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Author(s):
Karamooz Mahdiabadi, Morteza; Tiso, Paolo; Brandt, Antoine; Rixen, Daniel Jean

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A non-intrusive model-order reduction of geometrically nonlinear structural dynamics using modal derivatives

Morteza Karamooz Mahdiabadi, Paolo Tiso, Antoine Brandt, Daniel Jean Rixen

Institute for Mechanical Systems, ETH Zurich, Leonhardstraße 21, 8092 Zurich, Switzerland
Chair of Applied Mechanics, Faculty of Mechanical Engineering, Technical University of Munich, Boltzmannstr. 15, 85748 Garching, Germany

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Abstract
Non-intrusive model-order reduction methods are beneficial for reducing the computational costs of dynamic analysis of nonlinear finite element models, developed in programs that do not release nonlinear element forces and Jacobians (e.g., commercial software). One of the key aspects for developing a displacement-based non-intrusive reduced order model is a proper construction of the reduction basis, which has to be small in size, easy to compute, and must span the subspace in which the full solution lives. In this paper, we propose a non-intrusive model order reduction method based on modal derivatives stemming from a selected set of vibration modes of the linearized system. By definition, modal derivatives do not require the knowledge of the applied load. We name this load-independent basis. The method we propose is also simulation-free, meaning that no nonlinear dynamic simulations of the full model are required to construct the reduction basis. The method is tested with three examples of increasing complexity.

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1. Introduction

Lightweight structures appear in many different engineering applications, e.g., see [1–3]. When subject to operational loads, they often exhibit large deflections, that need to be modeled by geometrically nonlinear kinematics. As the solution of full Finite Element (FE) models of such structures is extremely time and storage consuming, Model Order Reduction (MOR) techniques need to be developed. MOR have been widely used in the field of structural dynamics, because they are efficient tools to speed up the solution of FE models. This is achievable by projecting the full-order model onto a reduced set of modes spanning a subspace of much smaller dimension than the size of the full-order model.

In general, two challenges exist to obtaining accurate NonLinear Reduced Order Models (NLROM) for complex FE structures. The first challenge is represented by how to efficiently compute the reduced nonlinear internal forces, where we define efficiency as ease of implementation and low offline and online computational costs. The second challenge is constituted by the “optimal” selection of the reduction bases, which are required to be both of limited size and convey the most important linear and nonlinear features of the response.

To address the first challenge, nonlinear MOR techniques for FE structures are generally classified into intrusive (or direct) and non-intrusive (or indirect) methods. In the intrusive methods, the nonlinear reduced internal forces are obtained by direct projection of the full-order equations to a reduced space using a pre-defined reduction basis (e.g. see [4–6]). However,
to employ intrusive methods for nonlinear MOR, direct access to the nonlinear element formulations are required. This is not the case of FE models built within commercial FE programs.

Alternatively, NLROMs can be developed in a non-intrusive manner, meaning the nonlinear reduced internal forces are obtained without requiring to handle the nonlinear elemental forces. Rather, they are identified based on inputs/outputs from the FE package [2,7,8]. To develop an NLROM with non-intrusive methods, the nonlinear internal forces due to geometric nonlinearity are assumed as a summation of quadratic and cubic polynomials of reduced coordinates multiplied with unknown coefficients, which are called Nonlinear Stiffness Coefficients (NSCs). The aim is then to identify these coefficients by prescribing a set of nonlinear static displacements (forces) to the FE package to solve for the corresponding reaction forces (displacements).

One widely used non-intrusive MOR approach is the Enforced Displacement (ED) (also known as STiffness Evaluation Procedure (STEP)) method, which was first introduced by Muravyov and Rizzi [9]. In this method, selected static displacements that trigger nonlinearity are prescribed to the model through the FE package. The FE package then computes the required reaction forces induced by the statically assigned displacements and output forces. An issue of this method is that the number of required static solutions increases as a cubic function of the reduced number of modal coordinates in the system. To alleviate this problem, Perez et al. [7] used the Tangent Stiffness (TS) matrices due to assigned static displacements, instead of reaction forces for identifying the NSCs. They showed that in this way the number of required nonlinear static solutions reduces significantly, leading to less offline computational costs for the development of NLROMs. In this paper, we refer to this method as the Enhanced Enforced Displacement (EED) method.

The accuracy of any NLROM highly depends on the correct selection of displacement vectors for the reduction basis. For instance, in the MOR of plate-like structures, choosing a set of truncated vibration modes as displacement basis, which are bending-dominated modes, does not deliver accurate results. Instead, also membrane-dominated modes have to be manually selected and appended to the input displacements to achieve accurate results. In the case of curved structures, this augmentation is not necessarily membrane-dominated, see [10], and the choice of the correct additional modes for the reduction basis is even more difficult, and put the ED and EED methods [11,2] to challenge.

In order to cope with this problem, Dual or Companion modes are proposed in [12] to capture the nonlinear interaction between the vibration modes (e.g. transverse-membrane coupling, also known as stretching effect). The idea is to enforce a set of selected static load cases related to the kept linear modes on the system and compute the corresponding nonlinear displacements. These displacements are then orthogonalized with respect to the linear modes kept in the basis and appended to them. By this, the basis is enriched in a quasi-systematic way to approximate the nonlinear response of the NLROM accurately. However, the main issue with the dual modes is that a relatively large number of representative load cases are required to trigger responses ranging from almost linear to strongly nonlinear. Moreover, the procedure for the selection of the important dual modes is in general load-dependent (i.e. the actual shape of the applied load on the full-order model is usually required) and based on trial-and-error to find the optimal scaling factors. This procedure delivers accurate results, e.g. see [7], but could be expensive for large structures.

In this paper, we propose to append non-intrusively computed Modal Derivatives (MDs) - instead of membrane or dual modes - to a basis of dominant vibration modes to form the reduction basis. To compute the MDs non-intrusively, two different strategies are outlined, namely for the cases when either the tangent stiffness or nonlinear reaction forces are available from a nonlinear static solver.

The first advantage of the proposed method over other available methods is that, by using MDs, one does not require manual selection of membrane-dominated modes required to accurately develop the NLROM. Instead, the most important nonlinear features of the full-order response are conveyed by non-intrusive MDs, which are systematically derived from the originating vibration modes (i.e. do not require trial-and-error selection) and are easy to compute. In fact, the computation of each MD requires the evaluation of the tangent stiffness matrix for two distinct configurations only. The second advantage is that the MDs are selected in a load-independent manner, meaning that no prior knowledge of the applied loads is required to generate them. The accuracy as well as the computational efficiency of the proposed method is evaluated by applying it to different FE models subject to various random sound pressure levels. The validation is carried out by comparing the Power Spectral Density (PSD) of the displacements of the NLROM and the full model.

We compare the results of our developed NLROMs with the ones obtained from the NLROMs using dual modes, which were shown to be very effective in capturing the nonlinear effects of the system, and they are arguably the reference method for non-intrusive ROM in nonlinear structural dynamics. As we will discuss in detail in Section 3.2, there exist two different versions of the dual mode selection methods, a load dependent [7] and a load-independent one [12]. The former produces very accurate results and usually outperforms the latter, but requires the knowledge of the applied load. As our method is load-independent, we compare to the procedure described in [12]. It should also be noted that the method proposed in [12] allows some degree of arbitrariness in the involved steps, which might slightly affect the results. The actual implementation and the selection of the required parameters is detailed in the results section.
2. Non-intrusive model order reduction

2.1. Governing equations for a geometric nonlinear ROM

In case of finite elastodynamics, the governing Equations Of Motion (EOM) for an FE model with linear material properties, large deformations (i.e. geometric nonlinearity) and n Degrees-Of-Freedom (DOFs) can be given in tensor form as (for more details see e.g. [13,14])

\[ \dot{M}_u(t) + \dot{D}_u(t) + f_{\text{int}}(u(t)) = f(t), \tag{1} \]

where the mass matrix is denoted by \( \mathbf{M} \), and \( f_{\text{int}} \) is the nonlinear internal force vector modeling geometric nonlinearity. In this paper, a linear viscous damping (denoted by \( \mathbf{D} \)) is adopted. The displacement and external force vectors are \( \mathbf{u} \) and \( \mathbf{f} \), respectively. Following standard notation, time derivative is indicated by \( \dot{\cdot} \). Note that in the remainder of this paper the time-dependency is omitted for the sake of simplicity, unless otherwise indicated.

In order to obtain a ROM for Eq. (1), a reduction basis must be defined, which maps the full-order model with \( n \) DOFs, to a reduced set of \( m \) generalized coordinates, with \( m \ll n \), namely

\[ \mathbf{u} \approx \mathbf{Vq}. \tag{2} \]

where \( \mathbf{V} (n \times m) \) is the reduction basis and \( \mathbf{q} (m \times 1) \) denotes the vector of the reduced set of generalized coordinates. The ROM is then achieved by introducing Eq. (2) to (1) and pre-multiplying it by \( \mathbf{V}^T \) (the superscript \( T \) denotes the transpose of the matrix) to enforce the generated errors due to model reduction orthogonal to the reduction basis. The \( i^{th} \) EOM of the NLROM reads

\[ \ddot{M}_i q_i + \ddot{D}_i q_i + \ddot{K}^{(1)}_i q_i + \ddot{K}^{(2)}_i q_i q_j + \ddot{K}^{(3)}_i q_j q_k q_p = \ddot{f}_i, \tag{3} \]

where

\[ \ddot{M} = \mathbf{V}^T \mathbf{M} \mathbf{V}, \tag{4} \]

\[ \ddot{K}^{(1)} = \mathbf{V}^T \mathbf{K} \mathbf{V}, \tag{5} \]

\[ \ddot{D} = \mathbf{V}^T \mathbf{D} \mathbf{V}, \tag{6} \]

\[ \ddot{f} = \mathbf{V}^T \mathbf{f}, \tag{7} \]

and \( \ddot{K}^{(2)}_i \) and \( \ddot{K}^{(3)}_i \) are the components of the reduced quadratic and cubic stiffness tensors (NSCs), respectively. Summation over repeated indexes is assumed. It should be noted that the nonlinear reduced internal force vector is approximated by a Taylor series expansion up to cubic terms and \( \mathbf{K} \) denotes the linearized stiffness matrix of the full-order model. The NSCs in Eq. (3) can be obtained either by direct projection of the full-order tensors (in case \( f_{\text{int}} \) is of polynomial cubic form, as is the case for solid elements and von-Karman shells) or by identification through non-intrusive methods. If the FE model of the structure is developed in a commercial FE analysis package, there is usually no access to the nonlinear stiffness tensors of the full model. Therefore, to obtain the NSCs of the reduced model, non-intrusive MOR approach must be applied, which will be explained next.

2.2. Enforced Displacement

The Enforced Displacement (ED) method was first developed by Muravyov and Rizzi [9], and later modified by Kim et al. [12] for the case where the linear stiffness matrix is also unknown. The ED method is based on enforcing a series of nonlinear static displacements to the FE model, and computing the corresponding forces required to create these displacements. When the applied displacement and the corresponding forces are known, then \( \ddot{K}^{(2)}_i \) and \( \ddot{K}^{(3)}_i \) can be computed by solving a series of linear algebraic equations.

Since the identification procedure here is based on static cases, Eq. (3) for the NLROM reduces to

\[ \ddot{K}^{(1)}_i q_i + \ddot{K}^{(2)}_i q_j q_i + \ddot{K}^{(3)}_i q_j q_k q_p = \ddot{f}_i, \tag{8} \]

In cases the linear stiffness of the full-order model is available, the first step consists of the construction of two different displacements from each single generalized coordinate, i.e.,

\[ u^{(a)} = q^{(a)} v_r, \quad a = 1, 2, \tag{9} \]

where \( u^{(1)} \) and \( u^{(2)} \) are the static displacement fields to be imposed to the FE package; \( q^{(1)} \) and \( q^{(2)} \) are different scaling factors for the \( r^{th} \) generalized coordinate \( (r = 1, 2, \ldots, m) \), and \( \mathbf{v}_r \) is the \( r^{th} \) column of the reduction basis matrix. We usually choose
the generalized coordinates as $q_r^{(1)} = -q_r^{(2)}$. By static imposition of the displacement fields of Eq. (9) to the structure, all the stiffness coefficients corresponding to two or three different generalized coordinates are zero (e.g. $K_{il}^{(2)} = K_{il}^{(3)} = 0, j \neq i \neq p$).

The modal forces corresponding to these displacements for the $i$th equation of (8) are then given by

$$\vec{f}_i^{(a)} = V_i^T \mathbf{f}^{(a)} a = 1, 2,$$

where $\mathbf{f}^{(1)}$ and $\mathbf{f}^{(2)}$ are the reaction forces due to the applied displacement in physical domain and $\vec{f}_i^{(1)}$ and $\vec{f}_i^{(2)}$ are the $i$th component of their counterparts in modal domain ($i = 1, 2, \ldots, m$). By substituting Eqs. (9) and (10) into Eq. (8) one obtains

$$K_{ir}^{(1)} q_r^{(a)} + K_{ir}^{(2)} (q_r^{(a)})^2 + K_{ir}^{(3)} (q_r^{(a)})^3 = \vec{f}_i^{(1)}, a = 1, 2,$$

which leads to the solution of all the unknown coefficients in the form of $K_{ir}^{(2)}$, and $K_{ir}^{(3)}$, in Eq. (11). Note that if the reduced linear stiffness is not obtained in advance from Eq. (5), then three static displacements per each generalized coordinate must be imposed in Eq. (9) to compute the unknowns [12].

In the next step, a combination of two columns of the reduction basis with different generalized coordinates are imposed on the system while all other modes are zero, namely

$$\mathbf{u} = [\mathbf{V}_r \mathbf{V}_s \mathbf{V}_t]^T \begin{bmatrix} q_r & -q_r & q_r \\ q_s & -q_s & -q_s \\ q_t & -q_t & -q_t \end{bmatrix}, s > r.$$

The corresponding forces required to induce the three displacements are then computed and projected in the modal domain to obtain the components of the modal force needed to be substituted in Eq. (8). This leads to three Eqs., which are utilized to obtain the unknown parameters $K_{ir}^{(2)}$, $K_{is}^{(3)}$, and $K_{it}^{(3)} (r \neq s)$ corresponding to the multiplication of two different modal coordinates ($q_r \neq q_s$). It should be mentioned here that due to the symmetry of the tensors, the elements of the nonlinear stiffness tensors $K_{ijl}^{(2)}$ and $K_{ijl}^{(3)}$ are only computed and stored for the case where $p > i > j$ (see [7] for more details).

The last step computes the NSCs corresponding to the combination of three different modes, i.e. $K_{ir}^{(3)} (r \neq s \neq t)$. To do so, combinations of three different generalized coordinates are employed as

$$\mathbf{u} = [\mathbf{V}_r \mathbf{V}_s \mathbf{V}_t]^T \{ q_r \ q_s \ q_t \}^T, t > s > r.$$

Similarly to the previous steps, these nonlinear static displacements are imposed to the full static model and the resulting forces are projected on $\mathbf{V}$. When inserted in Eq. (8), they generate the missing equations to find $K_{ir}^{(3)}$. The number $N_{ED}$ of the nonlinear static evaluations required to develop an NLROM with $m$ DOFs is

$$N_{ED} = 2m + 3mC_2 + mC_3,$$

where

$$mC_r = \frac{m!}{(m - r)!},$$

As can be seen from (15), $N_{ED}$ is of order $O(m^3)$. This constitutes a serious burden on the size of the NLROM. In order to reduce the order of required static solutions, the Enhanced ED (EED) method was proposed by [7]. We briefly describe it in the next section.

**Remark on generalized coordinates factor:** Theoretically, any magnitude of the generalized coordinates (scaling factors) $\mathbf{q}$ can be applied to the ED method. However, in practice these values must be large enough to trigger geometric nonlinearity but not too large to cause convergence problems of the FE solution. For instance, Mignolet et al. [15] have studied different scaling factors for transverse and membrane motion of shell-like structures. It is usually suggested to prescribe scaling factors, which result in maximum one thickness deformation of the structure for transverse-dominated modes and $\frac{1}{10}$ to $\frac{1}{100}$ of that for membrane-dominated modes.

### 2.3. Enhanced enforced displacement

The Enhanced Enforced Displacement method (EED) was first introduced by Perez et al. [7], and further validated for nonlinear structures under thermal, aerodynamic and acoustic loading conditions in [16,17]. The key idea of the EED method is employing the Tangent Stiffness (TS) matrix, instead of the restoring forces of the FE model, due to an imposed displacement. The TS matrix contains more information about the NSCs than the restoring forces, resulting in the identification of more coefficients from each nonlinear static analysis.

Let us assume that the FE package, in which the nonlinear full model is developed, releases the TS matrix for a given arbitrary configuration (this is the case for many commercial software, e.g. Abaqus, Nastran, etc.). The TS matrix of the full-order model from a nonlinear static analysis is then projected to $\mathbf{V}$ as

$$\mathbf{\hat{K}}(\mathbf{q}) = \mathbf{V}^T \mathbf{K}(\mathbf{Vq}) \mathbf{V},$$
where \( \mathbf{K}' \) and \( \mathbf{K} \) are the TS matrices for the reduced- and full-order models, respectively. The reduced TS matrix \( \mathbf{K}' \) is the Jacobian of the nonlinear restoring force vector with respect to generalized coordinates, namely,

\[
\mathbf{K}'(\mathbf{q}) = \begin{bmatrix}
\frac{\partial f_{\text{int}}}{\partial q_1} & \cdots & \frac{\partial f_{\text{int}}}{\partial q_m} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_{\text{int}}}{\partial q_{m-1}} & \cdots & \frac{\partial f_{\text{int}}}{\partial q_m}
\end{bmatrix},
\]

(17)

where its \( ir^{th} \) component is given by

\[
\mathbf{K}'_{ir} = \frac{\partial f_{\text{int}}}{\partial q_r} = \frac{\partial}{\partial q_r} \left[ \mathbf{K}^{(1)}_{ij} q_i + \mathbf{K}^{(2)}_{ij} q_i q_j + \mathbf{K}^{(3)}_{ijj} q_i q_j q_j \right] = \mathbf{K}^{(1)}_{ir} + \mathbf{K}^{(2)}_{irj} q_j + \mathbf{K}^{(3)}_{irjj} q_j q_j q_j q_j.
\]

(18)

Similar to the ED method, to obtain the first set of unknown NSCs, two static displacements are formed per each single generalized coordinate, as per Eq. (9). Then, the \( ir^{th} \) element of the resulting TS in modal domain yields

\[
\mathbf{K}^{(a)}_{ir} = \mathbf{K}^{(1)}_{ir} + \left[ \mathbf{K}^{(2)}_{irj} + \mathbf{K}^{(3)}_{irjj} \right] \left( q_j^{(a)} \right)^2, \quad a = 1, 2.
\]

(19)

Let us recall the assumption of Section 2.2 that the coefficients of the form \( \mathbf{K}^{(2)}_{ij} \) and \( \mathbf{K}^{(3)}_{ijj} \) are put to zero except the case where \( p > l > j \), due to symmetry of the tensors. This assumption splits Eq. (19) into three conditions, namely

\[
\mathbf{K}'_{ir}^{(a)} = \begin{cases} 
\mathbf{K}^{(1)}_{ir} + \mathbf{K}^{(2)}_{irj} \left( q_j^{(a)} \right)^2 & \text{if } j < r, \\
\mathbf{K}^{(1)}_{ir} + 2\mathbf{K}^{(2)}_{irj} \left( q_j^{(a)} \right)^2 + 3\mathbf{K}^{(3)}_{irjj} \left( q_j^{(a)} \right)^2 & \text{if } j = r, \\
\mathbf{K}^{(1)}_{ir} + \mathbf{K}^{(2)}_{irj} \left( q_j^{(a)} \right)^2 & \text{if } j > r.
\end{cases}
\]

(20)

For each condition, there are two unknowns corresponding to the quadratic and cubic NSCs, \( \mathbf{K}^{(2)}_{ij} \), where \( j \neq l \neq r \). Static displacements of the form

\[
\mathbf{u} = \mathbf{V}_l q_l + \mathbf{V}_r q_r,
\]

(21)

have to be enforced on the FE model to attain the TS matrices demanded for the last step. After transforming the TS to the modal domain, its \( ir^{th} \) component is written as

\[
\mathbf{K}'_{ir} = \mathbf{K}^{(1)}_{ir} + \left[ \mathbf{K}^{(2)}_{irj} q_j + \mathbf{K}^{(3)}_{irjj} q_j \right] + \left[ \mathbf{K}^{(3)}_{ijl} q_l + \mathbf{K}^{(3)}_{ijj} q_j q_j \right].
\]

(22)

The only unknown in this equation is \( \mathbf{K}^{(3)}_{ijj} \), which can then be computed.

As can be seen in the identification procedure of the EED method, the displacements from a combination of three generalized coordinates are no longer required to identify all the coefficients. The number \( N_{\text{EED}} \) of the required static cases for EED is given by

\[
N_{\text{EED}} = 2m + mC_2.
\]

(23)

Therefore, the order of nonlinear static analysis reduces from \( O(m^3) \) to \( O(m^2) \). The work flows of ED as well as EED are shown in the flowchart of Fig. 1. Furthermore, Fig. 2 shows the rate of growth of the required nonlinear static evaluations versus the number of generalized coordinates in the NLROM for ED and EED methods. This figure shows a significant reduction of the static solutions for EED compared to ED while the number of DOFs in the NLROMs have increased, which results in less computational costs for reducing the model.

### 3. Basis selection: simulation-free methods

As mentioned before, selection of an appropriate basis for nonlinear model reduction techniques is one of the key challenges. Generally, the following points must be considered while choosing a basis for NLROM:

- It must contain at least all the linear modes of the system in the frequency band of excitation and according to the applied load distribution;
- It must reproduce the main nonlinear features of the response;
- It should be of small size;
There are generally two types of reduction bases in the context of nonlinear ROM. The first class of bases is derived by data driven methods, which are computed by performing a dynamic simulation of the full-order model. Examples of these approaches are Proper Orthogonal Decomposition (POD) or Smooth Orthogonal Decomposition (SOD) (see [18–22,4]). These methods are not considered in this paper, as we aim at both avoiding the need of time integration of the full system and devise a basis construction that is not load dependent. To this end, we look at the so-called simulation-free methods, which do not require full dynamic simulation and yield a basis that is not dependent of the specific excitation. As such, the resulting

- It should be computed affordably.

Fig. 1. Flowchart of the steps to identify nonlinear stiffness coefficients of an NLROM using ED or EED.

Fig. 2. Number of required nonlinear static solutions for the ED and EED methods versus the number of generalized coordinates (or DOFs) in the NLROM.
ROM is potentially valid for different loading conditions. While methods along this line already exist, here we propose a new basis for reduction suitable for non-intrusive ROM methods.

3.1. Linear vibration modes

The original ED method in [9] proposed the use of linear Vibration Modes (VMs) as the basis for projection of the full-order linear matrices as well as identification of the NSCs. This basis has been further investigated in different works (see e.g. [11,23,24]). The VMs are the solution of the eigenvalue problem associated to the linearized system around the equilibrium, namely,

$$\left( K - \omega^2 \mathbf{M} \right) \mathbf{\varphi}_j = \mathbf{0},$$

where $\omega_j$ and $\mathbf{\varphi}_j$ are the $j$th eigenfrequencies and linear VM of the system, respectively. A set of $w$ vectors $\mathbf{\varphi}_j$, $j = 1, \ldots, w$ are then selected according to the frequency band of interest to form the reduction basis matrix as

$$\mathbf{V}_{VM} = \left[ \mathbf{\varphi}_1, \mathbf{\varphi}_2, \cdots, \mathbf{\varphi}_w \right],$$

where $\mathbf{V}_{VM}$ is mass-normalized. The number $w$ of VMs can be highly reduced if they are selected according to a specified spatial load distribution [11,23,24]. However, the basis $\mathbf{V}_{\text{VM}}$ leads to poor results if used for reduction, as $\mathbf{\varphi}_j$, $j = 1, \ldots, w$, do not represent well the nonlinear behavior. For instance, for a thin-walled structure, the low-frequency modes are typically of a bending nature, and as such, they do not feature axial displacements that are needed to represent the nonlinear bending-stretching coupling. Therefore, a few high frequency membrane-dominated modes should be computed and appended to $\mathbf{V}_{VM}$ to develop an accurate NLROM. Manual identification of the membrane-dominated modes for simple structures is a difficult but feasible exercise. However, for more complex geometries, this becomes impractical. Therefore, it is desirable to find an accurate basis in a systematic way.

3.2. Dual modes

In order to avoid the manual selection of membrane modes in the primary basis of ED (Section 3.1), dual modes are proposed (cf. [12]) and further investigated in many applications (e.g. see [25,26,16,17]). Dual modes are found by statically applying representative forces and solve for the corresponding displacements. All the static solutions are then collected and used, i.e., $d = 1$, but the procedure here described can be extended to the case of multiple dominant modes. We form a set of scaling factors $\mathbf{\alpha} = [\alpha_1, \ldots, \alpha_r]$ such that the static solutions $\mathbf{x}_r$, found as

$$\mathbf{f}_{\text{int}}(\mathbf{x}_r) - \mathbf{\alpha} \mathbf{K} \mathbf{\varphi}_d = \mathbf{0}, \quad i = 1, \ldots, r,$$

span from near linear to strong nonlinear range of deflections and with positive and negative signs. We now create a set of loads $\mathbf{G}^{(j)}$ as

$$\mathbf{G}^{(j)} = \text{diag}(\mathbf{\alpha}) \mathbf{K} \left( \frac{\varphi_d + \varphi_j}{2} \right), \quad j = 1, \ldots, w,$$

and let $\mathbf{g}^{(j)}$ be the $i$th column of $\mathbf{G}^{(j)}$. Then, for $j = 1, \ldots, w$:

1. Solve the static problem

$$\mathbf{f}_{\text{int}}(\mathbf{x}_r^{(j)}) - \mathbf{g}^{(j)}, \quad i = 1, \ldots, r,$$

and collect the solutions in $\mathbf{X}^{(j)}$ as

$$\mathbf{X}^{(j)} = \left[ \mathbf{x}_1^{(j)}, \ldots, \mathbf{x}_r^{(j)} \right].$$

2. Mass-orthogonalize $\mathbf{X}^{(j)}$ with respect to the retained vibration modes $\mathbf{\varphi}_1, \ldots, \mathbf{\varphi}_m$ as

$$\mathbf{x}_k^{(j)} = \mathbf{x}_k^{(j)} - \sum_{k=1}^{w} \left( \mathbf{\varphi}_k \mathbf{M} \mathbf{x}_k^{(j)} \right) \mathbf{\varphi}_k, \quad i = 1, \ldots, r,$$

and collect them into $\mathbf{X}^{(j)} = \left[ \mathbf{x}_1^{(j)}, \ldots, \mathbf{x}_r^{(j)} \right]$. 


3. Perform an SVD on $X^{(j)}$ and retain $k < r$ singular vectors $\hat{\psi}_i$, $i = 1, \ldots, k$ associated to the lowest $k$ singular values, as

$$\Psi^{(j)} = \text{SVD} \left( X^{(j)} \right),$$

where

$$\Psi^{(j)} = \begin{bmatrix} \hat{\psi}_1^{(j)} & \cdots & \hat{\psi}_r^{(j)} \end{bmatrix}, \quad \left| s_1^{(j)} < s_2^{(j)} < \ldots < s_i^{(j)} < \ldots < s_r^{(j)} \right.,$$

and $s_i^{(j)}$ is the $i$th singular value of $S^{(j)}$, and $R^{(j)}$ is the right matrix of the SVD (not used in this procedure).

4. Compute the linearized strain energy measures associated to each column of $\Psi^{(j)}$ (i.e. to each dual mode candidate), as

$$E_i^{(j)} = \sum_{s=1}^{r} \left( \beta_i^{(j)} \right)^2 \hat{\psi}_i^{(j)T} K \hat{\psi}_i^{(j)}, \quad \beta_i^{(j)} = \frac{\Psi^{(j)^T} X^{(j)}}{\hat{\psi}_i^{(j)T} \hat{\psi}_i^{(j)}},$$

and collect them into the vector $E^{(j)} = \left[ E^{(j)}_1, E^{(j)}_2, \ldots, E^{(j)}_r \right]$.

5. Retain the most relevant $z < k$ dual mode candidates $\psi_l^{(j)}, l = 1, \ldots, z$ associated to the largest entries of $E^{(j)}$, i.e.

$$E_{p_1}^{(j)} > E_{p_2}^{(j)} > \ldots, E_{p_z}^{(j)},$$

and let $V^{(j)}_{DM} = \left[ \psi_{p_1}^{(j)}, \psi_{p_2}^{(j)}, \ldots, \psi_{p_z}^{(j)} \right]$.


The collection of selected DM is then $V_{DM} = \left[ V^{(1)}_{DM} \ldots V^{(j)}_{DM} \ldots V^{(w)}_{DM} \right]$ and the reduction basis $V$ is given by

$$V = \left[ V_{VM} V_{DM} \right].$$

Note that for the numerical examples presented in this work, we found that when the orthogonalization procedure of step 3 to the kept linear vibration modes is performed after all final dual modes are collected for $j = 1, \ldots, w$ (i.e., after step 6), we can obtain slightly more accurate dynamic results.

Furthermore, Eq. (33) in this procedure reveals those left singular vectors $\hat{\psi}_i^{(j)}$, which are mostly present in the nonlinear displacements obtained from different scaling factors contained in $x$, and at the same time have the highest strain energy. According to [12], one singular vector is kept from each load set $j$, i.e., $z = 1$. Here, we also test the case of keeping two vectors ($z = 2$) per each load case $j$, and compare the results in Section 4. Alternatively, the dual modes can be selected regarding their contribution to a specific loading condition, as performed by Perez et al. [7]. This strategy can significantly increase the accuracy of the NLROM while preserving a small size. However, the issue with the procedure explained in [7] is that the modes are load-dependent, meaning that they are designed for a specific load case. Therefore, to compare the efficiency of this basis with our proposed alternative (non-intrusive model derivatives), we apply the procedure described above to select the dual modes, as done in [12].

An issue with the construction of the dual modes is that dominant mode(s) $\varphi_j$ are loosely defined in the literature, and a clear criteria for their selection is somehow lacking. We circumvent this problem by proposing the use of non-intrusive modal derivatives (MDs) for the reduction basis. MDs can be computed in a systematic way without requiring the user to define load cases’ scaling factors and to select dominant modes. The detail procedure to obtain non-intuitive MDs is illustrated in the next section.

3.3. Modal derivatives: a new basis for non-intrusive ROM

While linear vibration modes can efficiently approximate the motion of linear systems, they fail to reproduce well the nonlinear behavior. Still, the displacement vector $u$ can be written as a combination of eigenmodes (also called shape functions) that are a function of a reduced set of coordinates $q$ as

$$u \approx \sum_{i=1}^{w} \varphi_i(q) q_i.$$  

To account for the effect of change of the modes at least to second order, Idelsohn and Cardona [27] developed the displacement vector as a Taylor series expansion at $q = 0$, which gives

$$u = u_0 + \frac{\partial u}{\partial q} |_{q=0} q + \frac{1}{2} \frac{\partial^2 u}{\partial q \partial q} |_{q=0} : (q \otimes q) + \ldots$$

By introducing Eq. (36) into Eq. (37), we obtain
\[ \Delta \mathbf{u} = \mathbf{u}(\mathbf{q}) - \mathbf{u}_{eq} \approx 1 \sum_{i=1}^{w} \varphi_i(\mathbf{q} = \mathbf{0}) \mathbf{q}_i + \frac{1}{2} \sum_{i=1}^{w} \sum_{j=1}^{w} \frac{\partial^2 \varphi_i}{\partial \mathbf{q}_j^2} (\mathbf{q} = \mathbf{0}) + \frac{\partial \varphi_i}{\partial \mathbf{q}_j} (\mathbf{q} = \mathbf{0}) \mathbf{q}_i \mathbf{q}_j. \]  

(38)

This equation expresses that the nonlinear displacement of a system can be approximated as a superposition of linear eigenmodes computed at equilibrium, and corresponding derivatives. Note that (38) is a quadratic mapping between \( \mathbf{q} \) and \( \mathbf{u} \), which would not fit in the projection framework we describe. Therefore, by introducing a new generalized coordinate, \( \eta_r \), for each quadratic term, \( \varphi_i \mathbf{q}_i \), (38) is turned as a superposition of linear modes and their derivatives. The reduction basis \( \mathbf{V} \) in this case can be written as

\[ \mathbf{V} = \left[ \varphi_1, \varphi_2, \ldots, \varphi_w, \theta_{11}, \frac{1}{2}(\theta_{12} + \theta_{21}), \ldots, \frac{1}{2}(\theta_{ij} + \theta_{ji}), \ldots, \theta_{ww} \right]. \]  

(39)

where

\[ \theta_{ij} = \frac{\partial \varphi_i}{\partial \mathbf{q}_j}. \]  

(40)

is the \((ij)\)th MD of the system.

The main advantage of using MDs instead of membrane-dominated or dual modes is that they enrich the linear transverse-dominated basis in a systematic and load-independent manner. Furthermore, it will be shown in this paper that augmenting the linear modes with MDs presents accurate NLROMs for the examples that we investigated.

### 3.3.1. Calculation of modal derivatives

There are different ways to compute the MDs (e.g. see [28, 29, 27, 30]). In this work we consider two methods, which can be implemented non-intrusively. To obtain the \( \theta_{ij} \), (24) is differentiated with respect to the generalized coordinate \( q_i \), giving

\[ \left( \mathbf{K}^{(1)} - \omega^2 \mathbf{M} \right) \frac{\partial \varphi_i}{\partial q_j} = \left( \frac{\partial \omega^2}{\partial q_j} \mathbf{M} - \frac{\partial \mathbf{K}}{\partial q_j} \right) \varphi_i. \]  

(41)

The derivative of the eigenvalue, \( \frac{\partial \omega^2}{\partial q_j} \), is obtained by multiplying Eq. (41) with \( \varphi_i^T \). This value is then introduced to Eq. (41) leaving \( \theta_{ij} \) as the only unknown. However, the coefficient of the MD in the left hand side of Eq. (41) is singular according to Eq. (24). Therefore, to solve this equation for the MDs, Nelson [28] proposed a generalized solution of Eq. (41) as

\[ \frac{\partial \varphi_i}{\partial q_j} = \mathbf{v}_i + c_i \varphi_i, \]  

(42)

where \( \mathbf{v}_i \) and \( c_i \varphi_i \) are the particular and homogeneous solutions, respectively. According to the Nelson’s method, the solution of the MD is then split into the following form

\[ \frac{\partial \varphi_i}{\partial q_j} = \begin{cases} \mathbf{v}_i^{(1)} & \text{if } (r + 1) \text{th component of } \varphi_i \text{ is chosen such that it is zero for } \mathbf{v}_i, \\ 0 & \text{otherwise} \end{cases} \]  

(43)

where the superscripts (1) and (3) for the vectors denote the \( 1^{\text{st}} \) to \((r - 1)^{\text{th}}\) and \((r + 1)^{\text{th}}\) to \( n^{\text{th}} \) components of the vectors \( \mathbf{v}_i \) and \( \varphi_i \), respectively. Furthermore, the \( r^{\text{th}} \) component of \( \mathbf{v}_i \) and \( \varphi_i \) is chosen such that it is zero for \( \mathbf{v}_i \) (i.e., \( \mathbf{v}_i = 0 \)) and a nonzero value \( (\varphi_i) \) for \( \varphi_i \). Now the components of the vector \( \mathbf{v}_i \) can be obtained by substituting Eq. (43) into (41). The last unknown in Eq. (42) is the coefficient \( c_i \), which can be computed by employing the mass-normalization relation \( \varphi_i^T \mathbf{M} \varphi_i = 1 \), and by and differentiating the \( i^{\text{th}} \) mode with respect to \( q_j \). This gives

\[ 2 \varphi_i^T \mathbf{M} \frac{\partial \varphi_i}{\partial q_j} = 2 \varphi_i^T \mathbf{M} (\mathbf{v}_i + c_i \varphi_i) = 0 \rightarrow c_i = - \varphi_i^T \mathbf{M} \mathbf{v}_i. \]  

(44)

It should be noted that Nelson’s method can be used when all the eigenvalues are assumed to be distinct. In case multiple eigenvalues exist, the generalized inverse method can be used to obtain the MDs [31–33]. Lastly, the computed MDs have to be orthogonalized to linear modes and each other to avoid singularity problems in the basis.

### 3.3.2. Static modal derivatives

Regardless of which method is used to compute the MDs, a factorization of the dynamic stiffness matrix (Eq. 41) is required for each \( \varphi_i \), which can be computationally expensive. To circumvent this problem, Idelsohn and Cardona [29] approximated the MDs by neglecting the inertia terms. Accordingly, Eq. (41) is simplified to

\[ \mathbf{K}^{(1)} \frac{\partial \varphi_i}{\partial q_j} = \frac{\partial \mathbf{K}}{\partial q_j} \varphi_i. \]  

(45)
The MDs obtained from Eq. (45) are called Static MDs (SMDs). The computation of SMDs is easier than the MDs since the linear stiffness matrix $K^{(1)}$ needs to be factorized only once. A mechanical interpretation of the SMDs is discussed in [34].

### 3.3.3. Non-intrusive computation of (S) MDs

(S) MDs can be computed non-intrusively by a finite difference scheme and We consider two cases here for their computation. The first one is when the FE package releases the TS for each nonlinear static solution (this is the case for many commercial software, e.g. Abaqus, Nastran, etc.). Then, the derivative of the TS can be computed using for instance a central finite difference as

$$\frac{\partial K^{(\phi,q_i)}}{\partial q_j^{(h)}} \bigg|_{q_j=0} = \frac{K^{(\phi,q_i)}(u = \phi_j h) - K^{(\phi,q_i)}(u = -\phi_j h)}{2h},$$

where $h$ is a small perturbation in the direction of the $j^{th}$ mode (A numerical study for the selection of $h$ is performed in [35]).

The second case is when the FE package only releases the reaction forces due to nonlinear static displacements. In this case, the right hand side of Eqs. (41) and (45) can be written in terms of the second derivative of the nonlinear internal forces as

$$\frac{\partial K^{(\phi,q_i)}}{\partial q_j^{(h)}} = \frac{\partial^2 f_{int}}{\partial q_{j}^{(h)} \partial q_{i}^{(h)}} = \frac{\partial^2 f_{int}}{\partial q_{j}^{(h)} \partial q_{i}^{(h)}},$$

The numerical mixed derivatives of the nonlinear internal force obtained with central finite difference is given by

$$\frac{\partial^2 f_{int}}{\partial q_{j}^{(h)} \partial q_{i}^{(h)}} = \frac{f_{int}(u = \phi_j h + \phi_i h) - f_{int}(u = \phi_j h - \phi_i h) - f_{int}(u = -\phi_j h + \phi_i h) + f_{int}(u = -\phi_j h - \phi_i h)}{4h_i h_j}.$$  

Once the required tangent stiffnesses or nonlinear internal forces for each increment in Eqs. (46) and (48) are obtained from the corresponding nonlinear static solutions, the right hand side of Eqs. (41) (for MDs) and (45) (for SMDs) are calculated and, accordingly, the (S) MD are obtained in a non-intrusive manner.

### 3.3.4. Load-independent selection of modal derivatives

The total number $N_{MD}$ of added basis vectors in Eq. (39) due to (S) MDs is given by

$$N_{MD} = \nu C_2 + n,$$

which means that the number of (S) MDs increases as $\epsilon (\nu^2)$ with respect to the number of linear modes. Therefore, to still keep the NLROMs compact, only a few of the most significant (S) MDs should be kept. To select the most important (S) MDs, a few heuristic methods are available in the literature. In this work, we use a modified version of the method of Maximum Modal Interaction (MMI) [36,37]. The MMI criterion simply considers the modal evolution of a linear modal model, and takes the product of two arbitrary modal amplitudes as an indication of the potential interaction of these modes in the nonlinear regime. The pairs yielding the largest interactions are then giving the indexes of the MDs to be included in the basis. More specifically, a weighting matrix $W$ is defined such that its $i^{th}$ component is obtained as

$$W_{ij} = \int_0^T |q_i(t) q_j(t)| dt,$$

where $W_{ij}$ denotes the weighting value corresponding to the (S) MD $\theta_j$. Moreover, $q_i(t)$ is the response of the $i^{th}$ generalized mode in a linear time integration due to an external load. It should be noted that the original MMI method takes the same external loading as the load, which is aimed to be applied to the full and reduced systems for validation. However, this makes the selection of the (S) MDs load-dependent. In order to select the (S) MDs independently of the actual applied load, a random excitation with the desired frequency range of interest is used to excite the linear reduced model and compute the weighting matrix.

Consider the reduced EOM (3), which is linearized as

$$\hat{M} \hat{q}(t) + \hat{K}^{(1)} \hat{q}(t) = \hat{f}(t).$$

Now, we choose the components of the external load vector $f(t)$ as a Gaussian random signal in time with an arbitrary load distribution in space. This force randomly excites the model in a desired frequency band and its load distribution is not the same as the actual load exerted on the system. For this reasons, we refer to this selection of (S) MD as load-independent. For instance, in all our numerical examples, we take the same random amplitude for the forces/moments that are applied to all dofs. The linearized modal coordinates $q_i, i = 1, 2, \ldots, m$, are then integrated under this excitation and used in (50) for all time steps to obtain the components of the MMI matrix. It can be seen that the weighting matrix is symmetric,
while in general only SMDs are symmetric, and MDs are not (see [37]). This does not cause any problem for the selection of MDs, because if a MD $\theta_{ij}$ is selected to be in the basis, it will be added by its counterpart $\theta_{ji}$ to be symmetrized (as in Eq. (39)) and then added to the reduction basis.

4. Numerical examples

To examine the performance of the proposed non-intrusive MOR approach, three numerical examples featuring geometric nonlinearities are discussed here. The three examples are made of beam (flat and curved) and shell elements using Abaqus®. All models are excited by sound pressures in the transverse direction, which is randomly distributed in time and uniform in space. The frequency range of excitation of the loads is between 0–800 Hz and each example is excited with two different average Sound Pressure Levels (SPLs).

The NLROMs are developed using the proposed basis and compared with the full model as well as the NLROMs based on two previously developed bases for non-intrusive ROM, namely linear modes and dual modes. For all three examples, the first mode is chosen as the dominant mode to compute the dual modes. This choice is due to the highest modal contribution of the first mode in the response of the system (also proposed in [7]). The NLROMs are developed using both the ED and EED methods, to compare their offline computational costs. However, only the PSDs of the NLROMs that are developed by ED with different reduction bases are compared to each other, for the sake of brevity and also due to its time integration stability compared to EED. Additionally, the required derivatives to compute the non-intrusive (S) MDs are obtained using the TS matrices (Eq. (46)). The performance of the NLROMs are assessed by monitoring the displacement Power Spectral Density (PSD) of their nonlinear time response. The time integration of the NLROMs is performed using the implicit Newmark scheme, which we implemented in MATLAB® and the full model is implicitly integrated in Abaqus. For all the time integrations, a Rayleigh damping in the form $D = \alpha M + \beta K$ is used. All the simulations in this study are performed on a desktop PC with 32 GB RAM and Intel® Xeon® CPU (3.6 GHz). Lastly, to compare the time response of the NLROMs, the relative error of the selected DOFs are computed as

$$
\varepsilon_n = \left\| \mathbf{u}_{\text{full,}n} - \mathbf{u}_{\text{ROM,}n} \right\| / \mathbf{u}_{\text{average,}n} \times 100
$$

where $\mathbf{u}_{\text{full,}n}$, $\mathbf{u}_{\text{ROM,}n}$ and $\mathbf{u}_{\text{average,}n}$ are the time response of the full model, the ROM and the root-mean-square of the full model’s time signal, respectively, all for the $n^{th}$ DOF.

4.1. Beam models

4.1.1. Flat beam

The first investigated FE model is a two-side clamped beam, which is discretized with geometric nonlinear elements, and shown in Fig. 3. The material, geometry and FE model properties of this structure are illustrated in Table 1. The NLROMs of this model are developed using the proposed non-intrusive SMDs as well as MDs. For this example, two average SPLs of 135 dB and 150 dB are applied to the model. All the models are time-integrated for 5 s with the constant time increment of $5 \times 10^{-3}$. The coefficients of the Rayleigh damping are chosen such that the damping ratios for all modes in the excitation frequency are less than 1% ($\alpha = 6.8$, $\beta = 3.75 \times 10^{-6}$).

To have an insight into the shape of the linear modes and (S) MDs, the first six linear modes of the beam structure are depicted in Fig. 4, and the six SMDs stemming from the first three VMs are shown in Fig. 5. For a flat beam, the first VMs are transverse dominated, while the corresponding SMDs are in-plane dominated, as shown in Figs. 4 and 5, respectively. As the shapes of the MDs and SMDs are very similar to each other, the MD shapes are not shown here.

---

**Fig. 3.** Schematic of the clamped–clamped flat beam model.
Additionally, the MMI heuristic criterion for the first seven generalized DOFs are computed using the time integration of the linear reduced system and depicted in Fig. 6. The darkest squares indicate the mode pairs likely to interact the most when nonlinearities are present. Following this idea, the (S) MDs corresponding to such indexes are computed and inserted in the reduction basis. Since the excitation for computation of MMI is random, the selected (S) MDs might be slightly different for each time that the NLROM is developed. However, this slight difference in the selection of (S) MDs has a minimal effect on the nonlinear time responses.

4.1.2. Convergence check

To perform a convergence check study and also evaluate the numbers of required linear VMs as well as (S) MDs, the NLROMs with different number of generalized coordinates are developed and compared to the full-order model. Fig. 7 shows the PSD comparison of NLROMs under the SPL of 150 dB and using different numbers of linear modes and (S) MDs with the PSD of the full-order model for transverse motion of point A and in-plane displacement of point B (see Fig. 3). This is done since the maximum transverse and in-plane displacements of the beam model due to this load occurs near points A and B, respectively. The transverse PSD of the NLROMs developed by SMDs (Fig. 7a) as well as MDs (Fig. 7b) converge to the solution of the nonlinear full-order model by increasing the number of linear modes and corresponding (S) MDs in the basis (the number of used VM as well as (S) MDs are shown in the legend of each figure). Likewise, the accuracy of the in-plane PSDs

<table>
<thead>
<tr>
<th>Property (dimension)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Mass density (kg/m³)</td>
<td>7870</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>(2.05 \times 10^{11})</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>Geometry (l) (mm)</td>
<td>300</td>
</tr>
<tr>
<td>width (mm)</td>
<td>13</td>
</tr>
<tr>
<td>(t) (mm)</td>
<td>1</td>
</tr>
<tr>
<td>FE model (l)</td>
<td>Timoshenko (Abaqus B21)</td>
</tr>
<tr>
<td>Number of Elements</td>
<td>60</td>
</tr>
<tr>
<td>Number of DOFs</td>
<td>177</td>
</tr>
</tbody>
</table>

Fig. 4. The first six vibration modes of the flat beam model, (a) \(\varphi_{1} \mid \omega_{1} = 58.3\) Hz, (b) \(\varphi_{2} \mid \omega_{2} = 160.7\) Hz, (c) \(\varphi_{3} \mid \omega_{3} = 315.1\) Hz, (d) \(\varphi_{4} \mid \omega_{4} = 520.9\) Hz, (e) \(\varphi_{5} \mid \omega_{5} = 778.2\) Hz, and (f) \(\varphi_{6} \mid \omega_{6} = 1087.1\) Hz.

Additionally, the MMI heuristic criterion for the first seven generalized DOFs are computed using the time integration of the linear reduced system and depicted in Fig. 6. The darkest squares indicate the mode pairs likely to interact the most when nonlinearities are present. Following this idea, the (S) MDs corresponding to such indexes are computed and inserted in the reduction basis. Since the excitation for computation of MMI is random, the selected (S) MDs might be slightly different for each time that the NLROM is developed. However, this slight difference in the selection of (S) MDs has a minimal effect on the nonlinear time responses.

4.1.2. Convergence check

To perform a convergence check study and also evaluate the numbers of required linear VMs as well as (S) MDs, the NLROMs with different number of generalized coordinates are developed and compared to the full-order model. Fig. 7 shows the PSD comparison of NLROMs under the SPL of 150 dB and using different numbers of linear modes and (S) MDs with the PSD of the full-order model for transverse motion of point A and in-plane displacement of point B (see Fig. 3). This is done since the maximum transverse and in-plane displacements of the beam model due to this load occurs near points A and B, respectively. The transverse PSD of the NLROMs developed by SMDs (Fig. 7a) as well as MDs (Fig. 7b) converge to the solution of the nonlinear full-order model by increasing the number of linear modes and corresponding (S) MDs in the basis (the number of used VM as well as (S) MDs are shown in the legend of each figure). Likewise, the accuracy of the in-plane PSDs
of the NLROMs have been improved by increasing the numbers of linear modes and (S) MDs (Fig. 7c and d) present in the basis. These results confirm the convergence of the proposed approach.

Furthermore, the convergence check study is performed for the NLROMs relying on linear modes and dual modes (Fig. 8) under the same loading condition as the NLROMs based on (S) MDs. To obtain the dual modes of each NLROM, 10 scaling factors $\alpha_i$, ranging from $-2.5$ to $2.5$ are selected and their corresponding displacements are obtained for sequential POD anal-

**Fig. 5.** Displacement of the first six SMDs of the flat beam model in axial and transverse directions, (a) $\theta_{11} = \frac{u_1}{q_1}$, (b) $\theta_{12} = \theta_{21} = \frac{u_2}{q_2}$, (c) $\theta_{13} = \theta_{31} = \frac{u_3}{q_3}$, (d) $\theta_{22} = \frac{u_2}{q_2}$, (e) $\theta_{33} = \frac{u_3}{q_3}$, and (f) $\theta_{23} = \theta_{32} = \frac{u_2}{q_3}$.

**Fig. 6.** Maximum modal interaction matrix for the flat beam model with seven linear modes in the basis. Since this matrix is symmetric, only the lower triangle is shown.
ysis (see Section 3.2). As can be seen in Fig. 8, the NLROMs developed by linear and dual modes have converged to the full-order solution in a similar manner as the NLROM developed by VM and (S) MDs (Fig. 7).

Fig. 7. Convergence check of the flat beam’s displacement power spectral densities under SPL of 150 dB for (a) transverse DOF of point A using the NLROMs developed by SMDs, (b) transverse DOF of point A using the NLROMs developed by MDs, (c) membrane DOF of point B using the NLROMs developed by SMDs, and (d) membrane DOF of point B using the NLROMs developed by MDs.

Fig. 8. Convergence check of the flat beam’s displacement power spectral densities under SPL of 150 dB for (a) transverse DOF of point A and (b) membrane DOF of point B, using the NLROMs developed by dual modes.
4.1.3. Comparison of reduction bases

To further compare the performance of different bases, four NLROMs are developed, with the same number of basis vectors in the reduction basis, as:

- The first 10 linear vibration modes (NLROM-10VM).
- The first 5 linear vibration modes augmented with 5 dual modes (NLROM-5VM-5Dual).
- The first 5 linear vibration modes augmented with 5 static modal derivatives (NLROM-5VM-5SMD).
- The first 5 linear vibration modes augmented with 5 modal derivatives (NLROM-5VM-5MD).

These NLROMs are compared with the full-order response and the linear ROM with 10 vibration modes (LROM-10VM). The PSD comparison of the transverse DOF of point A and in-plane DOF of point B under two loading conditions with an average SPL of 135 dB and 150 dB are shown in Fig. 9. The maximum displacement for this example (under 150 dB) is 2.7 times the thickness. This figure confirms the accuracy of the NLROMs based on MDs compared to other NLROMs for both transverse and membrane DOFs, especially as the excitation level increases (see Fig. 9c and d). Additionally, the NLROM based on SMD has almost the same order of accuracy as the NLROM based on dual modes for transverse motion, while its accuracy for the membrane motion is higher than the NLROM based on dual modes. It should be noted that if the NLROM based on dual modes is developed by only selecting those modes and duals, which have the maximum contribution for this specific load distribution (load-dependent), one can obtain excellent results for this NLROM as shown in [38].

To better observe the improvement of the proposed method, the relative error for the first half second (from the whole five seconds) of the time integrations used in PSD computation of Fig. 9 are shown in Fig. 10 under both excitation levels for transverse direction of point A (Fig. 9a and c), as well as in-plane direction of point B (Fig. 9b and d). As can be seen from this figure, both NLROMs based on SMDs and MDs have less relative error for both directions compared to the NLROM based on dual modes. The NLROM based on MDs has the minimum relative error among others. The total number of generalized DOFs

![Fig. 9. Power spectral density comparison for the flat beam’s NLROMs developed by SVM-5SMD, 5VM-5MD, 5VM-5Dual and 10VM with the full-order model and the linear reduced model (10VM). The comparison is performed for (a) transverse DOF of point A under SPL of 135 dB, (b) membrane DOF of point B under SPL of 135 dB, (c) transverse DOF of point A under SPL of 150 dB, and (d) membrane DOF of point B under SPL of 150 dB.](image-url)
The developed NLROMs to accurately represent the full-order model in the whole range of 0–1000 Hz under mentioned loading conditions is only 10, which is approximately 6% of the total DOFs of the full-order model.

**4.1.4. Computational costs**

To compare the computational efficiency of the developed approach, the required time to develop the NLROMs (offline cost) as well as the required time to integrate the NLROMs and full-order model (online costs) are compared for the simulations used in Fig. 9. The offline computational cost comparison for building the four NLROMs used in Fig. 9 are shown in Table 2. As can be seen from this table, the accuracy achieved by (S) MDs does not increase the offline computational costs for development of the NLROMs. Since the developed NLROMs are potentially valid for a variety of load cases with excitation frequency of 0–800 Hz, the “one-time” offline cost to compute the NLROMs is amortized when they are used for time simulations under several loading conditions. Furthermore, the online computational costs for 5 s time integration of the NLROMs are compared with the full-order model in Table 3. As can be seen, the required online time for all four NLROMs

**Fig. 10.** Relative error comparison for the flat beam’s NLROMs developed by 5VM-5SMD, 5VM-5MD, 5VM-5Dual and 10VM with the full-order model and the linear reduced model (10VM). The comparison is performed for (a) transverse DOF of point A under SPL of 135 dB, (b) membrane DOF of point B under SPL of 135 dB, (c) transverse DOF of point A under SPL of 150 dB, and (d) membrane DOF of point B under SPL of 150 dB.

in the developed NLROMs to accurately represent the full-order model in the whole range of 0–1000 Hz under mentioned loading conditions is only 10, which is approximately 6% of the total DOFs of the full-order model.

**Table 2**

<table>
<thead>
<tr>
<th>NLROM</th>
<th>ED offline time (sec.)</th>
<th>EED offline time (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLROM-5VM-5Dual</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>NLROM-5VM-5SMD</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NLROM-5VM-5MD</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NLROM-10VM</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
are in the same order, because they all have the same number of the generalized DOFs. The required time for integration of these models are approximately 172 times faster than the full-order model, leading to highly efficient reduced models.

4.1.5. Curved beam

The second example of this work is a curve beam structure, because it has been shown in many works (see e.g. [39–41]) that it has a more complex nonlinear dynamic behavior than the flat beam due to the linear coupling between its transverse and in-plane motions. The schematic of this model is depicted in Fig. 11 and its material, geometry and mesh properties are illustrated in Table 4. The shape of the first six VMs of the model is depicted in Fig. 12 for both transverse and membrane directions. As shown in this figure, although these modes are transverse dominated, their membrane motion is also nonzero, which is unlike the flat beam model (Fig. 4). Moreover, the first six SMDs of the curved beam is shown in Fig. 13. In contrast to the SMDs of the flat beam (Fig. 5), these SMDs are not membrane-dominated anymore due to the coupling between the linear modes. see Fig. 14.

To select the (S) MDs in the reduction basis of the NLROMs, the MMI criterion is used for the first five modes. The (S) MD are then chosen according to the components of the MMI matrix with maximum values. Since the convergence check analysis is already performed for the flat beam, for the sake of brevity it is not repeated for the curved beam.

4.1.6. Comparison of reduction bases

To compare the performance of the proposed approach, two random excitations with SPLs of 158 dB and 168 dB are applied to the model for five seconds, and time integrations are performed with similar conditions as the flat beam. We choose the Rayleigh damping coefficients as $\alpha = 18.20$, $\beta = 2.95 \times 10^{-6}$, which lead to the damping ratios of less than 1% for all the modes in the frequency range of excitation. The NLROM of the curved beam model is developed using the following bases:

![Fig. 11. Schematic of the curved beam model.](image_url)
Fig. 12. The first six vibration modes of the curved beam, (a) $\phi_1$ | $\omega_1 = 179.4$ Hz, (b) $\phi_2$ | $\omega_2 = 298.6$ Hz, (c) $\phi_3$ | $\omega_3 = 477.6$ Hz, (d) $\phi_4$ | $\omega_4 = 582.1$ Hz, (e) $\phi_5$ | $\omega_5 = 879.0$ Hz, and (f) $\phi_6$ | $\omega_6 = 1214.0$ Hz. The transverse and in-plane displacements have different scaling for visual reasons.

Fig. 13. Transverse and in-plane displacements of the curved beam's first six SMDs, (a) $\theta_1 = \frac{\omega_1}{\omega_2}$ (b) $\theta_2 = \frac{\omega_2}{\omega_3}$ (c) $\theta_3 = \frac{\omega_3}{\omega_4}$ (d) $\theta_4 = \frac{\omega_4}{\omega_5}$ (e) $\theta_5 = \frac{\omega_5}{\omega_6}$, and (f) $\theta_6 = \frac{\omega_6}{\omega_7}$. The transverse and in-plane displacements have different scaling for visual reasons.
- The first 15 linear vibration modes (NLROM-15VM).
- The first 5 linear vibration modes augmented with 10 dual modes (NLROM-5VM-10Dual).
- The first 5 linear vibration modes augmented with 10 static modal derivatives (NLROM-5VM-10SMD).
- The first 5 linear vibration modes augmented with 10 modal derivatives (NLROM-5VM-10MD).

Fig. 14. Maximum modal interaction matrix for the curved beam model with five linear modes in the basis. Since this matrix is symmetric, only the lower triangle is shown.

Fig. 15. Power spectral density comparison for the curved beam’s NLROMs developed by 5VM-10SMD, 5VM-10MD, 5VM-10Dual and 15VM with the full-order model and the linear reduced model (15VM). The comparison is performed for (a) transverse (vertical) DOF of point A under SPL of 158 dB, (b) membrane (horizontal) DOF of point B under SPL of 158 dB, (c) transverse DOF of point A under SPL of 168 dB, and (d) membrane DOF of point B under SPL of 168 dB.
These NLROMs are compared with the full-order model and the linear ROM using 15 VMs modes (LROM-15VM) and under two mentioned excitations. Like the flat beam model, to obtain the dual modes of each NLROM, 10 scaling factors $\alpha_i$ ranging from $-2.5$ to $2.5$ are selected. Moreover, for each set $j$, $z = 2$ vectors are selected as final dual modes (see Section 3.2 for details).

Fig. 15 shows the PSD comparison of these models for the transverse direction of point $A$ and axial direction of point $B$ (see Fig. 11) under the two mentioned excitation levels (the maximum transverse and axial motion of the structure occurs near points $A$ and $B$, respectively). As can be seen, while the performance of NLROMs based on (S) MDs are in the same order as the other NLROMs for low level of excitation, for relatively higher excitation level, the NLROM developed by MDs is more accurate than others, especially for the membrane motion (Fig. 15d). This improvement can also be seen in Fig. 16, where the relative errors are shown for the first half seconds time responses that are used to compute the PSDs of Fig. 15 in both transverse and axial directions of points $A$ and $B$, respectively. The maximum displacement for this example is 1.1 times the thickness of the curved beam. Finally, one should note that the results obtained by these NLROMs can be improved significantly if the NLROMs are developed in a load-dependent manner, i.e., according to a specific load case, as for instance performed for dual modes in [42], for the case of a curved beam.

4.1.7. Computational costs

The offline computational costs to develop the investigated NLROMs by the proposed bases ((S) MDs), as well as the currently available simulation-free bases (linear and dual modes), are shown in Table 5. This table proves that the obtained results of the proposed bases accompany no increase in the offline computational costs compared to the NLROM developed by the dual modes. Furthermore, the online computational costs for 5 s time integration of the NLROMs, as well as the full-order model to obtain the results of Figs. 15c and d (under 168 dB SPL) are presented in Table 6. The required time to integrate all the NLROMs are in the same order, while they are at least 117 times faster than the full-order model. Furthermore,
since the developed NLROMs can be used for different loading conditions, the “one-time” offline computational costs are amortized in case of using them in several loading conditions.

4.2. Panel model

The last example of this paper examines a panel structure, which is when discretized, of much larger dimensions than the first two beam examples. The model of the panel, shown in Fig. 17, is built and analyzed in Abaqus. The structure is simply supported at all edges (as shown by black lines in Fig. 17) and is meshed by geometric nonlinear shell elements. The material, geometry and FE properties of this model are outlined in Table 7. The Rayleigh damping coefficients for this structure are set to $\alpha = 16.2$ and $\beta = 2.85 \times 10^{-6}$. This choice leads to damping ratios less than 1% for all the modes in the frequency range of

![Fig. 17. Schematic of the simply-supported panel model made of geometric nonlinear shell elements.](image)
excitation. All the time integrations for this example were performed with a constant time increment of $5 \times 10^{-5}$ s with 100,000 time steps.

As done for the beam examples, the first six linear modes of the plate are plotted in Fig. 18 while the six SMDs relative to the first three VMs are depicted in Fig. 19. As can be seen, all of the SMDs shown are membrane-dominated modes. The MDs are not shown, as their shapes are very similar to the SMDs. Furthermore, the MMI of the linear reduced model with 11 linear vibration modes under a random excitation is computed and shown in Fig. 20. Those (S) MDs, $h_{ij}$ (with their counterpart $h_{ji}$), are selected for the development of the NLROMs, based on the corresponding largest components in the MMI matrix, which is obtained under a random excitation.

4.2.1. Convergence check

As done for the beam example, a convergence check analysis for the developed NLROMs based on (S) MDs is performed to check the robustness of the proposed approach. In this analysis, the PSDs of the NLROMs under the random loading condition with average SPL of 155 dB are computed while the number of linear modes as well as (S) MDs are increased for the development of NLROMs. Fig. 21a and c illustrate the PSDs of the NLROMs developed by SMDs for transverse and membrane motions of points $A$ and $B$, respectively. Likewise, Fig. 21b and d depict the PSDs of the NLROMs based on MDs for the same points and same DOFs. The PSDs of both transverse and membrane DOFs converge to the one of the full-order solution by increasing the number of basis vectors in the NLROMs based on both MD and SMD. These results confirm the convergence as well as accuracy of the proposed approach.

The same analysis is performed but this time for the NLROMs, which are developed using different numbers of linear modes and dual modes in the basis, and the corresponding PSD results are shown in Fig. 22. To develop the dual modes for this example 10 scaling factors $\pi$ (See Section 3.2) ranging from $-2.5$ to $2.5$ are selected and the POD of their corresponding displacements are then analyzed. The NLROMs are improved by increasing the number of VMs as well as dual modes in

<table>
<thead>
<tr>
<th>Property (dimension)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material mass density ($\text{kg/m}^3$)</td>
<td>7870</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>$2.05 \times 10^{11}$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>Geometry length ($l$ mm)</td>
<td>800</td>
</tr>
<tr>
<td>width ($w$ mm)</td>
<td>400</td>
</tr>
<tr>
<td>thickness ($t$ mm)</td>
<td>2</td>
</tr>
<tr>
<td>FE model element-type</td>
<td>Shell 4-node (Abaqus S4R)</td>
</tr>
<tr>
<td>Number of Elements</td>
<td>3200</td>
</tr>
<tr>
<td>Number of DOFs</td>
<td>19202</td>
</tr>
</tbody>
</table>

Table 7. Geometry, material and FE model properties of the investigated panel model.

Fig. 18. The first six vibration modes of the panel structure, (a) $\varphi_1$ $\omega_1 = 150.5$ Hz, (b) $\varphi_2$ $\omega_2 = 240.6$ Hz, (c) $\varphi_3$ $\omega_3 = 391.2$ Hz, (d) $\varphi_4$ $\omega_4 = 513.1$ Hz, (e) $\varphi_5$ $\omega_5 = 602.6$ Hz, and (f) $\varphi_6$ $\omega_6 = 602.6$ Hz.
the basis, which confirms the convergence of the NLROMs developed by this basis. To develop an accurate NLROM with less number of DOFs, one can select only the VMs and duals, which are mostly relevant to a specific load case, as done in [7]. However, the aim of this work is to compare different selection strategies when modes are computed in a load-independent manner.

### 4.2.2. Comparison of the reduction bases

To further assess the efficiency of the proposed approach, four NLROMs with the same number of basis vectors but built by different ingredients are developed as follows:

- The first 22 linear vibration modes (NLROM-22VM).
- The first 11 linear vibration modes augmented with 11 dual modes (NLROM-11VM-11Dual).
- The first 11 linear vibration modes augmented with 11 static modal derivatives (NLROM-11VM-11SMD).
- The first 11 linear vibration modes augmented with 11 modal derivatives (NLROM-11VM-11MD).
The nonlinear response of these models under two different average SPLs of 145 dB and 157 dB are obtained and compared with the ones of the full-order and linear ROM (LROM-22VM) models. The PSD comparison of the transverse DOF of point A for the two loading levels are depicted in Fig. 23a and c and for the membrane DOF of point B are shown in Fig. 23b and d, respectively. The maximum displacement here is 0.8 times the thickness. As can be seen from these figures, for both
excitation levels and in both transverse and membrane directions, the proposed NLROMs based on (S) MDs are accurately following the full model’s response. Note that all of the bases in this work are selected in a load-independent manner, which means that the developed NLROMs can be potentially used for a variety of load distributions. Furthermore, Fig. 24 shows the relative error of the first half second (out of five seconds) of time responses, which are used to compute the PSDs of Fig. 23. In this figure, it is discernible that the relative error of the NLROM developed by MDs is smaller than the NLROMs developed by other bases. The results obtained for this example show that combining non-intrusive nonlinear model reduction method and modal derivatives allows building accurate NLROMs.

4.2.3. Computational costs
The last study for this example is evaluation of the computational costs for the proposed NLROMs and comparison of it with other bases. Therefore, as a sample, the required offline time to develop the NLROMs used in Fig. 23 are computed and listed in Table 8. One can note that the offline computational time of the proposed NLROMs based on (S) MDs are less than the one based on dual modes. This result together with the one displayed in Fig. 23, implies that while the accuracy of the NLROMs with the proposed bases have been improved, the offline computational costs for the development of the NLROMs are not compromised.

Furthermore, Table 9 reports the required online time for 5 s time integration of the NLROMs compared to the full-order model. Although the full model is integrated in Abaqus, which usually uses optimized algorithms and NLROMs are integrated in a research code in MATLAB, the online computational cost for NLROMs are at least 69 times faster than the full-model.

5. Remarks on non-intrusive MOR and basis selection

As demonstrated in Section 4, nonlinear reduced-order models using non-intrusive methods with the proposed basis can improve the computational performance and accuracy in dynamic response of nonlinear structures. However, we remark here some aspects, which could direct future research:
Table 8
Offline computational time for development of the panel's NLROMs.

<table>
<thead>
<tr>
<th>NLROM</th>
<th>ED offline time (min.)</th>
<th>EED offline time (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLROM-11VM-11Dual</td>
<td>193</td>
<td>127</td>
</tr>
<tr>
<td>NLROM-11VM-11SMD</td>
<td>78</td>
<td>29</td>
</tr>
<tr>
<td>NLROM-11VM-11MD</td>
<td>80</td>
<td>30</td>
</tr>
<tr>
<td>NLROM-22VM</td>
<td>91</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 9
Online time for time integration of the panel's NLROMs and full model for 5 s under SPL of 157 dB.

<table>
<thead>
<tr>
<th>Model</th>
<th>Online time (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full model</td>
<td>971</td>
</tr>
<tr>
<td>NLROM-11VM-11Dual</td>
<td>14</td>
</tr>
<tr>
<td>NLROM-11VM-11SMD</td>
<td>14</td>
</tr>
<tr>
<td>NLROM-11VM-11MD</td>
<td>14</td>
</tr>
<tr>
<td>NLROM-22VM</td>
<td>14</td>
</tr>
</tbody>
</table>

Fig. 24. Relative error comparison for the panel's NLROMs developed by 11VM-11SMD, 11VM-11MD, 11VM-11Dual and 22VM with the full-order model and the linear reduced model (22VM). The comparison is performed for (a) transverse DOF of point A under SPL of 145 dB, (b) membrane DOF of point B under SPL of 145 dB, (c) transverse DOF of point A under SPL of 157 dB, and (d) membrane DOF of point B under SPL of 157 dB.
The discussed methods are only applied to the shell-like structures in this paper; however, they could be applied to continuum elements without any restriction.

While augmentation of the reduction basis with the non-intrusive (S) MDs allows building accurate NLROMs, the resulting size of the generated NLROMs increases significantly. One possibility to avoid increasing the size of the reduction basis is to resort on nonlinear projection (e.g. quadratic manifolds, see [37,43]), which enslave modal derivatives (or the like) to the generalized coordinates of dominant vibration modes, implemented non-intrusively.

For our investigated examples, it has been observed that the NLROMs developed by the ED method using VMs with either dual modes or (S) MDs, always deliver stable time integration results. However, when the EED method of Perez et al. [7] is used with Abaqus static solutions, sometimes instabilities occur in the time integration of the resulting NLROMs. This can be due to inaccuracies induced by identifying NSCs with the tangent stiffness matrix (instead of reaction forces in ED) exported from Abaqus. This problem might be solved by using a “cleaning” procedure demonstrated in [44].

For a large nonlinear structure with several components, it is very expensive to reduce the FE model monolithically. Instead, non-intrusive MOR methods should be combined with dynamic substructuring approaches (e.g. see [45–50]) to expedite the construction of NLROMs and take advantage of parallel computation.

6. Conclusion

We propose the use of modal derivatives for non-intrusive model order reduction of geometrically nonlinear structures. One of the bottlenecks of the non-intrusive model order reduction techniques is selection of a proper reduction basis to accurately identify nonlinear stiffness coefficients of the reduced order model. The current available basis vectors in non-intrusive approach include linear vibration modes and dual modes [2,12,7], which deliver optimal results when selected in a load-dependent way. In this paper, we propose augmenting a basis of linear modes with non-intrusive modal derivatives. We have presented a non-intrusive formulation to compute the modal derivatives for both the cases when either only the nonlinear reaction forces are available, or also the tangent stiffness matrix is released from the finite element program. The presented method has the following advantages over the state-of-the-art:

- Modal derivatives do not require the knowledge of the external load shapes for construction. Therefore, it is a load-independent procedure.
- The computation of each modal derivative requires the evaluation of the tangent stiffness matrix associated to two configurations only, as opposed to the relatively large number of nonlinear static solutions required for dual modes. It must be noted, however, that the offline cost associated to the computation of the dual modes depends on the size of the vector of scaling factors \(a\), see Section 3.2. In this contribution, we followed the guidelines provided by the developers of the dual modes method for the selection of the scaling factors.
- We introduced a systematic way to select the modal derivatives from the set of linear vibration modes used to build the reduction basis, and therefore they do not need to be selected manually.

We applied the proposed method to three geometrically nonlinear FE models of straight and curved beams as well as panels, and compared the nonlinear random responses of the so obtained nonlinear reduced order model under different excitation levels with the nonlinear reduced order models that are developed with other existing methods. Our numerical experiments show that the proposed method delivers accurate results without compromising the offline and online computational costs when compared to state-of-the-art methods.

CRediT authorship contribution statement

Morteza Karamooz Mahdiabadi: Writing - original draft, Conceptualization, Methodology, Software, Visualization, Data curation. Paolo Tiso: Supervision, Writing - review & editing, Funding acquisition. Antoine Brandt: Software, Visualization. Daniel Jean Rixen: Supervision, Writing - review & editing, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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