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Modeling Crossroads in MATSim: the Case of Traffic-Signaled Intersections

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Abstract

In transportation simulation, the prediction accuracy of travel times on road segments can have substantial impacts on the simulation outcomes. The travel times are impacted, among other things, by traffic signals. Modeling traffic signals is not straightforward in large scale simulations, especially when data on their characteristics is not available. Due to this, in most of the applications using the agent-based transport simulation software MATSim, traffic signals are not explicitly modeled. This paper addresses this issue and proposes a method based on Webster's formula to compute delays experienced by motorists at traffic-signaled intersections without having access to the actual traffic signal data.

Within this study, results regarding the imputation of road flow capacities and estimating the impacts of congestion and crossroads on travel times will be presented and investigated. Evidence shows that Webster's approach can substantially improve the quality of the travel time estimates for the regional model of Zurich, Switzerland.

Keywords: Traffic lights; crossroads; MATSim; Webster's formula

1. Introduction

The goal of transportation simulation is to computationally reproduce travel conditions within a given area, in order to assess the effects of future policies, investments, and, more generally, changes in the infrastructure or in the services offered. For this purpose, it is of importance to calculate travel times as accurately as possible, because those will impact simulations' results, such as transport mode choice or destination choice. Travel times obviously depend on the length of the chosen routes and of the speed limits enforced in the network. They are also influenced by other parameters, such as congestion level, road geometry or zebra crossings. Road intersections, as well, cause additional delays to motorists. This is why a special attention has to be paid to the modelling of crossroads before actually running the simulation.

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This issue is only partially covered in the default settings of the agent-based transport simulation software MATSim (Horni et al. [17]). By default, no distinction is made in MATSim between the traffic-signaled intersections and the other ones. Additional modules presented in Grether [12] and Thunig et al. [27] offer however the possibility of adding traffic lights to the network and to implement different strategies for controlling the signal schedules. They rely on a detailed description of the intersections and on signal plans that had to be manually collected. Thus, those approaches could be counted as work within the field of microscopic scale simulation, whereas MATSim is usually used for mesoscopic scale applications.

This paper presents a contribution towards another possible way of modeling traffic-signaled intersections within the MATSim framework, which relies on Webster's formula (Webster [28]). This equation computes the optimal (in the sense of minimizing the average delay per motorist) traffic signal cycle length. The presented approach does not model each individual signal phase, but uses the average experienced delay, obtained through the Webster formula, to get more accurate estimates of the travel times. This study also provides a straightforward way of adding traffic lights' localizations into an existing MATSim network and compares empirical travel times, computed according to this new approach, to estimates obtained from two web APIs.

After presenting the best-known methods for traffic signal scheduling and the work already done within this field in MATSim, the present paper will guide the reader through the different implementation steps of this new approach and provide insights into validation results.

2. Background

2.1. Webster's approach for optimizing the cycle length of traffic signals

One of the first approaches aiming to optimize the traffic signals' schedule was described by [28]. The chosen minimization criterion is the average vehicle delay, and, in this work, the author develops a formulation for the optimal cycle length derived from equations approximating the expected vehicle delay.

The computation of this average delay was thoroughly studied in Rouphail et al. [25]. A first exact equation was proposed in Beckman et al. [4]; it is valid for fixed-time signals under the assumption of a binomial arrival process. This equation depends, among others, on the arrival and departure rates observed on the links, as well as on the overflow queue (i.e. the number of vehicles remaining in the queue from previous cycles). However, those variables can be difficult to estimate, and the assumptions underlying the formula do not necessarily cover a large range of real-world situations. This is why efforts have been made to approximate the expected vehicle delay by combining theoretical analysis and numerical simulations. The first approximate formula was developed in Webster [28]. According to this work, the average delay per vehicle d, given in seconds, can be approximated by

$$d \simeq \frac{C\left(1-\frac{g}{C}\right)^2}{2\left(1-\frac{g}{C}x\right)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{C}{q^2}\right)^{\frac{1}{3}} x^{2+5\frac{g}{C}}, \tag{1}$$

where

- C is the cycle length in seconds;
- g is the effective green time reserved to the link on which the vehicle approaches;
- x is the degree of saturation on this link, which means the ratio of the (hourly) flow on the link to the link capacity;
- q is the vehicle arrival rate in vehicles per second.

It has to be remarked that, if the first two terms of this expression have a theoretical justification, the last one was designed only so that the theoretical delays fit better with values obtained empirically.

Given this expression, Webster derives from it a formula for the optimal cycle length C_{opt} in seconds:

$$C_{\text{opt}} := \frac{1.5 \times L + 5}{1 - Y}, \tag{2}$$

where

- L is the total lost or unusable time during a signal cycle, in seconds. It corresponds to the sum of the time lost due to clearance times after each phase and of the time when all the traffic lights are red;
- Y is the sum of the saturation ratios x from Equation 1.

The effective green time, $G := C_{\text{opt}} - L$, can finally be split among the incoming links according to their contribution to the total flow.

Webster's formula has been successfully used in transportation analysis for decades and remains one of the prevailing approaches to optimize traffic signal operations. It has been adapted in several recent works. For instance, in Cheng et al. [8], the same optimization criterion was considered, but another approximate equation for the expected delay was used. Furthermore, with the recent rise in interest for more environmentally sustainable transportation systems, this approach was adapted in Calle-Laguna et al. [7] and Calle-Laguna et al. [6], considering new measures of traffic signals' effectiveness, namely fuel consumption and emissions of pollutants.

2.2. *Traffic lights in agent-based transport simulation*

In the microscopic traffic simulation software SUMO (Simulation of Urban Mobility) (Lopez et al. [23]), traffic signal plans can be automatically generated, using a customizable heuristic, defined in Lopez et al. [22]. For instance, with the parameters imposed by default, all the traffic lights have a fixed cycle length of 90 seconds and the green time is equally divided between all links. Traffic lights schedules, if available, can also be imported as "additional files".

In Flötteröd and Behrisch [10], the authors describe the implementation of Webster's approach within SUMO. The formula itself is slightly modified to make sure that constraints, such as minimal green time and minimal and maximal cycle lengths are respected. This approach is tested in two areas (the neighborhood of Berlin-Adlershof, Germany, and a single intersection located in Brunswick, Germany). The authors showed that, on these two study cases, implementing Webster's approach led to improvements in numerous indicators (trip duration, waiting time...).

As well as VISSIM (Fellendorf [9]), SUMO can be used to test other traffic signal control algorithms, as reported in Krajzewicz et al. [18] for SUMO and in Stevanovic et al. [26] for VISSIM, for instance. Most of these methods are based on decentralized and actuated approaches, thus suitable for microsimulation tools. On the contrary, Webster's approach rely only on flow estimates and seems hence adapted for mesoscopical simulations.

2.3. *Traffic lights in the agent-based transport simulation framework MATSim*

In MATSim, the traffic flow is controlled by first-in-first-out queue models relying on the roads' flow and storage capacities, speed limits and lengths. All those parameters (among others) are encoded in the network. The network itself is represented by a set of nodes, representing road crossings, and by a set of links, that represent road sections. Detailed information is provided about the edges whereas only the coordinates of the vertices are known. In MATSim's default implementation, the traffic is modeled mesoscopically: if the simulations detail the vehicles' movements through the network at the link level – for instance, one knows the instant when a car enters and leaves a link –, the events happening within a link are not modeled, and only one queue per link is created – thus, the traffic flow on one link is not split according to the agents' turning intentions.

A simple approach, proposed in Hörl [15], was employed to model the delay caused by intersections: each time an agent, travelling by car, crosses a road which is more important than the one they are currently driving on, their travel time on this particular link is increased by a certain value called the *crossing penalty*. This penalty is manually calibrated, depending on the scenario, and usually ranges from 3 to 8 seconds. Moreover, it is applied both to traffic-signaled and non traffic-signaled intersections.

In Grether [12] and Grether and Thunig [13], the authors lay the foundation of an implementation of traffic lights in MATSim at a microscopic level. A subgraph representing the existing lanes is added on top of each link. This subgraph reflects the structure of a link ending at a traffic-signaled intersection: at the beginning, there is only one lane and, at its end, different lanes can be present that model the distinct turn pockets. With this representation, a vehicle has to be located in the appropriate turn pocket to access its next planned link. This allows to physically separate the different queues, as this is done in real life, without modifying the routing approach – the shortest path algorithm captures the effect of the lanes.

With this representation, traffic signals can be placed at the end of the lanes, allowing for an accurate representation of the way a traffic-signaled intersection is operated in reality.

This approach has been successfully tested in Grether [12] first with fixed-time signal plans and then with actuated signals. In Thunig et al. [27], another implementation, this time based on Lämmer's algorithm – a decentralized, actuated approach presented in Lämmer and Helbing [20], Lämmer and Helbing [21] and Lämmer [19] – was proposed. Those three study cases are based on applications to the city of Cottbus, Germany. Note that all of them rely on fixed-time signal plans that were collected during a previous study.

One major drawback of this approach is that such data sets, gathering traffic signal schedules for a quite large area, are not always available. The present study addresses this issue by proposing a new method, based on Webster's approach, which can be employed without having access to already existing traffic signal schedules. It also has the advantage that it remains in the field of mesoscopic simulations: no complex modification of the network structure, potentially at the expense of runtime efficiency, is required; nonetheless, the exact events happening at the intersection level will not be simulated. However, this is not an issue in the context of the present study: the main objective of this paper is to present a way of estimating the impacts of the delays caused by traffic-signaled intersections on the total travel times in MATSim simulations.

3. Methodology

3.1. The Zurich scenario

The study case of Zurich (Switzerland) was chosen to implement and test Webster's approach into MATSim.

Zurich's scenario was created as a section of the larger Switzerland scenario, which is presented, as well as the cutting process, in Hörl [15] and in Hörl et al. [16]. It does not comprise all of Zurich's agglomeration, but the city itself and the neighboring cities within a radius of 30 kilometers around the city center.

The simulations were performed using a population sample of 10 percent. They were run using the version 1.0.6 of the eqasim framework [1]. Travel demand used in all simulations was obtained from an already converged simulation in regards to mode-choice decisions. Therefore, in the presented studies, in each iteration, during the re-planning stage, 5 percent of the agents had the opportunity only to change the route in order to observe the impacts of different methods on travel times.

3.2. Extracting the traffic signals' positions

The first step towards the implementation of Webster's formula into MATSim consists of collecting the locations of the traffic signals within the study area, and in adding them in some way into the existing network. Because this is not directly related to the present study, only the general ideas of this first step will be presented here. The Python scripts that were used will be made publicly available at https://github.com/eqasim-org.

1. One has to start by extracting the exact positions of traffic signals. This can be achieved with a single OSM request [2], for instance with the help of the QGis tool [3] and the QuickOSM module. The key-value pair that has to be used here is "highway-traffic_signals".
2. Then, the MATSim network is converted into a directed graph in Python using the library `NetworkX` [14]. The nodes representing crossroads are identified.
3. The output of the first step (a file containing the coordinates of all traffic lights) is superposed to the directed graph. Using a k-dimensional tree, it is possible to identify all the nodes such that there is at least one traffic lights within a circle of radius 20 meters centered around the node. The value of 20 meters was manually calibrated. All the selected nodes are tagged as traffic-signaled intersections.
4. Finally, for each traffic-signaled intersections, attention was paid to the incoming links. It was decided to group some of those links in pairs if they seem to belong to the same road (their name reported in the MATSim network is the same, or the angle they form is close to 180°). The reasons explaining why this last step was done will be described in the next paragraph.

3.3. Implementation of the Webster's approach

Traffic-signaled intersections have now been located and the necessary data about them is stored in a way that can be used by MATSim. Webster's formula, the equation that will be implemented to compute the delays at traffic-signaled intersections, depends to a large extent on the road saturation capacities. However, the flow capacities were determined using the module `pt2matsim` ([24]). This tool defines road capacities solely according to their road category in the OSM classification. Those capacities, however, are defined without knowing anything about the intersection in which the road ends and only reflect link characteristics. Traffic signals, in particular, are likely to significantly decrease the theoretical capacity by blocking the vehicles during a substantial share of the time.

A solution was to consider the intersection (seen as a node) capacity as the limiting parameter, rather than the link capacities. The value of 1800 vehicles/hour was chosen as this intersection capacity, to account for the two-second rule. In order to implement it into MATSim, it was assumed that the capacity of each link was equal to $1800 \times l$, where l is the number of lanes of the link.

In that way, Webster's formula for the optimal cycle length becomes

$$C_{\text{opt}} := \frac{1.5 \times L + 5}{1 - \frac{1}{1800} \sum_{\text{link } i} \frac{f_i}{l_i}}, \tag{3}$$

where f_i is the flow on the link i (the number of vehicles that left the link during the considered hour) and l_i is the number of lanes of the link. This equation can be interpreted in the following way: first, the flow on the incoming link is down-scaled according to the number of lanes since there is no conflict between vehicles entering the intersection from the same link. Secondly, these scaled flows are divided by the intersection capacity: those ratios represent the saturated flows, which are finally summed up together as in the original formula. The remaining of Webster's approach can then be used without any other change: the available green time is split among the links according to their contribution to the total flow. This could have been used to create precise signal plans, but this was not done as the focus of the present study lies in the delays caused by traffic signals, which can be computed only from the cycle lengths and green times.

As well as in Flötteröd and Behrisch [10], the formula was adapted so as to enforce constraints ensuring that minimal and maximal green times and cycle lengths were respected. Results from a baseline MATSim simulation were used to obtain the theoretical flows on the links.

Those hourly flows are used to compute, hour by hour, the optimal cycle length on each intersection, the green time devoted to each link and, ultimately, the average delay experienced by motorists on the links. At the end of the first paragraph of this section, it was mentioned that links belonging to the same road were grouped together. Concretely, during the computation of the optimal cycle length, the ratios are computed group by group – and not link by link – before being summed and the effective green time was split among the groups and not among the links. Individual link flows are only used to compute the delays, both directly and indirectly, in the saturation flow.

One of the main benefits of grouping together links belonging to the same road is to artificially reproduce the fact that green lights can be switched on, at the same time, on multiple links, if there is no conflict between those links and independently from the individual contribution of each link to the total flow, which means even if one direction is by far less used than the other. In the present approach, where the turn pockets are not modelled, it was considered to be the case for links that are facing each other or have the same name.

3.4. Enforcing the delays within the simulation

Once the delays are obtained through Webster's formula they can be enforced during the simulation. For a given link, the travel time is computed at each iteration: if an agent leaves this link by entering a link with a higher priority, the crossing penalty (defined in the introduction) is added to the time needed to go through the link. To enforce the new approach, this algorithm was completed with a simple test: if the link is part of an intersection, the corresponding delay is added to the default travel time instead of the crossing penalty. The hour of the day is known while computing those penalties; hence, the computation of delays based on hourly flows makes sense.

Moreover, at night, the traffic signals are often flashing yellow in Zurich. This was taken into account in the simulation: between 11 pm and 5 am, the modified algorithm was not applied and the default approach, with the crossing penalty only, was enforced.

4. Results and discussion

Two main simulations were run, for the Zurich scenario, using the methods and heuristics described in the previous sections, to assess the impacts of the delays experienced by motorists due to traffic signals and to queues that form at crossroads.

To compare the travel times in the different simulations, 5,000 origin-destination coordinate pairs were generated within the study area. A Java script was then used to compute the optimal route and the corresponding travel times according to simulation outputs, at different hours of the day (6AM, 8 AM, 12 AM, 6 PM and 8 PM), which correspond to different traffic situations (before the morning peak hours, the morning peak hours itself, day off-peak hours, evening peak hours and after the evening peak hours). Other travel time estimates, for the same origin-destination pairs and for the same hours of the day, were also computed by using the Bing Maps [5] API to serve as reference against which one can compare the estimates obtained with the different simulations. Achieving perfect accuracy in the travel times computed by MATSim is impossible, as they vary from one day to another (for instance because of accidents); the goal here is thus to obtain travel times which are similar, compared to estimates provided by Bing Maps, to estimates obtained through another web API, such as Google Maps ([11]). Moreover, the fact that only a 10% sample of the total population was used is likely to have biased the results as well.

Figure 1 shows the distribution of the expected delays, computed with Webster's approach, in the Zurich scenario, over the entire day. The figure shows that the immense majority of the expected delays are comprised between 7 and 20 seconds. One can compare this result with those from [10], where the authors estimated an average waiting time for motorists, in the isolated intersection in Brunswick, of 18.3 seconds.

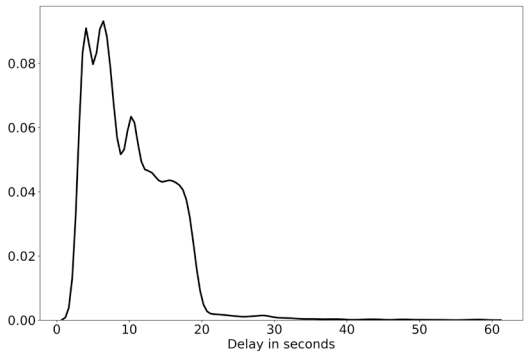


Fig. 1: Additional delays imposed at traffic signaled intersections in Zurich, shown as a probabilistic density function.

Figure 2 depicts the impacts of enforcing Webster's approach for the Zurich scenario, while keeping the crossing penalty unchanged at the default value of 3 seconds. One can first notice that the median of the distribution of the relative differences decreased when the traffic lights were enforced, dropping from 2.7% to -12.9%. To contrast this apparently dramatic drop, a last boxplot was added to Figure 2: the relative difference between estimates given by Google Maps ([11]). This shows that the median relative difference between estimates given by Bing and Google Maps is also close to -12.5%, and thus that Bing could tend to overestimate travel times. The low median of the obtained distribution with traffic lights is therefore not an issue. Moreover, one can see that both the interval between the first and ninth decile and the one between the first and third quartile shrink substantially when Webster's approach is enforced. This implies that, on average, travel time estimates provided with this method are more reliable than the ones obtained with the crossing penalty alone.

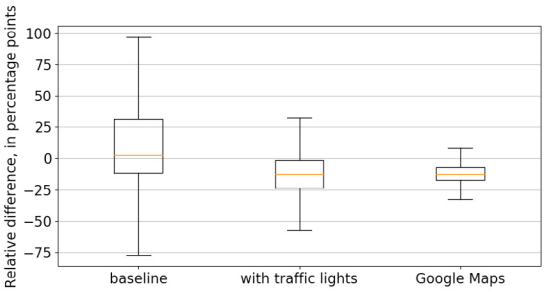
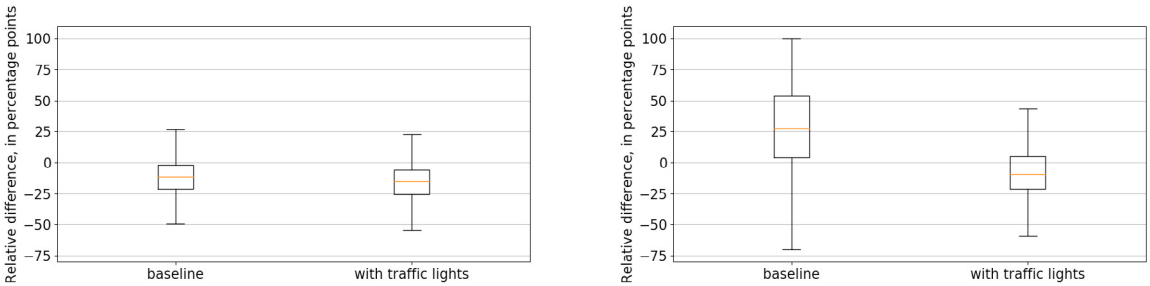


Fig. 2: Comparison of the relative difference in travel times estimated by MATSim, without ("baseline") and with traffic lights, and travel times estimates provided by Bing Maps, for the Zurich scenario.

Figure 3 show that the effect of imposing the delays caused by traffic lights during peak hours is much more significant than off peak hours. This observation suggests that enforcing a crossing penalty besides penalizing separately motorists in congestion situations, as it is done in the baseline simulation, might not be the most relevant approach. With Webster's method, as the computed delay simultaneously comprises the effects of the traffic lights and of congestion, the travel times estimates look more satisfactory, compared to the ones obtained from the Bing API.



(a) At 6 AM

(b) At 8 AM

Fig. 3: Comparison of the relative difference in travel times estimated by MATSim, without ("baseline") and with traffic lights, and travel times estimates provided by Bing Maps, for the Zurich scenario, at 6AM and at 8AM.

5. Conclusion

In this paper, a new heuristic to take into account traffic lights in MATSim simulations was proposed. This method uses Webster's formula to compute, at the mesoscopic level, the delays experienced by motorists due to traffic signals themselves and queues forming because of them.

While analyzing results obtained through this new approach, evidence was found that the approach used by default in order to take into account crossroads, which relies on the manually calibrated crossing penalty, in combination with flow capacities, might tend to over-penalize motorists, regarding travel times, in particular at peak hours. On the contrary, and even if no scientific proof is available yet, the presented heuristic seems to provide reliable travel time estimates, regardless of the hour of the day. Comparing those estimates to the ones provided by microscopic simulation tools, such as SUMO [23], would be worthwhile in order to provide more guarantees about the quality of those estimates.

Different parts in the article indicate potential future work. First, Webster's method is implemented in a static way: final results from one previous simulation are needed to compute the delays that will be enforced, and those delays are not updated iteratively during the simulation to represent the fact that motorists adapt their behavior to them. A dynamic implementation would allow motorists and traffic lights to adapt simultaneously to each other. Secondly, because of this, it is not straightforward to adapt this method to assess the impacts of changes in infrastructure: if a modification is done to the network, a first simulation will have to be performed with the modified network before

new delays can be computed and enforced. Besides that, as Webster's approach provides road flow capacity estimates, it could be interesting to integrate the presented approach in an iterative process, where flows and delays could be updated simultaneously, as well as to investigate the effects on travel times of changing solely the flow capacities or links entering an intersection, while setting the capacities of all other roads to the default value of 1,800 vehicles/hour.

The new approach is only suitable for traffic-signaled intersections. In the simulations that were performed for this study, the crossing penalty approach was thus still used for non traffic-signaled intersections. An additional improvement would be to propose another method to model in a more accurate way these non traffic-signaled crossroads. Doing so, a complete comparison between the default approach and the new one could be performed. The hope is to work in this direction in order to be soon able to offer a new MATSim module that would model all intersections and thus improve the quality of travel time estimates, when access to detailed traffic signal schedules is not available.

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