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A MACHINE LEARNING FRAMEWORK FOR ALLEVIATING BOTTLENECKS OF PROJECTION-BASED REDUCED ORDER MODELS

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Digital twins and virtual representations have become critical components in structural health monitoring applications of real-life engineering systems. These numerical surrogates should capture nonlinear effects and accurately recover the involved dynamics, whilst providing a substantial reduction of computational resources and a near real-time evaluation [7]. In this context, Reduced Order Models (ROMs) have emerged as efficient low-order representations, featuring in various monitoring applications ranging from vibration control to residual life estimation.

A dominant approach to derive physics-based ROMs is projection-based reduction. This exploits Proper Orthogonal Decomposition, or similar projection techniques, to approximate the subspace where the principal components of the dynamic response lie [2]. To achieve this, POD is applied on a series of response time series produced from the full-order model evaluation, henceforth termed as snapshots. This leads to the assembly of a basis, subsequently employed to project the governing equations in a linear subspace, thus enabling the propagation of the dynamics in a reduced coordinate space. Integrating the projected, low-order system of equations forward in time can potentially lead to substantial computational savings, while maintaining an accurate approximation, which additionally comes with physical connotation. The ROM is additionally coupled with a second-tier approximation termed hyper-reduction to address the bottleneck of evaluating the nonlinear terms on the reduced coordinate space [3].

Although this class of reduction strategies has been proven effective, both in terms of approximating nonlinear dynamic behavior and providing an efficient evaluation with respect to computational time, the derived ROMs suffer from two significant bottlenecks [6]. As already described, the ROM performance is directly related to the reconstruction error of the actual response manifold using the POD projection basis. Thus, the linear nature of the POD operator (or equivalent ones) imposes accuracy limitations, as it constrains the dynamics to evolve in a linear approximation of the original manifold. As a result, the nonlinear effects of the original, system behavior may not be adequately captured, especially in case of high sensitivity to system parameters. Several approaches have been proposed to circumvent this bottleneck [1, 2], though without directly addressing the limitations on the nature of the commonly adopted projection operator.

In such physics-based ROMs, the hyper-reduction scheme seems to pose the primary source of error propagation in time. Relying on either an additional POD-based reduction on the internal forces manifold [4], or on a sparse weighted evaluation of the nonlinear terms [3], it necessarily sacrifices the nonlinear mapping’s accuracy to achieve a substantial reduction in computational time [8]. As the nonlinear effects dictate the dynamics of the system, this second-tier approximation seems to be indispensable in terms of efficiency and, at the same time, comprise a critical, weak link of the ROM framework when aiming for higher accuracy.

Our work explores the potential of employing machine learning tools to derive a reduction framework able to address and potentially overcome the aforementioned bottlenecks, drawing inspiration from similar works in this context [5, 6, 7]. A conceptualization of our approach is depicted in Figure 1. We capitalize on two fundamental aspects. On the one hand, we tackle the reconstruction error minimization via use of autoencoders in order to approximate the nonlinear response manifold, as an initial step towards substituting POD-based projection. This attempts to define a more appropriate, nonlinear scheme to approximate the respective response manifolds leading to more accurate approximations.

At the same time, our efforts scrutinize the possibility of exploiting LSTM-based neural network schemes as a more accurate surrogate of the nonlinear mapping on the reduced space, as opposed to adoption of hyper-reduction. The update of the nonlinear terms in the equation of motion when integrating forward in time takes place on the low-order
Step 1: Parametric input states

Step 2: Time Integration of ROM

\[ \forall t_i, \quad \tilde{u}(t_i) \Rightarrow \tilde{V} \tilde{u}(t_i) \]

Step 3: Assemble matrices and evaluate Equations of Motion

\[ M_i(\tilde{u}(t_i)) + \tilde{c}_i(\tilde{u}(t_i), \dot{\tilde{u}}(t_i), p) = 0 \]

Step 4: Compute Residual on Equations and “predict” correction

\[ i f \quad R_i(\tilde{u}(t_i)) > \text{tol} \quad \Rightarrow \quad \tilde{u}_r(t_i) \]

Step 5: Employ updated solution to re-assemble nonlinear terms & matrices

\[ \tilde{u}_r(t_i) = V^T \tilde{c}_i(\tilde{u}(t_i), \dot{\tilde{u}}(t_i), p) \]

Notation:

- \( \mathcal{U} \): Full-order dimension
- \( N_p \): Number of training samples
- \( N_t \): Number of simulated timesteps
- \( M \): Mass matrix
- \( \Gamma \): External forcing
- \( \tilde{u} \): Response solution

Nonlinear terms still scale with full dimension

Use machine learning to derive surrogate mapping

LSTM-/AE-based surrogate

Avoid back-and-forth projection & hyper-reduction approximation of \( g \)

Achieve efficiency using surrogate mapping

Try to retain order of accuracy

Figure 1: Conceptual demonstration of approach under investigation

dimension, thus potentially implying a superior framework in terms of efficiency. The proposed ROM features are validated with respect to their ability to outperform the linear, POD-based ROM in a multi-degree of freedom shear frame under earthquake excitation featuring parametrized hysteretic nonlinearities. The generalization potential of the derived ROM as well as the accuracy trade-off are also discussed.

This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-NA0003525. SAND2021-3704 A

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