Inferring in situ stress variations from post-drilling borehole diametrical deformation

Master Thesis

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Inferring in situ stress variations from post-drilling borehole diametrical deformation

Master of Science in Applied Geophysics

by

Jinqiu Chong

August 16, 2021
Abstract

This research is to construct the workflow of using borehole shape deformation to determine the in-situ stress variations including the rotation of stress orientation and relative magnitude variations along depth. A variety of boreholes at the Bedretto Underground Laboratory for Geosciences (BULG) were studied in this research. The stress released after drilling caused the borehole deformation, and this deformation was measured by acoustic televiewer (ATV) logging. An actual borehole shape usually is not ideally elliptical. However, it can be fitted by the direct least square fitting method to determine the precise azimuth of its axis and the corresponding ratio between the short axis and long axis of the fitted ellipse. The azimuth variation of the deformed borehole infers the orientation change of the projection of in-situ stress on the borehole plane. The range of the azimuth of the fitted ellipse’s long axis is mainly between 90° to 150° relative to the high side of the borehole, and several significant rotations of the ellipse’s azimuth were also found in this research. The relative value of the maximum and minimum stress $S_{\text{max}}$ and $S_{\text{min}}$ can be derived from the borehole elliptical ratio. The fitted results are compared with the azimuth of breakout and the fracture density to verify their consistency. Abrupt changes of fitted azimuth were found at intervals with the presence of faults or at high fracture density zones, which are the major causes of geo-mechanical property’s changes in a homogenous medium. ATV data with high quality is required to reflect the actual borehole shape and determine the final fitting results. The extrapolation of the stress orientation determined by ellipticity is appliable in zones without breakout presence along the whole borehole depth. With the elliptical fitting of the deformed borehole, this stress determination method can be implemented in its actual application.
Acknowledgements

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Jinqiu Chong

August 4, 2021
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Chapter 1

Introduction

With the growing global need for energy, reservoir research is becoming increasingly important and necessary. The analysis of the borehole gives the knowledge of the orientation and magnitude of local stress around it. Knowledge of in-situ stress is valuable not only in the stabilization and analysis of boreholes but also in the development and exploration of reservoirs based on a profound understanding of the in-situ stress changes. In this research, the ATV travel time of the boreholes with the elliptical fitting process is used to help find out the local stress variation trend.

1.1 In-situ stress

The stress field inside the Earth's upper crust can be characterized as a tensor with six stress components. Shear stresses vanish in a principal coordinate system, and the stress field is fully represented by three principal stresses \( S_1, S_2, \) and \( S_3 \). By their magnitude, \( S_1, S_2, \) and \( S_3 \) are referred to as the major, intermediate, and minor principal stresses, respectively. The vertical stress \( S_v \), induced by the weight of the overburden is the stress component that is perpendicular to the Earth's surface (Zoback, 2010). Vertical stress \( S_i \), minimum horizontal stress \( S_{h\text{min}} \), and maximum horizontal stress \( S_{h\text{max}} \) define the in-situ stress in the Earth's upper crust. The local faulting style is determined by the relative magnitudes of these stress components (Anderson, 1905). For a vertical borehole, \( S_{h\text{max}} \) and \( S_{h\text{min}} \) are used to describe the stress on borehole walls, while in this research, all studied boreholes are deviated, and the stresses considered here are \( S_{\text{max}} \) and \( S_{\text{min}} \), which represent the projection of \( S_{h\text{max}} \) and \( S_{h\text{min}} \) on the borehole plane.

After the borehole drilling process, a part of the rock is removed from its original location, and this vacant part with missing rock cannot withstand the original far-field stress, resulting the fact that stresses concentrate around the borehole wall. As indicated in Figure 1.1 below, hoop stress \( \sigma_{\phi\phi} \) represents the circumferential stress around the borehole, which concentrates at the orientation of the minimum stress \( S_{\text{min}} \) and spread out along the orientation of the maximum stress \( S_{\text{max}} \). The stress around the borehole varies depending on its position and distance from the borehole wall. It is symmetric in terms of \( S_{\text{max}} \) and \( S_{\text{min}} \) distributions. The hoop stress is the cause of borehole shape deformation with no support inside the borehole. When hoop stress exceeds the rock strength, the breakout occurs (Zoback, 2010), as shown in Figure 1.2.
Figure 1.1: Hoop stress $\sigma_{\theta\theta}$ distribution around the borehole cross-section with stress $S_{\text{min}}$ and $S_{\text{max}}$ (edited from Zoback et al. (2003)). The color indicates the magnitude of hoop stress. Hoop stress with maximum magnitude is distributed along the azimuth of stress $S_{\text{min}}$ while the minimum magnitude is in the same direction as the stress $S_{\text{max}}$. As the distance from the borehole wall increases, hoop stress attenuates gradually.

Figure 1.2: When hoop stress exceeds the rock strength, the breakout occurs following the direction of the weakest stress. Red marks represent the location of the breakouts.

1.2 Borehole deformation

The borehole is meant to have an initial circular once it has been drilled in an ideal environment without any disturbance. Stress $S_{\text{max}}$ and $S_{\text{min}}$ are applied along the x and y axis, respectively, on the borehole wall with no support from the removed core. Wang et al. (2016) demonstrated that $P$, which represents one point on the circular borehole wall, becomes $P'(x, y)$ as the borehole deforms, satisfying the equation:

$$
\frac{x^2}{a \left(1 + \frac{3S_{\text{max}} - S_{\text{min}}}{E}\right)^2} + \frac{y^2}{a \left(1 + \frac{3S_{\text{min}} - S_{\text{max}}}{E}\right)^2} = \cos^2 \theta + \sin^2 \theta = 1
$$

(1.1)
where:
$S_{\text{max}}$ and $S_{\text{min}}$ are the stresses along x and y axis respectively, $S_{\text{max}} > S_{\text{min}}$.
a is the radius of the initial borehole.
$E$ is the elastic modulus of this cross-section.
$\theta$ is the angle between the radial direction of P and the x axis.

Equation 1.1 is a standard elliptical equation which shows how a circular borehole deforms into an elliptical shape under stress $S_{\text{max}}$ and $S_{\text{min}}$, as illustrated in Figure 1.3. The hoop stress here is not enough to cause the breakout to occur, only leading to the deformation of borehole shape.

Figure 1.3: Deformation of borehole shape under orthogonal stress $S_{\text{max}}$ and $S_{\text{min}}$. A solid circle represents the initial shape of the borehole after drilling. The dashed ellipse is the deformed shape, where $d_{\text{max}}$, the long axis of the ellipse, follows the azimuth of the minimum stress $S_{\text{min}}$ while $d_{\text{min}}$, the minor axis of the ellipse, is with the same orientation of the maximum stress $S_{\text{max}}$.

### 1.3 Research objectives

The goal of this research is to test and evaluate the use of borehole shape fitting with travel time of acoustic televiewer (ATV) logging on a variety of boreholes drilled in different locations at the Bedretto Underground Laboratory for Geosciences (BULG) by core drilling or hammer drilling, including CB1, CB2, CB3, ST1, ST2, MB4, MB5, MB7 MB8, and Welltec.

The raw data from the ATV logging is assigned to the high side (HS) direction and processed in WellCAD to obtain the travel time in all measured angles at each depth interval. Then a MATLAB program given in Appendix A, processes all calculations, including distance conversion from travel time, elliptical fitting, and fitting evaluation. As a result, it is able to help to better understand the in-situ stress distribution at BULG by fitting the borehole shape with the most approximate ellipse. The direction of the fitted ellipse’s minor and major axes shows the azimuth of the maximum and minimum stress. The ratio between the minor and major axes can also be used to determine the relative stress magnitude variation.

The drilling method, location, trajectory, and logging time of the analyzed boreholes are all described in Chapter 2. This chapter also includes a brief description of BULG as well as regional
geology in this area. The methodology for fitting a borehole shape and evaluating the fitting results are described in Chapter 3. The fitted results are displayed and compared among several boreholes in Chapter 4 and Appendix B. They are also compared to the distribution of breakouts and fault zones to verify the results and obtain more information on in-situ stress in the area of BULG. The relative stress along the depth was also discussed in this chapter. Chapter 5 gives the discussion on the influence of the drilling method and the azimuth variation pattern compared with the presence of breakout and fractures.
Chapter 2

Site description

2.1 Laboratory and regional geology

A side access tunnel in the Swiss Central Alpine region's Bedretto valley has been designated as an underground laboratory known as the Bedretto Underground Laboratory for Geosciences (BULG). Although the regional stress field around the Swiss Alps is not consistent, the $S_{Hmax}$ azimuth often falls within the NW quadrant (Ma et al., 2020). According to Kastrup et al. (2004), the stress regime changes from a slight preponderance of a strike-slip faulting regime in the foreland to a strong predominance of a normal faulting regime in the high parts of the Alps. Based on the regional $S_{Hmax}$ rotation pattern, a NW-SE direction of $S_{Hmax}$ is expected. The stress measurements conducted at BULG indicate that $S_{Hmax}$ direction is approximately N100E, albeit with some variations (Ma et al., 2020).

BULG is located in the Gotthard Massif, where Rotondo Granite is the most dominant rock type. Rotondo Granite is a relatively homogeneous massive light gray intrusion from the late phase of the Variscan Orogeny (Lützenkirchen et al., 2011), which leads to the fact that the mechanism behind this research is based on a homogeneous medium with relatively ideal properties.

2.2 Borehole Description

CB1, CB2, CB3, ST1, ST2, MB4, MB5, MB7, MB8, and Welltec are the ten boreholes at BULG logged with the Acoustic Televiewer tool and used in this investigation. Borehole information is shown in Table 2.1. In addition, the specific location of each borehole and its trajectory are indicated in Figure 2.1.
Figure 2.1: Side view of boreholes CB1, CB2, CB3, ST1, ST2, MB4, MB5, MB7, MB8, and Welltec and the BULG tunnel in local coordinate.

Table 2.1: Properties of studied boreholes.

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Position [TM]</th>
<th>Length [m]</th>
<th>Diameter [mm]</th>
<th>Inclination ['']</th>
<th>Global Orientation ['']</th>
<th>Drilling Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>CB1</td>
<td>2050</td>
<td>300</td>
<td>96</td>
<td>43.4-55.2</td>
<td>220.3-228.3</td>
<td>Core drilling</td>
</tr>
<tr>
<td>CB2</td>
<td>2043.3</td>
<td>219</td>
<td>96</td>
<td>36.9-45.1</td>
<td>221.9-230.2</td>
<td>Core drilling</td>
</tr>
<tr>
<td>CB3</td>
<td>2036.7</td>
<td>192</td>
<td>96</td>
<td>48.0-67.6</td>
<td>214.4-232.0</td>
<td>Core drilling</td>
</tr>
<tr>
<td>ST1</td>
<td>2030</td>
<td>399</td>
<td>215.9</td>
<td>34.1-45.1</td>
<td>214.1-236.9</td>
<td>Full hammer drilling</td>
</tr>
<tr>
<td>ST2</td>
<td>2065</td>
<td>349</td>
<td>215.9</td>
<td>38.9-55.6</td>
<td>218.1-227.2</td>
<td>Full hammer drilling</td>
</tr>
<tr>
<td>MB4</td>
<td>2016.6</td>
<td>250</td>
<td>152.4</td>
<td>40.9-49.4</td>
<td>216.6-231.3</td>
<td>Full hammer drilling</td>
</tr>
<tr>
<td>MB5</td>
<td>2023.3</td>
<td>223</td>
<td>152.4</td>
<td>44.0-47.8</td>
<td>219.6-226.9</td>
<td>Full hammer drilling</td>
</tr>
<tr>
<td>MB7</td>
<td>2009.1</td>
<td>150</td>
<td>175</td>
<td>63.9-69.8</td>
<td>244.9-255.0</td>
<td>Full hammer drilling</td>
</tr>
</tbody>
</table>
Multiple loggings were performed on the boreholes after drilling. The information obtained from ATV logging for each borehole is shown in Table 2.2 below.

The time it takes for an acoustic wave emitted by the probe to travel to the borehole walls and return is referred to as the ATV log travel time. ATV travel time then with travel velocity can be converted to distance to construct the real borehole shape. The ATV log offers a detailed image with a 360-degree overview of the borehole wall at each depth, which can be used to determine the presence and location of breakouts, fractures, and faults (Williams, 2002).

Azimuth, tilt, and caliper logs were also obtained during the capture of each ATV log. The azimuth log records the borehole's exact orientation, the tilt log tracks the borehole's deviation from the vertical axis, and the caliper log obtains the borehole's maximum and minimum diameter at each depth. Note that in most cases, the travel time is centralized first before its analysis. In this study, in contrast, the travel time is raw data without centralization, as centralization processes the data that decrease the sinusoidal trends, which in the end makes the shape of the borehole calculated from travel time more like a circular rather than an elliptical shape. This decrease in eccentricity makes it difficult to locate and calculate the directions of ellipse’s major and minor axes. Therefore, it should be noticed that the ATV travel time used in this study is raw data without centralization.

Table 2.2: ATV logging details of each borehole. The data used in this research are marked with (*). Vertical and angular resolution are both given from the chosen data.
<table>
<thead>
<tr>
<th>Item</th>
<th>Dates</th>
<th>Value</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST2</td>
<td>2020.07.30; 2020.08.01*; 2021.03.20; 2021.05.25</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>MB4</td>
<td>2020.05.27; 2020.06.04*</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>MB5</td>
<td>2021.03.22*</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>MB7</td>
<td>2021.05.03*</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>MB8</td>
<td>2021.03.11*</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>Welltec</td>
<td>2021.06.02*</td>
<td>0.01</td>
<td>1</td>
</tr>
</tbody>
</table>
Chapter 3

Borehole shape analysis methodology

Taking borehole CB1 as an example, CB1 has enough borehole depth and is relatively smooth with little disruptions during the core drilling, as detailed in the previous chapter. Directly from the ATV travel time image as Figure 3.1 shows, it is clear that, excluding the presence of breakouts and fractures, the color of ATV travel time changes in a 360-degree range at each depth, giving the first impression that boreholes from each depth are not in an ideal circular shape.

Figure 3.1: CB1 travel time angular variations from depth range 110 m to 130 m.

The equation below is used to convert the radius from travel time to distance:

\[
r(\theta, z) = \frac{\Delta t(\theta, z) \cdot v_a}{2} + r_p
\]

where \( \Delta t(\theta, z) \) is the initial travel time of waves emitted from the probe and back reflected by borehole walls, \( v_a \) is the acoustic velocity through the borehole fluid. Borehole fluid is water in all studied boreholes and set as 1480 m/s in calculations at all depths. \( r_p \) is the radius of the ATV probe, and in borehole CB1 measurement it is 9.5 mm. Figure 3.2 shows the transformation from ATV travel time to borehole radius and its corresponded diameter calculated from radius according to Equation 3.1. There is also a caliper record shown here as its comparison. It is clear to see that the calculated mean diameter from ATV is quite close to the caliper record, and they appear the similar trend along the depth. Note that the maximum, minimum, and mean diameter
are not obtained simply by adding the corresponding maximum, minimum, and mean radius but are calculated with the sum of two radii at the opposite direction (180° difference), which usually results in the calculated diameter greater than twice the minimum radius and less than twice the maximum radius.

Figure 3.2: ATV travel time, radius converted from travel time, diameter calculated from radius, and caliper record of CB1.

Figure 3.3 shows an illustration of a cross-section borehole shape. The borehole shape at 130 m is depicted on the upper plot. The expanded radius variations from 0° to 360° at the depth of 130 m are shown in the bottom plot. These two plots illustrate representative borehole shape deformations in zones without breakouts or fractures, which can be used to analyze in-situ stress distribution. However, in most cases, the initial deformation of a borehole shape can only provide an estimation because it is difficult to determine the exact azimuth direction of the maximum and minimum stress with an unregular borehole deformation. Furthermore, it was already proved in Chapter 1 that the borehole will deform into an elliptical shape with no material support inside. Therefore, it is logical to utilize an elliptical fitting algorithm on an actual borehole shape to quantitatively calculate the extent of deformation and the azimuth of the fitted ellipse's major and minor axis.
3.1 Borehole shape fitting

3.1.1 Direct least squares fitting of ellipses

The method of fitting implemented in this research is the direct least square fitting of ellipses, which was proposed and proved by Fitzgibbon et al. (1996), and improved by Halir (1999) to become numerically stable. Previous algorithms were either computationally expensive with iterative approaches or only fitted to general conics. Under the premise that $4ac-b^2=1$, the direct least square fitting of ellipses provides elliptical results by reducing the sum of squared algebraic distances from the real points to the ellipses. Furthermore, it is invariant to affine data transformations. Its outstanding robustness makes it effective in scenarios involving occlusion and noise. The method is non-iterative and computationally efficient, making it suited for use in a
MATLAB program to implement calculations.

An ellipse is a special case of a general conic which can be described by an implicit order polynomial (Halíř, 1999):

\[ F(x, y) = ax^2 + bxy + cy^2 + dx + ey + f = 0 \]  \hspace{1cm} (3.2)

With an ellipse-specific constraint

\[ b^2 - 4ac < 0 \]  \hspace{1cm} (3.3)

Point coordinate is represented as \((x, y)\). \(F(x, y)\) is the algebraic distance of point \((x, y)\) to the given conic. \(a, b, c, d, e, f\) are coefficients of the ellipse.

The polynomial can be expressed as

\[ F_a(x) = x \cdot a = 0 \]  \hspace{1cm} (3.4)

where

\[ a = [a, b, c, d, e, f]^T \]  \hspace{1cm} (3.5)
\[ x = [x^2, xy, y^2, x, y, 1] \]  \hspace{1cm} (3.6)

The fitting of a group of data points to a specific conic \(a\) is to minimize the sum of squared algebraic distances of these points to the conic, and it can be expressed as:

\[ \min_a \sum_{i=1}^{N} F(x_i, y_i)^2 = \min_a \sum_{i=1}^{N} (x_i \cdot a)^2 \]  \hspace{1cm} (3.7)

Setting

\[ D = \begin{pmatrix} x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^2 & x_iy_i & y_i^2 & x_i & y_i & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_N^2 & x_Ny_N & y_N^2 & x_N & y_N & 1 \end{pmatrix} \]  \hspace{1cm} (3.8)

The minimization becomes

\[ \min_a \|Da\|^2 \]  \hspace{1cm} (3.9)

The scale of coefficient \(a\) can be adjusted but still represent the same conic. To make sure that the solution is generated as an ellipse, the inequality constraint can be rewritten as

\[ 4ac - b^2 = 1 \]  \hspace{1cm} (3.10)

It can also be represented as

\[ a^T Ca = 1 \]  \hspace{1cm} (3.11)
where

\[
C = \begin{pmatrix}
0 & 0 & 2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]  \tag{3.12}

By applying the Lagrange multipliers, the minimization problem becomes

\[
S \mathbf{a} = \lambda C \mathbf{a}
\]  \tag{3.13}

\[
\mathbf{a}^T C \mathbf{a} = 1
\]  \tag{3.14}

where

\[
S = D^T D
\]  \tag{3.15}

The method above is unstable in the computation of its eigenvalues, and the optimal eigenvalues can even become a small negative number in some cases. Halír (1999) proposed to decompose the matrices of \( S, C, \) and \( \mathbf{a} \) to improve the fitting method. The problem then became

\[
M \mathbf{a}_1 = \lambda \mathbf{a}_1
\]  \tag{3.16}

\[
\mathbf{a}_1^T C_1 \mathbf{a}_1 = 1
\]  \tag{3.17}

\[
\mathbf{a}_2 = -S_3^{-1}S_2^T \mathbf{a}_1
\]  \tag{3.18}

The decompositions are as follows

\[
\mathbf{a} = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix}
\]  \tag{3.19}

where \( \mathbf{a}_1 = \begin{pmatrix} d \\ b \\ c \end{pmatrix} \) and \( \mathbf{a}_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix} \)

\[
C = \begin{pmatrix}
C_1 & 0 \\
0 & 0 \\
\end{pmatrix}
\]  \tag{3.20}

where \( C_1 = \begin{pmatrix} 0 & 0 & 2 \\
0 & -1 & 0 \\
2 & 0 & 0 \\
\end{pmatrix} \)

\[
D = (D_1 | D_2)
\]  \tag{3.21}

where
\[
D_1 = \begin{pmatrix}
x_1^2 & x_1 y_1 & y_1^2 \\
\vdots & \vdots & \vdots \\
x_i^2 & x_i y_i & y_i^2 \\
x_N^2 & x_N y_N & y_N^2
\end{pmatrix}
\quad \text{and} \quad
D_2 = \begin{pmatrix}
x_1 & y_1 & 1 \\
\vdots & \vdots & \vdots \\
x_i & y_i & 1 \\
x_N & y_N & 1
\end{pmatrix}.
\]

\[
S_1 = D_1^T D_1 \\
S_2 = D_2^T D_2 \\
S_3 = D_3^T D_3 \tag{3.22}
\]

\(M\) is the reduced scatter matrix which is represented as

\[
M = C_1^{-1} (S_1 - S_2 S_3^{-1} S_2^T) \tag{3.23}
\]

The solution to this problem is to find the appropriate eigenvector \(a_1\) of matrix \(M\) and then calculate the value of \(a_2\).

MATLAB code details to achieve this calculation can be found in Appendix A.2.

### 3.1.2 Depth interval resolution selection

A smoothing window on depth must be determined first to reduce noise and oscillations in results and to reduce the number of calculations required during fitting computations because the resolution of the initial ATV logging is 0.01 m in depth, which is too small for this study. A few depth intervals of sliding windows were tested in Figure 3.4 below, and the results show the azimuth of the fitted ellipses’ major axes. Given that the breakout selection has a resolution of 0.2 m, the depth intervals of 0.2 m and the multiples of 0.2 m (0.4 m and 0.8 m or even larger) were tested. It is apparent to notice that a depth interval of 0.2 m appears to have more noises on two sides of the main azimuth trend, which makes the trend unclear to determine the exact variation range. Results with a 0.8 m depth sliding window ignored some subtle changes. These small variations cannot be treated as noises but have to be considered as the actual trend changes with depth. In the end, a depth interval of 0.4 m was chosen in this study.
Figure 3.4 Azimuth of the major axes of fitted ellipses with different depth interval selections. The left figure is the result of the calculation with a 0.8 m interval. The middle figure is the result of the calculation with a 0.4 m interval. The right figure is the result of the calculation with a 0.2 m interval.

3.2 Fitting evaluation

3.2.1 Method

Same with the circular error and its least squares evaluation method, the fitting of ellipses can be evaluated with the similar principle. To build a coordinate system, use the center of the least squared ellipse as the origin point, major axis, and minor axis of the ellipse as the X and Y axis. Calculate the biased distance of the real data point $P_i(x_i, y_i)$ from the fitted ellipse at the same angular direction (Liu et al., 2002) by

$$\Delta r_i = \sqrt{x_i^2 + y_i^2} - r_i$$

where $r_i = ab \sqrt{a^2 \sin^2 \alpha_i + b^2 \cos^2 \alpha_i}$, $\alpha_i = \arctan\left(\frac{y_i}{x_i}\right)$.

The RMS value of $\Delta r_i$ calculated in equation 3.25 can be used as the indicator of the goodness of elliptical fitting. The lower the RMS is, the better the fitting will be.
\[ RMS = \sqrt{\frac{\sum_{i=1}^{N} \Delta r_i^2}{N}} \]  

(3.25)

### 3.2.2 Evaluation Results

The fitting result evaluation of borehole CB1 in relation to the fitted major axis azimuth and the ratio between the minor and major axes of the fitted ellipse is illustrated in Figure 3.5. There are essentially three zones when the RMS value changes abruptly. Between 0 and 30 m, it could be caused by the existence of the upper casing. The zones from 65 m to 70 m and 135 m to 210 m can be explained by the presence of the breakout zone, as described in the next chapter.

![Figure 3.5: CB1 borehole shape fitting evaluation compared with major axis azimuth and the ratio between the minor and major axis of the fitted ellipse](image)

Figures 3.6 and Figure 3.7 show a comparison between the real borehole shape with the fitted ellipse at depths of 130 and 150 m, respectively. The first figure is a typical example of a borehole shape in a normal zone without any breakouts or fractures, and the latter one is a representative of a borehole shape in a breakout zone, as two convex bulges of the borehole in approximal opposite directions can be observed clearly.
Figure 3.6: Comparison between real borehole shape and fitted ellipse at depth 130m.

Figure 3.7: Comparison between real borehole shape and fitted ellipse at depth 150m. Note that the magnitude of the real major radius is higher than the expected ellipse major axis.
Chapter 4

Results

The stress concentration around the borehole that causes borehole deformation determines whether the breakout occurs, as well as its azimuth and width. Breakouts only occur when hoop stress exceeds rock strength, which may not always be the case over the whole borehole length. Borehole shape deformation could provide information on the in-situ stress distribution over borehole intervals without breakouts. Because the borehole is not an ideal elliptical shape in the real situation, one of this method's issues is to determine the ellipse's long axis based on ATV travel time data. The solution to this problem is the elliptical fitting algorithm, as described in the previous chapter. To verify the results of borehole shape fitting, the actual breakouts based on ATV and OTV images are used as a comparison. In the following discussion, results of several boreholes are demonstrated, and the rest results for other boreholes are shown in Appendix B.

4.1 Borehole deformation correlation with breakout zones

The azimuth of the fitted major axis of the ellipse in some typical boreholes, including CB1, ST1, ST2, and MB8, as well as the actual breakout azimuth of each borehole, are illustrated in Figure 4.1 below. The boreholes selected to present here all have good ATV data quality. In addition, they are long in depth to present more features as possible. These boreholes are distributed with distances but relatively parallel to each other. As a result, the azimuths of the fitted ellipse long axis are quite close to the actual breakout azimuths and exhibit a similar variation trend in all figures.
CB1 by core drilling has a comparatively plain borehole wall, resulting in a relatively smooth azimuth variation trend with fewer outliers than the other three boreholes. The major azimuth trend varies from 45° to 125°. In CB1, there are primarily two breakout zones centered between 150 m to 185 m and 190 m to 210 m, respectively. The azimuths of the fitted ellipse major axis in both breakout zones are roughly 105° and 285°, both are quite close to the breakout azimuth. It demonstrates that the azimuth findings from elliptical fitting on borehole CB1 based on ATV travel time data are extremely trustworthy. There are three main breakout zones in borehole ST1. The fitted long axis has an azimuth trend range from 115° to 190°. The fitted azimuth in breakout zones is slightly smaller than the true breakout azimuth. However, they are still extremely close to each other and show generally steady azimuth fluctuations with depth. ST2 has four breakout zones and a fitted azimuth range from 130° to 160°. In all four breakout zones, the fitted azimuth is close to the breakout azimuth, despite the fact that the fitted ones are smaller. Only one breakout zone with a high breakout density exists in borehole MB8. As expected, the fitted azimuth and the breakout azimuth are also close. The fitted azimuth trend ranges from 100° to 205°, with a breakout zone of
about 130°.

In both smooth and unsmooth borehole wall situations, the trustworthiness of the elliptical fitting is very favorable, according to the comparison results between fitted azimuth and real breakout azimuth. Thus, it is reliable to obtain the azimuth distribution of S\(_{\text{max}}\) and S\(_{\text{min}}\) along the entire well depth using ellipticity as an indicator in borehole intervals without breakout.

### 4.2 Borehole deformation correlation with fracture zones

According to the previous regional investigation, the lithology along these boreholes is Rotondo Granite. The lithology does not vary with the depth. Therefore, except for the inaccuracy of ATV travel time measurements, any azimuth variation in the main trend of a borehole is assumed to be caused by the disturbance generated by natural fractures and fault zones or the appearance of washout and borehole failure. Shamir and Zoback (1992) concluded that active faults cause stress perturbation with the geo-mechanical changes.

The comparison between fracture density and fitted ellipse's long axis azimuth along depth is shown in Figure 4.2, using boreholes ST1 and ST2 as the study cases since they are both long in depth to present more visible details on azimuth variation than the other boreholes. In addition, they are truncated by several typical faults. The number of fractures per meter is used to represent the fracture density. Because the other major axis azimuth is determined by only adding the azimuth value (from 0° to 180°) with 180° and show the same azimuth variation trend with depth, only one azimuth trend is shown here. A zone with fracture density greater than 2 fractures/meter is treated as the fracture zone with high fracture density based on statistics of fracture number along with the depth. Borehole ST1 has a maximum fracture density of 8 fractures per meter at 390 m, and borehole ST2 has a maximum fracture density of 6 fractures per meter at 29 m and 300 m.

Figure 4.2 indicates a dramatic azimuth change from roughly 170° to 145° at a depth range from 108 m to 120 m in borehole ST1. This abrupt change is highlighted by two faults at the top and bottom of it. On borehole ST1, there are in total of 8 faults, the majority of which correspond to the azimuth abruptions along with depth. There are also some azimuth changes in a small range, and most of these changes appear in high fracture density zones. Borehole ST2 exhibits more azimuth variation changes along with depth than borehole ST1. The abrupt change in azimuth from 180° to 160° at 160 m is accompanied by the presence of a fault at the same position. Azimuth change from 100° to 130° at around 255 m depth has its relative fault presented. Other faults also correspond to azimuth changes, but the magnitudes of change are not as large as the prior two faults. Borehole ST2 features with several continuous azimuth variations in a relatively narrow depth range, such as zones from 150 m to 160 m, from 230 m to 250 m, and from 255 m to 270 m. These continuous azimuth variations always present at high fracture density zone.

By combining Figure 4.2 and Figure 4.1, it can be noticed that the depth that breakout appears usually corresponds with relatively high fracture density.
Figure 4.2: Fracture density variation compared with fitted ellipse major axis azimuth of borehole ST1 and ST2. Grey shades indicate the presence of faults.

4.3 Inferring relative stress from ellipticity ratio

Figure 4.3 shows a relation between the presence of breakout and the variation of the ellipticity ratio. The presence of the breakout zone is always accompanied with the decrease of ellipticity ratio, as the breakout elongated the major axis’s length in the borehole. This relation can be used to help to find out the approximate depth distribution range of breakouts.
Figure 4.3: Azimuth of the fitted ellipse’s major axis and ellipticity ratio of borehole ST2. Grey shades are breakout zones, and they all correspond to low ellipticity ratio intervals.

The estimation of the relative value of the maximum and minimum stress $S_{\text{max}}$ and $S_{\text{min}}$ can be derived directly from the borehole shape ratio. In Figure 4.4, CB1 borehole shape ratio tends to increase to 1 from depths of 20 m to 150 m, whereas the ratio decreases from depth 175 m to 300 m in the lower part of the borehole. The interval between these two trending parts is the breakout zone. According to the ratio variation trend, $S_{\text{max}}$ and $S_{\text{min}}$ close to the breakout zone exhibit few variances in magnitude. When close to the unstable low ratio zone between 100 m and 180 m, ST1 has a lower shape ratio, implying that the $S_{\text{max}}$ and $S_{\text{min}}$ have a significant value difference in this depth interval. The ratio increases when approaching the fracture zone from 270 m to 290 m, indicating that $S_{\text{max}}$ and $S_{\text{min}}$ are quite close to each other at a depth which is close to the fracture zone. The ratio trend in the breakout zone between 320 m to 330 m is identical to the one in the zone 100 m to 180 m as both are unstably distributed in the low ratio part. The ratio trend in ST2 increased and then decreased from depth 0 m to 150 m, indicating that the relative difference between $S_{\text{max}}$ and $S_{\text{min}}$ fell and then climbed as the depth increases. The ratio exhibits an approximately increasing trend below 210 m depth, which corresponds to the fact that $S_{\text{max}}$ and $S_{\text{min}}$ are gradually minimizing the difference between them. The ratio is particularly unstable in the upper part of borehole MB8, which could expect a disturbed zone caused not only by the existence of natural fractures but by failure and washout generated with the drilling procedure. The lower half of the borehole exhibits a generally growing ratio trend, implying that the difference between $S_{\text{max}}$ and $S_{\text{min}}$ decreases.
Figure 4.4: Fitted ellipticity ratio (short axis/long axis) of borehole CB1, ST1, ST2, and MB8.
The major axis of breakout represents the direction where breakout occurs, and it is the azimuth where $S_{\text{min}}$ orients. Therefore, it can be used as an indicator for stress distribution underground. In depth intervals where no breakout occurs, which is the case for most parts of the borehole, the deformed borehole shape as an ellipse can also become the indicator for stress orientation as they follow the same mechanism, which is the concentrated hoop stress causes the borehole wall change. The stress variation indication from borehole deformation can be considered as an extrapolation of the breakout indicator.

Borehole deformation requires the elliptical fitting to precisely give the azimuth of the major and minor axis of the borehole ellipse. One of the influences for the unideal borehole shape is from the drilling process. Crushing rock in hammer drilling is a kind of dynamic impact. The drill bit on the rock punching vibration is more likely to cause the breakage of the hole wall and the core, making it highly unfavorable for borehole wall sustaining (Jiang et al., 1999). Borehole CB1, CB2, and CB3 by core drilling have a relatively smooth borehole wall with minor disturbances. In contrast, the rest boreholes are drilled by the hammer which causes the unregular borehole shape at a small depth interval and a rough wall along with the depth. The depth interval in borehole shape fitting is selected as 0.4 m while the vertical resolution for ATV logging is 0.01m, which means a group of 40 ATV data is used to carry out the elliptical shape fitting in one depth interval. The selecting of the depth interval significantly lowers the uncertainty of the unregular borehole wall caused by hammer drilling at a small depth range. However, it is not the case that the larger the depth interval is, the better the fitting results will be. Figure 3.4 shows that too large interval selection will lead to the lack of details of the results in azimuth variations over depth.

The azimuth of fitted ellipse’s major axis in all boreholes, including CB1, ST1, ST2, and MB8 as illustrated in Figure 4.1 and other boreholes results in Appendix B demonstrates that the main orientation distribution of $S_{\text{min}}$ is at range from 90° to 150° relative with borehole’s high side. The specific azimuth for each borehole is slightly different because of the difference in the tilt and orientation of the borehole configuration. Usually, the azimuth with relatively constant value corresponds with the zones which have low fracture density, as in a relatively homogeneous medium with no lithology change and fractures presence, no variation in breakout azimuth is expected (Zoback et al., 1985). This is the same for the azimuth variation in no breakout zones. Depending on the slip tendency, zones with high fracture density might be either fracture zones or fault zones. If the slip tendency for the fractures present exceeds the coefficient of friction, the zone is considered as the fault zone, and slip on the fractures causes rotations of the azimuths (Paillet and Kim, 1987). From the observation of Figure 4.2, the presence of fault is usually accompanied by high fracture density, and a high fracture density zone has a corresponding breakout zone.

Considering that there are some inconsistencies of ATV log travel time data during the measuring,
it is very possible that the fitting result will be influenced because of it. Besides, any error in travel time will cause a significant difference in the result of shape fitting and the length of the axis when converting from time to distance. Therefore, the quality of ATV travel time data is essentially required during the elliptical fitting.
Chapter 6

Conclusion

The main conclusions in this thesis can be summarized as follows:

- The mean diameter in borehole CB1 calculated from ATV travel time is from 101 mm to 99 mm with depth increasing. The AVT data is reliable to calculate the actual borehole shape, as the diameter is quite close to the caliper result and have the similar variation trend.

- Following the same mechanism, we can extrapolate the breakout indication and use borehole shape ellipticity as the stress variation indicator along the whole borehole depth. The elliptical fitting gives the solution for boreholes without ideal elliptical shapes.

- Based on the method of direct least squares fitting of the ellipse, borehole shape at each depth interval can be fitted as an ideal ellipse to determine its azimuth of major axis. The fitted long major azimuth meets the azimuth of the breakout in borehole CB1, ST1, ST2, and MB8, which is at the range from 100° to 115° (HS). This match verified the reliability of the elliptical fitting method to determine stress orientation.

- The fitted azimuth range is from 100° to 150° (HS) in all tested boreholes at most parts of the depth. The abrupt variation in azimuth always comes with the presence of faults. In addition, relatively small azimuth variation is caused mainly by the high fracture density.

- RMS values to estimate the goodness of elliptical fitting in borehole CB1 is within the range between 0.0005 to 0.0015. The depth interval where values vary irregularly usually occurs at the breakout zone as the breakout shape is far from the ideal elliptical shape.

- The ratio between the fitted minor axis and major axis is within the range from 0.98 to 0.99. Zones with a ratio less than 0.93 can be treated as breakout zone or zones with washout or borehole failure occurrence. The estimation of the relative value for the maximum and minimum stress $S_{\text{max}}$ and $S_{\text{min}}$ can be made directly from the borehole shape ratio. The larger the ratio is, the less the difference between $S_{\text{max}}$ and $S_{\text{min}}$ is.

Borehole ellipticity fitting and determination of the azimuth of $S_{\text{min}}$ is a systematic process for now after this research and can be used in other boreholes to determine the orientation of stress in the
future. It can also be compared with the results from other stress tests to verify its accuracy. The method is only tested in the medium with relatively homogenous rock type, and more tests are worth to be done in the future to find out if it will also conclude a good result in a situation with complex rock types and less ideal rock properties. Because this method is simple to conduct and less time-consuming, which is a great improvement in stress determination without breakouts in a whole borehole length, further study in different kinds of environments is expected to expand its application field.

A comparison with DCDA (Diametrical Core Deformation Analysis) measuring the deformation of cores and indicates the in-situ stress from the length of the core’s axis, is expected in the future. This study is not only a test for these two methods to verify each other but helps to find out the relations between borehole deformation and core deformation, and the mechanism behind them.


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Appendix A

Appendix A.1 Ellipticity Analysis

clear
close all
clc

%% Original code by Rutger van Limborgh and corrected by Shihui Zhang
%% Modified and added by Jinqiu Chong

%% Default parameters
filename = '20200513_ATVSGAM_CB1_TT_HS_noncentralized.csv'; % Selecting traveltime dataset
z_min = 0; % Minimum depth of borehole (m)
z_max = 400; % Maximum depth of borehole (m)
dz = 0.4; % Interval distance (m)
ang = 1; % Angular resolution of traveltime data (deg)
v_a = 1500; % Acoustic speed through borehole fluid (m/s)
r_p = 0.0095; % Radius of the ATV probe

% Import data
TT_Data = table2array(readtable(filename,'HeaderLines',2));
TT_Data = TT_Data(TT_Data(:,2)>0,:);

% Setting looping parameters
z = (z_min:dz:z_max)'; % Creating array inbetween z_min and z_max with interval distance dz
\[ \theta = (0:1:(\text{size}(TT\_Data,2)-2))' \times \text{ang}; \] % Creating array containing angle corresponding to each column of data in TT\_Data

\[ \text{no}\_\theta = \text{size}(\theta,1); \] % nr. of columns of data corresponding to angles

\[ \text{no}\_dz = \text{numel}(z); \] % Counting total nr. of interval points in z

\[ \text{no}\_\text{Measurements} = \text{size}(TT\_Data,1); \] % Counting total nr. of measurement depths

\[ n = 1; \] % Setting initial value for iterations

%% Preparing data for ellipticity analysis

\[ \text{TopBottom} = \text{zeros}(\text{no}\_\text{dz},2); \]

\[ \text{for } i = 1:\text{no}\_\text{dz} \]

\[ \quad \text{Setting boundaries of the interval to analyse} \]

\[ \text{TopBottom}(i,1) = (z(i)-0.5*\text{dz}); \] % Upper interval boundary

\[ \text{TopBottom}(i,2) = (z(i)+0.5*\text{dz}); \] % Lower interval boundary

\[ \text{Intervals} = (TT\_Data(:,1) \geq \text{TopBottom}(i,1) \& TT\_Data(:,1) < \text{TopBottom}(i,2)); \] % Selecting measurement depths in interval

\[ \text{Index} = \text{find}(	ext{Intervals}); \]

\[ \text{Determining the amount of measurement depths in the interval} \]

\[ \text{no}\_\text{Index} = \text{size}(	ext{Index},1); \]

\[ \text{if } \text{no}\_\text{Index} > 0 \]

\[ \text{only intervals that contain data} \]

\[ \text{for } j = 1:\text{no}\_\text{Index} \]

\[ \quad \text{if } z(i) > 0 \] % Selecting only intervals that have a logical depth

\[ \quad \text{Interval\_Data\_TT\_depth}(n,1) = z(i); \] % Storing interval depths that will be analysed

\[ \text{end} \]

\[ \text{Interval\_Data\_TT\_values}(j,:) = TT\_Data(\text{Index}(j),2:end); \] % Storing all traveltime data for measurement depths present in interval
end

Average_Interval_Data_TT(n,:) = mean(Interval_Data_TT_values);

% Averaging traveltime data per interval

n = size(Average_Interval_Data_TT,1)+1;

% Calculating new looping value

else
end

end

%% Transferring travel time to borehole radius

Average_Interval_Data_r = ((Average_Interval_Data_TT * v_a * 10^-6)/2)+r_p;

% Transforming travel time data to borehole radius data using velocity

no_di = size(Average_Interval_Data_r,1);

% Calculating number of intervals for looping
Ellipticity_Data = zeros(no_di,6);

for i = 1:no_di

    Average_Interval_Data_r_use = Average_Interval_Data_r(i,:);

    % Selecting radius data for a specific interval
    XY_use = zeros(no_theta,2);
    for j = 1:no_theta
        XY_use(j,1) = sind(theta(j,1))*Average_Interval_Data_r_use(j);
        % Calculating x-value of data point
        XY_use(j,2) = cosd(theta(j,1))*Average_Interval_Data_r_use(j);
        % Calculating y-value of data point
    end

    % Fitting ellipse for set of x-y values
    A=EllipseDirectFit(XY_use); % Function EllipseDirectFit.m

    % Saving fitted ellipse coefficients
    a = A(1);
    b = A(2);

c = A(3);
d = A(4);
f = A(5);
g = A(6);

%% Calculating major and minor axis length
ax_1_use = 2*sqrt((2*((a*(f^2))+(c*(d^2))+(g*(b^2))-(2*b*d*f)-(a*c*g))/(((b^2)-(a*c))*(sqrt(((a-c)^2)+(4*b^2))-(a+c))))); %Calculating length of axis 1
ax_2_use = 2*sqrt((2*((a*(f^2))+(c*(d^2))+(g*(b^2))-(2*b*d*f)-(a*c*g))/(((b^2)-(a*c))*(-sqrt(((a-c)^2)+(4*b^2))-(a+c))))); %Calculating length of axis 2

%Determining ellipticity ratio
if ax_1_use > ax_2_use
    ax_ratio_use = ax_2_use/ax_1_use;
else
    ax_ratio_use = ax_1_use/ax_2_use;
end

%% Calculating deviation from 90 degrees of longest axis (Corrected by Shihuai Zhang)

% Option 1: Calculating deviation angle from horizontal; if I use this one I obtain 1 string of data points in the scatter plot, but it seems to be bounded by 45* and 135* (so 90+/−45).
% phi_use = 0.5*acotd((a-c)/(2*b));

%Option 2; Calculating deviation angle from horizontal; this one should be mathematically correct but results in 2 strings of datapoints in the scatter plot.

if abs(a)<abs(c)
    phi_use = 0.5*acotd((a-c)/(2*b));
else
    phi_use = 90 + (0.5*acotd((a-c)/(2*b)));
end
if b==0
    phi_use=0;
%Both option 1 and 2 can be plugged in to the rest of the code from here

%caculating orientation of long axis

ax_orientation = 90 - phi_use;

if ax_orientation < 0
    ax_orientation_use = 180 + ax_orientation;
else
    ax_orientation_use = ax_orientation;
end

% if ax_orientation_use<0
%    ax_orientation_use=abs(ax_orientation_use);
% end

% Recalculating values to have all data in between 0*-180*. For example, 350* -- 170*.
% if ax_orientation < 0
%    ax_orientation_use = 180 + ax_orientation;
% else
%    ax_orientation_use = ax_orientation;
% end

% Step used for checking data. Not needed for results
if a<c
    correct = 1;
else
    correct = 0;
end

% Storing Ellipticity data
Ellipticity_Data(i,1) = Interval_Data_TT_depth(i,1);
Ellipticity_Data(i,2) = ax_orientation_use;
Ellipticity_Data(i,3) = ax_orientation_use+180;
Ellipticity_Data(i,4) = ax_ratio_use;
Ellipticity_Data(i,5) = phi_use;
Ellipticity_Data(i,6) = correct;
if Ellipticity_Data(i,3) >360
    Ellipticity_Data(i,3)=Ellipticity_Data(i,3)-360;
end

% Determining long and short axis
if ax_1_use < ax_2_use
    bx = ax_1_use;
    axlong = ax_2_use;
else
    axlong = ax_1_use;
    bx = ax_2_use;
end

%% Calculating RMS value of fitted result
x = XY_use(:,1);
y = XY_use(:,2);
ind = 1:length(x');
deg = (0:ang:359)';  % Involved angles for each depth interval

% Center of ellipse
a1 = (c*d - b*f) / (b^2 - a*c);
b1 = (a*f - b*d) / (b^2 - a*c);
z = [a1/2; b1/2];

r = sqrt((y(ind) - z(2)).^2 + (x(ind) - z(1)).^2);  % Distance between real point to fitted ellipse center
th = ax_orientation_use - deg;  % The angle between the connection of fitted point to ellipse center, and the long axis of the ellipse
r_fit = eliseR(axlong/2, bx/2, th);  % Distance between fitted point to fitted ellipse center
dR = (r_fit - r);  % Difference between real and fitted distance to the center

% RMS of distance difference
fit_2(i) = dR(1, 1)^2;
for jj = 2:ang:360
    fit_2(i) = dR(jj, 1)^2 + fit_2(i);
end
fit(i) = sqrt(fit_2(i)/360);
%% Determining fitted ellipse shape at specific interval
if i<377&&i>375

alpha1=phi_use/180*pi;%Converting to radian
p=[cos(alpha1) -sin(alpha1);sin(alpha1) cos(alpha1)];%Converting matrix

% Coordinates before rotation
theta1=linspace(0,2*pi,50000);
xor=(axlong/2)*cos(theta1);
yor=(bx/2)*sin(theta1);
% Coordinates after rotation
x1=p(1,:)*[xor;yor];
y1=p(2,:)*[xor;yor];
% Coordinates after translation
x2=x1+z(1);
y2=y1+z(2);
% Coordinates of long axis
x_lx=linspace((-axlong/2)*cos(alpha1),(axlong/2)*cos(alpha1),100);
y_lx=x_lx*tan(alpha1);

%% Plotting fitted ellpise shape
figure() % Plotting ellpise shape
scatter(XY_use(:,1),XY_use(:,2),5,'filled') %Plotting real borehole shape
axis equal
title('CB1 Borehole shape at borehole depth 130m')
xlabel('x [m]')
ylabel('y [m]')
hold on
plot(x2,y2,'k-','linewidth',1.5) %Plotting fitted ellipse
hold on
plot(x_lx+z(1),y_lx+z(2)) %Plotting fitted long axis
legend('real borehole shape','ellipticity fit','long axis azimuth is ' ,num2str(ax_orientation_use),'Location','best')
title('CB1 Borehole shape at borehole depth 130m')
hold off

figure() % Plotting ellipse radius
scatter(deg,r*10^3,10,'filled');
legend('real radius','fitted radius')
title('CB1 Borehole radius at depth 130m')
xlabel('Deg [°]')
ylabel('Radius [mm]')
set(gca,'XTick',0:30:360);

end
end

%% Loading Breakout data

load 'Breakout_Interval_Data.csv'

%% Plotting long axis azimuth and breakouts

figure()
scatter(Ellipticity_Data(:,2),Ellipticity_Data(:,1),5,'b','filled','d');
hold on
scatter(Ellipticity_Data(:,3),Ellipticity_Data(:,1),5,'filled','d');
hold on
scatter(Breakout_Interval_Data(:,2),Breakout_Interval_Data(:,1),15,'o','filled','d');
hold off
legend('fitted azimuth','fitted azimuth+180','Breakout azimuth','Location','southwest')
title({'CB1 azimuth of fitted ellipse long axis and breakout azimuth'})
ax = gca;
ax.XTick = 0:90:360;
ax.XLim = [0 360];
ax.YLim = [0 400];
ax.YDir = 'reverse';
grid on
xlabel('azimuth (deg)');
ylabel('depth (m)');

%% Plotting RMS variations
fit=fit';
figure()
scatter(fit,Ellipticity_Data(:,1),5,'r','filled');
ax = gca;
ax.XAxisLocation = 'bottom';
ax.YAxisLocation = 'left';
ax.YLim = [0 400];
ax.YDir = 'reverse';
xlabel('RMS');
ylabel('depth (m)');
grid on
hold off
title({'CB1 ellipticity fit evaluation'})

%% Plotting ellipticity ratio

figure()
scatter(Ellipticity_Data(:,4),Ellipticity_Data(:,1),10,'b','filled','d')
title({'CB1 fitted ellipticity ratio (short axis/long axis)'})
xlabel('ratio [-]')
ylabel('Borehole Measured Depth [m]')
ax = gca;
ax.XLim = [0.85 1];
ax.YLim = [0 400];
ax.XAxisLocation = 'top';
ax.YDir = 'reverse';
ax.XGrid = 'on';
ax.YGrid = 'on';
ax.GridColor = 'k';
ax.GridAlpha = 0.5;
ax.XAxisLocation = 'bottom';

%% Borehole shape view
Radius_test = ((TT_Data(:,2:end) * v_a * 10^-6)/2)+r_p;
figure()
for ii = 15000:500:length(TT_Data)
    plot(sind(theta).*Radius_test(ii,:),cosd(theta).*Radius_test(ii,:),'k.'
hold on
end
axis equal
title('CB1 Borehole shape from high side view')
xlabel('x [mm]')
ylabel('y [mm]')
grid on

figure()
for ii = 1:500:length(TT_Data)
    a=rand(1,3);
    plot3(sind(theta).*Radius_test(ii,:)'*10^3,cosd(theta).*Radius_test(ii,:)'*10^3,TT_Data(ii,1)*ones(360,1), 'k.', 'color', a)
    hold on
end
set(gca, 'ZDir', 'reverse')
axis tight
grid on
title('CB1 Borehole shape from side view')
xlabel('x [mm]')
ylabel('y [mm]')

%% Plotting diameter variations from raw travel time data
figure()
subplot(1,3,1) %Plotting travel time variations
Average_Interval_Data_TT_mean=mean(Average_Interval_Data_TT,2);
Average_Interval_Data_TT_max=max(Average_Interval_Data_TT,[],2);
Average_Interval_Data_TT_min=min(Average_Interval_Data_TT,[],2);
scatter(Average_Interval_Data_TT_mean,Ellipticity_Data(:,1),5,'filled')
hold on
scatter(Average_Interval_Data_TT_max,Ellipticity_Data(:,1),5,'filled')
hold on
scatter(Average_Interval_Data_TT_min,Ellipticity_Data(:,1),5,'filled')
hold off
ax = gca;
ax.XGrid = 'on';
ax.YGrid = 'on';
ax.GridColor = 'k';
ax.GridAlpha = 0.5;
ax.YDir = 'reverse';
title({'CB1 Travel Time'})
xlabel('Travel Time [us]')
ylabel('Borehole Measured Depth [m]')
legend('Mean Travel Time','Maximum Travel Time','Minimum Travel Time','Location','southeast')

% Converting travel time to distance
Average_Interval_Data_r_mean=r_p+(Average_Interval_Data_TT_mean * v_a * 10^-6)/2;
Average_Interval_Data_r_max=r_p+(Average_Interval_Data_TT_max * v_a * 10^-6)/2;
Average_Interval_Data_r_min=r_p+(Average_Interval_Data_TT_min * v_a * 10^-6)/2;

subplot(1,3,2) %Plotting radius variations
scatter(Average_Interval_Data_r_mean*10^3,Ellipticity_Data(:,1),5,'filled')
hold on
scatter(Average_Interval_Data_r_max*10^3,Ellipticity_Data(:,1),5,'filled')
hold on
scatter(Average_Interval_Data_r_min*10^3,Ellipticity_Data(:,1),5,'filled')
hold off
ax = gca;
ax.XGrid = 'on';
ax.YGrid = 'on';
ax.GridColor = 'k';
ax.GridAlpha = 0.5;
ax.YDir = 'reverse';
title({'CB1 Radius'})
xlabel('Radius [mm]')
ylabel('Borehole Measured Depth [m]')
legend({'Mean Radius','Maximum Radius','Minimum Radius','Location','southeast'})

% diameter=2*mean(r_average)

for i=1:180

diameter(:,i)=Average_Interval_Data_r(:,i)+Average_Interval_Data_r(:,i+180);
end
diameter_mean=mean(diameter,2);
diameter_max=max(diameter,[],2);
diameter_min=min(diameter,[],2);

subplot(1,3,3) % Plotting diameter variations
scatter(diameter_mean*10^3,Ellipticity_Data(:,1),5,'filled')
hold on
scatter(diameter_max*10^3,Ellipticity_Data(:,1),5,'filled')
hold on
scatter(diameter_min*10^3,Ellipticity_Data(:,1),5,'filled')
hold off
ax = gca;
ax.XGrid = 'on';
ax.YGrid = 'on';
ax.GridColor = 'k';
ax.GridAlpha = 0.5;
ax.YDir = 'reverse';
title({'CB1 Diameter'})
xlabel('Diameter [mm]')
ylabel('Borehole Measured Depth [m]')
legend('Mean Diameter','Maximum Diameter','Minimum Diameter','Location','southeast')

Appendix A.2 Ellipse fitting

function a = EllipseDirectFit(XY)

x = XY(:,1);
y = XY(:,2);
D1 = [x .^ 2, x .* y, y .^ 2]; % quadratic part of the design matrix
D2 = [x, y, ones(size(x))];  % linear part of the design matrix
S1 = D1' * D1;             % quadratic part of the scatter matrix
S2 = D1' * D2;             % combined part of the scatter matrix
S3 = D2' * D2;             % linear part of the scatter matrix
T = - inv(S3) * S2';       % for getting a2 from a1
M = S1 + S2 * T;           % reduced scatter matrix
M = [M(3, :) ./ 2; - M(2, :) ; M(1, :) ./ 2]; % premultiply by inv(C1)
[evec, eval] = eig(M);  % solve eigensystem
cond = 4 * evec(1, :) .* evec(3, :) - evec(2, :) .^ 2; % evaluate a?Ca
a1 = evec(:, find(cond > 0)); % eigenvector for min. pos. eigenvalue
a = [a1; T * a1];

Appendix A.3 Distance of a point on an ellipse to its center

function R = eliseR(a, b, d)
% a long axis; b short axis; d angle relative to the long axis
R = a*b./sqrt((a.*sind(d)).^2+(b.*cosd(d)).^2);
end
Appendix A.4 Fracture density

clear
close all
clc

load 'CB1_fracture.csv'
depth=(0:1:300);
figure()
subplot(1,4,1)
% plot(MB8_fracture(:,1),depth);
h=barh(depth,CB1_fracture(:,1));
title({'CB1 Fracture density'})
xlabel('Fracture density [m^{-1}]')
ylabel('Borehole Measured Depth [m]')
grid on
ax = gca;
ax.YLim = [0 400];
ax.YDir = 'reverse';
Appendix B
Results for boreholes not included in main text
Figure B.1: Azimuth (HS) of fitted ellipse’s long axis of borehole CB2, CB3, MB4, MB5, MB7, and Welltec. Borehole CB2, CB3, and MB4 are with breakout azimuth data.
Figure B.2: Ratio of fitted ellipse of borehole CB2, CB3, MB4, MB5, MB7, and Welltec.
Figure B.3: Fit evaluation of borehole CB2, CB3, MB4, MB5, MB7, MB8, ST1, ST2 and Welltec