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COMPARISON OF DIFFERENT RISK MEASURES FOR PORTFOLIO-LEVEL EARTHQUAKE RISK ASSESSMENT

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Abstract

A risk measure quantifies the risk associated with a single asset (e.g., an individual building) or a group of assets (e.g., a building portfolio of a region) exposed to one or more sources of hazard during a given time horizon. These risk measures serve as objective functionals that define subsets of "acceptable" and "unacceptable" risks. In performance-based seismic design, a new building should fulfill a set of performance objectives not only to protect human life in rare earthquakes but also to limit direct (e.g., repair cost) and indirect (e.g., downtime, business interruption) financial losses in more frequent seismic events. These performance objectives are commonly formulated as limits on risk measures for individual buildings.

The potentially large spatial footprint of earthquakes and the increased concentration of population and values in dense urban areas call for an explicit consideration of seismic risk at a regional level, in particular when formulating performance objectives for new individual building structures. Subadditivity is a desired mathematical property of risk measures in this setting, because the sum of subadditive risk measures evaluated separately for each individual building is an upper bound on the joint risk measured for a portfolio of buildings.

The present study reviews different risk measures commonly employed in earthquake engineering and in the financial industry and discusses their mathematical properties with special emphasis on subadditivity. To illustrate the importance of subadditivity for earthquake engineering, a seismic loss analysis is performed for a given portfolio of buildings situated in a virtual hazard environment. Financial losses due to earthquake-induced building property damage are quantified for the individual buildings and for the portfolio of buildings using a set of risk measures. Given the defined hazard, vulnerability and exposure, the results show that quantile-based measures, such as the loss with a certain mean annual frequency of exceedance, are subadditive only for losses with a recurrence interval longer than 200 years. As a consequence, using quantile-based measures could lead to underestimation of portfolio-level financial losses for more frequent events.

Keywords: Risk Measures, Performance Based Design, Building Portfolio, Building Code, Regional Seismic Loss Analysis

1. Introduction

Earthquake events have proven to be one of the most devastating natural hazards threatening large regions and exposing inhabitants to potentially high risk of fatalities, direct and indirect financial costs, due to damage to the built environment, landslides or other cascading events, such as tsunamis or fire. Probabilistic quantification of earthquake risk has been used in the field of earthquake engineering for long time for various purposes, such as to raise public awareness and inform decision-makers [1] or to calibrate building code provisions for seismic design [2]. The latter provide minimal requirements to ensure that a new building has a "satisfactory seismic performance", thus they separate, implicitly or explicitly, subsets of "acceptable" and "unacceptable" risks. To define such subsets, the scattered outcomes of probabilistic risk analyses are reduced to a set of single quantities, the so-called risk measures.

Current seismic provisions aim to limit collapse risk for an individual building for rare and severe earthquake events [3], [4]. The Northridge (1994) and Great Hanshin (Kobe) (1995) earthquakes revealed the need to limit the damage and economic loss in addition to collapse prevention. This led to the first generation of performance based seismic design, described in the Vision 2000 report [5], where engineers aim to design a building not only for a single earthquake level (i.e. avoid collapse for a rare high intensity earthquake event) but define desired seismic performance limits for different levels of earthquake intensity (and, thus, different frequencies of occurrence). Going further, researchers at the Pacific Earthquake Engineering Research (PEER) Center developed a framework to estimate the exceedance frequency curves for different decision variables (e.g., fatalities or economic loss) taking into account all possible earthquake intensities and their frequency of occurrence [6], [7].

Earthquakes affect multiple buildings at the same time within a specific region close to the epicenter, leading to potentially high indirect costs in terms of permanent outmigration or long-term disruptions of important community services. As argued by various authors, a discussion on "satisfactory seismic performance" should not only focus on the performance of an individual building, but also on the regional seismic risk [8]–[10]. Probabilistic seismic risk assessment for portfolios of multiple buildings is, by now, well-established in the academic earthquake engineering community: see [11] for one of the first studies in this field or [12] for more recent work.

Estimation of regional seismic risk creates new challenges, requiring assessment of collapse and excessive losses on a regional scale considering the entire portfolio of structures in a region. Because seismic design is performed for a single building, it is important to understand how limits on individual building risks affect the joint building portfolio risk. A few studies in the scientific literature have revealed unexpected comparisons of earthquake-induced losses of individual buildings vs. those at a portfolio level. For example, results presented in [13] and [14] indicate that frequent regional losses are higher than what might be expected by adding the losses of individual buildings. As this study illustrates, these counter-intuitive results stem from quantifying earthquake-induced losses using risk measures that do not satisfy the subadditivity property.

The first section of the present study introduces the subadditivity property and reviews the mathematical definition of different risk measures. The importance of subadditivity is then illustrated in an example study where different risk measures are used to quantify the seismic risk for a building portfolio situated in a virtual hazard environment.

2. Risk Measures and the Subadditivity Property

The present study quantifies seismic risk in terms of direct financial losses induced by earthquake events. For brevity, these are referred to as "losses" hereafter. Specifically, the random variable L_i describes the loss to building i at location $\mathbf{x}_i \in \mathbb{R}^2$. L_i is a non-negative and continuous random variable with cumulative distribution function (CDF) $F_i(l)$ and complementary cumulative distribution function (CCDF) $G_i(l) = 1 - F_i(l)$. The latter will be referred hereafter as the loss exceedance probability curve or, simply, the loss curve.

For a portfolio of s buildings spread over an area A, the portfolio or regional loss is defined as $L_A = L_1 + \cdots + L_s$. A risk measure $\rho(\cdot)$ is a functional assigning a real number to a random variable, thus, it condenses the random variable into a single quantity, which serves as a surrogate for the underlying risk. Specifically, $\rho(L_\mathrm{A})$ assigns a real number to the regional loss random variable and $\rho(L_i)$ maps the random loss associated with the individual building i to a real number.

Artzner et al. (1999) [15] proposed four axiomatic properties that a risk measure should possess to allow for a coherent measurement of risk: (1) Monotonicity; (2) Translation invariance; (3) Subadditivity and (4) Positive homogeneity. The risk measures used in the present study differ only with respect to the third property. Specifically, given two loss random variables L_1 and L_2, a risk measure is subadditive if

$$\rho(L_1 + L_2) \leq \rho(L_1) + \rho(L_2)\,. \tag{1}$$

This means that a subadditive risk measure evaluated on the joint loss distribution is always lower or equal than the sum of the marginal risk measures. Therefore, given a subadditive $\rho(\cdot)$, the joint portfolio risk measure, $\rho(L_A)$, is always lower or equal than the sum of risk measures $\rho(L_i)$ evaluated separately for each individual building.

The importance of subadditivity in the context of earthquake engineering is (at least) twofold. First, it allows for a conservative approximation of regional, portfolio-level, loss without the assumption of any specific dependence structure among the elements of the portfolio (i.e., individual buildings). Second, it enables a decentralized risk management. Consider, for example, a local authority interested in limiting the seismic risk on a regional scale to a certain risk threshold $\bar{\mathrm{r}}$. To achieve this target, individual risk thresholds, $\bar{\mathrm{r}}_i$, are defined first such that $\sum \bar{\mathrm{r}}_i \leq \bar{\mathrm{r}}$. Then, verification of individual building risk $\rho(L_i) \leq \bar{\mathrm{r}}_i \ \forall\, i \in [1, s]$ ensures that the regional building portfolio risk

$$\rho(L_A) \leq \sum\nolimits_i \rho(L_i) \leq \bar{\mathrm{r}}\,. \tag{2}$$

However, if a risk measure $\rho(\cdot)$ is non-subadditive the regional building portfolio risk might be larger than $\bar{\mathrm{r}}$ even though the buildings satisfy individual risk thresholds $\bar{\mathrm{r}}_i$, i.e., are designed in accordance with the current seismic design provisions. Therefore, there is no guarantee that the aggregate, portfolio level, risk threshold is satisfied.

In what follows, the mathematical definitions of a set of four risk measures are provided, together with references to applications in the field of earthquake engineering. An interested reader is referred to [16] and [17] for a comprehensive overview. For the upcoming definitions, L refers to the random loss variable of either an individual building or the portfolio loss with a continuous and non-decreasing CDF $F(l)$ and CCDF $G(l) = 1 - F(l)$.

The **Expected Loss**, EL, is defined as:

$$\rho(L) \equiv \mathrm{EL}(L) = \int_{\mathbb{R}^+} G(l)dl\,. \tag{3}$$

Expected loss is a widely used risk measure in earthquake engineering: Numerous applications use EL for cost-benefit analysis to find optimal retrofit strategies (e.g. [18] and [19]), the Italian guidelines for seismic risk classification of structures included EL as one of the governing criteria [20], and recently O'Reilly and Calvi proposed an approach for conceptual seismic design based on expected loss [21]. In general, expected loss as a risk measure is appropriate for risk-neutral decision makers but inappropriate for risk-averse decision-makers, where more weight is given to low probability-high severity events, e.g. to the right tail of the loss distribution [22]. EL is an additive risk measure, because the expected value of a linear combination of random variables is equal to the linear combination of the expected values of these random variables, i.e. $\mathrm{EL}(L_1 + L_2) = \mathrm{EL}(L_1) + \mathrm{EL}(L_2)$, thus, it fulfills the subadditivity property as defined in Eq.1.

The **Value at Risk**, VaR_α, at confidence level $\alpha \in [0,1]$ is defined as the smallest number l such that the probability that $L > l$ is no larger than $(1 - \alpha)$:

$$\rho(L) \equiv \mathrm{VaR}_\alpha(L) = \inf\{l \in \mathbb{R}^+ : G(l) \leq 1 - \alpha\}. \tag{4}$$

VaR_{α}, in probabilistic terms, refers simply to the α-quantile of the CDF $F(l)$, thus, it offers information only about the severity of losses occurring with exceedance probability higher than or equal to $(1-\alpha)$, but neglects losses at smaller levels of exceedance. Furthermore, VaR_{α} is not subadditive in general, thus the sum of the marginal risks could underestimate the joint portfolio risk [23].

The **Expected Shortfall**, ES_{α}, at confidence level $\alpha \in [0,1]$ is defined as:

$$\rho(L) \equiv \mathrm{ES}_{\alpha}(L) = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_u(L)\, du\ . \tag{5}$$

Since L is a continuous random variable L, Expected Shortfall can also be expressed as the expected loss given that the loss exceeds VaR_{α}, $\mathrm{EL}\left(L \middle| L \geq \mathrm{VaR}_{\alpha}(L)\right)$. In the literature this measure is also referred to as Conditional Value-at-Risk (CVaR), Tail Value-at-Risk (TVaR) or Conditional Tail Expectation (CTE), which are equivalent to the definition in Eq.5 for continuous loss distributions [24]. In contrast to VaR_{α}, Expected Shortfall accounts for the severity of losses with probabilities of exceedance smaller than $(1-\alpha)$, thus, it is a "what-if" risk measure. Importantly, ES_{α} is a subadditive risk measure [25].

The **Loss-at-Frequency**, LaF_{α}, at confidence level $\alpha \in [0,1]$ is defined as:

$$\rho(L) \equiv \mathrm{LaF}_{\alpha}(L) = \inf\{l \in \mathbb{R}^{+} : \lambda(l) \leq 1-\alpha\}, \tag{6}$$

where $\lambda(l)$ is the mean frequency of exceedance of the loss level l. Opposite to the risk measures defined above, this measure maps losses with respect to the exceedance frequency and not to the exceedance probability curve $G(l)$. Thus, strictly speaking, it is not consistent with the formal definition of a risk measure. For low probability Poissonian events $\lambda(l) \approx P(1 - \text{No Event with } L > l \text{ in one year}) = G(l)$ and $\mathrm{VaR}_{\alpha}(L) \approx \mathrm{LaF}_{\alpha}(L)$ [26]. Loss-at-Frequency is arguably one of the most used measures within the earthquake engineering community to illustrate seismic risk exposure of individual buildings or spatially distributed building portfolios in terms of financial losses. Because of its close resemblance to VaR, it also fails, in general, to satisfy the property of subadditivity, which is also indicated by the results in [13] and [14] discussed in the introduction of the present study.

As pointed out above, only EL and ES satisfy the subadditivity property of a risk measure as defined in Eq.1, whereas subadditivity of VaR and LaF depends on the shape of the marginal loss curves and the dependence structure between them. Examples where non-subadditive behavior of VaR has been observed include: (1) Very skewed marginal loss distributions, that are independent or partially dependent; (2) Symmetric marginal loss distributions with highly asymmetric dependence structure; or (3) Independent but heavy-tailed marginal loss distributions [23]. The following case study aims to provide further insight into the potentially non-subadditive behavior of VaR and LaF risk measures, and compares to behavior of the subadditive risk measure ES.

3. Case Study

A regional seismic risk analysis is conducted for a building portfolio located in a virtual seismic hazard environment. This section is structured as follows: first, the assumed hazard environment and building portfolio are described, then the reader is introduced to the employed methodology to quantify seismically induced losses before the presentation of the results.

3.1 Virtual hazard environment and building portfolio

Consider a building portfolio located in a virtual hazard environment given as a rectangular 100x100 km area source zone. Earthquake magnitudes are assumed to follow a truncated Gutenberg-Richter distribution defined in the (5,7) magnitude range [27]. The parameters of this distribution are selected such that the slope b equals 1.0 and ν, the mean annual rate of exceeding the minimal magnitude anywhere within the source, equals 0.5. The present study quantifies the seismic loss of a single building at site i for seismic event k conditional on the ground-motion intensity measure $im_{i,k}$ at that site. The probabilistic structure of $im_{i,k}$ is given by ground motion prediction equations (GMPEs), which are typically expressed as

$$\log im_{i,k} = \overline{\log im_{i,k}}\,(M, R, \theta) + \eta_k + \epsilon_{i,k}\ , \tag{7}$$

where $\overline{\log im_{i,k}}$ is the mean of the logartihms of $im_{i,k}$ as a function of magnitude M, source-to-site distance R, and other site parameters θ, while η_k denotes the inter-event (between-event) residual and $\epsilon_{i,k}$ is the intra-event (within-event) residual of ground motion intensity measure. The residuals $\epsilon_{i,k}$ and η_k are usually assumed to be independent random variables, normally distributed with zero means and standard deviations σ_{intra} and σ_{inter}, respectively. In the case of a single source, a probabilistic seismic hazard analysis (PSHA) for a single site evaluates the mean annual frequency of exceeding a threshold im as:

$$\lambda(im) = \nu \cdot \int_r \int_m G(im|m,r)\ |dG(r|m)|\ |dG(m)|\ , \tag{8}$$

where $G(im|m,r)$ is derived based on Eq.7, $|dG(m)|$ is given by the truncated Gutenberg-Richter distribution of earthquake magnitudes defined above, and $G(r|m) = P(R > r|m)$ describes the probability of exceeding a certain source-to-site distance r. The present study employs the GMPE of Boore and Atkinson [28]. Fig.1a illustrates the virtual hazard environment, where the contours indicate values for peak ground acceleration (pga) associated with a mean annual frequency of exceedance of 0.21% (mean recurrence interval of 475 years). As indicated, the seismic hazard is uniform in an area of approximately 50x50km around the center of the area source. Fig.1b shows the hazard curve $\lambda(im = pga)$ at site i with location $\mathbf{x}_i = \{0,0\}$ km at the center of the areas source zone.

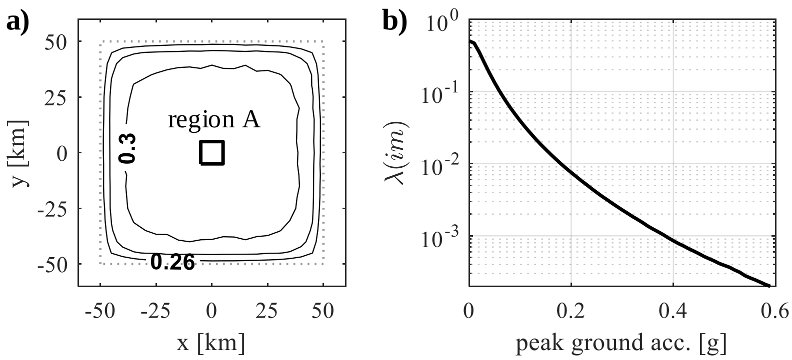

Fig. 1 – (a) Illustration of the virtual hazard environment, where the contours indicate values for pga [g] with a mean recurrence interval of 475 years and the dotted grey and solid black lines show the boundary of the area source zone and of region A respectively. (b) Seismic hazard curve for pga at the center of region A.

The building portfolio is located in a 5x5 km region A (Fig.1) and consists of 36 identical buildings spread evenly over region A. Equal soil conditions (vs30=760m/s) are assumed in region A. The buildings have a fundamental period of vibration T = 0.6s and are characterized by their equivalent single-degree of freedom (ESDOF) systems. The ESDOF system behavior is characterized by a linear-elastic/perfectly-plastic force-deformation response envelope. The base shear coefficient C_y^* of all ESDOF systems equals 0.25g. It is derived using $\mathrm{C}_\mathrm{y}^* = 2/3 \cdot \mathrm{sa}_\mathrm{d} \cdot \Omega/\mathrm{R}_\mathrm{I}$, where sa_d is the elastic, 5%-damped pseudo-acceleration at the fundamental period of vibration $Sa(\mathrm{T})$ associated with a 2% probability of being exceeded in 50 years, $\Omega = 2$ is a factor taking into account over-strength and the strength reduction factor $\mathrm{R}_\mathrm{I} = 3$ accounts for inelastic behavior.

To estimate seismic losses for the entire building portfolio, intensity measures at multiple sites are necessary and $G(\boldsymbol{im}|m, \boldsymbol{r})$ is commonly modelled as a multivariate normal distribution, where the inter-event residuals at all sites are constant (or fully correlated) for a given seismic event and correlation of intra-event residuals depend on the distance between sites. This spatial correlation is accounted for using the Jayaram and Baker model [29] in the present study.

3.2 Methodology to quantify financial losses

The methodology to quantify earthquake induced direct financial losses is adopted from [30] and [31] and is based on the PEER framework for performance based earthquake engineering. The most important steps to derive the risk measures for an individual building are illustrated first, followed by a description on how the joint risk measures are evaluated. For a more comprehensive outline of the methodology the reader is referred to the stated references. The total probability theorem is used to calculate the mean annual frequency of exceedance of a loss amount l for an individual building as:

$$\lambda(l) = \sum_{dm} \int_{im} G(l|dm)|dG(dm|im)||d\lambda(im)|\,, \tag{9}$$

where $L = l$ is the seismically induced financial loss, $DM = dm$ is a damage measure, $|dG(dm|im)| = P(DM = dm|im)$ is the so-called vulnerability function, $G(l|dm)$ is the cost function, a conditional CCDF expressing the probability that the financial loss is greater that l conditioned on a specific damage state, and $\lambda(im)$ is provided by a single-site PSHA as in Eq.8, where $IM = Sa(\mathrm{T})$.

Four discrete damage grades dg_k, where k $\in \{1,2,3,4\}$, are used as a discrete damage measure, which classify the overall building damage state into grades ranging from slight to very heavy/complete damage. Damage grade thresholds are defined as a function of the ESDOF displacement ductility demand $\mu_{\mathrm{lim,k}} \in \{0.7, 1.5, 0.5(1+\mu_{\mathrm{lim,4}}), \mu_{\mathrm{lim,4}}\}$ [32]. It is assumed that buildings suffer from very heavy/complete damage if the displacement ductility demand exceeds a value of $\mu_{\mathrm{lim,4}} = 5$. The vulnerability function $|dG(dm|im)|$ is modelled as a lognormal distribution, whose parameters are derived using the SPO2IDA tool [33]. Finally, the loss $G(l|dm) = \mathrm{PBPV} \cdot P(DR > dr|dg_k)$ is computed as a product of the present building property value (PBPV) and a damage ratio $DR \in [0,1]$, where the probability distribution of the damage ratio conditional on a certain damage grade is described using a beta distribution with parameters stated in [34].

Based on Eq.9, the risk measure $\mathrm{LaF}_{\alpha}(L_i)$ is then the smallest threshold l for which $\lambda(l) \leq 1-\alpha$. The risk measures VaR_{α} and ES_{α} are evaluated with respect to the occurrence exceedance probability (OEP) curve, which states the probability that a loss l is exceeded in at least one event in a time horizon t, also defined as the probability that the largest seismic loss triggered by a single earthquake event occurring in this time horizon exceeds l. Defining the time to the first excursion of loss level l as T_l and using the thinning property of the Poisson process, OEP for a certain time horizon t is formulated as

$$\mathrm{OEP}(l|t) = 1 - P\left(\left[\max_{0\leq t'\leq t} L_i(t')\right] \leq l|t\right) = P(T_l \leq t) = 1 - e^{-\lambda(l)\cdot t}\,. \tag{10}$$

Given Eq.10, $\mathrm{VaR}_{\alpha}(L_i)$ and $\mathrm{ES}_{\alpha}(L_i)$ are evaluated by replacing the general $G(l)$ with $\mathrm{OEP}(l|t = 1yr)$ in their definitions (Eq.4 and 5). For risk measure Expected Loss, the exceedance probability curve of cumulative seismic losses in a time horizon of one year is employed, which boils down to evaluating the area underneath the $\lambda(l)$ versus l curve. Formulated in this way, $\mathrm{EL}(L_i)$ refers to the average annual loss of an individual building, a measure commonly employed in earthquake engineering.

This direct earthquake-induced loss quantification methodology is implemented in a Monte-Carlo simulation to estimate regional, portfolio-level, losses. For an event k, with a magnitude sampled from $|dG(m)|$ and a random location within the area source, a spatially correlated ground motion field describes $im_{i,k}$ at the site of building i, conditional on which realizations of damage grade $dg_{i,k}$ and subsequently loss $l_{i,k}$ are sampled. The portfolio loss for s buildings located in region A (Fig. 1a) for event k is then estimated as $l_{A,k} = \sum_{i=1}^{s} l_{i,k}$. Denoting the number of simulated events by n, the mean annual frequency of exceeding a portfolio loss amount l_A is then approximated by:

$$\lambda(l_A) = \nu \cdot \frac{1}{\mathrm{n}} \cdot \sum_{k=1}^{\mathrm{n}} \mathbf{1}_{l_{A,k} > l_A} \ , \tag{11}$$

where ν is the mean annual rate of exceedance the minimal magnitude defined in Section 3.1 and $\mathbf{1}$ is the indicator function taking a value of one if $l_{A,k} > l_A$ and zero otherwise. Based on Eq.11 the risk measures $\rho \in \{\mathrm{LaF}_\alpha, \mathrm{VaR}_\alpha, \mathrm{ES}_\alpha, \mathrm{EL}\}$ are evaluated as in the case of a single building discussed above.

The outlined procedure takes into account all earthquake events possibly produced by the assumed seismic hazard environment, which corresponds to a time-based analysis in the terminology of FEMA P58 [35]. Conversely, in a scenario-based analysis, consequences are modelled conditional on the occurrence of a specified seismic scenario. The present study performs such analyses to estimate conditional loss exceedance curves, based on which scenario-based risk measures SEL, SVaR_α and SES_α are evaluated.

3.3 Results

The results for the case study presented above, first for a time-based analysis and, then, conditional on three earthquake scenarios are illustrated in this section, together with a sensitivity study generated by modifying the individual building vulnerability and the layout of the building portfolio.

Denote $\rho(L_A) = \rho(L_1 + \ldots + L_s)$ as a building portfolio risk measure in terms of a monetary equivalent of direct earthquake-induced damage in region A (Fig.1a), and $\rho^+(L_A) = \rho(L_1) + \ldots + \rho(L_s)$ as its approximation via the sum of the marginal risk measures. These quantities correspond to the left- and right-hand side of Eq.1, respectively. Whenever $\rho(L_A)$ is smaller or equal than $\rho^+(L_A)$ the measure is subadditive (under the assumed dependency structure among portfolio entities). To examine the behavior of different risk measures, the normalized subadditivity margin is defined as:

$$\delta_\rho^+ = \frac{\rho^+(L_A) - \rho(L_A)}{\rho(L_A)} \ , \tag{12}$$

where positive values of δ_ρ^+ indicate that an approximation via the sum of the marginal risks provides a conservative estimate of the joint risk, and negative values imply non-subadditivity for risk measure $\rho(\cdot)$ applied to a building portfolio in region A. In other words, it provides the percentage by which the joint risk measure $\rho(L_A)$ is under- or over-estimated when using $\rho^+(L_A)$.

All results in the following paragraph are normalized with respect to the present portfolio property value PPPV, e.g. the sum of all PBPVs of the buildings in the portfolio. Because the latter is assumed constant for all buildings, the process corresponds simply to normalizing by the number of buildings in the portfolio.

3.3.1 Time-Based Analysis

Fig.2 illustrates $\rho(L_A)$, $\rho^+(L_A)$ and $\delta_{\rho,A}^+$ for $\rho \in \{\mathrm{VaR}_\alpha, \mathrm{ES}_\alpha\}$ as a function of exceedance level (1-α). Because of its close similarity with VaR_α, results for Loss-at-Frequency are not shown separately. As shown in panel (a), Value-at-Risk is subadditive only for exceedance levels below 0.5%, which is approximately equal to the loss with a mean recurrence interval of 200 years, the so-called 200-year loss. For losses more frequent than the 200-year loss, the joint portfolio risk is underestimated when employing VaR_α or LaF_α as a risk-measure. On the other hand, panels (b) and (c) confirm the subadditivity of ES_α over the entire range of exceedance levels. It is worth noting that the normalized subadditivity margin for VaR_α varies from negative (non-conservative) to positive (conservative) as the exceedance level $(1-\alpha)$ decreases (i.e. the recurrence interval elongates), while it increases monotonically for ES. Note that because of additivity of Expected Loss $\mathrm{EL}^+ = \mathrm{EL}$. In this case study, the Expected Loss, i.e. the average annual loss, is 0.07% of PPPV.

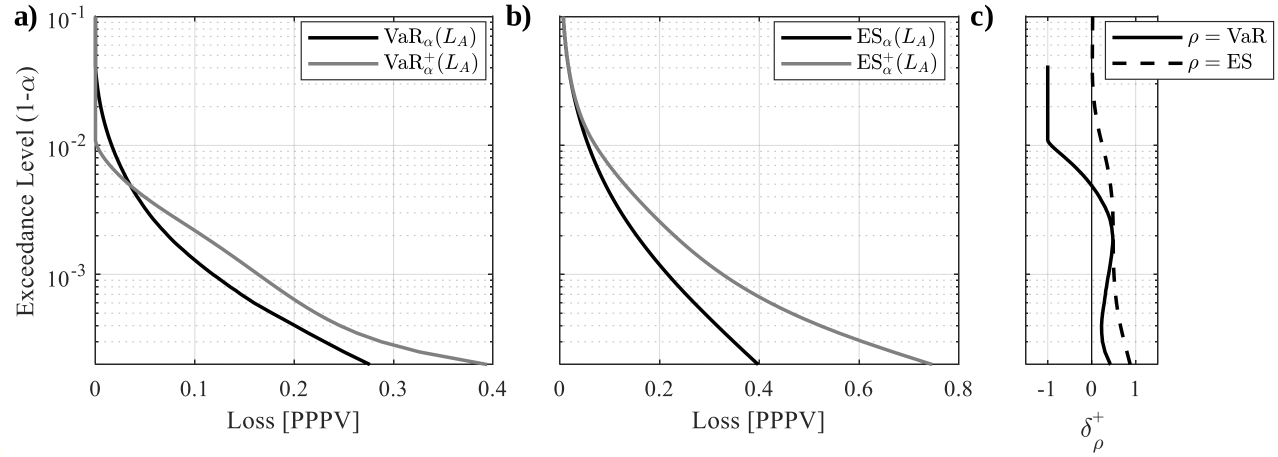

Fig. 2 – For the building portfolio situated in region A: (a) Value-at-Risk estimated on the joint loss curve VaR_α and its approximation via the sum of the marginal risk measures VaR^+_α as a function of exceedance level $(1-\alpha)$ (b) ES_α and ES^+_α as a function of $(1-\alpha)$ and (c) Normalized subadditivity margin δ^+_ρ . Note that, for graphical purposes, the independent variable $(1-\alpha)$ is on the y-axis and the dependent variable δ^+_ρ is on the x-axis.

3.3.2 Scenario-Based Analysis

A scenario is defined in terms of an earthquake magnitude $M = m$ and epicenter location $\boldsymbol{z} \in \mathbb{R}^2$, which defines the source-to-site distance $R = r$ for all buildings in the portfolio. Three scenarios are chosen based on a hazard disaggregation for the site located at the center of the building portfolio in region A. Specifically, a scenario is identified as the mode of the joint conditional probability density function $|dG(m, r|Sa(\mathrm{T}) > sa)|$, where the threshold sa is the elastic spectral acceleration with a mean recurrence interval of 72, 475 and 2500 years for the frequent, rare and very rare scenarios[1]. For numerical computation the joint distribution is discretized using bins of 0.2 and 5km for M and R, respectively. The magnitudes and distances of the identified scenarios are (5.5,7.5km), (6.1,7.5km) and (6.7,7.5km), respectively. Whereas the magnitudes are different, close events with distances between five and ten kilometers contribute most to the exceedance of all three sa thresholds. This is common for sites located in an area source dominating the seismicity of the hazard environment [36]. Scenario-based analyses are then performed for the magnitudes described above and an epicenter located at $\boldsymbol{z} = \{0, 7.5\}$ km.

Panels (a) – (c) in Fig.3 illustrate SVaR_α and SVaR^+_α for the three scenarios, whereas panel (d) compares the normalized subadditivity margin. The threshold exceedance levels below which SVaR^+_α is subadditive are 6%, 16% and 10% for the three scenarios. Results for an exceedance level of 15% (confidence level α=85%) indicate that portfolio risks $\mathrm{SVaR}_{0.85}$ are underestimated by 86% and 10% in the frequent (Fig.3a) and very rare (Fig.3c) scenarios, whereas they are overestimated by 4% in the rare scenario (Fig.3b). For the very rare scenario, the difference between $\mathrm{SVaR}_{0.85}$ and $\mathrm{SVaR}^+_{0.85}$ amounts to 2.5% of PPPV, which can easily be on the order of millions of dollars depending on the size and value of the building portfolio. Expected Losses $\mathrm{SEL} = \mathrm{SEL}^+$ amount to 0.8%, 3.9% and 11.8% of PPPV for the three scenarios. Results for SES_α are not shown separately. However, because of subadditivity, $\mathrm{SES}_\alpha \leq \mathrm{SES}^+_\alpha$ regardless of the specified confidence level.

[1] The scenario names refer to the frequency of exceedance of the threshold spectral acceleration sa. The "frequent scenario" is the most probable scenario leading to an exceedance of the sa with a mean recurrence interval of 72 years. This should not be confused with the recurrence interval of the scenario itself, which might well be much longer than 72 years.

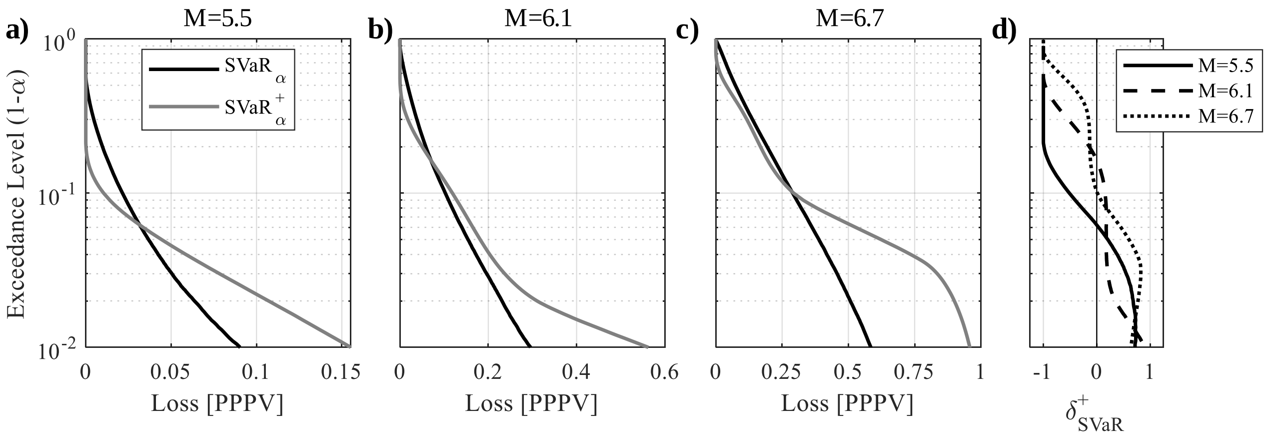

Fig. 3 – Spatially aggregated, scenario-based Value-at-Risk $SVaR_\alpha$ (black curves) and $SVaR^+_\alpha$ (grey curves) for scenarios with magnitudes of: a) 5.5 (frequent), b) 6.1 (rare) and c) 6.7 (very rare), all located at 7.5km away from the center of the building portfolio. Panel d) compares normalized subadditivity margins for the three scenarios. The dependent and independent variables are defined as in Fig.2.

3.3.3 Sensitivity

Two sets of sensitivity studies were conducted: (1) set V is related to the building vulnerability; and (2) set G to the geometry (layout) of the building portfolio. Fig.4 illustrates the normalized subadditivity margin δ^+ of Value-at-Risk risk measure wrt. the exceedance level $(1-\alpha)$ from a time-based analysis similar to that in Section 3.3.2. The base case (BC) is adopted from Section 3.3.2. Case V1 corresponds to an increase of individual building vulnerability by a 50% reduction (halving) of the base shear coefficient of the ESDOF system C_y^* and keeping the yield displacement constant. Conversely, in case V2 ESDOF C_y^* is increased by 50% (doubled) while keeping the same yield displacement, compared to the base case discussed above. The results in Fig.4a illustrate that the increased vulnerability in case V1 leads to a vertical shift upwards of the δ^+ versus $(1-\alpha)$ curve, whereas the opposite behavior is observed for case V2 with lower vulnerability. Vulnerability changes affect primarily the marginal loss distributions. Increasing vulnerability leads to less skewed marginal loss distributions, which translates to a less pronounced effect of non-subadditivity. Note that VaR in case V1 is subadditive for losses with a mean recurrence interval higher than 125 years. Conversely, decreasing vulnerability leads to higher skewness that results in a lower exceedance level threshold, corresponding approximately to a recurrence interval of 330 years.

The second set of studies focuses on the geometry (layout) of the building portfolio. In case G1, region A is increased to 25x25km, compared to the 5x5km used in the base case (BC). As indicated in Fig.1a, the site-specific seismic hazard is unchanged and thus the marginal loss distributions are the same as in the base case. Because the number of buildings is kept constant, average inter-building distance is increased from 2.5 km to 26.4 km, which reduces the correlation of intra-event residuals. In case G2, region A is decreased to 1x1 km and the average inter-building distance is only 0.5 km. The effect on the threshold exceedance level is small. As intuition suggests, the subadditivity margin increases substantially for the large area G1 at low exceedance levels.

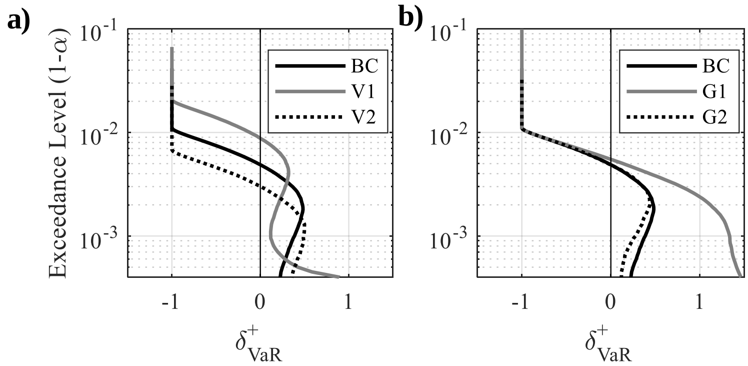

Fig. 4 – Normalized subadditivity margin vs. exceedance level $(1-\alpha)$ for: (a) Different vulnerabilities of the building portfolio and (b) different geometries of the building portfolio; the dependent and independent variables are defined as in Fig. 2.

4. Conclusion

The present study addresses the importance of the property of subadditivity for risk measures used in the field of regional seismic risk analysis. Subadditivity is of particular importance when one is interested in: (a) evaluating a conservative upper bound of portfolio risk by using marginal risk measures only; and (b) finding risk bounds for individual buildings that consistently limit portfolio risk. Results of the presented case study illustrate that quantile-based measures, such as Value-at-Risk or Loss-at-Frequency, are non-subadditive for losses more frequent than the so-called 200-year loss. When focusing on specific scenarios, such quantiles become subadditive only at low exceedance levels, e.g. lower than 10% and 15% depending on the scenario of interest. Thus, special care has to be taken when calibrating risk-based design guidelines for individual buildings with the aim to limit regional, portfolio-level, seismic risk. Expected Shortfall is an alternate subadditive risk measure, usable in this context.

The virtual building portfolio used in the present study to illustrate the importance of subadditivity is composed of buildings with equal use (importance), geometry, lateral load resisting system, and soil conditions. The utilized methodology quantifies building damage and losses focusing on displacement ductility demands on the ESDOF representation of the buildings rather than a component-based approach using a multi-degree of freedom representation of a building, where drift-sensitive and acceleration sensitive non-structural elements or building contents are treated separately [37, 38]. Furthermore, the dependency structure among the buildings in the portfolio is essentially given by the seismic hazard, whereas no additional dependency (for example, because of similar construction practice due to the same contractor or the same construction year) has been assumed. Modifying any of these assumptions will affect the shape of the marginal loss distribution and/or the dependency structure between them and thus, lead to different non-subadditive regions for VaR_α or LaF_α, whereas the subadditivity of Expected Shortfall risk measure holds regardless of these modeling assumptions, giving it a distinct advantage for regional earthquake risk assessment.

Based on the results of this study the authors conclude that subadditivity is paramount for quantification of the risk of seismic losses due to frequent earthquake events. Using non-subadditive risk measures, such as VaR_α or LaF_α, to characterize the seismic risk exposure of individual buildings or sub-portfolios could result in counter-intuitive and potentially misleading perception of the regional, portfolio-level, seismic risk.

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