Value-at-risk concept in a holistic risk model

Prof. Dr.-Ing. Gerhard Girmscheid, ETH Zurich, Institute for Construction Engineering and Management
(email: girmscheid@ibb.baug.ethz.ch)

All enterprises in the construction sector, particularly construction companies, face very high economical risks. This paper presents the risk load dimensions as the first part of the holistic, probabilistic enterprise risk management model to control the risk load, the risk coverage resources and the risk resistance of project oriented companies. The holistic company risk model (RMP-M) was created at the ETH Zurich, enacted by and in collaboration with the Swiss Association of General Contractors.

Keywords: Holistic risk model, risk load, value-at-risk

1. Introduction

The holistic, probabilistic overall enterprise process model (RMP-M) consists of three dimensions. In these model dimensions, the Cash flow Model / Asset risk model and the risk load resistance theorem will create the core of the RMP-M with the following structure (Fig. 1):
- Risk load dimension – bottom-up approach
- Risk resistance capacity dimension – top-down approach
- Risk load-resistance theorem – integrative approach

![Fig. 1: Structure of the holistic, probabilistic Risk Management Process Model (RMP-M) for project-oriented enterprises](image)

2. Risk load dimensions

The risk load dimensions differentiate risk loads at various levels of a company and are based on the following three concepts:
- Risk aggregation theorem
- Cash flow risk model (CRM) and Asset risk model (ARM)
- Risk load scenario model and risk load theorem

The levels of a company as outlined in Fig. 1 comprise e.g. (from bottom to top) the project level, the branch office level, the level of strategic business units (SBUs) and the level of the company overall. With the help of the risk aggregation concept, the risk costs of all projects of a branch office, of a strategic business unit and the total project risk of the company can be determined.

The cash flow and asset superposition concept is used to calculate the distribution functions of the operative probabilistic cash flows / assets at the various corporate levels. The resulting risk load is determined using the value-at-risk approach for various load levels (load scenarios).

After signing the construction contract every company has accepted particular risks (Fig. 2).

![Fig. 2: Probability density distribution functions of impact cost of risk in project](image)

Risk aggregation theorem

With reference to the central limit theorem of the stochastics and the Monte Carlo Simulation [1] the probability density function and cumulative distribution function of the risk cost in project \( j \) can be determined by the value of the expected risk cost \( R_{E,project,j} \) and the variance \( \sigma^2_{project,j} \) of risk cost. The probability density function in this case is quasi normally distributed independent of the density function of each risk if the number of risk is high enough (e.g. \( \leq 10 \)).

According to the central limit theorem of the stochastics the aggregated total risk cost and the variance of risk cost of the enterprise level can, according to [1], be expressed as follows:
\[ R_{\text{cost}} = \sum_{i} \sum_{j} \left( R_{i,j} \cdot R_{j} \right) \]
\[ \sigma_{\text{cost}} = \sum_{i} \sum_{j} \left( \sigma_{i,j} \right) \]

The related density and cumulative risk cost function:
\[ f(R_{\text{cost}}) = f(R_{\text{cost}}, \sigma_{\text{cost}}) \]
\[ F(R_{\text{project}}, \alpha) = \int_{R_{\text{cost}}}^{R_{\text{cost}}} f(R_{\text{project}}) \, dR \]

Fig. 3: Probability total risk cost density function and cumulative total risk cost distribution function of enterprise

**CR- and AR-model**

To design the CR- and AR-model for cash flow and profit impacted by risk, the reference point must be set at the start of the residual risk for the company. For this reason the CRM and ARM will be designed by transforming the cumulative total project risk distribution function into a cash flow risk function and a profit risk function by:

1. creating a mirror image at the f(R_{\text{total}})-axis of the total risk cost density function of the enterprise
2. transforming the coordination zero point of the mirror imaged total risk cost density function at the magnitude of
   \[ R_{\text{cost}} = R_{\text{cost}} + R_{\text{cost, total}} + \alpha \]
   \[ R_{\text{cost}} = R_{\text{cost}} + R_{\text{cost, total}} + \alpha \]

The new functions will be called (Fig. 4)

- CRM : F(CF_{project total}) - cash flow risk function
- ARM : F(G_{project total}) - profit risk function

**Risk load scenario model**

To protect the enterprises from excessive dangerous economical risk, the following generic risk load limits will be created which must be below the economical risk resistance capacity of a company (Fig. 4):

- Normal risk load limit
  The "normal risk load limit" represents a statistical acceptance limit of \( \alpha_{c} = 60\% \) and will only be exceeded in \( (1 - \alpha) = 40\% \) of all cases.

- Stress risk load limit
  The "stress risk load limit" represents a statistical acceptance limit of \( \alpha_{s} = 90\% \) and will only be exceeded in \( (1 - \alpha) = 10\% \) of all cases.

- Crash risk load limit
  The "crash risk load limit" represents a statistical acceptance limit of \( \alpha_{c} = 99\% \) and will only be exceeded in \( (1 - \alpha) = 1\% \) of all cases.

**Fig. 4: Cash flow risk function of CRM - Risk load scenario model**

**Risk load theorem** (Fig. 4)

According to Markowitz' portfolio theory, the risk load which endangers the anticipated cash flow or profit is called "value at risk".

Risk load at project j:
\[ VaR_{j} = (R_{j, project, \alpha} - R_{\text{total, project}}) \]

Risk load of SBU k:
\[ VaR_{k} = \sum_{j} VaR_{j} = \sum_{j} (R_{j, project, \alpha} - R_{\text{total, project}}) \]

Risk load of Enterprise:
\[ VaR = \sum_{k} VaR_{k} = \sum_{k} (R_{k, project, \alpha} - R_{\text{total, project}}) \]

**3. Conclusion**

The holistic probabilistic RMP-M offers a process instrument for companies to take risk consciously into account, and this in relation to the risk coverage, to maximize the profit. This paper represented the risk load dimension of the model. The risk resistance capacity dimension and risk-load-resistance theorem of the RMP-M will be presented by different papers.

**References**

Abstract

All enterprises in the construction sector, particularly construction companies, face very high economical risks. This paper presents a part of the holistic, probabilistic enterprise risk management model, which was created at the ETH Zurich, enacted by and in collaboration with the Swiss Association of General Contractors [1].

Keywords: Holistic risk model, risk load, value-at-risk

1. Introduction

The holistic, probabilistic overall enterprise process model (RMP-M) consists of three dimensions. In these model dimensions, the Cash flow model / Asset risk model and the risk load- resistance theorem will create the core of the RMP-M [2] with the following structure (Fig. 1):
- Risk load dimension – bottom-up approach
- Risk resistance capacity dimension – top-down approach
- Risk load-resistance theorem – integrative approach

The risk load dimension – bottom-up approach – differentiates risk loads at various levels of a company and is based on the following three concepts:
- Risk aggregation theorem
- Cash flow risk model (CRM) and Asset risk model (ARM)
- Risk load scenario model and risk load theorem

The risk coverage dimension – top-down approach – structures the monetary resources available to cover risks – known as risk coverage resources – to determine the risk load resistance capacity using a top-down approach. It incorporates the following three concepts:
- Risk coverage resources concept
- Risk resistance capacity concept
- Risk coverage resources distribution concept
The risk process control dimension – integrative approach – is pursuing three control concepts:
- Risk load - resistance theorem to control the risk equilibrium
- Opportunities – threats calculation to control the efficiency
- Redistribution concept of risk coverage resources to control the usage where needed

This paper is dealing only with the risk load dimension (bottom-up approach). The other dimensions will be subject to future papers.

![Diagram of Holistic Probabilistic Enterprise Risk Management](image)

**Fig. 1:** Structure of the Holistic, Probabilistic Risk Management Process Model (RMP-M) for project-oriented Enterprises

### 2. Prominent Previous Research

Mehr and Hedges [3] as well as Cristy [4] introduced fundamentals to risk management for insurance companies with broad statistical base. Oehler [5] and Schierenbeck [6] summarized risk procedure for banks in regard to defaulting loans and option pricing as well as to their time focus. Risk management for construction projects is described in Smith [7], Girmscheid/Busch [1], Chapman/Ward [8] and Flanagan/Norman [9]. Markowitz [10] introduced the portfolio theory from which the value-at-risk concept was developed. Risk management in insurance companies and banks is very well based on sound statistical material. Insurance companies make long-term risk decisions while banks, due to the stock market, in general make short-term decisions except in granting long-term loans. In the industry, market risks are dominating the production process risks. The RiskMetrics Group [11] developed benchmarks for corporate risks. In the construction industry, risks are predominantly caused by projects. Until now, neither in
practice nor in research a holistic probabilistic risk model is available to aggregate the contractually accepted project risk to various company levels and to derive the risk-bearing capacity of a company in regard to the insolvency-/bankruptcy-theory. This paper presents the first part, the risk load dimension, of the RMP-Model which was enacted by the Association of Swiss General Contractors.

3. Research Methodology

Construction process and enterprise management is part of the management sciences. The target is to construct new socio-technical realities. For this reason, the constructivist research paradigm [12] is applied to construct the bottom-up approach for risk aggregation and the risk load theorem, as well as the top-down approach to determine the risk resistance capacity and to merge the risk load-resistance theorem for the RMP-Model. This constructivist Risk Management Process Model is validated and reliabilitated by triangulation [13] by means of a theoretical framework and realization tests. For the theoretical framework of the whole RMP-Model, the system theory [14] and cybernetics [15] are applied. The cybernetic-system oriented RMP-M is based on mathematical modelling using the central limit theorem of the stochastics and the probabilistic simulations theories for the risk aggregation, insolvency-/bankruptcy-theory to derive risk load limits. The model was successfully tested by example calculation with data from the companies involved in the research project. Several Swiss general contractors have implemented this model.

4. Risk Load Dimensions – Bottom-up Approach

The levels of a company as outlined in Fig. 1 comprise for example (from bottom to top) the project level, the branch office level, the level of strategic business units (SBUs) and the level of the company overall. With the help of the risk aggregation concept, the risk costs of all projects of a branch office can be aggregated using a bottom-up approach to reveal the total risk of a branch office. The risk costs of all branch offices of a strategic business unit aggregate, in turn, to produce the total risk of the strategic business unit, and the risk costs of all strategic business units produce the total project risk of the company. The cash flow and asset superposition concept is used to calculate the distribution functions of the operative probabilistic cash flows/assets at the various corporate levels. The resulting risk load is determined using the value-at-risk approach for various load levels (load scenarios).

5. Risk Load Aggregation Theorem

Risk Aggregation at Project Level j
After signing the construction contract every company has accepted particular risks (Fig. 1). The risk cost/risk load will be calculated either by the deterministic “Praktiker method” [1] or by the probabilistic stochastic axioms as well as the probabilistic Monte Carlo Simulation (MCS) [16]. In case of the quasi normal distribution of the impact cost of each risk \(i\) in the interval \([T_{\min,i,j,k}, T_{\max,i,j,k}]\) in the project \(j\), the expectation value theorem of the stochastics can be used to determine the expected impact cost of risk \(i\) steadily distributed (Fig. 2) as follows:

\[
T_{E,i,j,k} = \frac{T_{\max}}{T_{\min}} \int_{T_{\min}}^{T_{\max}} f(T_{i,j,k}) \, dT_{i,j,k}
\]

To determine the expected risk cost of project \(j\), the addition theorem of the central limit theorem will be applied for independent quasi normally distributed impact cost at each single risk \(i\) under consideration of the occurrence probability of each risk \(i\). But in reality risks occur randomly, never all together at the same time arbitrarily within the interval of their probabilistic distribution. With reference to the central limit theorem of the stochastics [17] the probability density function and cumulative distribution function of the risk cost in project \(j\) can be determined by the value of the expected risk cost \(R_{E,\text{project } j,k}\) and the variance \(\sigma^2_{\text{project } j,k}\) of risk cost. The probability density function in this case is quasi normally distributed independent of the density function of each risk if the number of risk is high enough (e.g. \(\leq 10\)). The MCS is an excellent tool to be applied to aggregate the single risk to the total project risk or to super aggregate all the project risks on the strategic business unit or even on the company level. The MCS applied for risk analysis generates by random sampling (e.g. Latin Hypercube) random occurrences of risk in each scenario which represents one of the many simulation runs (calculation). The MCS uses two random sampling parameters. The first sampling parameter \(Z(W)_{i,j,k} = \{0 \lor 1\}\) determines the occurrence of the risk or not. In the case the occurrence probability \(P(W)_{i,j,k} = \alpha_{i,j,k}\) then in all total simulation runs \(l_1 = n_l \leq \infty\) the occurrence sampling parameter \(Z(W)_{i,j,k} = 0\) will occur \(1 - \alpha_{i,j,k}\) times and \(Z(W)_{i,j,k} = 1\) will occur \(\alpha_{i,j,k}\) times. The second sampling parameter is the frequency parameter \(Z(T_{i,j,k}) = \{Z(T_{i,j,k}) \in \Re \mid (0 \leq Z(T_{i,j,k}) \leq 1)\}\) which determines out of the cumulative risk impact cost distribution function \(F(T_{i,j,k})\) of each risk \(i\) the magnitude of the impact cost as the risk cost in the specific simulation run \(l_1\).

\[f(T_{i,j,k})\]
\[T_{i,j,k}\]
\[T_{i,j,k}\]
\[\ldots\]

\[f(T_{i,j,k})\]
\[T_{i,j,k}\]
\[\ldots\]

\[f(T_{i,j,k})\]
\[T_{i,j,k}\]
\[\ldots\]

\[\text{Fig. 2: } \text{Probability density distribution functions of impact cost of risk in project } j\]
Expected risk cost (mean value) of project $j$ with MCS:

$$\begin{align*}
R_{E,\text{project } j,k} &= \sum_{i=1}^{n} R_{E,i,j,k} \\
R_{E,i,j,k} &= \sum_{l=1}^{n} \left( T_{i,j,k} (P_{i,j,k,l}) * p_{i,j,k,l} \right) \cdot p_{i,j,k,l}(W)
\end{align*}$$

Variance of risk cost of project $j$:

$$\begin{align*}
\sigma_{\text{project } j,k}^2 &= \sum_{i=1}^{n} \sigma_{i,j,k}^2 = \sum_{i=1}^{n} \left( \sigma_{i,j,k}^2 = \sum_{l=1}^{n} \left( T_{i,j,k} (P_{i,j,k,l}) * p_{i,j,k,l} - T_{E,i,j,k} * p_{i,j,k,l} \right) \right)
\end{align*}$$

Probability density function of risk cost of project $j$ standardized to $A=1$:

$$f(R_{\text{project } j,k}) = \frac{\int_{m_{\text{project } j,k}}^{m_{\text{project } j,k}}} f(R_{\text{project } j,k}) dR$$

Cumulative distribution function of risk cost of project $j$:

$$F(R_{\text{project } j,k}) = \int_{m_{\text{project } j,k}}^{m_{\text{project } j,k}} f(R_{\text{project } j,k}) dR$$

**Fig. 3:** Probability density function and cumulative distribution function of risk cost in project $j$

Risk Aggregation Theorem at Enterprise Level

According to the central limit theorem of the stochastics the aggregated total risk cost and the variance of risk cost of the enterprise level can be expressed as follows [1], [2]:

$$\begin{align*}
R_{E,\text{total}} &= \sum_{k} \sum_{f} \sum_{i} \left( R_{E,i,j,k} \mid R_{E,i,j,k} = \sum_{l} \left( T_{i,j,k} (P_{i,j,k,l}) * p_{i,j,k,l} \right) * p_{i,j,k,l}(W) \right)
\end{align*}$$
The related density and cumulative risk cost function:

\[
\sigma_{total}^2 = \sum_{k} \sum_{j} \sum_{i} \left( \sigma_{i,j,k}^2 \right) \quad \text{and} \quad \sigma_{i,j,k}^2 = \sum_{l} \left( h_i \cdot T_{i,j,k} \cdot (p_{i,j,k,l} - T_{E,i,j,k}) \right) + \left( h_i \cdot p_{i,j,k,l} \right)
\]

The related density and cumulative risk cost function:

\[
f(R_{total}) = f(R_{E,total}; \sigma_{total}^2) \\
F(R_{total}) = F(R_{E,total}; \sigma_{total}^2)
\]

\begin{itemize}
  \item[i] Index for project risks \((0 \leq i \leq n)\)
  \item[l_1] Index for simulation runs at MCS \((l_1 = n_1 \leq \infty)\)
  \item[j] Index for numbers of projects at each SBU
  \item[k] Index for numbers of SBUs at the enterprise
\end{itemize}

\[f(R_{total})\]

\[A = 1\]

\[\sigma_{total}\]

\[R_{min total} \quad R_{E,total} \quad R_{max total}\]

\[1 \quad 0.5 \quad R_{min total} \quad R_{E,total} \quad R_{max total}\]

**Fig. 4**: Probability total risk cost density function and cumulative total risk cost distribution function of enterprise

### 6. Cash Flow Risk Model and Asset Risk Model

The cash flow risk model (CRM) and the asset risk model (ARM) should demonstrate how the total risk and in particular the total project risk associated with the different construction contracts will influence the cash flow and profit of the enterprise. With the CRM and ARM the company can also derive the probability confidence level for their risk premium to anticipate cash flow and profit targets. In project oriented enterprises such as construction companies the major risk exposure is directly related to the value creation / performance process of their contracted projects. For this reason, this paper will only focus on the project risks as the A-B-risk-categories of such companies. But the model can be extended to all kinds of economical risks. Once project analysis has been completed, project oriented companies will decide on the deterministic risk premium \(R_{kalk,project}\) depending on the general chosen probability confidence level as well as on the anticipated deterministic profit premium \(G_{kalk,project}\). Together with the holistic cumulative total risk cost distribution function, these two deterministic values form the base for the CRM and ARM. The CRM and ARM are designed to measure the
imposed risk load in reference to risk resistance capacity in terms of the dynamic cash flow and the company’s assets / profit. We assume for project oriented companies that the risk not related to projects can be neglected in reference to the magnitude of the project-related risks; however, the non operational risk can be considered analogous. Therefore we assume for the simplification of the presentation that the total enterprise risk load is equal to the total project risk load of the enterprise:

\[
R_{E, \text{project total}} \leq R_{E, \text{total}} \quad \text{and} \quad R_{\text{project total}} \leq R_{\text{total}} \quad \text{with} \quad \sigma^2_{\text{project total}} \leq \sigma^2_{\text{total}} \quad \text{and}
\]

\[
f(R_{\text{total}}) = f(R_{E, \text{total}}; \sigma^2_{\text{total}}) = f(R_{\text{project total}}) = f(R_{E, \text{project total}}; \sigma^2_{\text{project total}})
\]

\[
F(R_{\text{project total}}) = F(R_{E, \text{project total}}; \sigma^2_{\text{project total}})
\]

The CRM and ARM will be constructed by creating a target reference point in the cumulative project risk cost distribution function which can be called the profit-loss pivot point. At this pivot point it can be determined if in case of risk occurrence still a profit or already losses occur. Additionally with this target reference / pivot point the risk load resistance limits under consideration of the available risk coverage will be determined in the CRM and ARM. Where the project only considers the source of cash flow and profit in a project oriented enterprise, the target reference point is the deterministic sum either of cash flow premium \(CF_{kalk,\text{total}}\) or profit premium \(G_{kalk,\text{total}}\) and risk premium \(R_{kalk,\text{total}}\):

\[
R_{1}^{CF} = R_{kalk,\text{total}} + CF_{kalk,\text{total}} \quad \text{in the case of the CRM and}
\]

\[
R_{1}^{G} = R_{kalk,\text{total}} + G_{kalk,\text{total}} \quad \text{in the case of the ARM}
\]

To design the CR- and AR-model for cash flow and profit impacted by risk, the reference point must be set at the start of the residual risk for the company. For this reason the CRM and ARM will be designed by transforming the cumulative total project risk distribution function into a cash flow risk function and a profit risk function by

1. creating a mirror image at the \(f(R_{\text{total}})\)-axis of the total risk cost density function of the enterprise (Fig. 5)
2. transforming the coordination zero point of the mirror imaged total risk cost density function at the magnitude of (Fig. 6 - 9)

\[
R_{\text{trans}}^{CF} = R_{\text{min}} + R_{1}^{CF} \quad \text{and} \quad R_{\text{trans}}^{G} = R_{\text{min}} + R_{1}^{G}
\]

The new functions will be called
- CRM: \(F(CF_{\text{project total}})\) – cash flow risk function
- ARM: \(F(G_{\text{project total}})\) – profit risk function

\[
\sigma^2_{\text{project total}} \leq \sigma^2_{\text{total}}
\]
Fig. 5: Mirror image of risk cost density and cumulative function

\[ f(R_{\text{project total}}) = f(R_{E,\text{project total}}; \sigma_{\text{project total}}^2) \rightarrow f(-R_{\text{project total}}) = f(-R_{E,\text{project total}}; \sigma_{\text{project total}}^2) \]

\[ F(R_{\text{project total}}) = F(R_{E,\text{project total}}; \sigma_{\text{project total}}^2) \rightarrow F(-R_{\text{project total}}) = F(-R_{E,\text{project total}}; \sigma_{\text{project total}}^2) \]

Transformation process – horizontal shifting

Fig. 6: Profit risk density function of the ARM

\[ f(-R_{\text{project total}} + R_{\text{min}} + R_{\text{kalk,total}} + G_{\text{kalk,total}}) = f(G_{\text{project total}}) \]

Fig. 7: Cash flow risk density function of the CRM

\[ f(-R_{\text{project total}} + R_{\text{min}} + R_{\text{kalk,total}} + C_{\text{Fkalk,total}}) = f(C_{\text{Fproject total}}) \]
7. Risk Load Scenario Model and Risk Load Theorem

According to Markowitz’ [10] portfolio theory, the risk load which endangers the anticipated cash flow or profit is called “value at risk” of particularly in terms of
- cash flow – cash flow at risk (CFaR)
- profit / assets – earnings at risk (EaR)

The CFaR and EaR are the load vectors which endanger the anticipated deterministic cash flow and profit.

Risk Load Scenario Model

To protect the enterprises from excessive dangerous economical risk, the following risk load limits will be created which must be below the economical risk resistance capacity of a company:

- Normal risk load limit
  represents an excessive risk load exceeding the deterministic risk premium $R_{\text{kalk}}$ – which is considered in all contract prices – by $\text{VaR}_{\alpha}=60\%$. The “normal risk load limit” represents a statistical acceptance limit of $\alpha_N=60\%$ and will only be exceeded in $(1-\alpha) = 40\%$ of all cases.

- Stress risk load limit
represents an excessive risk load exceeding the deterministic risk premium $R_{\text{kalk}}$ – which is considered in all contract prices – by VaR $a = 90\%$. The “stress risk load limit” represents a statistical acceptance limit of $\alpha_S = 90\%$ and will only be exceeded in $(1 - \alpha) = 10\%$ of all cases.

- Crash risk load limit represents an excessive risk load exceeding the deterministic risk premium $R_{\text{kalk}}$ – which is considered in all contract prices – by VaR $a = 99\%$. The “crash risk load limit” represents a statistical acceptance limit of $\alpha_C = 99\%$ and will only be exceeded in $(1 - \alpha) = 1\%$ of all cases.

**Risk Load Theorem**

The derivation of the “cash flow at risk” (CFaR) at the different “risk load limit” stages can be derived out of Fig. 10:

$$\left(\text{CFaR}\right)_{n=\text{NSC}} = \left\{ \left(\text{CFaR}\right)_{n=\text{NSC}} = \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} - R_{\text{kalk;total}} \right\} \cdot (1)$$

$$\Rightarrow \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} > R_{\text{kalk;total}} \cdot (1)$$

$$\vee \left(\text{CFaR}\right)_{n=\text{NSC}} = 0$$

$$\Rightarrow \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} \leq R_{\text{kalk;total}} \cdot (1)$$

The “earnings flow at risk” (EaR) at the different “risk load limit” stages are derived analogously:

$$\left(\text{EaR}\right)_{n=\text{NSC}} = \left\{ \left(\text{EaR}\right)_{n=\text{NSC}} = \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} - R_{\text{kalk;total}} \right\} \cdot (1)$$

$$\Rightarrow \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} > R_{\text{kalk;total}} \cdot (1)$$

$$\vee \left(\text{EaR}\right)_{n=\text{NSC}} = 0$$

$$\Rightarrow \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} \leq R_{\text{kalk;total}} \cdot (1)$$

**Invariance of risk load theorem:**

The mathematical derivation of “cash flow at risk” (CFaR) as well as the “earnings at risk” (EaR) concludes to the invariance theorem in relation to the asset oriented and financial oriented values of the enterprises. Therefore:

$$\left(\text{EaR}\right)_{n=\text{NSC}} = \left(\text{CFaR}\right)_{n=\text{NSC}} = \left(\text{VaR}\right)_{n=\text{NSC}}$$

The “value at risk” is determined at the three risk load limits by:

$$\left(\text{VaR}\right)_{n=\text{NSC}} = \left(\text{VaR}\right)_{n=\text{NSC}} = \left(R_{\alpha;\text{project total}}\right)_{n=\text{NSC}} - R_{\text{kalk;total}} \cdot (1)$$
Risk load theorem:

Risk load at project j:

\[ \text{VaR}_j = (R_{\alpha_j, \text{project } j} - R_{\text{kalk, project } j}) \]

Risk load of SBU k:

\[ \text{VaR}_k = \sum_j \text{VaR}_j = \sum_j (R_{\alpha_j, \text{project } j} - R_{\text{kalk, project } j}) \]

Risk load of Enterprise:

\[ \text{VaR} = \sum_k \sum_j \text{VaR}_{j,k} = \sum_k \sum_j (R_{\alpha_j, \text{project } j} - R_{\text{kalk, project } j}) \]
8. Conclusion

The holistic probabilistic RMP-M offers a process instrument for companies to take risk consciously into account, and this in relation to the risk coverage, to maximize the profit. This paper represented the risk load dimension of the model. The risk load dimension enables the aggregation of risks at various company levels. With the constructed cash flow risk model (CRM) and asset risk model (ARM) the basic model is created to define the risk load / value-at-risk for the company. Further, the probabilistic CRM and ARM are structured in three generic risk load scenario levels. These risk load scenarios are representing different generic statistical acceptance limits. The generic acceptance limits are reciprocal to the occurrence probability. With this part of the holistic company risk model, the probabilistic risk load can be determined. The risk load dimensions are complementary to the holistic risk model RMP-M by two further dimensions: the risk resistance capacity dimension and risk-load-resistance theorem which will be presented by different papers.

References


