Simulation and compensation of thermal errors of machine tools

Doctoral Thesis

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Simulation and Compensation of Thermal Errors of Machine Tools

A dissertation submitted to the
ETH ZURICH

for the degree of
Doctor of Sciences

presented by
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accepted on the recommendation of
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Prof. Dr. O. Zirn, co-examiner

2012
Acknowledgment

This thesis would not have been the same without the invaluable support from a number of great people.

First and foremost I thank my advisor Prof. Dr. Konrad Wegener, head of the IWF, for offering me the Ph.D. position at his renowned lab, providing me with an inspirational working environment. His continuing support of my work, his stimulating suggestions, and his guidance since my days as a graduate student were priceless. My thanks also go to Prof. Dr. Oliver Zirn, head of the IPP at the TU Clausthal, for his interest in my work, his review and agreeing to co-examine this thesis.

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I appreciate the help of the project partners, which provided the machine tools used throughout this thesis. Their machines are the corner stone for enabling me to simulate and compensate the thermal behavior of machine tools as well as to validate the data by measurements.

Many thanks also go to Magdalena for being a good listener and her bearing with overhours. I dedicate this work to my family, especially to my parents and my brother who have always supported and encouraged me.

Markus Ess, March 2012
To my parents
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The list of symbols is divided in a list of commonly used subscripts that can be used in combination with any of the below-mentioned symbols. The list of subscripts can be found after the latin and greek symbols.

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<tr>
<th>Symbol</th>
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<tr>
<td>$A$</td>
<td>$m^2$</td>
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</tr>
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<td>$\frac{1}{m}$</td>
<td>Temperature gradient of shape functions with respect to global coordinates</td>
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<td>$\frac{K}{N}$</td>
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<td>$C_0$</td>
<td>$N$</td>
<td>Basic static load rating</td>
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<td>$\frac{kJgK}{1}$</td>
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<td>Coefficient for load dependent bearing friction.</td>
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<td>$\ddot{G}$</td>
<td>–</td>
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<td>EHD-lubrication: Central film thickness</td>
</tr>
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<td>EHD-lubrication: Minimal film thickness</td>
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<td>$kgm^2$</td>
<td>Moment of inertia at the center of mass of the body</td>
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<td>$\frac{W}{K}$</td>
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<td>$n$</td>
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<td>Power</td>
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<td>$Q^e$</td>
<td>$W$</td>
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<td>Symbol</td>
<td>Unit</td>
<td>Description</td>
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<td>--------</td>
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<td>( Q_n )</td>
<td>( W )</td>
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<td>( q_s )</td>
<td>( W/m^2 )</td>
<td>Heat flux per surface area</td>
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<tr>
<td>( q_v )</td>
<td>( W/m^3 )</td>
<td>Heat generation per volume</td>
</tr>
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<td>( R )</td>
<td>( K )</td>
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</tr>
<tr>
<td>( R_{el} )</td>
<td>( \Omega )</td>
<td>Electrical resistance</td>
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<td>Effective radius of roller pair = ( \frac{R_aR_b}{R_a \pm R_b} )</td>
</tr>
<tr>
<td>( r )</td>
<td>( m )</td>
<td>Radius</td>
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<td>( S )</td>
<td>–</td>
<td>Osculation = ( \frac{r_{ring}}{r_{rb}} )</td>
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<tr>
<td>( T )</td>
<td>( K )</td>
<td>Temperature</td>
</tr>
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<td>( T_{am} )</td>
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<tr>
<td>( t )</td>
<td>( s )</td>
<td>Time</td>
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<td>( t_m )</td>
<td>–</td>
<td>Ratio of pitch circle = ( \frac{r_m}{r_{rb}} )</td>
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<td>( { u } )</td>
<td>( m )</td>
<td>Vector of nodal displacement</td>
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<td>EHD-lubrication: Dimensionless speed parameter</td>
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<tr>
<td>( V )</td>
<td>( m^3 )</td>
<td>Volume</td>
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<td>( v )</td>
<td>( m/s )</td>
<td>Speed, subscript indicates direction</td>
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<td>( \dot{W} )</td>
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<td>( X )</td>
<td>( m )</td>
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<td>( P )</td>
<td>( W )</td>
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<td>( P_0 )</td>
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<td>Equivalent static bearing load</td>
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<td>( P_1 )</td>
<td>( N )</td>
<td>Equivalent dynamic bearing load</td>
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<td>( p )</td>
<td>( N/m^2 )</td>
<td>Pressure</td>
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<td>( Q )</td>
<td>( W )</td>
<td>Heat flux</td>
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<td>Radial load factor for the bearing (static load)</td>
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<td>( X_1 )</td>
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<td>Radial load factor for the bearing (dynamic load)</td>
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<td>( { x } )</td>
<td>( m )</td>
<td>Vector of position</td>
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<td>( Y )</td>
<td>( m )</td>
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### List of Symbols

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<td>Pressure-viscosity coefficient of the lubricant</td>
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<td>Lead angle of a ballscrew</td>
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<td>$K$</td>
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<td>$\Delta r_{gap}$</td>
<td>$m$</td>
<td>Air gap between rotor and stator</td>
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<td>$\delta$</td>
<td>$m$</td>
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<td></td>
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<tr>
<td>$y$</td>
<td>Y-direction</td>
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## List of Abbreviations

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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>ABK</td>
<td>Software: Achsbaukasten / Axis-construction kit</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial neural network</td>
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<tr>
<td>CAD</td>
<td>Computer aided design</td>
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<td>CFD</td>
<td>Computational fluid dynamics</td>
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<tr>
<td>CFRP</td>
<td>Carbon fiber reinforced polymer</td>
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<td>CNC</td>
<td>Computerized numerical control</td>
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<td>EHD</td>
<td>Elasto-hydrodynamic lubrication</td>
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<tr>
<td>ETVE</td>
<td>Environmental temperature variation error</td>
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<tr>
<td>FDM</td>
<td>Finite difference method</td>
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<td>FDEM</td>
<td>Finite difference element method</td>
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<td>FEM</td>
<td>Finite element method</td>
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<tr>
<td>GUI</td>
<td>Graphical user interface</td>
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<tr>
<td>IHCP</td>
<td>Inverse heat conduction problem</td>
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<tr>
<td>IPO</td>
<td>Interpolation cycle</td>
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<tr>
<td>ISO</td>
<td>International organization for standardization</td>
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<tr>
<td>LBB</td>
<td>Laser ball bar</td>
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<tr>
<td>MRA</td>
<td>Multi-regression analysis</td>
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<tr>
<td>MLCM</td>
<td>Modified lumped capacitance method</td>
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<tr>
<td>NC</td>
<td>Numerical control</td>
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<tr>
<td>PT – 1</td>
<td>First order lag</td>
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<td>PT – 2</td>
<td>Second order lag</td>
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<td>RA</td>
<td>Regression analysis</td>
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<td>TCP</td>
<td>Tool center point</td>
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<tr>
<td>TF</td>
<td>Transfer function</td>
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<tr>
<td>VMP</td>
<td>Software: Virtual Machine Prototype</td>
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<tr>
<td>WCP</td>
<td>Workpiece center point</td>
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Abstract

The following thesis deals with the simulation and compensation of thermal errors on machine tools using the finite element method (FEM). Whilst FEM has become increasingly important in analyzing the static behavior, its usage for thermal problems has so far been limited. This is because the thermal error on machine tools depends on a wide variety of effects, leading to a complexity that is hard to comprehend in its entirety.

The first problem is that machine tools consist of a large variety of components that influence the thermal behavior and as such it is necessary to understand the physics of every component. A variety of models are created for these components. These models are founded on known parameters and render further measurements for parameter identification unnecessary. This is of utmost importance during the design phase, where no prototype exists that can be used for measuring. Still it is desirable to perform reliable simulations in order to determine the quality of the design.

Common components are rolling bodies that are used in bearing, guideways and ball screws. Appropriate models are shown that consider the load and velocity of the rolling bodies in order to determine their friction loss and heat conductance. Feed drives and auxiliary components can also be found on every machine tool and hence models are derived that can be used to study their effect.

Even if the physics for these components are known it still is a difficult task to implement these in a FEM software. Some tasks that are connected to this are the implementation of the equations for the models in the FEM software, the knowledge of the loads and velocities and how to consider the positioning of the axes. A simulation software, called Virtual Machine Prototype (VMP) was developed in order to remedy the aforementioned problems. It is specifically design to simulate machine tools and to assist designers in creating more accurate and better machine tools. VMP consists of a multibody dynamics and thermomechanical FE code. Multibody dynamics are included to derive the loads on the components based on the NC path. The thermomechanical FE code automatically updates the model of the machine tool according to its current state and position.

The next step is to ensure that the simulation of the thermal problem can be performed in a matter of minutes. This is contrary to the common knowledge that the simulation of thermal problems can take many hours and even days. Whilst many researchers try to reduce the thermal system of equations, it will be shown that this is not necessary. By analyzing the physics behind thermal problems it is possible to write the problem in such a way that the solution takes little time.

Together with a reduction of the mechanical system of equations it is even possible to use FEM for the compensation of thermal errors on a machine tool. VMP will be connected to the CNC of a machine tool in order to compensate the thermal error in the whole working
Abstract

envelope of the machine tool. VMP uses the NC path to compute the state of the machine tool and then sends a command back to the CNC to move the axes in such a way that no relative error between the TCP and WCP occurs.

A lot of measurements have also been conducted in order to give a better understanding of the effects that have to be considered. These effects are made up from various internal and external heat sources, but also the design of the machine tool plays an important role.

The models that have been derived were verified with a variety of simulations. The results of the simulation have been compared with the measurements that were conducted. The compensation was also implemented on a machine tool and it was shown that the thermal error could be kept within 5\(\mu m\).
Kurzfassung


Eine der häufigsten Komponenten sind Wälzkörper, die in Führungen, Lagern und Kugelgewindetrieben verwendet werden. Geeignet Modelle wurden entwickelt, die die Last- und Geschwindigkeitsabhängigkeit der Wälzkörper berücksichtigen, um ihre Wärmeleitfähigkeit und Reibungsverluste zu bestimmen. Vorschubantriebe und Nebenaggregate sind weitere Elemente, die an jeder Werkzeugmaschine gefunden werden können. Folglich werden auch Modelle für diese Komponenten vorgestellt.


Der nächste Schritt ist es sicherzustellen, dass die Simulation der thermischen Probleme in wenigen Minuten durchgeführt werden kann. Dies ist im Widerspruch zur weitverbreiteten Meinung, dass die Simulation von thermischen Problemen mehrere Stunden, bis zu wenigen Tagen dauern kann. Während die Forschung ihr Augenmerk häufig auf Re-
Kurzfassung

duktionsmethoden richtet, wird hier gezeigt, dass diese für das thermische Problem nicht notwendig sind. Die Formulierung des thermischen Problems wird so gemacht, dass die Lösung nur einen geringen numerischen Aufwand hat.

Wird zusätzlich noch eine Reduktion des Gleichungssystems für das mechanische Verhalten vorgenommen, ist es möglich FEM zur Kompensation des thermischen Fehlers auf Werkzeugmaschinen zu verwenden. VMP kann hierbei mit der CNC einer Werkzeugmaschine verbunden werden, um den thermischen Fehler im ganzen Arbeitsraum zu kompensieren. VMP benutzt hierfür die NC-Bahn um den aktuellen Zustand der Maschine zu berechnen und schickt dann ein Signal zurück an die CNC. Dieses Signal bewegt die Achsen so, dass kein relativer Fehler zwischen dem Werkzeug und Werkstück entsteht.


Die Modelle, die hergeleitet wurden, werden anhand einer Vielzahl von Simulationen verifiziert. Die Ergebnisse der Simulationen werden dann mit den Messungen verglichen. Die Kompensationsmethode wurde auf einer Werkzeugmaschine implementiert und die resultierenden thermischen Fehler waren kleiner als 5\mu m.
1 Introduction and State of the Art

The thermomechanical deformation of machine tools, caused by external and internal heat sources, is one of the main contributors to the overall geometric error of the workpiece [1]. Machine tool manufacturers therefore try to reduce the thermal displacements and a lot of research has been done on this topic for the last decades. During this time a lot of findings and methodologies on the causes and the reduction of thermal displacements have been made. In order to reduce the errors on machine tools which are caused by thermal deformation, it is important to understand the influence factors. A large amount of different thermal influences exist, with the actual temperature distribution on the machine tool being a combination of all these effects and their histories. As well as the identification of these error sources one also has to measure them to quantify their influence on the overall accuracy of the machine tool. Standard methodologies which are state of the art are described in Sec. 1.2. With the ever increasing performance of electronics and information technologies new possibilities emerge. Simulation tools become more important in performing a more detailed analysis of the different effects and also try to model and predict the behavior of the machine tool, before it has been actually built. In order to achieve minimal thermal errors the first step is to use concepts that minimize the thermal errors, for example thermo-symmetric design. After that the design of the machine tool can be optimized, where simulation can help in decreasing development time of new components and increasing part quality. In order to achieve meaningful results, it is important to already have a good understanding of the system which should be analyzed prior to perform any simulation. As the understanding of thermal effects becomes better, different countermeasures can be taken to decrease their influence on the TCP. These countermeasures can be realized by means of “hardware”, for example by using chillers, or “software”, usually with compensational movements by the NC.

1.1 Thermal Influences on Machine Tools

In general thermal influences on machine tools can be classified in six basic groups shown in the thermal effects diagram [2]. These effects are:

- heat generated from the (cutting) process
- heat generated by the machine (drives,...)
- heat exchange due to tempering devices (cooling and heating)
- heat exchange with the environment
- the effect of people
- thermal memory from any previous environment
1 Introduction and State of the Art

Thermal effects diagram

![Diagram of thermal effects](image)

Figure 1.1: Thermal effects diagram; Originally from: [2]

All these effects lead to a thermomechanical deformation and as a result cause inaccuracies on the machine tool. A more detailed list of these effects and current research will be given in the following subsections, as well as the in the section of the thermal simulation of machine tools.

1.1.1 External Heat Sources and Sinks

External effects are attributed to the environment the machine is located in, but also the workpiece. The main effect on the machine is the variation of the environmental temperature during the day and night cycle, but also during longer periods as summer and winter over a year. Besides the temporal variation the spatial variation is also of importance, as the air temperature varies with the height above the ground, but gets also influenced by neighboring machines, opening/closing of the machine shops door and so
forth. Radiation from the sun can also seriously affect the machines temperature and environment.

For the workpiece which often comes from the storage the thermal memory is also important. Usually the temperature in the storage is different than on the shop floor and as a result the workpiece needs some time to acclimate to the temperature on the shop floor. This effect is also important to consider for high-precision parts which will be measured after a first production and then get finished in a further machining step. The temperature in the measuring room usually is controlled to $20^\circ C$ and hence often different from the temperature of the shop floor.

### 1.2 Measuring Thermal Deformation

As understanding and controlling thermal deformation is important for the accuracy of machine tools, a lot of work has been done on how to measure the relative errors between the tool and the workpiece. Standards provide an important guideline in the measurement and interpretation of thermal deformation, but also a lot of other measuring techniques have been explored.

#### 1.2.1 Standards


- Environmental temperature variation error (ETVE)
- Thermal distortion caused by rotating spindle
- Thermal distortion caused by linear motion of axes

As these standards are the base for every measurement on the thermal error on machine tools, also for those shown in Chap. 2 and the ones used for the verification in Chap. 9, a short explanation on the standardized measurement procedures is given. Before the test
procedures are started, the condition of the machine is important. The machine should be fully operational in accordance with the supplier’s/manufacturer’s instructions. Geometric tests ensuring the geometric accuracy and repeatability of the machine should be made prior. All auxiliary services should be operating and the axes should be in “Hold” position with no prior movement, until the machine had sufficient time to stabilize due to these heat sources. The test procedures are performed under no-load or finishing conditions, meaning that no machining process takes place during the measurement. Besides the displacements between the tool and the workpiece, temperatures at designated locations, such as spindles, measurement devices and ambient air are recorded too.

**ETVE** The ETVE (Environmental Temperature Variation Error) test, which is a drift test, is used to designate the effects of varying environmental temperatures on the positioning accuracy of the machine tool. It can also be used to estimate the thermally induced error during other measurements, whenever the environmental temperature is not constant. If possible the temperature should be controlled according to the environmental temperature changes which are considered being acceptable to keep sufficient accuracy or according to the environment of the shop floor, where the machine will be installed. One example of a test setup for a vertical spindle machining center is shown in Fig. 1.2. Other setups for different machine types can be found in the standard.

**Figure 1.2:** Measurement setup for the thermal drift during ETVE and spindle rotation [4]

Key: 1 ambient air temperature sensor; 2 spindle bearing temperature sensor; 3 test mandrel; 4 linear displacement sensors; 5 fixture; 6 fixture bolted to table;

As it can be seen in Fig. 1.2 five linear displacement sensors are being used, as well as two temperature sensors. Using the three linear displacement sensors (X1, Y1, Z1) the translation in X, Y and Z can be measured simultaneously. The additional two sensors (X2, Y2) can be used to calculate the rotation A and B. In order to make the interpretation
1.2 Measuring Thermal Deformation

of the results easier it is recommended to plot the displacements and temperatures in the same graph having its time abscissa as common.

**Rotating Spindle**  This test is performed in order to measure the resulting thermal distortion due to the heat generated when the spindle is rotating. It is recommended to use the same test setup as for the ETVE test, which is shown in Fig. 1.2. Ideally the linear displacement sensors are contact free, as capacitance sensors for example, so that the thermal distortion can be measured while the spindle is rotating. In this case, the effects of test mandrel run-out have to be removed by using a low-pass filter. These spindle distortions correspond to the X0C, Y0C and Z0C location errors. The rotations correspond to the orientation errors A0C and B0C. The orientation error C0C of the spindle can not be measured but is also not of interest in conventional machining processes as milling and grinding. The speed spectrum of the spindle during the measurement can be either variable or constant (as a percentage of the maximum speed). Of course the speed spectrum has to be documented. The constant speed is preferable to get a better understanding of the spindle behavior at different speeds, while the variable speed spectrum can be used to simulate a real machining process. The spindle shall run for at least four hours or until the error does not change more than 15% during the last 60min. After this the distortion has to be monitored for another hour, when the spindle is stopped, to measure the cooling.

**Linear Motion of Components**  By performing a pendular motion of the linear axes the effects of the heat generated by the machines positioning system which includes

- feed drives
- bearings
- ball screws
- guides

  can be studied. This test, unlike the previous two, is performed at two positions in order to analyze the elongation of the positioning system and deformation of the machine structure due to the heat. The elongation of the positioning system is especially important for indirect position control. The test setup is shown in Fig 1.3. It consists of two setups with five incremental probes, such that the thermal distortion can be measured at these two positions. The target positions should be close to the limits of travel. Other measurement setups can be found in the standard ISO 230-3:2005 [4].

  The rules for the duration of the test are the same as for the rotating spindle. The measurement starts at target 1, where the machine rest long enough (dwell time) to record the displacements. The slide of the machine then shall move to target 2, where again the displacements are being recorded. In the next step the movement is then reversed, towards target 1, where again measurements are recorded. This cycle is repeated until the end of the test duration. The traverse rate shall be a percentage of the maximum feed rate. Heat input changes with the feed rate and dwell time, consequently leading to different displacement measurements. Therefore these values have to be documented. To measure the cool down it is recommended to stop the machine in the middle of the travel and then take readings at both target positions every five minutes.
1.2.2 Alternative Methods

A ball bar was used in [6] to measure the spindle drift and the spindle tilts. To do so, the change of the ball bar length was measured at given time intervals during a spindle drift measurement. The ball bar performed a helical movement on a semi-sphere and a ball bar error equation was created, that relates the change of the ball bar length to the spindle errors. Similarly [7] used a ball bar to calculate the position errors of the machine tool. In this case the ball bar however performed two circular measurements and one at the top of the semi-sphere in order to maximize the sensitivity of the measuring system to the significant error components of the machine tool. In [8] a laser ball bar (LBB) was used to measure the spindle drift. Compared to a regular ball bar, which commonly uses optical scales, the LBB measures the displacement between the two precision spheres using a laser interferometer. The LBB was used to measure the spindle thermal drift on a two-axis lathe. In order to do so a sequential trilateration was used, where three sockets were used on the tool side and one at the spindle. During each measurement cycle the LBB had to be mounted in each of the three sockets on the tool side, in order to measure the change in length. This results in a cumbersome procedure which also takes a lot of time, namely four minutes, to take one measurement. Compared to the LBB, which is a contacting measurement and can not measure all three lengths needed for the trilateration, [9] uses three interferometers. Using the three interferometers with a cat’s eye mounted in the spindle the three lengths can be measured simultaneously, even during spindle rotation, as it is a non-contacting measurement. The drawback is that orientation errors of the spindle can not be detected. While in the ISO standard [4] the thermal error of a feed axis is only measured at discrete points of the axis a laser interferometer is used in [10] to measure the errors along the axis. The three translational errors (one position and two straightness errors) as well as the three rotational errors (pitch, yaw and roll) are considered along the feed axis and their change over time is measured.

Thermography is also becoming popular in understanding thermal effects, as it can be
used to visualize the temperature distribution and to identify important heat sources. The temperature distribution and its change with time was investigated in [11]. In [12] thermography was used to estimate the thermally induced positioning error in different positions of the feed drive. This is done to overcome the measurement of the thermal error at only two locations, as proposed in the standard ISO 230-3:2005 [4]. The 3D object points of a CAD model are projected onto the 2D thermography image and based on the temperature distribution along the ball screw and the thermal expansion coefficient, the displacement error can be calculated. Deviations between the displacements calculated from the results of thermography measurements and directly from interferometry were less than 20%. Further improvements are possible if the thermographic measurement is improved. The accuracy of thermographic measurements often suffers from the high reflectivity of blank steel and as such countermeasures have to be taken. A method that is proposed in [3] is to mask the blank steel with a tape of high emissivity.

1.3 Simulation of Thermal Deformation

The simulation of thermal deformation of machine tools started in the early 1970s with the emergence of microprocessor computers. The main methods used to simulate the thermal behavior of machine tools are finite difference (FDM) and finite element (FEM) methods. In the early days finite difference methods were often used to calculate the temperature distribution on a machine tool, as the method is easy to program and computationally less demanding than the finite element method. However it is usually only used to calculate the temperature distribution and not the thermal deformation. Over time a lot of other methods have also been proposed. Some of these models are based on a thermal network analysis, where masses are lumped on to the nodes of the thermal network [13] [14]. The thermal network then is coupled to a FE model which is used to calculate the deformation based on the result of the thermal network analysis. As lumping, especially when it is coarse, can lead to errors [15] proposes the modified lumped capacitance method (MLCM). This method is basically the same as the aforementioned, however each mass is modified by a coefficient in order to compensate for the errors which are due to the lumping. A similar method is FDEM [3], where the finite difference and finite element methods are coupled. In a first step the temperature distribution is calculated using FDM and based on the temperature distribution FEM is used to calculate the thermal deformation.

All of these methods have been utilized to either simulate complete machine tools or individual components and thermal effects. The following two subsections look at different research that has been done in the respective fields.

1.3.1 Simulation of Individual Components

Environment  Besides the simulation of the whole machine tool a lot of research has been done on simulating the individual sources for thermal errors on machine tools. One of these sources is the environment which can have a significant effect on the accuracy of the machine tool due to changes in the environmental temperature. Therefore it is necessary to have a good estimation of the convection coefficients on the machine tools structure. In [16] it was shown, by performing a sensitivity analysis, that the most important parameters for an accurate estimation of the thermal errors (when neglecting internal heat sources)
are the convection coefficient and the materials coefficient of expansion. The heat transfer coefficient depends on the temperature difference and inclination of the surface. As a result [17] and [18] proposed an automatism to estimate and assign the heat transfer coefficient in the model based on these factors. The effects of guards and machine covers, under which heat can accumulate, has also been studied in [19] and shown to cause a difference of up to 33% in the displacements when compared to a machine without covers.

**Linear axes** Thermally induced errors due to the movement of linear axes also have been thoroughly analyzed. In [20] the temperature distribution and the thermal error of a CNC machine center was analyzed. The focus was put on the ball screw and the simulations were supported by measuring the temperature at different points along the ball screw. While the previous work [20] used inverse analysis to compute the heat losses from the temperature measurements, [21] uses models for the calculation of the heat losses and convection from the ball screw. The frictional and thermal behavior of slide guides was analyzed in [22] and [15]. It was observed that even for slide guides the heat generated in the ball nut is larger than the one in the guide ways. The friction force was also dependent on the thermal deformation as the contact conditions change in the slide guides with the deformation of the guides. The change in the viscosity of the lubricant oil was rather small. As ball screws and guideways on linear axes are not separated heat sources, the effects of the interaction of these factors, as well as of the feed drive, should be analyzed in a complete simulation study. This was done in [23] and [24], where different cooling concepts for ball screw systems were compared.

**Bearings** One of the most important sources of heat in machine tools is the main spindle and its bearings. Because of this a lot of research can be found on the simulation of spindles. A lot of the research focus on the dynamic and static stiffness of the spindle, but some work can also be found for the thermal behavior of the spindle. The major mechanical components are the spindle bearings which have been studied in a series of publications. The influence of the heat generated in the spindle bearings of a grinding machine is considered in [25] as well as the contact resistance between the bearing and housing frame which depends on values as such as the contact pressure, area and roughness. The spindle bearing however does not consider effects as changes in the bearing stiffness due to centrifugal forces and preload due to thermal effects. These effects are also investigated in [26] and it was observed that changes in the rotational speed have severe effects on the stiffness of the system as the natural frequencies of the spindle system decrease with higher rotational speeds. One simplification made is in the temperature calculation as it is assumed that the shaft and the housing have a uniform temperature. From the active length and the difference in these two temperatures the thermal preload is computed. A dynamic and thermal FE model are coupled in [27] in order to consider the temperature distribution in the whole system. The bearing friction model also considers friction generated by rolling and spinning of the balls. Similar work was done shown by [28] with a non-linear FE model, which included the bearing clearance and possible loss of press fit between bearing rings and their surroundings. Heat transfer mechanics in a bearing are also shown, but no further details on how to calculate the individual effects is given. A hybrid model, using FEM for the headstock and FDM for the axially symmetric parts, was created in [29]. By using FDM ring elements for the axially symmetric parts as bearings, gears, etc. the mesh
1.3 Simulation of Thermal Deformation

density was decreased. The model could determine running clearance for different speeds and estimate bearing lifetime as well as study the effects of forced cooling in the bearings, heat generated in the bearings and heat generated in the belt drive.

**Electrical losses** Besides the bearings, electrical losses of the drives in the spindle act as major heat sources. These electrical losses also occur in the feed drives and as a result almost the same models can be used. A power flow model considering the losses in the bearing, air gap, stator and rotor was derived in [30]. As the motorized spindle was an AC induction motor, the slip was used to determine the distribution of power losses between the stator and rotor. In order to achieve this, measurements of the slip and the motor efficiency had to be made. Based on the power flow model a thermal FDM model of the spindle was derived in [31], which further included heat sinks, such as convection to cooling water and ambient air. Heat transfer through the bearing was also modeled and divided in three components namely convection to the air, conduction through the bearing and conduction through the gap between the bearing ring and frame.

1.3.2 Simulation of Machine Tools

Since in the beginning of the 1970s computational models were on the verge of being used for the analysis of machine tools early paper mainly explain the simulation methods and are used on idealized geometries of machine tools. This is also due to the fact that early computers were not capable of handling large models. In [32] an implicit finite difference method is used to compute the temperature distribution of machine tools. With the software the influence of different material properties on the temperature distribution was computed. The resulting structural deformations could not be computed. Finite elements were used in [33] to compute the thermal deformation of machine tools. The observed errors between measurement and simulation were less than 20%, with the main reason for the errors being the uncertainty in the boundary conditions. However the finite element methods proved to be practical to evaluate design changes and as a result improve the quality of the machine tool. Another software to analyse the thermal deformation of machine tool structures during the design stage was shown in [34], again using finite elements. The software is used to compute the thermal deformation of a single, thin-walled column due to the heat generated in the spindle bearings. Besides the simulation of the thermal deformation, the system matrices are also used to estimate the intensity of heat sources based on measured temperatures at the location of the heat source. At that time it was already discovered that in order to attain a satisfactory correlation between simulation and measurements the main limitation is to get accurate values for the heat sources and heat transitions (e.g. convection) [35].

The influence of changing operation conditions and the inter-dependencies of effects, for example the viscosity of the lubricant with the temperature, was considered in [36]. In [37] the finite difference method is used to compute the temperature field. Analytical formulations of the thermomechanical deformations on simple bodies are then used to compute the deformation of the machine tool, where the machine tool is made up of a series of simple bodies. The temperatures computed by the FDM model are used in the analytical formulations to estimate the thermal distortion on the machine tool. FEM is used in [19] to model different effects as spindle bearings, displacements at the spindle nose.
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due to different rotational speeds, as well as the mounting of the glass scale.

1.4 Reduction of Thermal Errors

The possibilities that exist to reduce thermal errors are summed up in a variety of papers ([38], [1], [39]). The methods shown in these papers will also be stated here. Basically all methods to reduce the thermal error can be classified in three categories according to [40]. These categories are stated below with more detail given in the subsequent subsections.

- Reducing temperature variations: for example by cooling or improved environmental conditions, as well as reduced heat generation
- Reducing thermal sensitivity: reducing the sensitivity of the machine tool structural loop to temperature changes
- Compensation of the errors: for example by means of software or hardware

1.4.1 Reducing Temperature Variations

One possibility to reduce the thermal error is to try and create an even temperature distribution on the machine tools structure. The lower the gradient is, the lower the thermal error will become. One of the possibilities to reduce the gradient is to reduce the heat that is generated in the elements of the machine tool. This, as explained in [41], can be achieved for example by reducing the masses of the machine tool structure. This is usually done in order to create energy-efficient machine tools, but it is also beneficial for reducing the losses that occur in a machine tool. With smaller masses less energy is needed to move them which in turn leads to smaller losses and lower temperatures on the machine structure. The most common option to reduce the temperature gradient however is to apply cooling to the machine tool and a lot of research and different possibilities exist to do so. Some of the approaches try to improve the removal of excess heat from machine elements. One of the approaches is shown in [40], where special cooling elements for spindles are designed. These cooling tubes try to make use of the Coanda-effect. When using the Coanda-effect, a fluid exits from a nozzle creating a primary stream. This primary stream drags along a secondary stream that can be up to twenty times larger than the primary one. Even if liquid cooling, due to the higher heat capacity, is more effective to remove heat, using the Coanda-effect improves the effectiveness of air-cooling. Temperature control of air in a lithography application is shown in [42]. In this context it is necessary to control the air in the machine enclosure to within $1m^\circ C$ in order to achieve the sub-nanometer tolerances for the evolving chip industry. Besides the temperature stability it was also shown that the temperature gradient inside the working volume of $7m^3$ is lower than $10m^\circ C$ in the main areas of the machine. Stabilization using oil showers is used in [43] and [44] as the oil shower can not only be used to remove heat from the machine tool, but also as an insulation from variations in the room temperature. In conventional chillers on-off control is often used which influences the oil temperature. Therefore a rather tight hysteresis is necessary [43] or a continuous control of the oil temperature [44] especially as the heat from the pump and the environment through the piping also have an influence. One of the benefits is that the temperature controlled oil can also be used for other parts
of the machine, such as the spindle for example. This is also used in [45], where the exit temperature of hydrostatic oil of the grinding spindle and linear bearings is kept within $\pm 0.1^{\circ}C$ for precision grinding of large free-form mirrors for telescopes. The influences and benefits of applying additional cooling to machine elements as ball nuts and feed drives was also studied using simulation in [24] and [46]. A different possibility to reduce thermal errors that does not directly reduce the temperature gradient on the machine, but modifies it, is the usage of heating and cooling elements. This is sometimes done when applying compensation to the machine tool ([47],[48],[49]). In order to reduce the TCP displacements certain elements of the machine tool can either be heated or cooled as e.g. the column of a vertical machining center. Even if this is more complicated then using the feed drives for compensation it has the benefit that angular errors can also be corrected on three-axis machines. It is also possible to retrofit machines with this type of compensation if the NC controller does not support compensations.

1.4.2 Reduction of Thermal Sensitivity

Besides the reduction of the gradients it is also possible to reduce the sensitivity of the machine tool to temperature changes. This means that the machine tool is designed in such a way that temperature changes do not lead to large deformations. This is often done by applying a thermo-symmetrical design to the machine tool. In [50] the mechanical constraints for the headstock of a lathe are placed in a way such that the center of the axis does not move during thermal expansion. Other possibilities to design non-sensitive machines are to allow thermal expansion in directions which do not affect the workpiece accuracy. Details on this can be found in books about thermal deformation on machine tools [51]. With new engineering materials emerging, it is also possible to use materials that have a low coefficient of thermal expansion. For carbon fiber reinforced plastics it is even possible to have a negative expansion coefficient. When applying these materials one has to keep their anisotropic nature in mind, making the design slightly more complex. Already in [52] an experimental spindle made from fiber reinforced plastic was measured and it was seen that the displacements were 15 times smaller than for a steel spindle. An aluminum housing with CFRP bandages was developed in [53] to compensate for the angular displacements, but not for the larger, axial ones. The CFRP bandages are equipped with temperature sensors and heating elements to create an adaptronic device. By heating the bandages, which have a negative coefficient of expansion, the displacements can be compensated. The influence of the tool holder was also removed by using super-invar which has a lower expansion than tool steel [54]. Using the super-invar tool holder the thermal effects could be reduced by up to 50% on a 400mm long workpiece. Besides all these materials, the most common alternative used to steel is polymer concrete. It is often said by polymer concrete companies that, due to its low heat conductivity and high heat capacity compared to steel, it is insensitive to environmental temperature changes. The low heat conductivity however often leads to accumulation of heat in the structure and necessitates the usage of cooling [55]. In [56] the thermal behavior of different material combinations of polymer concrete and steel were compared on typical machine tool components as headstocks, machine beds and machine tables. For machine beds it was shown that having a bed made only of polymer concrete is detrimental due to the low heat conductivity and high coefficient of expansion. Therefore steel inserts were recommended in order to improve the flow of heat and create a more even temperature distribution.
1.4.3 Compensation

Active compensation mainly uses the drives of the machine tool to correct the displacements which were caused by thermal effects. A large variety of ways exist to estimate the actual displacement. In general this can be divided in two classes of methods: Direct compensation means that the thermal error is directly measured, mostly by employing touch probes. This is the standard procedure used in work shops as it gives a direct measure of the thermal error which can be used by the CNCs operator to adapt the part program. As the TCP can not be measured directly during the machining process itself, this comes at the cost of reduced productivity, because the machine has to be stopped in order to take the measurements. Direct compensation of the thermal error can only be done at the point in time when the measurement is made and not during the machining. Indirect compensation tries to reduce the down time by removing the need of stopping the machining to take measurements. This is done by making use of the principle “cause and effect ” where the temperature is the cause and the thermal error the effect. Consequently, indirect compensation usually builds on temperature measurements, from which the thermal deformation at the TCP is estimated by employing different kind of models. These models usually have to be calibrated by prior measurements. The most common models used for indirect compensation are listed in the following paragraphs.

Artificial Neural Networks An artificial neural network (ANN) consists of a set of neurons that are connected with each other. The topology of an ANN consists of the number of neurons and the number of layers. Typical structures of ANN are feedforward networks, where the input directly leads to the output or recurrent networks which include feedback loops in order to implement an internal state and time delay into the neural network. To determine the weight between the nodes a training set is used. With this training set the ANN is trained to generate the desired output for a given input. In the compensation of thermally induced errors on machine tools usually feedforward networks ([57], [58]) are used which have measurements made by temperature probes as an input. The neural network then estimates the displacements at the TCP based on the temperature of the machine tool. As heat transfer is slow and the temperature sensors can not measure the temperature distribution [59] proposes to use recurrent ANN to implement the time delay needed for the heat flow. Using temperature sensors involves a lot of experimental work in order to find the locations and amount of temperature sensors necessary to achieve a good model. Usually ANN are equipped with more than ten temperature sensors which are needed in order to capture different time constants as the fast elongation of the spindle and the slow bending of the column, due to the heat of the spindle ([60], [61]). Therefore a lot of research has been done to reduce the number of sensors and find the ideal location. In [62] the number of temperature sensors could be reduced from 14 to four by training the ANN with a reduced number of inputs and determine the minimum number of sensors needed. An algorithm which automatically determines the minimum number of sensors was proposed in [63]. First the measurements are done with a large number of sensors mounted on heat sources, scales and the machine tool structure. The algorithm then checks which sensors have the highest transfer behavior from the input to the output. The final number of sensors is given either by a desirable target error or upper limit of sensors the machine tool manufacturer wants to employ. In the case of [63] the number of sensor could be reduced from 40 to six. A different approach is used in [64], where the
1.4 Reduction of Thermal Errors

ANN uses cutting data as cutting depth, spindle speed and so forth as an input. The only temperature sensor left is used to capture the environmental temperature. Typically ANN are used to compensate for the effect of spindle elongation, but sometimes also include linear axis ([65]), warm-up ([66]) or even the behavior of rotary axes ([62]). Thermal errors also are dependent on the position of the TCP in the working envelope meaning that a volumetric error compensation is necessary. In [58] the thermal drift of two spherical balls, placed at the opposing ends of the diagonal of the working envelope, is measured and used to estimate the thermal errors inside the working envelope. Error synthesis was used in [65] to model the thermal error at the TCP based on the distortion of the parts (column bending, linear axis elongation, etc.) of the machine tool. This however comes at the cost to find ways to identify and measure the deformations of the different parts of the machine tool. Most of the models have been derived for air-cutting, but in some cases measurements during real-cutting were used to derive the model. It could be seen that the thermal errors vary heavily on the cutting conditions [67]. This is due to the increased load on the spindle, chips and in the case of roughing the influence of the coolant. It was also shown that models trained by using air-cutting data, fail during real cutting conditions. This means that ANN only work when the training data, which covers the whole working range, is given as they fail to extrapolate to unknown conditions. while many paper also show that a reduction of the thermal error to about 10% is possible, others state that this is only possible when the ANN is applied to the training data and that on independent data sets the reduction is to about 20% ([63], [62]). In those cases it was also seen that simple linear models perform better than the ANN ([63]).

Regression Analysis Regression analysis (RA) is a statistical method to describe the relation between one dependent and several independent variables. It is therefore commonly used in prediction and forecasting values of the dependent variable. In the field of thermal errors on machine tool the dependent variable is the displacement of the TCP and the independent variables usually are temperatures, measured by probes mounted on the machine tools structure. In that respect it is comparable with the previously explained ANN. However, RA can not model internal states that describe a temporal behavior as can be done with recurrent neural networks. This is a major drawback especially as thermal errors largely depend on their history. The model used in the RA is based on the function which describes the relation of the variables as well as the parameters, usually found by the least squares method. The importance of selecting independent variables for the temperature sensors was discussed in [68] in order to prevent the multi-collinearity problem. If variables are not independent it can cause problems when using the least squares method to identify the parameters. Another problem is that too many sensors are used increasing cost and computation time. Because of the similarity to ANN [57] and [67] compared both methods in their capability to predict the thermal error on a machine tool. In [67] an unscreened MRA and stepwise MRA, where an automatic search was used to find the independent variables, were compared. It could be shown that under air cutting, for which the models were derived, both methods provided satisfactory results. For the extrapolation to real-cutting the stepwise MRA was more robust. Finally the results were compared to an ANN, where it could be seen that the MRA is more robust to conditions that differ from those during the model identification. In [57] the MRA model was expanded to include the dependence of the thermal error on the axis position. The comparison between the ANN
and MRA here showed that the ANN is more tolerant to the cut off of temperature measurements, but that both have competitive prediction accuracy. Real-cutting conditions were also included in [69]. The same author then compared in [70] MRA with nonlinear RA. In the nonlinear model it is switched between a linear and an exponential relationship between the temperatures and the thermal error, depending on the depth of cut. It can be shown that, whereas the MRA reduced the error by about 40%, nonlinear RA can reduce it by 60%.

**Physically-based Models** A large variety of models exists that, in contrast to ANN or RA, try to recreate and simplify the physical effects which are responsible for the thermal errors. In [7] a lumped capacitance method is used to compute the temperature distribution of the machine tool. This requires a lot of knowledge about the behavior of the machine tool as one has to know which parts of the machine can be lumped and how to identify the appropriate boundary conditions for the lumped bodies. Because of this a series of temperature sensors is used to support the model with measured data. However, the resulting model is quite small and as such can be easily computed in real-time. The displacements at the TCP can be computed by using the temperature distribution, stress-free theory and rigid body kinematics. Because of the modeling errors, introduced through the simplifications, when using lumped capacitances [71] proposes the modified-lumped capacitance method (MLCM). In this method parameters are introduced into the original equation system which can be chosen to fit the MLCM model to more accurate data from an numerically solved field model. A FDM model was used in [72] to compute and compensate for the thermal deformation of a hydrostatically supported precision spindle. The shearing in the hydrostatic bearing of the spindle caused an increase in the oil temperature of a few degree Celsius even if the input oil temperature to the bearing was controlled to a constant value. Based on the resulting temperature distribution, that was computed with the FDM model, a micropositioning-device containing a piezo actuator was used to compensate for the elongation of the spindle. A FDM model with a rather coarse resolution was also used in [73] to compute the temperature distribution on a machining and a turning center. Due to the low resolution the model could be integrated on a CNC and computed in real-time. The model also considers the losses in feed drives, variable positions and so forth due to the data from the NC. The deformation is based on bodies that have free thermal strain and thermal bending or camber. To directly compute the thermoelastic deformation FEM is used in [74] and [75]. For each axis of the machine a thermal model is created, which considers the external and internal heat sources. For these different heat-sources the steady-state deformation is computed. Then, using temperature probes, a linear correlation between the measured temperature of the heat sources and calculated deformation is used. This is done under the assumption of a static temperature field which means that the transient effects of heat flow are not considered. Using the machine kinematics the resulting thermal error, caused by the axis deformations, at arbitrary positions can be calculated. A mixture of FDM and FEM is used in [76] the transient thermal system is computed using FDM and in a staggered approach the resulting deformation is computed with FEM. A series of models exist for the various positions of the machine tool in the working envelope. Based on these positions the volumetric errors are computed and location and component errors of the axis derived. These errors then can, if supported by the NC of the machine, be compensated. Transfer functions (TF) have also been used to describe the thermal error on machine tools.
The thermal error of an ultraprecision air spindle has been investigated in [72]. Based on transfer functions the influence of the spindle speed and the ambient air on the spindle elongation can be estimated. Therefore only one temperature sensor is necessary for the TF from the ambient air temperature to the spindle elongation. The drawback of this method is that different spindle speeds cause for instance different convection coefficients. This not only results in a series of transfer functions for different spindle speeds, but also different transfer functions that relate the ambient air temperature to the thermal displacement. Extensive work on TFs has been done in ([48], [77], [78], [79], [80]). In this series of papers a so-called “fundamental generalized problem” (FGP) will be used. In this context it means that if no analytical description for a complex process exists, an approximation to it will be done by using the analytical solution to a similar, but simpler phenomenon. This simpler phenomenon was the heat conduction in a thin infinite plate with a convection boundary condition on the face and a radially symmetric central ring of heat generation. Parameters as heat generation, conductivity and so on were fitted in order to give the same TF as the physical system of the machine tool. By solving an inverse heat conduction problem (IHCP) it was also possible to estimate the heat generation of machine elements based on temperature measurements. The relation between the heat source and the temperature rise at a point in the vicinity of the source is given by a TF. The whole process of real-time compensation then takes the following form: measure selected temperatures, solve the IHCP and estimate the generated heat based on the heat calculate the temperature distribution with the FGP and then the displacement at the TCP. Transfer functions are also used in [81] and [82], but on a more phenomenological approach. There it is assumed that the thermal error from any load case can be described by PT1- and PT2-elements (first and second order time delay). Based on measurements which serve as calibration data the necessary parameters for the transfer function are identified. In [81] a three-axis machine tool is analyzed with the input to the model being the environmental temperature and the speed of the feed drives and spindle, thus requiring only one temperature sensor. For the load cases a transfer function is identified for each the heating phase and the cooling phase with five parameters per transfer function. Each axis and spindle is measured at 25%, 50%, 75%, 100% of the maximum speed. Considering all axes and the environmental temperature requires for a three-axis machine with a spindle in total $(3 + 1) \cdot 4 + 1 = 17$ individual measurements for the calibration of 495 parameters. The spindle behaviour is modelled in more detail in [82] which not only considers spindle speed, but also load. For each spindle speed of 20%, 40%, 60%, 80% of $n_{\text{max}}$ a measurement is made at four load cases, with 20%, 40%, 60%, 80% of $P_{\text{max}}$, summing up in 16 experiments. The resulting error on an arbitrary spindle loading however can be kept within a few microns.

### 1.5 Deficiencies of the State of Art

From the existing literature, it is obvious that a lot of thermal error research has been done. Work on this topic can be found since the beginning of the 20th century, but it gained importance with the emergence of computer controlled machines, when there is no operator who could directly measure the features on the parts produced. Even if the usage of the FEM method for the design of machine tools first came up more than 40 years ago hardly no research on a fully integrated FE model of machine tools exists. Most of the work only focuses on a single aspect of the machine tool.
One of the major topics in this thesis is to create a software and models for all relevant machine elements such that a FE model describing the whole machine tool can be derived and evaluated in a short amount of time. To do so requires models that describe the physical aspects as heat generation, heat transfer and stiffness. This means that not only thermal aspects, but also mechanical and even electrical effects have to be taken into consideration. The machine elements then have to be associated with each other, depending on the axis configuration, to consider their interaction. At the end a virtual machine tool (based on FEM) shall be created that replicates a real machine and all important physical effects. The purpose of every machine tool is to produce a part within the desired tolerances. To make it tangible for machine tool designers it is also desirable that the behavior of the virtual machine tool can be analyzed when producing a given part. The input for the virtual machine tool consequently has to be path data generated from the NC program which also is a novelty.

Considering compensation, it can be seen that most models are based on ANN or RA. These models are very time consuming to create because first optimal points for temperature sensors have to be found and then calibration data has to be generated from measurements. Usually they also fail to extrapolate to conditions for which they have not been calibrated. Some work has been done on physical models, but most are based on a severe simplification of the machine tool. Creating a reliable and accurate simplification of a machine tool requires a lot of knowledge which machine tool manufacturers usually do not have. If a virtual machine tool, based on the geometric data from the CAD model and information on machine elements from the manufacturers catalog, can be created, it becomes less complicated to create a robust and accurate compensation model. As structural information also is present in the FE model, the compensation model can also be evaluated at all points in the working envelope. No compensation model to date is based on a transient FEM simulation that can be incorporated on the machine tool and be evaluated in parallel while parts are manufactured. As for today this has not been tried because it is assumed that NC controls are not capable of handling large scale FE models. In this work it will be shown that a transient thermomechanical FE model of the machine tool indeed can be used for compensation.

With all the needs shown above the following novelties presented in this thesis can be summed up:

- **Macro-Models**: Efficiently-to-use models for all machine elements that are based on physical values and data from the manufacturers catalog instead of using parameter fitting
- **Based on NC-Data**: Use NC-data to consider position dependent effects, compute power losses and reduce the number of sensors necessary for the compensation model
- **Dynamic Loads**: Compute dynamic loads on machine elements due to NC-Data
- **Fully Integrated Machine Tool Model**: A virtual machine tool is created that not only considers all machine elements and machine internal effects, but also auxiliary components as chillers and so on.
- **Real-Time Capable FE model**: To use the FE model for the compensation of a real machine tool, it must be real-time capable
1.6 Outline

At first, a brief outline of the thesis should be given in order to achieve a better understanding of the subsequent chapters and their context.

Chap. 2 shows a summary of thermal error measurements that have been made on a variety of machine tools with different configurations. The measurements that have been made should introduce the reader to the complexity that is attributed to the thermal behavior of machine tools. It is shown that a large variety of machine configurations exists and that each configuration exhibits its own characteristics that lead to different thermal errors. It is also shown that the thermal error not only depends on the structural design of the machine, but also on auxiliary components and the environment. Besides raising the awareness to the complexity of the thermal behavior, it is also necessary to get a feel for the range of the thermal errors that can occur.

As was stated in Sec. 1.5 it is necessary to have a fully integrated machine tool model that considers all machine elements, machine internal effects, and auxiliary components. This is underlined by the previous measurements in Chap. 2. Consequently, a software was developed in Chap. 3 that can achieve that goal. To do so, it is essential to model the essential physics that influence the thermal error of the machine tools as power consumption, for example.

In Chap. 4 more detail is given on the algorithms that are used in the software VMP which was shown in Chap. 3. One of the novelties is that multibody dynamics are used to calculate the loads on the machine components. Based on these loads, the losses in the machine elements are computed and used as an input to the thermomechanical FEM simulation. Another technical innovation is that the whole machine behavior can be computed depending on the NC path of the machine tool and as such it is possible to compute the thermal errors that would occur when machining a part. Hence, Chap. 4 covers the aforementioned subjects “Based on NC-Data” and “Dynamic Loads” that were raised in the deficiencies of the start of the art.

The next step is to create a thermomechanical code that is real-time capable. Focus is especially given upon the formulation of the thermal system of equations as it depends on a large variety of parameters. As a result, the thermal system of equations is separated into a constant, non-varying part and a varying part. Only the varying part is updated at every time step and as such a high computation speed can be achieved.

The following chapters, namely Chap. 6, Chap. 7, and Chap. 8 concern the macro models that are used to describe the physics of typical components that are used on machine tools. Many components use rolling bodies and therefore it is necessary to model the heat conduction and heat generation in rolling bodies. Chap. 7 deals with the thermal and electrical models of electric drives mainly for the usage as feed drives, while Chap. 8 explains models that can be used for auxiliary components.

The software and the models are verified with a variety of measurements shown in Chap. 9. An example is shown for a moving linear axis, where the temperatures and displacement of measurements and simulations are compared.

Chap. 10 shows the implementation of a FEM-based thermal compensation. To achieve this, the mechanical system of equations has to be reduced and the software VMP has to
be connected to the NC, in order to get data from the path and to correct the relative error between the TCP and WCP.

Finally the software and the models are implemented as a compensation on a machine tool in Chap. 11. First a simple compensation is shown for the spindle before later the FEM based compensation is implemented in a machine tool.
2 Measurements of Thermal Deformation of Machine Tools

Thermal deformations of machine tools can have a lot of sources, which are attributed to quite complex phenomena that influence each other. The more precise machine tools have to become, the more important it is to understand the effects that occur. For most of these effects the ISO 230-3 standard [4], which introduces measurement setups and advises on how to measure different causes of thermal displacements, already exists. A variety of additional methodologies to measure temperatures and the displacements will be shown in this chapter. Different measurements on different scenarios of the machine in service will be shown in order to contribute to a better understanding. This will help to get an idea of the magnitude of the thermal errors for different load cases and the effects which are important. Besides aiding the understanding of thermal effects, the measurements are also used to verify the simulations in Chap. 9.

2.1 Measurement Devices

A series of different measurement devices have been used to record the temperature and the displacements. Each of these measurement devices has some benefits and drawbacks, making them suitable for different load cases.

2.1.1 Temperature

The temperatures are monitored by a large number of temperature probes. If one wants to measure the behavior of the whole machine a device is used that can handle up to twenty channels. This is done as every axis needs a probe for the ball screw, bearing, carriage and feed drive. In addition it is necessary to monitor the spindle and the coolant entry and exit temperatures, in order to measure the heat transfer from the spindle to the coolant. Further probes are needed to record the air temperature of the environment and within the machine itself. Thus it is obvious that a lot of temperature probes are necessary. It is also recommended to use thermal grease between the probes and the structure, to improve heat transfer.

Thermography is used to visualize the temperature distribution of structural parts. The advantage of thermography is that the temperature distributions can be made visible, while a probe can only measure the temperature at a single point. One of the drawbacks is that the accuracy is affected by the reflectivity of blank metal. As blank metal is highly reflective for infrared radiation, measures have to be taken in order to increase the accuracy. The easiest way to remedy this is by changing the emissivity of the material. This can be seen in Fig. 2.1, where crepe tape is used to mask blank metal parts of the machine, as recommended in [3]. The crepe tape has a high emissivity (and low reflectivity) and is
very thin. Reflections therefore are not a problem and do not influence the temperature
distribution, due to the low thickness of the tape. Images that have been recorded with
thermography are shown for example in Chap. 9 in Fig. 9.5.

2.1.2 Displacements

The displacements are always measured by incremental probes. For most measurements
that are shown in this chapter the setup according to ISO 230-3:2005 [4] is used. This can
also be seen in Fig. 2.1, which basically consists of two setups from Fig. 1.2. In this case
however a short tool is measured and only three incremental probes are used to measure
X, Y and Z. The setup in Fig. 1.2 uses five probes to additionally monitor two rotations.
All displacements which are shown in the following measurements are such that a positive
value means a larger workpiece.

For some of the measurements shown here the R-test [83] is used. The R-test uses a
precision ball and three incremental probes to measure the relative displacements, as can
be seen in Fig. 2.2. This makes it valuable in combination with machines that have rotary
axes. Thanks to the precision ball it is possible to measure the displacements in different
angular positions of the rotary axis.

The R-test is also used to measure the location errors of rotary axes due to thermal
errors. This can be seen in Fig. 2.3, where a precision ball is mounted on a rotary table.
The precision ball first is measured at position 1, which corresponds to 0°. The table is
then turned by 90°, to position 2, where the thermal error will be recorded again. This
thermal error is measured at four angular positions. Based on the measurements at these
2.2 NC-On: Machine Tool Warm-Up

In this measurement the machine tool is just switched on, without any axis or spindle movement. The heating of the machine tool during this measurement has different causes: The best-known one is the heating of drives, which start to be in control even when they are not doing any positioning. Especially drives which have to compensate for gravity tend to heat up significantly. Fig. 2.4 shows the linear displacements during a warm-up of a machine, recorded once with the auxiliary components switched on (dashed) and once without auxiliary components. It can be seen that it can take up to 24 hours until the machine has become stable. This can mean that, when the machine is switched on, the operator has to re-adjust the machine for quite a long time.

Besides the warm-up the auxiliary components also play an important role. As soon as they are turned on, they consume energy and therefore also produce losses. In the case of units as chillers and hydraulics, they not only emit waste heat to the surrounding air of the machine, but also influence the temperature of the media they are working with.
2 Measurements of Thermal Deformation of Machine Tools

Chillers cool/heat the coolant to a desired temperature, whereas in hydraulics the oil is heated due to the losses in the pump and in valves. The influence of these effects can be seen, when the same machine is warmed up without hydraulics and chillers for the cutting fluid and electrical cabinet. In order to allow a meaningful comparison, the environment was controlled to a defined temperature. Comparing the dashed and solid lines in Fig. 2.4, it can be seen that not only the time constant changes when there are no auxiliary units, but also the overall amplitude of the displacements changes by about 50%.

2.3 Environmental Temperature Variation Error

Results of an Environmental Temperature Variation Error (ETVE) test can be seen in Fig. 2.5. The environmental temperature on the machine shop varies heavily due to the day / night cycle, where the temperature rises during the day and then drops during the night. Due to the symmetric design of the machine, in this case a lathe, almost no displacements can be seen in the X-direction. This already shows the benefits of a thermo-symmetric design [51]. The Y-direction shows the strongest reaction to the environmental temperature and it is easy to see the relation between the change in the environmental temperature and the resulting displacements. As the machine is rather small, the reaction is only about $1 \text{µm/}^\circ C$. Usually a machine tool also reacts with a certain delay to the change in the environmental temperature, which is most obvious in the Z-displacement where one can see a slight delay of a few hours. This is especially obvious in the peaks of the displacements that are about three hours later than the peaks in the temperature. This is indicated by the two black lines in Fig. 2.5. Besides the change in the environmental temperature the machine also reacts to the changes in the coolant temperature, which causes the zig-zag behavior at a higher frequency (approximately one hour).
2.4 Linear Axes

ISO 230-3:2005 [4] shows how to measure the thermal displacements caused by moving linear axes. To achieve deeper understanding of the behavior of linear axes, direct and indirect measurement, and the fixation of the scales must be considered. A look will also be taken at the heating of linear axis machine elements, like guide carriages, ball nut and bearings.

2.4.1 Direct and Indirect Measurement

**Direct Measurement** Fig 2.6 shows the displacements of a machine tool, where the X-axis has performed a pendular movement with a feed of 3000 \( \text{mm/min} \) for about seven hours. Afterwards the machine cooled down, which was recorded as well, such that the heating and cooling behavior could be observed. As can be seen, the displacements stay almost zero, with the only effect visible being the temperature control of the coolant. The jump at the end of the pendular movement (at \( t=7h \)) is caused by the spindle lock being released. Fig. 2.7 shows the temperature increases of the various machine elements. It can be seen that the guide carriage temperature has almost no increase, whilst the ball nut, thrust bearing and feed drive temperature show considerable temperature rises. Measurements on other machine tools usually show similar behavior, with the temperature increase of the guide carriage having the smallest effect. Displacements due to moving linear axes tend to stay below ten microns, with the temperature increase of the guide carriage being lower than the one of the ball nut.

**Indirect Measurement** In comparison, Fig. 2.8 shows the displacements due to a pendular movement of a Z-axis, with 3000 \( \text{mm/min} \) for two hours (phase I) and 5000 \( \text{mm/min} \) from \( t = 2h \) to \( t = 6h \) (phase II), on a machine tool without a glass scale. In this case the ball screw
is used for an indirect position measurement. Consequently the heating and expansion of the ball screw has a considerable effect on the accuracy of positioning. As a result, the thermal error can become up to $90 \mu m$ when performing a pendular movement at $5000 \, \text{mm/min}$. During the cooling phase, after $t = 6h$ the displacements do not come back to zero due to the change in the environmental temperatures.

In general it can be said that the displacements, when using an indirect measurement system, are much larger than with a direct measurement system. As a result it is recom-
mended to always use a glass scale on the machine tool. In the case of a direct measurement system the thermal error can be drastically reduced even if the results will not be as good as in Fig. 2.6. With a direct measurement system it can often be observed that the direction of the largest displacement is not in the direction of the axis’ movement, but in a lateral direction and the magnitude is around $10\mu m$.

![Figure 2.8: Displacements caused by a pendular movement without a direct measurement](image)

### 2.5 Rotary Axes

Thermal displacements of rotary axes have so far rarely been measured, apart from [84]. In this work a series of rotary axes were also measured. Rotary axes can generally be classified into swivel axes and turning axes. In this section the location errors of the rotary axes, according to the ISO standard, are measured. A positive error on for example X0C means that the C-axis has moved in the positive direction of the X-axis.

#### 2.5.1 Tilting Spindle

As an example of thermal deformation, a tilting spindle, according to the configuration in Fig. 2.9 was measured. In this example the heat that is generated when performing tilting motions of the B-axis is analyzed. The spindle itself is mounted on the B-axis. The B-axis was turned with a feed rate of $f = 2000\, \text{°/min}$ between its limit positions and then the displacements were measured at three different B-axis positions. At the first position ($B = 0^\circ$) five incremental probes were used in order to measure the rotary displacements.

It can be clearly seen in Fig. 2.10 that the most important displacement is the one of the elongation in the direction of the tilting axis. For the B-axis this means that the machine expands in the negative Y-direction. This is because the spindle can expand freely away from the fixture, which is the bearing that performs the rotary motion. The displacements
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Figure 2.9: Machine setup with a tilting spindle

Figure 2.10: Displacements caused by a pendular movement of a tilting spindle

in the X- and Z-direction are also quite large. The twisting of the axis is negligible as the A and B rotatory displacements stay small.

2.5.2 Rotary-Turning Unit

Rotary-turning units are one of the standard elements on five-axis machines. Therefore it has been investigated which displacements are caused by moving both rotary axes. For these measurements the R-test and the procedure according to Fig. 2.3 is used. The axis configuration used is shown exemplary in Fig. 2.11.
2.5 Rotary Axes

Figure 2.11: Machine setup with a rotary-turning unit

![Diagram of machine setup with rotary-turning unit](image)

Figure 2.12: Displacements caused by a pendular movement with the rotary B-axis

In a first analysis the B-axis was turned with a feed rate of $f = 4500 \frac{^\circ}{\text{min}}$ for five hours. After five hours the displacements of the axes during the cooling period of the machine were measured. Fig. 2.12 shows the B-axis displacements that have been deduced from the displacement measurements at three angular positions, namely $B = 0^\circ$, $B = 45^\circ$ and $B = -45^\circ$. This was done as the B-axis has a limited travel range and can not perform a $360^\circ$ rotation, as shown in Fig. 2.3. The bearing of the rotary axis heats both the bed, where the rotating-turning unit is mounted, as well as the rotating turning unit itself. Because of this it can be seen that the B-axis moves in the positive Z-direction due to the heating of the bed (Z0B), but also that a radial elongation ($\Delta R$ away from the center of the B-axis) of the rotary-turning unit can be seen. Due to the symmetry of the unit almost no X-displacements occur (X0B).

The next step is the investigation of the turning unit. In this case the turning unit can be used as a regular feed axis, or as a spindle. Since it is dimensioned for the loads when used as a turning spindle there are almost no displacements seen when being used as a regular feed axis. Fig. 2.13 and Fig. 2.14 show the displacements when used as a spindle. In this case the displacements become much larger. Due to the friction in the bearings it
can be seen that the table is heated quite a lot. The heat causes a radial elongation of the table ($\Delta R$) and also a vertical growth ($\Delta Z$). Since the rotating turning unit itself is also mounted on the bed, it expands and moves away from it in the negative Y-direction (Y0C). The heat is mainly generated in the vicinity of the table, causing a positive rotation about the X-axis (A0C). This is probably caused by the heat that is generated of the torque bearing and by the thrust bearing that are mounted close to the table. As the heat slowly flows through the unit, the temperature gradient becomes smaller, also reducing A0C. Therefore the behavior of A0C resembled that of two PT1 elements, a fast and a slow one.
2.6 Spindle Drift

Spindle drift is an important effect, which has been well investigated and analyzed in literature. In general what happens is that a linear elongation in the direction of the spindle can be observed. The displacement that is compensated accounts for the structure from the thrust bearing to the TCP, including the tool. This elongation directly depends on the spindle speed and load. Because of this simple dependency between the rotation speed and elongation a lot of compensation models for spindles exist. These compensation models, generally, however only compensate for the displacement in the direction of the spindle. Lateral displacements and effects such as load dependency, due to process forces for example, as well as cooling conditions of the tool are often neglected.

Figure 2.15: Spindle drift with tilting spindle at $B = 0^\circ$, $B = 45^\circ$ and $B = 90^\circ$

When looking at Fig. 2.15 it can clearly be seen that displacements in other directions also occur and that they can be quite large. The displacements are measured for 25%, 50%, 75% and 100% of a spindle speed of $15000 \frac{rev}{min}$. This is caused by the spindle heat flowing into the structure of the machine tool. The spindle used was a tilting spindle and it can also be observed that the displacements have been measured at different angular positions. The use of this may not be plausible at first sight. However, when taking a look at the displacements at $0^\circ$ (−Z) and $90^\circ$ (−X), it can be seen that the displacements in the spindle direction are different.
Using the different angular positions it is possible to decompose the measured displacements into the spindle elongation and a movement of the swivel axis which is mounted on the machine. The decomposition is shown in Fig. 2.16. From this it becomes obvious that not all displacements measured at $B = 0^\circ$ are caused by the spindle, but that all heat flows have to be considered. This would be especially crucial if a spindle compensation is used. If the compensation model was solely created from the displacements measured at $B = 0^\circ$ it would cause errors at different B-axis positions.

2.7 Summary

A lot of measurements have been done on a variety of machine tools. For some readers it might now be interesting to see which load cases causes the largest thermal error. Of course this depends to a large extent on the design of the machine tool but some general conclusions can be made.

Two five-axis machine tools were also measured completely in order to compare the thermal errors caused by different load cases. Fig. 2.17 and Fig. 2.18 compare the ratios of the thermal errors. The value that has been used for each load case is the largest error that was recorded in the steady-state. For the B-axis in Fig. 2.17 this would be the $-20\mu m$ that have been measured as $\Delta Y$ after ten hours in Fig. 2.10. All of these values
are summed up for all load cases in order to determine the ratio. While the ratio differs between the two machines, it can be seen that NC-on, rotary axes and the spindle drift can have a large share.

The measurements that have been shown can also be used to summarize a series of effects. The following effects shall be remembered for a better understanding of thermal errors:

- The warm-up of a machine tool can take many hours and lead to quite large deformations. Auxiliary components can have a significant effect on the thermal displacements. Therefore it is beneficial to position them away from the machine to minimize interactions.

- Direct measurement systems drastically reduce the thermal displacements caused by moving linear axes (compare Fig. 2.6 and Fig. 2.8). If the machine tool is well designed, the heating caused by moving linear axes is rather small in this case. This can also be seen in Fig. 2.17 and Fig. 2.18, where each linear axis only has a small share when compared to other thermal errors.

- For rotary axes, heat is generated on both sides of the bearing. This causes a displacement of the rotary axis, but also a growth of the axis slide (Fig. 2.12). Rotary axes are only fixed at one location by their bearing, which acts as a fixture. Therefore the displacements that occur are caused by a simple elongation of the structural parts, away from their fixture. This can for example seen by the negative Y-displacement in Fig. 2.10 and also the Y0C error in Fig. 2.13, where the table moves away from the fixture of the rotary-turning unit to the bed. Rotary axes can also contribute a large share to the displacements, as is shown in Fig. 2.17 and Fig. 2.18. Another problem is that location errors of rotary axes can lead to additional errors during five-axis machining. The machine needs to know the exact location of the rotary axes in order to perform a five-axis motion. Changes in the location of the rotary axes will cause errors in the toolpath.

- Rotary axes can be used to measure displacements in different angular positions and identify the cause and location of certain thermal displacements, as was shown for
the spindle drift. This was used on a tilting spindle in order to verify the actual elongation of the spindle.

• Spindle drift is still one of the major causes of thermal errors. The main error is a elongation in the axial direction, away from the thrust bearing. Lateral errors are less dominated but also apparent.
3 Virtual Machine Prototype (VMP) - Software to Simulate Machine Tools

The underlying basis for all thermal simulations that are shown in the context of this work is software called Virtual Machine Prototype (VMP), which was written as a part of this thesis. The main goal of this software is to make it efficient for machine-tool designers to simulate and analyze machine tools, not only with respect to the thermal behavior, but also to the mechanical behavior and energy consumption.

![Diagram of Physics Responsible for Thermal Errors]

Figure 3.1: Physics responsible for thermal errors

This can be seen in Fig. 3.1, showing the underlying physics which are responsible for thermal errors on machine tools. One of the root causes for thermal problems on a machine tool are power losses. This means that VMP needs to be able to calculate the energy consumption of every component of the machine tool, in order to know the losses. Especially for the NC drives the losses are inherently coupled with the dynamics of the machine tool because accelerations result in loads on the machine elements. These loads subsequently influence the friction and heat generation. In the next step the thermal behavior is influenced by the heat transfer and the temperature distribution of the machine tool. The temperature distribution then causes the deformation of the structure. Therefore it is necessary to know the static behavior, more precisely the stiffness, of the machine tool. With the stiffness and the thermal forces it is possible to evaluate the thermal compliance of the machine tool. The deformation of the machine tool also depends on the kinematic chain and the position of the axis slides. A bending of a guide for example can cause position dependent errors. In the end only the relative displacement between the TCP and WCP is of interest, as this is causing the error on the workpieces. As a result one needs to be able to simulate the energy consumption and the static, the dynamic as well as the thermal behavior to model thermal errors.
In order to simplify this complex context, VMP consists of a special GUI, geared towards machine tools, and a mathematical kernel, consisting of FEM and multibody dynamics. The different possibilities and parts of VMP will be explained in the following sections.

3.1 Designing Machines with VMP

Due to its extensive capacities, VMP can be used at various stages along the design process. Fig. 3.2 shows the different stages of the design cycle of a machine tool, as well as where and how VMP can be used along this process. For early design stages, when little information is known, multibody dynamics can be used. With multibody dynamics, the statics and dynamics of the machine tool can be comprehensively analyzed. A rough FE model of the machine can already also be used to simulate the thermal behavior. Later in the design phase, when more information is known, FEM can be used to evaluate the machine tool and to further improve the design with respect to the mechanical, thermal and energy behavior of the machine. After the design of the machine is finished and a prototype exists, VMP can also be used to compensate for thermal errors (see Chap. 10). Another possible use of VMP is to identify sources of errors which have been discovered by measurements and optimize the machine tool with respect to its mechanical, thermal or energetic properties. The final step in the design phase is when the machine tool is in series production and possibly needs to be customized or certain errors occur on parts produced by the customer. In this case VMP can be used to analyze the effects of the customization or what part of the machine tool is responsible for the error on the manufactured parts.

In each of these phases (Fig. 3.2) the designer has the possibility to evaluate different properties of the machine tools. These are given by the physics shown in Fig. 3.1 and listed in the following subsections.

3.1.1 Static Behavior

For a machine tool it is important to know the static stiffness of the complete structure, between the tool and the workpiece. This can happen for example in the concept phase,
when little information is known about the machine tool. In this phase the structure may
not be very detailed, but the axis configuration is set. Using the multibody dynamics code
it can be analyzed what effects the distance between guideways, the center of gravity of a
slide, etc. have. Structural parts themselves are considered to be rigid and therefore their
influence can not be considered.

When the detailed design is known, where the structural parts are more or less defined
the machine tool can be analyzed using the FE code. In this part the stiffness of the
structural parts is also considered to calculate the overall static stiffness of the machine
tool.

3.1.2 Dynamic Behavior

Besides the evaluation of the static behavior, it is also important to understand the ma-
chine’s dynamics. This can be used to set the gain of the controllers and get higher
accelerations as well as less vibrations. With the help of modal analysis it is possible to
know where the eigenfrequencies of the machine tool are, which is important to know for
preventing chatter. Currently only multibody dynamics can be used in VMP to analyze
the dynamic behavior, but an extension to FEM is fairly easy.

3.1.3 Thermal Behavior

The thermo-elastic deformation of machine tools may contribute to up to 80% of the
overall geometric inaccuracies of the workpiece. Therefore it is important to understand
the effects, which are caused by waste energy, cooling devices, as well as the influence of
changes in the environmental temperature. The deformation of the machine tool, due to
these thermal effects, can only be simulated with numerical means, as the structure of
the machine plays an important part in how heat is distributed. The simulation of the
thermal sensitivity of the structure can already be started in the concept phase, when only
a rough sketch of the machine’s layout is available. In the design phase many more details
of the geometry and the components that are used are available. As a result the model
has to grow within the design phase, which is made possible by VMP and the usage of the
so-called macro elements.

3.1.4 Energy Consumption

To evaluate the thermal behavior of the machine it is necessary to know the energy losses
at every mechanical part. As a result the whole energy consumption of the machine tool
is also known. In combination with the thermal model it is also possible to evaluate the
energy consumption of the cooling devices, which switch off and on depending on the
temperature of the media. In Fig. 3.3 the power losses at different machine elements,
when the machine tool follows a given NC-path can be seen.

In the context of energy savings, ecological criteria often play only a secondary role
behind economic criteria. One reason for studying the power consumption is to dimension
the cooling system of the machine tool. The other reason is to have the power consumption
as an input parameter for the thermal behavior of the machine tool and thus to enhance
the accuracy. It can be checked whether too much cooling power was installed in the
machine, but also how much cooling power is necessary to keep the thermal deformations within a certain accuracy.

### 3.1.5 Compensation

VMP can also be used to create a compensation model for the thermal error on machine tools. As soon as the machine tool was designed and the simulation model validated by measurements on a real machine tool, it can be implemented in the NC controller. To do so, the simulation model has to be reduced and the reduced model can be used to calculate the thermal errors of the machine tool sufficiently fast so that it can be used for real-time computations. The compensation model corrects the thermal error by offsetting the coordinate system of the machine. Thus not only the accuracy of the machined part can be increased, but also the warm-up time needed to create good parts can be shortened. More details on how the compensation model works can be found in Chap. 10.

### 3.2 Software Layout and GUI

The GUI of the software is designed in a way, that makes it very efficient to create models of machine tools. The whole approach of creating models is geared towards machine tools, making it much easier to implement the specifics of a machine tool, compared to conventional simulation software.

In Fig. 3.4 the main window of the software is shown. Besides the 3D model of the machine tool, it is possible to see the main parts as the Bed and X-axis slide in the tree view on the left. Each part has a series of sub-properties, such as the geometry and boundary conditions, which can be a variety of machine tool elements, like guideways, feed drives etc..
If a boundary condition is selected, the designer can choose what kind of machine element should be attributed to the boundary condition. This selection can be made in the lower left corner of Fig. 3.4, where currently a guide element is selected. To the right one can then select further properties for the machine element. In the example shown, one can select whether it is a guideway or guide carriage and then select the axis to which the machine element belongs. In the case of a guide carriage one can also select the product name of the element. The name is connected to a database in which all the information necessary to calculate friction, heat conductivity and so on is stored. The user does not need to provide this data, but can solely select an element, not having to worry about the complex physical process taking place in the machine elements and the equations to describe them.

The elements for which models exist and which can be selected are shown in Tab. 3.1. Macro elements for the ball screw, bearing, guide and harmonic drive are used to model the mechanical and thermal behavior of these components. For the belt drive, cooling device and electric drive mainly thermal effects are computed. Finally the direct measurement system, tool and workpiece can be used to automatically evaluate the nodal displacements of these components and the resulting thermal error at the TCP.
Table 3.1: Macro elements of VMP

<table>
<thead>
<tr>
<th>Ball screw</th>
<th>Direct measurement system</th>
<th>Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing</td>
<td>Electric drive</td>
<td>Guide</td>
</tr>
<tr>
<td>Belt drive</td>
<td></td>
<td>Workpiece</td>
</tr>
<tr>
<td>Cooling device</td>
<td>Harmonic drive</td>
<td></td>
</tr>
</tbody>
</table>

Besides these elements the user has to specify the axis configuration for the machine kinematic (Fig. 3.5 and Sec. 4.5), cooling devices (Fig. 3.6 and Chap. 8) as well as the environmental temperature. The axis configuration is simply set by typing the order of the axis slides along the kinematic chain from the workpiece to the tool. This can be seen in Fig. 3.5, which shows the names of the axes slides in the upper left corner. These names are then entered in the corresponding order in the text field, shown in the lower part of the figure.

**Figure 3.5:** Definition of the axis configuration in VMP

In the case of the cooling device the user has to specify the controller, data about the cooling tank and the tubes which interact with the machine tools’ structure. The example in Fig. 3.6 shows the configuration for the controller of the chiller, where the hysteresis and the setpoint can be chosen. The setpoint can either be a fixed temperature or a variable one e.g. an environmental temperature. A similar configuration has to be made for the tank and the tubing, which are listed in the left of Fig. 3.6.
Definition of cooling devices:
*Controller, Coolant Tank and Tubing

**Figure 3.6:** Definition of cooling devices in VMP
4 Modeling Machine Tools

In this chapter a thorough approach to modeling machine tools will be given. The goal of this chapter is to derive a model which includes all major physical effects, but still can be created and solved efficiently. The previous chapter already introduced the software Virtual Machine Prototype, its features and its graphical user interface. In this chapter a detailed look at the methods and algorithms that are used by VMP to model machine tools is given. Therefore the general program flow will be described, before describing the program parts in detail.

4.1 Program Layout

In Fig. 4.1 the layout of the simulation software Virtual Machine Prototype is presented. The software starts by reading a file that contains the NC path. This movement includes the acceleration, velocity and position for each axis, discretized at the IPO cycle (cycle of the NC’s interpolator) of the controller.

The simulation software has three staggered parts, depending on the respective time scale of the problem. The range of these time scales is shown in Tab. 4.1. The dynamics of the machine tool, will be calculated at the IPO cycle, based on the movement. The thermal effects are slower and therefore larger time steps, will be used for the FEM simulation. This is necessary because of the higher complexity of the FEM simulation. The average time step for the FEM simulation is in the order of a few seconds.

<table>
<thead>
<tr>
<th>Dynamics</th>
<th>Thermal</th>
<th>Mechanical</th>
</tr>
</thead>
<tbody>
<tr>
<td>ms</td>
<td>s</td>
<td>s-min</td>
</tr>
</tbody>
</table>

The software retrieves the movement which will be made in the next time step of the FEM simulation and calculates the loads on all machine elements using a multibody dynamics model of the machine tool (Part 1 in Fig. 4.1). The dynamic forces which act on the machine elements are calculated based on the accelerations given by the movement of the machine tool. Using the results from the multibody dynamics simulation, load dependent values of each machine element can be calculated. These for example include losses and heat conductivity in machine elements with rolling bodies. In order to calculate the losses, the velocities of each element are used.

All axis positions, losses, heat conductivities and so forth use the IPO cycle time. Because of this, they will be averaged for the FEM time step and the FE model will be updated with all current values. After the thermal FEM simulation has finished (Part 2), the displacement of the TCP can be evaluated by the mechanical FE model. However, as
the update of the TCP displacement is not necessary at every time step, it will only be done periodically (Part 3). The whole computational cycle is then repeated for the next time step of the FEM simulation.
Figure 4.1: Layout of the simulation software
4.2 NC-Path

The first part of the simulation is that the NC-path for the next FEM simulation time step is read by the software. The time step, at which the acceleration, velocity and position are read, is made smaller and can be set to the IPO cycle time of the control system. The acceleration is then used to calculate the inertial load at each time step according to the acting accelerations. These loads on the coupling points will be used in Section 4.4 to calculate the load dependent friction. Given the velocity of the axis it is then possible to calculate the power loss together with the load dependent friction. The velocity is also necessary to calculate velocity dependent conditions, such as fluid film lubrication in the rolling bodies. The friction models resemble the well-known Stribeck curve in which dry, mixed and fluid friction are distinguished between, depending on the velocity of the friction partners.

The position of the axes is not used for the computation of the loads or losses. However it is necessary to update the position of the axes in the FE simulation model. For this the averaged position in every time step will be taken and the position of the individual axes will be updated. Based on the new position, the couplings between for example guideway and guide carriage will be updated, in order to integrate the power loss and heat conductivity of the coupling points at the appropriate axis position. This is shown in Fig. 4.2, where on the left side the coupling at location A is seen. On the right side the axis can be seen after a position change. The old couplings have to be disconnected, whilst the coupling at the new position is introduced.

![Figure 4.2: Change of position and appropriate coupling/decoupling of axes](image)

4.3 Multibody Dynamics

Multibody dynamic models are often used to analyze the dynamic behavior of machine tools. In a multibody model the machine tool is made up of a series of rigid bodies connected by springs and dampers. Each rigid body corresponds to the structure of an axis slide, whilst the springs and dampers are used to model guides and drives. This leads to small systems of equations of the machine tools’ dynamics (4.1) which can be solved in a very short amount of time, but still provide rather accurate results. Multibody dynamics can also be used to analyze the harmonic response and eigenfrequencies of a machine tool. The multibody dynamics model consists, of the following three parts: the masses and inertia of each rigid body, the springs which couple the bodies and the load acting on the bodies. As only a quasi-static analysis is performed, damping is not considered. These
parts will now be described in more detail. The conversion from a FE model to a multibody dynamics, with the above-mentioned parts, is shown in Fig. 4.3. The most obvious difference is that in the multibody dynamics model there is no more spatial discretization of the bodies, as indicated by the FE mesh.

\[
[M] \{\ddot{x}(t)\} + [D] \{\dot{x}(t)\} + [K] \{x(t)\} = \{F(t)\}
\] (4.1)

**Figure 4.3:** Conversion from a FE model to a multibody dynamics model

### 4.3.1 Mass and Inertia

The mass and inertia matrices of a rigid body can be calculated from a finite element mesh of the machine tool. Each element of the mesh is considered to be a single mass point. The matrices for a body are then derived by the sum of all mass points. The more elements the mesh has, the more accurate the solution will be. First, the volume and mass of each element, has to be calculated. The centroid of an element is given by the shape functions ((4.2) for the X coordinate) evaluated at the origin of the element coordinate system times the vector of the global node coordinates of the i-th element.

\[
X_c = \{N_0\}^T \{X_{n_i}\}
\] (4.2)

Based on the mass and the centroid of each single element, it is possible to derive the center of mass for the whole model. In this case the model usually corresponds to a single body of the machine tool. The center of mass is given by (4.3) to (4.5).

\[
X_{com} = \frac{\sum_{i=1}^{N} m_i X_{ci}}{\sum_{i=1}^{N} m_i}
\] (4.3)

\[
Y_{com} = \frac{\sum_{i=1}^{N} m_i Y_{ci}}{\sum_{i=1}^{N} m_i}
\] (4.4)

\[
Z_{com} = \frac{\sum_{i=1}^{N} m_i Z_{ci}}{\sum_{i=1}^{N} m_i}
\] (4.5)
4.3 Multibody Dynamics

With the mass of the element it is possible to calculate the moment of inertia with respect to the origin, as shown in (4.6) to (4.11).

\[ I_{xx} = \sum_{i=1}^{N} m_i \left( (Y_{c_i})^2 + (Z_{c_i})^2 \right) \] (4.6)

\[ I_{yy} = \sum_{i=1}^{N} m_i \left( (X_{c_i})^2 + (Z_{c_i})^2 \right) \] (4.7)

\[ I_{zz} = \sum_{i=1}^{N} m_i \left( (X_{c_i})^2 + (Y_{c_i})^2 \right) \] (4.8)

\[ I_{xy} = -\sum_{i=1}^{N} m_i \left( (X_{c_i}) (Y_{c_i}) \right) \] (4.9)

\[ I_{yz} = -\sum_{i=1}^{N} m_i \left( (Y_{c_i}) (Z_{c_i}) \right) \] (4.10)

\[ I_{xz} = -\sum_{i=1}^{N} m_i \left( (X_{c_i}) (Z_{c_i}) \right) \] (4.11)

It is now necessary to evaluate the inertia tensor, with respect to the center of mass of the model. Therefore Steiners rule will be applied, according to the following equations, with \( m \) being the total mass of the model.

\[ I'_{xx} = I_{xx} - m \left( (Y_{com})^2 + (Z_{com})^2 \right) \] (4.12)

\[ I'_{yy} = I_{yy} - m \left( (X_{com})^2 + (Z_{com})^2 \right) \] (4.13)

\[ I'_{zz} = I_{zz} - m \left( (X_{com})^2 + (Y_{com})^2 \right) \] (4.14)

\[ I'_{xy} = I_{xy} + mX_{com}Y_{com} \] (4.15)

\[ I'_{yz} = I_{yz} + mY_{com}Z_{com} \] (4.16)

\[ I'_{xz} = I_{xz} + mX_{com}Z_{com} \] (4.17)

\[ (4.18) \]

4.3.2 Stiffness

Stiffness for the guideways and ball screw can be found in the catalog of the manufacturers of the respective machine elements. Usually the values which are given will result in systems which are too stiff, as the mechanical connection of the machine element to the structure also has some elasticity, which is not included in the rigid body [85]. Commonly a factor of 1/3 to 2/3 shall be applied to the stiffness of the manufacturer’s catalog. Getting accurate values for the damping is still quite complicated and an active topic in research.

4.3.3 Loads

In the case of a thermomechanical analysis of machine tools the rigid body model will be used to calculate the load on each coupling point. This is necessary as the heat generated
in machine elements strongly depends on the loading. The rigid body software [86] includes the following load-cases:

- Gravity
- Acceleration
- Process force

The actual load on a machine element is a combination of all these effects. Acceleration loads for each time step can be calculated by combining the software with the NC path information (Section 4.2). An accurate representation of the process forces is quite complicated, but estimations could be made using the well-known Kienzle formula, for example.

### 4.4 Load Dependency

The computation of the load dependent effects in the machine elements consists of a series of steps. The first step is the computation of the load on each coupling point. As the multibody dynamics software does not yet consider frictional loads, these are added to the internal load at this point. The friction load of each machine elements has to be carried by the subsequent elements in the kinematic chain of the corresponding drive. In Fig. 4.4, a single linear axis is shown, for demonstration. In the gray circles the frictional forces from other components, that act as external loads on the selected part are shown.

![Figure 4.4: Components and their respective loads along an axis](image)

The guide carriages have a friction force, relating to the load and velocity on the rolling bodies inside the carriage. The friction forces of all guide carriages, as well as the inertial
load, then act on the ball screw. Progressing through the kinematic chain of the drive, each part has to bear the inertial load and the frictional force of the previous parts acting on it. In the end, the feed drive has to supply the whole torque acting on the axis. The models to calculate the friction and power losses are given in detail in Chapter 6 for rolling bodies and Chapter 7 for the electric drives.

4.5 Thermal FEM

A FEM model will be used to calculate the temperature distribution of the machine tools’ structure. Again, as the structure is made up of multiple axes, some considerations on how to appropriately model the structure are needed.

4.5.1 Modeling a Single Axis

Similarly to the multibody model, where each axis is represented by a body, each axis is modeled using a separate FE model. This means that for each axis a mesh has to be created and then the appropriate boundary conditions have to be set. Setting meaningful boundary conditions is the most demanding part when simulating machine tools. In a common FEM analysis the designer has to assign a value for each boundary condition. However, values for heat conductivity and losses in bearings for example are not well-known and assumptions have to be made and to be validated by a sensitivity analysis.

In the following context a list of models exist for common machine elements. The designer has to supply the specification of the machine element and the corresponding model will automatically assign physically correct boundary conditions, based on the current load case. The list of models, which will be explained in detail later (Chapters 6, 7, 8), includes the following machine elements:

- Guideways / guide carriages
- Ball screws / nuts
- Bearings
- Rotary axis
- Electrical drives: Feed drives and spindles
- Cooling devices

Besides these boundary conditions convection has to be applied on the free surfaces and the mechanical fixture has to be set at the bed.

4.5.2 Coupling of Axes

Machine tools consist of a tool and workpiece side, a machine bed and an individual axis configuration. The axis configuration is the foundation for the movement of the tool and the workpiece and is therefore crucial in representing the machine tool. These components have to be set in the model, in order to give a correct representation of the machine.
Given the schematic of a machine tool in Fig. 4.5 it can be seen that the axis configuration, from tool to the workpiece, can be described as

\[ t-X-b-w \]

where \( t \) stands for the tool, \( b \) for the bed and \( w \) for the workpiece. This axis configuration has to be supplied to the software and for each axis the corresponding FE model will be imported. After the whole machine tool is assembled the necessary boundary conditions and machine tool components, for example guideways and ball screws need to be set. Especially guideways and ball screws are important as they act as coupling elements between the different axis parts.

**Figure 4.5:** Schematic of a machine tool axis with its couplings between the bodies

For each guide carriage/ball nut a coupling node (see Fig. 4.6) is introduced into the system of equations. This coupling node, drawn in orange, acts as a bulk node to represent the rolling bodies, with all their thermal properties as heat capacity and conductivity. Due to the movement of an axis the part of the guideway/ball screw in contact with the balls will always change, indicated by the shaded areas in Fig. 4.6. Therefore it is necessary to determine the nodes in contact at each time step and update the thermal matrices with the new connections to implement the changed heat flow. For each coupling element a
network of parallel heat conductivities, depending on the number of nodes in contact, has to be calculated. The total conductivity of this network corresponds to the resultant heat conductivity of the coupling element.

4.6 Mechanical FEM

As the temperature distribution on the machine tool has been calculated by the thermal FEM, it is then possible to calculate the thermal deformation of the machine tool.

In order to do this it is again important to consider the coupling points of the carriages and guideways. Similar to the thermal FEM, coupling nodes are used, however this time one for each side of the contacting partner, as can be seen in Fig. 4.7.

![Figure 4.7: Mechanical coupling between a guide carriage and guideway](image)

A coupling node has six degrees of freedom, which are needed to define all movements of the coupling. To couple the guide carriage and the guideway, six springs are needed, three translational and three rotational ones, with stiffnesses according to the manufacturer’s data. As a guideway allows a linear movement, the spring in the direction of the guideway naturally has a stiffness of zero. A mechanical FE model only has three degrees of freedom for each node, which are needed for the translations; rotation is not included. The nodes on the guide carriage/ball nut are therefore tied to the coupling node, in order to create an elastic-rigid coupling. The rotations are then defined by using the offset to the coupling node. The corresponding nodes on the guideway/ball screw, which are in contact, are tied in the same way to the coupling node. As a result the guidance’s faces are made rigid and constrained to the movement of the coupling node. The method described here can also be found in [87].

Using this procedure the system of equations of the whole machine tool is assembled and the different axes can be connected. The last step to get the thermal error is to compute the relative displacement between the tool and the workpiece at the TCP. If a measurement system is installed, it is also necessary to evaluate the displacements between the glass scales and scanning heads.
5 Thermomechanical FEM

The following chapter should give an insight in how to handle the finite element analysis for thermomechanical problems. Due to the properties of machine tools certain simplifications can be made in the mathematical formulation of the problem. These simplifications however need a special treatment that differs slightly from the regular usage of FEM. The main goal of this treatment is to shorten the simulation time and to enable compensation based on FE models (Chap. 10). Usually control systems only have limited computational power, therefore the focus lies on achieving high computation speeds and sufficient accuracy by considering the mathematical properties of FEM and using simple models. One of the elementary ideas behind the following chapter is that the calculation speed of FEM can be drastically increased, if it is optimized for a given problem and combined with appropriate simplifications.

5.1 Thermal Model

The thermal behavior of a machine tool depends on a lot of factors that vary throughout time. As such it is obvious that the model has to be updated at every time step of the simulation. When having a closer look at the problem it becomes apparent that only certain parts of the machine need to be updated while others are constant. In the case of machines tools the boundary conditions as heat convection coefficients for example are changing, but the surface area on which convection takes place can be considered constant. The whole machine tool now can be considered to be made up of a list of domains, such as mechanical parts, areas with convection or heat generation and so forth. The concept is visualized in Fig. 5.1 with the X-axis of a machine tool being decomposed in its domains. These domains are the boundary where convection occurs (red), the inner region (yellow) and the couplings.

The proposed FE code uses a special structure for saving the system of equations, where each domain is separated in its geometry (area, volume,...) and its physical properties (density,...). The following subsections will show, how this knowledge can be used to reduce the time needed to solve a time-dependent FEM problem.

5.1.1 The FEM Equations at the Domain-level

In this section the structure of the different parts of the equations and in particular the separation of the geometric structure and the physical parameters for a domain will be explained.

The equation for the transient thermal FEM with implicit time integration can be formulated as in (5.1), similar to what is proposed in [88]. Shape functions are denoted with an \( \{N\} \) and \( [B] \) is the gradient of the shape function.
5.1 Thermal Model

Figure 5.1: Decomposition of an X-axis in its domains

\[
[C] \{\dot{T}\}^{t+\Delta t} + ([K_k] + [K_c]) \{T\}^{t+\Delta t} = \{Q\}^{t+\Delta t} + \{Q^e\}^{t+\Delta t} \tag{5.1}
\]

with

\[
[C] = \int_V \{N\} \rho c_p \{N\}^T dV \tag{5.2}
\]

being the heat capacitance matrix,

\[
[K_k] = \int_V [B] \lambda [B]^T dV \tag{5.3}
\]

the heat conductance matrix and

\[
[K_c] = \int_A h_{conv}^{t+\Delta t} \{N\} \{N\}^T dA \tag{5.4}
\]

the convection matrix.

On the right hand side,

\[
\{Q^e\}^{t+\Delta t} = \int_A h_{conv}^{t+\Delta t} \{N\} \{N\}^T \{T_{env}\} dA \tag{5.5}
\]

considers the influence of the heat convection from the environmental temperature and

\[
\{Q\}^{t+\Delta t} = \int_V \{N\} q_V dV^{t+\Delta t} + \int_A \{N\} q_A^{t+\Delta t} dA + \{Q_n\}^{t+\Delta t} \tag{5.6}
\]

considers prescribed heat fluxes over the elements volume, surfaces and nodes.

The temperature dependency of material properties can be neglected in comparison to the distribution of material properties within different batches as the temperature range
on a machine tool is rather small. The validity for this assumption is shown in [16] and [32].

Under the assumption that boundary conditions and material properties are constant in a domain, they can be taken out of the integral over the domain and therefore the system of equations can be divided into the integral and a coefficient. The evaluated integrals can be saved and whenever a change in coefficients occurs, it is not necessary to re-evaluate the integral.

\[
[K_c] = \int_A h^{t+\Delta t}_{conv} \{N\} \{N\}^T dA \rightarrow [K_c] = h^{t+\Delta t}_{conv} \int_A \{N\} \{N\}^T dA \quad (5.7)
\]

This is shown by the heat convection matrix (5.7). A practical part where this method is used is the ball screw. The heat convection coefficient, \(h_{conv}\), can be assumed to be equal over the whole ball screw and it is mainly dominated by the rotational speed. Therefore a velocity dependent heat convection coefficient is calculated and then applied on the whole convection boundary.

With this assumption it is possible to rewrite (5.1) in the form of (5.8).

\[
\rho c_p \int_V \{N\} \{N\}^T dV \{\dot{T}^{t+\Delta t}\} + \\
(k \int_V [B][B]^T dV + \sum_i \left( h^{t+\Delta t}_i \int_{A_i} \{N\} \{N\}^T dA_i \right) \{T\}^{t+\Delta t}) \\
= \sum_i h^{t+\Delta t}_i \int_{A_i} \{N\} \{N\}^T \{T_{env}\} dA_i + \sum_j q^{t+\Delta t}_j \int_{A_j} \{N\} dA_j + \\
q^{t+\Delta t}_V \int_V \{N\} dV + \{Q_n\}^{t+\Delta t} \quad (5.8)
\]

By looking at (5.8) it is obvious that each domain has to be stored separately because otherwise it would not be possible anymore to assign new coefficients. Therefore, in similarity to an element-by-element approach [89] the matrices also have to be stored for each domain.

### 5.1.2 Global System of Equations

The FE model of the machine tool has been divided in its domains for the different parts as internal geometry, boundary conditions and so forth. For each of these parts the integral over its region will be carried out in an initialization and then stored. Because of this the computational effort in the solver is drastically reduced as the only necessity left is to multiply the pre-evaluated matrix of the integral with the corresponding coefficients, as was shown in Sec. 5.1. In a final step the matrices of all domains are summed up to build the system of equations of the machine which then can be solved.
5.1 Thermal Model

**Figure 5.2:** Assembly of the thermal system of equations
The algorithm used for creating the global system of equations can be summarized as in Fig. 5.2. The processes marked inside the initialization phase are performed only once at the beginning of the simulation. Processes inside the solver, such as recalculating and assembling the matrices of the varying domains and assembling the whole system of equations, take place at every time step and are repeated throughout the whole simulation.

In a first step, during the initialization phase, it is decided whether the physical effects in a domain are constant or whether they can vary throughout the simulation time. For the constant domains the matrices will be assembled only once during the initialization phase (left branch of Fig. 5.2). The varying effects can be divided into two different groups. For those which have constant parameters over their region the surface or volume integral will be evaluated during the initialization phase and new coefficient is applied at every time during the solution phase (remember the above example of the ball screw). Then there are domains which have to be completely recomputed during the solution phase. An example for this is the coupling that is used to model the heat conductance through the rolling bodies. First a new coupling has to be created depending on the position (as explained in Sec. 4.5). An algorithm searches for all nodes for example on the guideway that are in contact with the guide carriage. These nodes then will be coupled by an equivalent thermal network that represents the heat conductivity of the rolling body according to its speed and load. The final step is to build the thermal system of equations for the whole machine tool. This is done by summing up all matrices of the constant and varying domains. After the system of equations has been assembled the solver can compute the temperatures at the new time step.

### 5.1.3 Mechanical System of Equations

The equations for the mechanical system are stored in a regular manner as its assumed that its properties are constant. This means that the stiffness matrix \( [K] \) will only be assembled once for the whole model and then stored in the memory. Structural analysis is a very common topic in FEM books and therefore the respective equations are just posted for completeness without any further explanation. The stiffness matrix, with \( [D_{\text{mat}}] \) being the material stiffness matrix, is

\[
[K] = \int_V [B_{\text{mech}}]^T [D_{\text{mat}}] [B_{\text{mech}}] \, dV \quad (5.9)
\]

The force caused by the thermal expansion, due to the coefficient of expansion \( \alpha \), is

\[
[F_{\text{therm}}] = \int_V [B_{\text{mech}}] [D_{\text{mat}}] \{N\}^T \{\Delta T\} \alpha \, dV \quad (5.10)
\]

### 5.2 Combining the Thermal and Mechanical Model

Calculation of the temperature profile and the thermoelastic deformation of the machine is based on a staggered algorithm. This approach has been also used in [76] and [3], where the thermal and mechanical model are not identical. In the former work, the thermal model is created using FDM and the mechanical model using FEM. The temperature data
from the FDM is interpolated onto the FE mesh to calculate the thermal forces and the deformation.

In this work, the thermal and mechanical model are not identical. While both models are created using FEM, the meshing is different, as well as the implementation of machine elements that use bulk nodes (see Sec. 4.5 and Sec. 4.6). Regarding the meshing, the main difference can be found in the shape function of the elements. The thermal model uses linear shape functions while the mechanical model is relying on quadratic shape functions. Detailed reasons to use this approach are explained in Appendix A. Generally one can say that it is better to use linear shape function on transient thermal problems to prevent numerical oscillations and quadratic shape functions for the mechanical behavior to get more accurate results on bending. Due to the usage of two different meshes, the results from the thermal simulation also have to be interpolated onto the mesh for the mechanical model. As all corner nodes of the element overlap this is rather easy. On these nodes the temperatures can be directly transferred to the mechanical model. The mechanical model also has a node in between the corner nodes, due to the quadratic shape functions. The temperature at this node simply is the average of both corner nodes.

5.3 Bulk Nodes

An important concept used throughout the modeling of the different machine elements are so called bulk nodes. Bulk nodes represent a degree of freedom that is not explicitly discretized as a part of the mesh. The bulk node however is added to the matrices of the system of equations. A simple example of a bulk node would be, if one models the exchange of heat between the machine and air temperature beneath the machine covers. Performing a CFD simulation would be too cumbersome, if one only wants an approximate result. Under the assumption that the temperature is uniform beneath the cover one can introduce a single degree of freedom, representing the air temperature. The equation added to the system would look like (5.11).

\[ \rho_{\text{air}} c_{\text{p,air}} V_{\text{air}} \dot{T}_{\text{air}} = h_{\text{conv}} (T_{\text{machine}} - T_{\text{air}}) \]  

(5.11)

The same concept is used for various rolling body elements and electric drives. Rolling bodies for example are not explicitly modeled. However, for the thermal system an additional node is introduced, representing the mass of the rolling bodies. This node is connected with the contacting elements to establish the heat conductivity.

Feed drives are entirely modeled using bulk nodes. This has been done because of two reasons:

- The internals of a feed drive are quite complex to model
- Only the flange temperature, where the feed drive and machine structure connect is of interest. No mechanical model of the feed drive is needed as its deformation has no influence on the TCP error

Chap. 7 shows the thermal equivalent network of a electric drive. All nodes explained there will be added to the system of equations without having to model and mesh the feed drive.
6 Rolling Bodies

Rolling bodies are an element which is used throughout different components of a machine tool, where relative motion between two elements is necessary. Because of this a general analysis on the physics of rolling bodies shall be given. These effects mainly concern the friction and heat conductance of rolling bodies. In the next step a closer look is taken at the different components as ball screws, ball / roller guides and bearings. The verification of the models will is in Chap. 9.

6.1 Principle Physics

A series of physical effects influence the behavior of rolling bodies. These effects are substantial in calculating the frictional losses and heat conductivity of rolling bodies.

6.1.1 Hertzian Pressure

Hertzian pressure [90] occurs in the contact of two elastic bodies. Using hertzian pressure one can calculate the size of the contact area, the pressure and the deformation of the elastic bodies. Hertzian pressure can be calculated for the contact of spheres, cylinders and plains. The assumptions under which the theories are applicable are the following:

- linear-elastic, homogeneous and isotropic materials
- small contact area compared to the contacting bodies
- no friction

The case of no friction is not exactly fulfilled, but generally it is assumed that the theory holds true for small friction coefficients.

Machine elements usually are preloaded in order to remove backlash. This is where the hertzian pressure becomes important as it is necessary to calculate the force on a rolling body, in order to get reasonable values for the friction and heat conductivity. Besides the preload external loads act on the rolling bodies. The real force on a rolling body therefore is influenced by the preload and the external load.

Let’s consider a single ball, contacting a raceway, to explain the basic effects. Later preloaded assemblies will be studied in Sec. 6.2, 6.3 of the corresponding machine elements.

The properties that affect the displacement of the ball center due to the pressure are the geometry of the contact, the material combination and the force. The basic equation for a ball is given by Hertz as in (6.1).
6.1 Principle Physics

\[ \delta = \frac{\psi}{\xi} \sqrt{\frac{9F^2 \sum c_i (1 - \nu^2)^2}{8E^2}} \]  

(6.1)

The contact geometry is defined by the curvatures \( c \) of both bodies. Depending on these curvatures \( c \) a value for \( \frac{\psi}{\xi} \) can be selected. For a combination of materials with different young moduli and poisson ratios (6.2) has to be considered. However, generally steel/steel contact can be used.

\[ \frac{1 - \nu^2}{E'^2} = \frac{1}{2} \left( \frac{1 - \nu_1^2}{E_1^2} + \frac{1 - \nu_2^2}{E_2^2} \right) \]  

(6.2)

6.1.2 Relative Motions

A series of relative motions exist in the contact of rolling bodies [91]. These relative motions usually are slip and boring as shown in Fig. 6.1. Slip can for example be caused by different elastic moduli of the contacting partners (Reynolds slip) or in the contact of curved bodies (Heathcote slip). Because of the curvature of the contact area, the distance between a contact and the rolling axis, for example \( d_1 \) and \( d_2 \) in the figure, is varying. Thus a sliding velocity distribution is given and only at two point no sliding occurs. On the other parts of the contact area either a positive or negative slip occurs. Boring can be seen in the right part of Fig. 6.1, where only in the middle of the contact there is zero relative velocity. Due to the rotary motion of the ball relative velocities occur in the rest of the contact area that increase with the distance from the axis of rotation.

![Image of relative motions](image)

Figure 6.1: Sliping (left) and boring (right) in a rolling body contact

6.1.3 Elastohydrodynamic Lubrication

Considering the local processes acting in a rolling contact, one has to distinguish between Coulomb friction and elastohydrodynamic lubrication. Depending on the speed of the rolling contact different friction types develop according to the well-known Strubeck curve. Elastohydrodynamic lubrication can be used to represent the conditions in a rolling contact at sufficient speed. A lot of research has been conducted with the main goal being the calculation of the lubrication film which depends on a variety of properties as rolling speed, load and the elasticity of the interacting materials. Based on a series of simulations and
measurements for different values of all parameters, formulas have been derived which can be used to calculate the minimal and central film thickness in the rolling contact. For elliptical conjunctions the equation for the central film thickness (6.3) can be found in [92].

\[ \tilde{H}_c = 2.96 \hat{U}^{-0.67} \hat{G}^{0.53} \tilde{W}^{-0.067} \left( 1 - 0.61 e^{-0.73 \tilde{k}} \right) \] (6.3)

In [93] the central film thickness was calculated for a line contact which can be used in roller bearings.

\[ \tilde{H}_{\text{min}} = 1.6 \hat{G}^{0.6} \hat{U}^{0.7} \tilde{W}^{0.13} \] (6.4)

The parameters \( \tilde{H}, \tilde{W}, \hat{U}, \hat{G} \) and \( \tilde{k} \) are all dimensionless parameters, representing the speed, load, material and ellipticity of the contact. It can be seen in the power of the different parameters that speed and material elasticity have the largest influence on the film thickness. The dimensionless parameters are defined the following way:

Dimensionless film thickness:

\[ \tilde{h} = \frac{h}{R_x} \] (6.5)

Dimensionless load parameter for elliptical conjunction for a ball and rectangular conjunction for a roller:

\[ \tilde{W}_b = \frac{F_{rb}}{E' R_x^2} \] (6.6)
\[ \tilde{W}_r = \frac{F_{rb}}{E' R_x l_{rb}} \] (6.7)

Dimensionless speed parameter:

\[ \tilde{U} = \frac{v \eta_0}{E' R_x} \] (6.8)

with, in general,

\[ v = \sqrt{v_x^2 + v_y^2} \] (6.9)

Dimensionless materials parameter with \( \alpha_{p,v} \) the pressure-viscosity coefficient of the lubricant:

\[ \tilde{\alpha} = \alpha_{p,v} E' \] (6.10)

Based on these equations for the film thickness it is desired to calculate the friction and heat generation inside the lubrication film. The film thickness and contact area also add to the thermal resistance between the raceway and the rolling body. In the case of full separation between both contact partners, the heat conduction through the lubricant film is simply given by the contact area, the film thickness and the conductivity of the oil ((6.11) from [94]).

\[ L_h = \lambda_{oil} \frac{A_{\text{hertz}}}{h} \] (6.11)

Friction is governed by the lubricant shear properties. Shearing occurs due to different velocities of the rolling bodies in contact, like slip and boring, as explained in Subsec.
6.1 Principle Physics

6.1.2. Friction forces in an isothermal rolling contact have been calculated for example in [95], [94] by using (6.12).

\[ F = \int_{A_{hertz}} \frac{\hat{h} \partial p}{2 \partial x} dA \] (6.12)

In real rolling contacts friction is more complicated due to the influence of temperature and pressure on the viscosity of the lubricant.

6.1.4 Heat Conductivity

The heat conductivity through rolling bodies is a quite complicated matter as not only a series of different heat transfer phenomena take place, but also that the rolling bodies usually move, leading to a heat transport. When looking at a single rolling body heat conductivity is mainly influenced by the resistance of the lubrication film and the resistance of the body itself. The heat conductivity through the lubricant film, depending on the contact area and film thickness, was given in (6.11). As for the conductivity through the body itself two cases have to be distinguished, namely the rolling body not moving, or when it is turning.

The first case, when the rolling body is not moving, is easier to describe and is explained in the following section. The most simple solution is the so called rod-model (6.13), where it is assumed that the heat enters through the contact area and then flows along that contact area over the whole diameter of the rolling body.

\[ L_{rod} = \frac{A_{hertz} \lambda}{d} \] (6.13)

This model however neglects the shape of the rolling body and that heat flow can distribute over the whole body. Thus (6.13) can be seen as a lower limit with heat conductivity in reality being higher. In [94] it is explained how to solve the heat conductivity equation over the whole rolling body. The boundary conditions are a constant heat entry/exit over the contact area and adiabatic boundary conditions on the rest of the rolling body. For example the heat conductivity for a roller in polar coordinates is given by the solution of (6.14) with the above mentioned boundary conditions.

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} = 0 \] (6.14)

Numerical analysis has been performed for the case of a rolling cylinder with the boundary conditions given in Fig. 6.2. The rolling body is rotating with a angular velocity \( \omega \) and a heat flux of \( q \) is entering at the outer contact area and exiting at the inner contact area. The contact angle between the area and the rolling body is indicated by \( \varphi \).

The results of two simulations for a cylinder with a radius of 3\( mm \) and a contact angle of 0.01\( rad \) are shown in Fig. 6.3 and Fig. 6.4. The heat flux \( q \) entering at the contact zone is 1W. Fig. 6.3 shows the temperature distribution in the rolling body when the angular velocity is zero. It can be seen that the heat flows not only directly from the hot to the cold side, but also distributes itself over the whole rolling body. From this it immediately
becomes obvious why the rod model in (6.13) results in a heat conductivity that is too low.

\[ L = \frac{q}{\Delta T_{\text{contact}}} \]  \hspace{1cm} (6.15)

An analytical solution for the case of a moving rolling body is quite complicated as the heat transport strongly influences the conductivity through the rolling body. This case has been investigated in [96] and [94]. In [94] the derivation of the heat conductivity is quite lengthy and based on the simplification that the rolling body can be regarded as a semi-infinite body. The reasoning for this simplification is that at high angular velocities...
6.2 Ball screw

Figure 6.4: Temperature distribution in a rolling cylinder with $\omega = 30 \text{rad/s}$

the heat is confined to a boundary layer and the center of the rolling body is not influenced (compare Fig. 6.4) In the end approximate solutions are given in (6.16) and (6.17) which were derived for common parameters in machine elements.

The heat conductivity from the inner to the outer ring for a single ball is given in (6.16)

$$L_b = 1.36\lambda (S - 1)^{-0.1012} r_b \tilde{W}^{1/3} b \left[ 1 + 0.59 (S - 1)^{-0.2058} \tilde{W}^{1/6} \sqrt{\frac{\omega r_t^2}{a}} \sqrt{t_m} \right]$$  \hspace{1cm} (6.16)

and for a single roller in (6.17)

$$L_r = 0.32\lambda l_r \tilde{W}^{0.08615} b \left[ 1 + 1.13 \tilde{W}^{1/4} \sqrt{\frac{\omega r_t^2}{a}} \sqrt{t_m} \right]$$  \hspace{1cm} (6.17)

The approximated solution of the conductivity of the lubrication which are based on the contact area and the film thickness (6.3), are given in (6.18) for the ball and (6.19) for the roller.

$$L_{h0} = 0.22\lambda o_l r_b \tilde{W}_p^{0.83} \tilde{U}^{-0.67}$$  \hspace{1cm} (6.18)

$$L_{h0} = 0.041\lambda o_l l_r \tilde{W}_r^{0.72} \tilde{U}^{-0.7}$$  \hspace{1cm} (6.19)

6.2 Ball screw

In this section an analysis of the effects of rolling bodies in ball screws shall be given.

6.2.1 Load on the Bodies

Effects of preloading have been thoroughly analyzed in[97] and [98]. The main principles shall be reproduced here in order to give a basic understanding for the reader. Effects are derived for a tensioned double nut, but derivation is similar for a single ball nut, where preload is created using an enlarged diameter of the rolling bodies.
When considering a ball screw the deformation on the side of the ball nut, as well on the side of the ball screw have to be considered. Therefore there is a influence of the geometries on both sides, which is included in the factor $c_k$, as seen in (6.20).

$$c_k = \frac{\psi_{bs}}{\xi_{bs}} \sqrt{\sum c_{bs}} + \frac{\psi_{nut}}{\xi_{nut}} \sqrt{\sum c_{nut}}$$  \hspace{1cm} (6.20)

Together with the factor $c_e$, given in (6.21), (6.1) will be changed to (6.22). In (6.22) the axial deformation of a ball due to an axial load is given. As (6.1) is in the normal direction of the ball, the contact angle and lead angle of the ball screw have to be considered which can be seen in Fig. 6.5. The axial force $F_a$ itself is divided by the number of balls $z_T$ which are in contact.

![Figure 6.5: Forces acting on the balls in the ball screw](image)

$$c_e = \sqrt{\frac{1 - \nu^2}{E^2}}$$  \hspace{1cm} (6.21)

$$\delta_a = 1.04c_e c_k \sqrt{\left(\frac{F_a}{z_T}\right)^2 \frac{1}{\cos^5 \beta \sin^5 \varphi}}$$  \hspace{1cm} (6.22)

However, (6.22) is still for a ball screw with out preload. Let’s consider a preloaded ball screw consisting of two ball nuts. First, derive the stiffness of the contact, by rearranging (6.22) to (6.23).

$$k = z_T \frac{0.943 \sin^{5/2} \varphi \cos^{5/2} \beta}{c_e c_k^{3/2}}$$  \hspace{1cm} (6.23)

Since there are two ball nuts the reaction to an axial force will be the following (6.24), where $F_1$ is the force on the balls in the first ball nut and $F_2$ the force on the balls in the second ball nut.

$$F_a = F_1 - F_2 = k_1 (\delta_{pr} + \delta_a)^3 - k_2 (\delta_{pr} - \delta_a)^3$$  \hspace{1cm} (6.24)

Based on this (6.24) the axial displacement for the preloaded ball screw can be derived and given as in (6.25). This is done by using the series expansion of the binomial (as e.g. in (6.40)) and then using only the linear part and neglecting higher order terms.

$$\delta_a = \frac{F_a}{3k \sqrt{\delta_{pr}}}$$  \hspace{1cm} (6.25)
6.2 Ball screw

In order to estimate the friction losses, the contact forces for each individual ball nut have to be calculated. This can be done by using the axial force which is known from the multibody dynamics simulation (see Sec. 4.3). The axial force is used together with (6.25) to calculate the axial displacement. As then can be seen in (6.24) the load $F_1$ and $F_2$ can be calculated for each side. The deformation of one single part of the nut caused by the preload, which is also necessary to know, can be calculated using (6.26).

$$\delta_{pr} = 1.04c_t^2 c_k \left( \frac{F_{pr}^2}{\sin^5 \varphi \cos^5 \beta} \right)^{1/3} z_t^{-2/3}$$  (6.26)

When looking at (6.24) it is important to note that there is a certain external force, at which one side of the ball nut will be unloaded. If $F_a$ exceeds that force all the external load has to be carried by the other side. The force at which this happens is, however, about 2.8 times higher than the preload.

6.2.2 Frictional Losses

Since ball screws with tensioned single nuts and tensioned double nuts exist, resulting in different contact geometries, different formulas have to be used to calculate the frictional losses. These formulas, taken from [94], give an approximation to the effects explained in Sec. 6.1. Using exponential functions, depending on the speed, for the coulomb friction (at low speed) and EHD section (at high speed), the Streebeck curve is approximated.

The speed coefficient $U$ for a ball screw is given in (6.27).

$$\dot{U} = \frac{\eta \omega t_m}{2E \cos \varphi}$$  (6.27)

In the case of a tensioned double nut with two point contact the coulomb friction, EHD friction and frictional losses are given by (6.28), (6.29) and (6.30). The contact angle should be between $40^\circ$ and $90^\circ$ with the lead angle between $1^\circ$ and $20^\circ$. For the double nut the friction and loss is calculated for both nuts depending on the respective load (see Subsec. 6.2.1). The total friction and load is given by the sum of the individual losses ($P_i / M_i$).

$$M_{C_i} = \frac{2.05\mu c \dot{W}_b^{0.091} (S - 1)^{-0.099} (\sin \beta + 0.041 \sin \varphi) t_m^{-0.05} F_i r_b t_m}{\cos \beta}$$  (6.28)

$$M_i = M_{C_i} e^{-6.4e+11 U^0.512 \dot{G}^{-0.2} W_b^{-0.516} F_i r_b t_m}$$  (6.29)

$$P_i = M_i \omega$$  (6.30)

For a tensioned single nut, resulting in a four point contact, the friction and losses are calculated by (6.31), (6.32) and (6.33). The contact angle should be between $30^\circ$ and $60^\circ$ with the lead angle between $1^\circ$ and $20^\circ$. 

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\[ M_C = 3.77 \mu_C \bar{W}^{0.229} (S - 1)^{-0.249} (\sin \beta + 0.295 \tan \varphi) t_m^{0.347} F_{rb} t_m \]  
(6.31)

\[ M = M_C e^{-6.8e11 \bar{U}} + \frac{9.16 \left(1 - e^{-6.8e11 \bar{U}}\right) \bar{U}^{0.493} \tilde{G}^{-0.297} \bar{W}^{0.5} F_{rb} t_m}{\cos \beta} \]  
(6.32)

\[ P = M \omega \]  
(6.33)

### 6.2.3 Heat Conductivity

For the thermal modeling of the ball screw it is also important to know the heat conductivity through it. The conductivity is composed of a variety of effects. These effects are the lubrication film, the conductivity through the ball (which was assumed to be adiabatic) and also conductivity between balls and so forth. The load \( F_1 \) and \( F_2 \), on the first and second nut, can be calculated according to the derivation in Sec. 6.2.1.

Using the conductivity through a single ball (6.16) and (6.18) for the conductivity through the lubrication film. The conductivity will be calculated for each nut depending on its load. The total heat conductivity through the ball screw then can be written as in (6.34). The parallel heat conductivity \( L_{||} \) is composed of the different components that contribute to the heat conductivity through the bearing and is a factor that has to be estimated from measurements.

\[ L = \sum_{i=1}^{2} z_i \left( \frac{L_{b_i} L_{h_i}}{2 L_{b_i} + L_{h_i}} \right) + L_{||} \]  
(6.34)

### 6.3 Guide Elements

In this section the losses in guide elements shall be described. For brevity only two examples will be shown, even if there are a lot of different combinations. These examples are ball and roller guides with four rows. Again three sections are shown, to calculate the loads, the friction and then heat conductivity through the roller.

#### 6.3.1 Load on the Bodies

In general, the load on the bodies in a guideway can be calculated according to the procedure shown in 6.2. However with rollers the Hertzian contact will be different as it will be a rectangular conjunction, opposed to the elliptical conjunction for balls. Therefore the calculation of the load on a roller guide with four rows will be shown.

**Roller Guide, 2-point Contact:** To calculate the stiffness, one has to first consider the force equilibrium in the X- and Y-direction, of all rows and an external force (6.35) and (6.36) on the guideway with four rows of rollers as shown in Fig. 6.6.

\[ F_x = F_{1x} - F_{2x} + F_{3x} - F_{4x} \]  
(6.35)

\[ F_y = F_{1y} + F_{2y} - F_{3y} - F_{4y} \]  
(6.36)
The relation for the Hertzian contact between the displacement and a force for steel rollers in a single row is given in (6.37).

\[ F = z_t \left( \frac{10^{5.0.8}}{3.97} \right)^{\frac{1}{3}} \delta^{\frac{1}{3}} \]  

(6.37)

The parameter \( k \) is given as in (6.38) with \( z_T \) being the number of carrying rolling bodies in a row.

\[ k = z_t \frac{10^{5.0.8}}{3.97} \]  

(6.38)

Inserting (6.37) in (6.35) and (6.36), under the consideration of the contact angle, yields the relation between a force on the guide carriage and the appropriate displacement of the rollers in each row. In (6.39) shows the combinations the equation for the load in Y-direction, with \( k \) being the factor considering the length from (6.37).

\[ \frac{F_y}{\cos \varphi} = k \left[ (\delta_{pr} + \delta_1)^{\frac{1}{3}} + (\delta_{pr} + \delta_2)^{\frac{1}{3}} - (\delta_{pr} + \delta_3)^{\frac{1}{3}} - (\delta_{pr} + \delta_4)^{\frac{1}{3}} \right] \]  

(6.39)

Again the binomial is expressed as a series expansion terminated after the first term as was done in [97] for the ball screw (Equ 6.40).

\[ (1 + x)^y = 1 + y \cdot x + \frac{y(y - 1)}{2} x^2 + ... \]  

(6.40)

Using (6.41) to (6.44) and (6.39) which relate the deformation normal to the roller to a global x and y displacement its possible to derive (6.45) and (6.46).

\[ \delta_1 = \delta_y \cos \varphi + \delta_x \sin \varphi \]  

(6.41)

\[ \delta_2 = \delta_y \cos \varphi - \delta_x \sin \varphi \]  

(6.42)

\[ \delta_3 = -\delta_y \cos \varphi + \delta_x \sin \varphi \]  

(6.43)

\[ \delta_4 = -\delta_y \cos \varphi - \delta_x \sin \varphi \]  

(6.44)

\[ \delta_x = \frac{F_x}{4.4k\delta_{pr}^{2} \sin^2 \varphi} \]  

(6.45)
The preload for a guideway is defined as the normal force in Y-direction at which the rollers are completely unloaded. Therefore the deformation due to the preload is given as in (6.47).

\[ \delta_{pr} = \frac{3.97}{10^{0.8}} \left( \frac{F_{0.9}}{2z_T \cos \varphi} \right)^{0.9} F_{pr} \]  

(6.47)

As now the deformation for each roller can be calculated, it is also possible to derive the normal force acting on it with (6.37) where:

\[ \delta = \delta_{pr} + \delta_i \]  

(6.48)

with \( i \) being the index of rows one to four.

**Ball Guide, 2-point Contact:** For completeness the same formulas will be given for guideways with balls as rolling elements with \( c_e \) and \( c_k \) defined analogous as for the ball screw.

\[ \delta_x = \frac{F_x}{6k\sqrt{\delta_{pr} \sin^2 \varphi}} \]  

(6.49)

\[ \delta_y = \frac{F_y}{6k\sqrt{\delta_{pr} \cos^2 \varphi}} \]  

(6.50)

\[ k = z_i \frac{1}{1.04c_e^2 c_k} \]  

(6.51)

\[ \delta_{pr} = \frac{1.04c_k c_e^2}{(2z_T \cos \varphi)^{\frac{3}{2}}} F_{pr}^2 \]  

(6.52)

The force on a single ball then is calculated with (6.53) where \( \delta_i \) is the same as in (6.41) to (6.44).

\[ F = \frac{k}{z_T} (\delta_{pr} + \delta_i)^{\frac{3}{2}} \]  

(6.53)

### 6.3.2 Frictional Losses

The frictional losses are again calculated based on [94]. First the dimensionless speed coefficient 6.54 is used with \( v \) being the velocity of the axis.

\[ \tilde{U} = \frac{\eta_0 v}{2E'_{rb}} \]  

(6.54)

For a single row in a roller guideway (6.55) to (6.57) are used to calculate the friction and power losses inside the row of a guide. The load \( F_i \) is calculated by the formulas that were previously given. The angle \( \Delta \varphi \), with values between 0.005rad and 0.02rad, is the deviation between the direction rolling direction and the orientation of the roller.
total power loss and friction is calculated by the sum of the individual components \( P_i / M_i \) for each of the four rows.

\[
F_{C_i} = 1.02 \mu_C \Delta \phi F_i \quad (6.55)
\]

\[
F_{F_i} = F_{C_i} e^{-7.7e11 U} + 85.7 \left( 1 - e^{-7.7e11 U} \right) \dot{U}^{0.598} G^{-0.352} W^{-0.575} F_i \quad (6.56)
\]

\[
P_i = F_{F_i} v \quad (6.57)
\]

In the case of a guide with the balls as rolling bodies (6.58) to (6.60) can be used to calculate the friction losses.

\[
F_{C_i} = 0.158 \mu_C \dot{W}_b^{0.408} (S - 1)^{-0.435} F_i \quad (6.58)
\]

\[
F_{F_i} = F_{C_i} e^{-8.1e11 U} + 4.31 \left( 1 - e^{-8.1e11 U} \right) \dot{U}^{0.538} G^{-0.2} W^{-0.569} F_i \quad (6.59)
\]

\[
P_i = F_{F_i} v \quad (6.60)
\]

### 6.3.3 Heat Conductivity

The heat conductivity through a guide element is calculated similar to Sec. 6.2.3. However this time the conductivity is given by the total conductivity that is given by the four rows (6.61). Depending on whether balls or rollers are used different equations for \( L_{rb} \) and \( L_{h0} \) have to be used as can be seen in Sec. 6.1.4.

\[
L = \sum_{i=1}^{4} \xi_i \left( \frac{L_{rb_i} L_{h_i}}{2L_{rb} + L_{h_i}} \right) + L_{||} \quad (6.61)
\]

### 6.4 Bearings

Ball screws are mounted using a thrust bearing on the driving side and a floating bearing on the driven side. In the subsequent sections the calculation of the heat induced in bearings by friction will be discussed. Bearings are a standard machine element which has been well investigated by the various manufacturers. They also give a series of formulas to calculate frictional losses, etc. Standardized values found in manufacturer catalogs, as from FAG and SKF, then can be used. Because of this and as there are limitless combinations of bearings it is easier to use the formulas of the manufacturers then to calculate for example the preload for each combination. However, if required, the same procedures as for the other machine elements can be employed.

#### 6.4.1 Frictional Losses

The loss caused by friction can be evaluated according to [99]. Appropriate coefficients are taken from the catalogs by e.g. FAG and SKF. Friction torque is mainly influenced by factors as revolution speed, load and the viscosity of the lubricant.

\[
M_{friction} = M_0 + M_1 \quad (6.62)
\]
The total friction torque $M_{\text{friction}}$ (6.62) is composed of a load-independent torque $M_0$ and a load-dependent torque $M_1$.

Besides the mean diameter of the bearing, revolution speed and viscosity play an important role as can be seen in (6.63). The viscosity also is a function of the temperature.

$$M_0 = f_0 \cdot 10^{-7} \cdot (\nu_{kv} \cdot n)^{2/3} \cdot d_m^3$$  \hspace{1cm} (6.63)

The mean diameter is calculated by using the inner and outer diameter of the bearing. The load-dependent friction torque $M_1$ is evaluated by using (6.64).

$$M_1 = f_1 \cdot P_1 \cdot d_m$$  \hspace{1cm} (6.64)

The values and equations to calculate $f_1$ and $P_1$ depend on the bearing type. Usually the equation take on the same form for different bearing types and contact angles, but varying coefficients are applied. The general form for $f_1$ is shown in (6.65).

$$f_1 = f_{1\text{factor}} \cdot \left( \frac{P_0}{C_0} \right)^{f_1\text{power}}$$  \hspace{1cm} (6.65)

The formula to calculate $P_0$ and its coefficients are found again in manufacturer’s data as for instance [100]. The axial force $F_a$ in (6.66) includes the preload and external axial forces. As known from bearing life calculation, $P_0$ is used for the static equivalent load.

$$P_0 = X_0 \cdot F_{ra} + Y_0 \cdot F_a$$  \hspace{1cm} (6.66)

Ideally there should be no radial force acting on the bearings as the guideways support them. However as the ball screw and bearing axis never perfectly align small radial forces will act on the bearing. Depending on the moving direction the axial force either increases or decreases the preload according to (6.67) and (6.68) in the case of two bearings.

$$F_a = F_{pr} + 0.67 \cdot F_{\text{bearing,ext}}$$  \hspace{1cm} (6.67)

$$F_a = F_{pr} - 0.33 \cdot F_{\text{bearing,ext}}$$  \hspace{1cm} (6.68)

---

**Figure 6.7:** Thrust bearing with external loads
As can easily be seen in Fig. 6.7 one bearing (red) is always loaded while the other is unloaded (blue). Whether the force acts from the left or right side does not change this fact. If the thrust bearing consists of three or more bearings appropriate coefficients have to be chosen. These can for example be found in the manufacturer catalog, as from BAG.

The load \( P_1 \) for the calculation of the frictional torque can be calculated according to (6.69), at least for most bearing types. The coefficients \( X_1 \) and \( Y_1 \) vary depending on the bearing type and appropriate values can be found in the manufacturer catalogs. As for the axial load it again has to be distinguished whether its a single bearing or a bearing pair. However, this can again be considered by the coefficients \( X_1 \) and \( Y_1 \).

\[
P_1 = \begin{cases} 
X_1 \cdot F_{ra} + Y_1 \cdot F_a, & P_1 > F_{ra} \\
F_{ra} & P_1 \leq F_{ra}
\end{cases}
\]  

(6.69)

### 6.4.2 Heat Conductivity

For the heat conductivity of bearings measurements have been made by FAG in [101] (as seen in [102]). The results of these measurements will be used here because these measurements were directly taken by the manufacturer for typical bearing types. As such they are believed to be more accurate than general theoretical formulas.

The common form for heat transfer between the inner and outer ring can be described by (6.70) which is also used for the bearing. In the case of a bearing the factor \( \lambda \) mainly depends on the number of load carrying bodies, the size of the bodies and the rotational velocity.

\[
Q = \lambda \cdot \Delta T = z_t \cdot \lambda_i (d_b, v_p) \Delta T
\]  

(6.70)

The heat transfer through a single rolling body is shown in Fig. 6.8 which can be approximated by (6.71). The peripheral speed \( v_p \) of the bearing can be calculated according to (6.72) with \( d \) being the nominal bore diameter.

![Figure 6.8: Heat conductivity of a rolling body; Originally from: [102]](image)
\[ \lambda_i (d_b, v_p) = \frac{1}{2400} \sqrt{14 + 2 \ln(v_p) - 2 \ln(d_b) \cdot d_b^2} \]  
\[ v_p = \frac{(d + d_b) \cdot n}{19099} \]
Electric drives on machine tools are used in different forms and for different purposes. Feed drives are used to position the axes according to the NC code with the smallest possible lag. Usually feed drives are permanent magnet synchronous machines. Sometimes asynchronous machines are also used when position requirements are not very high. In contrast to feed drives, most spindles use asynchronous machines with a recent trend towards permanent magnet synchronous machines to achieve a more slender design, also allowing threading and/or probing which requires defined spindle positions. Principles of electric drives can be found in a wide range of literature ([103], [104], [105]). The verification of the model for the electric drive is shown in Chap. 9.

In this chapter a generic thermal model for electric drives will be derived. This generic model can be used to compute the thermal behavior of spindles and feed drives. It will also be shown how to compute the losses of a permanent magnet synchronous machine. This is necessary to analyze the influence of feed drives on the thermal behavior of machine tools.

7.1 Thermal Equivalent Network

In this section a general thermal equivalent network shall be derived which can be used for all kind of electric drives occurring in machine tools. Usually the thermal effects of electric drives are modeled using one or two body systems, where the whole drive or stator and rotor are treated as a homogeneous body. In the case of machine tools sometimes a more detailed approach is needed for additional information as for example rotor temperatures in the case of a rotary feedthrough to prevent binding due to thermal expansions.

The basis for the thermal network is given by [106] which was created for totally enclosed fan cooled asynchronous motors. For bodies with internal heat generation often T-node equivalent circuit are used as the central temperature does not correspond to the average temperature. In a T-node circuit an additional resistance is introduced, compared to traditional thermal networks, to get the average temperature of the body [107]. A series of analogous thermal networks has been derived for the different types of electrical machines ([108], [109], [110]). Fig. 7.1 shows the general thermal network which was derived to model all types of electric drives.

The thermal network represents a radial cut of an electric drive. In the center there is the shaft and the rotor. In the case of a permanent magnet synchronous motor the rotor and shaft could be represented as a single body as no major losses are expected in the magnets. However in an asynchronous drive rotor losses have to be considered. Axial symmetry is not included because for a spindle the generated heat in the thrust and floating bearing is substantially different. The node from the rotor outwards, through the air gap, is the stator iron which is divided in stator teeth and stator yoke. In between the teeth there is the stator copper with the end windings. On the outside of the stator yoke the frame is...
reached. A variety of possibilities exist to remove the heat from the frame. In the case of a spindle the resistance of the spindle cooling has to be set. If the network is used to model a feed drive, then convection to air and especially the mounting is important as most of the heat will be dissipated to the machine structure through it.

7.2 Thermal Resistances and Capacitances

Thermal resistances and capacitances for a thermal equivalent network can be calculated quite efficiently. Considering a cutaway of a cylinder using the dimensions of Fig. 7.2 one can define radial, circumferential and axial resistance as shown in Tab. 7.1.

Table 7.1: Thermal resistances of a cylinder cutaway

<table>
<thead>
<tr>
<th></th>
<th>Radial</th>
<th>Circumferential</th>
<th>Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$\frac{\log\left(\frac{r_{o}}{r_{i}}\right)}{\Phi M}$</td>
<td>$\frac{r_{m}\Phi}{360(r_{o}-r_{i})\mu\Lambda}$</td>
<td>$\frac{l}{\pi(r_{o}^{2}-r_{i}^{2})\lambda}$</td>
</tr>
</tbody>
</table>

In some cases the calculation of the thermal resistance and capacitance is not as straightforward. For these cases more detail will be given in the following subsections.
7.2 Thermal Resistances and Capacitances

7.2.1 Air Gap

Between the stator and rotor there is a small air gap through which heat can flow in the radial direction. As the air gap is a feature of every electric drive, a lot of research has been invested in finding the heat flow through the air gap and many different expression can be found in the literature. Here it will be calculated using the modified Taylor number \( Ta_m \) (7.1) from which the Nusselt number (7.3) and heat transfer coefficient 7.4 can be derived [108]. In (7.1) \( r_{\text{gap}} \) denotes the average radius of the gap and \( \Delta r_{\text{gap}} \) the size of the air gap.

\[
Ta_m = \frac{\omega^2 r_{\text{gap}} \Delta r_{\text{gap}}^3}{\nu_{\text{ke,air}}} \quad (7.1)
\]

\[
Nu = \begin{cases} 
2, & Ta_m < 1740 \\
0.409 Ta_m^{0.241} - 137 Ta_m^{-0.75}, & Ta_m > 1740 
\end{cases} \quad (7.2)
\]

\[
h_{\text{conv}} = \frac{Nu \lambda_{\text{air}}}{2 \Delta r_{\text{gap}}} \quad (7.3)
\]

Now with the heat transfer coefficient being known, the thermal resistance of the air gap can be calculated as shown in (7.5). It shall be noted that, again, the heat conductivity and viscosity of air are temperature dependent. In a very accurate examination the size of the air gap would also be temperature dependent due to the thermal expansion of the rotor and shaft.

\[
R_{\text{airgap}} = \frac{1}{2\pi r_{\text{gap}} l_{\text{stator}} h_{\text{conv}}} \quad (7.5)
\]

7.2.2 Internal Air

Due to convection heat can also be exchanged between the internal air and the frame, end-windings and shaft. These values can vary if the electric drive is totally enclosed or fan cooled. Due to the complex geometries inside a motor it is hard to give an analytical description of the convection coefficient. Instead it is identified and fitted from measurements and model identifications on electric drives. This leads to a high uncertainty in the value, but usually the inner air has no significant effect on the temperatures because of the
low mass of air inside a motor. A function to calculate (7.6) the heat transfer coefficient is proposed in [110], where \( v_p \) is the peripheral speed of the rotor.

\[
h_{\text{conv}} = 15 + 6.75^{0.65} v_p^{0.65} \quad (7.6)
\]

### 7.2.3 Lamination

Between the stator copper and the iron there is lamination. The lamination is very thin and therefore is not considered as an independent node in the thermal network. However the materials used have a very low heat conductivity. Therefore they have to be included in the thermal resistance between the copper and iron. The thickness of the lamination layer, often made of Nomex, is not known, but often a thickness of 1mm is used [111].

### 7.2.4 Housing

A series of different housing types exist and each with a different convection coefficient. Convection from different housing types is given in [112] as (7.7) with different coefficient, shown in Tab 7.2.

\[
h_{\text{conv}} = k_{c1} \left( \frac{\Delta T_{H-A}}{d} \right)^{k_{c2}} \quad (7.7)
\]

\( d \) is the housing diameter and \( \Delta T_{H-A} \) is the temperature difference between housing and ambient.

### 7.3 Losses

Heat generated inside an electric drive can be divided into three types.

- **Ohmic losses** due to the current which is necessary to create the magnetic field.

- **Losses** inside the iron which occur due to the changes in the magnetic field, as well as eddy-current losses.

- **Friction losses** occur in the bearing and also inside the air gap.

Usually losses inside the air gap are small enough to be neglected.
7.3 Losses

7.3.1 Ohmic Losses

The formula for calculating copper losses which are caused by the resistance and current flowing through a conductor is well-known. In the case of an electric drive with three phases the formula is given by (7.8).

\[ P_{\text{copper}} = \sqrt{3} R_{\text{el}}(T) I_{\text{el}}^2 \quad (7.8) \]

In the case of deriving a model the hardest part is to get appropriate values for the resistance of a phase and the current. Usually values for the resistance can be found in the data sheet of the electric drive. It's however important to consider the temperature dependence of the electrical resistance [113].

\[ R_{\text{el}}(T) = R_{\text{el}0} \cdot \left( \frac{T + k_1}{T_0 + k_1} \right) \quad (7.9) \]

\( R_{\text{el}0} \) is the reference resistance measured at temperature \( T_0 \). \( k_1 \) is 234.5 for copper and 225 for aluminum.

Calculation of the stator current in permanent magnet synchronous drives can be achieved using the current-torque constant [114].

\[ M_{\text{drive}} \approx k_T(T) I_{\text{el}} \quad (7.10) \]

The strength of rare earth magnets also is temperature dependent and changes around 0.1...0.2% \( K \) [115]. Feed drives usually have small enough temperature changes to consider \( k_T \) as constant. However, it can have an impact if temperature increases are high.

7.3.2 Iron Losses

Iron losses consist of hysteresis and eddy current losses [116]. Hysteresis losses are linear and eddy-current losses quadratic of the frequency. To reduce eddy-current losses, the iron is made by a laminated sheet package, to stop the flow of the eddy-current at the interface of the sheets.

\[ P_{\text{iron}} = P_{\text{eddy}} + P_{\text{hysteresis}} \quad (7.11) \]

Giving an exact representation of the iron losses is a complicated procedure and is influenced by many parameters, for example the geometry of the iron core. Since these parameters are not known, the iron losses can be approximately calculated using (7.12) [114]. The exact value for the power depends on the hysteresis losses, which are linearly depend on the frequency, and eddy current losses which have a quadratic dependency of the frequency.

\[ P_{\text{iron}} = P_{\text{iron \_ rated}} \left( \frac{f}{f_{\text{rated}}} \right)^{1.5...2} \quad (7.12) \]

The iron losses at rated frequency \( P_{\text{iron \_ rated}} \) have to be known. Frequency dependence is given by the current frequency \( f \) and the rated frequency \( f_{\text{rated}} \).
7.3.3 Bearing Losses

In the data sheet one can usually find a value for the static friction of the electric drive. In a first step this value is sufficient to model the bearing losses. The static friction is given by the friction of both bearings. Therefore the bearing losses for a single bearing are calculated according to (7.13).

\[
P_{\text{bearing}} = \frac{M_{\text{static}} \omega}{2}
\]  
(7.13)
8 Cooling Devices

On machine tools cooling devices can often be found. Most commonly these cooling devices are used to create a flow through the main spindle in order to remove heat. These cooling devices usually have a two-level controller, with a certain hysteresis. In this chapter the way used to model cooling devices will be shown. The proposed model is rather simple, but sufficient to approximate the behavior of the cooling device.

8.1 Tube and Tank Element

The tube element is an element able to conduct heat and transmit a fluid between its inlet and outlet nodes. Inside the tube only heat transport of the fluid is considered, while heat conduction is neglected. This is admissible as the heat transport by mass flow is much higher than by conduction. Convection to the outside is accounted for by applying a film coefficient depending on the flow inside the tube. If a convection condition is set to interact with the tube element, the average temperature inside the tube will be used. For each tube element an additional degree of freedom is added to the thermal equation system.

Usually convection is modeled to an environmental temperature that is known. As the tube temperature is an unknown, it is necessary to incorporate the convection heat flow between the fluid and the structure into the left hand side of the thermal equation system. A FE method, in which convection between the two contact surfaces can be established, can be found in [117].

While heat flow is symmetric, mass flow is not. The fluid only flows from one node to the other, but can not flow in the other direction. As a result the system of equations becomes unsymmetric and an unsymmetric solver has to be used, increasing the computational demand.

![Figure 8.1: Tube element in contact with the structure](image)

The equation system of the tube element (visualized in Fig. 8.1) is shown in (8.1). Internal heat generation, due to viscous friction, or other heating effects can be applied as well. The assumption of a linear temperature increase and therefore usage of the average temperature is a simplification, but has only minor effects on the solution accuracy. To improve accuracy the number of tube elements along a cooling channel can be increased. The
temporal discretization then can be derived by applying the $\theta$-Method, which is explained in A.2.

$$mc_p \frac{\partial T_{\text{mean}}}{\partial t} = h_{\text{conv}} A (T_{\text{structure}} - T_{\text{mean}}) + \dot{m}c_p T_{\text{in}} - \dot{m}c_p T_{\text{out}} \quad (8.1)$$

$$T_{\text{mean}} = \frac{T_{\text{in}} + T_{\text{out}}}{2} \quad (8.2)$$

The tank element in principle is similar to the tube element (8.3). The only main difference is that it incorporates a chiller that can be turned on and off by a two-point controller, that can be used to regulate the tank temperature to a desired set-point. Depending on whether the chiller is on or off the coolant power will be extracted from the system. Losses caused by the pump in the tank are also incorporated in (8.3) by the expression $P_{\text{loss}}$.

$$mc_p \frac{\partial T_{\text{tank}}}{\partial t} = \dot{m}c_p T_{\text{in}} - \dot{m}c_p T_{\text{out}} + P_{\text{loss}} - P_{\text{cool}} \quad (8.3)$$

$$P_{\text{cool}} = \begin{cases} P_{\text{cool}}, & \text{chiller on} \\ 0, & \text{chiller off} \end{cases} \quad (8.4)$$

$$T_{\text{out}} = T_{\text{tank}} \quad (8.5)$$

8.2 Temperature Controller

The controller, used to switch the chiller in a tank on or off (see Fig. 8.2), is a simple two-point controller. The set-point can be chosen to be either:

- constant
- a given temperature profile
- a nodal temperature of the on-going simulation (which is equivalent to the value of a temperature sensor on a real machine)

The hysteresis value can also be chosen by the user. Depending whether the controller switches the chiller on or off, the chiller will be enabled and an additional cooling power added to the thermal equation system.

8.3 Examples

The simulation of cooling devices was implemented in order to perform a design analysis on a machine tool, which to a large extent is covered by the coolant. A schematic of the machine tool can be seen in Fig. 8.3.

The coolant is mainly in contact with the machine bed, while the axes are not influenced by it. Therefore the machine bed is influenced by the coolant, while the axes are exposed to changes in the ambient temperature. As a result it was desired by the machine tool
manufacturer to understand what the optimal coolant temperature would be when the ambient temperature is changing. Besides keeping the coolant at a constant temperature, temperature control of the coolant also was under consideration. In the case of a controlled temperature it should be analyzed what the optimal set point for the controller would be. The effects of the hysteresis of the control on the deformations were also studied.

Three different cooling strategies are analyzed and compared. For each strategy the coolant temperature has a different set-point. These set-points are:

- Constant temperature \( T = 20^\circ C \)
- Ambient temperature
- Structural temperature: X-axis slide

**Constant Temperature** The first case, depicted in Fig. 8.4 shows the displacements of the machine tool, when the ambient temperature is changing from \( 20^\circ C \) to \( 25^\circ C \) in twelve hours and then is kept at \( 25^\circ C \) for another twelve hours. In this case the coolant temperature is kept constant at the reference temperature of \( T = 20^\circ C \). The hysteresis of the controller was set to \( \pm 1^\circ C \). It can be seen that especially the Y-direction shows large displacements, due to the uneven temperature distribution between the bed and the axes. The remaining error is caused by the process heat of 3kW, which causes a slightly higher temperature on the table of the machine tool and an expansion of the bed. The displacements in the X-direction are almost zero, because of the structural symmetry of

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**Figure 8.2:** Two-point controller that switches on/off the chiller in the tank

**Figure 8.3:** Schematic of the machine tool
the machine in that direction and the Z-displacements are also smaller, as the thermal loop in that direction shows a more even temperature distribution.

**Ambient temperature** It was also analyzed whether the displacements could be reduced when the coolant is following the ambient temperature, or whether it would have no significant effect. The effects of the temperature control can be seen in Fig. 8.5. Especially the Y-displacements changed significantly and even changed their sign.

**Table 8.1:** Time constant of the machine

<table>
<thead>
<tr>
<th></th>
<th>Bed</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>0.74</td>
<td>4.55</td>
<td>3.48</td>
<td>6.15</td>
</tr>
</tbody>
</table>

To better understand the effects happening in Fig. 8.5, it is also helpful to have a look at the time constants. The time constants for the three axes and the bed are given in Tab. 8.1. For the axes, the time constant relates to a change in the ambient temperature, whereas for the bed it relates to a change in the coolant temperature. The time constant
of the bed is much smaller due to the higher heat transfer coefficient between the bed and the coolant.

The cooling power of the chiller is strong enough to apply a fast temperature change to the coolant. As the convection between the machine bed and the coolant is higher, than between the axes and the ambient temperature, the machine bed reacts faster to the temperature change. This is responsible for the change of the sign. The different time constants cause the “overshoot” in the displacements.

**Structural temperature**  The observation of the machine bed reacting quite fast to a temperature change of the coolant led to the idea, that the coolant temperature could be controlled to a temperature of the axes. This should lead to an even temperature distribution of the whole machine and consequently small deformations. In Fig. 8.6 the displacements can be seen when a temperature of the X-axis slide is used to generate the set-point for the temperature controller. The X-axis slide is used because its time constant is close to the average of the Y and Z slide. As such an even temperature distribution should be possible. It can be observed that now the displacements could drastically be reduced by enforcing a homogeneous temperature over the whole machine.

The last step was to investigate the influence of the hysteresis. This was done by performing another simulation where the hysteresis was set to ±0.1°C, shown in Fig. 8.7, using the third control scheme (setpoint follows X-axis temperature). By reducing the hysteresis, the zig-zag on the displacements could be removed, indicating that the hysteresis should be set as small as possible.

Every change in the coolant strategy will also influence the energy consumption of the machine tool. This is especially important in recent times when there is an increased demand on energy-efficient machines. Usually engineers will only try to reduce the cooling
power, but neglect the impact on the accuracy of the machine. In Fig. 8.8 the effects of the three different cooling strategies on the power consumption are shown. The power consumption for the first case, with the constant coolant temperature, is the highest. This is because the cooling device has to dissipate the process heat and the heat of the convection transfer between the bed and ambient air. The heat transfer from the ambient air to the bed is the larger than for the other two cases, because obviously the temperature difference is the largest one. For the second case, where the coolant should follow the ambient temperature, the heat transfer is the smallest. Ideally the bed has the same temperature as the ambient air and as such no heat transfer takes place. In the last case the cooling power increases slightly, as a small temperature gradient becomes present and the transferred heat has to be dissipated. When comparing the cooling power with the process heat, which is 3kW, it can be seen that the average cooling power is less. This is because the heat from the process is used to heat up the coolant. The reduction of the power consumption therefore depends on temperature profile of the ambient air.

In summary it can be said that by using improved cooling strategies it is not only possible to increase the accuracy of the machine tool, but also to influence its energy consumption. This example showed that the thermal error due to environmental temperature changes can be reduced by about 75% by choosing a proper cooling strategy. From Fig. 8.8 it can also be seen that measures regarding the thermal behavior influence the energy consumption and vice versa. In the example the power consumption was reduced by about 10%. Further energy savings are possible because the requirements on the climate control of the shop floor are less stringent.
Figure 8.7: Displacements due to environmental temperature change; Coolant temperature controlled to temperature of X-slide ±0.1°C;

Figure 8.8: Power consumption for the different cooling strategies
9 Model Verification

The models that have been shown throughout this thesis were also verified with a lot of measurements on real machine tools. A few of these measurements will be shown in this chapter to show the validity of the proposed methods. The verification of the individual models, as for rolling bodies (Chap. 6) and electric drives (Chap. 7), is performed on a machine tool.

In detail, Sec. 9.1 shows the machine tool that was used for the measurements. Measurements and simulations are compared for two load cases. Sec. 9.2 shows the results for a linear axis movement with an analysis of the temperature of the ball screw and the feed drive. An experiment was also performed for a rotary axis, as a five-axis machine tool was used for verification.

9.1 Test Setup

![Machine used for experiments](image)

The test setup was a five-axis machine tool with a schematic that can be seen in Fig. 9.1. The spindle is mounted on the B-axis which is connected to the Z-axis slide. All linear axes are on the tool side. The workpiece side consists of an A-axis. The resulting axis configuration is:

\[ t-B-Z-Y-X \text{-bed-A-w} \]

Incremental probes were used to measure the displacements. The measurement setup with the incremental probes can be seen in Fig. 9.2. The depicted measurement setup is a so called R-test [83] which measures the three translatory displacements of a precision ball.
9.2 Simulation of Linear Axis Movement

Additional Probes

![R-test setup to measure the thermal error at the TCP](image)

Figure 9.2: R-test setup to measure the thermal error at the TCP

Two additional incremental probes have been mounted in order to measure the rotatory displacements A and B.

Additionally temperatures were measured. A measurement system with a total of 20 probes was used to capture the temperature distribution of the machine tool. This is important to detect temperature changes and find causes for the TCP errors. The temperature probes were placed on the machine elements as guideways, ball screw nuts and feed drives to measure their temperature. The temperature is a result of the losses and by comparing simulations and measurements it is possible to verify the models. Besides the temperature probes an infrared camera was used to capture the temperature distribution of structural parts.

9.2 Simulation of Linear Axis Movement

Linear axis movement is one of the standard load cases for a machine tool and therefore simulations are performed to verify the models. The pendular movement of a X-axis is used to compare measurements and simulations of the machine elements. The resulting TCP error, caused by the load case, is also compared.

9.2.1 Measurement Cycle

The measurement cycle that was used to identify the effects of a linear axis movement was according to the ISO 230-3:2005 [4] standard. The pendular movement of the X-axis was performed over a stroke of 300mm for about five minutes and then a measurement of the displacements was made. The whole cycle was repeated over approximately six hours. Different feed rates were used in order to verify the models over a wide range of working conditions. These feed rates were \( F = 500 \text{mm/min} \), \( F = 1500 \text{mm/min} \) and \( F = 3000 \text{mm/min} \). In the best case the models should be able to extrapolate to all working conditions, only
based on manufacturer data, without the usage of any parameter identification. This is because no measurements can be made when the machine is in the design phase. So the designer has to rely on manufacturer data of the machine elements to perform simulations.

### 9.2.2 Temperatures

In the first step the simulated temperatures should be compared with the measured ones in order to verify that the models of the rolling bodies and electric drives, that were presented earlier, are correct. The values of temperature probes will be compared with the nodal temperatures of the thermal FE model which are located at the same point.

![Figure 9.3: Simulated (red) and measured (blue) temperatures of the motor flange](image)

Fig. 9.3 compares the simulated (red) and measured (blue) temperatures at the motor flange, so that the thermal model of the electric drive can be verified. The feed drive would include a temperature sensor itself, but its resolution is only 4°C and as such not high enough to monitor the small temperature changes that occur due to the pendular movement. This is because it is just used to ensure that the motor will not overheat.

In Fig. 9.3 the temperature of the flange are shown for the three feed rates, F=500mm/min, F=1500mm/min and F=3000mm/min, that have been mentioned previously. In general it can be seen that the measured temperatures rises almost linearly with the increase of the feed rate. Only the step from F=500mm/min to 1500mm/min does not lead to three times higher temperatures. Instead the temperature rises by a factor of four. Such non-linear dependencies show the complex behavior of thermal phenomena that have to be considered. However it can be shown that the simulation can cope with
these non-linearities quite well and that the error between the simulation and measurement usually stays below 25%.

![Graphs showing temperature increase with different feed rates](image)

**Figure 9.4:** Simulated (red) and measured (blue) temperatures of the ballscrew nut

Besides the electric drive, the models for the rolling bodies should also be verified. As an example Fig. 9.4 shows the temperature increase at the ball screw nut, due to the different feed rates. It can already be seen that the temperature increase here is much smaller than for the motor flange. The temperature increase of the guide carriages is not shown because usually their temperature increase is very small and often does not have a considerable effect on the thermal error of the machine tool.

While the simulations and measurements for F=1500mm/min and F=3000mm/min correlate quite well, this is no the case for F=500mm/min. Interestingly the measurement shows almost no temperature increase due to the pendular movement of the axis. A slight temperature increase is apparent in the simulation, but it is also rather low, in the region of 0.2°C.

The thermal FE model is also verified by the usage of an IR camera shown in Fig. 9.5. The simulated temperature distribution can be seen on the left and the measured on the right hand side. An important difference between the two images is that the feed drive itself is not visualized in the simulation, but only the motor flange. It should also be noted that the temperature scale for the simulation and measurement are different. This has been done because the simulation uses an initial temperature of 20°C, whereas the initial temperature in the measurement was higher (about 24°C). The focus here shall be put on the two white rectangles, that have been marked in the simulation and IR image. Number 1 focuses on the temperature of the ball screw nut, that is mounted on the X-axis.
Both the simulation and the measurement show a similar temperature distribution with a temperature increase of about 0.5°C. Rectangle number 2, shows a quite good correlation of the heat flow in the bed for the simulation and measurements. It can also be seen that the temperature in the region of the motor flange is much higher. Because the temperature of the motor flange has already been compared in Fig. 9.3 it will not be compared here again. This validates that the thermal FE code is working properly for simulating the temperatures caused by linear axis movements.

### 9.2.3 Displacements

The next step is to calculate the displacements that are caused by the temperature change. A first result of the linear TCP displacements in X-, Y- and Z-direction can be seen in Fig. 9.6. The displacements due to the pendular movement of the X-axis stay very small. As can be seen in Fig. 9.5 little temperature increase is introduced into the X-axis. The temperature flowing in the bed and the increased temperature of the ballscrew have no effect as they are detected and compensated by the direct measurement system. Results like these are an ideal case for a linear axis and usually displacements are much higher. It can be noted that the simulation also does not show major displacements and that the X- and Y-direction also have TCP errors of about ±2µm. The Z-displacement in the simulation however is much larger than the measured one. This is due to the heat of the ball screw nut that flows into the X-slide, causing an expansion into the vertical Z-direction.

As the same effect can be seen with the IR camera the question arises, why the simulation diverges from the measurement. The answer lies within the initial temperature distribution and the changes in the convection coefficient that happen due to the pendular movement of the axis. These effects will be incorporated in new simulations in order to get more accurate results.
The tower of the Y-axis is mounted on to the X-axis slide. As the feed drive of the Z-axis, which is mounted on top of the Y-axis slide, has to work against gravity it will heat up. In the measurements of Fig. 9.6 the machine tool had about a day to warm up, before the measurement was started. Due to the losses in the Z drive the heat could flow into the tower and heat it up, leading to a vertical expansion. As now the X-axis starts to move, the convection coefficients on the tower increase and more heat can convect to the cold environment. The Y-axis slide now will shrink, causing a negative Z displacement. This negative Z-displacement is counteracted by the expansion from the heat generated at the nut. Both displacements counteract each other leading to virtually no displacement visible in the measurement.

To show this behavior the next simulation therefore will not use an even initial temperature distribution, but a temperature distribution that was caused by the warm-up of the

**Figure 9.7:** Temperature distribution of the Y-axis at t=0h and t=6h; Compare with Fig. 9.1
machine tool. The heat convection coefficients this time will be made variable, in order to represent the forced convection that is caused by the axis movement. The results can be seen in Fig. 9.7. In the left image the temperature distribution at the start of the pendular movement can be seen. It is obvious that the temperature of the Z feed drive reaches more than $30^\circ C$ and heat is flowing in the structure of the tower. The right image shows the temperature distribution after six hours of the pendular movement. Due to the increased convection coefficient more heat is transferred to the environment and the temperature decreased. This is especially obvious if one compares the motor flange in the top left side of both images, which is marked by the white rectangles.

In all axis directions of the measurement the zig-zag movement, caused by the two point controller of the cooling device can be seen. The cooling device is not included in the simulation and thus one can not see this behavior in the simulations. Besides this the measured and simulated results agree very well now (Fig. 9.8). The Y-axis in both the simulation and measurement shows a positive trend, because the tower also shrinks laterally causing the positive displacements. The Z-axis also agrees well first becoming negative and then positive.

![Figure 9.8: Simulated (dashed) and measured (solid) displacements](image)

**9.3 Simulation of Rotary Axis Movement**

In case of a five-axis machine tool rotary axes can also be responsible for large thermal errors, as was shown in Chap. 2. Thus it is also important to model rotary axes. In this section results for a tilting spindle are compared.

**9.3.1 Measurement Cycle**

The test procedure was a pendular movement of the B-axis shown in Fig. 9.1. The B-axis moved between the angles of $-90^\circ$ and $45^\circ$ with a feed rate of $F = 2000 \text{ deg/min}$. Every 330s
9.3 Simulation of Rotary Axis Movement

The pendular movement is stopped, in order to measure the TCP error due to the thermal error that has been caused by the induced heat. The pendular movement is repeated for about eight hours.

The measurement setup that was used was the same as explained in Sec. 9.1. Before the cycle of the pendular movement was started a first measurement was taken. This was used to reset the error, such that changes in the measurement can be assigned to thermal errors. Temperature measurements in this case were a little more difficult due to the covers that are mounted on the B-axis. Because of the covers it was not possible to perform IR measurements directly on the B-axis structure.

9.3.2 Results

The mechanical setup of the B-axis consists of a harmonic drive, a belt drive and the corresponding feed drive. Inside the harmonic drive a bearing (see Fig. 9.1) is placed that connects the fixed part, the Z-axis slide, and the turning part, the tilting spindle. All of these three machine elements have macro elements, as explained in Tab. 3.1, that can be used to compute their behavior. The losses due to the friction in these machine elements causes a temperature increase of the B-axis structure.

![Figure 9.9: Structural temperature](image)

In Fig. 9.9 the temperature in the vicinity of the harmonic drive is compared. It can be seen that the simulation and measurement are in good agreement, especially over the first two hours. The temperature is measured on the right side of the harmonic drive, which is the Z-axis slide, shown in Fig. 9.1. The exact position of the temperature sensor is indicated in Fig. 9.10. Unfortunately it was not possible to position the sensor closer to the heat source due to the covers on the machine. Fig. 9.10 shows the simulated temperature distribution of the Z-axis, B-axis and the spindle. It can be seen that the harmonic drive acts as the major heat source with heat transfer to the left and the right. On the left side it can be seen that the structure cools down much faster. This is due to
the coolant circuit of the spindle which dissipates the heat. The same behavior can be seen in the measurement taken by the IR camera (Fig. 9.11). The actual structure of the B-axis is hidden under the cover, but it is still possible to note the heat increase that is causing their warm up. The heat dissipation of the spindle cooling circuit is also visible, as in Fig. 9.10.

The TCP-displacements that are caused by the pendular movement are shown in Fig. 9.12. The largest error can be found in the Y-direction. This is caused by the heating of the structural parts of the B-axis and Z-axis. The Z-axis guide carriage (see Fig. 9.10) can be regarded as an insulator and little heat flows through the carriage on to the Y-axis slide. The structural parts therefore move away from the Z-axis carriage in the negative Y-direction (Fig. 9.12). The simulation and measurement of the Y-displacement correlate quite well as a result of the accurate temperature simulation. The relative error between the simulation and measurement is about 10% to 20%.

The measurement also shows displacements in the X- and Z-direction. While the simulation and measurement show the same trend, they only correlate to about 30% for these two directions. This is possible caused by an unidentified heat source that is in the vicinity of the Y-axis slide. An indication for this heat source was seen in an increase of the am-
Figure 9.12: Measured (solid) and simulated (dashed) displacements;

ambient temperature on one side of the Y-axis slide. This unbalanced temperature influence causes a bending of the Y-axis around the B-axis, thus resulting in a positive X-error. The simulation does not include this heat source and as a result can not match the X- and Z-error as well as was the case for the Y-direction.
Thermal Compensation with FEM

After all parts have been described that are necessary to simulate machine tools, it is also desirable to compensate for the thermal errors. Because a FE model of the machine tool already has been created during the design phase it is also desirable to use it for compensation on the real machine tool. In this chapter it will be described how to implement a thermal compensation based on FEM with the help of the software Virtual Machine Prototype (VMP).

Thermal Space Error Compensation

The volumetric space error compensation is a common approach to compensate and measure the geometric error of machine tools in the workspace. An example of the volumetric space error is given in [118]. Using a 3D ball-plate the geometric errors in the whole workspace can be measured. The result from the measurements is the distorted space error grid from which, in the case of a three axis machine, the 21 error-components can be retrieved. The top of Fig. 10.1 for example shows the working envelope of a three axis machine tool and a volumetric grid of nodes at which the thermal errors are evaluated. The volumetric grid has three nodes in every axis direction (emphasized by the yellow points) and thus a total of 27 nodes. Using error-components is straightforward and intuitive for experts in the field of metrology which is why it was also used in [3] for thermal errors.

The compensation model used here also is based on a volumetric space error compensation. Each node corresponds to a certain position of the workpiece and tool and for each position the reduction method described in Subsec. 10.2 will be applied. If one considers calculation speed and accuracy as the main target, then it is better to work directly on the nodes, as not only information is lost when converting from the space grid to the 21 error-component model, but also computing time is needed for the conversion. In addition the CNC controller has to support compensation based on the 21 error-component model which is not possible on every CNC.

If one works directly on the volumetric grid, then the displacement at the TCP (red) is interpolated from the surrounding nodes. The TCP always can be found in a cell of the grid surrounded by eight nodes, as shown in the bottom of Fig. 10.1. Using the shape functions it is possible to evaluate the displacement at $u_{TCP}$ which is the current TCP position. The shape functions, with $g$, $h$, $r$ being the coordinate of the TCP inside the cell and $g_i$, $h_i$, $r_i$ the coordinate of the node (gray) are used as weighting functions. Note that the local coordinates range from -1 to 1. Thus $N_i$ always stays positive and the sum of all $N_i$ always is 1. This method works only well for translational axes. In the case of rotary axes the evaluation of the displacements become much more complicated. In that case they also depend on the axis angle.
10.2 Reduction of the Mechanical FE Model

The thermomechanical deformation of a machine tool relates to (10.3), where \([K]\) represents the stiffness matrix, \(u\) are the displacements and \(\{F_{\text{thermal}}\}\) the forces caused by the temperature distribution of the machine tool.

\[
[N_i] = \frac{1}{8} (1 + gg_i) (1 + hh_i) (1 + rr_i) \tag{10.1}
\]

\[
\{u_{TCP}\} = \sum_{i=1}^{8} N_i \{u_i\} \tag{10.2}
\]

10.2 Reduction of the Mechanical FE Model

To correct the thermomechanical deformation only the relative displacement between the tool and workpiece is of interest. Because of this it is not necessary to solve the displacements at all nodes of the FE model, but only for some nodes on the tool, the workpiece and, if available, the glass scale and scanning head of the direct measurement system. The nodes at the glass scale and scanning head are necessary because a direct measurement system will correct the thermal displacements which it can detect. In the case a direct measurement system is installed, it is important to consider the position of the axes as the scanning head moves along the glass scale. Depending on the position of the scanning head the nearest node of the glass scale will be used. For each scanning head and the opposing node on the glass scale the displacements also have to be calculated. The
displacements of nodes from the measurement system then will be projected onto the axis direction, in order to get the error which the measurement system can detect and correct.

If not only translational displacements, but also rotary displacements are needed, it is necessary to select more than one node for the tool and the workpiece. In that case three nodes are selected at the spindle nose and the workpiece side. Based on these three nodes it is possible to calculate a coordinate system which is fitted into these three nodes. Given the coordinate system the rotational displacement about its original position can be computed [119].

The inverse of the stiffness matrix will be calculated for the selected nodes. It is however not necessary to know all displacements, but only of selected nodes. As such it is not necessary to compute the whole inverse $[K]^{-1}$. The right hand side therefore is not the complete identity matrix, but $[I_{selected}]$ is a matrix that is assembled from the columns of the identity matrix, at the selected nodes. If the node of the tool for example has the index $i$, then $[I_{selected}]$ contains the $i$-th column of the identity matrix $[I]$. Applying (10.4) will calculate the inverse of the stiffness for these selected nodes.

$$[K][K_{selected}]^{-1} = [I_{selected}] \quad \text{(10.4)}$$

The inverse for the selected nodes then is multiplied by the matrix of thermal forces. As can be seen in (10.5), the computation of the thermal displacements will be reduced to a simple matrix multiplication which can be solved quite fast. A detailed description of this reduction method can be found in [120].

$$\{u\} = [K_{selected}]^{-1} \{F_{thermal}\} \quad \text{(10.5)}$$

Finally it is desired to calculate the TCP error at all nodes of the volumetric grid with as little computational effort as possible. This can be done by (10.6), where the subscript indicates the inverse for that part. The sign of the displacement depends on whether the scale / scanning head is mounted on the moving part of the axis. If so, the sign will be negative, as the axis will perform a movement to counter the detected displacement. The thermal FE system produces the vector for the change in temperature $\Delta T$, which also has less degrees of freedom than the vector of the forces $F_{thermal}$ as a node only has one temperature, but can have forces in three directions. Thus it is beneficial to further change the equation system to reduce the number of operations performed per matrix multiplication. The matrix $A$, which relates the temperature change to a force, can also be multiplied with the stiffness matrix. As the TCP in this case only has three translational displacements, the reduced matrix $R$ will have only have the size 3xm, where m is the number of degrees of freedom of the thermal FE system. Finally (10.8) can be used to easily calculate the TCP displacement from a given temperature change.

$$\{u_{node}\} = \left( [K_{Tool}^{-1}]^T - [K_{Workpiece}^{-1}]^T \pm [K_{Scale}^{-1}]^T \pm [K_{Sc.Head}^{-1}]^T \right) \{F_{thermal}\} \quad \text{(10.6)}$$

$$\{u_{node}\} = \left( [K_{Tool}^{-1}]^T - [K_{Workpiece}^{-1}]^T \pm [K_{Scale}^{-1}]^T \pm [K_{Sc.Head}^{-1}]^T \right) [A] \{\Delta T\} \quad \text{(10.7)}$$

$$\{u_{node}\} = [R] \{\Delta T\} \quad \text{(10.8)}$$

However, it is not possible to apply the reduction method in real-time to the current tool and workpiece position as the reduction method is quite costly. Instead a volumetric
space grid (Sec. 10.1) will be assembled in the workspace with a user-defined amount of grid nodes. The whole elastic equation set will be assembled and reduced for each node of the grid, in order to retrieve the matrix $R$. The reduced system then can easily be solved in real-time.

### 10.3 Implementation

The compensation based on the reduced FE model of the machine tool can now be implemented into the CNC of a machine tool. As a prototype a regular machine tool will be chosen. The machine tools’ CNC is a FANUC control running on a PC with Windows XP as an operating system. Having the CNC running on a Windows PC is beneficial in implementing an advanced compensation algorithm, as the one proposed here. VMP (Chap. 3) is running on the Windows PC and can send and retrieve data from the CNC with the usage of the FOCAS2 library from FANUC. A temperature measurement system is connected to the USB port of the machine and it can be accessed by VMP to get the environmental temperatures, as well as temperature boundary conditions. The goal of the compensation is to reduce the thermal disturbances that deteriorate the accuracy of the machine. In Fig. 10.2 one can see the scheme of the compensation strategy.

The compensation model interacts in a series of ways with the machine tools’ CNC. The NC code is transformed into tool path data by the CNC’s interpreter. This tool path data is ready by FOCAS2 and sent to VMP. The FE model needs to know the tool path that was driven in the last time step, as well as the velocities. According to Fig. 10.2 it can be seen that the tool path data, namely the positions and velocities, are transferred to the FE and compensation algorithm. The thermal errors at the TCP, calculated by the compensation, are then sent back to the CNC, shown in Fig. 10.2, and used to offset the machine tools coordinate system by the calculated amount. This way the thermal error can be corrected by changing the location of the coordinate system.

In detail, the FEM and compensation algorithm run in parallel on two threads. The one for the compensation algorithm is working at very small cycle times of approximately 250ms. This thread interacts directly with the CNC retrieving position and velocity data which is needed for the calculation of the displacements. It as well interpolates the position
10 Thermal Compensation with FEM

dependent TCP displacements according to (10.2) and sends a signal back to the CNC to change the coordinate system, according to the current thermal errors.

The second thread is running the FEM simulation of the machine tool according to Fig. 10.2. The time step for the FEM simulation is much larger because a lot more computational effort is needed to calculate the temperature distribution and the thermal error. The cycle time used here is 2.5s, but using large time steps is no problem for slow thermal effects. The calculations that take place in this thread are explained in Chap. 4 and shown in the flow chart of Fig. 4.1. The two threads interact by sending the tool path data to the FEM simulation and sending the computed thermal error grid, according to Subsec. 10.1, back.

In addition a series of temperature probes are connected to the CNC, which can also be seen in Fig. 10.2. The measured temperatures of these probes can be integrated into the FE model as temperature boundary conditions or as values for the environmental temperatures which interact with the machine tools structure.

10.4 Challenges

Every compensation model is faced with a series of challenges. These challenges mainly concern the accuracy and the robustness of the model. One problem concerning the accuracy is the resolution of the compensation and how new compensation values are fed to the machine. If the resolution is only in the range of microns and compensation values are updated abruptly, steps will occur in the workpiece, diminishing the surface quality.

Another problem with updating the compensation values are the boundary conditions as loads and environmental temperatures. If the compensation model should calculate the current state, but is updated only seldom, for example every five or ten minutes, it would be necessary to predict the boundary conditions. Otherwise the compensation would not be able to use the boundary conditions at the current time. Predicting the boundary conditions however is very complicated. In the case of temperatures that are measured using probes an extrapolation would be necessary. Extrapolation can often lead to wrong results, especially with changing boundary conditions and as such is not advisable. An example where such an extrapolation fails is in the case of chillers which cause a zig-zag pattern in the temperatures. In such a case a linear extrapolation can not detect a turning point in the zig-zag pattern thus causing an error. Thus a prediction and a correction step is necessary doubling the computational efforts. The data handling also becomes more complex because boundary conditions have to be stored for the correction step.

For the NC path, which is used for updating the position and computing the loads and losses, a look-ahead would be inevitable. This means that the NC path for the next two time steps should have to be evaluated in advance (see Fig. 10.3). If one wants to correct the phase that is marked by the red bracket, then it is necessary to have the data one time step earlier. This means that the data from the look-ahead (red arrow) has to be provided at the time that is marked by the red dot. The reason for this can be seen on the right side in the time scale of the compensation. The FE solver (gray box) needs some time to compute the volumetric grid before the TCP correction can take place (blue bar). It is obvious that such a look-ahead would also be hard to realize with a CNC as the look-ahead usually is only a few lines of NC code. Depending on how the NC code is written a
look-ahead might include toolpath data for a few milliseconds only, for example if a circle is converted to a series of linear interpolation commands. This can cause problems if the time step of the compensation is larger than what the look-ahead contains. Depending on the CNC it is also more difficult to access the look-ahead.

It now should be obvious that it is quite difficult to keep the compensation model at the exact same state as the real machine tool. That is the reason why another method was employed in the case of VMP. The compensation model is always lagging behind, but the update rate is increased in order to control the maximum error. The basic principle is shown in Fig. 10.4, explaining the compensation algorithm without a look-ahead. The FE solver (gray box) retrieves the toolpath data (red arrow) one compensation time step later. Thus the state of the compensation model is lagging one time step behind the real machine tool, as indicated by the state $t_1$.

Now the update rate of the compensation has to be increased in order to minimize the difference between the real and the computed state. This can be seen in (10.9) which calculates the update rate in such a way that the error is smaller than the resolution of the NC. The value $NC_{res}$ is the resolution of the NC, often $0.1 \mu m$, which imposes a limit on the accuracy of the compensation. The compensation can never be more accurate than the resolution of the NC. This value could alternatively be replaced with the desired accuracy of the compensation. If the maximum gradient of the displacements, $\dot{u}$, is known, then it is possible to calculate the update rate. The factor of one half is included because the TCP correction (blue box) uses the result from state $t_1$ for the whole time step. Thus the
maximum lag of the compensation is equal to twice the update rate of the compensation. In the case of a resolution of 0.1µm and a gradient of 10µm/m this would lead to an update rate of 18s.

\[ \Delta t = \frac{1}{2} \frac{NC_{\text{res}}}{\dot{u}} \] (10.9)

Even if this is a rather frequent update, especially due to the computational demands, it is believed to be more efficient than a prediction based algorithm. While it might be possible to have larger time steps with a prediction, it leads to higher computational demands and more complex data handling. The higher computational demands is given by the necessity of a corrector.

The exact way how VMP treats the different time scales is shown in Fig. 10.5. There are three different time scales present in VMP with the exact timings being a user-specified value. The retrieval of the toolpath data is done in the range of milliseconds. An average value of the toolpath data then is sent to the thermal FE code which then updates the model and solves it every few seconds. After some time the distortion of the volumetric grid can be calculated based on the current temperature distribution. The time step for the mechanical FEM can be based on (10.9) and is usually in the region of 30s to one minute. The computational demands that are caused by the small time steps of the thermal FE solver are not too severe, because the solver and method shown in Chap. 5 is very efficient. Updating, assembling and solving the thermal equation system is about 20-40 times faster than real time. Computing the volumetric grid deformation also takes little time due to the reduction that has been shown previously.

Using a compensation is only reasonable if the model can accurately predict the thermal error of the machine tool over a long range of time and during changing conditions. Thus a correction might be necessary from time to time. Such a correction could be realized using touch probes or temperature probes. In the case of a touch probe the measurement could be used to reset the error of the compensation model to zero.
A correction of the temperature distribution, using temperature probes, is much more complicated. The first challenge is to find the ideal location of the probes. Some suggest to place it where the sensitivity of the structure to temperature changes is the highest, while others recommend to place the probes in close vicinity to the heat sources / sinks. The ideal location could also be derived mathematically by using the FE model of the machine tool. If the temperature probes then are located on the machine tool a IHCP has to be solved, in order to correct the heat generation inside the models. It can be imagined that this is a complex problem that needs additional computational resources. As such a correction based on temperature probes currently is not included, but should be introduced in the future in order to improve the quality of the compensation.
11 Implementation of the Compensation

Compensation is also a major topic in this thesis and therefore Sec. 11.2 and Sec. 11.3 show a variety of compensation methods. First two simple compensation methods are shown that can be used to reduce the thermal error due to spindle drift. Especially Sec. 11.3 is of importance as it shows the compensation of thermal effects by FEM. The implementation of the FEM based compensation is based on the models and the compensation scheme that was shown before.

11.1 The Machine Tool

The different compensations have been derived for the same machine tool. The schematic of the machine tool that is compensated can be seen in Fig. 11.1. It consists of three linear axes, with the Z-axis being on the tool side and the X- and Y-axis on the workpiece side. The machine is separated in the working envelope and a machine room by a series of covers which are indicated by the dashed red line in Fig. 11.1. The X- and Y-axis are beneath the covers in the machine room, where there are the ancillary components as hydraulics, chillers and transformers. The stroke of the linear axes is given in Tab. 11.1. The machine itself is using a FANUC 160i-MB running on a Windows PC with a 1.5GHz CPU and 1GB of RAM. On this PC the compensation algorithm will be running as well.

The test setup to measure the thermal error and check the compensation consists of a series of temperature sensors and five incremental probes. The temperature sensors, mounted at a variety of different positions on the machine tool, are used to monitor the temperatures changes when the spindle is turning. Using the five incremental probes shown
11.2 Linear Spindle Compensation

Table 11.1: Stroke of the axes

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>300mm</td>
<td>220mm</td>
<td>180mm</td>
</tr>
</tbody>
</table>

in Fig. 11.2 it is possible to evaluate the three translatory displacements X, Y and Z as well as the rotations A and B. The rotation C around the spindle axis is not of interest, as the machine under investigation is used for grinding.

Figure 11.2: Setup for measuring the spindle drift

11.2 Linear Spindle Compensation

The main spindle of a machine tool is one of the largest contributors to the thermal error. Usually the largest fraction of the error occurs in the spindle direction. This makes it possible to use simple one dimensional compensation methods to correct for the elongation of the spindle. Two of these methods will be shown here in order to introduce the concepts of different compensation algorithms.

11.2.1 Test Procedure

In accordance with ISO 230-3:2005 [4] the drift is measured at 25%, 50%, 75% and 100% of the maximum spindle speed. At each speed the spindle is kept running for a few hours such that the system can reach a steady state.

The linear displacements measured at these spindle speeds can be seen in Fig. 11.3, which is divided in six phases (indicated by roman numerals), with the spindle speeds given in Tab. 11.2. It can be seen that the spindle rotation only starts after five hours. This has to be done because the coolant will start to run, as soon as the spindle is turned on. The coolant is controlled to a constant temperature of 19°C. Thus the coolant has a
different temperature than the machine and influences the machine temperature and the displacements. In order to only see the effects of the spindle drift, the coolant flows for a few hours, until it has no more changing effects on the displacements which are observed by the incremental probes. The effects of the coolant can especially be seen in Fig. 11.3 in X and Z during the first two hours. The spindle drift test can begin after the machine has become stable.

Table 11.2: Rotational speeds in \( \text{min}^{-1} \) during the spindle drift

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1800</td>
<td>3600</td>
<td>5400</td>
<td>7200</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

In phases II to V, the spindle is turned for 330 seconds and after each period the displacements are measured. This cycle is repeated for two hours at each spindle speed, before the speed is increased. It can also be observed that, as the spindle can not position the angle, the effect of the spindle runout is superposed on the thermal displacements caused by the spindle drift. This causes the large jumps in the measured spindle drift. At the end, in phase VI, the cooling down of the spindle is measured with the probes in continues contact with the mandrel.

The errors mainly occur in the Z-direction which corresponds to an elongation of the spindle. A slight influence can also be seen in the lateral direction, Y, but it is about four times smaller than in the spindle direction. The spindle drift is most obvious in phase II and III. Changes to higher spindle speeds do not result in a visible increase of the drift.

### 11.2.2 Sensor-Based Compensation

When comparing the changes in the temperature (see Fig. 11.4) and reading of the incremental probes (see Fig. 11.3) one can see that the temperature change at the spindle...
bearing correlates quite well with the spindle elongation. This fact is shown in Fig. 11.5, where the normalized spindle elongation and temperature of the spindle bearing is compared. As a result, it is possible to identify a linear relationship between the spindle bearing temperature and the spindle elongation.

This proves to be a simple but robust method to compensate for the spindle elongation. Of course, as can also be seen in Fig. 11.5 the correlation is not perfect, thus a small error remains. Therefore the compensation has to be reset at certain times in order to bring the temperature measurement and displacement together again.
11.2.3 Model-Based Compensation

If one does not want to install a temperature sensor, a model based on internal control data can also be used to compensate for the spindle drift. First of all it is necessary to identify a model that fits the measurement data and thus describes the real world data. When having a look at Fig. 11.6, it can be observed that the underlying thermal effects relate to a model of first-order. Thus it should be possible to describe the elongation of the spindle with such a model. Considering the underlying effects a model according to (11.1) comes to mind which makes the physical effects obvious. The spindle has a certain mass, $m$, and temperature $T$. The spindle has a convection heat transfer to the coolant of temperature $T_0$ and as soon as the spindle turns, a heat loss $P_{\text{loss}}$ will occur.

$$mc_p \frac{\partial T}{\partial t} = A_\alpha (T_0 - T) + P_{\text{loss}}$$  \hspace{1cm} (11.1)

As the spindle is assumed to perform a free deformation in one direction, it will have a linear expansion, $u$, which is directly proportional to the temperature, thus resulting in (11.2). The remainder of the work is to identify the three unknowns $k_0$, $k_1$ and $k_2$. The coefficient $k_0$ can simply be set to zero as the spindle deformation is zero when the spindle starts to turn. Another coefficient $k_1$ can simply be read out from the measurements in Fig. 11.3. The time constant $\tau$ needed to cool down the spindle relates to the inverse of $k_1$. For a first order system $\tau$ simply corresponds to the time needed to reach 63.2% of the final value. During the cool down in Fig. 11.3, in phase VI, this means that the deformation has returned from -0.75 to -0.28. The time needed to do so is approximately 40 min. The coefficient $k_2$ depends on the spindle speed and can be found by parameter identification. The resulting values for the coefficients can be found in Tab. 11.3.

$$\frac{\partial u}{\partial t} = k_0 - k_1 u + k_2$$  \hspace{1cm} (11.2)

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$k_1$</th>
<th>$k_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{1}{2300}$</td>
<td>0.0052, n=1800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0071, n&gt;1800</td>
</tr>
</tbody>
</table>

Table 11.3: Coefficients of the model based compensation

In order to implement the differential equation, (11.2), into the CNC of the machine tool it has to be discretized. The discretization of the derivative is given in (11.3). Using (11.3) and the Euler backward method, the whole compensation model (11.2) can be rewritten to (11.4), which is a very simple model. The result of this compensation model with the identified parameters is shown in Fig. 11.6. It can be seen that the compensation perfectly fits the measurement. However, this comes as no surprise as the parameters are directly identified from this measurement. In reality the spindle drift not only depends on the spindle speed, but also on the spindle load and the tool length, resulting in much more complex models. Of course this model also can not react to disturbances, such as in the environmental or coolant temperature. This becomes obvious when having a look at the first few hours of the measurement in Fig. 11.6, where displacements occur as the machine tool reacts to the coolant that started flowing through the machine. A properly
made sensor based compensation, as shown in Fig. 11.5, can detect and react to such disturbances.

\[
\frac{\partial u}{\partial t} = \frac{u^{t+\Delta t} - u^t}{\Delta t} \tag{11.3}
\]

\[
u^{t+\Delta t} = \frac{u^t + k_2 \Delta t}{1 + k_1 \Delta t} \tag{11.4}
\]

**Figure 11.6:** Comparison of measurement and model-based compensation

### 11.3 FEM-based Compensation

In this section VMP is used to create a FEM based compensation model for a machine tool. The FE model is created according to the materials of the previous chapters, the integration into the CNC is done according to Chap. 10. Using this method it is very easy to create a compensation model based on the simulations that have been made during the design phase. With the help of VMP a compensation model can efficiently be derived and the thermal error of the machine tool can be reduced by more than 50%.

#### 11.3.1 The Model of the Machine Tool

The compensation model consists of 16'000 degrees of freedom for the thermal system. The thermal FE model uses linear shape function while the mechanical FE model is using quadratic shape functions in order to get more accurate results. As such the mechanical FE model has more degrees of freedom. Before applying the reduction method it had 70'000 degrees of freedom. After the reduction the size of the reduced matrix \([R]\) becomes 3x70'000. The volumetric grid consists of 3x3x3 nodes which means that the model was reduced in three positions of every linear axis. In total 27 reduced matrices were assembled.
which need in total about 40MB of RAM. The memory demands therefore increase with the number of axis positions.

The thermal FE model is enhanced by a series of temperature probes. Three temperature probes record the environmental air temperatures which are used for the convection boundary conditions. One probe is placed in the working envelope, where the temperature is nearly constant. The other two probes are located in the machine room. Here there are feed drives and a variety of ancillary components that produce waste heat. As a result the air temperature in the machine room increases and interacts with the structure of the machine. Two probes are needed to capture the complex temperature distribution that is affected by the environment and waste heat from ancillary units. A variety of other probes are used to monitor the temperature of the feed drives. Even though the compensation model can calculate the temperatures of the feed drives (Chap. 7 and [121]), the temperature probes are used to get more accurate results.

In Chap. 10 it was explained that VMP uses three different time scales for the compensation. The cycle times for these time scales are user specified values. For the test which was shown here the following cycle times were used:

- Computation of volumetric grid deformation: 1min
- Computation of temperature distribution: 2.5s
- TCP displacement: 250ms

The maximum gradient of the displacements of the machine is about $10\mu m$. Every update of the volumetric grid deformation will cause a slight step in the displacements. If the grid is updated every minute, it will cause a step that is $0.17\mu m$ at most as shown in (11.5). Thus the step is rather small and the update time is considered to be enough. The thermal time step is in the order of seconds such that the current state of the machine can be modeled. VMP needs approximately 200ms to rebuild the thermal equation system (changes in the boundary conditions, axis position etc.) and solve the system to get the temperature distribution for the current time step. The toolpath data is retrieved every 250ms from the CNC. The interpolation of the TCP displacements, based on the TCP position inside the working envelope, is performed at the same cycle time. This cycle time may be a bit too slow, but currently is limited by library that is used to access the CNC.

$$\Delta x_{\text{step}} = \dot{u} \cdot \Delta t = 10 \mu m \cdot \frac{1}{60} h = 0.17 \mu m$$  

(11.5)

11.3.2 Test Procedure

In a first test three incremental probes have been used to measure the TCP error at a single position in the working envelope. The measurement setup that was used is the same as shown in Fig. 11.2, with the two probes (X2 and Y2) for the rotation removed. For a second test a cross-grid (Fig. 11.7) has been used to check the compensation model at different positions inside the working envelope. The four positions at which the displacements are measured are indicated by point 1 to 4.

With both measurement setups the thermal error at the TCP will be measured. During these measurements the FEM-based compensation is active and the thermal error of the
compensated machine tool is measured. The values of the compensation algorithm which are sent to the CNC are stored. As such it is possible to reconstruct the thermal error that occurs without the compensation.

The load case is the warm-up behavior of the machine tool. The first measurement setup is used to measure the error at a single position, but in every axis direction. With the cross grid only the error in the X- and Y-direction as indicated by the blue arrows can be measured. As soon as the machine is turned on the position of all four points is measured. These are used as the initial positions, without any thermal error. The TCP is then kept at position 1 for a 15 minutes. Next a measurement cycle starts, where the TCP moves to all four positions. At each position the TCP stays for about 10 seconds in order to measure the thermal error. The machine moves to point 1 and waits another 15 minutes to repeat the measurement cycle. This procedure is performed for about twenty hours until the machine is thermally stable.

### 11.3.3 Displacements

In a first test, the machine is turned on and kept at a single position inside the working envelope. Thus the warm-up behavior of the machine should be compensated. Fig. 11.8 shows the result of the measurement which has been made with three incremental probes. The TCP error without the compensation is given by the lines marked by the + sign. The compensated values are shown by the solid line. The compensation works very well for the X- and Y-axis, keeping the displacements inside a very small range over a time frame of 20h. The Z-axis however does not show equally good results, mainly because the compensation model does not show the positive Z displacement in the first few hours. The results still are promising as no prior identification of the FE model has been made and as such some boundary conditions might not be set properly.

With the cross grid position dependent effects in the XY-plane can be measured. Fig. 11.9 and Fig. 11.10 show the displacements at two points (point 2 and 4) in the working
envelope. Both positions are at the same Y- and Z-coordinate, but the X-axis has been moved by -150mm from point 2 to point 4. In this test again 50% to 75% of the error could be compensated. By comparing both Fig. 11.9 and Fig. 11.10 at t=21h the error in X changes from 0.6 to 0.4 without compensation, showing the position dependence of the X-error. With compensation no position dependent change in the X-error can be seen and it stays at 0.2. Thus the change in the X-error, due to the -150mm movement of the axis, was recognized by the model and could partly be compensated. As a result it can be said that the compensation can predict and remove position dependent effects.
Figure 11.9: Displacements at point 2 without (+) and with compensation

Figure 11.10: Displacements at point 4 without (+) and with compensation


12 Conclusion and Outlook

12.1 Conclusion

The measurements of the thermal behavior on a variety of machine tools show that thermal effects can cause rather large errors at the TCP. For example, the warm-up behavior is of interest as often machine tools need a certain time until they reach a stable state. Besides the warm-up, the environmental temperature variation as well as movement of linear and rotary axes can cause a serious deterioration of the accuracy.

The simulation methods shown in this work will help to efficiently analyze the previously mentioned effects, ensuring a better understanding and more accurate machine tools. With the methods that were described here it is much easier to simulate the thermal behavior of machine tools, compared to conventional FE simulation packages. The first step in doing so is providing a user-friendly GUI that is geared towards machine tool designers. With the help of the GUI the simulation model can be setup quickly.

The software VMP then performs all the simulation duties. Based on the NC-path multibody dynamics are used to calculate the load on the components as carriages, bearings etc. Given the load on, for instance, a carriage it can be further analyzed what the loading on each rolling body is. Based on this load the friction, power loss and heat conductivity of the carriage can be calculated. The NC path is also used to automatically update the position of the axis in order to represent position dependent effects and to introduce heat losses at the proper positions. The thermal FE model then will be updated with the current position and boundary conditions in order to represent the actual state of the machine tool.

The losses of feed drives play an important role during the movement of axes because their waste energy flows into the axis structure. Because of this a model for electric drives was also presented. This model directly uses the torque that is acting on the axis in order to calculate the resulting iron and ohmic losses. The excess heat of electric drives, for example in the spindle, is transferred to the coolant. Cooling devices sometimes also act to stabilize the machine against changes in the environmental temperature. As the energy consumption of the machine comes into focus, the question arises what the effects of switching off auxiliary components are. Because of this cooling devices were integrated into the simulation software. It was shown that by using a proper cooling strategy it is possible to reduce the error of the machine tool, due to varying environmental effects, by about 75%.

Not all thermal errors can be removed by improving the design of the machine. Therefore it is sometimes desired to correct the remaining errors by compensation. Depending on the complexity of the effects it can be time consuming to derive a compensation model. In this work a compensation model has been developed that is directly derived from the FE model. As such it is possible to create a compensation model based on the simulations that
have been made during the design phase. As the physics are modeled by the simulation it is also less time consuming than to identify a compensation model based on measurements. Using the compensation again 50% to 75% of the error could be removed.

12.2 Outlook

The models and equations which were shown can be used to calculate the energetic, mechanical and thermal behavior of a machine tool. In the scope of this work the verification of the models has been focused on the thermal behavior. As a future step it would be interesting to validate the calculations of the power losses with measurements of the energy consumption on a real machine tool. This would mean that the models would have to be enhanced to also calculate the effective power and not only the power loss.

Another possibility is the evaluation of the machine tools dynamics. While multibody dynamics are implemented and can be used to analyze the eigenfrequencies of the machine tool, it is also possible to include the modal analysis in the FE code of VMP for more accurate results. This would also need a better understanding of damping effects in the couplings as guides and ball screws.

The machining process also was not included in the scope of this work. A future step for the simulation, especially for the thermal one, would be to include the process heat that is generated and transferred to the machine either by chips or by heat flowing through the tool. In the context of the process cooling lubricant also is of major importance. The mathematical description of how coolant flows over the structure of the machine tool is quite complex and can be included in future works. Thermal errors however are often only important in the part finishing and as such the process heat and coolant do not play an equally important role as during roughing.

Regarding the FEM-based compensation in Sec. 11.3 it was shown that the warm-up of the machine could be compensated not only at a single position, but inside the whole working envelope. As the compensation method now has been tested and shown to work, it should be investigated in how to extend this method to rotary axes. Methods to compensate for the spindle elongation were also shown, but these were not FEM based. As a future step a spindle model could also be implemented into the FEM-based compensation.
A Appendix

Methods to improve the computational speed of FEM where shown in Chap. 5. As high calculation speeds are necessary to achieve real-time updates of the FE model, it is also necessary to use a coarse mesh discretization with as few degrees of freedoms as possible, while maintaining a sufficient accuracy. Because of this a number of measures have to be taken, which optimize the performance of the FEM for the specific case. A few hints on achieving a high accuracy are given in the subsequent sections, with respect to the spatial and temporal discretization.

A.1 Spatial Discretization

Grid quality is very important, as it influences the accuracy of the solution. The grid should be as regular as possible, in order to minimize the influence of distorted elements. If elements are distorted, errors occur in the coordinate transformation of the elements. Mesh size is another important issue, as shown in (A.1) ([122]), where the exact solution is given by $T$ and the numerical solution is defined by $\tilde{T}$.

$$\|T - \tilde{T}\| \leq Eh^\Gamma$$ (A.1)

It can be seen that the error of the solution depends on the mesh size, $h$, and the order of the shape function (influence on $\Gamma$). For a mesh of 3-node triangles with bilinear shape functions the coefficient $E$ depends on the second derivative of the error $e$ (A.2) with respect to the coordinates.

$$e = T - \tilde{T}$$ (A.2)

The derivation of $E$ for said problem can be found in [122]. As a result the curvature of the temperature (equals the rate of change of the heat flux) is important for the accuracy of the numerical solution, when using bilinear shape functions. A visual representation for the influence of the mesh size and the order of the shape functions is shown in Fig. A.1 for a one dimensional problem with linear shape functions. In case of linear shape functions, it is obvious that the error is large in areas of a high curvature of the exact solution (between $x$ and $x+h$).

Therefore it is advisory to use adaptive mesh refinement (h-refinement), implemented in most meshing tools, to reduce the grid size in areas of high gradient changes. Mesh refinement can be done a priori by using a steady-state solution of the corresponding problem. Usually machine tools do not exhibit high temperature differences, allowing rather coarse grids. Higher-order shape functions (p-refinement) could also be used as with an increased order, more complicated functions can be replicated.
A.2 Time Discretization

Time discretization is another important factor determining the solution speed and the accuracy. For the time-discretization usually the \( \theta \)-method is used, because of its simplicity, as shown in [123]. A term \( T \), for the temperature, can be dispatched according to (A.3) and the first order derivative is represented by (A.4).

\[
T = \theta T^{t+\Delta t} + (1 - \theta) T^t \\
\frac{\partial T}{\partial t} = \frac{T^{t+\Delta t} - T^t}{\Delta t}
\]  

(A.3) \hspace{1cm} (A.4)

The discretization of the transient heat transfer function then has the form of (A.5). \( Q^{\text{all}} \) includes all heat flows and a bar above a letter e.g. \( \bar{C} \) indicates the relation to prescribed temperature boundary conditions, while \( \tilde{C} \) shows the relation to free temperatures. Additionally one should note that \( C \) and \( \tilde{K}_k \) can be added together in advance, as they will not change during runtime and therefore it is possible to save some additional computational cost.

\[
\left[ \left( \frac{1}{\Delta t} C + \theta \tilde{K}_k \right) + \theta \tilde{K}_c \right] T^{t+\Delta t} = \left[ \left( \frac{1}{\Delta t} C - (1 - \theta) \tilde{K}_k \right) - (1 - \theta) \tilde{K}_c \right] T^t \\
- \frac{C}{\Delta t} \frac{T^{t+\Delta t} - T^t}{\Delta t} - (\tilde{K}_k + \tilde{K}_c) \left[ \theta T^{t+\Delta t} + (1 - \theta) \tilde{T} \right] \\
+ \theta Q^{t+\Delta t} + (1 - \theta) Q^t
\]  

(A.5)

In this case the value of the parameter \( \theta \) is important, because depending on its value the stability and accuracy of the scheme changes. Some values for it are shown in Tab. A.2.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>Scheme</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Forward (Explicit) Euler</td>
<td>( O(\Delta t) )</td>
</tr>
<tr>
<td>0.5</td>
<td>Crank-Nicholson</td>
<td>( O(\Delta t^2) )</td>
</tr>
<tr>
<td>0.878</td>
<td>Liniger</td>
<td>( O(\Delta t) )</td>
</tr>
<tr>
<td>1</td>
<td>Backward (Implicit) Euler</td>
<td>( O(\Delta t) )</td>
</tr>
</tbody>
</table>

Table A.1: \( \theta \)-values and accuracy for different time integration schemes
If the explicit scheme is used together with lumped heat capacitance matrix, (A.5) can be simply solved with a matrix multiplication and without the need of using a solver for the equation system. However, the explicit scheme is only conditionally stable and especially FEM have severe restrictions on the time-step as shown in the stability criterion (A.6), taken from [122], for a two-dimensional transient problem, with $r$ being the mesh-size ratio

$$r = \frac{a \Delta t}{h^2} \leq \frac{1}{12} \quad (A.6)$$

All schemes with $\theta \geq 0.5$ are stable, but still numerical oscillations can occur. By pure intuition it seems rather awkward, that a heat transfer equation can suffer from oscillations, but this is a result of the discretization. This effect shall be shown by deriving the amplification factor, as shown in [124]. Consider a first order homogeneous model equation, with $\lambda^h$ being the maximum eigenvalue:

$$\dot{d} + \lambda^h d = 0 \quad (A.7)$$

the discretization of (A.7) looks like

$$(1 + \Theta \Delta t \lambda^h) d_{n+1} = (1 - (1 - \Theta) \Delta t \lambda^h) d_n \quad (A.8)$$

Rewriting (A.8) to

$$d_{n+1} = A d_n \quad (A.9)$$

the influence of the amplification factor $A$ (A.10) becomes obvious.

$$A = \frac{1 - (1 - \Theta) \Delta t \lambda^h}{1 + \Theta \Delta t \lambda^h} \quad (A.10)$$

The stability requirement is $|A| \leq 1$ and the oscillation limit is $A = 0$, as the sign of $A^n$ changes with every step, if $A$ is smaller than zero. For any $\theta \neq 1$ the amplification can become smaller than zero, depending on the time step and the maximum eigenvalue $\lambda^h$. Therefore the implicit Euler is the only scheme, that does not suffer from oscillation, but it is only first order accurate in time. The value $\theta = 0.878$ was derived by [125], by a best-fit for a semi-infinite wall, with a prescribed surface temperature. In practice however one hardly deals with such ideal problems and in test examples it did not perform that well. By broad experience most accurate solutions can be retrieved by the Crank-Nicholson scheme, but it is prone to numerical oscillations, which becomes obvious when looking at the amplification factor because for $\theta = 0.5 \rightarrow A \approx -1$. Fig. A.2 shows the maximum values for $\Delta t \lambda^h$ for which the different schemes do not exhibit oscillations.

As [126] states most often unrealistic boundary conditions, such as sudden steps, excite oscillations are not much of a problem anymore and very accurate solutions can be obtained with it.

Considering a more practical and less theoretical approach, rough values for reasonable step sizes can be found in [127]. The Biot Number, a dimensionless ratio of convective and conductive thermal resistances, will be used as a reference.

$$Bi = \frac{h_{conv} h}{\lambda} \quad (A.11)$$
$h$ is the mean element width, $h_{conv}$ is the average film coefficient and $\lambda$ is the averaged conductivity. Expected values for the mean element width and the average film coefficient are $h \approx 0.02 [m]$ and $h_{conv} \approx 10 \left[ \frac{W}{mK} \right]$. As for the conductivity an average value for steel will be taken. In this case the Biot Number is roughly 0.0033 and in that case the time step can be estimated using (A.12).

$$\Delta t = \beta \rho cp(h) \frac{2}{k} = \beta \frac{(h^2)}{a} \quad (A.12)$$

$\rho$ is the averaged density and $c$ the averaged specific heat, while $\beta$ is a constant chosen, where $0.1 \leq \beta \leq 0.5$. The thermal diffusivity $a$ indicates whether a material is better at conducting thermal energy than storing it. The estimated time steps for steel are about $2.5 - 12[s]$, showing that large time steps, leading to fast calculation times, can be used.

**A.3 Shape-Functions and Numerical-Quadrature**

Numerical-quadrature influences the solution of the FEM calculations in different ways. In this case it is important to consider the effect of numerical quadrature on the transient thermal properties and the mechanical properties of the equation system.
Mechanical Properties  The most well-known is the influence on the stiffness. Since FEM is based on the principle of virtual work the calculated stiffness usually is too high. Especially bending problems are affected by this and therefore it usually is necessary to use many elements to retrieve an accurate solution. It is a well known fact that linear shape functions, in particular tetrahedrons, are too stiff. As an appropriate measure quadratic shape functions will be used, especially to describe the bending of thin plates, which occur often in welded structures.

Since it is desirable to use only very few elements, but still necessary to have an accurate solution, reduced integration is being used. Fig. A.3, taken from [128], shows a comparison between the analytical and FEM solutions for the deflection of a clamped beam. The first letter indicates the element type, tetrahedron or hexahedron, and the following number the amount of nodes. As mentioned above only elements with quadratic shape functions and different types of numerical integration are compared. It is clearly visible that by reduced integration, with 2x2x2 integration points, the analytical solution can be obtained with fewer elements, than with full integration (3x3x3). Another benefit is that the stiffness matrix can be calculated 3.5 times faster, because there are fewer integration points. However hour-glassing, or zero-energy modes, can appear, which results in wrong displacements. As hour-glassing can not propagate in elements with quadratic shape functions ([129]), no measures need to be taken, which is also the case in commercial FEM packages ([130]). The best option is to use the integration points specified in [131], which recommends 14 integration points. This method has the benefit of achieving results of the same quality as integration with 2x2x2 points, but without hour-glassing.
Figure A.3: Normalized deflection of clamped beam for different aspect ratios (height/length) and element types; From: [128]
Transient Thermal Properties  It has already been mentioned, that in transient calculations numerical oscillations occur, which have to be dealt with. If linear shape functions are used, the numerical quadrature points of the heat capacitance matrix can be set equal to the nodes of the element. As a result the heat capacitance matrix becomes lumped, with only entries on its diagonal and oscillations are significantly reduced. As can be seen in [132], this also has a slight impact on the convergence of the solution. However this is not of any major importance as the solution still converges quite fast.

![Graph showing temperature profile over time for semi-infinite wall](image)

**Figure A.4:** Comparison of quadratic (Tet 10) and linear (Tet 4) shape functions

Another benefit of using linear shape function can be seen in Fig. A.4, showing the solution for the temperature profile of a point in a semi-infinite wall with a prescribed surface temperature. First of all, the oscillations clearly have been damped out. Secondly, even if the mesh size of the linear shape functions only is half of the one for the quadratic shape functions, less degrees of freedom are being needed and the solution is more accurate (Tab. A.3).

<table>
<thead>
<tr>
<th>Shape Function</th>
<th>Avg. Mesh Size</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic</td>
<td>$\Delta x = 0.02m$</td>
<td>16865</td>
</tr>
<tr>
<td>Linear</td>
<td>$\Delta x = 0.01m$</td>
<td>13788</td>
</tr>
</tbody>
</table>

**Table A.2:** Comparison of degrees of freedom for quadratic and linear shape functions

Finally an expansion of the calculated temperatures is necessary, as they are calculated with linear shape functions, while the mechanical system, used to calculate the thermal...
deformation, uses quadratic shape functions. The expansion is simply done, by using the average of the two corresponding corner-nodes temperatures, for every midpoint-node.
List of Publications


Bibliography


[100] SKF. Interaktive Lagerungskatalog.


[130] SIMULIA. Abaqus 6.8.3 html documentation. ABAQUS.
