Doctoral Thesis

Electrostatic modification of the bending stiffness of adaptive structures

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Electrostatic Modification of the Bending Stiffness of Adaptive Structures

A dissertation submitted to the
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Doctor of Sciences ETH Zurich

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Abstract

Adaptive structures are characterized by their ability to reversibly modify their response to external mechanical stimuli so as to extend their operational space (i.e. the range of conditions under which they can operate). This feature differentiates adaptive structures from conventional ones, that are designed to maintain their properties invariant over time. Typically, adaptive structures are comprised of a host structure, one or more actuators, sensors to determine the actual state or trajectory of the structure and a control system. The bulk of the research done in this field focuses on the development of new transducing materials and systems to be used in material based actuators.

The present work focuses on the development of host structures with variable mechanical properties. The ability to modify the bending stiffness or the damping properties of a structure can be used to increase the functionality of the actuators in active structures or to suppress high amplitude vibrations in passive ones. The suppression of vibrations can be achieved either by shifting the eigenfrequency of the structure, thus avoiding resonance in the case of narrow band excitation (energy rejection) or by introducing a higher level of damping (energy dissipation).

The original contribution made by this work lies in the development and description of a structure that has the ability to adaptively modify its behavior based on a change in mechanical properties, rather than thanks to the action of one or more actuators. The change of the structure’s properties is caused by an apparent change of its topology.

Very limited literature on the use of electrostatic forces to modify the interaction between the layers of a multi-layer structure and thus its bending stiffness is available. This moderate level of knowledge on the working mechanism of multi-layer structures with variable bending stiffness, warrants the global approach to the discussion of this topic taken in this dissertation. After giving an overview on the current state of the art in variable stiffness structures, this work first describes the working principle at the base of the modification of the mechanical properties of simple structures, then it proceeds to confirming the validity of the description with the help of simple models and experi-
ments. Finally, it shows a practical application of the system in calculations that show the effect of the electrostatic modification of the mechanical properties of a Glass Fiber Reinforced Polymer (GFRP) bridge deck on its dynamic properties.

The method proposed for the adaptive modification of the bending stiffness and damping ratio of simple structures such as beams, consists of preparing structural elements made of multiple layers. These layers are provided with electrodes coated with a dielectric material at their surfaces. By applying an electrostatic field between adjacent layers, these are coupled by means of the electrostatic forces that accompany the field. If the amount of shear stress that can be transferred at the interface is large enough relative to the external transversal load that bends the structure, the structure will behave as if no interfaces existed between the layers.

A linear elastic analytical model considering interfaces within the system and a numerical model considering normal interlaminar stress and implementing contact and friction at the interfaces of the layers of a multi-layer beam are used to demonstrate the mechanism of the electrostatic stiffening of a multi-layer beam. A comparison between the shear stress distributions calculated with the two models shows that up to the maximum shear stress that can be transferred by means of friction at the interfaces, no difference can be made between true topology switch (i.e. an actual cancellation of the solid-solid interfaces in the structure) and electrostatic coupling of the layers. Once the shear stresses at the interface generated by external load exceed this limit, the system softens and displays high damping due to the friction between the contact surfaces. For small loads (i.e. below the elastic limit of the system), discrete stiffness states are predicted. Their number increases rapidly with the increasing number of layers of the system.

In the experimental section of this work, experiments on two different systems are presented. Both are sandwich beams with stiff faces and a compliant core. The first system is used to verify the relationship between normal electrostatic stress, shear stress and stiffness of the system. The mechanical properties of the second system used for this work are comparable to the ones of Empa’s cable stayed pedestrian bridge. The effect of the electrolamination of Carbon Fiber Reinforced Polymer (CFRP) strips on the dynamic behavior of a GFRP I-beam are investigated. As expected, the stiffening of the system can be demonstrated for low vibration amplitudes. At large amplitudes damping is the dominant effect of the electrolamination of the CFRP elements.

The change in mechanical properties due to the electrolamination of the CFRP elements onto the GFRP base structure measured in the experimental section is used to extrapolate the expected behavior of the GFRP deck of the pedestrian bridge that will
be upgraded to create an adaptive system based on the present work. The main result shown in the last section of this work is that the damping that is obtained through the interaction of the elements of the electrostatically tunable system is expected to have a more beneficial effect in terms of vibration damping than the stiffening of the bridge deck.


Der Beitrag dieser Arbeit zum Gebiet der adaptiven Strukturen besteht aus der Entwicklung und Untersuchung von Strukturen, welche ihr Verhalten dank einer Modifikation ihrer mechanischen Eigenschaften und nicht durch die Einwirkung von Aktuatoren ändern können. Die Änderung der mechanischen Eigenschaften wird durch eine scheinbare Änderung der Topologie der Struktur erzielt.

Zur Verwendung von elektrostatischen Kräften zur Änderung der Wechselwirkung zwischen Schichten einer Mehrschichten-Struktur ist wenig Literatur verfügbar. Dieser be-
grenzte Stand des Wissens zum Thema der Mehrschichten-Strukturen mit variabler Biegesteifigkeit rechtfertigt den umfassenden Umgang in der Diskussion der Materie, der für diese Dissertation gewählt wurde.


Die für die Veränderung der Biegesteifigkeit und des Dämpfungskoeffizienten vorgeschlagene Methode besteht darin, dass man mehrschichtige Strukturen erstellt, deren Grenzflächen jeweils mit einer Elektrode und einem darauf geschichteten Dielektrikum versehen sind. Durch anbringen eines elektrischen Potentials zwischen benachbarten Schichten, werden diese von den entstehenden elektrostatischen Kräften aneinander gekoppelt. Wenn die Kopplung im Vergleich zu den auf die Struktur wirkenden äußeren Kräften ausreichend stark ist, wird sich die Struktur wie eine durchgehende (also nicht mit Grenzflächen versehene) verhalten.


Im experimentellen Teil dieser Arbeit werden zwei Mehrschichten-System untersucht. In beiden Fällen handelt es sich um Sandwich-Balken mit steifen Aussenschichten.

Zum Abschluss wird die im experimentellen Teil dieser Arbeit ermittelte Änderung der mechanischen Eigenschaften einer elektrostatisch koppelbaren GFK-CFK Struktur verwendet, um das erwartete Verhalten des GFK Decks einer Fussgängerbrücke zu extrapolieren. Aus den Schätzungen, die mit Hilfe eines numerischen Modells durchgeführt wurden, geht hervor, die Dämpfung, welche aufgrund der Wechselwirkung an den Grenzflächen zwischen GFK Struktur und CFK Versteifungsselementen zustande kommt, erwartungsgemäß einen stärkeren Einfluss auf das dynamische Verhalten der Brücke haben wird, als die Erhöhung der Steifigkeit und somit der Eigenfrequenzen der Struktur.
List of Symbols

\begin{itemize}
  \item \( C \) capacitance of a parallel plate condenser
  \item \( D \) bending stiffness of a beam
  \item \( \delta \) thickness of a dielectric layer
  \item \( E \) elastic modulus (in mechanical equations), electric field (in electrostatic equations)
  \item \( E_b \) breakdown electric field
  \item \( G \) shear modulus
  \item \( \epsilon_0 \) permittivity of vacuum
  \item \( \epsilon_r \) dielectric constant of a material
  \item \( \epsilon_{xy} \) shear deformation
  \item \( \eta \) damping ratio
  \item \( \lambda \) logarithmic decrement
  \item \( P \) mechanical load
  \item \( Q \) shear force
  \item \( \sigma_{xy} \) shear stress
  \item \( \sigma_{xy}^f \) friction stress at an interface
  \item \( \sigma_{xy}^i \) shear stress in the \( i^{th} \) layer
  \item \( \sigma_{xy}^{int} \) shear stress at an interface
  \item \( \sigma_{yy} \) normal stress in \( y \) direction
  \item \( \sigma_{yy}^{el} \) electrostatic normal stress in \( y \) direction
  \item \( U \) electrostatic potential
  \item \( U_i \) interaminar electrostatic potential
  \item \( v(x) \) transversal displacement at the position \( x \)
  \item \( W \) energy
\end{itemize}
Chapter 1

Introduction

This chapter gives the rationale for the work presented in this dissertation. First, it shows where the contribution of structures with variable stiffness presented in this work is situated in the context of the research on adaptive structures. Furthermore, a brief overview of the state of the art in the field of variable stiffness host structures is presented in section 1.2, with a special attention to the energy requirements that are of paramount importance for most applications of adaptive structures. Finally, an overview of the goals of the present work is given.

1.1 Adaptive Structures

For a long time, engineering structures have been designed to be, ideally, invariant over time. Hence, the global properties of the structure were determined with the goal to satisfy at all times the extreme set of the requirements (marked in red in figure 1.1) determined by the application. This set is determined based on the conditions that are likely to be encountered during the service life of the structure, as shown in figure 1.1. Such requirements, that will be referred to as the design space, may include but not be limited to properties such as high ultimate strength, high stiffness, low weight, and many more. Some properties of the design space (the axes in figure 1.1) may be in conflict with others. The development of low density, high strength materials and composites (e.g. carbon fiber reinforced polymers) is a response to the need to fulfill such conflicting requirements. In the case of the development of lightweight structures with a high strength that are made by the use of high strength materials, the dynamic behavior of the systems represents a limiting factor to their performance.

While covering the complete space of requirements simultaneously is a viable approach
Figure 1.1: A time invariant system is designed to fulfill the envelope space of requirements at all times. The use cases represent different operational conditions that the structure will be exposed to during its operational life. Examples of load cases could be 'take off', 'high altitude cruise' or 'landing', in the case of an aircraft.

for a great number of applications, this is not always feasible and is bound not to provide optimum system properties in all situations. The need for lighter, more energy efficient, better performing structures calls for an optimization of the covered design space. For example, in many civil, automotive and aerospace engineering applications, the design of light, slender structures is necessary to achieve better performance. The use of high strength materials, such as fiber reinforced polymer composites makes it possible to satisfy the strength requirements of many such structures. Lightweight, slender structures tend to be more prone to vibrate with large amplitudes than heavier, massive ones. Furthermore, operating loads have a more marked effect on the vibratory properties of a lightweight system than they would have on heavy structures. This situation makes the implementation of time-invariant vibration mitigating measures more challenging and constitutes a rationale for the development of adaptive systems for the reduction of vibrations.

Adaptive structures are characterized by their ability to react to external stimuli (forces) in a way that allows them to achieve a performance that cannot be achieved using 'passive' materials, i.e. invariant structures.

A simple adaptive system is generally composed of a host structure, an actuator (more or less integrated into the structure) and a control system, as schematically shown in figure 1.2.

Through the action of the actuator, the control system minimizes the deviation of the
actual state of the system from the desired one. The trajectory of the system, i.e. its position as a function of time (thus the mechanical stress in the system) is one of the most common examples of state that is controlled using an actuator. The introduction of a time-dependent (controllable) component to the properties of the system (specifically to its energy level) allows for new degrees of freedom in the design process.

Apart of the continuous development of powerful control systems that allow for the management of increasingly fast and complex processes, the introduction of transducer-based actuators (as opposed to 'traditional' pneumatic, hydraulic and electrical motor based actuators, also referred to as geometric actuators) have given a strong stimulus to the development of smart structures.

### 1.1.1 Material Based Actuators

Transducer materials are materials that exhibit the ability to transform energy from one physical form to another by virtue of specific coupling mechanisms [5, 41] and are at the core of transducer material based actuators. More generalized definitions include the transformation of energy also within a domain [10]. Figure 1.3 outlines the best known multi-physical interactions and gives examples of materials and systems that exploit such couplings. Materials that are used to obtain specific effects thanks to the coupling between different physical domains (e.g. mechanical and electrical, thermal and electrical, etc.) are generally defined as 'smart', 'active' or 'adaptive' materials. Definitions of most of these terms are found in [41] and will be adopted for this work.

**actuator** An actuator can be seen as a system that establishes a flow of energy between an input (electrical) port and an output (mechanical) port. The actuator is
transducing some sort of input power into mechanical power. [41]

**active** In active actuators the work exchange can take any positive or negative value, $dW \geq 0$. For practical purposes, this means that active actuators can either increase or decrease the energy level of the controlled system. [41]

**semi-active** In semiactive actuators the work exchange can only be negative, $dW \leq 0$. In practice, this means that semiactive actuators can only dissipate energy as a consequence of mechanical interaction with the controlled system. [41]

**smart** A smart structure is comprised of transducers that are used both for sensing and actuating, thus allowing for the concomitant implementation of two of the functions needed for adaptive structures.

**adaptive** The term adaptive refers to structures and systems that are able to change their properties or behavior in order to adjust themselves to new conditions. Structures or systems comprising actuators as the ones listed above and control systems can be expected to be adaptive.

**passive adaptive** The concept of passive adaptive structure applies to non-actuated structures ($dW = 0$ or $dW \approx 0$) that can nevertheless adjust their behavior to new conditions.

In the case of mechanical smart structures, the output required from a transducer is generally in the mechanical domain and can be expressed in the form of the power conjugate variable force and velocity or torque and angular velocity [10]. Table 1.1 shows a comparison between transducer material based actuators and traditional actuators. The use of actuators in a structure implies the presence of an appropriate source of energy. While the nature and properties of such a source are not the subject of this work, the need to minimize the energy requirements of the ‘active’ components of an adaptive structures is a parameter of paramount importance for any application.

The most noticeable difference between traditional and transduction based actuators is that in the latter, the transformation of energy between domains takes place at the material level. In many cases, this leads to a better scalability in the direction of smaller systems. If the coupling between physical domains is bidirectional, the same device can be used as both a sensor and an actuator. The use of transducer materials for sensing and power generation thanks to this fact has also been reported, [4, 18, 29, 42].

The properties that characterize materials-based actuators make new applications that could not be realized using traditional actuators more practicable or even possible. A
short list of such applications includes among others the propulsion of vehicles [12], pumps [51], active prosthetics [6, 27], prestressing of civil engineering structures and building materials [28, 39], the active suppression of vibrations [2, 44–46], shape change [15, 36].

Some of the main advantages claimed for the use of active materials based actuators instead of more conventional electric or hydraulic actuators include lower weight and better integrability. In perspective given the lower level of mechanical complexity of transducer material based actuators, one could expect that eventually a high level of reliability should be achieved.
Table 1.1: Comparison of traditional vs. material based actuators (adapted from [41])

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<tr>
<th>Traditional actuators</th>
<th>Transducer material based actuators</th>
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<tr>
<td>• Based on geometrical transducers(^\d)</td>
<td>• Based on transducing materials (possibly in combination with geometrical concept)</td>
</tr>
<tr>
<td>• Off-the-shelf availability</td>
<td>• Designed for the application</td>
</tr>
<tr>
<td>• Good performance at normal scale</td>
<td>• Good for meeting miniaturization demands</td>
</tr>
<tr>
<td>• Lump ed approach: discrete components in motion control systems. Used in combination with external sensors</td>
<td>• Integrated and embedded approach: open to smart structure concepts. Pursuit of the smart actuator concept</td>
</tr>
<tr>
<td>• Conventional mechanical transmissions for (output) impedance matching</td>
<td>• New transmission designs based on hinges and friction</td>
</tr>
</tbody>
</table>

\(^\d\) Geometrical transducers are devices that owe their ability to transform energy from one domain to another to a geometrical feature. For example, a DC motor owes its ability to transform electrical energy into rotational mechanical energy to the geometry of the interaction between current and magnetic fields.

1.1.2 Host Structure

A considerable effort is put in the development of novel transducer materials and actuator systems for adaptive structures applications. Some of the couplings in figure 1.3 (marked in red) imply the modification of mechanical properties instead of transformation of energy from a non-mechanical domain into mechanical work and exhibit thus a passive adaptive or semi-active behavior. These effects are substantially different from the ones seen in active materials and can be used to develop passive adaptive host structures. This less intensively investigated aspect of adaptive structures is nevertheless a promising approach to the extension of the functionality of adaptive system that include actuators as well as for the development of purely passive adaptive systems.

In actuated systems, the control over the stiffness (i.e. the mechanical impedance) of the host structure allows for an improved efficiency in the use of the actuators, as schematically shown in figure 1.4. Only the selection of an appropriate transmission ratio makes it possible to lift the weight attached to the pulley. If the ratio is too high for a given actuator (e.g. an electrical motor), the energy will not flow into the mechanical system but rather be dissipated in form of heat in the actuator. Similarly, if the stiffness of a structure is too high, actuators integrated in the structure will not be able to carry
out their task in an efficient manner.

The schematic representation of an actuated system including a host structure with tunable mechanical impedance equivalent to figure 1.2 is thus extended as shown in figure 1.5.

Actuated structures with tunable mechanical impedance can be used for novel applications, such as the aeroelastically assisted shape change of structures [11]. A comprehensive presentation of the use of tunable stiffness elements in adaptive structures can be found in [20, 35]. The modification of the stiffness of a structure as a method to its morphing is presented in [25].

Also non-actuated systems can draw benefits from the adaptation of the host structure (see figure 1.6) as a means to exert control over the flow of energy from the environment (external perturbations) into the system. The primary goal is then not to change the energy level of the system \((dW \approx 0)\) but rather to change its mechanical impedance. Depending on the method chosen for the modification of the stiffness, a change in the energy level may be a more or less marked side effect. A few examples of structures with variable mechanical properties can be found in literature. One example of such work is found in the development of joints with variable friction parameters reported in [22–24] where the conditions at the boundaries of beam elements are modified by modulating the friction in the joints between elements. Also the modification of the stiffness of structures has been reported [17, 21, 31].

The coupling of damping devices to a structure such as a cantilever beam or the use of active systems currently represent the methods of choice for the purpose of reducing

![Figure 1.4: Mechanical impedance matching: (a) the actuator heats up as power is not transferred to the load and (b) the power can be transmitted to the load after pulley's impedance is matched to the actuator. [41].](image-url)
Figure 1.5: Schematic representation of an adaptive mechanical system with an extended set of components: structure, structure controller, actuator and actuator controller.

Figure 1.6: Schematic representation of a passive adaptive mechanical system consisting of structure and structure controller

the amplitude of its vibrations.

The modification of the mechanical properties of structural elements, such as their bending stiffness, can be of special interest for vibration mitigation applications. By modifying the bending stiffness of a structural element it is possible to influence its vi-
brary properties, such as the eigenfrequencies. This opens the way to application
of methods for the mitigation of vibrations by suppressing the adsorption of energy at
given frequencies.

The observation that high-amplitude vibrations are often caused by resonance phenom-
ena leads to the conclusion that, while the spectrum of the exciting forces can generally
not be influenced, it is in principle possible to shift the natural frequencies of a system,
so as to avoid the absorption of the high energy components of the exciting spectrum.
While this idea is almost trivial, its realization may be faced with serious technical diffi-
culties.
The vibratory properties of a body depend on its mass and its mechanical properties.
In the case of the bending natural frequencies of a beam, the eigenfrequencies are
described by:

$$\omega_k = \beta_k^2 \cdot \sqrt{\frac{D}{\rho}}$$  \hspace{1cm} (1.1)

where $\omega_k$ is the $k^{th}$ eigenfrequency of the system, $\beta_k$ is the $k^{th}$ solution of the equation
describing the vibration of the system, $D$ is the stiffness of the system and $\rho$ is its
density [52].

For the bending vibrations of a simple beam, the relevant mechanical property is the
bending stiffness $D$. Two parameters determine the bending stiffness of a structural
element, as shown in (1.2):

$$D = EI_z$$  \hspace{1cm} (1.2)

Where $E$ is the elastic modulus of the material the structure is made of and $I_z$ is the
second moment of area of the structure, a geometric property of the structure. Since
the mass of the system is generally expected to be invariant, the vibratory properties
will be tuned by modifying of the stiffness of the system.
1.2 Approaches to the Modification of the Mechanical Properties of the Host Structure, an Overview of the State of the Art

In the case of structural elements made of one bulk material, such as the simple cantilever beam shown in figure 1.7, a modification of the bending stiffness of the system can be achieved either by a modification of the elastic properties of the constituting material or by a change of the geometry of the system.

Different approaches to the modification of the bending stiffness of simple structural elements for passive adaptive structures will be presented and briefly discussed using hypothetical examples and examples from literature.

As outlined in section 1.1, adaptive structures are chosen over time invariant structures when the latter cannot fulfill the requirements set by a given number of use cases. Weight is often one of the most stringent constraints, so that energy storage and thus energy supply are likely to be among the critical factors that need to be addressed in the development of adaptive structures.

In the considerations made in this section, we estimated the energy demand needed to implement the solutions outlined in section 1.2.1 and 1.2.2. The energy to obtain a

Figure 1.7: Simple cantilever beam with cross-section $b \cdot h$ subject to a load $F$ at a distance $l$ from the origin.
certain increase in stiffness by modifying the elastic modulus or the second moment of area is compared to the energy input needed to obtain the maximum possible stiffness change in a cantilever beam using different methods.

The experiments are thought in comparison with a glass fiber reinforced polymer (GFRP) I-beam used for the experimental work presented in section 4.3 and shown in figure 4.7. The beams were thought to have the same initial bending stiffness as the GFRP beam. The energy needed to achieve the maximum stiffness that can be realized for the described stiffening methods was calculated. According to the manufacturer, the bending stiffness of the reference GFRP beam is $D = 71.3 \cdot 10^9 \text{Nmm}^2$ [16].

While the cases presented in this section are purely hypothetical and do not necessarily represent engineering grade solutions to the problem of modifying the bending stiffness of a structural element, they are meant to show the difficulty of developing energy efficient passive adaptive host structures for adaptive systems.

1.2.1 Modification of the Elastic Modulus

At a fundamental level, the elastic properties of solids are governed by the electrostatic attraction and repulsion forces between atoms of a material. The shape of the potentials that determine the elastic properties of the material is mainly determined by the nature of the bond (metallic, covalent, ionic etc.), the geometric distribution and the electronic properties of the atoms [33]. For most engineering materials, the options for a substantial modification of their elastic properties are extremely limited.

Significant changes in the elastic properties of solids are known mainly for certain solid-solid phase transformations, such as the $\gamma \leftrightarrow \alpha'$ phase transformation in NiTi shape memory alloys. In this case, the high temperature (body centered cubic, bcc) $\gamma$ phase has an E-modulus of approximately 80 GPa, compared to 35 GPa for the low temperature (face centered cubic, fcc) $\alpha'$ phase [7]. The noticeable change in elastic properties in this case is not due to changes in the interaction potentials at an atomic level but rather to the activation of twinning/de-twinning mechanisms in the fcc phase that are not available in the bcc phase.

Also well known is a marked reduction of the shear modulus $G$ of elastomers and partially crystalline thermoplastic polymers in correspondence with an increase of the temperature above the glass transition temperature $T_g$. Typical changes in shear modulus $G$ due to this second order transformation are of the order of 10 to 100 times. In both cases, the change in material properties is due to temperature driven processes and is thus subject to the laws of heat transfer.
**Energy Considerations**

In the case of the $\gamma \leftrightarrow \alpha'$ phase transformation, the energy necessary to achieve a stiffness increase of the order of three times can be estimated as follows:

$$W = W_{th} + W_{pt} = m \cdot \Delta T \cdot c_p + \Delta G_{pt}$$  \hspace{1cm} (1.3)

where $W_{th}$ is the thermal energy needed to heat the material to the phase transformation temperature, $W_{pt}$ is the phase transformation Enthalpy $\Delta G_{pt}$ for $\gamma \rightarrow \alpha'$, $m$ is the amount of material to be heated and to undergo the phase transformation, $T$ is the temperature difference between the initial temperature (e.g. room temperature) and the phase transformation temperature, $c_p$ and $\Delta G_{pt}$ are the specific heat capacity and the phase transformation free enthalpy, respectively. Considering that $c_p$ is approximately 0.5 J/g·K [7] for a $\Delta T$ of 60 K, $W_{th}$ is of the order of 30 J/g. Differential scanning calorimetry measurements show that $W_{pt}$ is also of the same order of magnitude (approximately 20 J/g) [7]. Assuming a total energy requirement of approximately 50 J/g, the change in stiffness is energetically quite expensive and, given the nature of the phenomenon, it is bound to be limited in speed by heat transfer processes.

A NiTi beam with an I cross-section and (54 mm wide, 108 mm tall, with a thickness $d=5.4$ mm, as shown in Figure 1.9) has a cross sectional area of approximately $113.7 \cdot 10^3$ mm$^2$ and a bending stiffness $D$ of approximately $71.3 \cdot 10^9$ Nmm$^2$, in its low temperature phase $\alpha'$. The volume of such a 2500 mm long beam is $2.843 \cdot 10^6$ mm$^3$ and the weight is approximately 18'340 g. The total energy needed to attain the stiffening of the beam by a factor of approximately 2.8 thanks to the $\alpha' \rightarrow \gamma$ phase transformation is thus of the order of 900 kJ.

In the examples outlined in this section, the change in elastic modulus takes place in connection with a thermodynamic first or second order transition. The occurrence of such phase transformations is thus mainly temperature controlled, although especially in the case of the $\gamma \leftrightarrow \alpha'$ phase transformation in NiTi alloys, mechanical stress also has an influence. Heat transfer mechanisms limit the rate at which such desired temperature driven changes in elastic properties can take place and thus the frequency response of the system.
1.2.2 Modification of the Moments of Area

Shape Change

For a simple, solid, cantilever beam with a rectangular cross-section, the bending stiffness is given by:

\[
D = E \cdot I_z = E \cdot \frac{bh^3}{12}
\]  

(1.4)

Where \( E \) is the elastic modulus of the material and \( I_z \) is the second moment of area of a rectangular cross-section like the one shown in figure 1.7, where \( b \) and \( h \) are the width and thickness of the cantilever, respectively.

Given an elastic modulus \( E \) for the beam material, the geometry of the cross-section of a cantilever has a significant influence on its bending stiffness, which has been extensively investigated.

The second moment of area \( I_z \) in 1.4 is calculated for an arbitrary cross-sectional ge-
ometry as:

\[ I_z = \int_A y^2 dS \]  \hspace{1cm} (1.5)

with \(dS = dx \cdot dy\) and the co-ordinate system with its origin in the center of gravity of the cross-section, as depicted in figure 1.7. For a given cross-sectional area, an appropriate selection of the geometry yields an increase in the stiffness of the beam. So, for the cross-sections shown in figure 1.8 we will obtain using (1.5) that \(I_{z,1} < I_{z,2} < I_{z,3}\), while \(A_1 = A_2 = A_3 = a^2\):

\[
I_{z,1} = \frac{a^4}{4\pi} \approx \frac{a^4}{12.5} \\
I_{z,2} = \frac{a^4}{3} \\
I_{z,3} = \frac{5.824a^4}{6} \approx a^4
\]  \hspace{1cm} (1.6)

A modification of the cross-sectional shape from cross-section 1 to cross-section 3 yields an increase in bending stiffness by a factor of approximately 12.5. This circumstance is well known and widely used to optimize the stiffness of structures while maintaining their mass constant. Nevertheless the development of a cantilever beam with variable stiffness based on the morphing of the geometry from cross-section 1 to cross-section 3 presents a number of practical difficulties if the structure is made of a commonly used engineering material.

More realistically, a fairly moderate geometry change can be achieved by modifying the height of the web of an I-beam:

Similarly to (1.6), (1.8) shows the effect of a modification of the geometry on the second moment of area of the I-beam shown in figure 1.9:

\[
A_{undeformed} = A_{deformed} \\
I_{z,undeformed} = 2bd\left(\frac{d + h_{web}}{2}\right)^2 + \frac{dh_{web}^3}{12} < I_{z,deformed} = 2bd\left(\frac{d + kh_{web}}{2}\right)^2 + \frac{dk^2h_{web}^3}{12}
\]  \hspace{1cm} (1.7)

The increase of the web height by a factor \(k\) leads to an increase in bending stiffness, as shown in (1.8). Keeping the volume of the web constant, the width of the web will then be reduced.

If the thickness of the flange is significantly smaller than the height of the web, the effect can be considered of the order of \(k^2\) for both terms of the right hand side of (1.8). The remarkable material properties of NiTi alloys allow for large (up to approximately 7%)
Figure 1.9: Effect of the modification of the cross section geometry on the second moment of area of a beam. A comparison of the areas and second moment of areas for the different cross sections is given in (1.7) and (1.8).

Superelastic (i.e. reversible) deformations, if suitable forces are applied to the flanges. Assuming $k \approx 1.07$, an increase of the stiffness of the order of 15% can be obtained. [8, 31] report about a method to modify the stiffness of a beam subject to bending by modifying the geometry of the system, in order to create a tunable vibration absorber.

Energy Considerations

Figure 1.10 shows the stress-strain diagram of a suitably selected alloy. The energy density needed for the superelastic deformation is approximately 17 MPa, based on the area under the stress strain curve. The volume of the web is approximately $1.31 \cdot 10^6$ mm$^3$ (97 mm x 5.4 mm x 2500 mm). The total energy needed for the geometry modification is given by the area under the stress-strain curve shown in Figure 1.10 is then approximately 22 kJ. The obtained increase in bending stiffness is of the order of 14.5%.
Figure 1.10: Stress strain curve for a NiTi alloy adapted from [7], after [43]

**Topology Modification**

Considering the properties of multi-layer composite structures reveals immediately that not only the overall geometry (shape) of a structure but also its topology has a substantial influence on its elastic properties.

The starting point of an example to illustrate the effect of topology change shall be again an I-beam. In its low stiffness configuration, the beam is divided in three layers (the two flanges and the web), as shown in the left hand side of figure 1.11.

As long as no interaction between adjacent layers is allowed, the bending stiffness of the described system can be written as the sum of the stiffness of the components:

\[
D_{\text{disconnected}} = E \cdot \left( \frac{b \cdot d^3}{12} + \frac{d \cdot h_{\text{web}}^3}{12} + \frac{b \cdot d^3}{12} \right)
\]  

(1.9)

with \(b\), \(d\), \(h\) and \(h_{\text{web}}\) as shown in figure 1.11.

Assuming the proportions between the measurements of the beam to be the same as
Figure 1.11: The modification of the topology of an I-beam causes a remarkable increase of the stiffness.

for the one used in section 4.3:

\[ d : b : h : h_{\text{web}} = 1 : 10 : 18 : 20 \]  \hspace{1cm} (1.10)

(1.9) becomes

\[ D_{\text{disconnected}} = E \left( \frac{5}{6}d + 486d + \frac{5}{6}d \right) = E \cdot 487.7 \cdot d^4 \]  \hspace{1cm} (1.11)

Once the components are connected at the interfaces (highlighted in red in figure 1.11), the contribution of the flanges becomes.

\[ D_{\text{flanges}} = E \cdot d \cdot b \cdot \left( \frac{h_{\text{web}}}{2} \right)^2 = E \cdot 1805d^4 \]  \hspace{1cm} (1.12)

Thus the stiffness of the connected I-beam (on the righthand side of figure 1.11) is:

\[ D_{\text{connected}} = E \cdot 2291d^4 \]  \hspace{1cm} (1.13)

The ratio of the stiffness of the beam in the two states (disconnected and connected) is:

\[ \frac{D_{\text{connected}}}{D_{\text{disconnected}}} = \frac{2291}{487.7} \approx 4.7 \]  \hspace{1cm} (1.14)

The difference between the two described cases is given by the ability of the interfaces between layers of a laminated beam to transmit shear stresses from one layer to the next.
The effect of topology on the stiffness of mechanical systems has already been reported in [49]. Also, reports of an equivalent system to what previously described by the authors can be found in [48], although the observed modification of the behavior of the system is attributed entirely to dissipative friction processes, rather than the modification of the topology of the system, from a mechanical point of view. The results reported in [3] indicate that the effects obtained by coupling layers of a beam exceed the dissipative effects in [48].

**Energy Considerations**

The modification of the topology of a multi-layer system is achieved by creating or canceling the interfaces between layers of the system. The creation of new interfaces requires energy, while the cancellation is an exothermal process. The order of magnitude of the energy needed for or released by the topology switching process equals, in theory, the surface energy of the material (e.g. 1.5-2 Jm$^{-2}$ for transition metals like Iron or Titanium, [1]). The thickness $d$ of the beam shown in figure 1.11 can be calculated for the low stiffness state using (1.9). In the case of a steel beam ($E = 210 GPa$) with the proportions discussed in section 1.2.2 and the aforementioned stiffness of $71.3 \cdot 10^9$ Nmm$^2$ the thickness $d$ will be 5.13 mm.

The energy needed or released for a connectivity change is then given by:

$$E_{connectivity} = 2 \cdot A \cdot e_{surf} = 4 \cdot 2.5m \cdot 0.00513m \cdot 1.5Jm^{-2} = 0.077J$$

(1.15)

Where $e_{surf}$ is the surface energy of the material, $A$ is the surface area of the interfaces that are created or canceled.

Based on this estimate, only approximately 0.08 J would be needed to create new surfaces and the same amount of energy would be released upon their cancellation. A comparison with other methods for the modification of the mechanical properties of structural elements, such as the ones outlined in this section shows that in theory the topology switch promises a very high pay-off in terms of amplitude of the stiffness variation in comparison with the energy needed to obtain it.

In reality, only upon cleavage of the material, new surfaces can be created at so little energy expense. Similarly, the cancellation of surfaces is in reality a process that requires a considerable amount of energy due to the need to provide the activation energy necessary to bond the surfaces. The energy necessary to bond metallic structural elements is either provided in form of thermal energy (in the case of welding soldering or brazing) or in form of chemical energy, if adhesive bonding is used. The separation of the bonded layers is technically achieved by machining the structure or by other high energy mechanical or thermal processes. These methods for the creation or cancella-
tion of interfaces are not compatible with the goal to adaptively modify the bending or torsional stiffness of a structure.

1.2.3 Summary

The modification of the mechanical properties of the host structure offers an additional degree of freedom in the design of adaptive systems. Structures with tunable mechanical properties can be used as such for vibration suppression applications or in combination with actuators for enhanced shape control applications.

The previous sections have described possible ways to obtain a modification of the bending stiffness of a simple structure.

Adaptive structures find often their application in systems where performance and weight are of paramount importance. The energy demand to obtain a given effect (in this case a modification of the stiffness) plays thus a central role in the assessment of the overall effectiveness of a new system.

Table 1.2 gives an overview of the two main parameters that were calculated for the presented examples: The increase in stiffness that can be achieved with each method and the energy needed to achieve it, under ideal circumstances.

The parameters listed in the table clearly show the potential of topology switching as a low energy approach to the modification of the bending of structural elements. Given its sensitivity to the distribution of the cross-sectional area in space, the torsional stiffness

<table>
<thead>
<tr>
<th>Method</th>
<th>Obtained increase in stiffness</th>
<th>Required energy [kJ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modification of the elastic modulus of NiTi (section 1.2.1)</td>
<td>2.7</td>
<td>900</td>
</tr>
<tr>
<td>Modification of the height of the web of a NiTi beam (section 1.2.2)</td>
<td>1.15</td>
<td>22</td>
</tr>
<tr>
<td>Modification of the topology of a steel beam (section 1.2.2)</td>
<td>4.7</td>
<td>80 * 10^{-6}</td>
</tr>
</tbody>
</table>

Table 1.2: Comparison of energy demand of the presented stiffening strategies
of the structure can be modified according to the same principle, but is not discussed here. The axial stiffness of structural elements cannot be modified using approaches that leverage on a change of the area distribution, since this property only depends on the cross-sectional area.

### 1.3 Goals and Structure of the Present Work

The primary goal of the present work is to present a method to realize the modification of the bending stiffness of simple adaptive host structures based on the topology switch approach.

The challenge of this work is to overcome the difficulties posed by the energy requirements set by the creation and cancellation of the interfaces within the system.

A method for the implementation of the stiffness modification equivalent to topology switching will be presented in chapter 2.

In order to appreciate the limits and possibilities of the method devised for the modification of the bending stiffness, a quantitative understanding of the mechanisms that govern it is necessary. The use of the analytical and numerical models presented in chapter 3 allows for such understanding without the experimental difficulties posed by the use of physical models.

The experiments described in chapter 4 will substantiate most of the aspects addressed in the two previous, with special attention for the demonstration of the modification of the vibrational properties of simple structures, and highlight the technical problems encountered in the implementation of the electrostatic modification of the bending stiffness.

An example of the application of multi-layer structures for vibration suppression will be presented in chapter 5.

Finally, the meaning of the present work will be summarized and discussed in chapter 6.
Chapter 2

Working Principle of the Electrostatic Tuning of the Stiffness

While true topology switching (i.e. the creation or cancellation of interfaces) cannot be realized without the use of large amounts of energy per unit interface area, the modulation of the contact stress at the interfaces, and thus of the shear stress transfer by means of friction, can yield an equivalent effect. In this chapter similarities between the shear stress transfer at the atomic level and at the meso-level are outlined at first. Then, the chapter shows how the application of electrostatic fields across the interfaces of a multi-layer structure will be used to mimic the creation and cancellation of interfaces.

2.1 Connectivity, Interfaces and Shear Stress Transfer

The previous section outlined how a change in the connectivity (or topology) of a multi-layer system can lead to a remarkably high change in mechanical properties for a nominally low energy input. In the context of this work, the term connectivity (of areas) refers to the presence or absence of interfaces within a cross section. ¹ In reality, the energy needed to divide a structure in many layers or join the layers of a structure to a monolith is much greater than the surface energy of the material the structure is made of. The main obstacle to the use of topology switching for the modification of the mechanical properties of a structural element is thus that there is no simple method to realize it while fulfilling following requirements:

¹The connectedness of a space is a concept of topology. In this context the terms ‘connectivity’ and ‘topology’ of a section will be used interchangeably.
• Limited energy requirements
• Small weight penalty
• Short reaction time

These requirements are central to the application of adaptive systems in high performance structures, as outlined in chapter 1.

In this chapter we will examine in more detail how the connectivity of a simple multi-layer system influences its mechanical properties. An alternative approach to a true topology switch for the modification of the properties of a structure will be presented.

2.2 Shear Stress Transfer and Stiffness

A closer look at the shear stress distribution in a multi-layer system shows that a change in connectivity of the system essentially influences the transmission of shear stresses at the interfaces between layers, as shown in figure 2.1.

Figure 2.1: Effect of connectivity switching on the shear-stress distribution in the cross-section of a cantilever beam
In a multi-layer beam subjected to a bending load, as in the top drawing of figure 2.1, the shear stress distribution is expected to have a zero value at the interfaces of the layers of the beam i.e. at positions $y = i \cdot h$ with $i = 1, 2, \ldots, n$, as detailed by the $\sigma_{xy}$ distribution. In the solid beam shown in the bottom figure, the $\sigma_{xy}$ distribution reaches a zero value only at $y = \pm \frac{n}{2} \cdot h$, i.e. at the outer faces. The difference between the two distributions accounts for the different bending behaviors of the two systems. It should be noticed that if subjected to a tensile stress in the x direction, the global behavior of the two systems will be the same, independent of the number and position of the interfaces in the system.

2.3 Shear Stress Transfer in Crystalline solids

The ability to transfer shear stresses across planes of a crystalline body is at the origin of the shear stiffness of materials such as metals, as shown in figure 2.2.

In order to induce a shear deformation $\epsilon_{xy}$ in a crystalline structure by applying a shear stress $\sigma_{xy}$, the crystal planes have to be shifted along one another as shown in the sequence (a) $\rightarrow$ (b) $\rightarrow$ (c) representing the idealized plastic deformation process by one lattice constant $d$ per plane. In order to do so, the distance $d$ of the planes will be increased up to $d'$. The periodic potential shown in the graphic under (b) is given by the electrostatic interaction between the atoms of the upper plane and the ones of the lower plane. The system reaches its maximum energy level when the relative position of neighboring planes is as shown in (b).

Small elastic shear deformations take place in proximity of the equilibrium state (a). For these deformations, the stress is known to be approximately proportional to the deformation, as indicated by the red line in the graphic under (b) [33]. Hence, the stress-deformation relationship for small deformations is linear:

$$\sigma_{xy} = G \cdot \epsilon_{xy}$$

(2.1)

Once the distance between the two planes is sufficiently large (D, in (d)), due to the creation of new surfaces within the material, the electrostatic interaction between the atoms on originally contiguous planes becomes so small (as indicated by the horizontal red line in the graphic under (e)) that no shear stress is needed in order to move the planes past one another.

\footnote{2In reality the carriers of plastic deformation are dislocations that are activated at significantly lower loads than would be needed for the homogeneous plastic shearing of the crystal.}
Figure 2.2: The resistance to shear of crystalline solids is given by the energy difference between the equilibrium state (a) and (c) and the state of maximum deviation of the crystal lattice parameter $d'$ (b). Adapted from [33]

2.4 Creation and Cancellation of Interfaces

The amount of energy needed to create new surfaces is in theory very modest [1], as shown in table 2.1. The reverse process, i.e. the joining of two surfaces is then an exothermal process.

Table 2.1: Surface energy of some metals used in engineering applications [1]

<table>
<thead>
<tr>
<th>Metal</th>
<th>Atomic Number</th>
<th>Surface Energy Calculated, $\sigma_c$ [J/m$^2$]</th>
<th>Surface Energy Measured, $\sigma_{exp}$ [J/m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titanium</td>
<td>22</td>
<td>1.79</td>
<td>1.44</td>
</tr>
<tr>
<td>Chromium</td>
<td>24</td>
<td>1.93</td>
<td>1.59</td>
</tr>
<tr>
<td>Iron</td>
<td>26</td>
<td>2.22</td>
<td>-</td>
</tr>
<tr>
<td>Nickel</td>
<td>28</td>
<td>2.35</td>
<td>1.44</td>
</tr>
<tr>
<td>Copper</td>
<td>29</td>
<td>1.66</td>
<td>1.12</td>
</tr>
</tbody>
</table>
Nevertheless, it is known that in reality much higher amounts of energy are involved in the switch from the connected to the disconnected state. Also, experience shows that although the endothermic process in which surfaces are created is in principle reversible, energy is needed both for dividing and for joining a body.

Observation of the geometric properties of real solid surfaces shown in figures 2.3 and 2.4 yields an insight into the reasons for the inapplicability of topology switching to engineering grade surfaces. The presence of surface defects hinders the contact between surface atoms [47] even on the smooth surfaces of cleaved single crystals.

![Figure 2.3: At an atomic level, even the smoothest real surfaces present terraces, ad-atoms, kinks, and other surface defects that make the real contact surface smaller than shown in the idealization of figure 2.2. Adapted from [47]](image)

Furthermore, almost every surface is known to be rough also on a larger scale than the one shown in figure 2.3. This means that most parts of the surface are not flat but form peaks or valleys. For engineering grade surfaces, the typical amplitude between peaks and valleys is of the order of about $1 \, \mu \text{m}$. Under normal conditions, the profile of the surface is essentially random, irrespective of its source and scale of size, as shown in figure 2.4. Hence, when two solids contact each other, the actual contact surface is substantially smaller than the apparent contact surface [47]. For common engineering materials, the cancellation of an interface cannot be achieved by contact between the surfaces.

The processes involved in the creation of new surfaces are generally more complex than the mere cleavage of a single crystal, as they generally involve plastic deformation of the material. This leads to a much higher energetic expenditure than needed just for the creation of new surfaces. Generally speaking, a high fracture energy is a desirable property of engineering materials, since the integrity of structures depends on it.

For the reasons outlined in this section, topology switching cannot be considered for the stiffness tuning of engineering structures in real applications.
2.5 Friction at Real Interfaces and Shear Stress Transfer

The contact interaction between solids is substantially different from the interaction between crystal planes moving relative to each other. The geometry of the surfaces that are in contact is random, thus no periodic potential governs the relative displacement between the surfaces. The actual contact area is generally so small that the electrostatic interactions between atoms of the two surfaces are not sufficient to make any significant contribution to the transfer of shear stresses across the interface.

The contact between solid surfaces has been studied intensively at different levels of detail, ranging from empirical observation to modeling of the processes at contact surfaces.

From a phenomenological point of view, it is known that when a normal force $N$ is applied to two solid surfaces that are in contact (see figure 2.5), friction between the surfaces is experienced. Friction is the force $F_f$ that is opposed to the relative motion of the surfaces in the direction imposed by an external force $F$.

Four basic empirical laws of friction have been know for a long time [47]:

1. There is a proportionality between the maximum tangential force before sliding and the normal force when a static body is subjected to increasing tangential load.
2. The tangential friction force is proportional to the normal force in sliding

3. Friction force is independent of the apparent contact area

4. Friction is independent of the sliding speed

If the forces shown in figure 2.5 are averaged over the apparent area of contact, points 1. and 2. can be written as:

$$\sigma_{xy}^f = \mu \cdot \sigma_{yy}$$  \hspace{1cm} (2.2)

Where $\sigma_{xy}^f$ is the average shear (friction) stress at the contact surfaces and $\sigma_{yy}$ is the average normal stress at the contact surfaces.

The development of structures with tunable bending stiffness proposed in this work is based on the assumption that for the purpose of calculating the bending stiffness of a structural element, the connectivity of the element can be stated in terms of the ability to transfer shear stresses across its cross-section independently of the mechanisms involved in the process.

In this work, the shear stress transfer functionality that in a solid body is guaranteed within the limits set by the strength of the material is replaced at selected locations.
by shear stress transfer through friction between solid surfaces. Up to the limit set by
the static friction between the surfaces, the shear stress in the cross section of the
component (e.g. a beam) subjected to bending forces can be transferred in full. The
governing parameters are the normal stress and the friction coefficient at the interface.
The application of normal forces at the interfaces of a multi-layer structure can be
achieved in various ways and can be modulated at a fairly high speed. The first two
requirements stated at the beginning of this chapter (low weight penalty and limited
energy consumption) need to be addressed by a suitable choice of the method used to
apply the needed force, as will be described in section 2.6.

2.6 Electrostatic control of the Shear Stress Transfer

For the purpose of the adaptive modification of the bending stiffness, we have seen
that a sufficiently large normal stress needs to be applied at the interfaces between
the layers of a multi-layer structure, in order to enable shear stress transfer at those
locations.

The application of an electrical field between the contact surfaces of adjacent layers is a
practical way to generate the normal stresses needed for this purpose. In order to apply
an electrical field, the two interfaces have to be electrically insulated by a dielectric layer,
as shown in figure 2.6.

![Image of electrostatic field](image)

Figure 2.6: The application of an electrostatic field between surfaces of the layers of
a multi-layer structure generates normal stress at the interface. Here, the layers are
assumed to be non-conductive, hence an electrode bonded to the layer is necessary at
the interface.
The use of strong electrostatic fields to generate normal stresses across thin dielectric layer is known in the field of Dielectric Elastomer Actuators (DEA). One difference between the system presented in this work and DEAs is that the dielectrics in DEAs are highly compliant in order to allow for the deformation of the system that leads to the actuation process. The electrodes have thus also to be compliant in order to fulfill compatibility requirements with the deformation of the dielectric. [37,38]

The normal stress \( \sigma_{yy}^{el} \) generated by the electrostatic potential \( U \) across the dielectric is:

\[
\sigma_{yy}^{el} = \frac{\epsilon_0 \epsilon_r U^2}{2\delta^2}
\]  
(2.3)

Where \( \epsilon_0 \) is the permittivity of vacuum, \( \epsilon_r \) is the dielectric constant of the dielectric layer, \( U \) is the applied potential, and \( \delta \) is the thickness of the dielectric layer. (2.3) shows that the stress increases proportionally to \( 1/2\delta^2 \), which indicates that thin dielectric layers are highly desirable.

When multiple dielectric materials fill the space between the electrodes of a capacitor, the system can be described as a series of capacitances, each with the thickness \( \delta_i \) and the dielectric constant \( \epsilon_r^i \). The effective dielectric constant of the system is calculated via the calculation of the capacitance of the system:

\[
\frac{1}{C_{tot}} = \sum_i \frac{1}{C_i}
\]
\[
\frac{1}{C_{tot}} = \sum_i \frac{\delta_i}{\epsilon_0 \epsilon_r^i A}
\]  
(2.4)

From (2.4) and the total thickness of the space between the electrodes, the equivalent dielectric constant \( \epsilon_r \) of the system can be obtained:

\[
\epsilon_r = \frac{\frac{\sum_i \delta_i}{\epsilon_0 A}}{\frac{\sum_i \delta_i}{\epsilon_r}}
\]
\[
\epsilon_r = \frac{\sum_i \delta_i}{\sum_i \frac{\delta_i}{\epsilon_r}}
\]  
(2.5)

The stress-potential relationship in a system with multiple dielectric layers can then be calculated substituting \( \epsilon_r \) from (2.5) into (2.3).
The use of very thin dielectric layers is limited by the breakdown field $E_b$, a physical property of the dielectric material indicating the maximum field strength above which the insulating properties of the material are lost. For a material with a breakdown field $E_b$ the minimum thickness of the dielectric layer is given by:

$$\delta_{\text{min}}(U) = \frac{U}{E_b} \quad (2.6)$$

Where $U$ is the potential across between the electrodes.

Together with the dielectric constant, $E_b$ determines the maximum normal stress that can be generated across the insulator:

$$\sigma_{y,y,\text{max}}^{\text{el}} = \frac{\epsilon_0 \epsilon_r U^2}{2\delta_{\text{min}}^2} = \frac{\epsilon_0 \epsilon_r E_b^2}{2} \quad (2.7)$$

It is noteworthy that (2.7) has the same form as the equation for the specific energy stored in the electrical field of a capacitor. Using the physical properties of a dielectric material commonly utilized for the production of capacitors, such as $\text{Al}_2\text{O}_3$, $\epsilon_r \approx 10$ and $E_b \approx 200 - 500 \text{kV/mm}$ the maximum normal stress will be of the order of 2-11 MPa. Assuming a fairly modest coefficient of friction $\mu$ of 0.2, the maximum stresses that can be transferred through friction at the interface will be of the order of 0.4-2.2 MPa. This compares to approximately 30 MPa shear strength for Aluminum or approximately 190 MPa for common stainless steel.

The energy needed to generate the estimated normal and friction stresses in the dielectric material between load bearing layers is of the order of 2-11 mJ/mm$^3$. Thus, from an energetic standpoint the use of electrostatic fields as a means to adaptively couple the layers of a multi-layer structure is an appealing approach. In the following chapters, numerical models and experiments will be used to show the potential and the limits of this type of tunable stiffness structures.
Chapter 3

Modeling of Structures with Tunable Stiffness

This chapter will deal with two aspects of the modeling of structures with tunable stiffness. In the first part, the effect of the interfaces of a multi-layer system on the shear stress distribution will be considered for bending loads. The system considered is a simple, homogeneous four-layer beam in which the shear stresses at all three interfaces are transferred by means of friction. The results obtained are especially interesting to show how friction-based shear stress transfer compares with shear stress transfer within a solid material. The calculations performed with a numerical model will show that once the maximum transferable shear stress at the interfaces is reached, the system softens considerably and then behaves linearly again. In this second part of the force displacement curve, the layers slip on one another. This is at the origin of a hysteresis behavior shown in the last section devoted to the local behavior of the system. The second part of this chapter presents the global behavior of the system within the elastic domain, i.e. before interface slipping starts occurring in a significant way. The effect of the virtual cancellation or creation of interfaces on the stiffness of a homogeneous multi-layer beam is shown: As a result of different interface activation patterns, the stiffness can be changed in discrete increments. The number of different stiffness values that can be realized increases with the number of layers in the system.
3.1 Local Behavior: Shear Stress Transfer by Means of Friction vs. Shear Stress Transfer in Solids

In this section, we present a numerical model of a homogeneous tunable multi-layer beam that considers contact and friction to describe the behavior of the studied structures. We will consider beams with a homogeneous cross-section, in which all layers are made of the same material and have the same width. The layers are the smallest unit of the system. They can be connected to a bundle of multiple layers by removing interfaces between them. The thickness of bundles is always a whole multiple of the thickness of a layer. This choice makes the modeling work more clear and the results more readily understood. The model is compared with analytical solutions for an equivalent system in which the connectivity of the layers is switched. In the analytical model, the layers are regarded as independent bodies that are subjected to loads such, that their deflection lines are identical.

The obtained model is used to better understand the processes taking place at the interfaces between layers, demonstrate the existence of discrete stiffness states and as a base for the selection of suitable dielectric materials for the generation of the electrostatic normal stresses needed for the shear stress transfer at the interface.

![Figure 3.1: Two cross sections with the same geometry and different topologies. The topologically connected cross section (right) has one neutral axis (n.a.), while the disconnected one (left) has four (n.a.1...n.a.4). On the bottom left the local coordinates for layer i are shown.](image)

3.1.1 Deflection and Shear Stress Distribution in a Multi-layer Beam

For the analytical description of the bending behavior of a multi-layer beam with rectangular cross section of total height $N \cdot h$ and width $b$, consisting of $k$ bundles of height
$n_i \cdot h$, following assumptions are made:

- The layers and the complete beam are modeled as Euler-Bernoulli beams, i.e. no shear deformation is considered. This is justified by the slenderness of the elements and the assumption that beams of an isotropic materials are considered. Shear compliant beams will be considered in chapter 4.

- The bending stiffness $D_i$ of the $i^{th}$ layer is given by:

$$D_i = E \cdot I_{z,i} = \frac{E \cdot b}{12} \cdot (n_i \cdot h)^3 \quad (3.1)$$

- The total stiffness of the beam is given by:

$$D_{tot} = E \cdot I_{z,tot} = \frac{E \cdot b}{12} \cdot \sum_{i=1}^{k} (n_i \cdot h)^3 \quad (3.2)$$

- The transversal displacement $v(x)$ of the system is:

$$v(x) = \frac{P}{D_{tot}} \cdot f(x) \quad (3.3)$$

Where $f(x)$ is a function, typically a polynomial, that describes the shape of the deflected beam and $P$ is a load scaling factor.

- The transversal displacement $v_i(x)$ is the same for each layer:

$$v_1(x) = ... = v_i(x) = ... = v_k(x) \quad (3.4)$$

From the previous assumptions, $P_i$, the load scaling factor used in the calculation of the loads acting on each layer, can be obtained:

$$P_i = P \cdot \frac{D_i}{D_{tot}} = P \cdot \frac{n_i^3}{\sum_{i=1}^{k} n_i^3} \quad (3.5)$$

The system bends like an array of $k$ parallel beams, each subject to a load proportional to the cube of their height, where the sum of the loads acting on each layer corresponds to the load applied to the component.

Using the shear force function $Q_i$:

$$Q_i(x) = D_i \cdot v''(x) = P_i \cdot f''(x) \quad (3.6)$$
The shear stresses in each layer become then:

$$\sigma_{xy,i}(x, y_i) = \frac{Q_i(x) \cdot ((\frac{nh}{2})^2 - y_i^2)}{2I_z,i} = \frac{6P \cdot f''(x) \cdot ((\frac{nh}{2})^2 - y_i^2)}{bh^3 \sum_{i=1}^{k} n_i^3}$$  \hspace{1cm} (3.7)

$y_i$ is a local coordinate in each layer representing the distance in y-direction from the neutral axis of each layer of $k$ layers. The maximum shear stresses in each layer of the multi-layer beam ($\sigma_{xy}(x, y_i = 0)$) is given by:

$$\sigma_{xy,i}(x, y_i = 0) = 3 \frac{P n_i^2 \cdot f''(x)}{b \cdot h \cdot \sum_{i=1}^{k} n_i^3}$$  \hspace{1cm} (3.8)

If all bundles in the system have the same thickness of $n$ layers, then the sum in the denominator of (3.8) can be written as $s \cdot n^3$, where $s$ is an integer thus yielding:

$$\sigma_{xy,i}(x, y_i = 0) = 3 \frac{P f''(x)}{b \cdot h \cdot s \cdot n}$$  \hspace{1cm} (3.9)

$s \cdot n \cdot h$ is the total height $H$ of the multi-layer beam. For a given load pattern, the maximum shear stress in a multi-layer beam in which all bundles have the same height, is independent of the height and number of the individual bundles.

The relations presented in this section are valid for a homogeneous multi-layer beam composed of $n$ layers of thickness $h$ that interact only by transferring transversal forces by contact. No other interaction (e.g. friction at the contact surface) is considered.

### 3.1.2 Numerical model of the friction based shear stress transfer at interfaces

The analytical model presented in the previous section describes how the system behaves when full or no shear stress transfer is provided at the interfaces between layers. A comparison between the electrostatically generated normal stresses (and the associated maximum transferable shear stresses) and the shear stresses at the position of the interfaces in a solid beam gives an indication whether the shear stress transfer demand at the interface between layers can be satisfied or not. In this section, a numerical model of the system is used to model the behavior of the system for the cases, where the shear stress exceeds the maximum friction stress transfer capability $\sigma_{xy,max}$ made available through the application of normal stresses at the interface.
For the purpose of investigating the bending behavior of a multi-layer system, a four layer beam was modelled using the COMSOL 3.4 Multiphysics modeling package. The modeled system shown in figure 3.2 had the properties listed in table 3.1.

Table 3.1: Main properties of the modeled system

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of layers</td>
<td>4</td>
</tr>
<tr>
<td>Material</td>
<td>steel (E=205 GPa)</td>
</tr>
<tr>
<td>Friction coefficient $\mu$</td>
<td>0.2</td>
</tr>
<tr>
<td>between surfaces</td>
<td></td>
</tr>
<tr>
<td>Layer dimensions $l \times b \times h$ [mm]</td>
<td>70 $\times$ 10 $\times$ 1</td>
</tr>
<tr>
<td>Load $q$</td>
<td>distributed: 100 $N m^{-1}$...500 $N m^{-1}$ (i.e. 10 kPa...50 kPa) on the top face</td>
</tr>
<tr>
<td>Boundary conditions</td>
<td>fixed on the left side of each layer, free otherwise</td>
</tr>
</tbody>
</table>

Additionally, a positive normal stress $\sigma_{yy,el}$ of 0 MPa or 3 MPa was applied to the contact surface pairs to couple them in a way to simulate the effect of the electrostatic normal stresses described above. A normal stress of 3 MPa corresponds to an electric potential of approximately 75 V over a 400 nm thick $\text{Al}_2\text{O}_3$ layer, i.e. a field of approximately 185 MV/m.

The system has 3 pairs of contact surfaces. Each of the pairs can be activated ($\sigma_{yy} =$
MPa) or inactivated ($\sigma_{yy} = 0$ MPa). The possible configurations of active and inactive interfaces are summarized in table 3.2, where a '0' in the left hand column means that the interface is not activated (i.e. no stress is applied to it) and a '1' means it is. In the second column, the thickness of each bundle, expressed in number of layers.

The contact forces were modeled based on a penalty barrier approach [40]:

$$\sigma_{yy,\text{contact}} = \begin{cases} 
T_n - b_n \cdot g & \text{if } g \leq 0 \\
T_n \cdot e^{-\frac{b_n g}{\sigma_n}} & \text{otherwise}
\end{cases}$$

(3.10)

Where $\sigma_{yy,\text{contact}}$ is the calculated contact pressure, $g$ is the distance between the two surfaces, and $T_n$ and $b_n$ are, respectively, the initial contact pressure and the barrier parameter with following values:

- $T_n = 1 MPa$
- $b_n \approx 10^{14} Pa/m$

The stress described by the penalty barrier function is represented graphically in figure 3.3

![Graph showing contact stress as a function of gap size](image)

Figure 3.3: Contact stress $\sigma_{yy,\text{contact}}$ as a function of the gap size as calculated using the penalty barrier approach described in (3.10)

The resulting contact stress was used to calculate the friction stresses. As for the
analytical model, no cohesion (i.e. friction at zero normal stress) was assumed for this model.

Table 3.2 shows the convention used to describe the connection pattern of the interfaces of a 4 layer beam.

Table 3.2: Interface activation configurations and bundle thicknesses $n_i$

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$n_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1,1,1,1</td>
</tr>
<tr>
<td>001,010,100</td>
<td>1,1,2 1,2,1 or 2,1,1</td>
</tr>
<tr>
<td>011,110</td>
<td>1,3 or 3,1</td>
</tr>
<tr>
<td>101</td>
<td>2,2</td>
</tr>
<tr>
<td>111</td>
<td>4</td>
</tr>
</tbody>
</table>

For the '111' beam (i.e. with fully connected layers), the behavior of the system was calculated for loads increasing from $q = 1N/m$ to $q = 500N/m$ in several increments, in order to investigate the slip behavior at the interface between layers.

### 3.1.3 Linear vs. Non-linear Behavior

Better understanding of the behavior of tunable multi-layer structures is obtained by observing the load displacement diagram for a beam in '111' configuration. The diagrams shown in figure 3.4 were obtained using the analytical model (solid line) and the numerical model presented in section 3.1.2 (markers).

The analytical model only describes the linear behavior of the system, since no slip at the interfaces between layers is allowed for. Its validity is limited to the load ranges where the shear stresses at the interfaces between layers of the beam are smaller than the maximum transferable shear stress, as defined in (3.8). The main advantage of the analytical model is that it gives a better understanding of the system and that it can be easily adapted to different geometries and load cases. The described numerical model accounts for slipping at the interfaces, as will be shown. This is a considerable advantage, compared to the analytical model. The disadvantage is that new instances of the model have to be set up, each time a new system is considered. The analytical model is used to verify the reliability of the output of the numerical model in the linear domain of the system.
For the analytical calculation of the deformation function \( v(x) = P \cdot f(x) \) in the load case described in the previous sections, the deformed shape of the beam is given by:

\[
f(x) = (x^4 - 4lx^3 + 6l^2x^2)
\]

(3.11)

The load scaling factor \( P \) is \( q/24 \).

Figure 3.4: Transversal displacement at \( x=l \), as calculated using the linear (analytical) model and the non-linear (numerical) model, as a function of the applied load \( q \)

The displacement calculated using the numerical model of the friction based system (‘+’ symbol in figure 3.4) shows an offset compared to the linear calculation. This is interpreted as an effect of the singularity of the transversal load at the fixation point of the cantilever at \( x=0 \) that leads to very high shear stresses in that region. If this offset is subtracted from the results (‘x’ symbol in figure 3.4), a very good agreement between numerical and analytical model is found until values of \( q \) of approximately 300 N/m. For loads exceeding this value, the deviation between results obtained from analytical and the numerical behavior increases markedly.
The shear stress distributions $\sigma_{xy}(x, y, q)$ calculated using the linear and the non-linear models are shown in figures 3.5 to 3.9. A comparison of the distributions for different levels of $q$ confirms that the onset of the non-linear behavior of the beam coincides with a marked deviation of the shear stress distribution in the numerical calculation from the linear model at $q \approx 320 \text{N/m}$. The numerical calculations show that the shear stress in the beam never exceed the value $\mu \cdot \sigma_{yy,\text{int}}$ at the interfaces, even for values of $q$ exceeding $320 \text{N/m}$. As the load increases, the portion of each interface where the actual shear stress deviates from the shear stress calculated with the linear model increases, as seen comparing figures 3.8 and 3.9.

In figure 3.9 can also be noticed how the shear stress obtained from numerical calculations at $x = 0.05\text{m}$ exceeds the one obtained from the analytical model calculations. This is explained as a result of the redistribution of shear stresses along the interface between the two central layers: The shear stress transfer demand near the fixation point exceeds the stresses that can be transferred by the friction at the interface. Such stresses are redistributed to locations at the interface where the demand is below the critical value $\sigma_{xy,\text{max}} = \mu \cdot \sigma_{yy,\text{el}}$.

Figure 3.4 clearly shows that once the load $q$ exceeds 320 N/m, the stiffness of the system no longer is a constant. A comparison of the shear stress distributions shown in figure 3.6 and 3.7 shows that the transition from full shear stress transfer to slipping...
Figure 3.6: Shear stress distribution in the cross section at different positions along the
length of the beam with a 111 interface activation pattern, for a load q=320 N/m. The
numerically calculated distribution at x=0.01 m (markers) shows a minor deviation from
the linear model (solid lines).

Figure 3.7: Shear stress distribution in the cross section at different positions along the
length of the beam with a 111 interface activation pattern, for a load q=360 N/m. The
numerically calculated distributions at both x=0.01 m and x=0.02 m (markers) show a
clear deviation from the linear model (solid lines).
Figure 3.8: Shear stress distribution in the cross section at different positions along the length of the beam with a 111 interface activation pattern, for a load q=400 N/m. The numerically calculated distributions at x=0.01 m and x=0.02 m (markers) show a remarkable deviation from the linear model (solid lines). Also at x=0.04 m the distribution does not correspond to the linear behavior.

Figure 3.9: Shear stress distribution in the cross section at different positions along the length of the beam with a 111 interface activation pattern, for a load q=500 N/m. None of the shear stress distributions calculated numerically (markers) coincides with the linear model (solid lines), although the results for x=0.04 m and x=0.05 m do not exceed $\sigma_{xy,\text{max}}$. 
is at the origin of the softening of the system. While this transition is not desirable for a system that is required to work in an elastic manner, the ability to dissipate energy through friction may be of interest for damping applications.

In order to show the hysteretic behavior of multi-layer systems when subjected to loads exceeding its elastic limit, a cyclic force-displacement diagram for stresses larger than shown in figures 3.5 to 3.9 was calculated using the numerical model and is shown in figure 3.10.

![Figure 3.10: The cyclic load-displacement diagrams calculated for three different levels of interfacial stress (3 MPa, 6 MPa and 9 MPa) show different amounts of hysteresis in the system. The load displacement diagram of the un-coupled system is drawn in black, for comparison. The uncoupled system does not have hysteresis, since no interaction between layers is allowed for.](image)

As expected, the higher the interfacial stress, the higher is the average stiffness of the system. Nevertheless it should be noted that the stiffness of the system in the first section of the diagram (the new curve starting at the origin of the coordinates system) is the same in all three cases. The onset of the softening is found at increasingly higher loads as the interfacial stress increases, since the higher shear transfer rate at the
interfaces allows for an extended linear domain.

An additional effect of the modification of the interfacial stress on the behavior of the change in the area contained within the hysteresis curves calculated for different levels of interfacial stress. As the average stiffness of the system increases, the deformation at the maximum load \( q = 1600 \text{ N/m} \) is substantially reduced, thus the ability to dissipate energy by relative motion of the layers is limited.

### 3.2 Global Behavior: Discrete Stiffness States in Homogeneous Multi-Layer Beams

Section 3.1.3 showed that the amount of shear stress at the interfaces determines the linearity of the behavior of the system. The activation of the interfaces (i.e. the application of an electrical potential between two neighboring layers) determines the amount of shear stress that can be transferred at the interface. In this section the effect of different activation patterns on the stiffness of the system is presented, under the assumption that the shear stresses do not exceed the maximum transferable stress

\[
\sigma_{xy,\text{max}} = \mu \cdot \sigma_{yy,\text{el}}
\]

Figures 3.11 to 3.16 show the transversal deformations and the shear stress distributions in beams with different interface activation patterns in the linear domain. As expected, with an increasing number of activated interfaces, the deformations of the beam decrease. A quantitative description of the decrease in deformation as a function of the interface activation pattern is contained in the ratio of the stiffness of the activated beam to the stiffness of the completely inactive beam \((D/D_{000})\), shown in table 3.3. The indices used in the table and in the rest of this section indicate whether at an interface shear stress transfer is permitted ('1') or not ('0'). So, for example the index '111' indicates that all three interfaces of the four layer system are locked, whereas the index '100') indicates that only one is locked. More examples are shown in figure 3.17.

For the numerical model results shown in table 3.3, the stiffness ratios are calculated based on the displacement of the tip of the beams.

As expected, the shear stresses tend to zero in proximity of the non-active interfaces, since no stress transfer is possible there. At the active interfaces, the shear stress is given by the distribution calculated over a beam of the corresponding height, as if no interface were present. Once the maximum transferable shear stress is reached at the interface, the parabolic function described in (3.7) no longer applies.
Figure 3.11: Transversal deformation functions for the beams with different interface activation patterns and a load $q=100 \text{ N/m}$, as calculated analytically (line) and numerically (markers).

A comparison between the numerical and the analytical results shows a very good level of agreement between the two models for the shear stress distribution as well as for the transversal deformation, within the limit given by (2.7).
Figure 3.13: Shear stress distribution in the cross section at x=0.01 m, ... 0.07 m for the beams with '010' interface activation pattern and a load q=100 N/m, as calculated analytically (line) and numerically (markers).

Figure 3.14: Shear stress distribution in the cross section at x=0.01 m, ... 0.07 m for the beams with '101' interface activation pattern and a load q=100 N/m, as calculated analytically (line) and numerically (markers).

The values for $D_{tot}$, $D_i$, $\sigma_{xy,i}(x, y)$ can be calculated using (3.1)...(3.7) under the assumptions listed in section 3.1.1. The values for the maximum shear stress $\sigma_{xy,i}(x, y_i = 0)$ are calculated using $f''''(x) = 24(x - l)$.

As long as no slipping at the interfaces takes place, that is as long as the shear stress...
Figure 3.15: Shear stress distribution in the cross section at x=0.01 m,... 0.07 m for the beams with '011' interface activation pattern and a load q=100 N/m, as calculated analytically (line) and numerically (markers)

Figure 3.16: Shear stress distribution in the cross section at x=0.01 m,... 0.07 m for the beams with '111' interface activation pattern and a load q=100 N/m, as calculated analytically (line) and numerically (markers)

transfer demand is satisfied by means of the friction at the interfaces, the stiffness of the system is varied in a discrete manner, as shown in table 3.3.

According to the analytical model calculations, when the system is divided in bundles of equal thickness, the maximum shear stress in each layer stays constant, as shown
by figures 3.12, 3.14 and 3.16. An uneven distribution of the bundle thickness across the beam leads to an increase in the maximum value of $\sigma_{xy}$, as shown in figures 3.13 and 3.15.

Table 3.3: Interface activation configurations, stiffness increase compared to the 000 configuration, comparison between analytical and numerical results for $q=100$ N/m

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$D/D_{000}$ analytical</th>
<th>$D/D_{000}$ numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>001, 100, 010</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>101</td>
<td>4</td>
<td>3.9</td>
</tr>
<tr>
<td>011, 110</td>
<td>7</td>
<td>6.7</td>
</tr>
<tr>
<td>111</td>
<td>16</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Within the limits set by the ability of the interfaces to transfer the shear stress, each bundle of layers that are coupled electrostatically can be considered as a solid beam of thickness $n_i \cdot h$ where $n_i$ is the number of layers in the bundle and $h$ is the height of a layer.

Figure 3.17: The stiffness of the beams with each connectivity pattern can be represented by an equivalent parallel connection of springs. The stiffness $D$ of each beam is expressed in multiples of $k$, where $k$ is the stiffness of one layer.

A total of 8 connectivity patterns can be obtained for the 4 layers beam in Figure 3.17. The figure shows that only 5 different stiffness values can be realized. Patterns ‘100’,
'010' and '001' can be considered equivalent. Patterns '110' and '011' also correspond to the same stiffness value.

If the shear stress transfer capability of each interface can be switched between 0 and 1, the number N of different connectivity patterns that can be obtained for a given number of layers n in the system is:

\[ N = 2^{n-1} \]  

(3.12)

For a system with 4 layers, as the one shown in figure 3.17 there are \(2^3\) possible connectivity combinations. The different stiffness levels for a 4 layers system is shown in figure 3.17. The number of different stiffness values that can be achieved is smaller than the number of connectivity patterns given by (3.12).

As outlined previously, the stiffness \(D\) of a multi-layer system can be varied in discrete steps by modifying its connectivity pattern. The knowledge of the stiffness levels that can be realized with a given multi-layer system is useful for the design of variable stiffness devices. As shown in Figure 3.17, the achievable stiffness values can be calculated considering each bundle of layers as a spring connected in parallel with the other bundles in the system. A bundle is a set of connected layers between two interfaces with no shear stress transfer capability. The stiffness of each bundle is proportional to its height cubed. The total stiffness is thus proportional the sum of the third power of the bundle heights.

These assumptions are realistic for small transversal loads, for which the shear stress at the interface, does not exceed \(\sigma_{xy,max}\). Under such conditions, it is justified to assume that a bundle of electrostatically coupled layers behaves as a solid bundle, as outlined in the previous section.

The computational effort needed to calculate the distribution of stiffness values increases with \(2^n\).

Figure 3.18 shows the stiffness values that can be realized with a 20 layers system and the frequency of each stiffness value. The diagram on the left hand side of the figure shows the stiffness calculated for each of the 524288 (\(2^{19}\)) connectivity patterns that can be realized in a system with 20 layers. If the state of the interface is represented with '0' for a non connecting interface (no stress transfer) and '1' for a connecting interface, each connectivity pattern can be identified by a 19 bit binary number. The pattern number on the x-axis of the diagram is the decimal equivalent of said binary number.

The diagram on the right hand side of Figure 3.18 shows how many patterns in a 20 layer system realize a particular stiffness value. While the points in this may intuitively be interpolated to a curve somewhat resembling a normal distribution, it should be
Figure 3.18: Stiffness distribution (left) and stiffness density (right) for n=20

noted that the number of patterns with stiffness values in between points drawn in the graph is effectively zero. For the high and the low stiffness values, gaps are present in which no connectivity pattern realizes certain values. For the high stiffness domain the gaps span over very large number of values that are not represented in the set stiffness states that can be realized by the system.

Figure 3.19 shows the stiffness value distributions of systems with 5, 10, 15, 20 and 25 layers, respectively. In order to make a comparison between the distributions for different n-values possible, the number of patterns possible a double logarithmic representation was chosen. The distributions show that the range over which the stiffness can be modified increases very rapidly with n. From a 25-fold increase in stiffness for n=5 the range grows to a 625-fold increase in stiffness for n=25. Also the number of possible states increases remarkably (approximately as $n^3/30$). The high and low ends of the distribution are more sparsely populated, both in terms number of patterns (y-axis of the graph) and in terms of distance between stiffness states that can be realized (x-axis of the graph).
3.3 Conclusions

The goal of this section was to present simple analytical and numerical models to describe and understand various aspects of the behavior of tunable multi-layer beams. The models provide useful information concerning the distribution of stresses in the structure. The understanding of the local behavior of the system can be used to estimate the limits within which such a structure behaves approximately like an elastic structure, without yielding due to slipping at the interfaces between layers. This can be of special interest for applications in which the suppression of large amplitude vibrations due to resonance phenomena is to be obtained by modifying the bending stiffness and hence the natural frequencies of the system. In such cases, the primary goal of the adaptation of the properties of a structure is to control its mechanical impedance, rather than introducing high damping. Conversely, for other applications, such as the harvesting of energy from structural vibrations, resonance effects are desirable, while dissipation due to friction at the interfaces is not.

The behavior of the system is hysteretic when the loads it is subjected to exceed the
ability of the interfaces to transfer the shear stresses in full. The numerical model shows that next to modifying the stiffness of the system, increasing the transfer of shear stress at the interfaces also has an effect on the area enclosed by the hysteretic curve and thus on the ability of the system to dissipate energy. This means that next to a modification of the stiffness, also a tuning of the damping behavior is possible in multi-layer systems.

The models also give some insight into the effect of the homogeneity of the thickness of the bundles of coupled layers on the value of the maximum shear stress in each cross section of the system. This knowledge is valuable for the design of multi-layer systems. The shear stresses that can be transferred at the interface between layers are limited by the maximum energy density that can be stored in the used dielectric material. Hence, interface activation patterns that lead to higher shear stress values for a given applied load, are more likely to cause the interfaces to yield than others.

Both, the analytical and numerical models confirm that a finite number of discrete stiffness levels can be achieved for each system. The number of stiffness levels increases with the number of layers in the system. This prediction does not account for dissipative effects such as inter-layer friction, that would lead to the coalescence of close states in systems with a large number of layers. The softening predicted by the non-linear calculations is expected to reduce the quality factor of the system and lead to a reduced tuning effect for the natural frequencies.

The calculations of section 2.6 show that using commonly available dielectric materials, such as alumina, titania and zirconia, the generated stresses are of a sufficient magnitude to effectively stiffen even in steel structures. Nevertheless the achievable interface stresses are far below the shear strength of typical engineering materials. Hence, simple models as the ones presented here, can be useful tools to help users make suitable design choices to obtain useful effects based on the electrostatic tuning of their bending stiffness.
Chapter 4

Experimental Work

This chapter presents the experimental work performed on structures with variable stiffness and variable damping. The examples chosen to demonstrate the effect of the electrostatic coupling of elements of a structure are two sandwich structures. The first structure considered in this chapter is a sandwich beam consisting of a highly compliant core and stiff faces. This structure has mainly a proof of concept value, and is used to show the similarity between the effect of lamination and electrostatic coupling on the stiffness of the beam. The second structure investigated in this chapter is a sandwich beam made of a GFRP I-beam as the core of the sandwich and two unidirectional CFRP bands as faces. The structure is used to demonstrate the effect of electrostatic coupling on the eigenfrequency and the damping ratio of a sandwich beam. Except for its size, this system is the same as the components that will be considered for the full scale demonstrator described in the last chapter of this work.

In the models presented in the previous chapter, the contact surfaces are assumed to be 'perfectly planar' which means that the apparent contact area between two adjacent layers extends to the whole length of the beam. This is possible if the individual layers do not present any curvature or other geometrical imperfections. In reality, most commercially available sheet materials, such as the metal sheet that would be used to prepare a set-up that replicates the modeled experiments present some level of curvature. Typically, sheets and bands are wound on coils after production and do present different amounts of curvature. The curvature is sufficient to create air gaps of the order of several tenths of a millimeter between two layers that are fixed at one end so as to create a cantilever beam. Under these conditions, the electrostatic stress between adjacent layers is not sufficient to close the gap between layers, as the attraction between electrodes decays in proportion to the square of the distance between them. Attempts to eliminate the curvature by means of a stress relaxation thermal treatment were not
successful. The use of extremely thin, compliant layers were faced with the problem that force needed to deform the structure in low stiffness state would be extremely small and thus quite difficult to apply and measure in a reliable way.

To obviate these problems, the experiments presented in this chapters will demonstrate the electrostatic coupling of the elements of a multi-layer structure using sandwich structures. These structures lend themselves for the demonstration of electrolamination for following reasons:

- Sandwich structures typically consist of 3 layers: 1 core and two faces. The limited number of interfaces reduces the complexity of the experimental set-up and the risk of electrical failure at the interfaces.

- The faces of a sandwich structure have generally a high tensile stiffness (thanks to the high modulus of the materials they are made of) and a negligible bending stiffness due to their very small thickness. This allows them to better conform to the topography of the core surfaces.

- Based on the sandwich theory, the difference in bending stiffness between the laminated and unlaminated sandwich structure is normally very large. Hence, even if the irregular surface topography reduces the effectiveness of the electrolamination, an effect on the global properties of the structures can be clearly detected.

In the following section the effect of electrolamination with respect to the global mechanical properties of sandwich beams are summarized. The experiments performed on two different sandwich structures are then described in sections 4.2 and 4.3.

4.1 Inhomogeneous Tunable Structures: Sandwich Beams

4.1.1 Mechanical properties of fully bonded sandwich structures

The above considerations lead to the the idea to perform all experiments on sandwich structures, a well known example of stiff, lightweight structural elements. In such structures, thin, axially stiff faces (typically made of CFRP or another high strength, high modulus material) are glued on a lightweight, low modulus thick core to obtain a composite structure with a high bending stiffness. The practical advantage of performing
the experiments on tunable stiffness sandwich structures is that the individual components can be very compliant in bending direction, thus obviating the problem described above.

The behavior of such structures has been extensively investigated. Some of the most important relations describing the properties of such structures in relation to the connectivity of the faces and the core are summarized in the following section. For common engineering applications it is a requirement that the stiff faces be properly bonded to the core, that means that a complete transfer of stresses between layers of the sandwich structure takes places. The flexural rigidity of a sandwich structure is given by:

\[
D_{\text{sandwich}} = \int E z^2 dS = 2 \times \frac{E_f b_f t_f^3}{12} + \frac{E_f b_f t_f d^2}{2} + \frac{E_c b_c t_c^3}{12} = 2 D_f + D_0 + D_c \tag{4.1}
\]

Where \(E_f\) and \(E_c\) are the Young’s moduli of the faces and core, respectively, \(t_f\) and \(t_c\) are the thicknesses of the faces and core, respectively and \(b_f\) and \(b_c\) the corresponding widths, and \(d = t_f + t_c\). \[4\] Under certain conditions, often met in sandwich structures, the first and the third term in (4.1) can be considered negligible and the stiffness can be approximated as:

\[
D_{\text{sandwich}} \approx \frac{E_f b_f t_f d^2}{2} \tag{4.2}
\]

The first term in (4.1) can be neglected (i.e. is less than 0.01 times the second term) if :

\[
3 \left( \frac{d}{t_f} \right)^3 > 100 \tag{4.3}
\]

The third term is less than 0.01 times the second term in (4.1) if

\[
\frac{6 E_f t_f d^2}{E_c t_c^3} > 100 \tag{4.4}
\]

Additionally, if the core of the sandwich is ‘soft’, a shear contribution to the transversal deformation has to be considered. With a shear stiffness \(S\)

\[
S = \frac{G_c b_c \ell^2}{t_c} \tag{4.5}
\]

the total deformation can be written as a sum of bending and shear deformation. In the case of a discrete transversal force \(P\) acting on the beam at a distance \(l\) from the origin, the total deformation \(v_{\text{sandwich}}\) can be written as:
\[ v_{\text{sandwich}} = v_{\text{bending}} + v_{\text{shear}} \]
\[ = \frac{P t^3}{6 D_{\text{sandwich}}} \left[ 3 \left( \frac{x}{l} \right)^2 - \frac{x}{l} \right] + \frac{P x}{S} \]  

(4.6)

The shear stress between faces and core is found to be

\[ \sigma_{xy}^{\text{int}} = \frac{T_x}{b_f D_{\text{sandwich}}} \cdot \frac{E_f b_f t_f d}{2} \]  

(4.7)

with \( T_x = dM_x / dx \), where \( M_x \) is the bending moment acting on the structure. Combination with (4.1) yields

\[ \sigma_{xy}^{\text{int}} \approx \frac{P}{b_f d} \]  

(4.8)

Local de-bonding between core and faces has also been studied as well as other modes of failure that are common for sandwich panels [53]. For practical purposes, de-bonding is regarded as a confined defect for which there is no adhesion between skin and core only in delimited areas, hence \( \sigma_{xy}^{\text{int}} = 0 \). If the delamination area increases it has been shown that the flexural stiffness of the sandwich will decrease [32].

### 4.1.2 Complete debonding

The extreme case of de-bonding is given when no shear stress transfer takes place between core and faces. Simple considerations and inclusion of the boundary conditions show that for a loose bundle composed of the same faces and core as considered in (4.1) and (4.2) the stiffness is

\[ D_{\text{bundle}} = \frac{2 E_f b_f t_f^3}{12} + \frac{E_c b_c t_c^3}{12} \]  

(4.9)

Again, for a core with a sufficiently low bending stiffness compared to the faces, i.e.

\[ E_c t_c^3 \ll 2 E_f t_f^3 \]  

(4.10)

the second term can be neglected in a first approximation, so that the ratio between the stiffness of a sandwich and the stiffness of a loose bundle can be estimated as
\[
\frac{D_{\text{sandwich}}}{D_{\text{bundle}}} = \frac{6E_I b f t f d^2}{2E_I b f t_f^2 + E_c b_c t_c^2} \approx 3\left(\frac{t_c}{t_f} + 1\right)^2 \quad (4.11)
\]

Taking the shear contribution to the displacement into account, the ratio of the displacements becomes a function of the position \(x\):

\[
\frac{v_{\text{sandwich}}}{v_{\text{bundle}}} = \frac{v_{\text{bending,sandwich}} + v_{\text{shear}}}{v_{\text{bending,bundle}}} = \frac{D_{\text{bundle}}}{D_{\text{sandwich}}} \left(1 + \frac{D_{\text{sandwich}}}{S} \frac{6}{3l x - x^2}\right) \quad (4.12)
\]

and for \(x = l\) follows:

\[
\frac{v_{\text{sandwich}}}{v_{\text{bundle}}} = \frac{D_{\text{bundle}}}{D_{\text{sandwich}}} \left(1 + \frac{3D_{\text{sandwich}}}{S l^2}\right) \quad (4.13)
\]

Depending on the measurements and elastic properties of the individual components, ratios \(D_{\text{sandwich}}/D_{\text{bundle}}\) of the order of 1000 can very easily be achieved, for the fully bonded in comparison with the fully de-bonded sandwich layers.

### 4.2 Proof of Concept with a 140 mm Cantilever Beam

#### 4.2.1 Experimental

The goal of the experiments presented in this section is to provide a demonstration of the electrostatic stiffening of a simple structure in static tests. For this purpose a sandwich beam (shown in figure 4.1) consisting of:

- two unidirectional (UD) carbon fiber reinforced polymer (CFRP) layers, coated on one side with a \(\sim 0.05\) mm thick Poly(vinylidene fluoride) (PVDF) film, \(\varepsilon_r = 8\)
- one electrically conductive, carbon-filled silicone elastomer core, \(\varepsilon_r = 2\)

was chosen as a testing object.

From a mechanical point of view, the choice of the core and face materials was dictated by the need to design a system in which the effect of changes in the shear stress transfer at the interfaces between core and faces of the sandwich beam would be observed.
unequivocally. Hence, the choice of a very low modulus material such as a silicone elastomer for the core of the sandwich was made. UD CFRP bands as the ones utilized for these experiment, have a high tensile stiffness and a very modest bending stiffness, due to their very small thickness. This combination of mechanical properties is expected to yield a markedly different behavior of the system in the bonded and unbonded state.

The main mechanical properties of the sandwich beam are listed in table 4.1.

Table 4.1: Properties of the mechanical components of the 'proof of concept' sandwich beam

<table>
<thead>
<tr>
<th>Property</th>
<th>Faces</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Carbon Fiber Reinforced Polymer</td>
<td>Carbon particle filled silicone elastomer</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>0.15</td>
<td>2.3</td>
</tr>
<tr>
<td>Width [mm]</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Young’s Modulus [GPa]</td>
<td>120</td>
<td>~0.004</td>
</tr>
<tr>
<td>Free Length [mm]</td>
<td>140</td>
<td>140</td>
</tr>
</tbody>
</table>

From an electrical point of view, the use of electrically conductive structural elements simplified the set-up of the experiment. The material originally chosen as the sole dielectric between the structural elements was the ~0.05 mm thick PVDF film. Due to the tendency of the silicone elastomer to adhere to the PVDF film layers, additional PTFE layers were applied to the surfaces of the core to obviate the problem. The thin dielectric layers were considered to have a negligible contribution to the stiffness of the system, due to their low elastic moduli.

With this configuration, an effective dielectric constant of $\varepsilon_r = 3.2$ and an effective dielectric layer thickness $\delta = 0.1$ mm (sum of the thicknesses of the two layers) are obtained, as calculated in (2.4) and (2.5).

The beam was positioned vertically and fixed 140 mm from the bottom end, where two small FeNdB permanent magnets were positioned. Their function was to make it possible to apply a transversal force by means of a time-variable magnetic field $B(I(t))$, generated by a current $I(t)$ that circulated in the coils. The coils were approximately in a Helmholtz configuration, which means that the distance between the coils was equal to their radius. In this configuration, a very homogeneous field in axial direction can be obtained. The current $I(t)$ was supplied by a Kepco bipolar operational power amplifier BOP 20-20 M driven by the analog output tension of a National Instruments 6036 E.
Figure 4.1: Test set-up for the measurement of force-displacement diagrams of the electrostatically tunable sandwich beam (left: schematic representation, right: photograph).

DAQ board. The transversal displacement of the composite beam was measured at two by means of two Micro-Epsilon optoNCDT laser displacement sensors (marked as LTD in figure 4.1). The displacement was measured with a sampling frequency of 64 Hz. The components of the sandwich beam were connected to a high voltage power supply (Stanford Research Systems PS 350), so that the necessary electrical potential $U_i$ could be applied between faces and core. The complete test setup was controlled from a PC via a LabView interface developed for this purpose.

### 4.2.2 Measurement Technique

The behavior of the sandwich beam was observed in a step-wise quasi static cantilever bending test. The force was applied at one position of the cantilever beam, 140 mm from its fixation point. The force exerted on the magnets by the magnetic field $B(I)$ as a function of the coil current $I(t)$ was measured by means of a load cell calibrated in a separate experiment. The lateral displacement of the beam was measured in a cantilever bending test, as a function of the position $x$ along its main axis, the force $P(I)$ and of the high voltage setting $U_i$ for the potential across the dielectric layer between faces and core. After the initial loading to $P_{\text{max}}$ several cycles $P_{\text{max}} \rightarrow -P_{\text{max}} \rightarrow P_{\text{max}}$ were measured. In addition, the transversal displacement of a similar beam with glued faces was measured for comparison.
4.2.3 Results and Discussion

Behavior of the system

Figure 4.2 shows the force-displacement diagrams measured at a distance of x=125 mm from the fixation point of the cantilever beam for $U_i$ settings between 0 V and 3000 V.

At lower voltage settings (0 V-1500 V) the curves can be roughly divided in three sections. The beam exhibits initially a high stiffness, while the force displacement behavior of the beam shows a considerably lower stiffness for larger forces. Between the high and the low stiffness portions, a transition domain can be observed. The transition between high and low stiffness becomes more diffuse with increasing voltage $U_i$. The measurement at 500 V does not show a remarkable effect on the behavior of the system. The slight reduction of the extension of the high stiffness portion of the force-displacement curve at higher voltages is evident.

Figure 4.2: Force-displacement curves for the initial loading as a function of the electrical potential across the face-core interface. Measured and calculated displacements for the fully bonded sandwich are in good agreement and show higher stiffness than an electrostatically stiffened bundle at high voltage (3000 V). The 2500 V measurement is not shown as it is practically identical with the 3000 V measurement.
displacement curve should rather attributed to experimental artifacts. The extension of the higher stiffness domain (between \( P = 0 \) mN and the begin of the softening) increases with further increasing voltage \( U_i \). For the higher voltage curves (2000 V and 3000 V) only very limited softening is visible, starting approximately at 5 mN load. The application of high voltage (3000 V) to the sandwich components, leads to an 18-fold decrease in lateral displacement at \( x=125 \) mm and \( P = P_{\text{max}} = 9.2 \) mN in comparison to the same system without the application of any electrical potential. The increase in voltage does not change the slope of the high stiffness section of the curve but increases the load at which softening can be observed. At each potential level \( U_i \), after the initial loading (shown in figure 4.2) two load cycles (from \( P_{\text{max}} \) to \(-P_{\text{max}}\) and back) were performed.

Figure 4.3 shows an example of such a cycling procedure for \( U_i = 1000 \) V. For better clarity, a comparison of the central portions of such measurements (from \(-P_{\text{max}}\) to \( P_{\text{max}}\) and back to \(-P_{\text{max}}\)) is shown in figure 4.4. As observed for the curves shown in figure 4.2, an approximately bi-linear behavior of the force-displacement curves is visible. The approximated slopes \( \frac{\Delta P}{\Delta v} \) for the two sections of each curve are shown in each plot. The intercept between the high stiffness branch and the low stiffness branch of the curves (marked with a dot in figure 4.4) was calculated as the intersecting point of the linear interpolation of the values measured for the two sections of the curves, hence com-

![Figure 4.3: Behavior of the system for two complete cycles (after the initial loading) at potential \( U_i = 1000 \) V.](image)
pensating for the transition zone between high and low stiffness and providing an ideal intersection point of the two lines. The linear interpolations were calculated assuming that the slopes of the two branches (high and low stiffness) are approximately constant over different $U_i$ values. This could be confirmed for the slope of the upper branches of the curves, where a $\Delta P/\Delta w$ value of 3.1 Nm$^{-1}$ could be calculated independently and with good confidence for $U_i$ values ranging from 0 V to 1500 V. The top section of the 2000 V curve seems to still be part of the transition to a lower stiffness value, which is assumed to explain the higher $\Delta P/\Delta w$ value.

The $\Delta P/\Delta w$ value for the bottom section of the curves (70 Nm$^{-1}$) could be calculated independently for the 2500 V (not shown here as it is practically identical with the 3000 V

Figure 4.4: Behavior of the system as a function of the electrical potential across the face-core interface. After the initial loading, the force-displacement diagrams show a hysteretic behavior. The stiffness of the system increases with increasing potential $U_i$. 

72
measurement) and 3000 V measurements. At lower voltages, this value is considered to provide a reasonable interpolation of the bottom branch, but could not be confirmed independently, due to the shortness of the interval on which the parameters for a regression could be calculated. The calculated force difference $\Delta P_s$ between $-P_{\text{max}}$ and the intersection point of the two interpolation lines described in the previous section is plotted against the voltage $U_i$ in figure 4.5. The magnitude of $\Delta P_s$ is interpreted as the load at which the shear stresses between faces and core exceed the maximum shear stress $\sigma_{xy,\text{max}}^{\text{int}}$ that can be transferred across the face-core interface due to the electrostatic normal stress $\sigma_{yy}^{\text{int}}$. The corresponding shear stresses at the core-face interface are calculated using (4.6).

According to (2.2) and (2.3), the shear stress at the face core interface is a quadratic function in $U_i$. The measured force-displacement diagrams in figure 4.4 clearly show a hysteretic behavior in measurements at lower voltages. The force displacement curves no longer pass through the origin of the coordinates, as shown in figure 4.4. This behavior is attributed to friction related processes. The area contained within each curve represents the energy dissipated by the friction processes in one load cycle. At first, the area slowly increases from 18.8 J to 24.7 J with increasing $U_i$ (0 V-1000 V) and then it decreases rapidly to 0.6 J at 3000 V as shown in table 4.3.

**Comparison with models**

In addition to the force-displacement curves measured for the varying parameter $U_i$, figure 4.2 shows also the force displacement diagram for the case in which the faces of the sandwich were glued to the core using epoxy resin. For comparison, the force displacement curve for the glued sandwich calculated using (4.6) is also plotted. In this case measured and calculated data are in good agreement. The displacements measured at 0 V are approximately 4 times smaller than predicted by (4.9), thus indicating

<table>
<thead>
<tr>
<th>$U_i$ [V]</th>
<th>$\Delta P_s$ [mN]</th>
<th>$\sigma_{xy,\text{max}}^{\text{int}}$ as calculated using $\Delta P_s$ for $P$ in (4.8) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.1</td>
<td>209</td>
</tr>
<tr>
<td>500</td>
<td>6.2</td>
<td>254</td>
</tr>
<tr>
<td>1000</td>
<td>8.6</td>
<td>352</td>
</tr>
<tr>
<td>1500</td>
<td>12.3</td>
<td>504</td>
</tr>
</tbody>
</table>

Table 4.2: Shear stresses calculated based on the sandwich theory
Table 4.3: Energy dissipated by the sandwich beam in one loading cycle.

<table>
<thead>
<tr>
<th>Tension across the dielectric layer $U_i$[V]</th>
<th>Area of the hysteresis curve [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18.8</td>
</tr>
<tr>
<td>500</td>
<td>21.1</td>
</tr>
<tr>
<td>1000</td>
<td>24.7</td>
</tr>
<tr>
<td>1500</td>
<td>22.3</td>
</tr>
<tr>
<td>2000</td>
<td>6.9</td>
</tr>
<tr>
<td>2500</td>
<td>0.8</td>
</tr>
<tr>
<td>3000</td>
<td>0.6</td>
</tr>
</tbody>
</table>

a higher stiffness than expected. The hysteresis in the curve indicates that already for $U_i=0$ V, a certain level of interaction between faces and core is present, in spite of the lack of an electrostatic normal force. To some extent, the difference can be explained by the stiffer behavior observed at both ends of the force displacement diagrams ($\Delta P > 0$) in figure 4.4. Nevertheless, from the slope of the top section of the $U_i=0$ curve a bending stiffness of approximately $2.4*10^{-3}$ Nm$^2$ was obtained, compared to $8.5*10^{-4}$ Nm$^2$ as predicted by the model.

This difference between measured and calculated behavior can thus be partly explained by an inaccuracy in the assumption that no interaction between faces and core takes place for $U_i=0$ V. Even a small contribution of the bending of the faces about the neutral axis of the sandwich would have a remarkable effect on the stiffness of the faces and core bundle.

Figure 4.6 shows the normalized (i.e. divided by the value measured or calculated at $x=140$ mm) displacements and fitted displacement functions for measurements performed at $U_i$ values of 0 V and 3000 V, respectively. For the measurements at 0 V, the displacement function for a bundle was used to approximate the $v(x)$ displacement function (lateral displacement as a function of the position along the main axis), whereas at 3000 V a sandwich behavior (i.e. also considering shear deformations) was assumed. The displacement functions $v(x)$ shown in figure 4.6 confirm an increase in the contribution of shear to the displacement for the high voltage measurement, as the reduced curvature of the beam indicates. With increasing voltage the overall stiffness of the system increases. The maximum displacement of the sandwich subjected to high voltage (3000 V) is approximately 0.12 mm at $x = 125$ mm, compared to 0.046 mm as predicted by the model (4.6) and measured for the fully bonded sandwich. A stiffness of
Figure 4.5: Force at which the regression lines calculated for the force-displacement diagrams shown in figure 4.4 intersect. The factor $3.2 \cdot 10^{-6} \text{mN V}^{-2}$ is in fairly good agreement with the value of $3.45 \cdot 10^{-6} \text{mN V}^{-2}$ calculated on the basis of electrostatic attraction forces using (4.8), (2.3) and (4.14).

Table 4.4 lists values for $\sigma_{yy}^{\text{int}}$ and an estimate of $\sigma_{xy}^{\text{int}}$ based on the electrostatic attraction at different voltages using (2.2) and (2.3), respectively, as well as the $\sigma_{xy}^{\text{int}}$ values obtained by using the $\Delta P_s'$ values from figure 4.4 in (4.8). A good degree of agreement between $\sigma_{xy}^{\text{int}}$, as calculated based on the electrostatic forces between faces and core and $\sigma_{xy}^{\text{int}}$, as obtained from the mechanical behavior of the system can be assumed.
Figure 4.6: Normalized bending lines at $U_i = 0$ and 3000 V respectively. The 0 V line appears to have a higher curvature than the 3000 V line due to a higher shear contribution (linear in $x$) to the displacement at higher voltages, as expected considering (4.6).

Table 4.4: Normal and shear stresses calculated using electrostatic attraction and sandwich theory.

<table>
<thead>
<tr>
<th>$U_i$ (V)</th>
<th>$\sigma_{yy}^{int}$, as calculated in (2.3) (Pa)</th>
<th>$\sigma_{xy}^{int}$, as calculated in (2.2), assuming $\mu=0.1$, (Pa)</th>
<th>$\sigma_{xy}^{int}$, as calculated using $\Delta P'$ for $P$ in (4.8), (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>500</td>
<td>354</td>
<td>35</td>
<td>45</td>
</tr>
<tr>
<td>1000</td>
<td>1417</td>
<td>142</td>
<td>143</td>
</tr>
<tr>
<td>1500</td>
<td>3187</td>
<td>319</td>
<td>295</td>
</tr>
</tbody>
</table>

### 4.3 Characterization of a Large (>2m) Tunable Glass Fiber Reinforced Beam

The system presented in the previous section served the purpose of demonstrating the effect of the electrostatic coupling of the faces and the core of a sandwich beam, but can hardly be of any practical use, due to its low stiffness.

The application example that will be presented in the next chapter calls for the modification of the mechanical properties of the lightweight glass fiber reinforced polymer (GFRP) elements that compose the deck of a cable stayed pedestrian bridge built in Empa’s Structural Engineering Research Laboratory. The adaptive upgrade of the com-
plete bridge deck is expected to be a labor intensive and expensive undertaking. The development of an intermediate size sample of tunable structural element deemed a reasonable way to prepare for the construction of the full-size demonstrator. In addition to characterizing the behavior of the tunable element, materials and methods for the upgrading were tested.

4.3.1 Experimental

In the set up used for the experiments, the bond between additional stiffening elements (CFRP plates, CarboDur M614, Sika, CH) and the core of the sandwich beam (GFRP I-beam, Fiberline Composites, DK) is given by electrostatic forces generated by an electrical field built up across a 12µm thick polyethyleneterephtalate (PET, $\epsilon_r \approx 1$, Amcor Flexibles, Burgdorf, CH) film that was coated with a thin aluminum film on one side (coating thickness unknown). The PET film was applied to the flanges of the GFRP beam using epoxide resin (Araldit Standard, Vantico AG, Basel, CH). The CFRP plates served at the same time as second pair of electrodes and stiffening member. This setup was chosen in order to optimize the electrostatic forces that could be obtained per unit voltage.

The multi-layer beam was fixed on one end and excited at the free end by means of the electromagnetic fields generated by a coil, acting on a 40 mm x 40 mm x 20 mm FeNdB permanent magnet that was attached to the end of the beam, 2240 mm from its fixation point. The brackets visible in the left hand side of Figure 4.7 were only used to keep the loose CFRP plates in place when the electrostatic field was turned off, ideally without exerting any normal forces. Two additional masses (1050 g each) were placed 2090 mm (position of the center of gravity) from the fixed end of the beam to help separate the first bending and torsion modes of the beam.

The currents necessary to induce the exciting alternate magnetic field, were generated by a bipolar operational power amplifier (BOP 20-20 M, Kepco, USA) driven in its current controlled mode by an arbitrary signal generator (33120 A, Agilent, USA).

The transversal accelerations at the end of the composite beam were measured by means of two accelerometers (PCB Piezotronic, Mod. 3701 G3 FA3 G, with a sensitivity of 1 V/g) positioned at the left and the right edges at the free end of the beam (sensor 1 and sensor 2, respectively). The current circulating through the coil was measured using a shunt resistor connected in series with the coil. The shunt had a resistivity of 0.01Ω (60 mV/6 A). The acceleration and the current circulating through the coil were measured with a sampling frequency of 512 Hz using an OROS 38 data acquisition sys-
Figure 4.7: Overview of the test set-up and cross section of a GFRP I beam as used in the experiments described in section 4.3.2

The components of the sandwich beam were connected to a high voltage power supply (PS 350, Stanford Research Systems, USA), so that the necessary electrical potential $U_i$ could be applied between faces and core.

The set-up could be prepared in a very short time. The application of the aluminized PET film, that served at the same time as electrode and dielectric layer, required only approximately half a day of the work of two people. The preparation of the electrical set-up (contacting the elements, setting up the elements, etc.) also required approximately half a day of work by two people. The mechanical set-up (fixation of the beam, preparation of the retaining brackets for the bottom CFRP UD band) was probably the most labor intensive part of the preparation work.

### 4.3.2 Shift of the Natural Frequency

The beam was excited with a 7-17 Hz linear chirp signal at six different current amplitudes ($I_0 = 0.5 \, \text{A} \ldots 3 \, \text{A}$). The vertical accelerations of the beam end and the coil current (assumed to be directly proportional to the generated electromagnetic-field, hence to the force exerted on the magnet at the end of the beam) were recorded as a function of time.
Figure 4.8 shows the typical acceleration amplitudes measured for $U_i=0$ V. The curves show that in the 7-17 Hz range the first bending and torsion modes are excited and are fairly close. In order to simplify the interpretation of the measurements, the torsional motion was filtered from the results, by averaging the signals of sensor 1 and sensor 2. As shown in the figure, this procedure effectively reduces the intensity of the torsion signal. In the following sections, only the bending behavior of the system will be discussed.

From the current and the acceleration data (average of sensor 1 and sensor 2), transfer functions of the system were calculated for different excitation current $I_0$ and potential levels $U_i$, as shown in the plots of figure 4.9.

Figure 4.9 shows the behavior of the system at six excitation levels ($I_0=0.5$ A ... 3 A) for different interlaminar potentials $U_i$. In the series of measurements performed at $I_0=0.5$ A, quite remarkable reduction of the vibration amplitudes of the beam (up to a factor of 5 in the transfer function for $U_i=300$ V) is recorded when the electrostatic coupling of stiffening elements to the core of the beam is activated. A widening of the transfer function peak is also observed for intermediate levels of $U_i$ (200 V and 300 V), while the tendency is reversed for higher potential values, where the maximum value of the transfer function also increases.

A similar pattern is observed in the spectra recorded at higher excitation amplitudes. The most remarkable difference between the measurement series performed at differ-
ent $I_0$ values is that at higher $I_0$ values, the peak values of the transfer functions $|G|$ for the activated beam are significantly lower than at low $I_0$ values, as summarized in figure 4.10. At the same time the reversal of the trend, as observed in the series measured at $I_0=0.5$ A is not observed for the highest excitation amplitudes.

Also, the maximum frequency shift for a given interlaminar voltage $U$ tends to decrease with increasing amplitude of the exciting force. This is caused by the softening of the system due to the interlaminar slip at higher transversal loads. The transfer function plots also show a clear difference between the lower and the higher excitation regimes in terms of peak value of the transfer function $|G|$, as summarized in the right hand plot of figure 4.10, where $\max(|G|)$ values are plotted as a function of interlaminar potential $U_i$ and excitation current amplitude. The resonance frequencies of the electrostatically coupled system are summarized for the measurement series at all excitation levels in the left hand plot of figure 4.10.

The transfer function drawn in black in the $I_0=1.0$ A plot of figure 4.9 shows the behavior of an equivalent GFRP-CFRP beam, in which the coupling between the elements was achieved using an epoxy-type adhesive. This curve represents the behavior of a system in which full shear stress transfer between the elements is obtained. The transfer function is characterized by a well formed, sharp resonance peak, indicating low energy loss due to damping and friction and by the significantly higher first bending resonance frequency than the electrostatically coupled system. The considerably lower eigenfrequencies registered in the electrostatically coupled system, compared to the system with glued CFRP elements indicate that a large potential for the improvement of the electrostatic coupling of the components of the beam is given. Ideally, it should be possible to attain the same level of stiffening of the beam using electrostatic coupling as with adhesive coupling. The reduction of the maximum values of the transfer functions and thus of the energy content of the system are explained by losses due to the damping behavior of the system, as detailed in the next section.

### 4.3.3 Damping

Section 4.3.2 shows that the application of an electrostatic field at the interface between the GFRP core beam and the CFRP stiffening UD bands has an effect on the first bending natural frequency of the structure. This property can be used to avoid the onset of resonance situations if the average frequency of a narrow band excitation coincided with the natural frequency of the unstiffened beam.

Figure 4.9 shows that due to the ability of the system to provide full shear stress trans-
Figure 4.9: The change of the transfer function measured at different $U_i$ levels and excitation current amplitudes indicates a change in the shear stress transfer at the interfaces of the system. The transfer function of a beam with laminated CFRP elements is plotted in black in the $I_0 = 1$ A graph for reference.
Figure 4.10: Overview of the eigenfrequency values of the system at different inter- 
 laminar voltage levels and current amplitudes (left) and of the maximum values \( \max(|G|) \) 
(right). For low excitation currents the lowest value of \( \max(|G|) \) is at intermediate voltage 
levels, while for high excitation currents, the minimum value coincides with the highest 
voltage levels that could be applied.

fer at the interfaces only within a small range, an approximately elastic behavior of 
the stiffened structure is observed at only high tensions and low excitation amplitudes. 
Specifically, only in the measurements with the lowest excitation amplitude (0.5 A) at the 
highest interlaminar potential, the shape of the resonance peak (with a higher maximum 
and sharper features) indicates a predominantly elastic behavior.

The energy dissipation in the system is due to material damping as well as to the friction 
that occurs at the interfaces between the layers. Since the normal stress is proportional 
to the square of the electrostatic field \( U^2 \) and the shear stress occurring at the interfaces 
depends on the amplitude of the vibration, the damping of the system is expected to 
vary depending on the amplitude of the excitation as well as on the voltage applied 
between faces and core of the sandwich.

Variable damping systems based on the use of magneto-rheological fluids are in use 
and under development for a variety of applications ranging from so called smart shock 
absorbers in cars to the damping of stay cable vibrations [50]. The development of opti- 
mal damping strategies using magneto-rheological dampers for stay cables is based on 
the observation that a setting of the damper that is too stiff will reduce the effectiveness 
of the device, while a setting that is too soft will not dissipate as much energy as would 
be possible for a given vibration velocity and amplitude.

The damping behavior of the system was investigated in experiments in which the sys- 
tem was excited with a given current amplitude \( I_0 \) (varying between 0.5 A and 3.0 A) at
its measured eigenfrequency for that excitation level. The excitation was then switched off and the acceleration at the free end of the cantilever was measured as it subsided. The set up for the experiments was the same as shown in figure 4.7.

The plots in figure 4.11 show the displacement at the free end of the beam and the envelope of the displacement curve for four of the experiments performed to investigate the damping behavior of the beam. The envelope of the curve corresponds to the instantaneous amplitude of the harmonic oscillation of the system. The envelope of the time displacement curve is calculated as the absolute value of the Hilbert transform $H(v(t))$ of the curve $v(t)$. In figure 4.11 it is plotted against the time $t$. The portion of the curve shown in the figure is the one used to study the damping behavior of the system. In the experiments, the excitation current was switched off immediately before the beginning of the shown plots.

The envelope of the amplitude-decay portion of the data was calculated for all the measured curves and is shown in figure 4.12, in its logarithmic form as a function of the number of cycles, to compensate for the different frequencies at which the measurement were performed. As expected based on the results shown previously, the curves measured with no interlaminar stress (0 V curves) present the highest displacement amplitudes. In figure 4.12 can be noticed how in the first graph on the top left (excitation current amplitude 0.5 A) the displacement amplitudes of the curves with low to intermediate interlaminar voltage ($U_i=0$ V ... $U_i=300$ V) show a faster decay than the high voltage curves.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.11}
\caption{Displacement as a function of time and in red its envelope (top) and excitation current as a function of time for a weakly excited beam and no interlaminar potential.}
\end{figure}
For the intermediate voltage values \(U_i=200\,\text{V}\) and \(U_i=300\,\text{V}\), the absolute value of the amplitude at the beginning of the decay curve is only marginally higher than the amplitude of the high voltage curves. Due to the higher slope of the former, the displacement amplitudes drop to lower values than for the high voltage levels \(U_i=400\,\text{V} \ldots U_i=800\,\text{V}\), within a few cycles from the beginning of the measurement. It follows that for small excitation amplitudes, the system displays a more efficient damping behavior at intermediate voltages than at the highest and lowest settings. At higher amplitudes, this behavior changes and the high voltage curves gradually (as the excitation amplitude grows) show a better performance in terms of overall reduction of the displacement amplitude than the low voltage curves.

In order to quantify the amount of damping present in a structure, it is customary to calculate the logarithmic decrement of the amplitude decaying vibration curve. This parameter is calculated under the assumption that the envelope of the curve has an exponential form, which is the commonly observed behavior for material damping. The logarithmic decrement \(\lambda\) of a decaying vibration is given by quotient of the amplitude \(v_0\) of the vibration at any given cycle and the amplitude \(v_n\) \(n\) cycles later as in (4.15).

\[
\lambda = \frac{1}{n} \ln \frac{v_0}{v_n}
\]  

(4.15)

In the plots shown in figure 4.12, an exponential decay of the vibration would be represented as a straight line with slope \(-\lambda\). Figure 4.13 shows the effect of the interlaminar voltage \(U_i\) on the shape of the decay curves. An exponential decay (plotted in red) represents the commonly measured material viscous damping behavior. A linear decay (plotted in green) describes a system in which energy is dissipated by coulomb friction.

As it can be seen in examples shown in figure 4.13, the exponential decay curves are in general a better approximation of the measured behavior than the linear decay. However, for the higher ratios of exciting current amplitudes to interlaminar voltage (namely \(I_0=0.5\,\text{A}\) with \(U_i=0\,\text{V}\), \(I_0=3.0\,\text{A}\) with \(U_i=0\,\text{V}\) and \(I_0=3.0\,\text{A}\) with \(U_i=300\,\text{V}\)) it can be seen that there is a higher level of disagreement between the exponential decay approximation and the measured decay than for lower ratios. This points to a higher contribution of friction damping to the overall damping of the system. Here, neither a linear decay model nor an exponential decay model correctly describe the behavior of the system. Nevertheless, there is a need to describe the damping behavior of the presented structures in order to be able to calculate its dynamic behavior. Most numerical packages for the mechanical modeling of structures support the calculation of the damping in structures based on an exponential decay, so a global logarithmic decrement will be calculated over a representative portion of the decay curves for use
Figure 4.12: Logarithm of the envelopes $\ln(H(v(c)))$ of the measured displacement as a function of number of cycles $c$ for excitation amplitudes between 0.5 A and 3.0 A and interlaminar voltages between 0 V and 800 V
Figure 4.13: A comparison of the measured decay envelopes with their logarithmic and linear approximation, between the start point and end point for decay curves measured at the lowest and at the highest excitation level.
in numerical models.

The relationship equivalent to 4.15, expressed for the continuously calculated envelope 
\( H(v(c)) \) is:

\[
\lambda = \frac{\Delta(\ln(H(v(c))))}{\Delta(\text{cycles})}
\]  

(4.16)

Using the continuous envelope of the decay curves, it is possible to calculate a continuous logarithmic decrement curve that gives us more detailed information about the damping behavior of the system, which is especially useful given the difficulty fitting the decay curves with the commonly used approaches.

The logarithmic decrement \( \lambda \) was calculated for the beam in all test conditions (coil current amplitude varying between 0.5 A and 3.0 A and interlaminar voltage varying between 0 V and 800 V). The results are presented in figure 4.14, where \( \lambda \) is plotted as a function of the number of cycles after the excitation current was switched off. The circles at the right edge of each plot represent the global logarithmic decrement, for comparison. The continuous logarithmic decrement curves shown in the figure are calculated for the same portion of the decay curves used to calculate the global decrement values. As it can be seen, the spread of the values increases remarkably for the higher excitation current values.

Finally, figure 4.15 gives an overview of the effect of electrostatic coupling on the two mechanical properties that are most relevant to the vibration behavior of the described GFRP-CFRP beam. The figure shows how the eigenfrequency values display a low domain, for the non activated system and after a fairly steep increase that begins at \( U_i \) values between 200 V and 400 V they reach a high plateau value. Correspondingly roughly at \( U_i \) values that mark the beginning of the increase in eigenfrequency, the logarithmic decrement values show a maximum. This picture is compatible with the general idea that the maximum damping is obtained when a good level of coupling between the components of the system is given, but not sufficient shear stress transfer is possible to obtain a full bond between them.

The values shown in figure 4.15 will be used as a base for the numerical calculations of the behavior of a single GFRP-CFRP beam and of a GFRP-CFRP bridge deck presented in chapter 5.
Figure 4.14: The graphs show the calculated instantaneous logarithmic decrement as a function of the number of cycles after the begin of the amplitude decay. Results for different excitation amplitudes $I_0$ and interlaminar potentials $U_i$ are shown. The circles on the right hand side of the graphs represent the global logarithmic decrements calculated based on the whole decay curves.
Figure 4.15: The effect of electrostatic coupling on the first bending eigenfrequency and the logarithmic decrement of the described beam is a function of the interlaminar potential $U_i$ and the excitation amplitude $I_0$, as shown in the two plots of this figure.

### 4.4 The Effect of the Interface Topography

Sections 2.4 and 2.5 showed that the deviation of the surface topography from a perfect plane at an atomic level and at a microscopic level have noticeable consequences on the interaction between interfaces. At a macroscopic level, the observation of the interfaces of a GFRP-CFRP beam used for preliminary tests, showed that after several million load cycles at varying amplitudes and interlaminar stress levels signs of wear of the dielectric layer applied to the GFRP structures were very localized, as shown in figure 4.16. In this section an estimation of the effect of the unevenness of the surfaces of the GFRP base structure on the ability of the CFRP stiffening elements to conform to the beam is made.

The beam shown in the figure was manufactured using a different technique than the one used for the beam presented in the previous sections. The dielectric layer was a 0.08 mm thick Polyvinylidene Fluoride (PVDF) film that was chosen for its higher dielectric constant ($\epsilon \approx 2...8$, depending on the polarization of the film). Since no metallized film was available, a separate electrode layer had to be applied between the GFRP host structure and the PVDF film to obtain the desired experimental set up. In order to optimize the electrostatic performance of the system, an electrically conductive adhesive was chosen to serve both as electrode and adhesive. This choice made it possible to nominally reduce the gap between the electrode applied to the GFRP beam and the CFRP elements to the thickness of the PVDF film. Due to the high viscosity of the silver particle loaded adhesive, it was not possible to obtain a sufficiently planar interface.
Figure 4.16: A view of the top flange of a GFRP beam, used in a tunable GFRP CFRP structure as described in the previous sections, after approximately two years of (non-continuous) operation. The areas marked in black presented clear signs of wear due to friction between the surface of the dielectric layer with the CFRP bands. All other areas appeared to be in pristine condition.

between the elements of the structures.

As a consequence of this situation, only a small fraction of the apparent area of contact (i.e. the length of the beam × the width of the flange) was in the condition to transfer shear stress between the components. The localization of the wear signs on the PVDF film surface points to the lack of planarity as a likely cause of decreased performance of the electrostatic coupling between the GFRP beam and the CFRP elements.

The ability of the CFRP elements to conform to the topography of the GFRP beam surface depends on their bending stiffness and on the radius of curvature of the topographical surface features. Although for the purpose of calculating the global stiffness of the structure, the bending stiffness of the CFRP elements can be neglected, this property is of interest in the local scale.
Peaks and valleys over a short distance (e.g. a few centimeters) are thus more likely to reduce the area of contact between the GFRP beam and the CFRP stiffening elements. The above observations indicate that local unevenness of the surface in this scale may represent a major source of loss in contact area, as shown in figure 4.16: Due to its stiffness, the CFRP elements cannot conform to such topographical features over a short distance, while 'long wave' features can be compensated for, as shown in figure 4.17.

In order to quantify the adverse effect of uneven surfaces on the contact stress between elements of a multi-layer stiffness, we shall consider a simple situation, as the one depicted in figure 4.18.

In figure 4.18, a CFRP element with bending stiffness $EI_{CFRP}$ is bent to conform to a local feature with radius $\rho$. The bending moment necessary to obtain the curvature $\kappa$ of the CFRP elements is given by the well known moment/curvature relation (4.17) [9]:

$$\kappa(x) = \frac{1}{\rho(x)} = \frac{M_b(x)}{EI_{CFRP}}$$  \hspace{1cm} (4.17)

Where $M_b$ is the bending moment acting on the element and $EI_{CFRP}$ is the bending
Figure 4.18: The CFRP stiffening element must adapt to the topography of the host structure, so that contact at the interfaces is possible and shear stress can be transferred between the two components by means of friction.

stiffness of the element and $\rho$ is the radius of the curve described by the neutral axis of the CFRP element. (4.17) means that for a given stiffness of the CFRP element, the bending moment needed for it to conform to a feature is proportional to the curvature of the feature.

From (4.17) and the situation depicted in figure 4.18, the order of magnitude of the distributed load that is necessary to maintain the curvature $\kappa$ can be easily calculated as $M_b = EI_{CFRP}/\rho$. The load applied to the CFRP elements, so that they conform to the surface of the GFRP beam and are pressed against it, is generated by the electrostatic field applied between the electrodes of the system. In the following calculations it will be assumed that the CFRP bands have already been deformed to conform to the surface of the structure. The distributed force needed to keep the bands in their deformed state is then calculated as a function of the bending stiffness of the CFRP elements and the length of the topographic feature the band is adapting to, for a fixed depth of the feature of 0.01 mm. Also, in order to eliminate unnecessary complexity, it is assumed that the profile of the topographic feature is described by the equation:

$$y(x) = c \cdot (x^4 - 2Lx^3 + L^2x^2)$$  \hspace{1cm} (4.18)

which corresponds to the bending line of a homogeneously loaded beam fixed at both
ends, as is the case for the CFRP band. Specifically, the deformed shape of the CFRP band subjected to distributed load $\sigma_{yy}^{el}$ is given by:

$$v(x) = \frac{\sigma_{yy}^{el}}{24 \cdot EI_{CFRP}} \cdot \left( x^4 - 2Lx^3 + L^2x^2 \right)$$

(4.19)

A few transformations of (4.19) yield the relationship between the distributed load $\sigma_{yy}^{el}$ necessary to keep the CFRP element deformed so as to conform to the surface profile, the length $L$ of the feature and the stiffness $EI_{CFRP}$ of the CFRP element, for a given maximum depth of the feature:

$$\sigma_{yy}^{el} = 384w_{max} \frac{EI_{CFRP}}{L^4}$$

(4.20)

Figure 4.19 shows the load needed to maintain the contact between a CFRP element with an indentation described by (4.18) and a depth $v_{max} = 0.1mm$, as described in figure 4.18.

The above considerations show that topography of the contact surfaces is thus an important parameter for the successful design of electrostatically coupled multi-layer structures. In order to increase the smoothness of the contact surfaces following measures should be adopted:

- The use of more refined methods such as autoclave curing to temporarily reduce the viscosity of the epoxide adhesive during curing (or the use of lower viscosity materials at room temperature).
- The application of smooth molds to the surface during curing will improve the quality of the surfaces, while applying a vacuum to remove excess resin.
- The use of metallized polymer films instead of a combination of plain polymer film and a metal particle loaded adhesive, would eliminate the need of such high viscosity adhesives.

## 4.5 Conclusions

The experiments presented in this chapter were designed so as to demonstrate the most remarkable properties that were predicted by the models of multi-layer systems.
In the first set of experiments (section 4.2 proof of concept), the ability to stiffen a simple structure by coupling stiffening elements electrostatically was proven using a structure with extreme properties (very compliant core and faces with very high axial stiffness). Also, the hysteretic behavior of the system, when loads are applied beyond the ability of the interfaces to transfer shear stresses was shown. This is of one of the properties that were predicted by the numerical model of a multi-layer beam. Furthermore it was possible to demonstrate the relationship that exists between maximum external load under which the system behaves linearly and the interlaminar stress.

The experiments described in section 4.3 served various purposes:

The preparation of the test set-up was an opportunity to improve the techniques for the manufacturing of electrostatically tunable devices. Next to the selection of appropriate materials, The procedure showed some of the difficulties that are encountered in the

Figure 4.19: The distributed load needed for the CFRP element to conform to the topographical features of the GFRP beam for a given feature depth increases steeply for short length features. The red line shows the approximate electrostatic stress that can be generated across an 80 µm thick PVDF film applying a potential of 3000 V or across a 12 µm thick PET film applying a potential of 800 V.
manufacturing of electrostatically coupled multi-layer beams. The main difficulty in the fabrication by hand is to obtain a sufficiently planar surface of the interfaces. In the case of the first beam prototype, an electrode needed to be applied to the surface of the insulating GFRP beam. In order to minimize the thickness of the insulating layer between the electrodes, a silver epoxide was chosen as an electrode that would serve also as an adhesive to mount the dielectric layer on the beam (see figure 4.7). The electrode was applied by hand, then the PVDF film was laid on it and pressed using a rubber roller to remove air inclusions.

This procedure yielded a fairly smooth electrode/insulator layer, but the topography of the electrode was still sufficiently pronounced. So, after about two years of operation it became clear from the wear marks that only one relatively small portion of the area of the dielectric layers was in contact with the CFRP elements, as shown in figure 4.16.

The second beam prototype that was used for the bulk of the presented experiments was manufactured using a metallized PET film and epoxide resin. While no autoclaving was used in the process, a qualitative improvement of the surface could be observed. Quantitative methods to measure the topography of the surface have yet to be applied.

Two aspects of practical relevance were studied in the presented experiments:

- At sufficiently small excitation amplitudes, a modification of the natural frequency of the element could be obtained while maintaining a similar quality factor as in the case of the unstiffened beam. This indicates that the effect of dissipative processes under these conditions is modest, as shown in figure 4.9.

- The dissipative effects suggested by the hysteretic behavior observed in the numerical simulations and in the experiments presented in section 4.2 could be confirmed by the observation of the dynamic behavior of the CFRP-GFRP beam at high excitation amplitude/interlaminar potential ratios. The measurements of the decay of the vibration amplitude presented in section 4.3.3 showed the electrostatic coupling of the CFRP elements to the beam also leads to a remarkably high damping of the system, under appropriate conditions. The effect of the friction damping is optimal only within a quite narrow window of vibration amplitudes, appropriate control systems will be necessary to optimize the energy dissipation over a wider window of amplitude, as shown by the results presented in figure 4.15.

The main goal of the experiments carried out on the CFRP-GFRP system was to demonstrate that the successful use of electrostatic coupling to suppress vibrations
is possible on elements that are used in real structures, such as the beam presented in the section. Based on the acquired information, the scaling up of this method will be performed on Empa’s GFRP pedestrian bridge deck, as described in the next section.
Chapter 5

A Practical Application of Elements with Tunable Mechanical Properties

The stiffness change and the damping properties determined for the CFRP-GFRP sandwich beam in the previous chapter are at the basis of the estimation of the properties of a composite bridge deck presented in this chapter. The goal of the application of electrostatic coupling of CFRP stiffening elements to the GFRP structure of a footbridge deck is the reduction of the amplitude of the vibrations that such lightweight structures are prone to. In the first section of this chapter, a homogenized finite element model of the beam discussed in the previous chapter is set up. The elastic and damping properties of the orthotropic beam are set so that the frequency response of the modeled beam corresponds to the measured frequency response. The second part of this chapter shows, based on calculations performed using a numerical model of the bridge in question, the effect that the modification of the elastic and damping is expected to have on the frequency response calculated for the bridge deck.

5.1 Suppression of the Vibrations of a GFRP Footbridge Deck

Light weight, slender bridges are prone to high amplitude vibrations due to their reduced mass and high degree of slenderness. A full scale pedestrian cable stayed bridge with a bridge deck composed of GFRP beams similar to the one described in section 4.3 was designed and built so as to recreate the conditions that are typically the origin of considerable vibration problems in pedestrian bridges. The structure shown in figure
5.1 is described in detail in [26] and is intended as a research platform that can be used in the investigation of vibration damping strategies and structural health monitoring systems.

The experiments described in the previous chapter have demonstrated that a combination of stiffening and increased damping is achieved when an electric field is built up between the interfaces. Under such conditions the system is stiffened. When the interfacial shear stresses between layers exceed the maximum shear stress that can be transmitted by means of friction, increased damping can be observed, as shown by the plots of figure 4.9. Under these conditions, interfacial slip occurs and energy is dissipated by Coulomb friction at the locations where high relative displacements are found.

This combined approach to the reduction of structural vibrations presents a few interesting advantages over traditional systems based on discrete devices such as conventional viscous dampers or tuned mass dampers:

- Depending on the nature of the excitation, vibrations can be suppressed either by a change of stiffness of the system (i.e. no energy transfer to or from the structure is allowed for) or by the introduction of friction damping into the system.

- Since the electrostatically coupled elements can be distributed over the whole length of the structure, a more efficient energy extraction is possible. The distributed nature of the system allows for a robust approach to damping that is less sensitive to the location of the nodes of vibration modes.

![Figure 5.1: Empa's full scale lightweight pedestrian bridge is used as a test platform for structural health monitoring and vibration mitigation developments. Left, general view. Right, detail from the bottom](image)
• Damping is only introduced as needed and the amount of damping can be controlled, if an appropriate control system is implemented.

The application of electrolaminated stiffening elements for the damping of lightweight structures such as the Empa laboratory footbridge is thus an appealing method for the suppression of their vibrations.

The implementation of this method for the damping of the deck of the footbridge is planned as a follow up to the work presented here. The plan is to embed the stiffening CFRP elements into the flange of the longitudinal GFRP beams shown in the right hand side picture of figure 5.1, as shown in the sketch of figure 5.2. Next to considerations made in the previous chapters, practical issues need to be addressed in order to put the plan into practice.

Figure 5.2: Embedded, electrostatically couplable CFRP elements for the stiffening and damping of the GFRP bridge deck
The experience gathered with the adaptive retrofitting of the GFRP beam shows that the quality of the interface between the host structure (GFRP beams with laminated electrodes and dielectric layer) is of great importance, especially in terms of the local planarity of the contact surfaces. Appropriate machining of the surfaces and lamination of the electrodes and dielectric materials will be necessary to obtain the maximal contact area (i.e. as close as possible to 100% of the apparent area of contact) between GFRP beam and CFRP stiffening elements. All joining details of the GFRP host structure (e.g. the screw connections in the flanges highlighted in the righthand photograph of figure 5.1) will have to be adapted to the new geometry of the beams (shown in figure 5.2), because of the space requirements set by the embedded CFRP elements.

In this pilot phase, the adaptive retrofitting of the bridge is expected to be labor intensive (and thus expensive), because all modifications to the existing structural elements will have to be performed by hand. Hence, it is necessary to estimate the benefits of the stiffening and damping on the vibratory behavior of the bridge obtained through the proposed retrofit prior to beginning its construction. A numerical model of the system is a viable way to obtain this information. In a first step, a model of the beam presented in the previous chapter is set up and used to estimate the mechanical parameters by comparing the measured and calculated values for the first bending eigenfrequency and adapting the mechanical parameters of the model accordingly. In a second step the behavior of the bridge is calculated by applying the estimated parameters to the numerical bridge model.

### 5.2 Models

As in section 3.1.2, the models presented in this section were calculated using Comsol 3.4. The package used for this work is not specialized for mechanical applications and thus only offers limited freedom in the selection of element types and mechanical material models. Since only the bending behavior will be considered for the calculations presented in this work, two dimensional models are sufficient to investigate the behavior of the structures under consideration. Given the geometry of the systems, plain strain is assumed for the calculations.

In order to reduce the complexity of the model and hence the computing time, homogenized models are used. In order to render the shear compliance of the structure that is derived from the shape of the cross section as well as the inherent properties of the materials, an orthotropic material model had to be used, so that independent input
for the tensile and shear properties could be given. The essential properties of the structures are given by two geometrical quantities (cross-sectional area $A$ and second moment of area $I$), their weight, and material properties of the orthotropic structure: $E_x$, $E_y$, $E_z$ and $G$ ($G_{xy}$), the elastic modulus in the direction of main axis of the beam, the two moduli perpendicular to the axis and the shear modulus, respectively. Given the bi-dimensional nature of the model, $G_{xz}$ and $G_{yz}$ are not considered in the model. The model is hardly sensitive to changes in $E_y$, $E_z$ for the load case considered here.

The Poisson ratios $\nu_{xy}$, $\nu_{xz}$, $\nu_{yz}$ were assumed to be 0.33.

### 5.2.1 Undamped Behavior of the Beam

In this section a numerical model of the beam presented in section 4.3 is described. The model is meant to represent the dynamic behavior of the structure in its two extreme states, i.e. when the stiffening CFRP elements are not bonded to the beam (low stiffness state) and when they are laminated to the beam using epoxide adhesive (high stiffness state). Additionally, in section 5.2.2 the effect of damping will be considered based on the properties calculated in section 4.3.3.

The homogenized model of the beam has the same bending stiffness $E_x \cdot I_z$, core shear stiffness $G \cdot A_{core}$, outer dimensions and weight per unit length as the beam used in the experiments, as shown in figure 5.3.

The eigenfrequencies calculated with this model are compared to the ones obtained in the presented experiments. The discrepancies between the results of the numerical
calculation and the experiment are resolved by a simple model updating [19] procedure, by which the elastic constants used in the model are modified so as to bring the model results to coincide with the experimental results (see figure 4.9, \( I_0 = 1.4 \)). Using this approach, the deviations of the behavior of the tested system from the expected behavior are thus ‘packed’ into the properties of the material defined in the model.

Additionally to the density of the beam, two discrete masses have to be considered in the model:

- The permanent magnet attached to the free end of the beam, used to obtain the excitation force in combination with the current coil placed under it (see figure 4.7).
- Two additional masses positioned near the web, 2090 mm from the fixation of the beam, used to separate the first bending mode from the first torsional mode of the beam, as explained in section 4.3.1.

At its fixation, the beam was clamped between two steel plates on a length of 250 mm. In order to guarantee the electrical insulation of the system from the environment, two thin PVC plates (2 mm thickness) were inserted between the beam and the clamping plates. The low stiffness of the PVC plates increased the compliance of the system thus contributing to the reduction of its bending eigenfrequencies. This property of the experimental set-up was accounted for in the numerical model by introducing two thin low stiffness plates \( E_{PVC} = 1 \text{GPa} \) in correspondence to the clamped area and setting the appropriate boundary conditions, as shown in figure 5.4.

The updated model will subsequently be scaled up and extended to estimate the behavior of Empa’s footbridge after the planned adaptive upgrade, as described in the previous section.

![Figure 5.4: Geometry and boundary conditions of the homogenized beam model. The mechanical properties assumed for the model components are listed in tables 5.1 and 5.2](image)
The properties of the homogenized beam are rendered in the model as shown in table 5.1. The measured and the calculated first bending eigenfrequencies \( f_{1\text{bending}} \) are shown at the bottom of the table the columns for the I-beam and the homogenized beam, respectively.

The eigenfrequency of the system is initially calculated using the parameters shown in table 5.1. The discrepancies between the calculated and the measured values for the first bending eigenfrequencies in the low (non laminated) and the high stiffness (laminated) states were resolved by updating the elastic constants \( E^h_x \) and \( G_{xyh} \) in the model until an acceptable agreement between the measured and the calculated eigenfrequencies was obtained. Both elastic constants were scaled by the same factor for the low stiffness state of the beam. To describe the stiffened state, only \( E^h_x \) was modified, as the shear compliance of the beam is concentrated in the web. The updated mechanical properties are listed in table 5.2. The corresponding calculated first bending eigenfrequencies after the updating of the material properties are in good agreement with the measured ones. Higher eigenfrequencies could not be determined experimentally, so that no comparison was possible. The updated model is therefore an acceptable representation of the actual beam.

The contribution of the beam and the CFRP elements to the overall stiffness of the system is calculated as follows: In the low stiffness state, the contribution of the CFRP elements is considered to be negligible, due to their very small second moment of area. The stiffness of the system in this state is considered exclusively due to the GFRP host structure. The difference between the frequency response in the low stiffness state and in the high stiffness state is attributed to the contribution of the CFRP elements. The updated elastic modulus of the CFRP bands is calculated dividing the stiffness change by the second moment of area of the laminated bands.

For both stiffness states, the elastic properties are lower than the ones assumed initially. In the low stiffness state, the difference between the elastic modulus stated by the manufacturer and the updated modulus is in proportion smaller (approximately 13%) than in the case of the CFRP bands (approximately 36%). In general, the deviations can be attributed to the short length of the fixation of the cantilever, leading to a higher compliance of the system, the inaccuracy of the stated material properties, and measurement inaccuracies.

Finally, it is possible to estimate a ratio between the stiffening obtained by electrolamination of the CFRP elements and the maximum attainable stiffening based on the homogenized model and the results shown in table 5.2. The maximal attainable eigen-frequency by electrolamination is 12.30 Hz, this corresponds to an increase of only 31%
Table 5.1: Properties of the mechanical components of the investigated sandwich beam, in the main axis, as declared by the suppliers and properties assumed in the numerical model, also in the main axis. The elastic moduli in the transversal directions ($E_{y}$ and $E_{z}$ were assumed to be 1/3 of the modulus in the main axis). $f_{1\text{, bending}}$ is the first bending eigenfrequency measured or calculated, for the I-beam or for the homogenized beam, respectively.

<table>
<thead>
<tr>
<th>Property</th>
<th>I-beam, faces not laminated</th>
<th>Model Beam, faces not laminated</th>
<th>I-beam, laminated faces</th>
<th>Model Beam, laminated faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [m]</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>Height [m]</td>
<td>0.123</td>
<td>0.123</td>
<td>0.123</td>
<td>0.123</td>
</tr>
<tr>
<td>Width [m]</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>$E_{\text{faces}}$ [Pa]</td>
<td>$2.10 \times 10^9$</td>
<td>-</td>
<td>$2.10 \times 10^9$</td>
<td>-</td>
</tr>
<tr>
<td>$E_{\text{beam}}$ [Pa]</td>
<td>$2.3 \times 10^9$</td>
<td>-</td>
<td>$2.3 \times 10^9$</td>
<td>-</td>
</tr>
<tr>
<td>$G_{\text{xy}}$ [Pa]</td>
<td>$3 \times 10^9$</td>
<td>-</td>
<td>$3 \times 10^9$</td>
<td>-</td>
</tr>
<tr>
<td>$I_{\text{faces}}$ [m$^4$]</td>
<td>$2 \times 1.37 \times 10^{-11}$</td>
<td>-</td>
<td>$2 \times 3.10 \times 10^{-7}$</td>
<td>-</td>
</tr>
<tr>
<td>$I_{\text{beam}}$ [m$^4$]</td>
<td>$3.1 \times 10^{-6}$</td>
<td>-</td>
<td>$3.1 \times 10^{-6}$</td>
<td>-</td>
</tr>
<tr>
<td>$A_{\text{web}}$ [m$^2$]</td>
<td>$6.48 \times 10^{-4}$</td>
<td>-</td>
<td>$6.48 \times 10^{-4}$</td>
<td>-</td>
</tr>
<tr>
<td>$EI_{\text{total}}$ [N m$^2$]</td>
<td>$7.13 \times 10^4$</td>
<td>$7.13 \times 10^4$</td>
<td>$2.01 \times 10^5$</td>
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<tr>
<td>$GA_{\text{total}}$ [N]</td>
<td>$1.95 \times 10^6$</td>
<td>$1.95 \times 10^6$</td>
<td>$1.95 \times 10^6$</td>
<td>$1.95 \times 10^6$</td>
</tr>
<tr>
<td>$I_{h}$ [m$^4$]</td>
<td>-</td>
<td>$9.3 \times 10^{-6}$</td>
<td>*</td>
<td>$9.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>$A_{h}$ [m$^2$]</td>
<td>-</td>
<td>$7.38 \times 10^{-3}$</td>
<td>-</td>
<td>$7.38 \times 10^{-3}$</td>
</tr>
<tr>
<td>$E_{h}$ [Pa]</td>
<td>-</td>
<td>$7.7 \times 10^9$</td>
<td>*</td>
<td>$21.6 \times 10^9$</td>
</tr>
<tr>
<td>$G_{h}$ [Pa]</td>
<td>-</td>
<td>$263.4 \times 10^6$</td>
<td>-</td>
<td>$263.4 \times 10^6$</td>
</tr>
<tr>
<td>Weight/unit length[kg/m]</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
</tr>
<tr>
<td>$f_{1\text{, bending}}$ [Hz]</td>
<td>$10.40(l_0 = 1A)$</td>
<td>11.36</td>
<td>$15.33(l_0 = 1A)$</td>
<td>17.92</td>
</tr>
</tbody>
</table>

$^1$The contribution of the faces to the overall moment of area depends on the location of the neutral axis (within the faces, in the inactive state or coinciding with the neutral axis of the beam in the activated state.)
Table 5.2: Properties of the mechanical components of the investigated sandwich beam, as calculated from the updated numerical model. The elastic moduli in the transversal directions \( E_{h}^{x} \) and \( E_{h}^{z} \) were assumed to be 1/3 of the modulus in the main axis. \( f_{1,bending} \) is the first bending eigenfrequency measured or calculated, for the I-beam or for the homogenized beam, respectively.

<table>
<thead>
<tr>
<th>Property</th>
<th>I-beam, faces not laminated</th>
<th>Model Beam, faces not laminated</th>
<th>I-beam, laminated faces</th>
<th>Model Beam, laminated faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [m]</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
<td>2.24</td>
</tr>
<tr>
<td>Height [m]</td>
<td>0.123</td>
<td>0.123</td>
<td>0.123</td>
<td>0.123</td>
</tr>
<tr>
<td>Width [m]</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>( E_{h}^{x} ) [Pa]</td>
<td>-</td>
<td>6.6.10^{9}</td>
<td>*</td>
<td>15.6.10^{9}</td>
</tr>
<tr>
<td>( G_{xy} ) [Pa]</td>
<td>-</td>
<td>223.9.10^{6}</td>
<td>-</td>
<td>223.9.10^{6}</td>
</tr>
<tr>
<td>( I_{h} ) [m^4]</td>
<td>-</td>
<td>9.3.10^{-6}</td>
<td>*</td>
<td>9.3.10^{-6}</td>
</tr>
<tr>
<td>( A_{h} ) [m^2]</td>
<td>-</td>
<td>7.38.10^{-3}</td>
<td>-</td>
<td>7.38.10^{-3}</td>
</tr>
<tr>
<td>( E_{beam} ) [Pa]</td>
<td>19.8.10^{9}</td>
<td>2.6.10^{9}</td>
<td>19.8.10^{9}</td>
<td>2.6.10^{9}</td>
</tr>
<tr>
<td>( G_{web,xy} ) [Pa]</td>
<td>2.6.10^{9}</td>
<td>135.10^{9}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( E_{faces} ) [Pa]</td>
<td>210.10^{9}</td>
<td>210.10^{9}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Weight/unit length[kg/m]</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
<td>2.92</td>
</tr>
<tr>
<td>( f_{1,bending} ) [Hz]</td>
<td>10.40(I_{0} = 1A)</td>
<td>10.42</td>
<td>15.33(I_{0} = 1A)</td>
<td>15.43</td>
</tr>
</tbody>
</table>

\(^{1}\) The contribution of the faces to the overall moment of area depends on the location of the neutral axis (within the faces, in the inactive state or coinciding with the neutral axis of the beam in the activated state.)
of the difference in stiffness between the beam with non laminated CFRP elements and the beam with laminated CFRP elements. This level of stiffening was attained at the lowest level of exciting current ($I_0 = 0.5A$). At high excitation levels, the shear stress transfer at the interfaces is not sufficiently effective to obtain the same effect as at low amplitudes. A more modest increase in stiffness is observed, while in return higher damping levels could be attained.

### 5.2.2 Damped Behavior of the Beam

The previous section showed how the elastic parameters of the beam were updated in the model, so as to obtain the same values for the 1st bending eigenfrequency as for the investigated system. The parameters obtained through this procedure will be used to estimate the behavior of the GFRP bridge deck in the next section. The previous chapter also showed that the increase in the logarithmic decrement $\lambda$ that can be achieved by activating the interfaces is noticeable and that it contributes to the reduction of the vibration amplitude of the beam in a significant way. The loss factor $\eta$ is the commonly used parameter to describe the damping behavior of a structure or material. The loss factor is calculated from the logarithmic decrement $\lambda$ as:

$$\eta = \frac{\lambda}{\pi}$$

Figure 4.9 shows that the strongest attenuation due to the application of the potential $U_i$ is observed for the highest excitation current $I_0$, with $U_0 = 500V$. In this section, the $\eta$ and the stiffening values in the numerical model for an acceptable fit of the calculated frequency response curves to the frequency responses measured at $I_0 = 3A$ with interlaminar voltages $U_0 = 0V$ and $U_0 = 500V$ will be determined. These data will then be used for calculating the effect of the activation of the GFRP-CFRP interfaces on the behavior of the bridge deck.

The maximal attainable eigenfrequency by electrolamination at $I_0 = 3.0A$ is 11.35 Hz, this corresponds to an increase of 17% of the difference in stiffness obtained by lamination. Based on the estimates shown in figure 4.15, the loss factor of the GFRP-CFRP beam can be varied approximately between 0.03 and 0.08, for $I_0 = 3.0A$. These values were used as a starting point for the calculation of the frequency response of the beam at $U_i = 0V$ and $U_i = 500V$, taking damping into account. The actual force acting on the beam at the position of the coil/permanent magnet assembly used to excite the beam is not known, as the system was not calibrated. This additional parameter was also
updated in a manual procedure. Figure 5.5 shows the frequency response curves calculated based on the $\eta$ values estimated in the previous section and an exciting force amplitude $F_0$ of 1 N. Based on the height and width of the low stiffness/low damping curve the loss factor for the low stiffness system was reduced to $\eta = 0.02$ and the exciting force to $F_0 = 0.53\, N$. The resulting $\eta$ value used to approximate the measurement performed at $U_i = 500\, V$ was 0.16.

As can be seen in figure 5.5, the frequency response curve calculated for the low stiffness system is a quite good approximation for the system at $U_i = 0\, V$. Instead, the high stiffness/high damping system is only partially approximated by the calculated frequency response curve, because the overall shape of the measured frequency response is non-linear. The reason for this behavior can be found in the onset of interlaminar slip once a certain amplitude is reached.

Nevertheless, the values for the increase in stiffness and the loss factor values found with the described procedure give satisfactory approximations of the behavior of the system at two different interlaminar voltage levels.

The values that will be used for the calculation of the frequency response of the GFRP bridge deck described in the next section are listed in table 5.3.

![Figure 5.5: Computed frequency response curves (dashed lines) before (left) and after (right) updating of the exciting force and damping parameters, compared to the measured frequency response curves (solid lines)
Table 5.3: Properties of the mechanical components of the investigated sandwich beam, as calculated from the updated numerical model considering damping.

<table>
<thead>
<tr>
<th></th>
<th>$U_i = 0V$</th>
<th>$U_i = 500V$</th>
<th>Laminated CFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta EI$</td>
<td>0%</td>
<td>17%</td>
<td>100%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.02</td>
<td>0.16</td>
<td>0.02</td>
</tr>
</tbody>
</table>

5.2.3 Bridge

The goal of the present work is to devise a method to effectively attenuate the vibration of structures by means of the electrolamination of stiffening elements. The numerical calculations presented in this section will show the effect that the vibration suppression method is expected to have on the dynamic behavior of the bridge deck. Since the calculations discussed in this section are meant as a preparation for the actual experiments, it is necessary to make a prediction of the expected behavior based on the experiments performed on the GFRP-CFRP beam. The extrapolations are based on the assumption that the same scaling factors for the material properties that were obtained through the model updating procedure used for the individual beam can be applied to the bridge deck. The increase in bending stiffness that can be obtained by laminating the CFRP elements to the GFRP structure is expected to have a smaller impact on the dynamic behavior of the deck than it had on the behavior of the single beam, due to the larger number of constraints given by the bearings and stay cables.

Figure 5.6 shows the simplified geometry used in the numerical model of Empa’s bridge. The geometrical model captures the most important features of the structure such as the overall size of the deck, the presence of three pairs of stays on the main span, the angles between the stays and the deck and the pylon respectively. Other features were rendered using equivalent properties, such as in the case of the three pairs of back stays that were replaced by one pair with the triple stiffness. The pylon was modeled only above the bridge deck, due to the limitations imposed by the two dimensional character of the model. Also, details like the pivoting anchorages of the stays in the bridge deck where disregarded. Especially the reduced length of the pylon and the stiff connection of the stays to the bridge deck lead to a stiffer overall behavior of the structure. The model is two-dimensional, since only the bending behavior of the bridge deck is considered. Accordingly, the geometry was discretized using triangular and rectangular 2D elements, as indicated in figure 5.6.

The bridge deck is made of two sets of longitudinal GFRP I-profiles similar to the beam
Figure 5.6: Simplified geometry used for the numerical modeling of Empa’s cable stayed pedestrian bridge. The red arrow marks the position at which the displacement is sampled to calculate the frequency response of the structure. In the red circles are examples of the 2D elements used to discretize the structures. Triangular and rectangular 2D elements were used.

described above and two sets longitudinal GFRP U-profiles. The joints along the longitudinal axis are not considered in the model, although they are expected to reduce the overall stiffness of the deck. The total length of the deck is 19.2 m. The height of the profiles is 200 mm and the width 100 mm for the I-profiles and 60 mm for the U-profiles. The thickness of all profiles is 10 mm. Unlike the beam described in the previous sections, the 1.4 mm thick CFRP elements are assumed to be embedded in the GFRP profiles as shown in figure 5.2. The width of the CFRP elements is assumed to be 60 mm. The homogenized bridge deck is assumed to be 1.6 m wide.

The mechanical properties of the GFRP bridge deck are summarized in table 5.4. The total bending and shear stiffness values of the beams, calculated based on the mechanical properties stated by the manufacturers of the GFRP beams and CFRP bands or found in literature are listed as $E_{th}$ and $G_{th}$, respectively.

The homogenized values for E-moduli and G-modulus used for the calculations take the updating factors calculated in sections 5.2.1 and 5.2.2 into account and are listed
in the table as $E_{upd.}$ and $G_{upd.}$, respectively. The row marked as $\Delta EI$ shows the change in bending stiffness in relationship to the maximum increase that could be obtained by laminating the CFRP bands: the stiffness of the deck with un laminated CFRP elements is defined as the base stiffness with a $\Delta EI$ of 0%. 100% corresponds to the value calculated for the laminated elements. The stiffness for the deck with electrolaminated CFRP elements is calculated as the base stiffness plus 17% of the difference between the two extreme stiffness values, based on the results shown in section 5.2.2. Additionally, the loss factors used for the calculations are listed at the bottom of the table.

Figure 5.8 shows the frequency response functions calculated for the bridge deck (at the position marked in figure 5.6) in three different states: unstiffened, with laminated CFRP elements and with CFRP elements electrolaminated at $U_i = 500 \text{V}$, based on the properties extrapolated as per table 5.4. As expected, the frequency shift achieved by the increase in bending stiffness is quite modest, due to the constraints set by the bearings and the stay cables. A shift of the eigenfrequency of approximately 0.4 Hz obtained by laminating the CFRP elements could only be an effective instrument for the reduction of the vibration amplitude of the bridge deck, if the spectrum of the exciting force is quite narrowly distributed.

While the response of the electrolaminated system shows an even more modest shift of the eigenfrequency (0.2 Hz) than the laminated system, the maximum acceleration amplitude is greatly reduced (by a factor of six to eight, compared to the other frequency response curves). As expected, the introduction of a significantly higher damping has a marked effect also on the highly constrained structure.
Table 5.4: Properties of the mechanical components of the investigated sandwich beam, as calculated from the updated numerical model. The elastic moduli in the transversal directions ($E_y$ and $E_z$ were assumed to be 1/3 of the modulus in the main axis). $f_{1,\text{bending}}$ is the first bending eigenfrequency calculated, for the homogenized bridge deck at different stiffness states.

<table>
<thead>
<tr>
<th>Property</th>
<th>Profiles, faces not laminated</th>
<th>Model Deck, faces not laminated</th>
<th>Profiles, laminated faces</th>
<th>Model Deck, laminated faces</th>
<th>Model Deck, electrolaminated faces (U,500 V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{\text{profiles}}$ [m$^4$]</td>
<td>7.9·10$^{-5}$</td>
<td>-</td>
<td>7.9·10$^{-5}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$A_{\text{webs}}$ [m$^2$]</td>
<td>7.2·10$^{-3}$</td>
<td>-</td>
<td>7.2·10$^{-3}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$I_{\text{faces}}$ [m$^4$]$^1$</td>
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<td>-</td>
<td>6.9·10$^{-6}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$E_{\text{profiles}}$ [Pa]</td>
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<td>-</td>
<td>28·10$^9$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$G_{\text{webs}}$ [Pa]</td>
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<td>-</td>
<td>3·10$^9$</td>
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<tr>
<td>$E_{\text{h}}$ [Pa]</td>
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<td>2.22·10$^6$</td>
<td>3.63·10$^6$</td>
<td>3.63·10$^6$</td>
<td>-</td>
</tr>
<tr>
<td>$G_{\text{h}}$ [Pa]</td>
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<td>2.16·10$^7$</td>
<td>2.16·10$^7$</td>
<td>2.16·10$^7$</td>
<td>-</td>
</tr>
<tr>
<td>$E_{\text{upd.}}$ [N m$^2$]</td>
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<td>-</td>
<td>2.80·10$^6$</td>
<td>2.04·10$^6$</td>
</tr>
<tr>
<td>$G_{\text{upd.}}$ [N]</td>
<td>-</td>
<td>1.8·10$^6$</td>
<td>-</td>
<td>1.8·10$^6$</td>
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<td>$I_{\text{h}}$ [m$^4$]</td>
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<td>0.32</td>
<td>-</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>$E_{\text{h}}$ [Pa]</td>
<td>-</td>
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<td>-</td>
<td>2.63·10$^9$</td>
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<td>$G_{\text{h}}$ [Pa]</td>
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<td>66.6</td>
<td>66.6</td>
<td>66.6</td>
<td>66.6</td>
</tr>
<tr>
<td>$\Delta E_{\text{I}}$ [ ]</td>
<td>-</td>
<td>0%</td>
<td>-</td>
<td>100%</td>
<td>17%</td>
</tr>
<tr>
<td>$\eta$ [ ]</td>
<td>-</td>
<td>0.02</td>
<td>-</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>$f_{1,\text{bending}}$ [Hz]</td>
<td>-</td>
<td>7.34</td>
<td>-</td>
<td>7.78</td>
<td>7.54</td>
</tr>
<tr>
<td>$a_{0,\text{max}}$ [ms$^{-2}$]</td>
<td>-</td>
<td>13.6</td>
<td>-</td>
<td>15.8</td>
<td>1.9</td>
</tr>
</tbody>
</table>

$^1$ The contribution of the faces to the overall moment of area depends on the location of the neutral axis (within the faces, in the inactive state or coinciding with the neutral axis of the beam in the activated state.)
Figure 5.8: Frequency response functions calculated based on the properties listed in table 5.4, for the bridge deck with unlaminated, electrolaminated and laminated CFRP stiffening elements, respectively. For the laminated and unlaminated cases, the loss factor $\eta$ was assumed to be 0.02 while for the electrolaminated case, the loss factor was assumed to be 0.16, as determined in the previous section.

5.3 Conclusions

In this chapter, two simple numerical models of the GFRP-CFRP cantilever beam investigated in chapter 4 and of Empa’s pedestrian bridge, respectively were presented. By comparing the response of the modeled beam with the measured response, the relevant mechanical properties of the materials implemented in the numerical model were adapted so as to obtain a good agreement between model and experiment. Subsequently, the properties were extrapolated for the GFRP bridge deck. The calculated first bending eigenfrequency for the unstiffened system (7.34 Hz) is in fairly good agreement
with the value of 6.59 Hz stated in literature [26], especially considered the simplifications made in the model that are expected to lead to a stiffer behavior than in the real structure.

As with any extrapolation, the correctness of the predicted behavior can only be confirmed by experiments. Nevertheless, the results shown in figure 5.8 are in agreement with the expectation that the frequency shift obtained by lamination of the CFRP elements would be modest. The results obtained from the calculations for the system equivalent to the electrolaminated structure give reason to expect that the damping introduced by the interaction between the GFRP structure and the CFRP stiffening elements will have a substantial and beneficial effect on the dynamic behavior of the bridge.
Chapter 6

Conclusions and Outlook

6.1 Conclusions

The development of low density, high strength materials and composites (such as carbon fiber reinforced polymers) has made the design and construction of structures with increasingly high performance possible. Due to their reduced mass and intrinsic damping, such systems are more prone to vibrations than before, thus making their dynamic behavior likely to be an important limiting factor for their operations. The goal of the present work was to demonstrate the use of electrostatic forces between layers of a multi-layer structure as a means to modify its mechanical properties and thus its dynamic behavior.

Very limited work has been published on the topic of multi-layer system with variable mechanical properties, to date. At the time when the present project was being defined, only the work of Tabata [48] explicitly mentioned the use of electrostatic fields in a multi-layer system as a method to modify its mechanical properties. Later, the work of Kornbluh [35] generally described the possibilities that might be opened by the development of materials or structures with variable mechanical properties. Only through personal communication with the author was it possible to confirm that work in the same direction as what is described in this report was at the base of the published report. At a later stage in the project two patents by the same authors were found, describing some practical aspects of the modification of the mechanical properties of structures [30, 34]. Lately, some additional work in the field of the modification of the bending stiffness of simple structures, typically based on the modification of the elastic modulus of this polymer layers intercalated between the stiffer load bearing layers of a structure [25] has been published.
The lack of any substantial information about the function of the electrostatic modification of the mechanical properties of multi-layers and experiments to support it, as well as of any description of a practical application of such systems, warrants the investigation described in this work. The main points of the work are addressed in the four central chapters:

- **Working Principle of the Electrostatic Tuning of the Stiffness:**
  In this chapter, a method for the modification of the mechanical properties of adaptive structures based on the electrostatic coupling of the layers of a multi-layer system is described.

- **Modeling of Structures with Tunable Stiffness:**
  In this chapter, an explanation of how the system works is given based on calculations made with analytical and numerical models.

- **Experimental Work:**
  In this chapter proof of the functionality of the electrostatic modification of the mechanical properties of a structure is given. Indirect confirmation of the predictions made based on the numerical and analytical models is given.

- **A Practical Application of Elements with Tunable Mechanical Properties:**
  Finally, in this chapter an outlook on the applications of the described method is given by making an estimate of the benefits that could be obtained thanks to the retrofitting of the GFRP deck of a pedestrian cable stayed bridge.

In chapter 2 the fundamental idea is presented, that if it were possible to introduce or remove interfaces in the cross section of a simple structure subjected to a bending load, it would be possible to obtain a marked change of the bending stiffness of the structure itself. This procedure would be equivalent to laminating the layers of a multi-layer structure and vice versa. If only the energy needed for the creation of new surfaces is considered, with surface energies of the order of 1 or 2 J/m² the process is also energetically very inexpensive. Furthermore, the reverse process (cancellation of the interfaces) is in principle exothermic. The amplitude of the range of stiffness values that can be realized by the switching of the topology can easily range in the order of a factor of $10^2$. The reality is that processes that involve the creation or cancellation of interfaces in a solid have generally an endothermic global balance and are ill suited for the kind of adaptive applications envisioned in this work. The modulation of the shear stress transfer at the interface of a multi-layer system is proposed as an alternative to true topology switching. This assumption means that under ideal conditions it is possible
to replicate the behavior of a system in which interfaces are created and canceled by switching the normal stress across interfaces on or off. The use of electrostatic fields for the generation of the necessary normal stresses is a logical choice thanks to the very limited weight penalty that has to be accepted and the ease of interfacing an electrostatic system with a control system. This formulation of the working principle of a structure with adaptable mechanical properties is the result obtained within this work and represent the starting point for the work presented in the other chapters.

The calculations described in chapter 3 are primarily meant to demonstrate, without the interference of unwanted effects that experimental data are inevitably affected by, that the modulation of the shear stress transfer at the interfaces of a multi-layer structure is equivalent to the switching of the topology of the structure from monolithic to multi-layer and vice versa. A numerical model is used to take the effect of normal and shear stresses at the interfaces into account. An analytical model shows the effect of the interfaces on the shear stress distribution in the structure. A comparison of the calculated shear stress distributions shows that as long as the shear stress at the interfaces is smaller than maximum transferable shear stress $\sigma_{xy,\text{max}} = \mu \cdot \sigma_{yy,\text{el}}$, the behavior of the interfaces at which a normal stress is applied is equivalent to the behavior of a system in which no interfaces are present. Hence, the hypothesis that the modification of shear stress transfer is mechanically equivalent to a modification of the topology of the system is confirmed.

At higher load levels, the presence of an interface subjected to a normal stress can be seen again, in terms of shear stress distribution as the shear stress cannot be transferred in full and slipping at the interfaces starts occurring. This situation is at the origin of the force-displacement hysteretic behavior of the system that causes energy dissipation to take place. The system chosen to show the effect of topology switching on the global and local behavior of a structure was the simplest possible in terms of geometry and selection of materials, since all layers of the system had the same geometry and were made of the same material. Electrostatically couple multi-layer structures are thus expected to display two interesting properties: the ability to modify their bending stiffness and, at high load levels, the ability to increase the damping ratio of the system.

The experimental verification of the proposed method for the modification of the mechanical properties of structural elements was focused on the investigation of sandwich beams due their higher sensitivity to the shear stress transfer between the core and the faces of the structure. Also, the high length to stiffness ratio of the faces makes the system fairly insensitive to the adverse effect of the lack of planarity of the interfaces presented in section 4.4. While it was not possible to determine directly the effect of the
face-core coupling on the shear deformation field, the results of the experiments performed on the CFRP-Silicone-CFRP sandwich could confirm the relationship between the applied potential $U_i$ and the external load $P$ (and thus the interfacial shear stress $\sigma_{xy,int}$) at which a softening of the system could be observed. This is interpreted as a reasonable indirect confirmation of the mechanism and calculation presented in the previous sections. The second set of experiments, performed on a fairly large CFRP-GFRP-CFRP sandwich beam, were geared towards the estimation of the impact of the electrolamination of stiffening elements onto a fairly rigid structure on its dynamic behavior. Additionally, this portion of the work yielded a useful base for the estimation of the range in stiffness and damping ratio that can reasonably be achieved with the chosen combination of core and face materials.

The system properties determined in the experimental part of this work, is finally used in the last part of this work that describes a real application of the developed method: the implementation of a novel vibration suppression method for the GFRP deck of Empa’s pedestrian bridge. In the first place, the stiffness and damping properties determined in the experiments performed on the CFRP-GFRP-CFRP beam were used in a homogenized numerical model to estimate the frequency response of the beam. Subsequently they were updated so as to resolve the discrepancies between the measured system and the original numerical model. Finally based on the updated properties of the model beam, the properties of the bridge deck model were calculated for three conditions of the target system: the unstiffened bridge deck, the bridge deck stiffened by laminating CFRP bands to it and finally the bridge deck with electrolaminated CFRP bands at conditions that yield the highest damping factor. Due to the fairly high level of constraint of the deck, the frequency shift obtained by laminating the CFRP bands is very limited. Depending on the band width of the exciting spectrum, the predicted frequency shift of approximately 0.5 Hz is likely not to be sufficient to suppress a high amplitude resonant vibration by disrupting the resonance situation. The ability of the system to realize quite high levels of damping represents a more viable approach to the suppression of structural vibrations.

The use of a coulomb friction based damping system leaves the method open to wear related issues, especially in view of the fact that the element subject to dry friction is also subject to a considerable electrostatic load. The danger of damaging the dielectric layer by the combined action of mechanical and electrostatic loads should be carefully considered, especially if thin dielectric layers are used to optimize the normal stress generated by the electrostatic field. The durability of the system, the effect of heat development as a consequence of the friction on the reversibility of the property changes of the interface system shall be taken in serious consideration in view of real life applica-
tions. Nevertheless, the behavior of the first of the two CFRP-GFRP-CFRP beams was operated for several months under fairly severe conditions and gave only marginal signs of deterioration, in spite of the unfavorable surface topography that lead to non-uniform contact between the elements. Furthermore, the planarity of the contact surfaces of the elements of the system is a property that has a strong influence on its performance.

In summary, the present work showed the development of a novel method for the modification of the stiffness and the damping ratio of structural elements. The work spans from the description of a concept for the modification of the mechanical properties of simple structures, to its experimental demonstration to the presentation of a real application of the system. Its working principle could be explained using simple models that describe the effect of normal stresses at the interfaces of the system. The performed experiments indicate that the proposed principle and model can describe the observed system behavior. Calculations performed to estimate the behavior of a structure that makes use of the proposed vibration suppression method show that the obtained increase of the damping ratio is more likely to have a remarkable effect on the vibration amplitude of the structure than the increase in stiffness, due to the high number of constraints in the structure.

6.2 Outlook

The present work has shown that the electrostatic modification of the stiffness and the damping properties of adaptive structures has the potential to become an effective, robust and relatively inexpensive method for the suppression of structural vibrations. Next to the use of polymeric films as dielectric layers, as in this work, there are many different materials and methods to exploit electrostatic stress for the implementation of this method. This leaves a great deal of freedom in the optimization of performance and costs for the realization of adaptive structures with variable stiffness and damping.

The issues that will need to be addressed in the development of electrostatically tunable structures span from materials optimization to the design of tunable structures on the macro scale.

At the smallest end of the scale is the optimization of dielectric layers for high energy density (and thus high normal stress) and resistance to wear. Possible approaches will include the use of particle toughening (e.g. with diamond particles that also have good dielectric properties or with layered alumino-silicate nanoparticles).

The micro- and nanostructuring of the contact surfaces promises to make a significant
contribution to the 'conversion' of normal stress into transferrable shear stress. Here the
hope is that by obtaining mechanical interlocking of surface features such as ridges, an
increased amount of shear stress can be transferred per unit normal stress. This will
extend the amount of external load under which the system can be expected to behave elastically, when the interfaces are activated. Also in this case, both the mechanical
and dielectric properties of the materials used to structure the surfaces will determine
the usability and reliability of the system.

On a larger scale, the planarity of the contact surfaces has a strong influence on the
effective area of contact between the surfaces of the system. The requirements set
to the planarity of the interfaces increase as the stiffness of the layers increases. The
use of more refined manufacturing processes and of dielectric materials specifically
designed for applications with high field strengths will lead to an improved performance
of the system.

The above mentioned steps towards an optimization of the interfaces system used to
implement variable stiffness and damping structures are necessary steps towards a
real life use of this type of novel intrinsically adaptive structures.

Especially for the implementation of the friction damping in connection with the interlaminar slip, the performance of the system could probably be enhanced by appropriately
modulating the electrostatic slip so as to maximize the dissipation of energy. A model
describing the quantitative relationship between dissipated energy, interlaminar stress
and deformation of the system will be needed, in order to implement a suitable control
system.

Furthermore, direct confirmation of the influence of the modulation of the interface in-
teractions on the shear stress distribution in the structure will be sought using methods
such as speckle interferometry or other image correlation methods, once the technol-
goical issues limiting the domain within which the multi-layer systems behave as a
monolithic element will be addressed.

Finally, in the field of basic considerations, the investigation of the use of the elec-
trostatic modification of shear stress transfer at a low scale should be considered. If
applied on structures consisting of a large number of layers, each a few µm thick, the
concepts presented in this work are expected to lead to novel devices with interesting
properties. High damping and high stiffness changes are expected from these devices
that could provocatively be called 'matrixless composite materials'. An impression of
the possibilities of such materials is given in figure 6.1.

While these aspects are considered, the design and demonstration of structures that
Figure 6.1: A proof of concept experiment showing the behavior of a structure made of approximately 50 layers of polymer film (coated on one side with aluminum) before and after an electrostatic field is applied. Such structures can potentially be used for morphing applications.

take advantage of the variable stiffness and variable damping concept can be carried out, based on currently available materials and processes.

The first step towards the application of the electrostatic modification of the mechanical properties of structures will be made with the implementation of the method on Empa’s pedestrian bridge. Within this project, technological aspects of the manufacturing of electrostatically tunable structures, such as the production of sufficiently planar contact surfaces and the application of suitable electrode/dielectric assemblies will be of special interest. It is difficult to predict, how closely related to a real life application this project is. Nevertheless, the construction of a demonstrator in the scale of Empa’s pedestrian bridge is a necessary step towards the development of ‘real life’ applications.

In the large scale, the use of variable damping to optimize the structural damping of wind turbine rotorblades is currently being considered. The wind turbine industry is potentially a good target for this application, as it makes use of technologically very advanced materials and solutions and is not yet as strictly regulated as, for example, the aero-space industry. Some efforts have already been made to increase the amount of structural damping of rotorblades [14] by optimizing the materials used for these structures. Possibly the additional damping that can be obtained with the method presented in this work could offer additional advantages in terms of fatigue resistance of the structure.

At a considerably smaller scale, some considerations have been made about the use of variable stiffness and variable damping in the cantilevers Atomic Form Microscopes. Considerations on the effect of variable damping are presented by Chang et. al [13].
The challenges posed by the fabrication of such devices as multilayer structures will have to be assessed in the first place and put in relationship to the potential benefits.
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Publications without peer review / Conference Proceedings


**Patents**