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CONTROL-ORIENTED MODEL VALIDATION

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Abstract.
Model validation is a means of assessing model quality with respect to experimental data. In a robust control context this amounts to determining whether or not a perturbation model is consistent with observed data. This paper offers a survey of recent results on model validation for $\mathcal{H}_\infty$ compatible perturbation models.
Several model paradigms and experiment frameworks are considered. The first is a frequency-domain setting for the data. More recent results deal with discrete-time models and time-domain experimental data. The most relevant framework, continuous-time systems and models, with discrete-time sampled data, is dealt with last. Each of the model invalidation tests offered involve tractable convex optimization problems.

1. INTRODUCTION

Model validation is the assessment of the quality of a given model with respect to experimental data. In essence, the problem is to determine whether or not the model is consistent with the experimental observation. As observed in [25], one can never "validate" models because of the impossibility of testing all experimental conditions and inputs. A model is said to be "invalidated" if a particular input-output datum is not consistent with the model.

Many modern multivariable control system design methods such as $\mathcal{H}_\infty$ and ℓ_1 optimal control (see [6,9,35,5,3]) begin with *perturbation models*. These models consist of a nominal input-output model together with a description of the uncertain dynamics and the noises and/or disturbances affecting the plant.

Traditional methods for model validation (see [18]) have involved whiteness and correlation tests of residuals. While these methods successfully deal with nominal models where any discrepancy between the true plant and the model is attributed entirely to stochastic noises and disturbances, they are not appropriate for perturbation models. There is a compelling need to develop tools to effectively marry classical identification methods with modern robust control techniques. It is this need that has driven much of the recent work on *control-oriented* system identification (see for example [8,23,12,10,19,1,27,11]).

In this paper we focus on model validation for $\mathcal{H}_\infty$ compatible perturbation models which is only one component of control-oriented system identification. While similar results are also available for ℓ_1 compatible perturbation models, we will not discuss these here. The remainder of this paper is organized as follows. After establishing notation in the next section, we discuss perturbation models in Section 3 and we formulate the general model validation problem in Section 4. Next, in

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Section 5, we discuss several extension theorems that form the basis for model validation. Following this, in Section 6, we offer model validation results in the context of frequency domain input-output data. More recent work dealing with time-domain data for discrete-time perturbation models is treated in Section 7. Section 8 presents the sampled data formulation of the model validation problem. This work has the most immediate connection with practical problems.

The discrete-time results offered in this paper are closely related to the recent work of Zhou and Kimura [38] where perturbation models are constructed from input-output data using similar extension theorems. There is also a close connection between model validation and robust parameter identification problems (see for example [14,15,36]), and failure detection problems (see for example [34]). There has also been some recent work on statistical methods for model validation of perturbation models (see [16]). We will not explore these connections in this paper.

This paper is a summary of the results in [32,31,24,29,26]. All proofs are omitted and the interested reader may consult the above references for details.

2. PRELIMINARIES

We will suppress the sizes of various matrices, vectors, and spaces throughout for clarity. For a sequence of vectors $v = \{v_0, v_1, \cdots, v_{N-1}\}$, let U denote the associated lower block Toeplitz matrix defined as

$$V = \begin{bmatrix} v_0 & 0 & 0 & \cdots & 0 \\ v_1 & v_0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ v_{N-1} & v_{N-2} & v_{N-3} & \cdots & v_0 \end{bmatrix} \qquad (1)$$

Let $\mathcal{S}^m$ denote the set of one sided sequences with elements in $\mathbf{R}^m$. Define the l-step truncation operator,
$\pi_l : \mathcal{S}^m \longrightarrow \mathcal{S}^m$, by $\pi_l(u_0, u_1, \cdots, u_{l-1}, u_l, \cdots) = (u_0, u_1, \cdots, u_{l-1}, 0, 0, \cdots)$. For continuous time signals, $u(t) \in \mathcal{L}_2[0,\infty)$, we define the truncation operator Π_T analogously.

Let ℓ_2 and $\mathcal{L}_2$ denote the usual Hilbert spaces of vector-valued square-summable sequences and square-integrable functions respectively equipped with the usual norms, $\|\|_2$. We shall deal exclusively with causal, stable operators with discrete-time and/or continuous-time inputs and outputs. For an operator H, we denote its induced 2-norm as $\|H\|_{i2}$. Also, H^* denotes the adjoint operator.

3. PERTURBATION MODELS

Many modern multivariable control design methods including $\mathcal{H}_\infty$ and ℓ_1 optimal control begin with a *perturbation model* for the physical plant. This perturbation model consists of a nominal input-output model together with a description of the uncertain dynamics Δ and the noises and/or disturbances n.

A general paradigm for perturbation models that has emerged over the last decade is the *linear fractional transformation (LFT)* model

$$y = F_u(P, \Delta) \begin{bmatrix} n \\ u \end{bmatrix}, \quad \Delta \in \boldsymbol{\Delta}, \; n \in \mathbf{N}, \qquad (2)$$

where $F_u(P,\Delta)$ denotes the feedback connection of the systems P and Δ as shown in Fig. 1. In this model, the relation between observed signals u and y, is described by a known, nominal linear plant P, perturbed by a unknown system Δ to account for model errors. In addition to the uncertainty Δ, the output may be further corrupted by an unknown disturbance d.

Bounds on the system uncertainty and disturbances are specified by the sets $\boldsymbol{\Delta}$ and $\mathbf{N}$, respectively. In this paper, we will consider norm-bounded, structured uncertainty sets of the form,

$$\boldsymbol{\Delta} := \{\Delta = \mathrm{diag}(\Delta_1, \ldots, \Delta_K), \|\Delta\|_{i2} \le 1\} \qquad (3)$$
$$\mathbf{N} := \{n = (n_1, \ldots, n_M), \; \|n_i\| \le 1\}. \qquad (4)$$

Other scaling factors and weights on the disturbances and uncertainties can be incorporated into the plant model P.

The LFT paradigm successfully encompasses a broad variety of modeling uncertainties including uncertain actuator and sensor dynamics, disturbances and measurement noise, real parametric errors etc. (see [22] for details). Current model validation procedures, however, are unable to address LFT models in their full generality. In the remainder of the paper, we will restrict our attention to uncertainty models where all the uncertainty components are LTI or all the components are LTV. Mixed and real-parametric uncertainty structures are considerably more difficult to handle, and will not be considered in this paper.

Remark 1 Since we are restricting our attention to bounded, deterministic disturbances n, the effects of the noise can be incorporated into the system uncertainty structure. As a consequence, we can, without loss of generality, ignore the disturbance set $\mathbf{N}$ entirely.

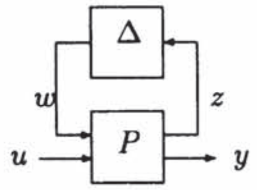

Fig. 1. Linear fractional uncertainty model with disturbances

The effect of the disturbances can be modeled as system uncertainty as follows. For each disturbance component n_i, one can construct an appropriate fictitious input v_i such that

$$\|n_i\| \leq 1 \Leftrightarrow \exists \|\Delta_i\|_{i2} \leq 1, n_i = \Delta_i v_i.$$

The disturbances n_i can then be incorporated into the system uncertainty by augmenting the input u with the signals v_i, and the uncertainty structure $\mathbf{\Delta}$ with the components Δ_i. In ignoring the disturbances n, the perturbation model (2) reduces to

$$y = F_u(P, \Delta)u, \quad \Delta \in \mathbf{\Delta}. \tag{5}$$

4. PROBLEM FORMULATION

The general model validation problem we treat in this paper is as follows. Consider the LFT perturbation model in Section 3, defined by a nominal plant P, a structured uncertainty set $\mathbf{\Delta}$ and a structured noise set $\mathbf{D}$. Given observed data (u, y), the validation problem is:

MV: Does there exist Δ and n such that

$$y = F_u(P, \Delta)\begin{bmatrix} n \\ u \end{bmatrix}, \Delta \in \mathbf{\Delta}, n \in \mathbf{N}\ ?$$

That is, the model validation problem is to determine if there the data (u, y) is consistent with the plant P, within the uncertainty bounds defined by the sets $\mathbf{\Delta}$ and $\mathbf{N}$. In the event such n and Δ exist we shall say that the perturbation model P is *not invalidated*. Otherwise, the perturbation model is said to be *invalidated*.

Unfortunately, for general structured uncertainties, the LFT model validation problem, MV, is computationally difficult. We will instead consider a related validation problem based on D-scaling. To state this problem, we will first ignore the disturbances set $\mathbf{N}$ as discussed in Remark 1. Also, corresponding to the uncertainty structure in (3), we define the set of *D-scales*,

$$\mathbf{D} := \{D = \text{diag}(d_1 I, \ldots, d_K I)\},$$

where the dimensions of the blocks correspond to the dimensions of the uncertainty blocks Δ_i in (3).

The D-scaled validation problem is: given a nominal plant P, uncertainty set $\mathbf{\Delta}$ and observed data (u, y),

MV-D: For all $D \in \mathbf{D}$, does there exist a Δ such that

$$y = F_u(P, \Delta)u, \ \|D\Delta D^{-1}\| \leq 1\ ?$$

The justification for considering the problem MV-D is as follows. First observe that, for any $\Delta \in \mathbf{\Delta}$,

$$\|D\Delta D^{-1}\| \leq 1, \quad \forall D \in \mathbf{D}.$$

Consequently, the feasibility of MV-D is a *necessary* condition for the feasibility of the validation problem MV.

The feasibility of MV-D, however, does not imply the feasibility of the validation problem MV. Nevertheless, in the context of validating models for robust control, there is little practical loss in considering the problem MV-D. The reason is that the standard procedure for robust control design is based, not on stabilizing against the set $\mathbf{\Delta}$, but rather providing robust stability against a larger set

$$\{\Delta \ : \ \|D\Delta D^{-1}\| \leq 1\} \tag{6}$$

for some $D \in \mathbf{D}$. Given this, it is only necessary to determine, if for any choice of D-scales, D, the model is consistent with Δ of the form (6). This is precisely the problem MV-D.

5. EXTENSION THEOREMS

Given a perturbation model, the input-output datum u, y places constraints on the inputs z and the outputs w of the uncertain dynamics $\Delta \in \mathbf{\Delta}$. The validation problem therefore reduces to the problem of determining whether or not there exists an operator $\Delta \in \mathbf{\Delta}$ that interpolates z to w subject to z and w being consistent with the equations for P and observed data (u, y). We refer to the problem of determining whether a there exists a Δ interpolating z to w as an *extension problem*. More precisely,

> Given a set of operators $\mathbf{\Delta}$ and signals w, z, does there exist $\Delta \in \mathbf{\Delta}$ such that $w = \Delta z$?

We now present several extension theorems. In each case, these theorems reduce the issue of existence of the requisite *operator* Δ to the existence of *signals* which satisfy certain computable constraints.

Our first extension theorem is in the context of frequency-domain data. Frequency-domain data is given in the form

$$(W, Z) := \{(W(e^{j\omega_n}), Z(e^{j\omega_n})),\ n = 0, \ldots, N-1\} \quad (7)$$

at a finite set of frequencies ω_n. Frequency-domain data is given in an analogous format for continuous-time signals.

Theorem 2 *Given frequency-domain sequences, (W, Z) as in (7), there exists an stable, causal LTI operator Δ with $\|\Delta\|_{i2} \leq 1$ such that*

$$W(e^{j\omega_n}) = \Delta Z(e^{j\omega_n}), \quad \forall n,$$

if and only if $|W(e^{j\omega_n})| \leq |Z(e^{j\omega_n})|$ for all n.

The following result offers a time-domain extension result for bounded LTI operators. Time-domain data is given in the form of a finite data record,

$$(w, z) := \{(w_m, z_n),\ n = 0, \ldots, N-1\}. \quad (8)$$

Theorem 5.1 *Given sequences, (w, z) as in (8), there exists a stable, causal, LTI operator Δ, with $\|\Delta\|_{i2} \leq 1$ such that $\pi_N w = \pi_N \Delta z$ if and only if*

$$W'W \leq Z'Z, \quad (9)$$

where W and Z are the block Toeplitz matrices associated with w_n and z_n.

For discrete-time LTV operators, we have the following extension result.

Theorem 5.2 *Given sequences, (w, z) as in (8), there exists a stable, causal, LTV operator Δ, with $\|\Delta\|_{i2} \leq 1$ such that $\pi_N w = \pi_N \Delta z$ if and only if if and only if*

$$\|\pi_n w\|_2 \leq \|\pi_n z\|_2, \quad \textit{for all } n = 1, \ldots, N.$$

The computational requirements imposed by this condition are significantly less than those imposed by the LTI perturbation condition (9).

We shall also need an extension theorem for continuous-time operators with sample and hold input-output data. This theorem is somewhat more complicated than the earlier ones. We begin by introducing some preliminary concepts. Define the sample operator S_T and the hold operator H_T as

$$S_T : \mathcal{L}_2 \to \ell_2 : y \to (y_0, y_T, y_{2T}, \cdots)$$
$$H_T : \ell_2 \to \mathcal{L}_2 : (u_0, u_T, u_{2T}, \cdots) \to u,$$

where $u(t) = u_k$, for $t \in [kT, KT + T)$. Fix continuous-time operators P_v and P_z, with P_v strictly causal, and consider the set of discrete-time operators,

$$\mathbf{\Omega} = \{S_T P_v \Delta P_z H_T \ :\ \|\Delta\|_{i2} \leq 1\}$$

Given a discrete-time, input-output datum, p_k, q_k, $k = 0, \ldots, N-1$ compute the sequences $\hat{v}$, $\hat{z}$ as

$$\hat{v} = (S_T P_v P_v^* S_T^*)^{-1/2} p \quad (10)$$
$$\hat{z} = (H_T^* P_z^* P_z H_T)^{-1/2} q \quad (11)$$

where * denotes the adjoint operator. We remark that the operator equations above are solvable and state-space formulae for conducting these computations are available in [30].

The extension theorem is now stated in terms of the l_2 sequences, $\hat{v}$ and $\hat{z}$, and bears some similarity to discrete-time extension theorem above.

Theorem 5.3 *Given sequences, p_k and q_k, $k = 0, \ldots, N-1$, construct $\hat{v}_k$, $\hat{z}_k$, as in (10) and (11). If $\hat{V}'\hat{V} \leq \hat{Z}'\hat{Z}$, then there exists a stable, LTI, operator $\Omega \in \mathbf{\Omega}$, such that, $\pi_{N-1} v = \pi_{N-1} \Omega z = \pi_{N-1}(S_T P_v \Delta P_z H_T) z$.*

This result becomes necessary and sufficient in the limit as $T \to 0$.

6. FREQUENCY DOMAIN METHODS

In the frequency-domain problem, we assume the nominal plant P and uncertainty Δ are LTI, The data for the problem consists of N vector-valued frequency-domain input-output samples,

$$(U, Y) := \{(U(e^{j\omega_n}), Y(e^{j\omega_n}))\ :\ n = 0, \ldots, N-1\} (12)$$

at frequencies of frequencies ω_n. As stated in Remark 1, we will ignore the disturbances, and assumed they are embedded in the uncertainty model.

In order that the frequency-domain data from the closed-loop system of P and Δ is well-defined, we make the following assumption.

Assumption 6.1 *Consider the LFT uncertainty model (5) defined by an LTI nominal plant P and uncertainty structure $\mathbf{\Delta}$ in (3). For all causal, LTI $\Delta \in \mathbf{\Delta}$, the feedback connection of P and Δ is well-posed and stable.*

In absence of this robust stability assumption, signals may not be bounded and the frequency-domain data may not exist. For validation of unstable plants, it is necessary to collect the data in closed-loop with a stabilizing compensator, and incorporate the compensator into the plant model P_0. Of course, this implicitly assumes that that the compensator is LTI.

To state the solution to the frequency-domain validation problem, we first partition the transfer function P,

$$\begin{bmatrix} z \\ y \end{bmatrix} = P(z) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} (z) \begin{bmatrix} w \\ u \end{bmatrix}.$$

Also, let $\mathbf{D}_{LTI}$ be the set of *frequency-dependent D-scales*,

$$\mathbf{D}_{LTI} := \left\{ D \ : \ \begin{array}{c} D \text{ is causal, stable LTI,} \\ D(e^{j\omega}) \in \mathbf{D},\ \forall\omega \end{array} \right\}$$

where

$$\mathbf{D} := \{ D = \text{diag}(d_1 I, \ldots, d_K I) \},$$

where each d_i is a complex scalar and the dimensions of the blocks correspond to the dimensions of the uncertainty blocks Δ_i in (3).

For $D \in \mathbf{D}_{LTI}$, define the frequency-dependent matrix,

$$G(e^{j\omega}) := \begin{bmatrix} G_{11} & G_{21}' \\ G_{21} & G_{22} \end{bmatrix} (e^{j\omega})$$

where

$$\begin{aligned} G_{11} &:= E'E - Z_0' D' D Z_0 \\ G_{21} &:= -P_{21}' E - D' D Z_0 \\ G_{22} &:= D'D - P_{11}' D' D P_{11} + P_{21}' P_{21} \end{aligned}$$

and $E = Y - P_{22}U$ and $Z_0 = P_{21}U$. We are now prepared to state our main result, which shows that the the D-scaled frequency-domain validation problem can be formulated as a linear matrix inequality (LMI).

Theorem 6.2 *Consider the LFT uncertainty model (5) defined by an LTI plant P and uncertainty set $\mathbf{\Delta}$ satisfying Assumption 6.1. Let (U, Y) be frequency domain data as in (12). Then the following are equivalent,*

a) For all $D \in \mathbf{D}_{LTI}$, there exists a causal, LTI, stable Δ with $\|D\Delta D^{-1}\| \leq 1$, and

$$Y(e^{j\omega}) = F_u(P, \Delta) U(e^{j\omega_n}), \quad \forall n.$$

b) For all n, there does not exist a $D \in \mathbf{D}$, such that $G(e^{j\omega_n}) > 0$.

The consequence of Theorem 6.2 is that the D-scaled frequency domain validation problem can be solved as an LMI. At each frequency ω_n, Theorem 6.2 requires that we check if there exists a D (dependent on ω_n) such that $G(e^{j\omega_n}) > 0$. The matrix $G(e^{j\omega})$ depends linearly on the matrix $D'D$, and therefore the feasibility problem may be evaluated as an LMI with $D'D$ as the decision variables. If, for any, frequency the LMI problem is feasible, the model is invalidated. Otherwise, the model is not invalidated.

The size of the LMI validation problem at each frequency-domain point is small. At each frequency, there are K decision variables, $d_1^2, \ldots, d_K^2$ where K is the number of uncertainty blocks. The constraint matrix $G(e^{j\omega})$ is $1 + n_z \times 1 + n_z$ where n_z is the dimension of the signal z. Observe that the LMI does not increase in size with higher state dimension, or the dimensions of the signals u and y.

Although the frequency-domain validation is computationally attractive, the main disadvantage is that the frequency-domain data must be obtained from time-domain experiments. This requires the use of large data sets and/or periodic excitation signals.

7. TIME DOMAIN METHODS

Consider again the LFT uncertainty model (5), where, in this case, the nominal plant P is LTV and given in state-space form,

$$\begin{aligned} x_{k+1} &= A_k x_k + B_{1k} w_k + B_{2k} u_k \\ z_k &= C_{1k} x_k + D_{12k} u_k \\ y_k &= C_{2k} x_k + D_{21k} w_k \end{aligned} \tag{13}$$

and initial condition $x(0) = x_0$. The initial condition is assumed to be bounded by some prescribed ellipsoid,

$$\mathbf{X_0} := \{ x_0 \ : \ x_0' Y_0^{-1} x_0 \leq 1 \}$$

for a given matrix $Y_0 > 0$. Given an initial condition x_0 and inputs u and w, we will denote the output of the system P by

$$(z, y) = P(w, u, x_0).$$

The data for the validation problem is given as a finite time-domain sequence with N vector-valued samples,

$$(u, y) := \{ (u_n, y_n),\ n = 0, \ldots, N-1 \}. \tag{14}$$

Now consider the uncertainty structure $\mathbf{\Delta}$ in (3), and corresponding to this uncertainty structure define the *time-varying D-scales*,

$$\mathbf{D}_{LTV} := \left\{ D \ : \ \begin{array}{c} D = (D_0, D_1 \ldots), \\ D_k \in \mathbf{D}, D_{k+1} \leq D_k \end{array} \right\},$$

where, similar to the frequency-domain case,

$$\mathbf{D} := \{D = \mathrm{diag}(d_1 I, \ldots, d_K I)\},$$

and each $d_i \geq 0$ is a real scalar and the dimensions of the blocks correspond to the dimensions of the uncertainty blocks Δ_i in (3). Any element $D \in \mathbf{D}_{LTV}$ can be regarded as a time-varying operator by pointwise multiplication,

$$(Dz)_k := D_k z_k.$$

The motivation for introducing the time-varying D-scales is the following Lemma.

Lemma 7.1 *Suppose Δ is any LTV operator. Then Δ is causal, stable and $\Delta \in \mathbf{\Delta}$ if and only if*

$$\|D \Delta D^{-1}\| \leq 1, \quad \forall D \in \mathbf{D}_{LTV}.$$

To state the solution to the time-domain validation problem, we introduce the following optimization problem. For all $D \in \mathbf{D}_{LTV}$,

$$\begin{aligned} F(D) := \max_{w, x_0} \ & \|\pi_N D z\|^2 - \|\pi_N D w\|^2 \\ & - x_0' Y_0^{-1} x_0 + 1 \\ \text{s.t. } & (z, y) = P(w, u, x_0) \end{aligned} \qquad (15)$$

This optimization problem is a standard $\mathcal{H}_\infty$ filtering problem. To evaluate $F(D)$, one must first solve a certain time-varying filter Ricatti equation. If the Ricatti equation does not admit a solution, the value $F(D)$ is infinite. In the case that the solution exists, the value of $F(D)$ can be computed from a simple time-varying linear filtering operation. This operation also yields the optimal w and x_0.

We are now prepared to state our main result.

Theorem 7.2 *Consider the LFT uncertainty model (5) with the LTV plant P in (13), and uncertainty structure $\mathbf{\Delta}$ in (3). Given time-domain data (u, y) as in (14),*

a) The function $F(D)$ is convex in D^2, and

b) For all $D \in \mathbf{D}_{LTV}$, there exists a causal, stable Δ with $\|D\Delta D^{-1}\| \leq 1$, and initial condition $x_0 \in \mathbf{X_0}$ such that

$$y = F_u(P, \Delta, x_0) u$$

if and only if $F(D) \geq 0$ for all $D \in \mathbf{D}_{LTV}$.

The consequence of this theorem is that the D-scaled time-domain validation problem with time-varying uncertainty blocks is equivalent to a convex programming problem. To determine if the model is invalidated by the data (u, y), one must simply minimize the convex function $F(D)$ over the time-varying D-scale $\mathbf{D}_{LTV}$. The model is invalidated if and only if the minimum is less than zero.

However, although the optimization problem is convex, the number of decision variables is large. If there are K uncertainty blocks and N data points, there are KN decision variables. In addition each function and gradient evaluation involves a linear time-varying filter operation. An exact solution of this problem, therefore, will likely be limited to modest size problems. Larger validation problems will require that the data is partitioned or the D-scale dependence on time is parameterized. Much further study will be required to fully assess the numerical aspects of this problem.

In certain special cases, such as additive uncertainty models, it can be shown that is only necessary to consider time-invariant D-scales which considerably reduces the problem size.

Certain classes of LTI uncertainty models can also be considered via convex programming, using the LTI extension theorem, Theorem 5.1. However, our experience has shown that evaluating the condition (9) is computationally prohibitive, unless the problems are extremely small.

8. SAMPLED DATA METHODS

We now consider model validation for continuous-time perturbation models with time domain input-output data. The input u is generated by the hold function, H_T, and we observe sampled values of the output y. This situation corresponds to that most commonly encountered in practice. Direct robust control system design for sampled-data perturbation models has been considered in [13,2,4,33].

For simplicity, in this section, we will only consider additive uncertainty models of the form

$$\begin{aligned} y = S_T \left(P_0 + P_w \Delta P_z\right) H_T u + S_T P_n n, \\ \|\Delta\| \leq 1, \ n \in \mathbf{N}, \end{aligned} \qquad (16)$$

where S_T and H_T are samples and holds as described in 5. Now define the residuals,

$$r_k = y_k - S_T P_0 H_T u_k,$$

where $y_k = S_T y$. The data (u, y) is consistent with the model (16) if and only if there exists v and n

$$r = S_T P_w w + S_T P_n n \, n \in \mathbf{N} \qquad (17)$$

$$w = \Delta P_z H_T u. \tag{18}$$

The constraint (17) is linear in w and n.

Using the techniques described in Section 5, we can rewrite the above constraints in discrete-time as,

$$r = (S_T P_w P_w^* S_T^*)^{1/2} \hat{w} + (S_T P_n P_n^* S_T^*)^{1/2} \hat{n} \tag{19}$$
$$\hat{z} = (H_T^* P_z^* P_z H_T)^{-1/2} u \tag{20}$$

Here $\hat{w}$ is constrained to lie in the convex set $\widehat{\mathbf{N}}$ which is the image of $\mathbf{N}$, under a linear isometry. We can now immediately apply Theorem 5.3 to establish the sampled-data model validation result.

Theorem 8.1 *Consider the perturbation model (16) together with the sampled input-output datum* (y_k, u_k), $k = 0, \ldots, N-1$. *If there exists sequences* $\hat{w}_k, \hat{n}_k$ *satisfying (19) and (20), and such that* $\hat{W}'\hat{W} \leq \hat{Z}'\hat{Z}$, *then the datum* does not invalidate *the perturbation model.*

A similar result can be obtained for time-domain validation of time-varying uncertainty models.

9. CONCLUSIONS

We have treated model validation problems with frequency-domain and with time-domain data for a broad class of perturbation models. We have also considered model validation for sample-data perturbation problems.

A number of important issues need to be addressed. The time-domain validation tests for LTI perturbation models are computationally unattractive. In this situation, until better algorithms are developed, the frequency-domain tests remain the method of choice. For LTV uncertainties, the time-domain validation is computationally feasible for problems with modest data records. Further work will be required to address larger size problems. The important problems of model validation for perturbation models with real parametric uncertainty remain open.

Model validation should be regarded as only one ingredient of the entire process of obtaining robust control oriented system models. Model validation is preceded by system analysis and understanding, physical modeling, and identification. If the uncertainty model is invalidated by the input-output data record then it becomes necessary to revisit the identification step and it may also be necessary to obtain additional data. In this event a revised perturbation model must be obtained, and this must be done bearing in mind the performance objective of closed-loop regulation. Systematic, effective methods for doing this must be developed and recent work on iterative closed-loop identification methods (see [28,37,17]) will doubtless play an important role here.

10. REFERENCES

[1] E.-W. Bai. In *Proc. IEEE Control Decision Conf.*, pages 1719–1724, 1992.

[2] B. Bamieh, M. A. Dahleh, and J. B. Pearson. *IEEE Trans. Auto. Contr.*, vol.38, no.5, pp. 717-732, May 1993.

[3] B. R. Barmish. *New tools for robustness of linear systems.* Macmillan, 1994.

[4] T. Chen and B. A. Francis. *Proc. 1991 ACC*, Boston, MA, pp.2704-9, Jun. 1991.

[5] M. A. Dahleh and I. J. D. Bobillo. *Control of Uncertain Systems — A Linear Programming Approach.* Prentice-Hall, 1995.

[6] J. Doyle, K. Glover, P. Khargonekar, and B. Francis. *IEEE Trans. Auto. Control*, AC-34:831–847, 1989.

[7] K. Glover, D. McFarlane. *IEEE Trans. Auto. Control*, AC-34:821-830, 1989.

[8] G. Goodwin, B. Ninness, and M. Salgado. In *Proc. Amer. Control Conf.*, pages 2400–2405, 1990.

[9] M. Green and D. J. N. Limebeer. *Linear Robust Control.* Prentice-Hall, 1995.

[10] G. Gu and P. P. Khargonekar. *IEEE Trans. Auto. Control*, 37:953–963, 1992.

[11] R. G. Hakvoort, R. J. P. Schrama, and P. M. J. Van den Hof. *Automatica*, 30(4):679–690, 1994.

[12] A. J. Helmicki, C. A. Jacobson, and C. N. Nett. *IEEE Trans. Auto. Control*, pages 1163–1176, 1991.

[13] P. T. Kambamba and S. Hara. *IEEE Trans. Auto. Contr.*, v.38, no.9, pp.1337-58, Sept. 1993.

[14] R. Kosut, M. Lau, and S. Boyd. In *Proc. IEEE Control Decision Conf.*, volume 6, pages 3162–3167, 1990.

[15] J. M. Krause and P. P. Khargonekar. *Automatica*, 26:113–124, 1990.

[16] L. Lee, and K. Poolla. *ASME Trans. J. Dynam. Syst. Meas. and Contr.* submitted, Sept. 1994.

[17] W. S. Lee, B. D. O. Anderson, R. L. Kosut, and I. M. Y. Mareels. *Int. J. of Adapt. Control & Sig. Proc.*, 7:183–211, 1993.

[18] L. Ljung. *System Identification, Theory for the User.* Information and System Sciences Series. Prentice-Hall, New Jersey, 1987.

[19] P. Mäkilä and J. Partington. *Proc. Amer. Control Conf.*, pages 70–76, 1991.

[20] Y. E. Nesterov and A. S. Nemirovsky. *Interior-Point Polynomial Algorithms in Convex Programming*. SIAM, Philadelphia, 1994.
[21] M. Newlin and R. S. Smith. *IEEE Trans. Auto. Control*, 1994. accepted for publication.
[22] A. K. Packard and J.C. Doyle. *Automatica*, vol. 29, no. 1, pp:71–110, 1993.
[23] P. J. Parker and R. R. Bitmead. In *Proc. IEEE Control Decision Conf.*, pages 348–353, 1987.
[24] K. Poolla, P. Khargonekar, A. Tikku, J. Krause, and K. Nagpal. *IEEE Trans. Auto. Control*, 39(5):951–959, 1994.
[25] K. R. Popper, *Conjectures and Refutations*, Routledge and Paul, London, 1969.
[26] S. Rangan and K. Poolla. In *Proc. Amer. Control Conf.*, 1995. submitted.
[27] R. Schied, D. S. Bayard, and Y. Yam. In *Proc. Amer. Control Conf.*, pages 1956–1958, 1991.
[28] R. J. P. Schrama. *IEEE Trans. Auto. Control*, 37:991–994, July 1992.
[29] R. Smith and G. Dullerud. *IEEE Trans. Auto. Control*, Sept. 1994. submitted.
[30] R. Smith and G. Dullerud. *Tech. Report, CCEC, UC Santa Barbara*, Nov. 1994.
[31] R. S. Smith. *Automatica*, 1994. submitted.
[32] R. S. Smith and J. C. Doyle. *IEEE Trans. Auto. Control*, 37(7):942–952, July 1992.
[33] W. Sun, K. Nagpal, and P. Khargonekar. *IEEE Trans. Auto. Contr.*, v.38, no.8, pp.1162-75, Aug. 1993.
[34] S. Thapliyal, Y. Faitakis, and J. Kantor. In *Proc. Amer. Control Conf.*, pages 3027–3028, 1994.
[35] The MathWorks, Inc., Natick, MA. *μ-Analysis and Synthesis Toolbox (μ-Tools)*, 1991.
[36] R. C. Younce and C. E. Rohrs. In *Proc. IEEE Control Decision Conf.*, pages 3154–3161, 1990.
[37] Z. Zang, R. R. Bitmead, and M. Gevers. In *Proc. IEEE Control Decision Conf.*, pages 279–284, 1991.
[38] T. Zhou and H. Kimura. *Syst. and Control Letters*, 20:167–178, 1993.