Conference Paper

Optimum route guidance in multi-region networks
A linear approach

Author(s):
Genser, Alexander; Kouvelas, Anastasios

Publication Date:
2020-01

Permanent Link:
https://doi.org/10.3929/ethz-b-000390155

Rights / License:
In Copyright - Non-Commercial Use Permitted
Optimum Route Guidance in Multi-region Networks: A Linear Approach

Alexander Genser*
Institute for Transport Planning and Systems, ETH Zurich
CH-8093, Switzerland
Tel: +41 44 632 75 19
Email: alexander.genser@ivt.baug.ethz.ch

Anastasios Kouvelas
Institute for Transport Planning and Systems, ETH Zurich
CH-8093, Switzerland
Tel: +41 44 633 66 95
Email: anastasios.kouvelas@ivt.baug.ethz.ch

* Corresponding author

Paper submitted to the special call “Improved flow modeling and advanced control strategies for large-scale urban traffic networks” issued by the Standing Committee on Traffic Flow Theory and Characteristics (AHB45) at the 99th Annual Meeting Transportation Research Board, Washington D.C., January 2020

Word count: 3886 words + 4 table(s) × 250 + 741 (references) = 5627 words; Figure count: 4

January 30, 2020
ABSTRACT
Dynamic congestion pricing is a useful tool not only to mitigate traffic congestion but also to influence people’s route choice. Dynamic tolls at the entrance of a protected region can give network users an incentive to reconsider their travel route and allow traffic management operators to direct a transportation network towards the system optimum. Computation of the optimal route guidance can be utilized as a benchmark for the pricing strategy of a corridor and significantly improve the performance of a transportation network. In recent work, decentralized control approaches have been applied, implementing a multi-region-network, where the urban regions are considered as homogeneous, and replicated with a representative Macroscopic Fundamental Diagram (MFD). The studies have tried to tackle the optimal route guidance problem by applying a Nonlinear Model Predictive Control (NMPC). Considering the shortcomings of local operating controllers and the well-known limitations of nonlinear optimization, we design a linear formulation of the optimization problem at the network level. This work evaluates the performance of a Linear Rolling Horizon Optimization (LRHO) which is applied to a multi-region-network. Results show a significant improvement in network performance by applying the determined dynamic routing compared to standard computation procedures of route choices. This work demonstrates that a linear formulation can be applied to the optimal route guidance problem, reduces the computational burden of centralized control approaches, and can potentially be utilized in real-time applications. Furthermore, the procedure can perform as a benchmark for traffic control with congestion pricing in future research.

Keywords: Multi-region-network modelling; Congestion pricing; Optimal route guidance; Linear rolling horizon optimization.
INTRODUCTION

The fact that more and more people are living in cities puts significant pressure on the mobility services of urban areas. One major challenge of today’s transportation systems is the mitigation of congestion. The traffic management domain has proposed a variety of different technologies in the last decades to tackle rising traffic demand. One well-known approach is perimeter control, allowing to reduce the user delay in a protected region significantly by controlling traffic lights at the region border (1, 2, 3). The properties of the Macroscopic Fundamental Diagram (MFD) allow maintaining the system at an optimal point, showing that operating at the critical density, allows serving the corresponding maximal traffic volume. Nevertheless, several approaches, such as perimeter control are not considering external effects which are mainly present when focusing on car traffic. Air pollution, noise, accidents, congestion, and space occupation are examples of costs the road users do not have to pay. Hence, this results in adverse effects on the performance of a traffic system, the environment, and the economy (4).

To reduce the user delay but also ensure the internalization of external effects, congestion pricing is a well-known approach, where users are charged for using the road network. The method can be implemented by HOT (High-Occupancy Toll)-lanes, where a specific vehicle occupancy is required, or the road user is willing to pay for the lane usage. Secondly, cordon-based congestion pricing approaches are available that charge the user for entering a protected region. Both methodologies are leading to a reduction in Total Time Spent (TTS). In our work, we are focusing on a dynamic cordon-based pricing system, where recent research and evaluation of implemented systems have shown that the pricing itself can be beneficial for mitigating traffic congestion (5). Furthermore, studies show that extending a system with dynamic toll approaches can further improve the performance of a transportation network (6, 7, 8, 9).

Although, a lot of research show case studies with an improvement of the TTS, an evaluation of the congestion pricing performance compared to the system optimum of a transportation network is still open to research. (10) collects a literature review on the role of travel information and stresses the importance to push a system from the user equilibrium (i.e., people are behaving selfishly in their route choice and try to maximize their benefit), to a more efficient system optimum (i.e., a part of the network users need to act in a non-selfish way by choosing an alternative route which might result in e.g., a longer travel time). One way to direct a transportation network towards the system optimum is by influencing the user’s route choice with travel information. Different systems have been utilized in the past to provide users with information about the current toll to enter a protected region (e.g., a website or mobile app that provides the current price or information systems on the highway displaying the current toll one would need to pay when the route does not change) (11). With the advancement of vehicle technology, toll data can even be provided in real-time to the user. Consequently, congestion pricing can not only be utilized as a general solution for reducing car traffic demand (i.e., the user’s mode choice or departure time is influenced) or the internalization of external effects. The user’s route choice can be affected, leading to a better distribution of traffic in the network and consequently to better system performance.

In the present work, we focus on a multi-region-network based on (12) to find the optimal route guidance solution (i.e., the system optimum). The defined urban regions are considered as homogeneous with different characteristics (i.e., size, capacity, average trip length) in the heterogeneous traffic network. A well-defined MFD characterizes every region with a recently proposed method by (13). The utilized nonlinear model was formulated as a linear program by applying several approximations proposed by (14) to obtain optimal route guidance with a Linear Rolling
Horizon Optimization (LRHO). Consequently, a methodology is designed, allowing to determine the optimal splitting rates. The obtained results are compared to two benchmark scenarios: (a) a simple splitting scenario where people are uniformly equally to the permitted paths between origin and destination (b) the standard Dijkstra route choice algorithm that considers the travel time on each permitted path for determining the splitting rates. The scenario comparison allows a performance evaluation of the proposed method with the Time Spent (TS) in every region and the TTS in the multi-region network.

The remainder of this paper organizes as follows: The methodology, i.e., the modeling of the simulation plant and the MFDs are introduced in Section 2. Section 3 elaborates the introduction of the Nonlinear Model Predictive Control (NMPC) problem and the formulation of the LRHO. Section 4 presents a case study with obtained results. The paper closes with a conclusion, as well as future research ideas, in Section 5.

MACROSCOPIC MULTI-REGION MODELING

A multi-region-network partitioned into homogeneous regions \( R \) is introduced, defined by \( R = \{1, 2, \ldots, K\} \), where \( K \) is the number of regions. Every region from \( R \) is modeled with a well-defined MFD, represented by the function \( G(N_I(t)) \). \( N_I(t) \) denotes the accumulation of a region at time \( t \). Consequently, the dynamic equations can be defined in continuous time as follows:

\[
\frac{dN_{II}(t)}{dt} = Q_{II}(t) - M_{II}(t) + \sum_{H \in N_I} M_{HIH}(t),
\]

\[
\frac{dN_{IJ}(t)}{dt} = Q_{IJ}(t) - \sum_{H \in N_I} M_{HIJ}(t) + \sum_{H \in N_I; H \neq J} M_{HIJ}(t),
\]

where the indices \( I \in R, H \in N_I \) and \( J \in R \) represent the origin region, stop-over region, and destination region, respectively. The variables \( N_{II}(t) \) and \( N_{IJ}(t) \) denote the accumulations of region \( I \) that have final destination region \( I \) and \( J \), respectively. \( N_I \) is a set containing all regions that are neighbors of \( I \). The internal demand within one region is defined by \( Q_{II}(t) \). Demands with origin \( I \) and destination \( J \) are denoted by \( Q_{IJ}(t) \). \( Q_{II}(t) \) and \( Q_{IJ}(t) \) are exogenous signals. Flows are computed by functions \( M_{II}(t) \) and \( M_{HIJ}(t) \) representing the internal flows in a region and the transfer flows from region \( I \) to \( H \) (with final destination \( J \)), respectively, defined as follows:

\[
M_{II}(t) = \frac{N_{II}(t)}{N_I(t)} G(N_I(t)),
\]

\[
M_{HIJ}(t) = \theta_{HIJ}(t) \frac{N_{IJ}(t)}{N_I(t)} G(N_I(t)).
\]

The variable \( \theta_{HIJ}(t) \) represent the route choices at time \( t \); for its computation, a scenario with fixed splitting rates \( (\theta_{HIJ}(t) = 1/\mathcal{N}_I) \), an implementation of the Dijkstra route choice algorithm and a model predictive program to find the optimal route guidance is utilized. The sequence of regions a user can traverse in the proposed model is not arbitrary. If the indices \( HIJ \) are parametrized with \( I = J \), paths are restricted (e.g. \( HIJ = 131 \)). This assumption does not allow for unrealistic path choices and improves the quality of the model. Note that the transfer flows need to be restricted by (5). The minimum of the incoming transfer flow or the maximum capacity of the region is considered, preventing a network region from accepting incoming flows that exceed the capacity.
Nevertheless, the constraint will be omitted throughout this work (please find the explanation in Section 3.1).

The elements of the set $\mathcal{R}$ are considered as homogeneous and can, therefore, be characterized by a well-defined MFD. Previous works are using mathematical relationships for modeling an MFD that is represented as an polynomial of degree $n$ (e.g., in (15) the approximation takes the form of $G(N_1(t)) = (aN_1^3 + bN_1^2 + cN_1)/\bar{L}$, where the coefficients $a, b, c$ are derived from measurement data and $\bar{L}$ denotes the average trip length). Furthermore, other approximations, such as an exponential function are used. However, the function parameters lack physical meaning and might introduce problems with the application of optimization procedures. Instead of assuming a functional relationship, another approach is to estimate the MFD from measurement data. Nevertheless, the quality of data or difficulties in data acquisition might lead to unreasonable approximations (13). In this work for the modeling of function $G(\cdot)$ the novel procedure developed by (13) is utilized; represented by an approximation of a trapezoidal diagram with the properties of smoothness, concavity, and continuity, defined by:

$$G(N_t) = -\lambda \ln \left( \exp \left( -\frac{q_{I_{n},t}}{\lambda} \right) + \exp \left( -\frac{q_{out}}{\lambda} \right) + \exp \left( -\frac{N_{t,jam}/L_{I,n} - N_{I,n}}{\lambda} \right) \right).$$

The function $G(\cdot)$ is the estimated outflow (veh/s) with respect to $N_t$. $q_{out}$ denotes the maximal outflow (capacity) in (veh/s). The parameter $L_{I,n}$ denotes the network length of a region in $\mathcal{R}$. The utilized approach for deriving the MFDs proposes a function that is dependent in the density $k$. Consequently, our approach needs to convert the input $N_t$ and jam accumulation $N_{I,jam}$ by applying $N_{I,jam}/L_{I,n}$ for the density $k$ (veh/m) and $N_{I,jam}/L_{I,n}$ for the jam density $\kappa$ (veh/m). $a$ and $b$ define the slopes of the free-flow and congestion propagation, respectively. $\lambda$ serves as the smoothing parameter. Please note that the variable names for the free flow speed $a$ and the congestion propagation $b$ are different from the work in (13), because the parameters are utilized for an urban region and not for a single intersection.

To model a realistic demand-supply system, the simulation plant receives reasonable demand patterns as trapezoids. A trapezoid is defined as a symmetric shape by specifying the rising time $t_r$ (s), the falling time $t_f$ (s) (where $t_r = t_f$), the time the demand remains constant $t_c$ (s), and the demand magnitude $Q_t$ in (veh/sec). Often these parameters are found by generating random numbers that satisfy the given application. In this work, an optimization procedure from (16) is utilized to find the parameters $t_r$, $t_f$, $t_c$, and $Q_t$, producing a desired simulation scenario (e.g., two congested and two uncongested regions). By setting a target accumulation on the MFD per region, different scenarios for testing the optimal route guidance determination can be generated efficiently (17).

**OPTIMAL ROUTE GUIDANCE WITH MODEL PREDICTIVE CONTROL**

The introduced multi-region model is formulated with several nonlinearities (e.g., the formulation of the MFD function $G(\cdot)$, the fraction of accumulations $N_{I_{n}}(t)/N_{I}(t)$, etc.). Hence, an NMPC is
applied in several other studies focusing on optimal control (12, 18, 19). This work formulates the problem as a linear model to allow the application of an LRHO. The formulations and assumptions are based on (14), which focuses on perimeter control in a multi-region model. The study provides proof that the approach is an accurate approximation of the nonlinear system and allows an efficient and fast solution of the global optimum. In the following subsections the nonlinear problem is introduced in Section 3.1, followed by the formulation of the linear approximation with the extensions for optimal route guidance and the LRHO in Section 3.2.

Nonlinear model predictive control

Model predictive control (MPC) is a widely applied control approach. At every sampling time step, an online optimization problem is solved for a finite horizon to obtain the best control strategy at the given time step. The solution of the optimization provides a set of optimal control actions for the finite time horizon; the first solution is applied to the simulation plant (20). In this work, the variables $\theta_{IJ}(t)$ determine the optimal route guidance. Hence, we are working with a nonlinear model and NMPC is utilized. The problem is formulated to maximize the internal and transfer flows in the multi-region model subject to the model equations as introduced in Section 2, as follows:

$$
\max_{N_{II}(k), N_{IJ}(k), \theta_{IJ}(k)} T_c \sum_{k=k_p}^{k_p+N_p-1} \sum_{I,J \in \mathcal{R}} [M_{II}(k) + M_{IJ}(k)], \\
\text{s.t. } N_{II}(k), N_{II}(k), M_{II}(k), M_{IJ}(k) = H(N_{II}(t), N_{IJ}(t)), \\
Q_{II}(t), Q_{IJ}(t), \theta_{IJ}(t)), \\
N_l(k) = \sum_{J \in \mathcal{R}} N_{IJ}(k), \\
k = k_p, k_p + 1, \ldots, k_p + N_p - 1 \\
N_c = \lfloor k/N_c \rfloor \\
\forall I, J \in \mathcal{R}, H \in N_l,$$

where $T_c$ defines the control sampling time, $H(\cdot)$ is a transfer function that discretizes equations (1) – (4), and $k$ is the discrete time step. $N_p$ is utilized as the finite horizon (in model steps) and the objective function tries to maximize all the flows in the system. Similar formulations and implementations of such a problem can be found in (12, 14). Note that the constraint to restrict transfer flows (Equation 5) is not included in the problem formulation. As we define an optimization problem to obtain the system optimum, essentially the obtained solution set implies that no region reaches the jam accumulation $N_{I,jam}$.

Model linearization and linear model predictive control

The application of an LRHO implies the utilization of a linear model. Since the introduced multi-region model is nonlinear, (14) proposes approximations that allow for a linear formulation. The reader can consider equation (4) showing the nonlinearities, i.e., the fraction of accumulations $N_{IJ}(k)/N_l(k)$, the MFD function $G_l(\cdot)$ and the products of the decision variables. First, the model parameters $\alpha_{II}(k)$ and $\alpha_{IJ}(k)$ are introduced which are updated every time a predicted solution is applied to the simulation plant; i.e., the parameters remain constant over the prediction horizon.
and are updated when rolling the prediction horizon. $\alpha_{II}(k)$ and $\alpha_{IJ}(k)$ are defined as follows:

\[
\alpha_{II}(k) = \frac{N_{II}(k)}{N_I(k)}, \tag{14}
\]

\[
\alpha_{IJ}(k) = \frac{N_{IJ}(k)}{N_I(k)}. \tag{15}
\]

Secondly, the MFD functions $G_{I}(\cdot)$ are approximated with a number of piece-wise affine (PWA) functions. $l = \{1, 2, \ldots, L\}$ denotes the index of the PWA function and $L$ the total number of functions, chosen for an accurate approximation. In the following the piece-wise linear MFD is indicated by $G^l_{I}(\cdot)$.

Thirdly, (14) introduces new decision variables

\[
f_{II}(k) = \theta_{III}(k)G^l_{I}(N_I(k))\alpha_{II}(k), \tag{16}
\]

and

\[
f_{III}(k) - G^l_{I}(N_I(k))\sum_{J \in \mathcal{R}} \theta_{IHJ}(k)\alpha_{IJ}(k), \tag{17}
\]

where $f_{II}(k)$ and $f_{III}(k)$ are defining the decision variables for the internal and transfer flows, respectively. The right side of equation (16) and (17) show the remaining nonlinearities by the product of the decision variables $\theta_{III}(k)$ and $\theta_{IHJ}(k)$, respectively. The introduction of $f_{II}(k)$ and $f_{III}(k)$ allow to complete the linearization of the problem. As in (14) the methodology was applied to find the optimal perimeter control, a transformation from $f_{II}(k)$ and $f_{III}(k)$ to the original control variables is used.

Nevertheless, the variables $f_{II}(k)$ and $f_{III}(k)$ are only considering the internal flows and the transfer flows to a neighboring region $H$; i.e., the information about the final destination $J$ is not available. In our approach to determine the optimal splitting rates $\theta_{III}(k)$ and $\theta_{IHJ}(k)$ this information is necessary to ensure that the sum of flow portions on every possible path from $I$ to $J$ is correct as well as for the transformation to the original decision variables. Therefore, we introduce one additional decision variable $f_{IHJ}(k)$ that is constrained with

\[
\sum_{J \in \mathcal{R}} f_{IHJ}(k) = f_{IH}(k), \forall I, H \in \mathcal{R} \tag{18}
\]

to ensure that the splitting rates can be constrained correctly and the calculation of the original decision signals $\theta_{III}(k)$ and $\theta_{IHJ}(k)$ can be obtained. Note that for $\theta_{III}(k)$ the result is not influencing the optimal route choice as the splitting rate corresponds to a user traveling from origin $I$, over $I$, to a final destination $I$. Hence, the splitting rate must be $\theta_{III}(k) = 1$. Nevertheless, the decision signals are included in the algorithm to validate the results.
Utilizing an LRHO can be introduced to solve the optimal route guidance problem:

$$\max_{N_I(k),f_{II}(k),f_{IH}(k)} \sum_{k=k_p}^{k_p+N_p-1} T_c \cdot \sum_{I \in \mathcal{I}} \left[ f_{II}(k) + f_{IH}(k) \right].$$  \hspace{1cm} (19)

s.t. \( N_I(k+1) = N_I(k) + T_c \left( Q_I(k) - f_{II}(k) - \sum_{H \in N_I} f_{III}(k) + \sum_{H \in \mathcal{H}} f_{HI}(k) \right) \)  \hspace{1cm} (20)

$$0 \leq f_{II}(k) \leq \alpha_{II} G_I^I(N_I(k))$$  \hspace{1cm} (22)

$$0 \leq f_{IH}(k)$$  \hspace{1cm} (23)

$$0 \leq N_I(k) \leq N_I,_{\text{jam}}$$  \hspace{1cm} (25)

$$k = k_p, k_p + 1, ..., k_p + N_p - 1$$  \hspace{1cm} (26)

$$\forall I, J \in \mathcal{R}, H \in \mathcal{N}_I$$  \hspace{1cm} (27)

Note that all the constraints in equation (19) - (27) are linear and consequently the problem can be solved with low computational power as a linear program.

**CASE STUDY**

This section presents a case study, where the modeling is based on an example of the city of Zurich. The region design is derived from analyzing the main traffic arterials of Zurich and the geographical reference of the available Loop Detectors (LD). The city center (denoted as \( R_1 \)) corresponds to an area of 1.5 (km\(^2\)) and has 113 available LDs. Consequently, the parameters for the MFD design are assumed with reasonable values as follows: Jam accumulation \( N_{I,\text{jam}} = 5000 \) (veh), average trip length \( \bar{L}_I = 1000 \) (m), and a network length \( L_{I,n} \) of 30 lane kilometers. \( R_2, R_3, \) and \( R_4 \) denote the border regions of the city center and are designed with an area of 5.0 (km\(^2\)) each. The number of detectors for regions \( R_2, R_3, \) and \( R_4 \) are 182, 277, and 135, respectively. The MFD for the border regions is designed with \( N_{I,\text{jam}} = 8000 \) (veh), \( \bar{L}_{2,3,4,n} = 2000 \) (m) and a road length \( L_{2,3,4,n} \) of 48 lane kilometers, respectively. Hence, the whole network is designed for a storage capacity of 29000 vehicles. The region design is depicted in Figure 1(a) and Table 1 depicts the introduced region parameters.

Considering the parameter design, we are proposing a four region network (Figure 1(b)), where the region \( R_1 \) is representing the city center. The derived inputs to determine the MFDs for

**TABLE 1 :** Parameters for the design of the city center (\( R_1 \)) and the border regions (\( R_2 - R_4 \))

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Unit</th>
<th>City center ( R_1 )</th>
<th>Border regions ( R_2 - R_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>( A )</td>
<td>[km(^2)]</td>
<td>1.50</td>
<td>5.00</td>
</tr>
<tr>
<td>Number of LDs</td>
<td>( n_{DT} )</td>
<td>[-]</td>
<td>113</td>
<td>182, 277, 135</td>
</tr>
<tr>
<td>Jam accumulation</td>
<td>( N_{I,\text{jam}} )</td>
<td>[veh]</td>
<td>5000</td>
<td>8000</td>
</tr>
<tr>
<td>Average trip length</td>
<td>( \bar{L}_I )</td>
<td>[m]</td>
<td>1000</td>
<td>2000</td>
</tr>
<tr>
<td>Network length</td>
<td>( L_{I,n} )</td>
<td>[m]</td>
<td>30.000</td>
<td>48.000</td>
</tr>
</tbody>
</table>
FIGURE 1: The region design of the city of Zurich and the multi-region-network model; (a) Every region is stated with an id (R₁ – R₄), the area A and the number of available LDs nDT; (b) The region R₁ is modeled as the city center and treated as a protected region with pricing (indicated by the double lines); R₂, R₃, and R₄ represent the boundaries to the center.

R₁ – R₄ are listed in Table 2. The maximum outflows q_out are considered with 4.50 (veh/s) and 6.00 (veh/s) for the city center R₁ and the border regions R₂, R₃ and R₄, respectively. The jam density κ is derived with N_{I,jam}/L_{I,n} and assigned to 0.16 (veh/m) for all regions. The smoothing parameter λ is set to 0.50 for the city center and 0.60 for the border regions. Note that the parameters a and b are the slopes of the free-flow (135.00 (m/s) and 219.38 (m/s), respectively) and the congested regime (48.21 (m/s) and 61.28 (m/s), respectively) of the outflow MFDs, and do not have a physical meaning. Figure 2 depicts the designed MFDs and the linear approximation for the city center and the border regions, respectively. The approximation granularity for each MFD is specified by the number of lines l = 20, considering the computational effort and the minimization of the approximation error.

To provide a relevant peak-hour simulation scenario for the optimal route guidance computation, representative demand patterns are derived. Therefore we defined target accumulations for every region and determined representative trapezoids parameters by solving an optimization problem. R₁ and R₂ represent a traffic situation in the congested regime, whereas R₃ and R₄ operate always in the non-congested states. Furthermore, the demand magnitudes show that R₁ and R₂ are contributing more to the accumulation trajectories, compared to R₃ and R₄. The derived demand

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>City center R₁</th>
<th>Border regions R₂ – R₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>[m/s]</td>
<td>135.00</td>
<td>219.38</td>
</tr>
<tr>
<td>q_out</td>
<td>[veh/s]</td>
<td>4.50</td>
<td>6.00</td>
</tr>
<tr>
<td>κ</td>
<td>[veh/m]</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>b</td>
<td>[m/s]</td>
<td>48.21</td>
<td>61.28</td>
</tr>
<tr>
<td>λ</td>
<td>[-]</td>
<td>0.50</td>
<td>0.60</td>
</tr>
</tbody>
</table>
FIGURE 2: The MFDs are designed according to assumptions related to the City of Zurich (region size, partitioning, etc.) and one can note that $R_2$, $R_3$ and $R_4$ are modeled as larger regions with higher capacity. The dashed lines are representing the linear fit of each MFD, respectively.

patterns $Q_{IJ}$ (veh/s) are depicted in Figure 3.

FIGURE 3: Traffic demand per region and pre-defined simulation horizon; configuration is for a 4X4 OD matrix, where $I$ specifies to the origin and $J$ the destination.

To prove the route guidance enhancement with a linear model approximation and the utilization of the LRHO method, we compare the proposed method to two benchmark scenarios: (a) the network users are distributed uniformly to the path possibilities, meaning that for a user traveling from $I$ to $J$, $\theta_{IHJ}(k) = 1/3 \forall H \in \mathcal{H}$ (referred as non-route choice (NRC)); (b) the splitting rates $\theta_{IHJ}(k)$ are computed with the well-known Dijkstra route choice algorithm by considering the current travel time on a path (referred as route choice (RC)). To provide a quantitative analysis of the accumulation results, an evaluation of the TS for every region is conducted. TS for every
region can simply be defined as

$$TS_I = \sum_{k=k_p}^{k_p-1} N_I(k).$$  \hspace{1cm} (28)$$

Furthermore, the time a region experiences congestion and the time it operates in the non-congested regime is analyzed. The results are depicted in Table 3 and will be compared to the accumulation outputs in the following.

**TABLE 3**: TS comparison and corresponding improvement of the NRC, RC, and LRHO scenario for $R_1 - R_4$, respectively. CS denotes the time the region experienced congestion; NCS the time a region was non-congested; IMPR states the improvement of each scenario, respectively.

<table>
<thead>
<tr>
<th></th>
<th>NRC</th>
<th></th>
<th></th>
<th>RC</th>
<th></th>
<th></th>
<th>LRHO</th>
<th></th>
<th></th>
<th>IMPR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[veh·h]</td>
<td>[h]</td>
<td>[h]</td>
<td>[veh·h]</td>
<td>[h]</td>
<td>[h]</td>
<td>[veh·h]</td>
<td>[h]</td>
<td>[h]</td>
<td>[%]</td>
</tr>
<tr>
<td>$TS_1 \cdot 10^6$</td>
<td>12.9</td>
<td>1.2</td>
<td>2.1</td>
<td>8.0</td>
<td>0.7</td>
<td>2.6</td>
<td>6.2</td>
<td>0.3</td>
<td>3.0</td>
<td>22.60</td>
</tr>
<tr>
<td>$TS_2 \cdot 10^6$</td>
<td>18.0</td>
<td>1.3</td>
<td>2.03</td>
<td>13.3</td>
<td>0.9</td>
<td>2.4</td>
<td>11.8</td>
<td>0.9</td>
<td>2.4</td>
<td>11.09</td>
</tr>
<tr>
<td>$TS_3 \cdot 10^6$</td>
<td>9.8</td>
<td>0</td>
<td>3.3</td>
<td>5.9</td>
<td>0</td>
<td>3.3</td>
<td>3.8</td>
<td>0</td>
<td>3.3</td>
<td>35.15</td>
</tr>
<tr>
<td>$TS_4 \cdot 10^6$</td>
<td>9.3</td>
<td>0</td>
<td>3.3</td>
<td>4.0</td>
<td>0</td>
<td>3.3</td>
<td>2.1</td>
<td>0</td>
<td>3.3</td>
<td>48.27</td>
</tr>
</tbody>
</table>

Figure 4(a) depicts the accumulation trajectories over time for scenario NRC with fixed splitting rates for the presented demand scenario. It is shown that $R_1$ and $R_2$ exceed the critical density (represented by the dashed lines in Figure 4). The regions are operating in the congested regime for 1.2 (h) and 1.3 (h), respectively. The TS in $R_1$ is 12.9 (veh·h·10⁶) and 18.0 (veh·h·10⁶) for $R_2$. $R_3$ and $R_4$ are close to critical density but no congestion occurs, i.e., the two border regions are operating in the non-congested regime the full time span of 3.3 (h). Nevertheless, the accumulation results are worse than the target accumulation that was used for determining the demand patterns (for the optimization problem the route choice algorithm was utilized). Figure 4(b) shows the accumulations, where the splitting rates are determined by the Dijkstra algorithm (RC). In every region a significant improvement is depicted. The time the regions $R_1$ and $R_2$ experience congestion are reduced to 0.7 (h) and 0.9 (h), respectively. The TS of $R_1$ reduces to 8.0 (veh·h·10⁶) and of $R_2$ to 13.3 (veh·h·10⁶) which corresponds to an improvement (compared to scenario (a) of 37.73 (%) and 26.46 (%), respectively. The border regions $R_3$ and $R_4$ are again operating in the non-congested state. The TS reduces by 40.51 (%) and 56.79 (%), respectively, which corresponds to a TS in $R_3$ of 5.9 (veh·h·10⁶) and 4.0 (veh·h·10⁶) in $R_4$. Finally we compare the results of the LRHO that are presented in Figure 4(c). The linear program aims for maximizing the flow in the multi-region-network and was designed with the following parameters: prediction horizon $N_p = 5$; control cycles $N_c = 5$; control time step $T_c = 20$sec. The LRHO results demonstrate a further improvement compared to the RC-scenario, especially in $R_3$ and $R_4$. The time $R_1$ and $R_2$ experience congestion are reduced to 0.3 (h) and 0.9 (h), respectively. The accumulations of $R_3$ and $R_4$ are further improved and again free-flow conditions can be shown. The TS can be improved by 22.60 (%), 11.09 (%), 35.15 (%), and 48.27 (%), for $R_1 - R_4$, respectively. In Figure 4(d) the aggregated trajectories are presented for every region and scenario, respectively. The accumulations demonstrate that $R_1$ and $R_2$ experience congestion, whereas $R_3$ and $R_4$ are operating in the non-congested regime. The performance improvement is finally evaluated with the $TTS$ (veh·h) in the network,
FIGURE 4: Accumulation trajectories $N_{ij}$ for (a) the non-route choice; (b) the route choice; (c) the LRHO scenario, respectively; (d) Aggregated accumulation results with no route choice (NRC), route choice (RC) and LRHO for $R_1 - R_4$. defined by

$$TTS = \sum_{k=k_0}^{K-1} \sum_{i \in \mathcal{I}} N_i(k).$$

(29)

Again, the area between the curves of the route choice scenario and the LRHO results is the total improvement of the system. It can be shown that the proposed methodology improves the accumulation trajectories in every region, but especially in $R_1$, $R_3$ and $R_4$. In $R_2$ no improvement could be achieved in the congested regime, but with decreasing demand the accumulation trajectory decreases faster compared to the benchmark scenarios. The overall improvement is listed in Table 4.

The TTS computation shows an improvement of 37.78 (%) that can be already achieved by simply introducing the Dijkstra route choice algorithm. By applying the LRHO method, the TTS can be further reduced by 23.38 (%) in the network. Furthermore, this improvement guarantees an optimal solution due to the linerization of the nonlinear optimization problem. Hence, it can be concluded that the proposed linearization method from (14) can also be applied to find the optimal
TABLE 4: TTS comparison and corresponding improvement of the NRC, RC, and LRHO scenario, respectively.

<table>
<thead>
<tr>
<th></th>
<th>NRC [veh·h]·10^6</th>
<th>RC [veh·h]·10^6</th>
<th>IMPR [%]</th>
<th>LRHO [veh·h]·10^6</th>
<th>IMPR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTS</td>
<td>50.11</td>
<td>31.18</td>
<td>37.78</td>
<td>23.89</td>
<td>23.38</td>
</tr>
</tbody>
</table>

route guidance in an urban multi-region-network.

CONCLUSION

The paper presents the optimal route guidance computation in a multi-region-network with homogeneous regions, characterized by well-defined MFD functions. The derived routing information is providing the network system optimum and can be utilized as a ground truth for the evaluation of dynamic congestion pricing systems and how users can be influenced in their route choice to distribute traffic demand more homogeneously. To relax the nonlinear optimization problem, a recent linearization methodology was implemented that allows the application of the LRHO. The proposed method from the literature was extended and utilized for obtaining the optimal splitting rates in the multi-region-network. The accumulation trajectories are utilized to show the system improvement of the methodology with the TTS as a performance indicator. The results are compared to a non-route choice (fixed splitting rates) scenario and a scenario with the utilization of the Dijkstra route choice algorithm. The proposed linear program reduces the TTS significantly and guarantees an optimal and fast solution as opposed to nonlinear formulations.

Future research should focus on a sensitivity analysis of the proposed controller settings. The performance evaluation can be further extended by comparing the TTS-improvement with a nonlinear system and also how sensitive the controller is to parameters such as the control time step and the prediction horizon. Based on recent research, the simulation plant should be extended with a trip length model (for now only average trip lengths are considered) that allows an extensive analysis of users travel times in the system. Furthermore, a weighting of the different regions can be applied in the optimization procedure to account for the different region parameters (i.e., size, storage capacity, etc.). This improves the quality of the modeling further and also contributes to a more detailed evaluation of the proposed methodology. In addition, the approach can also be tested by formulating an optimization problem to derive the user equilibrium that is necessary as a second benchmark for the evaluation of congestion pricing systems.
REFERENCES


**AUTHOR CONTRIBUTION STATEMENT**

The authors confirm contribution to the paper as follows: study conception and design: Alexander Genser, Anastasios Kouvelas; analysis and interpretation of results: Alexander Genser, Anastasios Kouvelas; draft manuscript preparation: Alexander Genser, Anastasios Kouvelas. All authors reviewed the results and approved the final version of the manuscript.