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How does in situ stress rotate within a fault zone? Insights from explicit modeling of the frictional, fractured rock mass

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Key Points:

- Stress variations in fault zones are explicitly simulated with a multilayer model including macroscopic fracture networks
- Fracture deformation and specific boundary conditions synergistically contribute to the stress variations and elastic property changes
- Fracture properties, pore pressure and fracture network affect stress variations by controlling the fracture deformation

Abstract

We quantitatively investigate the spatial stress variations within fault zones by explicitly incorporating macroscopic fracture networks in a multilayer fault zone model. Based on elastic crack theory, we first derive a unified constitutive relationship for frictional fractures, featuring elastic and plastic shear deformation and shear-induced normal dilatancy. To honor the progressively accumulated damage across a fault zone, we establish a fractured multilayer model including randomly-oriented frictional fractures with varying densities from layer to layer. Under the specific boundary conditions of a fault zone, the global mechanical response of each layer is quantitatively related to the deformation of the interior fractures. Stress variations and effective elastic property changes are systematically studied considering the influences of fracture properties and pore pressure. We show that the major principal stress always rotates toward a limiting angle of 45° with respect to the fault slip direction and that differential stress invariantly decreases with the fracture density. However, mean stress increases for an unfavorably-oriented fault and decreases when the regional major principal stress trends at a small angle (< 45°) to the fault slip direction. Accumulated damage also results in a decrease and increase in the effective Young's modulus and Poisson's ratio, respectively. The influences of fracture properties, pore pressure and fracture network can be attributed to their control on the fracture deformation components and relative proportion. Our model can predict continuous variations of stresses and effective elastic properties from intact country rock, through fractured damage zone, to the plastic fault core of a mature fault.

1. Introduction

Characterization of the in situ stresses is key to better understand the crustal deformation processes, such as earthquakes (Scholz, 2019) and subsurface engineering (Cornet et al., 2007; Ma and Zoback, 2017). In general, stress field within the intra-plate region is relatively uniform at tectonic scales (Zoback, 1992). However, the ubiquitous discontinuities (e.g., veins, joints, fractures, and faults) at various scales play a crucial role in modifying the local stress fields (Pollard and Segall, 1987). In particular, faults dominate the subsurface processes to great extent due to the jumbo size. They can act as either fluid conduits, advantageous to fluid flow in reservoirs (Zoback et al., 2012), or hazardous seismicity sources (NRC, 2013). Detailed information on the in situ stress conditions around faults is thus essential to relevant scientific and industrial activities.

The state of stress is often not adequately determined in the vicinity of faults due to the complexity of the stress conditions and limited measures for stress measurements (Stephansson and Zang, 2012). In general, stress fields at great depth can be only inferred from focal mechanism inversions (Michael, 1984; Vavryčuk, 2014) while from borehole/drillcore measurements at shallow depth (Sjöberg et al., 1997; Haimson and Cornet, 2003; Pierdominici and Heidbach, 2012; Funato and Ito, 2017).

These methods have provided ample field evidence that indicates significantly different regional far-field stresses from those near tectonic faults (Zoback et al., 1987; Lin et al., 2010) and reservoir-scale faults (Obara and Sugawara, 2003; Yale, 2003; Tamagawa and Pollard, 2008). For example, focal mechanism inversions of earthquake swarms near San Andreas Fault (SAF) indicate that the major principal stress ($S_1 = S_{Hmax}$) rotates generally from ~90° at far-field to a lower angle within the fault zones (Hardebeck and Hauksson, 1999; Provost and Houston, 2001), as shown in Figure 1. The exact reasons for such stress rotation are complicated. However, it is likely the result of a self-adjustment of the stress field, depending on the fault structure, rock rheology and pore pressure, as the fault is approached (Zoback, 2010; Faulkner et al., 2010; Stephansson and Zang, 2012).

A mature fault in the Earth's upper crust consists of a fault core and a surrounding fracture damage zone (Chester et al., 1993; Caine et al., 1996; Mitchell and Faulkner, 2009). The fault core is composed by highly comminuted rocks that accommodates most of the cumulative fault slip, whereas the damage zone manifests a decrease in fracture density with distance from the fault core without or with only slight displacement (Faulkner et al., 2003). As approach the interior of a fault, the cumulative damage becomes more pronounced, generally manifesting changing elastic properties. In Table 1, it is found that Young's modulus decreases from the host rock, through damage zone, to fault core, while Poisson's ratio is on the contrary. Based on such characterization, a multilayer model has been proposed to predict stress variations in a strike-slip fault damage zone (Faulkner et al., 2006). In this model, stress rotations are chiefly attributed to the elastic property changes as originally suggested by Casey (1980); but quantitatively related to the gradually increasing microcrack density as getting closer to the fault core. Therefore, cyclic measurements in laboratory (Faulkner et al., 2006; Heap et al., 2010) or crack theory (Healy, 2008) could be utilized to predict stress rotations resulting from accumulating microcrack damage.

The multilayer model has been mainly applied to the strike-slip faults oriented at high angles to the far-field S_{Hmax} (Faulkner et al., 2006). It predicts a rotation of S_{Hmax} from a large angle (~80°), with respect to the fault strike, to ~40° within fault zones, which seems perfectly consistent with the stress rotations associated with the SAF (Figure 1). On the other hand, it also predicts a continuously decreasing angle between S_{Hmax} and fault slip direction from a value less than 45°, together with increasing mean stress towards the fault core. In recent examples, two such fault zones at reservoir-scale have been identified at the Grimsel Test Site (GTS, Krietsch et al., 2018) and Bedretto Underground Laboratory for Geoenergies (BULG, van Limborgh, 2020), respectively, both of which show completely opposite trends of stress variations compared with previous predictions. For example, the subvertical fault zone at the GTS is oriented at ~15° with respect to the far-field S_{Hmax} (Figure 1). A series of elaborate stress measurements further unravel an increase in this angle up to ~45° near the fault plane, as well as significant magnitude drops in both horizontal stresses. Therefore, the multilayer model entails reexamination and further development to tackle the issues of stress variations near faults.

To this end, the goal of this study is to improve the understanding and prediction capability of stress variations around faults oriented at different angles with respect to S_1. With the multilayer model, we particularly focus on the description of individual layers by discrete fracture networks in terms of macrofractures, rather than microcracks. In addition, we explicitly incorporate the elastic and frictional properties into fractures and quantitatively relate their deformation to stress variations, as well as effective elastic properties. This paper is organized as follows. We first propose a unified constitutive relationship for frictional fracture under both loading and unloading conditions. Then we calculate the deformation of a group of fractal fractures embedded in each layer under specific boundary conditions, and relate it to the corresponding effective elastic properties and stress changes. In particular, we investigate the effect of fracture density, fracture stiffness, fracture friction, and pore pressure on the effective elastic properties and stress variations. Finally, we discuss the assumptions of the model, the stress rotation limit predicted by the classic multilayer model, and provide some implications for the spatiotemporal variations of stress within fault zone.

2 Constitutive Behavior of Frictional Fractures

In this section, the mechanical behavior of frictional fractures under plane strain condition (Zhang and Ma, 2020) is extended to include elastic deformation. In particular, complete constitutive relationship for frictional fracture is derived considering loading and unloading conditions, respectively, used to calculate the stress changes inside the multilayer model induced by fracture deformation in Section 3.

2.1 Stress-Displacement Relationship for Fractures under Loading Condition

We consider an elastic body in a state of plane strain with only in-plane displacements for the embedded fracture, shown by its cross-section in Figure 2a. The planar fracture has a normal unit vector **n** in the x_n coordinate direction, negligible thickness, and a length of l. The surrounding elastic matrix is linear, homogeneous, and isotropic with Poisson's ratio v and shear modulus G. Under remote (effective) normal and shear stresses (σ_n and τ), the relative normal and shear displacements (u_n and u_s) between its opposite sides can be quantified by the following theoretical derivations.

If the normal stress is compressive ($\sigma_n > 0$), the relative normal displacement is given as:

$$u_n = \sigma_n / k_n \tag{1}$$

where k_n is the normal stiffness of the fracture walls. When the fracture contains high pore pressure, the resultant normal stress σ_n acting on the fracture wall can be tensional ($\sigma_n < 0$). In this case, pure mode I deformation can be well quantified by elastic crack theory (Pollard and Segall, 1987), which gives the relative normal displacement as:

$$u_n = \sigma_n / k_m \tag{2}$$

with the matrix resistance to the opening of fracture walls k_m given by:

$$k_m = \frac{G}{2(1-\nu)}(\delta^2 - x_s^2)^{-0.5} \tag{3}$$

where δ is the half-length of the fracture and the local coordinate $x_s \in [-\delta, \delta]$.

Obviously, the matrix resistance k_m varies from fracture to fracture and changes along the fracture wall. The average matrix resistance for a specific fracture can be further obtained by integrating Eq. (3) across the whole fracture length:

$$\overline{k}_m = \frac{1}{2\delta}\int_{-\delta}^{\delta} \frac{G}{2(1-\nu)}(\delta^2 - x_s^2)^{-0.5}\, \mathrm{d}x_\mathrm{s} = \frac{2G}{\delta\pi(1\text{-}\nu)} \tag{4}$$

Therefore, Eq. (2) can be written as:

$$u_\mathrm{n} = \sigma_\mathrm{n}/\overline{k}_m \tag{5}$$

The relationships between normal stress and normal displacement in Eq. (1) and Eq. (5) are represented in Figure 2b.

On the other hand, shear deformation of a fracture is usually believed to be driven by the stress difference between the remote shear stress τ and the shear stress on the fracture τ_f (Pollard and Segall, 1987). In this paper, we extent the shear behavior of fractures by including the elastic shear deformation before the fracture reaches the frictional sliding. Specifically, when the shear stress on fracture τ_f is less than the frictional strength τ^μ (= $\mu\sigma_\mathrm{n}$), fracture is in the elastic stage, where the relative shear displacement is:

$$u_\mathrm{s} = \tau_f/k_\mathrm{s} \tag{6}$$

where k_s is the fracture shear stiffness, which is assumed to be constant over the whole fracture plane. Simultaneously, the elastic matrix also experiences the same shear deformation, leading to the matrix shear stress τ_m:

$$\tau_m = \overline{k}_m \cdot u_\mathrm{s} \tag{7}$$

Considering the concept of shear stress partition (Davy et al., 2018), the remote shear stress τ can be expressed as the sum of fracture shear stress τ_f and matrix shear stress τ_m:

$$\tau = \tau_f + \tau_m \tag{8}$$

By substituting τ_f and τ_m in Eq. (6) and Eq. (7), we further obtain:

$$u_\mathrm{s} = \frac{\tau}{k_\mathrm{s} + \overline{k}_m} \tag{9}$$

If τ_f reaches the frictional strength τ^μ (= $\mu\sigma_\mathrm{n}$), frictional sliding occurs and the shear motion is driven by the shear stress difference. Based on elastic crack theory (Pollard and Segall, 1987), the shear displacement is given as:

$$u_\mathrm{s} = \frac{\tau - \tau^\mu}{\overline{k}_m} = \frac{\tau - \mu\sigma_\mathrm{n}}{\overline{k}_m} \tag{10}$$

In Figure 2c, the shear stress - shear displacement law defined jointly by Eq. (9) and Eq. (10) is shown. In a straightforward way, we propose the following criterion to estimate whether fracture is in the elastic or the plastic stage:

$$\begin{cases} \tau < \tau^{\mu} \cdot \frac{\bar{k}_m + k_s}{k_s}, \ elastic \\ \tau \geq \tau^{\mu} \cdot \frac{\bar{k}_m + k_s}{k_s}, \ plastic \end{cases} \tag{11}$$

To reflect shear-induced dilatancy commonly observed in the brittle rock mass, we further utilize dilatancy factor β to relate the normal dilational displacement to the plastic shear displacement when frictional sliding occurs. The relative normal displacement defined in Eq. (1) can be modified as:

$$u_n = \frac{\sigma_n}{k_n} - \beta \cdot (u_s - \tau^{\mu}/k_s) \tag{12}$$

which further gives the resultant displacement vector:

$$\mathbf{u} = -u_n \mathbf{n} + u_s \mathbf{s} \tag{13}$$

It should be noted that the normal stiffness k_n for natural fractures is generally much larger than the shear stiffness k_s. Therefore, we can neglect the compressive normal displacement for sake of computational efficiency without much loss of accuracy, which has been numerically verified by Davy et al. (2018).

2.2 Stress-Displacement Relationship for Fractures under Unloading Condition

Unloading occurs on a fracture due to the decrease of remote shear ($\Delta\tau$) or normal ($\Delta\sigma_n$) stress. For a fracture in the elastic stage, the corresponding decrease of shear stress on the fracture $\Delta\tau_f$ is:

$$\Delta\tau_f = \frac{\Delta\tau}{k_s + \bar{k}_m} k_s \tag{14}$$

For a fracture in the plastic stage, however, two scenarios can be encountered during the unloading process: (1) The unloaded fracture is still in the plastic stage. The shear stress change on the fracture $\Delta\tau_f$ is:

$$\Delta\tau_f = \mu\Delta\sigma_n \tag{15}$$

(2) The unloaded fracture returns to the elastic stage. The shear stress decrease on the fracture $\Delta\tau_f$ now has a different expression:

$$\Delta\tau_f = \mu\sigma_n - \frac{\tau - \Delta\tau}{k_s + \bar{k}_m} k_s \tag{16}$$

It is hypothesized that the unloading shear stiffness is the same as the loading stiffness, as shown in Figure 2c. The shear stress decrease on the fracture defined from Eq. (14) to Eq. (16) can be adopted to calculate the recovered elastic shear displacement Δu_s during unloading:

$$\Delta u_s = \Delta\tau_f / k_s \tag{17}$$

With regard to the normal displacement, it is often assumed that the normal dilational component is plastic and irreversible. The recovered elastic normal displacement Δu_n is expressed by:

$$\Delta u_n = \Delta\sigma_n / k_n \tag{18}$$

3 Effective Properties of Fractured Rock Mass within Fault Damage Zone

We first briefly review the basic assumptions and boundary conditions of the multilayer model for fault damage zone (Faulkner et al., 2006). For each layer of the model, we then calculate the displacements and strains of individual fractures according to the fracture constitutive relationship, and relate them to the boundary strain response using effective medium theory. Due to the specific boundary conditions, we further propose a new method to update the stress field of each layer to accommodate the incremental strain component induced by fractures. In this way, it is able to obtain the final stress field and effective elastic properties of each layer.

3.1 Boundary Conditions of a Fault Damage Zone

Both Casey's model (1980) and its extended version, i.e., multilayer model (Faulkner et al., 2006), suggest that the damage zone is subject to constant strain along the slip direction and to constant stress in the off-fault direction. In Figure 3a, the conceptual multilayer model for a strike-slip fault zone is shown schematically as an example. Given far-field stresses, stress components for the outmost intact rock can be resolved based on Mohr diagram, as shown in Figure 3b. For these consecutive layers, normal and shear stresses (S_{xx} and S_{xy}) acting on the interfaces are constant for mechanical continuity, while constant strain ε_{yy} is applied in the fault slip direction assuming no slip between layers. Such boundary conditions also apply to dip-slip faults featuring a fault plane parallel to the intermediate principal stress.

Specifically, we set the outmost layer representing intact rock as a benchmark, which gives the following far-field stress components:

$$S_{xx} = \frac{S_1+S_3}{2} - \frac{S_1-S_3}{2}\cos 2\theta \tag{19}$$

$$S_{yy} = \frac{S_1+S_3}{2} + \frac{S_1-S_3}{2}\cos 2\theta \tag{20}$$

$$S_{xy} = \frac{S_1-S_3}{2}\sin 2\theta \tag{21}$$

where S_1 and S_3 are the far-field maximum and minimum principal stresses, respectively, θ is the angle between the fault slip direction and S_1. In the following, effective stresses ($\sigma_{xx} = S_{xx} - P_p$, $\sigma_{yy} = S_{yy} - P_p$, with pore pressure P_p subtracted from the total stresses) are used for calculation.

3.2 Updating the Stress Component on the Strain Boundary

When the model is subject to given external stresses, the mechanical behavior of each layer depends jointly on the elastic matrix and fracture deformation. In other words, the local deformation of individual fractures contributes to the total strain of the model. In Appendix A, we provide the derivations quantifying the fracture contribution (Eq. (A3)) to model's global strain based on effective medium theory. Note that the strain ε_{yy} in the fault slip direction is maintained as constant across all layers (Figure 3a). Therefore, the additional strain in the y-direction contributed by fractures entails the adjustment of strain in the elastic matrix. Equivalently, σ_{yy} should be modified to accommodate the strain increments (Zhang and Ma, 2020). Specifically, bisection method (see Appendix B) is used to search for the stress change in σ_{yy}, which gives an updated state of stress of each layer.

In Figure 4, we graphically present how the stress state evolves after updating in the scenarios of low θ (< 45°) and large θ (> 45°), respectively. Note that, all layers in the multilayer model are initially subject to the same boundary conditions as the outmost purely elastic layer. Therefore, the far-field stress/strain conditions are actually the initial conditions for each layer. In Figure 4a, the resolved σ_{yy} is initially larger than σ_{xx} so that fracture deformation induces compressive strain in the y-direction. Because of the constant strain boundary in the y-direction, however, σ_{yy} must decline (to σ'_{yy}) to decrease the corresponding strain component in the elastic matrix, accommodating the fracture contribution. As a result, it leads to an increase in θ (to θ'). By contrast, when the resolved σ_{yy} is smaller than σ_{xx} due to a high θ in the latter case, σ_{yy} increases (to σ'_{yy}) and thus θ decreases (to θ'). The boundary conditions require that the shear stresses between layers are continuously equal and maintained constant, which restrict the stress paths associated with y-direction to horizontal lines, as shown by the dashed arrows in Figure 4a and 4b. Moreover, it is worth noting that only the limiting condition that $\sigma_{yy} = \sigma_{xx}$ (zero differential stress) is allowed according to the minimum energy principle, which corresponds to plasticity with Poisson's ratio of 0.5 and Young's modulus of 0. Therefore, the maximum principal stress in either scenario always rotates towards the angle of 45° with respect to the fault slip direction.

4. Model Experimentation and Results

In this section, six sets of representative rock mass volumes are generated with different discrete fracture network densities to characterize different positions within a fault zone. Based on the aforementioned methods, the influences of fracture shear stiffness k_s, fracture frictional coefficient μ, pore pressure P_p, and the far-field angle θ are systematically investigated.

4.1 Generation of Discrete Fracture Networks

Field observations indicate that fault damage zones often exhibit a broad range of fracture lengths that can be quantified by a power law (Odling et al., 2004; Ostermeijer et al., 2020). Utilizing the classic double power law (Davy et al., 1990; Bonnet et al., 2001), it is able to describe the density fracture length distribution as:

$$n(l, L) = \alpha L^{D} l^{-a}, \;\; l \in [l_{min}, l_{max}] \tag{22}$$

in which $n(l, L)\mathrm{d}l$ is the number of fractures whose length is in the range $[l, l + \mathrm{d}l]$ in a representative volume with side length L, α is the density-related term, D is the fractal dimension, and a is the power law length exponent. Numerous 2D outcrop datasets suggest that D falls in the range of 1.3~2 and a varies between 1.3 and 3.5 (Renshaw, 1999; Bonnet et al., 2001). In particular, when $a = D + 1$, the fracture network is self-similar and its connectivity is scale invariant (Bour et al., 2002; Darcel et al., 2003).

Due to the complexity of fracture network related to fault damage zone and the limitation of field characterization, there is no generally agreed rule for the distributions of fracture length and orientation within fault damage zone (Wilson et al., 2003; Healy, 2008; Ostermeijer et al., 2020). In this study, we set D = 2 and a = 3 to generate six sets of representative rock mass volumes with scale-independent discrete fracture networks. For each set, the minimum length of fractures is set to 0.01L for normalization. Uniform distributions of fracture centers and orientation are assigned since fracture clustering and interaction are not considered here for simplicity. The density term α is set to 4, 3.2, 2.4, 1.6, 0.8, and 0.32, respectively, for each set to achieve decreasing fracture density. With these parameters, ten realizations are generated for each layer with specific fracture density, using the multiplicative cascade process (Darcel et al., 2003; Kim, 2007; Moein et al., 2019). In Figure 5, we further show the fractal analysis of the ten realizations of each layer, where the cumulative number $N(l)$ of fractures with length greater than l is well captured by the integrated form of Eq. (22):

$$\log N\left(>l/L\right) = \log\left(\frac{\alpha}{a-1}L^{D-(a-1)}\right) - \left(a-1\right)\log(l/L) \tag{23}$$

in which fracture length is normalized by the model size. The constant (a-1) of the second right-hand term represents a fractal dimension of the length distribution.

For all fractures, the normal stiffness is set to 12,000 GPa/m, which is several hundreds of times larger than the shear stiffness, as will be shown in the following section, and the dilatancy factor β is set to 0.05, corresponding to a constant dilation angle of 2.86°. Initially, all layers will be loaded instantaneously under the same stress level (S_1 = 150 MPa, S_3 = 75 MPa, P_p = 40 MPa, corresponding to a depth of ~4 km). Such a state of stress follows that the far-field crust is critically stressed with hydrostatic pore pressure (Zoback and Townend, 2001). A purely elastic layer without any fractures is set as reference to emphasize the effect of fracture density in the following analysis.

4.2 Effect of Fracture Shear Stiffness on Stress Variations

The large normal stiffness precludes remarkable contribution of the normal fracture deformation. Thus, we first explore the effect of fracture shear stiffness (k_s = 10, 20, 30 GPa/m) on stress variations with increasing fracture density, while keep the μ and P_p constant at 0.6 and 40 MPa, respectively. Figure 6a summarizes the stress rotations in four series of simulations in which θ =10°, 30°, 60° and 80°, respectively. For all cases, S_1 constantly rotates to the angle of 45° with the increase of fracture density. As k_s increases, θ' departs further from 45°. Apparently, the increase of k_s impedes the stress rotation. In general, the increase of fracture density brings higher uncertainty in our simulations. For constant fracture density, the uncertainty becomes more substantial when θ is closer to 45°. This is because the stress state is more symmetric when it is near the pure shear state (θ = 45°), implying that fractures of any orientations can deform preferentially.

To better show the stress variations, the mean values of principal stresses, for k_s = 20 GPa/m, are illustrated in Figure 6b as Mohr circles. It can be seen that the Mohr circles display distinct changes from case to case. For the case of θ = 10°, the stress variations manifest themselves a series of contracting Mohr circles, moving leftwards. In other words, both the mean stress and differential stress decrease with the increase of fracture density. As we assume frictional equilibrium at far-field, the local stress states in the fault zone suggest a more stable fault as we normally expect. To activate such a stable fault, a higher pore pressure or a lower frictional strength or a combination of both are needed. Note that this observation is completely opposite to what is predicted by conventional multilayer model (Faulkner et al., 2006) but consistent with the original Casey's model (Casey, 1980). As for another end-member (θ = 80°), Mohr circle also contracts with the fracture density but moves to a high stress level, indicating reduced differential stress and increasing mean stress as the inner fault is progressively approached. The scenario with high value of θ has been widely studied. The results here are well consistent with previous studies (Faulkner et al., 2006; Healy, 2008; Heap et al., 2010), which can provide static weakening mechanisms by invoking high pore pressure and low friction strength for such unfavorably-oriented active faults.

In addition, as quantified in Figure 6a, the middle two cases show a pure shear state for high fracture densities. Unlike the conventional multilayer model, the self-regulated process in our model does not allow S_1 to rotate past 45°. Therefore, S_1 is able to rotate at most to 45°. In this circumstance, the innermost layer corresponds to a plastic rheology as postulated by Coulomb plasticity (Byerlee and Savage, 1992; Lockner and Byerlee, 1993) or by material softening (Rice, 1992). Therefore, our model is able to combine the brittle fault zone, as the conventional multilayer mode, and soft gouge layer. All of cases, the local S_3 has a minimum when θ = 30° for a constant fracture density and the local S_1 has a maximum when θ = 60°. In particular, faults that are oriented at around 30° to S_1 are believed to be critical according to Coulomb failure theory together with laboratory-derived frictional coefficient (Byerlee, 1978). The results of θ = 30° case showing that every layer of the model is under a state of failure could demonstrate the criticality but still need to be corroborated by more field evidence. If it is the case in field, we would argue that stress variations may also occur in this critical condition.

4.3 Effect of Fracture Frictional Coefficient on Stress Variations

We further examine the effect of fracture frictional coefficient μ in this section. For all cases, k_s and P_p are set as 20 GPa/m and 40 MPa, respectively. The resultant stress rotations are shown in Figure 7. It can be seen that the decrease of μ from 0.6 to 0.4 has no significant effect on the stress rotation for the same fracture density, while a further reduction to 0.2 causes dramatic rotation especially at high fracture density levels. At θ = 30° and 60°, the decline of μ is relatively less influential. We note that, of all cases, S_1 in the case of θ = 60° first rotates to θ = 45° (when normalized fracture density is 0.8), suggesting that the model is prone to plasticity in this configuration. Therefore, a further increase in fracture density does not promote stronger rotation of S_1. In contrast, the case of θ = 30° exhibits a nearly plastic rheology in the innermost layer. Furthermore, remarkable μ effect can be found in the two end-members, which display drastic changes, as well as uncertainties, when $\mu = 0.2$.

In a different manner from Mohr diagrams, the mean stress and differential stress are plotted versus the fracture density and μ in Figure 8. Similar to Figure 6b, it is clearly seen from Figure 8a-d that the decline of differential stress is monotonic with fracture density in all cases. That is said, fracture deformation always consumes the strain energy of the model, causing differential stress relaxation. When $\theta = 45°$ no fracture deforms due to the equality of σ_{yy} and σ_{xx}, the principal stresses remain unchanged with fracture density. When $\theta < 45°$, the mean stress becomes less compressive with increasing fracture density, and it is more compressive when $\theta > 45°$ (Figure 8e-h). As a result, the principal stresses would rotate to be at 45° to the fault slip direction for all values of θ, except for 0° and 90° in which differential stress still relaxes but without stress rotations, i.e., contracting Mohr circles with one end fixed (e.g., Zhang and Ma, 2020). These results are in excellent consistency with those predicted by Casey's model (Casey, 1980) but with physics-based mechanisms.

4.4 Effect of Local Pore Pressure on Stress Variations

Besides the foregoing two factors, pore pressure effect is investigated by performing numerical tests with different pore pressure (40, 50, 60 MPa), while k_s and μ are set as 20 GPa/m and 0.6, respectively. Figure 9 shows that elevated pore pressure has trivial effect on the stress variations in our simulations. Since the fracture density has larger effect, the pore pressure effect is actually masked by the fracture density.

The negligible pore pressure effect in this series of simulations can be accounted for by the fracture constitutive behavior. As pointed out by Davy et al. (2018), the mechanical behavior of the fracture network is mainly controlled by the stiffness length l_s, which is defined as the fracture length for which the fracture shear stiffness equals to the average shear stiffness of matrix ($k_s = \bar{k}_m$):

$$l_s = \frac{4G}{\pi(1\text{-}\nu)k_s} \tag{24}$$

which is 0.141L for the simulations in this section. For fractures much shorter than l_s, frictional resistance is dominated by the matrix as $\bar{k}_m$ is much larger than k_s. For fractures much longer than l_s, the plastic limit in Eq. (11) can be simplified as τ^{μ} ($= \mu\sigma_{\mathrm{n}}$) since $k_s \gg \bar{k}_m$. As seen from Figure 5, only a small fraction of fractures are longer than l_s in this simulations. Therefore, the increase in pore pressure only makes the few long fractures slip frictionally while the remaining vast number of short fractures are completely unaffected. Equivalently, the increased pore pressure only contributes to subtle increase of plastic shear deformation in our simulations, even at a high fracture density level (we expand this argument in Section 4.5). It is thus expected to obtain more significant pore pressure influence with a lager fractal dimension *a* or in a weaker model.

It is worth noting that, the constitutive relationship of fracture we develop here aims for brittle deformation of damage zone. For mature fault core, weak and interconnected foliated microstructures, such as phyllosilicate foliae, might be pervasive (Collettini et al., 2019). Frictional sliding along fault-parallel fabric can be simply described by Coulomb criterion and Eq. (10), so that enormous effect of elevated pore pressure (especially overpressure) can be expected on stress rotations therein. Furthermore, Healy (2008) argued that, the deviatoric effect of pore pressure could exert an important influence on stress rotations when considering microcrack-related anisotropy, which, however, is beyond the scope of this study since fractures are randomly oriented.

4.5. Influences of Fracture Deformation on the Effective Elastic Properties

Our simulations show that the local stress state can be considerably modified, to different extent, by fracture network properties in various scenarios. Since all influence factors in previous sections essentially change the magnitudes of fracture deformation (i.e., global strain increments), we further investigate the influences of fracture deformation on the effective elastic properties. For each layer, we calculate its effective elastic properties by solving Hooke's law, i.e., Eq. (A5), but with the final global stress and strain. In Figure 10, effective elastic properties are plotted against the fracture-contributed strain in the fault slip (y) direction based on all foregoing numerical tests. We also provide the changes of effective elastic properties with fracture density of individual cases in Supporting Information. It can be seen that fracture deformation induces compression in the y-direction when θ is less than 45° while dilatancy emerges when θ is larger than 45°, which result in a decrease and increase in σ_{yy}, respectively, due to constant strain boundary as illustrated in Figure 4.

In general, fracture deformation (and the resultant strain increment in y-direction) increases with the fracture density, which leads to increasing Poisson's ratio and decreasing Young's modulus, as often observed in many fault zones (Table 1). We would argue that, the stress variations and modifications of the effective elastic properties are accompanying each other as approach the faut core. Both effective elastic parameters show nonlinear dependence on the fracture-related strain. The nonlinearity becomes more prominent when the fracture density gets higher. The data clusters at low fracture densities (i.e., normalized fracture density of 0, 0.08 and 0.2) indicate that the relative changes in both effective elastic parameters are dramatic. When the fracture density goes higher, the relative changes gradually mitigate but with higher uncertainty. For the same fracture density, in addition, more fracture deformation is induced when θ deviates further away from 45°, which requires larger stress adjustment to maintain the constant strain in the y-direction. When θ is closer to 45°, both effective elastic properties are more sensitive to the fracture deformation. In other words, small amount of fracture deformation results in significant changes of effective elastic properties. Therefore, it also accounts for why it is much easier for the two middle cases (θ = 30° and 60°) to reach a plastic state.

Other than fracture density, we further consider the individual influences of other factors. According to the constitutive relationship of fracture, we divide the fracture deformation into two parts: elastic and plastic deformation. Therefore, it is able to quantify the dependence of each type of deformation on fracture shear stiffness, frictional coefficient and pore pressure. In Figure 11, the total strain increment and the plastic component in *y*-direction, induced by fracture deformation, are plotted as functions of fracture density and each influence factor in the case of θ = 10°. As with Figure 10, the total strain increment constantly increases with fracture density. Regarding each fracture density level, the variation of each influence factor induces different degrees of deformation. Figure 11a illustrates the very control of k_s on the elastic fracture deformation, since all fractures are in the elastic stage (i.e., no frictional slip occurs) in this series of simulations. As k_s increases from 10 to 30 GPa/m, the elastic fracture deformation decreases correspondingly due to the higher resistance. In Figure 11b, it is found that most of the deformation is elastic when μ = 0.6 and 0.4. For certain fracture densities, slightly larger uncertainty of fracture deformation can be found for μ = 0.4 than 0.6. When μ decreases further to 0.2, both the total and plastic fracture strain increase remarkably. It demonstrates that the low level of μ forces many fractures into frictional sliding. The effect of μ, as well as the uncertainty, becomes more significant for the higher fracture density. This elucidates the observations in Section 4.3 that the local stress states of the inner layers are close to pure shear when μ = 0.2. The effect of μ could also explain why the increasing pore pressure brings about trivial changes in our simulations in Section 4.4, in addition to stiffness length. In this series of simulations, μ is set to 0.6. All of the fractures are well below the individual frictional strength. Even when pore pressure increases up to 60 MPa, which is close to the minimum principal stress (75 MPa), only a small fraction of plastic deformation is induced.

5. Discussion

5.1 Model Assumptions and Validity

Our simulations are performed based on the multilayer model extended from Casey's model (Casey, 1980), which has been previously used to assess the changes in stress state resulting from changes in elastic moduli (Faulkner et al., 2006; Healy, 2008; Heap et al., 2010). Instead of directly applying varying elastic properties to each layer, we explicitly relate the deformation of macroscopic fractures to the global mechanical behavior under the long-term quasi-static loads. We argue that both the stress rotations and accompanying elastic property changes result from the synergistic effect of fracture deformation and boundary constraints. Specifically, this bottom-up approach has the following primary assumptions about fractures: (1) The frictional fractures are preexisting without considering the complex seismic history; (2) These fractures exist at different scales, if not all scales, following power law; (3) Fractures are randomly distributed while the clustering and interaction are not considered.

Firstly, the off-fault damage is believed to be produced by earthquake ruptures due to transient loading of the stress waves emitted from rupture tips. Such coseismic damage generally occurs over a relatively short time scale. In the context of long-term mechanical response, therefore, we begin with preexisting fractures in our simulations without considering the complex dynamic mechanisms. Secondly, although there are only few field data on the length distribution of fractures within fault damage zones (e.g., Ostermeijer et al., 2020), fracture length is generally found to follow the power law (Bonnet et al., 2001) for different fracture systems in geological media. In addition, Knott et al. (1996) pointed out that secondary fault throw with fault damage zones can also be characterized by power law, and fault length has a linear relationship with the fault throw (Cowie et al., 1996). Consequently, power law distribution is reasonably hypothesized for fracture length within fault damage zones. It should be noted that, we only use self-similar and scale-invariant fracture network in this study, and the largest fracture length is limited by the model size L. Such model configuration gives trivial pore pressure effect due to the control of stiffness length, as elaborated in Section 4.4. More realistic results are expected by larger exponent. As observed by Mello et al. (2010), finally, the maximum strain rate direction rotates, when an earthquake rupture propagates at the sub-Rayleigh wave speed, from fault-parallel to fault-perpendicular direction. Such a dynamic fracturing pattern could support an isotropic damage model. We are also aware of the asymmetry on either side of a fault (Rice et al., 2005) and the heterogeneity along fault or perpendicular to fault (Wilson et al., 2003; Rempe et al., 2018; Ostermeijer et al., 2020), which, however, are beyond the scope of this study. Nonetheless, incorporating anisotropy induced by different fracture patterns in the multilayer model is trivial in the further study. Furthermore, fracture clustering and interaction may further intensify the stress rotations and elastic moduli weakening. Simulations without fracture clustering and interaction can act as an end-member for the estimation.

Regarding the mechanical loading, the far-field stress state is initially applied to all layers, under which the fault-parallel strain of the purely elastic (outmost) layer is set as the benchmark. Based on the specific boundary constraints, elastic moduli and their changes could be significantly different from those computed under stress boundary conditions. For example, a fractured rock mass could be characteristic of a bulk Poisson's ratio well above 0.5 under stress boundary conditions (Min and Jing, 2004), which is meaningless in the multilayer model. That is said, direct application of elastic moduli, measured under conventional stress boundary conditions, to the multilayer model could be problematic. We would therefore argue that the boundary constraints should not be ignored when using the multilayer model for fault damage zone. Since fracture deformation dominates the stress states and effective elastic properties, one would expect the stress variations and accompanying material weakening even if the unequal principal stresses are oriented orthogonal to the layering, which is different from previous models.

5.2 Stress Rotation Limit Imposed by the Conventional Multilayer Model

In the conventional multilayer model (Faulkner et al., 2006; Healy, 2008; Heap et al., 2010), S_1 is able to rotate past the limiting angle of 45°. For such applications, the mechanical equilibrium gives the solution to the final σ'_{yy} of each layer as:

$$\sigma'_{yy} = \frac{E_{\text{eff}}}{1-v_{\text{eff}}^2}\varepsilon_{yy} + \frac{v_{\text{eff}}}{1-v_{\text{eff}}}\sigma_{xx} \quad (25)$$

where E_{eff} and v_{eff} are effective Young's modulus and Poisson's ratio, respectively, of a specific layer, and the parameters ε_{yy} and σ_{xx} are prescribed as those of purely elastic layer, representing far-field. In our simulations, Eq. (25) also applies due to the same model configuration. In the following, we would like to re-examine this equation and its application to stress variations due to elastic moduli changes.

In the context of unfavorably-oriented faults ($\theta > 45°$), σ_{xx} is larger than σ_{yy} but the fixed strain ε_{yy} can be either compressive (positive) or dilatant (negative) according to the far-field stresses and elastic moduli. In previous studies considering microcrack damage effect, the outmost layer usually has a relatively small differential stress and high elastic moduli (derived from laboratory samples). Hence the fixed strain ε_{yy} is mostly compressive (positive). As revealed by the cyclic stressing experiments on different rock types (Heap et al., 2010), the elastic moduli evolution with increasing microcrack damage manifest a decrease in E_{eff} by 11 ~ 32% and a drastic increase in v_{eff} by 72 ~ 600%. Applying such data to stress variations as the fault core is progressively approached, the coefficients of the two right-hand terms in Eq. (25) both increase with v_{eff} regardless of the decreasing E_{eff}. This probably leads to a value of σ'_{yy} larger than σ_{xx}, especially when v_{eff} is close to 0.5, i.e., $v_{\text{eff}}/(1-v_{\text{eff}}) \approx 1$. Consequently, stress can rotate over the angle of 45° corresponding to a minimum differential stress as implied in Figure 4b, and the differential stress starts to increase. However, the exchange of the principal stress axes and the assumption of isotropic properties are incompatible under this circumstance.

The analysis also holds for the case with $\theta < 45°$, which is rarely studied previously. In this case, σ_{yy} is always larger than σ_{xx} and the fixed strain ε_{yy} is thus compressive (positive). Applying the laboratory-derived elastic moduli changes, σ'_{yy} could further increase, showing an increasing mean stress and a decreasing angle θ (Faulkner et al., 2006). Also note that, since the second right-hand term is monotonically increasing with v_{eff}, σ'_{yy} could decrease to some extent if the first right-hand term decreases more. This trend was originally presented by Casey (1980), who set a lower Young's modulus in the weak zone while maintained the Poisson's ratio constant. More generally, more substantially decreasing Young's modulus can be expected as in this study considering macroscopic fracture contribution. In particular, σ'_{yy} could decrease even to the value of σ_{xx} when v_{eff} = 0.5 and E_{eff} = 0 GPa, corresponding to the limit of $\theta = c$. In this case, the layer undergoes volume-preserving plastic flow. The plastic layer is stressed under constant compression σ_{xx}, and a small shear stress τ_{xy} suffices to induce plastic flow, leading to extrusion in the fault slip direction. Thus, the constant strain boundary condition is violated in this limiting case.

5.3 Spatial-temporal Variations of Stress Within Fault Zones

Our model exhibits it capability to predict continuous variations of local stresses and effective elastic properties from the purely elastic country rock, through fractured damage zone, to the plastic fault core of a fault. For faults that are characteristic of fractured damage zones, fracture density generally decreases as either an exponential (Chester and Logan, 1986; Vermilye and Scholz, 1998; Wilson et al., 2003) or a power (Johri et al., 2014) function of distance away from the fault core, either of which depicts a sharp decrease of fracture density. Despite that the fracture density - distance relationship is not explicitly incorporated, the effect of fracture density revealed by our model indicates that drastic changes in local stresses and effective elastic properties are restricted only to the regions near the fault core. In other words, the weakness of a poorly-oriented fault, tending to be active, mainly lies in the core zone and the adjacent limited damage zone.

Essentially, stress variations are controlled jointly by fracture deformation and the peculiar boundary conditions of a fault zone. However, the fracture constitutive behavior in this study is defined as time independent. In fact, the time dependency of fracture deformation could be significant (Scholz, 1968; Brantut et al., 2013), which may further promote stress rotations and weaken the fault strength in a temporal sense.

In addition, stress variations and effective elastic property changes in the current model are entirely attributed to the fracture deformation. More pronounced modifications are expected when further energy dissipation mechanisms are introduced in both fractures and matrices. For example, the chemically active fluids in the fault zone are able to change stiff mineral phase constituents into weak mineral phases (Wibberley, 1999; Schleicher et al., 2010; Collettini et al., 2019). Such fluid-assisted reaction-softening could weaken the protoliths (especially fault core) in a long term, e.g., interseismic period, generating time-dependent stress rotations. It is also noteworthy that, some other fluid-assisted mechanisms (e.g., mineral deposition and cementation) are operative that lead to healing of fractures and generate low-permeability barriers (Chester and Logan, 1986; Chester et al., 1993; Faulkner et al., 2006). Hence the trapped fluids can be pressurized and maintained at high pressures, also weakening the fault zone in a time-dependent manner until next earthquake occurs (Sleep and Blanpied, 1992).

6. Conclusions

In this paper, we numerically investigate damage-induced changes in stresses and effective elastic properties within a fault zone by explicitly incorporating macroscopic fracture networks in the classic multilayer model. By emphasizing the synergistic effect of the specific boundary conditions and fracture deformation, it is found that stress variations, as well as effective elastic property changes, within a fault zone are the common products of a self-adjusted process.

Frictional fractures are considered under plane strain condition, featuring elastic and plastic shear deformation and dilatancy in the normal direction. Based on the unified fracture constitutive relationship, a systematic parametric study is conducted to understand the effect of fracture properties (density, shear stiffness, friction) and pore pressure on stress variations. It is found that both the increase of fracture density and pore pressure and the decrease of fracture shear stiffness and friction promote the stress variations to different extents. On one hand, the damage-induced stress variations manifest themselves as principal stress rotations and magnitude changes. The simulation results reveal that the major principal stress always rotates toward the angle of 45°, accompanied by gradually decreasing differential stress. However, the change of mean stress depends closely on the fault setting. The mean stress shows an increasing trend with damage degree when the far-field major principal stress is oriented at an angle larger than 45° with respect to the fault slip direction, which on the contrary decreases when the angle is less than 45°. On the other hand, we also show that stress variations and effective elastic property changes accompany each other. As damage is accumulated, the effective Young's modulus continues decreasing while the effective Poisson's ratio increases.

The degrees of stress variations and changes in effective elastic properties are essentially proportional to the deformation of fracture network. It is demonstrated that fracture properties and pore pressure, even the fracture network property itself, are able to alter the elastic and plastic components of fracture deformation to different extents. Under the specific boundary conditions, these factors show remarkably different influences and uncertainties.

It is also worth emphasizing that we correlate a limiting condition with the plastic rheology wherein pure shear emerges. Therefore, a limiting angle of 45° is attained for the major principal stress rotation, and the effective Young's modulus and Poisson's ratio reach to their limiting values, i.e., 0 and 0.5, respectively. In other words, our model is able to predict continuous variations of local stresses and effective elastic properties from the purely elastic country rock, through fractured damage zone, to the plastic fault core of a mature fault. In addition, this model applies not only to strike-slip faults but also to dip-slip faults with fault plane parallel to the regional intermediate principal stress. the results of this study have important implications for estimating the state of stress and effective elastic moduli within a fault zone, especially when no focal mechanisms or direct stress measurements are available.

Appendix A Relation of Local Fracture Deformation and Global Strain

Instead of directly applying the varying elastic properties to different layers, we initially assign the resolved stress tensor $\boldsymbol{\sigma}$ of the outmost layer to all layers of the multilayer model. For one inner layer with certain fracture density, the remote normal and shear stresses (σ_n and τ) for a fracture with normal unit vector $\mathbf{n}$ can be related to the boundary stress tensor $\boldsymbol{\sigma}$ by:

$$\sigma_n = \mathbf{n}^T \cdot \boldsymbol{\sigma} \cdot \mathbf{n} \tag{A1}$$

$$\tau = \mathbf{n}^T \cdot \boldsymbol{\sigma} \cdot \mathbf{s} \tag{A2}$$

in which P_p is the pore pressure and the in-plane shear unit vector $\mathbf{s}$ defined in Figure

2a is adopted when the relative shear motion is right lateral, otherwise the opposite direction is used.

Based on the stress-displacement law, we can calculate the displacement vector **u** of the fracture, which is further quantitatively related to the global strain at the model boundary according to effective medium theory (Kachanov, 1992):

$$\Delta\boldsymbol{\varepsilon} = \frac{a}{A}(\mathbf{u}\otimes\mathbf{n}+\mathbf{n}\otimes\mathbf{u}) \tag{A3}$$

with A the cross-section area of the rock mass volume. Considering all fractures in the rock mass volume, the total global strain can be obtained by summing the elastic matrix strain and the additional strain contributed by fractures:

$$\boldsymbol{\varepsilon}_{\text{tot}}=\boldsymbol{\varepsilon}^{m}+\sum_i \frac{a_i}{A}(\mathbf{u}_i\otimes\mathbf{n}_i+\mathbf{n}_i\otimes\mathbf{u}_i) \tag{A4}$$

where the subscript i of the second right-hand term denotes the ith fracture under consideration, and $\boldsymbol{\varepsilon}^m$ is the elastic matrix strain given by Hooke's law under plane strain:

$$\boldsymbol{\varepsilon}^{m}=\begin{bmatrix}\varepsilon^m_{xx}\\ \varepsilon^m_{yy}\\ \varepsilon^m_{xy}\end{bmatrix}=\frac{1\text{-}\nu}{2G}\begin{bmatrix}1 & -\nu/(1\text{-}\nu) & 0\\ -\nu/(1\text{-}\nu) & 1 & 0\\ 0 & 0 & 1/(1\text{-}\nu)\end{bmatrix}\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} \tag{A5}$$

with the superscript 'm' representing the purely elastic (intact) matrix.

As indicated by Eq. (9) and Eq. (10), the shear displacement of any fracture is a function of the average matrix resistance and thus of the effective properties of the model, seeing Eq. (4). Generally, there are two theoretical methods to calculate the effective properties, depending on whether the fracture interaction is considered or not (Kachanov, 1992; Davy et al., 2018). One method is the Non-Interaction Approximation (NIA), which states that each fracture can be regarded as isolated and that its surrounding medium is maintained intact as the initial elastic matrix. The NIA allows a simple summation to calculate the effective properties while keeps high accuracy. Grechka and Kachanov (2006) showed that the NIA still holds up even at high fracture densities, as long as fracture locations are random. The second method, e.g., Self-Consistent Scheme (O'Connell & Budiansky, 1974) and Differential Scheme (Hashin, 1988), suggests that the impact of fracture interactions can be approximated by a reduced effective properties of the surrounding medium. For the ith fracture, therefore, the average matrix resistance can be calculated as:

$$\overline{k}_{m,i} = \frac{2G_{i-1}}{a_{i-1}\pi(1\text{-}\nu_{i-1})} \tag{A6}$$

in which $G_{i\text{-}1}$ and $\nu_{i\text{-}1}$ are the effective shear modulus and Poisson's ratio, respectively, of the rock mass volume resulting from the previous (i-1) fractures. In Section 4, randomly distributed fractures are generated within the elastic matrix. In order to investigate the influences of fracture properties on the effective properties and stress variations, we use the NIA for simplicity to quantify the fracture contribution.

Appendix B. Bisection method for stress update

Bisection method is utilized to iteratively determine the stress change $\Delta\sigma_{yy}$, before which a continuous function with variable $\Delta\sigma_{yy}$ is established in terms of constant strain:

$$f\left(\Delta\sigma_{yy,j}\right) = \varepsilon^{m}_{yy,0} - \left(\varepsilon^{m}_{yy,j} + \varepsilon^{frac}_{yy,j}\right) \tag{B1}$$

where $\varepsilon^{m}_{yy,0} = \varepsilon^{m}_{yy}$ is the fixed reference strain in the *y*-direction with '0' being the initial input. $\varepsilon^{m}_{yy,j}$ and $\varepsilon^{frac}_{yy,j}$ in the parenthesis represent the strain components in the *y*-direction of elastic matrix and contributed by fractures, respectively, at the *j*th iterative step. With the trial $\Delta\sigma_{yy,j}$ and a trial stress tensor,

$$\boldsymbol{\sigma}_i = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \left(\sigma_{yy} - \Delta\sigma_{yy,j}\right) \end{bmatrix} \tag{B2}$$

$\varepsilon^{m}_{yy,j}$ is calculated by Eq. (A5) with the trial stress tensor. Note that, the updated normal stress difference, i.e., differential stress $|\sigma_{xx} - \sigma_{yy}|$, is invariably decreased. Therefore, displacements of all fractures should be re-calculated under the trial stress tensor according to the unloading constitutive law in Section 2.2. Based on Eq. (A4), the new elastic matrix strain and new fracture displacements are further combined to update the global strain of each layer. This iteration will continue until Eq. (B1) reaches zero or a sufficiently small value.

More specifically, Figure B1 shows the numerical algorithm searching for the root of $f(\Delta\sigma_{yy}) = 0$. At the beginning of the iteration, it is necessary to define an interval $[m, n]$ allowing $f(m) \cdot f(n) < 0$. In the context of stress adjustment, we simply let $m = 0$, which is one end-member case with no constraint of strain boundary, and let $n = |\sigma_{xx} - \sigma_{yy}|$, which is the other end-member case where the representative volume is under an isotropic state of stress. Then each iteration performs these steps:

Step-1: Calculate the midpoint of the interval, $\Delta\sigma_{yy,j} = (m + n)/2$;

Step-2: Calculate the function value at the midpoint, $f(\Delta\sigma_{yy,j})$;

Step-3: If $|\Delta\sigma_{yy,j} - m|$ is sufficiently small or $f(\Delta\sigma_{yy,j}) == 0$, return $\Delta\sigma_{yy,j}$ and stop iterating;

Step-4: If Step-3 is not satisfied, examine the sign of $f(\Delta\sigma_{yy,j})$. Replace either m or n with $\Delta\sigma_{yy,j}$ in order to ensure the root is within the new interval. Then Set $j = j + 1$ and return to Step-1.

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