Inflation Forecast Contracts

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Abstract

We introduce a new type of incentive contract for central bankers: inflation forecast contracts, which make central bankers’ remunerations contingent on the precision of their inflation forecasts. We show that such contracts enable central bankers to influence inflation expectations more effectively, thus facilitating more successful stabilization of current inflation. Inflation forecast contracts improve the accuracy of inflation forecasts, but have adverse consequences for output. On balance, paying central bankers according to their forecasting performance improves welfare.

Keywords: central banks, incentive contracts, transparency, inflation targeting, inflation forecast targeting, intermediate targets

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1 Introduction

There is general consensus that central banks should publish forecasts about economic variables, and inflation in particular, but that these forecasts should not be viewed as commitments.\(^1\) In this paper we argue that the first part of this consensus is justified but that the second part is not. In particular, we propose making central bankers accountable for the accuracy of their inflation forecasts by introducing incentive contracts that reward central bankers for forecasting precision. We call these contracts inflation forecast contracts.

To assess our proposal, we make use of the standard New Keynesian framework (see Clarida et al. (1999) or Woodford (2003a)). In each period, the central banker issues an inflation forecast for the next period. In the absence of inflation forecast contracts, the central banker’s loss function equals the social loss function. If inflation forecast contracts are introduced, the central banker will also take into account the rewards he receives for precise forecasts.

We show that paying central bankers for the accuracy of their forecasts enhances welfare. Intuitively, inflation forecast contracts would lend credibility to the central bankers’ inflation forecasts by making it costly for central bankers to deviate from their forecasts. As a result, central bankers can use inflation forecasts to influence the public’s inflation expectations. This facilitates better stabilization of cost-push shocks because, in the New Keynesian Phillips curve, inflation also depends on expectations about future inflation.

Imagine, for example, a situation where a cost-push shock would drive up inflation. The conventional reaction by central banks would be to increase interest rates, thereby lowering output and hence also inflation. If central bankers are rewarded for the precision of their forecasts, they can use an additional channel to stabilize inflation by promising to ensure a low inflation rate in the future. The public knows that the

\(^1\) Currently, all major central banks release forecasts about key economic variables. For an overview of how transparent central banks are, see Eijffinger and Geraats (2006). Svensson (2009), among others, stresses that interest-rate forecasts should not be viewed as commitments.
central banker has a financial interest in fulfilling his promise, so the central banker can use inflation forecasts to steer the public’s inflation expectations and thus, in turn, inflation.

2 Related Literature

Our paper contributes to the large literature on inflation targeting. An early exposition of the experiences made by central banks adopting this monetary policy strategy can be found in Bernanke et al. (1999). The main advantages of inflation targeting are associated with anchoring inflation expectations and the furtherance of credibility and transparency.²

More particularly, our paper is related to Svensson (1997a), who introduced the notion of inflation forecast targeting. He shows that, in the presence of lags in monetary transmission, monetary policy is bound to be welfare-maximizing if the central bank’s optimal forecast corresponds to the inflation target. He argues that it is accordingly advantageous to use the inflation forecast as an intermediate target because it has the advantage of being easier to monitor by the public than inflation itself. Our paper differs from Svensson (1997a) in two ways. First, in our model the central banker sets his forecasts strategically to influence the public’s inflation expectations. Second, we consider the optimal design of contracts that make central bankers accountable for the accuracy of their forecasts.

Because the incentive contracts studied in this paper affect the central banker’s objective function, our contribution is also related to Woodford and Svensson (2005), who explore how the central bank’s loss function should be modified in order to minimize social losses from a timeless perspective. However, this procedure requires that the central bank change its own future loss function in each period. Moreover, it may result in alternative, inferior equilibria. In our approach, the central bank’s loss function is constant across periods, and the equilibrium is unique.

Gersbach and Hahn (2011) is complementary to the present paper. In a note, they exclusively address the question whether central banks should release interest-rate forecasts for the exogenously given psychological costs of deviating from forecasts previously made. In the present paper, we focus on inflation forecasts and endogenize the costs of deviating from forecasts through inflation forecast contracts.

The use of incentive contracts for central bankers was first proposed in the highly influential paper by Walsh (1995).³ In a neoclassical model with a classic time-inconsistency problem, Walsh (1995) identifies incentive contracts that lead both to an elimination of the inflation bias and to efficient shock stabilization. We consider incentive contracts that are contingent on the central banker’s forecasting performance and explore their consequences in a New Keynesian model.

Finally, inflation forecast contracts are related to Woodford (2003b). He shows that, even when interest-rate smoothing is not socially desirable per se, it is socially advantageous to assign an interest-smoothing objective to central banks. In our paper it is beneficial to make the central banker responsible for minimizing the deviations between his inflation forecasts and actual inflation, although highly accurate inflation forecasts have no direct implications for welfare.

### 3 Model

We consider the New Keynesian Phillips curve, presented in Clarida et al. (1999),

\[
\pi_t = \delta E_t[\pi_{t+1}] + \lambda y_t + \xi_t. \tag{1}
\]

We use \( \pi_t \) and \( y_t \) to denote (log) inflation and (log) output in period \( t \). \( E_t \) is the expectations operator. Parameter \( \lambda \) satisfies \( \lambda > 0 \), and \( \delta \) is the common discount factor \( (0 < \delta < 1) \). The cost-push shock \( \xi_t \) is given by an AR(1) process

\[
\xi_t = \rho \xi_{t-1} + \varepsilon_t, \tag{2}
\]

³Important further contributions to the theory of incentive contracts have been made by Beetsma and Jensen (1998, 1999), Jensen (1997), Lockwood (1997), and Svensson (1997b).
where $0 < \rho < 1$. The $\varepsilon_t$’s are i.i.d. and drawn from a normal distribution with zero mean and variance $v^2$. We refrain from complementing the model with an IS curve because the IS curve is irrelevant for our purposes.\(^4\)

Social losses in period $t$ are

$$l_t = \pi_t^2 + ay_t^2,$$

(3)

where $a > 0$. In each period $t$, the central banker publishes an inflation forecast $\pi_{t+1}^f$ for period $t + 1$.

For simplicity, we assume that monetary policy is conducted by an individual central banker.\(^5\) The central banker aims at minimizing social losses $l_t$. In addition, he may be held responsible for the accuracy of his own inflation forecasts. This can be achieved by means of an incentive contract that imposes costs $b \left( \pi_t - \pi_t^f \right)^2$ on the central banker if his forecasts fail to come about. Parameter $b$ ($b \geq 0$) is chosen by the contract. Effectively, a particular value of $b$ is associated with a particular salary decrease when forecasts are not accurate. To make sure that the central banker participates, the contract may also specify a fixed payment above his normal wage, resulting in an additional additive, policy-independent term in his loss function. As such a constant term in the loss function does not affect the central banker’s choice of monetary policy and forecasts, it will be neglected for the remainder of the paper.

As a consequence, total central banker losses in period $t$ are

$$l_t^{CB} = l_t + b \left( \pi_t - \pi_t^f \right)^2.$$ (4)

For $b = 0$, our model collapses to the case where the central banker minimizes social losses. Our main institutional design issue is which value of $b$ will minimize social losses and thus which type of inflation forecast contract is optimal.

\(^4\)For example, the IS curve could be specified as $y_t = -\sigma (i_t - \mathbb{E}_t \pi_{t+1} - \tau_t) + \mathbb{E}_t y_{t+1} + \mu_t$, with $\sigma > 0$, the nominal interest rate $i_t$, the natural real rate of interest $\tau_t$, and a demand shock $\mu_t$. The central banker could always stabilize $\mu_t$ by an appropriate adjustment of the interest rate. Therefore, without loss of generality, $y_t$ can be viewed as the central banker’s instrument.

\(^5\)Our analysis can be easily extended to the case where a committee rather than an individual central banker decides on monetary policy. Then, at each meeting, committee members would vote not only on the interest rate but also on an inflation forecast. All members would be paid according to the precision of the committee’s forecasts. Our results would continue to hold in such a framework.
With the help of the algorithm described in Söderlind (1999), we compute the discretionary solution for different values of $b$. In each period $t$, the central banker minimizes his losses by choosing current policy and the inflation forecast for the next period. In this, he takes the process by which the public forms its inflation expectations as given. This process, in turn, has to be consistent with the policy actually pursued by the central banker. In each period $t$, there are two predetermined variables ($\xi_t$ and $\pi^f_t$) and one non-predetermined one ($\pi_t$). The central banker’s instruments in period $t$ are $y_t$ and $\pi^f_{t+1}$. Appendix A gives more details on the application of Söderlind’s algorithm to our framework.

### 3.1 Plausible parameter values

Unless stated otherwise, we choose the parameter values used in Clarida et al. (2000) for quarterly data, i.e. $\delta = 0.99$, $\rho = 0.9$, and $\lambda = 0.3$. To select an appropriate value for $a$, we note that the social loss function can be derived from microeconomic foundations (see Woodford (2002)). In this case, $a = \lambda/\theta$ would hold, where $\theta$ is the elasticity of substitution in the Dixit-Stiglitz index of aggregate demand (see Woodford (2002), p. 22). A plausible value for $\theta$ is 11, which implies a mark-up of 10% over marginal costs. Consequently, $a = 0.3/11 \approx 0.03$. No assumption is needed about the size of $\nu^2$, which is the variance of the shock $\varepsilon_t$, because this parameter is immaterial to our results.

To demonstrate the generality of our findings, we compute some of our simulations for a broad range of plausible parameter values containing the abovementioned parameter constellation as a special case. In these simulations, we use a range of $\delta = 0.970...0.995$. Various studies find values from 0.05 (Taylor (1980)) to 1.22 (Chari et al. (2000)) for $\lambda$. For $\rho$, values between 0.0 and 0.9 are encountered in the literature (see Clarida et al. (1999)). We extend this range slightly to 0.00...0.95. Moreover, $a$, the weight of the output objective in relation to the inflation objective in the social loss function, can be plausibly assumed to be lower than 1. For $b$, which is chosen by the contract designer, we consider values up to 20.0. We summarize the set of plausible parameter values in Table 1.


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3.2 The impact of projections on expectations

As a preliminary step, we study the impact of inflation projections on the public’s inflation expectations. For this purpose, we note that Söderlind’s algorithm yields a $(1 \times 2)$ matrix $C$ that describes how the non-predetermined variable $\pi_t$ depends on the state variables $\xi_t$ and $\pi_f^t$ (see Appendix A):

$$\pi_t = C \left( \xi_t, \pi_f^t \right)'$$

With the help of $E_t \xi_{t+1} = \rho \xi_t$, this equation can be used to describe expectations about inflation

$$E_t \pi_{t+1} = C_{11} \rho \xi_t + C_{12} \pi_{t+1}^f.$$  \hspace{1cm} (6)

Hence the entries in $C$ describe how inflation expectations depend on the cost-push shock and the central banker’s inflation forecast.

With the Söderlind algorithm, it is straightforward to verify that $C_{12}$ converges to zero as $b$ goes to zero. Thus inflation forecasts have no impact on inflation expectations and other economic variables in the absence of an inflation forecast contract ($b = 0$).

For positive values of $b$, we obtain

**Numerical Finding 1**

For all parameter constellations in Table 1, $C_{12} > 0$ holds. For positive values of $\rho$, $C_{11} > 0$ also holds.

The finding $C_{12} > 0$ implies that, in line with Gersbach and Hahn (2011), an increase in the inflation forecast leads to higher inflation expectations. This is plausible because
the public knows that the central banker will find it costly to deviate from his forecast. Moreover, higher realizations of $\xi_t$ lead to higher inflation expectations for given inflation forecasts ($C_{11} > 0$) under the assumption of autoregressive cost-push shocks ($\rho > 0$).

To summarize, inflation forecast contracts enable the central banker to influence inflation expectations. Manipulating inflation expectations is potentially desirable because, in line with the New Keynesian Phillips curve, they impact on current inflation.

4 The Consequences of Inflation Forecast Contracts

4.1 The impact on inflation, output, and welfare

Next we turn to the implications that inflation forecast contracts have for welfare. In a first step, we explore how rewarding the central banker for the precision of his inflation forecasts affects inflation variance.

**Numerical Finding 2**

*For all parameter constellations in Table 1, the unconditional variance of inflation is reduced by inflation forecast contracts.*

The intuition for this finding is straightforward. Inflation forecast contracts enable the central banker to effectively anchor expectations about future inflation by choosing an appropriate inflation forecast, which stabilizes current inflation. As a result, inflation forecast contracts reduce inflation variance.

Next we examine how the introduction of inflation forecast affects output variance.

**Numerical Finding 3**

*For all parameter constellations in Table 1, inflation forecast contracts reduce the unconditional variance of output.*
Intuitively, the central banker cannot incorporate information about \( \epsilon_t \) into his inflation forecast in period \( t - 1 \). As he will later find it costly to deviate from this inflation forecast, he will not allow the shock \( \epsilon_t \) to have a strong impact on inflation. As a consequence, he will tolerate larger fluctuations in output in response to \( \epsilon_t \).

To summarize, inflation forecast contracts have ambiguous effects on welfare. They lower inflation variance but increase output variance. A priori, it is unclear which effect will dominate with regard to welfare. For small values of \( b \), i.e. a low weight on forecast deviations in the central banker’s loss function, we can establish a clear-cut result:

**Numerical Finding 4**

For all parameter constellations in Table 1, inflation forecast contracts lower social losses if \( b \) is sufficiently small but positive.

This finding has the important corollary that inflation forecast contracts always lead to welfare gains if the contract designer makes an optimal choice of \( b \).

### 4.2 Optimal level of \( b \)

Having demonstrated that inflation forecast contracts can always be used to enhance welfare, we now focus on the optimal design of these contracts and on the size of the resulting welfare gains. Accordingly, we examine the optimal weight on deviations from the inflation forecast target in the central banker’s loss function, \( b \).

For the benchmark parameter values, Figure 1 shows the welfare gains created by inflation forecast contracts over the benchmark case without incentive contracts. They are expressed as a fraction of the welfare gains that can be achieved by perfect commitment. The figure shows \( b \) on the horizontal axis. Two facts are remarkable. First, for an appropriate value of \( b \), large welfare gains are possible. Approximately 86% of

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7The optimal degree of commitment to an intermediate monetary target has been considered by Rogoff (1985). We perform a similar exercise for commitment to a forecast target.

8See Clarida et al. (1999), pp. 1681-3, for a specification of the solution under commitment.
the welfare gains achievable by optimal commitment can be attained by simple one-period-ahead, non-contingent inflation forecasts. Second, the optimal value of $b$ is rather high, with a maximum of welfare gains at $b \approx 7.3$. A high value of $b$ is socially beneficial because it enables the central banker to affect inflation expectations effectively through inflation forecasts, which in turn makes for effective stabilization of current inflation.

These findings demonstrate the desirability of incentive contracts, according to which central bankers’ wages depend on the accuracy of inflation forecasts. We emphasize that the desirability of inflation forecast contracts is not restricted to the parameter constellation considered in Figure 1. In line with Numerical Finding 4, rewarding central bankers for accuracy in their inflation forecasts improves welfare for the whole range of parameters specified in Table 1, provided that parameter $b$ is chosen optimally by the contract designer.

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9 More precisely, we compare unconditional losses in the inflation forecast scenario with those for optimal commitment from a timeless perspective (see Woodford (1999)).
4.3 Comparison to inflation contracts

Welfare comparison

In this section, we compare inflation forecast contracts with standard inflation contracts, i.e. incentive contracts rewarding central bankers not for the precision of their forecasts but granting bonus payments to central bankers for achieving the socially optimal level of inflation. More specifically, we assume that inflation contracts inflict additional costs $b'\pi^2_t$ on the central banker, where $b' \geq 0$ is a parameter that can be chosen by the contract designer.\(^{10}\)

As a result, the central banker’s per-period loss function is

$$l^C_{t} = l_t + b'\pi^2_t = (1 + b')\pi^2_t + ay^2_t. \quad (7)$$

Effectively, inflation contracts make central bankers more conservative by increasing the relative weight on deviations from the inflation target.

It is well-known that in the New Keynesian model the delegation of monetary policy to a conservative central banker yields welfare gains, even when central bankers are not pursuing an output target that exceeds the natural level of output (see Clarida et al. (1999)) so that the classic problem of an inflation bias is immaterial (Kydland and Prescott (1977)). This indicates the potential desirability of inflation contracts.

Figure 2 shows that rewarding central bankers for the precision of their forecasts generally leads to somewhat higher welfare, over and against the case where central bankers receive additional rewards for achieving the socially optimal level of inflation. In particular, the optimal level of $b$ in the former case guarantees higher welfare than the respective optimal level of $b'$ in the latter. The superior performance of wages contingent on forecasting accuracy is even more pronounced for lower values of $\rho$, as can be seen from Figure 3, which displays the welfare gains that can be achieved by both types of incentive contract for $\rho = 0.5$. This observation adumbrates a more general finding, to be discussed in the next Section 4.4. There we demonstrate that additional

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\(^{10}\)As in inflation forecast contracts, fixed wage increases can be specified in inflation contracts to satisfy the central banker’s participation constraint.
Figure 2: Welfare gains created by incentive contracts imposing additional costs on central bankers if (i) inflation differs from its socially optimal level (solid line) and (ii) inflation forecasts are not accurate (broken line). Parameter $b$ ($b'$) is shown on the horizontal axis. Other parameters: $\delta = 0.99$, $\rho = 0.9$, $\lambda = 0.3$, and $a = 0.03$

Figure 3: Welfare gains created by incentive contracts imposing additional costs on central bankers if (i) inflation differs from its socially optimal level (solid line) and (ii) inflation forecasts are not accurate (broken line). Parameter $b$ ($b'$) is shown on the horizontal axis. Other parameters: $\delta = 0.99$, $\rho = 0.5$, $\lambda = 0.3$, and $a = 0.03$
incentives for making accurate forecasts can improve monetary policy even for very low values of $\rho$, while in this case inflation contracts cannot improve monetary policy making.\textsuperscript{11}

\textit{Ease of contracting}

Compared to inflation contracts, which specify additional rewards for central bankers when inflation is close to the socially optimal rate, inflation forecast contracts can also be used if the socially optimal inflation rate is subject to shocks that cannot be contracted upon. This is an obvious consequence of the fact that only previous inflation forecasts and actual inflation rates are required to determine the wages of central bankers. Moreover, inflation forecast contracts can also be used if the central banker enjoys goal independence and can specify the inflation target himself. Inflation contracts cannot be applied in these cases.

\subsection*{4.4 Role of autocorrelated shocks}

One conceivable question is whether our results depend on the fact that the $\xi_t$’s are autocorrelated. But welfare gains from inflation forecast contracts can also be achieved if the $\xi_t$’s are independent and identically distributed, i.e. $\rho = 0$, as explained in this section.

Suppose $\rho = 0$ and a positive shock has occurred ($\xi_t > 0$). Then it might seem plausible for the central banker to forecast $\pi_{t+1}^f = 0$ because $E_t [\xi_{t+1}] = 0$. However, this is not the case, as such a choice would concentrate the entire losses stemming from $\xi_t$ in period $t$. Because per-period losses are convex, it is more efficient to distribute the impact of $\xi_t$ on inflation and output over several periods. This can be achieved by setting the inflation forecast below the target ($\pi_{t+1}^f < 0$), thereby reducing $E_t [\pi_{t+1}]$.

\footnote{\textsuperscript{11}Inflation forecast contracts may be inferior to inflation contracts for very low values of $\lambda$, as can be shown by considering $\lambda = 0.05$. In this case, the detrimental effect on output variance of the inflation forecast contracts becomes especially severe. Intuitively, a very low value of $\lambda$ means that the central banker has to engineer large swings in output in order to stabilize the impact of the present shock $\varepsilon_t$ on inflation.}

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which in turn lowers $\pi_t$ in period $t$. This procedure reduces social losses in period $t$ at
the expense of the social losses in period $t + 1$.$^{12}$

We conclude that a central banker with an inflation forecast contract can reduce ex-
pectations about future inflation below the long-term target and thus stabilize current
inflation after a positive cost-push shock. By contrast, a central banker with an inflation
contract cannot lower inflation expectations below the long-term target for inflation.
The public knows that he will always implement an inflation rate that on average is
identical to the inflation target. Hence inflation contracts cannot enhance welfare for
$\rho = 0$ in the absence of an inflation bias.

4.5 Impact on the accuracy of forecasts

In this section we examine the impact of the size of the costs incurred by deviations,
$b$, on the precision of the forecasts. As can be seen from Figure 4, the higher $b$ is, the
lower is the unconditional variance of the inflation forecast error $\pi_t - \pi_t^f$.

$^{12}$In the standard model, the commitment solution implies a mean-reverting price level (see Clarida et al. (1999)). In our model, the central banker mimics this solution to some extent by announcing inflation rates that are below target for positive cost-push shocks.
Thus we arrive at the plausible finding that rewarding the central banker for the precision of his forecasts will raise the accuracy of these forecasts. Two effects are responsible for this. First, if rewards for precise forecasts are high, the central banker will obviously be more interested in aligning inflation with the forecast. The second effect is more subtle. As we have shown, inflation forecast contracts lower the variance of inflation and thus make inflation more predictable. As a result, the accuracy of the central banker’s inflation forecasts will increase. Because inflation forecast contracts improve the precision of inflation forecasts, they may contribute to transparency in monetary policy.

Interestingly, the improvement in forecasting accuracy occurs although the precision of the central banker’s information is unaffected by the introduction of incentive contracts. If information acquisition were endogenous, incentive contracts would lead to additional improvements in the quality of forecasts by inducing the central banker to look for more precise information.

5 Conclusions

We have shown that rewarding central bankers for the precision of their inflation forecasts makes the inflation forecast an effective tool for influencing inflation expectations. As a consequence, inflation forecast contracts enable a more effective stabilization of inflation and reduce the error inherent in inflation forecasts. However, they also cause higher output variance.

With regard to welfare, the incentive contracts considered in this paper create a trade-off. They enable the central banker to influence the public’s expectations, which is socially desirable. However, they also reduce the central banker’s flexibility in responding to unexpected shocks. On balance, the beneficial effect of incentive contracts dominates for large sets of plausible parameters, and it is optimal to create large incentives for central bankers to adhere to their inflation forecasts.

Our model may also shed light on the apparent success of central banks that have adopted inflation targeting. One essential ingredient in the inflation targeting strategy
is the publication of inflation forecasts. It is not implausible that even in the absence of incentive contracts minor costs may accrue for central bankers when their forecasts fail to materialize. Then the release of inflation forecasts is socially desirable.

There are several useful extensions to our model. First, we might consider a central banker attempting to push output above its natural level. If monetary policy faced the problem of an inflation bias, the logic of our analysis suggests that incentive contracts contingent on the central banker’s forecasting performance will involve additional advantages.

Second, it is worth noting that the incentive contracts considered in this paper have no adverse effect on the stabilization of demand shocks. This can easily be verified by introducing demand shocks into our model and noting that they can always be stabilized perfectly, irrespective of whether incentive contracts are used.\(^{13}\)

Third, more complex incentive contracts may further improve the performance of central banks. However, these contracts would condition the remuneration of central bankers on current shocks and the output gap, which are difficult to measure.\(^ {14}\) Moreover, such contracts may not be feasible if the size and nature of economic shocks cannot be verified in court. By contrast, the incentive contracts considered in this paper are both simple and based on easily observable variables. Central banks routinely publish inflation forecasts, and prices can be measured with a comparably high degree of precision. Another important conclusion is that even the simple incentive contracts proposed in this paper would deliver a large proportion of the welfare gains that could be achieved by optimal commitment.

\(^{13}\)See footnote 4.

\(^{14}\)Beetsma and Jensen (1999) argue that state-contingent delegation is plausible to be more vulnerable to McCallum’s critique that delegation may not be time-consistent (see McCallum (1995)).
A Computation of Discretionary Solution

Using Söderlind’s notation, let $x_t := (\xi_t, \pi_t, \pi_t)'$. The predetermined entries of $x_t$ are $x_{1t} := (\xi_t, \pi_t)'$, and the non-predetermined entry is $x_{2t} := \pi_t$. The vector of policy instruments is $u_t := (y_t, \pi_t')'$. The evolution of $x_t$ can be written as

$$
\begin{pmatrix}
  x_{1t+1} \\
  \mathbb{E}_t x_{2t+1}
\end{pmatrix}
= A \begin{pmatrix}
  x_{1t} \\
  x_{2t}
\end{pmatrix} + Bu_t + (\varepsilon_{t+1}, 0, 0)',
$$

where

$$
A := \begin{pmatrix}
  \rho & 0 & 0 \\
  0 & 0 & 0 \\
  -\frac{1}{\delta} & 0 & \frac{1}{\delta}
\end{pmatrix}
\quad \text{and} \quad
B := \begin{pmatrix}
  0 & 0 \\
  0 & 1 \\
  -\frac{1}{\delta} & 0
\end{pmatrix}.
$$

The central banker’s loss function (see (4)) can be stated as

$$l^C_{CB} = x_t'Qx_t + 2x_t'Uu_t + u_t'Ru_t$$

with

$$Q := \begin{pmatrix}
  0 & 0 & 0 \\
  0 & b & -b \\
  0 & -b & 1 + b
\end{pmatrix}, \quad U := \begin{pmatrix}
  0 & 0 \\
  0 & 0 \\
  0 & 0
\end{pmatrix}, \quad \text{and} \quad R := \begin{pmatrix}
  a & 0 \\
  0 & 0
\end{pmatrix}.$$

When choosing $u_t$, the central banker has to take into account how expectations about the non-predetermined period-$(t + 1)$ variable are formed. The non-predetermined variable $x_2_{t+1} = \pi_{t+1}$ in period $t + 1$ will be a linear function of the predetermined variables in this period. Thus we can write $\pi_{t+1} = x_2_{t+1} = C_{t+1}x_{1t+1}$, where $C_{t+1}$ is a $(1 \times 2)$ matrix. Consequently, the expectations are given by $\mathbb{E}_t \pi_{t+1} = \mathbb{E}_t x_{2t+1} = C_{t+1} \mathbb{E}_t x_{1t+1}$. The central banker’s optimization problem leads to the Bellman equation:

$$x_{1t}'V_t x_{1t} + v_t = \min_{u_t} \left\{ x_{1t}'Qx_t + 2x_{1t}'Uu_t + u_t'Ru_t + \delta \mathbb{E}_t \left[ x_{1t+1}'V_{t+1} x_{1t+1} + v_{t+1} \right] \right\}$$

s.t. $\mathbb{E}_t x_{2t+1} = C_{t+1} \mathbb{E}_t x_{1t+1}$, Eq. (8) and $x_{1t}$ given.

This optimization problem can be solved recursively by the procedure introduced in Backus and Driffill (1986) and Oudiz and Sachs (1985) and implemented in Matlab by Söderlind (1999). We apply these Matlab routines.
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