Functional similarities in spatially correlated location choice models: An idea sketch

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**Abstract**

This work analyses the residential location choice problem. Location choice often is regionally clustered. The rules building up these clusters and the implementation in an appropriate model, investigates this work.

Additionally to structural and spatial similarities or neighbourhoods, this work especially incorporates functional ones. The functional component is important in residential location choice models. For example, functional commuting regions and travel times are considered as functional connections. Most studies examine one spatial level only. This study suggests a spatial neighbourhood as a three hierarchical level concept – functional commuting region, accessibility including travel time, and mail delivery areas – for application in a Swiss residential choice model. We bring the application problems and theoretical concepts together and think about the appropriate representation of a residential location choice model of Switzerland.

Most models use a multinomial logit (MNL) model to analyse this spatial problem, because of large set of alternatives. But the MNL model cannot count for unobserved similarities among alternatives. We give a short literature review about recent studies for residential choice models which allow spatial correlation between alternatives. This adoption is often done by GEV (generalized extreme values) models, a nested logit specification, or can be solved by implementing similarity measure into the utility function, or by a spatial probit model. These study focuses on functional closeness of a correlation coefficient not only assuming correlation in adjacent zones.

**Keywords**

Location choice model – spatial correlation – functional characteristics
1. Introduction

Why should we think about functional components implemented in a location decision choice model? We assume, that location choice models can be substantially improved while considering functional variables at the right place of the model. In this note, we focus on the residential choice process. A large numbers of studies showed that the commuting characteristics – possibly the most important functional criteria – have large influence on the residential decision process. Guo and Bhat (2007) discovers a slightly but not significantly better model of residential choice which explains choice on the assumption of a functional defined neighbourhood (see chapter 3 for more detail).

However, the formal factors of discrete location choice models are still very dominated in literature. Of course, formal factors have their legitimation with regard to the content. However, one main reason of their dominance is that functional data are generally rarer. They are more difficult to collect and more complicated to calculate and clearly definite functional data. They are person driven by individually different specification. Their limits are often fuzzy or subjective. Some arbitrary thresholds have to be set a priori processing these data.

This note gives a framework for following systematically testing of functional similarities in a residential choice model switching between theory and application. First ideas of implementation in a residential choice model of Switzerland with a national scope are suggested. The topic of spatially correlated alternatives is discussed in more detail (see chapter 3.3). The analyses will be applied to the Swiss individual Census data of the year 2000. Unfortunately, there is no other possibility to obtain more recent data with national scope which includes commuting information.

The implementation considers explicitly spatial problems of functional similarities in destination choice models on three different levels.

- What is the appropriate spatial resolution of analysing zones (see chapter 3.1)?
- How are large set of alternatives (around 3000 Traffic Analysing Zones in Switzerland) reduced by setting spatial constraints (see chapter 3.2)?
- What are the possibilities to handle alternatives which are spatially depending on each others (see chapter 3.3)?

This study provides a short theory background for all three aspects and comes up with practical solutions which are relatively easily implemented. At the moment, the use of further variables, which are included into the utility function, is not considered. Some functional variables are commonly integrated in location choice models e.g. applied in the Zürich-Area (e.g. Bürgle 2006).
2. Formal or functional attributes for destination choice

This categorisation of formal and functional attributes goes back on the regionalisation categorisation, which is explained in Montello (2003) “Functional regions are formed by patterns of interactions among separate locations on the earth. Spatially interaction is fundamentally the movement of matter or energy from place to place”. These interactions are activities which often have a focal point e.g. a centre or home place. However, we can also think about daily activity areas expressing daily schedules which have at least two spatial fixed locations as work and home place.

Formal attributes often are of thematic specification. They describe an entity, an individual or a unit. However, formal attributes are not only dot-shaped objects. They also have a spatial extend (e.g. administrative or urban boundaries), which are historically or morphologically fixed but not explicitly interaction driven by person’s activities.

2.1 Formal, functional and semi-functional characteristics in residential choice models

Generally, dwelling characteristics, land parcel, neighbourhood attributes and accessibility attributes are implemented in residential location choice models. Additionally, the choice is influenced by individual’s preference (e.g. Prashker et al. 2008). Guo and Bhat (2004) list different variables for each factor. The dwelling and land parcel characteristics are of formal or thematic nature. Variables are e.g. number of bedrooms or backyard size. The neighbourhood and accessibility characteristics contain some functional variables. A functional neighbourhood variable is e.g. the school catchment area. The commuting distance is an example for accessibility characteristics.

Mostly significant functional variable in workplace and residential location choice is the commuting distances. The first principle is that employment search is spatially systematic; then workers prefer jobs closer to their place of residence. However, the flexibility of the labour market is increasing and the willingness to leave home residence is small. That is shown in a stated preference study of Switzerland in the Zürich-Area (Erath and Axhausen, 2009). In the same study area Belart (2011) finds that the closeness to the previous home place is significant in a relocation process. Prashker et al. (2008) assumes that travel distance is the better indicator than travel time. Difference in short travel times are rather produced by the chosen traffic mode than by the spatial arrangement. Another commuting indicator when considering traffic modes is the number of transfer in public transport. These variables differ individually and they are often influenced by a priory choice decision e.g. mode choice or workplace choice or reverse. Generally, the location choice is strongly influence by physical
attributes. The discussion about the hierarchy structure of these choices is not the scope of this note. We focus on the spatial importance.

Accessibility factors of location choice models are considered in two ways. Firstly, accessibility is an individual concept and every household or person has its own perception. A household with two workers and two children specifically values the accessibility of a location. For example, a household with children is possibly interested more in being close to a nursery and public school and being embedded in a child-friendly neighbourhood providing lots of social contacts.

Secondly, Zondag and Pieters (2005) explains accessibility as a major factor that influences attractiveness of a certain location and argues that the reason why most people prefer to live in built-up areas is because of large potential variety of activities. We speak in this note rather about accessibility focusing on a potentially available functional connection e.g. proximity to one or more local amenities (e.g. hospital), travel time to the city centre or to the motorway. The individual use of this proximity is of large individual difference. Therefore, this categorisation is called the semi-functional attributes. The individual defined interactions as the exact home-work distance is a functional attributes. Both categorisations of accessibility are of importance. The result of Zondag and Pieters (2005) shows that households are less likely to move from a more accessible location.

2.2 Descriptive analysis of functional and semi-functional characteristics of the Swiss Census

Following investigations aim to test the ideas of this note on the base of the individual Swiss census data of the year 2000. Therefore, we have a look at the availability of functional or semi-functional data in this dataset and shortly compare the result for validation to findings in other studies.

Prashker et al. (2008) shows that average distances and travel times from home to work is increasing with income, numbers of cars, education level and lower particularly with young or old age. Seremons and Koppelmann (2001) are considering travel time and are looking at gender specific commuting distance of different household compositions, where households with children have larger gender difference than without.

In 2000, similar and not surprising pattern are detected on household level in Switzerland (see Figure 1) and in a gender specific view (see Figure 2).

- Mean home to work distance of a household is decreasing with a higher number of children up to eight children per household. The increase of commuting distance with more than 8 children can be explained by rare data and a changing family structure.
• Generally, collective households have lower average home to work distances than “classical” family household. These collective households are located nearer to the city centre, where also jobs are close. The home to work distance for family households is increasing up to five workers in the household. The shorter commuting distance of higher number of workers per family and not-family households shows, that their social activity pattern of large family households gets more similar to that one of a collective household.

• Women are generally more flexible in moving than men. During their lives, they move larger distance away from their birth place than men. Because this data does not represent cohorts, we need also to keep in mind that the cultural and traditional environment could have changed over the years.

• Young women have longer commuting distances than men. By the age of 24, this habit changes to shorter commuting distances of women, what coincidence with the family founding process.

• Women earlier leave home than men. However, people between 20 and 34 years have one common phase of relocation of longer distances in their lives. Before and after this phase the relocation, distances of moving are relatively short. That implies the importance of the previous home location in a relocation process.

Figure 1   Household specific home-work distance

Source: Census data 2000 (Federal Office of Statistics)
Figure 2  Gender specific mobility behaviour

Source: Census data 2000 (Federal Office of Statistics)
3. Implementation of functional similarities in location choice model

This note relinquishes to give a general framework of discrete location choice models. A short overview you find e.g in Schuessler and Axhausen (2009) or Erath and Axhausen (2009). We focus on certain theoretical aspects, which are of importance implementing functional attributes in location choice models. Generally, they can be used as functional or semi-functional variables in the utility function of discrete choice models but plays also an important role on aggregation level; choice set generation, and spatially correlation of alternatives.

3.1 Aggregation level

An important aspect of location choice alternatives is that they can be defined for different levels of scale (spatial resolution). In spatial analyses this problem is known as modifiable areal unit problem (MAUP). „This scale problem arises because of uncertainty about the number of zones needed for a particular study” (Openshaw, 1977). A location choice model often applies three spatial levels. Firstly, there is the level of Traffic Analysis Zones (TAZ): this aggregation level has the advantage to maximize consistency with existing travel models and other data (e.g. Bekhor and Prashker 2008). Secondly, more detailed models for micro-simulation use, work with parcel level data with a coverage of an urban area. This level often represents the land ownership relations. The access to such data can be hindered by a combination of proprietary and political problems or because this data are relatively expensive (e.g. Waddell et al. 1998). Thirdly, the spatially smallest level of choice alternative is an apartment or house. This spatial level is applied in housing market analyses (e.g. Belart 2011). To choose the appropriate spatial resolution is dependent on the scope and scale of the study. The data availability is steadily improving and the computer capacities to work with larger datasets are increasing - still being the limiting factor in many studies.

Beside the hierarchical aggregation Guo and Bhat (2007) discuss about the concept of neighbourhood and the aspects of a more functional or formal aggregation techniques. A neighbourhood can generally be defined formal e.g. as historical administrative boundaries of a quarter or with similar structural attributes (e.g. Manaugh et al., 2010). This aggregation level seldom meets the functional criteria nowadays, where people undertake many of their activities as shopping or meeting friends. However functional neighbourhood are often a multi level structure, several activities are overlapping and boundaries are fuzzy and the limit is difficult to define (often arbitrary). Guo and Bhat (2007) tested three alternative aggregation levels of residential choice models: 1) The administrative boundary 2) A radial approach 3) A
shortest network approach. The first two are more a formal definitions. The last criterion is the most functional and showed generally the best results.

To avoid the problem of choosing one single level Guo and Bath (2004) suggest a MSL model what allows to combine attributes on different hierarchical level. Administrative boundaries are only used which are hierarchally structured and not overlapping. For functional multi level analyses this approach is not satisfactory.

**Suggestion for application in a Swiss national residential choice model**

In further studies, we undertake a national wide residential choice model. Because of the national scope it is not useful to undertake this analysis on a small aggregation level. Comparable studies use the TAZ level. This level goes back on the administrative boundaries in Switzerland. A more functional aggregation level is the zip-code aggregation. This aggregation level is used by the mail delivery service and has a functional background. In comparison to the TAZ, the zip-codes areas are in cities of smaller scale and in rural area larger. These functional areas are more comparable between, containing a post office, a small centre with e.g. a school or a bakery store.

**3.2 Choice set generation**

In location choice models the universal choice set is often extremely large and contains many alternatives that are irrelevant for the choice of the decision-maker. The choice set should be appropriately reduced to an individual sub-set of known alternatives and to decrease the burden of computer time. The functional travel-time is an important constraint in spatial choices. Till and Horowitz (1997) investigate different probability function of time constraints going back on the idea of space-time prisms of Hägerstrand (1970). The space-time prisms approach eliminate all alternatives that cannot be reached within the time-budget of the individual and, thus, obtain a very low choice probability. The space-time prism approach is better suited than the traditional way of including all alternatives within a circle. Scott (2006) tested two version of choice set generation, firstly a method of overlaps and secondly a method of shortest network path. The shortest network path method is suggested if accuracy is needed. The structure of the traffic network is evident for these constraints (e.g. Kwan and Hong, 1998).

Horni et al. (2010) and Pagliarea and Timmermans (2009) give an overview about this deterministic choice sets and the more probabilistic or stochastic choice set formation. The approaches listen here focus on the deterministic approach, which is attractive because of simplicity of implantation. By all means, the accuracy of the choice set generation is elementary.
Suggestion for application in a Swiss national residential choice model

The sampling procedure could consider the travel time from the work location as well as the travel time from the previous location and applying a weighted sampling. This technique is depending on a single node or focal point perspective. Residential choice process is often depending on different home-work distances of a household. Household member possibly do not work at the same place and their common residential place lies somewhere between. Areas are developed by Killer and Axhausen (2011) based on social network methods with a multi nodal perspective depending on all commuting linkages within this area. These commuting regions are rather large where more spatially distributed job possibilities exist. The use of these multi nodal regions should be tested and could additionally be combined with a travel distance constraints where the sub-set. The percentage of randomly chosen alternative is halved by the closeness of the region to the main region. The percentage of small regions is starting with 100% in larger regions e.g. Zürich by around 40% to maintain choice sets of 100-200 alternatives. the commuting region, still contains too many alternatives.

Figure 3  Commuting regions with overlapping cities and areas between

3.3 Spatially correlated alternatives

The commonly used discrete choice model for location choice is the multinomial logit model (MNL) proposed by McFadden (1974). It is based on the assumption that the random term, often called error term, is identically and independently and Gumbel distributed. MNL models offer computational tractability for large choices sets. A prominent disadvantage of the MNL model is the independence from irrelevant alternatives (IIA) property: The probability to choose an alternative should not depend on the existence or the characteristics of other choice
alternatives. However, location choice models suffer from potential violations of the IIA property arising from spatially correlated choice alternatives that can lead to inconsistent parameter estimates. This dependence can be positively or negatively correlated.

On one hand, individuals often consider spatial units that are near to each other. They can be adjacent to one another, or just closer. These similar alternatives can be described as “proximal cluster” sharing some characteristics as socio-demographic composition of residents, income levels, density and pattern of development, proximity to services and shopping opportunities (Sener et al. 2008). Because of the similarity of the surrounding alternatives the probability to chose one alternative out of this cluster decreases. On the other hand, the spatial influence of one alternative to another can also be an advantage. If an alternative is close to another alternative containing many amenities it is more possible to be chosen. The chose of an alternative is depending on the accessibility of its surrounding alternatives.

Schuessler and Axhausen (2007) discuss the methods the overcome the IIA properties in three categories: Subdividing alternatives into nests, opening the variance-covariance structure, and introducing a factor in the deterministic part of the utility function. For this discussion we borrow that framework and look at approaches which focus on the spatially correlation problem for location choice models in more recent work. Smirnov (2010) gives an overview about spatial problems in discrete choice models mainly focusing on spatial dependence between individuals. This discussion is not part of this note.

3.3.1 Generalised Extreme Values models

Most location choice models assume a Multinomial Logit (MNL). These models allow for some spatial dependence between alternatives, which is commonly resolved by a nested logit model (NL) (e.g. McFadden, 1978). The NL model is a special case of the GEV (generalized extreme values) model, with rather more restrictive assumptions. The NL model assumes a choice structure with similar clusters. The Nested Logit (NL) model relaxes the assumption of zero covariance between clusters, but covariance has to be equal among all alternatives in a common nest. However, the NL model is not without any limitations. For example the structure of each cluster or nest of alternatives must be specified a priori.

In the past few years, several discrete choice models were developed based on the Generalized Extreme Value (GEV) theorem of McFadden (1978). The GEV models are able to capture unobserved similarities among alternatives. The flexibility of GEV structures also allows the modelling of spatial location choice problems, where the utilities of various location choice alternatives may be correlated with one another. Bhat and Guo (2004) proposed a GEV-based model formulation called the Spatially Correlated Logit (SCL) model
that results in a closed-form expression. The SCL model is applied to analyse residential location choice.

The SCL model is founded on the following GEV formulation (Bhat and Guo, 2004):

\[
G(e^{V_{n1}}, \ldots, e^{V_{nL}}) = \sum_{i=1}^{L-1} \sum_{j=i+1}^{L} ((\alpha_{i,j} e^{V_{ni}})^{1/\mu} + (\alpha_{j,i} e^{V_{nj}})^{1/\mu})^\mu 
\]

(1)

\(V_{ni}\) is the deterministic component of the utility of the location alternative i for individual n.

\(\mu\) is a parameter of dissimilarity for the correlation between spatial units.

\(\alpha_{i,j}\) represents the allocation parameter that characterizes the portion of alternative i that belongs to local cluster.

The development started with the assumption of structural correlation within clusters of adjacent zones. This focus changed to more functional correction in recent studies (see Formula 2-4). The allocation parameter (\(\alpha\)) first is defined by the number of neighbouring zones (w) (Bhat and Guo 2004). Then Bhekor and Prashker (2008) brought up the idea to use the boundary line (L) of adjacent zones. Zones with long common boundaries are assumed to be more similar than zones that share a small common length. In Formula 4, the alternatives k has not to be adjacent. The parameter \(\phi\) is a distance (d) decay function. It is negative, so that correlation between alternatives reduces if distances between them increase.

\[
\alpha_{i,j} = \frac{w_{ij}}{\sum_k w_{ik}}
\]

(2)

\[
\alpha_{i,j} = \frac{L_{ij}}{\sum_k L_{ik}}
\]

(3)

\[
\alpha_{i,j} = \frac{\exp(\phi \ln d_{ij})}{\sum_k \exp(\phi \ln d_{ik})} = \frac{d_{ij}^\phi}{\sum_k d_{ik}^\phi}
\]

(4)

The main limitation of the first two SCL models is that it accommodates spatial correlation alternatives that share a common border. Formula 4 and a further approach by Chen et al. 2009 use travel time distance of non adjacent zones relaxes this constant coefficient assumption and allow a distance-based correlation between nonadjacent zones. This is more a functional than formal correction of the IIA property.

### 3.3.2 Dependency measures

The most general formulation and an extended list of different similarity measures of destination choice model to include as adjustment term in the utility of an alternative can be found in Axhausen and Schuessler (2007). Advantages of this approach are that dependencies has not to be set a priori, and its elegance and simplicity resulting in relatively short computing time.

The dependency of several alternatives in residential choice models can not be described satisfactory by their similarity only. The dissimilarity has even a large effect. Therefore, we prefer to speak about a dependency measure covering similarity and dissimilarity in this
study. Spatial dependency measures listened in Schuessler and Axhausen (2009) and Carscetta et al. (2005) are added here:

- The simplest measure is just considering the average distance from an alternative to all other alternatives proposed by Borgers and Timmermans (1987). This approach does not taking into account the importance or utility of the different alternatives. Thereby, independence of an alternative is low with a height measure value.

- Fotheringham (1983) developed the Competing Destination (CD) model, which weights the distance with the according utility of the corresponding alternative. Thereby, an accessibility measure is included into the utility function in a simple way. The higher the measure of an alternative the more it is depending on other alternatives. The underlying assumption is that the dependence of an alternative decreases its probability to be perceived as a separate alternative.

- The dominance rule in spatial choice process is considered by Cascetta et al. (2005). This probabilistic choice set generation process can also be interpreted as a dependency measure. Dominating attributes are defined in a first step and then the number of dominance degree can be determined for the other dominated attributes. This dominance variables can be defined Boolean or with a dominance ranking of the alternatives. The lowest rank of is the alternative with the largest dominance effect. This ranking can be implemented as a parameter in the utility function. Cascetta et al. (2005) suggest dominance rules such as lower land price or shorter distance to the workplace.

- Bernardin et al. (2009) looks at spatial competition and agglomeration effects taking into account that alternatives of different supply correlated positively and alternative with similar supply negatives. It is proposed to introduce two separated parameters into the utility function. It is an accessibility formulation which includes a variable of dissimilarity. The formulation of this dissimilarity measure depends on the attraction, the number of facilities (number of stores of a different categorisation) and the number of times a store of this categorisation is visited. The broader concept of this approach is convincing, the estimation of the dissimilarity factor is a challenge and not easily transferable to residential choice models.

### 3.3.3 Open the Covariance structure

One problem with multinomial logit models is the IIA assumption, where alternatives assumes to be independent distributed from each other, i.e. the covariance matrix is restricted to be a diagonal matrix. One alternative to break down the IIA assumption therefore consists in allowing to be correlated with each other and that is the multinomial probit model.
However, the formulation of the probit model is complex and its choice probabilities do not have a closed form. It can not be solved analytically and requires simulation for estimation as well as application.

McMillen (1992) considers especially the autocorrelation in probit models. Commonly-employed spatial autocorrelation models imply spatial errors, but this causes probit to be inconsistent. McMillen (1992) proposes and illustrates the use of two categories of estimators for probit models with spatial autocorrelation. One category is based on the EM algorithm, and requires repeated application of a maximum-likelihood estimator and is appropriate for models with spatially dependent errors (SDE models) or the spatial autoregressive model (AR). A specified weight matrix is introduced into the error term. The other category, which can be applied, is the spatial expansion method, only requires weighted least squares. Furthermore, the SDE and AR models are limited to small data sets. One iteration in the estimation of this category of model involves calculating the expected value of the latent dependent variable, and estimating new coefficients by maximizing the log-likelihood function. Given these advantages, the spatial expansion model seems preferable to the SDE and AR models for most applications. Recent work (e.g. Wang et al. 2011) implements even the temporal dimension additionally to the spatial in similar models.

**Suggestion for application in a Swiss national residential choice model**

We try to include a dependency measure into the residential choice model because of the simplicity of the model implementation and the consistency with large datasets. Three different accessibility measures are calculated including positive or negative dissimilarity and similarity.

The accessibility measure by Tschopp et al. (2005) is extensively used in Swiss studies. It includes travel costs from one to all other municipality in respect to a decay function and these costs are weighted by the number of workplaces and population as possible facilities. This approach is accordingly extended: Not all municipalities are included into the accessibility function. The municipalities of the three different characteristics (positive or negative dissimilarity, and similarity) are calculated and included separately as accessibility measure into the utility function. We use some data that represents the periphery-centrality structure of Switzerland for this categorisation (e.g. the ARE-municipality type attribute or the land price could be considered) to determine the similarity and dissimilarity. A high value of accessibility of similarity value according to municipalities with similar land prices or ARE-municipality types, decrease the possibility to choose this alternative. A positive dissimilarity measure includes all municipalities with a lower land-price or lower periphery-centrality index. It is the dominance variable of this alternative. It is more likely chosen. In contrast, the negative dissimilarity measures is the effected of being dominated by others and
seldom chosen. For measuring the difference of dissimilarity of the different municipalities the scale is ranked and the accessibility measure accordingly weighted.

4. Conclusion and Outlook

This note is the starting point of a research going on during the next year. New questions can be raised by the application and the model results will probably lead to some correction of this preceding ideas.

The three different aspects of a location choice model, aggregation level, choice set generation, and correlated alternatives, are not clearly separable and influencing each other. Therefore, a test plan should carefully be constructed for following work in order that the difference of the models can be detected.

It will be a challenge to properly evaluate all these ideas. We will start by a regular model. This regular model is based on a TAZ aggregation level, applying a random choice set generation, and containing the simple accessibility by Tschopp et al. (2005). This regular model is compared to this suggested more functional model by its model fit. Possibly, the model fit of this functional model is not that good, because alternatives are more restrictively built and chosen and therefore have less variation.
References


