Comparison of plate and solid spectral element modeling of composite delamination for guided wave simulations

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Title: Comparison of plate and solid spectral element modeling of composite delamination for guided wave simulations

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ABSTRACT

As composite laminates have become essential to numerous modern industries, structural health monitoring (SHM) strategies have to be implemented in order to manage the aging of structures made out of composite materials. In this paper, we present and compare two approaches involving solid and plate Spectral Elements (SE) in the time domain, with respect to their ability of modeling composite delamination: a task which lies in the context of model-based data processing for non destructive evaluation techniques with Guided Waves (GWs). To this end, we review the multi-layer flat shell delamination model [1], and propose a single-layer alternative by extending the Layered Solid Spectral Element (LSSE) [2] method by means of partition of unity enrichment functions [3]. These approaches are then compared in a numerical example with different damage scenarios. Finally, conclusions on the benefits and draw-backs of these two strategies are presented.

INTRODUCTION

Composite laminates have driven tremendous progress in the aerospace, automotive and wind energy industries. Their positive qualities mainly consist in a high strength to weight ratio, as well as resistance to corrosive processes and fatigue-related damage. Delamination is a frequent concern in the exploitation of these materials. It can be caused by even small impacts during manufacturing or operation and is hardly detectable by visual inspection. To overcome this, damage detection methods relying on GWs can be employed, which typically require transient analysis of several numerical realizations of the damaged structure. This procedure poses two main challenges, namely the requirement for high resolution models, imposed by the high frequency of GWs, and the necessity of adequately modeling the laminates cross section.

The first challenge can be addressed with the SEM, due to its favorable properties in both, space and time discretizations: in space, high order Ansatz polynomials can be used; while in time, fast explicit solvers can be readily exploited by virtue of the nodal

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Figure 1: Different SE models for laminated composite plates. The shown solid elements are of linear order along the cross section purely for clarity of the image, and laminae are represented by dashed lines. (a) Laminae-conforming SE meshing of the cross section. (b) LSSE: A single solid SE spans the whole cross section. (c) The composite Plate is modeled by a flat shell SE.

quadrature method [4, 5], which renders the mass matrix diagonal by construction. The second challenge stems from the fact that again large models are necessary in order to capture the complex displacement profile resulting from the anisotropy of the material, as suggested by the use of laminae-conforming SE in Fig. 1(a). Therefore, the models shown in Fig. 1(b)-(c) are typically used, in an effort of gradually reducing the number of Degrees Of Freedom (DOFs) while conceding the smallest possible loss in accuracy. If a Layered Solid Spectral Element (LSSE) formulation is adopted (Fig. 1(b)), the discontinuity of material properties can be resolved via an integration rule which accounts for each ply, and the displacement field along the cross section will be a continuous polynomial according tho the chosen SE order. Finally, if a plate or shell theory is applied (Fig. 1(c)), the dependency of strains and displacements in the vertical direction is resolved analytically, thus reducing the model to two dimensions.

PROBLEM STATEMENT

For the context of this work, let us consider a domain $\Omega$ in the Cartesian coordinates system of $d = 3$ dimensions. The problem at hand is described by the elastodynamics equations, which can be expressed in weak form as:

$$
\int_{\Omega} \rho \ddot{u}(t) \cdot v \, d\Omega + \int_{\Omega} \sigma(u(t)) : \epsilon(v) \, d\Omega = \int_{\Gamma} p_s(t) \cdot v \, d\Gamma
$$

(1)

where $u(t)$ represents the displacement solution at time $t$, $\rho$ is the material density, $v$ is the trial function, $\sigma$ is the Cauchy stress tensor, and $p_s(t)$ is a surface traction acting on the domains boundary $\Gamma$. We should mention that the effect of material damping is neglected in this work, and that the delamination surface $\Gamma_c \in \Omega$ is considered as a free boundary. Importantly, we should also note that Hooke’s Tensor $C_\theta$, governing the stress-strain relation:

$$
\sigma = C_\theta \epsilon
$$

(2)

is generally discontinuous in stacking direction due to the different materials employed as laminae and/or their different orientations.
SPECTRAL ELEMENT MODELING OF COMPOSITE DELAMINATION

Due to the previously mentioned qualities, Patera’s SEM [6] has seen widespread adoption in numerous fields of computational mechanics, including structural dynamic analysis. We refer the reader to [7, 8, 5] for a presentation and discussion of this method, which might be summarized as a finite element method where the Ansatz space is supported at Chebyshev or Gauss-Lobatto nodes. Composite material modeling with the SEM in essence consists in using the resulting shape functions to discretize Eq. [1] and adopting measures to account for material anisotropy in the integration of the weak form. In this contribution, LSSE are used for the solid case, while Mindlin–Reissner first-order shear deformation theory is employed in the shell formulation [9].

Delamination with Plate Elements

With the reduction to two topological dimensions, the use of shells can greatly simplify procedures such as damage-conforming meshing and node duplication. Therefore, delamination can be represented by two shell layers that are connected only at the boundary of the damaged area, as shown in Fig. 2. These elements are computed by considering the corresponding, reduced, number of laminae, and their offset form the neutral axis of the plate. This approach, which is detailed in Ref. [1], has the great advantage of preserving the computational efficacy of the model, as the increase in nodes is usually modest, and the beneficial properties of the mass matrix are maintained.

Delamination with Solid Elements

On the other hand, the aforementioned procedure is not well suited for solid elements, for two main reasons. Firstly, damage-conforming meshing would require reverting to the layer-wise model displayed in Fig. 1(a), which is inefficient. Secondly, singularities in the stress field near the tip of the delamination cannot be captured by the polynomial Ansatz space, and thus some accuracy will be lost, in spite of the very large model size. EXtended Finite Element Models (XFEM) [3] elegantly overcome these issues by providing mesh-independent damage descriptions in the form of signed distance functions (a.k.a. level sets [10]), and including damage-related features in the solution space by
means of enrichment functions. This method is only summarized here. More details can be found in Ref. [11]. With this approach, the displacement assumption in the reference system can be extended as follows:

\[ u(\xi, t) = \sum_{i=1}^{n} N_i(\xi) u_i(t) + \sum_{j=1}^{n_{\text{enr}}} N_j(\xi) F_j(\phi(\xi), \psi(\xi)) a_j(t) \]  (3)

where \( n_{\text{enr}} \) is the number of enriched nodes within the element, \( F_j \) is the enrichment function applied to node \( j \), and \( a_j(t) \) is the corresponding enrichment parameter belonging to the solution vector \( u_s \). Although sophisticated choices of enrichment functions are available [12], for simplicity we employ a single enrichment function, according to a topological enrichment scheme [13]:

\[ F_j = \sqrt{r} \sin \frac{\theta}{2}, \quad \text{with:} \quad r = \sqrt{\phi^2 + \psi^2}, \quad \theta = \arctan \frac{\phi}{\psi}. \]  (4)

We denote \( \phi(\xi) \) and \( \psi(\xi) \) as the normal and tangent level sets, respectively [14], which, in practice, are interpolated in the reference system via the element shape functions [15, 16].

In the global Cartesian system, the delamination surface is then defined as:

\[ \Gamma_c = \{ x | x \in \Omega, \phi(x) = 0, \psi(x) \leq 0 \}. \]  (5)

Figure 3 displays a section of the integration mesh along with the the zero iso-surfaces of the level sets for the damage configuration that will be studied in the next section. In enriched elements, partitions conforming with both, the laminae and the crack are deployed, due to the fact that \( F_j \) introduces an additional discontinuity in the integration of the weak form. In absence of damage, laminae-conforming hexahedral element partitions suffice, as in the LSSE method. We should also mention that, due to these special integration rules, nodal quadrature of the mass matrix can only be applied on pristine elements (and only as long as \( \rho \) is constant across laminae). To overcome this, we adopt a novel moment fitting procedure similar to Ref. [17], which is under ongoing study, to restore the diagonal property of the mass matrix.
NUMERICAL EXAMPLE

Figure 4 shows a square laminated plate consisting from four Glass-epoxy laminae stacked in the configuration \( \theta_s = [45/-45/45/-45] \), whose engineering constants have been taken from Ref. [18] and are resumed in Table I. Four sensor-actuators \( S_{1-4} \) are deployed, and actuation is modeled by a vertical unit load \( p_0 = 1 \text{ [N]} \) acting at \( S_1 \), and modulated by the Hann window:

\[
p(t) = p_0 \sin(\omega t) \sin^2\left(\frac{\omega t}{2n}\right), \quad t \in \left[0; \frac{n}{f}\right]
\]

with \( \omega = 2\pi f, f = 50 \text{ [kHz]} \) and \( n = 3[-] \). In Fig. 4, both delaminated regions are represented by ellipses of diameters 20 [mm] and 10 [mm], centered at \( D_1 \) and \( D_2 \) over the \( x-y \) plane. In both cases, the damage affects the upper two laminae. To ease the comparison, we consider these two cases separately, although, in principle, both models would allow for coexisting delaminations. The compared SE models are of order \( p = 3 \) for solids and \( p = 5 \) for shells. For solids, this choice is justified by its efficiency in terms of storage memory usage [19], as well as the necessity for providing an adequate polynomial assumption for the solution along the cross section. For shells instead, this choice is driven by the improved accuracy, avoidance of locking behavior, and the availability of high-performance mesh-coloring algorithms [9]. Both instances of

![Figure 4: Four-layered composite plate of dimensions \( l_x = 500 \text{ [mm]}, l_y = 500 \text{ [mm]}, l_z = 4 \text{ [mm]} \). Sensors \( S_{1-4} \), of which \( S_1 \) is used for actuation. Two different damage scenarios are represented by elliptically-shaped delaminations \( D_1, D_2 \).](image)

<table>
<thead>
<tr>
<th>( E_1 ) [GPa]</th>
<th>( E_2, E_3 ) [GPa]</th>
<th>( G_{12}, G_{13} ) [GPa]</th>
<th>( \nu_{12}, \nu_{13} ) [-]</th>
<th>( \nu_{23} ) [-]</th>
<th>( \rho ) [kg/m(^3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.40</td>
<td>17.70</td>
<td>5.83</td>
<td>0.278</td>
<td>0.400</td>
<td>2400</td>
</tr>
</tbody>
</table>
the SEM are implemented in-house and are solved explicitly with the Central Difference Method (CDM).

**Results**

In Fig. 5, a selection of time histories at $S_2$ and $S_4$ is presented. From the pristine case (Fig. 5(a)) it can be established that the initial pulse reaches the sensors after roughly $7 \cdot 10^{-5} [s]$, followed by a window of rest, before reflections from the boundaries are recorded. One can observe that shell and solid models are in very good agreement, which speaks to the similarities between both formulations: although shells rely on first order shear deformation theory, a shear correction factor is employed to account for parabolic shear strain distribution, an assumption that can be matched by a solid element of third order along the thickness. Due to the similarities between pristine simulations, the difference observed in the delamination case can be attributed to the somewhat important differences in the modeling of this damage. In Figs. 5(c) and 5(d) it can be seen that, at $S_1$ and $S_2$, delamination $D_1$ can be detected due to minor reflections preceding the ones from the boundary. The proximity of the damage to $S_1$, indicated by the comparatively shorter arrival time of the corresponding reflections, constitutes in general useful insight for damage localization. It is interesting to note that both damage models are consistent in this regard, as they would presumably deliver comparable arrival times, despite the different magnitudes and frequencies of the reflections. For the case of $D_2$, a better agreement of the models can be observed at $S_2$ (Fig. 5(b)), which coincides with the wider side of the delamination being reflective.

![Figure 5: Comparison of solid and shell SEM. Time histories for a selection of sensors and pristine (a), as well as delaminated (b-d), scenarios.](image-url)
CONCLUSIONS

In this contribution, we review and compare two different applications of the SEM in the modeling of delamination. The numerical investigations reveal an excellent agreement in simulation of a pristine material, with qualitatively comparable results achieved for the damaged case. In light of these results, we suggest that the choice between multi-layered shells or enriched LSSE depends on the intended application. If a spectral shell implementation is readily available, then delamination can be modeled with minor interventions in the code and some preprocessing work. Moreover, the presented comparisons are a testament to the remarkable effectiveness of shell formulations in the modeling of GW. The limitations of this approach are a restriction to the capabilities of the meshing software of choice (Gmsh in our case), to the orientation of the damage (which must be parallel to the midsection of the plate) and to the representation of anti-symmetric Lamb wave modes.

On the other hand, the presented enriched solid formulation is less efficient in this setup due to the absence of an embedded bending assumption, which dominates the solution of this type of wave. Secondly, its implementation is more involved due to the need for element partitioning and mass lumping routines, which must also guarantee the stability of explicit time integration. However, a more versatile model is achieved in this fashion: all sorts of damage orientations are possible, and can be generated at runtime, without the need for external unstructured meshing routines. Lastly, it is important to keep in mind that all investigated models are based on a polynomial displacement assumption along the cross section, however, in general, a piece-wise continuous field might also be assumed. Although multiple layers of SE can be used to include such an assumption in the solution of the weak form, both of the investigated methods enable to drastically reduce the model size, and thus have an enhanced applicability in a SHM context.

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