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Introducing the 3D-MFD network design problem

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Optimal pricing and investment in a multi-modal city - Introducing the 3D-MFD network design problem

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The three-dimensional macroscopic fundamental diagram (3D-MFD) is a physically consistent functional relationship between the accumulation of buses and cars and each modes average speed in an urban network that captures interactions among vehicles. The 3D-MFD network design problem (3D-MFD-NDP) builds upon advances in 3D-MFD estimation that explicitly link design variables of urban transportation networks to the shape of the 3D-MFD. This advancement allows to study investment effects in urban transport networks directly without separate traffic simulations.

The 3D-MFD-NDP aims to find the optimal investment in transport network infrastructure and pricing such that the behavioral response minimizes total travel time and system subsidy. Mathematically, the 3D-MFD-NDP is a bi-level optimization problem formulated as a mathematical problem with equilibrium constraints (MPEC). At the upper level, the design variables are road network length, bus service frequency, share of dedicated bus lanes, car and bus prices and the system's subsidy. At the lower level, traffic distributes across modes and routes following Wardrop's equilibrium principle. The 3D-MFD-NDP is applied to greater Zurich to study the effects of urban scale pricing and investment.

Key words: MPEC, MFD, congestion, bus, pricing, investment

History:

1. Introduction

We build our transportation infrastructure for carrying people and goods. However, too many vehicles on the road at the same time leads to congestion, makes journey speeds unsatisfactory and increases negative external costs. At the same time, replacing cars with buses can benefit the overall flow of passengers. However, the optimal solution is not only having buses, as bus journeys require users to walk to and from the stop and accept waiting time at a bus stop, which can extend travel times substantially. Therefore, how much traffic and which combination of buses and cars is optimal for a city and how should it be priced? This question is key to transport planning and has been raised since the second half of the 20th century (Smeed 1968).

This question relates to the trade-off of congestion externalities and human preferences for (not) sharing the vehicle, e.g. a bus or ride sharing. Consequently, this question aligns at least with four relevant strategic and tactical decision dimensions of urban transportation. Most notably, the question relates to the traditional network design problem (NDP), i.e. how network size and design affect congestion and travel choices (e.g. Boyce 1984, Magnanti and Wong 1984, Friesz 1985, Migdalas 1995, Yang and Bell 1998), but also how a bus network should be designed and operated (e.g. Patz 1925, Sonntag 1977, Salzborn 1972, Schéele 1980, Holroyd 1965, Ceder and Wilson 1986, Guihaire and Hao 2008, Kepaptsoglou and Karlaftis 2009, Ibarra-Rojas et al. 2015). A further strategic decision is the allocation of dedicated infrastructure to certain modes, e.g. buses or high occupancy vehicles (e.g. Black, Lim, and Kim 1992, Currie, Sarvi, and Young 2004, Menendez and Daganzo 2007, Gonzales and Daganzo 2012, Gonzales et al. 2010, Zheng and Geroliminis 2013). Farahani et al. (2013) summarized the literature on urban transportation NDPs and identified only a few numbers of studies that consider simultaneously several strategic and tactical decisions, i.e. more than one of the above three dimensions. Fourth and last, adequate or optimal pricing for infrastructure use and mobility (e.g. Pigou 1920, Small and Verhoef 2007, Parry 2009, Parry and Small 2009, Anas and Lindsey 2011, de Palma and Lindsey 2011, Kraus 2012, Tirachini and Hensher 2012, Tirachini, Hensher, and Rose 2014, Tirachini, Hensher, and Bliemer 2014), which has been frequently cited as a powerful tool to influence behavioral responses.

Ultimately, for cities to understand how to obtain the best possible mobility, all four dimensions (road network design, bus network design and operations, dedicated lanes, and pricing) have to be combined. So far, methodological opportunities to do so seem to be missing. However, the recently introduced macroscopic fundamental diagram (MFD) and in particular its bi-modal extension to the 3D-MFD (Geroliminis and Daganzo 2008, Geroliminis, Zheng, and Ampountolas 2014, Loder et al. 2017) allow to cover and combine all four relevant dimensions. First research provides promising results (e.g. Zheng and Geroliminis 2013, Dantsuji, Fukuda, and Zheng 2017, Zhang et al. 2018), but a coherent approach to all four dimensions is missing, most likely as no physically

consistent functional form for the 3D-MFD exists that does not require the 3D-MFD estimation in a separate traffic simulator anymore. Here, we use a recently proposed functional form for the 3D-MFD that captures explicitly the physical properties of the network (Loder et al. 2019). Consequently, the 3D-MFD needs no separate re-simulation anymore and the speeds for each mode can be directly obtained from the network topology and vehicle accumulations of both modes. Therefore, we combine this functional form with strategic, tactical and pricing decisions of urban car and bus networks and introduce it as the 3D-MFD-Network Design Problem (3D-MFD-NDP), formulated as a mathematical problem with equilibrium constraints (MPEC).

The idea of this 3D-MFD-NDP is a bi-level optimization problem where at the upper level infrastructure and pricing decisions are made to minimize the total system cost of the network (i.e. total travel time plus subsidy) and at the lower level traffic distributes across routes following Wardrop's equilibrium principle. The design variables of the urban transportation system are road network length, bus service frequency, share of dedicated bus lanes, car and bus prices and the system's subsidy. In other words, the 3D-MFD-NDP is looking for the solution of the network design and pricing variables such that behavioral response minimizes the total time expenditures and subsidy of the system, subject to constraints that existing demand is assigned to the network and monetary expenditures equal revenue.

This paper is organized as follows. In Section 2 we introduce the mathematical formulation of the 3D-MFD-NDP and discuss each constraint. In Section 3 we illustrate the feasibility of the proposed model with a case study to greater Zurich (CH). Here, we first discuss in Section 3.1 the model calibration and, then, in Section 3.2 we investigate the optimal investment and pricing strategy before we summarize our findings and conclude the paper in Section 4.

2. Model

In this 3D-MFD-NDP, we are looking for a network configuration and mobility pricing strategy that minimizes total system cost, subject to the constraints that the existing demand is assigned to the network, that monetary expenditures for infrastructure equal revenue, and that the system's physical constraints are satisfied. Consequently, we formulate the 3D-MFD network design problem as a mathematical program with equilibrium constraints (MPEC) (Luo, Pang, and Ralph 1996), because the upper level objective is to minimize total travel time costs and subsidy in the network and the lower level objective is a multimodal user equilibrium traffic assignment. The design (free) variables of the 3D-MFD-NDP are (i) network length L , (ii) share of dedicated bus lanes η_b , (iii) the bus service frequency or headway H , (iv) the fixed price of mobility tools π^{fix} , i.e for the car or public transport season-ticket, (v) the variable costs per unit distance π^{var} , (vi) and the system's (public) transport subsidy S .

In the 3D-MFD-NDP, the morning commute is mathematically described where commuters move from their residential location (inside and outside the city), i , to their workplaces in the central business district, j . The transportation systems offers a set of \mathcal{M} transport modes with two elements $m \in \{\text{bus}; \text{car}\}$. Based on the travel cost, commuters choose their mobility tool t out of set \mathcal{T} of available mobility tools. They can have either only a car, only a public transport season ticket or both. The price or cost structure of both mobility tools has a fixed and variable component, i.e. π^{fix} and π^{var} . This allows, for example, to consider different pricing schemes such as a public transport season-ticket without any variable costs or a distance-dependent pricing for public transport without any fixed costs. In this model, we formulate the constraint that only that number of commuters who use a certain mode is limited by the number of commuters who have access to that mode with their set of mobility tools.

From a spatial perspective for MFD-based applications, urban road networks are partitioned into small and homogeneously congested traffic zones (Ji and Geroliminis 2012). Multiple zones allow to account for spatial heterogeneity in the allocation of space, e.g. comparatively more dedicated space to buses in the central districts compared to more distant zones. Consequently, the morning commute is modeled as shown in Figure 1. The city is divided into several MFD zones or reservoirs as shown in Figure 1a, where each zone has internal flows and exchanging flows. In this analysis, we then follow the regional path perspective for the trips (Yildirimoglu and Geroliminis 2014, Yildirimoglu, Ramezani, and Geroliminis 2015, Batista, Leclercq, and Geroliminis 2019), where agents choose regional path r out of \mathcal{R} paths for each origin-destination pair. Regional paths do not correspond to a specific route matched to the road network, but describe the distribution of trip lengths for a certain origin-destination pair in a specific zone k (Batista and Leclercq 2019). However, with a static assignment used in this analysis, we focus on the mean of this distribution. We assume that the number of trips for each origin-destination pair, n_{ij} , the trip distances, d_{ijmr} , and the shares of each trip in each zone k , θ_{ijkmr} , are known and fixed.

In this section, we first introduce the objective function in Section 2.1, then we provide the set of equilibrium constraints in Section 2.2, the physical capacity constraints in Section 2.3, and the economic constraints in Section 2.4. Thereafter, we formulate the entire 3D-MFD-NDP in Section 2.6. Note that values and variables with an overline denote the observed values in the model calibration. Further, we formulate the equilibrium constraints as a mixed complementarity problem (MCP), following Rutherford (1995), Nagurney (1993) and van Nieuwkoop (2014). We use the \perp to indicate complementarity between an equation and the associated variable.

2.1. Objective function

The upper level objective y of the 3D-MFD-NDP is defined in Eqn. 1. Here, N_{ijmr} are the passenger flows between origin i , destination j , on mode m , using route r , and T_{ijmr} is the corresponding

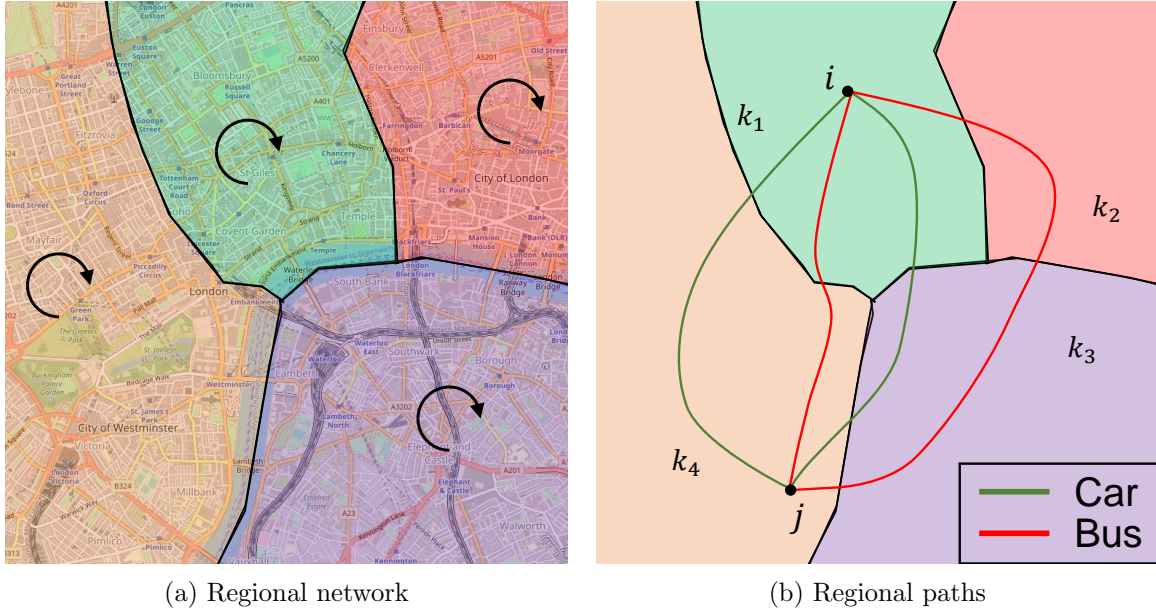


Figure 1 Network idea for the MFD based traffic assignment. (a) illustrates the partition of an urban area into reservoirs with cars circulating in the reservoirs. (b) shows the idea of regional paths across several regions.

travel time for each passenger. The second part of the objective function equals to the total system subsidy S that is used to fund the city’s mobility assets: road infrastructure and buses. To value both parts with the same units, we transform travel time to monetary terms using the city’s value of time VOT .

$$y = VOT \sum_{ijmr} N_{ijmr} T_{ijmr} + S \quad (1)$$

The costs for mode use will be accounted for in the income balance in Section 2.2. They are not accounted for in Eqn. 1 for two main reasons. First, they have to be paid by agents anyway to cover the infrastructure expenses and thus only the subsidy matters. Second, this model does not include a mechanism for the agents to generate income to reflect their trade-offs in time and money.

2.2. Equilibrium constraints

We adopt a multimodal traffic assignment, formulated as a stochastic user equilibrium (SUE) following Wardrop’s first principle of the user equilibrium based on perceived travel cost. We assume that agents choose mode m and route r with the lowest perceived costs, \check{C}_{ijmr} . Thus, a particular route and mode between origin i and destination j is only chosen if its perceived path costs \check{C}_{ijmr} along that route are equal to the minimum path costs, i.e. $M_{ij} \equiv \min_{mr} \check{C}_{ijmr}$. In other words, $N_{ijmr} > 0$ only if its path costs are equal to the minimum cost M_{ij} . If costs exceed M_{ij} , the route is not used, i.e. $N_{ijmr} = 0$. This feature is captured in the complementary condition of Eqn. 2.

$$\check{C}_{ijmr} - M_{ij} \geq 0 \quad \perp \quad N_{ijmr} \geq 0 \quad (2)$$

The perceived paths costs are defined according to Eqn. 3, where C_{ijmr} are the actual path costs as defined by Eqn. 4 and μ_R is the associated scale parameter. Note that Eqn. 3 is adopted from Chen (1999) and describes a simultaneous route and mode choice multinomial logit model.

$$\check{C}_{ijmr} = C_{ijmr} + \frac{1}{\mu_R} \log(N_{ijmr}) \quad (3)$$

The path costs C_{ijmr} are the generalized cost of travel and combine as given by Eqn. 4 the in-vehicle travel time T_{ijmr} for both modes and shadow prices resulting from the capacity constraints, i.e. for parking ρ^P , car ownership ρ^C , bus passenger capacity ρ^B , and season ticket ownership ρ^T . The associated constraints are introduced in Section 2.3. Note that the monetary costs for using a particular mode and mobility tools are separately considered in the mobility tool ownership constraints in Section 2.5. Further, the path costs contain for buses a general waiting time defined as half the headway H_i in departure zone i . Preferences of commuters and other factors that influence the choice as well are captured in $\bar{\varphi}_{ij}$ that requires calibration from data.

$$\begin{aligned} C_{ij,\text{car},r} &= T_{ij,\text{car},r} + \rho_i^C + \rho_j^P \\ C_{ij,\text{bus},r} &= T_{ij,\text{bus},r} + \frac{H_i}{2} + \rho_i^B + \rho_i^T + \bar{\varphi}_{ij} \end{aligned} \quad (4)$$

Travel times T_{ijmr} are calculated with Eqn. 5 that is the sum of travel times within each region along each route r . Here, V_{km} is the journey speed of mode m in sub-region k , $\theta_{ijkmr} d_{ijmr}$ equals the trip distance in region k . For buses, the travel time contains the in-vehicle time including dwelling behavior.

$$T_{ijmr} = \sum_k \theta_{ijkmr} \frac{d_{ijmr}}{V_{km}} \quad (5)$$

We derive the journey speeds from the 3D-MFD (Geroliminis, Zheng, and Ampountolas 2014, Loder et al. 2019). The 3D-MFD links the current accumulation of cars, $A_{k,\text{car}}$, and buses, $A_{k,\text{bus}}$, to the average speed of mode m in region k as formulated by Eqn. 6. The shape of the 3D-MFD results from the features and topology of the road and bus networks. Consequently, when changing the network design variables of the 3D-MFD-NDP, the 3D-MFD will change and thus affect the speeds in the network.

$$V_{km} = \text{3D-MFD}_{km}(A_{k,\text{car}}, A_{k,\text{bus}}) \quad (6)$$

In our traffic assignment model, we cannot use Edie's (1963) definition to calculate the accumulation or density of vehicles as there is no distinct time interval. Therefore, we calculate each modes' vehicle accumulation differently. For the accumulation of cars, $A_{k,\text{car}}$, we assume a vehicle occupancy of one passenger and obtain the accumulation by Eqn. 7, where $\theta_{ijk,\text{car},r}$ is the fraction of trip length of that particular route going through region k .

$$A_{k,\text{car}} = \sum_{ijr} \theta_{ijk,\text{car},r} N_{ij,\text{car},r} \quad (7)$$

We derive the accumulation of buses, $A_{k,\text{bus}}$, from the structure and design of the bus network (adopted from Daganzo (2010)) as given by Eqn. 8. Here, α_k is an exogenous parameter describing the design of the bus network in each region for which $0 < \alpha_k \leq 1$ holds. Close to its lower bound, α_k describes a hub-and-spoke network, while at its upper bound it describes a grid network. Values in between are hybrid networks where one can see α_k as the fraction of network exhibiting a grid network. As in many cities bus lines are partially overlapping, we introduce z_k that quantifies how many bus lanes are overlapping on the bus infrastructure B_k . In case no bus lines are overlapping, $z_k = 1$, if two bus lines are overlapping on the entire network, then $z_k = 2$ and so on. Last, $V_{c,\text{bus}}$ is the design commercial speed of buses in the network.

$$A_{k,\text{bus}} = z_k \frac{2B_k}{H_k} \frac{3\alpha - \alpha^2}{1 + \alpha^2} / V_{c,\text{bus}} \quad (8)$$

Lastly, Eqn. 9 then provides the conservation of passenger flows for each origin and destination pair. M_{ij} is the associated complementary variable. Importantly, as we are formulating the 3D-MFD-NDP based on regional paths, there is no requirement to explicitly model the in- and outflows at each node as in a link based assignment.

$$n_{ij} - \sum_{mr} N_{ijmr} = 0 \quad \perp \quad M_{ij} \geq 0 \quad (9)$$

2.3. Physical system (capacity) constraints

The static traffic assignment model has a set of four inequalities that describe the physical constraints of the system and which have associated shadow price variables that factor into the path costs. The first constraint describes the parking supply in each zone as given by Eqn. 10. Here, \bar{P}_j is the parking supply in zone j , an exogenous parameter. Consequently, the total arrival car passenger flow cannot exceed the parking supply. If the parking demand exceeds the parking supply, then a non-zero shadow price, ρ_j^P , will ensure that the number of arriving cars is restricted to the parking supply.

$$\bar{P}_j - \sum_{ir} N_{ij,car,r} \geq 0 \quad \perp \quad \rho_j^P \geq 0 \quad (10)$$

The second and third inequality captures the model's property that all departing trips of a zone of a certain mode must not exceed the availability of mobility tools in that zone. In other words, the number of car trips starting in i cannot be greater than the number of available cars in i as given by Eqn. 11. Similarly, the number of outbound bus passenger trips cannot exceed the number of public transport season-tickets, or in brief *abos*¹, in that zone as formulated in Eqn. 12. In these equations, Q_{ijt} describe the shares of mobility tool ownership, where the elements of set t are having only a car, an abo or having both. The calculation of Q_{ijt} is discussed along with Eqn. 20. When the inequality becomes binding, the respective shadow prices ρ_i^C and ρ_i^T become non-zero.

$$\sum_j (Q_{ij,car} + Q_{ij,both}) n_{ij} - \sum_{jr} N_{ij,car,r} \geq 0 \quad \perp \quad \rho_i^C \geq 0 \quad (11)$$

$$\sum_j (Q_{ij,abo} + Q_{ij,both}) n_{ij} - \sum_{jr} N_{ij,bus,r} \geq 0 \quad \perp \quad \rho_i^T \geq 0 \quad (12)$$

The fourth inequality describes that the passenger capacity of the bus system is limited to a total passenger accumulation of Z_k as formulated in Eqn. 13. The total bus passenger flows have to be always less or equal to that capacity. When supply equals demand, public transport users experience additional waiting time ρ_k^B in their departing zone as all arriving buses are full.

$$Z_k - \sum_{ijr} \theta_{ijk,bus,r} N_{ij,bus,r} \geq 0 \quad \perp \quad \rho_k^B \geq 0 \quad (13)$$

The total bus passenger capacity in a region Z_k results from the accumulation of buses in each region according to Eqn. 14. Note that overline variables denote observed calibration values.

$$Z_k = \bar{Z}_k \frac{A_{k,bus}}{\bar{A}_{k,bus}} \quad (14)$$

2.4. Economic constraints

This set of constraints restricts solutions to the 3D-MFD-NDP where the revenue from mobility (ownership π_t^{fix} and use π_{tm}^{var}) and the subsidy S equals the operational costs O for the city's mobility assets. This operational costs O for the city's mobility assets, i.e. the costs for the provision of roads and bus operations, are calculated with Eqn. 15. Here, c_k^{road} and c_k^{bus} are the unit prices for the provision of infrastructure and buses, respectively. Consequently, the totals then depend on the size of the network L_k and number of buses $A_{k,bus}$.

¹ Abbreviation of the German word Abonnement.

$$O = \sum_k c_k^{bus} A_{k,bus} + c_k^{road} L_k \quad (15)$$

Then, the income balance of costs, revenue and subsidy is mathematically expressed in Eqn. 16, where Q_{ijt} corresponds to the shares of mobility tool ownership for that particular origin-destination pair (see derivation in Section 2.5). Here, we assume that the total revenue counts towards the available budget for infrastructure spending, although in reality this is too simplistic as public funding and budgets are usually more complex (see our discussion in the calibration of the model in Section 3.1).

$$\sum_{ijt} \pi_t^{fix} n_{ij} Q_{ijt} + \sum_{ijmr} \pi_{tm}^{var} N_{ijmr} d_{ijmr} + S = O \quad (16)$$

The costs for buses c_k^{bus} and roads c_k^{road} can be subject to (dis-) economies of scale. Therefore, we account for this by the formulations for both costs in Eqn. 17 and 18, respectively. The cost functions are calibrated to market values that are indicated by an overline to the cost function's elasticity ε . In case $\varepsilon = 0$, the costs are constant, while for $\varepsilon > 0$ diseconomies of scale, and for $\varepsilon < 0$ economies of scale result.

$$c_k^{road}(L_k) = \bar{c}_k^{road} \exp(\varepsilon^{Road} \log(L_k/\bar{L}_k)) \quad (17)$$

$$c_k^{bus}(A_{bus,k}) = \bar{c}_k^{bus} \exp(\varepsilon^{Bus} \log(A_{bus,k}/\bar{A}_{bus,k})) \quad (18)$$

2.5. Mobility tool ownership constraints

The shares of mobility tool ownership Q_{ijt} change with the prices of the chosen portfolio t , π_{ijt}^{total} , in a two stage logit-based choice. In this choice environment, individuals have three mobility tool portfolio options to choose from: only a car, only an abo, i.e. bus season-ticket, or both. This simplifies the complexity of choices for mobility tools typically available to individuals, as shown, for example, in Switzerland (Becker et al. 2017, Loder and Axhausen 2018). Note that for readability we omit the set indices ij for Q and π^{total} in the following two equations.

We define the utility functions for the logit model as given in Eqn. 19. The alternative specific constant (ASC) is related to the calibrated market share \bar{Q}_{ijt} . Utility changes with changes relative to the calibration prices with scale parameter μ^M that captures the price elasticity of mobility tool ownership. Eqn. 20 then presents the two-stage logit-based model to obtain the shares of mobility tool ownership. The first stage determines the shares of having both or not both mobility tools (car and abo), while the second stage determines the shares between car and abo owners along

those not having both mobility tools. Note that the formulation of a logit model ensures that the shares always add up to one.

$$\begin{aligned}
u_{\text{both}} &= \log(\bar{Q}_{\text{both}}) + (\pi_{\text{both}}^{\text{total}}/\bar{\pi}_{\text{both}}^{\text{total}} - 1) / \mu^M \\
u_{\text{not both}} &= \log(1 - \bar{Q}_{\text{both}}) \\
u_{\text{car}} &= \log(\bar{Q}_{\text{car}}) + (\pi_{\text{car}}^{\text{total}}/\bar{\pi}_{\text{car}}^{\text{total}} - 1) / \mu^M \\
u_{\text{abo}} &= \log(\bar{Q}_{\text{abo}}) + (\pi_{\text{abo}}^{\text{total}}/\bar{\pi}_{\text{abo}}^{\text{total}} - 1) / \mu^M
\end{aligned} \tag{19}$$

$$\begin{aligned}
Q_{\text{both}} &= \frac{\exp(u_{\text{both}})}{\exp(u_{\text{both}}) + \exp(u_{\text{not both}})} \\
Q_{\text{car}} &= (1 - Q_{\text{both}}) \frac{\exp(u_{\text{car}})}{\exp(u_{\text{abo}}) + \exp(u_{\text{car}})} \\
Q_{\text{abo}} &= (1 - Q_{\text{both}}) \frac{\exp(u_{\text{abo}})}{\exp(u_{\text{abo}}) + \exp(u_{\text{car}})}
\end{aligned} \tag{20}$$

The average price π_{ijt}^{total} or total cost of ownership for mobility tool portfolio t between i and j is calculated with Eqn. 21. Here, F_{ijtm} gives the fraction of using mode m with mobility tool set t (defined below in Eqn. 22) and the last term in parentheses gives simply the average trip distance. Recall that the fix costs or price per mode m with mobility tool portfolio t is π_t^{fix} , while the variable and distance-depending prices for mode m and mobility tool portfolio t is π_{tm}^{var} . For simplicity in this analysis, we do not distinguish between different price or cost components and subsume all taxes, fares etc. under the term costs. Further, for cars we do not account for the variable operating costs (as we do not study commuters income), but only the fixed costs, e.g. car registration and fees for parking at the destination.

$$\pi_{ijt}^{\text{total}} = \pi_t^{\text{fix}} + \sum_m \pi_{tm}^{\text{var}} F_{ijtm} \left(\sum_r \frac{d_{ijmr}}{|\mathcal{R}|} \right) \tag{21}$$

The fraction F_{ijtm} using mode m with mobility tool set t is defined according to Eqn. 22. I_{tm} is an indicator function that equals one if $t = \text{abo} \ \& \ m = \text{bus}$ or $t = \text{car} \ \& \ m = \text{car}$, and equals zero otherwise. In other words, when owning only either an abo or a car, only the respective mode can be used, i.e. $F_{ijtm} \equiv 1$, and the other mode cannot be used, i.e. $F_{ijtm} \equiv 0$. Only when having both mobility tools, F_{ijtm} can be different from zero or one as shown in Eqn. 22. Then, F_{ijtm} is simply the number of mode m commuters over all commuters for each origin-destination pair having both mobility tools.

$$F_{ijtm} = \begin{cases} \frac{\sum_r N_{ijmr} - n_{ij} \sum_{t'} I_{t'm} Q_{ijt'}}{Q_{ijt} n_{ij}}, & \text{if } t = \text{both} \\ I_{tm}, & \text{otherwise} \end{cases} \tag{22}$$

2.6. Problem formulation

The objective function y in Eqn. 1 and all constraints defined in Eqns. 2 - 22 allow to formulate the 3D-MFD-NDP as formulated in Eqn. 23: The 3D-MFD-NDP is looking for the solution of the network design and pricing variables that reduces the total system costs (travel time and subsidy), subject to the constraints that the existing demand is assigned to the network, that the physical capacity constraints are satisfied and that monetary expenditures equal revenue.

$$\begin{aligned}
 & \text{minimize} && y \\
 & \text{subject to} && (2) - (9) \quad \text{solving MCP} \\
 & && \text{and} \quad (10) - (14) \quad \text{solving capacity constraints} \\
 & && \text{and} \quad (20) - (22) \quad \text{solving ownership constraints} \\
 & && \text{and} \quad (15) - (18) \quad \text{solving economic constraints}
 \end{aligned} \tag{23}$$

The model formulation in Eqn. 23 provides an opportunity to understand the interaction of multi-modal vehicle physics and behavioral response.

3. Case study

3.1. Calibration

The 3D-MFD-NDP requires calibration to an observed point, not only because of \bar{Q}_{ijt} , $\bar{\pi}_{ijt}^{\text{total}}$, and $\bar{\varphi}_{ij}$, but also because to provide meaningful starting values and to determine a meaningful solution space.

In this paper, we calibrate the 3D-MFD-NDP to the morning commute in the greater region of Zurich, Switzerland. Figure 2 shows the extent of the case study area. We partition the network into two regions: Zones 1-12 denote the city where the commuters live and work and zones 101-111 are zones where commuters live and have to commute into the city, i.e. zones 1-12 to work. We investigate pricing effects for all commuters, i.e. living in all zones, but investigate investment effects only to the infrastructure in the city, i.e. zones 1-12. In this model, zones 1-12 each exhibit a 3D-MFD as formulated by Loder et al. (2019) to obtain the speeds as given in Eqn. 6. The speeds for zones 101-111 are considered to be fixed, i.e. independent of demand. The zones 101-111 are added to the model to capture the influence of suburban commuting into the city.

For the origin and destination matrix n_{ij} we use the commuting matrix of a synthetic Swiss population for the agent-based simulation MATSim (Bösch, Müller, and Ciari 2016). We re-scale the total arrivals in each of the twelve inner zones of MATSim's commuter matrix to correspond to the work place totals used by the national transport model (NPVM).

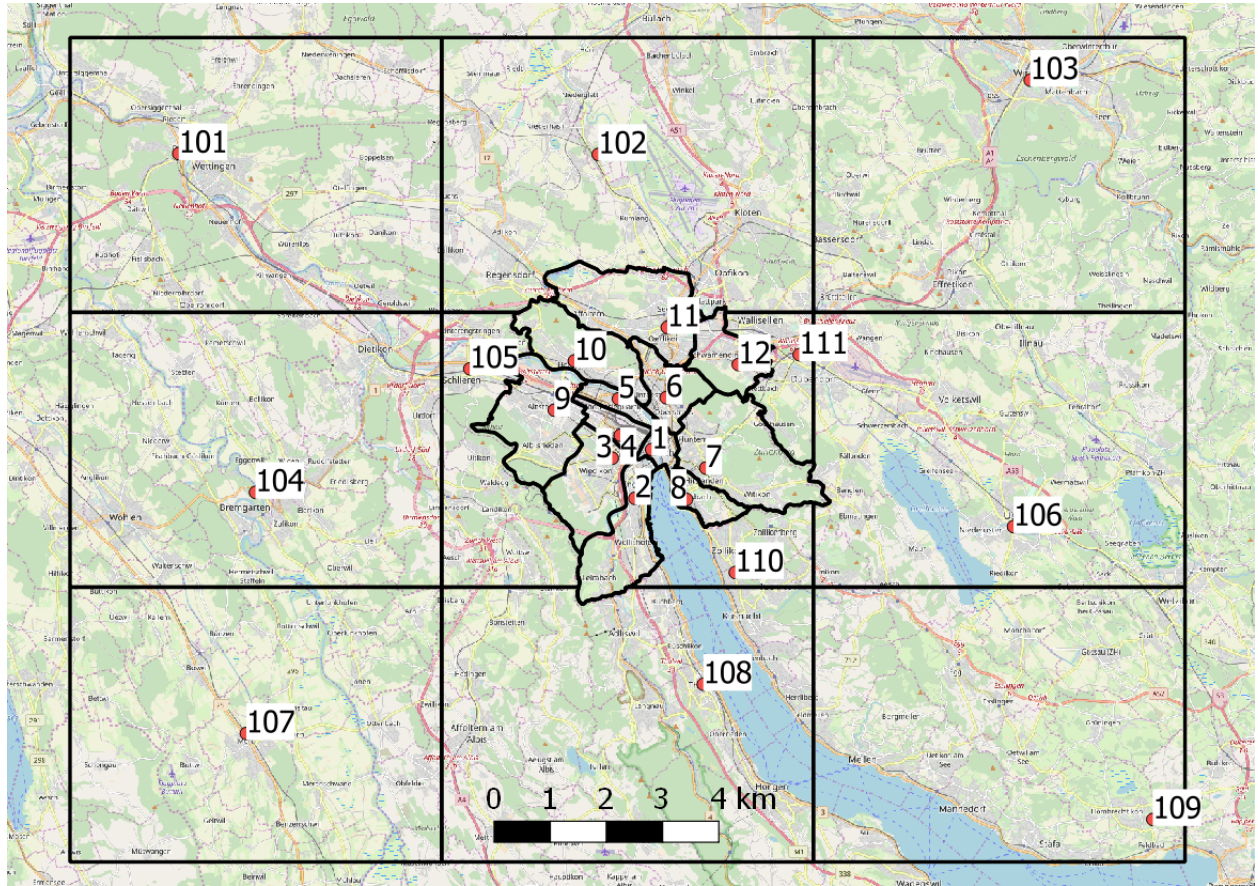


Figure 2 Zonal system for the case study.

We obtained spatial information on the regional paths for both modes from the Google directions API: For each origin and destination pair, we requested the shortest route including alternative routes without using the motorway and calculated θ_{ijkmr} thereof. We calibrated the 3D-MFD speed functions based on OpenStreetMap data and measurements based on the data used by Loder et al. (2017). From the 2015 Swiss travel survey we obtained the mode shares of outbound trips of each region (Swiss Federal Statistical Office and Swiss Federal Office for Spatial Development 2017). With all this information, we solved for each origin and destination pair a nonlinear programming problem minimizing the squared difference between the observed mode share and resulting mode choice from Eqn. 9 to calculate $\bar{\varphi}_{ij}$ of the path costs.

For Zurich, we consider the following mobility tool portfolio and pricing situation: When having a car, commuters face fix costs as well as variable costs per unit distance. When having a bus season-ticket (or as defined here “*abo*”), commuters face only fix costs, but no variable costs. In other words, it is not possible to purchase single ride or distance-dependent bus tickets. This situation reflects the situation in Switzerland where most public transport commuters own a season-ticket. When having both mobility tools, commuters have to pay the costs of both single mobility tools.

Parameter or observed value	Unit	Value
\bar{c}^{bus}	(CHF/day bus)	3100 ¹
\bar{c}^{road}	(CHF/day km)	1900 ²
ε^{bus}	(-)	{0; -0.2} ⁷
ε^{road}	(-)	{0; 0.2}
$\bar{\pi}_{abo}^{fix}$	(CHF/day)	3 ³
$\bar{\pi}_{abo}^{var}$	(CHF/km)	0 ⁴
$\bar{\pi}_{car}^{fix}$	(CHF/day)	5 ⁵
$\bar{\pi}_{car}^{var}$	(CHF/km)	0.1 ⁶
μ^R	(-)	10
μ^M	(-)	-0.5 ⁹
VOT	(CHF/h)	25 ⁸

¹ Zurich's VBZ has annual expenditures of 600 million CHF for their operation of 470 vehicles in 2016.

² Swiss average of annual expenditures of cities <https://opendata.swiss/de/dataset/statistik-der-schweizer-stadte-strassenrechnung>

³ Costs for annual ZVV pass (season-ticket) for three zones approx CHF 1200, divided by 365 working days.

⁴ Set to zero as we focus on the option of season-tickets.

⁵ Assuming fix annual taxes, fees etc. of CHF 800, and annual parking costs of CHF 1.200, in total CHF 2.000, divided by 365 working days.

⁶ For urban traffic, assuming 8 litre per 100km, a fuel price of CHF 1.30 per litre.

⁷ Bösch et al. (2017) reported average discounts of 20 % for fleet operators.

⁸ Estimated by Schmid (2019).

⁹ Average fuel price elasticity estimated by Erath and Axhausen (2010).

Table 1 Price and cost information for the calibrated 3D-MFD-NDP.

We obtain from the 2015 Swiss travel survey the shares of mobility tool ownership \bar{Q}_{ijt} based on all commuters living in the case study zones. Note that we assign all commuters without a car or abo (season-ticket) to the abo category. For most origin-destination pairs, this share was less than ten percent of the total origin-destination demand.

In Table 1, we summarize the economic and behavioral parameters used for the model and its benchmark. Note that in solving the 3D-MFD-NDP MPEC, we will solve it once with economies of scale of investment in roads and buses as well as assuming no economies of scale. We further comment in Table 1 on where or how the values are obtained.

In Table 2 we summarize the benchmark model's most relevant performance indicators. For the situation in Zurich as shown in Figure 2 it is important to explain how the mobility tool revenue relates to the public budget. We define for inbound commuters that they only pay a fraction of their mobility tool expenses to the city's transport budget. The fraction is determined by the fraction of their trip length in the city. We further define that 60 % of the ticket revenue goes to the budget, while we consider the remaining part is going to the railway operators (not considered in this analysis) and to the cantonal transport agency. Similarly, we assume for the car that only 25 % of the revenue is directed to the city's budget, while the remaining revenue goes to private companies (e.g. petrol stations) and other tax purposes (e.g. the CO₂ tax).

Observed value	Unit	Value
$\sum \bar{L}$	(km)	1400 ¹
$\sum \bar{A}_{\text{bus}}$	(bus)	1040 ²
\bar{H}	(h)	0.125
\bar{S}	(CHF/day)	48'000 ³
Share of car trips	(-)	0.33 ⁴
\bar{Q}_{car}	(-)	0.18
\bar{Q}_{both}	(-)	0.35
\bar{Q}_{abo}	(-)	0.47

¹ Includes the city's and the canton's road network, the infrastructure of public transport and the space on the high capacity roads. The calibration of the 3D-MFD to observed speeds, L is overestimated by around 200 km.

² This vehicle number is twice as much as the bus operator owns, but this results from the problem that we cut bus routes at the zones from Figure 2 and consequently count vehicles several times.

³ Calculated as the difference between infrastructure costs and revenue from mobility tools.

⁴ Calculated based on travel kilometers.

Table 2 Transport network calibration values for the 3D-MFD-NDP.

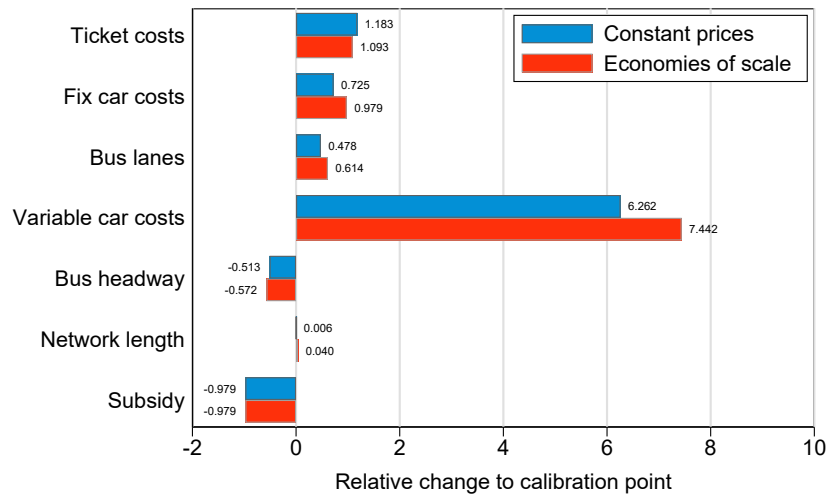
Lastly, we have to calibrate or define meaningful upper and lower bounds for all design variables. Arguably, the model is calibrated to the current situation and solutions far off this situation, e.g. building twice as many roads, are physically not feasible. Therefore, we allow only for changes in the length of the road network of $\pm 10\%$ of its length. We further limit the length of dedicated bus infrastructure to 90% of the actual network length of buses because some interactions with cars, e.g. at intersections are unavoidable. For the prices, we set the following upper values $\pi_{\text{abo}}^{\text{fix}} = 20$ (CHF / day), $\pi_{\text{car}}^{\text{fix}} = 50$ (CHF / day), and $\pi_{\text{car}}^{\text{var}} = 0.1$ (CHF / km). We further bound the number of bus services with a headway between 1 and 12 minutes.

3.2. Scenario analysis

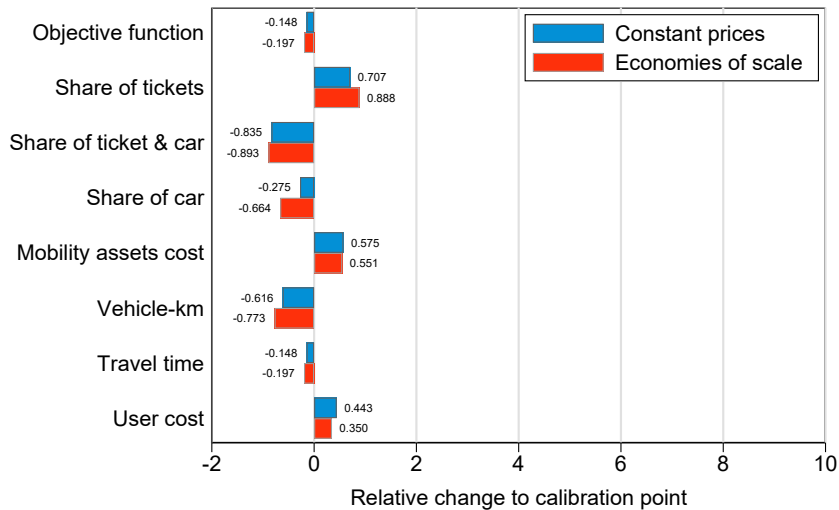
In this section, we solve the MPEC for the 3D-MFD-NDP to illustrate feasibility of the model. We investigate the optimal system configuration and its costs compared to the calibration point, introduced in the previous section.

In this scenario analysis, we set all design variables, i.e. network length, dedicated bus lanes, bus frequency, fixed costs for car and bus season-ticket (the abo option), variable car costs as well as the subsidy) as free variables. In Figure 3a we show the relative changes of the design variables compared to the calibration point and in Figure 3b several global indicators of the total system performance. In general, we find that having constant prices and economies of scale leads to a similar response pattern with only small differences between both cases. However, the existence of scale economies lead to a larger reduction in travel time.

In Figure 3a we find that costs for the car increase sharply, so are the costs for the season-tickets. These increases compensate for the subsidy reduction to its lower bound, but also to shift demand



(a) Design variables.



(b) System response.

Figure 3 Solution to the 3D-MFD-NDP with all design variables as free variables. (a) shows the changes of the design variables relative to the calibration point. (b) shows the changes of the system response relative to the calibration point.

from cars to the bus system as seen in the changed shares of mobility tool ownership as seen in Figure 3b. The substantial increase for the variable car costs can also be seen as congestion pricing. For the bus system, the optimal solution suggests to increase the share of dedicated bus lanes as well as the bus frequency. In presence of scale economies, both effects are strengthened. Notably, the total network length experiences almost no changes, but as the share of dedicated bus lanes is increasing, less space for cars is available. The subsidy is in both scenarios driven to its lower bound because this variable can be reduced without consequence of any other budget at the expense of higher prices for mobility tools. However, note that in reality the subsidy is paid by the

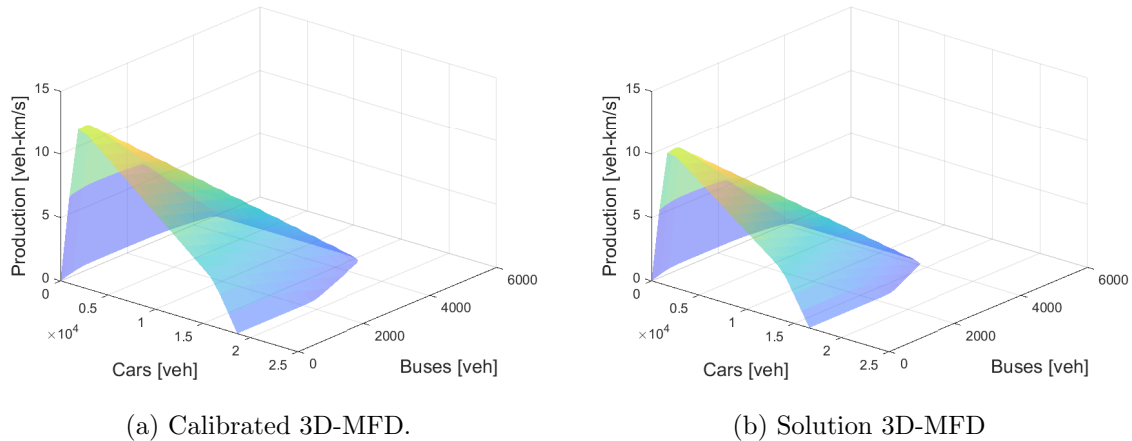


Figure 4 Comparison of resulting 3D-MFDs for zone 1 of the case study. (a) shows the 3D-MFD for the calibration point. (b) shows the 3D-MFD resulting from the 3D-MFD-NDP without presence of economies of scale.

overall public budget with results from tax revenue paid by agents, but this loop is omitted in the 3D-MFD-NDP.

The consequences of the optimal solution are savings in travel time of almost 20 % in presence of scale economies, a more than 70 % reduction in car kilometers travelled, and increased costs for the mobility assets (road infrastructure and the operations of buses) and user costs. With scale economies present, the cost increase is intuitively smaller.

Lastly, to illustrate the system changes on the shape of the 3D-MFD we plot the calibrated and resulting 3D-MFD for zone 1 (see Figure 2) in Figure 4. It can be clearly seen that the reduction of 15 % in road infrastructure decreased the total capacity of the zone’s road network. Further, we find that separating both modes in the 3D-MFD-NDP solution reduces the impact of production losses due to cross-modal interactions, especially in the congested branch.

4. Conclusions

In this paper, we introduced the multi-modal macroscopic fundamental diagram network design problem (3D-MFD-NDP) formulated as a mathematical program with equilibrium constraints (MPEC), which is built around a recently introduced functional form for the 3D-MFD (Loder et al. 2019). The 3D-MFD-NDP is looking for the solution of the network design and pricing variables that behavioral response minimizes the total system costs of the urban transportation system, subject to the constraints that the existing demand is assigned to the network and then and that monetary expenditures equal revenue. The proposed 3D-MFD-NDP provides a macroscopic and multi-modal approach to combine road network design, bus network design and operations, allocation of dedicated lanes, and mobility pricing in a single optimization problem. The design

variables of the proposed 3D-MFD-NDP formulation are road network length, bus service frequency, share of dedicated bus lanes, car and bus prices and the system's subsidy.

The 3D-MFD-NDP describes important and timeless conflicts and trade-offs in strategic transportation planning. Consequently, the results of increasing car costs and improving public transport for a better system performance do not surprise. Nevertheless, the 3D-MFD-NDP helps to understand the direction and magnitude of changes required for an improved or even optimal performance. The formulated 3D-MFD-NDP focuses on key design variables for urban transportation and their interactions and thus support planners and decision makers to derive quantitative results. Future research that extends the idea of the 3D-MFD-NDP should consider wider economic impacts of distributional effects and accessibility to account both the rich and the poor.

This model has limitations. First, we use a static traffic assignment for two reasons: (i) its simplicity is widely been used for long-term planning (ii) the functional form for the 3D-MFD describes its lower envelope (Loder et al. 2019) and does not capture in its present formulation the dynamics of a network. Thus, using a dynamic traffic assignment would not only improve the model's power for policy making, but would also allow to include traffic control strategies (e.g. Haddad and Geroliminis 2012) and the effects of transit priority (e.g. Guler and Menendez 2014, Guler, Gayah, and Menendez 2016). Second, the 3D-MFD-NDP's solution strongly depends on the calibration of the infrastructure and bus cost curves. Without reliable estimates, the solution space is not only falsely defined but also provides inaccurate results and then in turn inappropriate policy implications. As these cost functions are usually hard to obtain - as it is the case for Zurich as well - the model requires a careful data preparation and calibration phase. Third, the topological design of the bus and road network as well as the effects of other modes, e.g. pedestrians, motorcycles and bicycles is not part of the optimization in this formulation of the 3D-MFD-NDP, because it is still ongoing research how these dimensions influence the shape of the MFD. Thus, once this knowledge is available, future research can re-formulate the 3D-MFD-NDP. Fourth, focusing on the morning commute is obvious, as it is a situation where the infrastructure is operating usually at capacity and the boundary condition is a rather empty network. However, this perspective is too simplistic because it ignores urban logistics, leisure travel etc. Thus, accounting for these traffic contributors in future research would improve the quantitative implications that the 3D-MFD-NDP provides.

In closing, the 3D-MFD-NDP presents here the first approach to bundle several important strategic urban and transport planning decisions into a single optimization problem. The proposed 3D-MFD-NDP is simple and computationally fast as it does not requires separate traffic simulations, but it still accounts for many interactions and feedback of physical properties and behavioral responses. Thus, urban planners and policy makers can use the 3D-MFD-NDP to generate and compare various different policy scenarios subject to their city-specific constraints to identify the

optimal investment pricing strategy for their city. As the objective is to minimize total system costs - the social optimum - the proposed 3D-MFD-NDP proves a macroscopic and multi-modal tool to derive strategies to mobility for everyone.

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