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Designing a Demand Responsive Timetable for MRT Services

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Mass rapid transit systems (MRT) have become the most important public transport mode in many major cities globally, given its advantages in larger capacity, faster velocity and higher reliability compared with other modes. Hence, various projects directed to improve service quality of MRT systems by optimizing operation strategies have been conducted worldwide. Among numerous methods to adjust operation parameters for MRT services, timetable optimization has emerged as the simplest and most effective cure. On one hand, a demand responsive timetable has the potential to improve service quality by reducing both waiting time and crowdedness on trains. On the other hand, for transit operators, a demand responsive timetable will improve operation stability and hence reliability. In this study, we propose an optimization model to solve timetable scheduling considering fine grained dynamic demand data. The objective is to minimize the total waiting time for all the passengers. Historical demand data extracted from smart card system is used as inputs. Then, a predefined demand responsive timetable will be generated with the optimization model. Since operational constraints such as the number of available trains and minimal headway are considered as boundary conditions, we ensure that the proposed schedule actually has the potential to be implemented in actual operation.

A case study on a representative MRT service is conducted and presented in this paper. In the absence of effective/operated timetable, simulations on the optimal timetable obtained from this proposed model and simplified timetable as extracted from Google Maps are performed based on detailed travel demand extracted from smart card transactions. The result demonstrates the applicability of the proposed model in designing a demand sensitive timetable and its potential to minimize waiting times. However, simplified assumptions concerning potential operational constraints when switching directions or manpower scheduling, restrain the immediate application of the obtained results.

Keywords: timetable, MRT, public transit
INTRODUCTION

Rapid transit systems (RTS) have become the most important public transport mode in many major cities around the world, given its advantages in larger capacity, faster velocity and higher reliability compared with other transit modes. Hence, issues regarding the service reliability of MRT (Mass Rapid Transit) systems attract attentions worldwide, especially on operation strategies. Although various of methods have been proposed to adjust operation parameters for MRT services, such as adjusting velocity, increasing or decreasing dwell time and designing new timetables, timetabling is accepted by most transit operators as the most straightforward and effective cure. On one hand, for passengers, a reliable and demand responsive timetable has the potential in improving their travel experience by reducing both waiting time and crowdedness on trains. On the other hand, for transit operators, a reliable operational timetable will make more balanced assignment on passenger demand, furthermore, it can also lower the operation cost at certain service level. Therefore, both passengers and transit operators will profit from implementing a well-designed timetable.

In general, timetable of transit services is the schedule determined by vehicle frequencies and headways, and it should be constructed based on passenger demand. Ceder (1) has addressed the problem on how to determine bus frequencies and proposed schemes to choose headways in given time periods with the load profile collected from passenger counts. As a continuation of (1), an automated procedures for setting bus time timetables efficiently was established based on Automatic Data Collection systems (ADCS) (2). Afterwards, Ceder et al. (3) extended the timetabling problem from single bus service to networks with transfer stations taken into account, focusing on maximizing the number of buses arrive at transfer stations simultaneously, which can reduce waiting time for transfer passengers. This problem is formulated as a mixed integer linear programming and solved using heuristic algorithm. With the intention to obtain an optimal timetable given certain number of vehicles, an optimization model is proposed by minimizing passengers’ total schedule delay (4). In this model, passengers are assumed to have an ideal time to travel, and the delay cost varies from traveling earlier or later than the ideal time to travel. However, regarding the observation from actual operations of bus services, it is difficult to implement these methodologies effectively, as a result of the variabilities in bus operation which trigger bus bunching and service irregularity (5).

Compared with bus services, rail transit services are much more stable regarding velocity, dwell time and regularity. Hence, railway transit is a more appropriate transit mode for researchers to conduct studies on service timetabling. For example, Chang and Chung (6) proposed a model to describe train operation, in which both flexibility of train rescheduling and the process of defining timetable were considered. This model is able to provide a quick response in regulation and constructing new timetable when incident occurs using genetic algorithm. Ding and Chien (7) also focused on the irregularity caused by stochastic variations from passenger demand and traffic conditions. A real-time control strategy is introduced to maintain headway regularity by minimizing total headway variance. This strategy is further tested with simulating a LRT (Light Rapid Transit) service in Newark, New Jersey. Eberlein et al. (8) formulated the vehicle holding problem as a deterministic quadratic programming for a loop network with equal scheduled headways, besides, this model is tested using data collected from a LRT service. In regard to MRT services, efforts are also made on train regulation problems, in particular on fighting against disturbance using automatic and adaptive control (9, 10). In (9), a dual heuristic dynamic programming model is proposed based on the predefined cost function. Lin and Sheu (11) further discussed this problem since the dual heuristic dynamic programming is sensitive to the errors in modeling. To further
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develop the control strategies, an adaptive optimal control algorithm is presented which is able to learn from traffic data with artificial neural networks. However, in these studies, more attention are paid on maintaining service regularity, but not on designing a demand responsive timetable which may reduce the disturbance fundamentally.

Based on the periodic event-scheduling problem in graph model, Liebchen ([11]) obtained a timetable with shorter waiting time for passengers. This timetable has been implemented on Berlin subway system in daily operation and it is reported that both passengers and transit operators profit from the designed timetable. With the same intention as ([3]), Wong et al. ([12]) also addressed the timetabling problem with transfers taken into account, and proposed a mixed integer programming model which aims to achieve a coordinated timetable and smooth transfers by minimizing the transfer waiting time. This strategy is tested on Mass Transit Railway in Hong Kong. However, these contributions are limited in considering passenger demand and train capacity, which have significant impact on passengers’ travel experience.

The implementation of smart card based automatic fare collection system has generated large quantities of data recording passenger activities with detailed time and location information. It has been publicly recognized that the potential benefit of smart card data on improving public transit planning and operation is enormous ([13]). Smart card data offer an excellent opportunity to identify the demand pattern regarding both spatial and temporal variation ([14]). Moreover, it also allows to reconstruct transit operation scenarios to test the proposed methodologies to improve public transit services.

This present study deals with timetabling problem with dynamic demand taken into consideration. As one of the input parameters of the timetabling procedure, the dynamic demand comes from former studies. Various distinct demand pattern will be assigned to stations along the service. After obtaining the demand profile, a predefined timetable will be constructed. In other words, the intention of this study is to construct a planning timetable which is demand responsive and has potential to be applied with further refinements into real operation.

In the remainder of this paper, we will first introduce the formulation of the optimization model thoroughly in Section 3. The characteristics of the MRT systems in Singapore based on previous research and assumptions of the proposed model are described and introduced in this section. In Section 4, the information about smart card data and a case MRT service to test the proposed timetabling model are introduced. Afterwards, given the data and case service, the initial passenger demand profile is generated considering both direct trips and transfer trips. Section 5 begins with introducing the optimal timetable obtained from the proposed model. In the absence of effective timetable from operators, the designed optimal timetable is analyzed in comparison with a simplified timetable extracted from Google Maps. Finally, conclusion and discussion on further research and applications are presented in Section 6.

MODEL FORMULATION

In this section, the characteristics on MRT operation are described firstly based on previous studies. Then, a deterministic mathematical programming problem is formulated based on some assumptions on trains and passengers, which tries to define the departure time of each train and waiting profile of passengers to minimize the given objective function representing total waiting time.
MRT System Characteristics

In this study, we focus on one single MRT track with $S_n$ stations along the route, and $K_n$ trains departing within a day. Our purpose of this study is to define the departure schedules of all the trains. In a previous study, [Sun et al. (14)] proposed a method to extract trains’ trajectories using smart card data, which also shows the stability of MRT operation regarding operating velocity. Therefore, to simplify this problem, the trajectories of trains are assumed to be straight lines paralleling with each other.

Concerning the MRT operation, there also exists a headway threshold $[h_{min}, h_{max}]$ for two successive trains. The lower value $h_{min}$ guarantees the safe operating distance, while the upper value $h_{max}$ represents the maximum headway which is acceptable based on intended service level. In this study, the concept of time slots of fixed duration length is introduced to facilitate the demand sensitive timetable optimisation problem. All the time dependent parameters and variables in this model are associated with a specific time slot. Then, considering the applicability of the new defined departure time of each train, it is assumed that all the trains depart following a headway interval with length of $n$ time slots, where $h_{min} \Delta t \leq n \leq h_{max} \Delta t$, $\Delta t$ is the length of each time slot.

Given these assumptions, time slots could be further defined considering both temporal and spatial information. As shown in FIGURE 1, each time slot is associated with certain length of time with the time offset of each station taken into account. At the departure terminal (Station 1), time slots are defined by cutting the operation time into segments of equal durations. For other stations, the operation time is postponed according to the corresponding offset of the station. The modeling system is further illustrated in FIGURE 1. Trains depart at the beginning of defined time slots: Train 1 departs at the beginning of time slot 1 and Train 3 departs at the beginning of time slot 3. Increasing offsets are given to each station along the route.

![FIGURE 1 : MRT system characteristics](image-url)
Assumptions

Regarding the passenger demand, the exact number of passengers arriving at each station is provided by the smart card data. Therefore, it is possible to obtain the demand dynamics of passengers accessing the system at each station. In this study, for each station, passengers’ arrival time is aggregated in time slot and the arrival rate is assumed to be uniformly distributed within each time slot. Hence, the proposed model tries to minimize the total waiting time of all passengers with subject to such aggregation. Another assumption is made on capacity of trains to simplify the model. We assume that the capacity of each train is infinite. This assumption is obviously unrealistic from an operational perspective. However, it is expected that the timetable provided by the optimization model will balance the occupancy of trains and hence minimise the potential implication of such a simplification. Nevertheless, this assumption needs to be revisited and is discussed in Section 5.

For the instance shown in FIGURE 1, passengers arrive in time slot 1 and 2 will board on Train 2, while passengers arrive in time slot 3–7 will board on Train 3 and so on. Then, given an operation timetable, it is possible to define the waiting time for each time slot and each station. Taking the passengers arriving at Station 2 in time slot 4 as an example, this passenger flow will wait for another 3 time slots to board on Train 3 arriving at the beginning of time slot 8. Formulated in more general terms, we define the waiting profile of passengers as the additional number of time slots for them to wait. It can also be noted that passengers associated with the same time slot will be assigned the waiting profile given the schedule of the next train.

Above all, the assumptions for the proposed model are as follows:

1. The fleet of trains run at the same velocity. The headway for two successive trains does not vary through the whole journey.
2. For operation’s convenience, trains depart with \( n \times \Delta t \)'s headway, where \( \Delta t \) is the length of each time slot. Minimum and maximum headway constraints are also established.
3. For each station and within each time slot, passengers arrive uniformly.
4. There is no capacity constrains for trains.
5. Time slots are defined with same length based on the operating time of MRT services.
6. The first train depart at the beginning of times slot 1, while the last train depart at the beginning of the last time slot.
7. The train operation is modeled as single track, unidirectional system.

Problem Formulation

The following notations shown in TABLE 1 are used in this proposed model.

Based on Assumption (3), for passengers arriving in given time slot \( t \) at any station \( s \in S \), they will choose the same waiting profile. Thus, if waiting profile \( p \) is chosen, the number of time slots they have to wait is:

\[
w_{t,p}^s = w_{t,p} = p, \forall s \in S
\]

Hence, considering the uniform arrival of passengers, the total waiting time can be formulated as:

\[
W_{t,s} = B_{t,s} \times \sum_{p \in P} y_{t,p} \left( w_{t,p} + \frac{1}{2} \right)
\]
TABLE 1: Formulation Notations

<table>
<thead>
<tr>
<th>Sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>set of time slots, $T = {1, 2, \cdots, T_n}$</td>
</tr>
<tr>
<td>$S$</td>
<td>set of MRT stations, $S = {1, 2, \cdots, S_n}$</td>
</tr>
<tr>
<td>$P$</td>
<td>set of waiting profile, $P = {0, 1, 2, \cdots, P_n - 1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>index of time slots, $t \in T$</td>
</tr>
<tr>
<td>$s$</td>
<td>index of MRT stations, $s \in S$</td>
</tr>
<tr>
<td>$p$</td>
<td>index of waiting profile, $p \in P$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_n$</td>
<td>number of time slots</td>
</tr>
<tr>
<td>$S_n$</td>
<td>number of stations along the route</td>
</tr>
<tr>
<td>$K_n$</td>
<td>number of services departing within a day</td>
</tr>
<tr>
<td>$P_n$</td>
<td>number of waiting profiles, $P_n &lt;&lt; T_n$</td>
</tr>
<tr>
<td>$B_{t,s}$</td>
<td>number of passengers entering station at in time slot $t$ at station $s$</td>
</tr>
<tr>
<td>$N_{\text{max}}$</td>
<td>maximum headway for two successive trains (in number of time slots)</td>
</tr>
<tr>
<td>$N_{\text{min}}$</td>
<td>minimum headway for two successive trains (in number of time slots)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>binary, equals to 1 if there is a train departing at the beginning of time slot $t$, 0 otherwise</td>
</tr>
<tr>
<td>$y_{t,p}$</td>
<td>binary, equals to 1 if passengers arrive in time slot $t$ use waiting profile $p$, 0 otherwise</td>
</tr>
</tbody>
</table>

Considering all the time slot and stations in this proposed model, with the notations defined in TABLE 1, the objective function which represents the total waiting time in this system is formulated as:

**Objective Function**

$$\min \sum_{s \in S} \sum_{t \in T} W_{t,s}$$  

The objective function is convex since it is a linear combination of the decision variable $y_{t,p}$. The objective (Equ 3) is minimized with the following constraints:
1 **Constraints**

\[
\begin{align*}
\sum_{t \in T} x_t &= K_n \quad \text{(4)} \\
x_1 &= 1 \quad \text{(5)} \\
x_{T_n} &= 1 \quad \text{(6)} \\
\sum_{p \in P} y_{t,p} &= 1 \quad \forall t \in T \quad \text{(7)} \\
x_s &\geq y_{t,p} \quad s = t + w_{t,p} \leq T_n, \forall t, \forall p \in P \quad \text{(8)} \\
x_t &\in \{0,1\} \quad \forall t \in T \quad \text{(9)} \\
0 &\leq y_{t,p} \leq 1 \quad \forall t \in T, \forall p \in P \quad \text{(10)} \\
\sum_{t} x_t &\geq 1 \quad \forall t \in \{1,2,\cdots T_n - N_{\text{max}}\} \quad \text{(11)} \\
\sum_{t} x_t &\leq 1 \quad \forall t \in \{1,2,\cdots T_n - N_{\text{min}}\} \quad \text{(12)}
\end{align*}
\]

Constraint (4) defines the total number of trains departing within a day, which is determined by the vehicle stock and availability of train drivers. Constraints (5) and (6) define the departure time of the first and last trains, which is also determined by the operation time period of MRT services. Constraints (7) guarantee that there is one waiting profile adopted by each group of passengers. Constraints (8) ensure that commuters can board only if a train is available. Constraints (9) and (10) ensure feasible solutions of this problem. In Constraints (11) and Constraints (12), the maximum and minimum headway are defined in number of time slots.

To further test the performance of this model, a case study is performed based on real demand records from smart card data on a selected MRT service.

**DATA PREPARATION & CASE STUDY**

In this section, the data used to design the new timetable is illustrated firstly. Then, a case MRT service is introduced and selected to conduct the numerical experiment. Afterwards, we describe our method to obtain the initial demand, which is the most significant input parameter to the optimization model.

**Smart Card Data**

The smart card based automated fare collection system has been introduced in many major cities around the world. In Singapore, it has been introduced for more than 10 years since April, 2002. Although the original intension of this system is to provide a convenient, automated fare collection scheme in public transportation, it also generates large quantities of data, recording passengers’ tapping in and tapping out activities. Compared with of this particular dataset is that it comprises precise spatio-temporal information for both tapping in and tapping out activities.

To generate comprehensive demand profile of weekdays as inputs for the proposed model, considering data from Friday has different characteristics compared with other weekdays, the data from Monday to Thursday is used to define a typical weekday.
Case MRT Service

In this study, a representative MRT service is selected to test the proposed optimization model in constructing a demand responsive timetable.

To conduct this case study, the information on tapping in time, boarding and alighting stations extracted from smart card data records are used to generate the initial demand for this case service. Other information such as location of stations along the routes and characteristics of stations are accessed from supplementary sources.

Initial Demand

Smart card data enable researchers to investigate demand pattern and characteristics with exact count data for each station. Some researchers have been using such data to identify people’s travel behavior. By analyzing the smart card data collected in Outaouais, Canada, different trip habits based on predefined user type, and variability of trips against time have been identified (15).

Utsunomiya et al. (16) pointed out that the demand pattern varies with day in week, especially for weekdays and weekends, therefore, it is reasonable for transit operators to make different operation schedules within a week. Park et al. (17) studied the demand characteristics of different transport modes, in particular on rapid transit systems, in Seoul, South Korea. Sun et al. (14) investigated the demand pattern of the case MRT service and extracted passengers’ spatio-temporal density and trains’ trajectories based on the demand. In this present study, we use effectively observed dynamic travel demand patterns to create different demand characteristics to each individual station. In this study, smart card data collected from 4 days (Monday-Thursday) is used to estimate the initial demand of a typical weekday.

Smart card dataset provides us the origin and destination profile of each passenger together with the exact tapping in/out timings, however, the transfer activities are not recorded since transfer passengers do not have to tap their card again at interchange stations. Thus, for trips that involve multiple lines, both transfer station and transfer time need to be predetermined. In this study, we use the MATSim transport simulation toolkits to reconstruct such information from the smart card dataset (18). To this end, the observed tapping in time, origin and destination are combined to so-called activity plans and simulated at once. The router that is needed for such simulation is designed to minimise generalised travel cost which in this case is composed of both expected travel time and distance but does not take into account crowdedness.

To generate time slot based demand, we have to define the offset to each station. Based on the method proposed in (14), the value of offset can be calculated using regression. With this offset data, the demand can be aggregated according to time slots as explained in FIGURE 1. In this case study, the operation time starts at 5:30 and ends at 23:30 and the duration of each time slot is \( \Delta t = 30 \) s. Therefore, a total number of 2161 time slots are defined.

FIGURE 2 shows the demand variation of 24 selected stations against the predefined time slots. In fact, the input parameter \( B_{t,s} \) in the model can then be obtained from the demand variation. It can be observed that the demand rather low and stable during the most time of the day, but shows distinct morning and evening peaks with arrival rates up to around 1800 pax/30s. When comparing the demand patterns between stations, it becomes obvious that each station has its unique demand distribution varying with time of day. For all the stations, demand presents burst in morning and evening peaks, however, with different strength, shape and happening time.

After obtaining this initial demand \( B_{t,s} \), we have introduced all the input parameters to the proposed optimization model. The general input parameters are defined in TABLE 2. This
numerical experiment is performed using CPLEX on a PC with Intel Core i7 dual 2.8GHz CPU and 8GB RAM.

**TABLE 2 :** Input parameters of case study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_n$</td>
<td>2161 (5:30-23:30)</td>
</tr>
<tr>
<td>$S_n$</td>
<td>29</td>
</tr>
<tr>
<td>$K_n$</td>
<td>175</td>
</tr>
<tr>
<td>$P_n$</td>
<td>30</td>
</tr>
<tr>
<td>$N_{max}$</td>
<td>30 (900s)</td>
</tr>
<tr>
<td>$N_{min}$</td>
<td>7 (210s)</td>
</tr>
</tbody>
</table>

**RESULTS AND ANALYSIS**

In this section, the optimal solution of this case study is presented. A comparison is conducted between the simplified timetable as extracted from Google Maps and the optimal solution using simulation under the passenger profiles from smart card data records on Monday. The assumption of not considering capacity constraints in the optimization model is revisited in the following.

**Optimal Timetable**

The designed timetable is extracted with the average demand from Monday to Thursday, therefore, the optimal solution is suitable to typical weekday’s scenarios. In this case study, the number of train services is considered as a constraint and is derived from the simplified schedule extracted from Google Maps. FIGURE 3 shows the comparison of headway variation between the published

**FIGURE 2 :** Demand variation against time slots
and optimal timetable. It can be seen that the timetable obtained from the proposed model is more dynamic regarding to headway variation. This variation provides the potential to reduce the average waiting.

Nevertheless, capacity constraint of trains is significant to real performance of the optimization model. Thus, comparison is conducted between the published timetable and the optimal solution using simulation.

**Comparison with a Peak-offpeak Timetable**

In the absence of the effectively operated timetable, we use the a simplified schedule derived as published by GoogleMaps to conduct a comparison study. Simulation is performed using the dynamic passenger demand profiles for a Monday in April 2011. The boarding and alighting profiles of 507,589 passengers are simulated under the "first arrive, first board" strategy. Results are summarized in TABLE 3, regarding both the published and optimal timetable, under different capacity constraint scenarios. Without considering capacity constraint, the value of average waiting time decreases by 4.7% relative to the published timetable. The optimal timetable does not improve service level significantly in this situation.

However, with the capacity constraint taken into consideration, the value of average waiting time decreases 35.4% compared to the published timetable. In fact, it can be implied from the result that most of the waiting time comes from passenger who wait more than one train to be able to board. For one passenger, the additional waiting means the number of additional trains he/she has to wait if it is impossible to board on the first train due to capacity constraints. By applying the optimal timetable, the total number of additional waiting for all passengers decreases by 84.3% relative to the published timetable.

To make the comparison more comprehensive, the trajectories and passenger load indicated by color are analyzed with the two timetables, as shown in FIGURE 4. For the published timetable shown in FIGURE 4a, the indication of occupancy density shows that the actual morning peak begins earlier than the scheduled train frequency increase, while the actual evening peak last also
TABLE 3: Simulation result under different scenarios

<table>
<thead>
<tr>
<th></th>
<th>Timetable</th>
<th>Peak-Offpeak</th>
<th>Proposed</th>
<th>Peak-Offpeak</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of passengers (pax)</td>
<td>507,589</td>
<td>507,589</td>
<td>507,589</td>
<td>507,589</td>
<td></td>
</tr>
<tr>
<td>Train capacity (pax/train)</td>
<td>∞</td>
<td>∞</td>
<td>2,000</td>
<td>2,000</td>
<td></td>
</tr>
<tr>
<td>Average waiting time (min)</td>
<td>3.070</td>
<td>2.925</td>
<td>4.997</td>
<td>3.228</td>
<td></td>
</tr>
<tr>
<td>Additional waiting (times)</td>
<td>0</td>
<td>0</td>
<td>191,036</td>
<td>29,925</td>
<td></td>
</tr>
</tbody>
</table>

longer than the period with increased frequency. Overall, this leads to additional waiting because of overcrowding, traveler need to wait for subsequent trains with available space during peak hours. The optimised timetable, on the other hand is more sensitive to the actual demand variations. Therefore, peak hour demand is more equally absorbed as shown in FIGURE 4b which results in substantial reduction of the total waiting time.

As has mentioned, in the proposed optimisation model potential capacity constraints are not considered. In fact, there are two reasons to make this assumption. On one hand, the current operation parameters (number of trains and timetable) can match the current demand in most cases, therefore, if the minimum headway and maximum headway are constrained, the maximum occupancy will be similar to actual operations. On the other hand, it allows a straightforward and simple model formulation which ensures that the model can be solved easily.

To test the sensitivity of the solution with regard to the constraint of the number of services, FIGURE 5 shows how the average waiting time varies as a function of the total number of services. It can be seen, with the increasing number of services, the improvement on average waiting time is not very distinct if capacity constraints are neglected. However, when capacity constraints are considered, the simulated average waiting time is much higher with less number of services. Under this circumstances, the average waiting time decrease significantly and converge to optimal waiting time obtained from the proposed model. There is no difference between these two scenarios if the number of services is greater than 190. For the published service frequency, which is 175 services daily, the difference is 4.7% as has mentioned. Therefore, it is reasonable to ignore the capacity constraint in the model formulations as the potential gains are minor but keeps the model much easier to solve.

CONCLUSION AND FUTURE RESEARCH

In this paper an optimization model to design a new operational timetable which is responsive to the demand variation was presented. After obtaining the initial parameters for this model, the optimal solution could be obtained with acceptable amount of computation time using the CPLEX software package.

The model has the following novel characteristics regarding the MRT operation in reality: (a) departure time/headway solved as decision variable is time-slot based, which is applicable to transit operation in reality; (b) the optimal headway variation against time can be used as guidance to transit operators; (c) the demand of boarding passengers could be extracted from smart card data with its own characteristics, since different day in week has different variation profile; and (d) the model is much simpler and more straightforward when the capacity constraint is not considered, whereas the optimal solution obtained from the proposed model has the potential to satisfy the capacity constraint in most cases.
FIGURE 4: Trajectory/Occupancy Comparison

The proposed optimization model is evaluated with simulating a case MRT service. After obtaining the optimal timetable, different scenarios are compared regarding the scenarios which operate according to simplified and optimal timetables with/without capacity constraint. The result obtained from the case study indicates that it is inappropriate to compare the performance without considering capacity constraints. With capacity constraint considered, the average waiting time and number of additional waiting are reduced 35.4% and 84.3% respectively, relative to the simplified timetable as published on Google Maps. The trajectory plots with occupancy indicated by colors also show that the simplified timetable do not perform as good as the demand responsive one in occupancy, headway variation and number of additional waiting time. Although the schedule as published on Google Maps is not the real operated one from our observations in reality (the effectively operated timetable is also flexible to some extend to demand variations) this model still shows great potential in creating a demand responsive timetable using optimization. The computation time required for this case study is less than 30s on a PC, which shows great capability
FIGURE 5: Simulated waiting time with & without capacity constraint

There are some limitations of the proposed model: (a) for train operation, there are some constraints exist in scheduling, depot arrangement, fleet management and driver schedules, which are not taken into account in this present model; (b) the dispatching management based on operation constraints such as train turning at terminals remains unconsidered; (c) the proposed model may work well for typical weekday and weekend since the demand variation in this study is obtained from smart card data, however, it will fail when special events or incidents happen since it is difficult to capture the pattern and significant characteristics of passenger demand. These limitations offers opportunities for future research on this topic, considering the uncertainty and sensitivity in model parameters. More operational constraints could be taken into account to describe the MRT operation more precisely.

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