On Goldbach conjecture

On Goldbach Conjecture

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Abstract

The Goldbach Problem is reduced to three smaller problems according to the concept of left, middle, and right numbers [1]. The left and right numbers are present in the Goldbach pairs. The solutions to the three problems are similar and simpler.
A condition for the satisfaction of the Goldbach conjecture as well as a main equation for the number of Goldbach pairs, which is valid for any even number, are derived.
A new theorem is presented which is the deterministic equivalent of the theory of large numbers in probability theory. With this theorem one gets formulas for Goldbach pairs, composite pairs and mixed pairs. It is shown that the results satisfy the main equation.
It is also shown that the formula of Goldbach pairs which is positive also has a positive derivative which shows that the Goldbach conjecture is always valid and the number of Goldbach pairs is increasing.

1 Introduction

In [1] it is shown that the odd numbers from 3 to ∞ (1 is excluded as a special number) can be partitioned as follows:

1. Left numbers $7+6k$ $k=0\dots\infty$
2. Middle numbers $3+6k$ $k=0\dots\infty$
3. Right numbers $5+6k$ $k=0\dots\infty$

The same is with even numbers $4+6k$, $6+6k$ and $8+6k$. The reason for this partition is that a left even number has right primes as Goldbach pairs. A middle even number has Goldbach pairs consisting of one left and one right prime. A right even number has left primes as Goldbach pairs. Simple Algebras for addition and multiplication are depicted in tables 1 and 2 respectively. Accordingly, the Goldbach problem is reduced to three small, similar and simpler problems.

Table 1: Addition - Simple Algebra

+	**L**	**M**	**R**
L	R	L	M
M	L	M	R
R	M	R	L

Table 2: Multiplication - Simple Algebra

×	**L**	**M**	**R**
L	L	M	R
M	M	M	M
R	R	M	L

1.1 Left and right composites [1]

The left composites are given by

p^2+6pk, $\qquad p=5,7,11,13,\dots$ (all odd primes except 3)

an infinite number of arithmetic series with elements $25, 49, 55, 85, 91, 115, 121, \ldots$.
The corresponding left primes are the rest: $7, 13, 19, 31, 37, 43, 61, 67, 73, 79, \ldots$.
Similarly, the right composites are an infinite number of arithmetic series given by:

$$\begin{array}{ll} p^2 + 6pk + 4p & \text{if } p \text{ is left and} \\ p^2 + 6pk + 2p & \text{if } p \text{ is right,} \end{array}$$

with elements $35, 65, 77, 95, 119, 125, \ldots$ and the corresponding right primes $5, 11, 17, 23, 29, 41, 47, 53, 59, 71, \ldots$.

2 Main Equation

Let x be even. If x is right then consider the range of elements under x from 7 to $x - 7$. The number of elements is given by

$$P + C = \frac{x - 8}{6},$$

where P is the number of left primes and C the number of left composites in the range.
If x is left then consider the range of elements under x from 5 to $x - 5$. The number of elements is given by

$$P + C = \frac{x - 4}{6}.$$

In this case P and C are the number of right primes and the number of right composites in the range respectively. For middle x, P and C are a combination of both left and right primes and composites from 5 to $x-5$. The number of elements in this case is given by

$$P + C = \frac{x - 6}{3}.$$

Now if the series of primes and composites up to x is folded in the middle (at $x/2$) one gets opposite numbers which give Goldbach pairs G, composite pairs D and mixed pairs E. Thus for Goldbach pairs to exist we have the inequality

$$\frac{P + C}{2} > C - D.$$

From this we get the number of Goldbach pairs

$$G = \frac{P - C}{2} + D. \tag{1}$$

The main equation (1) is valid for left, middle or right even x. Thus it is valid for the three problems.

Example 1. Computation of the number of composite pairs for three example values of x:

$x = 128$:	$P = 13$,	$C = 7$	$G = 3$	$D = 0$
$x = 256$:	$P = 27$,	$C = 15$	$G = 8$	$D = 2$: $35 + 221$, $95 + 161$
$x = 512$:	$P = 45$,	$C = 39$	$G = 10$	$D = 7$ $85 + 427$, $121 + 391$, $169 + 343$, $187 + 325$, $217 + 295$, $247 + 265$, $253 + 259$.

3 Reflection Theorem

Theorem 3.1. *If the composites or primes are reflected from a neutral point 2^n with $n \to \infty$, which does not share any of the prime factors with the composites, then the reflected members will meet the primes and composites in the same ratio as their numbers.*

Proof. The proof is straight forward: Because 2^n is neither biased towards composites (no prime factors other than 2) nor primes and the composites and primes are determined by the same law, then the theorem is valid. □

The choice of 2^n leads to the least number of Goldbach pairs, as it does not share prime factors with composite numbers. This logic proof is enforced by the fact that the results of the application of this theorem can be shown to satisfy the main result of (1). This can be considered as the deterministic analog of the theory of large numbers in probability theory. From this theorem we get:

$$\begin{aligned} G &= \frac{P^2}{2(P+C)}, \\ D &= \frac{C^2}{2(P+C)}, \\ E &= \frac{PC}{(P+C)}. \end{aligned} \tag{2}$$

Then

$$G + D + E = \frac{P+C}{2}$$

which is in the middle of the folded prime and composite series to x. G and D in (2) satisfy the main result in (1).

As $x \to \infty$

$$\begin{aligned} P &= \frac{x}{2\ln x}, \\ C &= \frac{x}{6} - \frac{x}{2\ln x}, \end{aligned}$$

for even, left or right x.

Then

$$G = \frac{3x}{4\ln^2 x} \tag{3}$$

For even, middle x

$$\begin{aligned} P &= \frac{x}{\ln x}, \\ C &= \frac{x}{3} - \frac{x}{\ln x}. \end{aligned}$$

For middle x a reflection point of $3 \cdot 2^n$ is used instead of 2^n.

4 Goldbach Conjecture

For its history and development see [3]. The Goldbach conjecture has been proven numerically till a high value of even x. For example the result in [2] is given till $4 \cdot 10^{18}$. However, as for $x \to \infty$

$$G = \frac{3x}{4\ln^2 x},$$

which is positive.

As $x = 2^n$, with no factors other than 2, this gives the least number of Goldbach pairs. That means Goldbach conjecture is valid as $x \to \infty$. Since the differential

$$\frac{dG}{dx} = \frac{3(\ln x - 2)}{4(\ln^3 x)}$$

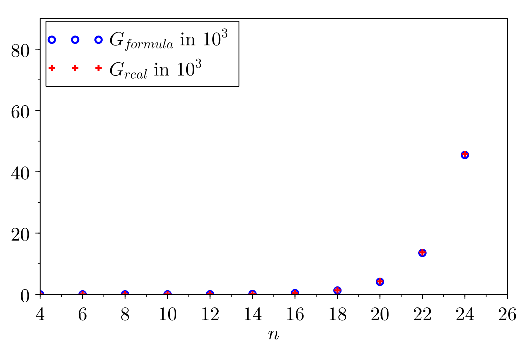

Figure 1: Number of Goldbach Pairs (in 1000) and calculated value with (3) for even n

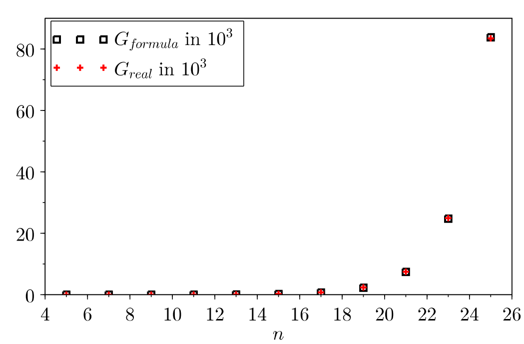

Figure 2: Number of Goldbach Pairs (in 1000) and calculated value with (3) for odd n

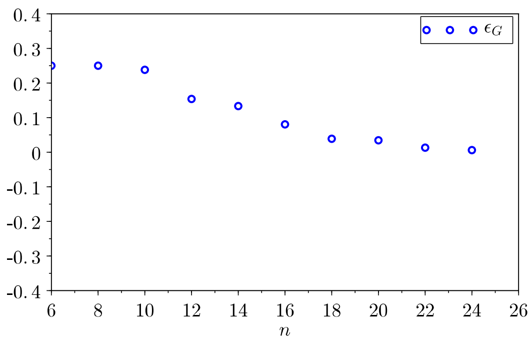

Figure 3: Relative error: Goldbach Pairs and calculated value with (3) for even n

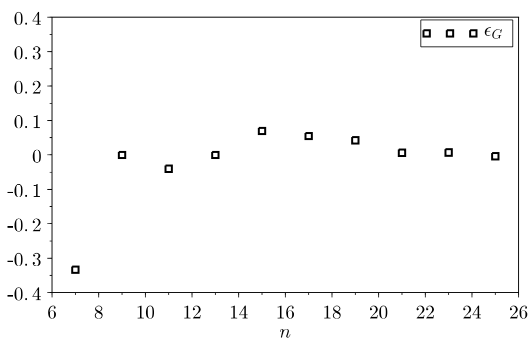

Figure 4: Relative error: Goldbach Pairs and calculated value with (3) for odd n

is also positive, this means that G is an increasing function.

Table 3 shows the different values till 2^{25} and figures 1 and 2 show the corresponding G, which was calculated using Ivan Ianakiev's Algorithm, and the calculated $G_{formula}$ according to equation (3). For small x, (3) is an approximation. As is depicted in figures 3 and 4, the relative error becomes small for large n.

Remark 1. For odd n: If $x-1$ is prime or composite it is not included in P_L or C_L and if $x-3$ is prime or composite it is not included in P_R or C_R. If $x-1$ or $x-3$ is prime, it is not included in G.

Remark 2. For even n: If $x-3$ is prime or composite it is not included in P_L or C_L, or G if it is prime.

Conclusions

Using the decomposition of the odd primes in left, middle and right numbers one can reduce the Goldbach problem to three similar and simpler problems. The main equation is the solution which is valid for any even x. The deterministic analog of the theory of large numbers in probability theory leads to the formula for the number of Goldbach pairs as $x \to \infty$ which is positive and its derivative is positive. This shows Goldbach conjecture to be valid.

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Table 3: Computation of Numbers of left/right Primes and composites, Goldbach Pairs according to (3) and it's real counterpart. Where n is the exponent of the limit $Z = 2^n$, P_L/P_R the number of left/right primes respectively, C_L/C_R the number of left/right composites respectively (see remarks 1 and 2), $G_{Formula}$ the evaluation of (3), G_{real} the real number of Goldbach Pairs, ΔG the absolute difference between the formula and real number and ϵ_G the relative error.

n	$Z = 2^n$	P_L	C_L	P_R	C_R	P	C	$G_{Formula}$	G_{real}	ΔG	ϵ_G
4	16	1	0	2	0	3	0	1.56	1	−1	−100.00 %
5	32	3	1	4	0	7	1	2	1	−1	−100.00 %
6	64	6	3	9	1	15	4	2.78	4	1	25.00 %
7	128	13	7	15	5	28	12	4.08	3	−1	−33.33 %
8	256	25	16	27	15	52	31	6.24	8	2	25.00 %
9	512	45	39	49	35	94	74	9.87	10	0	0.00 %
10	1024	81	88	88	82	169	170	15.98	21	5	23.81 %
11	2048	151	189	156	184	307	373	26.42	25	−1	−4.00 %
12	4096	276	405	285	397	561	802	44.4	52	8	15.39 %
13	8192	504	860	521	843	1025	1703	75.67	76	0	0.00 %
14	16 384	938	1791	959	1771	1897	3562	130.49	150	20	13.33 %
15	32 768	1745	3715	1765	3695	3510	7410	227.34	244	17	6.97 %
16	65 536	3259	7662	3281	7641	6540	15 303	399.62	435	35	8.05 %
17	131 072	6103	15 741	6145	15 699	12 248	31 440	707.98	749	41	5.47 %
18	262 144	11 465	32 224	11 533	32 157	22 998	64 381	1263.01	1314	51	3.88 %
19	524 288	21 676	65 704	21 711	65 669	43 387	131 373	2267.11	2367	100	4.23 %
20	1 048 576	40 984	133 777	41 038	133 724	82 022	267 501	4092.14	4238	146	3.45 %
21	2 097 152	77 732	271 792	77 877	271 647	155 609	543 439	7423.38	7471	48	0.64 %
22	4 194 304	147 866	551 183	148 078	550 972	295 944	1 102 155	13 527.73	13 708	180	1.31 %
23	8 388 608	281 932	1 116 168	282 229	1 115 871	564 161	2 232 039	24 753.95	24 928	174	0.70 %
24	16 777 216	538 755	2 257 446	539 113	2 257 089	1 077 868	4 514 535	45 468.2	45 745	277	0.61 %
25	33 554 432	1 031 627	4 560 777	1 032 060	4 560 344	2 063 687	9 121 121	83 806.98	83 467	−340	−0.41 %