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A parametrized Reduced Order Model for rapid evaluation of flaws in Guided Wave testing

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ABSTRACT

Data from guided wave propagation in structures, produced by piezoelectric elements, can offer valuable information regarding the possible existence of flaws. Numerical models can be used to complement the attained data for refining the potential for flaw characterization. Unfortunately, evaluation of these models remains computationally expensive, especially for small defects, due to the short wavelength required for detection and, the in turn fine discretization in time and space. This renders real–time simulation infeasible, rendering GW–approaches less attractive for inverse problem formulations, where the forward problem needs to be solved several times.

We propose an accelerated computation method, which exploits the properties of guided waves interacting with defects, where an extra band of waves is created, whose phase is differentiated, depending on the location of the flaw (e.g. notch) within the medium. To expedite the actual simulation for the inverse problem, the system is parametrized in terms of the location of the flaw and, in an offline phase, is repeatedly solved to produce snapshots of the system’s response. The snapshots are used to create a physics–informed interpolation of the solution of the wave propagation problem for different flaw locations. The gained information is then used in an inverse setting for localising the defect using an evolution strategy as a means to stochastic, derivative-free numerical optimization. The method is demonstrated in simulations of a 2D slice of a thin plate.

INTRODUCTION

Structural Health Monitoring (SHM) is perceived as the process of continuous monitoring of structures for the purpose of diagnostics and flaw detection, as well as prognostics of flaw evolution. SHM enjoys industrial adoption with applications in the
aerospace [1,2], wind energy [3,4], and power vessels [5] domains. SHM methodologies can be classified according to Rytter’s scheme into four levels [6]. The fundamental level is the detection of flaws, followed by the localisation and the quantification of damage, while the highest level involves the estimation of residual life–time.

As GW-based methods can exploit relatively cheap sensors, they can offer coverage of large areas, at a quite dense instrumentation for detection of small defects [7, 8]. In their review, Mitra and Gopalakrishnan [7] have identified 6 fields for further research in GW-based SHM, including the reduction of computational costs for guided wave simulations. Towards this direction, this contribution provides a scheme for rapid evaluation of response in GW propagation under varying defect configurations. The method is founded on the concept of interpolation in the frequency domain.

In this contribution, the problem of wave propagation in elastic wave guides is first described, followed by description of the proposed method for expediting simulations through interpolation. The potential of the method is demonstrated on the localisation and quantification of notched defects in a 2D-slice of a flat plate, with a critical assessment and future resarch directions discussed in the concluding remarks.

PROBLEM FORMULATION

Weak form

In Figure 1, a domain \( \Omega \) with a homogeneous, linear-elastic material is illustrated. Part of the boundary \( \Gamma \) might be excited dynamically. The system response to an arbitrary excitation can be determined using the so–called weak form, which can be obtained by minimizing the potential \( \Pi \) in the domain \( \Omega \).

\[
\int_{(\Omega)} D \nabla^S u \nabla^S \delta u d\Omega + \int_{(\Omega)} \rho \ddot{u} \delta u d\Omega = \int_{(\Gamma_n)} t \delta u d\Gamma + \int_{(\Omega)} b \delta u d\Omega \tag{1}
\]

where \( D \) denotes the constitutive matrix, \( u \) the vector of deformations, \( \rho \) the density, \( t \) the vector of traction forces, and \( b \) the body forces. \( \nabla^S \) equals the symmetric part of the \( \nabla \) operator (\( \frac{1}{2} (\nabla + \nabla^T) \)). Discretizing (1) e.g. with finite elements, a discrete form of the differential equation can be obtained:

\[
K u(t) + M \ddot{u}(t) = f(t) \tag{2}
\]
This framework establishes the foundation for modelling the phenomenon of wave propagation, as used for the purpose of GW-based flaw detection. In structures with two parallel surfaces, overlap of reflecting waves results in formulation of Lamb waves [9], which tend to lose less energy due to geometrical damping, thus traveling longer distances. This trait makes them particularly interesting within the context Structural Health Monitoring (SHM).

**Inverse Problem Formulation**

Herein, defect localization and quantification is performed solving an inverse problem, whereby the combination of damage parameters that leads to the best match between simulated and measured response data is sought. In the present case, the parameters used are geometrical features of damage, such as the location and depth of the notch, while the measured response consists of displacement time–histories at sensor locations. The inverse problem can be formulated as an optimisation problem:

$$\mu^* = \min_{\mu \in \mathbb{R}^{N_\mu}} f(\mu)$$  \hspace{1cm} (3)

where $\mu$ is the parameter vector, $N_\mu$ is the number of parameters, $\mu^*$ is the estimated parameter vector, and $f$ is the objective function, typically some norm of the difference between measured and simulated response.

While several definitions of the objective function can be used, herein it is assumed that measurements from the undamaged system are available and the following definition is employed:

$$f(\mu) = \| (U_m - U_{m,p}) - (U_s(\mu) - U_{s,p}) \|_2$$  \hspace{1cm} (4)

where $U$ is a vector with the displacement time history at the sensors, and subscripts m, s, and p relate to measured, simulated, and pristine plate response.

**OFFLINE-ONLINE SCHEME FOR EXPEDITED SIMULATION**

The calculation of the numerical solution for high-frequency wave propagation problems is numerically expensive and becomes an obvious hindrance during the solution of an inverse problem. To avoid the repetition of expensive evaluations, we introduce a method which divides the evaluations into an offline part (before the inverse problem is solved) and an online part (while the inverse problem is solved). In the offline phase, the system is evaluated several times for different parameter values to produce training data for the reduced model, which can be evaluated efficiently for the solution of the inverse problem in the online phase.

**Dynamic excitation and window functions**

For solution of the problem, it is advantageous to transform this into the frequency domain via use of the Fourier transformation. Equation 2 is then transformed as:

$$K \hat{u}(\omega) - \omega^2 M \hat{u}(\omega) = f(\omega)$$  \hspace{1cm} (5)
can be separated into a spacial vector $f_s$ and a frequency dependent scalar $f(\omega)$ so that $f(\omega) = f_s \cdot f(\omega)$, under the condition that $f_s$ can not be altered. $f_s$ is the vector of spatial assignment of external excitation to the meshed nodes. However, the magnitude and the dynamical behaviour of the excitation can be changed through $f(\omega)$. Therefore, we reformulate equation 5 and insert $f(\omega) = f_s \cdot f(\omega)$:

$$\hat{\mathbf{u}}(\omega) = (K - \omega^2 M)^{-1} f_s \cdot f(\omega)$$

Given a reference solution $\hat{\mathbf{u}}(\omega) = \hat{\mathbf{u}}_1(\omega)$ for one realisation of the excitation $f(\omega) = f_1(\omega)$, the solution $\hat{\mathbf{u}}(\omega) = \hat{\mathbf{u}}_2(\omega)$ for any other excitation $f(\omega) = f_2(\omega)$ can be obtained as $\hat{\mathbf{u}}_2(\omega) = \hat{\mathbf{u}}_1(\omega) \frac{f_2(\omega)}{f_1(\omega)}$, provided that $f_1$ assumes non zero values for the frequency range of interest. The term $(K - \omega^2 M)^{-1} f_s$ is termed the Frequency Response Function (FRF).

As the Fast Fourier Transformation (FFT), essentially computes a Fourier series approximation, it assumes a periodic function as input. This is generally not true for an arbitrary system response, as deformations are not necessarily the same for the first and last time step, introducing a jump or a kink in the response, which leads to the so-called Gibbs phenomenon [10]. To avoid such issues, we apply a window in the time domain response. Since we assume the system to be at rest at $t = 0$, a sufficiently smooth response can be obtained with a window function that assumes a value of zero at the end of the signal. To this end, our adopted window function is defined as follows

$$W(t) = \begin{cases} 
1 & \text{if } t <= t_{\text{begin}}, \\
\frac{1}{2} \cdot \left( \cos \left( \frac{(t-t_{\text{begin}}) \cdot 2\pi}{t_{\text{end}}-t_{\text{begin}}} \right) + 1 \right) & \text{if } t > t_{\text{begin}} \& t < t_{\text{end}}, \\
0 & \text{if } t >= t_{\text{end}}
\end{cases}$$

where $t_{\text{begin}}$ describes the threshold at which the cosine transition begins and $t_{\text{end}}$ describes the end of the computed response, after which the signal is 0-padded to increase its smoothness. The smooth cosine transition offers a function, which the Fourier transformation can approximate adequately.

The nature of the reference excitation $f_1$ is important for ensuring accurate results over a broad frequency range. A typical selection is a single impulse:

$$f_1(\omega) = \begin{cases} 
1 & \text{if } t = 0, \\
0 & \text{otherwise}
\end{cases}$$

which excites all frequencies equally.

Geometrical features of the flaws

The above method allows to represent the response for almost arbitrary excitation inputs. However, the solution of the inverse problem of (3), requires repeated solution under different geometrical parameters of the flaw, represented with the parameter vector $\mu$. To this end, the system is solved for a set of parameter values using the defined reference (impulse) excitation of (8), with the response then transferred to the frequency domain. Subsequently, the FRF of the system can be computed and interpolated for different values of the parameter vector. After interpolation, the results can be adopted for a
### Table I: Plate Dimensions and Material Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>1.6 mm</td>
</tr>
<tr>
<td>Length</td>
<td>600 mm</td>
</tr>
<tr>
<td>Distance actuator-sensor</td>
<td>110 mm</td>
</tr>
<tr>
<td>Density $\rho$</td>
<td>2700 kg/m$^3$</td>
</tr>
<tr>
<td>P-wave velocity $c_p$</td>
<td>5055 m/s</td>
</tr>
<tr>
<td>S-wave velocity $c_s$</td>
<td>3135 m/s</td>
</tr>
<tr>
<td>Young’s modulus $E$</td>
<td>$69 \cdot 10^9$ Pa</td>
</tr>
<tr>
<td>Poisson ratio $\nu$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

![Figure 2](image.png)

**Figure 2:** Investigated specimen: Notch location varies between actuator and sensor. All distances are in [mm]

The interpolation model is created from a regular grid of parameters, consisting in
Figure 3: Mesh around a notch

Figure 4: Error of the interpolation model for a notch depth of 0.4 mm. The error is calculated by the norm of difference of the vertical deformation at the sensor in frequency domain.

notch location ($\mu_1$) and notch depth ($\mu_2$), as displayed in figure 2. The notch locations are specified every 2 mm, between the actuator and sensor. The shallowest notch is 0.2 mm deep and the deepest is 1.4 mm with samples taken at 0.1 mm intervals. In total, this results in a grid of 51 by 13 entries, for which the full order model has to be evaluated. In Figure 4, the error of the interpolation model is reported for different notch locations. As can be seen, the error is almost zero at sample locations, while it does not exceed a value of 2% for interpolated locations. Time histories of the sensor deformation for two different frequencies can be found in figure 4. The speedup of the interpolation model with respect to the full order model using our in–house MATLAB code is approximately 1700 (from several minutes to less than a second). This enables to solve the system several times within the optimization process.

Figure 5: Comparison of reduced model and full model for a notch with location $\mu_1 = 49.9$ mm and depth $\mu_2 = 0.4$ mm. Excitation with 500 kHz (a) and 750 kHz (b) modulated pulse with 5 cycles. This notch configuration is not in the training set. $d$ describes the vertical deformation at the sensor.
TABLE II: ERROR IN THE ESTIMATED LOCATION AND DEPTH OF THE NOTCH FOR DIFFERENT EXCITATIONS. IN THE LAST COLUMN, THE OBJECTIVE FUNCTIONS FROM ALL THREE CASES ARE ADDED. ALL VALUES IN [mm]

<table>
<thead>
<tr>
<th>True</th>
<th>250 kHz</th>
<th>500 kHz</th>
<th>750 kHz</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \mu_1$</td>
<td>$\Delta \mu_2$</td>
<td>$\Delta \mu_1$</td>
<td>$\Delta \mu_2$</td>
</tr>
<tr>
<td>35.7</td>
<td>0.41</td>
<td>-0.5</td>
<td>0.10</td>
<td>-0.5</td>
</tr>
<tr>
<td>45.7</td>
<td>0.41</td>
<td>-0.3</td>
<td>0.13</td>
<td>-8.4</td>
</tr>
<tr>
<td>45.7</td>
<td>0.81</td>
<td>-0.6</td>
<td>0.26</td>
<td>-0.7</td>
</tr>
<tr>
<td>62.3</td>
<td>0.53</td>
<td>-0.4</td>
<td>0.13</td>
<td>-0.4</td>
</tr>
<tr>
<td>93.1</td>
<td>0.29</td>
<td>-0.1</td>
<td>0.05</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Notch Detection

To detect the notch, the problem of (3) was solved using Particle Swarm Optimization (PSO) [12], with results for different frequencies and notch locations reported in table II, in terms of the differences between actual and estimated parameter values. Several observations can be made, among which the most important is probably that the results are frequency dependent, with some frequencies yielding significantly increased errors for certain notch locations. Since it is not always possible to predict which frequencies will provide the best localisation, an alternative approach is introduced, where the mean of several frequencies is used as an objective function. This gives good results in our tests as shown in column ”sum” of table II. A second finding is that the prediction for the depths of the notch undergoes large variation. In particular, for the notch at $\mu_1 = 45.7$ the variation in between different frequencies but also the variation of the sum is significant. This can be attributed to the fact that the response is relatively insensitive to small variations in the notch depth. On the other hand, the precision of the predicted notch location for the interpolation model is higher than the grid points would allow without interpolation (see figure 4), thus justifying the use of interpolation. Finally, it should be mentioned that, the run-time of the optimisation using the proposed interpolation model is in the range of 2-10 minutes, rendering the approach very attractive for online application.

CONCLUDING REMARKS

In this contribution, a method for model-based notch localization using ultrasonic guided waves is presented. Initially, interpolation of the system transfer functions is employed to build a model able to represent the response over a wide range of notch locations and depths at a fraction of the computational time required by the original finite element model. Subsequently, optimisation is employed to match the computed response with measured data, yielding the damage location as a result.

The employed model can significantly speed up simulations, rendering the approach attractive for more realistic applications, such as the detection of cracks or delaminations in aluminum or composite plates, using advanced numerical models [13]. As a potential drawback of the approach, the interpolation model requires extensive calculations in the offline phase for different notch locations, as a full grid of parameters is needed. To
alleviate this, in future work, interpolation will be replaced by more advanced methods, such as Gaussian Process Regression, which only require limited training data.

**ACKNOWLEDGEMENTS**

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