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Working Paper

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Publication date:
2021-11

Permanent link:
https://doi.org/10.3929/ethz-b-000516401

Originally published in:
Economics Working Paper Series 21/365

CER-ETH – Center of Economic Research at ETH Zurich

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Working Paper 21/365
November 2021

Economics Working Paper Series

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Capital Flows and Endogenous Growth*

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This version: November 18, 2021

Abstract

So-called "uphill capital flows", i.e. flows of physical capital from relatively poor to rich countries, are a new phenomenon with yet unclear impact. We develop a unified framework incorporating economic institutions, human capital and physical capital to study the interaction of international capital flows and growth. Analytically, we study conditions under which a positive change of a country's economic institutions can attract inflows of physical capital from abroad, leading to long-term growth via the accumulation of human capital. Our mechanism shows how a small initial difference in the level of institutions can lead to substantial divergence in income over time. We derive conditions under which a country receives inflows of capital over time and increases its investment in human capital. Finally, we provide simulations to illustrate our results.

Keywords: Growth, International Capital Flows, Inequality, Institutions, Human Capital

JEL Classification: E02, F21, F43, O41, O43

*I would like to thank Alexandra Brausmann, Treb Allen and the participants of the Astute Modeling Seminar at ETH Zurich for helpful comments.

1 Introduction

Motivation

Are international capital flows a cause for growth in inequality between countries? In light of persistent income differences between industrialized and developing countries, and the fact that capital seems to flow from capital-poor to capital-rich countries, this question becomes relevant. Whether developing countries, experiencing capital inflows, can benefit from capital inflows and whether they should open up to financial markets are further policy issues.

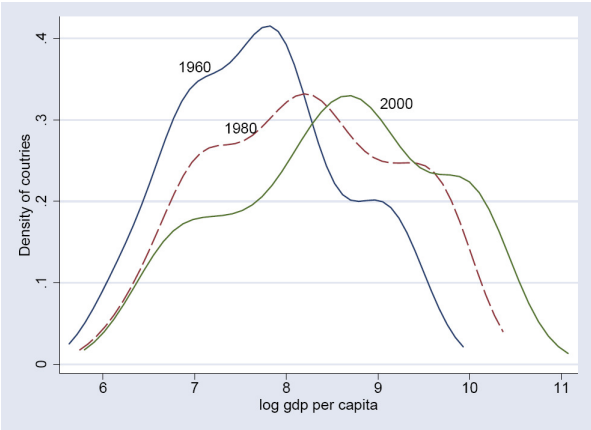

Figure 1: Log GDP per capita, Source: Acemoglu (2009).

Figure 1 shows the growing inequality between countries, using log GDP per capita for three points in time, 1960, 1980 and 2000. The mean of the distribution is moving to the right over time, indicating a general increase in prosperity worldwide. However, the variance is also increasing, as the decreasing peak from 1960 and the widening of the distribution show. This observation is in contrast to predictions of the neoclassical model, as income differences between countries seem to increase rather than to decrease.

Another phenomenon is the occurrence of "uphill capital flows", i.e. flows of capital from poor countries to rich ones. Such flows can be observed in Figure 2, which is taken from Prasad et al. (2007). The authors state "*[n]ot only is capital not flowing from rich to poor countries in the quantities the neoclassical model would predict—the famous paradox pointed out by Robert Lucas—but in the last few years it has been flowing from poor to rich countries*". They refer to the trend beginning around the year 2000, after which capital-importing countries (deficit countries)

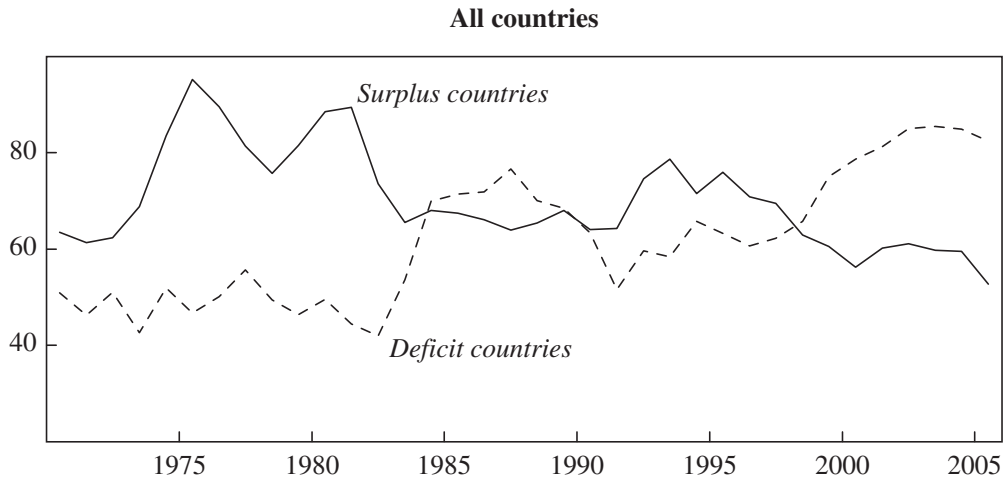

Figure 2: Relative GDP per capita of capital exporters and capital importers. Source: Prasad et al. (2007).

comprise more and more countries with a relatively large GDP.

While uphill capital flows are a rather new observation in comparison to the increasing inequality from Figure 1, the question arises whether such flows reinforce the divergence of countries. A closely related question, namely the impact of capital flows on growth, is studied by Kose et al. (2009). Their study contains a review of the recent empirical literature, finding that the literature "[...] *provides little robust evidence of a causal relationship between financial integration and growth.*" However, the authors do not claim that international capital flows have no effect on growth. The flows rather have indirect effects which might play out over a long time horizon and, among others, can take the form of increased competition and the development of a stronger financial market.[1] Hence, Kose et al. (2009) conclude "[...] *that it is not just capital inflows themselves, but what comes along with the capital inflows, that drives the benefits of financial globalization for developing countries.*"

A concrete theoretical interaction as described by Kose et al. (2009) has not been modeled up to now, and such a model is our goal in this study. We provide a framework where international capital flows are related to and interact with well-

[1]The findings of Kose et al. (2009) are corroborated in the survey by Edison et al. (2002). However, there are studies that find a clear positive impact of increased financial integration, such as Henry (2003).

known causes of growth. This way, we describe some ideas concerning structures of international capital flows and endogenous growth. We aim at providing a framework for future empirical research. Following North and Thomas (1973), we assume that causes of growth can be either proximate, such as accumulation of factors and technological change, or fundamental, such as political and economic institutions. Hereby "fundamental" means that without well designed institutions, no growth-driving accumulation of factors is possible in an economy.[2] We align those two causes of growth with international capital flows in a unified framework. Our main idea is that better economic institutions, which will be the fundamental cause for growth, attract international capital flows. These, in turn, set the proximate causes in motion. The proximate source of endogenous growth will be the accumulation of human capital.

Our choice to combine economic institutions and human capital is based on the idea shown in Figure 3. Since there is little direct impact of capital flows on growth, we argue that capital can only have an impact via variables that interact with growth themselves. Hence, our approach requires variables correlating with international capital flows *and* growth.

There is a consensus that economic institutions and the accumulation of human capital are paramount for economic growth[3]. Their correlation with international capital flows has been demonstrated in several studies, for economic institutions in work like Olsen et al. (2000), Alfaro et al. (2008) and Kose et al. (2009), and for human capital in Lucas (1990) and Caselli and Feyrer (2007), for instance.[4]

After having chosen the variables that are relevant for our approach, the question remains how these variables interact with each other. We build on the idea that good economic institutions enable the accumulation of factors, such as physical and human capital, or the raise of the technology level. In turn, the accumulation

[2]For a thorough discussion about the reasons and ways, in which institutions are a fundamental cause for growth, see Acemoglu et al. (2004).

[3]There is an extensive literature, respectively, for the case of human capital and for the case of institutions. For the former, some examples are work like Lucas (1989), Romer (1989), Barro (1991) and Mankiw et al. (1992). Pelinsecu (2014) provides a survey, including more recent work. For the latter, some examples are work like Acemoglu et al. (2001), Acemoglu et al. (2002), and Hall and Jones (1999). Glaeser et al. (2004) provide a critical overview.

[4]For an overview of other determinants of international capital flows, see Taylor and Sarno (1997).

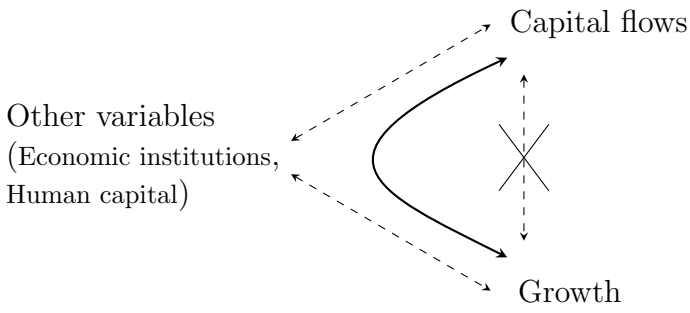

Figure 3: Interaction of international capital with growth. Source: Author's design.

of factors then leads to growth. We take this narrative and re-position it in an international setting to formulate the following mechanism:

1. Better economic institutions allow agents to reap the benefits of their investment in physical capital. This interpretation is in line with Acemoglu et al. (2004) that "*[e]conomic institutions matter for economic growth because they shape the incentives of key economic actors in society, in particular, they influence investments in physical and human capital* [...]".

2. The country with a larger return to capital will attract capital flows.

3. Capital inflows increase wages and hence agents have enough income to reduce labor supply and invest some time in the formation of human capital.

4. Human capital is not depreciated but accumulated, allowing for long-term growth.

Approach and results

We use an OLG model with three generations, consisting of children, adults and seniors. Children can either work or form human capital through schooling. Adults work, save and decide whether to school their children. Seniors only consume. We have two countries and a single good, produced with physical and human capital, embodied in labor. Capital can move freely between the countries while labor cannot. Both countries have economic institutions, determining the net return on physical capital and initially, arbitrage ensures that returns are equal across countries. Then, a country experiences an improvement in its institutions and a

subsequent increase in its return to capital. Capital flows in and wages increase. We examine whether the wage increase is sufficient for adults to send their children to school. Hereby, schooling leads to the formation of human capital that can be used without costs in the next period.

We demonstrate how a change in institutions, coupled with international capital flows, can generate differences in economic growth. While the country with better economic institutions moves to a balanced growth path with increasing levels of human capital, the other remains in a zero growth steady state. Furthermore, we study the existence of different steady states and convergence to them.

Throughout our work, we interpret capital flows as private flows of foreign direct investment (FDI). By doing this, our model can replicate the empirical findings of Gourinchas and Jeanne (2013) and Alfaro et al. (2014), where the authors find that public capital flows behave differently than private ones. Only private capital flows to countries with higher growth rates. In a simulation, the country with increasing investment in education experiences increasing productivity growth, measured as an increasing stock of human capital and attracts more capital. Hence, we establish a positive correlation between productivity growth and inflows of capital.

Relation to the literature

Our work is closely related to studies that use the neoclassical growth model in an international setting, such as Barro et al. (1992), Gourinchas and Jeanne (2006), and Aguiar and Amador (2011) in which the authors study the impact of capital flows on the convergence of an economy to a steady state. While Aguiar and Amador (2011) focus on explaining the behavior of governments, Barro et al. (1992) and Gourinchas and Jeanne (2006) consider an economy where the steady state is a balanced growth path with a constant growth rate. In both studies, the production function accounts for human capital formation and growth is exogenous, driven by technological progress. These studies show that inflows of capital have a rather small effect on the convergence path. While our findings are consistent with this result, our approach differs in three crucial ways: First, we do not assume a constant world rate of return. Instead, the flows of physical capital and the accumulation of physical and human capital in a period will determine, whether a country can also attract capital inflows in the next period. Second, we do not study the speed of convergence, but we are interested in the question,

whether a country can escape a poverty trap and converge to a steady state with a constant rate. We provide analytical conditions for such an escape. Third, we consider an endogenous growth model in which growth stems from the accumulation of human capital.

Another study that analyzes a neoclassical growth model is Davenport (2018). However, Davenport (2018) studies the importance of expectation formation for international capital flows and growth. The authors are able to replicate substantial current account imbalances but assume an exogenous growth process, where countries catch up to a technological frontier. We endogenize the growth process and relate it to the structure of capital flows, making them interdependent.

Howitt (2000) provides an alternative to the neoclassical model approach. He constructs a multi-country model with elements of the Solow-Swan-model and the Schmupeterian growth model to explain cross-country differences in growth. Countries that invest in R&D will grow at a positive rate, while those who are not able to do so, are stuck with the same output. Howitt (2000) abstracts, however, from international capital flows, which are now incorporated in our work. Furthermore, we highlight the importance of economic institutions and include the accumulation of human capital instead of technological progress as the main driving force of long-term growth.

Our study is also related to Bell et al. (2019), where the authors construct an OLG model with endogenous growth. As in this work, growth is driven by the accumulation of human capital through schooling and the effects of exogenous shocks are studied in a closed economy. We take some of the central assumptions and reassess them in an international setting with capital flows, including a central role for institutions.

Finally, this study is related to other works about international capital flows that, either focus on the international asset structure, such as Caballero et al. (2008) and Tille and Van Wincoop (2010), or on the international trade structure, such as Jin (2012). While our model incorporates international assets, it does it in a simple way, allowing us to focus on the interaction of capital flows and growth and to show the causes for international inequality.

Structure

The remainder of the paper is structured as follows. Section 2 presents a simple endogenous growth model with human capital accumulation. In Section 3 institutions are introduced and Section 4 introduces intarnational capital flows. Section 4 presents the full model, including international capital flows. Section 6 contains a simulation and while Section 7 offers an extenstion. Section 8 concludes.

2 Simple model

The Productive Sector

We consider an economy with only a single good that is used either for consumption or investment. It is produced in two different sectors. The first is the capital-intensive sector that employs physical capital and human capital of adults.[5] The second is the child-labor sector that employs human capital of children. Their respective output is given by

$$Y_t^2 = AK_t^\alpha (H_t^2)^{1-\alpha} \quad \text{and} \tag{1}$$

$$Y_t^1 = H_t^1, \tag{2}$$

where A is some constant total factor productivity parameter, K_t physical capital, and H_t^2 the stock of human capital in the capital-intensive sector and H_t^1 the one in the child-labor sector.[6] We assume a representative firm for the capital-intensive sector that borrows both types of capital from households at the capital rental rate R_t and at the wage rate w_t^2. Under perfect competition, profits and the demand functions of the firm are

$$\Pi_t^2 = Y_t^2 - R_t K_t - w_t^2 H_t^2, \quad R_t = \alpha \frac{Y_t^2}{K_t}, \quad \text{and} \quad w_t^2 = (1-\alpha)\frac{Y_t^2}{H_t^2}. \tag{3}$$

We also assume perfect competition in the child-labor sector and have

$$\Pi_t^1 = Y_t^1 - w_t^1 H_t^1 \quad \text{with} \quad w_t^1 = 1. \tag{4}$$

[5] We assume that capital is complementary to the skills of adults, while it is not to the skills of children. This assumption appears plausible, as children lack the physical strength to operate machinery and the intellectual maturity to work with other types of equipment.

[6] We follow the approach by Docquier et al. (2007).

Since, $w_t^1 = 1$ we will write w_t^2 simply as w_t.

Households

A household consists of three generations that are alive at the same time: children, adults, and seniors. Children and seniors only consume and do not take any economic decision. Additionally, children can use their time either for work or for education. The former earns a wage income for the family, while the latter increases the children's human capital in the next period. Consequently, the education decision involves a trade-off between wage income today and more income tomorrow. Whether and how much schooling takes place is decided by adults. They maximize their own utility, supply their own human capital to firms, decide whether and how much to school children, and how much should be saved.

Children have the level of human capital ηL^1, where L^1 is the amount of labor and η is the fixed level of human-capital per child. The adults' stock of human capital is given by $\phi_t L^2$, with L^2 being the fixed amount of labor supplied by adults and ϕ_t the amount of human capital per worker. This amount increases with schooling: If children only supply the share $(1 - e_t)$ to the labor market and spend the share e_t in school, the human capital stock per worker grows, i.e. $\phi_{t+1} > \phi_t$. Hereby, we only impose positive returns to schooling: $\partial\phi_{t+1}/\partial e_t > 0$. Also, we assume that the current stock of human capital can be inherited without schooling, so that ϕ_{t+1} is always at least as large as its predecessor. For instance, if children receive education of size e_t^* then

$$\phi_{t+1}^* = \int_0^{e_t^*} \frac{\partial\phi_{t+1}}{\partial e_t} de_t + \phi_t.$$

The stock of human capital, supplied inelastically by adults to the capital-intensive sector is $\phi_t L^2$. We therefore have $H_t^2 = \phi_t L^2$ and the stock of human capital that is supplied by children to the child-labor sector is $(1 - e_t)\eta L^1$. For simplicity, we will write H_t^2 as H_t from now on.

We model the utility of an agent in period t as in Bell et al. (2019) so that the adults receive utility from consumption in t, as well as consumption in $t + 1$. We assume a linear benevolence term that enters the utility function and depends on the level of education of children in $t + 1$.[7] Hence, the life-time utility for an adult

[7] We choose this structural form as it allows for an analytical approach.

in t reads

$$U_t^2 = \log[c_t^2] + \beta \log[c_{t+1}^3] + \beta\phi_{t+1}.$$

Adults receive their own wage income and that of children to whom, in turn, they give the share γ_1 of the total wage income. Seniors give a share of their capital return γ_3 to adults, as an additional form of income. The motivation for this behavior can be either altruism or the fact that seniors are not alive for the entire time period and bequest some of their income to adults. Hence, the income of adults is a combination of the remainder of wage income and the received capital income. Their consumption, then, is what is left of this income after savings, leading to the following utility function:

$$\begin{aligned} U_t^2 = &\log\left[(1-\gamma_1)(w_t H_t + (1-e_t)\eta L^1) + \gamma_3 K_t R_t - s_t\right] \\ &+ \beta \log\left[(1-\gamma_3) R_{t+1} s_t\right] + \beta\phi_{t+1}. \end{aligned} \tag{5}$$

We make an additional assumption, that the income which is paid to children is only used for consumption and does not enter the savings decision. The rational behind this assumption is that it simplifies the analysis strongly. However, it can be argued that parents only then send their children to work when it is necessary, i.e. when the family goes hungry otherwise. They do not ask their children to work in order to save and accumulate capital. We relax this assumption in an extension in Section 7. In the following, we will refer to the income of adults that accrues from capital and adult human capital as "income", while the sum of capital income, wage paid to adults, and wage paid to children will be called "total income". Hence, maximizing with respect to savings yields

$$\frac{\partial U_t^2}{\partial s_t} = \frac{-1}{(1-\gamma_1) w_t H_t + \gamma_3 R_t K_t - s_t} + \frac{\beta}{s_t} = 0. \tag{6}$$

Note that due to our assumption from above the term $(1-e_t)\eta L^1$ is missing from

this expression. Maximizing with respect to education yields

$$\frac{\partial U_t^2}{\partial e_t} = \frac{-(1-\gamma_1)\eta L^1}{(1-\gamma_1)(w_t H_t + (1-e_t)\eta L^1) + \gamma_3 R_t K_t - s_t} + \beta\phi'_{t+1} \leq 0 \quad \text{for} \quad e_t \geq 0, \tag{7}$$

$$\frac{\partial U_t^2}{\partial e_t} = \frac{-(1-\gamma_1)\eta L^1}{(1-\gamma_1)(w_t H_t + (1-e_t)\eta L^1) + \gamma_3 R_t K_t - s_t} + \beta\phi'_{t+1} \geq 0 \quad \text{for} \quad e_t \leq 1, \tag{8}$$

Solving (6) for s_t, we obtain

$$s_t = \frac{\beta}{1+\beta}\left((1-\gamma_1)w_t H_t + \gamma_3 R_t K_t\right). \tag{9}$$

Plugging this expression into (7) yields

$$\beta\phi'_{t+1} = \frac{(1+\beta)\eta L^1}{w_t\phi_t L^2 + (1+\beta)\eta(1-e_t)L^1 + \gamma R_t K_t}, \quad \text{with} \quad \gamma = \frac{\gamma_3}{1-\gamma_1} \tag{10}$$

if the expression holds with an equality sign. From Equation (10) we observe that investment in education depends on the marginal effect of education, given by ϕ'_{t+1}. Also, wealthier households, i.e households with a larger stock of human capital, $\phi_t L^2$ and ηL^1, and more capital income $R_t K_t$ are more likely to invest in human capital, as they suffer relatively less from the income loss associated with schooling. This loss is given by the numerator ηL^1.

Now we use the demand for human and physical capital to substitute w_t and R_t,

$$\beta\phi'_{t+1} = \frac{(1+\beta)\eta L^1}{Y_t^2((1-\alpha)+\alpha\gamma) + (1+\beta)Y_t^1},$$

so that wealthier countries, with larger Y_t^2, are more likely to invest in human capital and are more likely to grow. We can interpret this as "history matters" or as the potential occurrence of a poverty trap in which countries with low endowment of both types of capital will not invest in education. Also, if child labor is a relevant source of income and the ratio of children's productivity to output is large, investment in education is unlikely since the opportunity costs in terms of forgone output are too large. Next we discuss the role of institutions in this economy.

3 Institutions

When modeling institutions, we follow the approach of Acemoglu et al. (2004) that institutions shape the incentives to invest, with one way of doing so being the protection from expropriation.

We model the risk of expropriation by assuming a government in the economy, which impounds a constant share of capital returns in each period, so that households only receive $\psi R_t K_t$. To distinguish $1-\psi$ from a simple tax on capital income, we additionally assume that the expropriated income does not create any welfare. The government does not provide any public goods or corrects any market failures with it. Instead, one could imagine that $\psi R_t K_t$ increases the consumption of some government agents that form a vanishingly small share of the population that is negligible for total welfare. Put simply, the amount of goods $(1-\psi)R_t K_t$ is lost.[8]

With these assumptions, it is straightforward to study how institutions shape the outcome of the economy. Assume two different types of institutions, denoted by $\underline{\psi}$ and $\overline{\psi}$, with $\underline{\psi} < \overline{\psi}$. The set of institutions with higher quality has a larger value of ψ. Now, consider Equation (10) and multiply the term $R_t K_t$ with $\underline{\psi}$, so that the right hand side becomes

$$\frac{(1+\beta)\eta L^1}{w_t \phi_t L^2 + (1+\beta)\eta(1-e_t)L^1 + \underline{\psi}\gamma R_t K_t}.$$

With a low level of institutions, such as $\underline{\psi}$, the denominator on the right hand side of the equation is relatively small, leading to no education. Under higher quality institutions, i.e. when $\underline{\psi}$ is replaced by $\overline{\psi}$, the opposite might be the case. The increase in institutions also affects the incentive to invest in physical capital, as savings are given by

$$s_t = \frac{\beta}{1+\beta}\left[(1-\gamma_1)w_t H_t + \gamma_3 \overline{\psi} R_t K_t\right].$$

[8] Our approach is similar to Klein (2005).

4 International capital flows

As we pointed out above, better economic institutions can lead to investment in education and subsequently to growth. Yet, even with a high level of ψ, the marginal loss of schooling, which is given on the right hand side of (10), might be larger than the marginal benefit. One way to reverse that relationship is to decrease the marginal cost by providing the economy with a larger stock of physical capital K_t. Such an increase in K_t might stem from inflows of capital from another country. To model this possibility, we assume two countries indexed by $I \in \{A, B\}$, with their respective level of institutions ψ_I. Capital is internationally mobile, while labor is not. We allow for international investment and flow of goods. Also, we consider a fixed exchange regime, with an exchange rate of 1. By doing so, we abstract from nominal issues and focus on real variables.

At first, both countries are identical and have the same endowment of both types of capital and institutions. They find themselves in a zero growth steady state without capital flows, as the arbitrage condition

$$\psi_A R_t^A = \psi_B R_t^B \tag{11}$$

holds. We begin in a situation where neither of the two countries has invested in education, so that in the two countries the following holds:

$$\beta \phi'_{t+1} \leq \frac{(1+\beta)\eta L^1}{w_t \phi_t L^2 + (1+\beta)\eta L^1 + \psi \gamma R_t K_t}. \tag{12}$$

The right hand side of the inequality is positive, so that $e_t = 0$ holds as an equilibrium and the total stock of human capital in a country remains at its initial level. We omitted the country-specific superscript I, as both countries are identical. The stock of physical capital is fixed and is given by

$$K = \left(\frac{\beta A}{1+\beta} \left[(1-\gamma_1)(1-\alpha) + \alpha \psi \gamma_3 \right] \right)^{\frac{1}{1-\alpha}} H, \tag{13}$$

where H is the initial stock of human capital of adults in the economy. Hence, we define the zero growth steady state in the following way:

Definition 1. *In a zero growth steady state, international capital flows are absent and countries do not invest in education, i.e.* (11) *and* (12) *hold. Furthermore, the stock of physical capital is constant and given by* (13)*. Wages, interest rates, profits and output are constant and given by* $(1) - (4)$*.*

Next, we allow for heterogeneous institutions across countries.

Change in institutions

In period t, before agents have decided about education, Institutions in country A improve from ψ^A to $\widetilde{\psi}^A$, so that we have $\widetilde{\psi}_A R_t^A > \psi_B R_t^B$. Hence, capital flows from country B to A until returns are equalized again, affecting the education decision.

The exact sequence of events is shown in Figure 4. Initially, we have a positive

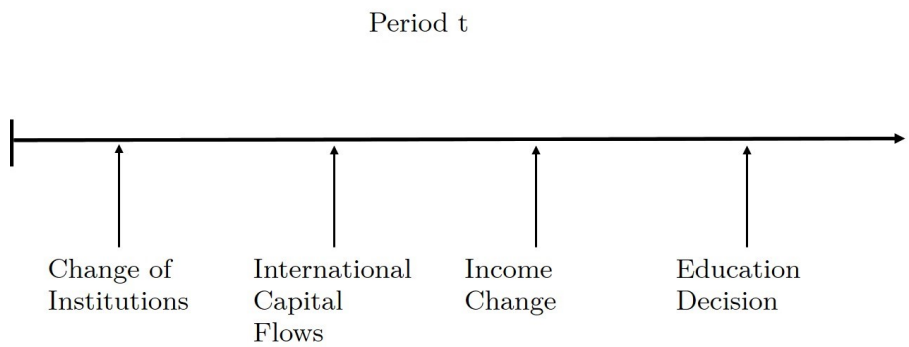

Figure 4: Sequence of events.

change in the quality of institutions in country A. The improvement leads to an inflow of capital and a change in income. As this is the income before adults decide whether children should receive a positive amount of schooling, we call this income their pre-education income. If agents decide to school children, the total supply of human capital, given by $H_t + (1 - e_t)\eta L^1$, decreases.

5 The full model

Given that capital markets pay the marginal return of capital we have

$$\widetilde{\psi}^A(\widetilde{K}_t^A)^{\alpha-1} = \psi^B(\widetilde{K}_t^B)^{\alpha-1} \Leftrightarrow \widetilde{K}_t^A = \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}} \widetilde{K}_t^B, \tag{14}$$

where $\widetilde{K}_t^A$ and $\widetilde{K}_t^B$ are the capital stocks after capital movements from B to A. It holds that $\widetilde{K}_t^B = K_t^B - (\widetilde{K}_t^A - K_t^A)$, where we assume that initially, $K_t^B = K_t^A$, and where we use the result for $\widetilde{K}_t^A$ to obtain

$$\widetilde{K}_t^B = 2K_t^B - \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}} \widetilde{K}_t^B \Leftrightarrow \widetilde{K}_t^B = 2\left(1 + \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{-1} K_t^B.$$

Intuitively, the new capital stock $\widetilde{K}_t^B$ depends on the ratio of institutions. The number 2 on the right hand side stems from the assumption that initial capital stocks are equal and the factor with exponent -1 shows how capital is allocated between countries. Using (14), we obtain

$$\widetilde{K}_t^A = 2\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}} \left(1 + \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{-1} K_t^A$$

for the new capital stock in A. One of the factors on the right hand side is increasing in $\widetilde{\psi}_A$, while the other is decreasing. The increasing part stems from the arbitrage condition which yields that better economic institutions in A must increase the capital stock in A compared to the one in B. The decreasing part stems from the fact that better economic institutions in A mean that the final capital stock in B is already smaller than it initially was, leaving less capital to flow to A. These two forces offset each other in the limit, as

$$\lim_{\widetilde{\psi}_A \to \infty} \widetilde{K}_t^A = 2K_t^A,$$

which is necessary, as the total initial endowment of capital is $2K_t^A$. Of course, $\widetilde{K}_t^A$ is strictly increasing in the relative level of institutions, as one can see by slightly

rewriting the expression above,

$$\widetilde{K}_t^A = 2\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{-1}{1-\alpha}}\right)^{-1} K_t^A.$$

Note the flow of capital does not lead to a change in ownership of capital. The capital going to B to A still belongs to agents in B even if it is productive in A. This oberservation will be crucial for determining how these flows affect income in B.

Income change

So far, a change in institutions triggers international capital flows. These flows impact the income of households in both countries. For country A, we observe the following: Before capital flows, its capital income was given by $\psi_A R_t^A K_t^A$, after capital flows, it is given by $\widetilde{\psi}_A \widetilde{R}_t^A \widetilde{K}_t^A$ and we form the ratio of both

$$\frac{\widetilde{\psi}_A \widetilde{R}_t^A \widetilde{K}_t^A}{\psi_A R_t^A K_t^A} = \frac{\psi_B}{\psi_A} 2^{\alpha-1}\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}.$$

This term can be either larger or smaller than one. Its size depends on three things: First, on the improvement in institutions vis-à-vis the status quo ψ_A, second, on the improvement relative to the institutions of country ψ_B and third, on the initial allocation of capital, which is now represented by the number 2. These three issues decide whether the improvement of institutions is so strong that the net return rises although the marginal return to capital decreases in light of capital inflows. Under our initial assumption that $\psi_B = \psi_A$, the income ratio is 1 if no change occurs, i.e. $\widetilde{\psi}_A = \psi_B$. If $\widetilde{\psi}_A > \psi_B$ then the income from capital increases above 1, as the factor on the right hand side is a strictly increasing function in the ratio of institutions $\frac{\widetilde{\psi}_A}{\psi_B}$.

Now let us study the relative change in labor income of adults, given by

$$\frac{\widetilde{w}_t^A H_t^A}{w_t^A H_t^A} = \left(\frac{\widetilde{K}_t^A}{K_t^A}\right)^{\alpha} = 2^{\alpha}\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{-1}{1-\alpha}}\right)^{-\alpha}.$$

We use $\widetilde{w}_t^A$ to denote the wage after capital flows and recall that there is no positive

level of education yet. We see that the ratio is 1 for $\widetilde{\psi}_A = \psi_B$. For $\widetilde{\psi}_A > \psi_B$, it is larger than one because the expression on the right hand side is increasing in $\widetilde{\psi}_A/\psi_B$, as we have shown above.

Now let us turn to the income of agents in the capital-exporting country B. The capital income of agents in B is the same as in A, and thus increases. To see, recall that the capital that has moved from B to A still belongs to residents in B. The wage income, however, decreases as the total local capital stock used in production in B becomes smaller. This is reflected in the following ratio:

$$\frac{\widetilde{w}_t^B H_t^B}{w_t^B H_t^B} = \left(\frac{\widetilde{K}_t^B}{K_t^B}\right)^{\alpha} = 2^{\alpha}\left(1 + \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{-\alpha},$$

which is smaller than 1. With capital and wage income moving in opposite directions, we study the net effect on income in B. The disposable income of adults without earnings of their children, $I_t^B = (1-\gamma_1)\widetilde{w}_t^B H_t^B + \psi_B \gamma_3 \widetilde{R}_t^B K_t^B$, can be written as

$$I_t^B = A(H_t^B)^{1-\alpha}(\widetilde{K}_t^B)^{\alpha}\left((1-\alpha)(1-\gamma_1) + \alpha\psi_B\gamma_3(\widetilde{K}_t^B)^{-1}K_t^B\right),$$

or after substituting $\widetilde{K}_t^B$

$$\begin{aligned} I_t^B =& A(H_t^B)^{1-\alpha}(K_t^B)^{\alpha}2^{\alpha}\left(1 + \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{-\alpha} \cdot \\ & \left((1-\alpha)(1-\gamma_1) + \frac{\alpha}{2}\psi_B\gamma_3\left(1 + \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)\right). \end{aligned}$$

Taking the derivative of I_t^B with respect to $\widetilde{\psi}_A$ provides the first proposition.

Proposition 1. *Given an initially symmetric allocation of capital between country A and B, the income in B decreases after capital flows if and only if*

$$\frac{\psi_B\gamma}{2} < \left(1 + \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{-1}. \tag{15}$$

The pre-education income in country A increases unambiguously.

We provide a short proof in the Appendix and in the following we will assume that (15) holds and that income in B decreased. Proposition 1 implies that the three factors, we encountered before, determine the change in income. First, the relative increase in institutions is crucial, as can be seen from the right hand side. Second, the initial level of institutions, represented by ψ_B on the right hand side, is also important. Third, the number 2, on the left hand side of the inequality, shows that the initial allocation of capital determines the outcome as well. If we did not assume that the initial stocks of capital are equal, the number 2 would be replaced by the inverse of $K_t^B/(K_t^B + K_t^A)$.

To clarify, let us consider two examples. In the first, country B is relatively rich in capital and has good institutions, i.e. ψ_B is relatively large. If country A strongly improves its institutions with $\widetilde{\psi}_A$ significantly larger than ψ_B, then the income in country B actually increases. Country B loses some wage income, though, because it is rich in capital and can reap the benefits of investment. It is more than compensated by the larger return on capital and larger capital income. In the second example, Country B has a low share in world capital and comparatively weak institutions. If, in this case, country A improves its institutions slightly, country B will actually incur a decrease in income. For a more general discussion of the income change in B see the Appendix.

Finally, we provide the income net of the earnings of children in A,

$$I_t^A = A(H_t^A)^{1-\alpha}(K_t^A)^{\alpha}\Gamma_t^A \quad \text{with} \tag{16}$$

$$\Gamma_t^A = 2^{\alpha}\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{-1}{1-\alpha}}\right)^{-\alpha}\left((1-\gamma_1)(1-\alpha)+\frac{\alpha\widetilde{\psi}_A\gamma_3}{2}\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{-1}{1-\alpha}}\right)\right). \tag{17}$$

We have established that wage and capital income in country A increase and consequently lead to an increase of income. With larger income, savings in A also increase.

Education decision

A change in income might alter the decision about schooling. To see this, consider Equation (12), where we assumed that before capital flows, neither country invests

in education. The marginal benefits on the left hand side of the inequality are equal to or smaller than the marginal income loss at zero education. The marginal loss will be even larger if the country's income has decreased. This is what has happened in B by our assumption earlier. Thus, we are certain that country B does not invest into education after the change in institutions in A.
We have seen that income increases in A. It enters the denominator of (12) allowing the right hand side to be actually lower than $\phi'_{t+1}(e_t = 0)$. If we additionally assume that the marginal benefits of education are constant, there exists a value for $e_t^* \in (0, 1]$ such that

$$\beta\phi'_{t+1}(e_t^*) = \frac{(1+\beta)(1-\gamma_1)\eta L^1}{A(\phi_t L^2)^{1-\alpha} K_t^\alpha \Gamma_t^A + (1+\beta)(1-\gamma_1)(1-e_t^*)\eta L^1} \tag{18}$$

is fulfilled. Writing the ratio $\frac{\widetilde{\psi}_A}{\psi_B}$ as χ, we note that Γ_t^A increases in χ. To see this, recall the expression for I_t^A from above. It is written as the product of income before capital flows, $A(H_t^A)^{1-\alpha}(K_t^A)^\alpha$, times the factor Γ_t^A. This factor must be larger than one, as we have shown that income in A does increase due to capital inflows. Also, we have shown that both components of income increase in χ, so that Γ_t^A must too. Alternatively, we can rewrite Γ_t^A as

$$2^\alpha(1-\alpha)(1-\gamma_1)(1+\chi^{\frac{-1}{1-\alpha}})^{-\alpha} + \alpha 2^{\alpha-1}\widetilde{\psi}_A\gamma_3(1+\chi^{\frac{-1}{1-\alpha}})^{1-\alpha},$$

noting that $\widetilde{\psi}_A = \chi\psi_B$. We see that both summands increase in χ. For further analysis, it is useful to study whether households might choose a level of e_t that lowers the family income below its initial level, i.e. the level that prevailed before the inflow of capital. This can occur, as investment in education reduces the supply of human capital and also the earnings of children. We find the following:

Proposition 2. *Under the types of education function that allow for a zero growth steady state, i.e. those with constant, decreasing or sufficiently slowly increasing returns to schooling, the country that experiences capital inflows will have a larger income than before. This even holds when agents in that country sacrifice some income in order to school their children.*

Next we study the set of optimal e_t^*.We find a sufficient condition under which, given the capital inflows, no level of education reduces the income of adults below

the level of the beginning of the period. This condition is

$$\frac{2^{\alpha}\left(1+\chi^{\frac{-1}{1-\alpha}}\right)^{-\alpha}\left[(1-\alpha)(1-\gamma_1)+\alpha\widetilde{\psi}_A\gamma_3(1+\chi^{\frac{-1}{1-\alpha}})\right]}{(1-\alpha)(1-\gamma_1)+\alpha\widetilde{\psi}_A\gamma_3} \geq \frac{(1-\gamma_1)\eta L^1}{(H_t^A)^{1-\alpha}(K_t^A)^{\alpha}}. \tag{19}$$

If it is fulfilled, then e_t can take any value in $(0,1]$, depending on the formation of human capital, given by $\phi'_{t+1}(e_t)$. If it is not fulfilled, then there exists a $\overline{e}_t$, such that no value for $e_t \in [\overline{e}_t, 1]$ is optimal, as otherwise, income is reduced below its initial level. This holds independently of $\phi_{t+1}(e_t)$. Whether this condition is fulfilled depends on the relative importance of the adult labor force and the size of capital inflows, and thus on the improvement of institutions. A country that improves its institutions dramatically will attract a sizable amount of capital and thus be able to choose from a variety of education levels. For a country that improves its institutions marginally, we observe relatively low levels of education being implemented, if any. We summarize

Lemma 1. *If Expression* (19) *is fulfilled the upper bound for the optimal level of* e_t *is 1.*

Next period

Now, we will study the impact of institutional change, capital flows and schooling on the two countries in the next period. Above, we established an equilibrium in which capital flows in every period from B to A. In this equilibrium, B does not invest in education while country A does, allocating the share e_t of its children's time to the formation of human capital. Hence, in $t+1$, country A is endowed with a larger stock of total human capital, given by $\phi_{t+1}L^{2,A} + \eta L^{1,A}$. Also, as shown above, income in A is larger than before. This implies

$$s_t^A > s_{t-1}^A \Rightarrow K_{t+1}^A > K_t^A \quad \text{and} \quad s_t^B < s_{t-1}^B \Rightarrow K_{t+1}^B < K_t^B,$$

as savings are a constant share of income, due to logarithmic utility. Given smaller savings and physical capital, but a constant amount of human capital, the rental rate of capital increases in B. In A, we observe an increase in both types of capital, having opposite effects on the interest rate. Hence, it is not clear whether R_{t+1}^A is smaller, equal or larger than R_{t+1}^B.

Now the following holds:

$$R^A_{t+1} = \alpha A(\phi_{t+1}L^2)^{1-\alpha}(K^A_{t+1})^{\alpha-1} \quad \text{and} \quad R^B_{t+1} = \alpha A(\phi_t L^2)^{1-\alpha}(K^B_{t+1})^{\alpha-1}, \quad \text{with}$$

$$K^A_{t+1} = \frac{\beta A}{1+\beta}\left(H^A_t\right)^{1-\alpha}\left(K^A_t\right)^{\alpha} 2^{\alpha}(1+\chi^{\frac{-1}{1-\alpha}})^{-\alpha}\left((1-\alpha)(1-\gamma_1) + \frac{\alpha}{2}\widetilde{\psi}_A\gamma_3(1+\chi^{\frac{-1}{1-\alpha}})\right),$$

and

$$K^B_{t+1} = \frac{\beta A}{1+\beta}(H^B_t)^{1-\alpha}(K^B_t)^{\alpha}2^{\alpha}(1+\chi^{\frac{1}{1-\alpha}})^{-\alpha}\left((1-\alpha)(1-\gamma_1) + \frac{\alpha}{2}\psi_B\gamma_3\left(1+\chi^{\frac{1}{1-\alpha}}\right)\right) \quad .$$

Hence, $\widetilde{\psi}_A R^A_{t+1}$ is larger than $\psi_B R^B_{t+1}$ if

$$\chi\left(\frac{H^A_{t+1}}{K^A_{t+1}}\right)^{1-\alpha}\left(\frac{K^B_{t+1}}{H^B_{t+1}}\right)^{1-\alpha} > 1.$$

We substitute K^A_{t+1} and K^B_{t+1} to obtain:

$$\left(\chi\frac{\gamma_B}{\gamma_A}\right)^{1-\alpha}\left(\frac{H^A_{t+1}}{(H^A_t)^{1-\alpha}(K^A_t)^{\alpha}}\right)^{1-\alpha}\left(\frac{H^B_{t+1}}{(H^B_t)^{1-\alpha}(K^B_t)^{\alpha}}\right)^{\alpha-1} > 1, \tag{20}$$

$$\gamma_A = (1-\alpha)(1-\gamma_1) + \frac{\alpha}{2}\widetilde{\psi}_A\gamma_3\left(1+\chi^{\frac{-1}{1-\alpha}}\right), \quad \text{and} \tag{21}$$

$$\gamma_B = (1-\alpha)(1-\gamma_1) + \frac{\alpha}{2}\psi_B\gamma_3\left(1+\chi^{\frac{1}{1-\alpha}}\right), \tag{22}$$

where γ_B and γ_A can be interpreted as the sum of income shares. Labor income of adults has the same share in both countries, $1-\alpha$, while capital has different shares in A and B. So whether the net interest rate in country A is also larger in period $t+1$ depends on the ratio of institutions, as is shown by χ and by the ratio of γ_B and γ_A. Human capital in $t+1$ in country A affects the interest rate positively, while the opposite holds for human capital in t, as it leads to more income in t. The same argument holds for the stock of physical capital in that period.

We can rewrite the above expression, using that $K^A_t = K^B_t$ and $H^B_{t+1} = H^B_t$, where we assumed the former and have shown that there is no investment in education

in B due to decreasing income. This yields

$$\left(\chi\frac{\gamma_B}{\gamma_A}\right)^{1-\alpha}\left(\frac{H_{t+1}^A}{H_t^A}\right)^{1-\alpha}\left(\frac{H_t^B}{H_t^A}\right)^{\alpha(\alpha-1)} > 1. \tag{23}$$

Turning to the term $\chi\gamma_B/\gamma_A$, we know that $\chi > 1$ and $\gamma_B > \gamma_A$, so that the product is larger than one. To see this, note that the following holds:

$$\frac{\widetilde{\psi}_A}{\psi_B} < \chi^{\frac{1}{1-\alpha}} = \left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}} \quad \text{for all} \quad \widetilde{\psi}_A > \psi_B.$$

Next, we turn to the factor in the middle, H_{t+1}^A/H_t^A. This ratio of human capital of adults in $t+1$ to the amount of human capital of adults in t is larger than one, as the human capital stock of tomorrow will be larger than the one of today. The exact size of this term depends on the productivity of schooling. Hence, the ratio reflects the way in which education in country A in period t can ensure capital inflows in period $t+1$: Education increases H_{t+1}^A, allowing for a larger R_{t+1}^A. The last factor depends negatively on H_t^B so that a larger stock of human capital in B reduces the chances of outflow of capital in $t+1$.

In $t+1$, the capital market clears if the following condition is met:

$$\widetilde{\psi}_A\left(\frac{H_{t+1}^A}{\widetilde{K}_{t+1}^A}\right)^{1-\alpha} = \psi_B\left(\frac{H_{t+1}^B}{\widetilde{K}_{t+1}^B}\right)^{1-\alpha} \quad \Rightarrow \quad \widetilde{K}_{t+1}^A = \chi^{\frac{1}{1-\alpha}}\left(\frac{H_{t+1}^A}{H_t^B}\right)\widetilde{K}_{t+1}^B.$$

The market clearing condition has changed, now including the ratio of two different levels of human capital. We denote the ratio H_t^A/H_t^B as H_t from now on. As country A has invested in education, we observe $H_{t+1}^A > H_t^B$. Country B has not invested, remaining at the level of human capital H_t^B. While the difference in institutions is still relevant, as shown by χ, the new ratio of human capital $H_{t+1} = \frac{H_{t+1}^A}{H_t^B}$ increases the difference between capital stocks in the two countries.

To solve for the post-flow capital stocks $\widetilde{K}_{t+1}^A$ and $\widetilde{K}_{t+1}^B$, we note two things. First, from the savings decision above, we know that

$$K_{t+1}^A = K_{t+1}^B\frac{\gamma_A}{\gamma_B}\chi^{\frac{\alpha}{1-\alpha}}H_t^{1-\alpha} \quad \text{wih} \quad H_t = \frac{H_t^A}{H_t^B}. \tag{24}$$

Second, the international sum of physical capital after flows must equal the sum

of physical capital after flows, implying

$$\widetilde{K}^B_{t+1} = K^A_{t+1} + K^B_{t+1} - \widetilde{K}^A_{t+1}.$$

We substitute the market clearing condition and (24) to obtain

$$\widetilde{K}^B_{t+1} = \left(1 + \frac{\gamma_A}{\gamma_B}\chi^{\frac{\alpha}{1-\alpha}}\right)\left(1 + \chi^{\frac{1}{1-\alpha}}H_{t+1}\right)^{-1} K^B_{t+1} \quad \text{and}$$
$$\widetilde{K}^A_{t+1} = \left(1 + \frac{\gamma_B}{\gamma_A}\chi^{\frac{-\alpha}{1-\alpha}}\right)\left(1 + \chi^{\frac{-1}{1-\alpha}}H^{-1}_{t+1}\right)^{-1} K^A_{t+1},$$

where we use that $H_t = 1$. In contrast to the equation that related $\widetilde{K}^B$ and K^B_t, the number 2 is replaced by the left factor. This factor is smaller than 2 and thus shows that, in period $t+1$, country B has a smaller share of the global capital stock than in t. This is due to $\gamma_A/\gamma_B < 1$ and the fact that $H_{t+1} > 1$. While this implies that $\widetilde{K}^B_{t+1} < K^B_{t+1}$, it is not clear whether $\widetilde{K}^B_{t+1}/K^B_{t+1}$ is larger or smaller than $\widetilde{K}^B_t/K^B_t$. Let us also consider the education decision for period $t+1$. In equilibrium, the optimal amount of schooling e^*_{t+1} must fulfill,

$$\beta\phi'_{t+2}(e^*_{t+1}) = \frac{(1+\beta)\eta L^1}{A(\phi_{t+1}L^2)^{1-\alpha}(K^A_{t+1})^\alpha\Gamma^A_{t+1} + (1+\beta)(1-\gamma_1)(1-e^*_{t+1})\eta L^1}, \tag{25}$$

with

$$\Gamma^A_{t+1} = (1-\gamma_1)\left(1 + \frac{\gamma_B}{\gamma_A}\chi^{\frac{-\alpha}{1-\alpha}}\right)^\alpha \left(1 + \chi^{\frac{1}{1-\alpha}}H_{t+1}\right)^{-\alpha} \chi^{\frac{\alpha}{1-\alpha}}(H^A_{t+1})^\alpha \cdot$$
$$\left[1 - \alpha + \alpha\widetilde{\psi}^A_t\gamma\left(1 + \frac{\gamma_B}{\gamma_A}\chi^{\frac{-\alpha}{1-\alpha}}\right)^{-1}\left(1 + \chi^{\frac{1}{1-\alpha}}H^A_{t+1}\right)\chi^{\frac{-1}{1-\alpha}}(H_{t+1})^{-1}\right], \tag{26}$$

where the total endowment with both types of capital in A has increased, i.e. $\phi_t < \phi_{t+1}$ and $K^A_t < K^A_{t+1}$.

Next, we turn to Γ^A_{t+1} which is structurally similar to Γ^A_t. However, the factor 2 is replaced by $1 + \frac{\gamma_B}{\gamma_A}\chi^{\frac{-\alpha}{1-\alpha}}$, which is smaller than 2, and the term $\chi^{\frac{1}{1-\alpha}}$ is replaced by $\chi^{\frac{1}{1-\alpha}}H_{t+1}$, with $H_{t+1} > 1$. This makes the derivative $\partial\Gamma^A_{t+1}/\partial\chi$ not-trivial, as the terms γ_B, γ_A, and H_{t+1} all depend on χ. We cannot say whether income in A, in period $t+1$ before children are educated, is actually larger than the one in period t. However, a condition, allowing for such a case, can be found in the following

way: First, we write income without the earnings of children in its general form,

$$I_{t+1}^A = (1-\alpha)(1-\gamma_1)A(H_{t+1}^A)^{1-\alpha}(\widetilde{K}_{t+1}^A)^{\alpha} + \alpha\widetilde{\psi}_A\gamma_3 A(H_{t+1}^A)^{1-\alpha}(\widetilde{K}_{t+1}^A)^{\alpha-1}K_{t+1}^A.$$

This expression holds for $t+1$ and t. It is clear that I_{t+1} increases in H_{t+1}^A and K_{t+1}^A. Forming the derivative with respect to $\widetilde{K}_{t+1}^A$, we find

$$\frac{\partial I_{t+1}^A}{\partial \widetilde{K}_{t+1}^A} = \alpha A\left(\widetilde{K}_{t+1}^A\right)^{\alpha-1}\left(H_{t+1}^A\right)^{1-\alpha}\left[(1-\alpha)(1-\gamma_1) + (\alpha-1)\widetilde{\psi}_A\gamma_3\frac{K_{t+1}^A}{\widetilde{K}_{t+1}^A}\right] > 0,$$

as $\widetilde{\psi}_A < 1$, $K_t^A/\widetilde{K}_t^A < 1$ due to capital inflows and that γ_3 is unlikely too different from $1-\gamma_1$. We know that $H_t^A < H_{t+1}^A$ and that $K_t^A < K_{t+1}^A$. However, it is not clear whether $\widetilde{K}_t^A < \widetilde{K}_{t+1}^A$. $\widetilde{K}_{t+1}^A$ can be written as a share of the total supply of physical capital K_{t+1}

$$\widetilde{K}_{t+1}^A = \left(1 + \frac{1}{\chi^{\frac{1}{1-\alpha}}H_{t+1}}\right)^{-1} K_{t+1}. \tag{27}$$

Country A obtains a larger share of total world capital. Yet, we have not established whether the global total stock of physical capital has increased, i.e. whether $K_t < K_{t+1}$. Studying the marginal changes of income in A and B we find that world income increases in t if the following condition holds:

$$\frac{\gamma_3}{1-\gamma_1}\widetilde{\psi}_A > \frac{\chi-1}{1+\chi^{\frac{1}{1-\alpha}}}. \tag{28}$$

For a proof, see the Appendix. This inequality provides a lower bar for the value of $\widetilde{\psi}_A$. However, it is easily fulfilled if we consider small changes in institutions, so that χ is relatively close to 1.

With this, country A is richer after capital flows in $t+1$ than after flows in t, leading to a higher optimal level of education e_{t+1}^*. Hence, the stock of human capital will increase more strongly from $t+1$ to $t+2$ than it did from t to $t+1$. We summarize in the following proposition:

Proposition 3. *If Inequality* (28) *holds, then world income increases in t. This is a sufficient condition for country A to increase its level of education in $t+1$.*

More general discussion

So far, we have seen that in period t and $t+1$, two relevant things occur. First, capital flows from country B to A and second, A invests in education, and does it in $t+1$ even more than in t. We study now, whether this can also happen in the following periods, i.e. whether there is a path where two conditions are met in each period. First, country A experiences capital inflows and second, it increases its level of education e_t, so that it converges to $e_T^* = 1$, in some period T. The first condition can only be met if

$$\chi^{\frac{1}{1-\alpha}} \left(\frac{H_{t+2}^A}{H_{t+2}^B} \right) \frac{\Gamma_{t+1}^B \left(H_t^B\right)^{1-\alpha} (K_{t+1}^B)^\alpha}{\Gamma_{t+1}^A \left(H_{t+1}^A\right)^{1-\alpha} (K_{t+1}^A)^\alpha} > 1 \tag{29}$$

is fulfilled in $t+2$ and in every following period. For this condition to hold, it is important for country A to accumulate human capital sufficiently quickly, as K_{t+1}^A is increasing due to country A's larger wealth and K_{t+1}^B is decreasing. Also the factors Γ_{t+1}^A and Γ_{t+1}^B diverge, equalizing interest rates across the two countries.

A sufficient condition for country A to increase its education level is $1 < \Gamma_t^A < \Gamma_{t+1}^A$. It can be generalized for the following periods. If $\Gamma_{t+1}^A < \Gamma_{t+2}^A$ and so on, then $e_{t+1}^* < e_{t+2}^*$ will hold, as can be seen from (25). The factor Γ_{t+1}^A increases over time if $\widetilde{K}_{t+1}^A$ increases, which in turn, becomes larger over time if income rises more quickly in A than income decreases in B, in a given period. This is expressed by the following condition:

$$\begin{aligned} &(H_{t+1}^A)^{1-\alpha}(K_{t+1}^A)^\alpha \Gamma_{t+1}^A - (H_{t+1}^A)^{1-\alpha}(K_{t+1}^A)^\alpha \Gamma_{t+1}^{A,0} \geq \\ &\left|(H_{t+1}^B)^{1-\alpha}(K_{t+1}^B)^\alpha \Gamma_{t+1}^{B,0} - (H_{t+1}^B)^{1-\alpha}(K_{t+1}^B)^\alpha \Gamma_{t+1}^B\right|, \end{aligned} \tag{30}$$

with $\Gamma_{t+1}^{I,0} = (1-\alpha)(1-\gamma_1) + \alpha\gamma_3\psi_I$. Slightly rewritten, we have

$$\frac{Y_{t+1}^{2,A}}{Y_{t+1}^{2,B}} \left(\Gamma_{t+1}^A - \Gamma_{t+1}^{A,0}\right) > \left|\Gamma_{t+1}^{B,0} - \Gamma_{t+1}^B\right|, \tag{31}$$

which is a generalization of (28). $Y_{t+1}^{2,A}$ is country A's potential output before capital flows and education. We see that it is beneficial for A if country B is relatively poor, i.e. $Y_{t+1}^{2,B}$ is rather small. We summarize our findings in the following proposition:

Proposition 4. *If returns to education are non-increasing or increase sufficiently*

slowly and in every period (29) *and* (31) *are fulfilled, there exists a path for country A along which it experiences inflows of capital in every period and increases its level of education.*

What happens, however, if (29) does not hold? To answer this question, let us consider the following. From period $t = 0$ until some period $\widetilde{T}$, capital has been flowing from B to A, and A has been investing in education, without reaching the state of full education, so that $e_{\widetilde{T}-1} < 1$. Let us first assume, that in $\widetilde{T}$, the return on capital is equal in both countries, so that there are no capital flows in this period. If capital inflows matter for country A, it might be the case that $\widetilde{K}^A_{\widetilde{T}-1} > \widetilde{K}^A_{\widetilde{T}}$, i.e. country A had more capital available for production in the previous period than in the current one, due to previous capital inflows. This might reduce the incentive to invest in education in period $\widetilde{T}$ in comparison to $\widetilde{T}-1$. However, we can exclude the possibility that $e_{\widetilde{T}}$ drops back to zero, as country A has been accumulating human capital over the previous periods, which encourages investment in education. Also, human capital is larger in the current period than in $\widetilde{T}-1$, hindering any conclusion about the relative size of $e_{\widetilde{T}}$ compared to $e_{\widetilde{T}-1}$. If capital inflows are negligible in comparison to country A's capital stock, such considerations do not matter. As before, country A will follow its path, increasing its stock of human capital. The opposite will hold in B. The country does not invest in education, as it is poorer in $\widetilde{T}$ than in the initial period t, where it already did not invest in education. Taken together, these effects enable (29) to hold as an inequality.

Now assume, that the return in B is actually larger than in A and that capital flows from A to B. It is more likely now that A invests less in education than in the previous period with $e_{\widetilde{T}} < e_{\widetilde{T}-1}$, but it is quite unlikely that A will completely stop investing, as the accumulation of human capital and physical capital has been increasing income. If inflows in B are large, providing B with more capital than it initially had, so that $K^B_t < \widetilde{K}^B_{\widetilde{T}}$, B might begin to invest in education. Even if it does, it will most likely invest less than country A, so that interest rates will equalize in the following period. However, by investing in education, B might also converge to the full education steady state.

Steady state

Let us turn to the existence of a balanced growth path. While the economy can exhibit different steady states, we concentrate on the steady state in which country

A grows at the largest possible positive rate while country B does not. We define such a state as follows:

Definition 2. *Along a balanced growth path, human capital and physical capital grow at the same rate. In country, A this rate is positive and as large as possible. In B, it is zero. In the long run, the stocks of human capital and physical capital become so large that agents in A neglect the effect of capital inflows from B and the stock of children's human capital in their decisions.*

With this definition, we obtain for country A

$$\frac{K^A_{t+1}}{H^A_{t+1}} = k^A_{t+1} = \frac{\beta A((1-\alpha)(1-\gamma_1) + \alpha\gamma_3\widetilde{\psi}_A)}{(1+\beta)\Phi}(k^A_t)^\alpha \quad \text{with} \quad \Phi = \frac{\phi_{t+1}(1)}{\phi_t(1)} \quad \forall t. \tag{32}$$

The constant ratio of the two forms of capital is $k^A = [\beta A((1-\alpha)(1-\gamma_1)+\alpha\gamma_3)/(1+\beta)\Phi]^{1/(1-\alpha)}$. For the derivation of this equation, see the Appendix. Additionally, we assume that the steady state defined above was reached along a transition path, where country B never invested in education. This might be due to country B's rental rate always being smaller than A's, i.e. Inequality (29) always held along the convergence, or country B experienced capital inflows which did not stimulate income sufficiently to ensure investment in education. Regardless of the transition path, we show that there exists a steady state where country A invests in education and country B does not.

Such a steady state cannot be one without capital flows, as country B has been exporting capital and has seen a continuous reduction in income along the assumed path. Hence, in some arbitrary period T, where A is already in its steady state, B faces a capital stock that is smaller than its initial endowment. With K^B_T being smaller than K^B_t and with a policy function for K^B_t that is concave, as can be seen from the savings decision, country B will accumulate capital in the absence of international capital flows. This is not consistent with a steady state, and it is not clear how this accumulation would affect the overall dynamics either.

Instead, we demonstrate the existences of a steady state with the following features. At the beginning of every period, the market clearing condition is not fulfilled, so that a time-independent amount of capital flows from B to A, clearing the arbitrage condition. In every period, country B possesses the same amount of capital K^B,

of which a constant fraction flows out. Furthermore, country B's stock of capital used in production is also constant, i.e. $\widetilde{K}^B$ is fixed. Denoting the steady state ratio of capital in A by k^A, the rental rate in A is given by $A(k^A)^{\alpha-1}$, which we write as R^A. Hence, K^B and $\widetilde{K}^B$ must fulfill

$$\widetilde{\psi}_A R^A > \psi_B \left(\frac{H^B}{K^B}\right)^{1-\alpha}, \quad \widetilde{\psi}_A R^A = \psi_B \left(\frac{H^B}{\widetilde{K}^B}\right)^{1-\alpha}, \quad \text{and}$$

$$K^B = \frac{\beta A}{1+\beta}\left[(1-\alpha)(1-\gamma_1)\left(H^B\right)^{1-\alpha}\left(\widetilde{K}^B\right)^{\alpha} + \alpha\gamma_3\widetilde{\psi}_A R^A K^B\right].$$

We solve for

$$\widetilde{K}^B = H^B\left(\chi R^A\right)^{\frac{-1}{1-\alpha}}$$

and substitute this expression to obtain

$$K^B = \frac{\beta A}{1+\beta}\left[(1-\alpha)(1-\gamma_1)H^B\left(\chi R^A\right)^{\frac{-\alpha}{1-\alpha}}\right]\left(1-\frac{\alpha\beta\gamma_3\widetilde{\psi}_A A}{1+\beta}R^A\right)^{-1}. \tag{33}$$

The term in the second factor on the right hand side is very likely to be positive, unless R^A is very large, for which we do not see any reason. Next, we verify whether K^B is such that the capital market clearing condition is violated at the beginning of each period, leading to

$$\frac{1}{\alpha\gamma_3\widetilde{\psi}_A} > R^A > \frac{1}{\left[(1-\alpha)(1-\gamma_1)\frac{\widetilde{\psi}_A}{\psi_B} + \alpha\gamma_3\widetilde{\psi}_A\right]}. \tag{34}$$

We find a lower and an upper bound on the long-term interest rate R^A, where the upper bound comes from the condition $K^B > 0$ and the lower bound can be found by plugging (33) into $\widetilde{\psi}_A R^A > (H^B)^{1-\alpha}(K^B)^{\alpha-1}$. The lower bound only differs with respect to the term $(1-\alpha)(1-\gamma_1)\frac{\widetilde{\psi}_A}{\psi_B}$ from the upper bound, so that the range of admissible values is not huge. We summarize in the following proposition:

Proposition 5. *A steady state as described in Definition* 2 *exists if the long-term interest rate in country A, R^A, lies between the boundaries given by* (34).

With two countries, it is not clear whether the two arrive at their respective steady state simultaneously or whether, for instance, A reaches k^A before country B

reaches K^B. One can imagine that at some T, A is in its respective steady state while B is not, i.e. $K_T^B > K^B$. Then, we know from above that capital outflows will reduce overall income in B, leading to smaller savings, so that eventually $K_t^B = K^B$. The opposite, namely that B reaches its steady state before A, is not possible, however. The reason is that the capital stock in B depends on the return in A, R^A. If R^A, and hence the effective capital stock in A, k_t^A, changes, this will have an impact on the capital stock in B. Let $K_T^B = K^B$ for some T, but $k_T^A \neq k^A$ and $R_T^A \neq R^A$. The return in A will change from period T to $T+1$, thus moving K_T^B away from K^B.

6 Simulation

To study whether we can actually obtain two diverging countries, we provide a simulation exercise. Our choice of parameters can be found in Table 1. We set α and β to the standard values of 1/3 and 0.85. The institutional parameters ψ_B and $\widetilde{\psi}_A$ take values of 0.85 and 0.95, to study a change in institutions that is consequential. ηL^1 is set smaller than $\phi_0 L^2$, reflecting that adults can supply more human capital to markets. Similar to Bell et al. (2019), we set $\gamma_1 = 0.4$ and $\gamma_3 = 0.25$. Finally, the initial level of human capital per adult ϕ_0 is set to 8. The reason is that this value allows for a an initial zero-growth steady state, where both countries find it optimal not to invest in education at all. Also, with this level of human capital, a relatively small change in the level of institution allows for divergence of the two countries.

We use the following production function for the formation of human capital:

$$\phi_{t+1} = (1 + \delta \log(1 + e_t))\phi_t. \tag{35}$$

Hence, we have decreasing returns to e_t, but linearity in ϕ_t, allowing for a balanced growth path with $e = 1$. We set δ to 0.03, which leads to a growth rate of 2.05% along the path which is in line with estimates for the United States but also developping countries such as Kenya.[9]

The left upper panel of Figure 5 contains the main output of the simulation for

[9]For the former see for instance Kohlscheen and Nakajima (2021) and for the latter Connell et al. (2000) and Sachs and Warner (1997).

Parameter	Value	Variable
$1-\alpha$	2/3	Factor share of human capital of adults
A	1.24	TFP-parameter
β	0.85	Discount factor
γ_1	0.4	Share of wage income going to children
γ_3	0.25	Share of capital income going to adults
δ	0.03	Productivity of human capital formation
ϕ_0	8	Initial stock of human capital per adult
ηL^1	0.56	Human capital per child
ψ_B	0.85	Initial level of institutions
$\widetilde{\psi}_A$	0.95	New level of institutions
L^2	1	Labor supplied by adults

Table 1: Parameter values.

the model. We compare the evolution of physical capital for the two countries. The full line shows the evolution for country A and the dotted line for B. We see that country A and country B diverge indeed. Both begin in the zero growth steady state, with K_0 given by (13). Then we observe for country A a concave convergence over the first few periods, before its stock of physical capital begins to grow exponentially. For country B, we find that the capital stock slightly declines at first and remains almost constant afterwards.

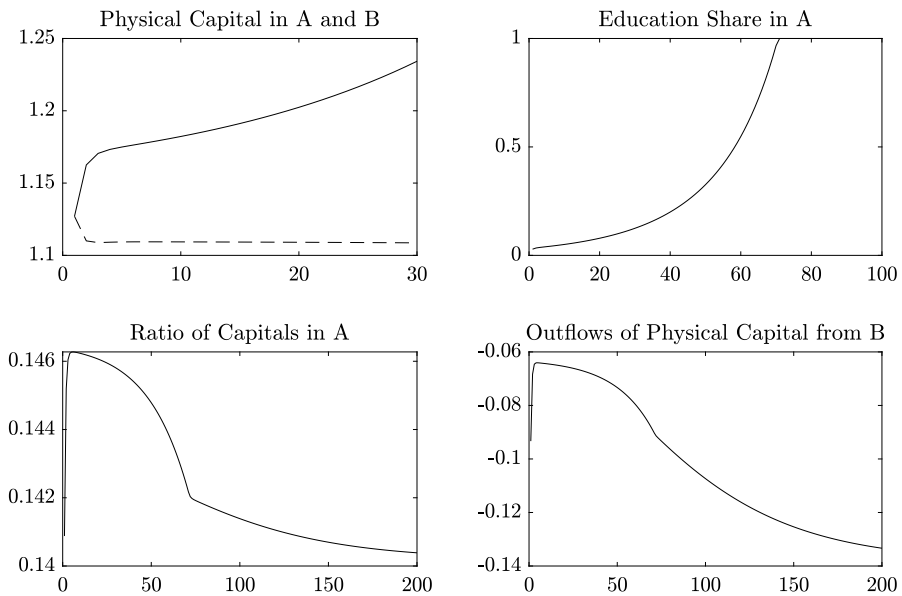

Figure 5: Simulation for the case that country A improves its economic institutions.

Country A's increase of physical capital is accompanied by a rising education share, as can be seen in the right upper panel of Figure 5. The share of time that children spend in school, e_t^A, is exponentially increasing until it reaches 1, shortly after the 70th period. Thereafter, it remains constant at that value. Investment in education in country B remains at $e^B = 0$, and is not shown.

The dynamics of the ratio of capitals in A, K_t^A/ϕ_t^A can be seen in the left lower panel of Figure 5. It reflects our previous findings. At first, the ratio of capital increases, due to the strong accumulation of physical capital, which we observed in the left upper panel. Then, country A accumulates human capital faster than physical capital, decreasing the ratio. Finally, after country A has reached the point in time, where $e_t^A = 1$, we see a convergence to the new steady state.

A theoretical prediction that we made was that country B converges to a steady state where it exports the same amount of physical capital in every period. In the right lower panel of Figure 5 we see that this indeed is the case. While flows of physical capital to country B mirror the dynamics of K_t^A/ϕ_t^A, they are always negative, indicating that B experiences outflows. The outflows decrease at first, due to A's fast accumulation of physical capital, which reduces A's return. However, outflows increase as A invests more in education. Furthermore, when country A reaches the state of full education with $e_t^A = 1$, outflows continue to increase but at a significantly lower rate, and start to converge. The convergence is quite slow, going on for over 200 periods.

7 Extension: Income of children and savings

Unlike before, we now consider the income of children to be used for savings. With this assumption, the optimality conditions (9) and (10) become

$$s_t = \frac{\beta}{1+\beta}\left((1-\gamma_1)(w_t H_t + (1-e_t)\eta L^1) + \gamma_3 \psi R_t K_t\right)$$

and

$$\frac{\beta \phi'_{t+1}(e_t)}{1+\beta} = \frac{\eta L^1}{(1-e_t)\eta L^1 + w_t H_t + \psi \gamma R_t K_t} \quad \text{with} \quad \gamma = \frac{\gamma_3}{1-\gamma_1},$$

where we write the second condition with an equality sign for convenience, although it might hold with a smaller than sign if $e_t = 0$ or an greater than sign if $e_t = 1$. The only difference to the main model, apart from including the third component into the savings decision, is that the term $1 + \beta$ now appears on the left-hand side of the condition. Consequently, the capital stock in the zero growth steady state K now is implicitly given by

$$K\left(1 - \frac{\beta A}{1+\beta}\left(\frac{H}{K}\right)^{1-\alpha}[(1-\gamma_1)(1-\alpha) + \alpha\psi\gamma]\right) = (1-\gamma_1)\eta L^1. \tag{36}$$

Since the child labor sector is unaffected by capital flows, our results for the change in income due to capital flows in both countries in period t carry over. Furthermore, our results about the education decision are the same as well.

The results of this section differ more strongly from the previous ones when we look at the period after the change in institutions occurred. In this period $t+1$, the capital stocks at the beginning of the period, i.e. before international capital flows, are

$$K^A_{t+1} = \frac{\beta}{1+\beta}\left[(1-\gamma_1)(1-e^*_t)\eta L^1 + 2^\alpha(1+\chi^{\frac{-1}{1-\alpha}})^{-\alpha}\gamma_A A(H^A_t)^{1-\alpha}(K^A_t)^\alpha\right] \quad \text{and}$$
$$K^B_{t+1} = \frac{\beta}{1+\beta}\left[(1-\gamma_1)\eta L^1 + 2^\alpha(1+\chi^{\frac{1}{1-\alpha}})^{-\alpha}\gamma_B A(H^B_t)^{1-\alpha}(K^B_t)^\alpha\right],$$

where γ_A and γ_B are defined in (21) and (22). Let us again study the condition under which country A experiences inflows of capital in this period:

$$\chi^{\frac{1}{1-\alpha}}\left(\frac{H^A_{t+1}}{H^B_{t+1}}\right)\left(\frac{(1-\gamma_1)\eta L^1 + \gamma_B A(H^B_t)^{1-\alpha}(K^B_t)^\alpha}{(1-\gamma_1)(1-e^*_t)\eta L^1 + \gamma_A A(H^A_t)^{1-\alpha}(K^A_t)^\alpha}\right) > 1. \tag{37}$$

We discuss the size of the three expressions that make up the left hand side in turn: First, there is $\chi^{\frac{1}{1-\alpha}}$, for which we know that it is larger than 1. Second, we have the ratio of human capital stock of adults H^A_{t+1}/H^B_{t+1}, which is also larger than one. Country A invest in education in t, while B does not so that $H^A_t < H^A_{t+1}$ and $H^B_t = H^B_{t+1}$. Third, we find the ratio of income after capital flows and after education to be relevant. Even now, with the inclusion of the earnings of children and the education level, we know that this ratio is smaller than 1. The reason is twofold: On the one hand, we have that γ_B is smaller than one, i.e. country B

has after the outflows of capital less income than initially. On the other hand, we have shown that country A, while investing in education, chooses a value for e_t^* such that its income does not decrease below the initial level, i.e. the one before capital flows. Unlike before, we find that education can ensure inflows of capital to country A not in one but in two ways. While an efficient schooling technology can increase H_{t+1}^A more strongly, a larger level of education in t decreases the savings and thus the accumulation of capital in that period.

To determine the capital stocks in both country after capital flows, we use the following relation between K_{t+1}^A and K_{t+1}^B

$$K_{t+1}^A = K_{t+1}^B \frac{\chi^{\frac{\alpha}{1-\alpha}}\gamma_A}{\gamma_B} + \frac{\beta(1-\gamma_1)\eta L^1}{1+\beta}\left(1 - e_t^* - \frac{\chi\gamma_A}{\gamma_B}\right).$$

We use this relation together with

$$K_{t+1}^A + K_{t+1}^B = \widetilde{K}_{t+1}^A + \widetilde{K}_{t+1}^B \quad \text{and} \quad \widetilde{K}_{t+1}^A = \chi^{\frac{1}{1-\alpha}} H_{t+1} \widetilde{K}_{t+1}^B,$$

where we denote H_{t+1}^A / H_{t+1}^B as H_{t+1}. We obtain

$$\widetilde{K}_{t+1}^B = \left(1 + \chi^{\frac{1}{1-\alpha}} H_{t+1}\right)^{-1} \left[\left(1 + \frac{\chi\gamma_A}{\gamma_B}\right) K_{t+1}^B + \frac{\beta(1-\gamma_1)\eta L^1}{1+\beta}\left(1 - e_t^* - \frac{\chi\gamma_A}{\gamma_B}\right)\right] \tag{38}$$

and

$$\begin{aligned}\widetilde{K}_{t+1}^A = &\left[\left(1 + \frac{\gamma_B \chi^{\frac{-\alpha}{1-\alpha}}}{\gamma_A}\right)\left(K_{t+1}^A - \frac{\beta(1-\gamma_1)\eta L^1}{1+\beta}\left(1 - e_t^* - \frac{\gamma_A \chi^{\frac{\alpha}{1-\alpha}}}{\gamma_B}\right)\right) + \right. \\ &\left. \frac{\beta(1-\gamma_1)\eta L^1}{1+\beta}\left(1 - e_t^* - \frac{\gamma_A \chi^{\frac{\alpha}{1-\alpha}}}{\gamma_B}\right)\right]\left(1 + \chi^{\frac{-1}{1-\alpha}} H_{t+1}^{-1}\right)^{-1}.\end{aligned} \tag{39}$$

It still holds that the total income of a family in country A increases in H_{t+1}^A, K_{t+1}^A, and $\widetilde{K}_{t+1}^A$. While we know that $H_t < H_{t+1}^A$ and $K_t^A \leq K_{t+1}^A$, we do not know whether $\widetilde{K}_{t+1}^A$ is larger or smaller than its predecessor $\widetilde{K}_t^A$. One way to find out would be to take the derivative of (39) with respect to χ. However, since a change in χ not only impacts $\widetilde{K}_t^A$ directly, but also over e_t^* and H_{t+1}, the derivative would be difficult to analyze. The other way is the one we used before. First, we realize that $\widetilde{K}_{t+1}^A$ is still given by (27). Hence, an increase in total world income in t

is a sufficient condition for country A to increase its level of education in $t+1$ compared to period t. The income of country B at the end of period t is

$$(1-\gamma_1)\eta L^1 + \Gamma_t^B (H_t^B)^{1-\alpha}(K_t^B)^\alpha, \tag{40}$$

with Γ_t^B given by (43). The income in country A, at the end of t, is

$$(1-\gamma_1)(1-e_t^*)\eta L^1 + \Gamma_t^A (H_t^A)^{1-\alpha}(K_t^A)^\alpha, \tag{41}$$

with Γ_t^A given by (17). As before, the world income increases if and only if the change of (40) with respect to a change in χ is smaller in absolute value than the change in (41). Using the derivatives for Γ_t^A and Γ_t^B that we obtained before, we arrive at the following boundary condition for the change of e_t^*

$$\left|\frac{\partial e_t^*}{\partial \chi}\right| \leq \frac{2^\alpha \alpha (1+\chi^{\frac{-1}{1-\alpha}})^{-\alpha}}{(1-\gamma_1)\eta L^1}\left[\frac{(1-\gamma_1)(1-\chi)}{(1+\chi^{\frac{1}{1-\alpha}})} + \frac{(\widetilde{\psi}_A + \psi_B)\gamma_3}{2}\right](H_t^A)^{1-\alpha}(K_t^A)^\alpha,$$

where we used that $K_t^A = K_t^B$ and $H_t^A = H_t^B$. This condition imposes restrictions on e_t^* that stand in opposition to those from (37). In Condition (37), we saw that large values of the optimal level of education e_t^* are beneficial for capital flows to A in $t+1$, since they reduce the accumulation of physical capital. Now, we see that these values should not be too large, as they otherwise reduce the accumulation so strongly that the global stock of capital in $t+1$ is smaller than in t. Hence, only with intermediate values of e_t^* it is possible for country A to receive capital flows in two consecutive periods and also to increase its level of education.

In this model, the conditions under which country A receives capital flows and increases its level of education in all future periods are given by (37), which has to hold for every period after $t+1$, and by

$$\begin{aligned}&(H_{t+1}^A)^{1-\alpha}(K_{t+1}^A)^\alpha \Gamma_{t+1}^A - (H_{t+1}^A)^{1-\alpha}(K_{t+1}^A)^\alpha \Gamma_{t+1}^{A,0} - (1-\gamma_1)\eta L^1 e_{t+1}^* \geq \\ &\left|(H_{t+1}^B)^{1-\alpha}(K_{t+1}^B)^\alpha \Gamma_{t+1}^{B,0} - (H_{t+1}^B)^{1-\alpha}(K_{t+1}^B)^\alpha \Gamma_{t+1}^B\right|,\end{aligned} \tag{42}$$

which is (30) with the additional term $-(1-\gamma_1)\eta L^1 e_t^*$ on the left hand side. Our discussion about what might happen if these conditions do not hold carries over from the main part.

Next, let us look at the second steady state we discussed in the section above, i.e. the steady state along which country A grows, while country B does not. Since, along this growth path, children in country A receive the largest possible amount of schooling $e^* = 1$, Expression (32) holds also for this model and country A finds itself in the same steady state.

For country B the constant amount of locally used capital is

$$K^B = \frac{\beta A}{1+\beta}\left[(1-\alpha)(1-\gamma_1)(H^B)^{1-\alpha}(\widetilde{K}^B)^{\alpha} + \alpha\gamma_3\widetilde{\psi}_A K^B + \frac{(1-\gamma_1)\eta L^1}{A}\right],$$

with

$$\widetilde{K}^B = H^B(\chi R^A)^{\frac{-1}{1-\alpha}}.$$

It is not possible to further discuss K^B, as it can be only solved for numerically. However, the discussion about which country might first reach this steady state carries over.

8 Conclusion

Institutions, human capital and flows of physical capital all seem to have an impact on long-term growth. In this study, we provide a unified framework, where these forces interact, and that might explain increasing inequality between countries. We propose the idea that better economic institutions attract international capital, more physical capital increases households' income and thus allows for the formation of human capital. Human capital, then, drives growth. We analytically provide conditions under which a country can undergo these steps towards long-term growth and show that in our model endogenous separation between countries, due to an initial difference in institutions, can occur. The country that was able to improve its institutions benefits from inflows of capital and converges to a path with a positive growth rate, while the other country remains at a zero growth steady state.

There are different avenues that further research can take. First, we assume simple sharing rules for family income and do not incorporate a pension system. Implementing the latter might yield interesting insights. Second, one can extend the model to incorporate more countries and a multitude of institutional parameters

to see which institutional changes are especially vital in enabling growth.

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9 Appendix

9.1 Proof of proposition 1

To see that the condition provided above is necessary, consider the term $2\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)^{-1}$. This term is smaller than one, as $\widetilde{\psi}_A > \psi_B$ by assumption. This implies that the term $\left((1-\gamma_1)(1-\alpha)+\frac{\alpha}{2}\psi_B\gamma_3\left(1+\left(\frac{\widetilde{\psi}_A}{\psi_B}\right)^{\frac{1}{1-\alpha}}\right)\right)$ must be larger than one, which is only possible if (15) holds.

9.2 General discussion of income change in B

We write the income in B after capital flows but before any education decision, on the left hand side of the following inequality:

$$(1-\gamma_1)\widetilde{w}_t^B H_t^B + \gamma_3\psi_B\widetilde{R}_t^B K_t^B \geq (1-\gamma_1)w_t^B H_t^B + \gamma_3\psi_B R_t^B K_t^B.$$

The right hand side is the income before capital flows occur. Substituting wages and interest rates before and after capital flows yields

$$G\left(\frac{\widetilde{K}_t^B}{K_t^B}\right) \geq 1 \qquad \text{with}$$

$$G\left(\frac{\widetilde{K}_t^B}{K_t^B}\right) = \left(\frac{\widetilde{K}_t^B}{K_t^B}\right)^{\alpha} \frac{(1-\alpha)(1-\gamma_1)+\alpha\psi_B\gamma_3\frac{K_t^B}{\widetilde{K}_t^B}}{(1-\alpha)(1-\gamma_1)+\alpha\psi_B\gamma_3}.$$

We define $\widetilde{K}_t^B/K_t^B$ as x, so that the above expression is 1 for $x = 1$, i.e. all local capital is utilized in production. The derivative of $G(x)$ is

$$\alpha(1-\alpha)x^{\alpha-1}\left(1-\gamma_1-\frac{\psi_B\gamma_3}{x}\right).$$

It has a global minimum at $x^{min} = \psi_B\frac{\gamma_3}{1-\gamma_1}$ and is strictly increasing to the left and right of it, making $G(x)$ strictly convex. If γ_3 is similar to $1-\gamma_1$ the minimum lies to the left of $x = 1$, with $G(x^{min}) < 1$. Due to the strict convexity and the fact that $\lim_{x\to 0} G(x) \to \infty$, there must be an $\tilde{x}$ between $[0, \psi_B\gamma]$, such that $G(\tilde{x})$ is equal to 1. Hence, income in country B increases or decreases, depending on the size of capital flows and the subsequent ratio of capital used in local production to the total amount of initial capital. If capital outflows are very large, reducing $\widetilde{K}_t^B$ strongly relative to K_t^B, then country B will actually experience an increase in its income since the exported capital still belongs to the residents in B. One determining factor is certainly the change in institutions in A. The larger the change, the larger capital outflows from B will be, and the larger capital outflows from B, the better the chances for B to benefit in net from it. Let us assume for the following that income in B decreases.

9.3 Proof of proposition 2

We write the (18) as $\phi'_{t+1}(e_t^*) = D(e_t^*, \Gamma^A)$, where $D(.)$ is a function of the optimal value of education and of the factor that scales output. Note that before capital flows, it held that $\Gamma^{A,0} = (1-\alpha)(1-\gamma_1) + \alpha\gamma_3\widetilde{\psi_A}$. In the initial steady state, we had $\phi'_{t+1}(0) \leq D(0, \Gamma^{A,0})$. After the inflow of capital, we assume that $\phi'_{t+1}(0) > D(0, \Gamma_t^A)$ holds. This allows for some e_t^* such that $\phi'_{t+1}(e_t^*) = D(e_t^*, \Gamma_t^A)$. Now assume that the education function has a constant marginal product, i.e $\phi'_{t+1}(0) = \phi'_{t+1}(e_t^*)$. Then, $\phi'_{t+1}(e_t^*) = D(e_t^*, \Gamma_t^A) > D(0, \Gamma_t^A)$ and $D(e_t^*, \Gamma_t^A) \leq D(0, \Gamma^{A,0})$, so that the family income does not decrease below the initial level. This holds even more strongly, when ϕ_{t+1} has decreasing returns. To see this, note that $\phi'_{t+1}(0) > \phi'_{t+1}(e_t^*) = D(e_t^*, \Gamma_t^A)$. It follows that $D(e_t^*, \Gamma_t^A) < D(0, \Gamma^{A,0})$. Thus, the family income will be strictly larger for every optimal e_t^*. The opposite holds for increasing returns to education. In this case, $\phi'_{t+1}(0) < \phi'_{t+1}(e_t^*) = D(e_t^*, \Gamma_t^A) > D(0, \Gamma_t^A)$ and potentially $D(e_t^*, \Gamma_t^A) > D(0, \Gamma^{A,0})$. It is possible that adults invest

so much in education that they have less income than before. However, in the case of increasing returns to education, $\phi'_{t+1}(e_t)$ must grow more slowly than the right hand side in e_t. Otherwise, there cannot be a steady state in which a country does not invest in education and the only optimal choice becomes $e_t = 1$.

9.4 Change in world income in t

We observe Γ_t^B which is given by

$$\Gamma_t^B = 2^\alpha (1+\chi^{\frac{1}{1-\alpha}})^{-\alpha} \left((1-\alpha)(1-\gamma_1) + \frac{\alpha}{2}\psi_B\gamma_3(1+\chi^{\frac{1}{1-\alpha}}) \right), \tag{43}$$

which, as we know, decreases in χ. The marginal change is

$$2^\alpha \alpha (1+\chi^{\frac{-1}{1-\alpha}})^{-\alpha} \left[-\frac{1-\gamma_1}{1+\chi^{\frac{1}{1-\alpha}}} + \frac{\psi_B\gamma_3}{2} \right].$$

Next, we study the derivative of Γ_t^A. It reads

$$2^\alpha \alpha \left(1+\chi^{\frac{-1}{1-\alpha}}\right)^{-\alpha} \chi^{-1} \left(\frac{1-\gamma_1}{1+\chi^{\frac{1}{1-\alpha}}} + \frac{\widetilde{\psi}_A\gamma_3}{2} \right).$$

and it is larger than $\partial\Gamma_t^B/\partial\chi$ in absolute terms if the following expression holds[10]

9.5 Policy function for capital

The policy function for capital can be derived in two ways. We assume that only A grows, so that $K_t^A \gg K_t^B$ and $\phi_t L^2 \gg \eta L^1$ for $t \to \infty$. The inflow of capital from B has virtually no effect on the overall income in A and the cost of sending children to school becomes negligible. Hence the total stock of human capital in A is very close to the utilized stock of human capital, so that we have $\widetilde{K}_t^A = K_t^A$ and $H_t^A = H_t^A$.

The first way to derive the equation is by using this assumption and consider

[10] To be precise, this is the relevant condition if $\frac{1-\gamma_1}{1+\chi^{\frac{1}{1-\alpha}}} > \frac{\psi_B\gamma_3}{2}$. In the opposite case, $|\frac{\partial\Gamma_t^A}{\partial\chi}| > |\frac{\partial\Gamma_t^B}{\partial\chi}|$ is always fulfilled, regardless of parameter values.

country A as a closed economy, so that

$$K_{t+1}^A = \frac{\beta A}{1+\beta}(H_t^A)^{1-\alpha} K_t^\alpha \left((1-\alpha)(1-\gamma_1) + \alpha\gamma_3 \widetilde{\psi}_A\right),$$

where $H_t^A = \phi_t L^2$. Dividing both sides by H_{t+1}^A delivers the results.

The second way is to begin with the general savings equation, in some period, where $K_t^A \neq K_t^B$, i.e. the initial allocation of physical capital is not the same, but it holds that $K_t^A + K_t^B = K_t = \widetilde{K}_t^A + \widetilde{K}_t^B$. With the stock of human capital also differing across countries, we have

$$\begin{aligned} K_{t+1}^A =& \frac{\beta A}{1+\beta}(\phi_t L^2)^{1-\alpha} K_t^\alpha \left(1 + \chi^{\frac{-1}{1-\alpha}} H_t^{-1}\right)^{-\alpha} \cdot \\ & \left[(1-\alpha)(1-\gamma_1) + \alpha\widetilde{\psi}_A \gamma_3 \left(1 + \chi^{\frac{-1}{1-\alpha}} H_t^{-1}\right) \frac{K_t^A}{K_t}\right]. \end{aligned}$$

As country A grows, it holds almost the entire share of international capital, so that $K_t^A \approx K_t$. Also, H_t becomes very large and the term $H_t(1 + \chi^{\frac{1}{1-\alpha}} H_t)^{-1}$ converges to 1, yielding the policy function.

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