Doctoral Thesis

Rheometry for large particle fluids and debris flows

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Publication Date:
2005

Permanent Link:
https://doi.org/10.3929/ethz-a-005004664

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RHEOMETRY FOR LARGE PARTICLE FLUIDS AND DEBRIS FLOWS

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH
for the degree of
Doctor of Sciences

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2005
Acknowledgements

Many people have contributed in different ways to the successful completion of the present work. In particular I would like to thank the following persons:

I deeply thank Prof. Dr. Ing. Hans-Erwin Minor who gave me the opportunity to do research in the fascinating field of the flow properties of mud and debris flows. He shared my motivation and gave important hints in relevant issues of the present study. I appreciated very much that he always found time when I required for.

Sincere thanks go to my supervisor and co-examiner Dr. Gian Reto Bezzola. Based on his sharp analysis concerning technical issues as well as the manuscript text the present work profited a lot. I am further deeply thankful for his psychological support at a difficult stage of the work. During and beyond the thesis Gian Reto was an important former for me.

The origin of the thesis was the collaboration with co-examiner Dr. Peter Fischer from the Institute of Food Science of ETH Zurich. The collaboration was launched within a project of physical and numerical debris flow simulation at VAW. His open-minded nature and his willingness to make the laboratory apparatus and especially the ball measuring system available to us made the present study possible. I am very thankful for that as well as for all the technical conversations we had in the inspiring atmosphere of the above institute.

By the same token I would like to express my deep gratitude to co-examiner Prof. Erich Windhab, leader of the institute, who accompanied me in crucial theoretical issues of the present study.

I owe great thank to Dr. Philippe Coussot who gave important hints for the consolidation of the conversion theory elaborated in the present study.

The workshop and electronic team of VAW was one key of success. Dani Gubser, Stefan Gribi, Georg Meier, Rolf Meier, Bruno Schmid, Walter Schmid, Robert Pöschl, Walter Guhl and Roger Lörtscher helped me to install the set-ups for the different rheometric tests and apparatus. A special thank goes to Bruno Zimmermann who assisted me during the experiments and helped to manage the large sample volumes. We usually ended up to be covered with mud from head to foot after a working day.

The BML viscometer experiments were conducted at the concrete consulting company TFB in Wildegg. Thank you Dr. Andreas Griesser for the assistance during the experiments and the discussions we had.

I namely thank Dr. Günther Kahr of the laboratory of clay minerals of ETH Zürich for the clay analysis of the different sediment material and for the introduction of the Kasumeter.
Acknowledgements

It was a lucky coincidence when I discovered the fresh deposit of a debris flow in the Maschänserrüfe torrent during my holidays in spring 2001. It was then thank to the unbureaucratic acting of the responsables of the Kieswerk Untervaz that the material was excavated the other day in order to guarantee an undisturbed material sample for the present investigation. I thank Martina Kunz, Enrico Tempesta and Daniel Devan-théry for sorting out stones and boulders. By the same way I thank the local forestman Mr. Hemmi for communicating and sharing observations of debris flows in this torrent.

Cordial thanks are given to Andreas Rohrer for the draw and final design of many figures and to Bernhard Etter for the photographs. I sincerely thank Victor Bailey for corrections of the manuscript text regarding the english language. I further thank Claudio Dalri e Matteo Pinotti for the italian summary and Bernard Cuche for corrections concerning the french summary.

My great motivation during the thesis was built and maintained by numerous and ongoing discussions I had/have with the following people about the physics of debris flows and about the usefulness of the ball measuring system. I first thank Christian Tognacca, former debris flows researcher at VAW. I further thank P. Vollmöller, R. Iverson, J. O’Brien, T. Davies, C. Ancey, E. Bardou, R. Chhabra, S. Springman, P. Burlando, L. Vuillet, D. Laigle, M. Roth, W. Gostner, J. Seiler, C. Graf, B. Mc Ardell, M. Zimmermann, D. Weber, D. Rickenmann, A. Johnson, R. Garcia, A. Armanini, M. Arattano, H. Suwa, M. Jäggi and the unknown reviewers of the journal and conference articles.

I want to express my thanks to Patricia Requena and Monika Weber for creating a very comfortable working atmosphere in the office. In the same way I thank all the further collaborators for creating the good atmosphere at VAW.

Sincere thanks are given to Jürg and Barbara who enabled me to study the very interesting subject of rural engineering at ETH Zürich and EPF Lausanne. Likewise I thank my brothers Franz and Adrian as well as my friends for all we have shared so far.

I finally deeply thank my wife Deborah for her large support and for giving birth to our daughter Jasmine Tabea. The thesis is dedicated to Jasmine Tabea.
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Abstract

The present study focused on existing and novel rheometric tools for the efficient determination of the rheological behaviour of large particle fluids with particular interest in the application to debris flows.

The main goal was the examination of the ball measuring system (BMS). Implemented in a standard rheometer, the BMS consists of a sphere that is dragged at specified speeds across a sample of 0.5 liter volume with the help of a small sphere holder. Accordingly torques due to drag exerted on the sphere and its holder as well as corresponding speeds are measured within a wide range.

Based on the present study it could be shown that the system principally allows the determination of the flow curve, relaxation time and fluid fatigue of fluids containing particles up to 10 mm grain size. The existing theory for the conversion of measured into rheological data was improved by (i) respecting the characteristics of different types of fluids (Newtonian, Power Law and Yield Stress Fluids) and (ii) by considering the laminar flow regime ($Re \leq 1$) as well as the transitional regime ($1 < Re < 100$). In case of the Yield Stress Fluids it was shown that the new semi-empirical relationships correspond well with the yield stress criterion derived by different authors for spheres starting to move in this type of fluid.

For comparison, a variety of debris flow material mixtures and other suspensions were investigated with the BMS as well as with other rheometric tools, such as the large scale rheometer of Coussot&Piau, the BML viscometer, the Slump Test, the Inclined Channel Test, the Inclined Plane Test and the Kasumeter. Overall the flow curves and the yield stresses obtained with the BMS agreed well with the results of the other systems.

For the determination of the flow curve in daily application, the ball measuring system is recommended for particle fluids up to 10 mm grain size, and the BML viscometer for fluids up to 30 mm grain size. The yield stress is determined efficiently with the Slump Test, either based on the slump height or based on the profile of deposit.

With regard to debris flows, a distinction between granular debris flows, mud flows and viscous debris flows is given in the present study. For granular debris flows where a porefluid (water, clay and silt) is distinguished from the larger particles, rheology is considered to be relevant on the level of the porefluid. For the latter the rheological properties are determined based on standard measuring systems in conventional rheometers. For mudflows where a muddy phase (water, clay, silt, sand and ev. gravel) is distinguished from the larger particles, the rheological properties of the mud can be determined with the help of the rheological apparatus and tests recommended in the section above. For viscous debris flows where the entire mixture of water and all sediments behaves more or less as one phase, two rheometric methods are given: One is based on field and laboratory investigations which requires the apparatus and tests recommended above. The other method is entirely based on field investigations and requires observations of the flowing debris flow.
Zusammenfassung

Die vorliegende Arbeit beschäftigte sich mit bestehenden und neuen Messsystemen zur Bestimmung der rheologischen Eigenschaften von Grobpartikelfluiden mit besonderem Interesse hinsichtlich der Anwendung auf Murgänge.

Das Hauptziel der Arbeit lag in der Prüfung des Kugelmesssystems (BMS). Modular einbaubar in einem Standardrheometer besteht das BMS aus einer an einer schmalen Halterung befestigten Kugel, die - exzentrisch rotierend - bei definierten Geschwindigkeiten durch ein Probefluid von 0.5 l Volumen gezogen wird. Entsprechend werden die auf die Kugel aufzubringenden Momente sowie die zugehörigen Geschwindigkeiten in einem grossen Messbereich gemessen.

In der Arbeit wurde gezeigt, dass das BMS die Bestimmung der Fließkurve der Erholungszeit und der Ermüdung von Grobpartikelfluiden mit Körnern bis zu 10 mm Durchmesser erlaubt. Die bestehende Theorie zur Umwandlung der gemessenen Grössen in rheologische Grössen wurde verbessert und erweitert durch (i) Differenzierung nach Fluidtypen (Newton, Power Law und Grenzschubspannungsfluid) und (ii) unter Berücksichtigung der laminaren Kugelumströmung (Re < 1) wie auch des Übergangs bereiches (1 < Re < 100). Im Falle der Grenzschubspannungsfluide konnte gezeigt werden, dass der neue Ansatz gut mit dem von verschiedenen Autoren definierten Kriterium für den Bewegungsbeginn von Kugeln in solchen Fluiden korrespondiert.


Für die effiziente Bestimmung der Fließkurve von Grobpartikelfluiden bis zu 10 mm Korn durchmesser wird das BMS, bis zu 30 mm Korn durchmesser der BML Viskometer empfohlen. Der Slump Test empfiehlt sich zur Bestimmung der Grenzschubspannung, wo letztere entweder aus der Slumphöhe oder aus dem Ablagerungsprofil ermittelt wird.

Résumé

En vue de l'application aux laves torrentielles ce travail se concentre sur la rhéométrie pour déterminer les propriétés rhéologiques de fluides formés de gros grains.

Le but principal du travail est l'analyse du système de boule (BMS). Implémenté dans un rhéomètre conventionnel, le BMS est constitué d'une sphère entraînée par un bras rotatif. L'ensemble – en rotation excentrique – est mu à des vitesses définies à l'intérieur d'un échantillon de 0.5 l de volume. Le couple exercé sur l'ensemble boule et bras est mesuré dans une grande plage en même temps que la vitesse.

Ce travail montre que le BMS permet de déterminer la courbe d'écoulement, le temps de repos et la fatigue de fluides contenant des particules jusqu'à un diamètre de 10 mm. La théorie existante de la conversion des grandeurs mesurées en grandeurs rhéologiques a été améliorée par la considération (i) de différents types de fluides (newtonien, «power law », fluide à seuil) ainsi (ii) que différents régimes (laminaire (Re ≤ 1), transitionnel (1 < Re < 100)). Dans le cas des fluides à seuil, la nouvelle approche corrobore le critère de seuil donné par différents auteurs pour des sphères se mouvant dans ce type de fluide.

Pour comparaison, de nombreux mélanges d'eau et de sédiments de laves torrentielles ainsi que différentes suspensions ont été analysés avec le système BMS et d'autres systèmes comme le rhéomètre à grande échelle de Coussot&Piau, le viscomètre BML, le « slump test », le casumètre et des tests dans un chenal incliné et sur un plan incliné. Les courbes d'écoulement et les seuils de contrainte obtenus avec le BMS concordent bien avec les résultats obtenus avec ces autres systèmes.

Pour une détermination optimale de la courbe d'écoulement de fluides contenant des grains jusqu'à un diamètre de 10 mm, le système BMS est recommandé. Pour un diamètre allant jusqu'à 30 mm le viscomètre BML convient. Le seuil de contrainte est déterminé efficacement avec le « slump test », basé soit sur la hauteur du « slump », soit sur le profil du dépôt.

Quant aux laves torrentielles, on distingue la lave granulaire, la coulée de boue et la lave visqueuse. Pour la lave granulaire, le concept de rhéologie n'est applicable qu'au fluide intergranulaire (eau et sédiment fin). Pour celui-ci la courbe d'écoulement peut être déterminée avec des systèmes de mesure standards dans un rhéomètre conventionnel. La coulée de boue est composée d'une phase liquide (boue: eau et sédiment d'un mélange argile-gravier) et d'une phase solide (pierres, blocs). Les propriétés de la boue peuvent être déterminées avec les appareils et systèmes décrits dans le paragraphe précédent. Enfin pour la lave visqueuse où le mélange eau et sédiments se comporte comme une seule phase liquide, deux méthodes rhéométriques sont proposées. La première est basée sur des analyses de terrain et des analyses de laboratoire, tandis que la deuxième est uniquement basée sur des analyses de terrain. Toutefois dans ce cas il faut pourvoir observer la lave en mouvement.
Riassunto

La tesi seguente tratta l’uso di esistenti e innovativi dispositivi reometrici per una efficiente determinazione del comportamento reologico di fluidi a particelle grossolane con’un accenno particolare sull’applicazione nel campo delle colate detritiche.

L’obiettivo principale dello studio era indagare il “ball measuring system (BMS)”. Il sistema (BMS) è composto da una sfera, solidale ad un supporto, la quale è mossa ad una specifica velocità, all’interno di un campione di materiale di circa 0.5 litri. Simultaneamente, il momento torcente richiesto per il movimento della sfera e la sua velocità sono misurati in un grande campo di misura.

Lo studio evidenzia come il BMS permette la determinazione del reogramma e del tempo di rilassamento per fluidi che contengono particelle grossolane di diametro superiore a 10 mm. Le teorie esistenti, devote alla conversione dei dati misurati in parametri reologici, sono state migliorate considerando: (i) le caratteristiche di differenti tipologie di fluidi (newtoniani, pseudoplastici, dilatanti o dotati di tensione di soglia) e nel considerare (ii) il regime laminare (Re ≤ 1) tanto che il regime transizionale (1 < Re < 100). Nel caso di fluidi dotati di tensione di soglia è stato possibile mostrare come le nuove semi-empiriche relazioni proposte corrispondono bene al criterio del innizio moto di sfera in tali fluidi derivato da diversi autori.

Per confronto sono state analizzate altri tipi di misture e di sospensioni utilizzando sia il BMS che altri sistemi reometrici, come il grande reometro di Coussot&Piau, il viscometro BML, il Slump Test, il kasumeter e gli sperimenti con canale inclinato e su piano inclinato. I reogrammi e i valori di tensione di soglia ottenuti con il BMS sono in accordo con i risultati degli altri sistemi reometrici.

Per la determinazione efficiente dei reogrammi, il BMS è raccomandato per fluidi con particelle grandi fino a 10 mm, mentre il viscometro BML è raccomandato per fluidi con particelle grandi fino a 30 mm. La tensione di soglia viene accuratamente determinata per mezzo dello Slump Test osservando o la variazione in altezza del provino o la formazione dello stesso a fine prova.

Riguardo le colate di detriti (debris flow) si possono distinguere le colate detritiche granulari, le colate di fango e le colate di detrito viscose. Per le colate detritiche granulari, in cui è possibile individuare una matrice ed una parte più grossolana, la reologia è rilevante solo per il fluido intergranulare (acqua e sedimento argilloso-siltuoso). In questo caso il reogramma si determina con dei consueti sistemi reometrici. Per le colate di fango, dove si distingue una fase liquida (il fango: acqua e sedimento argilloso-ghiaioso) da una fase solida (massi, blocchi), la reologia può essere ricavata con le indicazioni già fornite precedentemente. Per le colate di detrito viscose, in cui le diverse fasi si confondono in una unica pseudofase, sono proposti due metodi: Il primo include delle analisi sia di campo che di laboratorio compiuta con le attrezzature e le metodologie già descritte. Il secondo metodo include solamente delle analisi di campo attraverso l’osservazione diretta del flusso di una colata di detriti.
1 Introduction

The present study focuses on rheometric systems and methods for the determination of the rheological properties of fluids containing particles with maximum grain sizes equal or larger than 1 mm.

Such large particle fluids are widely encountered on a daily basis and the knowledge about their rheological properties is of great and increasing importance. The relevant fields hereby are namely:

Natural hazards: Physical and numerical modeling of debris flows, avalanches, lavas, etc. for hazard zone mapping and for the investigation of mitigation measures.

Building materials: Optimization of flow properties of plaster, concrete, gypsum, paints, etc. in production and application.

Gas and oil industry: Optimization of technical equipment for the drilling and the transport of soil materials, slurries and mud.

Sewage treatment and technology: Optimization of slurries for pumping and transport.

Food industry: Optimization of flow properties of yoghurts, ice cream, sauces, etc. with regard to industrial processing and customer acceptance.

In the present study debris flows and the application of rheometry for debris flows are of primary interest.

Debris flows endanger people and animals and cause damage in hilly and mountainous areas. In a given catchment area of a debris flow torrent the direct hazard zones are usually settlements and traffic lines situated on (i) steep hillslopes (debris flow initiation area), (ii) along the channel (debris flow transit area) and (iii) on the fan (debris flow deposition area).

Debris flows may also cause indirect hazard by the transport of large material volumes into the downstream river and creating a natural dam in the river. This dam can cause large backwater which may provoke bank collapse and flooding in the adjacent area of the river.

More than 2000 people died in 2003 worldwide due to debris flow or combined debris flow – flood events (SwissRe 2004). In the European alps 1 to 2 people die every year due to debris flows. During the storm of October 2000 in Southern Switzerland 16 people lost their life due to debris flows (BWG 2000). The total annual cost of direct
damage due to debris flows in Switzerland is 20-35 million Euro (Bezzola & Minor 2002).

In order to protect people, animals and infrastructure from damage, (i) non-structural measures such as hazard zone mapping and warning systems as well as (ii) structural measures such as reforestation, dikes, breakers, retention basins, etc. must be considered depending on the specific situation.

For both types of measures debris flow modeling is becoming a more and more important task. With the help of numerical and in some cases also physical modeling (i) hazard zones can be delineated and (ii) the efficiency and design of structural measures can be tested and improved.

For the simulation of the flow and deposition process, a rheological model can be a very useful physical concept for some types of debris flows. Beside the debris flow volume and other parameters, the flow curve parameters - expressing the shear rate - shear stress relation - are required when a rheological model is considered. By contrast, efficient rheometric apparatus and methods which are used for the determination of these parameters hardly exist or were not evaluated, respectively.

With the present study existing and new measuring systems, apparatus and methods which are appropriate for large particle fluids and debris flows are tested. Here the ball measuring system is of primary interest. Implemented in a standard rheometer, the system could be an efficient rheometric tool for the investigation of fluids containing particles up to 10 mm grain size. However the system must be first analysed thoroughly. In particular the conversion of measured data into rheological data, first derived by Tyrach (2001), must be improved in order to obtain reliable rheological data for particle fluids. Then, the system might even have some potential to be expanded to a large scale device where fluids containing particles up to 60 mm grain size could be investigated.

*Study overview:*

The study begins with an introduction into rheology and debris flows. By distinguishing different types of debris flows, it is shown on which level a rheological concept is useful for the simplified description of the many physical processes usually involved in debris flows. The actual state of research in rheometry for large particle fluids and debris flows is then summarized (chapter 2).

The specific problems to be solved, the goal and the methodology of the present study is given in chapter 3.
Chapter 4 contains the examination of the ball measuring system. Among others the theory of the conversion of measured into rheological parameters is improved here.

The application and examination of other rheometric systems for large particle fluids is the subject of chapter 5.

The ball measuring system and the other examined rheometric systems are then compared as far as rheological data and their application in practice is concerned (chapter 6).

Finally two methods are introduced and applied for the determination of the flow curve parameters of viscous debris flows (chapter 7) before the study closes with the conclusions (chapter 8).
2 Basics and state of the art

2.1 Rheology

2.1.1 Introduction and definitions

Rheology is the science of the flow and deformation behaviour of materials considering both liquids and solids. In the present study the viscous flow behaviour of fluids is of primary interest. This section thus aims to provide a short introduction of the principles and definitions concerning the physical and mathematical description of the flow behaviour of fluids with regard to the application used in the present study. The description of deformation behaviour and given elasticity is omitted here.

The flow behaviour of a given fluid is expressed either by the flow curve, which expresses the relationship between the shear rate $\dot{\gamma}$ and the shear stress $\tau$, or by the viscosity curve, which expresses the relationship between the shear rate $\dot{\gamma}$ and the viscosity $\eta$. All three parameters $\dot{\gamma}$, $\tau$ and $\eta$ are defined with the help of the following two-plate model (fig. 2.1):

![Two-plate model](image)

**Figure 2.1** The two-plate model for the definition of the rheological parameters.

Here the fluid is set between two plates which are separated by a small gap $H$. The bottom plate is at rest. The upper plate with the surface area $A$ is moved with the velocity $v$ by applying the force $F$. Accordingly the fluid in between is sheared. It is required that the fluid adheres to the plates surface (no slip) and the shear flow is a [laminar layer flow](#) and not a turbulent flow. Respecting these two conditions as well as

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1 Any material that is able to flow is called a fluid (Mezger 2000)
a small gap $H$ the velocity distribution within the fluid is linear and the rheological parameters are defined as follows:

\begin{align}
\dot{\gamma} &= \frac{v}{H} \\
\tau &= \frac{F}{A} \\
\eta &= \frac{\tau}{\dot{\gamma}}
\end{align}

Depending on the range of shear rate $\dot{\gamma}$ considered and depending on the fluid type the flow curve/viscosity curve can exhibit a simple to a very complex behaviour. The mathematical description therefore also can span from simple to highly complex constitutive equations. With regard to the present study the shear rate usually ranges between $0.1 \leq \dot{\gamma} \leq 100 \text{ s}^{-1}$ and the applied fluids involve oils, polymer solutions and sediment-water-mixtures. Accordingly the following fluid classification is given here based on the flow curve/viscosity curve characteristics for this range of the shear rate $\dot{\gamma}$ (fig. 2.2):

1. **Newtonian fluid:** $\eta = \text{constant}$
2. **Non-Newtonian fluids:** $\eta \neq \text{constant}$
3. Power Law Fluid (PLF) with shear-thinning behaviour
4. Power Law Fluid (PLF) with shear-thickening behaviour
5. Yield Stress Fluid (YSF) with shear-thinning behaviour
6. Bingham Fluid: Simplified Yield Stress Fluid (YSF)

**Figure 2.2** Simplified classification of fluids based on the characteristic of the flow curve (left) and the viscosity curve (right).
For Newtonian fluids the flow curve is described by a linear function:

\[ \tau = \mu \cdot \dot{\gamma} \tag{2.4} \]

where \( \mu = \text{constant} \) is the Newtonian viscosity. Examples of Newtonian fluids are water, oils, glycerine, milk, etc.

Non-Newtonian fluids are divided here into Power Law Fluids (PLF) and Yield Stress Fluids (YSF)\(^2\). The flow curve of PLF is described by a power-law function:

\[ \tau = m \cdot \dot{\gamma}^n \tag{2.5} \]

where \( m = \text{the power-law consistency coefficient} \) and \( n = \text{the power-law index} \). If \( n < 1 \) the power-law fluid behaves as a shear-thinning fluid. If \( n > 1 \) it behaves as a shear-thickening fluid. Examples of shear-thinning fluids are polymère solutions or shampoos. Examples of shear-thickening fluids are highly concentrated dispersions of starch or ceramic.

A Yield Stress Fluids (YSF) is characterized by the presence of a yield stress \( \tau_y \), which expresses the stress that must be overcome to set a fluid into motion. If the applied stress is lower than \( \tau_y \) no fluid motion takes place. The flow curve of a YSF is usually described either by a Herschel-Bulkley or a Casson model. The Herschel-Bulkley model function is:

\[ \tau = \tau_y + m \cdot \dot{\gamma}^n \tag{2.6} \]

where \( m = \text{the Herschel-Bulkley consistency coefficient} \) and \( n = \text{the Herschel-Bulkley index} \). A simplification of the Herschel-Bulkley model is the Bingham model. Here \( n \) reduces to \( n = 1 \) and \( m \) reduces to \( m = \mu_B \), where \( \mu_B \) is the Bingham viscosity.

The Casson model function is:

\[ (\tau)^{0.5} = (\tau_y)^{0.5} + (\mu_c \cdot \dot{\gamma})^{0.5} \tag{2.7} \]

where \( \mu_c \) is the Casson viscosity parameter.

YSF typically described by the Herschel-Bulkley model are sediment water mixtures rich of fine material. YSF typically described by a Casson model are yoghurt, tomato purée, molten chocolate or blood.

\(^2\) Note that there are many more non-Newtonian fluids classified depending on the mathematical description of the flow curve/viscosity curve. A good overview is given for example in Mezger (2000) or in Roberts et al. (2001).
2.1.2 Shear flow in particle fluids

The rheometric systems used for the determination of the flow curve of particle fluids are introduced in sections 2.5 to 2.7. Here some principal aspects are mentioned when a particle fluid is under shear.

If a particle fluid is sheared the following side effects may occur: wall slip and segregation (particle settling and particle migration). Both effects contradict the concept of the shear flow defined before, since (i) the fluid is not fully sheared and (ii) the grains leave a given flow layer and enter into another one. If such effects occur the measured flow curve might not represent the property of the entire fluid. It is important to detect such effects or to know how relevant these effects are for a given particle fluid under a given shear flow. This is still difficult because:

- Measuring systems which give insight into the shear flow of particle fluids and to detect such effects directly usually hardly exist or are still under development (Armanini et al. 2003, Coussot et al. 2003).

- An unlimited variety of particle fluids occur worldwide in different fields of application which renders a classification and attribution of these side effects to specific types of particle fluids very difficult.

Nevertheless wall slip effects, which is often linked with particle migration, can be often detected indirectly by a step in the flow curve (see example in Appendix D.3).

The use of roughened plate surfaces to avoid wall slip can be helpful in some cases but might lead to secondary effects in other cases. The latter are namely (i) slip within the fluid and (ii) irreversible particle migration (Nguyen & Boger 1992, Coussot 1997).

Compared to wall slip the detection of segregation (particle settling and particle migration) is less developed. Extent of particle segregation depends on the sediment concentration, grain size distribution, type/shape of particles within the fluid as well as of the shear rate and the applied rheometric system.

It is generally known that particle settling decreases when the sediment concentration in the fluid increases (Vanoni 1975). In very concentrated particle fluids which are rich of fine material, particle settling can be often rendered negligible within the limited duration of the shear flow (Coussot 1997).

For an unsheared particle fluid composed by two phases, a Yield Stress Fluid-liquid and grains, it is possible to determine whether the grains settle or not based on the following relation (Chhabra & Richardson 1999):
Here $g$ is the acceleration due to gravity, $d$ is the grain diameter, $\rho_s$ is the density of the grains, $\rho_f$ is the density of the liquid, $\tau_y$ is the yield stress of the liquid and $Y$ is the dimensionless static equilibrium number. $Y$ was determined in various studies as 0.04 to 0.2 (Chhabra & Richardson 1999). By contrast when the particle fluid is sheared eq. 2.8 is invalid and grains which were in static equilibrium tend to settle if the density of the grains $\rho_s$ is larger than the density of the liquid $\rho_f$ (Coussot 1997).

The aspect of particle settling during shear flow is further discussed for the specific case of debris flows in section 2.3.

### 2.2 Debris flows

**2.2.1 Phenomenon**

A debris flow is a mixture of sediment and water moving down a channel or irregular surface due to gravity. In contrast to bed load transport or hyperconcentrated flows the transported sediments are distributed over the entire flow depth. Depending on mixture composition stones and boulders are preferably transported close to the bed, homogeneously over the entire flow depth or close to the surface (inverse grading). Often a debris flow surge shows the following characteristic features along its longitudinal section (fig. 2.3).

The front is composed mainly by blocks and stones which are either being accumulated directly from the bed or migrating from the body to the front. Usually the largest flow depth of the surge is attained in the front. The front is followed by the body, the most voluminous part of the debris flow. The end of the flow is defined by the tail. Both, body and tail, are usually characterised by decreasing flow depths and decreasing sediment concentrations.

Regarding the transversal section of a debris flow the formation of lateral levees can be often observed (fig. 2.3). The content of blocks and stones is usually larger in the levees than in the interior of the debris flow. After the passage of the surge the levees normally remain deposited.

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Example: For $\tau_y = 10$ Pa, $\rho_f = 1500$ kg/m$^3$ and $\rho_s = 2650$ kg/m$^3$ the maximum size of grains which do not settle in the unsheared fluid is $d = 4$–20 mm.
In addition debris flows may also erode material when they flow downstream. In some cases erosion can be very significant and lead to a multiplication of the initial volume. However the degree of erosion usually varies greatly from one surge to another and depends strongly on the composition of the debris flow mixture and the presence of erodable material. Additionally, due to intake of erodable material and fragmentation of solid particles during flow, the grain size distribution can alter from the initiation zone down to the deposition zone.

In contrast to sediment depositions induced by bed load transport or hyperconcentrations the deposition of a debris flow is clearly delineated and its shape is similar to a tongue. In the case that a debris flow stops on a irregular surface the deposition is usually spread into several fingers (fig. 2.4).
Figure 2.4  Debris flow catchment area of Val Varuna (CH) from initiation down to deposition zone (Photo A. Godenzi, Chur).
Debris flows occur as a landslide on small hillslopes of only some 10 m$^2$ but also form in large catchment areas of several 10 km$^2$. Accordingly debris flow volumes $V$ span a range of 100 to more than 1'000'000 m$^3$. In special cases (volcanic lahars e.g.) volumes of up to 70'000'000 m$^3$ were observed (Rickenmann 1999). Similar to the volumes, the peak discharges $Q_p$ varies from some m$^3$/s to several 1'000 m$^3$/s, for volcanoe lahars up to several 10'000 m$^3$/s. The mean velocity $v$ of debris flows span a range of 1 to 30 m/s. However velocities reach rarely more than 15 m/s.

2.2.2 Initiation

Two main mechanisms can be distinguished which cause the initiation of debris flows:

1. Surface runoff
2. Failure

**Surface runoff** means the progressive erosion of partly or fully saturated bed material by clear water running over the material. Depending on clear water discharge, degree of saturation in the bed material, bed slope and availability of erodable material, a debris flow with distinctive discharge and sediment concentration is progressively developed (Takahashi 1991, Tognacca 1999). Usually this initiation mechanism is found in torrent channels or on scree slopes where clear water is collected in a rocky zone above the scree slope (fig. 2.5). An important case is also the dam collapse of a natural or artificial water reservoir in a torrent or river due to overtopping.

**Failure** is a soil-mechanical process. Depending on the inclination of the sediment mass and the mechanical properties of the sediment (cohesion, internal friction angle) the sediment mass may fail when the sediment is more and more saturated with water. Increase of water saturation degree, ongoing with an increase of pore water pressure, is usually due to heavy rainfall, in some cases also due to rapid snow melt (Suwa 2003, Hemmi 2004, Kolenko et al. 2004), groundwater upwelling (Lin&Chang 2003) or earthquakes (Liao&Chou 2003). Failure of a sediment mass usually occurs on steep hill slopes and represents the initiation mechanism of landslides. In some cases it also occurs within vast channel deposits and landslide induced dams in channels due to seepage failure (Liao&Chou 2003). Note that the released mass may additionally be liquified due to contraction during slope failure, ongoing with a decrease of initial porosity and increase of pore water pressure (Iverson 2000).
2.2.3 Flow characteristics

Sediment concentration and grain size distribution are the relevant parameters which determine the flow characteristics of a debris flow whereas other parameters such as bed slope, particle shape, type of clay mineral, etc. are of minor importance. Davies (1988) gives a very useful scheme upon which the specific flow behaviour can be determined (fig. 2.6). Here the debris flow is simplified to a mixture composed by water, fine material ($d < 0.1 \text{ mm}$) and coarse material ($d > 60 \text{ mm}$).
According to fig. 2.6 the flow is dominantly turbulent for a small volumetric sediment concentration \( C_v \) whereas for a large sediment concentration \( C_v \) the flow is dominantly laminar. For a low content of fine material inertial stresses (turbulence and particle collisions) dominate over viscous stresses of the fluid, which is composed by water and fine material. By contrast, for a large content of fine material viscous stresses dominate over inertial stresses. For small to medium concentrations \( C_v \) and a small content of fine material the debris flow behaves as a Newtonian fluid whereas for a larger content of fine material and larger concentrations \( C_v \) it behaves as a non-Newtonian fluid.

Within a given debris flow surge the flow characteristics may change from the front to the tail due to changes in grain size distribution and sediment concentration \( C_v \).

### 2.2.4 Classification

Different classification schemes have been elaborated for debris flows in the last decades. Usually the classifications highlight the dominant mechanism during the flow process (Bagnold 1954, Iverson 1997, Bardou et al. 2003 e.g.). Some authors combine

\[ C_v = \frac{V_s}{(V_s + V_w)} \] where \( V_s \) = volume of sediments (solids), \( V_w \) = volume of water
the mechanisms with important parameters such as sediment concentration $C_v$ and grain size distribution and attribute characteristic debris flow types (Davies 1988, Coussot & Meunier 1996).

For the present study the classification of Coussot & Meunier (1996) is used and adapted in order to show characteristic debris flow types depending on the sediment concentration of the debris flow and the content $p$ of fine material ($d \leq 0.04$ mm) within the complete grain material (fig. 2.7). Likewise the classification distinguishes the different debris flow types from other natural mass movements such as bed load transport, hyperconcentrated flow, rock fall/avalanche and block type landslides.

According to fig 2.7 the following characteristic debris flow types are distinguished: Granular debris flow, viscous debris flow and mudflow.

A granular debris flow is composed of water, a small amount of fine material and a large amount of coarse material. Usually the granular debris flow is divided into a fluid phase (pore fluid) constituted of water and the very fine material and a solid phase constituted of the coarser material (Takahashi 1991, Iverson 1997). In a granular debris flow interactions between the coarser particles, such as collisions and friction as well as interactions of the pore fluid with the coarser particles are dominant.

![Figure 2.7 Adapted classification of debris flows (A-D) and other mass movements induced by gravity (E-H) following Coussot & Meunier (1996).](image-url)
By contrast this is not the case in a viscous debris flow where the complete flow behaves more or less as one homogeneous viscous phase and the flow of the entire mixture is dominantly laminar. Here the sediment concentration \( C_v \) of the debris flow is large and the content \( p \) of fine material within the complete grain material is larger than 10%. Further, the grain material is usually poorly sorted\(^5\), so that the space between larger particles is filled with smaller particles from the macroscopic scale (block fraction) down to the microscopic scale (clay fraction). Due to the large content \( p \) of fine material the coarser particles are surrounded by a mixture of fine material and water. This, as well as the large sediment concentration dampen the collisions between the coarser particles and make the entire debris flow appear more or less as one homogeneous viscous phase.

In comparison a mudflow is constituted of a still larger amount of fine material and a smaller amount of coarser particles compared to a viscous debris flow. In a mudflow a fluid phase (the mud), which is composed of water and finer sediments can be distinguished from a solid phase made of the coarser particles. Depending on the content of coarser particles, interactions between coarser particles frequently occur or are negligible. However, compared to a granular debris flow interactions between coarser particles are of less importance here. Depending on the sediment concentration within the mud, the inclination of the flow bed and the flow depth, the mudflow exhibits a laminar or turbulent flow behaviour.

The viscous-granular debris flow is a transition and shows features of both the granular and the viscous debris flow.

Fig. 2.8 shows examples of the deposition of the three characteristic debris flow types.

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\(^5\) Poorly sorted means that each grain class is sufficiently represented in the given grain material (see grain size distribution example in fig. 3.1)
Figure 2.8  Deposition of characteristic debris flows.
Usually a link between the geological conditions of the catchment area and the debris flow type exists. Granular debris flows more occur in zones of mother stones (granite, gneis, etc.) whereas viscous debris flows and mudflows more occur in zones of weatherable sediments (schists, black marls, löss, moraines, etc.).

But it must be mentioned that, due to complex geological conditions in a given catchment area, different types of debris flows may occur, even from one surge to another. A debris flow may also mutate into a hyperconcentrated flow or vice versa, or mutate from a granular debris flow into a viscous debris flow due to changes in geology, sediment concentration and fragmentation of grain material along its flow path.

In the European Alps granular and viscous debris flows are more common than mud flows.

2.3 Rheology and debris flows

2.3.1 Velocity field within debris flows

The vertical distribution of velocities $v$ of a debris flow moving down a channel or plane with an inclination angle $i$ is schematically represented in fig. 2.9. Usually the flow depth can be divided into a zone of strongly increasing velocities adjacent to the bed (thickness $H$) and a zone of almost stable velocities adjacent to the surface. Due to the increase of velocities in the zone adjacent to the bed the debris flow is sheared and an analogy to the shear zone defined with the two-plate model (fig. 2.1) is therefore assumed. In the following it is discussed whether this zone corresponds to the laminar layered shear zone defined in section 2.1 for the different debris flow types.

![Simplified vertical velocity distribution in debris flows](image)

*Figure 2.9  Simplified vertical velocity distribution in debris flows.*
2.3.2 The shear zone for different debris flow types

**Granular debris flows:**

In granular debris flows intergranular friction and collisions are a dominant feature within the flow process. Due to collisions, particles hardly move in laminar layers but move in variable depths between the flow bed and the surface. Due to collision and friction, velocity fluctuation of the particles are very large and the velocity field shown in fig. 2.9 is only an average picture of the very complexe and time-dependent velocity distribution in granular debris flows. As a consequence the shear zone defined in section 2.1 can not be observed on a macroscopic scale. Nevertheless, the laminar layered shear zone may exist on a microscopic scale in the pore fluid (fig. 2.10). The pore fluid is usually constituted of water and particles of the clay and silt fraction (Iverson 1997) but might eventually also include larger particles up to \( d = 0.1 \) mm grain size (Davies 1988)\(^6\). Hence, within granular debris flows the rheological concept is only relevant on the level of the pore fluid but not on the level of the prototype debris flow.

**Viscous debris flows:**

In viscous debris flows which are characterized by a high sediment concentration, a large content of fine material and a generally poorly sorted grain material, the flow appears more or less as one viscous phase. Usually laminar flow dominates over turbulence and the criteria of a laminar layered shear flow is well fulfilled. However, due to the large grain size \( d \) of stones, blocks or boulders, the thickness of a specific shear layer is variable with time and position. As the shear zone is large and not confined to a small gap \( H \), the shape of the velocity distribution is not linear but convex (fig. 2.9, 2.10). The shape of the velocity distribution hereby expresses the rheological behaviour of the debris flow (Coussot 1997). To conclude, the rheological concept is an appropriate physical concept for the flow and deposition process of highly concentrated viscous debris flows containing particles from the clay up to the block and boulder fraction (\( d \leq 200 - 1000 \) mm).

If the total sediment concentration is decreased towards a hyperconcentrated flow and the grain size distribution altered towards a well sorted one, particle settling, particle collision and turbulence become more and more important. In this case the laminar layered shear zone disappears and the rheological concept is not fully appropriate.

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\(^6\) The aspect of the grain size \( d \) is discussed in more detail below in the case of the mudflow.
Figure 2.10  Laminar layered shear zone within different types of debris flows.

**Mudflows:**

Mudflows are characterized by a muddy phase (water and finer particles) and a solid phase (coarser particles). Depending on the sediment concentration of the mud, the flow bed inclination and the flow depth the flow regime is laminar or turbulent. It is laminar for large sediment concentrations, small bed inclinations and small flow depths. It is turbulent for small sediment concentrations, large bed inclinations and large flow depths (Coussot 1997). Collisions between coarser particles usually play a minor role in laminar flow but become relatively more important in the case of turbulence.
The concept of the laminar layered shear zone (the rheological concept) holds for the mud in the case of laminar flow. In the case of turbulence of the mud and collisions between coarser particles, additional physical concepts must be introduced to account for the latter processes (O’Brien & Julien 1985, Julien & Lan 1991 e.g.).

Grain size of particles belonging to the mud:
Given that the rheological concept is useful for the muddy phase, it must be established up to which grain size $d$ particles belong to the mud, respectively from which grain size $d$ particles belong to the solid phase. O’Brien & Julien (1988) attributed roughly the clay and silt fraction ($d \leq 0.06\text{ mm}$) to the mud, whereas Malet et al. (2003) consider particles up to $d = 20\text{ mm}$ grain size constituting the mud. In case of O’Brien & Julien (1988) the grain size $d$ is a theoretical value which corresponds with the fall velocity of particles in clear water (Vanoni 1975, p. 25) and the duration of a debris flow event (the particle with the grain size $d$ settles less than half of the flow depth for the duration of the debris flow event; see also Iverson 1997, p. 253). The grain size $d$ found by Malet et al. (2003) comes from field observation and corresponds with the fact that particle settling is reduced when the sediment concentration in the fluid is increased (Vanoni 1975, p. 27), or is hindered when the grain size distribution is poorly sorted and the sediment concentration is large (Coussot 1997). The author of the present study considers the value indicated by O’Brien & Julien (1988) as too small, because the sediment concentration of the mud is usually rather large. It is therefore recommended to operate with grain sizes $d \leq 0.1 - 20\text{ mm}$ for the particles of the mud. If the sediment concentration is very large and the grain size distribution of the mud is poorly sorted up to the gravel fraction, a value of $d = 20\text{ mm}$ is appropriate. For smaller concentrations and a more bimodal grain size distribution of the complete grain material (rich of clay, silt and fine sand, poor of coarse sand and gravel, rich of stones) a value of $d = 0.1\text{ mm}$ is appropriate.

The use of the rheological concept:
To summarize, the use of the rheological concept for the simplified description of debris flow physics varies strongly with the considered type of debris flow. The rheological concept is very useful for viscous debris flows and mudflows in laminar flow regime. For granular debris flows the rheological concept is often inappropriate for the description of the macroscopic behaviour and usually useful only on the level of the pore fluid.

In viscous debris flows the rheological behaviour (flow curve) must be determined for the prototype debris flow containing particles up to $d = 200 - 1000\text{ mm}$ grain size. By contrast for mudflows the flow curve must be determined usually for the mud
containing particles up to \( d = 0.1 - 20 \text{ mm} \) grain size. For granular debris flows the determination of the flow curve is limited to the pore fluid which contains particles not larger than \( d = 0.1 \text{ mm} \).

2.4 Physical concepts for debris flows

2.4.1 The work of Bagnold

Many theoretical concepts of debris flows which were elaborated decades later, often rely on the pioneer work of Bagnold (1954).

Experimental set-up and definitions:

Bagnold used a variety of dispersions of buoyant-neutral wax spheres of \( d = 1.3 \text{ mm} \) diameter suspended in liquids such as water or a glycerin-water-alcohol mixture. He tested the dispersions in a shear apparatus of two vertical concentric cylinders (gap between cylinders \( H = 10.8 \text{ mm} \), cylinder length \( L = 50 \text{ mm} \), radius of the inner cylinder \( R_i = 46.4 \text{ mm} \)) (fig. 2.11).

*Figure 2.11 The shear apparatus used for the experiments of Bagnold (1954)*
Here the inner cylinder was at rest and the outer cylinder rotated at different rotational speeds $\Omega$. With the apparatus the shear stress $\tau$ was determined based on measurements of the torque $T$ induced at the inner cylinder and the geometric configuration of the apparatus (see relation between $\tau$ and $T$ in 2.5). Additionally the normal stress $\sigma$ of the sheared mixture was measured with the help of pressure difference measurements between the following two chambers: the first chamber is the inside of the inner cylinder which was filled with water. The second chamber was the space between the outer and inner cylinder which was filled with the different dispersions.

The experiments were conducted at 10 different solid concentrations $C_v$ of the wax spheres ($0.135 < C_v < 0.623$) in water and at one concentration ($C_v = 0.555$) in the glycerin-water-alcohol liquid, with the volumetric solid concentration $C_v$ defined as:

$$C_v = \frac{V_s}{V_s + V_f}$$

(2.9)

where $V_s$ is the volume of the solids and $V_f$ is the volume of the liquid. Bagnold introduced the linear grain concentration $\lambda$ which is defined as the ratio of grain diameter to the mean free distance within the dispersion (see Bagnold 1954, p. 50):

$$\lambda = \frac{1}{\left(\frac{C_0}{C_v}\right)^{\frac{3}{5}} - 1}$$

(2.10)

where $C_0 = 0.74$ is the maximum possible solid concentration for spheres of identical diameter $d$.

Bagnold defined the following dimensionless number $N_B$ (Bagnold number), which expresses the ratio between inertial stresses (stresses induced by collisions of the spheres) and viscous stresses (stresses induced by the viscosity of the liquid):

$$N_B = \frac{\lambda^2 \cdot \rho_s \cdot d^2 \cdot \dot{\gamma}^2}{\lambda^{1.5} \cdot \mu \cdot \dot{\gamma}} \cdot \frac{\lambda^{0.5} \cdot \rho_s \cdot d^2}{\mu} \cdot \frac{\mu}{\mu} \cdot \frac{\mu}{\mu} \cdot \frac{\mu}{\mu}$$

(2.11)

where $\mu$ is the Newtonian viscosity of the liquid, $\rho_s$ is the density of the solid spheres and $\dot{\gamma} = \nu/H$ is the shear rate. Note that this is an apparent shear rate because the gap $H = 1.08$ cm between the two cylinders is rather large and the velocity distribution within the gap thus not linear (fig. 2.12).

**Results:**

Based on the experimental results two main regimes were distinguished depending on the value of the Bagnold number $N_B$. 
In the **grain-inertia regime**, defined for $N_B \geq 450$, shear stress $\tau$ and normal stress $\sigma$ were found to depend on the square of the shear rate $\dot{\gamma}$. For $\dot{\gamma} < 12$ it was found:

$$\tau = a \cdot \sin \alpha \cdot \rho_s \cdot \lambda^2 \cdot d^2 \cdot \dot{\gamma}^2$$  \hspace{1cm} (2.12)

$$\sigma = \frac{\tau}{\tan \alpha}$$  \hspace{1cm} (2.13)

where $a = 0.042$ and $\tan \alpha = 0.32$ are empirical constants and $\alpha$ is the dynamic friction angle.

In the **macro-viscous regime**, defined for $N_B \leq 40$, shear stress $\tau$ and normal stress $\sigma$ were found to depend linearly on the apparent shear rate $\dot{\gamma}$. For the entire range of $\lambda$ it was found:

$$\tau = b \cdot \lambda^{1.5} \cdot \mu \cdot \dot{\gamma}$$  \hspace{1cm} (2.14)

$$\sigma = \frac{\tau}{\tan \alpha}$$  \hspace{1cm} (2.15)

where $b = 2.25$ and $\tan \alpha = 0.75$ are the corresponding empirical values.

In the **transition regime**, defined for $40 \leq N_B \leq 450$, the dependence of the shear stress $\tau$ and the normal stress $\sigma$ from the apparent shear rate $\dot{\gamma}$, progressively increased from linear to the square.

*The review of Hunt et al. (2002):*

The experiments of Bagnold were fundamentally reviewed by Hunt et al. (2002) who analysed the raw data of the experiments of Bagnold. They showed that based on the Bagnold's results the shear stress $\tau$ does not depend on the square but only on the 1.5 power of the apparent shear rate $\dot{\gamma}$ in the grain-inertia regime. More, they proved that the results were influenced by the experimental facility. Due to the short cylinder length $L$ relative to the cylinder gap $H$ ($L/H = 4.6$), strong secondary flows perpendicular to the shear flow were induced along the bottom plate and the cylinders for high rotational speeds $\Omega$ (fig. 2.12). As a consequence the dependence of the shear stress on the shear rate by the 1.5 power is not due to collisions of the wax spheres but due to the secondary flows. Hunt et al. thus concluded that without these secondary flows, a macro-viscous regime and thus a linear dependence of the shear stress $\tau$ on the shear rate $\dot{\gamma}$ is obtained for all experiments.
They propose that the shear stress $\tau$ is determined by treating the dispersions as a Newtonian fluid with a corrected viscosity $\mu'$ that depends on the solids concentration. It can be expressed by the Krieger-Dougherty relation (Krieger 1972):

$$\mu' = \frac{\mu}{1 - \frac{C_v}{C_o}}$$  \hspace{1cm} (2.16)

Hunt et al. defined $C_o = 0.68$ by considering some deformation of the wax spheres and found $k = 3.5$ for Bagnold's data.

The relationship between the normal stress $\sigma$ and the shear rate $\gamma$ could not be reviewed because no raw data were available. Hunt et al. (2002) assumed that, due to the secondary flows, the $\sigma$-data were also influenced and thus, at least the relationship indicated for the grain-inertia regime must be flawed.

**Other similar investigations:**

Savage & McKeown (1983) did similar experiments like Bagnold except that the inner cylinder was rotated and the effects of cylinder wall roughness were also investigated. The cylinder gap $H$ was 17.5 mm, the ratio $L/H = 5.1$ and the diameter $d$ of the grains (polystyrene beds) varied between 1 and 1.8 mm.

Although obtained on a larger stress level, the results verified the original statement of Bagnold that the shear stress is dependent on the square of the shear rate in the grain-inertia regime. Nevertheless they mentioned that this has not to be necessarily due to
grain collisions but could also be due to side effects such as turbulence of the test fluid, which corroborates the findings of Hunt et al. (2002).

In addition, the quadratic dependence of the shear stress on the apparent shear rate was obtained by Daido (in Takahashi 1991) who performed similar experiments like those of Bagnold and Savage & McKeown.

To summarize, while Hunt et al. focus the dependence of the shear stress from the shear rate without secondary flows induced by the apparatus boundary, Bagnold, Savage & McKeown and Daido derived the shear stress dependence for the shear flow and the secondary flows induced by the boundaries of the experimental apparatus.

With reference to the modeling of debris flows, the fluid-grain model of Bagnold (1954) is a theoretical basis for physical debris flow concepts developed decades later.

### 2.4.2 The concept of Takahashi for granular debris flows

In a period that spanned more than one decade, Takahashi (1991) developed a full theory for granular debris flows to explain the initiation due to surface runoff, the flow itself and the deposition process by considering erosion/deposition and segregation processes during the flow.

The theory elaborated for the flow of granular debris flows in the inertial regime is based on extensive laboratory flume experiments and relies on the theories of Bagnold (1954) Shen & Ackermann (1982), Tsubaki and Myamoto (in Takahashi 1991) and Savage & McKeown (1983) who found the shear stress \( \tau \) to depend on the square of the shear rate \( \dot{\gamma} \).

**Flow velocity in the inertial regime:**

Based on the momentum equations for both, solids and liquids, and not considering turbulence and viscosity of the interstitial fluid, the shear stress \( \tau \) of the entire mixture can be expressed as:

\[
\tau = g \cdot \sin i \cdot \int_{z}^{h} ((\rho_s - \rho_f) \cdot C_v + \rho_f) \cdot dz \tag{2.17}
\]

where \( i \) is the channel inclination angle and \( h \) the flow depth. Combining eq. 2.16 and eq. 2.12 developed by Bagnold and assuming \( C_v(z) = \text{const.} \), it is:

\[
\tau = g \cdot \sin i \cdot \int_{z}^{h} ((\rho_s - \rho_f) \cdot C_v + \rho_f) \cdot dz = a \cdot \sin \alpha \cdot \rho_s \cdot \lambda^2 \cdot d^2 \cdot \dot{\gamma}^2 \tag{2.18}
\]
Integrating eq. 2.18 with the boundary condition \( v(z = 0) = 0 \) an expression can be found for the flow velocity \( v \) depending on the vertical position \( z \):

\[
v(z) = \frac{2}{3} \cdot d \cdot \left[ \frac{g \cdot \sin i}{a \cdot \sin \alpha} \cdot \left( C_v + \left(1 - C_v\right) \frac{\rho_f}{\rho_s}\right)^{0.5} \cdot \left(\frac{C_v}{C_P}\right)^{\frac{1}{3}} - 1\right] \cdot (h^{1.5} - (h - z)^{1.5}) \tag{2.19}
\]

In order to determine the empirical constants \( a \) and \( \alpha \) Takahashi made experiments in a 7 m long, 0.2 m wide and 18° inclined rectangular flume with a rigid bed. He used different concentrated mixtures of water and uniform particles \( (d = 4 \text{ mm}, \rho_s = 2.65 \text{ g/cm}^3) \) or water and a light weight material \( (d_{s0} = 1.5 \text{ mm}, \rho_s = 1.74 \text{ g/cm}^3) \) and measured the flow velocity at different depths \( z \). A good correlation between measured and calculated velocity (eq. 2.18) was obtained in both cases for \( a = 0.04 \) and \( \tan \alpha = 0.6 \) compared to Bagnold's \( a = 0.042 \) and \( \tan \alpha = 0.32 \) obtained in the shear apparatus.

Performing experiments on an erodable bed using mixtures of water and nearly uniform particles \( (d = 5.1 \text{ mm}, \rho_s = 2.65 \text{ g/cm}^3) \), a value \( a = 0.35 - 0.5 \) was found to fit the measured velocities best, which is one order of magnitude larger than in the case of a rigid bed. Takahashi observed that the value of \( a \) mainly depended on the degree of saturation within the erodable bed and concluded that a value \( a = 0.4 \) must be considered in the case of an unsaturated bed.

**Sediment concentration \( C_v \) of coarse particles in granular debris flows:**

Based on the experiments with coarse material Takahashi developed the following formula for the concentration \( C_v \) of transported coarse particles as a function of the bed inclination angle \( i \), respecting the boundary conditions \( i < 26^\circ \) and \( C_v < 0.9 \cdot C_0 \):

\[
C_v = \frac{\rho_f \cdot \tan i}{(\rho_i - \rho_f) \cdot (\tan \phi - \tan i)} \tag{2.20}
\]

where \( \phi \) is the internal friction angle of the coarse particles.

Armanini et al. (2003) showed that eq. 2.20 doesn't hold true for non granular mixtures containing a considerable amount of fine material and flowing down on mild to moderately inclined channels \( (0^\circ < i < 12^\circ) \).

**The numerical code:**

The theory of Takahashi for granular debris flows in the inertial regime as well as other theories concerning deposition, particle segregation, etc. are implemented in the numerical code of Takahashi et al. (1993). The code was tested with the help of various
flume experiments using well sorted grain mixtures, each composed of particles of five different characteristic grain sizes.

Development of hydrograph, mean solid concentration and mean grain diameter from the front to the tail as well as development of debris flow deposition and local deposition depth $h(x,y)$ on the fan were well predicted. So was the distribution of the grain sizes $d$ across the flow depth (particle segregation). Insufficiently predicted was the distribution of the solid concentration across the flow depth. It must be noted further that the model was only severely tested for flume inclinations of $i = 18^\circ$ respectively fan inclinations of $i = 5^\circ$. However the model of Takahashi is well established in practice and used for the analysis of complex catchment areas (Nakagawa&Takahashi 1997) or the evaluation of debris flow countermeasures (Nakagawa et al. 2003).

2.4.3 The concept of Iverson for landslides and granular debris flows

Iverson&Denlinger (2001) focus on the landslides and landslide induced debris flows and developed a theory for the initiation due to failure as well as for the flow/slip and deposition process. As for the initiation, the classic Mohr-Coulomb-criteria for soil-failure is adopted (Iverson et al. 1997, Springman et al. 2003):

$$
\tau = (\rho - P) \tan \phi' + c'
$$

where $\sigma = \rho g h$ is the normal pressure, $P$ is the pore fluid pressure, $\phi'$ is the internal friction angle and $c'$ is the cohesion.

The general view of Iverson concerning the physics of the flow process:

As for the flow and deposition process Iverson (1997) gives a very general introduction of all the physical processes involved in debris flows including the principles of thermodynamics until the final dissipation of the energy into heat. In contrast to many other authors Iverson claims that for the inertial regime, not a shear stress - shear rate relationship should be highlighted but the grain fluctuation energy expressed by the granular temperature $T_g = (v')^2$, as already proposed earlier by Jenkins&Savage (1983) for granular flows. Here $(v')$ is the fluctuation of the solid velocity about its mean value due to collisions. However such a classic kinetic approach does not meet one main principle of the Navier-Stokes equations (the fluid is incompressible). Thus other than the Navier-Stokes equations should be considered to integrate the aspect of thermodynamics for debris flows (Hutter et al. 1996). This aspect remains unsolved so far.

Iverson (1997) stresses further that due to the high solid concentration, not collisional but frictional processes dominate in many natural debris flows. He therefore extended
The dimensionless number $N_s$ originally derived by Savage&Hutter (1989) for granular flows to mixed flows composed by grains and a liquid. $N_s$ represents the ratio of collisional stresses to gravitational grain contact stresses that produce Coulomb friction:

$$N_s = \frac{\rho_s \cdot d^2}{(\rho_s - \rho_f) \cdot g \cdot h \cdot \gamma^2}$$

(2.22)

where $h$ is the flow depth (depth of friction layer). Savage&Hutter (1989) found that for $N_s > 0.1$ grain collision may affect flow dynamics significantly. Based on some small documented examples, Iverson (1997) and Iverson&Denlinger (2001) showed that this value is not attained and that friction therefore dominates over collision in many natural debris flows.

The theoretical concept:

As a consequence, the theory developed by Iverson&Denlinger (2001) for the flow and deposition of debris flows is based on Coulomb friction and the development of the porefluid pressure $P$.

In order to quantify the dynamics of the porefluid pressure $P$, large-scale flume experiments in a 88 m long, 2 m wide and 31° inclined channel followed by a 7 m long, 5 m wide and 2.5° inclined plane were performed (Iverson&LaHusen 1993) and the basal porefluid pressure $P_{bed}$ measured (Iverson 1997). Whereas in the debris flow front $P_{bed}$ was zero, a specific value of $P_{bed}$ was measured in the body depending on the flow depth $h$, the hydraulic diffusivity $D_{hy}$ of the mixture and the duration of steady flow $t_{st}$ (Iverson&Denlinger 2001).

In order to model the flow and deposition within complexe three-dimensional terrains depth-averaged momentum equations in the two dimensions $x$, $y$ are used and constitutive relations were established which account for the different stresses $\tau$ and $\gamma$ of the solids and the porefluid, which result in the different dimensions $x$ and $y$. To summarize, the solid and pore fluid stresses $\tau$ and $\gamma$ depend on the following parameters:

$$\tau_s, \tau_f = f(\varphi', \varphi_{bed}, \rho, \rho_p, \mu_p, P_{bed}); \quad P_{bed} = f(h, D_{hy}, t_{st})$$

(2.23)

where $\varphi'$ is the effective internal friction angle, $\varphi_{bed}$ is the basal friction angle of the flow bed, $\rho$ is the density of the entire debris flow, $\rho_p$ is the density of the pore fluid and $\mu_p$ is the Newtonian viscosity of the pore fluid.

Note that Iverson (1997) assumes that the pore fluid is composed by water and particles smaller than 0.06 mm diameter and that the pore fluid is a Newtonian fluid.
The theory was tested with the help of small scale laboratory and large scale flume experiments (Denlinger & Iverson 2001). With reference to the experiments using dry sand spreading on a 31° inclined plane and depositing on a horizontal plane, a very good agreement was obtained between the physical experiment and the numerical prediction for the local flow and deposition depth $h(x,y)$ as well as for the propagation time $t$. With reference to the grain-water-mixture experiments conducted in the large scale flume a fairly good agreement was obtained for the flow depth $h$ and for the propagation time $t$. By contrast, extension of deposition and local deposition depth $h(x,y)$ on the 2.5° inclined spreading plane were not well predicted.

The theoretical concept of Bozhinskiy and Nazarov (2000) bases on a similar distinction between the fluid and the solid phase as the one of Iverson. The concept has been recently implemented in a numerical code at WSL (Forschungsanstalt für Wald, Schnee und Landschaft, Birmensdorf, Switzerland) where it is now being tested based on extensive data of natural debris flow events in the Illgraben torrent in Southern Switzerland.

2.4.4 The concept of O'Brien for mudflows and muddy hyperconcentrated flows

O’Brien & Julien (1985) proposed a semi-rheological model for the simulation of the flow and deposition process of mudflows and muddy hyperconcentrated flows which are rich of fine material. Both type of flows are regarded as two-phase flows. One phase is the fluid (mud) which is composed of water and finer particles. The other phase is the solid phase which is made of the coarser particles. The following quadratic rheological model is proposed for the ensemble of the fluid and the solid phase:

$$\tau = \tau_y + \mu_b \cdot \dot{\gamma} + \zeta \cdot \dot{\gamma}^2 \quad (2.24)$$

where $\tau$ is the yield stress and $\mu_b$ is the Bingham viscosity of the fluid phase (mud). $\zeta$ is the turbulent-dispersive parameter which accounts for both, (a) the turbulent stress induced within the fluid phase and (b) the grain inertia stress induced by collisions within the solid and within the fluid phase, respectively. Hence, $\zeta$ is composed of a turbulent term (a) and a collisional term (b):

$$\zeta = \rho \cdot l_m + a \cdot \rho_s \cdot \lambda^2 \cdot d^2 \quad (2.25)$$

where $\rho$ is the density of the entire mixture and $l_m$ is the mixing length (Bezzola 2003, p. 4-8). Note that the second term of eq. 2.25 represents the collisional term originally derived by Bagnold, except that the parameter $\sin \alpha$, which was originally considered to express the ratio between normal and shear stress, was omitted.
Unlike in water, where $l_m$ is a function of the universal Karman-constant, the mixing length $l_m$ in mud is unknown for a given sediment concentration $C_v$. As a consequence, it is impossible to determine the turbulent-dispersive parameter $\zeta$ directly. By implementation of the quadratic rheological model in a numerical code, the turbulent dispersive parameter $\zeta$ is therefore replaced by a Manning roughness coefficient $n_m$ (O’Brien et al. 1993).

As far as the yield stress $\tau_y$ and the Bingham viscosity $\mu_B$ of the fluid phase is concerned, the two parameters can be obtained through conventional rheometric measurements if the maximum grain size of the mud is smaller than approximately $d_{\text{max}} = 0.25 \text{ mm}$ (see 2.5). O’Brien & Julien (1988) determined the two parameters $\tau_y$ and $\mu_B$ for variably concentrated mudflow samples of several catchment areas in the Colorado Mountains by using a concentric cylinder system in a standard rheometer. Based on this vast investigation they found that both, $\tau_y$ and $\mu_B$, increase exponentially with the increase of the solid concentration $C_v$ of a given sample material.

It must be stressed that O’Brien & Julien (1988) used the Bingham model to fit the rheological data of the different mudflow samples. The use of the Bingham model might be a simplification, because many other studies (Chhabra & Uhlherr 1988, Hemphill et al. 1993, Coussot 1997 e.g.) showed that the general Herschel-Bulkley model predicts more accurately the rheological behaviour of mud. Such a simplification could reduce the accuracy of predictions of the flow and, specifically of the deposition process (Bergomi 2001). However, the commercially available numerical code of O’Brien et al. (1993) is very well established in practical applications and, due to the lack of other commercial codes, also applied for granular and viscous debris flows.

2.4.5 The concept of Coussot for viscous debris flows

Coussot (1994ab, 1997) developed a full rheological model for the simulation of the flow and deposition process of viscous debris flows. Viscous debris flows are defined by a portion larger than 10% of fine material ($d \leq 0.04 \text{ mm}$) relative to the complete grain material, by a generally high sediment concentration and by a usually poorly sorted grain material. Accordingly the viscous debris flows appear as one unique fluid where effects of particle collision and particle settling are of minor importance. The flow regime is dominantly laminar. Two characteristic zones can be usually observed over the entire flow depth: A shear zone adjacent to the flow bed and a rigid zone adjacent to the flow surface (see also fig. 7.2).

By the use of different rheometric systems Coussot (1992, 1995) investigated sediment-water mixtures containing the fine material portion ($d \leq 0.1 \text{ mm}$) and the fine and coarse
material portion \( (d \leq 20 \text{ mm}) \) of viscous debris flows of five different catchment areas. For both, the mixtures containing the fine material portion and the mixtures containing the fine and coarse material portion, he obtained a flow curve (shear rate \(-\text{shear stress relation})\) which can be best fit by a Herschel-Bulkley model:

\[
\tau = \tau_y + m \cdot \dot{\gamma}^n
\]

(2.26)

where \( \tau_y \) is the yield stress, \( m \) is the Herschel-Bulkley consistency coefficient and \( n \) is the Herschel-Bulkley index.

The Herschel-Bulkley index was found to be roughly \( n = 0.33 \) for both the mixtures containing the fine material portion and the mixtures containing the fine and coarse material portion. By contrast \( \tau_y \) and \( m \) strongly depended on the sediment concentration and other material characteristics such as grain size distribution and particle shape.

Based on these extensive rheological investigations Coussot inferred that the Herschel-Bulkley-model in eq. 2.26 is also applicable for the flow and deposition process of the prototype debris flow. Coussot et al. (1998) presented a method how to extrapolate the three parameters \( \tau_y, m \) and \( n \) for the prototype debris flow based on rheological measurements of mixtures containing different portions from the complete material (chapter 7).

The Herschel-Bulkley model is implemented in the numerical codes of Laigle & Coussot (1994, 1997) for one-dimensional flow and Laigle (1996a) for two-dimensional flow and deposition. Comparison between laboratory flume experiments showed that both, flow depth \( h(x) \) and propagation time \( t \), were very well predicted in the case of one-dimensional channel flow. As for the two-dimensional spreading and deposition on a variably inclined plane \( (5^\circ < i < 11^\circ) \), extent and depth \( h(x,y) \) of deposition was very well predicted. By contrast propagation time \( t \) was not predicted with satisfaction. For practical application it thus remains still difficult to assess the impact force of the spreading and stopping debris flow. Further, where due to small surface roughness (asphalte, wet grassland) bed slip takes place, inaccurate results are obtained. Apart from these restrictions the theoretical concept is convincing and the code widely used in practical application (Laigle & Marchi 2000, Laigle et al. 2003 e.g.).

2.4.6 Other physical concepts

The concepts introduced above are considered as the keystones of physical debris flow theories, but of course do not represent the complete set of theories developed worldwide. Knowing that there are many more concepts it is worth to briefly mention some which are important in practical use or which have some future potential.
Flow process:

Possibly the most simple concept for the simulation of the flow process is one that integrates a generalized flow resistance accounting for both the resistance due to the boundary (flow bed, side walls) and the inner resistance due to the mixture. Such a concept is normally applied for clear water hydraulics and can be summarized as:

\[ v = c \cdot h^a \cdot \sin^b (i) \]  

(2.27)

where \( v \) is the depth-averaged flow velocity, \( c \) is the generalized flow resistance term, \( h \) is the flow depth, \( i \) is the bed inclination angle and \( a \) and \( b \) are empiric parameters. As an example in the case of the Manning-Strickler equation \( c \) reduces to \( 1/n_m \), \( a = 2/3 \), \( b = 0.5 \). In the case of the Chézy-equation \( c \) is the Chézy resistance parameter and the empirical parameters are \( a = 0.5 \), \( b = 0.5 \).

Due to the lack of commercial numerical codes this simple flow model is widely used in practical application. In catchment areas lacking historical documentation and where the flow behaviour of debris flows is instationary and varies largely, this simple model provides a rough estimation of the flow characteristics (Arattano & Franzi 2003).

Strengths are undertaken in the determination of the resistance parameter \( c \) in order to relate \( c \) with other parameters such as the relative flow depth, debris flow concentration, etc. (Rickenmann & Weber 2000). However due to the lack of a Coulomb friction or a yield stress term this concept does not enable the simulation of debris flow deposition.

Encouraging concepts are currently being developed at the University of Trento. Based on an innovative laboratory flume where the mixture can be recirculated in order to produce an endless debris flow and with the help of ultrasound Doppler technique, the three-dimensional (x-y-z-) velocity field and velocity fluctuation is measured (Armanini et al. 2003). By variation of the flume inclination and the investigated mixtures, different but stationary flow regimes (rigid bed debris flow, immature and mature debris flow, plug flow) are obtained and different flow layers for each flow regime observed. Based on the different regimes and flow layers, specific models are being developed so as for viscous debris flows (Armanini et al. 2003) as well as for granular debris flows (Armanini 2004). Measuring the velocity fluctuation, the aspect of the granular temperature \( T_g \) is also considered. However, due to the restriction of the Navier-Stokes equation for incompressibility, \( T_g \) can not be included directly in the equations but is replaced by an added mass term.

The theory of Ancey & Evesque (2000) is also based on a variety of experiments of different dense granular material in variably inclined channels. The theory focuses on the particle network and the influence of the normal pressure within granular materials.
and also includes the aspect of granular temperature $T_g$ by the use of empirical relationships.

**Segregation process:**

A comparably rarely investigated field remains the modeling of the *segregation process* within debris flows. Takahashi (1980) derived some semi-empirical relationships for the inverse grading and for the migration of larger particles to the front for some chosen materials on defined channel inclinations. More fundamentally the inverse grading process was analysed by Kern (2000) for granular materials. He found that only in steady flow regimes and for some specific combinations of Coulomb friction and energy absorption by the grains, inverse grading takes place.

To summarize, for the numerical simulation of debris flows the rheological parameters must be determined for a variable portion from the complete debris flow material depending on the debris flow type. In the coming sections 2.5 to 2.7 the existing rheometric systems, apparatus and methods, which allow to determine these parameters, are introduced.

### 2.5 Rheometry for non particle and fine particle fluids

Rheometry means the measurement of the rheological behaviour. For the determination of the flow curve/viscosity curve, rheometric systems usually base on the two-plate model presented in 2.1. The fluid to be investigated is sheared between (a) two concentric cylinders, (b) a cone and plate or (c) two parallel plates (fig. 2.13). Usually the inner cylinder (a), the cone (b), respectively the upper plate (c) are rotated in order to obtain a shear zone within the gap $H$. A torque $T$ is applied to rotate the cylinder/cone/upper plate at a specific rotational speed $\Omega$.

In analogy to the two-plate model the rheological parameters $\dot{\gamma}$ and $\tau$ are obtained based on the measured values $T$ and $\Omega$ as well as the geometric features of the specific system.

For the *Concentric Cylinder System (CCS)* it is:

$$\dot{\gamma} = \frac{2 \cdot \pi \cdot \Omega \cdot R_i}{(R_o - R_i)} = \frac{2 \cdot \pi \cdot \Omega \cdot R_i}{H} \quad ; \quad \tau = \frac{T}{2 \cdot \pi \cdot R_i^2 \cdot L} \quad (2.28)$$

where $R_i$ is the radius of the inner cylinder, $R_o$ is the radius of the outer cylinder and $L$ is length of the inner cylinder.
Figure 2.13 Different geometric configurations of the two-plate model within rheometric systems. (a) Concentric Cylinder System (CCS), (b) Cone and Plate System (CPS), (c) Parallel Plate System (PPS).

For the Cone and Plate System (CPS) it is:

\[ \dot{\gamma} = \frac{2 \cdot \pi \cdot \Omega \cdot r}{H(r)} \quad \tan \alpha \]

\[ \tau = \frac{3 \cdot T}{2 \cdot \pi \cdot R^3} \]  

(2.29)

where \( \alpha \) is the cone angle and \( R \) is the radius of the cone and the bottom plate.

Regarding the Parallel Plate System (PPS) the shear rate \( \dot{\gamma} \) varies in function of the radius \( r \) because it is \( \dot{\gamma} = 2 \cdot \pi \cdot \Omega \cdot rH \). The rheological parameters are defined at the periphery of the plates (\( r = R \)). Here it is:

\[ \dot{\gamma}(R) = \frac{2 \cdot \pi \cdot \Omega \cdot R}{H} \; ; \; \tau(R) = \frac{3 \cdot T}{2 \cdot \pi \cdot R^3} + \frac{\dot{\gamma}(R)}{2 \cdot \pi \cdot R^3} \cdot \frac{dT}{d\dot{\gamma}(R)} \]  

(2.30)

For a Newtonian Fluid it is approximately \( \tau \approx 2 \cdot T/(\pi R^3) \).

The three geometric systems (CCS, CPS, PPS) are placed in standard rheometers where torques \( T \) and rotational speeds \( \Omega \) are measured with a high resolution over a wide measuring range. Due to the small size of the measuring systems used in standard systems, small sample volumes of only several milli- or centiliters are usually required which enables efficient measuring (fig. 2.14).
The small size of the measuring systems in standard rheometers has disadvantages as far as the maximum grain size $d_{\text{max}}$ in particle fluids is concerned. Usually it is recommended to consider a ratio gap width over maximum grain size “$H/d_{\text{max}}$” larger than at least 10 (Van Wazer et al. 1963 in Chhabra & Richardson 1999) in order to guarantee a shear flow that consists of at least 10 layers. The gap width $H$ in a CCS of a standard rheometer is usually not larger than $H = 2.5$ mm, which limits the maximum grain size in particle fluids to $d_{\text{max}} = 0.25$ mm.

### 2.6 Rheometry for large particle fluids

#### 2.6.1 Definition

Large particle fluids are defined here as fluids containing particles with maximum grain sizes equal or larger than $d_{\text{max}} = 1$ mm.
2.6.2 Large scale devices of standard measuring systems

The apparatus of Bagnold and others for testing artificial solid-liquid material:

As introduced in 2.4.1 Bagnold (1954) developed a large scale CCS ($R_i = 46.2$ mm, $H = 10.8$ mm, $L = 50$ mm, $V_{\text{sample}} = 0.18$ liter) for the analysis of dispersions of identical sized and buoyancy neutral wax spheres ($d = 1.3$ mm) within Newtonian fluids. Similar sized CCS apparatus were developed also by Savage & McKeown (1983) and Daido (in Takahashi 1991). It is important to stress that all these CCS large scale devices do not necessarily consider the laminar layered shear flow defined in rheology for the entire measuring range. Due to the small ratio of cylinder length and gap width ($L/H = 4\text{-}5$) secondary flows can be induced at the apparatus boundary for liquid materials and large rotational speeds $\Omega$ (Hunt et al. 2002). This influences the measured torques $T$ used to calculate the shear stress $\tau$ for this measuring range and falsifies the real flow curve of a given fluid.

The wide-gap CCS of Major & Pierson and Coussot & Piau:

Major & Pierson (1992) developed a large scale CCS ($R_i = 359$ mm, $H = 31$ mm, $L = 62$ mm, $V_{\text{sample}} = 10$ liter) for the analysis of the fine material part of a debris flow ($d \leq 0.06$ mm) as well as for the analysis of different artificial mixtures ($d \leq 2$ mm) composed by the fine debris flow material and variable contributions of sand. All the mixtures were medium to highly concentrated ($0.44 < C_v < 0.66$).

Due to the wide gap $H$ and a yield stress $\tau_y$ measured in the mixtures the shear rate $\dot{\gamma}$ can not be obtained with eq. 2.28. Instead the shear rate $\dot{\gamma}$ must be determined according to the method of Nguyen & Boger (1987) which is introduced later in this study (5.1.3). Major & Pierson obtained in most mixtures, except the sand-rich mixtures, a clear flow curve which was well fitted with either the Herschel-Bulkley-model or the simplified Bingham model. For shear rates $\dot{\gamma} < 5$ s$^{-1}$ the picture was not always clear: In the experiments with the sand-rich mixtures torque fluctuations, time-dependent behaviour (variation of the torque with time) and hysteresis effects were observed and reproducibility of the results was moderately good to poor. These phenomena were attributed to strong friction, particle reorganisation and cluster formation within densely packed sediment-water mixtures at low shear rates. Note that these observations are similar to those made during triaxial tests in soil-mechanics.

Coussot & Piau (1995) developed a mobile large scale CCS ($R_i = 385$ mm, $H = 200$ mm, $L = 770$ mm, $V_{\text{sample}} = 500$ liter) for the field investigation of debris flow material mixtures containing particles $d \leq 20$ to $25$ mm. Due to the wide gap $H$ of this device,
shear rates are also obtained according to the method of Nguyen&Boger (1987) in the case of Yield Stress Fluids (5.1.3).

Referring to the investigated portion $d \leq 20$ to $25$ mm of viscous debris flows of five different catchment areas, clear flow curves were obtained which were well fitted with the Herschel-Bulkley-model. For the same sediment portion of granular debris flow material similar results to those reported by Major&Pierson (1992) for the sand-rich mixtures were obtained.

Note that the ratio of cylinder length and gap width is also small ($L/H = 2-4$) in the case of the large scale devices of Major&Pierson and Coussot&Piau. Thus secondary flows could be expected similar to the those found by Hunt et al. (2002) in the experiments of Bagnold. It is important to mention that in comparison to the experiments of Bagnold, here (i) Yield Stress Fluids were tested which means that the shear zone usually did not develop over the entire gap width, (ii) the mixtures were always medium to highly concentrated and (iii) slight accumulation of larger particles on the bottom of the outer cylinder took place. All points (i) to (iii) reduced or prevented the development of secondary flows.

The two large scale CPS of Phillips&Davies:

Phillips&Davies (1991) constructed two large scale CPS with a cone angle $\alpha = 30^\circ$. In both devices the cone was situated at the bottom plate and the cone was inverted compared to the situation in standard measuring systems. The side was closed in order to prevent the test fluid from outflow. The first device had a radius $R = 200$ mm and required a sample volume of $V_{\text{sample}} \sim 8$ liter. Removing particles larger than $d = 35$ mm grain size, material mixtures of two different debris flows were investigated at different sediment concentrations $C_s$ for shear rates $\dot{\gamma} \leq 16$ s$^{-1}$. Even though shear stresses varied much over the specified shear rates, clear bands of flow curves were obtained. The second device had a radius $R = 1000$ mm and required a sample volume of $V_{\text{sample}} \sim 1000$ liter. Here debris flow material mixtures containing particles $d \leq 120$ mm grain size were investigated at shear rates $\dot{\gamma} \leq 4$ s$^{-1}$. No clear rheological behaviour could be derived with the second instrument because measured shear stresses varied too much over the small range of shear rates. In a later study of Contreras and Davies (2000), coarse-grained debris flow material mixtures containing particles $d \leq 20$ mm grain size were investigated with the first device. The experiments also showed a strong variation of shear stresses for shear rates $\dot{\gamma} \leq 7$ s$^{-1}$, which inhibited to obtain a clear flow curve. Nevertheless they identified a hysteresis between the yield stress $\tau$ obtained for the up curve (flow initiation) and down curve experiments (flow stopping). Similar to Major&
Pierson (1992) they obtained a time-dependent behaviour of the measured torque $T$ at the different specified speeds $\Omega$.

**Magnetic resonance imaging (MRI):**

A large scale apparatus based on a CCS and CPS for the rheological investigation of fluids containing particles $d \leq 2$ mm grain size was developed by Coussot et al. (2003). The two devices are equipped with magnetic resonance imaging (MRI) which allows to draw the velocity profile within the shear gap. This enables to detect side effects such as wall slip and to derive directly the flow curve of a given fluid based on the velocity distribution within the shear gap. It further has some potential to prove the homogeneity of the test fluid during measuring.

### 2.6.3 Viscometers

**Standard scale and large scale CCS with altered geometry of inner cylinder:**

The instruments base on the principle of a CCS. By contrast, instead of the inner cylinder, a specific stirring tool is used. The geometry of the tool usually deviates more or less from the simple geometry of a cylinder surface (fig. 2.15). Many instruments have been developed over the last decades to estimate the rheological behaviour of concrete containing aggregate particles roughly up to $d = 30$ mm grain size. In the BML viscometer of Wallevik & Gjørv (1990) a tooth-ring geometry was used (fig. 5.8 and fig. 5.9). Tattersal & Bloomer (1979) developed a viscometer using a two-point impeller, which consists of two excentric angled blades. Tattersal & Banfill (1983) used H-shaped impellers. In order to minimize particle settling during measuring Banfill (1994) further developed the impeller by using blades of improved shape.
Note that these are only some examples and the list of authors is not complete. Important is that these viscometers normally allow a rough estimation of the yield stress $\tau_y$ and the development of the torques $T$ against the rotational speed $\Omega$. But they often do not allow the determination of the flow curve unless the instruments have been calibrated extensively. This is usually due to the specific geometry of the impellers which render the flow field within these instruments in a complex and 3-dimensional manner. As a consequence such a system often does not meet the requirements of the rheology for a laminar layered 1-dimensional shear flow.

*Flow and stoppage in tubes, channels and on planes:*

It is possible to obtain the flow curve of any fluid by the examination of the flow in a tube, in a channel or on a plane. In these cases the shear zone either develops in radial inward direction from the tube boundary, or in vertical direction from the channel or plane bed, respectively from the channel wall. For the flow curve determination such a rheometric system is rarely applied, because the shear zone and the velocity profile can be determined only at the surface and the boundary but not within the fluid (Whipple 1997, Hübl&Steinwendtner 2000, Parsons et al. 2001) and the range of shear rate where the flow curve can be obtained is usually limited (Coussot&Boyer 1995). Only the recently developed ultrasound Doppler velocimetry has some potential herein, as it is possible to measure the 3-dimensional velocity profile within the entire flowing fluid (Armanini et al. 2003).

For now these rheometric systems are usually applied to determine the yield stress $\tau_y$ after fluid stoppage. In this case $\tau_y$ is not dependent on the velocity profile but only dependent on the fluid density, the geometry of the viscometer and the deposition depth (plane, channel) respectively the final pressure height (tube) of the fluid. Several apparatus and tests were developed for this purpose, such as the Kasumeter — a tube viscometer (Schulze et al. 1991), the Inclined Channel Test (Johnson 1970, Whipple 1997, Coussot 1994a) and the Inclined Plane Test (Coussot et al. 1996).

An additional rheometric system for the determination of the yield stress of highly concentrated sediment water mixtures is the Slump Test, which is often applied in concrete research. Here the fluid is filled in a two-side open hollow cylinder or cone placed on a horizontal surface. The cylinder is lifted and the fluid slumps. Based on the slump height, the fluid density and the geometry of the cylinder/cone the yield stress is determined (Pashias&Boger 1996, Schowalter&Christensen 1998).

Some of the above systems will be subject of the experiments of the present study and will be presented in more detail in chapter 5.
2.6.4 The ball measuring system (BMS)

The ball measuring system bases on a sphere that is dragged across a given fluid with different specified velocities while the required drag forces are measured (fig. 2.16). The main difference between this system and the systems introduced before is, that it does not rely on the classic shear flow, but is based on the drag flow around the sphere. Therefore, a relation between the drag flow and the classic shear flow (fig. 2.1) must be known when the flow curve is determined with this system.

Müller et al. (1999) developed a ball measuring system which can be installed in conventional rheometers (Physica Messtechnik GMBH 1999). Here the sphere is fixed on a holder and rotated excentrically in a round container. Accordingly torques $T$ are measured when the sphere is rotated with specified rotational speeds $\Omega$ (fig. 2.16 and fig. 4.1).

Assuming a linear relationship between the shear rate $\dot{\gamma}$ and the rotational speed $\Omega$, a linear relationship between the shear stress $\tau$ and the torque $T$ was deduced for laminar flow around the sphere (Tyrach 2001). Tyrach determined the relating linear factors with the help of well known non particulated Newtonian and Power Law Fluids. The system has potential because with the present device – implemented in a conventional rheometer - the flow curve of large particle material up to $d_{\text{max}} = 5 - 10$ mm grain size can be obtained efficiently and over a wide measuring range. However, two main problems were observed when the system was applied for some characteristic and various concentrated particle fluids (Schatzmann et al. 2003a, 2003b): (1) Different flow curves were obtained using different sphere sizes. (2) Reliable results were obtained only for laminar flow around the measuring sphere.
2.7 Rheometry for debris flows

In section 2.3 it was shown that for granular debris flows the rheology is relevant for the porefluid which is composed of water and particles up to roughly $d_{\text{max}} = 0.1 \text{ mm}$ grain size. Consequently the flow curve can be determined using standard measuring systems (CCS, CPS, PPS) in conventional rheometers.

As for the mudflow, the flow curve must be measured for the mud that is composed of water and particles up to $d_{\text{max}} = 0.1 - 20 \text{ mm}$ grain size. Due to the grain size $d$ of the largest particles standard rheometers usually can not be used. Only one of the large scale CCS- or CPS- devices or a viscometer presented earlier could be used instead.

Regarding viscous debris flows the flow curve must be determined for the prototype debris flow including the grain material from the clay up to the block and boulder fraction ($d_{\text{max}} = 200 - 1000 \text{ mm}$). Coussot et al. (1998) proposed a method, where different portions of the prototype debris flow are analysed separately by using appropriate rheometric systems in the laboratory as well as in the field. Based on the results obtained for the different portions, the flow curve parameters of the prototype debris flow are extrapolated. The method is further explained and applied in 7.1.

Genevois et al. (2000) determined the development of the viscosity $\eta$ from the front to the tail of a debris flow in the Acquabona catchment area (Italy) based on online measurements of the flow depth $h$, the surface velocity $v$ and the density $\rho$. This enabled them to determine the shear stress by $\tau = \rho \cdot g \cdot h \cdot \sin i$. Further they calculated an apparent shear rate $\dot{\gamma} = v/h$ and obtained the viscosity as $\eta = \tau/\dot{\gamma}$. The principal idea of the method is explained and applied in 7.2.
3 The study

3.1 The problem

In order to model the flow and stopping of viscous debris flows, mud flows, concrete, slurries, food products, etc., the rheological behaviour of fluids containing particles larger than 1 mm must be determined. By contrast rheometric systems which allow the flow curve determination of such fluids are rare and commercially not available. Further, the few large scale devices of standard measuring systems developed for large particle fluids exhibited the following difficulties:

- Large sample volumes up to several hundred liters are necessary for experimentation. Preparation of these volumes are time- and cost-intensive. Sediment concentration and mixing technique is difficult to control during preparation and experimentation.
- Measuring range of rotational speed $\Omega$ and torque $T$ is often limited which restricts the range of $\gamma$ and $\tau$ for which the flow curve can be determined.

The ball measuring system is a potential rheometric system that could fill some of these gaps. Using a sample volume of only 0.5 liter it could be an efficient tool for large particle fluids containing particles up to $d_{\text{max}} = 10$ mm grain size. Implemented in a conventional rheometer it further allows measurements in a wide range of the rotational speed $\Omega$ and the torque $T$. However, two main problems must be overcome in order to make it useful for practical applications: The existing theoretical approach for the conversion of the measured parameters $\Omega$ and $T$ into the rheological parameters $\gamma$ and $\tau$ of Tyrach (2001) must be improved in order to guarantee that (1) identical flow curves are obtained using different sphere sizes and (2) in order to extend the parameter conversion also for non laminar flow around the measuring sphere.

In the case these problems are solved the ball measuring system has some potential to be expanded to a large scale device: A device requiring a still manageable sample volume of less than 100 liters would allow to investigate fluids containing particles up to $d_{\text{max}} = 60$ mm grain size.

3.2 Goal of the study

The first goal of the present study is to improve the conversion theory for the ball measuring system. Based on the deficiencies of the present theory (Tyrach 2001) mentioned above, the following specific aims are defined for the conversion of
measured data of rotational speed $\Omega$ and torque $T$ into rheological data of shear rate $\dot{\gamma}$ and shear stress $\tau$.

1. To obtain identical flow curves using different sphere sizes.

2. To extend the range of application to non-laminar flow around the sphere.

The second goal is the comparison of the ball measuring system with other rheometric apparatus and tests currently used for large particle fluids. Here both (1) rheological data (flow curve and yield stress) as well as (2) applicability in daily practical use (measuring range, measuring efficiency, etc.) are compared.

The third goal is to prove the applied rheometric apparatus and tests as well as other techniques for the application to debris flows.

### 3.3 Methodology

#### 3.3.1 Masterplan

The overall concept is the measurement of a variety of non particle-, fine particle- and large particle fluids with the ball measuring system and other rheometric apparatus and tests.

The flow curves of the non particle- and fine particle fluids ($d_{\text{max}} \leq 0.25 \text{ mm}$) are measured in standard measuring systems and in the ball measuring system. They are used as reference curves in order to relate the classic shear flow with the sphere drag flow in the ball measuring system, respectively to derive an improved theory for the conversion of measured data into rheological data.

The flow curves and yield stresses of the different large particle fluids ($d_{\text{max}} > 1 \text{ mm}$) obtained with the ball measuring system and the additional rheometric apparatus and tests are used for the comparison of the rheological data.

The ball measuring system is subject of chapter 4. First, the principles of measurement are introduced and the data scatter of the measurements investigated. The existing theoretical approach for the conversion of measured into rheological data is then explained before the new approach of the present study is introduced. A comparison of flow curve data of the existing and the new approach is made before some important aspects such as the reliability of the new approach, the definition of the shear rate, the sphere Reynolds number in non-Newtonian fluids and the standard drag curve of the ball measuring system are discussed.
In a second step additional rheometric apparatus and tests for large particle fluids are applied (chapter 5). For each system the measuring procedure and the way of data acquisition is shown before the theory of data conversion and finally the rheological results are presented.

The third step is the comparison of the ball measuring system with the other rheometric tools for large particle fluids (chapter 6). Here the comparison of flow curve and yield stress data is shown. The comparison of rheological data is followed by an overall comparison of all the systems which includes additional aspects such as data reliability, allowable maximum grain size $d_{\text{max}}$ in the fluid, measuring range and measuring efficiency. This finally enables recommendations be given for the use of appropriate rheometric tools for large particle fluids in general and the application to debris flows in particular.

Chapter 7 contains the estimation of the flow curve parameters of a viscous debris flow. The existing extrapolation method of Coussot et al. (1998) is applied and compared with an alternative approach relying only on field observations.

### 3.3.2 Applied rheometry

**For non particle and fine particle fluids ($d_{\text{max}} \leq 0.25 \text{ mm}$):**

<table>
<thead>
<tr>
<th>System</th>
<th>Abbreviation</th>
<th>Rheometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentric Cylinder System</td>
<td>CCS</td>
<td>DSR Rheometer</td>
</tr>
<tr>
<td>Cone and Plate System</td>
<td>CPS</td>
<td>DSR Rheometer</td>
</tr>
<tr>
<td>Ball Measuring System</td>
<td>BMS</td>
<td>Physica MCR 300</td>
</tr>
</tbody>
</table>

**For large particle fluids ($d_{\text{max}} \geq 1 \text{ mm}$):**

<table>
<thead>
<tr>
<th>System</th>
<th>Abbreviation</th>
<th>Rheometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball Measuring System</td>
<td>BMS</td>
<td>Physica MCR 300</td>
</tr>
<tr>
<td>CCS large scale rheometer (Coussot&amp;Piau)</td>
<td>CCS-LS</td>
<td></td>
</tr>
<tr>
<td>BML viscometer</td>
<td>BML</td>
<td></td>
</tr>
<tr>
<td>Kasumeter</td>
<td>KAS</td>
<td></td>
</tr>
<tr>
<td>Inclined Channel Test</td>
<td>ICT</td>
<td></td>
</tr>
<tr>
<td>Inclined Plane Test</td>
<td>IPT</td>
<td></td>
</tr>
<tr>
<td>Slump Test</td>
<td>ST</td>
<td></td>
</tr>
</tbody>
</table>

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7 Commercially available
3.4 Test fluids

3.4.1 Newtonian fluids

Silicon Oils with the viscosities $\mu = 60, 30, 12.5, 2, 1, 0.05$ Pa s from Wacker-Chemie GmbH were used. The theoretical and measured viscosity values $\mu$ are summarized in table D.1 of appendix D for the investigated temperature of 20$^\circ$ C. For this temperature the density $\rho$ of the oils is 975 kg/m$^3$, except for the oil with a viscosity $\mu = 0.05$ Pa s for which is $\rho = 965$ kg/m$^3$ (Wacker 1993).

3.4.2 Power Law Fluids

Two different materials were used to prepare polymer-water solutions which behave like Power Law Fluids (PLF): Guar and Carboxymethylcellulose (CMC).

*Characteristics of the material:*

Guar is the pulverised endosperme of the guar plant which is cultivated mainly in USA, Pakistan and India. Guar is a galacto-mannan and belongs to the group of hydrocolloids. It is used as a setting component for various purposes (food industry e.g.). The guar used in the present study was the Guar - CB 94 from Meyhall Chemical AG.

Carboxymethylcellulose (CMC) is like guar a hydrocolloid which is used for various purposes in food industry. Somewhat different to guar, CMC is based on cellulose forming the cell walls of bioplants where the cellulose can be extracted relatively easy. Cellulose added with variable parts of alcali and chlor acetic acid form a variety of CMC which exhibit different hydrocolloidal behaviour when mixed with water. The CMC used in the present study was the Carboxymethylcellulose Sodium Salt – Nr. 21904 (ultra high viscosity) from Fluka Chemie GmbH.

*Mixture preparation for rheometric tests:*

Two guar-water solutions were prepared: Guar 1 % and Guar 2 %. As for the 1 %-Guar-solution 1 gram guar powder was stepwise added to 99 gram of deionized water while stirring the entire solution with a paddle mixer and a rotating magnete on the bottom of the beaker. In order to enable a complete dissolving, the solution was heated up to a temperature of 60$^\circ$ C. At this temperature mixing time lasted 2 hours. The 2 %-Guar-solution was prepared by adding another weight percent guar powder stepwise to the already existing Guar 1 % - solution. Mixing technique was the same but mixing time lasted 6 hours.
Two CMC-water solutions were prepared: CMC 1% and CMC 2%. Preparation of the solutions was analogue to the guar solutions. With reference to the 2%-CMC-solution mixing time lasted 4.5 hours.

3.4.3 Yield Stress Fluids I: Debris flow material mixtures

Characteristics of the sediment material:

The solid material was taken from a fresh deposit of a small debris flow which occurred on May 3, 2001 in the Scalärarüfle near Trimmis/Chur in Eastern Switzerland. The geology of the area is dominated by formations of schists, a sediment susceptible to weathering. As a result large volumes of loose sediment with a considerable amount of fine material are produced every year which encourage the formation of debris flows.

4 m$^3$ of an undisturbed$^8$ frontal deposition tongue of the debris flow were excavated three days after deposition (see Appendix A).

The excavated material was brought by lorry to a gravel sorting factory where the material dried naturally during one month. Then a full grain size analysis of the 4 m$^3$ material was conducted requiring a total of six person days (Appendix A):

All the 210 blocks and stones larger than 100 mm were analysed individually by measuring the three main diameters $a$, $b$ and $c$. Assuming that the grains have a nearly ellipsoidal shape, the characteristic grain diameter $d$ was then determined with the formula for the equivalent sphere diameter $d_s$:

$$d_s = \left(\frac{a \cdot b \cdot c}{3}\right)^{\frac{1}{3}}$$

(3.1)

The following ratios where obtained for the present debris flow material: $d/a = 0.64 \pm 0.12$, $d/b = 0.98 \pm 0.15$ and $d/c = 1.70 \pm 0.44$. These values differ from the values $d/a = 0.68$, $d/b = 0.91$ and $d/c = 1.30$ obtained by Whittaker & Jäggi (1986) for the laboratory investigation of block ramps in rivers. This demonstrates that the particle shape of debris flow material might differ considerably from the particle shape of fluvial material. Thus, in the absence of a sieve analysis for particles larger than 100 mm, not only one axis and considering one predefined ratio of $d/a$, $d/b$ or $d/c$ but all the axis $a$, $b$, $c$ should be measured and eq. 3.1 be used or, where possible, the particles weighed in order to determine the characteristic grain diameter $d$.

The grain size distribution of the fraction $0.063 \leq d \leq 100$ mm was obtained through sieve analysis and the distribution of $d \leq 0.063$ mm by the areaometer test (time of

$^8$ No rainfall occurred meantime so that the fine material was not washed out.
The study

Sedimentation within clear water for specific fine material fractions. Both was done by Griso Prüflabor AG, Untervaz.

Finally the grain size distribution of the complete debris flow material was obtained by the superposition of the distributions of the three different analysis (1) to (3).

The grain size distribution of the complete material is shown in fig. 3.1a. The material is characterized by a considerable content of fine material: Particles smaller than 0.04 mm represent 9% of the total material. Because this value is close to the specific value of 10% defined by Coussot (1994a) for a viscous debris flow, a rheological investigation of the material was considered useful.

The particles of the block, stone and gravel fraction are dominantly flat and angular. With focus on the sand, silt and clay fraction these general features remained similar, even though for the very small particles (d ≤ 0.25 mm) particle shape could not be assessed as precisely as for the very large particles (see photos in Appendix A).

With reference to the solid density ρs, Steiger (1999) obtained ρs = 2.74 g/cm³ for the solid material of the catchment area by analyzing the particles smaller than 0.5 mm in the glass pyknometer. By contrast, measuring the weight and the volume of 30 stones (120 mm < d ≤ 150 mm) by immersing them in a water bath, a solid density ρs = 2.60 ± 0.62 g/cm³ was obtained. According to Steiger (2001) this discrepancy in the solid density ρs is due to the fact that, within the large and very large particles (mainly in the gravel, stone and block fraction), cavities of crevices and fissures exist which are not filled with water during the latter experiment. This evidently results in a smaller solid density ρs. In order to work with a unique value for all grains the solid density was defined as ρs = 2.7 g/cm³ in the present study.

A spectral analysis of the clay fraction was done by the laboratory of clay minerals at ETH Zurich. Using the program "Autoquan" the following composition of clay minerals was found: Calcite (46.5 ± 4.9 %), Illite (24.0 ± 1.4 %), Quarz (26.0 ± 7.1 %), Chlorite (3.5 ± 0.7 %). Based on this composition it can be concluded that the clays of the present debris flow material are hardly or not swelling at all. Due to the strong presence of illit the material exhibits lubricating effects when mixed with water (Kahr 2001). As a consequence thixotropic effects are not expected with the present debris flow material.
Figure 3.1  Grain size distribution of May 3 – 2001 – debris flow in Maschänserrüfe. a) Full grain size distribution of complete material. b) Partial distributions of the portions used for the rheological analysis in the present study.

Mixture preparation for rheometric tests:

Different material portions out of the complete grain material were selected for the present study: Material with grain sizes \(d \leq 0.25\) mm, \(d \leq 1\) mm, \(d \leq 5\) mm, \(d \leq 7\) mm, \(d \leq 10\) mm and \(d \leq 25\) mm. The grain size distribution that corresponds to each portion is shown in fig. 3.1 b).

The sediment material of a given portion was mixed with rain water (pH = 4.75 ± 0.25). Debris flow material mixtures of different volumetric solid concentration \(C_s\), were pre-
pared for each portion, by adding different quantities of sediments to a given quantity of water, according to the definition of the volumetric sediment concentration $C_v$:

$$C_v = \frac{M_s}{V_s + V_w} = \frac{\rho_s}{\rho_s + \rho_w} \cdot \frac{M_s}{2.7}$$

(3.2)

where $V_s$ = the volume of the added sediments (solids), $V_w$ = the volume of the added water, $M_s$ = the mass of the added solids [kg], $M_w$ = the mass of the added water [kg], $\rho_s = 2.7$ = solid density [kg/l] and $\rho_w = 1$ = density of water [kg/l].

Treating a specific mixture of a given portion as a partial fluid of the complete debris flow, a relation can be derived between the sediment concentration $C_v$ of the mixture and the sediment concentration $C_v_{DF}$ of the prototype debris flow (Coussot et al. 1998):

Analogue to eq. 3.2 the sediment concentration $C_v_{DF}$ of the prototype debris flow is expressed as:

$$C_v_{DF} = \frac{V_{s_{DF}}}{V_{s_{DF}} + V_w}$$

(3.3)

where $V_{s_{DF}}$ is the sediment volume of the prototype debris flow including all particles from the clay up to the block fraction and $V_w$ is the volume of the water within the prototype debris flow. Focusing a unit volume of the prototype debris flow it is:

$$V_{s_{DF}} + V_w = 1$$

(3.4)

Eq. 3.4 introduced in eq. 3.3 gives:

$$C_v_{DF} = \frac{V_{s_{DF}}}{V_{s_{DF}} + V_w} = \frac{V_{s_{DF}}}{1} = V_{s_{DF}}$$

(3.5)

Based on eq. 3.5, eq. 3.4 can be expressed as:

$$V_w = 1 - C_v_{DF}$$

(3.6)

From the grain size distribution of fig. 3.1 a) we know that:

$$V_s = p \cdot V_{s_{DF}}$$

and with eq. 3.5 $V_s = p \cdot C_v_{DF}$

(3.7)

where $p$ is the content of a given material portion within the complete debris flow grain material. Introducing eq. 3.6 and eq. 3.7 into eq. 3.2 the following relationship between the sediment concentration of a given material portion $C_v$ and the sediment concentration of the prototype debris flow $C_v_{DF}$ is obtained:

$$C_v = \frac{p \cdot C_v_{DF}}{1 + C_v_{DF} \cdot (p - 1)}$$

respectively

$$C_v_{DF} = \frac{C_v}{p + C_v \cdot (p - 1)}$$

(3.8)
Range of sediment concentration $C_v$ for the different material portions:

The range of sediment concentration $C_v$ considered for each material portion spanned the range where the mixtures flew more or less as one phase and where particle settling did not prevail. This range was assessed for the portion $d \leq 5$ mm with the help of the Inclined Channel Test (ICT). For this portion the range was $0.474 \leq C_v \leq 0.607$.

Based on the content $p = 0.316$ (fig. 3.1 a)) of the portion $d \leq 5$ mm within the complete material the corresponding range of sediment concentrations $C_v\ DF$ of the prototype debris flow where the latter flows as one phase was calculated as $0.74 \leq C_v\ DF \leq 0.83$ using eq. 3.8.

For this range of $C_v\ DF$ the range was calculated for all other portions, except the portion $d \leq 5$ mm, based on the content $p$ (fig. 3.1 a) and using eq. 3.8.

An overview of all the debris flow material mixtures prepared for the present study is given in table 3.4.

Mixing technique:

Appropriate mixing procedures were applied depending on the sample volume $V_{\text{sample}}$ necessary for a given rheometric system:

(1) For $V_{\text{sample}} \leq 1$ liter the defined quantity of water was placed in a container and the required quantity of sediments added to the water at once. Then the container was closed and shaken for 2 minutes.

(2) For $V_{\text{sample}} \leq 60$ liter the defined quantity of water was placed in a cylindrical container of 0.5 m diameter and the sediments added stepwise to the water. A mobile propeller of 120 mm diameter and 150 rpm was used to stir the mixture until all the sediments were added. Once all sediments had been added, the entire mixture was stirred for 10 minutes.

(3) For $V_{\text{sample}} \leq 650$ liter the highest concentrated mixture was prepared first. For this mixture the defined quantity of water was placed in a cylindrical container of 1.2 m diameter and the sediments added stepwise to the water. A four-wing propeller of 300 mm diameter and 25 rpm fixed in the container center as well as a mobile propeller of 120 mm diameter and 150 rpm along the container wall were used to stir the mixture until all the sediments were added which lasted 60 minutes. Then the entire mixture was stirred another 60 minutes with both propellers. For the lower concentrated mixtures the additional quantity of water was added to the existing mixture and the new mixture stirred for 30 minutes with both propellers.
3.4.4 Yield Stress Fluids II: Clay-Dispersions and Suspensions

A. Clay-Dispersions:

Characteristics of the sediment material:

Opalit, a clay-rich material of a sediment layer between the Dogger and Lias formation, excavated by Opalit AG/Holderbank was used as sediment material. It is a sealing material used for the construction of bulkheads, deposition sites and biotops and consists of particles \( d \leq 0.25 \) mm. The grain size distribution obtained with the arenometer test is shown in fig. 3.2. The shape of the particle was dominantly angular (Appendix A).

According to the geotechnical analysis of Stiefel & Stockmeyer (1995) the solid density of the Opalit is \( \rho_s = 2.74 \pm 0.01 \) g/cm\(^3\). The clay minerals are constituted by muscovite/illite (30 %), kaolinite (20 %), quartz (18 %), calcite (10 %), chlorite (10 %), clay minerals with changeable layers (kaolinite-montmorillonite-smectite) (5 %) and others (7%).

Mixture preparation for rheometric tests:

Four clay-dispersions of different sediment concentrations \( C_{v,\text{fines}} \) were prepared: \( C_{v,\text{fines}} = 0.3, \ C_{v,\text{fines}} = 0.275, \ C_{v,\text{fines}} = 0.25, \ C_{v,\text{fines}} = 0.225 \). Because the clay-dispersions are also used later as basic liquid for the clay-gravel-suspensions, the notation \( C_{v,\text{fines}} \) instead of \( C_v \) is used here; fines = particles with \( d \leq 0.25 \) mm grain size.

The following mixing technique was applied: A large volume of 350 liter was prepared. The defined quantity of water (pH 7.8) required for the highest concentrated mixture \( (C_{v,\text{fines}} = 0.3) \) was placed in a large cylindrical container and the fine material stepwise added to the water.

A four-wing propeller of 200 mm diameter fixed in the container center was used to stir the mixture until all the sediments were added which lasted 30 min. Then the entire mixture was stirred another 120 minutes. As for the lower concentrated mixtures, the necessary quantity of additional water was added to the higher concentrated mixture prepared before, and the new mixture stirred for 30 minutes.

B. Clay-Gravel-Suspensions:

Clay-gravel-suspensions with defined quantities of gravel added to specific clay-dispersions were prepared.
Characteristics of the solid material:
The gravel had a solid density $\rho_s = 2.65 \text{ g/cm}^3$. The particle shape of the gravel was ellipsoidal and rounded (Appendix A).

Mixture preparation for rheometric tests:
Gravel of grain size $3 \leq d \leq 10 \text{ mm}$ in linear distribution was used. The gravel was added at different concentrations ($C_{v\text{gravel}} = 0.1, 0.2, 0.3, 0.4$) to the clay-dispersions ($C_{v\text{fines}} = 0.225, 0.275$) in order to obtain 8 different suspensions with specific total concentrations $C_{v\text{tot}}$ where $C_{v\text{tot}}$ is defined as:

\[ C_{v\text{tot}} = C_{v\text{fines}} + C_{v\text{gravel}} \quad (3.4) \]

The grain size distributions of the different clay-gravel suspensions are shown in fig. 3.2. The volume of each suspension was $0.8\text{ l}$. The clay-dispersion was placed in a container and the gravel added. Then the container was closed and shaken for 1 minute.

C. Mixed Suspension:
One mixed suspension with defined quantities of sand and gravel added to a specific clay-dispersion was prepared.

Characteristics of the solid material:
The gravel and sand had a solid density $\rho_s = 2.65 \text{ g/cm}^3$. The particle shape of the gravel was ellipsoidal and rounded, while the sand exhibited both rounded and angular particle shape (Appendix A).

Mixture preparation for rheometric tests:
Sand of grain size $0.25 < d \leq 2 \text{ mm}$ and gravel of grain size $2 < d \leq 10 \text{ mm}$ was added to a clay-dispersion with a concentration of $C_{v\text{fines}} = 0.205$ in order to obtain a highly concentrated suspension with a sediment concentration of $C_{v\text{tot}} = 0.683$. The grain size distribution of the mixed suspension is shown in fig. 3.2 c). The total volume of the mixed suspension was $600\text{ l}$. Accordingly mixing technique (3) introduced for the debris flow material mixtures was applied. After the sand and gravel had been added final mixing lasted 30 minutes.
Figure 3.2 Grain size distribution of a) Clay-Dispersion and Clay-Gravel-Suspensions with $C_v_{	ext{fines}} = 0.225$, b) Clay-Gravel-Suspensions with $C_v_{	ext{fines}} = 0.275$ and c) Mixed Suspension.
3.5 Experiment Overview

The following tables give a summary of all investigated fluids (mixtures), the type of obtained rheological data (flow curve, yield stress) and the applied rheometric systems.

The temperature of the fluids investigated with the different rheometric systems was generally controlled at \(20^\circ \pm 1^\circ\) C.

It must be added here, that the effects of temperature (variation of temperature) on the rheological behaviour of the particle fluids were not investigated in the present study. This is principally because the influence of the sediment concentration \(C_v\) on the rheological behaviour of such fluids is much larger than the influence of the temperature as already shown by Coussot (1992).

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<table>
<thead>
<tr>
<th>Material</th>
<th>(\mu) [Pas]</th>
<th>Flow curve: BMS CCS CPS</th>
<th>measured in:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ ]</td>
<td>[Pa s]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silicon Oil 1</td>
<td>X</td>
<td>x</td>
<td>march 2002</td>
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<td>nov 2003</td>
</tr>
<tr>
<td>Silicon Oil 12.5</td>
<td>x</td>
<td>x</td>
<td>nov 2003</td>
</tr>
<tr>
<td>Silicon Oil 60</td>
<td>x</td>
<td>x</td>
<td>nov 2003</td>
</tr>
</tbody>
</table>

*Table 3.1 Rheometry used for the investigation of the Newtonian Silicon Oils. \(\mu =\) Newtonian viscosity.*

<table>
<thead>
<tr>
<th>Material</th>
<th>(C_m) [-]</th>
<th>(\eta_0) [Pa s]</th>
<th>Flow curve: BMS CCS CPS</th>
<th>measured in:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Guar 1% 0.01</td>
<td>15</td>
<td>x</td>
<td>x</td>
<td>march 2002</td>
</tr>
<tr>
<td>Guar 2% 0.02</td>
<td>300</td>
<td>x</td>
<td>x</td>
<td>march 2002</td>
</tr>
<tr>
<td>CMC 1% 0.01</td>
<td>6</td>
<td>x</td>
<td>x</td>
<td>nov 2003</td>
</tr>
<tr>
<td>CMC 2% 0.02</td>
<td>100</td>
<td>x</td>
<td>x</td>
<td>nov 2003</td>
</tr>
</tbody>
</table>

*Table 3.2 Rheometry used for the investigation of the non-Newtonian polymers. \(C_m =\) concentration by weight, \(\eta_0 =\) zero viscosity (viscosity at very low shear rates \(\dot{\gamma}\)).*
<table>
<thead>
<tr>
<th>$C_v$</th>
<th>$C_{vDF}$</th>
<th>Flow curve:</th>
<th>Yield Stress:</th>
<th>measured in:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BMS</td>
<td>CCS</td>
<td>CCS-</td>
</tr>
<tr>
<td>[-]</td>
<td>[-]</td>
<td></td>
<td></td>
<td>LS</td>
</tr>
<tr>
<td>$0.74$</td>
<td>$0.349$</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>$0.77$</td>
<td>$0.386$</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.429$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.83$</td>
<td>$0.479$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.815$</td>
<td>$0.453$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d \leq 0.25,mm$:</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>$0.74$</td>
<td>$0.418$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.77$</td>
<td>$0.458$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.502$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.815$</td>
<td>$0.526$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.83$</td>
<td>$0.552$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d \leq 1,mm$:</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.751$</td>
<td>$0.488$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.766$</td>
<td>$0.508$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.794$</td>
<td>$0.549$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.808$</td>
<td>$0.571$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.829$</td>
<td>$0.605$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.74$</td>
<td>$0.474$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.77$</td>
<td>$0.514$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.558$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.815$</td>
<td>$0.582$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.83$</td>
<td>$0.607$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d \leq 5,mm$:</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.74$</td>
<td>$0.488$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.77$</td>
<td>$0.529$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.573$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.815$</td>
<td>$0.596$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.83$</td>
<td>$0.621$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d \leq 7,mm$:</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.74$</td>
<td>$0.511$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.77$</td>
<td>$0.551$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.595$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.83$</td>
<td>$0.642$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$d \leq 10,mm$:</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.74$</td>
<td>$0.588$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.78$</td>
<td>$0.615$</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.8$</td>
<td>$0.643$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.81$</td>
<td>$0.658$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$0.82$</td>
<td>$0.673$</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$d \leq 25,mm$:</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 3.3** Rheometry used for the investigation of the debris flow material mixtures.  
$C_v = $ volumetric sediment concentration of the investigated mixture.  
$C_{vDF} = $ corresponding volumetric sediment concentration of the prototype debris flow (eq. 3.2).
<table>
<thead>
<tr>
<th>$C_v_{\text{finest}}$</th>
<th>$C_v_{\text{gravel}}$</th>
<th>$C_v_{\text{tot}}$</th>
<th>Flowcurve:</th>
<th>Yield Stress:</th>
<th>measured in:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-]</td>
<td>[-]</td>
<td>[-]</td>
<td>BMS CCS</td>
<td>KAS ICT IPT</td>
<td></td>
</tr>
<tr>
<td>Clay - Dispersions, $d \leq 0.25$ mm:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.225 0 0.225 x x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25 0 0.25 x x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.275 0 0.275 x x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3 0 0.3 x x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay - Gravel - Suspensions, $d \leq 10$ mm:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.225 0.1 0.325 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.225 0.2 0.425 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.225 0.3 0.525 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.225 0.4 0.625 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.275 0.1 0.375 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.275 0.2 0.475 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.275 0.3 0.575 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.275 0.4 0.675 x</td>
<td>x</td>
<td>feb 2001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixed Suspension, $d \leq 10$ mm:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.205 0.683 x x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>march 2003</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4 Rheometry used for the investigation of the clay-dispersions and the suspensions. $C_v_{\text{finest}}$ = volumetric sediment concentration of fine material ($d \leq 0.25$ mm), $C_v_{\text{gravel}}$ = volumetric sediment concentration of gravel ($3 < d \leq 10$ mm), $C_v_{\text{tot}}$ = total volumetric sediment concentration of dispersion/suspension.
Seite Leer / Blank leaf
4 The ball measuring system

4.1 General features

The ball measuring system (BMS) used in the present study was implemented in the Paar Physica MCR 300 rheometer. It consists of a cylindrical container with the radius $r_c = 57.5 \text{ mm}$ and the height $h_c = 48 \text{ mm}$ where a sample fluid of 0.5 liter is introduced.

![Ball measuring system installation](image)

*Figure 4.1 The ball measuring system installed in the Physica MCR 300 Rheometer (top) and its geometrical configuration (bottom).*

9 The BMS is one of several measuring systems that can be modularly implemented in that rheometer.
An eccentrically rotating sphere with given diameter $D$, fixed onto a 0.6 x 3 mm thin holder, is dragged through the fluid with fixed rotational speeds $\Omega$ (fig. 4.1). The distance between sphere and container wall is $s_w = 17$ mm and the one between sphere and container bottom is $s_b = 22$ mm. Spheres of variable diameter $D$ (8, 12, 15 mm) were used in the present study. Depending on the sphere diameter $D$ the radius of the center sphere path $r$ and the length of the immersed holder part $s_{im}$ vary (table 4.1).

In order to drag a specific sphere through a given fluid at a fixed speed $\Omega$, a torque $T$ must be applied. The speed $\Omega$ is controlled and measured in a range of 0.001 rps to 5 rps, and the torque $T$ is measured in a range of $10^{-7}$ Nm to 0.1 Nm.¹⁰

<table>
<thead>
<tr>
<th>Lengths</th>
<th>Surface areas</th>
<th>Volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ [mm]</td>
<td>$r$ [mm]</td>
<td>$s_{im}$ [mm]</td>
</tr>
<tr>
<td>15</td>
<td>33</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>34.5</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>36.5</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 4.1 Geometric configuration of the ball measuring system used in this study.

### 4.2 Principles of measurement

#### 4.2.1 Basic BMS experiment

The basic BMS experiment consists in measuring the torque $T$ and the rotational speed $\Omega$ at a specified fix speed $\Omega$, while the sphere makes one full rotation. In the present study 40 pairs of $T$- and $\Omega$-data were recorded during one sphere rotation.

In order to attain a prescribed speed $\Omega$, the sphere must be accelerated. Acceleration requires an additional torque beside the torque required to drag the sphere across a given fluid. For low and medium rotational speeds ($\Omega \leq 0.045$ rps), the torque due to acceleration is small and transformed into a torque undershoot by the control mechanism of the rheometer (fig. 4.2 a). Here torque data is influenced only at the very beginning of the experiment. At higher speeds ($\Omega > 0.045$ rps) the torque due to

¹⁰Note that this is the ordinary measuring range for the BMS. The Paar Physica MCR 300 rheometer principally enables measurements in the following range of the speed $\Omega$: $10^5 < \Omega < 50$ rps.
The ball measuring system

The ball measuring system

acceleration is larger and torque data is affected by overshoot measurements at the first part of each experiment (fig. 4.2 b, c). Magnitude of torque overshoot was found to depend solely on the specified speed $\Omega$ and the given sphere size $D$, whereas the number of torque data affected by acceleration depended partly also on the fluid characteristics (Appendix B).

After acceleration a steady drag flow around the sphere and sphere holder is achieved (fig. 4.2). In this regime the measured torque $T$ is only dependent on the rheological properties of the fluid.

It should be noted that only speed and torque data after the sphere acceleration regime respectively within a steady drag flow regime can be used for any further rheological investigation. As a consequence, the acceleration regime must be separated from the steady drag flow regime. An overview of the separation of acceleration and steady drag flow regime for the fluids of the present study is given in Appendix B1.

Further phenomena observed during measuring:

In liquid non particle and particle fluids it was observed that for very high speed of $\Omega = 2.25$ to 4.5 rps the accelerating sphere generated a wave in front of the sphere. This did not allow to record stable torque data $T$ in the steady drag flow regime (example in fig. 4.2 c): debris flow material mixture $d \leq 0.25$ mm, $C_v = 0.349$) and resulted in a relatively high standard deviation of the measured torque $\sigma_T$ (4.2.2). For the less liquid fluids such a wave was not observed and the standard deviation of the torque $\sigma_T$ was much smaller (4.2.2). However a tendency of increasing or decreasing torque data along the sphere path was found in some cases, which gives reason to question if for the very high speeds $\Omega$ the definition “steady drag flow” is still appropriate (Appendix B2).

When investigating silicon oils ($\mu \geq 2$ Pa s) the oil was lifted up along the sphere holder for higher rotational speeds $\Omega$, which is defined as viscous overstream. While this effect was observed for $\Omega \geq 1.35$ rps for the medium viscous oil $\mu = 2$ Pa s, this effect appeared already at $\Omega = 0.135$ rps for the highly viscous oil $\mu = 60$ Pa s (Appendix B). It might be that the measured torque is slightly falsified due to this effect. The data where this effect was observed were not considered for the model building of the conversion of measured into rheological parameters (4.3.3).

Important issue:

Due to the fact that sphere acceleration induced torques are produced when the speed $\Omega$ is altered to a specified fix value, it is not possible to perform direct flow curve experiments by sweeping stepwise from one controlled $\Omega$- or $T$- value to the next value.
within one sphere rotation, as this is possible with conventional measuring systems. Thus the final flow curve obtained with the BMS must be based on the composition of experiments performed at different prescribed speeds \( \Omega \).

![Diagram showing torque vs sphere rotation for different conditions](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>D &lt;= 0.25 mm</th>
<th>D &lt;= 1 mm</th>
<th>D &lt;= 5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>SilOil ( \mu = 2 ) Pa s</td>
<td>0.349</td>
<td>0.429</td>
<td>0.429</td>
</tr>
<tr>
<td>SilOil ( \mu = 12.5 ) Pa s</td>
<td>0.349</td>
<td>0.429</td>
<td>0.429</td>
</tr>
<tr>
<td>CMC 1%</td>
<td></td>
<td>0.502</td>
<td>0.502</td>
</tr>
<tr>
<td>CMC 2%</td>
<td></td>
<td>0.558</td>
<td>0.558</td>
</tr>
<tr>
<td>Debris flow material mixture</td>
<td>0.349</td>
<td>0.429</td>
<td>0.429</td>
</tr>
</tbody>
</table>

**Figure 4.2** Basic BMS experiments conducted with the sphere \( D = 12 \) mm: Measured torque \( T \) along the path of one sphere rotation. a) \( \Omega = 0.045 \) rps (medium speed), b) \( \Omega = 1.35 \) rps (high speed), c) \( \Omega = 4.5 \) rps (very high speed).
**Number of experiments:**

In the present study experiments were performed twice at all speeds $\Omega$ for $\Omega < 2.25$ rps. Because steady drag flow covers only 20 of the 40 data points in an experiment conducted at a high speed $\Omega$, four instead of two experiments were conducted for $\Omega \geq 2.25$ rps.

### 4.2.2 Fluctuation of BMS data

The BMS has been developed for the rheological investigation of large particle fluids. Thus in the next step it must be established how strongly the measured BMS data fluctuate for different maximum grain sizes $d_{\text{max}}$, grain size distributions and sediment concentrations $C_v$ in the fluid.

It was observed that the fluctuation of the torque $T$ increased when the maximum grain size $d_{\text{max}}$ within the fluid increased (fig. 4.3).

![Debris flow material mixtures:](image)

<table>
<thead>
<tr>
<th>Debris flow material mixtures:</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d \leq 0.25$ mm, $C_v = 0.429$</td>
<td>■</td>
<td>■ ■</td>
</tr>
<tr>
<td>$d \leq 1$ mm, $C_v = 0.502$</td>
<td>■ ■</td>
<td>■ ■ ■</td>
</tr>
<tr>
<td>$d \leq 5$ mm, $C_v = 0.559$</td>
<td>■ ■ ■ ■</td>
<td>■ ■ ■ ■ ■ ■</td>
</tr>
<tr>
<td>$d \leq 10$ mm, $C_v = 0.595$</td>
<td>■ ■ ■ ■ ■ ■ ■</td>
<td>■ ■ ■ ■ ■ ■ ■</td>
</tr>
</tbody>
</table>

*Figure 4.3. Torque fluctuation within one sphere rotation in debris flow material mixtures of different maximum grain size for $\Omega = 0.449$ rps using the sphere $D = 12$ mm.*

Beside the fluctuation of the torque $T$ also fluctuation of the speed $\Omega$ was observed in large particle fluids. Fluctuation of both, the torque $T$ and the speed $\Omega$ was specifically pronounced when the content of the large particles within the fluid was high (fig. 4.4).
Figure 4.4. Example of the fluctuation of the rotational speed $\Omega$ and the torque $T$ for Clay-Gravel-suspensions with $C_{v_{\text{fine}}}=0.225$ (Concentration of fine material) and different concentrations $C_{v_{\text{gravel}}}$ of rounded gravel ($3 \text{ mm} < d < 10 \text{ mm}$).

Standard deviation of torque and speed data:

Based on the torque- and speed data set of the two (respectively four) experiments executed for each fluid at a prescribed rotational speed $\Omega$, the mean values of the speed and the torque $\Omega$ and $T$ as well as the standard deviation $\sigma_{\Omega}$ and $\sigma_{T}$ were determined.

A good correlation between the relative maximum particle size $d_{\text{max}}/D$ and the standard deviation $\sigma_{\Omega}$ and $\sigma_{T}$ was found (fig. 4.5).

For $d_{\text{max}}/D \leq 0.125$ both, $\sigma_{\Omega}$ and $\sigma_{T}$, remained smaller than 10% in every BMS experiment when $\Omega \leq 2.7$ rps. Only for the very high speed $\Omega = 4.5$ rps a higher torque deviation was obtained in specific cases: The values of $\sigma_{T} > 10\%$ here correspond with the fluids where the formation of a wave - induced by the accelerating sphere - was observed, which impeded the generation of a steady sphere drag flow. Consequently the data for which is $d_{\text{max}}/D < 0.125$ and $\sigma_{T} > 10\%$ were excluded from further data analysis.

Regarding the values for $d_{\text{max}}/D \geq 0.125$, a strong increase of $\sigma_{T}$ and a very strong increase of $\sigma_{\Omega}$ was obtained. Torque and speed standard deviation span one or two decades roughly between 1% and 100% here. The following two parameters are introduced in order to give a clearer picture of the standard deviation of BMS measurements: Sediment concentration $C_v$ and grain size distribution.
Fig 4.6 shows that the standard deviation of the rotational speed $\sigma\Omega$ is strongly related to the sediment concentration $C_v$. This is mainly due to increasing friction and local or temporary jamming between the sphere, particles and the container boundary along the sphere path. These effects are amplified with the presence of a large content of gravels in the mixture:
Figure 4.6. Average standard deviation of rotational speed, $\sigma\Omega$, and torque, $\sigma T$, against sediment concentration $C_v$ for different large particle fluids.

<table>
<thead>
<tr>
<th>$C_v$ fines</th>
<th>$C_v$ gravel [-]</th>
<th>$C_v$ tot [-]</th>
<th>$\sigma\Omega$ [%]</th>
<th>$\sigma T$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.225</td>
<td>0.1</td>
<td>0.325</td>
<td>2.5</td>
<td>13.4</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.425</td>
<td>4.7</td>
<td>29.4</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.525</td>
<td>15.4</td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.625</td>
<td>23.7</td>
<td>23.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C_v$ fines</th>
<th>$C_v$ gravel [-]</th>
<th>$C_v$ tot [-]</th>
<th>$\sigma\Omega$ [%]</th>
<th>$\sigma T$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.275</td>
<td>0.1</td>
<td>0.375</td>
<td>4.9</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.475</td>
<td>10.1</td>
<td>16.6</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.575</td>
<td>29.5</td>
<td>23.5</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.675</td>
<td>61.5</td>
<td>23.9</td>
</tr>
</tbody>
</table>

Table 4.2 Average standard deviation of measured rotational speed, $\sigma\Omega$, and torque, $\sigma T$, for clay-gravel suspensions with variable content of gravels.
In poorly sorted material, such as the investigated debris flow material mixtures, an average standard deviation of \( \sigma \Omega < 10\% \) was found for sediment concentrations \( C_v < 0.6 \) (fig. 4.6). By comparison a much higher deviation of \( \sigma \Omega < 60\% \) resulted for the bimodal clay-gravel-suspensions for the same range of sediment concentrations \( C_v \) (table 4.2).

As far as the standard deviation of the torque \( \sigma T \) is concerned, a correlation with the sediment concentration \( C_v \) is not evident. Here the relative maximum particle size \( d_{\text{max}}/D \) is the parameter that mainly determines the value of \( \sigma T \). Nevertheless it must be noted that the \( \sigma T \)-values tend to be higher for lower concentrations \( (C_v < 0.5 - 0.55) \) than for slightly higher concentrations \( (C_v < 0.55 - 0.6) \), but tend to increase again as the concentration is increased further \( (C_v > 0.6) \). The higher \( \sigma T \)-values at lower concentrations are probably due to impacts of large particles onto the sphere and sphere holder, which are relatively higher in lower concentrated than in higher concentrated mixtures. It is concluded that the importance of impacts is progressively reduced by increasing \( C_v \), because larger particles are increasingly connected to each other and build a framework that reacts as a whole with the dragging sphere and sphere holder. As friction and local or temporary jamming becomes more and more important by a further increase of \( C_v \), it is concluded that torque fluctuation mainly results from the automated adaptation of the rotational speed (acceleration and decceleration) by the rheometer in order to maintain the prescribed speed \( \Omega \).

To summarize, for \( d_{\text{max}}/D \) smaller than 0.125, the standard deviation was less than 5\% in case of the rotational speed and less than 10\% in case of the torque. For \( d_{\text{max}}/D \) larger than 0.125, the standard deviation of the rotational speed \( \sigma \Omega \) depends strongly on the sediment concentration \( C_v \) and the grain size distribution. For \( C_v \) smaller than 0.6, \( \sigma \Omega \) was less than approximately 10\% in poorly sorted mixtures and less than approximately 60\% in gravel rich mixtures. For \( C_v \) larger than 0.6, \( \sigma \Omega \) increased up to 100\%. The standard deviation of the torque \( \sigma T \) was found to be dominantly dependent on the relative maximum grain size \( d_{\text{max}}/D \) and was usually smaller than 30\%, but remained always smaller than 50\%.

### 4.2.3 Determination of fluid relaxation time and fluid fatigue

**Definitions:**

As introduced in section 4.2.1, the basic BMS experiment consists in measuring \( T \) and \( \Omega \) at specified rotational speeds \( \Omega \), while the sphere makes one full rotation. However it is also possible to measure \( T \) and \( \Omega \) at specified rotational speeds \( \Omega \), while the sphere makes more than one rotation. In this case the sphere is dragged through an undisturbed
sample fluid for the first rotation, whereas for the following rotations the sphere is
dragged through an eventually disturbed sample due to the influence (namely within the
flow path) of the first rotation. Degree of disturbance of the sample after the first
rotation depends on the fluid property and on the specified rotational speed $\Omega$. The latter
is related to rotation time $t_{rot}$ as follows:

$$t_{rot} = \frac{1}{\Omega}$$

(4.1)

where the rotation time $t_{rot}$ is the required time to make one rotation along the sphere
path. Obviously the higher the rotational speed $\Omega$ is, the shorter is the time $t_{rot}$ for one
sphere rotation. The shorter $t_{rot}$ is, the shorter is the time for a given fluid to relax from a
disturbance of a previous rotation. With reference to the BMS, relaxation means the
filling of fluid into the flow path of the sphere after its previous rotation and the
rebuilding of its original interconnected aggregate structure (fluid relaxation). In the
case the original structure is not rebuilt anymore, the fluid endured fatigue. In the
following a method is presented how to determine the fluid relaxation time $t_{relax}$ and the
detection of fluid fatigue by the use of the BMS:

Method:

Experiments are performed where the sphere rotates several times at different specified
rotational speeds $\Omega$. Torques $T$ are measured for the first rotation, for the following
second rotation as well as for the rotations 2 to 10. The total number of rotations is 10.
For rotational speeds where the measured torques $T$ do not differ between the first, the
second and all the following rotations, the fluid has completely relaxed. For rotational
speeds where the torque values of the second rotation and all the following rotations are
smaller than the value of the first rotation, the fluid has not completely relaxed. For
rotational speeds where the torque value for the rotations 2 to 10 is smaller than the
value of the second rotation, the fluid endured fatigue.

Example:

A clay-dispersion with a medium solid concentration $C_{v \text{fines}} = 0.25$ is the test fluid.
Torques $T$ are measured for the first, the second as well as for all the following nine
rotations at different rotational speeds $\Omega$ (table 4.3, figure 4.7).

Almost identical values of the torque $T$ are obtained for the first rotation, the second
rotation as well as for rotations 2 to 10 as long as $\Omega \leq 0.449$ rps. Hence the fluid does
fully relax and endured no fatigue here. For rotational speeds $\Omega \geq 1.348$ rps torque
values $T$ decrease, so as for the second rotation as well as for rotation 2 to 10. Therefore
the fluid has not fully relaxed and endured some fatigue. As \( \Omega = 0.449 \) rps corresponds with \( t_{rot} = 2.2 \) s (eq. 4.1) and \( \Omega = 1.348 \) rps corresponds with \( t_{rot} = 0.74 \) s, fluid relaxation time \( t_{relax} \) must lie somewhere around 1s for the specific fluid.

<table>
<thead>
<tr>
<th>( \Omega ) [rps]</th>
<th>( t_{rot} ) [s]</th>
<th>Rotation 1</th>
<th>Rotation 2</th>
<th>Rotation 2 - 10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( T ) [mNm]</td>
<td>( \sigma T ) [mNm]</td>
<td>( T ) [mNm]</td>
</tr>
<tr>
<td>0.045</td>
<td>22.2</td>
<td>3.02</td>
<td>0.062</td>
<td>2.99</td>
</tr>
<tr>
<td>0.135</td>
<td>7.4</td>
<td>3.18</td>
<td>0.068</td>
<td>3.13</td>
</tr>
<tr>
<td>0.45</td>
<td>2.2</td>
<td>3.51</td>
<td>0.074</td>
<td>3.60</td>
</tr>
<tr>
<td>1.35</td>
<td>0.74</td>
<td>4.58</td>
<td>0.092</td>
<td>4.00</td>
</tr>
<tr>
<td>4.5</td>
<td>0.22</td>
<td>6.40</td>
<td>0.207</td>
<td>5.80</td>
</tr>
</tbody>
</table>

Table 4.3 Determination of fluid relaxation time and fluid fatigue using a medium concentrated clay-dispersion (\( C_{v,\text{fines}} = 0.25 \)): Comparison of measured torques \( T \) obtained for the first rotation in an undisturbed sample, for the following second rotation as well as for following 9 rotations in a disturbed sample for different rotational speeds \( \Omega \) mNm = 10\(^{-3}\) Nm.

Figure 4.7 Determination of fluid relaxation time and fluid fatigue using a medium concentrated clay-dispersion (\( C_{v,\text{fines}} = 0.25 \)).
4.3 Theory: Conversion of measured data into rheological data

4.3.1 Introduction

The content of the present chapter is the derivation of a relation between the drag flow around the sphere and sphere holder and the classic shear flow defined by the two-plate model in order to convert the measured parameters rotational speed $\Omega$ and torque $T$ into the rheological parameters shear rate $\dot{\gamma}$ and shear stress $\tau$ (fig. 4.8).

![Figure 4.8 Principal task when the ball measuring system is used for classic rheological investigation: To derive the relation between the drag flow around sphere and sphere holder and the classic shear flow.](image)

4.3.2 The existing approach of Tyrach

Tyrach (2001) developed an approach for the conversion of measured rotational speed $\Omega$ and torque $T$ into the rheological parameters shear rate $\dot{\gamma}$ and shear stress $\tau$ for a laminar flow regime around the dragging sphere. The approach is based on a dimensional analysis and based on the assumption that the shear rate $\dot{\gamma}$ is linearly dependent on the rotational speed $\Omega$. The conversion procedure includes three main steps:

1. Dimensional analysis and determination of the BMS system number $C$
2. Determination of the relation between $\dot{\gamma}$ and $\Omega$
3. Determination of the relation between $\tau$ and $T$

1. **Dimensional analysis and determination of BMS system number $C$**:

In an idealized system (no sphere holder and container boundary), where an eccentrically rotating sphere drags through a viscous fluid, six physical parameters act and three dimensions appear (table. 4.4). The six physical parameters are the torque $T$, the
rotational speed $\Omega$, the sphere diameter $D$, the radius of the center sphere path $r$, the fluid density $\rho$ and the viscosity $\eta$ of the fluid.

<table>
<thead>
<tr>
<th>Physical parameters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ $\Omega$ $D$ $r$ $\eta$ $\rho$</td>
</tr>
<tr>
<td>Dimensions:</td>
</tr>
<tr>
<td>[kg] 1 1 1 1</td>
</tr>
<tr>
<td>[m] 2 1 1 -1 3</td>
</tr>
<tr>
<td>[s] -2 -1 -1</td>
</tr>
</tbody>
</table>

Table 4.4 Physical parameters and their dimensions in case of sphere drag flow.

According to the Buckingham Pi-Theorem three dimensionless numbers are necessary to describe the system of six acting parameters and three dimensions:

To characterize the flow field the sphere Reynolds number $Re$ expressing the ratio of inertial and viscous forces is considered:

$$Re = 2 \cdot \pi \cdot \frac{\Omega \cdot r \cdot D \cdot \rho}{\eta}$$  (4.2)

To characterize the drag resistance the dimensionless drag coefficient $c_D$ is:

$$c_D = \frac{T}{r}$$  (4.3)

To characterize the geometry of the system the geometry number $G$ is defined as:

$$G = \frac{r}{D}$$  (4.4)

Multiplying the Reynolds number (eq. 4.2) and the drag coefficient (eq. 4.3), a dimensionless system number $C$ is obtained:

$$C = Re \cdot c_D = \frac{4}{\pi^2} \cdot \frac{T}{\Omega} \cdot \frac{1}{D \cdot r^2} \cdot \frac{1}{\eta}$$  (4.5)

For the laminar flow regime ($Re < 0.1 - 1$) it is $C = Re \cdot c_D = 24$ (Stokes flow, Clift et al. 1978) so that eq. 4.5 becomes:

$$T' = 6 \cdot \pi^2 \cdot \Omega \cdot D \cdot r^2 \cdot \eta$$  (4.6)

Due to self-rotation of the sphere in the BMS an additional torque is produced (Flügge 1963):

$$T'' = 2 \cdot \pi^2 \cdot \Omega \cdot D^3 \cdot \eta$$  (4.7)
The total torque then is given by:

\[ T = T' + T'' = 2 \cdot \pi^2 \cdot Q \cdot D \cdot \eta \cdot \left( 3 \cdot r^2 + D^2 \right) \]  

(4.8)

If eq. 4.8 is introduced into eq. 4.5, it can be shown that in the laminar flow regime the system number \( C \) is only dependent on the geometry number:

\[ C = \text{Re} \cdot c_d = 24 + 8 \cdot \frac{D^2}{r^2} = 24 + \frac{8}{G^2} \]  

(4.9)

24 is the Stokes flow number valid for a non-rotating sphere in an infinite flow field of a Newtonian fluid. The ratio \( 8/G^2 \) is attributed only to the self-rotation of the sphere.

The value of \( C \) are experimentally determined by using Newtonian fluids and measuring the torque \( T \) at different rotational speeds \( Q \) in the laminar flow regime and using eq. 4.5. Then the values are compared with the theoretical values obtained by eq. 4.9 (table 4.5).

<table>
<thead>
<tr>
<th>BMS</th>
<th>Theory (eq. 4.9)</th>
<th>Experiment (eq. 4.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C )</td>
<td>( C )</td>
</tr>
<tr>
<td></td>
<td>( D ) ( s_{im} ) ( r ) ( G )</td>
<td>( C ) ( C ) ( \Delta C )</td>
</tr>
<tr>
<td>[mm]</td>
<td>[mm] [mm] [-]</td>
<td>[-] [-] [-]</td>
</tr>
<tr>
<td>15</td>
<td>11 33 2.200</td>
<td>25.7 44.3 18.6</td>
</tr>
<tr>
<td>12</td>
<td>14 34.5 2.875</td>
<td>25.0 48.9 23.9</td>
</tr>
<tr>
<td>8</td>
<td>18 36.5 4.563</td>
<td>24.4 51.5 27.1</td>
</tr>
</tbody>
</table>

Table 4.5 Comparison of theoretical and experimentally determined system number \( C \) for the BMS. The experimental value \( C \) was determined with the help of the Silicon Oils (\( \mu = 60, 12.5, 2 \) and \( 0.05 \) Pa s).

A strong difference between the theoretical and the experimental value of \( C \) resulted. Tyrach (2001) attributed the difference \( \Delta C \) to the influence of the sphere holder. He verified this assumption by determining the \( \Delta C \)-value at different immerging depths \( s_{im} \) of the sphere holder and plotting a linear function at the experimental data points \( \Delta C \) and \( s_{im} \) (Appendix C).

To conclude, for the following step 2 the experimentally determined system number \( C \) must be applied, because the experimental value expresses the specific boundary setting of the BMS (sphere-sphereholder-container) for a given sphere size \( D \).
2. Determination of the relation between $\dot{\gamma}$ and $\Omega$:

The BMS specific viscosity $\eta$ is given by transforming eq. 4.5 and considering the experimentally determined system number $C$:

$$\eta = \frac{1}{C} \cdot \frac{4}{\pi^2} \cdot \frac{T}{\Omega} \cdot \frac{1}{D \cdot r^2}$$  \hspace{1cm} (4.10)

Tyrach (2001) assumed a linear relationship between the shear rate $\dot{\gamma}$ and the rotational speed $\Omega (\dot{\gamma} = K_\Omega \Omega)$. With the constant $K_\Omega$ relating the two parameters $\dot{\gamma}$ and $\Omega$, eq. 4.10 can be transformed into:

$$\eta = \frac{1}{C} \cdot \frac{4}{\pi^2} \cdot \frac{T \cdot K_\Omega}{\dot{\gamma}} \cdot \frac{1}{D \cdot r^2}$$  \hspace{1cm} (4.11)

For the conversion of speed into shear rate a Power Law Fluid is required, for which the viscosity is given by:

$$\eta = m \cdot \dot{\gamma}^{n-1}$$  \hspace{1cm} (4.12)

Tyrach (2001) measured a specific Power Law Fluid (Guar 2% - solution) in the BMS and in a reference system (CPS). He then overlayed the viscosity curve of the BMS (eq. 4.11) with the viscosity curve of the reference system (eq. 4.12) and found the value of $K_\Omega$ graphically for the best curve overlay.

Tyrach (2001) verified the value of $K_\Omega$ a posteriori with the help of the measurements of other Power Law Fluids, such as CMC-solutions and other Guar-solutions. As a result, the value of $K_\Omega$ was found to differ by $\pm 3\%$.

3. Determination of the relation between $\tau$ and $T$:

It is per definition (eq. 2.3):

$$\tau = \eta \cdot \dot{\gamma}$$  \hspace{1cm} (4.13)

Introducing eq. 4.11 into eq. 4.13 the shear stress $\tau$ can be obtained by:

$$\tau = K_\Omega \cdot \frac{1}{C} \cdot \frac{4}{\pi^2} \cdot \frac{1}{D \cdot r^2} \cdot T = K_T \cdot T$$  \hspace{1cm} (4.14)

It follows that the shear stress $\tau$ is linearly dependent on the measured torque $T$ as long as $C$ is constant. As seen before, $C$ is constant in the laminar flow regime ($Re < 1$). The relating constant $K_T$ can be calculated directly once the constants $C$ and $K_\Omega$ have been determined.

The constants $K_\Omega$ and $K_T$ which were determined for the BMS in the Physica MCR 300 Rheometer are listed in table 4.6:
Table 4.6  Constants $K_\Omega$ and $K_T$ relating the BMS raw parameters $\Omega$ and $T$ with the rheological parameters $\dot{\gamma}$ and $\tau$ according to the theory of Tyrach.\textsuperscript{11}

$K_T' = K_T \cdot r \cdot (A_s + A_h)$ is the dimensionless constant of $K_T$ where $r$ is the radius of the sphere path and $A_s$, $A_h$ are the surfaces of the sphere and the sphere holder, respectively (see according values in table 4.1).

Discussion of the approach of Tyrach:

The system number $C$:

The values of the experimentally determined system number $C$, which express the sphere-sphere holder-container setting of the BMS, are generally large compared to the theoretical values of $C$ for a sphere dragged in an infinite boundary.

Tyrach claimed that the difference in the value $\Delta C$ is only attributed to the sphere holder, because he fitted a linear function to some specific measurements of the immersing depth $s_{im}$ of the holder and the value of $\Delta C$ (Appendix C). The author of the present study largely shares this view. But the author here proposes to plot a non linear function at the data points in Appendix C in order to express a very limited but increasing influence of the container boundary with increasing immersing depths $s_{im}$, beside the strong influence of the sphere holder.

There are no formulas yet available to calculate the influence of the sphere holder in the laminar flow regime independently for the specific geometry of the present sphere holders, as such is possible for other dragging bodies, such as spheres, ellipsoids or bars with quadratic sections, etc. (Clift et al. 1978, p.87). However, the relatively large ratio of sphere surface and sphere holder surface, $A_s/A_h (= 1.5 \text{-} 9$; table 4.1) compared to the ratio of $C_{\text{theor}}/\Delta C$ ($\approx 1$; table 4.5) indicates that, for the sphere holder, form drag (due to pressure distribution) must play a relatively more important role than skin friction (viscous drag) compared to the sphere. For a sphere, the ratio between skin friction and form drag is 2:1 for laminar flow within Newtonian fluids (Clift et al. 1978).

\textsuperscript{11} Values of the constants $K_\Omega$ and $K_T$ which are presently implemented in the BMS Software of the Physica MCR 300 Rheometer.
The constants $K_\Omega$ and $K_T$:

The constant $K_\Omega$ found with the approach of Tyrach relies on the assumption that the shear rate $\dot{\gamma}$ is a linear function of the rotational speed $\Omega$. The assumption was not proved by Tyrach.

The constant $K_T$ depends on the geometric configuration of the BMS as well as on the system number $C$ and the constant $K_\Omega$. $C$ and $K_\Omega$ were determined with Newtonian and Power Law Fluids. The constant $K_T$ thus does not consider possible properties of Yield Stress Fluids (usually the fluid type for medium to highly concentrated particle fluids) for which the BMS was principally designed for.

An improved approach should thus (1) show the relation between the shear rate $\dot{\gamma}$ and the rotational speed $\Omega$ and (2) consider the aspect of Yield Stress Fluids.

### 4.3.3 An improved approach based on Metzner-Otto-theory

The approach bases on the power characteristic of the measuring system. The latter is used for the derivation of the relationship between the shear rate $\dot{\gamma}$ and the rotational speed $\Omega$. Once this relationship is known, a relationship between the shear stress $\tau$ and the torque $T$ can be found afterwards. In the present study a conversion of measured data into rheological data was derived for a laminar flow around the sphere ($Re < 1$) as well as for the transitional regime ($1 < Re < 50 - 300$). In analogy to the approach of Tyrach, the conversion procedure includes the following three main steps:

1. Power characteristic and determination of the BMS system number $C_1$
2. Determination of the relation between $\dot{\gamma}$ and $\Omega$
3. Determination of the relation between $\tau$ and $T$

#### 1. Power characteristic and determination of the BMS system number $C_1$:

The power $P$ of a stirring system at a constant rotational speed $\Omega$ is:

$$P = T \cdot 2 \cdot \pi \cdot \Omega$$  \hspace{1cm} (4.15)

The dimensionless power number (Newton number) $Ne$ is described as follows (Metzner&Otto 1957, Windhab 2001):

$$Ne = \frac{P}{\Omega^2 \cdot D^5 \cdot \rho} = \frac{T \cdot 2 \cdot \pi}{\Omega^2 \cdot D^5 \cdot \rho}$$  \hspace{1cm} (4.16)

where $D$ is the characteristic diameter. For the BMS, $D$ is the sphere diameter.

Further, the sphere Reynolds number $Re$ is:
Multiplying the power number (eq. 4.16) with the Reynolds number (eq. 4.17), a characteristic BMS system number $C_I$ is obtained:

$$C_I = \text{Re} \cdot \text{Ne} = 4 \cdot \pi^2 \cdot \frac{T}{\Omega} \cdot \frac{r}{D^4} \cdot \frac{1}{\eta}$$

Note that $C_I$ is, analogue to the system number $C$ in the approach of Tyrach, proportional to $T/\Omega \eta$. $C_I$ only distinguishes by the subterm $\pi^2 \cdot r/D^4$ from $C$ where the subterm of the same dimensions is $1/r/D^2$.

In the present study $C_I$ was determined with the help of different Newtonian Silicon Oils ($\mu = 60, 30, 12.5, 2, 1, 0.05$ Pa s) measured in the BMS and partially referenced in the CPS and CCS (Appendix D). $C_I$ turned out to be constant for $\text{Re} \leq 1$, but increases strongly for $\text{Re} > 1$ (fig. 4.9). Increase of $C_I$ is related to the transition from laminar to turbulent flow (wake formation and onset of turbulence, Clift et al. 1978).

For $\text{Re} \leq 1$ an average value of $C_I$ was determined based on the set of $C_I$- and $\text{Re}$- data. For $\text{Re} > 1$ a polynom function was fit to the $C_I$- and $\text{Re}$- data (table 4.7).

<table>
<thead>
<tr>
<th>$D$ [mm]</th>
<th>$Re \leq 1$:</th>
<th>$Re &gt; 1$:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_I = \text{const.}$</td>
<td>$C_I = \sum_{j=0}^{j=4} a_j \cdot \text{Re}^j$</td>
</tr>
<tr>
<td></td>
<td>$D$</td>
<td>$C_I$</td>
</tr>
<tr>
<td>15</td>
<td>46775.9</td>
<td>4.44E+04</td>
</tr>
<tr>
<td>12</td>
<td>113384.6</td>
<td>1.05E+05</td>
</tr>
<tr>
<td>8</td>
<td>491381.6</td>
<td>4.60E+05</td>
</tr>
</tbody>
</table>

Table 4.7 The system number $C_I$ for the different spheres $D$ in function of the sphere Reynolds number $Re$. 
2. Determination of the relation between $\gamma$ and $\Omega$:

Power Law Fluids are used for the derivation of the shear rate $\dot{\gamma}$ based on the power characteristics of the measuring system. The viscosity $\eta$ of a Power Law Fluid is given in eq. 4.12.

Further, eq. 4.18 can be transformed into an expression for the viscosity $\eta$:

$$
\eta = 4 \cdot \pi^2 \cdot \frac{T}{\Omega} \cdot \frac{r}{D^4} \cdot \frac{1}{C_1}
$$

(4.19)
With eq. 4.11 introduced in eq. 4.19, the following expression for the shear rate \( \dot{\gamma} \) is found:

\[
\dot{\gamma} = \left(4 \cdot \pi^2 \cdot \frac{T}{\Omega} \cdot \frac{r}{D^4} \cdot \frac{1}{C_1} \cdot \frac{1}{m}\right)^{\frac{1}{n-1}}
\] (4.20)

In the present study a 1 %-CarboMethylCellulose-solution (CMC 1 %) and a 1 %-Gaur-solution (Guar 1 %) were used as Power Law Fluids. Based on reference measurements in the CCS the consistency coefficient \( m \) and the index \( n \) were determined (Appendix D). Based on measurements in the BMS the torque \( T \) was determined for the different specified speeds \( \Omega \). Considering the system number \( C_i \) of table 4.7 the shear rates were then calculated using eq. 4.20.

It must be noted that the value of \( C_i \) is a priori unknown for the applied Power Law Fluids, because \( \eta \) and \( \text{Re} \) are unknown at a given speed \( \Omega \). Thus, the Reynolds number \( \text{Re} \) must be estimated first and \( C_i \) calculated based on this first estimation. In the present study the adapted Re-number for Power Law Fluids defined by Atapattu et al. 1995 was used for the first estimation (see eq. 4.34). The shear rate \( \dot{\gamma} \) is then calculated with eq. 4.20 and \( \eta \) calculated based on the reference curve (Appendix D). The Reynolds number \( \text{Re} \) is then recalculated (eq. 4.17). For recalculated Reynolds numbers \( \text{Re} \) smaller than 1, calculation stops because \( C_i \) is constant. For recalculated Reynolds numbers \( \text{Re} \) larger than 1, \( C_i, \dot{\gamma}, \eta \) and \( \text{Re} \) is calculated in several rounds (iterations) until the last calculated values don’t differ from the values calculated one round before (fig. 4.10).

\begin{figure}[h]
\centering
\includegraphics{procedure.png}
\caption{Procedure for the determination of the shear rate \( \dot{\gamma} \) in the BMS.}
\end{figure}

After this procedure data of shear rate \( \dot{\gamma} \) is plotted against data of rotational speed \( \Omega \) (fig. 4.11):
With reference to the results obtained for all three spheres, the shear rates of the two different polymer fluids diverge at low speeds ($\Omega < 0.03$ rps). This is probably due to a
different adhesional/slipping behaviour of the two fluids on the sphere and sphere holder surface occurring at very low speeds. For increased speeds ($\Omega > 0.03$ rps) data of both fluids correspond well. At the highest speeds ($\Omega > 2$ rps) shear rates of both fluids diverge again which corresponds to Reynolds numbers $Re > 10 - 20$. In this regime divergence is assumed to be due to some elastic behaviour of the polymers being different for the investigated CMC 1 % and the Guar 1 %.

To build the relation between the shear rate $\dot{\gamma}$ and the rotational speed $\Omega$ only the data with no adhesional/slipping and elastic effects are considered first ($\Omega > 0.03$ rps). Second, to avoid any possible effect of non laminar flow in this relation, only data within laminar flow ($Re < 1$) are included first. In this range $\Omega > 0.03$ rps and $Re < 1$ (which approximately corresponds to $\Omega < 0.6$ rps) a linear relation between $\dot{\gamma}$ and $\Omega$ is found:

$$\dot{\gamma} = K_\Omega \cdot \Omega$$  \hspace{1cm} (4.21)

The average coefficient $K_\Omega$ and its standard deviation were calculated based on the corresponding set of $\dot{\gamma}$- and $\Omega$- data for each sphere. The $K_\Omega$- values including the standard deviation are listed in table 4.8 and the fit curves are shown in fig. 4.11.

<table>
<thead>
<tr>
<th>$D$ [mm]</th>
<th>$K_\Omega$ [-]</th>
<th>$\sigma K_\Omega$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>22.8</td>
<td>12.3</td>
</tr>
<tr>
<td>12</td>
<td>25.6</td>
<td>6.6</td>
</tr>
<tr>
<td>8</td>
<td>25.6</td>
<td>10.6</td>
</tr>
</tbody>
</table>

Table 4.8 Constant $K_\Omega$ relating the rotational speed $\Omega$ with the shear rate $\dot{\gamma}$ and standard deviation $\sigma K_\Omega$.

It must be stressed that in contrast to the approach of Tyrach - where the linearity between the shear rate data $\dot{\gamma}$ and the rotational speed $\Omega$ was assumed - the linearity was found here after analysis of $\dot{\gamma}$- versus $\Omega$- data. Thus, the linearity assumed by Tyrach was confirmed with the present approach.

Fig. 4.11 shows that the linearity roughly lasts up to $\Omega = 3$ rps, which corresponds to $Re = 10 - 20$ for the spheres $D = 15$ mm and $D = 12$ mm. For higher speeds ($\Omega > 3$ rps) the data deviate from this linearity. As for the smallest sphere $D = 8$ mm, data deviate from linearity already for $\Omega > 1$ rps which corresponds to $Re > 1$. Note that for the larger spheres the deviation of the shear rate from the linear function is positive, whereas for the smallest sphere this deviation is negative.
The divergence of $\dot{\gamma}$ and $\Omega$-data obtained for the two fluids ($I$) and the different trend of deviation from linearity obtained by the use of different spheres ($2$) renders an interpretation difficult, whether there is an influence of the non laminar flow regime on the linear relation between $\dot{\gamma}$ and $\Omega$ obtained for the laminar flow regime. For the following third step the linear function obtained for the laminar flow regime (eq. 4.21) and the calculated coefficients $K_\Omega$ listed in table 4.8 are used for both the laminar and non laminar flow regime. The topic of the character of the shear rate $\dot{\gamma}$ is discussed further in section 4.6.

3. Determination of the relation between $\tau$ and $T$:

The relation between $\tau$ and $T$ is investigated for the following characteristic groups of fluids:

Silicon Oils: Newton Fluids (NWT)

Polymers: Power Law Fluids (PLF)

Particles: Yield Stress Fluids (YSF)

As for the "Silicon Oils", oils of viscosity $\mu = 60, 30, 12.5, 2, 1, 0.05$ Pa s, as for the "Polymers", CMC 1 %, CMC 2 %, Guar 1 % and Guar 2 % were measured in the BMS and referenced in the CCS and CPS (Appendix D). As for the "Particles", debris flow material mixtures containing particles smaller than $d = 0.25$ mm grain size were measured in the BMS and referenced in the CCS considering mixture concentrations $C_v = 0.345, C_v = 0.386, C_v = 0.429$. Additionally clay-dispersions with particles smaller than $d = 0.25$ mm grain size and the concentrations $C_v = 0.225, C_v = 0.25, C_v = 0.275, C_v = 0.3$ were measured in the BMS using the sphere $D = 12$ mm and referenced in the CCS (Appendix D).

Based on the measurement of speed $\Omega$ in the BMS the shear rate $\dot{\gamma}$ was calculated with the empirically determined coefficient $K_\Omega$ (table 4.8) for all fluids. The shear stress $\tau$ was then calculated for every BMS-shear rate $\dot{\gamma}$ based on eq. 2.4, 2.5 and 2.6 respecting the parameter values $\mu, m, n$ and $\tau_\gamma$ obtained for the different fluids with the help of the reference measurements (Appendix D).

Plotting calculated shear stresses $\tau$ against measured BMS torques $T$, a roughly linear relationship between the two parameters is found for $Re \leq 1$ over the entire range of measured torques $T$. By contrast, data with $Re > 1$ tend to deviate from this linearity (fig. 4.12). Hence, the relation between $\tau$ and $T$ must be derived in dependence of the Reynolds number Re.
The relation between the ratio $\tau/T$ and $Re$ for the different BMS spheres is shown in fig. 4.13:

For a given sphere and a given fluid group, except the Polymers 2%, the ratio $\tau/T$ is roughly constant in the laminar flow regime ($Re \leq 1$). In the transitional regime ($Re > 1$) $\tau/T$ decreases.

For the Polymers 2%, a relation between sphere drag flow and classic shear flow can hardly be deduced due to probable elastic behaviour of these fluids. However inaccuracy decreases with increasing sphere size $D$ for this type of fluid.
Figure 4.13  Relation of the ratio $\tau/T$ and the Reynolds number $Re$ for the different investigated fluid groups and for the three BMS spheres.
Consequently, except for the Polymers 2 \%, data of each fluid group were fit with appropriate functions depending on the flow regime. For the laminar flow regime ($Re \leq 1$) the function is:

$$
\tau = K_T \cdot T
$$

(4.22)

For the transitional regime ($Re > 1$) it is:

$$
\tau = \left[ k_1 + k_2 \cdot e^{-k_3 \cdot Re} \right] \cdot T
$$

(4.23)

The average value of the coefficient $K_T$ of eq. 4.22 was calculated based on the corresponding $\tau \cdot T$ data set for each sphere and each fluid group. The coefficients $k_1$, $k_2$ and $k_3$ of eq. 4.23 were calculated by using the least-squares method of Gauss. All coefficients and their standard deviations/errors are listed in table 4.9.

<table>
<thead>
<tr>
<th>$Re \leq 1$:</th>
<th>Silicon Oils (NWT), Polymers 1% (PLF)</th>
<th>Particles (YSF)</th>
<th>$D$</th>
<th>$K_T$</th>
<th>$K_T'$</th>
<th>$\sigma K_T$ ($\sigma K_T'$)</th>
<th>$K_T$</th>
<th>$K_T'$</th>
<th>$\sigma K_T$ ($\sigma K_T'$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[mm]</td>
<td>[m$^3$]</td>
<td>[-]</td>
<td>[%]</td>
<td>[m$^3$]</td>
<td>[-]</td>
<td>[%]</td>
<td>[m$^3$]</td>
<td>[-]</td>
<td>[%]</td>
</tr>
<tr>
<td>15</td>
<td>12600</td>
<td>0.3261</td>
<td>3.8</td>
<td>10700</td>
<td>0.2769</td>
<td>3.5</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>14900</td>
<td>0.2834</td>
<td>3.2</td>
<td>15000</td>
<td>0.2853</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>18500</td>
<td>0.2221</td>
<td>2.9</td>
<td>22800</td>
<td>0.2737</td>
<td>5.4</td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$Re &gt; 1$:</th>
<th>$D$</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$\sigma k_1$</th>
<th>$\sigma k_2$</th>
<th>$\sigma k_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[mm]</td>
<td>[m$^3$]</td>
<td>[m$^3$]</td>
<td>[-]</td>
<td>[%]</td>
<td>[%]</td>
<td>[%]</td>
<td></td>
</tr>
<tr>
<td>Silicon Oils (NWT)</td>
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<td>10052</td>
<td>0.033435</td>
<td>10.5</td>
<td>3.3</td>
<td>9.9</td>
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<td>7.1</td>
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<td>8</td>
<td>5553</td>
<td>13910</td>
<td>0.052913</td>
<td>13.7</td>
<td>5.7</td>
<td>16.5</td>
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<td>Polymers 1% (PLF)</td>
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<td>8513</td>
<td>4254</td>
<td>0.115740</td>
<td>9.4</td>
<td>20.7</td>
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<td>4927</td>
<td>9440</td>
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<td>126.9</td>
<td>62.8</td>
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<td>8068</td>
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<td>13.1</td>
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<tr>
<td>Particles (YSF)</td>
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<td>8377</td>
<td>0.009679</td>
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<td>8</td>
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<td>11635</td>
<td>0.055850</td>
<td>10.4</td>
<td>9.8</td>
<td>27.0</td>
<td></td>
</tr>
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Table 4.9 Coefficients $K_T$ ($K_T'$) and $k_j$ (including standard error $\sigma K_T$, $\sigma K_T'$, $\sigma k_j$) for the calculation of the shear stress $\tau$ based on the measured torque $T$ and the Reynolds number $Re$.

12 Only the data of the debris flow material mixtures are considered in the coefficients.
Note that the coefficients $K_T$, $k_1$ and $k_2$ have the dimension $[m^3]$. In order to eliminate the dimensions the coefficients can be multiplied with the radius of the center sphere path $r$ and the total of sphere surface ($A_s$) and sphere holder surface ($A_h$), (see specific values in table 4.1). In the case of the constant $K_T$, the dimensionless constant $K_T'$ is then:

$$K_T' = K_T \cdot r \cdot (A_s + A_h)$$ (4.24)

The results obtained after data fitting can be summarized as follows:

**Laminar flow regime ($Re \leq 1$):**

Identical coefficients $K_T$ are found for the Newtonian Silicon Oils and the non-Newtonian Polymers 1% for a given sphere. By contrast the $K_T$-value obtained for the debris flow material mixtures differs from the one obtained for the Silicon Oils and Polymers (fig. 4.13, table 4.9). It is postulated that this is due to the presence of a yield stress $\tau_y$ in this type of fluid. This hypothesis was later verified with the help of the yield stress criterion (4.6.2). Hence it can be concluded that depending on the type of fluid investigated in the BMS specific coefficients must be considered.

Regarding the different concentrated *clay-dispersions*, which are also Yield Stress Fluids but which can exhibit thixotropic behaviour (Schatzmann et al. 2003b), roughly the same value of the coefficient $K_T$ is obtained ($K_T = 15'300 \, m^3$) as the one for the debris flow material mixtures ($K_T = 15'000 \, m^3$). By comparison data scatter around the average value is considerably higher in the case of clay-dispersions (fig. 4.13, $D = 12 \, mm$). It is assumed that this larger scatter is due to thixotropic effects of the latter dispersions, which were not systematically controlled and recorded during measuring.

**Transitional regime ($Re > 1$):**

Depending on the type of fluid, a specific relation of the ratio $\tau/T$ and $Re$ results. The ratio $\tau/T$ of the Newtonian Silicon Oils drops much earlier with increasing $Re$ than does the ratio $\tau/T$ of the particle fluids. The polymers indicate to deviate from constancy of $\tau/T$ already for $Re > 0.1$. But the trend of the decrease of $\tau/T$ is not clear. The polymers 1% indicate a similar trend as the particle fluids, whereas the more elastic polymers 2% tend to follow the trend of the Silicon Oils.
Standard error of calculated shear stress after conversion:

The standard error $\sigma \kappa_n$, $\sigma \kappa_r'$, respectively the standard error of the calculated shear stress $\sigma \tau$ is smaller than 6% in every group of fluid in the laminar flow regime ($Re \leq 1$). In the case of particle fluids, the standard error is tendentially smaller for the larger spheres than for the smaller spheres. This is probably due to the relatively stronger influence of the sphere holder in the case of the smaller sphere, which creates a more irregular flow around the sphere - sphere holder - tool by interaction with particles.

By comparison, the standard error of the conversion coefficients in the transitional regime ($Re > 1$) is worse. Adding the standard errors $\sigma k_j$ of the three coefficients $k_1$, $k_2$ and $k_3$ (based on the error propagation of Gauss), the standard error of the calculated shear stress $\sigma \tau$ principally depends on the Reynolds number $Re$. However, due to the negative sign in the exponent of the fit function (eq. 4.23) the influence of the Reynolds number remains small. Independent of the sphere size $D$, $\sigma \tau$ is less than 10% for $Re < 50$ and less than 15% for $Re > 50$ in the case of the Silicon oils. In the case of the Polymers, $\sigma \tau$ is less than 80% for $Re < 50$ and less than 120% for $Re > 50$. In the case of the Particles, $\sigma \tau$ is less than 15% for both, $Re < 50$ and $Re > 50$.

It is important to stress that the coefficients and the accuracy of the data conversion derived for the different groups of fluids rely on the specific fluids considered in this study. This is notably important in the case of the particle fluids (Yield Stress Fluids) in the transitional regime ($Re > 1$). Here the $\tau T$ – data of the clay-dispersion indicate another relationship compared to the debris flow material mixtures. Thus for the transitional regime the present empirical coefficients $k_j$ are considered as a first order approach which must be tested with further fluids.

4.3.4 Control of conversion theories

Before applying the conversion theory to large particle fluids, the existing theoretical approach of Tyrach (2001) and the new approach of the present study are applied to the fluids which were measured in both (1) the BMS and (2) the standard measuring systems. This enables to control the empirical coefficients derived for the new approach and to compare it with the existing approach of Tyrach (2001).

By the use of the new approach the flow curve data of the BMS fit the reference curve obtained in the standard measuring systems well for any group of fluid, each used sphere and for the entire range of shear rates and shear stresses (figure 4.14 a,b).
Considering the approach of Tyrach (2001)\textsuperscript{13} this is not always the case. For the Silicon Oils and Polymers the data fit the reference curve well if Re ≤ 1 - 10. For Re > 10 data diverge because Tyrach did not derive any relationship for the transitional flow regime.

Regarding the Silicon Oils and Polymers in the laminar flow regime (Re ≤ 1), the approach of Tyrach indicates slightly lower values compared to the new approach. This is probably because (1) Tyrach used only a few fluids for the derivation of his approach. Further, after having investigated numerous particle fluids in the present study, (2) abrasion could have altered the surface roughness of the sphere and the holder in a manner that slightly different coefficients of conversion resulted for the non particle fluids investigated after the particle fluids.

![Reference curve](image)

**Fig. 4.14 a) BMS flow curves obtained with the new approach and obtained with the approach of Tyrach in comparison with the reference curves obtained in standard measuring systems.**

\textsuperscript{13} Use of empirical coefficients of table 4.6.
As for the *Particles* (debris flow material mixtures) in the laminar flow regime (Re ≤ 1 - 10), flow curves on a lower shear stress level are obtained using smaller spheres with the approach of Tyrach (2001) compared to the new approach. This is not surprising as Tyrach did not consider this type of fluid and their particularities (namely the presence of a yield stress $\tau_y$) for the conversion of torques $T$ into shear stresses $\tau$.

Similar to the Silicon oils and Polymers the data obtained with the approach of Tyrach (2001) for the *Particles* diverge from the reference curve in the transitional regime (Re > 1 - 10). This again is due to the fact that Tyrach did not consider the influence of this flow regime in his approach.
4.4 Application of conversion theories to large particle fluids

In figure 4.15 a) and b) the BMS flow curves of different debris flow material mixtures are shown for both, the approach of Tyrach and the new approach. The following portions are represented: $d \leq 1$ mm, 5 mm, 7 mm and 10 mm. For each portion, flow curves of mixtures are shown which represent specific sediment concentrations $C_v$. For reasons of readability, the flow curve data of only three out of five or more investigated concentrations $C_v$ are shown for each portion $d$. The following results were obtained:

![Graph showing flow curves for different portion sizes](image)

*Fig. 4.15 a) Flow curves of variably concentrated debris flow material mixtures (portions $d \leq 1$ mm and $d \leq 5$ mm). Comparison of results obtained with the new approach and the approach of Tyrach.*
Fig. 4.15 b) Flow curves of variably concentrated debris flow material mixtures (portions \(d \leq 7\) mm and \(d \leq 10\) mm). Comparison of results obtained with the new approach and the approach of Tyrach.

Flow curves using different spheres:

With reference to the mixtures of the portion \(d \leq 1\) mm, identical flow curves are obtained for the different sphere sizes \(D\) applying the new approach. In comparison different flow curves are obtained when using different spheres and applying the approach of Tyrach. The values of the shear stress \(\tau\) obtained with the medium sphere \((D = 12\) mm\) are 20\% lower and those obtained with the smallest sphere \((D = 8\) mm\) are 35\% lower compared to those obtained with the largest sphere \((D = 15\) mm\).
With reference to the mixtures of the portions $d \leq 5 \text{ mm}$, $d \leq 7 \text{ mm}$ and $d \leq 10 \text{ mm}$, the flow curves obtained with the different spheres and applying the new approach are not completely identical. One is inclined to say that higher values of the shear stress $\tau$ are obtained here using smaller spheres compared to larger spheres. The values of the shear stress $\tau$ obtained with the medium sphere ($D = 12 \text{ mm}$) are 10% higher and those obtained with the smallest sphere ($D = 8 \text{ mm}$) are 20% higher compared to those obtained with the largest sphere ($D = 15 \text{ mm}$) as far as the mixtures $d \leq 5 \text{ mm}$ and $d \leq 7 \text{ mm}$ are considered. For the mixtures $d \leq 10 \text{ mm}$ the corresponding values are 40% ($D = 12 \text{ mm}$), respectively 30% ($D = 8 \text{ mm}$).

The approach of Tyrach provide flow curves which are on a slightly lower level of the shear stress $\tau$ compared to the new approach. Tendencially smaller values of the shear stress $\tau$ are obtained using smaller spheres compared to larger spheres as far as the mixtures $d \leq 5 \text{ mm}$ and $d \leq 7 \text{ mm}$ are considered. For the mixtures $d \leq 10 \text{ mm}$, the values of the shear stress $\tau$ obtained with the medium sphere ($D = 12 \text{ mm}$) are 15% lower and those obtained with the smallest sphere ($D = 8 \text{ mm}$) are 10% higher compared to those obtained with the largest sphere ($D = 15 \text{ mm}$).

Transitional regime ($Re > 1$):

For the more liquid mixtures the values of the shear stress $\tau$ increase much stronger at higher shear rates in the case of the approach of Tyrach than in the case of the new approach. The corresponding data belong to the transitional regime ($Re > 1$). Thus, the stronger increase of the shear stress $\tau$ obtained with the approach of Tyrach compared to the new approach is obviously due the fact that in the approach of Tyrach the transitional regime is not considered.

The difference between the shear stress values obtained with the different spheres is tendencially increased in the transitional regime compared to the laminar flow regime. This confirms the relatively smaller accuracy of the data conversion in the transitional regime compared to the laminar flow regime.

4.5 Rheological properties of fine and large particle fluids

The flow curve data obtained with the new approach were fitted with the general Herschel-Bulkley-model function (eq. 2.6) using the Least-Squares method of Gauss. The Herschel-Bulkley model was chosen as it proved to be very appropriate for the rheological characterization of medium to highly concentrated sediment-water mixtures (Chhabra & Uhlherr 1988, Coussot 1992, Hemphill et al. 1993). The fit results for each
The obtained results can be summarized as follows:

**Influence of the sphere size $D$ on the yield stress $\tau_y$:**

Fairly identical values of the yield stress $\tau_y$ were obtained for particle fluids up to a maximum grain size of $d_{\text{max}} = 1 \text{ mm}$ using different sphere sizes $D$. By comparison, tendentially larger values of the yield stress $\tau_y$ were obtained when smaller spheres were used instead of larger ones (table 4.10). This confirms the observations already made in section 4.4 when analyzing the flow curves obtained with the different spheres. The latter tendency was more pronounced in the rather concentrated mixtures ($C_v > 0.55$). For the highly concentrated mixed suspension ($C_v = 0.683$) the $\tau_y$-value obtained by the smallest sphere ($D = 8 \text{ mm}$) was 80% larger, the one obtained by the medium sphere ($D = 12 \text{ mm}$) 35% larger compared to the value obtained by the largest sphere ($D = 15 \text{ mm}$).

<table>
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<tr>
<th>Relative yield stress</th>
<th>$d$ [mm]</th>
<th>(\leq 0.25)</th>
<th>(\leq 1)</th>
<th>(\leq 5)</th>
<th>(\leq 7)</th>
<th>(\leq 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_y (D = 12 \text{ mm}) / \tau_y (D = 15 \text{ mm})$</td>
<td>[%]</td>
<td>107</td>
<td>103</td>
<td>112</td>
<td>101</td>
<td>127</td>
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<tr>
<td>$\tau_y (D = 8 \text{ mm}) / \tau_y (D = 15 \text{ mm})$</td>
<td>[%]</td>
<td>101</td>
<td>100</td>
<td>120</td>
<td>108</td>
<td>119</td>
</tr>
</tbody>
</table>

*Table 4.10 Relative values of the yield stress $\tau_y$ depending on the sphere size $D$ obtained for the debris flow material mixtures of the different portions $d$.***

**Trend of the Herschel-Bulkley index $n$:**

The Herschel-Bulkley index $n$ tends to increase for larger values of $d_{\text{max}}$ in the fluid (table 4.11). While for the fine particle fluids ($d_{\text{max}} = 0.25 \text{ mm}$) it is roughly $n = 0.4$, the value increases up to $n = 0.9$ for the large particle fluids ($d_{\text{max}} = 10 \text{ mm}$). This tendency found in both the debris flow material mixtures and the clay-dispersions/suspensions is in contrast to Coussot (1992). He analyzed the material of viscous debris flows of different catchment areas and found the Herschel-Bulkley-index to be approximately $n = 0.33$ for the mixtures containing fine particles ($d \leq 0.04 \text{ mm}$) as well as for the mixtures containing both fine and larger particles ($d \leq 25 \text{ mm}$). In contrast to this study, Coussot used a parallel plate (PPS) and a cone and plate system (CPS) for the fine material and the CCS large scale rheometer for the large particle material (Coussot & Piau 1995). At this point it is difficult to judge whether this difference is due to the investigated material or due to the applied rheometry. The application of the latter
system and the comparison of the obtained rheological data with the corresponding data of the BMS will give more insight (chapter 5 and 6).

<table>
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<tr>
<td>$n$ [-]</td>
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<tr>
<td>$\sigma n$ [%]</td>
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</tbody>
</table>

<table>
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<tr>
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<th>Clay-Gravel-Suspension:</th>
<th>Mixed Suspension:</th>
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</thead>
<tbody>
<tr>
<td>$d$ [mm]</td>
<td>$d$ [mm]</td>
<td>$d$ [mm]</td>
</tr>
<tr>
<td>$\leq 0.25$</td>
<td>$\leq 10$</td>
<td>$\leq 10$</td>
</tr>
<tr>
<td>$n$ [-]</td>
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<td>0.94</td>
</tr>
<tr>
<td>$\sigma n$ [%]</td>
<td>71</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 4.11 Development of the Herschel-Bulkley index $n$ within different type of particle fluids.

Some shear stress data $\tau$ obtained at higher shear rates $\dot{\gamma} \geq 50$ s$^{-1}$ ($\dot{\Omega} \geq 2.25$ rps) indicate a rather strong stress increase (Appendix E, figures E.1 to E.10). Although no clear dependence and explanation could be found, the phenomenon preferably occurred in the less concentrated mixtures and by the use of the smaller spheres.

4.6 Discussion

4.6.1 Boundary effects in the BMS

Particle settling:

The measuring sphere drags through the sample fluid roughly at medium container depth. Even though the sample fluid was stirred before every experiment, settling of particles takes place within measuring time. Whereas it is not expected that the settling particles influence the torque measurements, one must be aware of an inverse gradient of sediment concentration over the entire container depth (fig. 4.16). Such a gradient depends on the overall sediment concentration in the fluid sample, grain size distribution, grain shapes and physico-chemical properties of the finest particles.\(^{14}\) With

\(^{14}\) The occurrence of particle settling in fluids with a bimodal grain size distribution can be estimated based on the yield stress $\tau$ of the fine material-water mixture and the grain size $d$ of the large particles (eq. 2.8).
the BMS the measurements are used to determine the rheological properties for a given sample fluid. In fluids inclined to particle settling it is not clear whether at the defined sphere depth the average fluid properties or the properties of a less concentrated fluid are measured. Settling and gradient of sediment concentration was not determined for the numerous mixtures of this study, but is kept in mind when the BMS data are compared with the data of the other applied rheometry.

![Diagram of sphere holder and container boundary](image)

Fig. 4.16  *Sediment concentration* \( C_v \) in the particle fluids investigated with the ball measuring system.

**Sphere holder and container boundary:**

Obviously not only the sphere itself but also the holder as well as the container boundary with its relatively small distances from the bottom and the wall to the sphere influences the drag flow and contribute to the total measured torque \( T \). The sphere holder is 0.6 mm thin and 3 mm long in the direction of the sphere path and the immersed length varies between \( s_{im} = 11 \) mm for the \( D = 15 \) mm sphere and \( s_{im} = 18 \) mm for the \( D = 8 \) mm sphere. The distance between the wall and the sphere is \( s_w = 17 \) mm, the distance between the bottom and the sphere is \( s_b = 22 \) mm. While the influence of this boundary system can be quantified for each sphere dragging across well defined Newtonian non particle fluids (table 4.5), some unpredictable effects remain when measuring in particle fluids and Yield Stress Fluids:

One effect in particle fluids is namely the temporary transport of a larger particle on top of the sphere while being supported backwards by the holder. As in this case the dragging body is not only the sphere and the holder but the sphere, the holder and the large particle, increased larger torque values are measured in this case. A second effect is the temporary jamming of particles between the sphere, the container boundary and/or the holder in high to highly concentrated mixtures.
Both effects are difficult to determine based on the data sets of torque $T$ and speed $\Omega$ and can be covered by the natural data fluctuation measured in large particle fluids. One possibility to detect such an effect is the comparison of the torque data of the first and the second experiment at a prescribed rotational speed. If data of both experiments cover the same data range, no such boundary effect is assumed to occur. In the case a significant discrepancy between the torque data of the first and the second experiment appears, one must assume such an effect (see example in fig. 4.4: $\Omega = 1.35$, $C_{\text{v,gr}} = 0.3$; here a large number of the torque data of experiment 1 are considerably higher than the torque data of experiment 2).

Another uncertainty is the influence of the container boundary on the Yield Stress Fluids investigated in this study. The flow field of a sphere dragged across a Yield Stress Fluid is characterized by a sheared zone around the dragged sphere and a non-sheared zone beyond the sheared zone (Beris et al. 1985, Chhabra & Uhlherr 1988). Generally, size and shape of the sheared zone depends on the sphere velocity, the rheological model parameters of the fluid as well as the confining boundary (fig. 4.17). Atapattu et al. (1995) measured size and shape of the sheared zone in a variety of experiments using different spheres dragging across Yield Stress Fluids in tubes, whereas Beaulne & Mitsoulis (1997) made numerical predictions and found a good agreement between experimental and numerical results.

Based on those results, the size of the sheared zone was estimated in the case of the present BMS using the spheres and Yield Stress Fluids of this study. It was estimated that the sheared zone must normally have spread to the container wall and bottom. For the smaller spheres ($D = 8$ and $12$ mm) dragged at low rotational speeds (sphere velocities) across the rather concentrated Yield Stress Fluids, it might be that this was not always the case. Because of the lack of appropriate technical equipment to measure the boundary of sheared and unsheared zone in the present BMS, this aspect could not be further analyzed.

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15 For very low sphere velocities unsheared polar caps and islands within the sheared zone may appear additionally (fig. 4.17 b)
The ball measuring system

Fig. 4.17  Sheared (in white) and unsheared zone (in gray) around a sphere dragging with different velocities \( v \) across a Yield Stress Fluid (\( \tau_y = 46.5 \text{ Pa, } m = 23.9 \text{ Pa s}^n, n = 0.5 \)) in a tube (ratio of tube diameter \( D_t \) and sphere size \( D = 3:1 \)). Results of numerical predictions of Beaulne & Mitsoulis 1997. a) \( v = 6 \text{ m/s} \), b) \( v = 0.001 \text{ m/s} \).

4.6.2 Reliability of data conversion

Regarding the application to Yield Stress Fluids the reliability of both the existing data conversion approach of Tyrach (2001) and the new approach is investigated and discussed here.

The yield stress criterion:

A necessity to control the reliability of the data conversion in the laminar flow regime (\( \text{Re} \leq 1 \)) for this type of fluids is to investigate whether the yield stress criterion is fulfilled or not: In a system where a sphere is dragged across a Yield Stress Fluid (YSF), the following dimensionless yield stress number \( Y \) can be formulated (Beris et al. 1985, p. 224):

\[
Y = \frac{1}{2} \frac{\tau_y \cdot A_y}{F}
\]  
(4.25)
where \( \tau \) is the yield stress, \( A_s \) the area of the sphere surface and \( F \) the externally applied force.

For laminar flow (\( \text{Re} \leq 1 \)) the following relationship was found for the BMS (eq. 4.22 and eq. 4.24):

\[
\frac{\tau}{T} = K_T = \frac{K_T'}{r \cdot (A_s + A_h)}
\]

(4.26)

Due to the fact that the sphere is dragged with the help of a holder the area of the sphere holder surface, \( A_h \), is considered in eq. 4.26 in addition to the area of the sphere surface, \( A_s \). Replacing the torque \( T \) and the radius \( r \) of the center sphere path by the force \( F \), eq. 4.26 can be rewritten as:

\[
K_T' = \frac{\tau \cdot (A_s + A_h)}{F}
\]

(4.27)

In the creeping region (\( \text{Re} \ll 1 \)) \( \tau \) may be replaced by the yield stress \( \tau \). In this case eq. 4.27 is comparable with eq. 4.25 and it can be concluded:

\[
K_T' = 2 \cdot Y
\]

(4.28)

Most important is that in the case of the critical state between motion and no motion \( K_T' \) or \( Y \) should take a specific value, independent of the sphere size and independent of the investigated Yield Stress Fluids (Chhabra & Richardson 1999, p. 212). Regarding the corresponding \( K_T' \) - values obtained in the BMS for the Yield Stress Fluids (table 4.9), \( K_T' \) roughly takes the same value for all three spheres. Given that the \( K_T' \) - values are included in the new empirical formula for the conversion of raw data into rheological data we can conclude that the new approach fulfills the yield stress criterion. By contrast this is not the case for the approach of Tyrach (2001) where the \( K_T' \) - values (table 4.6), which originate from non Yield Stress Fluids differ for the three spheres. Therefore the use of the new approach developed in this study is recommended when measured BMS data of Yield Stress Fluids are converted into rheological data.

Comparing the \( Y \)-value of the present study with the same value obtained by other authors, the BMS value of \( Y \sim 0.14 \) corresponds with the values of 0.18 ±0.06 obtained by the observation on the motion/no motion of spheres under free fall conditions and the observation of the residual force upon the cessation of the flow (Chhabra & Richardson 1999). It corresponds well with the value of 0.143 obtained by numerical predictions of the movement of a sphere across Yield Stress Fluids (Beris et al. 1985, Beaulne & Mitsoulis 1997).
Transitional flow regime (Re > 1):

No data conversion formula is provided by Tyrach for this regime but an approach was developed in this study for the different type of fluids. Concerning the particulated Yield Stress Fluids, a specific relation between the ratio \( \tau T \) and the sphere Reynolds number Re was developed for each sphere with the help of different concentrated mixtures of fine debris flow material. For the sphere \( D = 12 \text{ mm} \) this relation is challenged by data of the clay-dispersions which indicate a different course of \( \tau T \) versus Re-data. Thus future work should focus on the analysis of other particulated Yield Stress Fluids within this flow regime. So far the use of the data conversion formula derived in this study is recommended.

Fluids containing particles larger than \( d = 1 \text{ mm} \) grain size:

While for fluids containing particles up to \( d = 1 \text{ mm} \) grain size, fairly identical flow curves were obtained for the different spheres using the new data conversion approach, this is not any more the case for fluids containing particles larger than \( d = 1 \text{ mm} \) grain size. Here the smaller spheres tendentially provide flow curves on a higher shear stress level than do larger spheres. The same tendency was obtained by comparing the yield stress \( \tau_y \) after data fitting (table 4.10). Shear or yield stress difference between the different spheres was more pronounced in highly concentrated than in less concentrated mixtures. No final explanation but only some assumptions could be found for this general phenomenon.

One assumption is that the influence of the sphere holder on the measurement of the torque \( T \) is larger in large particle fluids, especially when they are concentrated, compared to fine particle fluids: In case of large particle fluids, the flow around the holder is not always well defined but might be disturbed due to temporary blockages between larger particles and the holder (fig. 4.18) as already explained in 4.6.1. The immersed part of the sphere holder \( s_{im} \) is longer for smaller spheres than for larger spheres (fig. 4.16, table 4.1). Therefore the influence of the sphere holder on the flow in large particle fluids is larger for smaller spheres than for larger spheres.

An additional consideration is the fact that the sphere holder of the smaller sphere is closer to the container wall than in case of the larger sphere (fig. 4.16, table 4.1). This fact might contribute to increase the probability for temporary blockage and jamming effects in concentrated large particle fluids and therefore relatively increase the value of the torques \( T \) in case of the smaller compared to the larger spheres.
As the assumptions for the difference in the flow curve level using different spheres and testing large particle fluids could not be proven, the flow curves obtained with the different spheres are treated equally in the further parts of this study: The flow curves obtained with the three spheres are regarded as an ensemble and the flow curves are shown in a band, respectively (chapter 6).

### 4.6.3 The system number $C$ and $C_I$

The system number $C$ (Tyrach) or $C_I$ (present study) is required for the determination of the relationship between $\dot{\gamma}$ and $\Omega$. While $C$ includes the drag resistance (eq. 4.5), $C_I$ includes the power characteristic (eq. 4.18) of the BMS for a given sphere – sphere holder – container - configuration. Both, $C_I$ and $C$, are proportional to $T/(\Omega \eta)$.

In contrast to Tyrach who used the viscosity-curve overlay technique to determine the relationship between $\dot{\gamma}$ and $\Omega$, the shear rate $\dot{\gamma}$ can be also derived based on the drag resistance considering the system number $C$: In this case eq. 4.12 is inserted into eq. 4.10 and one obtains:

$$\dot{\gamma} = \left( \frac{4}{\pi^2} \frac{T}{\Omega} \frac{1}{r \cdot D^2} \frac{1}{C} \frac{1}{m} \right)^{\frac{1}{n-1}}$$

(4.29)

Remember the following expression found for the shear rate $\dot{\gamma}$ considering the system number $C_I$ representing the power characteristic of Metzner&Otto (eq. 4.20):

$$\dot{\gamma} = \left( 4 \cdot \pi^2 \frac{T}{\Omega} \frac{r}{D^4} \frac{1}{C_I} \frac{1}{m} \right)^{\frac{1}{n-1}}$$

(4.30)
Because \( \dot{\gamma} \) in eq. 4.29 and eq. 4.30 is proportional to \( T/(\Omega \cdot C_f^{(1/(n-1))}) \) and \( T/(\Omega \cdot C_t^{(1/(n-1))}) \), respectively, and both \( C_f \) and \( C_t \) are proportional to \( T/(\Omega \cdot \eta) \), it is independent whether the drag resistance path (eq. 4.29) or the power characteristic path (eq. 4.30) is considered for the derivation of the relationship between \( \dot{\gamma} \) and \( \Omega \).

Tyrach obtained slightly different values for the constant \( K_{\Omega} \) relating \( \dot{\gamma} \) and \( \Omega \) (table 4.6) considering the drag resistance path compared to the values of the present study (table 4.8) which is based on the power characteristic path. The differences are solely attributed to the facts that Tyrach applied the viscosity-curve overlay technique and used other reference fluids and reference systems compared to the present study.

To conclude, from a theoretical point of view the difference between the approach of Tyrach and the new approach is not found in the derivation of the relationship between \( \dot{\gamma} \) and \( \Omega \) (step 1 and step 2) but principally in step 3 of the parameter conversion: The conversion of measured torques \( T \) into shear stresses \( \tau \).

### 4.6.4 The BMS shear rate

After the relationship between \( \dot{\gamma} \) and \( \Omega \) was found and the shear rates calculated with the relating empirical constant \( K_{\Omega} \) for any new fluid, the calculated shear rates are representative shear rates. This is generally because the relationship between sphere drag flow and shear flow is always empirical. Two explications (A and B) will help for understanding. Briefly in advance and very important, the final result - the flow curve parameters (\( \tau_s, m, n \)) - are not affected by this actual fact.

**A. The constant \( K_{\Omega} \) is a simplified concept:**

The empirical constant \( K_{\Omega} \) is a best fit of a data scatter of shear rates \( \dot{\gamma} \) against rotational speeds \( \Omega \). In the present study the fit was obtained in a limited measuring range (\( \Omega > 0.03 \) rps, \( \text{Re} < 1 \)) for two different Power Law Fluids (CMC 1% and Guar 1%). But the empirical constant \( K_{\Omega} \) was also used for \( \text{Re} > 1 \) and particularly for \( \text{Re} > 10 \) where the few data points deviate from the linearity expressed by the constant \( K_{\Omega} \) (fig. 4.11). As it is unknown whether this deviation is true or due to elastic effects of the used CMC 1% the BMS shear rate \( \dot{\gamma} \) must be considered as a representative shear rate.

In the following the representative character of the shear rate will be shown (fig. 4.19). Not knowing whether for \( \text{Re} > 10 \) (\( \Omega \geq 2 \) rps) the deviation of the \( \dot{\gamma}/\Omega \) -data from the linearity is true or influenced by elastic effects of the CMC 1%, a three parameter curve fit (\( \dot{\gamma}/\Omega = f(\text{Re}) \)) is given in order to represent this deviation (fig. 4.19 a).
Fig. 4.19 Comparison of flow curve data of different concentrated debris flow material mixtures (d < 0.25 mm) for $\gamma/\Omega = K \Omega$ and $\gamma/\Omega = f(Re; Re > 10)$ for the sphere $D = 15$ mm: a) $\dot{\gamma} - \Omega$ - relation, b) $\tau/T$ - Re - relation, c) flow curve.
Considering this curve fit now for the conversion of measured torques $T$ into shear stresses $\tau$ for particle fluids, a different relationship between the ratio $\tau/T$ and Re results for $\text{Re} > 10$ ($\Omega \geq 2 \text{ rps}$) compared to the relationship obtained in the case $\gamma/\Omega = K_\Omega = \text{constant}$ (fig. 4.19 b). Likewise a new curve fit is found in order to express the new relationship $\tau/T = f(\text{Re})$ for the calculation of the shear stress $\tau$ at any Reynolds number Re.

As an example the effect on the flow curve data is shown when the alternative function $\gamma/\Omega = f(\text{Re}; \text{Re} > 10)$ is applied compared to the existing function $\gamma/\Omega = K_\Omega = \text{constant}$ (fig. 4.19 c)): Using the alternative function, much larger shear rate values are obtained compared to the case when the existing function is used. The values of the shear stress $\tau$ behave likewise. Because of this gap between the shear rate/shear stress - values obtained with the two functions the shear rates and shear stresses obtained in the BMS are in fact representative shear rates and representative shear stresses.

Most important is that there is no significant effect on the final flow curve when the first or the alternative function is considered, which is demonstrated by fig. 4.19 c): The data obtained with the alternative function ($\gamma/\Omega = f(\text{Re}; \text{Re} > 10)$ are on the same flow curve like the data of the first function ($\gamma/\Omega = K_\Omega = \text{constant}$).

**B. The constant $K_\Omega$ was derived based on non-Yield Stress Fluids but is applied to Yield Stress Fluids:**

The $\gamma - \Omega$ relation which mounded into the constant $K_\Omega$ was determined using Power Law Fluids and bases on the system number $C_I$ which was determined with Newtonian fluids. By contrast, the BMS is designed for the flow curve determination of particle fluids which are often Yield Stress Fluids (YSF). As the empirical relationship between $\tau$ and $T$ derived for particulated YSF differs from the $\tau - T$ - relationship derived for non-YSF, it is assumed that the $\gamma - \Omega$ - relationship must differ similarly. This assumption is supported by the fact that the flow field around the dragging sphere in a Yield Stress Fluid is principally different from the one in a non-Yield Stress Fluid as outlined in 4.6.1.

In the following a concept is introduced how to determine the $\gamma - \Omega$ - relationship for Yield Stress Fluids in order to obtain representative shear rates which might deviate less from the “true” shear rates obtained in standard measuring systems (CCS, CPS).

**A proposal for the determination of the $\gamma - \Omega$ - relationship considering the characteristics of Yield Stress Fluids:**

Step 1: Determination of the BMS system number $C_I$ (see section 4.3.3).
Step 2: Determination of the relation between $\dot{\gamma}$ and $\Omega$: Eq. 4.19 gives an expression for the viscosity $\eta$ in the BMS:

$$\eta = 4 \cdot \pi^2 \cdot \frac{T}{\Omega} \cdot \frac{r}{D^4} \cdot \frac{1}{C_1}$$

(4.31)

For a known Yield Stress Fluid which is described by a Herschel-Bulkley model function (eq. 2.6) the viscosity $\eta$ is expressed as:

$$\eta = \tau_y + m \cdot \dot{\gamma}^n$$

(4.32)

Inserting eq. 4.31 into eq. 4.32 the following equation results:

$$\tau_y + m \cdot \dot{\gamma}^n = 4 \cdot \pi^2 \cdot \frac{T}{\Omega} \cdot \frac{r}{D^4} \cdot \frac{1}{C_1} \cdot \frac{1}{\dot{\gamma}}$$

(4.33)

Based on eq. 4.33 the shear rate $\dot{\gamma}$ is found by iteration.

Practically, different Yield Stress Fluids (Herschel-Bulkley-Fluids) are measured in the BMS and in a reference system in order to determine the values of $\tau_y$, $m$, $n$, $T$ and $\Omega$ beside the known values of $C_1$, $D$ and $r$. The shear rate data $\dot{\gamma}$ is then obtained with eq. 4.33 for every specified rotational speed $\Omega$. Afterwards the shear rate data $\dot{\gamma}$ is plotted against the speed $\Omega$ and the relationship between $\dot{\gamma}$ and $\Omega$ derived accordingly.

At least two rounds of $C_1$-determination (step 1) and determination of the $\dot{\gamma}$- $\Omega$-relationship (step 2) are necessary to consider only the behaviour of the Yield Stress Fluids in the $\dot{\gamma}$- $\Omega$-relationship (table 4.12):

<table>
<thead>
<tr>
<th>Round 1:</th>
<th>Applied Fluids:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1:</td>
<td>$C_1$- determination</td>
</tr>
<tr>
<td>Step 2:</td>
<td>$\dot{\gamma}$- $\Omega$ - relationship</td>
</tr>
<tr>
<td>Newtonian Fluids (NWT)</td>
<td>Yield Stress Fluids (YSF)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Round j: (Iteration)</th>
<th>Step 1:</th>
<th>$C_1$- determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2:</td>
<td>$\dot{\gamma}$- $\Omega$ - relationship</td>
<td></td>
</tr>
<tr>
<td>Yield Stress Fluids (YSF)</td>
<td>Yield Stress Fluids (YSF)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.12 Procedure for the determination of the $\dot{\gamma}$ - $\Omega$- relationship considering the characteristics of Yield Stress Fluids.

In the first round $C_1$ can be determined only with Newtonian fluids. Hence, after the first round the shear rate $\dot{\gamma}$ is also partly dependent on the characteristics of the Newtonian fluids.

After the $\dot{\gamma}$- $\Omega$ -relationship has been derived in the first round, $C_1$ is determined based on the viscosity $\eta$ of the Yield Stress Fluids in the second round, because an estimation of $\dot{\gamma}$ and thus of $\eta$ is known after the first round. Depending on the difference between
the $\dot{\gamma} - \Omega$ - relationship derived in the first round and in the second round, more than two rounds (iterations) are necessary to derive the final $\dot{\gamma} - \Omega$ - relationship.

4.6.5 The sphere Reynolds number in non-Newtonian fluids

In 4.6.4 it was shown that the shear rates and shear stresses obtained in the BMS have representative character. Accordingly the viscosity $\eta$ is a representative viscosity in case of the non-Newtonian fluids (eq. 2.3). For the Newtonian fluids the viscosity $\eta$ is constant ($\eta = \mu = \text{const.}$) and therefore does not require the conotation “representative”.

The viscosity $\eta$ enters the sphere Reynolds number Re. Hence the sphere Reynolds number Re is “true” for the Newtonian fluids but representative for the non-Newtonian fluids.

The determination of the sphere Reynolds number in non-Newtonian fluids is an unsolved problem and has been investigated and debated for decades (Ansley & Smith 1967, Beris et al. 1985, Chhabra & Uhlherr 1988 e.g.). So far it has been most convincingly solved by the fundamental study of Atapattu et al. (1995) and the further work outlined in Chhabra & Richardson (1999): Instead of the classic Reynolds number a dimensionless dynamic parameter $Q^*$ is introduced which does not depend on the unknown viscosity $\eta$ but depends on the known\(^{16}\) model parameters $m$, $n$ and $\tau_y$ which express the flow curve of the given fluid. In the following this concept of the adaptation of the Reynolds number for non-Newtonian fluids is introduced:

For Power Law fluids the Reynolds number is expressed as $Re^*$:

$$Re^* = \frac{v^{2-n} \cdot D^n \cdot \rho}{m}$$

(4.34)

where $m$ is the consistency coefficient and $n$ the Power-Law index. For Yield Stress Fluids the yield stress $\tau_y$ must be considered additionally. In a first step a dimensionless Herschel-Bulkley-number (Bingham number) $B^*$ is therefore defined:

$$B^* = \frac{\tau_y}{m \cdot \left( \frac{v}{D} \right)^n}$$

(4.35)

The combination of $Re^*$ and $B^*$ yields a dynamic parameter $Q^*$:

$$Q^* = \frac{Re^*}{1 + B^*}$$

(4.36)

\(^{16}\) The model parameters $m$, $n$ (and $\tau_y$) are obtained with the help of any flow curve measuring system and fitting an appropriate model function (Power Law, Herschel-Bulkley) to the measured flow curve data.
Note that $Q^*$ is a general dimensionless flow parameter for Newtonian and non-Newtonian fluids. For a Power Law Fluid (PLF) $Q^*$ reduces to $Re^*$ and for a Newtonian fluid (NWT) $Q^*$ reduces to the classic Reynolds number $Re$ (eq. 4.2).

The advantage of the definition of the dynamic parameter is that $Q^*$ can be determined for a sphere moving across a known non-Newtonian fluid, whereas with the standard sphere Reynolds number $Re$ this is impossible, as long as no conversion between the sphere velocity (rotational speed) and drag force (torque) into the rheological parameters shear rate and shear stress was derived.

In the next step a comparison is made between the representative Reynolds number $Re$ obtained by the use of the new data conversion approach (4.3.3)\textsuperscript{17} and the dynamic parameter $Q^*$ introduced here. The comparison includes the non-Newtonian fluids measured in both, in the BMS and in a reference system (fig. 4.20).

Over the entire measuring range the Reynolds number $Re$ obtained in the present study correlates well with the dynamic parameter $Q^*$ proposed by Atapattu et al. (1995). However depending on the type of fluid and the used sphere some deviation is observed. The deviation is quantified by the ratio $Re/Q^*$ in table 4.13. With reference to the Polymers 1 % (PLF), the deviation is most pronounced (+25 %) in the case of the largest sphere ($D = 15$ mm) but negligible for the smallest sphere ($D = 8$ mm). With reference to the particulated fluids (YSF), which are the debris flow material mixtures with $d \leq 0.25$ mm, the deviation ranges from +55 % ($D = 15$ mm) to -10 % ($D = 8$ mm).

<table>
<thead>
<tr>
<th>Type of fluid</th>
<th>$D = 15$ mm</th>
<th>$D = 12$ mm</th>
<th>$D = 8$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polymers 1% (PLF)</td>
<td>1.25 ± 0.13</td>
<td>1.17 ± 0.10</td>
<td>0.96 ± 0.08</td>
</tr>
<tr>
<td>Particles (YSF)</td>
<td>1.55 ± 0.10</td>
<td>1.35 ± 0.09</td>
<td>0.90 ± 0.06</td>
</tr>
</tbody>
</table>

Table 4.13 Ratio $Re/Q^*$ for the different non-Newtonian fluids and sphere sizes $D$.

To conclude, the deviation between the Reynolds number $Re$ obtained in this study and the dynamic parameter $Q^*$ is always less than 100 % which is small. At this point it is not possible to decide whether the Reynolds number $Re$ obtained in this study or the dynamic parameter $Q^*$ more precisely represents the true Reynolds number for the investigated non-Newtonian fluids. Only an additional derivation of the $\dot{\gamma} - \Omega$ relationship as presented before (section 4.6.3) could answer whether $Re$ of the present study or $Q^*$ more accurately represents the true Reynolds number.

\textsuperscript{17} $Re = 2 \cdot \pi \cdot \frac{Q \cdot r \cdot D \cdot \rho}{\eta}$ (eq. 4.2); $\eta = \frac{\tau}{\dot{\gamma}}$ (eq. 2.3); $\dot{\gamma} = f(\Omega)$, $\tau = f(T)$ (table 4.8, 4.9)
Fig. 4.20  Comparison between the Reynolds number $Re$ obtained with the approach of the present study and the dynamic Parameter $Q^*$ defined by Atapattu et al. (1995). Particles = debris flow material mixtures ($d \leq 0.25$ mm).
4.6.6 The BMS standard drag curve

In the above section no final answer was found whether the sphere Reynolds number $Re$ obtained with the new BMS data conversion approach (4.3.3) or the dynamic parameter $Q^*$ is more representative for the true Reynolds number of the non-Newtonian fluids. In order to show the BMS standard drag curve, it was decided to consider the Reynolds number $Re$ obtained with the new BMS data conversion approach\(^{18}\). The drag coefficient $c_D$ is calculated using eq. 4.3.

Fig. 4.21 shows the standard drag curve for the three different spheres and the different type of fluids. As expected, the BMS standard drag curve exhibits a linear behaviour of drag coefficient $c_D$ and Reynolds number $Re$ for $Re < 1$. This linear behaviour is expressed by the multiplication of $Re$ and $c_D$ respectively the system number $C$ (eq. 4.5). In analogy to the relationship of $\pi T$ and $Re$ obtained in section 4.3.3, the value of $C$ varies depending on the fluid type and depending on the sphere size $D$, where the $C$ expresses the sphere - sphere holder – container configuration (table 4.14).

<table>
<thead>
<tr>
<th>Type of fluid</th>
<th>$D = 15\text{mm}$</th>
<th>$D = 12\text{mm}$</th>
<th>$D = 8\text{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon Oils (NWT)</td>
<td>44.9</td>
<td>48.8</td>
<td>52.6</td>
</tr>
<tr>
<td>Polymers 1% (PLF)</td>
<td>44.9</td>
<td>48.8</td>
<td>52.6</td>
</tr>
<tr>
<td>Particles (YSF)</td>
<td>52.9</td>
<td>48.4</td>
<td>42.7</td>
</tr>
</tbody>
</table>

Table 4.14 System number $C = Re \cdot c_D$ for the different group of fluids and the different spheres $D$ (expressing the sphere – sphere holder – container configurations) in the laminar flow regime.

In the transitional regime ($Re > 1$) the behaviour of $c_D$ and $Re$ is comparable with the behaviour of $\pi T$ and $Re$ presented in 4.3.3: The non particulated NWT and PLF tend to indicate a larger value of the drag coefficient $c_D$ than the particulated YSF. This might lead to the conclusion that (1) YSF and/or (2) particles dampen the onset of turbulence. (1) corresponds with Chhabra and Uhlherr (1988) who found that the beginning of the transitional regime is retarded for spheres moving across Bingham fluids.

\[ Re = 2 \cdot \pi \cdot \frac{\Omega \cdot r \cdot D \cdot \rho}{\eta} \quad (eq. 4.2); \quad \eta = \frac{\tau}{\dot{\gamma}} \quad (eq. 2.3); \quad \dot{\gamma} = f(\Omega), \tau = f(T) \quad (table 4.8, 4.9) \]
Fig. 4.21  BMS standard drag curve for Newtonian and non-Newtonian fluids. Particles = debris flow material mixtures (d ≤ 0.25 mm).
5 Application of other rheometry for large particle fluids

5.1 CCS large scale rheometer

5.1.1 System overview

In order to determine the flow curve of large particle fluids $d \leq 10$ mm and $d \leq 25$ mm the original CCS large scale rheometer of Coussot & Piau (1995) was used (fig. 5.1, 5.2). As the apparatus is explained in detail in the mentioned publication, the apparatus characteristics are only briefly summarized here: The rheometer bases on a concentric cylinder system with $R_o = 585$ mm, $R_i = 385$ mm, $H = 200$ mm, $L = 720$ mm for which a sample volume $V_{\text{sample}} = 500$ liter is required. The rotating inner cylinder is driven by a hydraulic motor which is served by an ordinary combustion engine. Rotational speeds $\omega$ are measured via a tachometer on the cover of the inner cylinder.

![Geometric scheme of the CCS large scale rheometer of Coussot & Piau.](image)

Figure 5.1 Geometric scheme of the CCS large scale rheometer of Coussot & Piau.
Torques $T$ are measured with the help of strain gauges fixed between the inner cylinder and the crankshaft. For the tests carried out in this study a completely new process controller for the electronic transformation of the strains into torques $T$ was used as the existing one was in bad condition.

In order to prevent the fluid from slippage, steel bars - 35 mm in depth in radial direction – are fixed on the outer cylinder. The inner cylinder surface was covered with a mesh of expanded metal (net of rhomboids of 20 mm side length and 6 mm thickness).

Conducting one flow curve experiment required 240 person minutes: 30 minutes to measure the apparatus resistance (2 persons), 15 minutes to fill the fluid into the apparatus (2 persons), 45 minutes for the experiments (2 persons) and 60 minutes to empty and clean the apparatus (1 person).

5.1.2 Measurements and observations

Apparatus resistance torque $T_0$:

Due to friction between rubber seal and the platform of the outer cylinder, an apparatus resistance torque $T_0$ occurs which must be substracted from the total torque $T_{tot}$ measured during flow curve experiments. The $T_0$-values were determined always before a flow curve experiment was conducted, respectively after the entire apparatus had been cleaned thoroughly from the remainings of a previous flow curve experiment (the inner cylinder was lifted and sediment deposits on the platform as well as near the crankshaft were removed). The development of $T_0$-values for the different apparatus resistance experiments are shown in fig. 5.3.
Large and variable values of approximately $100 < T_0 < 200 \text{ Nm}$ were noted in the different experiments of the present study. A tendency of $T_0$-dependence on the rotational speed $\Omega$ was observed in some single experiments but could not be derived for the sum of all experiments.

This general picture contrasts to Coussot&Piau (1995) who notes a very small and almost stable value of $T_0 = 18 \pm 2 \text{ Nm}$ for the entire range of rotational speeds $\Omega$. Reasons for this deviation might be found in the condition of the apparatus after having been used in several projects after the construction of the apparatus in 1989/90.

Due to the large variation obtained in the $T_0$-experiments, and due to the difficulties to estimate the real apparatus resistance at a given speed, a band of $100 < T_0 < 200 \text{ Nm}$ was considered for the following flow curve experiments.

**Measuring large particle fluids:**

**Torque measurements $T_{tot}$:**

The fluid was filled in the CCS large scale rheometer while the inner cylinder rotated at a low rotational speed of $\Omega = 0.005 \text{ rps}$. Afterwards experiments were started at $\Omega = 0.005 \text{ rps}$. By acceleration of the motor the speed was increased in 8 to 10 steps roughly up to $\Omega = 0.2 \text{ rps}$ before the speed was decreased in 3 to 4 steps back to roughly the same starting speed. The total torque $T_{tot}$ was measured at each step of the rotational speed $\Omega$ for 120 s with a frequency of 100 Hz.
Investigating different large particle fluids of $d \leq 10 \text{ mm}$ and $d \leq 25 \text{ mm}$, the measurement curves exhibited the following principal behaviour (fig. 5.4):

a) Dependence of $T_{tot}$ from $\Omega$.

b) Independence of $T_{tot}$ from $\Omega$.

c) Hysteresis between up and down curve.

In case c) the hysteresis must be due to significant changes within the fluid (particle settling and/or particle migration towards the inner or outer cylinder) or within the apparatus (apparatus resistance $T_0$) during measuring. Based on continuous torque measurements it was observed that the average total torque $T_{tot}$ increased during the measuring time at a given rotational speed $\Omega$ (fig. 5.5). However it is not possible to analyze to what extent the increase of $T_{tot}$ is due to changes within the fluid and to what extent it is due to changes of the apparatus resistance $T_0$. Therefore no methods to quantify eventual particle settling and migration were applied here.

Fig. 5.5 also shows the oscillations of the measured torque $T_{tot}$ which are due to eccentric rotation of the inner cylinder. The number of oscillations correspond to the number of cylinder revolutions at a given speed $\Omega$. 

![Figure 5.4](image.png)

*Figure 5.4* Development of measured torque $T_{tot}$ with rotational speed $\Omega$ for the CCS large scale rheometer of Coussot&Piau: a) Dependence of $T_{tot}$ from $\Omega$. b) Independence of $T_{tot}$ from $\Omega$. c) Hysteresis between up and down curve.
To conclude, only the measurement curves of type a) enabled the determination of the flow curve. Due to independence of $T_{\text{tot}}$ from $\Omega$ in case b) and due to the large hysteresis in case c) the range of shear stresses $\tau$ but not real flow curves were determined for these specific mixtures.

Before the flow curve can be determined, the apparatus resistance torque $T_0$ must be subtracted from the measured total torque $T_{\text{tot}}$ in order to obtain the effective torques $T$ at the different specified speeds $\Omega$.

**Sediment concentration $C_v$ a priori and a posteriori:**

The sediment concentrations $C_v$ of the investigated mixtures correspond to the concentrations $C_v$ a priori when the mixture was prepared. Loss of water due to evaporation during mixture preparation was balanced out based on periodic evaporation measurements (weight and volume measurements of the container) in the hall where the mixture was prepared. This was not possible outdoor where the measurements with the CCS large scale rheometer were executed. High outdoor temperatures of 30° to 35° and inevitable heating of the apparatus by the sun led to a loss of water in some experiments.

---

19 Debris flow material mixtures ($d \leq 10$ mm, $C_v = 0.642$) and Mixed Suspension ($d \leq 10$ mm, $C_v = 0.683$)

20 Debris flow material mixtures ($d < 25$ mm, $C_v = 0.673$; $d \leq 25$ mm, $C_v = 0.658$; $d < 25$ mm, $C_v = 0.643$)
that could not be quantified and balanced out during measuring. After the mixture had
been filled back in the preparation container and stored in the hall, the sediment
concentration \( C_v \) a posteriori was determined again. Table 5.1 shows the values of \( C_v \) a
priori and a posteriori for the different investigated mixtures. Whereas in most mixtures
the difference remained negligible, only in one mixture a considerable difference larger
than \( \Delta C_v = 0.1 \) resulted.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( C_v ) a priori</th>
<th>( C_v ) a posteriori</th>
<th>( \Delta C_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debris flow material mixture, ( d \leq 10 \text{ mm} )</td>
<td>0.642</td>
<td>0.646</td>
<td>0.04</td>
</tr>
<tr>
<td>Debris flow material mixture, ( d \leq 10 \text{ mm} )</td>
<td>0.595</td>
<td>0.602</td>
<td>0.07</td>
</tr>
<tr>
<td>Debris flow material mixture, ( d \leq 25 \text{ mm} )</td>
<td>0.673</td>
<td>0.673</td>
<td>0</td>
</tr>
<tr>
<td>Debris flow material mixture, ( d \leq 25 \text{ mm} )</td>
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<td>0.661</td>
<td>0.03</td>
</tr>
<tr>
<td>Debris flow material mixture, ( d \leq 25 \text{ mm} )</td>
<td>0.643</td>
<td>0.658</td>
<td>0.15</td>
</tr>
<tr>
<td>Mixed Suspension, ( d \leq 25 \text{ mm} )</td>
<td>0.683</td>
<td>0.683</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 5.1 \( C_v \) a priori and \( C_v \) a posteriori for the different large particle fluids investigated with the CCS large scale rheometer.*

5.1.3 Flow curve determination

*Theory:*

The CCS large scale rheometer of Coussot&Piau (1995) is a classic wide gap system.
This means that the velocity distribution within the large gap of \( H = 200 \text{ mm} \) is not
linear (fig. 5.6).

![Visualization of the velocity distribution in the experiment of the debris flow material mixture (d ≤ 25 mm, \( C_v = 0.643 \)) at \( \Omega = 0.052 \text{ rps} \) with the help of confetti. Left: 5 s after confetti were put in line. Right: 33 s after confetti were put in line.](image)
Thus the shear rate $\dot{\gamma}$ is different at every radial position within the gap (fig. 5.7). Further, when measuring Yield Stress Fluids (YSF), often not the entire but only part of the mixture within the gap is sheared (fig. 5.2, 5.7).

![Diagram of shear flow of a Yield Stress Fluid (YSF) in a wide gap concentric cylinder system for the case $R_i < r_{crit} < R_o$.](image)

**Figure 5.7** Shear flow of a Yield Stress Fluid (YSF) in a wide gap concentric cylinder system for the case $R_i < r_{crit} < R_o$.

Theoretically the flow curve could be determined simply based on the velocity distribution visualized on the flow surface by calculating the shear rate $\dot{\gamma}$ and the shear stress $\tau$ at distinctive radial positions. As, due to possible particle settling and migration, the average velocity distribution over the entire cylinder length $L$ is assumed to be different to the one visualized on the flow surface the method proposed by Nguyen & Boger (1987) is applied here.

Investigating Yield Stress Fluids (YSF) it must be examined in a first step, whether the fluid is sheared within (1) the entire gap $H$ or (2) only in a zone that extends from the radius of the inner cylinder $R_i$ to a critical radius $r_{crit}$, where the shear stress $\tau$ is equal to the yield stress $T_y$ (fig. 5.7). Per definition it is:

$$T(\Omega) = \tau_i \cdot 2 \cdot \pi \cdot L \cdot R_i^2 = \tau_o \cdot 2 \cdot \pi \cdot L \cdot R_o^2 = \tau_y \cdot 2 \cdot \pi \cdot L \cdot r_{crit}^2$$

(5.1)

Based on eq. 5.1 the critical radius $r_{crit}$ can be calculated as:
5 Application of other rheometry for large particle fluids

If \( r_{\text{crit}} > R_o \), the shear zone spreads over the entire gap. If \( R_i < r_{\text{crit}} < R_o \) the shear zone only spreads to the critical radius \( r_{\text{crit}} \). Application of eq. 5.2 requires knowledge of the value of the yield stress \( \tau \). Based on the \( \tau \) - data obtained with other rheometric systems, \( R_i < r_{\text{crit}} < R_o \) resulted for any investigated \( T(\Omega) \) considering only the two mixtures which showed a clear dependence of \( \Omega \)- and \( T \)-data\(^{21} \).

The **second step** is the determination of the shear rate \( \dot{\gamma} \) at the periphery of the inner cylinder.

Considering the basic equation for concentric cylinder systems, and for the case \( R_i < r_{\text{crit}} < R_o \) the shear rate \( \dot{\gamma} \) is derived as follows (Appendix F):

\[
\dot{\gamma}_i = 2 \cdot \tau_i \cdot \frac{d\omega}{d\tau_i} \tag{5.3}
\]

where \( \omega = (2 \cdot \pi) \cdot \Omega \) is the angular velocity and \( \tau_i \) is the shear stress at the inner cylinder which is calculated with eq. 5.1.

Usually \( \tau \) - and \( \omega \)-data are scattered when particle fluids are investigated in a wide gap system. Therefore a method must be applied to determine the gradient \( d\tau/d\tau_i \) at each specified \( \omega \). Borgia&Spera (1990) recommend a least square regression for the scattered \( \tau \) - and \( \omega \)-data including an error analysis for the calculated \( \dot{\gamma} \)-data, whereas Chhabra&Richardson (1999) recommend a graphical solution. Here the least square regression method of Borgia&Spera was applied. Accordingly the regression should be performed with the following function:

\[
\tau_i = a_0 + \sum_{j=1}^{l} a_j \cdot \omega^j \tag{5.4}
\]

where \( a_0 \) and \( a_j \) are coefficients to be determined during regression. Similar to earlier application of formula 5.4 (Borgia&Spera 1990, Major&Pierson 1992), \( j \) was set = 2 in the present study (fig. 5.8).

\(^{21}\) Debris flow material mixtures \((d \leq 10 \text{ mm, } C_v = 0.642)\) and Mixed Suspension \((d \leq 10 \text{ mm, } C_v = 0.683)\)
Results:

For the debris flow material mixtures \((d \leq 10 \text{ mm}, C_v = 0.642)\) and the mixed suspension \((d \leq 10 \text{ mm}, C_v = 0.683)\) the flow curve was determined. Due to the large uncertainty of the value of the apparatus resistance \(T_0\) \((100 - 200 \text{ Nm})\), the flow curve is represented in a band.

For all other mixtures with no clear dependence of \(T_1\) and \(\omega_0\)-data, the flow curve determination was not considered useful. In this case a range of the shear stress \(\tau\) is indicated: The bottom line of the \(\tau\)-range corresponds with the minimum \(T\)-value measured in the experiments, considering the larger value of the apparatus resistance \(T_0\) \((200 \text{ Nm})\). The upper line corresponds with the maximum \(T\)-value measured in the experiments, considering the smaller value of the apparatus resistance \(T_0\) \((100 \text{ Nm})\).

The flow curve band and the shear stress range are shown in fig. 5.9. The flow curve could be roughly determined in the range of shear rate \(1 \leq \dot{\gamma} \leq 100 \text{ s}^{-1}\).

As expected, the shear stress range is usually much larger than the flow curve band, due to the cumulated uncertainty of the torque resistance \(T_0\) and the hysteresis measurements.

---

\[ \tau = a_0 + a_1 \cdot \omega + a_2 \cdot \omega^{0.5}; \quad a_0 = 373 \text{ Pa}, a_1 = -1.783 \text{ Pa s}, a_2 = 2.52 \text{ Pa s}^{0.5} \]

\[ \]
The Herschel-Bulkley model (eq. 2.6) was fitted to the flow curve data using the least square regression of Gauss. In most cases either no final solution was found, or negative values of the Herschel-Bulkley viscosity parameter $m$ as well as of the Herschel-Bulkley index $n$ were obtained for the best fit. This is in contrast to Coussot & Piau (1995) who found the Herschel-Bulkley index $n$ to be around 0.33 for the flow curve data of such type of mixtures. Looking for a solution and avoiding negative parameter values, an alternative data fit was performed where the index $n$ was predefined as $n = 0.33$ (table 5.2). As shown in fig. 5.10, the fit curve tendentially overestimates the shear stresses at low and high shear rates ($\dot{\gamma} < 1-3$ s$^{-1}$, $\dot{\gamma} > 10-30$ s$^{-1}$), but slightly underestimates them at medium shear rates ($1-3 < \dot{\gamma} < 10-30$ s$^{-1}$).

Figure 5.9  Range of shear stresses $\tau$ and band of flow curves for the different mixtures investigated with the CCS large scale rheometer.
Mixture | Herschel-Bulkley-fit: $\tau$  | $m$  | $n$  | $R$ (correl.)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[Pa]</td>
<td>[Pa s$^n$]</td>
<td>[-]</td>
<td>[_]</td>
</tr>
<tr>
<td>A</td>
<td>354</td>
<td>64</td>
<td>0.33</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>59</td>
<td>0.33</td>
<td>0.91</td>
</tr>
<tr>
<td>B</td>
<td>445</td>
<td>112</td>
<td>0.33</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>591</td>
<td>121</td>
<td>0.33</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Table 5.2 Results of Herschel-Bulkley-fit (eq. 2.6) with a predefined index $n = 0.33$ for the flow curve data of (A) debris flow material mixture ($d \leq 10$ mm, $C_v = 0.642$) and (B) mixed suspension ($d \leq 10$ mm, $C_v = 0.683$). Upper row: Results for $\tau(T_0 = 200$ Nm). Lower row: Results for $\tau(T_0 = 100$ Nm).

Figure 5.10 Flow curve data and Herschel-Bulkley-fit curve for debris flow material mixture and mixed suspension.

To summarize, the CCS-large scale rheometer did not allow an accurate determination of the flow curve in the present study. The main reasons are the unknown and variable apparatus resistance $T_0$ as well as the unknown extent of particle settling and migration during a flow curve experiment. Further, the chosen measuring procedure (120 s measuring time for each speed $\Omega$) was not optimized. For rotational speeds $\Omega$, where measurements are done for several revolutions of the inner cylinder, it might possibly be better to reduce the measuring time to the time required for one cylinder revolution, in order to keep particle settling and migration as limited as possible.
5.2 BML Viscometer

5.2.1 System overview

The BML (Building Material Learning) viscometer was developed by Wallevik & Gjorv (1990) at the Norwegian Building Material Learning Department. It is a commercial instrument for testing the rheological properties of concrete. Basing on the principles of a concentric cylinder system, the rotating outer cylinder is a conventional hollow cylinder with longitudinal ribs in order to avoid wall slip. The inner cylinder is a tooth-ring shaped tool where the central part of the inner cylinder tool \( L = 150 \text{ mm} \) is connected with a load cell in order to measure the required torques \( T \) (fig. 5.11 and 5.12). The upper and bottom part of the inner cylinder tool are fixed and thus separated from this central part. This configuration enables to measure the rheological properties in an average depth, while possible influences induced by the bottom boundary are absorbed by the fixed bottom part of the inner cylinder tool. But it is not clear whether within this central zone, due to a possible gradient of sediment concentration over the cylinder depth induced by particle settling, the average fluid properties are measured.

With the BML viscometer large particle fluids up to \( d_{\text{max}} = 32 \text{ mm} \) grain size can be investigated. Depending on \( d_{\text{max}} \) smaller or larger outer cylinders are usually used. For the investigation of the debris flow material mixtures with \( d_{\text{max}} = 10 \text{ mm} \) and \( d_{\text{max}} = 25 \text{ mm} \) an outer cylinder with a radius \( R_o = 145 \text{ mm} \) was used. Given the radius \( R_i = 100 \text{ mm} \) of the inner cylinder tool, a gap width \( H = 45 \text{ mm} \) results. Use of this specific outer cylinder required a sample volume of 20 l. In order to investigate the influence of the gap width \( H \), a smaller cylinder with radius \( R_o = 125 \text{ mm} \) and gap width \( H = 25 \text{ mm} \) was additionally tested for one given fluid \((d_{\text{max}} = 10 \text{ mm})\) which required a sample volume of 16 l.

The BML viscometer measuring procedure is fully automated: Once the start button on the apparatus is pressed the inner tool penetrates the fluid in the outer cylinder. By stepwise increasing \( \Omega \), the outer cylinder is rotated at the following rotational speeds \( \Omega \): \( 0.09 \rightarrow 0.15 \rightarrow 0.21 \rightarrow 0.28 \rightarrow 0.34 \rightarrow 0.40 \rightarrow 0.46 \text{ rps} \). Then the speed is decreased and a control measurement is done at \( \Omega = 0.31 \text{ rps} \). Each \( \Omega \)-step lasts 6 s. While the first 2 s are used to adjust the speed \( \Omega \) the torque \( T \) is measured in the last 4 s. A computer is used for data sampling but only the average \( T \)-value is provided for each prescribed \( \Omega \).
Figure 5.11 Geometric scheme of the BML viscometer of Wallevik & Gjørv.
5.2.2 Measurements and observations

Different debris flow material mixtures were investigated with the BML viscometer in the present study. Two minutes after the stirred up mixture was poured into the outer cylinder the measurements were started. Each mixture was tested twice, some were tested three or four times. After the first experiment was conducted the specific mixture was at rest for 1 min and not stirred again. The second experiment was then conducted. The same procedure was followed for an eventual third and fourth experiment.

The evolution of the torque $T$ against rotational speed $\Omega$ is shown in fig. 5.13 for the different investigated mixtures. No significant difference in the $T$-values was found between the different experiments for the given mixtures. Some slight trend was observed that the following experiments yielded slightly higher $T$-values than the experiment executed earlier. On average the increase was 2% but remained always
smaller than 5%. In the higher concentrated mixtures the increase was slightly larger than in the lower concentrated mixtures. Based on these observations it can be concluded that particle settling and migration remained a negligible process in the experiments performed with the BML viscometer.

For the debris flow material mixture \(d \leq 10 \text{ mm}, C_v = 0.595\) also a smaller outer cylinder \((H = 25 \text{ mm})\) was used beside the standard cylinder \((H = 45 \text{ mm})\). Using the smaller cylinder, \(T\)-values of 95% relative to the \(T\)-values obtained with the standard cylinder were measured. Thus the gap width \(H\) proved not to influence the torque measurements significantly for the specific mixture.

![Graph showing T-values obtained in the BML viscometer at specified rotational speeds \(\Omega\). Results of different concentrated and coarsed debris flow material mixtures: a) \(d \leq 10 \text{ mm}\), b) \(d \leq 25 \text{ mm}\).](image-url)
As far as the lowest concentrated mixtures are concerned \((d \leq 10 \text{ mm}, \ C_v = 0.551\) and \(d \leq 25 \text{ mm}, \ C_v = 0.588\), the torque \(T\) did not exhibit any significant increase by the increase of the rotational speed \(\Omega\). Most probably this is due to the limited accuracy for \(T\) – measurements in the region of 0.5 Nm, considering that the BML viscometer is designed for \(T\) – measurements between 0.5 and 66 Nm.

### 5.2.3 Flow curve determination

**Theory:**

Wallevik & Gjorv (1990) consider the assembly of the tooth-ring shaped tool and the fluid between the teeth as an inner cylinder when immersed in the test fluid. With reference to the above results this assumption is justified as it was found that particle migration, and thus exchange of material between the inner space of the tooth-ring shaped tool and the outer space, is negligible. It can be therefore assumed that the shear zone begins at the periphery of the inner cylinder \((r = R_i)\) and spreads outwards in radial direction (Wallevik 1998). Hence the BML viscometer can be regarded as a wide gap concentric cylinder system.

Therefore the same procedure for the shear rate determination as already applied for the CCS large scale rheometer of Coussot & Piau was also applied here (5.1.3):

**First Step:** Application of eq 5.2. Based on the yield stress \(\tau\) estimated a priori and determined a posteriori, it was found that the shear zone did not spread over the entire gap but always spread to some critical radius \(r_{\text{crit}}\) for any investigated mixture and any applied rotational speed \(\Omega\).

**Second Step:** The shear rate \(\dot{\gamma}\) was therefore determined using eq. 5.3 by applying eq. 5.4 for the analysis of the the gradient \(d\sigma/d\dot{\gamma}\).

**Results:**

The calculated flow curve data are shown in fig. 5.14 for the different coarsed and concentrated debris flow material mixtures \((d \leq 10 \text{ mm} \text{ and } d \leq 25 \text{ mm})\).

For the less concentrated mixtures no useful flow curve data were obtained: The flow curve data were either (1) obtained in a too small range of shear rates \(\dot{\gamma}\) or (2) did not indicate a clear rheological behaviour in the given range of shear rates \(\dot{\gamma}\).

---

\(^{23}\) Based on the assumption that the test fluids are Bingham fluids the software of the BML viscometer calculates and outputs the yield stress \(\tau\), and the Bingham viscosity \(\mu_B\) using the Reiner-Rivling equations for concentric cylinder systems (Wallevik 1998). These parameters were not used in the present study, because the mixtures of the present study did not show any Bingham like behaviour.
Figure 5.14  Flow curve data, Herschel-Bulkley fit curve as well as yield stress $\tau_y$ applying eq. 5.5 for the BML viscometer. Results of different concentrated and coarsed debris flow material mixtures: a) $d \leq 10\ mm$, b) $d \leq 25\ mm$.

For the higher concentrated mixtures useful flow curve data were obtained in the range of medium shear rates $10 < \dot{\gamma} < 50\ (80)\ s^{-1}$. By contrast no flow curve data were obtained at smaller shear rates which renders a precise determination of the yield stress $\tau_y$ difficult. In order to determine the yield stress $\tau_y$ more precisely, measurements must be done at smaller shear rates or rotational speeds, respectively. However, due to the fully automated measuring procedure the speed can only be altered with specific technical adaptations.

In order to determine the yield stress $\tau_y$ the Herschel-Bulkley model was fitted to the flow curve data (fig. 5.14, table 5.3). The determination of the yield stress $\tau_y$ can not be
determined precisely with the latter fit due to the lack of data at low shear rates. Therefore the alternative method described below was considered additionally.

**Alternative method for the determination of the yield stress \( \tau_y \):**

For the determination of the shear rate \( \dot{\gamma} \) according to the concept of Nguyen & Boger (1987), data of the shear stress at the inner cylinder \( \tau_i \) are plotted against data of angular velocity \( \omega_i \) as outlined in 5.1.3. Then a regression is performed based on the non linear function of eq. 5.4. While \( a_j \) are coefficients which are dependent on both, the shear stress \( \tau_i \) and the angular velocity \( \omega_i \), the coefficient \( a_0 \) is independent on \( \omega \) but solely dependent on \( \tau \). It can be thus concluded that (see also Major & Pierson 1992):

\[
\tau_y \equiv a_0
\] (5.5)

From both, eq. 5.5 and the Herschel-Bulkley-fit, two values for the yield stress \( \tau_y \) were obtained for some mixtures. Together both values indicate the data range of the yield stress expected with the BML viscometer (table 5.3).

<table>
<thead>
<tr>
<th>Material portion</th>
<th>Eq. 5.5: ( C_v )</th>
<th>( \tau_y )</th>
<th>Herschel-Bulkley-fit:</th>
<th>( \tau_y )</th>
<th>m</th>
<th>n</th>
<th>R (correl.)</th>
<th>Data points</th>
</tr>
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<td></td>
<td>( \text{[-]} )</td>
<td>( \text{[Pa]} )</td>
<td>( \text{[Pa]} )</td>
<td>( \text{[Pa s}^n \text{]} )</td>
<td>[\text{-}]</td>
<td>[\text{-}]</td>
<td>[\text{-}]</td>
<td>[\text{N}^\circ]</td>
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<td>0.990</td>
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<td>1.6</td>
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<td>0.979</td>
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<td></td>
<td>0.551</td>
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<td></td>
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<tr>
<td>( d \leq 25 \text{ mm} )</td>
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</tbody>
</table>

**Table 5.3** Yield stress \( \tau_y \) obtained with eq. 5.5 and parameter values of Herschel-Bulkley-fit for the different debris flow material mixtures investigated with the BML viscometer.

### 5.3 Kasumeter

#### 5.3.1 System overview

The Kasumeter is a capillary viscometer. In the present study the technique was used for the determination of the yield stress \( \tau_y \) only. The used Kasumeters were built in analogy to the Kasumeter of Schulze et al. (1991) which was designed for the yield stress...
determination of clay-dispersions on construction sites. It consists of a vertical cylinder and a horizontal outlet capillary fixed in the bottom region of the cylinder (fig. 5.15 and 5.16).

Figure 5.15 Principle of the Kasumeter.

Figure 5.16 Kasumeters used in the present study. Left: Standard scale. Right: Large scale.

While the outlet of the capillary remains closed with a tap, the test fluid is filled in the cylinder (and the capillary) up to an initial height \( h_i \) close to the top. The capillary is then opened and the fluid flows out while the level \( h(t) \) of the fluid in the cylinder decreases with the time \( t \). The outflow stops when a final stagnation level \( h_f \) of the fluid in the cylinder is achieved.

Two different sizes of Kasumeters were designed. The standard scale Kasumeter had the same dimensions as the one developed by Schulze et al. (1991): The vertical
cylinder measures 100 mm in diameter and 300 mm in height and requires a sample volume of 2.3 l. The length of the outlet capillary is \( L_c = 200 \) mm and the capillary diameter is \( D_c = 9 \) mm. The standard scale Kasumeter was used to test fine particle fluids containing particles smaller than \( d = 0.25 \) mm.

The large scale Kasumeter measured 290 mm in diameter and 800 mm in height and required a sample volume of 50 l. Length of the outlet capillary was \( L_c = 300 \) mm. In order to guarantee that the stagnation level is always higher than the top level of the outlet capillary, the diameter of the capillary was varied between \( D_c = 38.5, 25 \) and 16 mm depending on the expected yield stress \( \tau_y \) of the test fluid (section 5.3.3). The large scale Kasumeter was used to test large particle fluids containing particles smaller than \( d = 5 \) mm or \( d = 10 \) mm. Both Kasumeters are made of plexiglas. In order to avoid wall slip the inside of the capillaries was covered with sandpaper.

Conducting a Kasumeter experiment required 15 to 75 person minutes: 2 minutes to fill the fluid, 0.5 to 60 minutes for the experiment and 10 minutes to empty and clean the apparatus.

5.3.2 Measurements and observations

The stagnation level \( h_f \) was measured with a laser distance meter and controlled with a vertical measuring band fixed on the cylinder.

In the present study different concentrated clay-dispersions (\( d \leq 0.25 \) mm), clay-gravel-suspensions (\( d \leq 10 \) mm) and debris flow material mixtures (\( d \leq 5 \) mm) were tested. Experiments were repeated in case of the clay-dispersions and clay-gravel-suspensions. In case of the debris flow material mixtures experiments were executed once.

The experiments with the clay-dispersions and clay-gravel-suspensions lasted between 0.5 and 3 minutes depending on the sediment concentration and the size of the applied capillary. The end of the experiment was characterized by a relatively clear and abrupt stop of outflow; the end was defined at the moment when the continuous outflow stopped and an interrupted outdropping of the fluid (in much less quantity) started.

A clear definition of the end of the experiment was difficult in case of the debris flow material mixtures. In this case a very slow transition from continuous into interrupted outflow was observed which lasted between 20 and more than 60 minutes. During this transition the outflow was more and more characterized by a plug flow spreading to the capillary boundary. Such a plug flow means that the wall shear stress \( \tau_w \) is below the yield stress \( \tau_y \) of the fluid. Therefore wall slip must occur to push out the fluid in the end of this transition regime. Slip effects of this kind are a very common phenomenon in
capillary rheometry and are usually eliminated or reduced by the use of a rough wall (Nguyen & Boger 1992). The use of a sandpaper in the present study turned out to be not sufficient to avoid this phenomenon for the debris flow material mixtures. Avoiding this phenomenon by using wall roughness in the order of the grain size of the fluid particles \((d = 1 - 5 \text{ mm})\) would be technically difficult to achieve in capillaries of limited diameter.

When emptying the cylinder after the experiments using the less concentrated debris flow material mixtures \((C_v \leq 0.508)\), it was observed that larger particles accumulated at the bottom of the vertical cylinder. Thus particle settling took place during the long experiment period. The process was not quantified but turned out to be much more accentuated for the smallest concentration \((C_v = 0.488)\) than for the slightly higher one \((C_v = 0.508)\). Particle settling was not observed within the clay-gravel-suspensions. Here the yield stress of the liquid (clay-dispersions) was always higher than the gravitational stress of the gravel particles (eq. 2.8).

5.3.3 Yield stress determination

Theory:

For fully developed laminar flow of time-independent inelastic fluids the force balance in a capillary is (fig. 5.17):

\[
\tau_w \cdot 2 \cdot \pi \cdot D_c \cdot L_c = \pi \left( \frac{D_c}{2} \right)^2 \cdot [(P + \Delta P) - P]
\]

with \(\tau_w\) = wall shear stress, \(P\) = pressure force and \(\Delta P\) = pressure force difference between the entrance and the end of the capillary.

![Figure 5.17 Principle scheme of acting pressures P, \(\Delta P\) and wall shear stress \(\tau_w\) for laminar flow \(Q\) in a capillary of length \(L_c\) and diameter \(D_c\).](image)

\[\text{Figure 5.17 Principle scheme of acting pressures P, } \Delta P \text{ and wall shear stress } \tau_w \text{ for laminar flow } Q \text{ in a capillary of length } L_c \text{ and diameter } D_c.\]
If the capillary outflow $Q$ stops, $\Delta P$ reduces to the static pressure difference $\Delta P = \rho \cdot g \cdot h_f$. In this case the wall shear stress $\tau_w$ becomes to equal the fluid yield stress $\tau_y$:

$$\tau_y = \tau_w(Q = 0) = \frac{1}{4} \frac{D}{L_c} \rho \cdot g \cdot h_f$$  \hspace{1cm} (5.7)

**Results:**

Table 5.4 and 5.5 indicate the yield stress $\tau_y$ obtained with eq. 5.7 for the investigated fine particle and large particle fluids using both, the standard scale and the large scale Kasumeter.

<table>
<thead>
<tr>
<th>$C_{v\text{fines}}$</th>
<th>$C_{v\text{gr}}$</th>
<th>$C_{v\text{tot}}$</th>
<th>Type of Kasumeter</th>
<th>$D_c/L_c$</th>
<th>$\tau_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-]</td>
<td>[-]</td>
<td>[-]</td>
<td></td>
<td>[-]</td>
<td>[Pa]</td>
</tr>
<tr>
<td>Clay-Dispersion</td>
<td>0.225</td>
<td>0.225</td>
<td>Standard</td>
<td>0.045</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>0.225</td>
<td>0.225</td>
<td>Large scale</td>
<td>0.128</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>Large scale</td>
<td>0.128</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>0.275</td>
<td>0.275</td>
<td>Large scale</td>
<td>0.128</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.3</td>
<td>Large scale</td>
<td>0.128</td>
<td>124</td>
</tr>
<tr>
<td>Clay-Gravel-Suspension</td>
<td>0.225</td>
<td>0.1</td>
<td>0.325</td>
<td>Large scale</td>
<td>0.128</td>
</tr>
<tr>
<td></td>
<td>0.225</td>
<td>0.2</td>
<td>0.425</td>
<td>Large scale</td>
<td>0.128</td>
</tr>
</tbody>
</table>

*Table 5.4 Yield stress $\tau_y$ obtained with the standard and the large scale Kasumeter for the clay-dispersions and clay-gravel-suspensions.*

<table>
<thead>
<tr>
<th>$C_v$</th>
<th>$D_c/L_c$</th>
<th>$\tau_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-]</td>
<td>[-]</td>
<td>[Pa]</td>
</tr>
<tr>
<td>0.488</td>
<td>0.053</td>
<td>10</td>
</tr>
<tr>
<td>0.508</td>
<td>0.083</td>
<td>36</td>
</tr>
<tr>
<td>0.549</td>
<td>0.083</td>
<td>59</td>
</tr>
<tr>
<td>0.571</td>
<td>0.083</td>
<td>89</td>
</tr>
<tr>
<td>0.605</td>
<td>0.128</td>
<td>251</td>
</tr>
</tbody>
</table>

*Table 5.5 Yield stress $\tau_y$ obtained with the large scale Kasumeter for the different debris flow material mixtures ($d \leq 5$ mm).*

The scale of the Kasumeter did not significantly influence the results as the yield stresses $\tau_y$ obtained with the standard and the large scale Kasumeter differed only by $\pm 10\%$. 
Performing five experiments using the clay-gravel-suspension ($C_{v \text{rel}} = 0.325$) a standard deviation of the Kasumeter measurement of 8.5% was obtained. Schulze et al. (1991) report a standard deviation of 5%.

To sum up, the Kasumeter enabled the estimation of the yield stress $\tau_y$ with an error of approximately $\pm 10\%$ for fine and large particle fluids which did not encourage wall slip. For particle fluids which encourage wall slip, such as the present debris flow material mixtures, the error was assumed to be larger because the end of the flow process was difficult to assess.

5.4 Inclined Channel Test

5.4.1 System overview

The Inclined Channel Test (ICT) was originally applied for the study of the development of the wall shear stress of slowly flowing and deposing Yield Stress Fluids in channels of arbitrary sections (Coussot 1994a,b).

The stopping of the flow is a special case herein which enables to determine the yield stress $\tau_y$ directly from the characteristic deposition depth $h_0$, the fluid density $\rho$ and the channel geometry.

The experimental set up consists of a long uniform and inclined channel and a tank above the channel entrance (fig. 5.18). While the gate at the tank outlet is closed the test mixture is filled into the tank and the mixture stirred up again before the experiment is started. The gate is then slowly opened in order to produce a uniform laminar channel flow $Q$ which is decreasing with time $t$. Once the channel flow stops completely a characteristic deposition depth $h_0$ is observed in the channel.

In the present study a rectangular channel of 2.1 m length and 0.2 m width, inclined by the angle $i = 5.71^\circ$ was used (fig. 5.19). The flume bed consisted of PVC. The sidewalls of the channels consisted of glass covered with a transparent protection foil.

The end of the channel was followed by an inclined ($i = 1.26^\circ$) large plane, which was used for the deposition of the first part of the test fluid. The zone for the $h_0$ - measurement in the channel spanned over 1.2 m. The zone was situated 0.3 m below the reservoir gate and 0.6 m above the end of the channel in order to avoid any measurements close to the tank outlet and in the backwater of the slope transition.
Figure 5.18 Principle of the Inclined Channel Test.

Figure 5.19 Set up of the Inclined Channel Test in the present study.
In some experiments combined Inclined Channel Tests and Inclined Plane Tests were performed. In this case the channel was used for the Inclined Channel Tests and the plane was used for the Inclined Plane Tests (section 5.5).

Depending on the expected deposition depth $h_0$ of a given mixture in the channel, and depending on whether single Inclined Channel Tests or combined tests were conducted, the sample volume $V_{\text{sample}}$ varied between 30 and 100 liter.

Conducting one experiment required 60 person minutes: 5 minutes to fill the fluid into the tank and to agitate the fluid, 5 minutes for the experiment, 20 minutes for the measurement and 30 minutes for the tank and channel cleaning.

### 5.4.2 Measurements and observations

Experiments were conducted for debris flow material mixtures $d \leq 5 \text{ mm}$ and $d \leq 10 \text{ mm}$ as well as for the mixed suspension. For each mixture the experiment was repeated. The characteristic deposition depth $h_0$ was measured by point gauge in defined intervals of 0.2 m. At a given channel section $h_0$ was measured at three points: in the channel center and 30 mm from the sidewalls.

Fig. 5.20 shows the $h_0$-measurements recorded in the channel for two test mixtures. The standard deviation of the $h_0$-measurement ranged between 2 and 8 % for all investigated mixtures.

![Graph](image)

*Figure 5.20 Deposition depth $h_0$ measured in the Inclined Channel Test for two different concentrated debris flow material mixtures $d \leq 10 \text{ mm}$. Channel entrance at 2.1 m, Channel end at 0 m.*
Observations during the experiments:

(1) No evident shear zone appeared close to the sidewalls as this is usually observed in these experiments (Coussot 1997, Parsons et al. 2001). The mixtures flew down the channel as a plug which spread over the entire channel width. Obviously this must be due to wall slip along the sidewalls because the glass walls, covered with a foil, were not rough enough. On the channel bed a direct observation was only possible at the front part of the test fluid. Here no shearing was observed, which is probably due to the smooth PVC used for the channel bed.

Coussot (1997) does not indicate the appropriate roughness but stresses the importance of the no-slip condition at the channel boundary. The used Kaolin-dispersions in the experiments of Coussot obviously did not exhibit wall slip on the smooth channel surface. This is because, in contrast to the debris flow material mixtures of the present study, the water in the Kaolin-clay is bound and does not drain and therefore does not produce a water film between the boundary and the fluid.

Parsons et al. (2001) made experiments with different sediment-water mixtures containing particles up to $d_{\text{max}} = 5$ mm grain sizes in a rough channel where a well-sorted sand ($d_{50} = 1$ mm) was glued on the channel bed and channel wall.

(2) Once the supplying tank was emptied, it was observed that the fluid was flowing in the downstream channel region whereas in the upstream channel region the fluid was already at rest. It could be observed that the fluid first stopped at the very beginning of the upstream channel and then continuously stopped from the upstream to the downstream end of the channel within a time span of one to several minutes depending on the investigated mixture.

5.4.3 Yield stress determination

Theory:

Coussot (1994a) showed that the yield stress $\tau_y$ can be determined with the well known shear stress - flow depth - relationship, as long as $h_0/b < 1$:

$$\tau_y = \rho \cdot g \cdot R_h \cdot \sin(i); \quad (5.8)$$

where $\rho$ is the fluid density, $i$ is the channel slope and $R_h$ is the hydraulic radius. In case of a rectangular channel the hydraulic radius $R_h$ is:

$$R_h = \frac{h_0 \cdot b}{h_0 + 2 \cdot b} \quad (5.9)$$

where $h_0$ is the deposition depth and $b$ is the channel width.
Results:

Table 5.6 lists the average value of the yield stress \( \tau_y \) obtained with the Inclined Channel Test for the different debris flow material mixtures as well as for the mixed suspension. The deviation of the results between the first and the second experiment was on average 4\% but remained always smaller than 10\%. In two cases more than two experiments were conducted. Also here the standard deviation obtained from the different experiments was smaller than 10\%.

Considering additionally the standard deviation of the measured \( h_0 \) – values which ranged between 2 and 8\%, it can be concluded that the Inclined Channel Test enabled the determination of the yield stress \( \tau_y \) with an error of approximately \( \pm 10\% \).

<table>
<thead>
<tr>
<th>Debris flow material mixtures</th>
<th>Mixed suspension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( d \leq 5 \text{ mm} )</td>
</tr>
<tr>
<td></td>
<td>( C_v )</td>
</tr>
<tr>
<td>[\text{-}]</td>
<td>[\text{Pa}]</td>
</tr>
<tr>
<td>0.605</td>
<td>74</td>
</tr>
<tr>
<td>0.571</td>
<td>53</td>
</tr>
<tr>
<td>0.549</td>
<td>32</td>
</tr>
<tr>
<td>0.508</td>
<td>19</td>
</tr>
<tr>
<td>0.488</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 5.6 Yield stress \( \tau_y \) obtained with the Inclined Channel Test for different debris flow material mixtures and the mixed suspension.

5.5 Inclined Plane Test

5.5.1 System overview

The Inclined Plane Test (IPT) first studied by Uhlherr et al. 1984 (from Coussot & Ancey 1999) was further developed by Coussot et al. (1996) and Wilson & Burgess (1998) in order to study the deposition behaviour of Yield Stress Fluids and to relate the profile of deposit with the yield stress \( \tau_y \) of the fluid.

In the Inclined Plane Test a given fluid volume is slowly introduced at a fixed location on an inclined plane. Driven by gravity, the fluid spreads downslope as well as to a decreasing extent in lateral directions forming finally a heap with a specific 3-dimensional geometry. Accordingly characteristic profiles of deposits are obtained at the deposition border depending on the angle \( \theta \) (fig. 5.21). If a large volume is used, the
heights $h$ of any profile mound into a characteristic asymptotic fluid depth $h_0$ at a given distance from the deposition border.

In the present study the Inclined Plane Test was used for the investigation of fluids containing particles of $d \leq 10$ mm grain size. The test was combined with the Inclined Channel Test: After the fluid had passed the inclined channel (5.4), it spread over a $i = 1.26^\circ$ inclined test plane (fig. 5.22). The plane was unrestricted in downstream and lateral directions but confined in backward direction. The plane surface was made of a uniform granular paint with a surface roughness equivalent to $d = 2$ mm.

Conducting one experiment required 60 person minutes: 5 minutes to fill the fluid into the tank, 5 minutes for the experiment, 20 minutes for the measurement and 30 minutes for the channel and plane cleaning.

Figure 5.21 Principle scheme of the Inclined Plane Test performed in the present study.
5.5.2 Measurements and observations

For the investigated debris flow material mixtures and the mixed suspension single experiments were conducted.

The profile of deposit, which is the flow depth $h$ versus the distance $x$ from the deposit border, was measured in the direction of the highest slope. This was done with a thin steel plate cutting the fluid perpendicular to the plane, while the profile of deposit was directly printed on both sides of the plate. The plate was equipped with a coordinate system on both sides which enabled to read the flow depth $h$ in function of the distance $x$ from the deposit border in increments of 10 mm in the range $0 \leq x \leq 180$ mm.

The measurements are shown and discussed in 5.5.3 in combination with the theory for the yield stress determination.

5.5.3 Yield stress determination

Theory:

Yield stress determination based on the asymptotic fluid depth $h_0$:

Analogue to the Inclined Channel Test the yield stress $\tau_y$ can be determined based on the asymptotic fluid depth $h_0$ in the deposition center (Coussot 1994a):

$$\tau_y = \rho \cdot g \cdot h_0 \cdot \sin(i)$$  \hspace{1cm} (5.10)

Note that in contrast to the Inclined Channel Test the asymptotic fluid depth $h_0$ is only reached if a sufficiently large sample volume is used. In the case of highly concentrated fluids exhibiting a large yield stress value, very large volumes are usually necessary in case the plane inclination $i$ cannot be increased significantly (Coussot & Ancey 1999).
Investigating such type of fluids the Inclined Channel Test or the following theoretical concept must be applied for the determination of the yield stress $\tau_y$.

**Yield stress determination based on the profile of deposit:**

Coussot et al. (1996) derived a first theoretical approach for the relation between the yield stress $\tau_y$ of a non-thixotropic fluid and the profile of deposit at any angle $\theta$ on the inclined plane (fig. 5.21). The theory was further developed by Wilson & Burgess (1998) in the directions of non largest slope. However in the case of the largest slope ($\theta = 0$) both theories are reduced to a relatively simple relation already described by Liu & Mei (1989):

$$X^* = \varepsilon \cdot H^* - \ln \left[ 1 + \varepsilon \cdot H^* \right]$$

with the dimensionless length numbers $X^*$ and $H^*$ defined as:

$$X^* = \frac{\rho \cdot g \cdot x \cdot (\sin i)^2}{\tau_y \cdot \cos i}$$

$$H^* = \frac{\rho \cdot g \cdot h \cdot \sin i}{\tau_y}$$

where $h$ is the height of deposition at the distance $x$ from the deposition border (fig. 5.21).

Considering eq. 5.12 and 5.13 in eq. 5.11 and considering the downstream profile for which is $\varepsilon = -1$ eq. 5.11 is transformed into:

$$\frac{\rho \cdot g \cdot \sin i}{\tau_y} \cdot (x \cdot \tan i + h) = -\ln \left[ 1 - \frac{\rho \cdot g \cdot h \cdot \sin i}{\tau_y} \right]$$

The value of $\tau_y$ can now be found by best fitting of $h$- and $x$- data.

Note that the above theory was derived under the condition of the long-wave theory: It requires that the extension of the deposition is much larger than the fluid depth $h(x)$ in the heap center. In the present study the fulfilment of the condition was fair to middling, as the diameter of the deposition in lateral direction exceeded the fluid depth $h(x = 180 \text{ mm})$ by a factor 4 (highly concentrated mixtures) to 16 (low concentrated mixtures).

**Results:**

Due to the limited sample volume, the limited extension and inclination of the experimental plane (extension and inclination) and due to the rather concentrated
mixtures investigated here, the asymptotic fluid depth $h_0$ was usually not reached. Therefore only the second method for the determination of the yield stress based on the profile of deposit was applied here.

Expressed by the dimensionless length numbers $X^*$ and $H^*$, fig. 5.23 compares the experimental with the theoretical profile of deposit for the different investigated mixtures.

![Graph](image)

**Figure 5.23** Experimental $X^*$- and $H^*$- data in comparison with theory (eq. 5.11) for the different investigated large particle fluids.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$C_v$</th>
<th>$\tau_y$</th>
<th>$\sigma\tau_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debris flow material mixture, $d \leq 10$ mm</td>
<td>0.642</td>
<td>265</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>0.595</td>
<td>61</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>0.551</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>Mixed Suspension, $d \leq 10$ mm</td>
<td>0.683</td>
<td>538</td>
<td>8</td>
</tr>
</tbody>
</table>

**Table 5.7** Average yield stress $\tau_y$ and standard error $\sigma\tau_y$ obtained with the Inclined Plane Test for the different investigated mixtures.
The experimental data fit the theoretical curve well. Some fluctuation of the data about the theoretical curve is visible but no trend along the profile can be observed.

Table 5.7 lists the average $\tau_y$ values as well as the standard errors $\sigma_{\tau_y}$ obtained after best fitting of $h$- and $x$- data using the “least squares regression method” of Gauss. Based on the few single experiments conducted in the present study, the yield stress obtained with the Inclined Plane Test was determined with a standard error smaller than 20%.

5.6 Slump Test

5.6.1 System overview

The Slump Test (ST) was originally developed for the estimation of the yield stress $\tau_y$ of fresh concrete on construction sites. Nowadays, the application of the test has been extended to investigate also other fluids, such as strongly flocculated suspensions e.g., which are difficult to test in conventional measuring systems (Pashias et al. 1996).

For the Slump Test a conical or cylindrical container is required, which is open at the top and the bottom. The container is placed on a horizontal plane and filled with the test fluid. Then the container is lifted vertically which allows the fluid to collapse under its own weight and to radially spread on the plane (fig. 5.24).

![Principle of the Slump Test](https://via.placeholder.com/150)

*Figure 5.24 Principle of the Slump Test.*
Usually the yield stress is estimated based on the slump height $S$, which is the difference between the initial fluid depth (usually the container height $H_c$) before the experiment and the final fluid depth $h_f$ after the experiment (Pashias et al. 1996 for a cylindrical mold, Schowalter & Christensen 1998 for a conical mold). Applying the theory derived for the Inclined Plane Test, it is also possible to determine the yield stress based on the profile of deposit (Coussot et al. 1996).

In the present study debris flow material mixtures $d \leq 0.25$ mm, $d \leq 1$ mm, $d \leq 5$ mm and $d \leq 10$ mm were investigated with the Slump Test. A cylindrical PVC-container with a height $H_c = 89$ mm and a diameter $D_c = 103.5$ mm was used which required a sample volume of 0.75 l. At the bottom end of the sidewalls a rubber ring was fixed in order to avoid eventual drainage of water/fluid between the container and the plane. The plane was made of PVC and had a smooth surface.

Conducting one experiment required 20 person minutes: 1 minute to fill the fluid into the container, 1 minute for the experiment, 15 minutes for the measurement and 3 minutes for the plane and container cleaning.

Figure 5.25 Final deposition of debris flow material mixtures after the Slump Test. Left: $d \leq 5$ mm, $C_v = 0.558$ (slant view). Right: $d \leq 5$ mm, $C_v = 0.474$ (top view).

5.6.2 Measurements and observations

For a given mixture experiments were always repeated. After the experiment the final fluid depth $h_f$ and the profile of deposit was measured.

The slump height $S$ was determined by simply measuring the final fluid depth $h_f$ in the center of the deposition and substracting $h_f$ from the cylinder height $H_c$.

The profile of deposit was obtained similar to the Inclined Plane Test: The deposited fluid was cut perpendicular to the horizontal plane with a thin steel plate while the
profile of deposit was directly printed on the plate. The profile was measured at three different locations (each 120°) along the 360° circle of the radial deposition. The plate was equipped with a coordinate system based on which the flow depth $h$ was measured in function of the distance $x$ from the deposit border in the range $0 \leq x \leq 80$ mm.

The measurements are shown and discussed in section 5.6.3 in combination with the theories for the yield stress determination.

**Observations made during the experiments:**

1) When the container was lifted the mixture immediately slumped and spread radially on the horizontal plane and stopped after a short time of a few seconds.

2) For the more concentrated mixtures ($C_v > 0.55$) the deposition showed the characteristics usually expected in slump tests (Pashias et al. 1996, Ancey & Jorrot 2001): An unyielded rigid part in the center with the final depth $h_f$ and a diameter approximately equal to the diameter $D_c$ of the container, is radially surrounded by a yielded part of smaller depth (fig. 5.25). For the less concentrated mixtures ($C_v < 0.55$) a distinction between a rigid center part and a fluidized outer part was not observed. The entire deposition material appeared to be yielded.

3) A few minutes after the end of the experiment clear water drained out of the deposition forming a thin water film on the deposition surface and border.

### 5.6.3 Yield stress determination

**Theory:**

**Yield stress determination based on the slump:**

For the application of a cylindrical container the following formula was developed by Pashias et al. (1996) in order to determine the yield stress $\tau_y$ based on the slump height $S$:

$$\frac{S}{H_c} = 1 - 2 \cdot \frac{\tau_y}{\rho \cdot g \cdot H_c} \left( 1 - \ln 2 \cdot \frac{\tau_y}{\rho \cdot g \cdot H_c} \right)$$

(5.15)

where the yield stress $\tau_y$ is found by iteration.

The authors found a good agreement of eq. 5.15 with experimental results for the dimensionless slump height $S' = S/H_c$ in the range $0.4 < S' < 0.75$: error $= \pm 10\%$. For $S' > 0.75$ no data were tested, for $S' < 0.4$ the error is larger than $\pm 10\%$. 
Yield stress determination based on the profile of deposit:

Based on the theory derived for the Inclined Plane Test Coussot et al. (1996) derived the following simplified relation in the case of a horizontal plane ($i = 0^\circ$):

$$ h = \sqrt{\frac{2 \cdot \tau_y \cdot x}{\rho \cdot g}} \quad (5.16) $$

where $h$ is the height of deposition at the distance $x$ from the deposition border (fig. 5.24). Practically the average value of $\tau_y$ is found after best fitting of $h$- and $x$- data with the theoretical equation 5.16.

Note that eq. 5.16 only holds if the long-wave approximation is respected (Wilson & Burgess 1998, Coussot & Ancey 1999). This requires that the deposition extension should be much larger than the deposition height in the deposition center. In the present study the deposition diameter outweighed the center height by a factor 8 (highly concentrated mixtures) to 40 (low concentrated mixtures). It is thus concluded that condition of the long-wave approximation was well fulfilled.

**Results:**

Table 5.8 shows the yield stress $\tau_y$ obtained from the slump height $S$ and the one obtained from the profile of deposit for the different debris flow material mixtures.

<table>
<thead>
<tr>
<th>Debris flow material mixture</th>
<th>Slump (eq. 5.15)</th>
<th>Profile of deposit (eq. 5.16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portion $C_v$</td>
<td>$S' = S/H_c$</td>
<td>$\tau_y$</td>
</tr>
<tr>
<td>$d \leq 0.25$ mm</td>
<td>0.349</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>0.386</td>
<td>6.2</td>
</tr>
<tr>
<td></td>
<td>0.429</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>0.479</td>
<td>55</td>
</tr>
<tr>
<td>$d \leq 1$ mm</td>
<td>0.458</td>
<td>8.5</td>
</tr>
<tr>
<td></td>
<td>0.502</td>
<td>25</td>
</tr>
<tr>
<td>$d \leq 5$ mm</td>
<td>0.474</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>0.514</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>0.558</td>
<td>40</td>
</tr>
<tr>
<td>$d \leq 10$ mm</td>
<td>0.511</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>0.551</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>0.595</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>0.642</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 5.8 Yield stress $\tau_y$ obtained with the slump test for different debris flow material mixtures. $\tau_y$ obtained from the slump height $S$ (eq. 5.15) and from the profile of deposit (eq. 5.16).
Results obtained from the slump (eq. 5.15):

The yield stress $\tau_y$ obtained from the slump height $S$ is 70 - 100 % higher than the yield stress $\tau_y$ obtained from the profile of deposit for the low to medium concentrated mixtures ($C_v = 0.511$, $C_v = 0.551$) of the portion $d = 10$ mm. The corresponding dimensionless slump height $S'$ is 0.88 respectively $S' = 0.83$ which is far out of the range for which eq. 5.15 was proved to be valid ($0.4 < S' < 0.75$; Pashias et al. 1996). Stress differences of 40 % and 10 % were obtained for the medium concentrated mixtures of the two portions $d \leq 5$ mm and $d \leq 10$ mm ($C_v = 0.558$, $C_v = 0.595$) where the dimensionless slump height was $S' = 0.75$, which is at the upper limit of the $S'$-range indicated for validity of eq. 5.15.

The values of $S' > 0.75$ correspond with the observation that the mixture was entirely yielded after deposition whereas for $S' = 0.75$ the unyielded and deeper center part could be optically distinguished from the yielded and more shallow part surrounding the center part (fig. 5.25). In the case where the fluid is entirely yielded the slump height $S$ cannot be determined successfully and thus eq. 5.15 should not be applied.

To summarize for $S' > 0.75$ only the yield stress data obtained from the profile of deposit are considered for the comparison of rheological data (chapter 6). In the case of $S' \leq 0.75$ the yield stress data obtained by both methods are considered.

Results obtained from the profile of deposit (eq. 5.16):

Fig. 5.26 shows the experimental and fitted/theoretical profile of deposit for the different investigated mixtures. Unlike the Inclined Plane Test the experimental data do not follow that well the fitted/theoretical profile (eq. 5.16): At the deposition border the height $h$ increases more significantly than predicted. By contrast the height $h$ does not increase as significantly as predicted for a distance $x$ from the border larger than about 30 mm.

For each portion $d$ this difference between experimental data and theoretical prediction is more apparent in case of the lower compared to the higher concentrated mixtures. This is also expressed by the values of the standard error $\sigma_{\tau_y}$ which increase from the higher towards the lower concentrations $C_v$ for the different portions $d$ (table 5.8).

From Baudez et al. (2002) it is known that the plane surface roughness does not influence the slump height $S$ of the unyielded central part significantly. Such was not answered yet in the case of the profile of deposit of the yielded surrounding part. It is assumed here that due to the smooth surface the spreading fluid is not only sheared but partially slides on the horizontal plane. Sliding could significantly alter the deposition behaviour of a given fluid and thus result in a different profile of deposit.
Application of other rheometry for large particle fluids

Figure 5.26 Experimental $x$- and $h$- data in comparison with theory (eq. 5.16) for the different debris flow material mixtures: a) $d \leq 0.25$ mm, b) $d \leq 1$ mm, c) $d \leq 5$ mm, d) $d \leq 10$ mm.

This argument claims that sliding has a stronger influence on the profile of deposit in the lower than in the higher concentrated mixtures. This is partly confirmed by the fact that in the lower concentrated mixtures the entire mixture is yielded and may be thus subject of sliding, whereas in the higher concentrated mixtures only part of the entire
mixture is yielded and may be subject to sliding. However, future work should focus on this particular aspect.
6 Comparison of rheometry

6.1 Comparison of rheological data

6.1.1 Flow curve

The flow curves of large particle fluids up to \( d = 10 \text{ mm} \) grain size were determined with the ball measuring system (BMS), the CCS large scale rheometer of Coussot & Piau (1995) (CCS-LS) and the BML viscometer of Wallevik & Gjørv (1990). For the fluids up to \( d = 25 \text{ mm} \) grain size the CCS-LS and the BML were applied.

While the BML viscometer provides one flow curve for a given fluid, the BMS and the CCS-LS provide a flow curve range or a stress range. For the BMS the flow curve range is given by the use of the different sphere sizes. For the CCS-LS the flow curve range is given by the unknown apparatus resistance. Only a stress range is indicated where the CCS-LS - measurements revealed no clear dependence of the torque and speed data.

For the CCS-LS the flow curve band could only be determined for two mixtures (debris flow material mixture \( d \leq 10 \text{ mm}, C_v = 0.642 \) and mixed suspension). For the remaining mixtures the flow curve was not determined due to a manifest hysteresis between the up and down curve experiment (5.1). In this case only a stress range, indicating the range where the flow curve is expected, is given in the following plots.

The flow curves were determined in the range of \( 0.1 < \dot{\gamma} < 30 \text{ to } 100 \text{ s}^{-1} \) for the BMS, in the range \( 1 < \dot{\gamma} < 100 \text{ s}^{-1} \) for the CCS-LS and in the range \( 10 < \dot{\gamma} < 100 \text{ s}^{-1} \) for the BML viscometer.

Results:

Absolute values:

Overall the different measuring systems yield flow curves which are in the same range of shear stresses \( \tau \) (fig. 6.1 and 6.2). Only for the medium concentrated debris flow material mixture \( (d \leq 10 \text{ mm}, C_v = 0.595) \) the \( \tau \)-range indicated by the CCS-LS is on a higher level.

The shape of the flow curve between the different systems is not identical. At higher shear rates \( (\dot{\gamma} \geq 10 - 50 \text{ s}^{-1}) \) the ball measuring system provides a stronger stress increase compared to the other systems.
Figure 6.1 Comparison of flow curves of different large particle fluids ($d \leq 10\text{ mm}$) using the Ball Measuring System (BMS), the CCS large scale rheometer of Coussot & Piau (CCS-LS) and the BML viscometer (BML). a) Debris flow material mixture $C_v = 0.595$, b) Debris flow material mixture $C_v = 0.642$, c) Mixed Suspension $C_v = 0.683$.

Accuracy of the flow curve data:

No full indication about the accuracy of the flow curve data of the BML viscometer is possible because information about the error of the measured torque at a given speed is not available (5.2). However due to the fact that the difference in measured torque data between the first and the following experiments was always smaller than $\pm 5\%$, the accuracy of the BML viscometer is classified good.

Considering the CCS large scale rheometer of Coussot & Piau the flow curve band is relatively small, and thus the accuracy good in the few cases where the flow curve could be determined.
Figure 6.2  Comparison of flow curves, respectively range of shear stress $\tau$, of debris flow material mixtures ($d \leq 25$ mm) using the CCS large scale rheometer of Coussot&Piau (CCS-LS) and the BML viscometer. a) $C_v = 0.643$, b) $C_v = 0.658$, c) $C_v = 0.673$.

For the BMS the flow curve band is defined by the flow curves of the three spheres. Referred to the flow curve of the medium sphere ($D = 12$ mm) the variation of the flow curve obtained by the other spheres is $\pm 15-25\%$ which gives a moderately good accuracy. This variation is due to the fact that the smaller spheres yielded higher stress values than the larger spheres in mixtures (1) containing particles larger than $d_{\text{max}} = 1$ mm grain size and in mixtures which are (2) highly concentrated (chapter 4).

Discussion:

The fact that all three measuring systems provide flow curve data more or less in the same range of shear stresses $\tau$ strengthens the reliability of these three systems within their corresponding ranges of application ($d_{\text{max}}$, $\dot{\gamma}$-range, $C_v$).

An interpretation for the relatively higher range of shear stress obtained by the CCS-LS for the debris flow material mixture ($d \leq 10$ mm, $C_v = 0.595$) is difficult. One might
assume that this is due to particle settling followed by the intrusion of fine sediment material below the rubber seal into the interior of the inner cylinder of the CCS-LS (which was observed after apparatus cleaning) and the corresponding increase of the apparatus resistance.

An explanation for the notable stress increase at high shear rates for the ball measuring system in comparison with the other systems is also difficult. The range of the shear rate (\(\dot{\gamma} \geq 10 - 50 \text{ s}^{-1}\)) where the stress increase is observed corresponds with the beginning of the transitional regime (\(1 < \text{Re} < 15\)). It is possible that for this specific regime the conversion of measured data into rheological data derived for the BMS with the help of fine particle fluids is different from the conversion in the case of large particle fluids.

To conclude, the advantage of the ball measuring system compared to the other two systems is that flow curve data can be obtained in a wide range of the shear rate \(\dot{\gamma}\) and in a wide range of the sediment concentration \(C_v\). The latter holds especially for lower concentrations (\(C_v < 0.55 - 0.6\)) where the other systems did not allow to determine any useful flow curve data. The disadvantage of the ball measuring system is that the flow curve determination is limited to mixtures containing particles not larger than \(d_{\text{max}} = 10 \text{ mm grain size}\). The disadvantage of the CCS-LS is the unknown apparatus resistance (bad apparatus condition) and the unknown extent of particle segregation during the experiments which often rendered the determination of the flow curve impossible.

### 6.1.2 Yield stress

The following rheometric systems were considered for the comparison of the yield stress \(\tau_y\):

The ball measuring system (BMS), the CCS large scale rheometer of Coussot&Piau (CCS-LS), the BML viscometer (BML), the Kasumeter (KAS), the Inclined Channel Test (ICT), the Inclined Plane Test (IPT) and the Slump Test (ST).

Depending on the slump process the yield stress \(\tau_y\) of the Slump Test was determined either from the profile of deposit (ST-PD) or from the slump height \(S\) (ST-SL) (5.6).

The comparison includes the different debris flow material mixtures as well as the clay dispersions, clay-gravel-suspensions and the mixed suspension considering the following material portions: \(d \leq 0.25 \text{ mm}, d \leq 1 \text{ mm}, d \leq 5 \text{ mm}, d \leq 10 \text{ mm}, d \leq 25 \text{ mm}\).

The accuracy of the yield stress obtained from a specific system is either indicated by a band width or by an error bar. The band width indicates the range of the yield stress (CCS-LS, BML). The error bar indicates the standard error obtained after data fitting of
flow curve data with the Herschel-Bulkley model (BMS), or the standard error obtained for the deposition depth (ICT, IPT, ST-PD) and stagnation depth (KAS), respectively.

Results:

Absolute values (d ≤ 0.25 to 10 mm):

For the mixtures of the portions d ≤ 0.25, 1, 5, 10 mm a fairly good agreement of the yield stress $\tau_y$ is found between the Slump Test, the Inclined Plane Test, the BML viscometer, and the BMS (fig. 6.3 to 6.6): In 21 out of 24 cases the yield stress $\tau_y$ deviates less than 30% from the values obtained by the medium BMS sphere ($D = 12$ mm)$^{24}$. In 3 cases the deviation is between 30 and 50% (BML, ST-PD). Note that roughly the same deviation of less than 30% is obtained for the two other spheres ($D = 8$ mm, $D = 15$ mm) compared to the medium sphere ($D = 12$ mm sphere) as far as the mixtures $d ≤ 10$ mm are concerned. As for the mixtures $d ≤ 5$ mm, $d ≤ 1$ mm and $d ≤ 0.25$ mm the deviation for the two other spheres is roughly less than 10%.

Within this comparison the values of the BML viscometer are higher (+20%) whereas the values of the Inclined Plane Test and the Slump Test are tendecially lower (-5%) than the values of the medium BMS sphere ($D = 12$ mm). Note that for the mixtures of the portion $d ≤ 10$ mm the larger BMS sphere ($D = 15$ mm) gives values which are lower (-20%) and the smaller sphere ($D = 8$ mm) gives values which are slightly higher (+10%) than the value of the medium sphere ($D = 12$ mm). For the mixtures of the portions $d ≤ 5$ mm, $d ≤ 1$ mm and $d ≤ 0.25$ mm these differences are reduced to approximately -10 to -5% ($D = 15$ mm) and +5% ($D = 8$ mm).

For the mixtures of the portion $d ≤ 10$ mm the CCS large scale rheometer (CCS-LS) provides $\tau_y$-values which are on average 25% higher (variation between -20% and +80%) than the values of the BMS ($D = 12$ mm sphere).

For the mixtures of the portions $d ≤ 0.25$ mm and $d ≤ 5$ mm the Kasumeter yields $\tau_y$-values which are 60% higher (variation between +15% and +160%) than the comparable values of the BMS ($D = 12$ mm sphere), independent of the type of fluid and independent of the sediment concentration $C_v$ (fig. 6.5 and 6.8).

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$^{24}$ The medium BMS sphere ($D = 12$ mm) is considered as a good reference because its $\tau_y$-value usually lies between the one obtained for the largest sphere ($D = 15$ mm) and the smallest sphere ($D = 8$ mm).
Figure 6.3 Yield stress $\tau_y$ for the debris flow material mixtures $d \leq 0.25$ mm at different sediment concentrations $C_v$. Comparison between the Ball Measuring System (BMS) and the profile of deposit after the Slump Test (ST-PD).

Figure 6.4 Yield stress $\tau_y$ for the debris flow material mixtures $d \leq 1$ mm at different sediment concentrations $C_v$. Comparison between the Ball Measuring System (BMS) and the profile of deposit after the Slump Test (ST-PD).
Figure 6.5  Yield stress $\tau_y$ for the debris flow material mixtures $d \leq 5$ mm at different sediment concentrations $C_v$. Comparison between the Ball Measuring System (BMS), the profile of deposit after the Slump Test (ST-PD), the slump height after the Slump Test (ST-SL), the Kasumeter (KAS) and the Inclined Channel Test (ICT).
Figure 6.6 Yield stress $\tau_y$ for different large particle fluids $d \leq 10$ mm at different sediment concentrations $C_v$. $0.5 < C_v < 0.65$: Debris flow material mixtures. $C_v = 0.683$: Mixed Suspension. Comparison between the Ball Measuring System (BMS), the profile of deposit after the Slump Test (ST-PD), the slump height after the Slump Test (ST-SL), the Inclined Channel Test (ICT), the Inclined Plane Test (IPT), the CCS large scale rheometer of Coussot & Plaum (CCS-LS) and the BML viscometer (BML).
Figure 6.7 Yield stress $\tau_y$ for the debris flow material mixtures $d \leq 25 \text{ mm}$ at different sediment concentrations $C_v$ obtained with the BML viscometer.

Figure 6.8 Yield stress $\tau_y$ for different concentrated clay-dispersions ($d \leq 0.25 \text{ mm}$) and clay-gravel-suspensions ($d \leq 10 \text{ mm}$). Comparison between the ball measuring system (BMS) and the Kasumeter (KAS).

Clay-Dispersions: $C_v = 0.225$ to $0.300$. Clay-Gravel-Suspensions: $C_v = 0.325$ ($C_{v \text{ fines}} = 0.225$, $C_{v \text{ gravel}} = 0.1$) and $C_v = 0.425$ ($C_{v \text{ fines}} = 0.225$, $C_{v \text{ gravel}} = 0.2$).
By contrast for the two investigated clay-gravel suspensions (portion $d \leq 10 \text{ mm}$) tendentially smaller values (-5 %) were obtained for the Kasumeter compared to the BMS sphere $D = 12 \text{ mm}$ (fig. 6.8). For the mixtures of the portions $d \leq 5 \text{ mm}$ and $d \leq 10 \text{ mm}$ the Inclined Channel Test exhibited a very different course of $\tau_y$-values in function of the sediment concentration $C_v$ compared to all other applied systems. While for high concentrations $C_v$ the $\tau_y$-values are much smaller (-70 to -80 %) than the comparable values of the BMS ($D = 12 \text{ mm}$ sphere), they are roughly equal at medium concentrations but much larger at low concentrations (+40 to +60 %).

**Absolute values ($d \leq 25 \text{ mm}$):**

Due to the hysteresis in the up and down curve experiments of the CCS large scale rheometer of Coussot&Piau measured for these mixtures, the determination of the flow curve and thus of the yield stress $\tau_y$ was not possible. Hence, only the data of the BML viscometer are shown (fig. 6.7).

**Accuracy of the $\tau_y$-values:**

In the case where no hysteresis occurred in the CCS-LS measurements the width of the yield stress range is 25 to 30 % referred to the higher of the two $\tau_y$-values, which gives a moderately good accuracy.

In the case of the BML viscometer the band width is 10 to 25 % which gives a moderately good to good accuracy.

The accuracy of the $\tau_y$-value obtained with the Kasumeter depends on the investigated type of mixture. For the clay-dispersions the standard error was found to be roughly 10 % which indicates a good accuracy. For the debris flow material mixtures favouring wall slip the standard error was not determined. However as the end of outflow was difficult to assess, it is assumed that the accuracy of the yield stress is worse.

The accuracy of the $\tau_y$-value obtained with the Inclined Channel Test and the Inclined Plane Test is good because the standard error was determined as 10 %.

The accuracy of the $\tau_y$-value obtained with the Slump Test using the profile of deposit depends on the sediment concentration $C_v$. The accuracy is moderately good to good at medium concentrations (standard error 10 to 30 %) but poor at low concentrations (standard error 50 to 100 %). For the $\tau_y$-value derived from the slump height $S$, the accuracy was not investigated.

To turn back to the BMS, the three different spheres are regarded as an ensemble: For the mixtures $d \leq 5 \text{ mm}$, $d \leq 1 \text{ mm}$ and $d \leq 0.25 \text{ mm}$ the deviation of the larger
(\(D = 15\) mm) and smaller (\(D = 8\) mm) sphere from the medium sphere (\(D = 12\) mm) is roughly less than 15\% As for the mixtures \(d \leq 10\) mm the deviation is roughly less than 30\%. Note that these values correspond well with the standard deviations of the torque - measurements found in 4.2.2.

**Discussion:**

The fact that the yield stress data obtained from the Slump Test, the Inclined Plane Test, the BML viscometer, the CCS large scale rheometer and the BMS are not significantly different strengthens the reliability of the rheological data and strengthens the reliability of these rheometric tools.\(^{25}\) By contrast the yield stress data obtained from the Inclined Channel Test and the Kasumeter are significantly different. The reasons are discussed in the following:

**Inclined Channel Test:**

While all other measuring systems show roughly the same dependence of the \(\tau_y\)-value from the sediment concentration \(C_v\), the Inclined Channel Test shows a different one: The \(\tau_y\)-values of the Inclined Channel Test are much smaller at high concentrations (\(C_v > 0.6\)) but considerably larger at low concentrations (\(C_v < 0.55\)).

In 5.4 it was reported that wall and bed slip must have occurred during the Inclined Channel Test because plug flow spread over the entire channel width and a shear zone was not observed. In case of wallslip the measured shear stresses drop primarily at low shear rates whereas the influence is usually significantly lower and disappears at high shear rates. Because the Inclined Channel Test is usually performed at low shear rates, it is postulated that the yield stress values obtained with the Inclined Channel Test in the present study are falsified.

The discrepancy between the \(\tau_y\)-value obtained with the Inclined Channel Test and the \(\tau_y\)-values obtained with the other rheometric systems decreases with decreasing

\(^{25}\) The BML viscometer and the CCS large scale rheometer of Coussot & Piau were already subject of a comparison of different concrete rheometers (Banfill et al. 2000): Based on 12 concrete mixtures of different grain size distribution, maximum particle size (\(d_{\text{max}} = 6.3\) mm, 16 mm, 20 mm), sediment concentration and additifs a total of four rheometers/viscometers were compared in this study. The CCS large scale rheometer exhibited values for the yield stress \(\tau\) which were usually double but in some cases up to four times larger than the values obtained from the BML viscometer. In contrast to the present study, where the flow curve was determined independently and an appropriate model function fitted afterwards, in that study the mixtures were defined as Bingham fluids with the model function \(\tau_y = \mu \dot{\gamma}\) and the conversion from measured into rheological parameters derived accordingly. Thus at this point it is unclear whether that predefinition of the rheological model function or the mixtures themselves led to such a large difference in the yield stress \(\tau\) for the two apparatus in that study.
sediment concentrations ($C_v = 0.6 \rightarrow C_v = 0.55$). This might indicate that wall slip is more pronounced in highly concentrated mixtures compared to less concentrated ones. But it is important to raise the question why the $\tau_y$-values of the Inclined Channel Test become larger compared to the $\tau_y$-values obtained with the other systems when the sediment concentration $C_v$ is decreased further ($C_v < 0.55$). At this point it is unclear whether this phenomenon is also due to wall slip or due to the ratio $h_0/d_{\text{max}}$. In the latter case the layer of the largest particles (thickness $d_{\text{max}}$) is not significantly different from the layer of the complete mixture (thickness $h_0$) and the ratio of deposition depth and maximum particle size $h_0/d_{\text{max}}$ lies somewhere between 1 and 2.4 for sediment concentrations $C_v < 0.55$. Such a small ratio $h_0/d_{\text{max}}$ not in agreement with the conditions usually recommended in sheared fluids where the layer of the complete mixture should outweigh the size of the largest particle by a factor of at least 10 (Van Wazer et al. 1963 in Chhabra & Richardson 1999).

**Kasumeter:**

Except for the lowest concentrated debris flow material mixture ($d \leq 5$ mm, $C_v = 0.488$) and the two investigated clay-gravel-suspensions, the Kasumeter indicates $\tau_y$-values which are significantly larger compared to those obtained with the BMS sphere $D = 12$ mm.

This difference cannot be simply explained by the uncertain definition about the end of the experiment which was the case for the debris flow material mixtures, because the difference of $\tau_y$-value also occurred in the case of the clay-dispersions, where the end of the experiment was clearly defined.

A first answer to prove is the use of a reduced capillary length $L_c' = L_c - 2D_c$ instead of the real capillary length $L_c$ in the $\tau_y$-formula (eq. 5.7), as proposed by Schulze et al. (1991), due to the observation that, at its end, the capillary is not entirely filled with the test fluid. This answer fails because the use of $L_c'$ in eq. 5.7 leads to a larger $\tau_y$-value rather than a smaller one.

Therefore it is argued that the flow from the vertical cylinder into the capillary must be partly disturbed or accompanied by an additional resistance which is not yet considered in the formula to calculate the yield stress $\tau_y$ (eq. 5.7). Blockage of the capillary entrance by larger particles is not assumed to be the reason for the additional resistance, because the difference of $\tau_y$-value was obtained in both the debris flow material mixtures containing particles up to $d_{\text{max}} = 5$ mm grain size and the clay-dispersions containing particles up to $d_{\text{max}} = 0.25$ mm grain size only. Future work should thus clarify the exact reason for this difference of the yield stress $\tau_y$ obtained in the Kasumeter.
Nevertheless, an explanation is required why for three out of eleven mixtures the yield stress $\tau_y$ obtained with the Kasumeter did not differ from the one obtained with the other rheometric systems. A second effect must thus have occurred in case of these three mixtures in order to balance the unknown first effect:

In case of the lowest concentrated debris flow material mixture ($d \leq 5$ mm, $C_v = 0.488$) sedimentation of the larger particles occurred during the experiment. As a consequence not the representative but a less concentrated mixture with an altered grain size distribution passed the capillary. For this altered mixture the yield stress $\tau_y$ was determined. This yield stress $\tau_y$ must be evidently lower than the yield stress $\tau_y$ of the real mixture.

In case of the clay-gravel-suspensions with a bimodal grain size distribution (fines and gravels), it is assumed that the gravels migrated into the next flow path towards the tube center, while the liquid composed of water and the fines (clay-dispersion), remained at the capillary boundary. This effect, which becomes most significant when the ratio of capillary to particle diameter $D_c/d$ is small, leads to a reduction of the stress measurement (Nguyen&Boger 1992).

6.2 Comparison in practical application

In the following, all rheometric systems which were applied for large particle fluids in the present study are compared. The comparison bases on the fluids/mixtures investigated in the present study and includes all important aspects such as "Type of rheological data (flow curve or yield stress $\tau_y$)", "Test fluid characteristics", "Measuring range", "Reliability of rheological data", "Required sample volume as well as required mixing and experimental time" (table 6.1):
Table 6.1 General comparison of rheometry applied for the large particle fluids of the present study. Part A.

<table>
<thead>
<tr>
<th>Rheometry</th>
<th>Type of rheological data</th>
<th>Test fluid characteristics</th>
<th>Measuring range</th>
<th>Reliability of rheological data</th>
<th>Sample volume ($V_{\text{sample}}$), time of mixture preparation and experimental time (*)</th>
</tr>
</thead>
</table>
| BMS       | Flow curve + Yield stress | $d_{\text{max}} = 10$ mm  | $0.1 < \dot{\gamma} < 100$ s$^{-1}$ | Re $< 1$, $\dot{\gamma} < 30$ s$^{-1}$: Good | $V_{\text{sample}}$: 0.5 l  
Mixture preparation: 10'  
Experiment: 30' / sphere |
|           |                          | Small to large $C_r$      | $0.5 < \tau < 2000$ Pa  | Re $> 1$, $\dot{\gamma} > 30$ s$^{-1}$: Modest |
|           |                          |                           | Re $< 300$ ($D = 15$ mm)  | |
|           |                          |                           | Re $< 100$ ($D = 12$ mm)  | |
|           |                          |                           | Re $< 50$ ($D = 8$ mm)    | |
| CCS-LS   | Flow curve + Yield stress | $d_{\text{max}} = 25$ mm  | $1 < \dot{\gamma} < 100$ s$^{-1}$ | $\dot{\gamma} < 30$ s$^{-1}$: Good | $V_{\text{sample}}$: 500 l  
Mixture preparation: 120'  
Experiment: 240' |
|           |                          | Only large $C_r$          | $200 < \tau < 8500$ Pa    | $\dot{\gamma} > 30$ s$^{-1}$: Modest |
|           |                          |                           | In case of hysteresis measurements: Poor |
| BML       | Flow curve + Yield stress | $d_{\text{max}} = 25$ mm  | $10 < \dot{\gamma} < 50 - 80$ s$^{-1}$ | $\dot{\gamma} < 30$ s$^{-1}$: Good | $V_{\text{sample}}$: 10 - 30 l  
Mixture preparation: 20'  
Experiment: 10' |
|           |                          | Medium to large $C_r$     | $20 < \tau < 5000$ Pa     | $\dot{\gamma} > 30$ s$^{-1}$, $\tau < 50$ Pa: Modest |
|           |                          | (Flow curve), Small to large $C_r$ | | |
|           |                          | (Yield stress)            | | |
| KAS       | Yield stress             | $d_{\text{max}} = 10$ mm  | Standard scale ($D_1/L_c = 0.045$) | Modest - Poor | $V_{\text{sample}}$: 10 - 25 l  
Mixture preparation: 20'  
Experiment: 5' - 60' |
<p>|           |                          | Small to medium $C_r$     | $\tau &lt; 40$ Pa  | |
|           |                          |                           | Large scale ($D_1/L_c = 0.128$) | |
|           |                          |                           | $\tau &lt; 300$ Pa          | |</p>
<table>
<thead>
<tr>
<th>Rheometry</th>
<th>Test fluid characteristics</th>
<th>Measuring range</th>
<th>Reliability of rheological data</th>
<th>Sample volume $V_{sample}$, time of measurement $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICF</td>
<td>Small to large $C_r$</td>
<td>$d_{max} = 10\text{ mm}$</td>
<td>In case of bed-wall slip: Modest-Poor</td>
<td>$V_{ICF} = 10 - 100\text{ ml}$, mixture preparation: 20-30' Experiment: 60'</td>
</tr>
<tr>
<td>IPT</td>
<td>Small to large $C_r$</td>
<td>$d_{max} = 10\text{ mm}$</td>
<td>Depends on channel inclination $\theta$ and channel width $h$, respecting $k/b &lt; 11$ For $\tau &lt; 250\text{ Pa}$ For $\tau &gt; 5.7\text{ Pa}$ $b = 0.2\text{ m}$</td>
<td>$V_{IPT} = 10 - 100\text{ ml}$, mixture preparation: 20-120' Experiment: 60'</td>
</tr>
<tr>
<td>ST$^{-}$</td>
<td>Medium to large $C_r$</td>
<td>$d_{max} = 10\text{ mm}$</td>
<td>Depends on plane inclination $\theta$ and the long-wave approximation For $\tau = 13\text{ Pa}$ $V_{sample} &lt; 500\text{ Pa}$</td>
<td>$V_{ST^{-}} = 11\text{ ml}$, mixture preparation: 10' Experiment: 30'</td>
</tr>
<tr>
<td>PD</td>
<td>Medium to large $C_r$</td>
<td>$d_{max} = 10\text{ mm}$</td>
<td>Depends on the sample volume For $V = 11\text{ ml}$</td>
<td>$V_{PD} = 11\text{ ml}$, mixture preparation: 10' Experiment: 30'</td>
</tr>
<tr>
<td>SI</td>
<td>Medium to large $C_r$</td>
<td>$d_{max} = 10\text{ mm}$</td>
<td>For dimensionless slumps $SV/FY = 0.1 &lt; SV &lt; 0.75$ For $H = 0.09\text{ m}$ For $\tau &lt; \tau &lt; 600\text{ Pa}$</td>
<td>$V_{SI} = 11\text{ ml}$, mixture preparation: 10' Experiment: 30'</td>
</tr>
</tbody>
</table>

Table 6.1 General comparison of rheometry applied for the large particle fluids of the present study. Part B.
6.3 Recommendations

6.3.1 Large particle fluids

Based on the above comparison (6.2) the following rheometric systems are recommended for the analysis of large particle fluids \((d_{\text{max}} \geq 1\text{mm})\) in daily practical and laboratory application:

\textit{Flow curve determination:} \hspace{1cm} \textit{Yield Stress determination:}

\begin{align*}
1 \text{ mm} &< d_{\text{max}} \leq 5 \text{ mm}: \\
& \quad \bullet \text{ Ball measuring system} \\
5 \text{ mm} &< d_{\text{max}} \leq 10 \text{ mm}:^a \\
& \quad \bullet \text{ Ball measuring system} \\
& \quad \bullet \text{ BML viscometer} \\
10 \text{ mm} &< d_{\text{max}} \leq 30 \text{ mm}:^b \\
& \quad \bullet \text{ BML viscometer}
\end{align*}

\textit{Legend:}

\begin{itemize}
  \item[a] Compared to the BML viscometer \((10 \leq \dot{\gamma} \leq 100 \text{ s}^{-1})\) the ball measuring system enables the determination of the flow curve in a wide range of the shear rate \((0.01) \ 0.1 \leq \dot{\gamma} \leq 100 \text{ s}^{-1}\), which provides at the same time a good estimation of the yield stress \(\tau_y\). Further, low up to highly concentrated mixtures can be investigated with the ball measuring system, whereas with the BML viscometer only medium to highly concentrated mixtures can be investigated. However, the investigation of liquid fluids is limited to shear rates \(\dot{\gamma} \leq 30 \text{ s}^{-1}\) in the BMS (4.2). In contrast to the BMS the measurements with the BML viscometer are more efficient and the raw data less scattered.
  \item[b] The CCS large scale rheometer of Coussot & Piau is not recommended here because (i) it is not a commercially available apparatus, (ii) the application is limited to high and highly concentrated mixtures only and (iii) fluid preparation, measuring procedure and apparatus cleaning is very intensive.
  \item[c] The Slump Test is the most efficient test for the yield stress determination compared to all other measuring systems. In addition to what was shown by the present study, the test can be easily applied also for mixtures up to \(d_{\text{max}} = 30 \text{ mm}\) grain size.\end{itemize}
the use of a larger test cylinder. For low concentrated mixtures the yield stress is
determined based on the profile of deposit (Coussot et al. 1996). For medium to highly
concentrated mixtures the yield stress $\tau_y$ is determined based on the slump height $S$
respecting the condition for the dimensionless slump height $S' \leq 0.75$ (Pashias et al.
1996).

### 6.3.2 Debris flows

**Granular debris flow:**

As shown in chapter 2, the rheological concept is useful for the characterization of the
pore fluid containing particles up to roughly $d_{\text{max}} = 0.1$ mm grain size. Hence the flow
curve and if present, the yield stress, can be determined with the help of standard
rheometers, respectively standard measuring systems (CCS, CPS or PPS). Note that
effects of wallslip and particle migration might occur by the application of these
measuring systems and must be therefore taken into account.

**Mudflow:**

The fluid phase of a mudflow, for which a rheological treatment is useful, is characterized by a mixture of water and sediment particles up to roughly $d_{\text{max}} = 0.1 - 20$ mm
grain size. Thus for the flow curve determination the ball measuring system or the BML
viscometer are appropriate. If not already determined with the help of these systems, the
yield stress can be obtained efficiently with the Slump Test.

In addition to laboratory measurements the yield stress of the fluid phase can be
determined based on the deposition in the field using the formula developed for the
Inclined Channel Test (in case of channel deposition) or the Inclined Plane Test (in case
of deposition on a plane). Ensure that in the wider area of the location of the measure¬
ments the boundary (bed and wall) of the plane/channel is rough in order not to include
measurements which are influenced by possible wall slip effects.

**Viscous debris flow:**

The viscous debris flow is characterized by a mixture of water and sediment particles
ranging from the clay and silt up to the stone and block fraction ($d_{\text{max}} = 200 - 1000$ mm)
which behaves more or less as one phase. For the entire mixture the rheological
parameters must be extrapolated based on laboratory measurements of different portions
of the entire mixture and/or based on field observations. This is the topic of the
following chapter 7 where the method is explained in detail.
7 Estimation of rheological parameters for viscous debris flows

7.1 A method based on field and laboratory investigations

7.1.1 Objective

The method proposed herein enables to estimate the Herschel-Bulkley parameters $\tau_y$, $m$, $n$ of the prototype debris flow which are required when the flow and stopping process of viscous debris flows in laminar flow regime is modelled with a numerical code (Laigle 1996a e.g.).

7.1.2 Theory

The method has been developed by Coussot et al. (1998) for viscous debris flows where the content of fine material ($d \leq 0.04 \text{ mm}$) relative to the complete grain material is roughly larger than 10 %.

The principal idea is that there is an analogy between the rheological behaviour of debris flow material mixtures which are analysed in the laboratory and the prototype debris flow. The debris flow material mixtures are obtained by adding to a defined volume of water, specific sediment portions from the complete grain material of the prototype debris flow. The debris flow material mixtures are thus regarded as partial fluids of the prototype debris flow.

The analogy between the debris flow material mixtures and the prototype debris flow is expressed by a specific value for the Herschel-Bulkley-index $n$ and by a constant ratio of the Herschel-Bulkley viscosity coefficient and the yield stress $m/\tau_y$. Thus, (1) knowing the yield stress $\tau_y$ of the prototype debris flow based on field observations and (2) determining $\tau_y$, $m$ and $n$ of debris flow material mixtures based on laboratory measurements, the Herschel-Bulkley-index $n$ and the Herschel-Bulkley viscosity coefficient $m$ of the prototype debris flow are extrapolated.

The method is divided into the following main steps:

1. Determination of the yield stress $\tau_y$ of the prototype debris flow in the field
2. Material excavation and determination of full grain size distribution
3. Preparation of debris flow material mixtures and flow curve determination
4. Extrapolation of rheological parameters to prototype debris flow
7.1.3 Example

In the following the method is applied for the debris flow which occurred on May 3 2001 in the Maschänserüfe in Trimmis near Chur/Switzerland. For this debris flow the deposition material was excavated and laboratory investigations were conducted in the present study.

The content of fine material \( d \leq 0.04 \) mm relative to the complete grain material is 9 % (fig. 3.1a). Because this value is close to the minimum value of 10 % proposed by Coussot (1996) the application of the method is considered useful.

1. Determination of the yield stress \( \tau_y \) of the prototype debris flow in the field:

It is referred to the deposition of the debris flow as shown in fig. A.1 of Appendix A. The asymptotic deposition depth \( h_0 \) of the 4 m large and 20 m long deposition tongue varied between 0.6 and 0.75 m and the rough (stony) deposition plane was inclined under an angle \( i = 6.8 \) to 8.5°.

As the undisturbed deposition was discovered two days later, it was not possible to determine the real sediment concentration \( C_{v, DF} \) of the debris flow. For reasons of simplicity it is assumed that this concentration was somewhere between 0.77 < \( C_{v, DF} \) < 0.8 which corresponds to a debris flow density of \( \rho = 2309 \) kg/m³ to 2360 kg/m³.

Using the well known formula already presented in eq. 5.10, the yield stress \( \tau_y \) is calculated as:

\[
\tau_y = \rho \cdot g \cdot h_0 \cdot \sin(i)
\]

(7.1)

With the above values the yield stress is calculated as \( \tau_y = 2000 \pm 500 \) Pa.

Note that the value of the yield stress \( \tau_y \) can also be estimated based on the profile of deposit (eq. 5.14) or based on the profile of deposit of levées and/or further methods described in Coussot (1996). However, except eq. 5.14, the other methods are theoretically less founded and should be thus applied only if the first methods cannot be applied.

Eq. 5.14 was not applied here because the front part of the deposition in the direction of the steepest slope fell into a small erosion reach (fig. A.1 Appendix A).

---

\(^{26}\) The inclination \( i \) was determined with the help of an accurate topographic map based on terrestic survey. In the absence of a map also a simple inclinometer can be used.

\(^{27}\) Note that the range of sediment concentration where the debris flow flows more or less as one phase was assessed as 0.74 < \( C_{v, DF} \) < 0.83 (chapter 3).
2. *Excavation and determination of full grain size distribution:*

The excavation of the deposition material and the procedure for the determination of the grain size distribution of the complete debris flow material is documented in Appendix A and section 3.3.3. The full grain size distribution is shown in fig. 3.1 a).

3. *Preparation of debris flow material mixtures and flow curve determination:*

Based on the complete debris flow material the following sediment portions were considered for the different debris flow material mixtures:

\[ d \leq 0.25 \text{ mm}, \quad d \leq 1 \text{ mm}, \quad d \leq 5 \text{ mm}, \quad d \leq 7 \text{ mm}, \quad d \leq 10 \text{ mm} \quad \text{and} \quad d \leq 25 \text{ mm}. \]

The material portions were obtained by sieving from the complete grain material (for the grain size distributions of each portion refer to fig. 3.1 b)).

Based on the total concentration of the prototype debris flow ranging between \( C_{v,DF} = 0.77 \) and \( 0.8 \) the concentrations \( C_v \) of the different debris flow material mixtures were determined with eq. 3.8, which is given here again:

\[
C_v = \frac{p \cdot C_{v,DF}}{1 + C_{v,DF} \cdot (p - 1)} \quad (7.2)
\]

where \( p \) is the content of the specific portion within the complete grain material (for \( p \) refer to fig. 3.1 a).

Considering two concentrations of the prototype debris flow \( (C_{v,DF} = 0.77, \quad C_{v,DF} = 0.8) \), a total of 12 debris flow material mixtures resulted for the selected material portions (table 7.1). These 12 mixtures were subject of laboratory investigation using appropriate rheometric tools. Based on the recommendations given in section 6.3, the ball measuring system (BMS) and the BML viscometer (BML) were considered here.

The measuring procedure and the flow curve determination for both rheometric systems are shown in chapter 4 (BMS) and 5.2 (BML). The results of the flow curve fit using the Herschel-Bulkley model are summarized in table 7.1.
Estimation of rheological parameters for viscous debris flows

<table>
<thead>
<tr>
<th>Mixture</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ [mm]</td>
<td>≤0.25</td>
<td>≤1</td>
<td>≤5</td>
<td>≤7</td>
<td>≤10</td>
<td>≤25</td>
</tr>
<tr>
<td>$C_v$ [-]</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>$C_{v,DF}$ [-]</td>
<td>0.386</td>
<td>0.458</td>
<td>0.514</td>
<td>0.529</td>
<td>0.551</td>
<td>0.615</td>
</tr>
<tr>
<td>Applied Rheometry</td>
<td>BMS</td>
<td>BMS</td>
<td>BMS</td>
<td>BMS</td>
<td>BMS</td>
<td>BML</td>
</tr>
</tbody>
</table>

| $\tau_f$ [Pa] | 5.6 | 7.5 | 12.0 | 13.7 | 19.5 | 25 |
| $m$ [Pa s$^n$] | 2.4 | 1.9 | 0.84 | 0.66 | 0.80 |
| $n$ [-] | 0.35 | 0.44 | 0.87 | 0.90 | 1.1 |
| $m/\tau_f$ [s$^{-1}$] | 0.43 | 0.25 | 0.07 | 0.05 | 0.04 |

<table>
<thead>
<tr>
<th>Mixture</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ [mm]</td>
<td>≤0.25</td>
<td>≤1</td>
<td>≤5</td>
<td>≤7</td>
<td>≤10</td>
<td>≤25</td>
</tr>
<tr>
<td>$C_{v,DF}$ [-]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$C_v$ [-]</td>
<td>0.429</td>
<td>0.502</td>
<td>0.558</td>
<td>0.573</td>
<td>0.595</td>
<td>0.643</td>
</tr>
<tr>
<td>Applied Rheometry</td>
<td>BMS</td>
<td>BMS</td>
<td>BMS</td>
<td>BMS</td>
<td>BMS</td>
<td>BML</td>
</tr>
</tbody>
</table>

| $\tau_f$ [Pa] | 13.5 | 22.7 | 38.9 | 49.8 | 78.7 | 88 |
| $m$ [Pa s$^n$] | 5.3 | 4.8 | 1.9 | 2.9 | 3.5 | 1.6 |
| $n$ [-] | 0.35 | 0.51 | 0.80 | 0.80 | 0.86 | 0.83 |
| $m/\tau_f$ [s$^{-1}$] | 0.39 | 0.21 | 0.05 | 0.06 | 0.05 | 0.02 |

Table 7.1 Herschel-Bulkley parameter values obtained after data fit for the debris flow material mixtures 1 to 12 investigated with the ball measuring system (BMS) and the BML viscometer (BML). The values indicated for the BMS are average values obtained for the three spheres (Appendix E).

4. Extrapolation of rheological parameters to prototype debris flow:

The values of the Herschel-Bulkley index $n$ and the ratio $m/\tau_f$ obtained for the different debris flow material mixtures and shown in table 7.1 are subject of the extrapolation.

Considering all mixtures, no constant value resulted, neither for the Herschel-Bulkley index $n$ nor the ratio $m/\tau_f$. Thus, it is recommended to proceed with a band width for each of the two parameters. Based on the intuitive consideration that the rheological analogy between the debris flow material mixtures and the prototype debris flow is better for the coarser mixtures (debris flow material mixtures 3-6 and 9-12) than for the finer mixtures (debris flow material mixtures 1-2 and 7-8), the parameter band is defined as follows:

a) $n = 1, \quad m/\tau_f = 0.05$

b) $n = 0.65, \quad m/\tau_f = 0.1$

With knowledge of the yield stress of the prototype debris flow ($\tau_f = 2000$ Pa) the Herschel-Bulkley coefficient $m$ can now be calculated based on the specific ratio $m/\tau_f$.

The calculated Herschel-Bulkley parameter values of the prototype debris flow are listed in table 7.2 and the flow curves shown in fig. 7.1. In case a) the debris flow is reduced to a Bingham fluid ($n = 1$), whereas in case b) it is a Herschel-Bulkley fluid with a slight shear-thinning behaviour.
7 Estimation of rheological parameters for viscous debris flows

<table>
<thead>
<tr>
<th>$\tau_y$</th>
<th>$n$</th>
<th>$m/\tau_y$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Pa]</td>
<td>[-]</td>
<td>[s$^n$]</td>
<td>[Pa s$^n$]</td>
</tr>
<tr>
<td>a) 2000</td>
<td>1</td>
<td>0.05</td>
<td>100</td>
</tr>
<tr>
<td>b) 2000</td>
<td>0.65</td>
<td>1</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 7.2 Herschel-Bulkley parameter values estimated for the prototype debris flow.

![Flow curve range for the prototype debris flow determined with the method based on field and laboratory investigations.](image)

Figure 7.1 Flow curve range for the prototype debris flow determined with the method based on field and laboratory investigations.

7.2 A simplified method based on field observations

7.2.1 Objective

The method enables to estimate the Bingham parameters $\tau_y$ and $\mu_B$ of the prototype debris flow which are required when the flow and stopping process of viscous debris flows is modelled with a numerical code which bases on a simplified Bingham model (O’Brien et al. 1993 e.g.).

Note that the method here can also be applied for numerical codes which are based on a Herschel-Bulkley model. In this case the more sophisticated Herschel-Bulkley fluid is reduced to the simplified Bingham fluid.

7.2.2 Theory

The method is a direct application of the theories of Johnson (1970) and Coussot (1997) concerning the flow and stopping process of Yield Stress Fluids in idealized flow geometries.

The yield stress $\tau_y$ of the debris flow is determined based on the deposition in the field with the technique already described in the method above (7.1).
The Bingham (viscosity) parameter $\mu_B$ is estimated based on the yield stress $\tau_y$ as well as the observed flow depth $h$ and surface velocity $v_s$ of the same debris flow. The derivation for the calculation of the Bingham (viscosity) parameter $\mu_B$ is given in the following:

The flow depth $h$ of a Yield Stress Fluid (YSF) is roughly divided into a plug zone with the depth $h_{plug}$ and a shear zone $h_{shear}$ (fig. 7.2). For the present method the velocity distribution within the sheared zone is simplified to a linear distribution (fig. 7.2).

With knowledge of the yield stress $\tau_y$ and for the case without effects of lateral boundaries, the thickness of the plug zone $h_{plug}$ can be estimated as:

$$h_{plug} = \frac{\tau_y}{\rho \cdot g \cdot \sin(i)}$$

(7.3)

with $i$ = inclination angle of the flow bed. In the case where observations are made in a confined channel, effects of lateral boundaries must be considered. For the simplified case of a semicircular channel it is (Johnson 1970):

$$h_{plug} = \frac{2 \cdot \tau_y}{\rho \cdot g \cdot \sin(i)}$$

(7.4)

Based on the calculated plug thickness $h_{plug}$, the thickness of the shear zone $h_{shear}$ is calculated as:

$$h_{shear} = h - h_{plug}$$

(7.5)
with \( h = \text{observed flow depth} \). For a simplified linear velocity distribution the apparent shear rate \( \dot{\gamma} \) relative to the observed velocity \( v_s \) is (provided that the flow bed is rough and thus bed slip can be neglected):

\[
\dot{\gamma} = \frac{v_s}{h_{\text{shear}}} \tag{7.6}
\]

Assuming an identical density \( \rho \) over the entire flow depth \( h \), the bed shear stress \( \tau_{\text{bed}} \) is:

\[
\tau_{\text{bed}} = h \cdot \rho \cdot g \cdot \sin(i) \tag{7.7}
\]

The Bingham viscosity parameter \( \mu_B \) can now be calculated as:

\[
\mu_B = \frac{\tau_{\text{bed}} - \tau_{\gamma}}{\dot{\gamma}} \tag{7.8}
\]

The method is divided into the following main steps:

1. Determination of the yield stress \( \tau_{\gamma} \) in the field
2. Observation of the flowing debris flow
3. Determination of the Bingham parameter \( \mu_B \)

### 7.2.3 Example:

The method is applied for the same debris flow being already subject of the first method (chapter 7.1). Remember that the density of the debris flow is roughly \( \rho = 2300 \text{ kg/m}^3 \).

1. **Yield stress determination in the field:**

The procedure and calculation was explained in detail in the method introduced before (7.1.3) where the yield stress was calculated as \( \tau_{\gamma} = 2000 \pm 500 \text{ Pa} \).

2. **Observations of the flowing debris flow:**

Observations of the flow depth \( h \) and the surface velocity \( v_s \), are usually provided by local people (foresters, farmer, etc.), seldom by automatic measurements in a instrumented debris flow torrent. For the specific debris flow analyzed with the above method (7.1) no observations were made. Here the following assumptions are given:

- Inclination of the flow bed \( i = 10^\circ \) (no effects of side boundary, rough flow bed)
- Flow depth \( h = 0.75 \text{ m} \)
- Surface velocity \( v_s = 2 \text{ m/s} \)
3. Determination of the Bingham parameter \( \mu_B \):

Based on eq. 7.3 and eq. 7.5 to 7.8, the following results are obtained:

\[
\begin{align*}
    h_{plug} &= 0.51 \text{ m}, \\
    h_{shear} &= 0.24 \text{ m}, \\
    \dot{\gamma} &= 8.4 \text{ s}^{-1}, \\
    \tau_{bed} &= 2940 \text{ Pa} \\
\end{align*}
\]

\( \rightarrow \quad \mu_B = 110 \text{ Pa s} \)

Based on the calculated Bingham parameter \( \mu_B \) the flow curve of the prototype debris flow is shown in fig. 7.3.

\begin{figure}[h]
    \centering
    \includegraphics[width=0.5\textwidth]{flow_curve.png}
    \caption{Flow curve of the prototype debris flow determined with the simplified method based on field observations: \( \tau_f = 2000 \text{ Pa}, \mu_B = 110 \text{ Pa s} \).}
\end{figure}

7.3 Discussion

7.3.1 Estimation of rheological parameters of a specific debris flow

One of the two presented methods (7.1 and 7.2) for the estimation of the flow curve parameters must be applied when the flow and deposition process of a specific viscous debris flow is modelled with a numerical code or in a physical model.

Both methods require the determination of the yield stress \( \tau_f \) based on geometric features of the debris flow deposition in the field. This can be usually done anytime after the event as long as no other processes (floods, excavation e.g.) alter the original geometry of the deposition (skeleton).

Applying the first method (7.1) for the determination of the other flow curve parameters (Herschel-Bulkley-parameters \( m \) and \( n \)), it is indispensable to excavate undisturbed deposition material for laboratory analysis. This means that the grain material from the clay up to the silt or sand fraction may not be washed out by rainfall or by small floods passing or running over the deposition after stoppage of the debris flow.

The first method requires intensive work (excavation, determination of grain size distribution, laboratory investigations) of approximately two person weeks. The reliability of the extrapolated flow curve parameters \( m \) and \( n \) is still relatively poor. This is
because the grain material within the coarsest analyzed debris flow material mixtures 
\(d \leq 25 - 30\) mm often represents less than 50% of the complete grain material of the 
prototype debris flow. As a consequence, the shear stresses measured in the coarsest 
debris flow material mixtures are one decade smaller (several hundred Pa) than the 
shear stresses present in the prototype debris flow (several thousand Pa).

In comparison, the second method (7.2) requires information about the flow depth and 
the surface velocity of the debris flow in order to determine the flow curve parameter 
(Bingham parameter) \(\mu_b\). However this information is not always available and the 
quality of the information is often poor. Further, the second method relies on the 
Bingham model which is a simplification of the Herschel-Bulkley-model. This might 
lead to relatively less accurate results in the numerical simulation of the flow and 
deposition process (see also 2.4.4).

It is worth to stress again that both methods rely on a rather large generalization of the 
real physics within a viscous debris flow. As introduced in chapter 2, a debris flow is 
usually characterized by changes in grain size distribution and sediment concentration 
from the front to the tail and from the initiation down to the deposition area. 
Accordingly the values of the flow curve parameters develop with position and time. 
Application of the above methods in a given case thus requires experience and a careful 
appraisal of the specific debris flow and the given catchment area.

7.3.2 Estimation of rheological parameters for hazard zone mapping

For the hazard assessment within hazard zone mapping projects, not only one specific 
but several debris flows of different return periods are usually considered. Viscous 
debris flows of different return periods are characterized by different volumes, different 
maximum discharges and different rheological parameters, which must be estimated in 
advance. The most crucial parameter is the debris flow volume, followed by the 
rheological parameters and the maximum discharge. While the volume and the 
maximum discharge is usually estimated based on historical data and/or based on field 
investigations in the catchment area, the following principles are given here for the 
estimation of the rheological parameters:

- Definition of a yield stress range \(\tau_y\) based on the deposition features of different 
past events.

- Definition of a yield stress range according to field experiences in other 
catchment areas of viscous debris flows (Laigle 1996b):
\[
500 - 1'500 \leq \tau_y \leq 5'000 - (10'000) \text{ Pa}
\]
• Definition of range of Herschel-Bulkley parameters $m$ and $n$ based on rheological analysis of different undisturbed deposition samples within the catchment area. Very intensive work.

• Definition of range of Bingham parameter $\mu_B$ based on diverse observations of flow depth $h$ and flow velocity $v$ of past events.

• Definition of range of Herschel-Bulkley parameters $m$ and $n$ according to field experiences in other catchment areas of viscous debris flows (Laigle 1996b, Laigle et al. 2003): $n = 0.33$, $0.1 \leq m/\tau_y \leq 0.35$ s$^{0.33}$.

The debris flow density usually ranges from $\rho = 2'000$ to $2'400$ kg/m$^3$. The ratio $\tau_y/\rho$ often varies between 0.8 and 2.2 (Laigle et al. 2003) but seldom varies between 0.25 and 4 m$^2$/s$^2$ (Laigle 1996b).

Finally it must be noted that in a catchment area where - due to the presence of large quantities of fine material - usually viscous debris flows occur, also other types of flow, such as fluvial transport, hyperconcentrations or even granular flows may occur occasionally. For these type of flows, others than the rheological concept must be considered additionally, as outlined in chapter 2.
8 Conclusions

8.1.1 Summary and recommendations

*Goal of the study:*

The present thesis has focused on rheometric systems and methods for the determination of the rheological properties of large particle fluids \((d_{\text{max}} \geq 1\text{mm})\) in general and of debris flows in particular.

The main object of the study was the investigation of the ball measuring system and the improvement of the theory for the conversion of measured into rheological data.

The second object was the comparison of the ball measuring system with other rheometric systems for large particle fluids as far as (i) rheological data and (ii) practical application was concerned.

The third object was to prove the application of the examined rheometric systems and further rheometric techniques for the rheological investigation of debris flows.

*The ball measuring system:*

The standard experiment consists in measuring the torque \(T\) while the excentrically rotating sphere drags across the test fluid at a prescribed rotational speed \(\Omega\) for one sphere rotation.

For the flow curve determination experiments of one sphere rotation must be conducted at different prescribed rotational speeds \(\Omega\). Experiments with one and with several rotations at different prescribed rotational speeds \(\Omega\) allow the determination of fluid relaxation time and fluid fatigue.

Based on the experiments it could be shown that the investigation of particle fluids up to \(d_{\text{max}} = 10\text{ mm}\) grain size is possible with all the used sphere sizes \(D (D = 15, 12\text{ and } 8\text{ mm})\). However, the standard deviation of the measured torque \(T\) increased with the increase of the relative maximum grain size \(d_{\text{max}}/D\). The standard deviation of the rotational speed \(\Omega\) mainly increased with an increase of the sediment concentration \(C_v\), but increased further with an increase of the relative maximum grain size \(d_{\text{max}}/D\) as well as with an increase of the content of large particles in the sediment material.

For the conversion of measured speed \(\Omega\) and torque \(T\) into the rheological parameters shear rate \(\dot{\gamma}\) and shear stress \(\tau\), a new semi-empirical approach has been elaborated. For the conversion of the speed \(\Omega\) into the shear rate \(\dot{\gamma}\) the approach is based on the specific
power characteristics (Metzner-Otto-theory) for each sphere-sphere holder-container-configuration. Compared to the earlier approach of Tyrach (2001) which is based on the drag characteristics, it was found irrelevant whether the power characteristics or the drag characteristics is considered in the theory for the conversion of the speed $\Omega$ into the shear rate $\dot{\gamma}$. Important is that in the conversion of the torque $T$ into the shear stress $\tau$ characteristic types of fluids such as Newtonian, Power-Law and Yield Stress Fluids are distinguished and distinctive relationships between $T$ and $\tau$ derived accordingly. As a consequence the new approach fulfills the yield stress criterion for spheres starting to move in Yield Stress Fluids. This is not the case for the approach of Tyrach were only Newtonian fluids were considered. Additionally relationships were derived for both the laminar flow regime ($Re \leq 1$) and the transitional regime ($Re > 1$) with the new approach which is in contrast to the earlier approach where only the laminar regime was considered.

For the future investigation of a given fluid it is therefore recommended to use the relationships developed in the present study. These are namely:

\[
Re \leq 1 \text{ and } Re > 1: \quad \dot{\gamma} = K_\Omega \cdot \Omega \\
Re \leq 1: \quad \tau = K_T \cdot T \\
Re > 1: \quad \tau = \left[k_1 + k_2 \cdot e^{-k_3 \cdot Re}\right] \cdot T
\]

Applying the specific relationships to Yield Stress Fluids containing particles equal or larger than $d_{max} = 5$ mm grain size, slightly different flow curves were obtained depending on the used sphere size $D$.

As a consequence the flow curve can be shown as a single curve for fluids containing particles equal or smaller than $d_{max} = 1$ mm grain size. For fluids containing particles equal or larger than $d_{max} = 5$ mm grain size, the flow curve must be indicated in a band. The following two methods are recommended for the determination of the band:

1. Flow curves obtained with the largest and the smallest sphere ($D = 15$ mm and $D = 8$ mm). (2) Flow curve obtained with the medium sphere ($D = 12$ mm) and indication of an error band of $\pm 20\%$ ($d_{max} = 5$ mm), $\pm 30\%$ ($d_{max} = 7$ mm) and $\pm 40\%$ ($d_{max} = 10$ mm).

Application and comparison of rheometry for large particle fluids:

For the flow curve determination the CCS large scale rheometer of Coussot&Piau and the BML viscometer of Wallevik&Gjorv were used in addition to the ball measuring system. For the determination of the yield stress the Kasumeter, the Inclined Channel
Test, the Inclined Plane Test and the Slump Test were used in addition to the above apparatus. The investigated large particle fluids (all Yield Stress Fluids) were either (i) debris flow material mixtures, (ii) clay-gravel-suspensions or (iii) a specific mixed suspension.

For the flow curves the range of shear stresses obtained with the three different rheometric systems was roughly identical for both the debris flow material mixtures as well as the mixed suspension. This enhances the quality of the rheological data and strengthens the reliability of the applied rheometric apparatus.

For the yield stress \( \tau_y \), a variation of usually less than \( \pm 30\% \) from the results of the ball measuring system \( (D = 12\, \text{mm}) \) was obtained for all measuring systems, except for the Kasumeter and the Inclined Channel Test. The difference of the results obtained with the Inclined Channel Test is probably due to wall slip (in case of medium to large sediment concentrations \( C_v \)) or due to a relatively small ratio of deposition thickness and maximum grain size \( h_0/d_{\text{max}} \) (in case of small sediment concentrations \( C_v \)). The difference of the results obtained with the Kasumeter was assumed to be due to disturbances in the flow from the vertical cylinder into the relatively short horizontal outlet capillary.

With regard to practical application the rheometry used herein showed large differences as far as the measuring range, the required time and man power for the conduction of the experiments as well as the preparation of the sample volume was concerned. As a result the following rheometric systems/apparatus are recommended for daily practical application:

**Recommendations for the application of rheometry for large particle fluids:**

**Flow curve determination:**

1 mm < \( d_{\text{max}} \) ≤ 5 mm:

- Ball measuring system

5 mm < \( d_{\text{max}} \) ≤ 10 mm:

- Ball measuring system
- BML viscometer

10 mm < \( d_{\text{max}} \) ≤ 30 mm:

- BML viscometer

**Yield Stress determination:**

\( d_{\text{max}} \leq 30\, \text{mm} \):

- Slump Test
**Determination of rheological parameters for debris flows:**

Rheology is considered as an essential physical concept for the numerical or physical modeling of the flow and stop process of **viscous debris flows and mud flows**.

Regarding **granular debris flows** rheology is - from a theoretical point of view - an inappropriate concept and considered only relevant on the level of the pore fluid containing particles up to \( d_{\text{max}} = 0.1 \text{ mm grain size} \). The flow curve parameters of the pore fluid can be determined based on measurements in conventional rheometric systems using standard rheometers.

For **mud flows**, where a muddy phase can be distinguished from a solid phase, the application of rheology is limited to the muddy phase in laminar flow. For turbulent flow of the muddy phase and grain inertia of the solid phase, additional physical concepts must be taken into account. The maximum grain size within the mud is strongly dependent on the sediment concentration and grain size distribution of the mud and varies roughly between \( d_{\text{max}} = 0.1 \) and \( 20 \text{ mm} \). Hence the ball measuring system and the BML viscometer are regarded as useful and efficient rheometric tools for the flow curve determination of the mud.

For **viscous debris flows** the shear flow spans over the entire grain material up to the block and boulder fraction \( (d_{\text{max}} = 200 \text{ to } 1000 \text{ mm}) \). In the present study two methods were applied for the estimation of the flow curve parameters:

The first method includes (i) the study of the deposition features in the field and (ii) intensive rheological analysis of the debris flow material in the laboratory. Here different portions \( (1 \text{ mm} \leq d_{\text{max}} \leq 20 \text{ to } 30 \text{ mm}) \) from the complete material must be analysed for which the application of the ball measuring system and BML viscometer is recommended. Based on field and laboratory analysis the flow curve parameters of the prototype debris flow are finally extrapolated.

The second method is entirely based on field observations. Beside the study of the deposition features, the method requires direct observations during the debris flow event at one or several flow sections.

Both methods allow a reasonable estimation of the flow curve parameters. If possible it is recommended to apply both methods in order to compare the resulting flow curves.

The use of a large scale device of the ball measuring system or the BML viscometer, allowing the investigation of debris flow material mixtures containing particles larger than \( d_{\text{max}} = 30 \text{ mm grain size} \), could significantly enhance the quality of the extrapolation in case of the first method.
8.1.2 Outlook

**Rheometric systems/apparatus for large particle fluids:**

The following further studies are recommended in case of the ball measuring system:

1. The empirical relationships for the conversion of measured torque into rheological shear stress data developed for particulated Yield Stress Fluids must be tested and possibly further developed with the help of other sediment water mixtures (other grain size distributions) than used in the present study. This is notably important in the transitional flow regime (Re > 1) where the clay-dispersions indicated another course of this relationship than the debris flow material mixtures.

2. The flow curves obtained for the different sphere sizes D did not coincide for fluids containing particles up to \( d_{\text{max}} = 5 \) to \( 10 \) mm grain size. This was assumed to be due to the length of the immersed part of the sphere holder \( s_{\text{im}} \) and the distance between the sphere holder and the container wall which was not equal for the different applied spheres and which lead to a different jamming behaviour in such fluids depending on the sphere size \( D \). In order to avoid this phenomenon it is thus worth to test a larger container where the distances between the sphere/sphere holder and the container boundary are sufficiently large and to test the influence of the immersing length \( s_{\text{im}} \) on the rheological results of such fluids by variation of \( s_{\text{im}} \).

3. The shear rate \( \dot{\gamma} \) and Reynolds number Re in the ball measuring system have representative and not true character. A method is given in 4.6.4 how to possibly reduce the gap between the representative and true value of the shear rate \( \dot{\gamma} \) and the Reynolds number Re, respectively.

**BML viscometer:** In collaboration with the manufacturer the measuring range of the rotational speed \( \Omega \) should be extended towards both low and high rotational speeds \( \Omega \). Accordingly the range of shear rates \( \dot{\gamma} \) can be extended, which improves the determination of the flow curve and the yield stress \( \tau_y \).

**Slump Test:** The experimental profile of deposit did not follow well the predicted profile, especially in the case of the more liquid mixtures. Further work should focus on the investigation of the influence of the roughness of the spreading plane on the profile of deposit and the influence of particle migration during the test.

**Inclined Channel Test:** For a better understanding of wall slip effects and the minimum deposition thickness \( h_0 \) in Yield Stress Fluids containing large particles, future work should focus on the influence of (i) different boundary roughness and (ii) the
relative maximum grain size $d_{\text{max}}/h_0$ on the yield stress $\tau_y$. Investigations should be performed for different particle fluids as well as for different sediment concentrations $C_v$ of a given particle fluid.

**Determination of rheological parameters for viscous debris flows:**

In order to enhance the quality of the extrapolation method (7.1), one might consider constructing a large scale device of the ball measuring system or the BML viscometer.

Expanding the present devices to a scale that debris flow material mixtures up to $d_{\text{max}} = 100$ mm grain size can be investigated, requires sample volumes of around 500 liters. Experience with the large scale concentric cylinder system of Coussot & Piau (1995) showed that (i) sample volumes of this order requires a lot of time and manpower and that (ii) control of sediment concentration and temperature of such a large sample becomes very difficult.

It is therefore recommended to develop a large scale device of either the ball measuring system or the BML viscometer for the analysis of material up to $d_{\text{max}} = 60$ mm grain size. This would require a sample volume of around 100 liters which can be still managed and controlled with reasonable time and effort.

Instead of developing a large scale laboratory device of the ball measuring system, consideration should be given to install a large measuring sphere directly into a debris flow torrent and to measure the rheological parameters directly in the field. However one must be aware of the very large forces occurring in debris flows and the difficulty to guarantee the stability of the measuring sphere under these circumstances.
Notation

A  | surface area [m^2]
A_s | surface area of sphere [m^2]
A_h | surface area of sphere holder [m^2]
a, a_f | empirical constant used for different applications [-]
b | empirical constant used for different applications [-]
B* | Herschel-Bulkley (Bingham) number [-]
C  | system number (drag flow characteristics) [-]
C_l | system number (power characteristics) [-]
C_v | volumetric sediment concentration/solid concentration [-]
C_0 | max. volumetric package concentration for spheres of identical size [-]
C_v_DF | volumetric sediment concentration of debris flow [-]
C_v_fines | volumetric concentration of fine material (d ≤ 0.25 mm) [-]
C_v_gravel | volumetric concentration of rounded gravel (d = 3 to 10 mm)[-]
C_v_tot | total volumetric concentration of clay-gravel suspension [-]
c | generalized flow resistance term for Chézy/Manning-Strickler equations
c' | cohesion [Pa]
c_D | drag coefficient [-]
D  | sphere diameter in the ball measuring system [m]
D_c | capillary diameter, container diameter [m]
D_t | tube diameter [m]
D_hy | hydraulic diffusivity [m^2/s]
d  | grain size [m]
d_max | maximum grain size [m]
d_s | equivalent sphere diameter [m]
F  | force [N]
g | acceleration due to gravity [m/s^2]
H  | gap width [m]
H_c | container height in Slump Test [m]
H* | dimensionless height in the Inclined Plane Test and in the Slump Test [-]
h  | flow depth, local thickness of fluid [m]
h_0 | deposition depth [m]
h_f | fluid stagnation depth (Kasumeter), final material depth (Slump Test) [m]
i | inclination angle [°]
j | indices used for empirical constants
K_T | constant relating T with τ in the laminar flow regime for the BMS [m^3]
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_T$</td>
<td>dimensionless value of the constant $K_T$ for the BMS [-]</td>
</tr>
<tr>
<td>$K_{\Omega}$</td>
<td>Constant relating the rotational speed $\Omega$ with the shear rate $\gamma$ [-]</td>
</tr>
<tr>
<td>$k, k_j$</td>
<td>empirical coefficients used for different applications [-]</td>
</tr>
<tr>
<td>$L$</td>
<td>length of cylinder within concentric cylinder system [m]</td>
</tr>
<tr>
<td>$L_c$</td>
<td>length of capillary [m]</td>
</tr>
<tr>
<td>$l_m$</td>
<td>mixing length [m]</td>
</tr>
<tr>
<td>$M$</td>
<td>mass [kg]</td>
</tr>
<tr>
<td>$M_s$</td>
<td>mass of solids [kg]</td>
</tr>
<tr>
<td>$M_f$</td>
<td>mass of fluid [kg]</td>
</tr>
<tr>
<td>$M_w$</td>
<td>mass of water [kg]</td>
</tr>
<tr>
<td>$m$</td>
<td>Power Law- or Herschel-Bulkley- consistency coefficient [Pa s$^n$]</td>
</tr>
<tr>
<td>$N_B$</td>
<td>Bagnolds number: collisional vs. macro-viscous stresses [-]</td>
</tr>
<tr>
<td>$N_S$</td>
<td>Savage number: collisional vs. Coulomb-friction stresses [-]</td>
</tr>
<tr>
<td>$n$</td>
<td>Power Law index, Herschel-Bulkley index [-]</td>
</tr>
<tr>
<td>$n_m$</td>
<td>Manning surface roughness parameter [$m^{1/3}$]</td>
</tr>
<tr>
<td>$p$</td>
<td>content of grain material portion within complete grain material [-]</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure, pore-fluid pressure [Pa]</td>
</tr>
<tr>
<td>$P_{\text{bed}}$</td>
<td>basal pore-fluid pressure [Pa]</td>
</tr>
<tr>
<td>$Q$</td>
<td>flow, discharge [$m^3/s$]</td>
</tr>
<tr>
<td>$Q_p$</td>
<td>debris flow peak discharge [$m^3/s$]</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>dynamic parameter [-]</td>
</tr>
<tr>
<td>$R$</td>
<td>coefficient of correlation [-]</td>
</tr>
<tr>
<td>$R_i$</td>
<td>radius of the inner cylinder [m]</td>
</tr>
<tr>
<td>$R_o$</td>
<td>radius of the outer cylinder [m]</td>
</tr>
<tr>
<td>$Re$</td>
<td>sphere Reynolds number [-]</td>
</tr>
<tr>
<td>$Re^*$</td>
<td>sphere Reynolds number for Power Law Fluids [-]</td>
</tr>
<tr>
<td>$r$</td>
<td>radius, radius of center sphere path in the BMS [m]</td>
</tr>
<tr>
<td>$r_c$</td>
<td>container radius [m]</td>
</tr>
<tr>
<td>$S$</td>
<td>slump height [m]</td>
</tr>
<tr>
<td>$S^*$</td>
<td>dimensionless slump height [-]</td>
</tr>
<tr>
<td>$s_b$</td>
<td>distance between sphere and container bottom [m]</td>
</tr>
<tr>
<td>$s_w$</td>
<td>distance between sphere and container wall [m]</td>
</tr>
<tr>
<td>$s_{\text{im}}$</td>
<td>length of immersed holder part [m]</td>
</tr>
<tr>
<td>$T$</td>
<td>torque [Nm]</td>
</tr>
<tr>
<td>$T_0$</td>
<td>torque due to apparatus resistance [Nm]</td>
</tr>
<tr>
<td>$T_{\text{tot}}$</td>
<td>total torque [Nm]</td>
</tr>
<tr>
<td>$t$</td>
<td>time [s]</td>
</tr>
<tr>
<td>$t_{\text{st}}$</td>
<td>duration of steady flow [s]</td>
</tr>
</tbody>
</table>
Notation

$V$ volume [m$^3$]

$V_{\text{sample}}$ sample volume [m$^3$]

$v$ velocity [m/s]

$v_s$ surface velocity [m/s]

$X^*$ dimensionless distance in the Inclined Plane Test [-]

$x$ distance from the border of deposition [m]

$x,y,z$ coordinates

$Y$ static equilibrium number for spheres in Yield Stress Fluids [-]

$\alpha$ dynamic friction angle, cone angle in the CPS [$^\circ$]

$\varphi$ internal friction angle [$^\circ$]

$\varphi'$ internal friction angle considering effective stresses [$^\circ$]

$\varphi_{\text{bed}}$ basal friction angle [$^\circ$]

$\dot{\gamma}$ shear rate [s$^{-1}$]

$\dot{\gamma}_i$ shear rate at the inner cylinder [s$^{-1}$]

$\dot{\gamma}_{\text{max}}$ maximum possible shear rate to measure with a DSR rheometer [s$^{-1}$]

$\eta$ dynamic viscosity [Pa s]

$\eta_0$ zero viscosity (viscosity at very low shear rates) [Pa s]

$\lambda$ linear grain concentration [-]

$\mu$ Newtonian fluid viscosity [Pa s]

$\mu'$ Newtonian viscosity of a fluid containing solids of identical size [Pa s]

$\mu_p$ Newtonian viscosity of the pore fluid [Pa s]

$\mu_B$ Bingham viscosity parameter [Pa s]

$\mu_C$ Casson viscosity parameter [Pa s]

$\rho$ density [kg/m$^3$]

$\rho_s$ solid density [kg/m$^3$]

$\rho_f$ fluid density [kg/m$^3$]

$\rho_w$ density of water [kg/m$^3$]

$\sigma$ standard deviation [%], normal stress [Pa]

$\tau$ shear stress [Pa]

$\tau_i$ shear stress at the inner cylinder [Pa]

$\tau_w$ wall shear stress [Pa]

$\tau_y$ yield stress [Pa]

$T_g$ granular temperature [m$^2$/s$^2$]

$\Omega$ rotational speed (frequency) [rps = s$^{-1}$]

$\omega$ angular velocity [rad s$^{-1}$]

$(v')$ fluctuation of the solid velocity around its mean value [m/s]

$\theta$ direction angle of profile of deposit in the Inclined Plane Test [$^\circ$]
### Abbreviations:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>BML</td>
<td>BML (Building Material Learning) viscometer</td>
</tr>
<tr>
<td>BMS</td>
<td>Ball Measuring System</td>
</tr>
<tr>
<td>CCS</td>
<td>Concentric Cylinder System</td>
</tr>
<tr>
<td>CCS-LS</td>
<td>CCS large scale rheometer of Coussot&amp;Piau</td>
</tr>
<tr>
<td>CPS</td>
<td>Cone and Plate System</td>
</tr>
<tr>
<td>ICT</td>
<td>Inclined Channel Test</td>
</tr>
<tr>
<td>IPT</td>
<td>Inclined Plane Test</td>
</tr>
<tr>
<td>KAS</td>
<td>Kasumeter</td>
</tr>
<tr>
<td>NWT</td>
<td>Newtonian Fluid</td>
</tr>
<tr>
<td>PLF</td>
<td>Power Law Fluid</td>
</tr>
<tr>
<td>PPS</td>
<td>Parallel Plate System (Plate and Plate System)</td>
</tr>
<tr>
<td>ST</td>
<td>Slump Test</td>
</tr>
<tr>
<td>ST-SL</td>
<td>Slump Test considering the slump height</td>
</tr>
<tr>
<td>ST-PD</td>
<td>Slump Test considering the profile of deposit</td>
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<tr>
<td>YSF</td>
<td>Yield Stress Fluid</td>
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</table>
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Born  
6. Januar 1969

Family status  
moved, 1 daughter (2004)

Profession  
dipl. Kultur-Ing. ETH

Professional Life

since November 1997

Executive officer, project leader and deputy of the head of the river engineering division (since winter 2003/2004) at the Laboratory of Hydraulics, Hydrology and Glaciology (VAW) of ETH Zürich.

Main activities:

- Simulation of clear water, mudflow and debris flow, sediment transport and floating wood in physical and numerical models.
- Expertises concerning bed and bank stability and erosion in torrents and rivers.
- Thesis in the field of rheometry for large particle fluids and debris flows.

September 1997 – October 1997

Executive officer at Umwelt Controlling & Consulting Wälti (UCW)/Rafz (70%) as well as at the foundation PanEco/Berg a. Irchel (30%).

Main activities:

- Environmental impact study „SBB-line Zurich HB-Wipkingen“: Editing (UCW)
- Expertise of a contaminated land (UCW)
- Concept study for the organization of volunteer work (PanEco)
Professional Life  
**August 1995 - August 1997**
Trainee, project manager and deputy of coordinator in the rural development project „Puka“ in Albania funded by the Swiss Agency for Development and Cooperation (SDC).

Main activities:
- Development and realization of strategies for the rehabilitation of land exposed to erosion
- Training for teachers and students in environment education
- Others

Education  
**October 1990 - April 1995**
Study of rural engineering at the ETHZ in Zürich and the EPFL in Lausanne.

During the semester breaks work at Zurlinden Surveying in Zurzach as well as work at various mountain farmers in the cantons Uri, Berne, Lucerne and Tessin organized by Caritas.

1988 - 1990
Work at different gardening enterprises in the region of Baden as well as work at the landscape architecture offices Zulauf&Partner/Baden and Atelier Stern&Partner/St. Gallen.

Military service

1976 - 1988
Primary, secondary and high school (type C) in the region of Baden.

Grown up in Untersiggenthal and Turgi close to the river mouth of Aare, Reuss and Limmat

Outside Profession
- Family
- Chamber music (Violoncello, Piano)
- Hiking, Orienteering, Skitouring
Appendix A

Photo documentation of grain material

A1  Debris flow grain material

A2  Grain material used for clay-dispersions, clay-gravel-suspensions and mixed suspension
A.1 Debris flow grain material

Figure A.1 Excavation site of debris flow material: Tongue of "May 3/2001 - debris flow" in Scalärinarife near Trimmis/Chur. Arm of crane used for excavation (in red).

Figure A.2 Sorting out stones, blocks and boulders larger than \( d = 100 \text{ mm} \).
Figure A.3  Debris flow material $d \leq 100$ mm (in front) and $d > 200$ mm (behind).

Figure A.4  Debris flow material $5 < d \leq 25$ mm (left) and $1 < d \leq 5$ mm (right).

Figure A.5  Debris flow material $0.25 < d \leq 1$ mm (left) and $d \leq 0.25$ mm (right).
A.2 Grain material used for clay-dispersions, clay-gravel-suspensions and mixed suspension

Figure A.6 Grain material \( d \leq 0.25 \text{ mm} \) used for clay-dispersions, clay-gravel-suspensions and mixed suspension.

Figure A.7 Gravel \( 3 < d \leq 10 \text{ mm} \) used for the clay-gravel-suspensions.

Figure A.8 Grain material \( 0.25 < d \leq 1 \text{ mm} \) (left) and \( 1 < d \leq 10 \text{ mm} \) (right) used for the mixed suspension.
Appendix B

Steady drag flow in the BMS

B1 Separation between acceleration and steady drag flow regime

B2 Steady drag flow for very high rotational speeds
B.1 Separation between acceleration and steady drag flow regime

Figure B.1  Scheme of points along the sphere path in the ball measuring system where average torque and speed measurements were recorded within one rotation of the sphere.

Explications:

For the present study the settings of the MCR 300 rheometer were given as follows: 40 data points (40 pairs of average torque and speed data by a measuring frequency of 100 Hz) are recorded along the sphere path for one rotation (fig. B.1).

Due to sphere acceleration up to a defined speed $\Omega$ the first data points can’t be used. Only data points within a steady sphere drag flow after acceleration regime correspond with the rheological behaviour of the fluid and can be thus considered for any further data treatment.

In the following tables B.1 to B.3 the data points which correspond to the steady sphere drag flow are indicated for the different spheres, the different investigated fluids and the different rotational speeds $\Omega$. Example: 6-40 means that the data points 6 to 40 belong to the steady sphere drag flow regime whereas the data points 1 to 5 belong to the acceleration regime.
<table>
<thead>
<tr>
<th>Material:</th>
<th>$\Omega$</th>
</tr>
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<tbody>
<tr>
<td></td>
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<td>11 - 40 21-40** 26-40**</td>
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<tr>
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Table B.1  Data points considered as steady drag flow data along the sphere path for the sphere $D = 15 \text{ mm}$. 1 = first data point, 40 = last data point (fig. B.1).
* = Observation of wave formation by accelerating sphere.  ** = Observation of viscous overstream effect.
<table>
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<th>0.027</th>
<th>0.0449</th>
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<th>0.27</th>
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<td>6 - 40</td>
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<td>6 - 40</td>
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<td>16 - 40</td>
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<td>21-40</td>
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<tr>
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<td>21-40**</td>
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<td></td>
</tr>
<tr>
<td>CMC 1%</td>
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<td>18 - 40</td>
<td>21 - 40</td>
<td>23 - 40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMC 2%</td>
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<td>16 - 40</td>
<td>21 - 40</td>
<td>21 - 40</td>
<td>26 - 40</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>16 - 40</td>
<td>21 - 40</td>
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<tr>
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<td>16 - 40</td>
<td>21 - 40</td>
<td></td>
<td>21</td>
<td>21-40</td>
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</table>

Table B.2 Data points considered as steady drag flow data along the sphere path for the sphere $D = 12$ mm. $1 = \text{first data point, } 40 = \text{last data point (fig. B.1).}$

* = Observation of wave formation by accelerating sphere. ** = Observation of viscous overstream effect.
<table>
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<th>Material:</th>
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<tr>
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</tr>
<tr>
<td>All mixtures</td>
<td>3 - 40</td>
</tr>
<tr>
<td><strong>Clay-Dispersions and Suspensions</strong></td>
<td></td>
</tr>
<tr>
<td>All mixtures</td>
<td>6 - 40</td>
</tr>
<tr>
<td><strong>Non Particle Material</strong></td>
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</tr>
<tr>
<td>Silicon Oil $\mu = 0.05$ Pa s</td>
<td>3 - 40</td>
</tr>
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<td>Silicon Oil $\mu = 1$ Pa s</td>
<td>3 - 40</td>
</tr>
<tr>
<td>Silicon Oil $\mu = 2, 12.5$ Pa s</td>
<td>3 - 40</td>
</tr>
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<td>Silicon Oil $\mu = 30$ Pa s</td>
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<tr>
<td>Silicon Oil $\mu = 60$ Pa s</td>
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</tr>
<tr>
<td>CMC 1%</td>
<td>6 - 40</td>
</tr>
<tr>
<td>CMC 2%</td>
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<tr>
<td>Guar 1%</td>
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</tr>
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<td>Guar 2%</td>
<td>6 - 40</td>
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<th>Material:</th>
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<tbody>
<tr>
<td></td>
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<td></td>
<td>[rps]</td>
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<td><strong>Debris Flow mixtures</strong></td>
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</tr>
<tr>
<td>$d \leq 1$ mm, $C_v \leq 0.458$</td>
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</tr>
<tr>
<td>Remaining mixtures</td>
<td>11 - 40</td>
</tr>
<tr>
<td><strong>Clay-Dispersions and Suspensions</strong></td>
<td></td>
</tr>
<tr>
<td>All mixtures</td>
<td>11 - 40</td>
</tr>
<tr>
<td><strong>Non Particle Material</strong></td>
<td></td>
</tr>
<tr>
<td>Silicon Oil $\mu = 0.05$ Pa s</td>
<td>11 - 40</td>
</tr>
<tr>
<td>Silicon Oil $\mu = 1$ Pa s</td>
<td>11 - 40</td>
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<tr>
<td>Silicon Oil $\mu = 2, 12.5$ Pa s</td>
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<td>CMC 2%</td>
<td>16 - 40</td>
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<tr>
<td>Guar 1%</td>
<td>11 - 40</td>
</tr>
<tr>
<td>Guar 2%</td>
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Table B.3  Data points considered as steady drag flow data along the sphere path for the sphere $D = 8$ mm. $I =$ first data point, $40 =$ last data point (fig. B.1). $*$ = Observation of wave formation by accelerating sphere. $** =$ Observation of viscous overstream effect.
B.2 Steady drag flow for very high rotational speeds

Figure B.2 Development of the measured torque $T$ along one sphere rotation for the debris flow material mixtures $d \leq 1$mm of different concentrations $C_v$ at the very high rotational speed of $\Omega = 2.25$ rps.

Figure B.3 Development of the measured torque $T$ along one sphere rotation for the debris flow material mixtures $d \leq 1$mm of different concentrations $C_v$ at the very high rotational speed of $\Omega = 4.5$ rps.
Explications to fig. B.1 and fig. B.2:

The development of the torque data for the very high speeds $\Omega = 2.25$ to $4.5$ rps is exemplified with the help of the debris flow material mixtures mixtures $d \leq 1$mm for different sediment concentrations $C_v$ (fig. B.1 and fig. B.2).

For the very high speeds $\Omega = 2.25$ to $4.5$ rps the torque data of the low to medium concentrated mixture ($C_v = 0.458$) tend to slightly increase in the course of the steady drag flow regime while the torque data of the medium to high concentrated mixture ($C_v = 0.502$) doesn’t show any particular tendency here. The torque data of the highly concentrated mixture ($C_v = 0.552$) tend to slightly decrease for the very high speed $\Omega = 4.5$ rps.

Therefore the question is raised if for the very high speeds $\Omega$ the definition “steady drag flow” is still appropriate.
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Appendix C

Influence of sphere holder and container boundary in the BMS
Figure C.1 Dependence of the value $\Delta C$ on the immerging depth $s_{im}$ of the sphere holder for the sphere $D=15$ mm measured by Tyrach (2000) in a Silicon Oil. $\Delta C$ is added to the theoretical value of the system number $C = 24 + 8/G^2$, where the theoretical value corresponds to a laminar flow around a rotating sphere that is free of any boundary influence of sphere holder and container.

Continuous line: Data fit of Tyrach (2000) expressing a dependence of the $\Delta C$- value only on the immerging depth $s_{im}$.

Dashed line: Data fit expressing a strong dependence of the $\Delta C$- value on the immerging depth $s_{im}$ and a very limited dependence on the container boundary (proposition of the author of the present study).

\[ \Delta C = 1.0531s_{im} \] (Tyrach 2000)

\[ \Delta C = 1.576 + 0.8385s_{im} + 0.006748s_{im}^2 \] (proposition of Schatzmann)
Appendix D

Reference measurements in standard measuring systems

D1 Silicon Oils
D2 Polymers
D3 Particles
D.1 Silicon Oils

The following Newtonian Silicon Oils were measured in standard measuring systems using the DSR rheometer in order to prove the characteristic viscosity $\mu$ already indicated by the producer (table C.1): $\mu = 60, 12.5, 2, 0.05 \text{ Pa s}$. For these oils the measured value was used in the application of chapter 4. In the case where the viscosity $\mu$ was not proved ($\mu = 30, 1 \text{ Pa s}$), the theoretical values indicated by the producer were used (table D.1).

<table>
<thead>
<tr>
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<th>Theoretical value</th>
<th>Measured value</th>
<th>Measuring system</th>
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<td>$\mu$</td>
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<td>[Pa s]</td>
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<tr>
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<td>0.0496</td>
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<td>CCS</td>
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Table D.1 Newton viscosity $\mu$ of Silicon Oils applied in the present study.

For the oil $\mu = 0.05 \text{ Pa s}$ the Concentric Cylinder System (CCS) was used (fig. 2.13), as the very liquid oil could be easily filled in this system. The dimensions of the CCS were $R_0 = 16 \text{ mm}, H = 1.25 \text{ mm} \text{ and } L = 44.4 \text{ mm}$. By contrast the more viscous oils $\mu = 60, 12.5, 2 \text{ Pa s}$ were investigated in the Cone and Plate System (CPS) as it was difficult to fill them into the CCS avoiding bubbles of air. The dimensions of the CPS (fig. 2.13) were $R = 25 \text{ mm} \text{ and } \alpha = 5.7 \degree$. The plate surfaces of both systems were smooth.

Up and down curve stress sweep experiments were performed with both systems by altering the stress in adequate increments. The flow curve/viscosity $\mu$ was determined in a range of shear rates $0.1 < \dot{\gamma} < \dot{\gamma}_{\text{max}} \text{ s}^{-1}$. Due to the maximum shear stress of $\tau = 320 \text{ Pa}$ measurable with the DSR rheometer $\dot{\gamma}_{\text{max}}$ was $5 \text{ s}^{-1}$ for the oil $\mu = 60 \text{ Pa s}$, $25 \text{ s}^{-1}$ for the oil $\mu = 12.5 \text{ Pa s}$, $160 \text{ s}^{-1}$ for the oil $\mu = 2 \text{ Pa s}$ and limited as $\dot{\gamma}_{\text{max}} = 300 \text{ s}^{-1}$ for the oil $\mu = 0.05 \text{ Pa s}$.

---

1 Values issued by the producer (Wacker GmbH, 1993) at 20°C
2 Average values measured in the present study at 20°C
For each investigated oil the \( \mu \)-value obtained (i) in the entire range of shear rate \( \dot{\gamma} \) and (ii) for the three independent up or down curve experiments did not deviate more than \( \pm 2 \% \) from the average value of all experiments. The latter value is indicated in table D.1.

### D.2 Polymers

The different polymer-solutions CMC and Guar were measured in the same CCS already used for the Silicon Oils. Three up and down curve stress sweep experiments were performed for each solution in the range \( 0.01 < \dot{\gamma} < 300 \text{ s}^{-1} \). For each solution the flow curve data obtained from the different independent up and down curve experiments did not deviate more than \( \pm 5 \% \) from the average flow curve of all experiments.

<table>
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<th>CMC 2%</th>
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<td>R</td>
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<td>[s(^{-1})]</td>
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<td>R</td>
<td>( \dot{\gamma} )</td>
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<td>[-]</td>
<td>[-]</td>
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<td>0.590</td>
<td>0.9999</td>
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<td>0.9997</td>
<td>2.89 to 3.83</td>
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<td>27.4 to 40.7</td>
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<td>1.0000</td>
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*Table D.2* Power-law viscosity coefficient \( m \) and index \( n \) as well as correlation coefficient \( R \) of data fit obtained in the specific ranges of shear rates \( \dot{\gamma} \) for the different polymers.
Then a power-law function (eq. 2.5) was fitted to the data of the average flow curve of all experiments. The fit was done in confined ranges of shear rates $\dot{\gamma}$ in order to obtain a very accurate reproduction of the flow curve for any shear rate $\dot{\gamma}$ (table D.2).

D.3 Particles

Three debris flow material mixtures ($C_v = 0.349, 386, 429$) and four clay-dispersions ($C_v_{\text{fines}} = 0.225, 0.25, 0.275, 0.3$) containing particles $d \leq 0.25 \text{ mm}$ were investigated in the DSR rheometer with the same CCS already used for the Silicon Oils.\(^3\) One up and down curve experiment was performed for each mixture/dispersion. For both type of particle fluids the flow curve data of both experiments agreed well, except at low shear rates ($\dot{\gamma} < 1\text{-}10 \text{ s}^{-1}$). Here wall slip and in the case of the clay-dispersions also thixotropic effects occurred (Schatzmann et al. 2003a, Schatzmann et al. 2003b). Wall slip was due to the smooth surface of the inner and outer cylinder and was detected by the flow curve step within $1 < \dot{\gamma} < 10 \text{ s}^{-1}$: As figure D.1 shows, wall slip was much more pronounced in the debris flow material mixtures than in the clay-dispersions.

![Figure D.1](image_url)

**Figure D.1** Examples of measured and fitted flow curve in the case of wall slip in the concentric cylinder system using smooth cylinder surfaces. Measured = data of up curve experiments.

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3 Tests were also performed using (i) a CCS with the dimensions $R_0 = 16 \text{ mm}, H = 2.5 \text{ mm}$ and $L = 44.4 \text{ mm}$ and (ii) a PPS with the dimensions $R = 12.5 \text{ mm}$ and $H = 1 \text{ mm}$. Both systems turned out to be less useful because in (i) the exact shear rate at a given speed is less certain and the probability of secondary flows perpendicular to the shear flow increased and in (ii) particle segregation and alteration of the original shear layer during measuring was found (Schatzmann et al. 2003a).
As a consequence only the data of the up curve experiment beyond the influence due to wall slip were considered ($\dot{\gamma} > 1\text{-}10\text{ s}^{-1}$) for the flow curve data fit. Here the data were fit with the general Herschel-Bulkley model (eq. 2.6). The results are listed in table D.3.

### Debris flow material mixtures:

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<tr>
<th>$C_v$</th>
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<th>$n$</th>
<th>$R$</th>
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<td>[Pa s$^{n}$]</td>
<td>[-]</td>
<td>[-]</td>
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<tr>
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<td>2.7075</td>
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### Clay-Dispersions:

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<th>$n$</th>
<th>$R$</th>
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<td>[Pa]</td>
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<td>[-]</td>
</tr>
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<td>0.2288</td>
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<td>64.551</td>
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*Table D.3* Parameter values of Herschel-Bulkley fit as well as correlation coefficient $R$ of data fit obtained for the different particle mixtures.
Appendix E

Fit of Herschel-Bulkley model to BMS flow curve data of particle fluids
In the following tables E.1 to E.3, the results of the fit of the flow curve data using the Herschel-Bulkley model (eq. 2.6) and applying the Least-Squares method of Gauss are listed for the different fine and large particle fluids investigated with the ball measuring system (BMS). The flow curve and the fit curve for the different particle fluids are then shown in fig. E.1 to E.10. Note that the BMS flow curve data considered for the data fit were obtained with the new data conversion approach of the present study (4.3.2).

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<th>m</th>
<th>n</th>
<th>( \sigma_{\tau_r} )</th>
<th>( \sigma_K )</th>
<th>( \sigma_n )</th>
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<tbody>
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<td>[mm]</td>
<td>[Pa]</td>
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<td>[%]</td>
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Table E.1  Results of Herschel-Bulkley fit for the rheological data of the different debris flow material mixtures investigated with the ball measuring system.
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Table E.1 Results of Herschel-Bulkley fit for the rheological data of the different debris flow material mixtures investigated with the ball measuring system.
### Table E.1
Results of Herschel-Bulkley fit for the rheological data of the different debris flow material mixtures investigated with the ball measuring system.

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<td>0.78</td>
<td>7.9</td>
<td>82.9</td>
<td>22.6</td>
<td>0.9919</td>
</tr>
</tbody>
</table>
Table E.3 Results of Herschel-Bulkley fit for the rheological data of the mixed suspension investigated with the ball measuring system.

<table>
<thead>
<tr>
<th>$d$ [mm]</th>
<th>$C_v$ [-]</th>
<th>$D$ [mm]</th>
<th>$\tau_\gamma$ [Pa]</th>
<th>$m$</th>
<th>$n$</th>
<th>$\sigma_{\tau_\gamma}$ [%]</th>
<th>$\sigma_m$ [%]</th>
<th>$\sigma_n$ [%]</th>
<th>$R$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10</td>
<td>0.683</td>
<td>15</td>
<td>389.66</td>
<td>55.91</td>
<td>0.58</td>
<td>10.5</td>
<td>72.0</td>
<td>34.0</td>
<td>0.9809</td>
</tr>
<tr>
<td>12</td>
<td>531.41</td>
<td>1.91</td>
<td>1.63</td>
<td>4.9</td>
<td>179.2</td>
<td>31.7</td>
<td>0.9849</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>695.87</td>
<td>70.98</td>
<td>0.69</td>
<td>9.1</td>
<td>76.2</td>
<td>30.7</td>
<td>0.9831</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure E.1  Measured and fitted flow curve for the debris flow material mixtures of the portion $d \leq 0.25$ mm investigated with the ball measuring system in march 2002 ($C_v = 0.349, 0.386, 0.429, 0.479$) and in june 2003 ($C_v = 0.453$). $C_v = \text{volumetric sediment concentration}$. $D = \text{sphere size}$. 
Figure E.2 Measured and fitted flow curve for the debris flow material mixtures of the portion $d \leq 1$ mm investigated with the ball measuring system in June 2003. $C_v =$ volumetric sediment concentration. $D =$ sphere size.
Figure E.3  Measured and fitted flow curve for the debris flow material mixtures of the portion $d \leq 5$ mm investigated with the ball measuring system in June 2003. $C_v =$ volumetric sediment concentration. $D =$ sphere size.
Figure E.4  Measured and fitted flow curve for the debris flow material mixtures of the portion $d \leq 5 \text{ mm}$ investigated with the ball measuring system in November 2001. \( C_v = \text{volumetric sediment concentration.} \ D = \text{sphere size.} \)
Figure E.5 Measured and fitted flow curve for the debris flow material mixtures of the portion $d \leq 7$ mm investigated with the ball measuring system in June 2003. $C_v$ = volumetric sediment concentration. $D$ = sphere size.
Figure E.6  Measured and fitted flow curve for the debris flow material mixtures of the portion $d \leq 10$ mm investigated with the ball measuring system in June 2003. $C_v =$ volumetric sediment concentration. $D =$ sphere size.
Figure E.7 Measured and fitted flow curve for the clay-dispersions \((d \leq 0.25\ mm)\) investigated with the ball measuring system using the sphere \(D = 12\ mm\). \(C_v\) = volumetric sediment concentration.
Figure E.8 Measured and fitted flow curve for the clay-gravel-suspensions investigated with the ball measuring system using the sphere $D = 12$ mm. Basic fluid: Clay-dispersion ($C_v = \text{volumetric sediment concentration} = 0.225$). $C_{v_{\text{gravel}}} = \text{volumetric solid concentration of the gravel (3 mm} \leq d \leq 10 \text{ mm)}$ added to the clay-dispersion.

Figure E.9 Measured and fitted flow curve for the clay-gravel-suspensions investigated with the ball measuring system using the sphere $D = 12$ mm. Basic fluid: Clay-dispersion ($C_v = \text{volumetric sediment concentration} = 0.275$). $C_{v_{\text{gravel}}} = \text{volumetric solid concentration of the gravel (3 mm} \leq d \leq 10 \text{ mm)}$ added to the clay-dispersion.
Figure E.10 Measured and fitted flow curve for the mixed suspension ($C_v =$ volumetric sediment concentration = 0.683) investigated with the ball measuring system. $D =$ sphere size.
Appendix F

Derivation of the shear rate for Yield Stress Fluids in wide-gap concentric cylinder systems
I: Derivation of the basic equation for concentric cylinder systems (Krieger & Maron 1952):

A concentric cylinder system is considered, as schematically shown in fig. 2.13: $R_i$ is the radius of the inner cylinder, $R_o$ the radius of the outer cylinder and $L$ is the length of the inner cylinder. In case of a rotating inner cylinder a torque $T$ must be exerted to rotate the inner cylinder with the angular velocity $\omega$. Under laminar flow the shear rate $\dot{\gamma}$ at a distance $r$ from the axis of rotation is:

$$\dot{\gamma} = r \frac{d\omega}{dr} \quad (F.1)$$

Based on the measured torque $T$ the shear stress $\tau$ is:

$$\tau = \frac{T}{2 \cdot \pi \cdot r^2 \cdot L} \quad (F.2)$$

Differentiation of eq. F.2 gives:

$$\frac{d\tau}{dr} = \frac{2 \cdot T}{2 \cdot \pi \cdot r^3 \cdot L} = \frac{2 \cdot \tau}{r} \quad (F.3)$$

Per definition it is (see 2.1):

$$\dot{\gamma} = f(\tau) \quad (F.4)$$

Considering eq. F.4 and the ratio $dr/r = -2\cdot d\tau$ of eq. F.3 in eq. F.1, eq. F.1 becomes:

$$d\omega = -\frac{1}{2} \frac{f(\tau)}{\tau} \, d\tau \quad (F.5)$$

Integration of eq. F.5 with the bottom limit at the inner cylinder ($R_i$) and the upper limit at the outer cylinder ($R_o$) leads to the basic equation for concentric cylinder systems with a rotating inner cylinder:

$$\omega(R_i) = \omega = -\frac{1}{2} \int_{\tau(R_i)}^{\tau(R_o)} \frac{f(\tau)}{\tau} \, d\tau \quad (F.6)$$

Note that in case of a rotating outer cylinder, eq. F.6 has a positive sign.

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4 Note that the angular velocity is defined as $\omega = 2 \cdot \pi \cdot \Omega$, with $\Omega =$ rotational speed (frequency, rps).
II: Derivation of the shear rate $\dot{\gamma}$ for Yield Stress Fluids in wide gap concentric cylinder systems (Nguyen & Boger 1987):

As already outlined in 5.1.3 one must principally distinguish between the case, the cylinder gap $H$ is (1) partly sheared ($R_i < r_{crit} < R_o$) or (2) fully sheared ($r_{crit} > R_o$), where $r_{crit}$ is determined as:

$$r_{crit} = \sqrt[4]{\frac{T(\Omega)}{2 \cdot \pi \cdot \tau_y \cdot L}}$$  \hspace{1cm} (F.7)

The shear rate is always referred at the periphery of the inner cylinder: $\dot{\gamma} = \dot{\gamma}_i$. Thus eq. F.4 becomes:

$$\dot{\gamma}_i = f(\tau_i)$$  \hspace{1cm} (F.8)

**Case (1): Partly sheared condition (fig. F.1):**

![Figure F.1](image)

*Figure F.1  Shear flow of a Yield Stress Fluid (YSF) in a wide gap concentric cylinder system for the case $R_i < r_{crit} < R_o$.***
As the shear zone only spreads to $r_{crit}$, the upper limit in the integral of the basic equation for concentric cylinder systems (F.6) is $\tau_y$. Considering eq. F.8, eq. F.6 becomes:

$$\omega = \frac{1}{2} \cdot \dot{\gamma}_i \cdot \int_{r(R_i)}^{r(R_h)} \frac{1}{\tau} d\tau$$ \hspace{1cm} (F.9)

Integration of eq. 5.9 gives:

$$\omega = \frac{1}{2} \cdot \dot{\gamma}_i \cdot \left[ \ln \tau_y - \ln \tau_i \right]$$ \hspace{1cm} (F.10)

Differentiation of eq. F.10 gives the following expression for the shear rate $\dot{\gamma}$:

$$\dot{\gamma}_i = 2 \cdot \omega \frac{d\omega}{d(\ln \tau_i)} = 2 \cdot \tau_i \frac{d\omega}{d\tau_i}$$ \hspace{1cm} (F.11)

**Case (2): Fully sheared condition:**

The basic equation F.6 can not be solved exactly. Therefore, different approximation formula were elaborated by Krieger&Co-workers for the determination of the shear rate $\dot{\gamma}$. A good summary of these formula is given in Nguyen&Boger (1987).