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# A Physics-based Reduction with Monitoring Data Assimilation for Adaptive Representations in Structural Systems

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**ABSTRACT**
Digital twin representations have become an indispensable tool for delivering data-informed virtualizations of operating systems, especially in structural health monitoring applications. In this context, challenges arise when the response often shifts beyond regular operating conditions due to extreme events such as earthquakes or structural damage. Our work proposes a reduced order modeling for adaptive digital twins, for systems undergoing damage, condition deterioration or experiencing stochastic excitation. Our approach initiates by featuring a projection-based Reduced Order Model (ROM), relying on Proper Orthogonal Decomposition (POD) and local subspaces to form a low-cost surrogate of the parametrized high fidelity system that retains a physical connotation. However, extreme events induce loading conditions and model states that challenge the accuracy of such representations. To this end, we propose adopting the derived ROM as a forward simulator and adapt the projection basis on-the-fly during operation via a Gaussian Processes Regressor (GPR) scheme. During operation, the ROM framework receives response monitoring information from a sparse number of nodes. It employs a suitable condition indicator to highlight the potential low precision of the initial surrogate. Subsequently, the GPR-based scheme utilizes the monitoring input to reconstruct the current deformed configuration of the whole system in an online manner. In turn, this approximation serves as a damaged mode that enriches the projection-based ROM and enables online adaptivity. This coupling yields a ROM equipped with critical features for health monitoring applications such as (near) real-time basis refinement, signaling potentially irreversible consequences, and estimation of the uncertainty in the enrichment mode and the adapted ROM prediction.

**Keywords**: adaptive GPR-ROM, monitoring data assimilation

## INTRODUCTION

Dynamical systems may experience complex nonlinear phenomena, which extend beyond behavior that is expected during operational conditions. For tackling loading and response regimes that correspond to unusual or extreme events, data assimilation techniques, driven by availability of monitoring observations, could provide a form of performance adaptivity, either on-the-fly or afterward.

This notion of adaptivity in a ROM context has already been discussed, for example, in [1, 2], where suitable error estimators are utilized in the input parameter space to assemble an adaptive basis construction scheme. Performance enrichment and basis update techniques during the online phase have also been suggested in [3] via low-rank matrix updates, in [4, 5] by means of full-order simulations, or in [6, 7] employing local refinements and vector sieving strategies.

Our approach proposes an alternative methodology that relies on online monitoring information from sparse structural degrees of freedom, as opposed to assuming knowledge over the entire response vector. Our work features ROMs as efficient surrogates and attempts to address the aforementioned challenge of adaptive response prediction under extreme events like structural damage or state deterioration. In essence, we utilize an initially deployed ROM and then adopt a Gaussian Processes Regressor (GPR) to adapt the projection basis during operation. This fusion allows for an adaptive ROM through an online projection basis refinement, while estimating the uncertainty involved.

## FRAMEWORK FORMULATION

As a first step, to derive the adaptive framework, our work follows the approach described in [8] to assemble an initial ROM. In short, we formulate a low-order system equivalent to the full-order model (FOM) through a Galerkin projection and Proper Orthogonal Decomposition (POD).

The constructed ROM is equipped with an indicator representing an alarm. The condition indicator is constructed on the basis of the noisy response of a few selected system degrees of freedom (dofs). The discrepancy between the measurement and the respective ROM prediction at the selected dofs is utilized to indirectly evaluate the precision of the ROM approximation. In our work, the Mahalanobis distance is employed as a comparative measure, applied on the residuals between the ROM prediction and the actual response on the monitored dofs. The alarm threshold is defined exploiting the respective chi-distribution statistics with a suitable significance level for each case.

Based on previous work in [9], artificial measurement data at the chosen sensor locations are produced and the corresponding vector $\mathbf{d}_k \in \mathrm{R}^{n_d}$ from the selected dofs is polluted with Gaussian white noise to generate the noisy output $\tilde{\mathbf{d}}_k \in \mathrm{R}^{n_d}$ at every timestep $k$ following:

$$\tilde{\mathbf{d}}_k = \mathbf{d}_k + \delta \sigma_d \mathbf{r}_k \tag{1}$$

where $n_d$ is the number of measured quantities, $\delta$ denotes the noise level, $\sigma_d \in \mathrm{R}^{n_d \times n_d}$ is a diagonal matrix containing the standard deviations of the signals and $\mathbf{r}_k \in \mathrm{R}^{n_d}$ is a vector of random values drawn independently from the standard normal distribution. This setup aims to account for modelling and measurement noise in an actual system.

In an alarm event, the Gaussian Processes Regression (GPR) receives the noisy residuals as input and estimates the residual response on the entire physical space. In turn, the output is utilized as an additive correction to the ROM prediction. The resulting approximation represents an instance of a damaged configuration of the system, equivalent to some sort of POD mode, and functions as a projection basis enrichment that captures the system's behavior during a damage event.

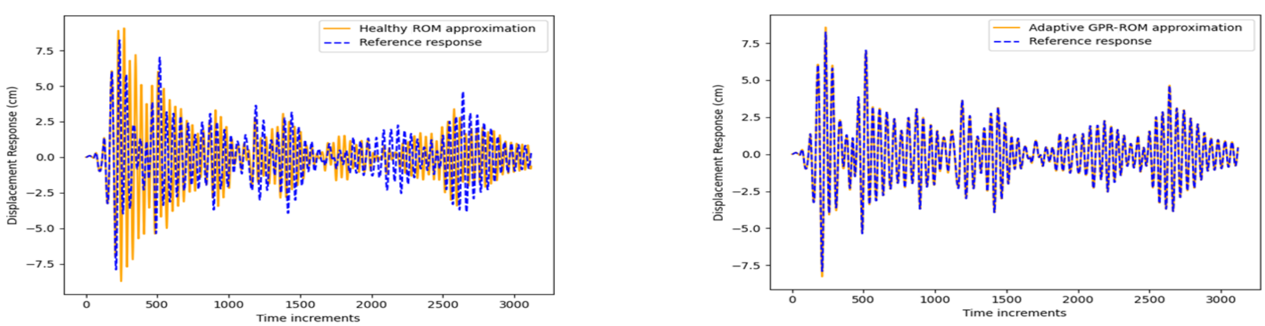
The ROM subspace is thus updated on-the-fly delivering an adaptive GPR-ROM. The GPR scheme is trained similar to the ROM, relying on a Radial Basis Function kernel. Additionally, the training leverages local correlations between coordinates representing physical response correlations between neighboring degrees of freedom. This implies that the response in each coordinate is approximated, exploiting information from the most suitable monitoring channels.

**ANALYSIS**

The adaptive nature of the proposed ROM is validated using the following proof of concept case studies: The first is a $10m$ steel cantilever beam, whereas the second is a two story plane frame out of steel with a story height of $4m$ and a width of $10m$. Both structures are discretized using beam elements and excited using Gaussian noise. In all the couplings of the frame and in the boundary node of the cantilever beam Bouc-Wen springs are assembled, enforcing a hysteretic behavior based on the following:

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{x} + (1-\alpha) k \mathbf{z} \qquad \dot{\mathbf{z}} = \frac{A\dot{\mathbf{x}} - (1.0 + \delta_\nu \int_0^t \mathbf{z}\dot{\mathbf{x}} dt)(\beta |\dot{\mathbf{x}}| \mathbf{z} |\mathbf{z}|^{w-1} - \gamma \dot{\mathbf{x}} |\mathbf{z}|^w)}{1.0 + \delta_\eta \int_0^t \mathbf{z}\dot{\mathbf{x}} dt}$$

where $\mathbf{R}$ denotes the restoring forces, $\mathbf{x}$ the displacements and $\mathbf{z}$ represents the hysteretic term. The rest are characteristic parameters of the Bouc-Wen spring, controlling the shape of the response curve. A detailed description can be found in [8].

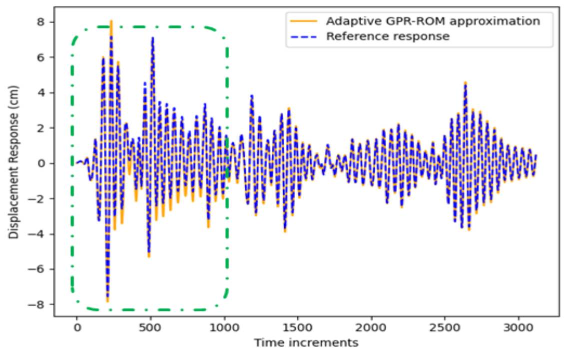



**(a)** Healthy ROM approximation **(b)** Adaptive GPR-ROM approximation

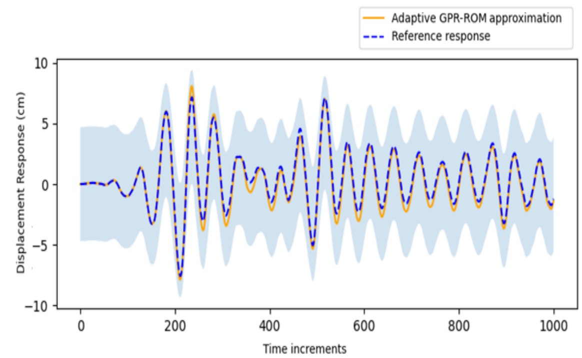
**Fig. 1** Performance of the adaptive GPR-ROM in capturing the response under a damage scenario.

Regarding the cantilever beam, a linear ROM is initially deployed. Subsequently, the Bouc-Wen spring is activated during operation representing damage. This process is controlled by varying parameters $a$ and $k$. Figure 1b highlights the ability of the GPR-ROM to capture the response of the damaged structure by performing on the fly adaptations of the projection basis. On the contrary, the initially deployed healthy ROM fails to capture the damage effect in Figure 1a.

For the plane frame, the initial ROM is trained based on the nonlinear behavior of the structure. During operation, stiffness degradation and strength deterioration effects are activated by tweaking $\delta h$ and $\delta v$. In this more advanced case study, the GPR-ROM provides a sufficiently accurate approximation as depicted in Figure 2a. Additionally, the confidence bounds of the prediction in Figure 2b offer an initial uncertainty estimation and function as envelope curves for the actual response.




**(a)** Adaptive GPR-ROM approximation




**(b)** Magnified view and confidence bounds of the prediction

**Fig. 2** Performance of the adaptive ROM in capturing the response under a condition deterioration scenario.

## CONCLUSIONS

In our work, we derive an adaptive low-order representation with the ability to update its projection basis on the fly to address challenges of response estimation during operational conditions that extend beyond expected behavior. This is achieved utilizing a condition indicator and a Gaussian Processes Regressor that reconstructs the residual response in all physical nodes of the system by relying on sparse monitoring measurements. The output approximation is subsequently used for basis enrichment. This fusion allows for an adaptive ROM, able to perform basis refinement, signaling of potentially irreversible consequences, and uncertainty estimation features critical for health monitoring. Potential extensions involve utilizing the ROM as a forward simulator assembled in an inverse setup that aims to estimate features of the system's state [10].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