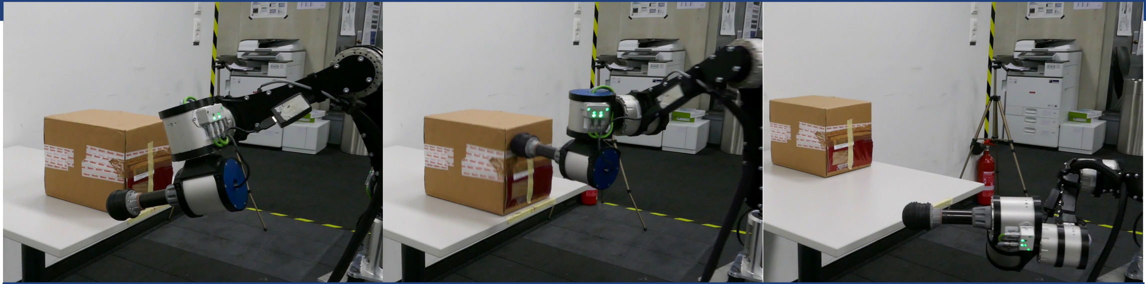


Contact-Implicit Trajectory Optimization for Dynamic Object Manipulation

Jean-Pierre Sleiman, Jan Carius, Ruben Grandia, Martin Wermelinger, Marco Hutter

Robotics Systems Lab, ETH

Introduction

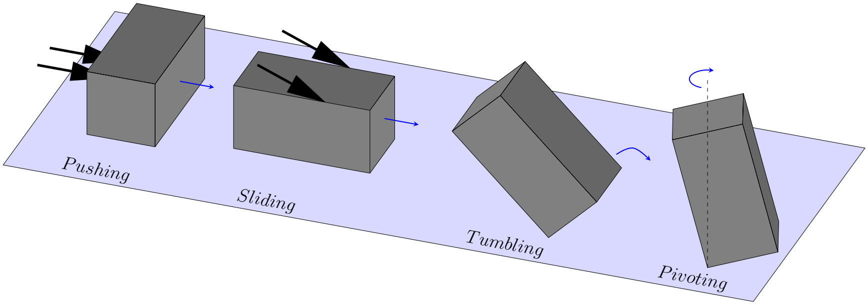
There has been a shift in attention from industrial robotics towards the development of robots that are capable of a more dexterous interaction with their environment.

Locomotion

Dynamic Manipulation

Dynamic Manipulation

Nonprehensile (graspless) manipulation includes phases where the manipulator loses possibility of contact with the object before task completion



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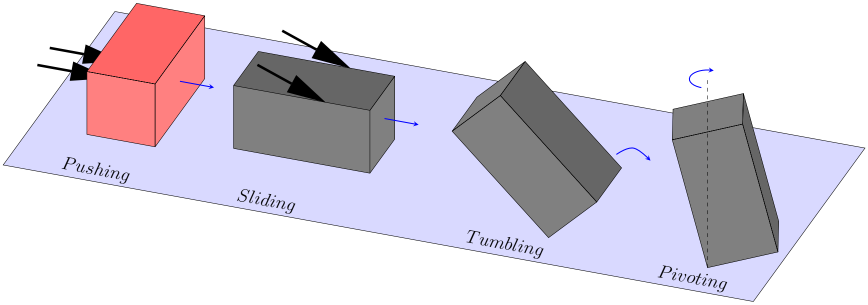
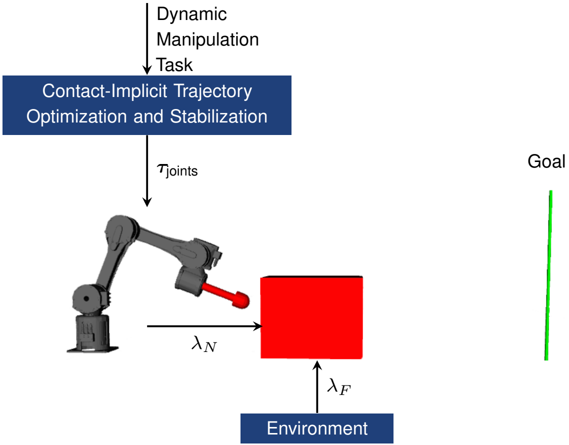
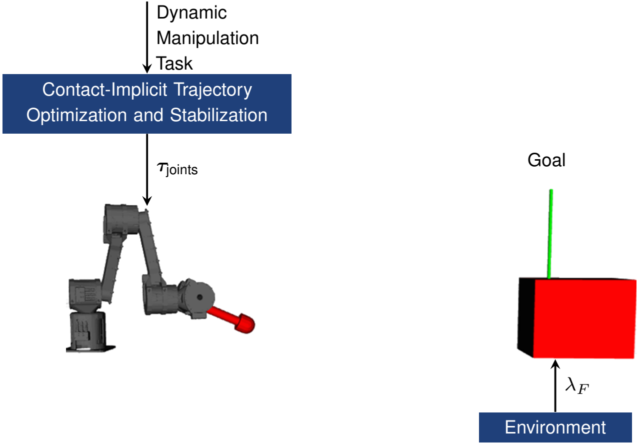


Motivation & Problem Statement



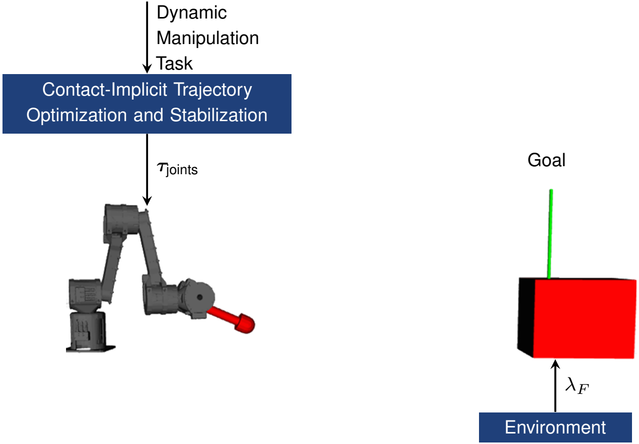
Motivation & Problem Statement

1. Solve the optimal control problem efficiently for hopes of applying it within an MPC framework



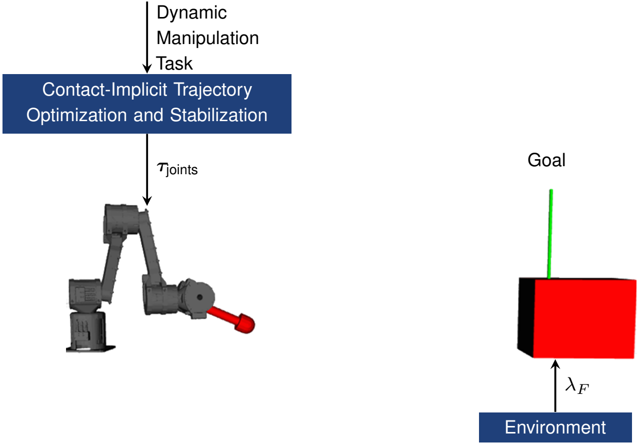
Motivation & Problem Statement

1. Solve the optimal control problem efficiently for hopes of applying it within an MPC framework
2. Ensure dynamic feasibility and physically consistent contact behavior
3. Avoid specifying a hand-crafted contact schedule



Motivation & Problem Statement

1. Solve the optimal control problem efficiently for hopes of applying it within an MPC framework
2. Ensure dynamic feasibility and physically consistent contact behavior
3. Avoid specifying a hand-crafted contact schedule
4. Ensure ease of transferability onto real hardware without any post-optimization modifications



Optimal Control Problem

Trajectory optimization techniques aim to solve the following nonlinear optimal control (NLOC) problem:

$$
\begin{cases}
\min\limits_{\boldsymbol{\tau}(\cdot)} \quad J(\boldsymbol{x}(t), \boldsymbol{\tau}(t), t) = m(\boldsymbol{x}(T)) + \int\limits_0^T L(\boldsymbol{\tau}(t), \boldsymbol{x}(t))\mathrm{d}t \\
\boldsymbol{s.t.} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{\tau}(t)) \qquad \textbf{(nonlinear, smooth ODEs)} \\
\qquad \boldsymbol{h}_{min} \leq \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{\tau}(t)) \leq \boldsymbol{h}_{max} \qquad \textbf{(inequality constraints)} \\
\qquad \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{\tau}(t)) = 0 \qquad \textbf{(equality constraints)} \\
\forall t \in [0, T]
\end{cases}
$$

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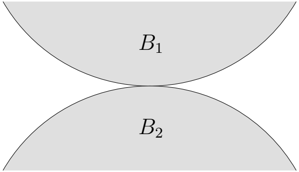
How to incorporate the multi-contact behavior into the dynamics?

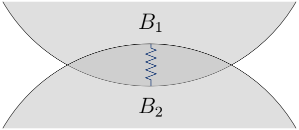
Non-Smooth Contact Dynamics

Modeling

A reliable model describing multi-body contacts has to be chosen before simulating the system dynamics



(a)



(b)

Contact dynamics can be modeled using either **(a)** a hard contact-model or **(b)** a soft contact-model

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→ *How to simulate the hybrid dynamics resulting from a hard contact-model?*

Non-Smooth Contact Dynamics

Simulation

Two prominent schemes used for simulating hybrid dynamical systems are: **event-driven** and **time-stepping** techniques

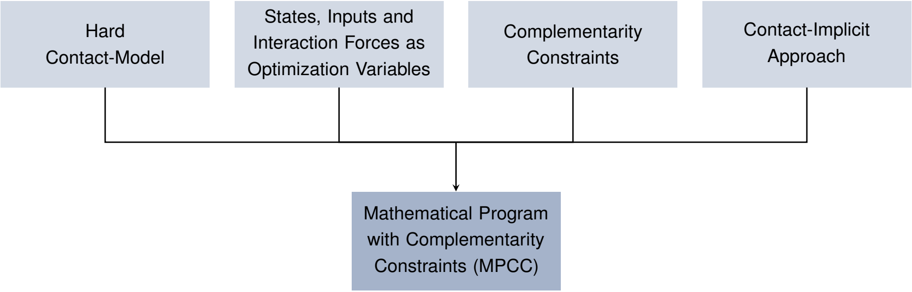
Courtesy of Blender Guru and Phymec

The forward simulation of multi-contact dynamics can be eventually formulated as a so-called **linear complementarity problem (LCP)**

Non-Smooth Contact Dynamics

Trajectory Optimization

Unlike the **multi-phase approach**, **contact-implicit optimization (CIO)** does not require a prespecified contact schedule!



Multiple Shooting

Transcription Process

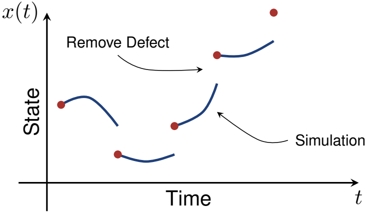
- Integral expressions replaced with Riemann sums, defines the cost function
- System dynamics integrated with a Runge-Kutta scheme, defines the state transition function

↓

Multi-staged Program

$$
\begin{cases}
\min\limits_{z_1,\ldots,z_N} \quad F_N(\boldsymbol{z}_N) + \sum\limits_{k=1}^{N-1} F(\boldsymbol{z}_k) & \\
s.t \quad \boldsymbol{E}_k \boldsymbol{z}_{k+1} = \boldsymbol{c}(\boldsymbol{z}_k) & \forall\ k = 1, \ldots, N-1 \\
\qquad \boldsymbol{S}_1 \boldsymbol{z}_1 = \boldsymbol{z}_{init} & \\
\qquad \boldsymbol{S}_N\ \boldsymbol{z}_N = \boldsymbol{z}_{final} & \\
\qquad \underline{\boldsymbol{z}}_k \leq \boldsymbol{z}_k \leq \bar{\boldsymbol{z}}_k & \forall\ k = 1, \ldots, N \\
\qquad \underline{\boldsymbol{h}}_k \leq \boldsymbol{h}(\boldsymbol{z}_k) \leq \bar{\boldsymbol{h}}_k & \forall\ k = 1, \ldots, N
\end{cases}
$$

Discretize



Optimize ⟶ FORCES PRO

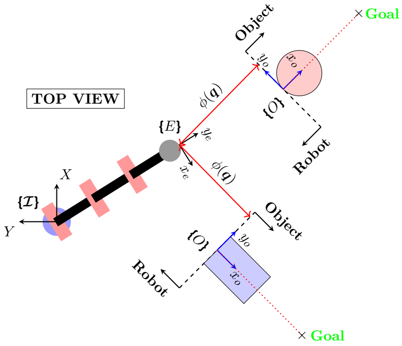
Contact-Implicit Optimization

1. The cost function minimizes energy and penalizes the robot's wrist joint-velocities

$$J(\boldsymbol{z}_k) = \sum_{k=1}^{N} \Delta t \cdot \left(\frac{\boldsymbol{\tau}_k^T \boldsymbol{R} \boldsymbol{\tau}_k}{\tau_{max}^2} + \frac{\dot{\boldsymbol{q}}_{r_k}^T \boldsymbol{Q} \dot{\boldsymbol{q}}_{r_k}}{\dot{q}_{max}^2} \right)$$

2. The manipulation task is specified in terms of final boundary conditions in the optimization program

3. The signed distance function $\phi(\boldsymbol{q})$ is defined such that a negative distance indicates that the end-effector is penetrating the object



Contact-Implicit Optimization

- Normal contact force λ_N, contact impulse Λ_N, and percussion P_N between robot and object
- Frictional force λ_F, frictional impulse Λ_F, and percussion P_F between environment and object
- Variables λ_n and λ_f defined in relation to the percussions: $P_N = \Delta t \cdot \lambda_n$ & $P_F = \Delta t \cdot \lambda_f$

Underactuated Dynamics & Impulse-Momentum Equations

$$\begin{cases} \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{S}^T \boldsymbol{\tau} + \boldsymbol{J}^T(\boldsymbol{q}) \cdot \begin{bmatrix} \lambda_N \\ \lambda_F \end{bmatrix} \\ \boldsymbol{M}(\boldsymbol{q})(\dot{\boldsymbol{q}}^+ - \dot{\boldsymbol{q}}^-) = \boldsymbol{J}^T(\boldsymbol{q}) \cdot \begin{bmatrix} \Lambda_N \\ \Lambda_F \end{bmatrix} \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \dot{\boldsymbol{q}}_{k+1} = \dot{\boldsymbol{q}}_k + \Delta t \cdot \boldsymbol{M}_k^{-1} \left(\boldsymbol{S}^T \boldsymbol{\tau}_k - \boldsymbol{h}_k + \boldsymbol{J}_k^T \cdot \begin{bmatrix} \lambda_{n_k} \\ \lambda_{f_k} \end{bmatrix} \right) \\ \boldsymbol{q}_{k+1} = \boldsymbol{q}_k + \Delta t \cdot \dot{\boldsymbol{q}}_{k+1} \end{cases}$$

Complementarity Constraints

$$0 \leq \phi(\boldsymbol{q}_k) \perp \lambda_{n_k} \geq 0$$

$$\Updownarrow$$

$$\begin{cases} \phi(\boldsymbol{q}_k) \geq 0 & \text{(no penetration)} \\ \lambda_{n_k} \geq 0 & \text{(unilateral contact force)} \\ \lambda_{n_k} \cdot \phi(\boldsymbol{q}_k) = 0 & \text{(no force at a distance)} \end{cases}$$

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Contact-Implicit Optimization

- Positive scalar quantity v_d signifying the speed along the desired direction of motion
- Pre- and post-impact relative separation velocities γ^- / γ^+
- Newton's restitution coefficient ϵ

Newton's Restitution Law of Impacts

$$\phi(\boldsymbol{q}) = 0: \qquad 0 \leq P_N \perp \left(\gamma^+ + \epsilon\gamma^-\right) \geq 0$$

$$\Downarrow$$

$$P_{N_k} \cdot (\gamma_{k+1} + \epsilon\gamma_k) = 0$$

$$\Delta t \cdot \lambda_{n_k} \cdot \left(\dot{\phi}_{k+1} + \epsilon\dot{\phi}_k\right) = 0$$

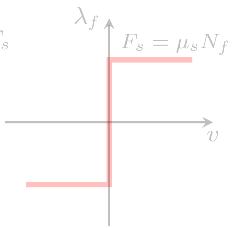
$$\Delta t \cdot \lambda_{n_k} \cdot (\boldsymbol{J}_N(\boldsymbol{q}_{k+1})\dot{\boldsymbol{q}}_{k+1} + \epsilon\boldsymbol{J}_N(\boldsymbol{q}_k)\dot{\boldsymbol{q}}_k) = 0$$

Coulomb Friction Model

$$\begin{cases} \lambda_f = \lambda_n & \text{if } v_d = 0 \;\&\; \lambda_n < F_s \\ \lambda_f = F_s & \text{if } v_d > 0 \end{cases}$$

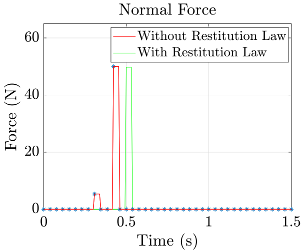
$$\Uparrow$$

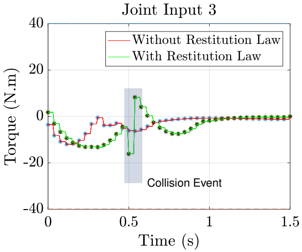
$$\begin{cases} 0 \leq v_{dir} \perp (F_s - F_f) \geq 0 \\ (F_s - F_f)(\lambda - F_f) = 0 \end{cases}$$



Without Newton’s Restitution Law

With Newton’s Restitution Law





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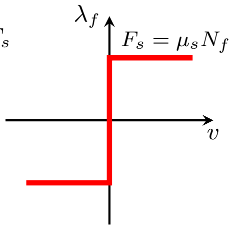
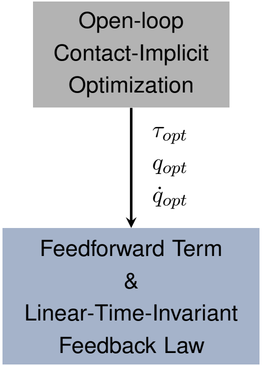


Contact-Implicit Optimization

The optimal input sequence τ_{opt_k} is an open-loop control policy. Therefore, **trajectory stabilization** is still needed.

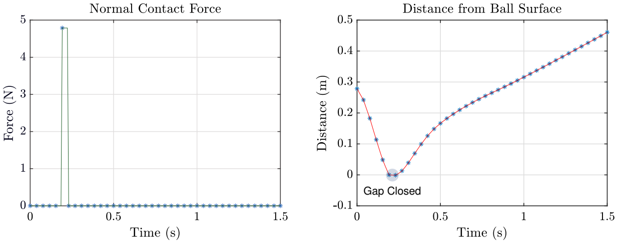
$$\tau_i(t) = \underbrace{\tau_{opt_i}(t)}_{feedforward\ term} + \underbrace{k_{p_i}\left(q_{opt_i}(t) - q_i(t)\right) + k_{d_i}\left(\dot{q}_{opt_i}(t) - \dot{q}_i(t)\right)}_{feedback\ term}$$

where $\tau_{opt}(t)$ is obtained by a zero-order-hold while $q_{opt}(t)$ and $\dot{q}_{opt}(t)$ by linear interpolation



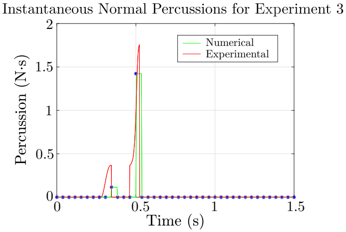
Robot-Door Manipulation Task

Robot-Ball Manipulation Task



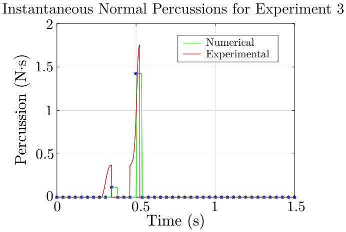
Robot-Block Manipulation Task

- Compatibility between optimal contact force and measured one



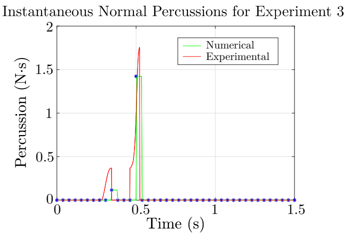
Robot-Block Manipulation Task

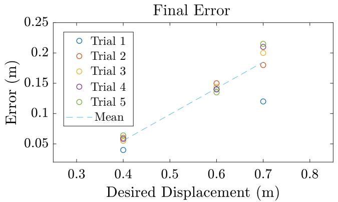
- Compatibility between optimal contact force and measured one
- Convergence rate two orders of magnitude higher than that of previous works



Robot-Block Manipulation Task

- Compatibility between optimal contact force and measured one
- Convergence rate two orders of magnitude higher than that of previous works
- Satisfaction of manipulation task with inaccuracy proportional to desired displacement





Thank you for your attention!

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Any Questions?