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Author(s):
Chin, Jun Xing, Hug, Gabriela

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Formation of ad hoc microgrids for prompt critical load pickup during blackouts by leveraging stochastic distributed energy resources

Jun-Xing Chin1 | Gabriela Hug2

1 ETH Zurich, Future Resilient Systems, Singapore-ETH Centre, Singapore
2 Power Systems Laboratory, ETH Zurich, Zurich, Switzerland

Abstract
Extreme weather events that are increasing in frequency and impact can cause extended disruptions to power grids. To increase the resilience of power grids, focus is shifting from infrastructure hardening to quickening the restoration of distribution systems after a disaster. One method to achieve this is through the use of microgrids leveraging distributed generation in islanded mode. While much work has been done in using microgrids for system restoration, the stochastic nature of distributed energy resources (DERs) have so far been neglected. Here, a data-driven approach to leverage stochastic DERs to form ad hoc microgrids to supply local critical loads prior to wide-scale system restoration is presented. The proposed method is tested on a modified IEEE European LV Test Feeder. Results show that considering the stochastic nature of the resources during microgrid formation greatly affects their supply adequacy confidence levels.

1 | INTRODUCTION

Power grids are susceptible to high-impact and low probability extreme weather events, which can disrupt electricity supply for extended periods of time, resulting in loss of critical services for days and even potentially weeks. A report in 2012 has shown that 78% of outages in the United States between 1992 and 2010 were caused by extreme weather events [1], while in 2015, [2] estimates that the economic costs of these outages range between 18 to 33 billion US dollars per year. More recently in February 2021, extreme cold weather in central US resulted in widespread power outages in the Electric Reliability Council of Texas, Inc. region of Texas, US, affecting millions of consumers [3]. While investigations are ongoing, it is estimated that at the peak of the outage, approximately 51 GW of generation [3] was forced offline simultaneously against an estimated peak demand of 76.8 GW [4]. Unfortunately, the frequency and impact of these extreme weather events are expected to rise with climate change [5, 6]. Thus, also increasing the number of weather related outages, for example, in the United States where there was a 70% increase in such outages between 2010 and 2019 compared to the decade before [7]. Despite efforts in hardening infrastructure, given their nature, it is inevitable that power grids remain vulnerable and exposed to natural disasters. As such, the power industry is shifting resilience efforts to focus on quick restoration of distribution systems after a disaster [8].

One method in expediting the restoration process in distribution grids is through the use of distributed generators (DGs) within islanded microgrids (MGs), which has been the subject of many recent works, for example [9–14]. In [9], the authors proposed a method to form MGs for critical load (CL) supply during post-disaster real-time operations using a distributed multi-agent scheme that includes situational awareness to account for damaged nodes. The authors in [10] proposed a method for MG formation for CL supply through self-sustained islands that are robust to post-restoration failures, and extended their method to consider DG availability and post-restoration path reliability in [13]. While in [11], a method that extends existing MGs to supply CLs external to them is presented. The authors in [12] and [14] considered the system transient dynamics such as inrush currents when restoring supply to CLs through sequential load pickup using MGs. Notwithstanding the effectiveness in CL restoration using...
MGs demonstrated in these works, the restoration process itself is time consuming, especially in determining the post-disaster condition of the distribution grid. This necessitates an interim load pickup strategy before wide-scale system restoration.

According to the International Energy Agency, the installed capacity of residential solar photovoltaics (PV) is projected to reach 93 GW globally by the end of 2020, and 173 GW by 2025 [15]. Further, residential solar PV systems are also increasingly being paired with ESSs, with both resources potentially capable of supplying islanded MGs without synchronous generators [16], when installed with grid-forming inverters. These islanded MGs can then be leveraged to supply the CL demand during blackouts. To the best of our knowledge, only [17] directly considers the stochasticity of the distributed energy resources (DERs) and their correlations during CL pickup using MGs in real-time post-disaster operations. Other works that do consider DER uncertainty during post-disaster system restoration are DER type specific and rely on forecasts for PV generation, for example, [18], or on scenario-based stochastic optimisation of wind generation, for example, [19, 20]. Forecasts and scenarios are less readily available for distribution grids, particularly for residual capacity-based sources. Most existing works also typically use convolution-based techniques when combining DERs, ignoring their correlations [17]. The work in [17] is limited to CL pickup within fixed MGs that may be networked together to improve resilience, while [19, 20] study the formation and scheduling of MGs during real-time post-disaster system restoration. Further, these works only considered wind and solar PV generation as sources for local supply, without including other types of DERs, such as the residual supply capacity from energy storage systems (ESSs) and combined heat and power (CHP) plants.

Moreover, existing works related to MG formation for improving system resilience in the literature either study real-time post-disaster restoration (and scheduling), or preventive scheduling directly related to specific foreseeable events, for example, in [21] against windstorms or in [22] against floods. To the best of the authors’ knowledge, there is a lack of work on proactively planning for contingent ad hoc MGs to enhance system resilience by enabling fast CL pickup during blackouts as part of normal operations.

Here, we propose a method to form intermediate ad hoc MGs using stochastic DERs of any type to supply local CLs, such as street lighting, traffic lights, communications infrastructure without backup generators, and water pumps. We envision that these ad hoc MG groupings are determined on a regular basis during normal operations, and are executed directly after a blackout that may result from a natural disaster. As such, the MG formation problem should also minimise the probability of failure to execute due to node and line damage in addition to ensuring supply adequacy. These ad hoc MGs are intended to provide prompt power restoration to CLs to bridge the time interval between a blackout and a proper wide-scale system restoration, including those that leverage MGs presented in the literature. The contribution of this paper is a method to produce feature-dependent contingency plans to form ad hoc MGs during blackouts, which act as stop-gap measures to expedite CL pickup before situational awareness can be established. These contingent ad hoc MGs are planned during the course of normal operations but not executed, and the underlying DERs are normally operating independently. This strategy is a form of operational resilience [23] that can be categorised under the resist and re-establish phase of a resilience curve [24].

The rest of the paper is organised as follows: Section 2 discusses the problem considered; Section 3 details the method used in forming the ad hoc MGs; Section 4 presents details related to the implementation of the proposed method; implementation of the proposed method using a case study is presented in Section 5; and Section 6 concludes the paper with an outlook for future work.

2 | PROBLEM FORMULATION

We consider a feeder in a distribution grid that has a set of \( N \) nodes, denoted as \( \mathcal{N} \), with a subset of these nodes \( \mathcal{N}_{DG} \) hosting stochastic DERs, and a set of lines \( \mathcal{L} \) that can be switched individually. These DERs, for example, residual supply from local generators, ESSs and renewable energy resources, can be leveraged to quickly supply the set of nodes that serve local CLs \( \mathcal{N}_{CL} \subseteq \mathcal{N} \) during blackouts through the formation of pre-determined ad hoc MGs before wide-scale service restoration begins. The problem considered is the formulation of contingency plans for the formation of ad hoc MGs that quickly restore power to CLs prior to establishing situational awareness and beginning wide-scale system restoration. Hence, the goal is to form MGs that maximise the total amount of CLs supplied, while minimising the MG sizes to enable quick execution, minimise the time required for coordination, and reduce the probability of failure to implement due to damaged nodes during a disaster.

These ad hoc MGs are determined by solving an optimisation problem that maximises the CL supplied, while encouraging smaller MG sizes, and ensuring that the CLs can be supplied with a certain level of confidence \( \varepsilon \), that is,

\[
\begin{align*}
\text{maximise} & \quad \Gamma(p_{CL}) - \Phi(N_{MG}) \\
\text{s.t.} & \quad \Pr(p_{CL}^{drop}) \leq \varepsilon.
\end{align*}
\]  

The function \( \Gamma(p_{CL}) \) gives the amount of CL supplied by the MGs, \( \Phi(N_{MG}) \) is a function that penalises MG size, \( \Pr(p_{CL}^{drop}) \) is the probability of supply inadequacy in the MGs, and \( p_{CL}^{drop} \) is a vector of peak power demand of the CLs in the set \( \mathcal{N}_{CL} \). As the available power from DERs is expected to change with time, for example, seasonality and condition of the DER (soiling, deterioration, etc.), the optimisation problem should be computationally tractable such that it may be repeated at regular intervals.

For the rest of the paper, we denote sets with calligraphic letters, for example, \( \mathcal{A}, |\mathcal{A}| \) as the cardinality (size) of set
\[\mathbf{A}, \text{ and denote vectors and matrices with bold letters, for example, } \mathbf{A}.\]

### 3 | FORMATION OF AD HOC MICROGRIDS

Here, the formulation of the microgrid formation problem considering stochastic DERs is presented.

#### 3.1 | Node assignment and formation of microgrids

To formulate the ad hoc MG formation optimisation problem, let the binary variables \(x_{ij}\) represent the membership of node \(i\) in a particular microgrid \(j\) \((x_{ij} = 1\) if node \(i\) is in microgrid \(j))\); \(\mathcal{G}_{MG}\) be the set of indices of possible standalone MGs, that is, the set of nodes with grid-forming DERs, \(\mathcal{G}_{MG} \subseteq \mathcal{N}_{DER}\). The binary variables \(g_j\) indicate whether MG \(j\) is active (formed), and \(p^{CL}_j\) is the CL peak demand at node \(i\). Additionally, assume that all the DERs are able to provide sufficient reactive power support. Then, the MG formation optimisation problem omitting the power balance is given by

\[
\text{maximise } \sum_{j \in \mathcal{G}_{MG}} \left( \sum_{i \in \mathcal{N}_{CL}} x_{ij} p^{CL}_j + \sigma g_j - \phi \sum_{i \in \mathcal{N}} x_{ij} \right),
\]

s.t. \(g_j \leq \sum_{i \in \mathcal{N}_{CL}} x_{ij}, \quad \forall j \in \mathcal{G}_{MG},\) \(\ldots\)

\[
g_j \leq \sum_{i \in \mathcal{N}_{CL}} x_{ij}, \quad \forall j \in \mathcal{G}_{MG},
\]

\[
\sum_{j \in \mathcal{G}_{MG}} x_{ij} \leq 1, \quad \forall i \in \mathcal{N},
\]

where \(\sigma\) and \(\phi\) are weighting coefficients. The function

\[\Gamma(p^{CL}) := \sum_{i \in \mathcal{N}_{CL}} \sum_{j \in \mathcal{G}_{MG}} x_{ij} p^{CL}_j\]

gives the total CL assigned to active MGs and thus supplied by them. Additionally, maximising the number of active MGs (second term) and penalising the total number of nodes assigned across all MGs (third term) lead to smaller MGs with fewer DERs and nodes in each MG. Note that \(\sigma\) and \(\phi\) need to be chosen such that they do not reduce the total amount of CL supplied as that is the main objective. Constraints (3) and (4) ensure that active MGs consist of at least one DER and one CL, while (5) ensures that nodes cannot be assigned to more than one MG. Here, we only considered CLs and assumed that all CLs are of equal importance. Nonetheless, normal loads can easily be included by introducing weighting factors that prioritise the different load types and CLs.

### 3.2 | Stochastic power balance

Ideally, the non-convex power flow constraints are included in the optimisation problem to ensure that line limits, voltage drops and losses are accounted for. However, this greatly affects computational tractability given the mixed-integer nature of the proposed optimisation problem. Linearised power flow approximations that work well on distribution grids, for example, the linearised DistFlow method [25], do not give accurate estimates for line losses, which is crucial when considering supply adequacy. Hence, we take the following pragmatic approach, which does not require the inclusion of power flow equations. Fortunately, voltage drops in the system can be handled during MG operation through the DER set-points, and should be within limits assuming voltages are within limits during normal operation. Moreover, the ad hoc MGs typically lead to shorter distances between generation and supplied demand, which generally reduces the observed voltage drops. Nonetheless, line losses that directly impact the adequacy of supply in the MGs still need to be accounted for in the optimisation problem. To balance computational tractability with the feasibility of the ad hoc MGs, we include a line loss margin, \(l_{loss}\), in the power balance equations. This margin can be computed either from historical operational data, or from power flow analyses of the grid under normal conditions.

The probabilistic power balance constraint in (1) can be formulated by means of chance constraints. This is done by including security margins in the constraints; iteratively solving the optimisation problem and updating these security margins until they converge. However, it is not straightforward to compute and include these margins while ensuring convergence given the non-convex nature of the optimisation problem. As such, we leave this for future work, and propose a combinatorial approach that is simpler, but less scalable, instead. Given a set of DER profiles, we define \(\mathcal{P}(\mathcal{N}_{DER})\) as the power set of \(\mathcal{N}_{DER}\) that contains all possible combinations of the DERs (i.e., all subsets), \(P^G_k\) as the power supplied at the \(\varepsilon\) quantile obtained from the combined probability distribution function (PDF) of the subset \(C_k \in \mathcal{P}(\mathcal{N}_{DER})\) of DERs, and \(c_{kj}\) as binary variables that represent whether subset \(k\) is assigned to MG \(j\). Then, the power balance constraints considering a DER supply adequacy confidence level of \(1 - \varepsilon\) can be expressed as:

\[
\sum_{i \in \mathcal{N}_{CL}} x_{ij} p^{CL}_j \leq \sum_{k=1}^{N_c} c_{kj} P^G_k(1 - l_{loss}), \quad \forall j \in \mathcal{G}_{MG},
\]

\[
|C_k| c_{kj} \leq \sum_{i \in C_k} x_{ij}, \quad \forall j \in \mathcal{G}_{MG},
\]

\[
\sum_{k=1}^{N_c} c_{kj} \leq 1, \quad \forall j \in \mathcal{G}_{MG},
\]

where \(\varepsilon\) is the probability of the event that the power supply is less than or equal to the demand.
where $\mathcal{N}_\epsilon := 2^{\mathcal{N}'_{\text{DER}}}$ is the cardinality of $\mathcal{P}(\mathcal{N}'_{\text{DER}})$. In order to determine the generation of a subset $C_k$, we cannot simply sum the power supplied at the $\epsilon$ quantile of each DER’s PDF as this value pertains to a convolution of the DER PDFs with typically unknown correlations. While solar PV or wind generation are highly correlated within a close proximity, the same may not hold true for other DER sources. As such, the combined PDFs of all subsets $C_k$ need to be estimated in order to obtain their $\epsilon$ quantile power supply values.

There are various methods to estimate (model) the uncertainty of the power output of each DER in order to derive the PDF of their combined supply. The authors in [26] provide a comprehensive overview of DER uncertainty modelling methods. Specifically, each DER’s supply uncertainty can be modelled using parametric methods, for example, Weibull distributions for wind generation [27], Beta [28] or Saunier [29] distributions for PV generation, or Gaussian mixture models for both PV and wind generators [17]. However, parametric methods are less readily available for non-mainstream DERs such as residual supply from ESSs. Moreover, due to the stochastic nature of the DERs themselves, the assumption that the DER outputs follow a known family of distributions may not always be true [26]. Hence, an alternative approach is to use non-parametric methods such as boundary optimisation, interval arithmetic [26], or histogram-based methods to model the DER uncertainty. The former two are interval based and not readily applicable for the risk-based approach required for this work. More importantly, the precise and effective modelling of output uncertainty from multiple DERs remains a research challenge [26]. Therefore, we opted to use a simpler histogram-based data-driven approach here by directly estimating the combined PDF of a DER subset from its time series data without first estimating their individual PDFs, which allows for their correlations to be implicitly modelled. First, the power supplied by all DERs in a subset are summed to result in a time series of that DER subset’s power supply. Then, the PDF for that DER subset is estimated, allowing for its $\epsilon$ quantile power supply to be computed and used in the optimisation problem. Note that for simplicity, we consider that the DER time series data logs the minimum DER power output over the measurement interval. This ensures that by considering the CL peak demand against the minimum DER power supply, the desired supply adequacy levels can be achieved.

Here, constraint (6) ensures that there is sufficient DER supply after subtracting line losses in each MG to achieve the desired supply adequacy level, while (7) ensures that the supply from a DER subset can only be included in an MG if all associated DERs are assigned to that MG. Additionally, each MG is limited to one combination of DERs to ensure that the correlation between the DERs implicitly included when estimating their combined PDFs are accounted for; this is modelled by (8). Explicit constraints on each DER subset being assigned to only one MG are not required as this is implicitly handled by (3). Reactive power balance is not considered in the optimisation problem as we assume that all DERs are able to provide sufficient reactive power support for all connected loads meeting grid code power factor requirements. To improve tractability for larger grids, power set $\mathcal{P}(\mathcal{N}'_{\text{DER}})$ can be replaced by subsets of the DER combinations for each MG, excluding some combinations based on the grid structure and characteristics of the DERs. These subsets of DER combinations may be determined through clustering techniques and is left for future work. Alternatively, instead of individual DERs, one could aggregate multiple DERs into groups in the optimisation problem to improve tractability. While we focus only on stochastic DERs here, the proposed method can easily be extended to include both stochastic DERs and CLs at the cost of scalability.

Energy balance constraints are important when considering energy constrained DERs such as ESSs (state-of-charge) and CHPs (fuel supply levels). Given that they are also stochastic in nature, these constraints can theoretically be included in a similar manner as the stochastic power balance constraints by imposing an operational duration limit on the ad hoc MGs. The method proposed here is meant as a stop-gap contingency plan while more time consuming restoration methods that include ascertaining energy adequacy and grid conditions are executed. Moreover, time series data on energy availability from consumers would be lacking in reality. Hence, the issue of energy balance has been omitted in this paper, and is left to real-time system restoration processes.

### 3.3 Connectivity of microgrids

In order for the MGs to be feasible, they should be fully connected. This needs to be enforced by additional constraints. Let the directed graph $(\mathcal{N}, \mathcal{L})$ represent the distribution feeder network, where the direction of the edges $\mathcal{L}$ are defined arbitrarily. Then, connectivity of the MGs can be achieved by adopting a hybrid version of the single-commodity flow method presented in [30] and [31], which is used to ensure connected sub-graphs. This method is based on the fact that an arbitrary commodity unit that has no physical meaning needs to be able to ‘flow’ from a source node to each node within a fully connected MG. A quantity $y_{ij}$ of commodity units equivalent to the cardinality of each MG are injected at their source nodes, with each node in the MG ‘absorbing’ one unit of the commodity. The MG is fully connected if all the injected units are completely ‘absorbed’ by the nodes without violating the nodal balance and branch flow constraints. We fix the source injection points for each MG as in [30] to improve scalability, but use bidirectional flow variables as proposed in [31]. The source injection nodes are fixed at unique grid-forming DERs for each MG, that is, given that the number of possible MGs are defined by the number of grid-forming DERs, we set their injection points at the corresponding DER. As such, each grid-forming DER node is ‘tagged’ to a particular MG, acting as a source only if it is assigned to its particular MG, and treated as a regular node otherwise.

Define $\mathbf{H}$ as the branch incidence matrix of the nodes, where $H_{(i,j)}$ indicates the $i$th column of the branch incidence matrix, whose $l$th value is 1 if line $l$ enters node $i$, $-1$ if it leaves it, and 0 otherwise. Further, let $\varphi_{i,j}$ represent the commodity flow over
line / in MG $j$, and $\varphi_{i,j}$ be a collection of flow variables across all branches in MG $j$. Then, the constraints, which are modified from [30, 31],

$$y_j = \sum_{i \in \mathcal{N}} x_{i,j}, \quad \forall j \in \mathcal{G}_{MG},$$

(9)

$$\mathbf{H}^T_{(i,j)} \varphi_{(i,j)} + y_j = x_{i,j}, \quad \forall j \in \mathcal{G}_{MG}, \quad i = j,$$

(10)

$$\mathbf{H}^T_{(i,j)} \varphi_{(i,j)} = x_{i,j}, \quad \forall i \in \mathcal{N}, \quad \forall j \in \mathcal{G}_{MG},$$

(11)

$$-\Phi_{F_{i,j}} \leq \varphi_{i,j} \leq \Phi_{F_{i,j}}, \quad \forall i \in \mathcal{L}, \quad \forall j \in \mathcal{G}_{MG},$$

(12)

$$-\Phi_{F_{i,j}} \leq \varphi_{i,j} \leq \Phi_{T_{i,j}}, \quad \forall i \in \mathcal{L}, \quad \forall j \in \mathcal{G}_{MG},$$

(13)

$$\Phi_{F_{i,j}} = \left( \mathbf{H}_{F_{i,j}} \mathbf{x}_{(i,j)} \right) \mathbf{y}_j, \quad \forall i \in \mathcal{L}, \quad \forall j \in \mathcal{G}_{MG},$$

(14)

$$\Phi_{T_{i,j}} = \left( \mathbf{H}_{T_{i,j}} \mathbf{x}_{(i,j)} \right) \mathbf{y}_j, \quad \forall i \in \mathcal{L}, \quad \forall j \in \mathcal{G}_{MG},$$

(15)

where $x_{i,j} := \mathbf{H}_{F_{i,j}} \mathbf{x}_{(i,j)}$. The values for $m$ and $M$ used for linearising the bilinear terms are intuitive to derive: choosing $m = 1$ and $M = |\mathcal{N}|$ ensures that the bilinear terms are either equal to zero, or the size of the MG, both of which are valid bounds for the flows.

In addition to the correlations between the stochastic DERs, their PDFs are usually also known and need to be estimated. Moreover, each DER could be characterised by multiple PDFs, which are feature dependent, for example, the time-of-day and season for PV generation, and additionally day-of-the-week for residual supply from sources dependent on consumer load profiles [33]. Hence, the ad hoc MG groupings may have to be dependent on these features, and thus, multiple runs of the proposed MG formation method are required. To obtain the $\varepsilon$ quantile generation (supply) of the DER subsets, we implement a straightforward data-driven approach that makes no assumption on the PDF types. Given historical time series of grid injections by the DERs, we first group the data based on a number of features that define their PDFs, for example, time-of-day, day-of-the-week, but ignore longer term features such as seasonality. These features can be obtained through data-analysis and machine-learning techniques. The PDFs of the DERs are then estimated from the grouped data by considering historical windows of fixed lengths, which when repeated on a rolling basis (e.g. monthly), allows the implicit modelling of longer term features, such as seasonality and changes in DER condition and consumer behaviour. The omission of longer term features reduces the number of different ad hoc MG groupings that need to be considered at any given time. The estimation of the DER PDFs and the formation of the ad hoc MGs are repeated at regular intervals to reflect the longer term changes in DER PDFs. Note that the correlation between the DERs are not explicitly estimated, but captured when summing their time series data for a given time period before computing the $\varepsilon$ quantile values. As each DER’s PDFs may be characterised by different values for the features such as time-of-day or ambient temperature, a careful combination of these different values is required when grouping the time series data to ensure that all the DER PDFs are estimated using data that correspond to the same time periods. The selection of these feature values is left to works focusing on the analysis of such data.

### 4 IMPLEMENTATION AND DER PDF ESTIMATE

The bilinear terms in (14) and (15) are products between a binary and an integer variable, which can be linearised by using the Big-M approach [32]. We illustrate this for (12) by replacing it with the following constraints:

$$y_j - M(1 - x_{i,j}) \leq \Phi_{F_{i,j}} \leq y_j, \quad \forall i \in \mathcal{L}, \quad \forall j \in \mathcal{G}_{MG},$$

$$m x_{i,j} \leq \Phi_{F_{i,j}} \leq M x_{i,j}, \quad \forall i \in \mathcal{L}, \quad \forall j \in \mathcal{G}_{MG},$$

where $x_{i,j} := \mathbf{H}_{F_{i,j}} \mathbf{x}_{(i,j)}$. The values for $m$ and $M$ used for linearising the bilinear terms are intuitive to derive: choosing $m = 1$ and $M = |\mathcal{N}|$ ensures that the bilinear terms are either equal to zero, or the size of the MG, both of which are valid bounds for the flows.

In addition to the correlations between the stochastic DERs, their PDFs are usually also known and need to be estimated. Moreover, each DER could be characterised by multiple PDFs, which are feature dependent, for example, the time-of-day and season for PV generation, and additionally day-of-the-week for residual supply from sources dependent on consumer load profiles [33]. Hence, the ad hoc MG groupings may have to be dependent on these features, and thus, multiple runs of the proposed MG formation method are required. To obtain the $\varepsilon$ quantile generation (supply) of the DER subsets, we implement a straightforward data-driven approach that makes no assumption on the PDF types. Given historical time series of grid injections by the DERs, we first group the data based on a number of features that define their PDFs, for example, time-of-day, day-of-the-week, but ignore longer term features such as seasonality. These features can be obtained through data-analysis and machine-learning techniques. The PDFs of the DERs are then estimated from the grouped data by considering historical windows of fixed lengths, which when repeated on a rolling basis (e.g. monthly), allows the implicit modelling of longer term features, such as seasonality and changes in DER condition and consumer behaviour. The omission of longer term features reduces the number of different ad hoc MG groupings that need to be considered at any given time. The estimation of the DER PDFs and the formation of the ad hoc MGs are repeated at regular intervals to reflect the longer term changes in DER PDFs. Note that the correlation between the DERs are not explicitly estimated, but captured when summing their time series data for a given time period before computing the $\varepsilon$ quantile values. As each DER’s PDFs may be characterised by different values for the features such as time-of-day or ambient temperature, a careful combination of these different values is required when grouping the time series data to ensure that all the DER PDFs are estimated using data that correspond to the same time periods. The selection of these feature values is left to works focusing on the analysis of such data.

### 5 CASE STUDY AND DISCUSSION

To illustrate our proposed method, we use a modified IEEE European LV Test Feeder with 142 nodes and two additional meshed lines as a test case. In this modified network, there are 12 CLs that need to be supplied by intermediate MGs during a blackout before the system restoration process begins. Figure 1 illustrates the modified distribution feeder, while Table 1 summarises the peak power demand of these CLs. We considered two cases with 12 grid-forming DERs (i.e. $\mathcal{G}_{MG} = \mathcal{N}_{DER}$) each for the case study:

(i) Scenario 1 (S1)—This scenario consists of DERs in the form of generation from residential PV using hourly
profiles for the Singapore climate obtained from the PVWatts application developed by the National Renewable Energy Laboratory [34]. We studied the time range between 10:00 and 15:00, and assumed that the corresponding data is characterised by the same PDFs. In this scenario, there is insufficient PV generation to support the peak CL load beyond a 75\% confidence level.

(ii) Scenario 2 (S2)—The DERs are based on the availability of residual supply from ESSs or CHP generation after satisfying non-critical residential consumer demand using hourly profiles from the Irish Smart Meter Dataset [35]. The stochastic nature of these DERs is due to the underlying stochastic non-critical local demand. We used PDFs that describe the demand after work hours before occupants went to bed, that is, between 20:00 and 01:00, and assumed that the DERs (including ESSs) are able to supply their rated power for the entire duration in this time window. Hence, the residual supply is the DER rated power minus the underlying residential load profile. There is sufficient supply to support the peak CL load beyond a 95\% confidence level in this scenario.

Due to data scarcity (1 year for PV profiles, and approximately 1.5 years for the load profiles), we did not consider features other than time-of-day when estimating the PDFs to avoid over-fitting. This paper aims to present a method that considers the stochastic nature of DERs when forming ad hoc MGs. Hence, while important in reality, the effect of features that define the different PDFs are outside its scope. Table 2 summarises the mean (sample average) supply available from the DERs and their variances for both scenarios. The ad hoc MGs were formed with $\sigma = 1$ and $\phi = 1e^{-3}$, which were chosen based on the order of magnitude of the first objective function term (related to the peak CL demand) to reduce MG size, but avoiding the reduction of the load picked up; and $l_{loss} = 4\%$, which was estimated using a power flow analysis of the distribution feeder under normal conditions with the PandaPower package in Python [36].

Figures 2–5 illustrate the ad hoc MGs formed for Scenarios 1 and 2 with mean (sample average typically used in deterministic settings) and 5\% quantile DER generation (minimum available supply in 95\% of the cases), respectively. The DERs in both scenarios were designed such that they are able to fully supply the CLs based on mean supply availability, as can be seen in Figures 2 and 4. For Scenario 1, this is achieved by combining multiple DERs in larger MG groups, while in Scenario 2, most DERs are able to support at least one local CL independently, reducing the need for synchronisation and coordination. As the desired supply adequacy confidence level is increased, the MG groups become larger to leverage residual generation from neighbouring DERs (see Figures 3 and 5); and in the case of Scenario 1, some CLs have to be dropped, as seen in Figure 3.
It follows that larger MGs would entail larger execution risks as there are higher chances of damage to the lines and nodes during a natural disaster, similar to the path reliability discussed in [13]. Hence, depending on the susceptibility of the nodes and lines to damage during a natural disaster, the grid operator may wish to incentivise smaller ad hoc MGs by increasing $\phi$ to improve their success in restoring power to the CLs.

Power flow analyses of the ad hoc MGs were conducted using the $\xi$ quantile supply from the DERs to verify their feasibility (line losses, and node voltage) in PandaPower. Voltage deviations at all nodes were found to be within 0.01 p.u. of nominal voltage in all of the ad hoc MGs.

For simplicity and ease of numerical analysis, we assumed that the CLs draw constant power equivalent to their peak power demands when assessing the probability of supply adequacy $Pr(p_{CL_{\text{drop}}})$ of the ad hoc MGs. In instances where there is insufficient DER power supply, we assumed that the CLs are either fully supplied or turned off, that is, each individual CL’s demand cannot be partially met. Next, the available power from the DERs are computed by deducting line losses based on values from the previous power flow analyses. The total CL demand served (in kWh) is then calculated.
TABLE 3  Selected numerical results of ad hoc MGs

<table>
<thead>
<tr>
<th>Case</th>
<th>Max LL [%]</th>
<th>Max $\Pr(p_{\text{drop}}^{CL})$</th>
<th>Total CL served [kWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 Mean</td>
<td>1.16</td>
<td>0.35</td>
<td>95,862</td>
</tr>
<tr>
<td>S1 50%</td>
<td>0.89</td>
<td>0.34</td>
<td>95,882</td>
</tr>
<tr>
<td>S1 75%</td>
<td>0.57</td>
<td>0.21</td>
<td>94,214</td>
</tr>
<tr>
<td>S1 90%</td>
<td>1.98</td>
<td>0.10</td>
<td>59,072</td>
</tr>
<tr>
<td>S1 95%</td>
<td>2.13</td>
<td>0.04</td>
<td>52,852</td>
</tr>
<tr>
<td>S2 Mean</td>
<td>1.87</td>
<td>0.13</td>
<td>160,630</td>
</tr>
<tr>
<td>S2 50%</td>
<td>0.26</td>
<td>0.32</td>
<td>143,188</td>
</tr>
<tr>
<td>S2 75%</td>
<td>1.52</td>
<td>0.13</td>
<td>160,684</td>
</tr>
<tr>
<td>S2 90%</td>
<td>2.70</td>
<td>0.07</td>
<td>163,576</td>
</tr>
<tr>
<td>S2 95%</td>
<td>1.51</td>
<td>0.03</td>
<td>165,396</td>
</tr>
</tbody>
</table>

by matching the CL demand to the available power in the MGs. Table 3 summarises the maximum MG line losses at $\epsilon$ quantile DER supply, the maximum probability of insufficient supply, and the total demand served for the different desired supply adequacy confidence levels. Note that the total demand served is not comparable across Scenarios 1 and 2 as the total number of assessed hours are different (2190 and 3217, respectively).

As can be seen in Table 3, the probabilities of having insufficient supply were below the desired levels in all cases. For Scenario 1, the mean values of the DER PDFs were close to their median values, while for Scenario 2 the mean values were closer to the 25% quantile, as can be seen from the results in Table 3. There was no noticeable trend in the line losses despite the MGs becoming larger for higher required confidence levels. The higher line losses in certain MG groupings were due to the significant energy transfers over lines with higher impedance, which was not considered in the optimisation problem. More importantly, assuming that DER supply is sufficient for all CLs, lower probabilities of dropping the CL demand resulted in more load served, as demonstrated in Scenario 2. This is due to the fact that a lower probability of dropping CL demand equates to the CLs being supplied for more hours, that is, more total CL load served. With higher supply adequacy confidence level requirements, the MGs are formed with less instances where there is insufficient DER supply to meet the CL demand, and hence a lower probability of dropped CL demand. However, when there is insufficient DER supply at the lower PDF quantiles, a lower probability of supply inadequacy in the ad hoc MGs could result in less load served, as some CLs were not assigned to the MGs in order to maintain supply confidence levels. This can be seen for Scenario 1, where there is a marked trade-off between the total load served and the probability of MGs not being self-sufficient. The Pareto curve in Figure 6 shows this trade-off for Scenario 1. As can be seen, the demand served can be increased at a cost to reliability of supply until all CLs are assigned to an ad hoc MG. After this point, there is a decline similar to Scenario 2; instances where the CLs cannot be supplied by the smaller MG groups increase, thus reducing the overall demand met. Accordingly, system operators would need to weigh the importance of overall supply adequacy versus total load served when faced with supply limitations.

FIGURE 6  Pareto curve plotting the maximum probability of inadequate MG supply against the total CL served in Scenario 1. The total CL served decreases beyond a probability of CL dropped, when all CLs are assigned to an MG, but with inadequate supply, similar to Scenario 2.

### 6  CONCLUSION AND OUTLOOK

Here, we presented a method for forming intermediate ad hoc microgrids that can be executed immediately following a blackout to supply critical loads by using stochastic DERs. These ad hoc microgrids are intended to bridge the interval between grid supply interruption and the commencement of wide-scale system restoration. Results from the case study have shown that the stochastic nature of the DERs greatly affect the supply adequacy probabilities of the microgrids. However, increasing supply adequacy may reduce the total critical load served. Thus, system operators will need to balance between the two objectives. Nonetheless, it is clear from the case study results that it is vital to consider the stochastic nature of DERs when leveraging them to supply critical loads through the formation of microgrids. Future work will focus on improving the scalability of the proposed method through the use of chance constraints or subsets of the DER combinations, and the consideration of stochastic critical load profiles.

### ACKNOWLEDGEMENTS

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$H_{k}^{()}$ Branch incidence matrix of the nodes
$H_{k}^{()}$ “From” branch incidence matrix
$H_{k}^{()}$ “To” branch incidence matrix
$C_{k}$ DER subset $k$
\( \mathcal{G}_{MG} \) Set of possible standalone MG indices
\( \mathcal{L} \) Set of all lines in the distribution grid
\( \mathcal{N} \) Set of all nodes in the distribution grid
\( \mathcal{N}_{CL} \) Subset of nodes that serve local CLs
\( \mathcal{N}_{DER} \) Subset of nodes hosting stochastic DERs
\( \mathcal{N}_{MG} \) Subset of nodes assigned to MGs
\( \mathcal{P}(\mathcal{N}_{DER}) \) Power set of \( \mathcal{N}_{DER} \)
\( \phi \) Weighting coefficient for penalising MG size
\( \Phi_{Fi,j} \) Variables constraining commodity flow over line \( l \) if the “from” node of line \( l \) is not in MG \( j \)
\( \Phi_{Ti,j} \) Variables constraining commodity flow over line \( l \) if the “to” node of line \( l \) is not in MG \( j \)
\( \sigma \) Weighting coefficient for number of MGs
\( \varphi_{i,j} \) Commodity flow over line \( l \)
\( c_{k,j} \) Binary variables assigning subset \( C_k \) to MG \( j \)
\( g_{j,k} \) Binary variables indicating if MG \( j \) is formed
\( l_{loss} \) Line loss margin
\( m, M \) Constants used in the Big-M linearisation approach
\( N_{CL} \) Number of all possible DER subsets
\( P_{CL} \) Peak power demand of CL \( i \)
\( p_{G,k} \) Quantile power supply of DER subset \( k \)
\( P_{CL}^{max} \) Probability of dropping CL demand
\( S_{i,j} \) Binary variables that determine MG membership
\( y_j \) Quantity of commodity units in MG \( j \)

**ORCID**

Jan-Xing Chin [https://orcid.org/0000-0002-3760-3505](https://orcid.org/0000-0002-3760-3505)

Gabriela Hug [https://orcid.org/0000-0002-4312-616X](https://orcid.org/0000-0002-4312-616X)

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