Improving the SLAM Convergence with a Relative Map Filter

Conference Paper

Author(s):
Martinelli, Agostino; Siegwart, Roland

Publication date:
2004

Permanent link:
https://doi.org/10.3929/ethz-a-010090430

Improving the SLAM Convergence with a Relative Map Filter

Agostino Martinelli and Roland Siegwart
Swiss Federal Institute of Technology Lausanne (EPFL)
CH-1015 Lausanne, Switzerland

Abstract. This paper presents an approach to solve the SLAM problem in the stochastic map framework based on the concept of the relative map. The idea consists in introducing a map state, which only contains quantities invariant under translation and rotation. This is the only way in order to have a decoupling between the robot motion and the landmark estimation and therefore not to rely the landmark estimation on the unmodeled error sources of the robot motion. The approach is general and can be applied for several kind of landmark. However, only the case of point landmark is considered here. For this special case, the structure of the proposed filter is deeply examined and a comparison with the joint vehicle-landmark approach (absolute map filter) is carried out theoretically and through accurate simulations. The main result shown about this new approach is the map convergence in large environment even when the odometry is affected by undetected systematic errors or by large or unmodeled non-systematic errors.

1 Introduction

In the SLAM problem a mobile robot has to be able to autonomously explore the environment with its on board sensors, gain knowledge about it, interpret the scene, build an appropriate map and localize itself relative to this map.

Many approaches have been proposed to solve the SLAM problem both in the framework of the metric and the topological navigation.

A very successful method is the stochastic map approach. After the first precise mathematical definition of the stochastic map [13] early experiments ([4], [8]), have shown the quality of fully metric simultaneous localization and map building: the resulting environment model permits highly precise localization that is only bounded by the quality of the sensor data. However, these approaches suffer from some limitations. Firstly, they rely strongly on odometry. For automatic mapping this makes the global consistency of the map difficult to maintain in large environments where the drift in the odometry becomes too important. Furthermore, they represent the robot position with a single Gaussian distribution. This means that an unmodeled event (i.e. collision) could cause divergence between the ground truth and the estimated position from which the system is unable to recover (lost situation). In [1] it has been shown that by taking into account all the correlations the global consistency is better maintained. However, this is not sufficient as confirmed by another work [2] where a solution is proposed by extending the absolute localization to include a localization relative to local reference frames.

In [6], the convergence of a filter which estimates the robot configuration and the absolute location of the landmarks by adopting a Kalman filter (absolute map filter, AMF), is theoretically proven. However, the proof is based on strong and practically infeasible assumptions. The map convergence is not proven when these hypothesis are not fully satisfied. Moreover, Julier and Uhlmann proved that the AMF yields an inconsistent map, even for the special case of a stationary vehicle with no process noise [7]. This problem can occur when a new landmark is introduced in the map. Indeed, even in the case when the dynamics and the observation are linear, the Kalman filter cannot be considered optimal since the introduction of a new landmark is related to a very non-linear process (e.g. maximum range in the observation).

The aim of the method introduced in this paper in the frame-work of the stochastic map approach to SLAM, is to minimize the loop consistency problem (i.e. the global convergence and consistency of the built map in large environment - large meaning when the size of the environment is much larger than the range of the adopted exteroceptive sensor). The basic idea consists in introducing a map state which only contains quantities invariant under translation and rotation. This is the only way in order to have a decoupling between the odometry and the landmark estimation and therefore not to rely the landmark estimation on the unmodeled error sources in the robot motion. Moreover, since the new elements introduced in the map during the navigation are not correlated with the old elements when they are not observed together, this approach does not suffer from the inconvenient pointed out by Julier and Uhlmann [7].

A relative map was already introduced by Newmann ([11] and [12]). He used two filters in the estimation, called the relative map filter and the geometric projection filter. The second one provides a means to produce a geometrically consistent map from the relative map, by solving a set of linear constraints. Both filters are optimal since the dynamics and the observation are both linear and they are based on the Kalman Filter. However, the elements used in this approach are invariant for translation only, not for rotation. Our approach is to take invariant elements, but for both translation and rotation and to apply a Kalman Filter for the estimation, contrasting to [5], who used the same invariants in combination with a non-optimal filter. Only the case of point landmark is here considered although the same idea could be applied to other kind of landmark. In section 2 we discuss the main drawbacks of the AMF. The proposed filter is presented in section 3 for the case of point landmark. The results obtained through accurate simulations are displayed in section 4 where also a comparison with the absolute map filter is shown. Finally, conclusions and future research are given in section 5.

2 Drawbacks in the absolute map filter

In the absolute map filter the robot configuration and the location of each landmark are registered in one common global reference frame. A Kalman filter is used to estimate the state containing the previous global coordinates and its covariance matrix.

$$X = [X_v^T \; p_1^T \; ... \; p_N^T]^T \tag{1}$$

$$P = \begin{bmatrix} P_{vv} & P_{v1} & ... & P_{vN} \\ P_{v1}^T & P_{11} & ... & P_{1N} \\ ... & ... & ... & ... \\ P_{vN}^T & P_{1N}^T & ... & P_{NN} \end{bmatrix} \tag{2}$$

where $X_v = [x, y, \theta]^T$ is the robot configuration, p_i is the absolute location of the i^{th} landmark, P_{ij} is the cross-covariance between the i^{th} and j^{th} landmark location and P_{vi} is the cross-covariance between the i^{th} landmark and the vehicle configuration. The ”state transition equation” for the state X restricted to the map part (p_i) is the identity. Concerning the vehicle part, this equation is determined by the drive system of the robot. The Kalman filter is used to fuse the information coming from this transition equation with the information coming from an observational equation. This equation models the observation coming from an external sensor and provides a vector depending on the state given in the equation (1).

$$Z = h(X, w) \tag{3}$$

where w is a vector of temporally uncorrelated observation errors with zero mean and covariance matrix R.

The convergence of the absolute map filter is proved in [6] (theorems 1, 2 and 3 in the paper). However, the proof of these theorems is based on two very strong and infeasible assumptions.

- Perfectly modeled Odometry;
- Linear Observation

The first assumption means that the odometry is perfectly calibrated and the non-systematic errors are perfectly approximated by a Gaussian zero mean vector. A typical problem which arises from an imperfect calibration (for instance, due to an uncertainty on the wheel diameter) is a drift in the built map. Clearly, the Kalman filter fuse the odometry data with the data coming from the external sensor without considering this uncertainty and this produce inevitably a drift in the built map. The problem is that even a knowledge of the wheel diameter with a very high accuracy, produces such a problem (in the section 4 we show that an accuracy of 0.1% is still too low for the convergence of the map in large environment). Even worse are the problems caused by the non-systematic errors. Indeed, there are many error sources (for instance a robot collision) which are faulty modeled by a Gaussian statistics. Again, the Kalman filter fuse the odometry data with the data coming from the external sensor and the result is an error in the built map and in the robot configuration which could not be compensated any more.

The second assumption means that the function h in the equation (3) is linear in X. Concerning point landmark, this can be true only if the orientation is a priori known which means that the robot configuration X_v only contains the $(x\ y)$ coordinates. This assumption is widely used to prove the convergence theorems. The main problem with this assumption is that also the Jacobian of the function h with respect to the state X, $H = \nabla_X h$, depends on the predicted X. Indeed, the key point in the proof, is that H does not depend on the state (equations (26) and (27) in the paper).

3 The Structure of the Relative Map Filter

A possible way to solve the previous drawbacks is obtained by introducing a filter whose state only contains quantities invariant under translation and rotation. This is the idea characterizing the relative filter introduced here. Once the relative map has been estimated through this filter and the absolute location of a set of landmarks is known (e.g. by using the first

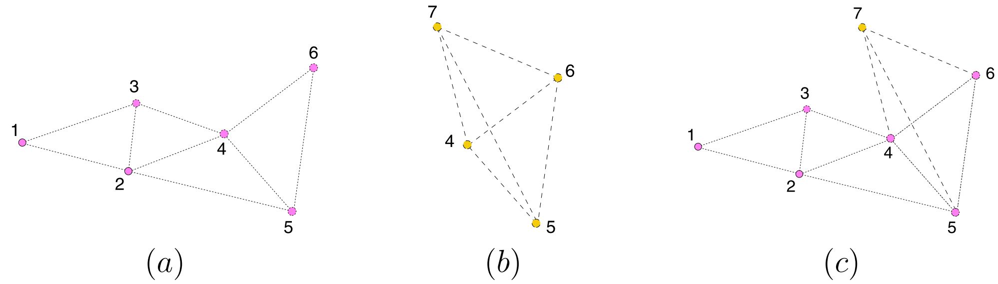

Figure 1: Relative Map before the observation (a), the observation (b), and the relative map obtained by fusing the information coming from the old map and the observation (c). In all the three figures the map state only contains the indicated distances between the landmarks

observation) it is possible to build the absolute map. Therefore, the entire method contains two algorithms. The former estimates the relative map, the latter builds the absolute map. In the sections 3.1 and 3.2 respectively we describe the two algorithms.

3.1 The Relative Map Filter

The state estimated through this filter only contains the distances between the point landmarks. Clearly, the distance is a quantity invariant under translation and rotation, i.e. it is independent of the robot configuration. Let denote with d the state and with P its covariance matrix. In fig. $1a$ the vector d contains the marked distances between the 6 landmarks. Clearly, not all of the distances between the 6 landmarks are stored in d because not all the landmarks were observed together at the same time. At a given time step, the observation consists of a set of distances between the landmarks observed by the robot through its external sensor (fig. $1b$). Clearly, these distances may be already observed (i.e. can be in the vector d) or may not. Let introduce the following notation:

$$d_{old} = [u, w_{old}]^T \qquad d_{obs} = [w_{obs}, v]^T \tag{4}$$

where d_{old} is the state estimated at a given time step and d_{obs} is the observation at the same time step, containing a set of distances between the landmarks observed by the robot. u contains the distances which are not re-observed (i.e. which do not appear in the vector d_{obs}) and w_{old} contains the distances re-observed (denoted by w_{obs} in the vector d_{obs}). Finally, v contains the distances observed for the first time at the considered time step. The covariance matrix of the previous vectors are:

$$P_{old} = \begin{bmatrix} P_{uu} & P_{uw} \\ P_{uw}^T & P_{ww} \end{bmatrix} \qquad P_{obs} = \begin{bmatrix} R_{ww} & R_{wv} \\ R_{wv}^T & R_{vv} \end{bmatrix} \tag{5}$$

We adopt the following notation to denote the estimated quantities, obtained by fusing the old state with the observed one (the new estimated distances are depicted in fig. $1c$).

$$d_{new} = [u_{new}, w_{new}, v_{new}]^T \qquad P_{new} = \begin{bmatrix} Pn_{uu} & Pn_{uw} & Pn_{uv} \\ Pn_{uw}^T & Pn_{ww} & Pn_{wv} \\ Pn_{uv}^T & Pn_{wv}^T & Pn_{vv} \end{bmatrix} \tag{6}$$

We obtain the new estimation for the state and its covariance matrix by applying the equations of the Kalman filter. Observe that the observation is linear in the state (is the identity) and therefore the Kalman filter is optimal.

$$u_{new} = u + P_{uw}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{obs} - w_{old}\right) \tag{7}$$

$$w_{new} = w_{old} + P_{ww}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{obs} - w_{old}\right) \tag{8}$$

$$v_{new} = v + R_{vw}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{old} - w_{obs}\right) \tag{9}$$

$$Pn_{uu} = P_{uu} - P_{uw}\left(P_{ww} + R_{ww}\right)^{-1} P_{wu} \tag{10}$$

$$Pn_{uw} = P_{uw} - P_{uw}\left(P_{ww} + R_{ww}\right)^{-1} P_{ww} \tag{11}$$

$$Pn_{uv} = 0 \tag{12}$$

$$Pn_{ww} = P_{ww} - P_{ww}\left(P_{ww} + R_{ww}\right)^{-1} P_{ww} \tag{13}$$

$$Pn_{wv} = R_{wv} - R_{ww}\left(P_{ww} + R_{ww}\right)^{-1} R_{wv} \tag{14}$$

$$Pn_{vv} = R_{vv} - R_{vw}\left(P_{ww} + R_{ww}\right)^{-1} R_{wv} \tag{15}$$

Instead of the equations (8) and (13) it is possible to use the following equations:

$$w_{new} = w_{obs} + R_{ww}\left(P_{ww} + R_{ww}\right)^{-1}\left(w_{old} - w_{obs}\right) \tag{16}$$

$$Pn_{ww} = R_{ww} - R_{ww}\left(P_{ww} + R_{ww}\right)^{-1} R_{ww} \tag{17}$$

They are derived by observing the symmetry of the filter with respect to the change ”observation” $\leftrightarrow$ ”old state”. Observe that the coincidence of the previous equations could be easily proved also by using the inversion lemma.

3.2 Recovering the Absolute Landmark Location

We adopt a simple linear method to recover the absolute landmark locations starting from the absolute location of three or more landmarks and the state estimated by the previous filter, which contains the distances between the landmarks. At a given time step the absolute locations of a set of landmarks are available (we assumed that the absolute coordinates of at least three landmarks are known at the beginning; these coordinates could be provided by the first observation). The aim is to estimate the location of a new landmark denoted by j. We extract from the previous set a subset containing the landmarks whose distance from the landmark j is provided by the relative filter. Let denote the locations of these landmarks by (x_i, y_i) and the distance between the landmark j and the landmark i of this subset by d_i. We assumed that the number of the elements of this subset is equal to n. If $n < 3$ the absolute location of the landmark j cannot be provided. Clearly, we have for the landmark i

$$d_i^2 = (x_j - x_i)^2 + (y_j - y_i)^2 \tag{18}$$

We therefore obtain a linear system in the unknowns x_j and y_j by considering all the differences $d_i^2 - d_1^2$. By applying recursively this method for all the landmarks (all j) it is possible to get their absolute location at each time step.

4 Results

In fig $2a$ the environment adopted in our simulations is displayed. The dotted line represents the actual robot trajectory (which is always the same in all the simulations). The adopted unit is the meter for both the axes. The cross represent the actual landmark location.

The data association problem is not considered here both in the case of the absolute and relative map (i.e. the observations are always associated with the right landmark).

The simulated external sensor is a laser range finder. It provides the distance of the objects around the robot with an angular resolution equal to $1deg$, a maximum range equal to $25m$ and the provided distance is a Gaussian quantity whose mean value is the actual one and the variance is equal to $(0.03m)^2$.

Finally, the frequency of the filter (both in the case of the absolute and the relative filter) is always one cycle for each meter traveled by the robot.

In the figures 2 and 3 the trajectory estimated by the filter is marked with the symbol o while for the landmark location estimation the symbol $*$ is adopted. The figures display the landmark positions as estimated at each time step of the navigation (for this reason it is possible to have multiple points per landmark).

Figures $2b - c$ and $3a - b$ concern the absolute map filter. The influence of the odometry error on the convergence of the filter is shown. The adopted model to characterize the odometry error is the one proposed by Chong and Kleeman [3]. The differential drive is considered. The translation of the right/left wheel as estimated by the odometry sensors are assumed to be Gaussian random variables satisfying the following relation:

$$\delta\rho^{R/L} = \overline{\delta\rho}^{R/L} + \nu^{R/L} \qquad \overline{\delta\rho}^{R/L} = \delta\rho^{aR/L}\delta_{R/L} \qquad \nu^{R/L} \sim N(0, K_w|\delta\rho^{aR/L}|) \tag{19}$$

In other words, both $\delta\rho^R$ and $\delta\rho^L$ are assumed Gaussian random variables, whose mean values are given by the actual values (respectively, $\delta\rho^{aR}$ and $\delta\rho^{aL}$) corrected for the systematic errors (which are assumed to increase linearly with the distance traveled by each wheel), and whose variances also increase linearly with the traveled distance. Moreover, it is assumed that $\delta\rho^R$ and $\delta\rho^L$ are uncorrelated. With respect to the Chong-Kleeman model, only one parameter (K_w) is here adopted to characterize both the variances for the right and left wheel. Finally, in the Chong-Kleeman model also the distance between the wheels is affected by a systematic error. However, in our simulations, we do not consider this systematic component, which, in any case, has an influence only on the AMF.

In fig. $2b - c$ a perfect odometry calibration is assumed (i.e. the systematic parameters δ_R and δ_L are assumed to be known without uncertainty). Therefore, there is only the influence of the non-systematic errors. In fig. $2b$ $K_w = 10^{-10}m$ (which means that the standard deviation in each wheel translation after one meter of navigation is equal to $10^{-5}m$). The adopted value

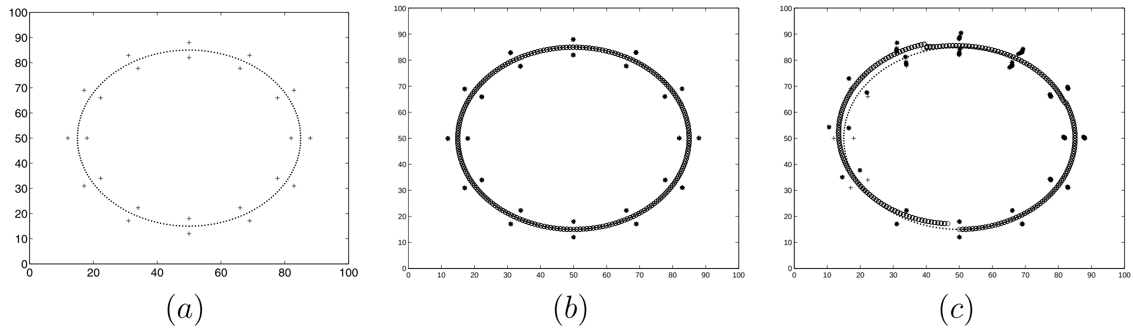

Figure 2: The considered environment with the actual trajectory and the actual position of the landmarks (a). The estimated trajectory and landmarks position obtained through the AMF when the odometry is perfectly calibrated with the non-systematic error obtained by considering $K_w = 10^{-10}m$ (b) and $K_w = 10^{-8}m$ (c)

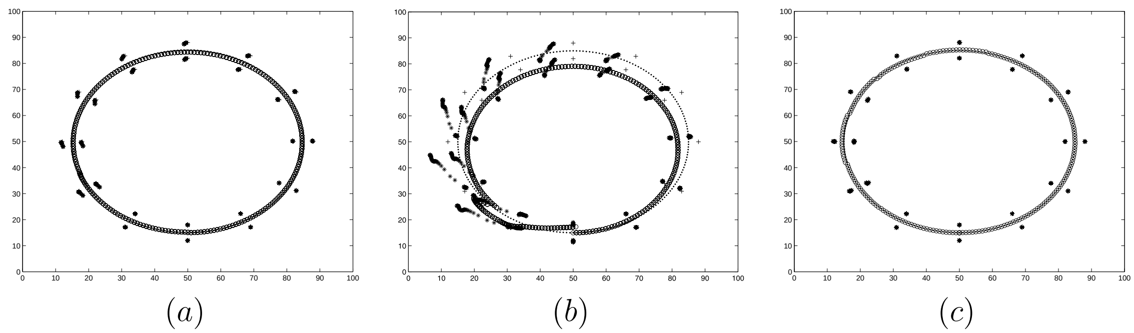

Figure 3: The estimated trajectory and landmarks position obtained through the AMF by introducing a systematic error. $\delta_L = 1.0001$ and $\delta_R = 1.0$ (a) and $\delta_L = 1.001$ and $\delta_R = 1.0$ (b). The estimated trajectory and landmarks position obtained through the Relative Filter (c)

in fig 2c is $K_w = 10^{-8}m$. It is possible to conclude that the convergence of the filter starts to be faulty as K_w becomes larger than $10^{-8}m$. Observe that this value is smaller than the value experimentally estimated ([10] and [9]).

Fig. 3$a - b$ show the results obtained for $K_w = 10^{-10}m$ and by changing the uncertainty on the systematic parameters (i.e. on the wheel diameters). We conclude that the convergence of the filter is good only when the uncertainty on the wheel diameter is better than 0.01%.

Fig. 3c shows the results obtained through the relative map filter. The absolute location of the first three landmarks is obtained through the first observation. In this case the odometry is not adopted and the error coming from a systematic and non-systematic component does not affect the built map. Clearly, the odometry could play an important role in a real implementation to solve the data association problem here not considered.

5 Conclusions and Future Research

In this paper we presented an approach to solve the SLAM problem in the stochastic map framework based on the concept of the relative map. The idea consists in introducing a map state which only contains quantities invariant under translations and rotations. This is the only way in order to have a decoupling between the robot and the landmark estimation and therefore not to rely the landmark estimation on the unmodeled error sources in the robot motion. A comparison with the absolute map filter is carried out both theoretically and through accurate simulations. We conclude that the convergence of the absolute map filter is true only if several infeasible hypothesis are satisfied. The proposed approach does not require these hypothesis. Clearly, the main assumption done in this paper is that the observation is better than the odometry (better meaning that the error model is better known). If, as often happens, other more precise sensors than the odometry are available with a well-known error model, it is much better to avoid the odometry in the estimation process, as shown in this paper through simulations. In the proposed approach, the only error source, which could create a divergence in the long term, is the gaussian assumption adopted in the statistical knowledge of the external sensor.

We are implementing this approach on a real platform. We are also extending the approach to the case of more general landmark. In particular, we are deriving the equations for this relative filter in the frame-work of the SP-model [13].

Acknowledgments

This work has been supported by the European project RECSYS (Real-Time Embedded Control of Mobile Systems with Distributed Sensing)

References

[1] Castellanos, J. A., J. D. Tards, et al. (1997). Building a Global Map of the Environment of a Mobile Robot: The Importance of Correlations. IEEE International Conference on Rob. and Aut. (ICRA), Albuquerque.

[2] Castellanos, J. A., M. Devy, et al. (2000). Simultaneous Localization and Map Building for Mobile Robots: A Landmark-based Approach. IEEE International Conference on Rob. and Aut. (ICRA), San Francisco.

[3] Chong K.S., Kleeman L., “Accurate Odometry and Error Modelling for a Mobile Robot,” *International Conference on Robotics and Automation*, vol. 4, pp. 2783–2788, 1997.

[4] Crowley, J.L., (1989). World Modeling and Position Estimation for a Mobile Robot Using Ultrasonic Ranging. IEEE International Conference on Robotics and Automation (ICRA), Scottsdale, AZ.

[5] M.Csorba, J.K.Uhlmann and H.F.Durrant-Whyte“A subOptimal Algorithm For Automatic Map Building,” *American Control Conference*, p. 537 -541, Albuquerque, New Mexico, USA 1997.

[6] Dissanayake, Newman, Clark, Durrant-Whyte and Csorba, 2001, A Solution to the Simultaneous Localization and Map Building (SLAM) problem, IEEE Trans. On Rob. And Aut. Vol 17, No.3, June 2001

[7] S.Julier and J.K.Uhlmann, (2001). A Counter Example to the theory of Simultaneous Localization and Map Building. IEEE International Conference on Rob. and Aut. (ICRA), Seoul, Korea - May 21-26, 2001

[8] J.J. Leonard, H.F. Durrant-Whyte, “Directed Sonar Sensing for Mobile Robot Navigation,” *Kluwer Academic Publishers*, Dordrecht, 1992.

[9] Martinelli A, “The odometry error of a mobile robot with a synchronous drive system”, *IEEE Trans. on Robotics and Automation* Vol 18, NO. 3 June 2002, pp 399–405

[10] Martinelli A, Tomatis N, Tapus A. and Siegwart R., “Simultaneous Localization and Odometry Calibration” *International Conference on Inteligent Robot and Systems (IROS03)* Las Vegas, USA

[11] P.M.Newman, “On the Structure and Solution of the Simultaneous Localization and Mapping Problem,” *PhD thesis*, Australian Centre for Field Robotics, University of Sydney, 1999.

[12] P.M.Newman and H.F.Durrant-Whyte, “An efficent solution to the slam problem using geometric projection,” *Sensor Fusion and Decentralized Control in Robotics Systems*, Boston, USA, 2001.

[13] Smith, Self, et al. (1988) ”Estimating uncertain spatial relationships in robotics” *Uncertainty in Artificial Intelligence 2* Elsevier Science Pub: 435-461.