Optimum Route Guidance in Multi-region Networks: A Linear Approach

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**MOTIVATION**

Dynamic congestion pricing is a useful tool not only to mitigate traffic congestion but also to influence people’s route choice and direct a transportation network towards the system optimum. Recent works have tried to tackle computation of the system optimum (i.e., the optimal route guidance) problem by applying a nonlinear control strategy. Considering the shortcomings of local operating controllers and the well-known limitations of nonlinear optimization, this work has the objectives to:
- design a linear formulation of the optimization problem at the network level,
- evaluate the performance of a Linear Rolling Horizon Optimization (LRHO),
- provide optimal route guidance as a benchmark for novel congestion pricing approaches.

**MODEL LINEARIZATION**

As the dynamic model holds several nonlinearities, the following steps are taken for linearization:

1. Introduction of second-level decision variables to consider splitting rate constraints
2. Introduction of new parameters
3. Constraint for restricting transfer flows omitted
4. Parameters are held constant over the prediction horizon
5. Soft constraints are changed to hard constraints
6. Loop Detectors (LD) geographical reference

**CASE STUDY - CITY OF ZURICH**

Case study based on an example of the city of Zurich with a region design based on:
- analyzing the main traffic arteries of Zurich and
- the geographical reference of the available Loop Detector (LD).

**MACROSCOPIC MULTI-REGION MODELING**

Dynamic model:

\[
\frac{dM(t)}{dt} = \sum_{j \in J} Q_j(t) - M_j(t) + \sum_{i \in I} M_{ij}(t), \quad i \in I, R \in R, j \in J, i \in I, \quad R = \mathbb{R}
\]

\[
\frac{dM_{ij}(t)}{dt} = \sum_{j' \in J} Q_{ij'}(t) - M_{ij}(t) + \sum_{i' \in I} M_{i'j}(t), \quad i \in I, R \in R, j \in J, i \in I, \quad R = \mathbb{R}
\]

with:

\[
M_j(t) = \frac{N_j(t)}{N_j} G(N_j(t)), \quad M_{ij}(t) = \frac{N_{ij}(t)}{N_j} G(N_j(t)),
\]

\[
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\]

\[
G(N_j) = -\ln \left( \exp \left( -\frac{N_j}{N_{ij}} \right) \right) + \exp \left( -\frac{N_j}{N_{ij}} \right)
\]

**RESULTS AND CONCLUSION**

Evaluation of the optimization procedure was done with the Total Time Spent (TTS) for each ITS application.

- The derived routing information leads to the network system optimum
- Optimization procedure was done with the Total Time Spent (TTS)
- With a region design based on
- The geographical reference of the available Loop Detector (LD)
- The geographical reference of the available Loop Detector (LD)

**REFERENCES**
