Modeling the acoustic emissions generated during dynamic fracture under bending

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ABSTRACT

We quantitatively analyze the dynamic fracture of a glass tube under bending and model the associated generation of acoustic emissions. In our experiments, precisely controlled dynamic fracture is induced by focusing flexural waves. High-speed videos show the crack propagating rapidly through 85% of the cross section, before slowing down considerably. The resulting acoustic emissions are modeled by computing the diffraction of the incident flexural waves at the traction-free crack surface. We observe that longitudinal and flexural waves are generated during the fracture process, with very good agreement between experiments and simulations.

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1. Introduction

During a fracture process, stress waves are generated that propagate in the surrounding body. Often, these stress waves are referred to as acoustic emissions (AE), which is a wider term for elastic waves in solids that are generated through sudden or continuous stress redistribution (Grosse et al., 2008). Acoustic emissions are a helpful tool in structural-health monitoring as they provide information about the inside of bulk material. In standard AE testing, the number of AE hits can reveal information on the existence of cracks or the loading level of a structure (Grosse et al., 2008). When evaluating single acoustic emissions with quantitative methods, one can gather further information, such as the location (Ernst and Dual, 2014; Ernst et al., 2016), the type and the size of a crack (Gutkin et al., 2011). However, the quantitative interpretation of single acoustic emissions requires understanding not only of the propagation of the wave from source to receiver, but also of the underlying fracture mechanisms that produce the AE. This is also the main reason why the quantitative analysis of AE remains challenging.

In experimental studies, AE signals were measured in a variety of materials, ranging from metals (Grosse et al., 2008), and concrete (Ohtsu, 1996; Aggelis, 2011) to glass (Kim and Sachse, 1986; Kim and Sachse, 1986; Gross et al., 1993), plastics (Jacobs et al., 1991), and composites (Gutkin et al., 2011). For the acquired AE signals, many different parameters were studied to characterize the signal and the corresponding failure mechanisms (Surgeon and Wevers, 1999; Hensman et al., 2010; Gutkin et al., 2011; Aggelis, 2011). Even though the classification of huge amounts of data has become much easier, e.g. with neural networks, the understanding of the underlying physical processes that produce the emission signal remain just as important.

In theoretical works, acoustic-emission sources were modeled with dislocation vectors and the radiation patterns were evaluated with moment tensor analysis. Typically, penny-shaped cracks in an infinite 3D continuum were considered (Ohtsu and Ono, 1984; Ohtsu and Ono, 1989). Alternatively, the AE source was modeled as a stress-boundary-condition on the crack surface (Lysak, 1996). More recent publications show the representation of AE sources in finite-element models with mono-pole, and di-pole sources, which emulate the radiation patterns from analytical theory that are also observed in experiments (Prosser et al., 1999; Hamstad et al., 1999). Sause et al. implemented finite-element models where the acoustic emissions are produced by the opening of a crack in the pre-loaded structure (Sause and Horn, 2010; Sause and Richler, 2015).

Still, the validation of AE models remains difficult in so far, as the repeatable generation of AE signals in a test specimen is challenging. A common resort is to emulate the acoustic emission with the wave pulse generated by a pencil-lead break on the surface of the test specimen (Gorman, 1991; Gary and Hamstad, 1994; Hamstad et al., 1996; Zelenyak et al., 2015; Le Gall et al., 2018). Originally, this method was introduced by Hsu to calibrate AE
sensors (Breckenridge et al., 1975; Hsu and Breckenridge, 1981). While certainly very useful, it remains questionable how far a pencil-lead break can model a real acoustic emission, as the fracture does not take place in the test specimen itself, but is only represented by an external point load.

Acoustic emissions produced during fracture were not only studied within the scope of structural-health monitoring and non-destructive testing, but also within fracture mechanics. Here, the motivation focuses more on the understanding of the fracture process itself, of which the acoustic emissions are a direct result. Miklowitz observed second fracture phenomena in tensile fracture experiments, which were caused by the stress waves generated during the first fracture (Miklowitz, 1953). A phenomenon that is maybe more commonly known to happen in spaghetti under bending (Audoly and Neukirch, 2005). Similarly, Phillips (1970), Kolsky (1973), and Kinra (1976) studied the stress waves produced during the tensile fracture of glass beams. In other experiments, Bodner (1973), Kinra and Kolsky (1977), and Schindler and Kolsky (1983) studied the stress waves produced by dynamic fracture of a glass beam under bending. Again, phenomena of multiple fracture due to the released stress waves were observed (Schindler, 1981; Schindler and Sayir, 1984). In most bending experiments, the load was produced either statically by classical three- and four-point-bending setups (Kinra and Kolsky, 1977; Schindler and Kolsky, 1983), or by impact loading (Colton and Herrmann, 1975; Marur et al., 1995; Yokoyama and Kishida, 1983). In many cases, the resulting stress waves were modeled with rather simple analytic approaches (Miklowitz, 1953; Freund and Herrmann, 1976) but also FE models (Yokoyama and Kishida, 1983), both of which showed good agreement with the measured waveforms.

In recent work, the authors presented a method to induce dynamic fracture in a glass tube by focusing flexural waves (van Gemmeren et al., 2018). The flexural waves are excited at one end of the glass tube by an electromechanical transducer and focus at an arbitrarily chosen location after having been reflected multiple times at the ends of the tube. Thus, a strong bending-moment pulse is created at the desired location along the tube by means of constructive interference between multiple wave packets. At this location, the amplitude of the bending-moment pulse surpasses the bending strength of the glass tube, which induces dynamic fracture. In this experiment, no external loads are applied during the fracture process, so no stress concentrations from load transmission are present. Still, high loading rates can be achieved. Moreover, the loading can be controlled precisely via the wave focusing, so that repeatable acoustic emissions can be generated.

In the present study, we investigate the stress waves/acoustic emissions produced during the dynamic fracture of the glass tube due to flexural-wave focusing. We analytically model the fracture process as the transition from wave transmission to wave reflection. We then simulate the resulting flexural and longitudinal waves and compare them to experimental results. On the one hand, this provides insight into the fracture process under flexure, as the loading is extended to the dynamic regime without requiring any external impaction. On the other hand, we show reproducible acoustic emissions due to an internal fracture event and a straightforward way of modeling them. This may serve as a benchmark for other AE modeling approaches.

In Sec. 2, we show the modeling of the fracture process and the acoustic emissions, as well as the experimental setup. In Sec. 3, we firstly present our observations on the crack dynamics and then discuss the results from our acoustic-emission measurements and simulations. We finish by giving a short conclusion in Sec. 4.

2. Methods

In our experiment, we induce dynamic fracture in a glass tube through the constructive interference of two propagating flexural-wave pulses. To model the generation of acoustic emissions, we firstly consider the situation of a propagating wave pulse in a one-dimensional waveguide that breaks instantly fast. Secondly, we consider the fracture process to be of finite duration and discuss transmission, reflection and mode conversion phenomena at the crack. Lastly, we discuss the numerical modeling of flexural and longitudinal waves and present the experimental setup.

2.1. Instantaneous fracture

The situation of a propagating wave pulse in a one-dimensional waveguide encountering a crack is sketched in the time–space map shown in Fig. 1a. A wave pulse propagating into positive x-direction encounters a crack that opens instantaneously at time \( t_c \), the location \( x_c \). The crack subsequently divides the spatial domain into two parts. The part of the wave pulse that arrives at the fracture location \( x_c \) before fracture has occurred (\( t < t_c \)) is not obstructed and can propagate onward, i.e. it is transmitted. However, the part of the wave pulse that arrives after fracture has occurred (\( t \geq t_c \)) encounters a free boundary at \( x_c \) and is reflected. Hence, the opening of a crack results in the separation of the propagating wave pulse into two parts which propagate into opposite directions.

The situation of two wave pulses \( A \) and \( B \) propagating in opposing directions is shown in Fig. 1b. Through the instantaneous fracture process, both pulses are split into two parts each, one being transmitted, the other one being reflected. So, if one would measure the deflections at the lower side of the crack (\( x < x_c \)), one would expect to see the superposition of three different wave pulses: First, the entire incident wave pulse \( A \) propagating to the fracture location, second, the transmitted, early part of the wave pulse \( B \), and third, the reflected, later part of the wave pulse \( A \), both of which propagate away from the site of fracture.

In reality, the crack propagation cannot occur instantly, but it will take the crack a certain amount of time \( \Delta t_c \) to traverse the cross section. Hence, there will be a transition period between complete transmission and complete reflection during that simultaneous transmission and reflection can be expected. Moreover, mode conversion can occur at the partially or fully developed crack.

![Fig. 1. Time–space map illustrating the interaction of propagating wave pulses with the opening of a crack. a) The part of the incident wave pulse that arrives at the fracture location \( x_c \) before fracture has occurred (\( t < t_c \)) is transmitted. The later part is reflected. b) Situation of two pulses \( A \) and \( B \) propagating into opposite directions.](Image)
2.1.2. Gradual fracture

In our modeling of the gradual fracture process, we conceptually follow the work of Miklowitz (1953), Phillips (1970), and Kinra (1976). In their studies on the stress waves produced during tensile fracture, they assumed the normal stress on the crack surface to be zero, whereas the normal stress on the remaining uncleaved area was assumed to retain the static distribution that was present before fracture occurred. From this stress distribution in the fracture plane, they computed the resultant normal force and bending moment acting on the two sides of the beam. Kinra and Kolsky (1977), and Schindler and Kolsky (1983) also suggested a similar approach to model the stress waves produced during fracture under static bending.

Similar to the aforementioned work, we assume the crack to propagate along a plane, that is the crack plane, perpendicular to the axis of the tube, which is well justified by experimental observations. Further, we assume the crack front to be a line parallel to the z-axis with coordinate \( y_{cf} \) as shown in Fig. 2. Moreover, we do not solve for the dynamics of the crack front but use an ansatz function \( y_{cf}(t) \) for the location of the crack front, which is based on experimental observations (see Section 3.1.1). The computation of the crack dynamics would require a full-field-fracture-mechanics simulation, which is not within the scope of this work as we are mostly interested in the stress waves generated during the fracture process. Further, we neglect all stress waves ahead of the crack tip, as their contribution to the overall acoustic emissions was found to be small in other experiments (Miklowitz, 1953; Phillips, 1970; Kinra, 1976).

Contrary to Kinra, Kolsky, and Schindler, we do not assume the stress distribution in the remaining cross section to be constant throughout the duration of the fracture process. Instead, the stress distribution is time-dependent and dictated by the incident bending-wave pulses. Following Timoshenko beam theory, this is \( \sigma_{xy,t} = -yM/I \) for the normal stress and \( \tau_{xy,t} = Q/A \) for the shear stress, where \( M \) is the bending moment, \( I \) the second moment of inertia, \( Q \) the shear force, and \( A \) the cross-sectional area.

We consider a bending-wave pulse propagating in positive x-direction hitting a partial crack with crack-front coordinate \( y_{cf} \). We cut the beam at the fracture plane into two parts. On the right-hand side \( (x > x_c) \), the stress distribution in the intact part of the cross section \( (y < y_{cf}) \) has to be that of the incident wave, whereas it has to be zero on the crack surface \( (y > y_{cf}) \):

\[
\sigma_{xx,t} = \begin{cases} 
0 & \text{for } y > y_{cf}, \\
-yM/I, & \text{for } y < y_{cf},
\end{cases}
\]

\[
\tau_{xy,t} = \begin{cases} 
0 & \text{for } y > y_{cf}, \\
-Q/A, & \text{for } y < y_{cf},
\end{cases}
\]

![Fig. 2. a) Cross section of the tube with outer radius \( R \) and inner radius \( r \). The crack front is assumed to be a line parallel to the z-axis with coordinate \( y_{cf} \). b) Stress distributions \( \sigma_{xx,t} \) and \( \tau_{xy,t} \) of a transmitted bending-wave pulse. The crack initiates on the tensile side at \( y = R \). On the crack surface the stresses are zero.](image)

From this, we can compute the resultant normal force \( N_{tr} \), bending moment \( M_{tr} \), and shear force \( Q_{tr} \) acting on the right-hand side of the beam.

On the left-hand side of the beam \( (x < x_c) \), part of the incident wave pulse is reflected at the traction-free crack surface. For the reflected wave, the boundary conditions are

\[
\sigma_{xx,te} = \begin{cases} 
-yM/I, & \text{for } y > y_{cf}, \\
0, & \text{for } y < y_{cf},
\end{cases}
\]

\[
\tau_{xy,te} = \begin{cases} 
Q/A, & \text{for } y > y_{cf}, \\
0, & \text{for } y < y_{cf},
\end{cases}
\]

The stress distribution on the left-hand side is the superposition of the stress distribution of the incident bending-wave pulse with the stress distribution on the right-hand side. So, we restrict our further computations to the right-hand side of the beam, i.e. the transmitted waves.

For a beam with annular cross section, the resultant normal-force component \( N_{tr} \) of the transmitted wave is

\[
N_{tr} = \int_{y_{intra}}^{y_{int}} \frac{My}{I} \, dy,
\]

which evaluates to

\[
N_{tr} = \begin{cases} 
N_{tr} \left( 1 - \eta^2 \right)^{3/2}, & \text{for } \eta > \gamma, \\
N_{tr} \left( 1 - \eta^2 \right)^{3/2} - \left( \gamma^2 - \eta^2 \right)^{3/2}, & \text{for } \eta < \gamma,
\end{cases}
\]

with \( \eta = y_{cf}/R; \gamma = r/R \), and

\[
N_0 = -\frac{8M(1-\gamma^2)}{3\pi R(1-\gamma^2)}.
\]

Similarly, the resultant bending moment of the transmitted wave is

\[
M_{tr} = \int_{y_{intra}}^{y_{int}} \frac{My'^2}{I} \, dy.
\]

For \( \eta \leq -\gamma \), this is

\[
M_{tr} = \frac{M}{\pi(1-\gamma^2)} \left[ \frac{\pi}{2} + \arcsin(\eta) - \eta\sqrt{1-\eta^2}(1-2\gamma^2) \right],
\]

for \(-\gamma < \eta < \gamma \)

\[
M_{tr} = \frac{M}{\pi(1-\gamma^2)} \left[ \frac{\pi}{2} + \arcsin(\eta) - \eta\sqrt{1-\eta^2}(1-2\gamma^2) - \frac{3\pi/4 - \gamma^4 \arcsin(\eta/\gamma)}{2} + \frac{\gamma^2 \arcsin(\eta/\gamma)}{2} - \eta^2 \sqrt{\gamma^2 - \eta^2}(\gamma^2 - 2\eta^2) \right],
\]

and finally for \( \eta \geq \gamma \)

\[
M_{tr} = \frac{M}{\pi(1-\gamma^2)} \left[ \frac{\pi}{2} + \arcsin(\eta) - \eta\sqrt{1-\eta^2}(1-2\eta^2) - \pi\gamma^4 \right].
\]

For the resultant shear force \( Q_{trans} \) it follows

\[
Q_{tr} = \int_{y_{intra}}^{y_{int}} \frac{Q}{A} \, dy.
\]

For \( \eta \leq -\gamma \), this is

\[
Q_{tr} = \frac{Q}{\pi(1-\gamma^2)} \left[ \frac{\pi}{2} + \arcsin(\eta) + \eta\sqrt{1-\eta^2} \right],
\]

for \(-\gamma < \eta < \gamma \)

\[
Q_{tr} = \frac{Q}{\pi(1-\gamma^2)} \left[ \frac{\pi}{2} + \arcsin(\eta) + \eta\sqrt{1-\eta^2} + \frac{-\gamma^2 \arcsin(\eta/\gamma) - \eta^2\sqrt{\gamma^2 - \eta^2}}{2} \right],
\]
and for $\eta \geq \gamma$

$$Q_n = \frac{Q}{\pi(1-\gamma^2)}\left[\frac{\pi}{2} + \arcsin(\eta) + \eta\sqrt{1-\eta^2-\pi^2}\right].$$ (15)

Lastly, the reflected counterparts are

$$N_{rF} = N_n,$$ (16)

$$M_{rF} = M_n - M,$$ (17)

$$Q_{rF} = Q_n - Q.$$ (18)

The resultant normal force $N_n$, bending moment $M_n$, and shear force $Q_n$ are depicted in Fig. 3. The transmission ratios of the bending moment and shear force go from zero (i.e., total reflection) for a fully developed crack ($\eta = -1$) to one (i.e., total transmission) for no crack ($\eta = 1$). The transmission ratio of the normal force, on the other hand, is zero for $\eta = 1$ and $\eta = -1$, but non-zero in between. This shows that a part of the bending-wave pulse is converted to a longitudinal wave due to the asymmetry of the fracture process. By computation of the resultant forces and moments acting on the cross section, we essentially restrict ourselves to the fundamental wave modes (i.e., longitudinal and flexural waves). The higher-order shell modes are neglected as they are mostly evanescent and only have a small influence on the acoustic emissions.

### 2.2. Simulation of flexural waves

Having discussed the interaction of the propagating flexural-wave pulses with the opening of the crack, it remains to compute the specific loading at the fracture location $x_c$ due to a wave pulse focusing at $x_F$ and the resulting acoustic emissions that are recorded at the measurement locations $x_{M1}$, $x_{M2}$. Note that the distance between focal point and fracture location is due to experimental uncertainties and typically small: $|x_c - x_F| < 5$ mm.

We model the glass tube with Timoshenko beam theory to simulate the propagation of the flexural waves. The corresponding governing equations are

$$\begin{align*}
GAK_1 \left( \frac{\partial^2 \phi}{\partial t^2} - \phi \right) &= \rho A \ddot{w}, \\
EI \left( \frac{\partial^2 \phi}{\partial t^2} \right) + GAK_1 \left( \frac{\partial w}{\partial t} \right) &= \rho IK_2 \ddot{\phi}.
\end{align*}$$ (19)

Here, $w(x, t)$ is the lateral deflection of the beam axis, $\phi(x, t)$ the rotation of the cross section, $G$ the shear modulus, $A$ the cross-sectional area, $\rho$ the density, $E$ the Young’s modulus, $I$ the second moment of inertia, and $K_1$ and $K_2$ shear coefficients (Doyle, 1997). We solve these differential equations in the frequency domain with the spectral-elements method proposed by Doyle (1997) and explained in greater detail (van Gemmeren et al., 2018). All quantities in the frequency domain are denoted by a superposed caret. Given the nodal shear force $\hat{Q}_n$ and moment $\hat{M}_n$, the nodal displacement $(\hat{w}_n, \hat{\phi}_n)^T$ of a semi-infinite beam element can be computed by solving the linear system of equations

$$\hat{\mathbf{K}} \begin{pmatrix} \hat{w}_n \\ \hat{\phi}_n \\ \hat{Q}_n \\ \hat{M}_n \end{pmatrix} = \begin{pmatrix} \hat{Q}_n \\ \hat{M}_n \end{pmatrix},$$ (20)

where $\hat{\mathbf{K}}$ is the dynamic-stiffness matrix. Subsequently, the deflection $w(x, t)$, rotation $\phi(x, t)$, shear force $Q(x, t)$, and bending moment $M(x, t)$ can be computed at any point along the beam element by evaluating the corresponding shape functions.

We start the spectral-element simulation at the focal point $x_F$, that is the point where the left- and right-traveling wave pulses interfere constructively, because there the wave forms are well known. Specifically, the waveform of the bending moment is in the shape of the Ricker wavelet (also known as Mexican–hat wavelet) as shown in Fig. 5. Further details on the shape of the focused wave pulse are given in Sec. 2.4.

With these boundary conditions at $x_F$ (point 1 in Fig. 4), we can compute the waveforms of the two incident wave pulses A and B at the two measurement locations $x_{M1}$ and $x_{M2}$ before they have encountered the crack (points 2 and 3 in Fig. 4). Note that we have to go backwards in time for this simulation. So, we first time-reverse the boundary conditions at $x_F$, which corresponds to taking the complex conjugate in the frequency domain (Prada et al., 1991). Then, we compute the deflection at $x_{M1}$ and $x_{M2}$, respectively, which we have to time-reverse again to get back to the usual time-direction.

Similarly, we compute the shear force $Q$ and bending moment $M$ produced by the incident pulse B at the site of fracture $x_c$ (point 4 in Fig. 4). Here, the fracture process splits the wave pulse into a transmitted and a reflected part, as discussed in Section 2.1.2 and shown in Fig. 5. The boundary conditions for those two wave parts are determined in the time domain with the equations derived beforehand (Eqs. (9)–(15), (18)). Thus, we can compute the waveform of the transmitted pulse $B_T$ at $x_{M2}$ and the one of the reflected pulse $B_R$ at $x_{M1}$ (points 5 and 6 in Fig. 4).

For the right-traveling wave pulse A, we proceed likewise. Starting from the Ricker wavelet, we compute the shear force and bending moment at the fracture location $x_c$ (points 7 in Fig. 4). We derive the boundary conditions of the transmitted and reflected

![Fig. 3. Normalized amplitudes of the transmitted normal force $N_n$, shear force $Q_n$, and bending moment $M_n$ with respect to the normalized crack-front location $\eta$. Here, $\eta = -1$ corresponds to a fully developed crack and $\eta = 1$ to no crack. The ratio between the inner and outer radius of the tube is $\gamma = r/R = 0.75$.](image)

![Fig. 4. Schematic time–space map indicating the simulation procedure for flexural waves. Solid arrows indicate steps going forward in time. Dashed arrows indicate steps going backwards in time. The numbered, white points indicate the simulation steps.](image)
wave pulses and evaluate the waveforms of the transmitted pulse $A_t$ at $x_{M1}$ and the one of the reflected pulse $A_r$ at $x_{M2}$ (points 8 and 9 in Fig. 4).

Thus, we have arrived at the three superimposed flexural wave components that can be detected on both sides of the fracture location: the incident pulse, the transmitted pulse, and the reflected pulse.

### 2.3. Simulation of longitudinal waves

The asymmetry of the fracture process leads to the generation of longitudinal waves in the glass tube. For low frequencies, these waves are only weakly dispersive so that fairly good results can be achieved even without a wave-propagation simulation. To obtain a better accuracy also for higher frequencies, we model the glass tube as a circular cylindrical shell and compute the dispersion relation for axially-symmetric waves as proposed by Mirsky and Herrmann (1957) (see Fig. 6). The boundary condition for the longitudinal waves (i.e., the normal force) is derived with Eq. (6) from the bending moment of the incident pulses at the fracture location. The bending moments of the two incident wave pulses $A$ and $B$ at the fracture location have already been computed with the simulation of the flexural waves in Sec. 2.2.

### 2.4. Experimental setup

For the dynamic-fracture experiments, we use a 1.5 m long glass tube (Schott AR-Glas, Schott AG, Germany) with an outer diameter of 4 mm and a wall thickness of 0.5 mm. The tube is placed horizontally on several foam supports, and flexural waves are excited with an electromechanical transducer consisting of a spherical magnet and an electromagnetic coil. The surface of the tube is slightly sanded with grade-60 sandpaper to reduce the bending strength to approximately 27 MPa, which corresponds to a critical bending moment of $117 \times 10^{-3}$ Nm. The focused bending-moment pulse has the shape of a Ricker wavelet with a pulse width of 100 μs (see Fig. 5), which corresponds to a center frequency of 2.5 kHz with a 3 dB bandwidth of 1.9 kHz. The amplitude of the focused wave pulse is around $184 \times 10^{-3}$ Nm but can vary slightly due to the variability of the experimental specimens. Details on the computation of the excitation pulse that generates the bespoke bending-wave pulse at the desired location are given in van Gemmeren et al. (2018). Typically, fracture occurs within a distance of $\Delta x_c = |x_c - x_0| < 5$ mm around the focal point. The location of the focal point $x_0$ is set to be in the middle of the beam to maximize the time window before reflections of the AE arrive.

For the flexural waves, the lateral deflection of the tube is measured with two laser Doppler vibrometers (Polytec OFV 303 sensor head with a VD-02 velocity decoder, Polytec PSV 400 sensor head with a VD-09 velocity decoder, Polytec GmbH, Germany) on both sides of the focal point at a distance of ± 10mm. The measurement locations are chosen as close as possible to the fracture location to minimize the effect of damping and dispersion on the acoustic emissions. For the laser vibrometers, a resolution of $1 \text{ m s}^{-1} \text{V}^{-1}$ and $0.2 \text{ m s}^{-1} \text{V}^{-1}$ with a maximum detectable frequency of 1.5 MHz is used. To register the longitudinal waves produced during fracture, we measure the longitudinal displacement at the tip of the beam without transducer. The measurement location is chosen on the line $y = 0$ to avoid detecting the flexural motion of the tube. Again, we use the Polytec PSV 400 laser Doppler vibrometer with a resolution of $0.2 \text{ m s}^{-1} \text{V}^{-1}$.

Furthermore, we recorded the fracture process with a high-speed camera (Photron Fastcam SA-Z, Tokina AT-X M100 AF Pro D macro lens) at a frequency of 252 kHz. To this end, the glass tube was illuminated from below and a small area around the focal point was recorded from above.

### 3. Results and discussion

#### 3.1. Crack dynamics

The recording of the fracture process with a high-speed camera allowed us to investigate the duration and the dynamics of the crack propagation. The high-speed videos show that the crack is
initiated at the side of the beam that is under tension. Two crack fronts are visible, one on the topside of the tube, the other on the backside, indicated by crosses and diamonds in Fig. 8 (see also supplemental movie S1). The final location of the crack front is not always on the edge of the tube at \( y = -R \) due to small misalignments of tube and camera (see also Fig. 9).

In the initial stage, the crack propagates with a relatively high and rather constant velocity of 500 m/s in the \( y-z \)-plane. The crack slows down considerably after having traversed approximately 85% of the cross section, having arrived on the side of the beam that is under compression. The duration of the initial phase of high crack-speed is about 8 \( \mu \)s, whereas the duration of the entire fracture process is typically around \( \Delta t_c = 80 \mu \)s. Thus, the duration of the fracture process is shorter than the width of the focused wave pulse, which is 100 \( \mu \)s (compare Fig. 5).

For the simulations, we approximate the propagation of the crack front with a piecewise linear function where the first segment represents the initial, fast phase and the second segment captures the slow, remaining duration of the fracture process (see Fig. 9).

In some instances, the direction of crack propagation begins to turn parallel to the \( x \)-axis during the final phase of the fracture process (see Fig. 8). This is in accordance with the observation of a small tongue on the crack surface after the experiment. We found this phenomenon to be stronger when the distance between focal point and fracture location is larger. Moreover, the crack always tends to propagate towards the focal point.

Our observation of initially fast followed by slow crack growth corresponds qualitatively to the findings of Kinra, Kolsky, and Schindler for fracture under four-point bending (Kinra and Kolsky, 1977; Schindler and Kolsky, 1983). However, they observed that the crack does not propagate all the way through the cross section but turned parallel to the axis of the beam. The fracture process was completed only by the arrival of a reflected tensile wave-pulse. In our experiments, the crack goes through the entire cross section without stop. We attribute this difference in crack dynamics to the difference in loading: Three- and four-point-bending experiments are generally displacement-controlled. As the crack propagates, the stiffness of the beam reduces, causing a reduction of the loading on the crack. In our experiments, the loading on the crack is produced by incident wave pulses, which maintain an almost constant loading level throughout the fracture process. This corresponds more to a load-controlled situation.

### 3.2. Acoustic emissions

#### 3.2.1. Flexural motion

Fig. 10 shows the flexural motion at the locations \( x_{M1} = 0.76 \) m and \( x_{M2} = 0.74 \) m when the focal point is set to \( x_F = 0.75 \) m and the tube breaks at \( x_F = 0.749 \) m. The shape of the reference curve,
where no fracture occurs, closely resembles the deflection produced by the Ricker wavelet, as $x_M^L$ and $x_M^F$ are close to the focal point $x_F$. In the case where dynamic fracture occurs, the time traces remain identical until the time instant of fracture, where we can observe a sharp peak ($t = 0$ ms, Fig. 10) followed by a low-frequency pulse. The acoustic emissions recorded on both sides of the fracture location look almost identical because the loading is close to symmetrical in space and time.

A number of parameters has to be determined to match the AE simulation to the measurements. The geometry and material parameters, as well as the measurement locations $x_M^L$, $x_M^F$, and the focal point $x_F$ are known in advance. The amplitude of the focused wave pulse $M_F$, the precise location of fracture $x_C$, and the time of fracture initiation $t_C$ are specific to each experiment. The amplitude of the focused wave pulse $M_F$ is determined from the reference measurement where no fracture occurs (see Fig. 10). This is to account for the variation of the amplitude due to the variability of the experimental specimens and transducers. The location of fracture $x_C$ is determined with a measuring tape after the fracture experiment. Lastly, the time of crack initiation $t_C$ is determined iteratively by fitting the arrival time of the flexural AE in the simulation to the experiment. The time $t_C$ we obtained in this manner was typically 8 ms before complete focusing took place, that is, as soon as the critical load was reached. Having set these parameters, the simulation closely follows the measured flexural motion, in particular the sharp peak produced during fracture. Low-frequency differences between simulation and experiment originate mostly from imperfections in the wave-focusing procedure that lead to a slight distortion of the Ricker wavelet. Moreover, some high-frequency oscillations seen in the experiment are absent in the simulation, because higher-order shell-wave modes were not modeled.

3.2.2. Longitudinal motion

The longitudinal waves produced during dynamic fracture are detected by measurement of the axial velocity at the right tip of the glass tube with a laser Doppler vibrometer (Fig. 7). Here, no longitudinal motion can be detected before fracture occurs (see Fig. 11). Shortly after fracture, a longitudinal-wave pulse with a duration of approximately 80 $\mu$s is measured. Thereafter, the slightly dispersed reflections of the same pulse can be detected as it propagates back and forth through the fractured tube part. The longitudinal-wave pulse typically shows a fast rise to the maximum amplitude followed by a slow decay to zero superimposed with some higher-frequency ripples. The duration of the measured longitudinal-wave pulse (80 $\mu$s) agrees very well with the duration of the fracture process recorded with the high-speed camera.

The modeling of the longitudinal-wave pulse shows good agreement in shape but overestimates the amplitude by about 30%. This discrepancy cannot be explained by material damping but indicates the normal force might have a tensile component that was not accounted for in the model. This normal-force component might be due to tensile stress concentrations at the crack tip, higher-order scattering effects of the moving crack tip (Kinra, 1978), or energy lost to higher modes. To identify which of these effects is dominating, it would be interesting to model the fracture process more closely including the stress waves ahead of the crack tip. This would probably call for a finite-element simulation, as analytical solutions have been developed mostly for plane problems. The high-frequency ripples observed in the experiment were also introduced in the model by including the dispersion relation of axially symmetric waves in a circular cylindrical shell.

While the overall duration, shape, and amplitude of the longitudinal acoustic emissions was consistent throughout the experiments, we observed a higher variability compared to the flexural waves. We suppose this is because the shape of the longitudinal-wave pulse is dominated by the crack dynamics. Contrarily, the shape of the flexural waves is dominated by the incident wave pulses, which are more easily controlled and more uniform. Thus, a refined modeling of the longitudinal waves would require a more elaborate treatment of the fracture process and the stress waves ahead of the crack tip.

4. Conclusion

In this work, we studied and modeled the acoustic emissions produced during the dynamic fracture of a glass tube under bending. Dynamic fracture was induced by focusing flexural waves to a high-amplitude, localized bending-moment pulse. The fracture process was found to consist of an initial phase with relatively high crack-velocities, where the crack traverses 85% of the cross section, followed by a final phase with much lower crack velocities. The acoustic emissions that resulted from the fracture process produced both longitudinal and flexural motion of the glass tube.

For the modeling of the acoustic emissions, we assumed a piecewise linear function for the crack length with respect to time. In our model, we assumed the traction in the un-cleaved cross section to be that of the incident wave pulse, whereas it was set to zero on the crack surface. From this, we derived the transmitted and reflected fractions of an incident flexural-wave pulse and computed the resultant normal force, shear force, and bending moment at the site of fracture. The propagation of the resulting acoustic emissions was modeled with Timoshenko beam theory for the flexural component and axis-symmetric shell theory for the longitudinal component.

In previous work on dynamic fracture under bending, it was shown that the direction of crack propagation turns parallel to the axis of the beam, leaving a part of the cross section un-cleaved (Kinra and Kolsky, 1977). Contrarily, in our experiments, the crack cleaves the cross section on a mostly straight path without stopping. We observed turning of the crack-propagation direction only in some instances, when the fracture did not occur directly at the focal point. We attribute this difference in crack dynamics to our more load-controlled and dynamic type of loading compared to the displacement-controlled loading in previous work (Kinra and Kolsky, 1977; Schindler and Kolsky, 1983). However, a dedicated analysis and modeling of the crack dynamics (under flexural wave loading) remains to be done and could possibly reveal under which conditions a turning of the crack does or does not take place.
In the context of acoustic emissions, our work shows an example of reproducible acoustic emissions that are created through an internal fracture process and not through external loading such as the pencil-lead break. Thus, our experiments are suitable to test and verify modeling approaches for acoustic-emission sources. Here, we modeled the generation of acoustic emissions by introducing a traction-free surface during the fracture process, similar to other studies (Lysak, 1996). Conveniently, this modeling approach can be transferred easily to other, more complex situations with finite-element models (Sause and Richler, 2015). Our simulation showed that acoustic emissions can be represented well with this simple modeling approach, but that stress concentrations at the moving crack tip can have a non-negligible effect.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ijsolstr.2020.07.012.

References

