Analytical eddy current loss model for foil conductors in gapped cores

Thomas Ewald, Jürgen Biela
Laboratory for High Power Electronic Systems (HPE), ETH ZÜRICH
ewald@hpe.ee.ethz.ch
https://www.hpe.ee.ethz.ch

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Abstract
For modelling and optimizing gapped high-frequency inductors, the calculation of eddy current losses in foil windings due to the two-dimensional fringing field caused by air gaps in the core is important. The winding loss models must offer a high accuracy when calculating the 2D field distribution and must be computationally efficient in order to enable several thousand calculations in a short time required during the optimization. This article proposes an analytical model based on the magnetic vector potential formulation that can predict the eddy current losses in foil windings due to the fringing field of an arbitrary number of air gaps. The analytical model is used to derive a closed-form loss formula, which is verified by FEA simulations.

1 Introduction
High-frequency effects resulting in non-homogeneous magnetic field distributions are one of the main causes of high winding losses in magnetic components in power electronic converters. In the design stage of magnetic components, there is a need to predict these losses in order to avoid overheating and redesigns. In the field of converter design, there is a strong trend towards virtual prototyping [1] in order to keep development time and cost low. The most important aspect in virtual prototyping of power electronic systems is the accurate modelling. Hence, analytical models are needed for predicting the winding losses in magnetic components. Since several thousand designs are calculated in the optimization process, modelling methods that offer a minimum of computational effort with reasonable accuracy are preferred.

Fig. 1: a) 2D magnetic field caused by an air gap and the definition of the geometrical parameters of an idealized inductor core window. b) Zoomed section around the air gap of the core window with the air gap modelled as a surface current density.
Although most effects in magnetics are well known, not for all of them analytical models exist. Existing models include the analytical calculation of the skin and proximity effect [2–4]. Another very important effect, the fringing effect in gapped inductors [5], is much harder to cover with analytical models. The reason is the two-dimensional magnetic field distribution inside the core window. Fig. 1a shows the magnetic field inside a core window with an air gap. When considering the fringing field of gapped inductors, especially foil windings cause problems in the calculation, because of the feedback of the conductor currents on the magnetic field itself must be considered.

Finite element analysis (FEA) is the most commonly applied approach to calculate the 2-D field but can result in computation times of several hours or days for automated optimization tasks [6]. To overcome this disadvantage different methods have been proposed in [7–11] using FEA simulations to parameterize empirical formulas, that can be used afterwards without performing time-consuming field simulations. However, the applicability of these empirical formulas is limited.

Analytical models can be found in [12–16], which, however are inaccurate. Either the field distortion due to the current feedback is neglected [12, 13, 15], or the models make significant geometrical simplifications [14, 16]. In [14] only one conductor is considered, whereas [16] neglects insulation distances between individual conductors. Both consequently assume the fringing field to decay in one massive conductor block. In addition, neglecting the insulation represents a rough approximation since the position of the conductor layers deviates significantly from the actual distance with increasing distance to the air gap [17]. [16] considers only one air gap. Methods considering multiple, arbitrary air gaps are existing [18, 19]. Unfortunately, all of these methods are semi-numerical, since they rely on numerical methods to calculate the magnetic field. This results in computation times in the range of 1/10 of the corresponding FEA [18], which is still not sufficiently fast for optimization purposes.

To precisely compute the elevated winding losses in foil conductors caused by the skin, proximity and fringing effects in gapped inductors, this paper proposes an analytical model based on the magnetic vector potential formulation. In section 2 a model is derived for calculating the inhomogeneous current density distribution in each foil conductor, considering skin, proximity and fringing effects. The new model considers also the insulation distance between the conductors, as well as an arbitrary number of symmetrical air gaps. In section 3, the results of section 2 are used to derive a closed-form formula for the loss per unit length calculation with short computation times. The model and the loss per unit length formula are then verified in section 4 by FEA simulations.

2 Model Derivation

To minimize the computational effort for calculating the winding losses, an analytical solution for the magnetic field \( \vec{H} \) and the current density distribution \( \vec{J} \) in each foil conductor is required. This solution can be obtained by directly solving Maxwell’s equations, based on the following assumptions:

1. The surrounding core material is ideal with infinite permeability (\( \mu_r \to \infty \)).
2. Quasi-Magnetostatic calculations, so that Ampere’s law becomes \( \nabla \times \vec{H} = \vec{J} \).
3. Harmonic time dependency, so that \( d/dt \to j_0 \).
4. The core window is rectangular and infinitely long in \( z \)-direction, so that all fields do not depend on \( z \). This assumption is valid, as long as the radius of the curvature of the winding is significantly larger than the actual conductor thickness [20].

In Fig. 1a the geometry and the definitions of the dimensions, that are used in the model, are given. It is assumed that the inductor has a single winding around the center leg with \( N \) turns. The rectangular core window is divided into \( 2N+1 \) regions: i.e. in \( M = N+1 \) non-conductive regions (marked from \( A_1 \) to \( A_M \)), and in \( N \) conductive regions (marked from \( C_1 \) to \( C_N \) and colored with a copper shade). Every region has its own reference coordinate system in \( x \)-direction. The reference coordinate system of the \( m \)-th non-conductive region starts at \( u_m \) and the one for the \( n \)-th conductor at \( v_n \), where \( m \) is the number of the non-conductive region and \( n \) is the number of the conductor, respectively. Additional dimensions are the foil thickness \( d_f \), the distance between foils \( d_t \), the distance from center leg to the first conductor \( d_{k,1} \), the distance from last conductor to limb \( d_{k,0} \), the foil height \( h_f \), and the air gap height \( h_g \).

The current, the current density, and the magnetic vector potential are defined to have only components in \( z \)-direction. Since the magnetic field is perpendicular to the magnetic vector potential, it has only components in the \( xy \)-plane. Under the given assumptions the Helmholtz equation for the magnetic
vector potential simplifies to \( \nabla^2 \vec{A} = -\mu \vec{J} \) [21]. Since the magnetic vector potential and the current only have a \( z \)-component, the vector potential simplifies to a magnetic scalar potential. In all non-conductive regions the current density in \( z \)-direction is \( J_z = 0 \), hence the magnetic scalar potential formulation has to satisfy the Laplace equation

\[
\nabla^2 A_z = 0
\]  

in these regions. In the conductors, the scalar diffusion equation \( \nabla^2 J_z = \gamma^2 J_z \) has to be satisfied [22]. In addition, the equation \( \mu J_z = -\gamma^2 A_z \) must be fulfilled under the assumption that the magnetic vector potential and the scalar electric potential can be summarized [23]. Thus, the magnetic scalar potential has to satisfy the scalar diffusion equation

\[
\nabla^2 A_z = \gamma^2 A_z
\]  

inside the conductors. The influence of the air gaps on the magnetic field is modelled as surface current densities and the core is closed with high permeable material, as shown in Fig. 1b). The surface current flows in the opposite direction as the current in the winding. The magnetic field around the air gaps is decomposed into a spatial Fourier series. The decomposition uses cosine functions exclusively, what requires a strict symmetry of the core window to the \( x \)-axis. Unsymmetrical cases would also include sine terms [13], but are not part of the model in this paper. The magnetic scalar potential that satisfies (1) and (2) is given in [14]. It is a combination of standard solutions, for example given in [24], and has the form:

\[
A_z = \begin{cases} 
C_{m,0} e^{-\gamma(x-u_m)} + N_{n,0} e^{\gamma(x-v_n)} & \text{for } x < u_m \\
\sum_{k=1}^{\infty} \left\{ C_{m,k} e^{-p_k(x-u_m)} + D_{m,k} e^{p_k(x-u_m)} \right\} \frac{\cos(p_ky)}{p_k} & \text{for } x > u_m \end{cases}
\]  

The magnetic scalar potential inside the conductors has to satisfy the diffusion equation (2), thus, the unknown attenuation constant \( \xi_k \) is determined as \( \xi_k^2 = \gamma^2 + p_k^2 \). The \( x \) and \( y \) components of the magnetic field are calculated with \( \mu \vec{H} = \nabla \times \vec{A} \) as:

\[
\begin{align*}
\mu H_x &= -\sum_{k=1}^{\infty} \left\{ C_{m,k} e^{-p_k(x-u_m)} + D_{m,k} e^{p_k(x-u_m)} \right\} \sin(p_ky) \\
\mu H_y &= \sum_{k=1}^{\infty} \left\{ \frac{C_{m,0} e^{-\gamma(y-v_n)} - N_{n,0} e^{\gamma(y-v_n)}}{\gamma} \right\} + \sum_{k=1}^{\infty} \left\{ \frac{C_{m,k} e^{-p_k(x-u_m)} - D_{m,k} e^{p_k(x-u_m)}}{p_k} \right\} \cos(p_ky)
\end{align*}
\]  

At the core yokes the tangential component of the magnetic field must be zero for all \( x \) due to the infinite permeability of the core material, i.e. \( H_x(x, y = -h_w/2) = H_x(x, y = h_w/2) = 0 \), so that \( p_k = 2\pi k N_g / h_w \), where \( N_g \) is the number of air gaps. Note, that in the above formulations for the magnetic field in (4) the sum terms for \( k \neq 0 \) represent the field caused by the air gaps. The corresponding coefficients \( C_{m,k}, D_{m,k}, M_{n,k} \) and \( N_{n,k} \) are defined as ‘ac spatial coefficients’ in the following, because they describe the spatial harmonics of the Fourier decomposition. All terms containing coefficients with index \( k = 0 \) denote the 1D field in a closed transformer core window without air gaps, equivalent to known derivations, e.g. [2]. Since these terms do not contain any spatial harmonics, the coefficients are defined as ‘dc spatial coefficients’. These definitions are in accordance with [16].

Based on the Maxwell equations it can be derived, that the tangential component of the magnetic field, in this case \( H_y \), must be continuous at an arbitrary boundary. In addition, it can be derived, that the normal component of the magnetic flux density, \( B_x \), must be continuous at an arbitrary boundary:

\[
\begin{align*}
B_x^{(A)} &= B_x^{(C)} \\
H_y^{(A)} &= H_y^{(C)}
\end{align*}
\]  

There the superscripts denote conductive (\( C \)) and non-conductive regions (\( A \)). Considering that the relative permeability of air and copper is approximately the same, this results in the condition that the normal component of the magnetic field, \( H_t \), must also be continuous. The tangential components of the magnetic field and the normal components of the magnetic flux density are shown in Fig. 1b) for
Clarification.

All the unknown coefficients $C_{m,k}$, $D_{m,k}$, $M_{n,k}$ and $N_{n,k}$ are determined with the continuity conditions for the magnetic field at the $2N$ adjacent boundaries of non-conductive and conductive regions and at the window walls.

Solving the dc spatial coefficients

For the case $k = 0$ in (4), $D_{m,0} = 0$ by definition. In this case, the magnetic field does not have an $x$-component, and the $y$-component is:

$$
\mu H_y = \begin{cases} 
-C_{m,0} \\
\gamma (M_{n,0} e^{-\gamma(x-v_n)} - N_{n,0} e^{\gamma(x-v_n)})
\end{cases}
$$

(6)

The coefficients $C_{m,0}$ for the non-conductive regions can be determined based on the fact, that in the non-conductive regions, the magnetic field strength is constant and independent of the position $(x, y)$. Nevertheless every non-conductive region has to satisfy Ampere’s law:

$$
C_{m,0} = \frac{\mu}{h_w} (M - m) I
$$

(7)

There $I$ is the peak amplitude of the sinusoidal current. With all $C_{m,0}$ being known, the continuity condition of the $y$-component of the magnetic field can be used to set up a set of equations. For each conductor two boundaries are defined. The first boundary is defined at $x = v_n$, where $m = n$ and the second boundary at $x = v_n + d_l$, where $m = n + 1$. The equations at these boundaries are:

$$
M_{n,0} - N_{n,0} = -\frac{1}{\gamma} C_{n,0}
$$

$$
M_{n,0} e^{-\gamma d_l} - N_{n,0} e^{\gamma d_l} = -\frac{1}{\gamma} C_{(n+1),0}
$$

(8)

The coefficients $M_{n,0}$ and $N_{n,0}$ are determined by solving the equation system (8), what results in:

$$
M_{n,0} = \frac{C_{(n+1),0} - C_{n,0} e^{\gamma d_l}}{2\gamma \sinh(\gamma d_l)}
$$

$$
N_{n,0} = \frac{C_{(n+1),0} - C_{n,0} e^{-\gamma d_l}}{2\gamma \sinh(\gamma d_l)}
$$

(9)

With (7) and (9), all coefficients to model the dc part of the magnetic scalar potential and the magnetic field in the core window are determined. The dc part is equivalent to existing 1D field calculations.

Solving the ac spatial coefficients

For the case $k \neq 0$ only the ac spatial components of the magnetic field must be considered. Since all ac spatial components have to satisfy the constraints for every $k$, the solution can be determined depending on $k$. The magnetic fields $k$-th components are given by (4) as:

$$
\mu H_x = \left\{ \begin{array}{l}
C_{m,k} e^{-\xi_{k}(x-v_n)} + D_{m,k} e^{\xi_{k}(x-v_n)} \\
M_{n,k} e^{-\xi_{k}(x-v_n)} + N_{n,k} e^{\xi_{k}(x-v_n)}
\end{array} \right\} \sin(pk_y)
$$

$$
\mu H_y = \left\{ \begin{array}{l}
C_{m,k} e^{-\xi_{k}(x-v_n)} - D_{m,k} e^{\xi_{k}(x-v_n)} \\
M_{n,k} e^{-\xi_{k}(x-v_n)} - N_{n,k} e^{\xi_{k}(x-v_n)}
\end{array} \right\} \cos(pk_y)
$$

(10)

For $N$ conductors and $M = N+1$ non-conductive regions, in total $4N+2$ linearly independent equations have to be found to calculate all coefficients $C_{m,k}$, $D_{m,k}$, $M_{n,k}$, and $N_{n,k}$. Using the same boundaries as before, $4N$ equations are obtained due to the continuity of normal and tangential components of the
The magnetic field at adjacent boundaries. Those are:

\[ C_{n,k} e^{-p_{4d_t}} + D_{n,k} e^{p_{4d_t}} = M_{n,k} + N_{n,k} \]

\[ C_{n,k} e^{-p_{4d_t}} - D_{n,k} e^{p_{4d_t}} = \frac{\xi_k}{p_k} (M_{n,k} - N_{n,k}) \]

\[ M_{n,k} e^{-\xi_k d_t} + N_{n,k} e^{\xi_k d_t} = C_{(n+1),k} + D_{(n+1),k} \]

\[ \frac{\xi_k}{p_k} (M_{n,k} e^{-\xi_k d_t} - N_{n,k} e^{\xi_k d_t}) = C_{(n+1),k} - D_{(n+1),k} \]

The first of the two missing equations is obtained, when the y component of the magnetic field is decomposed into a series of spatial harmonics [14, 16]. Then, for \( x - u_1 = 0 \), every spatial harmonic \( k \) of \( H_y \) has to satisfy the following condition:

\[ \int_{-h_y/2N_g}^{h_y/2N_g} \frac{1}{\mu} (C_{1,k} - D_{1,k}) \cos^2(p_k y) \, dy = \int_{y_1-h_y/2}^{y_1+h_y/2} -\sum_{i=1}^N \frac{I_i}{N_g h_g} \cos(p_k y) \, dy \]

Using the normalized sinc function, the solution to (12) is:

\[ C_{1,k} - D_{1,k} = -\frac{2\mu}{k\pi N_g h_g} \frac{\left( p_k \frac{h_y}{2} \right)}{\frac{N}{2}} \sum_{i=1}^N I_i = -\frac{2\mu}{h_w} \frac{\left( p_k \frac{N_g h_g}{h_w} \right)}{\frac{N}{2}} \sum_{i=1}^N I_i = B_w \]

The last equation is obtained at the window wall where \( x = u_M + d_{x,o} \). For every \( k \) and an arbitrary \( y \), \( H_y \) must be zero, hence:

\[ C_{M,k} e^{-p_{4d_t,o}} - D_{M,k} e^{p_{4d_t,o}} = 0 \]

The resulting equation system (11) can be uniquely solved, although an analytical solution is, especially for larger \( N \), very complex. The system was solved analytically in [14, 16] for \( N = 1 \), already resulting in a tremendously convoluted solution. Thus, for a system with \( N > 1 \) the \( (4N+2) \times (4N+2) \) linear equation system

\[ K_k c_k = b_k \]

is solved numerically. The matrix for the calculation of the spatial harmonic coefficients \( C_{m,k}, D_{m,k}, M_{n,k} \) and \( N_{n,k} \), is given as

\[
K_k = \begin{bmatrix}
1 & -1 & 0 & \cdots & 0 \\
e^{-p_{4d_t,i}} & e^{p_{4d_t,i}} & \cdots & \cdots & 0 \\
e^{-p_{4d_t,i}} & -e^{p_{4d_t,i}} & \cdots & \cdots & 0 \\
0 & 0 & e^{-\xi_k d_t} & \cdots & \cdots & 0 \\
0 & 0 & -\xi_k p_k & e^{\xi_k d_t} & \cdots & \cdots & 0 \\
0 & 0 & 0 & e^{-p_{4d_t}} & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & 0 & e^{-p_{4d_t,o}} & -e^{p_{4d_t,o}} \\
\end{bmatrix}
\]

Note, that the insulation thickness of the first non-conductive domain is \( d_{x,i} \) and not \( d_t \). Moreover, the matrix \( K_k \) has a repetative structure, that allows to automatize its generation for an arbitrary number of conductors. The equation system (11) consists of 4 equations, relating to 6 coefficients. Therefore, for every \( n \) the equation system results in one submatrix of size \( 4 \times 6 \), that can be found in \( K_k \). The first submatrix starts from the second row and ends in the fifth row of \( K_k \). For every additional conductor there is one additional submatrix in \( K_k \). The vector of coefficients and the solution vector are given as

\[
c_k = [C_{1,k} \quad D_{1,k} \quad M_{1,k} \quad N_{1,k} \quad C_{2,k} \quad \cdots \quad D_{(N+1),k}]
\]

\[
b_k = [B_w \quad 0 \quad \cdots \quad 0]
\]
With the solution vector $c_k$ given in (17) the coefficients $M_{n,k}$ and $N_{n,k}$ are determined, that are necessary to model the ac part of the magnetic scalar potential, the magnetic field, and consequently the current density in the conductors.

### 3 Calculation of the Power Loss per Unit Length

Using the equations derived in section 2 for the magnetic field and the scalar vector potential, the conduction losses of a single foil exposed to air gaps can be calculated by applying Poynting’s theorem [23]. Since it is assumed, that the electric and the magnetic field do not vary with the $z$-coordinate, the overall losses are obtained by multiplying the losses per unit length $P'_n$ with the individual conductor length $l_n$.

Assuming a given conductor length, only the per unit length losses have to be determined by integrating the Poynting vector over the conductor edges $\partial S_n$ in the $xy$-plane, followed by a multiplication with the conductor length. The Poynting vector $\vec{E} \times \vec{H}$ ($\vec{H}^*$ denotes the complex conjugate of $\vec{H}$) is reformulated using $\vec{E} = \vec{J}$ and $\mu \vec{H} = -\nabla \times \vec{A}$, so that the losses per unit length and per conductor, in terms of the scalar magnetic potential, are given as:

$$P'_n = -\frac{1}{2} \Re \left\{ \frac{\gamma^2}{\mu \delta} \int_{\partial S_n} A_z (H_x^* \hat{e}_x - H_y^* \hat{e}_y) \, d\delta_n \right\}$$  \hspace{1cm} (18)

By using the fact, that the $x$-component of the magnetic field at the yokes is forced to zero, (18) can be simplified to an integration along both conductor edges in $y$-direction. Using $\gamma^2 = j \omega \mu$ results in the following integral representation for the losses per unit length per conductor:

$$P'_n = \frac{\omega}{2} \Re \left\{ \int_{-\frac{d_f}{2}}^{\frac{d_f}{2}} A_z (v_n + d_f,y) H_x^* (v_n + d_f,y) - A_z (v_n,y) H_x^* (v_n,y) \, dy \right\}$$ \hspace{1cm} (19)

So far, existing models using the magnetic vector potential formulation do not give a closed-form expression for the conduction losses [25], or explicitly use numerical integration [17] to solve (19). In the considered cases, numerical integration takes up to 95 % of the total computing time. This motivates the analytical solution of (19) to obtain a closed-form loss per unit length expression. Solving the integral of the $n$-th conductor, with (3) and (4), the losses per unit length are:

$$P'_n = P'_{n,0} + \sum_{k=1}^{\infty} P'_{n,k}$$ \hspace{1cm} (20)

Using $\Delta = d_f / \delta$ and the skin depth $\delta = \sqrt{1/\pi j \sigma \mu}$, $P'_{n,0}$ and $P'_{n,k}$ are given as:

$$P'_{n,0} = \frac{\omega \rho_{hw}}{\mu} \left( (|M_{n,0}|^2 e^{-\Delta} + |N_{n,0}|^2 e^{\Delta}) \sinh(\Delta) + 2 \Re \left\{ M_{n,0}^* N_{n,0} e^{j \Delta} \right\} \sin(\Delta) \right)$$ \hspace{1cm} (21)

$$P'_{n,k} = \frac{\omega \rho_{hw}}{2 \rho_{l}^2 \mu \delta} \left( \zeta_{1,k} \left( |M_{n,k}|^2 e^{-\zeta_1 \Delta} + |N_{n,k}|^2 e^{\zeta_1 \Delta} \right) \sinh(\zeta_2 \Delta) + 2 \xi_{2,k} \Re \left\{ M_{n,k}^* N_{n,k} e^{j \zeta_1 \Delta} \right\} \sin(\zeta_1 \Delta) \right)$$ \hspace{1cm} (22)

with

$$\zeta_{1,k} = \left( \left( \frac{\rho_{l}^4 \delta^4}{4} + 1 \right)^{\frac{1}{2}} - \frac{\rho_{l}^2 \delta^2}{2} \right)^{\frac{1}{2}} \quad \zeta_{2,k} = \left( \left( \frac{\rho_{l}^4 \delta^4}{4} + 1 \right)^{\frac{1}{2}} + \frac{\rho_{l}^2 \delta^2}{2} \right)^{\frac{1}{2}}$$ \hspace{1cm} (23)

It can be observed, that the overall losses for each conductor are separated into two terms. $P_{n,0}$ takes the current distribution into account, that is caused by the skin and the proximity effect (this is equivalent to Dowell’s formula [2]). The terms $P_{n,k}$ reflect the additional losses caused by field distortion due to the air gaps, each for the respective spatial harmonic $k$. The resulting closed-form formula is verified in section 4 by FEA simulations. Note, that adding up all $N$ per unit length contributions and scaling the resulting losses per unit length with an overall mean turn length will not yield the correct result. This is caused by the fact, that the influence of the air gap field is attenuated drastically after the first few conductors. The losses in the conductors closest to the air gaps are significantly increased, but the corresponding length of the conductors (in the considered geometry) is comparatively short. A suitable
length scaling of each individual conductor is mandatory to properly reflect this fact in the overall loss calculation:

\[ P_{\text{total}} = \sum_{n=1}^{N} l_n P'_n \]  

(24)

There, the individual conductor length is \( l_n \), which must be derived from the geometry of the core that is used.

### 4 Verification and discussion of the proposed Model

To verify the proposed model and the loss calculation procedure, the losses per unit length \( P' \) for an example inductor are calculated and compared with results from FEA simulations. Fig. 1a) defines the parameters of the core window of the inductor. The respective parameter values for two test cases (one window with a single air gap, one window with 3 symmetrical air gaps) are given in Table I. Fig. 2a) shows the resulting normalized current density distribution \( \vec{J}_z(x,y) \), calculated with the proposed model. For verification purposes, several FEA simulations over a wide frequency range from 1 Hz to 10 MHz have been performed. Fig. 2b) shows, that the proposed model matches FEA simulations over a wide frequency range. Fig. 2c) shows the corresponding relative deviation of the proposed model from the FEA simulations.

**Table I: General specifications of test cases and operating point.**

<table>
<thead>
<tr>
<th>( I ) (peak, A)</th>
<th>( N )</th>
<th>( N_g )</th>
<th>( h_t ) (mm)</th>
<th>( h_g ) (mm)</th>
<th>( d_t ) (mm)</th>
<th>( d_{s,1} ) (mm)</th>
<th>( d_{s,0} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1, 3</td>
<td>25</td>
<td>1</td>
<td>0.65</td>
<td>0.65</td>
<td>1</td>
</tr>
</tbody>
</table>

One of the basic assumptions of the above derivations is that the foil conductors fill the core window from the bottom yoke to the top yoke. This assumption is typically not fulfilled in reality, so that errors in the calculated losses occur. In order to be able to estimate the resulting deviation, the losses for an inductor with distances between winding and yokes are calculated with a 2D FEA model. The additional distance \( h_i = (h_w - h_f) / 2 \) is depicted in Fig. 3a). The influence of this effect on the loss calculation is examined by varying the window height in the FEA model. The window is enlarged, opposed to reducing the conductors in height, because for \( h_w / h_f > 1.5 \) and a fixed conductor height, the geometry approaches

---

Fig. 2: **a)** Normalized current density distribution \( \vec{J}_z(x,y) \) in the five conductors of the inductor shown in Fig. 1a) but with 3 air gaps, calculated with the proposed model for \( f = 10 \text{kHz} \). **b)** Power loss per unit length \( P' \) in W/m, calculated with FEA (circles) and (20) (straight line). **c)** Deviation \( \nu = 100\% \cdot (P'_{\text{model}} - P'_{\text{FEA}}) / P'_{\text{FEA}} \) as a function of the frequency. The purple line indicates \( f = 10 \text{kHz} \) for which \( d = \delta \).
the case of having no yokes above and below the winding. Such a case is described in [26] as a separate outside-window-portion in a ‘double-2D’ modelling approach. The resulting relative error, calculated with FEA, is depicted in Fig. 3b). It is notable, that the deviation in the considered case does not exceed 2%. Depending on the deviation, it may be sufficient to not consider a separate outside-window model in the loss calculation. In general, the deviation depends on the conductor geometry and the number of conductors. For conductors with \( h_f \gg d_t \), an error less than 2% can be expected. If the conductors have a more quadratic cross-section or even \( d_t > h_f \) [16], higher deviations must be expected. When a comparatively high number of conductors is considered, the skin effect at the top and bottom conductor edges, which is not considered in the analytical model, results in higher deviations. In [2], this is taken into account by introducing a porosity factor. This porosity factor effectively changes the electrical conductivity, and the penetration depth of the magnetic field, respectively [27]. In the proposed model, the introduction of such a porosity factor would influence the air gap field as well, which would result in errors in the air gap field contribution. Therefore, a porosity factor is not introduced in this approach.

Another important aspect is the expected computing time of the model. For determining the time, the model is calculated 100 times for each of a wide range of number of turns (\( N = 1 \ldots 100 \)) and the obtained times are averaged per \( N \). The result is shown in Fig. 3c). For \( N < 20 \) the computing time is below 10 ms, which is reasonable for optimization purposes. Above that, the computing time scales rather badly, since the equation system that must be solved scales with \( 4N+2 \). Therefore, the models computation time depends directly on the number of turns. In addition, when no sinusoidal excitation is given, a necessary Fourier analysis of the exciting waveform will further increase the computing time. In [28], an investigation is conducted, that shows that the magnetic field attenuates significantly in the first conductor. This knowledge can be used to estimate only the additional loss contribution due to a 2D field around the air gaps in the foil closest to the gaps. However, this is only possible if \( d_t > \delta \). Fig. 2a) shows that not only the first foil is affected by the 2D field, if \( d_t = \delta \). In fact, when the foil thickness is optimized for a specific frequency, \( d_t < \delta \) has to be expected [20]. Tests with several different foil thicknesses showed, that in a realistic scenario, the 3-5 foils closest to the air gaps are affected. However, the equation system (15) can only be solved for the complete geometry, considering all conductors, since the boundary conditions are set at the core window walls. Therefore, considering the field in all foils is necessary.

5 Conclusion

In this paper, an analytical model for the current density distribution, as well as an analytical formulation for the eddy current losses per unit length due to a 2D magnetic field distribution in foil windings exposed to an arbitrary number of symmetrically distributed air gaps in the center leg is provided. The model considers the eddy currents in foil conductors caused by skin, proximity and fringing effects. It has the advantage of minimizing the computational effort while being accurate in comparison to FEA models. The deviation compared with FEA simulations for two example inductors (one air gap and three air gaps) is around \( \pm 1 \% \) over the tested frequency range from 1 Hz to 10 MHz. This makes the model applicable in optimization tasks considering gapped inductors with foil windings. The achieved computation times are in the range of milli seconds for sinusoidal exciting waveforms and a reasonable number of conductors. Longer computing times are to be expected if other exciting waveforms have to be considered by means.
of Fourier analysis. The number of conductors also influences the computing times.

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References


