. 6.

$$v = v_f (1 - f_s)^n f_r = v_f (1 - f_s)^{n+1}$$

(6)
Importantly, [Herman and Prigogine (1979)] pointed out that $f_s$ is proportional to a power law with exponent $h$ of the density to jam density ratio. In an empirical study, [Lu et al. (2018)] reported that $h \approx 1$, making computation easier. Here, however, we carry the $h$ further as it can be context specific.

$$f_s = \left( \frac{T_s}{T} \right) \propto \left( \frac{k}{\kappa} \right)^h$$

(7)

The fundamental equation of the two-fluid theory results from Eqns. 4 and 6 and is given by Eqn. 8 (see for derivation [Herman and Prigogine (1979)]). This equation establishes a relationship between the total travel time per unit distance $T$, the running time per unit distance $T_r$, the free flow speed $v_f$ as well as the network parameter $n$. We illustrate the functional behavior of the two fluid theory in Figure 1 for different values of $n$. For higher values of $n$, the fraction of stop time out of the total trip time decreases.

$$\log T_r = \frac{n}{n+1} \log T + \frac{1}{n+1} \log \left( \frac{1}{v_f} \right)$$

(8)

With the MFD expressed by Eqn. 1, we can algebraically derive formulae for $T, T_r, T_s$. The total trip time per kilometer or pace, $T$, is simply obtained by the inverse of the space-mean speed in the MFD as shown by Eqn. 9.

$$T(k) = \frac{k}{-\lambda^0 \ln \left( \exp \left( -\frac{v_f k}{\lambda^0} \right) + \exp \left( -\frac{Q}{\lambda^0} \right) + \exp \left( -\frac{(\kappa-k)w}{\lambda^0} \right) \right)}$$

(9)

Then, we obtain the running time per kilometer, $T_r$, by using Eqns. 5, 7 and $T(k)$ from Eqn. 9.

$$T_r(k) = \left( 1 - \left( \frac{k}{\kappa} \right)^h \right) T(k)$$

(10)
Last, we obtain the stopping time per kilometer, $T_s$, by subtracting the running time from the total trip time as given by Eqn. 11.

$$T_s (k) = T (k) - T_r (k)$$  \hspace{1cm} (11)

In their empirical work, Herman and Prigogine (1979), Ardekani and Herman (1985) and Ardekani et al. (1992) estimated $n$ econometrically from measurements of $T_r$ and $T$. As we obtained formulae for $T$ and $T_r$, we can derive $n$ analytically. Thus, after some algebra, we can solve Eqn. 8 for $n$, resulting in Eqn. 12.

$$n = \frac{1}{\log v_f - \log T_r}$$  \hspace{1cm} (12)

In the following, we define $\lambda^0$, $n^0$, $T^0$, $T_r^0$ and $T_s^0$ with superscript 0 to denote the case of a steady-state traffic stream without any cross-modal disturbances, i.e. interaction with other modes. Let us then consider that the interactions with other transport modes create additional delays $\Gamma$. We assume that these delays either affect only the stopping time with $\Gamma_s (k)$ or the running time $\Gamma_r (k)$. We discuss these functions in detail later in this section. Here, $\Gamma$ is a scalar function with $\mathbb{R}^m \rightarrow \mathbb{R}$. Accordingly, the two-fluid travel time variables can be rewritten for the case with additional delays with Eqns. 13-15.

$$T_r (k) = T_r^0 (k) + \Gamma_r (k)$$  \hspace{1cm} (13)

$$T_s (k) = T_s^0 (k) + \Gamma_s (k)$$  \hspace{1cm} (14)

$$T (k) = T_r^0 (k) + \Gamma_r (k) + T_s^0 (k) + \Gamma_s (k)$$  \hspace{1cm} (15)
With the additional interaction delays $\Gamma$ on pace or travel times quantified (for a given $\lambda^0$ and $k$), we can identify the network performance measures $n$ and $\lambda$ in the presence of cross-modal interactions. We use the inverse hat to indicate that the measure includes cross-modal delays. Then, for $n$, we use Eqn. 12 and Eqn. 13 to obtain Eqn. 16.

\[
n(k) = \frac{1}{\log v_f - \log (T^0_r (k) + \Gamma_r (k))} - \frac{\log (T^0_r (k) + \Gamma_r (k)) - \log (T^0_r (k) + \Gamma_r (k) + T^0_s (k) + \Gamma_s (k))}{\log (T^0_r (k) + \Gamma_r (k) + T^0_s (k) + \Gamma_s (k))}
\] (16)

Then, to obtain $\lambda$, we equate in Eqn. 17 the space-mean speed of the $\lambda$ trapezoidal function from Eqn. 1 and the speed of the two-fluid theory from Eqn. 6. Note that the information of $k$ is now carried along with $n$ and that the right-hand side of Eqn. 17 is similar to the inverse of Eqn. 9 but the $\lambda^0$ is replaced by $n$ to calculate the interaction effects.

\[
v_f \left(1 - \left(\frac{k}{\kappa}\right)^h\right)^{n(k)+1} = -\lambda \frac{\ln \left(\exp \left(-\frac{v_f k}{\lambda}\right) + \exp \left(-\frac{Q}{\lambda}\right) + \exp \left(-\frac{(n-k)w}{\lambda}\right)\right)}{k}
\] (17)

Eqn. 17 can simply then be solved as a root problem using a mathematical software when transformed into Eqn. 18. The only unknown is $\lambda$.

\[
0 = -\lambda \frac{\ln \left(\exp \left(-\frac{v_f k}{\lambda}\right) + \exp \left(-\frac{Q}{\lambda}\right) + \exp \left(-\frac{(n-k)w}{\lambda}\right)\right)}{k} - v_f \left(1 - \left(\frac{k}{\kappa}\right)^h\right)^{n(k)+1}
\] (18)

The problem formulated in Eqn. 18 must be solved for each mode $m$ separately and because of the high non-linearity of model, we propose to solve Eqn. 18 for each demand situation separately, i.e. for all possible values of $k$, instead of assuming constant $n$ or $\lambda$ values over all densities.
3 Delay functions

In the following, we focus on identifying delay functions, i.e. $\Gamma (k)$ for an urban network with given MFDs for each mode. Here, we firstly consider that the interactions between modes are uniformly distributed over time (we discuss later spatial heterogeneity of delays). The methodology presented above is generic and allows to use any formulation of $\Gamma_s (k)$ and $\Gamma_r (k)$ functions for all modes that can be represented with a MFD, but here we focus on bicycles, buses, and cars.

We use the following notation: $\Gamma_{c\rightarrow b}$ describes the interaction stopping delays caused by cars on buses. We use the $\rightarrow$ operator to indicate which mode affects which other mode. Where we do not provide the $\rightarrow$ operator, $\Gamma$ corresponds to the total interaction delay caused by all other modes combined. Intuitively, the interaction delay functions depend on the network topology, i.e. in case all modes run on dedicated infrastructures the interaction delays are zero, while they are non-zero when their infrastructure is (partially) overlapping.

As mentioned before, $\Gamma (k)$ has two mechanisms: stopping delays $\Gamma_s (k)$ and running delays $\Gamma_r (k)$. We assume additivity of delays within each mechanism as formulated in Eqn. 19 for the additional stopping delays for cars $\Gamma_{s}^{a}$ (k), i.e. we calculate additional delays pairwise, and their sum is then the total interaction delay. In other words, this assumes no combined or second order effects, e.g. from bicycles and buses on cars.

$$\Gamma_{s}^{c} (k) = \Gamma_{s}^{b\rightarrow c} (k_{c}, k_{b}) + \Gamma_{s}^{v\rightarrow c} (k_{c}, k_{v})$$  \hspace{1cm} (19)

In Table 1 we summarize for each of the two mechanisms the delay model used for quantifying the interactions between all considered modes. In total, we use four different delay models: A continuous multiclass fundamental diagram (FD) taken from Bliemer (2001), a discrete multiclass FD proposed by Wierbos et al. (2018), a platoon dispersion model as proposed by Robertson (1969), and a bus dwelling behavior model based on Daganzo (2010). We do not use a continuous multiclass model for the interactions that involve bicycles as we assume that in the congested case of cars and buses, bicycle speeds do not converge to that of the other modes, i.e. bicycles can sneak through the vehicle queues. Therefore, we provide a separate discrete multiclass FD.
<table>
<thead>
<tr>
<th>Interaction case</th>
<th>Running delay</th>
<th>Stopping delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c \rightarrow b$</td>
<td>Continuous multiclass FD</td>
<td>Delay scaling</td>
</tr>
<tr>
<td>$b \rightarrow c$</td>
<td>Continuous multiclass FD</td>
<td>Bus dwelling behavior</td>
</tr>
<tr>
<td>$b \rightarrow v$</td>
<td>Discrete multiclass FD</td>
<td>Bus dwelling behavior</td>
</tr>
<tr>
<td>$v \rightarrow b$</td>
<td>Discrete multiclass FD</td>
<td>Bicycle platoon dispersion</td>
</tr>
<tr>
<td>$c \rightarrow v$</td>
<td>Discrete multiclass FD</td>
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</tr>
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<td>$v \rightarrow c$</td>
<td>Discrete multiclass FD</td>
<td>Bicycle platoon dispersion</td>
</tr>
</tbody>
</table>

Table 1: Assignment of delay models to the mechanisms of running and stopping delays.

For the additional running delays, the general modeling idea is to express the additional delay as the difference in pace between the pace from the multi-modal FDs and the unimodal FDs. In other words, the difference between the case where both modes are interacting and the case without interaction. For the interaction stopping delays, the general modeling idea is to quantify the sources of additional stopping delays caused by the interactions. In this case, we do not need to subtract anything from these delays as they are fully additional, i.e. in the uninterrupted case, no such delays are to be expected.

4 Accounting for heterogeneity of delays

The interaction delays are not spatially and temporally homogeneous: In an urban network with $d$ lanes, buses and bicycles usually only use a single lane, the curbside lane. Further, buses have a given headway and thus their bottleneck nature is only activated when a bus is present. Thus, we need to scale interaction delays by the probability of interaction to obtain the expected value of interaction delays. Importantly, the required scalars are case specific and need to be defined for each interaction situation separately. For the tri-modal case of bicycles, buses and cars, we consider $\rho_l$ to capture the spatial probability (or heterogeneity) induced by the number of lanes and $\rho_b$ the temporal probability (or heterogeneity) of bus interactions. Accordingly, both are scalars to the respective delays.

First, for the spatial heterogeneity of interaction delays, we assume that $d - 1$ lanes of the network are dedicated for cars while only one lane is mixed. Consequently, the interaction effects will fully affect the mixed lane, while the other lanes are less affected. However, due to spillover effects, the $d - 1$ lanes are not completely unaffected. Thus, we assume that the network-wide probability of interaction is a convex function, decreasing with the number of lanes. We propose Eqn. \[20\] as a possible functional representation of this behavior by using the square root. However, future research is required to validate this
assumption. For the case of bicycle-bus interactions, where both use the curbside lane, we consider here only the mixed lane, i.e. \( \rho_l = 1 \).

\[
\rho_l = \frac{1}{\sqrt{d}} \tag{20}
\]

Second, we define the temporal probability of interaction, \( \rho_b \), on a link where vehicles of mode \( m \) are affected by stopping buses. We assume that bus stops are spaced at distance \( p \) in the network, the headway of buses is \( H \) that is a function of bus density \( k_b \), \( \gamma \) is the total time a bus stops, calculated using the commercial bus speed equation by Daganzo (2010), and \( \alpha \) is the design of the bus network as proposed by Daganzo (2010), for which \( 0 < \alpha \leq 1 \) holds. At its lower bound it describes a hub-and-spoke network, while at its upper bound it describes a perfect grid network. Values in-between indicate hybrid structures. Here, we consider that the temporal probability of encountering a stopping bus at a bus stop follows the binomial distribution as given in Eqn. 21 as there is a chance of encountering a bus at one of \( p \) stops per kilometer, at two of \( p \) stops or at every \( p \) stops.

Note that the first expression in parentheses in Eqn. 21 is the binomial coefficient. The probability of encountering a bus at a bus stop is given by \( \gamma/H \). The minimum number of bus stops a car will encounter per kilometer in a network is given by \( \lfloor \alpha/p \rfloor \). Importantly, this is a strict simplification of multi-modal urban traffic as it as assumes independence of interactions.

\[
\rho_b(k_b) = \sum_{j=1}^{\lfloor \alpha/p \rfloor} \binom{\alpha/p}{j} \left( \frac{\gamma}{H(k_b)} \right)^j \left( 1 - \frac{\gamma}{H(k_b)} \right)^{\lfloor \alpha/p \rfloor - j} \tag{21}
\]

5 Calibration to London

The calibration of the tri-modal MFD as previously introduced to a specific context requires data. While speed measurements of buses and cars are widely available, e.g. Loder et al. (2017) and Loder et al. (2019), to calibrate the unimodal MFDs for cars and buses as well as the continuous multiclass FD, no data set exists that allows to calibrate the bicycle interaction models. Therefore, we collected suitable video data in London.
Figure 2: Experimental sites for the measurements of bicycle interactions. (a) Measurement locations for the calibration of the bicycle interaction models along Stamford Street in London. Background map courtesy of [https://osmaps.ordnancesurvey.co.uk/](https://osmaps.ordnancesurvey.co.uk/). (b) Travel time corridor along Sarphatistraat in Amsterdam. Background map courtesy of [https://www.openstreetmap.org/](https://www.openstreetmap.org/). (c) Locations between which travel times of cars and bicycles were recorded. There are several locations to account for in and outflow from site streets. The measurements used for the calibration are between locations II and III.
At the marked locations 1 to 5 in London, we measured the bicycle dispersion at intersection that causes additional delays to cars and buses. Figure 2 shows the bicycle stop boxes at the five locations for measuring the additional delays. For the calibration of the discrete fundamental diagram, we measured travel times along the two marked corridors in Figure 2 in London (Stamford Street) and in Amsterdam (Sarphatistraat). We had to rely on two different experimental sites because the Amsterdam site saw too few cars to identify an interaction effect of bicycles on cars, while the London site saw too few bicycles to identify an interaction effect of cars on bicycles. In detail, the Stamford Street in London bicycles, buses and cars share the same lane. Data is collected in both directions in the first week of June 2019. The Sarphatistraat is reconstructed such that cyclists have priority but cars can ride along. Data is collected in the westbound direction between Alexanderplein and Weesperplein. Data has been collected in the morning peak between 8-9am on the 5th of June, 2018.

The results of these measurements are illustrated in Figure 4. Figure 4(a) shows the additional delays for cars at intersections as a function of the number of bicycles, Figure 4(b) shows effects of bicycle presence on car traffic for (based on measurements from London), and Figure 4(c) shows the effects of car presence on bicycle traffic (based on the measurements from Amsterdam). In all situations, we observe the expected behavior that additional delays are increasing with the extent of interactions with bicycles or cars. Figure 5 then shows preliminary results for the estimated tri-modal MFDs, where we show the influence of modal interactions on the car flow. The expected and intuitive capacity loss for car traffic due to more interactions with other modes is observed.

6 Discussion

In this paper, we proposed the first analytical way to link microscopic or local disturbances caused by other modes to car traffic at the network level. This is achieved by linking the two-fluid theory of urban traffic [Herman and Prigogine (1979)] to the travel times derived from a recently formulated functional form for the MFD [Ambühl et al. (2018)]. For each interaction case between bicycles, buses, and cars, we proposed functions or models to derive the additional delays. We further showed with empirical measurements from London how the interactions with bicycles can cause substantial delays to cars and buses. However, it is important to always consider the space efficiency of the different
Figure 3: Details of experimental sites in London for bicycle platoon dispersion.
Figure 4: Measuring interaction delays between bicycles and cars. (a) Additional stopping delays for cars at intersections caused by bicycles. (b) Car speeds on Stamford Streets, by direction. The horizontal line describe the sample means. Two sample t-test suggests that means are different. (c) Bicycle speeds on Sarphatistraat, by car density.
Figure 5: Car MFDs with interaction effects. (a) and (b) show the bus interaction effects on car flow; (c) and (d) show the bicycle interaction effects on car flow.

modes, as passengers riding bicycles or bus usually require less space than cars (e.g. as discussed in Zheng and Geroliminis (2013)).

Future research will include focus on the influence of the bus network design, i.e. Daganzo’s \( \alpha \) Daganzo (2010), on the shape of the 3D-MFD (the case of bus-car interactions, and the analysis of optimal modal split from a traffic flow perspective on shared space. Future research can also explore how the issue of traffic homogeneity in the network (see Ji and Geroliminis (2012), Saeedmanesh and Geroliminis (2016) or Saeedmanesh and Geroliminis (2017) for discussion) can be described with the presented framework.

In closing, the proposed framework is a very general and flexible framework that can model a variety of multi-modal networks from the developing to the developed world. If analytical formulae for the additional delays cannot be derived, e.g. between tuk-tuks, jaywalking persons, and stray dogs, the relationships for \( \Gamma \) can be econometrically derived from data or based on probabilistic approaches. Consequently, the proposed framework can still be applied in strategic planning and operations and help those cities to improve their transportation system.
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Author contribution statement

Methodological idea and initial research design by AL. Bicycle delay model by AL, HB and KWA. Bus delay model by AL and MM. AL, HB, MW and VK conceived bicycle data collection and model analysis. AE and MO organized the data collection. LB developed the mathematical program. AL, MM, KWA wrote the manuscript. All authors revised the manuscript.
7 References


