Endogenous persistence with recursive inattentiveness

Author(s):
Dräger, Lena

Publication Date:
2011-07

Permanent Link:
https://doi.org/10.3929/ethz-a-006543954

Rights / License:
In Copyright - Non-Commercial Use Permitted
Endogenous Persistence with Recursive Inattentiveness

Lena Dräger
Endogenous Persistence with Recursive Inattentiveness

Lena Dräger*

July 14, 2011

Abstract

The DSGE model with endogenous and time-varying sticky information in Dräger (2010) is extended by allowing agents’ recursive choice between forecasts under rational or sticky information to affect the model solution. Dynamic equilibrium paths generate highly persistent series for output, inflation and the nominal interest rate. Agents choose predictors in a near-rational manner and we find that the share of agents with rational expectations reacts to the overall variability of aggregate variables. The model can generate hump-shaped responses of inflation and output to a monetary policy shock if the degree of inattentiveness is sufficiently high. Finally, feedback from agents’ degree of inattentiveness to the model solution affects the determinacy region of the model. The Taylor principle is then only a necessary condition for determinacy, and monetary policy should target the output gap as well in order to ensure a unique and stable solution.

Keywords: Endogenous sticky information, heterogeneous expectations, DSGE models, persistence.

JEL classification: E31, E37, E52.

*University of Hamburg and KOF Swiss Economic Institute, ETH Zurich. Contact author: Lena.Draeger@wiso.uni-hamburg.de

The author would like to thank Ulrich Frötsche, Ingrid Größl, Jan-Oliver Menz, Alexander Meyer-Gohde, Jan-Egbert Sturm, Artur Tarassow as well as seminar participants at the University of Hamburg and the Royal Economic Society 2011 conference for very helpful comments and Paul De Grauwe for sharing his code. I thankfully acknowledge financial support from the German Research Foundation. All remaining errors are mine.
1 Introduction

Models in modern macroeconomics aim at reproducing stylized facts found in empirical data, while at the same time providing rigorous micro-foundations for macroeconomic relations. Stylized facts regarding aggregate inflation and output found in postwar U.S. data include their high persistence over time and the hump-shaped, delayed responses to a monetary policy shock.\(^1\) However, as noted in Rudd and Whelan (2005), the New Keynesian Phillips curve with rational expectations cannot account for these empirical findings: With forward-looking expectations, the model cannot generate persistence in inflation as shocks are accounted for immediately. It can thus only be reconciled with empirical facts when including a lagged endogenous term. However, while appealing for instance to habit formation or rule-of-thumb price setting, this procedure remains ad hoc and is thus subject to the Lucas critique.

In their models with sticky information, Mankiw and Reis (2001, 2002, 2003, 2007) propose an alternative to fully rational expectations: They assume that all agents in the economy are rational, but underlie an exogenous probability \(\lambda\) of not being able to update to the most recent information set each period, due to the costs related to acquiring and processing new information. Only when they can update do agents form fully rational expectations, otherwise they remain inattentive towards new information and forecast with an outdated information set. The authors claim that their model replicates the stylized facts, yielding both persistence in aggregate data and hump-shaped responses to a monetary policy shock.

In Dräger (2010), we extend the sticky information model by endogenizing the probability of being able to update to the new information set, i.e. the share of agents with rational expectations each period. Employing a switching mechanism derived in a seminal paper by Brock and Hommes (1997), we allow agents to choose between costly rational expectations and

forecasts under costless, but outdated information. We assume that agents evaluate their mean squared forecast errors and switch to the rational predictor once losses from forecasting with outdated information become too high. Hence, the share of agents with rational expectations, \( \lambda_t \), becomes endogenous and time-varying. We thus incorporate endogenous sticky information into a DSGE model with flexible prices, where we simulate agents’ choice of predictors given equilibrium time paths for aggregate variables.

While we are able to reproduce the hump-shaped response of inflation to a monetary policy shock in Dräger (2010), we do not find any significant persistence in simulated data for aggregate output, inflation and nominal interest rates. It thus seems that also in sticky information models, persistence can only be generated when adding either lagged endogenous variables or assuming autocorrelated shocks. In this paper, we extend the model in Dräger (2010) by allowing for feedback from agents’ predictor choice to the model equilibrium. As in Brock and Hommes (1997), the model is then solved recursively, where the optimal share of agents with rational expectations in the current period, \( \lambda_t \), influences the model solution for the next period, when agents again decide between predictors, yielding \( \lambda_{t+1} \) and so on. We thus get a dynamic equilibrium path for aggregate output, inflation and nominal interest rates with recursive inattentiveness.

Allowing for feedback from agents’ switching between forecasts to the model solution yields a highly persistent time series for aggregate output, without assuming autocorrelated demand or cost-push shocks or habit per-

---

2 Empirical evidence of persistent heterogeneity in inflation expectations and frequent switching between predictors is given in Maag (2010) and Pfajfar and Zakelj (2009).

3 A number of papers have analyzed robustness of the results in Mankiw and Reis (2001, 2002, 2003, 2007); Coibion (2006) evaluates robustness of responses to a monetary policy shock in a DSGE model with sticky information for both consumers and firms. The author finds that parameter specifications regarding real rigidities and monetary policy objective function affect the result of hump-shaped impulse responses after a monetary policy shock. Trabandt (2007) finds that results of the initial sticky information model in Mankiw and Reis (2002) are robust in a larger DSGE model, but a hybrid New Keynesian Phillips curve fares equally well. By contrast, comparing estimates of DSGE models with sticky information or sticky prices, Andres et al. (2005) cannot reproduce the hump-shaped responses even with sticky information, while Korenok (2008) finds that the sticky price model statistically dominates the sticky information model.
sistence i.e. rule-of-thumb pricing.\footnote{Note, however, that we allow for interest rate smoothing by the central bank and assume that the technology shock driving natural output $\hat{y}_t^n$ follows a first-order autoregressive process.} With respect to inflation, however, we find that although the model simulation implies a persistent trend, the inflation series shows rather high short-run volatility. This is due to the standard deviation of cost-push shocks on inflation, which we initially set equal to the standard deviation of demand shocks on output. Reducing the size of the cost-push shock generates more persistence in inflation and a higher degree of inattentiveness towards inflation.

Previous results from Dräger (2010) remain robust also with feedback from agents’ switching to the model: Agents are still found to behave near-rationally as in Akerlof and Yellen (1985) and Akerlof et al. (2000), in the sense that they pay closer attention to recent changes in output and inflation if the variability of the forecasted variable rises as otherwise losses from forecasting with outdated information are small. In addition to our earlier results, we find more interaction between inattentiveness towards output and inflation. Regarding impulse-responses of output and inflation to a monetary policy shock, we find that both show a hump-shaped response once the degree of inattentiveness is sufficiently high. However, compared to the response of inflation, a higher degree of inattentiveness is needed to obtain a hump-shaped response of output after a monetary policy shock.

There are a number of approaches in the literature related to ours. While to our knowledge this is the first model analyzing endogenous inattentiveness over time, Branch et al. (2006, 2009) derive the optimal degree of inattentiveness by firms in a model with sticky information as in Ball et al. (2005). The authors show that a symmetric Nash equilibrium of inattentiveness exists, where firms minimize a quadratic loss function relating their firm-specific price under an individual degree of inattentiveness to the optimal price given some fixed economy-wide $\lambda$. The authors assume that firms have to pay a fixed cost relative to $\lambda^2$ in order to process new information. Our approach differs from theirs in that we analyze agents’ predictor choice over time and allow for feedback of agents’ switching between predictors to the model so-
We are thus able to evaluate the effect of heterogeneous expectations on the dynamic equilibrium path of the economy.

Analyzing persistence of inflation with boundedly-rational inflation expectations, Lansing (2009) evaluates the hybrid New Keynesian Phillips curve with a time-varying parameter on lagged inflation. This parameter is given by the Kalman gain from a filter describing agents’ optimal inflation forecast as an exponentially weighted moving average of past inflation, thus assuming a form of bounded rationality regarding inflation expectations. The author finds that his model set-up generates low-frequency swings in inflation from expectational feedback, resulting in a near-random walk behavior of inflation. Similarly, Ball (2000) presents a model with near-rational inflation expectations as in Akerlof and Yellen (1985): When forming expectations, agents optimally use past values of inflation, but ignore other variables that might affect inflation rates. This generates strong persistence in actual inflation, where the author notes that the model fits U.S. data well both for the period 1879-1914, when inflation was stationary, and for the period 1960-2000, when inflation was highly persistent. Furthermore, endogenous persistence in output and inflation is also generated in the DSGE model by De Grauwe (2008, 2010), where agents can choose between simple heuristic predictors in the switching mechanism proposed by Brock and Hommes (1997).

Analyzing the relation between professional inflation forecasts and those of the general public from survey data for the UK, Easaw and Golinelli (2010) find empirical evidence of near-rationality and inattention as in Akerlof et al. (1996, 2000). Assuming that the general public may absorb professional forecasts through the media and social transmission or ignore it, the authors report that professional forecasts are incorporated faster into own expectations when these lie below the reference value of the professional prediction. Inattentiveness by professional forecasters is further evaluated by Andrade and Le Bihan (2010) using the ECB Survey of Professional Forecasters. The authors find persistent disagreement between forecasters and evidence that new information is not incorporated into forecasts systematically, while forecasters also differ in their speed of updating. However, the data cannot be reconciled with a sticky information model because professional forecasters
on the one hand seem to have strongly persistent forecast errors, while on the other hand disagreeing relatively little.

Finally, our model also relates to the literature on rational inattention founded by Sims (2003). Assuming that agents have a limited capacity to process information, only a fraction of all information that arrives can be incorporated into forecasts. Mackowiak and Wiederholt (2009, 2010) as well as Paciello and Wiederholt (2011) present DSGE models with rational inattention of firms, solving for the equilibrium degree of inattention and analyzing optimal monetary policy. Further approaches with rational inattention can be found in Adam (2007, 2009). While we assume in contrast to the literature on rational inattention that agents can form rational expectations once they pay the cost for it, our model incorporates aspects of rational inattention in that we assume agents are capable of assessing their forecast errors. Hence, agents are aware of some aggregate information, even if they consequently choose not to incorporate it into their expectations due to the related processing costs.

The remainder of the paper is structured as follows: After the introduction, we briefly present the model in section 2. Results of the model simulations with recursive inattentiveness are given in section 3, where we analyze persistence of the variables, the nature of recursive inattentiveness, responses to monetary policy shocks and the stability of the model. Finally, section 4 summarizes and concludes.

2 A Model with Endogenous and Time-Varying Inattentiveness

2.1 The Model

We analyze a model with endogenous sticky information, building on the one derived in Dräger (2010). Extending the models with sticky information by Mankiw and Reis (2001, 2002, 2003, 2007), we recursively derive the share of rational agents each period as an endogenous and time-varying expression.
In that sense, our approach differs from the one in Branch et al. (2009), who solve for the constant equilibrium degree of inattentiveness by firms. The model equations are briefly reviewed here and we refer the reader to Dräger (2010) for detailed derivations.

Our model takes the form of a New Keynesian DSGE model with flexible prices and heterogeneous expectations. Heterogeneity arises because agents have the choice each period between paying the cost for the newest information set necessary to form rational expectations (what we term the ‘rationality cost’), and using an older, costless information set to form expectations on output and inflation. While aggregate information may be publicly available, the rationality cost captures all costs related to acquiring and processing this information into agents’ forecasts. Each period, thus, a share of agents has rational expectations, while the rest of the population is subject to sticky information, forecasting with information from the date when they last paid for new information. Note that we assume that all agents know the relevant model and are computationally able to form rational expectations, so that the only deviation from full rationality may be the use of outdated information. An expression for aggregate heterogeneous expectations is then derived as follows:

\[
\tilde{E}_t(x) \equiv \lambda_t E^{RE}_t(x) + (1-\lambda_t) E^{SI}_t(x) = \lambda_t E_t(x) + (1-\lambda_t) \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}(x),
\]

(1)

where \(E^{RE}\) and \(E^{SI}\) denote expectation operators under rational and sticky information, respectively, and \(\lambda_t\) is the time-varying share of rational agents in period \(t\). Note that the sticky information expectation operator comprises expectations of all agents that do not update in period \(t\), but instead use information from some time in the past. Their forecasts receive less weight, the older their information set is. Since all agents are computationally capable of producing rational forecasts, they switch to being rational as soon as they pay the rationality cost in a given period. Conversely, they belong to the sticky information group if they do not update and hence continue to
use their information set from the previous period.

We then derive the Euler equation with heterogeneous expectations of households, where \( \hat{x} \) denotes the deviation of \( x \) from its steady state:

\[
\hat{c}_t = \tilde{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{\pi}_t - \tilde{E}_t \hat{\pi}_{t+1}),
\]  

(2)

Under the assumption that markets clear, output \( \hat{y}_t \) is derived in a New Keynesian IS curve with heterogeneous expectations, where \( u_t \) denotes an i.i.d. demand shock:

\[
\hat{y}_t = \lambda_t \left( E_t \hat{y}_{t+1} + \frac{1}{\sigma} E_t \hat{\pi}_{t+1} \right) + (1 - \lambda_t) \sum_{j=0}^{\infty} (1 - \bar{\lambda})^j E_{t-1-j} \left( \hat{y}_{t+1} + \frac{1}{\sigma} \hat{\pi}_{t+1} \right) - \frac{1}{\sigma} i_t + u_t
\]  

(3)

Next, we derive an expression for aggregate prices \( \hat{p}_t \) of firms, assuming flexible prices and the same heterogeneity with respect to expectations as for households:

\[
\hat{p}_t = \tilde{E}_t [\hat{p}_t + \psi (\hat{y}_t - \hat{y}_t^n) + e_t],
\]  

(4)

where \( e_t \) is an i.i.d. cost-push shock. The expression \( (\hat{y}_t - \hat{y}_t^n) \) denotes the output gap, defined as the deviation of output \( \hat{y}_t \) from natural output \( \hat{y}_t^n \). This is the optimal output that would occur under flexible prices and fully rational expectations and which is driven by an i.i.d. technology shock \( z_t \):

\[
\hat{y}_t^n = \frac{1 + \eta}{\sigma + \eta} \hat{z}_t
\]  

(5)

After some algebra, we get the sticky information Phillips curve with heterogeneous expectations from (4), where switching between predictors in the previous period influences the inflation rate \( \pi_t \) in the current period:
\[
\pi_t = \frac{\psi \lambda}{1 - \lambda} (\hat{y}_t - \hat{y}_t^n) + \frac{\lambda}{1 - \lambda} e_t + \lambda_{t-1} E_{t-1} \left[ \pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t \right] \\
+ (1 - \lambda_{t-1}) \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-2-j} \left[ \pi_t + \psi \Delta (\hat{y}_t - \hat{y}_t^n) + \Delta e_t \right]
\]

(6)

Finally, the model is closed by specifying that monetary policy sets nominal interest rates \(\hat{i}_t\) according to a Taylor rule with interest rate smoothing, targeting actual inflation and the output gap as in Mankiw and Reis (2007):

\[
\hat{i}_t = \mu_i \hat{i}_{t-1} + (1 - \mu_i) \left( \mu_\pi \pi_t + \mu_{ygap} (\hat{y}_t - \hat{y}_t^n) \right) + \eta_t,
\]

(7)

where \(\eta_t\) denotes an i.i.d. monetary policy shock.

An expression for the time-varying degree of inattentiveness captured by the share of rational agents, \(\lambda_t\), is then derived by adapting the switching mechanism derived in a seminal paper by Brock and Hommes (1997). This approach assumes that agents continuously evaluate the accuracy of their forecasts and switch to being rational once the losses from forecasting with outdated information exceed the costs for the new information set. Conversely, if the gains from rational forecasts are not sufficient to outweigh the rationality costs, agents refrain from using rational expectations and switch to forecasting with sticky information by not updating in the current period.

In order for agents to be able to evaluate their forecast accuracy, we assume that some information on aggregate variables arrives continuously, but due to the related costs agents may choose not to process it into their forecasts. In that sense, our definition of sticky information differs slightly from the one in Mankiw and Reis (2001, 2002, 2003, 2007) and is closer to the concept of rational inattention in Sims (2003). Nevertheless, deriving microfoundations for the sticky information model, Reis (2006) also finds that the optimal degree of inattentiveness is a function of the volatility of shocks as well as the difference between profits under full or sticky information, thus using aggregate conditions to derive a fixed \(\lambda\).
In line with the literature on heterogeneous expectations and also the approach in Branch et al. (2009), we define agents’ mean squared forecast errors as the metric of forecast accuracy. These are given with respect to the variable \( \hat{x} \) with rational or sticky information by the following expressions:

\[
V_{t}^{RE} = -\sum_{k=0}^{\infty} \left[ \omega_k (\hat{x}_{t-k} - E_{t-k} \hat{x}_{t-k})^2 + K^{RE} \right]
\]

\[
V_{t}^{SI} = -\sum_{k=0}^{\infty} \left[ \omega_k \left( \hat{x}_{t-k} - \lambda \sum_{j=k-1}^{\infty} (1 - \lambda)^j E_{t-j} \hat{x}_{t-j-k} \right)^2 \right],
\]

where \( K^{RE} \) is the rationality cost of obtaining up-to-date information. We define \( K^{RE} \) relative to the mean squared forecast error under sticky information and assume a baseline value of 50%. The weights \( \omega_k \) are assumed to be geometrically declining and sum to one, defined as \( \omega_k = (1 - \rho) \rho^k \), with \( 0 < \rho < 1 \) measuring the degree of agents’ memory of past mean squared forecast errors.

Finally, following Brock and Hommes (1997), the time-varying degree of inattentiveness is defined by a multinomial logit map, deriving the probability of choosing the rational predictor as a function of its relative desirability, i.e. its measure of forecast accuracy \( V^{RE} \). This approach is frequently used in discrete choice theory, see Manski and McFadden (1981). Since in our model agents form expectations regarding output and inflation, we define two switching mechanisms regarding the share of agents with rational output and inflation expectations, respectively. This allows us to account for the different effects of shocks in the economy on output and inflation and their different weights in the central bank’s Taylor rule. We thus get for the time-varying degree of inattentiveness regarding output, \( \lambda_{t}^{y} \), and inflation, \( \lambda_{t}^{\pi} \):

---

5 Robustness of the model with respect to changing values of \( K^{RE} \) is analyzed in Dräger (2010). Generally, a higher rationality cost induces a lower share of agents with rational expectations and vice versa.

6 Note that we assume that agents inherit knowledge of the past forecast accuracy of their predictor when switching between forecasts under fully rational or sticky information.
\[ \lambda^y_t = \frac{\exp(\gamma V_{y,t}^{RE})}{\exp(\gamma V_{y,t}^{RE}) + \exp(\gamma V_{y,t}^{SI})} \]  
(10)

\[ \lambda^\pi_t = \frac{\exp(\gamma V_{\pi,t}^{RE})}{\exp(\gamma V_{\pi,t}^{RE}) + \exp(\gamma V_{\pi,t}^{SI})} \]  
(11)

where the parameter \( \gamma \) is called the ‘intensity of choice’ and measures the degree to which agents will be influenced in their choice of predictor by its past forecasting performance.

### 2.2 Equilibrium Dynamics

The model solution is found recursively over the simulation horizon: As in Dräger (2010), we use the algorithm by Meyer-Gohde (2010) to numerically solve for the system of linear rational expectation equations with an infinite sum of lagged expectations. The algorithm combines a Generalized Schur Decomposition to solve for the undetermined coefficients of the MA(\( \infty \)) recursive law of motion with an approximation to the infinite sum of lagged expectations by calculating matrices of limiting coefficients. The model solution is different from the one in Dräger (2010), however, in that we allow for feedback from agents’ switching decision between expectation operators to the evolution of the model economy. As in Brock and Hommes (1997), the model is thus solved recursively over time, yielding simulated time paths for aggregate variables and time-varying inattentiveness.

Specifically, the timing of events is as follows: Starting from an initial simulation of the model with fixed degree of inattentiveness, agents evaluate the performance of their forecast model in period \( t \) and decide whether to switch predictors. The degree of inattentiveness is then found via the multinomial logit map given in equations (10) and (11). The new values of \( \lambda^y_t \) and \( \lambda^\pi_t \) are incorporated into the model equations and influence its solution in the next period. Given the new solution and an exogenous vector of shocks, the model simulation for period \( t + 1 \) is found. Again, agents evaluate their forecast performance and decide on their predictor, thus defining the degree of inattentiveness \( \lambda^y_{t+1} \) and \( \lambda^\pi_{t+1} \). These feed back into the model solution for
period $t + 2$ and so forth.

The existence of an equilibrium with endogenous inattentiveness is proven by Branch et al. (2009) for a model with a sticky information price setting curve as in Ball et al. (2005). The authors model endogenous inattentiveness as firms’ optimal choice of $\lambda$ via a loss function describing expected profit losses when deviating from an economy-wide degree of inattentiveness $\overline{\lambda}$. The equilibrium $\lambda^*$ is then given as a symmetric Nash equilibrium: It is defined by the fixed point of the map describing firms’ best-response function as the value of $\lambda$ that minimizes the loss function and the costs of updating defined relative to $\lambda^2$. The authors show that a symmetric Nash equilibrium of this kind exists, but highlight the fact that multiple equilibria may be present.

For the case of models with recursively time-varying shares of agents using a particular predictor, Brock and Hommes (1997) analyze equilibrium dynamics in a cobweb model, where agents choose between rational and adaptive expectations. The authors find that if a cost to rational expectations is introduced and if the intensity of choice, $\gamma$, is sufficiently high, complicated equilibrium dynamics may arise. Specifically, for high values of $\gamma$, the system is close to or has a homoclinic orbit and corresponding strange attractors. A homoclinic orbit is defined as the intersection of the stable and the unstable manifold of the steady-state saddle point equilibrium. If additionally the Jacobian of the saddle point at the homoclinic orbit has two eigenvalues whose product is absolutely smaller than one, there exist values around the homoclinic orbit for which the system has a strange attractor. This implies a complex and potentially chaotic long-run dynamic behavior of the system.

From an economic perspective, this means that for sufficiently high values of $\gamma$, agents have a high propensity to switch to their optimal predictor each period. In the cobweb model with rational and adaptive expectations by Brock and Hommes (1997), if the economy is in a stable phase, most agents will use the cheap adaptive predictor. This causes prices to move away from their steady state and an unstable phase begins. In order to stabilize profits, agents will then be willing to pay the costs for rational expectations, which in turns moves prices back to the steady state as most agents switch to the rational predictor. The equilibrium dynamic path of the model thus consists
of irregular switching between phases where most agents are adaptive and prices fluctuate and phases with predominant rationality and prices close to the steady state.\footnote{Note that for $\gamma = +\infty$, in each period all agents choose the optimal predictor so that the system converges to a (locally unstable) saddle point equilibrium steady state, see Brock and Hommes (1997).} We analyze dynamic equilibrium paths of our model with endogenous sticky information in section 3.4.

## 3 Results

In this section we present results from numerical simulations of the model with endogenous and time-varying inattentiveness. We define the model as quarterly and simulate over 1500 periods, where the first 500 periods initialize the model and produce lagged expectations and are dropped consequently. Calibration parameters are chosen in line with those in Dräger (2010) and correspond closely to the calibration in McCallum (2001), where the model is defined as quarterly. We refer the reader to Dräger (2010) for a discussion of the parameters and of alternative calibrations. In line with Dräger (2010), we initially set the standard deviation of demand and cost-push shocks on output and inflation equal at $\tau_y = \tau_\pi = 0.03$ percentage points. Additionally, we assume no autocorrelation in the shocks, except for the technology shock on natural output $\hat{y}_t^n$.

### 3.1 Endogenous Persistence

Allowing for feedback from agents’ decision between rational or sticky information expectations, we simulate equilibrium time paths of output, inflation and the nominal interest rate. As described in the previous section, these should be understood as dynamic equilibria, which are computed recursively over time as the equilibrium response of the model economy to shocks and time-varying degrees of inattentiveness.

Dynamic equilibrium time paths of output $\hat{y}$ and inflation $\pi$ are shown together with the time-varying share of agents having rational output and inflation expectations, respectively, in Figures 1 and 2.
From Figure 1 we see that allowing for feedback from agents’ switching decision to the model economy produces considerable persistence in the time path of output, as the share of agents with rational output expectations fluctuates between zero and one. This is important because it suggests that the model is able to generate strong inertia of aggregate variables simply by endogenizing the choice of predictor each period. Hence, it seems that our model with endogenous and time-varying inattentiveness can reproduce an important stylized fact, namely the persistence of aggregate output usually found in empirical data, see for instance Fuhrer and Moore (1995) and Gordon (1997). By contrast, most standard DSGE models have to assume either autocorrelated shocks or the presence of lagged endogenous variables due to rule of thumb price setters and habit formation in consumption. Indeed, we showed in Dräger (2010) that the standard sticky information model is not able to yield persistence either when not assuming autocorrelation in the shocks.

Regarding the dynamic equilibrium path of inflation, Figure 2 shows that simulating the model with the initial calibration yields a path for inflation showing a persistent trend, but rather high short-run variability. This seems at odds with empirical findings of a relatively high degree of persistence also in (quarter-on-quarter) inflation, albeit being somewhat smaller than that of aggregate output. Furthermore, while we initially calibrated the standard deviations of the shocks on output and inflation to be equal, several studies assume cost-push shocks on inflation to be smaller in absolute size than the demand shocks on output. For instance, in what we take to be our baseline calibration, McCallum (2001) sets $\tau_\pi = 0.002$ and $\tau_y = 0.03$. Therefore, we reduce the size of the cost-push shock, setting $\tau_\pi = 0.015$ percentage points.\(^8\) The adjusted calibration gives a significantly more persistent time path.
path of equilibrium inflation. While the share of agents with rational inflation expectations, $\lambda_{\eta}^{\pi}$, still deviates between zero and one, we see that the smaller size of the cost-push shock leads agents to increasingly opt for the cheaper sticky information predictor.

Table 1 summarizes sample statistics of aggregate output, inflation and nominal interest rates across model specifications. With the initial calibration, both output and nominal interest rates are highly persistent and close to a random walk, while we find no significant autocorrelation in inflation due to the high degree of short-term variation. By contrast, reducing the standard deviation of cost-push shocks to $\tau_\pi = 0.015$ increases persistence of inflation significantly with a serial correlation coefficient of about 0.5.

### 3.2 Recursive Inattentiveness

After analyzing dynamic equilibrium time paths for aggregate variables of the model, we turn to evaluating recursive inattentiveness. Table 2 presents sample statistics of $\lambda_{y}^{\eta}$ and $\lambda_{\pi}^{\eta}$ for the two cost-push shock calibrations.

We find that the share of agents with rational output expectations is not affected significantly by changing the size of the cost-push shock on inflation. Overall, agents deviate between full rationality and full inattentiveness regarding output, while on average about 50% of agents use either predictor for an average of 2.6 quarters before switching again.\(^9\) Regarding the degree of inattentiveness towards inflation, reducing the size of the cost-push shock lowers the mean share of agents with rational inflation expectations from

---

9The average cycle length of switching between predictors is defined as the average time that $\lambda_{y}^{\eta}$ or $\lambda_{\pi}^{\eta}$ do not deviate from their values in the previous period by more than a threshold of 0.001 and is calculated in quarters.
about 44% to about 22%.\textsuperscript{10} This is not surprising, as a smaller shock on inflation will make the inexpensive sticky information predictor more attractive compared to costly rational expectations. However, a smaller $\tau_{\pi}$ also reduces the average switching frequency regarding inflation predictors from nearly 3 to 1.4 quarters. Comparing these results to the ones obtained in Dräger (2010), it seems that allowing for feedback from agents’ predictor choice to the model induces agents to switch more frequently, especially with respect to inflation expectations. The result that on average agents seem to be more rational with respect to output than to inflation remains robust. This is due to the fact that as agents know that the central bank places a larger weight on stabilizing inflation relative to the output gap, they can ‘delegate’ rationality to the central bank and thus concentrate more on current output movements.\textsuperscript{11}

Finally, we analyze the relation between agents’ choice of predictors and the macroeconomic conditions in the model economy. Table 3 presents correlation coefficients of $\lambda_{y_t}$ and $\lambda_{\pi_t}$ with the level and variance of output, inflation and nominal interest rates. In line with our results in Dräger (2010), we find that the degree of attentiveness is strongly correlated with the variance of the variable to be forecasted. In that sense, agents in our model behave near-rationally as in Akerlof and Yellen (1985) and Akerlof et al. (2000). Thus, they increasingly opt for costly rational expectations as the variability of the forecasted variable rises, and remain inattentive towards new developments in the variable otherwise. Interestingly, allowing for feedback from predictor choice to the economy does not significantly affect the degree of correlation between $\lambda_{y_t}$ and $Var(\hat{y}_t)$, while the correlation of $\lambda_{\pi_t}$ with $Var(\pi_t)$ rises from about 0.4 to about 0.6.

\textsuperscript{10}Note that our mean values of $\lambda^y$ and of $\lambda^\pi$ with the larger cost-push shock fit well with empirical estimations of the overall probability of updating sticky information for U.S. data of $\lambda$ between 0.44 and 0.71 in Kiley (2007). By contrast, the mean value of 0.22 obtained for $\lambda^\pi$ with a smaller cost-push shock is closer to estimates of about 0.3 found in Carroll (2003) for the U.S. and in Döpke et al. (2008a,b) for a panel of European countries.

\textsuperscript{11}This effect is reduced as the Taylor rule coefficients $\mu_{\pi}$ and $\mu_{ygap}$ converge, see Dräger (2010).
Also in line with our results in Dräger (2010), attentiveness towards inflation is positively correlated with the variance of nominal interest rates, suggesting a strong link between monetary policy and inflation: As monetary policy becomes more active, agents interpret this as a signal to pay closer attention to recent inflation developments. However, once recursive inattentiveness influences dynamic equilibrium outcomes of the model, we find that also attentiveness towards output is increasingly influenced by variation in inflation and nominal interest rates. Although this effect is smaller than the link between inflation expectations and interest rates, it shows that the dynamics of the model become more complex once we allow for feedback from endogenous inattentiveness to the model. Especially with $\tau_\pi = 0.015$, the correlation between $\lambda_y$ and the variances of inflation and of nominal interest rates is close to 10%.

### 3.3 Monetary Policy with Recursive Inattentiveness

After evaluating the statistical properties of simulated series for aggregate variables and recursive inattentiveness, we turn to analyzing the effects of a monetary policy shock. Specifically, we are interested in whether the model can generate the delayed, hump-shaped, responses of both output and inflation after a monetary policy shock found empirically for instance in Christiano et al. (2005) and Rotemberg and Woodford (1997). Since our model has a dynamic equilibrium with time-varying parameters $\lambda_y$ and $\lambda_\pi$, however, overall impulse-responses in terms of the $MA(\infty)$-coefficients cannot be derived, since the MA-representation of the model changes each period. Therefore, over 500 simulation periods, we plot impulse responses of both output and inflation to a monetary policy shock with time-varying degree of inattentiveness $\lambda_y$ and $\lambda_\pi$, shown in Figure 3.

![Figure 3 here](image)

From Figure 3 we see that over the simulation period, output and inflation show both peaked and hump-shaped response functions after a monetary policy shock. However, while impulse responses of inflation mostly show
the hump-shaped pattern, the simulation for output suggests that hump-shaped responses are relatively less frequent. In order to inquire into the differences between responses of output and inflation, we approximate effects of a monetary policy shock on output and inflation by fixing the share of agents with rational inflation expectations at $\lambda^\pi = 0.5$, while letting the share of agents with rational output expectations vary between zero and one. Conversely, the effect of a monetary policy shock on inflation is simulated for varying values of $\lambda^\pi$, keeping $\lambda^y$ fixed at 0.5.\(^\text{12}\)

< Figure 4 here >

Figure 4 shows impulse responses of output to a one-standard-deviation monetary policy shock for varying degrees of inattentiveness towards output. Even with full rationality ($\lambda^y = 0$), we see that an unexpected increase in nominal interest rates causes output to fall below its steady state value for about 10 quarters. This is because the model still assumes inattentiveness towards inflation, which leads to an overall slower adjustment process after the shock. However, we cannot generate a hump-shaped response of output to the monetary policy shock. Setting the degree of inattentiveness towards output at 50%, the negative response of output to the shock is considerably smaller, as only half of the population learns about it in the current period, and the adjustment process becomes more persistent. Finally, assuming that all agents use information from the last period or older ($\lambda^y = 1$), we are able to replicate the hump-shaped response of output to a monetary policy shock found in empirical data. The negative effect of the monetary policy shock is mitigated even further and has its strongest impact in the second quarter after the occurrence of the shock.

< Figure 5 here >

Impulse responses of inflation to a one-standard-deviation monetary policy shock are presented in Figure 5. Similar to our results for impulse-responses of output, with full rationality towards inflation ($\lambda^\pi = 0$) a positive

\(^{12}\)All simulations are carried out with $\tau^\pi = 0.015$.\)
shock to nominal interest rates reduces inflation significantly below its steady state with a gradual adjustment of about 7 quarters. Note that the strongest effect of the shock materializes in the second, period after the shock because inflation in the sticky information Phillips curve with flexible prices is affected by inattentiveness in the previous period, see equation (6). In contrast to our results for impulse-responses of output, we find a hump-shaped response of inflation to a monetary policy shock already when assuming that 50% of all agents forecast with sticky information. The negative effect of the unexpected increase in interest rates is reduced considerably, and inertia of the adjustment process is increased. Finally, with all agents forecasting inflation under sticky information, the hump-shaped response is even more pronounced: A monetary policy shock has its strongest impact on inflation up to 5 quarters after the shock. Overall, we thus find that hump-shaped impulse-response functions can be reproduced for output and inflation when all agents use the sticky information predictor, while a hump-shaped response of inflation is found even for $\lambda_\pi = 0.5$. In periods of relatively high inattentiveness by agents, a monetary policy shock will thus have more persistent effects.

### 3.4 Stability of the Model

After analyzing equilibrium dynamics of aggregate variables and recursive inattentiveness in our model with endogenous sticky information, we check for stability of the steady state and evaluate conditions for determinacy of the model.

As noted in Brock and Hommes (1997), endogenous switching between predictors in a dynamic equilibrium may lead to complex and potentially chaotic dynamics if the intensity of choice, $\gamma$, is sufficiently high. This may result in the occurrence of a homoclinic orbit with strange attractors, implying that the system does not converge to its steady state after an initial shock. In order to check for the existence of strange attractors, we run a number of simulations, where either output or inflation are subjected to an initial shock. After the shock, the model is simulated for 1000 periods and
we collect the final attractors that output, inflation, interest rates and the shares of rational agents converge to. Figure 6 plots attractors of output and inflation for simulations across a range of Taylor rule coefficients $\mu_{ygap}$ and $\mu_\pi$ and a range of initial shocks with standard deviations $\tau_y$ and $\tau_\pi$.

From Figure 6 we see that both output and inflation converge to their steady states of zero after being subjected to a range of positive and negative shocks. This result remains robust across changing values of Taylor rule coefficients for the output gap ($0 \leq \mu_{ygap} \leq 2$) and inflation ($1 < \mu_\pi \leq 2$), where we respect the Taylor principle by ensuring that monetary policy reacts more than one-for-one to changes in inflation. Convergence to the zero steady state occurs also for nominal interest rates and the shares of rational agents converge to values close to zero.13 We thus find that our model with endogenous sticky information does not show any system-inherent chaotic long-run dynamics, as after an initial shock all aggregate variables return to their steady state values and agents thus opt for a constant degree of inattentiveness. Hence, although the dynamic equilibrium paths of the aggregate variables and recursive inattentiveness in our model seem similar to the switching behavior described in Brock and Hommes (1997), dynamics die out quickly if the system is no longer subject to exogenous shocks. Our result is in contrast to De Grauwe (2008, 2010)’s DSGE model where agents choose between simple heuristic predictors: The author finds that chaotic strange attractors may arise for sufficiently large shocks on output and inflation if monetary policy is not credible, resulting in endogenous cyclical movements of output and inflation.

Finally, we evaluate determinacy of the model across a range of Taylor rule coefficients. In Dräger (2010) we analyze determinacy with a fixed degree of inattentiveness given by $\lambda^y = \lambda^\tau = 0.5$, since it is assumed that agents’ choice between predictors does not influence the model solution. Given a constant share of agents with rational expectations, our model can reproduce the result

13We omit graphical representation of these results for reasons of space limitations, but the results can be obtained from the author upon request.
in Meyer-Gohde (2009) who finds that determinacy in a sticky information model depends solely on the Taylor principle.

Because we assume that in the limit all agents have rational expectations, we can use the well-known eigenvalue accounting method by Blanchard and Kahn (1980) to check for the existence of a unique and stable solution to the model. Here, we thus evaluate the number of unstable eigenvalues across Taylor rule coefficients for varying degrees of inattentiveness towards inflation and output.

Figure 7 plots the number of unstable eigenvalues of simulations with $0 \leq \mu_\pi \leq 2$ and $0 \leq \mu_{g\, gap} \leq 2$, where we solve each combination of Taylor rule coefficients for all values of $0 \leq \lambda^\pi \leq 1$ and $0 \leq \lambda^y \leq 1$ and collect the number of unstable eigenvalues. With six endogenous variables in our model, a unique and stable solution exists if the number of unstable eigenvalues is exactly equal to the number of endogenous variables. With more unstable eigenvalues than endogenous variables, the system does not yield a stable solution, while with less unstable eigenvalues multiple equilibria may arise.

As shown in Figure 7, for all values of $\mu_\pi$ and $\mu_{g\, gap}$ analyzed here, there exist combinations of inattentiveness towards output and inflation for which the model yields exactly six unstable eigenvalues, so that a unique and stable solution emerges. However, if monetary policy does not respond more than one-to-one to changes in inflation ($\mu_\pi \leq 1$), there exist also combinations of $\lambda^\pi$ and $\lambda^y$ with multiple solutions to the model system. Hence, the result that monetary policy should respect the Taylor principle remains robust when allowing for feedback from time-varying inattentiveness to the model. Nevertheless, it seems that with recursive inattentiveness restrictions for determinacy regarding the central bank’s response to the output gap matter as well: Accounting for changes in $\lambda^\pi$ and $\lambda^y$, a unique solution for all coefficients $\mu_\pi > 1$ exists only if the central bank targets the output gap with at

---

Supplementary note:

Endogenous variables include inflation, output, nominal interest rates, natural output driven by a technology shock, the output gap as the difference between output and natural output, and the change of the output gap.
least a coefficient of 0.5 (the baseline value of our calibration) and is never feasible if the central bank puts zero weight on the output gap.\textsuperscript{15} This result is mostly due to the interaction between the model and the share of agents with rational inflation expectations. Interestingly, as the Taylor rule coefficient on inflation increases from 1 to 2, multiple equilibria may emerge for an increasing range of coefficients on the output gap below 0.5. It seems that the indeterminacy region increases in the form of a step function: For values of $\mu_\pi = [1.1, 1.2]$ the model generates multiple equilibria with values of $\mu_{y_{gap}} = 0$, for $\mu_\pi = [1.3, 1.4]$ multiple equilibria can be avoided when setting $\mu_{y_{gap}} > 0.1$ and so on. This suggests that as monetary policy reacts more forcefully to changes in inflation, in order to avoid multiple equilibria it should increasingly target the output gap as well.

4 Conclusion

Building on the model derived in Dräger (2010), we present simulation results from a DSGE model with recursive inattentiveness. Extending the models of sticky information in Mankiw and Reis (2001, 2002, 2003, 2007), we endogenize the probability that agents may update to the new information set. Employing a switching mechanism from Brock and Hommes (1997), agents decide on their degree of inattentiveness towards inflation and output each period by choosing optimally between losses under forecasts with sticky information and a fixed cost of updating to the new information set. While in Dräger (2010) it was assumed that agents choose predictors given the model simulation, we extend this approach by allowing for recursive feedback from predictor choice to the model solution. This yields a dynamic equilibrium path with an endogenous and time-varying share of agents with rational expectations.

We find that when changes in the degree of inattentiveness influence the

\textsuperscript{15}Note that this result depends on the range of Taylor rule coefficients on inflation tested here. If monetary policy targets inflation with a coefficient larger than 2, a coefficient on the output gap larger than 0.5 might be necessary to ensure determinacy. However, since most models assume a reaction coefficient to inflation of about 1.5 as in our calibration, we restrict the analysis to the range $0 \leq \mu_\pi \leq 2$. 

21
model solution, simulated time series for output and nominal interest rates exhibit very strong persistence with autocorrelation close to a random walk. Inflation in our model shows a persistent trend, but relatively strong short-run fluctuations. However, for reasonable cost-push shocks on inflation, the simulated series has an autocorrelation coefficient of about 0.5, close to empirical values for quarter-on-quarter inflation in the U.S. Hence, it seems that our model with recursive inattentiveness can replicate the stylized fact of strong persistence in aggregate inflation and output data as highlighted by Fuhrer and Moore (1995) without resorting to the assumption of autocorrelated shocks or lagged endogenous variables.

All main results from the earlier analysis in Dräger (2010) remain robust also when allowing for interaction between agents’ switching and the model solution. We still find that on average agents choose to pay more attention to output than to inflation. While the share of agents with rational expectations is positively correlated with the forecasted variables, the share of rational inflation expectations is also strongly correlated with the variance of interest rates, emphasizing the link between monetary policy and attentiveness towards inflation. However, with feedback from predictor choice we additionally find that also the share of agents with rational output expectations is to some degree correlated with the variance of inflation and of nominal interest rates. Agents in our model thus behave near-rationally as in Akerlof and Yellen (1985) and Akerlof et al. (2000), paying more attention to recent developments of output and inflation in times of high volatility in the economy and ignoring smaller changes. Note that in a related model with near-rational inflation expectations, Ball (2000) also finds that the model generates strong persistence in inflation.

With respect to the stylized fact of a hump-shaped response to a monetary policy shock emphasized in Christiano et al. (2005) and Rotemberg and Woodford (1997), we find that our model can reproduce a hump-shaped impulse-response of both output and inflation when the degree of inattentiveness is sufficiently high. In a stylized exercise with fixed degrees of inattentiveness, we find a hump-shaped response of inflation already when half the population employ the rational predictor, while a hump-shaped response
of output is only found when all agents use outdated information.

Finally, evaluating stability of the model we find that all variables converge to their steady states after an initial shock for a range of Taylor rule coefficients and for positive and negative shocks of varying size. We thus conclude that the potential problem of chaotic long-run dynamics with strange attractors highlighted by Brock and Hommes (1997) does not arise in our model with recursive inattentiveness, at least for reasonable shocks. Regarding conditions for determinacy of the model, accounting for agents’ switching between predictors reduces the size of the determinacy region. While we still find that the Taylor principle is a necessary condition for a unique and stable solution of the model, multiple equilibria may nevertheless arise for some combinations of $\lambda^y$ and $\lambda^r$ if monetary policy puts too little weight on the output gap. If the output gap is targeted at least with a coefficient of 0.5, the model is determinate for all degrees of inattentiveness. As the Taylor rule coefficient converges towards its minimal value close to 1, smaller coefficients on the output gap become feasible as well.

While a number of approaches, such as Ball (2000), De Grauwe (2008, 2010) and Lansing (2009), can generate high persistence of inflation and output in models with near-rational or heuristic expectations, our model has the advantage of nesting fully rational expectations as a special case. Hence, the model generates persistence from expectational feedback, but includes the option of full rationality. Agents will be willing to take this option if the losses from forecasting with outdated information outweigh the rationality cost.

5 Appendix

5.1 Figures
Figure 1: Output and Time-Varying Inattentiveness across Models Specifications

\[ \lambda^y \text{ with } \tau = \tau^z = 0.03 \]

\[ y \text{ with } \tau^y = \tau^z = 0.03 \]

\[ \lambda^y \text{ with } \tau^z = 0.015 \]

\[ y \text{ with } \tau^z = 0.015 \]
Figure 2: Inflation and Time-Varying Inattentiveness across Model specifications

\[ \lambda^\pi \text{ with } \tau_y = \tau_\pi = 0.03 \]

\[ \lambda^\pi \text{ with } \tau_\pi = 0.015 \]

\[ \pi \text{ with } \tau_y = \tau_\pi = 0.03 \]

\[ \pi \text{ with } \tau_\pi = 0.015 \]
Figure 3: Simulated Responses to a Monetary Policy Shock

Figure 4: Impulse Responses of Output to a Monetary Policy Shock
Figure 5: Impulse Responses of Inflation to a Monetary Policy Shock

![Figure 5: Impulse Responses of Inflation to a Monetary Policy Shock](image)

Figure 6: Steady State Attractors of Output and Inflation

![Figure 6: Steady State Attractors of Output and Inflation](image)
Figure 7: Determinacy across Taylor Rule Coefficients

Simulated with $\tau_\pi = 0.015$. 
5.2 Tables

Table 1: Sample Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Standard Deviation</th>
<th>AR(1) Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{y}$</td>
<td>$\tau_y = \tau_\pi = 0.03$</td>
<td>0.1587</td>
<td>0.9776</td>
</tr>
<tr>
<td></td>
<td>$\tau_\pi = 0.015$</td>
<td>0.1580</td>
<td>0.9767</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$\tau_y = \tau_\pi = 0.03$</td>
<td>0.0416</td>
<td>0.0530</td>
</tr>
<tr>
<td></td>
<td>$\tau_\pi = 0.015$</td>
<td>0.0264</td>
<td>0.4594</td>
</tr>
<tr>
<td>$i$</td>
<td>$\tau_y = \tau_\pi = 0.03$</td>
<td>0.0372</td>
<td>0.9361</td>
</tr>
<tr>
<td></td>
<td>$\tau_\pi = 0.015$</td>
<td>0.0355</td>
<td>0.9757</td>
</tr>
</tbody>
</table>

Note: Values from simulating the model 1000 times over 1000 periods.

Table 2: Time-Varying Inattentiveness

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\lambda^y$</th>
<th>$\lambda^\pi$</th>
<th>$\lambda^y$</th>
<th>$\lambda^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_y = \tau_\pi = 0.03$</td>
<td>$\tau_\pi = 0.015$</td>
<td>$\tau_y = \tau_\pi = 0.03$</td>
<td>$\tau_\pi = 0.015$</td>
</tr>
<tr>
<td>Min.</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0069</td>
</tr>
<tr>
<td>Max.</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>Mean</td>
<td>0.5616</td>
<td>0.5360</td>
<td>0.4445</td>
<td>0.2198</td>
</tr>
<tr>
<td>Std.</td>
<td>0.4425</td>
<td>0.4429</td>
<td>0.4628</td>
<td>0.2751</td>
</tr>
<tr>
<td>Av. Cycle</td>
<td>2.616</td>
<td>2.608</td>
<td>2.992</td>
<td>1.413</td>
</tr>
</tbody>
</table>

Note: Mean values from simulating 1000 times over 1000 periods.
The average cycle length is calculated in quarters.
Table 3: Time-Varying Inattentiveness and Macroeconomic Variables

<table>
<thead>
<tr>
<th>Correlation with</th>
<th>$\lambda^y_t$ $\tau_y = \tau_x = 0.03$</th>
<th>$\lambda^y_t$ $\tau_y = \tau_x = 0.015$</th>
<th>$\lambda^\pi_t$ $\tau_y = \tau_x = 0.03$</th>
<th>$\lambda^\pi_t$ $\tau_y = \tau_x = 0.015$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level $\pi_t$</td>
<td>-0.001 0.000</td>
<td>0.000 0.000</td>
<td>0.000 0.007</td>
<td></td>
</tr>
<tr>
<td>Variance $\pi_t$</td>
<td>0.064 0.098</td>
<td>0.603 0.654</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level $\hat{y}_t$</td>
<td>-0.001 0.000</td>
<td>0.000 -0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance $\hat{y}_t$</td>
<td>0.310 0.307</td>
<td>0.016 0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level $\hat{i}_t$</td>
<td>0.000 0.000</td>
<td>0.000 0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance $\hat{i}_t$</td>
<td>0.108 0.103</td>
<td>0.444 0.479</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Values from simulating 1000 times over 1000 periods.
References


