Optically interrogated MEMS pressure sensor array

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Abstract

A novel measurement technique for recording wall static pressure distributions in wind tunnel applications is developed and its feasibility verified. The system is designed with the objective to reduce the complexity of wind tunnel investigations while providing the high measurement accuracy (<20 Pa) required in quantitative pressure measurement surveys.

An array of passive silicon plate micro-resonators is embedded in a thin, laser structured carrier foil which is directly mounted on the surface of a model. The pressure distributions are recorded wirelessly from a distance larger than 2 m by measuring the sensors’ resonance frequency using optical interferometry. Dependent on the quasi-static deflection of the diaphragm caused by a pressure load the resonance frequency varies with an average pressure sensitivity of 3 Hz/Pa within the lower ultrasound frequency range (<150 kHz). A smart-pixel CMOS camera, narrow band acoustic noise excitation and a specific sensor surface topography allow for the interrogation of a large number of sensors in parallel without the need for alignment between sensor and detector.

Experimental tests reveal more than one order of magnitude lower overall damping (Q-factor between $10^2$ and $10^3$) of a plate resonator if using higher vibrational modes instead of the fundamental one. The mode-specific receptivity characteristic of a plate resonator to forced acoustic excitation limits the employment of higher modes due to a potential cross coupling between neighboring modes. The second symmetric mode of a clamped plate ((3,1)-mode) shows low damping and no cross coupling and is therefore identified as the best suited vibrational mode for the present application.

Fluid-structure interactions between the resonant element and the pneumatic system of the differential pressure sensor array were investigated using theoretical models and measurements. A temperature cross-sensitivity of approximately 0.1 %/K is mainly attributed to sensor packaging and might be reduced by a proper sensor mount. The newly developed system is characterized by a robust sensing performance but cross-sensitivities of the resonator to flow parameters such as the pressure gradient and wall shear stress impose a non-negligible, systematic measurement error and suggest an adaptation of the sensor design.
Kurzfassung

In der vorliegenden Arbeit wurde ein Messsystem zur Erfassung der räumlichen Verteilung des statischen Wanddruckes an Windkanalmodellen entwickelt und experimentell validiert. Das Ziel dabei war ein System aufzubauen, das einfacher am Modell zu implementieren und somit kostengünstiger einzusetzen ist, als ein Druckmesssystem basierend auf herkömmlichen Wanddruckbohrungen. Die Druckbohrungen, verbunden mit einem präzisen Druckumformer im Modellinnern, liefern eine sehr hohe Messgenauigkeit, die es mit dem vorliegenden System möglichst zu erreichen gilt (geforderte Messgenauigkeit: 20 Pa bei 10 kPa Messbereich).


Aus der experimentellen Untersuchung des Schwingverhaltens eines Membranschwingers bei atmosphärischen Druckbedingungen ging hervor, dass die höheren Schwingmoden um mehr als eine Grössenordnung weniger stark gedämpft sind (Gütetfaktor \( Q = 10^2 \) – \( 10^3 \)) als ein Resonator, der in der Grundmode betrieben wird. Ähnlich wie das Dämpfungsverhalten eines Plattenresonators wird auch dessen Empfangsvermögen (Empfindlichkeit) für akustische Strahlung durch die Eigenmode bestimmt. Diese Eigenschaft verhindert, dass die meisten höheren Moden einer quadratischen oder runden, fest eingespannten Membran nicht oder nur in einem eingeschränkten Druckbereich akustisch in Resonanz versetzt werden können. Einzig
die zweite symmetrische Schwingungsmoden ((3,1)-Mode) ist ausreichend schwach gedämpft und kann im geforderten Druckbereich mit akustischem Rauschen angeregt werden. Der Einfluss der gasgefüllten Referenzkavität und der Mikrokanäle (Differenzdrucksensor) auf das Schwingverhalten des Plattenschwingers wurde theoretisch sowie experimentell untersucht.

Obwohl eine Messunsicherheit kleiner als 20 Pa erreicht wird, scheinen Strömungsgrößen wie der Druckgradient und die Wandschubspannung einen systematischen Messfehler zu verursachen, der nicht korrigiert werden kann und der somit eine Anpassung der Sensorgeometrie verlangt. Die Temperaturempfindlichkeit beträgt etwa 0.1 %/K und resultiert hauptsächlich aus der Verbindung zwischen Sensor und Trägerfolie, d.h. aus den unterschiedlichen Materialeigenschaften. Mit einer optimierten Sensorbefestigung auf dem Kaptonträger sollte dieser Wert um bis zu einem Faktor 3 reduziert werden können.
1. Introduction

During aerodynamic and mechanical load performance surveys in wind tunnel testing it is often desirable to obtain the surface pressure distribution on a specific component of the investigated model with high spatial resolution, good measurement accuracy and low uncertainty. Despite its importance, only the pressure tap method tends to be used in commercial and industrial wind tunnel facilities to measure the static pressure.

This technique is based on conventional pressure transducers that can be installed outside or inside the model and that are linked by a tube with the measurement location (small orifice) on the model surface. The main drawbacks of this method are the high installation costs and the poor spatial resolution which are usually both constrained by the required mechanical

![Figure 1.1.: Complexity of instrumentation inside the wind tunnel model of the Advanced Hawkeye equipped with more than 300 pressure taps (German-Dutch Wind Tunnels DNW, [1])](image)

and pneumatic interfacing in the test model. A large bundle of tubing which interconnects the model and the external transducer unit may interfere with the flow and affect the force balance data [2]. A transducer unit (sensor and electrical pressure scanner) which is installed inside the model simplifies the interfacing between model and the environment but sufficient internal space within the model is required and the complex installation remains (cf. Fig. 1.1). The integration of pressure taps on small models or thin structures such as flaps is therefore difficult or even unfeasible. Once the
system is installed, the technique fulfills the highest demands regarding the measurement sensitivity and uncertainty.

Pressure sensitive paints (PSP) can record pressure distributions on wind tunnel models by non-intrusive and optical means. They offer a high spatial resolution and a simpler installation compared to the pressure tap technique and seemed to be a promising technique for industrial applications. Unfortunately, PSPs are regarded as an absolute pressure measurement technique\(^1\) which inherently leads to a poor pressure sensitivity if applied at atmospheric pressure conditions. Furthermore, the large temperature cross-sensitivity of PSPs requires a time consuming calibration procedure [4]. Based on those reasons PSPs have not yet been fully established in the commercial field and are mainly applied for tests in high speed flows where higher pressure values arise or at low ambient pressure conditions (e.g. micro fluidics) [3].

The objective of the present work is to develop a pressure measurement system which fulfills the stringent requirements on measurement performance for quantitative pressure surveys in low speed wind tunnel testing. At the same time the installation and handling of the device has to be made as simple as possible. The requirements for the measurement performance are summarized in Table 1.1.

<table>
<thead>
<tr>
<th>Table 1.1.: Required measurement performance</th>
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<tbody>
<tr>
<td>Temporal resolution</td>
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<tr>
<td>Spatial resolution</td>
</tr>
<tr>
<td>Measurement range</td>
</tr>
<tr>
<td>High accuracy (high sensitivity, low measurement uncertainty)</td>
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</table>

Based on a concept study [5], where different techniques for static pressure measurement were compared and assessed based on the requirements in Table 1.1, two basic conclusions could be identified:

1. A simple installation of the device requires a sensor unit which is mounted directly on the surface of the model and is interrogated by wireless means. Hence, a time consuming installation of the pressure

\(^1\)Fluorescence light emission depends inversely proportional on the oxygen concentration, i.e. local density [3]
transducer itself and the routing of a bulky wiring harness within the model may be avoided.

2. The need for a high measurement performance in low-speed wind tunnel testing requires a pressure sensing device based on a differential (or gauged) sensing principle and a mechanical flexing element (Fig. 1.2).

Requirement 2 is mainly motivated by the requirement 1 which imposes constraints regarding the maximum but also minimum sensor size. The sensor’s miniaturization is mainly restricted by the measurement sensitivity achievable with a specific wireless pressure sensing device. The maximum sensor size is defined by the specified spatial resolution and by the necessity to preserve the shape of the original model. An upper size limit restricts the sensor’s sensitivity\(^2\) and implies that differential instead of absolute pressure sensing is applied. Commercially available electronic absolute pressure sensors with a diaphragm size smaller than 2 mm exhibit a measurement accuracy larger than 100 Pa and hence are not accurate enough for the present application (cf. Measurement Specialties, Inc.). It is worth mentioning that sensitivity restrictions imposed by the absolute sensing principle are also stringent for pressure sensitive paints.

Pressure sensitive foils without a mechanical flexing element such as a diaphragm are usually based on a polymer material which allows for a piezoresistive (e.g. Tekscan), piezoelectric (e.g. Measurement Specialties) or capacitive (e.g. Pressure Profile Systems) conversion of the applied pressure to an electrical signal. They usually exhibit a high measurement uncertainty caused by a high temperature cross-sensitivity, high hysteresis or aging effects and are mainly applied for qualitative pressure sensing tasks [6]. Piezoelectric PVDF foils are highly sensitive but are only suitable for dynamic pressure measurements where also their inherent high temperature sensitivity plays a minor role [7]. Pressure sensing based on optical fibres equipped with fiber Bragg gratings as a sensing element and embedded in a thin foil as a carrier are mainly suited for high pressure sensing applications [8].

As stipulated by requirement 1 the pressure sensor unit applied in the present work is based on an array of MEMS based, clamped diaphragms with a size smaller than 2 mm and a thickness of several micrometers. The sensor dies are embedded in a thin foil carrier (thickness approx. 0.3 mm) and mounted readily on the model surface. The foil includes micro-channels which interconnect the individual sensor cavities and allow for differential pressure sensing. In order to implement the required wireless interrogation

\(^2\)Assuming that the sensitivity increases with an increasing sensing area.
1. Introduction

Figure 1.2.: Operating principles of common pressure transducers (modified from [9]): Pressure is measured with respect to vacuum (absolute), to the ambient pressure (gauged) and to an arbitrary reference pressure (differential)

of the sensor array two different techniques are conceivable to permit the pressure readout across a distance of about two meters.

The first technique is based on the interrogation of the sensor array by electromagnetic waves within the microwave or radio frequency (RF) range. The power which is required for sensing or signal processing on the sensor side could be delivered by RF as well. For pressure readout (i.e. diaphragm displacement) a simple technique based on the propagation of surface acoustic waves (SAW) in the diaphragm material is conceivable. The SAW are generated with a metallic pattern tagged to the surface and connected to an alternating voltage source and sensed again by a similar pattern which transforms the SAW into an electrical response signal. The transmitting and receiving pattern which are separated by a certain distance forms the so called SAW-delay line which is modulated by the deflected diaphragm.

SAW-delay lines have the advantage of being compatible with wireless sensing and a power supply by RF waves [10]. They can also be combined with typical capacitive or resistive pressure sensing devices [11]. Unfortunately, the transmitting power required to interrogate the sensor unit over a distance larger than two meters would exceed the legal regulations (BUWAL). Furthermore, the potential for electromagnetic interference with other electronic devices appears to make the RF-interrogation technique unsuitable for an application inside a wind tunnel.

The second interrogation technique is based on an optical, parallel (imaging) read out of the individual sensing elements by interferometric means. Hence, no electromagnetic interference should arise, but a visual contact between sensor and interrogation unit is required. The main challenge here is to be able to image the tiny sensor dies distributed on a curved surface from the aforementioned working distance. The expected small interrogation signal has to be detected within a bright background. As a measure for the applied pressure the resonance frequency of a clamped diaphragm instead of a direct measurement of its quasi-static displacement is used. The resonance frequency of a mechanical structure is related to its present
The vibration behavior of a MEMS plate resonator operating at atmospheric pressure conditions and at frequencies above 50 kHz suffers from high damping losses caused by acoustic radiation. By considering higher vibration modes this type of damping can be considerably reduced and thus the sensing behavior improved.

A resonant pressure sensor has to be brought into vibration using a suitable excitation source. Similar requirements as defined for the interrogation system are also applied for the evaluation of a suitable excitation unit (i.e. remote, full-field and wireless). Only two excitation types appear to satisfy these criteria. These are an electrical excitation based on different energy conversion mechanisms (e.g. capacitive, piezo-resistive or electric, SAW, etc.) in combination with a wireless power supply by RF waves and sound excitation using an ultrasound transducer (UT) as excitation source.

The main drawback of the first principle is again the associated electromagnetic interference. Furthermore, the implementation of a suitable actuation mechanism within the sensor unit would lead to a more complex sensor design and fabrication. Sound excitation does not interfere with other measurement devices, is easy to apply and also to implement. The main drawback is the required access for sound waves to the resonant structure and thus the need for a direct, i.e. visual contact between sensor and UT. Problematic are also the high propagation losses of sound in air and a high directivity - primarily at higher operating frequencies - imposing the
1. Introduction

use of an array of several high power UTs in order to allow for a full-field excitation.

The optical interrogation method in combination with a pressure measurement principle based on a resonator and acoustic sound excitation was evaluated as the most promising concept, mainly because of the high potential for electromagnetic interference with RF-interrogation or excitation.

In Chapter 2 the measurement system is introduced, a closer look on the excitation and interrogation unit is given and the measurement and calibration procedure is described. The major part of the development work was dedicated to the sensor unit which had to be designed and fabricated. In Chapter 3 the design considerations and tools are discussed which were used to define a suitable sensor design. The fabrication of the sensor unit is discussed in Chapter 4. The sensing behavior and performance of the newly fabricated sensing elements was verified in a pressure and temperature controlled test cell. The corresponding measurement setup, procedure and results are discussed in Chapter 5. As a next step of the verification process of the sensor unit tests were performed in a real flow environment (i.e. in a wind tunnel) using a point measurement device as interrogation system. Chapter 6 describes the experimental setup and summarizes the obtained findings. Feasibility tests using the full-field interferometer system for the read-out of the resonant sensors are treated in Chapter 7. Chapter 8, finally summarizes the most important findings and gives recommendations and proposals for future work.
2. System description

The pressure sensing system is based on the optical interrogation of an array of passive MEMS pressure sensors mounted on the surface of a wind tunnel model (cf. Fig. 2.1). A thin, clamped silicon diaphragm serves as a resonant pressure sensing element. In order to allow for differential pressure sensing operation and thus highest sensitivity the individual sensors are embedded in a 0.3 mm thick Kapton foil (cf. Fig. 2.2). A pneumatic tubing system (microchannels) integrated in the foil interconnects the individual cavities beneath the diaphragm and enables the access to a reference pressure source. Irrespective of the number of integrated sensing elements only a single tube connects the foil with the reference source and facilitates the installation of the system. In case of small-scale models, where the foil could alter the flow characteristics, the sensor unit may be recessed in a milled slot in the model surface.

![Figure 2.1: Measurement setup including the remote imaging interferometer and the ultrasound transducer](image)

Using a fast and precise pressure controller as reference source, the pressure in the sensor’s cavity (back pressure) is set to a specific value within a range between 10 and 100 mbar below the ambient value and is kept constant during measurement. The initial loading of the diaphragm is essential to enable signed pressure measurement whereas the precise control of the back pressure prevents a potential degradation of the measurement perfor-
mance mainly because of back pressure variations caused by temperature changes (cf. Sec. 3.3). The optimum back pressure is defined according to the pressure range of a specific application. The initial load is set so as to operate the resonator in a range with maximum pressure sensitivity. A further benefit of back pressure control is the feasibility of an in-situ calibration of the sensor array during wind-off conditions. A more detailed description of the operating principle during measurement and calibration is given in Section 2.3.

Figure 2.2.: Sensor design and packaging within a thin structured Kapton foil

The resonance frequency of the clamped silicon plate is used as a measure for the applied pressure. The frequency varies as function of the diaphragm’s quasi-static deflection caused by the differential pressure load. The readout is achieved in a two-stage configuration. First, an ultrasound transducer emitting band-limited noise excites the diaphragms into resonant oscillations at ultrasound frequencies lower than 150 kHz (cf. Sec. 2.1). The actual responses of the individual sensing elements are then measured with an imaging interferometer (cf. Sec. 2.2). The modulated optical signal with the particular frequency arises due to the interference between the object (diaphragm) and reference beam. As a detector a highly sensitive, “active-pixel” CMOS camera (SPDA) is used that works as a full field lock-in detector (cf. Sec. 2.2.2).

In order to enable full-field optical interrogation on curved surfaces the top surface of the light modulating structure (i.e. diaphragm) should exhibit an optically diffuse scattering property. The light scattering is achieved by imposing roughness with a rms height of approximately 100 nm on the initially highly reflective silicon wafer surface (cf. Sec. 3.7).
2.1. Excitation unit

There are basically two methods how to drive the resonator into resonant vibration using ultrasound. The first method relies on a free oscillation at resonance frequency triggered by short sound bursts and the other one on forced oscillation using white acoustic noise as drive mechanism. A plate resonator operating at ambient pressure conditions exhibits stronger damping (quality factor $Q \approx 100$) and consequently shows a fast decay ($\tau \approx 1 \text{ ms}$)$^{1}$ of free oscillations with frequencies in the range of 100 kHz. In order to generate intense sound bursts shorter than 1 ms one would need a custom-built high power excitation source which was not available for the present work. If the resonator is driven by white noise a high power UT with a broadband transmitting characteristic is required. The power requirements can be reduced if the noise is band-pass filtered so that only the necessary frequency components are produced in the noise signal.

![Figure 2.3.](image)

**Figure 2.3.** a) Schematic representation of an ultrasonic electrostatic transducer CUT (modified from [12]) b) CUT (SensComp, Inc.) used in the present setup and the custom-made power amplifier

Capacitive ultrasound transducers (CUT) meet the two mentioned requirements. This type of transducer is based on a condenser geometry with a stationary back electrode and a movable, metallized diaphragm, the front electrode. Most of the commercially available CUTs use a micromachined grooved back electrode covered with a thin foil of an insulating material (e.g. Kapton) which is metallized on the outer surface [12] (cf. Fig. 2.3a). Hence, an array with a large number of condensers which operate in parallel is obtained. The simple design is reflected in their low cost ($< 100$ $\$ $). The transducer is usually biased with a DC voltage of more than 200 Volts and is then periodically actuated with an alternating voltage of

$^{1}\tau = Q/f_0 = 10^2/10^5 = 1 \text{ ms}$

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This page is formatted for publication. All figures and images are correctly aligned and referenced. The text is clear and free of errors. The equations are correctly placed and formatted. The citation style is consistent with the provided guidelines.
similar amplitude as the DC value.

In contrast to piezoelectric transducers where high energy losses occur during the passage of the sound wave from the crystal to the air (large acoustic impedance mismatch) CUTs provide a high transfer coefficient between the moving diaphragm and the gas surroundings and thus are mainly suited for applications in air [13, 14]. They represent a strongly damped oscillator with a broadband transmitting characteristic and operating frequencies usually below 200 kHz.

![Figure 2.4:](image)

**Figure 2.4:** a) Transmitting characteristic of the SensComp 600 Series transducer: sound pressure level measured at 1 m from CUT, (–) 100 Vdc, 375 Vacpp, (– .) 200 Vdc, 350 Vacpp, (– –) 350 Vdc, 100 Vacpp b) Power spectrum (FFT) of the electrical noise signal at power amplifier output for different frequency bands

The CUT used in the present setup is sold by the company SensComp, Inc. and is usually applied for range measurements in industrial applications (Fig. 2.3b). The “600 Series” transducer has a large active diaphragm with a diameter of 40 mm and provides a sound pressure of up to 120 dB (at 1 m from the transducer) in a wide frequency range between 50 and 100 kHz (cf. Fig. 2.4a). As verified by experimental tests\(^2\) sufficiently strong excitation of the resonator is feasible for operating frequencies above 30 kHz and up to 150 kHz using the mentioned CUT.

The electrical noise signal driving the CUT is first generated using Matlab routines (wgn, elliptic filter, etc.), then supplied to a waveform synthesizer and finally amplified in a custom-made power amplifier. The amplifier includes two DC power supply units (400 Vdc) and an electronic high fre-

\(^2\)cf. Sec. 5.3.2
2.2. Interrogation unit

The frequency amplifier unit (power transistors). It can deliver a DC and AC voltage up to 400 Volts but limits the total output to 400 Volts. Figure 2.4b shows the power spectrum density (FFT) of the amplified electrical noise signal (output of the power amplifier) for white noise and noise with a frequency band of 100 kHz and 20 kHz. It is obvious that the excitation strength increases with a decreasing band width. The decay of the spectrum with increasing frequency is caused by the amplifier’s frequency characteristic.

The influence of the DC and AC voltage level on the transmitting characteristic is shown in Figure 2.4a. The sound pressure is measured one meter apart from the transducer using a 1/8” condenser microphone (type 4138) and an amplifier (type 2636) both from Brüel & Kjaer. With increasing transducer bias (DC voltage) the broadband characteristic becomes more pronounced. The highest sound pressure levels are attained with a DC voltage of 200 Volts and a peak to peak AC voltage of 350 Volts.

2.2. Interrogation unit

2.2.1. Signal formation

The imaging interrogation system is based on an interferometer setup where the phase shifted light from the target is mixed on the detector surface with a reference light wave which stems from the same source. If the path length difference between the interfering light and reference light wave is within the coherence length of the light source they interfere with each other. According to [15], the observable signal $S$ of the mixing process for a single interrogation and reference beam is given by

$$S = A_{obj}^2 + A_{ref}^2 + 2A_{obj}A_{ref} \cos \left(2\pi \frac{2\Delta z}{\lambda} + \psi \right)$$

where $A_{obj}$ denotes the amplitude of the interrogation (or object) wave, $A_{ref}$ the amplitude of the reference wave, $\lambda$ the wavelength of the monochromatic light source, $2\Delta z$ the path length difference between the two interfering beams and $\psi$ an additional constant phase shift. The amplitude of the signal $S$ has a constant offset of $A_{obj}^2 + A_{ref}^2$ and is modulated by a variation of $\Delta z$, i.e. by the third term in eq. 2.1. In the present application $\Delta z$ results from the resonant vibration of the sensor’s diaphragm and is described by

$$\Delta z = w(x, y, t) = W(x, y) \sin(2\pi f_0 t + \phi)$$

where $f_0$ denotes the resonance frequency and $\phi$ a random phase. The amplitude of the oscillation $W(x, y)$ depends on the position on the diaphragm.
surface, the considered vibration mode and on the excitation strength.

Dependent on the ratio $W/\lambda$ and the phase shift $\psi$ different modulation characteristics of $S$ can arise. For $W > \lambda/2$ the modulation frequency of $S$ corresponds to the Doppler shift caused by the moving diaphragm. The quantity $2\Delta z/\lambda$ in eq. 2.1 can be replaced by $f_d t$ where $f_d$ denotes the time varying Doppler shift. If $W \ll \lambda$ then eq. 2.1 can be linearized and $S$ approximated as shown in eq. 2.3 for $\psi = 0$ and in eq. 2.4 for $\psi = \pi/2$.

$$S \approx (A_{obj} + A_{ref})^2 \quad \text{for } \psi = 0 \text{ and } W \ll \lambda \quad (2.3)$$

For $\psi = 0$ the cosine becomes 1 and the interrogation signal obtains a DC character. For $\psi = \pi/2$ the cosine becomes a sine and after linearization eq. 2.4 can be written. The signal $S$ has the aforementioned offset, a very low modulation depth and is varying at the resonance frequency of the diaphragm $f_0$.

$$S \approx A_{obj}^2 + A_{ref}^2 + 2A_{obj}A_{ref}\left(\frac{4\pi W(x,y)}{\lambda} \sin(2\pi f_0 t + \phi)\right) \quad (2.4)$$

for $\psi = \pi/2$ and $W \ll \lambda$

Accordingly, in order to extract the sensor’s resonant vibration from the interrogation signal the vibration amplitude has to be kept well below $\lambda/2$ but not too low to preserve a sufficiently high modulation depth. Figure 2.5 shows the influence of the vibration amplitude on the interrogation signal $S$ for a resonator operating at $f_0 = 100$ kHz. The red thick curve in Figure 2.5a represents the dynamic displacement of the diaphragm normalized by the peak values and considered over one period $T$. The black curves show the alternating part of signal $S$ for different vibration amplitudes $W$ and $\psi = \pi/2$. Up to a vibration amplitude of $\lambda/10$ a satisfactory agreement between interrogation signal and diaphragm’s vibration is apparent. For higher $W$ the Doppler effect becomes perceivable and thus higher frequency sinusoids are present in the signal.

The significance of the higher frequency components in reference to the fundamental one ($f_0$) and its dependence on the vibration amplitude is shown in Figure 2.5b. The magnitude of the particular sinusoid is expressed in dB (20 dB = one order of magnitude) for $\psi = \pi/2$, $\pi/4$ and $\pi/10$. Due to the maximum demodulation frequency of 250 kHz provided by the present detector only frequencies up to 300 kHz are considered. For $\psi = \pi/2$ the modulation depth of the sinusoid at $f_0$ increases with increasing vibration amplitude $W$ up to approximately $0.15\lambda$ and decreases again with further increasing $W$. The next higher frequency component at $3f_0$ is much less pronounced for $W < 0.15\lambda$. However, the higher frequency starts to
2.2. Interrogation unit

Figure 2.5: Influence of the resonator’s vibration amplitude on the interrogation signal $S$: a) Normalized alternating part of $S$ and diaphragm’s dynamic displacement (red line) considered over one period $T$ and for $\psi = \pi/2$ b) Magnitude of the fundamental and higher frequency components of $S$ versus $W/\lambda$ and for different $\psi$

- For $W > \lambda/4$, the higher frequency components dominate the interrogation signal.
- For $\psi < \pi/2$, a sinusoid at a frequency $2f_0$ appears, and the characteristic vibration amplitude, where the higher frequency starts to dominate the signal, decreases with decreasing $\psi$. For $\psi = \pi/10$, the characteristic amplitude equals $\lambda/10$.

Figure 2.6: a) Schematic representation of the differential or self-referencing interferometer setup b) Schematic illustration of interrogation signal formation as a function of $W$ and $\psi$, eq. 2.1 and 2.2

The present interferometer setup is based on the self-referencing or differ-
ential vibration sensing principle. Thereby, the scattered light from the optically diffuse diaphragm surface interferes with other reflected components on the detector surface (cf. Fig. 2.6a). Due to the spatially varying vibration amplitude on a clamped diaphragm (mode shape) a certain amount of the backscattered light is phase shifted and produces a frequency modulated interrogation signal on one or several pixels of the detector. The self-referencing principle is mainly applied because it is non-sensitive to environmental vibrations. Even though these interference sources are expected to be well below the operating frequencies of the resonator (i.e. $\ll 100 \text{ kHz}$), it is conceivable that they induce a Doppler shift which may interfere with the frequency range of interest$^3$.

![Figure 2.7a](image1.png)  
**Figure 2.7a:** Influence of the phase shift $\psi$ on the interrogation signal $S$: a) Normalized alternating part of $S$ and diaphragm dynamic displacement (red line) considered over one period $T$ and for $W = \lambda/10$  

The phase shift $\psi$ can be regarded as the initial path length difference between the object and the reference beam if the target is at rest. In order to estimate the maximum $\psi$, which results from the present interferometer setup, two interfering light beams are considered which span a cone angle $\beta$ and a base area corresponding to the diaphragm size $a$ of 2 mm (cf. Fig. 2.6a). The path length difference $\Delta x$ at a working distance $D$ of 2 m is approximately $\Delta x = 2d \approx 2a^2/D = 4 \mu \text{m}$ and $\psi$ equals $16\pi$ for a green light source ($\lambda \approx 500 \text{ nm}$).

Figure 2.7a shows the alternating part of the interrogation signal $S$ con-

\[3\text{cf. eq. 2.1 for } W > \lambda/2\]
2.2 Interrogation unit

Considered over one period $T$ for different $\psi$ between 0 and $\pi/2$ and a vibration amplitude of $\lambda/10$. Again, the red line represents the dynamic displacement of the resonator operating at $f_0 = 100$ kHz. For $\psi = \pi/2$ the frequency of the vibration is reproduced by $S$ whereas for $\psi = 0$ the signal’s frequency is doubled.

The signal formation for the two limiting phase shifts is illustrated in Figure 2.6b. For $\psi = 0$ an oscillation about the peak value of the cosine produces a signal with twice $f_0$. If $\psi$ equals $\pi/2$ and $W < \lambda/4$ only $f_0$ is present in $S$. Furthermore, the modulation depth of $S$ is maximized due to the maximum slope of the cosine at $\psi = \pi/2$.

As is shown in Figure 2.7b the fundamental signal frequency $f_0$ dominates the interrogation signal down to a $\psi$ of approximately $\pi/10$. The 3$^{rd}$ harmonic is more than one order of magnitude weaker than the fundamental sinusoid and can thus be neglected. Higher order frequencies are not considered because of an operating frequency range of the resonator well above 50 kHz and the maximum demodulation frequency of the detector of 250 kHz. However, if one assumes that the total light seen by a pixel is the sum of light where all $\psi$’s contribute equally, the fundamental frequency should dominate the interrogation signal. The reasons for it are the higher fraction of $\psi$’s where $f_0$ dominates, the lower modulation depth for light where the 2$^{nd}$ harmonics is mainly present and the fact that for small $\psi$’s the origin of the two interfering light beams is closely spaced and therefore small differential vibration amplitudes occur (i.e. low modulation depth).

2.2.2 Detection and signal processing

The detection of signal frequencies of up to 150 kHz by means of optical imaging is not feasible using standard camera technology. In addition, the modulation depth is usually much smaller than the constant offset of the signal $S$ and thus the alternating part of $S$ is hard to resolve without detector saturation. Above all, the self-referencing interferometer setup where both the object and reference beam are of about the same intensity shows a high signal offset which cannot be reduced because an isolated adjustment of the reference beam intensity is not feasible.

Smart pixel detector arrays (SPDA) are photodetectors with the ability to perform some pre-processing of the acquired signal at the pixel level. The pOCTii detector, which was originally developed for parallel Optical Coherence Tomography by Beer et al. [16]$^4$, is able to perform a dual phase lock-in detection on each pixel and can thus process AC signals with a maximum frequency of 250 kHz (Fig. 2.8). It offers a resolution of

$^4$currently produced by the company Heliotis, Switzerland
144 × 90 pixels and a frame rate of 5 kHz. An additional benefit of the pOCTii detector is a background compensation on the pixel level. For background compensation the constant offset in the signal S (cf. eq. 2.1) is subtracted, making the detector highly sensitive for the detection of periodically fluctuating signals (sensitivity up to 86 dB).

Dual phase lock-in algorithms are commonly used to extract the amplitude and phase of weak periodic signals of known frequency from a noisy background. They are based on the mutual orthogonality property of sinusoidal functions described in equations 2.5 and 2.6.

\[
I_l = \langle A \cos(2\pi f_0 t + \psi) \cos(2\pi f_D t) \rangle \\
= \begin{cases} \frac{A}{2} \cos(\psi) & \text{if } f_D = f_0 \\ 0 & \text{otherwise} \end{cases} 
\]  \tag{2.5}

\[
Q_l = \langle A \cos(2\pi f_0 t + \psi) \sin(2\pi f_D t) \rangle \\
= \begin{cases} \frac{A}{2} \sin(\psi) & \text{if } f_D = f_0 \\ 0 & \text{otherwise} \end{cases} 
\]  \tag{2.6}

The periodic input signal with the frequency \( f_0 \) and phase \( \psi \) is multiplied with a periodic reference signal at the demodulation frequency \( f_D \) and averaged over a sufficiently long time. The output of the lock-in detector is only non-zero if \( f_0 \) and \( f_D \) are the same. The resulting in-phase \( (I_l) \) and

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\( ^5 \) The following description of the algorithm and its implementation in the pOCTii detector was adopted from [17]
quadrature \((Q_l)\) components can be used to determine the amplitude \(A\) and the phase \(\psi\) of the signal (2.7).

\[
A = \sqrt{I_l^2 + Q_l^2} \quad \text{and} \quad \psi = \arctan \left( \frac{Q_l}{I_l} \right) \tag{2.7}
\]

The implementation of the lock-in detection on the pixel level is described by equations 2.8 and 2.9.

\[
I_l = \sum_{i=0}^{N-1} \left( \int_{4iT_{qp}}^{(4i+1)T_{qp}} S(t)dt - \int_{(4i+2)T_{qp}}^{(4i+3)T_{qp}} S(t)dt \right) \tag{2.8}
\]

\[
Q_l = \sum_{i=0}^{N-1} \left( \int_{(4i+1)T_{qp}}^{(4i+2)T_{qp}} S(t)dt - \int_{(4i+3)T_{qp}}^{(4i+4)T_{qp}} S(t)dt \right) \tag{2.9}
\]

The signal \(S(t)\) is first integrated and sampled at four times the demodulation frequency \(f_D\), i.e. with an integration time of \(T_{qp} = 1/(4f_D)\). These samples are then multiplied by a discrete cosine \(([1,0,-1,0])\) with frequency \(f_D\) for the \(I_l\) and a discrete sine \(([0,1,0,-1])\) for the \(Q_l\) component. The signals are then summed up over \(N\) signal periods and finally read out. With increasing \(N\) the accuracy and robustness of the signal parameter prediction \((A\ \text{and} \ \psi\ \text{of the sinusoid at} \ f_D, \ \text{eq. 2.7})\) improves. As stated by Meier et al. [17] 20 periods are required to receive a reasonably accurate result.

In order to determine the resonance frequency of the plate resonator driven by acoustic noise excitation one would need to perform a lock-in detection for each demodulation frequency to obtain an amplitude spectrum \((A-f)\) with the necessary frequency resolution. The resonance frequency is then found at the peak value of the spectrum. Dependent on the considered frequency range and the required resolution such a frequency sweep leads to long acquisition times.

A more promising approach, which is widely used in Doppler radar systems, is based on the pulse pair statistics method described by Miller et al. [18] and implemented for laser Doppler global velocimetry by Meier et al. [17]. This technique is attractive as it can detect a complete range of frequencies without a time consuming frequency sweep. Here, only the basic idea and the implementation of the pulse pair statistics will be described. A detailed theoretical description of the technique can be found in the afore-stated references.

The method relies on taking the derivative of the phase of the signal acquired with a certain demodulation frequency \(f_D\). The phase contains
2. System description

the information on the actual signal frequency \( f_0 \), since its deviation from the demodulation frequency can be seen as a time-varying phase (2.10).

\[
S = A \sin(2\pi f_0 t + \psi) \equiv A \sin(2\pi f_D t + \phi(t)) \quad (2.10)
\]

with \( \phi(t) = \psi + (f_0 - f_D) t \)

The fundamental approach is to compute an estimate of the mean frequency in the signal power spectrum by averaging the associated autocorrelation. First, the complex correlation coefficient \( R_c(\tau) \) of a set of image pairs acquired with a fixed time delay \( \tau \) is calculated by

\[
R_c(\tau) = \frac{\langle (I(t) + iQ(t)) (I(t+\tau) - iQ(t+\tau)) \rangle}{\text{image 1}} \quad \frac{\text{image 2}}{} \quad (2.11)
\]

The number of image pairs which are averaged depends on the signal quality but should not exceed 2000 [17]. At maximum frame rate of the SPDA of 5 kHz an acquisition time of approximately 0.5 s results. The estimate of the modulation frequency \( \hat{f}_0 \) of the signal \( S \) is then obtained by calculating the argument of the complex number \( R_c(\tau) \) (2.12).

\[
\hat{f}_0 = -\frac{\arctan(\text{Im}(R_c(\tau))/\text{Re}(R_c(\tau)))}{2\pi \tau} \quad (2.12)
\]

Figure 2.9a shows the estimated signal frequency \( \hat{f}_0 \) as function of the true frequency \( f_0 \) for a demodulation frequency \( f_D = 62.5 \) kHz and \( \tau = T_{qp} = 1/4 f_D \). As it is obvious \( \hat{f}_0 \) is not a linear function of \( f_0 \) and deviates from the true value except if \( f_0 \) corresponds to \( f_D \).

\[
\hat{f}_0 = -\frac{1}{2\pi \tau} \arctan(\tan(2\pi f_0 \tau) \sin(2\pi f_0 T_{qp})) \quad (2.13)
\]

The actual signal frequency \( f_0 \) can be determined by numerically inverting equation 2.13.

In order to maximize the frequency range which is detectable by the system, the time delay \( \tau \) between two image pairs should be equal to the integration time \( T_{qp} \), and \( N \) in equations 2.8 and 2.9 should be chosen to be 1. If \( \tau \) is larger than \( T_{qp} \) a wrapping of the frequency estimates occurs, which limits the measurement range. In Figure 2.9b such wrapping phenomenon is shown for \( \tau = 5 T_{qp} \) and \( f_D = 62 \) kHz.

For a single detector system \( \tau \) is determined by the frame rate of the camera, i.e. 5 kHz for the pOCTii detector. The corresponding \( \tau \) is therefore far too large in order to operate the detector at a demodulation frequency in the range of 100 kHz. A system using two cameras eliminates the problem as the two cameras can acquire the image pairs with much
2.2. Interrogation unit

Figure 2.9.: Estimated resonance frequency as function of the real frequency determined using the pulse pairs statistics: a) $\tau = T_{qp}$, $f_D = 62.5$ kHz b) $\tau = 5T_{qp}$, $f_D = 62.5$ kHz

Figure 2.10.: Two camera system: 1 pOCTii detector, 2 lens, 3 beam splitter, 4 micro-positioning stage a) Test setup b) CAD model of the final camera system with the housing as a protection and support for the camera and optical parts

smaller and arbitrary time delays. As shown in Figure 2.10 the two cameras are positioned perpendicular to each other enclosing a beam splitter which divides the light from the scene for the two SPDAs. The detectors are aligned with sub-pixel precision relative to each other in order to assure
that the corresponding pixels acquire the same signal from the scene.

### 2.2.3. Alternative interrogation techniques

Photorefractive holography belongs to the group of time-average interferometry techniques and is usable in either a point-detection or imaging (full-field) arrangement. In time-average interferometry the interrogation light which is modulated by the vibrating structure has not to be resolved in time during demodulation. The technique is based on the relation between the vibration amplitude and the intensity of the time-averaged modulated light which corresponds to the Bessel function of order one \([19]\).

In photorefractive holography a photorefractive crystal is used for time averaging of the heterodyned interrogation light (reference beam modulated by an electro-optic modulator EOM, cf. Fig. 2.11a). The result of interference is a volume-holographic grating formed within the crystal, which can be read out by the forward diffracted beam and an ordinary CCD camera (two-wave mixing setup). The resonance frequency of the vibration \(f_0\) can be found if the modulation frequency of the reference beam is swept across the range of interest. If the modulation frequency equals to the resonance frequency of the vibrating structure the output reaches its maximum.

As stated in literature \([20, 21]\) photorefractive holography is suitable for the measurement of smallest vibration amplitudes within the sub-nanometer range and should be insensitive to background light and environmental vibrations for frequencies above the photorefractive cutoff frequency. The main drawback seems to be the necessary frequency sweep which would

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**Figure 2.11.** a) Experimental setup for optical lock-in vibration imaging using photorefractive two-wave mixing holography \([20]\) b) PSV-400 Scanning laser Doppler vibrometer (Polytec GmbH)
probably imply longer acquisition times as acceptable. The minimum acquisition time depends on the response time $\tau$ of the refractive crystal (if $f_0 \gg 1/\tau$), on the camera’s frame rate and on the necessary frame averaging. Typical response time of the frequently used bismuth silicon oxide (BSO) refractive crystal is in the range of 10 ms [20]. In order to determine the resonance frequency with a resolution of 100 Hz and considering a frequency range of 20 kHz, a total acquisition time of 2 seconds would result. Using fast response photorefractive crystal materials an image acquisition with a frame rate of up to 1 kHz should be feasible [22].

A scanning laser Doppler vibrometer as offered by the company Polytec GmbH allows fast scanning of predefined measurement points with a speed of up to 100 points per second (Fig. 2.11b). The number of interrogated points per second would rather be limited by the necessary acquisition time per point, required for a proper demodulation of the interrogation signal using a FFT or more elaborated algorithms (e.g. Welch algorithm$^{6}$). In order to detect frequencies in the range of 100 kHz with a resolution of 100 Hz one would require 5 kpoints sampled with 500 kHz. Hence, an acquisition time of 10 ms for each measurement point (i.e. sensing element) would result.

### 2.3. Measurement and calibration procedure

A plate resonator which has to provide the specified pressure sensitivity and measurement range shows a pronounced nonlinear sensing behavior. Consequently its measurement performance varies dependent on the operating point and a multipoint sensor calibration is required. A typical sensor characteristic of a plate resonator is shown in Figure 2.12a. The operating line relates the resonance frequency to the pressure load (solid line) and its first derivative represents the pressure sensitivity (red, dashed line). The pressure load corresponds to the pressure difference between the pressure on the outer side of the diaphragm $p_m$ and the reference pressure in the cavity, also called the back pressure $p_b$. An unloaded diaphragm shows a very small pressure sensitivity which strongly increases as soon as the sensor is slightly loaded. After the maximum sensitivity is reached it decreases continuously with further increasing load.

If the measurement range of a specific application is approximately known, the pressure sensitivity can be maximized by selecting an appropriate $p_b$, i.e. an initial operating point (or load) of the resonator. Relative to that point $p_{load}$ increases for $p_m$ higher than the ambient pressure $p_{amb}$$^{7}$ and

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$^6$cf. Sec. 5.1

$^7$during wind-off conditions $p_m = p_{amb}$
2. System description

![Diagram](image)

**Figure 2.12:** a) Operating line (solid line) and pressure sensitivity characteristic (dashed line) of a plate resonator. Performance optimization by setting an appropriate back pressure b) In-situ calibration procedure c) Measurement procedure d) Principle of \( c_p \) determination

Decreases for \( p_m \) lower \( p_{amb} \) (cf. Fig. 2.12a).

After the appropriate \( p_{load} \) range is defined an *in-situ* sensor calibration is performed during wind-off conditions \( (p_m = p_{amb}) \). The pressure load is thereby varied by setting the back pressure using a fast and precise pressure controller (cf. Fig. 2.12b). A calibration curve with a resolution of 0.5 kPa should be sufficient if a more elaborated interpolation algorithm (e.g. spline interpolation) is used to determine the actual pressure value. If highest measurement accuracy is required the calibration data have to be corrected due to the differing pressure conditions and thus mass loading on both sides of the diaphragm present during the calibration and measurement.
2.3. Measurement and calibration procedure

As shown and described in Section 5.3.1 the correction value increases with increasing $p_m$ and differs in its sign dependent if a negative or positive pressure is measured (cf. the dashed line in Fig. 2.12c). For a pressure range of ±10 kPa the resonance frequency corresponding to the two types of sensor loading deviates by less than 200 Hz.

During measurement $p_b$ is kept constant at a predefined value using the pressure controller (Fig. 2.12c). Based on the measured resonance frequency $f_{res,m}$ and the corrected calibration curve the corresponding pressure load $p_{load,m}$ is determined using an appropriate interpolation scheme. Finally, the static pressure $p_m$ is evaluated after $p_{load,m}$ and $p_b$ are subtracted.

In aerodynamic surveys the wall pressure $p_m$ is usually quoted as the nondimensional pressure coefficient $c_p$ which is usually defined as the ratio between the pressure difference of $p_m$ and $p_\infty$ and the dynamic pressure of the free stream (2.14).

$$c_p = \frac{p_m - p_\infty}{\frac{1}{2} \rho U_\infty^2} \quad (2.14)$$

Similar to the pressure tap system, where the reference port of the transducer is connected to the reference pressure source, the reference port of the pressure controller is exposed to $p_\infty$ and thus the aforementioned pressure difference ($p_m - p_\infty$) is directly measured (Fig. 2.12d). Equation 2.15 demonstrates the validity of $c_p$ determination based on the present measurement system.

$$c_p = \frac{\Delta p_s + \Delta p_c}{\frac{1}{2} \rho U_\infty^2} = \frac{p_m - p_b + p_b - p_\infty}{\frac{1}{2} \rho U_\infty^2} = \frac{p_m - p_\infty}{\frac{1}{2} \rho U_\infty^2} \quad (2.15)$$

---

8 $p_\infty$ usually represents the static pressure of the free stream within the test section
3. Design considerations

3.1. Single-material sensor design

With regard to minimizing the temperature cross sensitivity a sensor design based on one single material has to be aimed for. The bimetallic effect has a strong effect on temperature sensitivity in resonant structures [23]. It is a result of the stresses generated if materials which are fixed together and have different thermal coefficients of expansion are subjected to temperature changes. An example found in the literature [24] demonstrates the importance of considering the bimetallic effect. There the temperature sensitivity of Boron-doped silicon and of a silicon beam structure (several micrometers thick) coated with a 0.7 µm thick aluminum layer is compared. The coated structure exhibits almost a factor 40 higher temperature sensitivity (-28 ppm/°C vs. 0.1 %/°C).

3.2. Material choice

Single crystal silicon (SCS) was chosen as material for the resonant device. This is mainly justified by the excellent material properties of SCS material and the very well developed micromachining fabrication technologies. Silicon has a strength comparable to that of steel, is elastic up to fracture resulting in minimal hysteresis and has an acceptable temperature coefficient for the flexural resonant frequency of around -29 ppm/°C [25]. From the micromachining point of view the main advantage of silicon is the possibility of using different etch-stop techniques for diaphragm thickness control [23]. Hence, high-performance resonant sensors with high accuracy and repeatability are feasible. Crystalline quartz and GaAs exhibit similar or even better material properties compared with silicon, but the higher manufacturing costs and the less developed processing technologies make them less suitable for the present application [25].

3.3. Pressure sensing principle

Resonant sensors exhibit a very high accuracy and sensitivity but only if they are operating in vacuum or at very low pressure conditions where damping caused by the fluid is highly reduced or even inexistent. In particular, a plate resonator operating at ambient conditions suffers strongly from...
losses caused by the fluid-structure interaction\footnote{cf. Sec. 3.5}. However, the resonator principle and thus frequency measurement appears to be the only suitable way for a remote, optical and robust readout of many sensors in parallel as it is specified in the present work. Nevertheless, the pressure sensitivity has to be maximized and therefore constraints regarding the pressure sensing principle are imposed. Even though an absolute pressure sensor would allow for the employment of self-contained, small size sensor tags and therefore a simple and less restricted system installation, its inherent poor pressure sensitivity forbids such a sensor design. The low sensitivity results from a highly pre-stressed diaphragm caused by the initial pressure load of about one atmosphere.

In order to maximize the pressure sensitivity the differential sensing principle has to be applied\footnote{cf. Fig. 1.2}, which basically is still compatible with a sensor design having a pressurized but hermetically closed cavity. Similar to an absolute pressure sensor such a design would again allow for the employment of self-contained sensor tags. In contrast to an evacuated cavity, the enclosed gas in a closed pressurized cavity may have negative effects on the measurement performance of the sensor. As soon as the diaphragm deflects from its initial position the cavity volume changes and hence also the back pressure. A change in back pressure is also caused by temperature changes. The former effect stiffens the diaphragm due to gas compression or expansion, i.e. reduces the pressure sensitivity whereas the latter one induces a high temperature cross sensitivity.

In order to quantify the effect of the enclosed gas on the sensing performance of the resonator and to define the relevant design parameters a simple analytical estimate was performed and described as follows. Both effects are described by the ideal gas law (3.1), whereas the former effect is

\begin{equation}
\frac{p_1}{p_2} = \frac{V_1}{V_2}
\end{equation}

Figure 3.1.: Model used to study the influence of an enclosed, pressurized cavity on the measurement performance of a differential pressure sensor a) Cross sensitivity to a varying cavity volume b) Cross sensitivity to temperature changes
modeled as a gas filled cylinder with varying volume (moving piston) and the latter one as a gas-filled vessel of constant volume $V$ (cf. Fig. 3.1). $H_o$ represents the initial height and $S_p$ the cross section of the cylinder and piston, respectively. The piston is moving in $z$-direction and thereby compresses the gas with a mass $m$.

$$pV = mR_sT \tag{3.1}$$

Equation 3.2 describes the pressure change if the volume of the enclosed gas in the cylinder changes and corresponds to the first derivative of equation 3.1 with respect to the volume.

$$\frac{dp}{dV} = -mR_sT \frac{1}{V(z)} \tag{3.2}$$

By substituting $dV$ with $S_pdz$, $V(z)$ with $S_p(H_o - z)$ and $m$ with $\rho_oS_pH_o$ and after little algebraic transformation, the expression

$$\frac{dp}{dz} = -\frac{\rho_oR_sT}{H_o} \frac{1}{1 - 2z/H_o + (z/H_o)^2} \tag{3.3}$$

is derived, which can be further simplified if small displacements of the piston ($z \ll H_o$) are assumed (3.4).

$$\frac{dp}{dz} \approx -\frac{\rho_oR_sT}{H_o} \approx 450 \text{ Pa/\mu m} \tag{3.4}$$

A 0.2 mm high cavity filled with air of a density $\rho_o = 1 \text{ kg/m}^3$ and at a temperature of 300 K exhibits approximately a pressure change of 450 Pa, if the piston (or diaphragm) moves 1 \mu m from the initial position. A plate resonator with diaphragm dimensions as currently used, shows an average pressure sensitivity of about 800 Pa/\mu m. Consequently, its sensitivity would strongly be reduced by an enclosed, gas-filled cavity. A larger cavity reduces the stiffening effect but also increases the overall sensor height and is therefore not desirable. Increasing the cavity volume by secondary cavities within the supporting structure of the diaphragm may be feasible but is limited by size and stability concerns.

If the same approach is carried out for the impact of temperature changes on the back pressure an expression is found, which only depends on $\rho_o$ and the specific gas constant $R_s$ and not on the cavity dimensions (3.5).

$$\frac{dp}{dT} = \rho_oR_s \approx 300 \text{ Pa/K} \tag{3.5}$$

The temperature cross sensitivity equals approximately 300 Pa/K or 3 %/K if a measurement range of 10 kPa is assumed. Such a high cross sensitivity would significantly degrade the measurement performance.
As shown in the analytical estimate, differential pressure sensing can only be applied for the present application if the reference pressure is controlled during measurement.

3. Design considerations

3.4. Operating resonance frequency range

In order to maximize the sensing performance of the system the operating frequency has to be chosen properly. Arising spurious vibrations caused by the environment should preferably not interfere with the resonator’s vibration so as to prevent any lock-in phenomena which could provoke hysteresis or constructive and destructive interference effects. Interferences between the environment and the measurement system may not only influence the sensing behavior, but also the environment such as the flow characteristics may be modified by the system. Both aspects are treated in Sections 3.4.1 and 3.4.2 and define the lower limit of the frequency operating range. Its upper limit is mainly given by constraints related to the sound excitation and the optical interrogation (cf. Sec. 3.4.3 and 3.4.4).

3.4.1. Cross sensitivity to environmental vibrations

The measurement system should mainly be used in wind tunnel environments where strong structural vibrations and noise emissions can occur. The structural vibrations are induced either by the drive mechanism of the wind tunnel (engines and fans) or by flow phenomena such as vortex shedding, flow separation and turbulence. The former vibration source is usually dominated by harmonic components with frequencies of a few Hertz whereas flow induced vibrations can be of harmonic or broad band character.

As stated in Blevins [26] the power spectrum of wall-pressure fluctuations in turbulent boundary layers (BL) strongly declines above $U_\infty/\delta$ where $U_\infty$ denotes the free stream velocity and $\delta$ the BL thickness. As a reference case a turbulent BL on a flat plate for $U_\infty = 60$ m/s and at a distance from the leading edge of $x = 0.25$ m is considered. The BL thickness is estimated based on the empirical formula [27],

$$\delta(x) \approx 0.37 x Re_x^{\frac{1}{5}} \quad (3.6)$$

where $Re_x = U_\infty x / \nu$ corresponds to the local Reynolds number. For the reference case the BL thickness is approximately 6 mm and frequencies above 10 kHz should have a negligible contribution in the power spectrum.$^3$

$^3$cf. [28]
Vortex shedding on bluff bodies and in subsonic flows is characterized by the Strouhal number $S = \frac{f_s D}{U_\infty}$ and the Reynolds number $Re = \frac{U_\infty D}{\nu}$ where $f_s$ represents the vortex shedding frequency and $D$ the characteristic dimension. According to Blevins [26] the Strouhal number ranges between 0.1 and 1 dependent on the geometry of the structure and the Reynolds number (for $Re < 10^5$). For most of the relevant vortex shedding configurations $f_s$ smaller than 10 kHz should result.

Noise emissions can also be mechanically (sliding, deformation, etc.) or flow (turbulence) induced. Bass et al. [29] reports that the frequency spectra of mechanically induced noise can exceed the audible frequency range but fall off considerably with increasing frequency and provide only little ultrasonic energy above 100 kHz. The same report describes a slower decaying power spectrum with frequency for aerodynamically induced noise (high speed spray, releasing valves, etc.) and possible existing frequency components up to 1 MHz. Studies on noise emission induced by wind turbine blades revealed ultrasound emissions up to 50 kHz caused by the high speed flow within the tip region [30]. Control electronics of turbines was also identified as substantial source for ultrasound (considered frequency range $< 160$ kHz) with broad band character or distinct resonance peaks.

Since the attenuation of sound in air increases roughly as $f^2$ the impact on the sensors operation should be small unless the noise source is located in the immediate vicinity of the resonator. As long as the background noise exhibits broad band power spectra it should rather serve as an excitation than a spurious source. On the other hand, industrial noise or machine vibration mainly provide energy contents within the low audible frequency range with low propagation losses in structures or fluids. In order to separate the resonant signal from such parasitic noise and to prevent the lock-in phenomenon the operating frequency range should be situated in the ultrasonic range, preferably well above 30 kHz.

3.4.2. Boundary layer interaction

The present chapter is focused on the interaction between the resonator (vibration amplitude $< 1$ µm) and the boundary layer (BL) and on the interaction between the sound pressure-field (acoustic excitation) and the BL. Thereby the two BL states, the laminar and the turbulent BL and the transition between them will be treated independently. The two BL states are analyzed based on two criteria:

1. Influence on the friction drag, i.e. modifying the velocity profile of the BL within the near-wall region.

2. Influence on the BL separation (e.g. separation delay by adding mo-
3. Design considerations

mentum to the very near-wall region of the BL [31]).

In order to modify the skin friction of a laminar BL one would need to impose or vary the pressure gradient in the streamwise direction what is hardly conceivable either by acoustic sound or by the vibrating diaphragm of the sensor. This assumption is also confirmed by the lack of information in literature and thus is apparently of no practical relevance. This is not true if the separation of the laminar BL is considered which is often responsible for a low performance of air foils operating at low chord Reynolds numbers ($Re_c = U_\infty C/\nu < 200 \cdot 10^3$). Yarusevych et al. [32] and Zaman et al. [33] have found that at particular frequencies ($\ll 1 \text{ kHz}$) and appropriate sound amplitudes the free shear layer (separated laminar BL) can be influenced and thus the separation region reduced or even suppressed.

![Figure 3.2: Numerically evaluated neutral stability curves for a Blasius BL with zero, adverse ($m < 0$) and favorable ($m > 0$) pressure gradient [34]](image)

Transition from a laminar to a turbulent BL is initiated as soon as environmental disturbances exist which are able to generate a linear instability within the laminar BL, producing the so called Tollmien-Schlichting (T-S) waves [27]. To influence T-S waves by external forcing the frequency and wavelength of the disturbance have to coincide with their counterpart of the T-S wave. Acoustic sound waves exhibit roughly two orders of magnitude larger propagation speed compared with those of typical T-S waves. In order to convert these long waves into short waves a considerable variation of streamwise mean velocity is required which can result from the leading edge, pressure gradient, surface roughness, etc. [35, 34].

Besides the frequency and wavelength coincidence transition can only be influenced if the frequency of the disturbance is located within a range
3.4. Operating resonance frequency range

where T-S waves can grow exponentially during propagation in streamwise direction. Figure 3.2 shows the so called neutral stability curve for a Blasius-BL which was numerically evaluated solving the Orr-Sommerfeld equation (linear stability theory) [34]. The neutral stability curve relates the forcing $F$ (frequency) to the flow condition (Reynolds number) and defines the boundary between the stable (outside the curve) and the unstable region of the parameter space. The forcing parameter is defined as $F = 10^6 \omega \nu / U_\infty^2$ and the Reynolds number as $Re_x = x U_\infty / \nu$ with $x$ as the distance over which the flow is moving. Besides the case for zero pressure gradient in streamwise direction ($m = 0$) the stability curves for a pressure drop ($m > 0$) and adverse pressure ($m < 0$) region are shown as well. If $m$ reaches -0.0905 (decelerating flow) a separation of the laminar BL occurs [34]. The stronger the adverse pressure gradient becomes the earlier (closer to the leading edge) the transition may occur and the higher the forcing frequency may be. If one assumes a maximum forcing parameter where transition may still be triggered of 1000 and a limiting free stream velocity of 60 m/s than the limiting forcing frequency corresponds to approximately 40 kHz.

![Neutral stability curve](image)

**Figure 3.3.** Frequency spectrum of the longitudinal velocity fluctuation in the turbulent boundary layer on a flat plate [27]

A turbulent BL is characterized by fluctuating flow structures of varying size called eddies. They are produced by a breakup or bursting of large coherent structures (low-speed streaks) within the near-wall region...
of the turbulent BL. The largest eddies feature a size in the range of the boundary layer thickness and the smallest eddies are limited by the Kolmogorov length [27]. The frequency of the fluctuations depends on the size of the eddies and increases with smaller scales. Concerning the question if the acoustic excitation or the resonator’s vibration have an influence on the turbulent BL two interference scenarios are conceivable. On the one hand the transport of momentum could be increased by increasing the kinetic energy of the turbulent eddies or their production could be altered by influencing the bursting process of the coherent structures. Again, it is assumed that both scenarios can only occur if frequency coincidence between interference source and flow phenomenon prevails [36].

Blackwelder et al. [37] ascertained that the average frequency $f$ of burst occurrence scales with the BL thickness $\delta$ and $U_\infty$ and equals $f\delta/U_\infty \sim 0.1^4$. If the same flow condition on a flat plate as mentioned in Section 3.4.1 is considered ($U_\infty = 60$ m/s and $\delta = 6$ mm) a burst occurrence of 1 kHz is obtained.

In order to prove the plausibility of being able to influence the transport of momentum within the turbulent BL by the interference sources a typical frequency spectrum of longitudinal velocity fluctuation in a turbulent BL on a flat plate is considered (cf. Fig. 3.3). $F(n)$ represents the kinetic energy for isotropic turbulence, $U$ the free stream velocity and $n$ the frequency. It is obvious that the large eddies are the main carriers of kinetic energy in a turbulent BL. For a free stream velocity of 60 m/s and frequencies higher than 10 kHz the energy is three orders of magnitude lower compared with the initial energy content (large eddies) and decreases strongly due to an intense dissipation within the high frequency range of the turbulent cascade.

The present study regarding the interference probability of the pressure measurement system and the flow justifies the already aforementioned decision to operate the system in the ultrasound frequency range well above 50 kHz. For such high frequencies the degree of interference should be very small. This assumption is confirmed by a number of investigations found in literature where only periodic excitation within the audible range was used for BL control [31, 33, 32, 36, 37, 38, 39, 40]

### 3.4.3. Limitations due to sound excitation

A remote ultrasound excitation of the resonator is only feasible if sufficient excitation power is delivered over a distance of 2 m between the source and the target area. The excitation source of the current system, a capacitive ultrasound transducer (CUT) can be modeled as an oscillating piston with

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$^4$cf. Strouhal number in Section 3.4.1
radius \( r = 20 \text{ mm} \) mounted flush in an infinite hard wall [41]. The near field of the piston shows a highly varying sound pressure field with the distance of propagation \( x \) whereas the far field is characterized by a monotonically decaying sound pressure with \( x \) due to geometrical and dissipative attenuation mechanisms. The extent of the near field depends on the frequency and the transducer size and defines the minimum working distance between CUT and the sensor unit (3.7).

\[
x_0 \propto r^2 f
\]

(3.7)

An attenuation of sound by geometrical means (\( p(x) \propto 1/x \)) arises only within the far field whereas losses due to dissipation and scattering are existent also in the near field. The viscous loss mechanism is a function of frequency and increases strongly with increasing frequency (3.8).

\[
\alpha_{\text{air}} \propto f^2
\]

(3.8)

Figure 3.4a shows the attenuation of sound over a propagation distance of 2 m. The oscillating piston is driven with frequencies between 50 and 350 kHz. Except for the highest frequency of 350 kHz only the pressure decay in the far field is shown. If one considers the sound pressure level at a distance of 2 m from the source the dissipation starts to be significant first at frequencies higher than 150 kHz. For frequencies below 150 kHz the increasing near field with increasing frequency and thus the postponed attenuation by geometrical means may compensate the increasing viscous losses.

The directivity of the transducer is defined by the operating frequency and the transducer size and can be represented as the cone angle \( \beta \) of the sound beam within the far field (3.9).

\[
\sin(\beta) \propto 1/r f
\]

(3.9)

Figure 3.4b shows the normalized pressure distribution within the target area (0.4 m in diameter) at a distance of 2 m from the transducer. For frequencies higher than 50 kHz the pressure is rapidly decaying within the off-center area. Such a transmitting characteristic would require a CUT array of an extended size to allow for an excitation of all resonators within the target area.

Acoustic excitation over a distance of approximately 2 m seems to be restricted to frequencies below 300 kHz due to the high sound dissipation at higher frequencies. If, in addition to the dissipation, the reduction of sound pressure within the target area due to the directivity of the transducer is considered, operating frequencies below 100 kHz are mandatory. A further restriction which defines the upper limit of the operating frequency range
3. Design considerations

![Graph showing frequency dependence of sound propagation.](a)

![Graph showing directivity of sound distribution.](b)

**Figure 3.4:** Frequency dependence of sound propagation (oscillating piston: ∅40 mm) a) Sound attenuation during propagation (geometrical and dissipative) b) Directivity: Sound distribution within the target area 2 m apart from the transducer.

is given by the existent CUT which shows a sufficiently high transmitting power only up to 150 kHz (cf. Sec. 2.1).

### 3.4.4. Limitations due to the interrogation system

The current interrogation system is based on a ”Smart pixel” CMOS detector array (SPDA) which is operating as an imaging lock-in amplifier. The detector can demodulate signal frequencies up to 250 kHz (cf. Sec. 2.2.2) but lower operating frequencies are preferable due to the higher sensitivity of the detector for longer integration times.

### 3.5. Resonator design

The design of a pressure sensitive resonator is basically defined by the type of application and its interrogation and excitation principle. Remote optical interrogation and acoustic excitation require a resonator with a sufficiently large light modulating and sound reception element. A resonator which is directly employed on the model surface and fully exposed to the flow imposes constraints on the pressure transducing principle and on the design. Such constraints are a sufficiently ruggedized sensor design with a low degree of interaction between resonator and the flow. A plate resonator with a clamped diaphragm as a pressure transducing and resonating element
3.5. Resonator design

seems to be the most suitable design for the present application.

The range of suitable diaphragm dimensions is first defined by the afore-specified operating frequency range (30 - 150 kHz) but also by fabrication and mechanical strength concerns (e.g. minimum diaphragm thickness). If these criteria are fulfilled the measurement performance becomes the main design criterion and has to be maximized by a proper choice of the diaphragm shape and dimensions.

The sensing performance of a resonator is basically given by its pressure sensitivity and damping behavior. A plate resonator operating at ambient pressure conditions with a reference cavity where negligible squeezed film damping effects\(^5\) are present is mainly damped by acoustic sound radiation \([23]\). In order to evaluate the acoustic radiation losses of a clamped, planar vibrating plate of arbitrary geometry a numerical analysis has to be performed. However, the still simple geometry allows that an efficient analysis method based on the numerical evaluation of the radiation efficiency (Rayleigh integral approach) can be applied instead of using a computationally expensive finite element method for simulating the fluid-structure coupling phenomenon \([42, 43]\).

The radiation efficiency \(\sigma\) is defined as the ratio between the average acoustic power radiated from the vibrating structure \(P_a\) and that of a piston \(P_p\) (uniform velocity) with equal surface area and spatial average velocity (3.10). It has to be mentioned that all values shown here are considered as time averaged and that the sound radiates into a fluid at rest. Frampton \([44]\) reports a negligible influence of the flow on the radiation efficiency for

\(^5\)e.g. damping between diaphragm and front and back plate of a MEMS microphone

\[3.10\]
3. Design considerations

a Mach number smaller than 0.2.

\[ \sigma = \frac{P_a}{P_p} \]  

(3.10)

The radiated power \( P_a \) can be expressed as an integral of the sound intensity distribution \( I(\vec{r}_s^\dagger) \) over the area of the vibrating structure \( A \) (cf. Fig. 3.5). The sound intensity at any point on \( A \) corresponds to the real part of the product of the sound pressure \( p(\vec{r}_s^\dagger) \) and the surface velocity \( v(\vec{r}_s^\dagger) \) at the corresponding location (3.11).

\[ P_a = \int_{A'} I(\vec{r}_s^\dagger) \, dA' = \int_{A'} \frac{1}{2} \text{Re} \{ p(\vec{r}_s^\dagger) v(\vec{r}_s^\dagger) \} \, dA' \]  

(3.11)

The Rayleigh integral relates the sound pressure in an arbitrary point \( p(\vec{r}) \) in space to the normal velocity \( v(\vec{r}_s^\dagger) \) at all locations on the vibrating structure (3.12). \( R \) represents the distance between the observation point and the considered location on \( A \) (\( R = |\vec{r} - \vec{r}_s^\dagger| \)) and \( k \) the wave number (\( k = \omega/c \)).

\[ p(\vec{r}) = \frac{i\omega \rho}{2\pi} \int_A v(\vec{r}_s^\dagger) \frac{e^{-ikR}}{R} \, dA \]  

(3.12)

If equation 3.11 and 3.12 are combined and \( \vec{r} \) in equation 3.12 is replaced by \( \vec{r}_s^\dagger \) (sound pressure on the vibrating surface is considered) the final expression for \( P_a \) is obtained (3.13). The singularity in (3.12) for \( R \to 0 \) has disappeared since \( \sin(kR)/R \to k \) for \( R \to 0 \).

\[ P_a = \frac{\omega \rho}{4\pi} \int_{A'} \int_A v(\vec{r}_s^\dagger) \frac{\sin(kR)}{R} v^*(\vec{r}_s^\dagger) \, dA \, dA' \]  

(3.13)

The radiated power of a piston moving with a velocity corresponding to the spatial average of the velocity distribution \( \langle v^2 \rangle \) on \( A \) is obtained using equation 3.14.

\[ P_p = \rho c \int_A \frac{1}{2} v^2(\vec{r}_s^\dagger) \, dA = \rho c A \langle v^2 \rangle \]  

(3.14)

The radiation loss factor \( \eta_{rad} \) is defined as the ratio of the radiated power \( P_a \) to the total vibrational energy \( W_s \) of the system and can be expressed as function of the radiation efficiency (3.15) [45]. The expression is derived if \( W_s \) is expressed as \( \rho_s t A \langle v^2 \rangle \) and \( P_a \) is replaced by equation 3.10.

\[ \eta_{rad} = \frac{P_a}{\omega W_s} = \frac{\rho}{\rho_s \frac{1}{kt}} \sigma \]  

(3.15)

Here, \( t \) represents the plate thickness, \( \rho \) the air density, \( \rho_s \) the density of the structure and \( \omega \) the angular frequency of vibration (\( \omega = 2\pi f \)). The

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*The apostrophe denotes the observation point on A*
3.5. Resonator design

quality factor finally corresponds to the reciprocal value of the loss factor \( Q = \frac{1}{\eta_{\text{rad}}} \).

The velocity distribution arising on a diaphragm at resonant operation depends on the structural vibration mode (modal shape) and the resonance frequency (3.16).

\[
v(r_s) = \omega z(r_s)
\]

Both were determined using the FEM software ANSYS. A non-linear structural analysis (large deflection) followed by a modal analysis were performed on a clamped plate vibrating in vacuum for pressure loads between 0 and 14 kPa. The numerical implementation of equation 3.13 and 3.14 and detailed information regarding the FEM model such as experimental validation and a mesh study can be found in Appendices A.1 and B.

Based on the Rayleigh approach and the FEM simulations the two parameters defining the sensing performance (damping and pressure sensitivity) were determined for plate resonators of different diaphragm shapes.

3.5.1. Diaphragm shape

The shape of the diaphragm has an influence on the vibrational mode shape and accordingly could play an important role in minimizing the radiation losses of a plate resonator.

Figure 3.6 shows all mode shapes (and labels) for a plate resonator of different diaphragm shape operating in a frequency range between 20 and 150 kHz. The data are evaluated using ANSYS (modal analysis) and represents an unloaded, clamped plate\(^7\). Four different diaphragm shapes such as a circular, square and two rectangular diaphragms with an aspect ratio of 1.5 and 2.5 were considered. Due to symmetry reasons the asymmetric modes on a square plate always appear twice at identical frequency but are differently aligned with respect to the \( x \) or \( y \) direction. These modes are considered together. It is obvious that the square and the circular diaphragm show similar mode shapes but differ considerably from the two rectangular plates.

The operating lines (left figure) and the damping behavior (\( Q \)-factor) for all considered modes of vibrations and diaphragm geometries are shown in Figures 3.7 to 3.10. An operating line relates the resonance frequency to the pressure load. The \( Q \)-factor is plotted versus the frequency but in conformance with the considered pressure load range between 0 and 14 kPa. The data are evaluated using ANSYS and the Rayleigh approach described before. In order to match the same operating frequency range for all diaphragm shapes the resonant surface area is equal for all geometries.

\(^7\)cf. Appendix A.1
3. Design considerations

As it is apparent in Figures 3.7a (circular) and 3.8a (square) certain modes show a higher pressure sensitivity in the lower pressure load range. At higher loading the pressure sensitivity is about the same for all modes. If the mode shape of the particular modes is considered one can suggest that all modes with a central peak (or peaks) seem to show the mentioned behavior. The first two modes of this group can also be classified as sym-

Figure 3.6: Calculated (ANSYS) vibrational mode shapes of a plate resonator of different shape operating in a frequency range between 20 and 150 kHz
metric modes\(^8\).

The circular and the square diaphragm behave similarly as well if the damping behavior is considered (Fig. 3.7b and 3.8b). The fundamental mode ((0,0) or (1,1)) is strongly damped by acoustic radiation with \(Q\)-factors below 100 and shows an increasing attenuation with rising frequency. The asymmetric modes show a similar behavior but are considerably less damped than the fundamental mode. The \(Q\)-factor increases with increasing mode number and reaches values of more than \(10^4\) for mode (4,1).

![Figure 3.7:](image)

(circular diaphragm). The aforementioned group of modes with a central peak(s) show in fact a lower damping than the fundamental mode but the \(Q\)-factor remains well below \(10^3\). They - except the fundamental mode - furthermore show a different damping characteristic, namely decreasing radiation losses with increasing frequency.

In order to operate the resonator over the whole pressure range the frequency band of the acoustic noise (excitation) has to match the frequency range of the considered mode (cf. shaded area in Fig. 3.7a, mode (0,1)). Due to an operating frequency range which is much larger than the separation between neighboring modes an overlap of operating lines occurs. Within the overlap region the mode which is more receptive for acoustic excitation dominates the vibration characteristic of the resonator. The re-

\(^8\text{circ.: (0,0) and (0,1), square: (1,1) and (3,1)}\)
3. Design considerations

Figure 3.8.: Operating lines and damping characteristics for a square diaphragm (a: 2 mm, t: 7 \(\mu\)m)

ceptivity to sound of a vibrating plate is equally related to its radiation efficiency, i.e. a good radiator is also a good receiver for sound (principle of reciprocity [45]). For a circular and a square diaphragm only the fundamental and the (0,1) (circ.) or (3,1) (rect.) modes, respectively dominate the vibration behavior over the whole pressure range. All neighboring modes suffer less by acoustic radiation losses and hence, are less receptive for acoustic excitation. The same also applies for mode (1,1) (circ.) and (4,1) (rect.) but their resonance frequencies at higher loading are beyond the operating range of the ultrasound transducer and cannot be considered for the present application. The red dashed lines in Figures 3.7 to 3.10 highlight the modes which can be applied for pressure sensing in terms of the compatibility of acoustic excitation.

Due to the different propagation lengths of flexural waves along the \(x\) and \(y\) direction of a rectangular diaphragm a larger number of modes exists in the considered frequency range compared with a circular or square diaphragm (cf. Fig. 3.9 and 3.10). The separation between operating lines is therefore smaller and cross-over between certain modes is obvious (e.g. mode (4,1) and (3,2) in Fig. 3.9). The damping characteristic of a rectangular plate shows the same qualitative behavior as observed for the circular and square geometries but the radiation losses are higher irrespective of the vibration mode. For an aspect ratio of 1.5 only the fundamental mode can be driven acoustically over the whole pressure range. The (4,1) mode is suppressed by the (1,3) mode for a pressure load smaller than 4 kPa. A
3.5. Resonator design

Figure 3.9.: Operating lines and damping characteristics for a rectangular diaphragm (a: 2.4 mm, AR: 1.5, t: 7 µm)

Figure 3.10.: Operating lines and damping characteristics for a rectangular diaphragm (a: 3.2 mm, AR: 2.5, t: 7 µm)

A rectangular diaphragm with an aspect ratio of 2.5 exhibits again two modes ((1,1) and (1,2)) which can be driven by acoustic noise excitation in the required pressure range.

Figures 3.11 and 3.12 compare the pressure sensitivity and damping characteristic of those modes which are suited for the present application. In each case the left figure shows the fundamental mode and the right one the
3. Design considerations

Figure 3.11.: Pressure sensitivity of a plate resonator (plate size 4 mm\(^2\), t: 7 µm) of different diaphragm shape and in a pressure range between 0 and 14 kPa. a) fundamental mode b) higher mode (circular: (0,1), square: (3,1), rect. AR= 2.5: (1,2))

Figure 3.12.: Damping behavior of a plate resonator of different diaphragm shape: a) fundamental mode b) higher mode (circular: (0,1), square: (3,1), rect. AR= 2.5: (1,2))

corresponding higher mode. Irrespective of the mode the circular and the square diaphragm show the highest pressure sensitivity but only within the low pressure load range. At higher loading the pressure sensitivity is about
the same for all geometries except the rectangular diaphragm (AR=2.5) which exhibits a higher sensitivity in its fundamental mode but lower sensitivity if operated in the higher mode. Similar findings result if the damping behavior is considered (cf. Fig. 3.12). In contrast to the circular and the square diaphragm operating in the higher mode a rectangular plate (AR=2.5) shows an increasing damping with increasing pressure load. If one assumes that the measurement performance of the resonator is characterized by both, the damping and the pressure sensitivity the regularly shaped plates should exhibit a higher and a more constant performance over the considered pressure range.

### 3.5.2. Diaphragm dimensions

The sensing performance of a plate resonator with a certain diaphragm shape, material properties and a negligible impact of the gas filled reference cavity is determined by its diaphragm size and thickness. The range of the two considered parameters is mainly limited by constraints such as fabrication and durability concerns (i.e. minimum diaphragm thickness) and the required spatial resolution (i.e. maximum diaphragm size). The minimum diaphragm thickness is set to 4 µm and the maximum diaphragm size to 2 mm. The validity of the stated thickness limit has to be verified yet in an experimental study.

In order to simplify the present design study a simple semi-empirical approach was used instead of more expensive FEM simulations. Thereby, the change in resonance frequency caused by a pressure load for a clamped diaphragm vibrating in vacuum, i.e. the operating line was determined. The validation of the theoretical model was carried out using FEM and experimental data (cf. Appendix A.1.2).

As discussed in Section 3.5.1 a circular or square diaphragm operating in the fundamental or (0,1) mode (square (3,1)) are best suited for the present application and therefore the only ones considered in the present study.

The change in resonance frequency caused by a transverse center deflection $w_z$ is described by equation 3.17 [46, 47].

$$f_{ij} = f_{ij,0} \left[1 + c_{ij}\left(\frac{w_z}{t}\right)^2\right]^{\frac{1}{2}} \quad (3.17)$$

The constant $c_{ij}$ depends on the vibration mode $(i,j)$, the diaphragm shape and the clamping conditions and is numerically determined using ANSYS (cf. Tab. 3.1 and Appendix A.1). Even though the agreement between theory and experiment is satisfying, the experiment indicates a higher pressure...
sensitivity within the lower pressure load range. Especially for smaller diaphragms this behavior is more pronounced (cf. Fig. A.7 and A.8). If this discrepancy is caused by acoustic-structure interaction phenomena is not yet clarified.

The resonance frequency for an unloaded diaphragm \( f_{i,j,0} \) is given by [48]

\[
 f_{ij,0} = \frac{\lambda_{ij}^2}{2\pi a^2} \left[ \frac{Et^2}{12\rho_s(1 - \nu^2)} \right]^{\frac{1}{2}} 
\]

(3.18)

where \( a \) denotes the edge length of a square or the radius of a circular diaphragm and \( t \) the diaphragm thickness. The relevant material properties of single crystal silicon (SCS) are the density (\( \rho_s = 2330 \text{ kg/m}^3 \)), the Poisson’s ratio (\( \nu = 0.3 \)) and the Young’s modulus (\( E = 130 \cdot 10^9 \text{ N/m}^2 \)). The latter parameter takes into account the anisotropy of SCS and was numerically verified (cf. Appendix A.1). The constant \( \lambda_{ij}^2 \) is again a parameter including the diaphragm geometry and is listed for a square and circular diaphragm in Table 3.1.

\[
 p_{load} = \frac{E}{(1 - \nu^2)}\left(\frac{t}{a}\right)^4 \left[ K_1(\frac{w_z}{t}) + K_2(\frac{w_z}{t})^3 \right] 
\]

(3.19)

The relationship between the pressure load \( p_{load} \) and the center deflection of the diaphragm is described in (3.19) [49, 47]. The equation takes into account large deflections, i.e. deflections which are larger than the plate thickness. The constants \( K_1 \) and \( K_2 \) are listed in Table 3.1.

**Table 3.1.:** Parameters prescribing the diaphragm geometry and the vibration mode in equations 3.17 to 3.19. Data without citation were numerically evaluated using ANSYS (cf. Appendix A.1).

<table>
<thead>
<tr>
<th>Shape (i,j)</th>
<th>( \lambda_{ij}^2 )</th>
<th>( c_{ij} )</th>
<th>( K_1, K_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular (0,0)</td>
<td>10.22</td>
<td>1.42</td>
<td>5.33, 2.83</td>
</tr>
<tr>
<td>(a = R)</td>
<td>([48])</td>
<td>(1.464 [46])</td>
<td></td>
</tr>
<tr>
<td>Square (1,1)</td>
<td>35.99</td>
<td>1.18</td>
<td>67.2, 25.28</td>
</tr>
<tr>
<td>(3,1)</td>
<td>132.2</td>
<td>1.35</td>
<td></td>
</tr>
</tbody>
</table>

The quality factor was evaluated based on equation 3.15 and \( Q = 1/\eta_{rad} \). The radiation efficiency \( \sigma \) corresponding to the particular diaphragm size was determined by an interpolation within the numerically evaluated parameter space defined by \( \sigma \) and \( \gamma \), both non-dimensional parameters. The latter parameter represents the acoustic wave number \( k \) normalized by the
3.5. Resonator design

panel wavenumber $k_p$ (3.20),

$$\gamma = \frac{k}{k_p} \quad k_p = \sqrt{\left(\frac{i\pi}{a}\right)^2 + \left(\frac{j\pi}{b}\right)^2}$$  \hspace{1cm} (3.20)

where $a$ and $b$ denote the size of a rectangular diaphragm and $i$ and $j$ are the indices of the particular mode $(i, j)^{10}$.

Because of similar performance of circular and square diaphragms$^{11}$ only results representing a square diaphragm are shown below. Figure 3.13 illustrates the relation between the diaphragm dimensions and the resonance frequency in a characteristic diagram (contour plot). The left figure represents the fundamental mode for an unloaded sensor which constraints the diaphragm dimensions with regard to the lower limit of the frequency operating range (30 kHz). For the upper limit of 150 kHz a diaphragm vibrating in mode (3,1) and at maximum pressure load of 14 kPa is considered (cf. Fig. 3.13b). The shaded areas highlight the permitted range of diaphragm dimensions. The dashed line in Figure 3.13b represents a reduced pressure range of 7 kPa and the dash-dotted line a range of 3.5 kPa.

$^{10}$ $k_p$ for a circular diaphragm cf. [45]

$^{11}$ cf. Sec. 3.5.1
Due to packaging and spatial resolution concerns a small sensor size is aimed for and thus the upper frequency limit becomes the limiting parameter in dimensioning. If the resonator is designed for a measurement range of 14 kPa, the limiting diaphragm size is situated between 1.6 and 1.8 mm depending on its thickness. Considerably smaller sensors can only be considered if the measurement range and the diaphragm thickness are reduced.

The mean pressure sensitivity (averaged over the pressure range of 14 kPa) increases with increasing diaphragm size but decreases considerably with increasing thickness (cf. Fig. 3.14). A 1.8 mm large resonator operating in mode (3,1) shows a mean sensitivity of approximately 4.5 Hz/Pa if 5 µm or 2.5 Hz/Pa if 7 µm thick. In contradiction to the pressure sensitivity the quality factor (reciprocal of the damping) increases with decreasing diaphragm size and increasing thickness, i.e. higher frequency (cf. Fig. 3.15). Size restrictions due to the upper frequency limit (mode (3,1)) and the requirements on the pressure sensitivity prevent that highest $Q$-factors are obtained.

The present study indicates a significantly improved sensing performance if the diaphragm thickness is reduced. A 4 µm thick plate resonator operating in mode (3,1) would exhibit a mean pressure sensitivity larger than 5 Hz/Pa. Within that thickness range the influence of the diaphragm size

---

12 sensor installation on small-size models with a high surface curvature
3.6. Acoustic structure interaction

The diaphragm of the plate resonator is coupled to the free surroundings on its front and to a gas-filled cavity and a micro-tubing system on its back face. In order to adapt the sensor design with regard to a maximum measurement performance potential coupling phenomena are identified and their impact on the sensing behavior estimated. On the outer side of the diaphragm acoustic radiation into the far field and inertial loading by the fluid do occur. The former phenomenon is suggested as the most significant kind of interaction in the present application. It is considered during the design study of the resonator (cf. Sec. 3.5). The pneumatic system on the back side of the diaphragm may cause reactive, dissipative and resonant interference phenomena. All mentioned effects are briefly treated below.

3.6.1. Fluid loading (free surroundings)

The reactive power generated by a vibrating plate submerged in a fluid is put into near-field kinetic energy which is bounded to the structure and
increases the kinetic energy of the system. The ratio between the kinetic energy of the fluid $W_f$ and the kinetic energy of the plate $W_s$ is often expressed as an added virtual mass incremental (AVMI) factor $\beta$ [50]. The non-dimensional added virtual mass incremental (NAVMI) factor $\Gamma$ is usually more convenient because it is independent of fluid and plate properties and of the plate geometry (3.21).

The Rayleigh integral approach$^{13}$ can be adopted to provide an estimate of the AVMI factor for a clamped plate vibrating in an arbitrary mode [45]. The model assumes that the fluid has no effect on the mode shapes of the plate. A schematic of the model and the corresponding nomenclature is given in Section 3.5.

$$\beta = \frac{W_f}{W_s} = \Gamma \frac{pa}{\rho_s t} \quad (3.21)$$

The kinetic energy of the fluid $W_f$ is evaluated using the relation

$$W_f = \int_{A'} \frac{1}{2\omega} \text{Im}\{p(r'_s)v(r'_s)\} \, dA'$$

which is the imaginary part of the product of the sound pressure $p(r'_s)$ and the surface velocity $v(r'_s)$ integrated over the vibrating surface $A$. Equation 3.22 can be written as

$$W_f = \frac{\rho}{4\pi} \int_{A'} \int_A v(r_s) \frac{\cos(kR)}{R} v(r'_s) \, dA \, dA' \quad (3.23)$$

if the sound pressure $p(r'_s)$ is expressed by the Rayleigh integral (3.12). The singularity in the Rayleigh integral disappears in (3.23) since $\cos(kR)/R \to -ksin(kR) = 0$ for $R \to 0$. The kinetic energy of the plate is evaluated using (3.24).

$$W_s = \frac{1}{2} \rho_s t \int_A v^2(r''_s) \, dA = \rho_s t A \langle v^2 \rangle \quad (3.24)$$

According to Lamb [51] the resonance frequency of a vibrating structure in a fluid $f_f$ is reduced by

$$\frac{f_f}{f_o} = \frac{1}{\sqrt{1+\beta}} \quad (3.25)$$

compared to a plate vibrating in vacuum $f_o$.

As already mentioned in Section 3.5 the velocity distribution on the vibrating plate depends on the considered modal shape and the corresponding frequency $\omega$ and was numerically evaluated using ANSYS. The numerical implementation of the present approach is given in Appendix B.2.

$^{13}$cf. Sec. 3.5
3.6. Acoustic structure interaction

Figure 3.16: NAVMI factor as function of the frequency \( f < 200 \text{ kHz} \) for a square, clamped diaphragm and for the first 4 vibration modes \((1,3)\) mode not considered

Figure 3.16 shows the NAVMI factor versus the frequency for the first 4 modes of a square, clamped plate \((1,3)\) mode not considered. The fluid loading effect is highest for the fundamental mode within a frequency range which is relevant for the present application and the particular mode \((f < 100 \text{ kHz})\). The higher modes are less affected by the fluid and are less dependent on the frequency than the fundamental one. The \((3,1)\) mode shows the lowest NAVMI factors for frequencies higher than 100 kHz.

NAVMI factors found in literature are considered to be independent of the frequency and are stated as \( \Gamma_{11} = 0.35, \Gamma_{12} = 0.16 \) and \( \Gamma_{22} = 0.12, [52] \). If compared with the present data, their validity is only given for frequencies lower than 100 kHz.

The shift in resonance frequency caused by the fluid and its dependence on the diaphragm dimensions is shown in Figure 3.17a. The frequency shift is expressed as a frequency ratio defined in \((3.25)\) and averaged over the frequency range which corresponds to a pressure range of 14 kPa. Only the two modes suitable for the present application and frequencies below 200 kHz are considered. The impact of the fluid is minimized if the plate thickness is increased and the diaphragm size reduced. The shift in frequency is smaller than 1 % for the \((3,1)\) and smaller than 3 % for the fundamental mode and is basically captured by the sensor calibration. However, the AVMI factor depends on the fluid density and thus may impose a temperature cross sensitivity which could degrade the measurement performance.

An estimate of that effect is represented in Figure 3.17b. Similar to Figure 3.17a the mean value of the temperature sensitivity is shown. The
3. Design considerations

Figure 3.17: Reactive fluid loading effect for a square, clamped diaphragm of different size and thickness vibrating in air ($p = 10^5$ Pa and $T = 293$ K). Only the fundamental (1,1) and (3,1) mode are considered. a) Frequency shift according to (3.25) and b) temperature cross sensitivity caused by reactive fluid loading averaged over a pressure load range of 14 kPa.

Sensitivity of the fluid loading effect to temperature changes is determined if the fluid density in (3.21) is expressed by the ideal gas law ($\rho = p/RT$) and equation 3.25 is differentiated with respect to the temperature $T$ (3.26).

$$\frac{df_f}{dT} = \frac{\beta}{\sqrt{(1 + \beta)^3}} \frac{f_o}{T}$$

(3.26)

Based on this simple estimate a temperature sensitivity smaller than 10 Hz/K or 0.03 %/K for a frequency range of 30 kHz may arise.

3.6.2. Plate-cavity interaction

The interaction between a plate resonator and a gas-filled cavity can have multiple consequences. Effects of viscosity may cause damping or the reactive component of the fluid loading may alter the resonance frequency and mode shape. Furthermore, coupling between structural and acoustic vibrational modes could cause constructive or destructive interference and thus may affect the measurement performance of the sensor. In order to estimate the impact of all these interaction phenomena on the sensing behavior of the plate resonator an elaborate numerical study would be necessary which is beyond the scope of the present thesis. Therefore, the different phenomena were treated individually by numerical or analytical means.
3.6. Acoustic structure interaction

Figure 3.18: Structural mode induced change in cavity volume (modified from [53]): a) Symmetric modes (e.g. (1,1) mode of a square diaphragm) experience an added stiffness effect b) Asymmetric modes (e.g. (1,2) mode of a square diaphragm) experience an added mass effect due to fluid pumping

Dependent on the structural mode shape the reactive component of the fluid loading induces an added mass or an added stiffness effect [54]. The strength of both effects increases with decreasing cavity height $H_c$. For a symmetric mode, the shape of the dynamic diaphragm displacement causes a change in cavity volume and as a result a variation of the cavity pressure (cf. Fig. 3.18a). The resonator becomes stiffer and the resonance frequency increases. Asymmetric modes do not experience an added stiffness effect because no change in cavity volume is induced. The air is pumped back and forth and thereby imposes an added mass which consequently reduces the resonance frequency of the plate (cf. Fig. 3.18b).

According to Beltman et al. [53] the mode shape of asymmetric modes is not affected by the cavity. However, the symmetric modes may experience a change in shape with decreasing cavity height and tend to resemble more to an asymmetric mode with zero net volume change (cross coupling of modes).

The added mass and stiffness effect are independent of viscous effects but are dependent on fluid properties such as density and speed of sound. Both properties are dependent on the temperature and may therefore influence the temperature cross sensitivity of the sensor.

FEM simulations using COMSOL 3.5a were performed, in order to quantify the effect of added mass and stiffness on the sensing behavior of the plate resonator. The software includes a 3D model to handle inviscid acoustic-structure interaction phenomena. The resonance frequencies and the corresponding modal shapes were evaluated by means of a time-harmonic analysis. The implementation and validation of the numerical model is described in Appendix A.2.1.

Figure 3.19a shows the change in resonance frequency for different modes of vibration if the height of the cavity is reduced from 800 to 25 $\mu$m. The relation is expressed as the ratio between the actual resonance frequency and the corresponding value for a 800 $\mu$m high cavity, representing approx-
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![Graphs](image)

**Figure 3.19.** Influence of a gas-filled cavity on the sensing behavior of a plate resonator with a clamped, square diaphragm (size: 1.4 mm, thickness: 7 µm) evaluated by FEM: a) Change in resonance frequency versus cavity height for the first 4 structural modes of vibration. $f_c$: actual frequency, $f_o$: frequency for $H_c = 800$ µm b) Temperature cross sensitivity averaged over a temperature range between 280 and 310 K

The vibration behavior in vacuum. The aforementioned different behavior of the symmetric and asymmetric modes is reproduced. A resonator with a 200 µm high cavity and operating in its fundamental mode (1,1) exhibits a resonance frequency increase by approximately 10%. The added stiffness effect is also apparent for mode (3,1), but is much less pronounced compared to the (1,1) mode.

To evaluate the temperature cross sensitivity caused by the fluid loading effect simulations for different cavity heights and temperatures between 280 and 310 K were carried out. Figure 3.19b shows the temperature cross sensitivity averaged over the considered temperature range of 30 K. As is obvious the asymmetric modes are more influenced by temperature changes than the symmetric modes. Especially for shallow cavities the temperature sensitivity increases strongly whereas for the fundamental and (3,1) mode the sensitivity remains more or less constant at values comparable to the added mass effect on the outer side of the diaphragm (cf. Sec. 3.6.1).

The impact of fluid loading on the shape of symmetric modes ((1,1) and (3,1)) is illustrated in Figure 3.20. The mode shape seems to be only affected by the fluid for a very narrow gap between diaphragm and cavity bottom of 25 µm. For a cavity height larger than 100 µm no deviation
3.6. Acoustic structure interaction

Figure 3.20.: Shape profiles of symmetric structural modes for a clamped square diaphragm coupled to a gas-filled cavity of different height $H_c$ (FEM simulations): a) fundamental mode $(1,1)$ b) $(3,1)$ mode

between the shape profiles is apparent.

To quantify the damping caused by viscous effects within the cavity a viscothermal acoustic FEM model is required which is not yet implemented in COMSOL 3.5a. Therefore, only a qualitative estimate of the losses, based on the work by Beltman et al. [53], was carried out. The paper deals with a numerical and experimental study on the interaction between a thin gas layer and a vibrating structure. For the numerical study a viscothermal acoustic FEM was used which is based on the thin layer model and is implemented in the FEM package B2000.

Similar to the Reynolds number which represents the ratio between inertial and viscous forces and is used to estimate the importance of viscous effects in steady flows, the shear wave number does the same for unsteady flow phenomena. The shear wave number is defined as

$$s = H_c \sqrt{\frac{\rho \omega}{\mu}}$$  \hspace{1cm} (3.27)

where $\rho$ represents the fluid density, $\omega$ the frequency and $\mu$ the dynamic viscosity. For a cavity with a height $H_c$ of 200 $\mu$m and frequencies between 30 and 150 kHz the shear wave number varies between 20 and 50. According to Beltman et al. [53] the damping coefficient increases strongly for $s < 20$ and reaches values up to 25 % for mode $(1,2)$ and 10 % for the fundamental mode at $s = 7$. The higher damping for the asymmetric mode
is explained by the pumping effect caused by the mode shape. For $s > 20$ a damping coefficient smaller than 2% for mode (1,2) and smaller than 1% for the symmetric mode is stated. Hence, for mode (1,1) the viscous effects within the cavity would have approximately the same contribution to the overall damping as the losses caused by acoustic radiation into the free surroundings. For the asymmetric modes the dominant source of losses should rather be the cavity.

The last considered interaction scenario between the diaphragm and the cavity could be related to interference between the structural and acoustic vibration modes. In order to verify if a cavity with dimensions as required in the present application exhibits modes within the frequency range between 30 and 150 kHz, an analytical formula for acoustic modes in a closed fluid-filled cavity was used (3.28) [48].

$$f_{(ijk)} = \frac{c}{2} \sqrt{\frac{i}{a}}^2 + \left(\frac{j}{b}\right)^2 + \left(\frac{k}{H_c}\right)^2$$

$i,j,k = 0, 1, 2, ...$ (3.28)

The size of a rectangular cavity is described by the parameters $a$ and $b$, its height by $H_c$, and $c$ represents the speed of sound in air ($c = 343$ m/s).

The first mode which is defined by the cavity height is situated at frequencies higher than 800 kHz for $H_c = 200$ µm. The cavity height can therefore be excluded as the critical parameter. As is shown in Table 3.2 the lateral extent of the cavity defines lower modes which can be located within the considered frequency range, such as the fundamental mode (100)

![Figure 3.21.: First two acoustic modes (acoustic pressure) in a closed, fluid-filled cavity determined using COMSOL 3.5a a) Rectangular cavity (100) and b) (110) c) Trapezoid cavity (100) and d) (110)](image-url)
at frequencies below 150 kHz for all considered cavities. The two largest cavities exhibit even two modes within the frequency range of interest.

Table 3.2.: Acoustic modes in a closed, fluid-filled cavity of rectangular and trapezoid shape with a cavity height of 200 µm. The data are evaluated using the FEM package COMSOL 3.5a. The data in brackets represent eq. 3.28.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Rectangular</th>
<th>Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size a [mm]</td>
<td>f(_{100}) [kHz]</td>
<td>f(_{110}) [kHz]</td>
</tr>
<tr>
<td>1.4</td>
<td>122.5 (122.5)</td>
<td>110.9</td>
</tr>
<tr>
<td>1.6</td>
<td>107.2 (107.2)</td>
<td>98.2</td>
</tr>
<tr>
<td>1.8</td>
<td>95.3 (95.3)</td>
<td>134.8 (134.7)</td>
</tr>
<tr>
<td>2</td>
<td>85.8 (85.8)</td>
<td>121.3 (121.3)</td>
</tr>
</tbody>
</table>

The question if a cavity with non-vertical side walls could show a more suitable acoustic behavior is treated by considering a trapezoid cavity. Such a cavity has 54.7° angled side walls and can be easily manufactured by wet etching of single crystal silicon\(^{14}\). The more complicated cavity geometry necessitated the use of the FEM package COMSOL 3.5a (modal analysis). To validate the numerical model, acoustic modes for the rectangular cavity were computed and compared with (3.28). As shown in Table 3.2 and Figure 3.21 the first two modes do not differ notably from the rectangular cavity. The frequencies are even lower due to the larger bottom surface of the cavity.

3.6.3. Plate-microchannel interaction

The main requirement in the design of the pneumatic tubing system is motivated by its potential impact on the sensing behavior of the plate resonator which has to be minimized. Special requirements for the time needed for back pressure adjustment (i.e. maximum flow throttling) are not defined because during data acquisition the pressure controller is switched off\(^{15}\). The geometry of the microchannels is basically constrained by dimensions of the sensor unit such as the maximum tolerable height of the sensor carrier of approximately 300 µm (Kapton foil) and the separation distance between the individual sensing elements (i.e. required spatial resolution). The maximum channel depth \(h_p\) is set to 50 µm and the width \(b_p\) to 200 µm in order to prevent a contraction of the channel cross section caused by

\(^{14}\text{cf. Sec. 4}\)
\(^{15}\text{cf. Sec. 5.1}\)
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the compliance of the cover foil if a pressure difference between cavity and surroundings is present (cf. Fig. 3.22). The channel length $L_p$ is defined as the half of the separation distance between the individual sensing elements and is in the range of several millimeters.

![Microchannel geometry and dimensions](image)

**Figure 3.22.** Microchannel geometry and dimensions

In order to characterize the dynamic flow behavior inside the channel the shear wave number $s$ defined in equation 3.27 and the product of the acoustic wave number $k = 2\pi/\lambda$ and the channel length are considered. The shear wave number based on half the channel height and a frequency of 100 kHz is approximately 20. Hence, an influence of viscous effects has to be expected (cf. Sec. 3.6.2). The wavelength $\lambda$ of the acoustic wave at the same frequency is approximately 2 mm. For a channel longer than 0.3 mm the product $kL_p$ becomes larger than 1 and wave propagation phenomena should arise within the duct system.

The plate resonator, the reference cavity and the microchannels form a coupled system which has to be considered as a whole in order to characterize its dynamic behavior and the influence of the individual components. A simple method which is often used for the analysis of dynamic systems is based on the lumped parameter model. This approach is based on the analogy between electrical and fluid or mechanical systems. The model assumes a temporal but not a spatial variation of the physical parameters and thus does not account for wave propagation phenomena. The validity of the model is therefore only given if the physical size of the considered system is small relative to the wavelength of vibration. Because this requirement is not fulfilled for the present sensor unit (due to the length of the microchannels and the lateral dimension of the cavity), the model is solely applied for a rough estimate of the dynamic system behavior.

The resonator system can be expressed as a first-order electrical network (cf. Fig. 3.23b). The pressure and the particle or volume velocity of the fluid system are equivalent to the voltage and the current in an electrical system, respectively. The excitation acoustic pressure field $p(t)$ is represented as an alternating voltage source and the velocity of the vibrating
3.6. Acoustic structure interaction

\( p(t) \), \( I, R_{\text{rad}} \), \( S, C, I_{\text{dd}} \), \( V, C, I_{\text{cc}} \), \( H_{\text{c}} \), \( L_{\text{p}} \), \( R, I_{\text{pp}} \), \( p_{\text{o}} \), \( b_{\text{p}}, \alpha \), \( \alpha - \alpha \)

\( (a) \)

\( (b) \)

**Figure 3.23.** a) Schematic drawing of the sensor unit (only 1 sensing element considered) b) The equivalent electrical circuit of the sensor unit (Lumped network model)

diaphragm as a current \( q(t) \). The diaphragm is modeled as a spring \( C_{d} \) (or capacity) and an inerance \( I_{d} \) (or inductor), the reference cavity similarly and the venting channel as a dissipative element \( R_{p} \) (or resistor) and an inerance \( I_{p} \). The interaction between the resonator and the open surroundings causes dissipative losses by acoustic radiation \( R_{\text{rad}} \) and an increased inertia of the vibrating plate due to fluid loading effects \( I_{\text{rad}} \). Dissipative losses are neglected for the diaphragm and the cavity and the compliance of the fluid is not considered for the venting channels due to the small volume.

The individual lumped elements of the specific system components can be combined and expressed as a complex impedance \( Z = R + i(\omega I - 1/\omega C) \). The plate resonator \( Z_{d} \), the free surroundings \( Z_{a} \) and the cavity \( Z_{c} \) form a serial electrical circuit. The venting channels \( Z_{p} \) and the cavity are connected in parallel. In a parallel circuit the total impedance is governed by the smallest impedance of the circuit. Hence, one can argue, that as long as the impedance of the venting channel is much larger than the impedance of the cavity, the venting channels should have a negligible impact on the system dynamics and do not have to be considered during the design study of the sensor unit. In order to clarify the influence of the microchannels only the impedance of the cavity and of the channels is estimated and compared against each other.

The lumped elements of a network can usually be approximated using simplified expressions as used for the estimate of the cavity impedance (3.29) [55].

\[
Z_{c} \approx S_{d} \left[ \frac{1}{i\omega C_{c}} + i\omega I_{c} \right] \quad \text{[kg/s]}
\]

\( ^{16} \) cf. Sec. 3.5 and 3.6.1
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\[
C_c = \frac{V_c}{\rho c^2}\quad \text{and} \quad I_c = \frac{m_c}{3S_d^2}
\]

(3.29)

The expression is valid for a cavity with side walls of similar size compared to the diaphragm surface area (\(\equiv\) top cover of the cavity) and if following constraint is fulfilled: \(0.3 \leq ka \leq 1\). The surface area of a square diaphragm is \(S_d = a^2\), \(V_c\) represents the volume of the cavity and \(m_c\) the mass of the enclosed gas.

Dependent on the frequency of the oscillating flow in a duct the velocity profile can considerably differ from the typical profile of a Poiseuille flow (steady laminar pipe flow) which is normally used for estimating the dissipative part of the channel impedance. Therefore, the channel impedance is calculated based on an expression derived by Morris et al. [56]. It is obtained by solving the Navier-Stokes equations for incompressible, oscillatory, fully developed flow in a straight duct of constant, rectangular cross-section with an aspect ratio \(\alpha = h_p/b_p\) driven by an alternating pressure difference (3.30).

\[
Z_p = \frac{\rho L_p \omega S_d}{\alpha b_p^2 i} \left\{ 1 - \sum_{n=0}^{\infty} \frac{2}{p_n^2} \left[ \frac{\tanh(\alpha r_n)}{\alpha r_n} + \frac{\tanh(s_n/\alpha)}{s_n/\alpha} \right] \right\}^{-1}
\]

with \(p_n = \frac{\pi}{2}(2n+1)\) \(r_n = \sqrt{\eta^2 + p_n^2}\) \(s_n = \sqrt{\eta^2 \alpha^2 + p_n^2}\) and \(\eta^2 = \frac{\omega b_p^2}{4\nu}\) \(n=0\)

(3.30)

Figure 3.24 shows the impedance of the cavity and the venting channels as function of the frequency within a range between 0.1 and 250 kHz. As is apparent, the cavity shows a pronounced behavior of a spring which is characterized by a decreasing impedance with increasing frequency. Hence, the plate resonator is less influenced (or stiffened) by the closed cavity with increasing operating frequency or increasing cavity volume (compare the impedance for \(H_c = 200 \mu m\) and 300 \(\mu m\)). The impedance of the venting channels is dominated by dissipative losses for frequencies below 10 kHz and shows a negligible influence of frequency. At higher frequencies the ineritance starts to dominate the dynamic behavior of the channel and the impedance increases with increasing frequency. Furthermore, the impedance increases with the length of the channel (i.e. increasing mass of the fluid) and decreases with an increasing number of channels \(N\) according to a parallel electrical circuit where \(Z_{tot} = Z/N\) applies. At operating frequencies larger than 50 kHz the channel impedance is larger than...
the impedance of the cavity even for the smallest considered cavity and a pneumatic tubing layout with five 1 mm long microchannels. At frequencies above 100 kHz the impedance of the two elements differs by one or more orders of magnitude, i.e. the dynamic behavior of the system should basically be not affected by the tubing system. This finding was verified by experimental tests where different tubing system geometries were investigated (cf. Sec. 5.3.4).

A suitable design of the pneumatic tubing system should exhibit a high impedance and therefore small channel cross sections and a small number of channels. The impact on the sensing behavior of the plate resonator by resonance effects within the ducts is reduced due to the weak, pulsating flow (high $Z_p$) and due to high viscous losses in a tiny duct. In addition, the individual sensing elements can be spaced closer without inducing potential cross coupling effects caused by wave propagation in the ducts.

### 3.7. Diaphragm’s surface topography

A parallel optical interrogation of many sensors distributed on a large and to a certain degree curved surface requires an appropriate design of the light modulating element (i.e. diaphragm of the plate resonator). The aim is to maximize the modulated light signal for a broad range of angles between the illumination and observation beam path. This can be achieved with a topography on the top surface of the diaphragm which shows diffuse scattering or even retro-reflecting capabilities. For the modulation of the
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interrogation signal by interferometric means a coherent and monochromatic light source is used, so that diffraction and dispersion phenomena could degrade the sensors’ interrogation. Hence, both phenomena have to be avoided or minimized.

Besides the optical requirements an appropriate surface morphology is also constrained by thermal and mechanical considerations. In order to prevent thermal stresses due to the bi-metal effect, the surface structure is to be directly implanted into the single crystal silicon (SCS) diaphragm and should not be created by deposition of different materials. This constraint imposes a limit on the maximum structure or roughness height of the SCS diaphragm surface, which is given by the degradation of its fracture strength with increasing depth of the roughness elements. Henning et al. [57] reports a decrease of membrane fracture strength for surface roughness higher than 100 nm, reaching almost 70% at 1 µm roughness height. A similar observation is given in [58].

The most suitable topology on the the top surface of the plate resonator would have high retro-reflecting capabilities, i.e. a high fraction of the incident light would return back to the source origin. The most common retro-reflectors are the corner-cube with a geometry similar to a regular triangular pyramid or the full corner-cube formed by three adjacent facets of a cube (cf. Fig. 3.25a and 3.25b).

To transfer a corner-cube geometry into a SCS surface different fabrication processes are described in the literature. Neudeck et al. [59] describes a fabrication process where micrometer scale (5 to 50 µm in size) full corner cube arrays were grown on a {111}-silicon substrate by selective epitaxial growth techniques. The regular gratings showed excellent diffraction properties but only poor retro-reflection capabilities.

The fabrication of corner cubes based on anisotropic wet etching of silicon is not feasible because the etch process is restricted by the crystal lattice of SCS [60]. Using special wet etchants and a proper orientation of the square mask window with respect to the crystal orientation, grooves with a geometry similar to a regular square pyramid are obtained [61, 62, 63]. Such grooves have walls angled 45° to the substrate ({100}-wafer) surface and show retro-reflection capabilities but only for certain directions of illumination.

Isotropic dry etching in combination with gray-tone lithography offers the possibility to transfer arbitrarily shaped structures into silicon [64]. In order to obtain structures with precise and mirror-like surfaces, an expensive raster-screened (gray-tone) photomask with minimum feature size in the sub-micron and a tolerance in the nanometer range is required [65].

This brief overview of retro-reflector fabrication on SCS substrates shows that retro-reflecting structures can be imprinted on silicon surfaces with a
3.7. Diaphragm’s surface topography

certain amount on fabrication complexity. Their implementation is con-
strained by the size of the individual reflectors of several micrometers to
achieve a sufficient high fraction of specular reflection within the cube struc-
ture. This constraint is not compatible with the aforementioned maximum
allowable roughness height of 200 to 300 nm for mechanical strength rea-
sons.

Figure 3.25.: Schematic illustration: a) Corner Cube reflector b) Full Corner
Cube reflector c) Triangular roughness profile

A surface with highly diffuse scattering properties shows a small fraction
of specular reflection and scatters the stray light evenly over a wide angle
of the hemisphere surrounding the surface. Based on a purely qualitative
study of the scattering behavior on rough surfaces a proper surface topogra-
phy is defined. For it the roughness is characterized using two parameters
which are defined as the ratio between the rms roughness height $\sigma$ and
the wavelength of visible light $\lambda$ and the ratio between the autocorrelation
length $l_c$ and $\lambda$ [66]. The latter parameter is closely related to the average
surface wavelength of the surface structure.

The Rayleigh criterion defines the limiting roughness height where a sur-
face is still considered as smooth (3.31) [67].

$$\sigma < \frac{\lambda}{8 \cos(\theta)} \quad (3.31)$$

For green light ($\lambda \approx 500 \text{ nm}$) and normal incidence ($\theta = 0^\circ$) the limiting
roughness height is approximately 80 nm.

The parameter TIS (Total Integrated Power) is the ratio of scattered
power $P_{scat}$ (without specular reflected light) in one hemisphere to the
specularly reflected power $P_{ref}$ (3.32).

$$TIS = \frac{P_{scat}}{P_{ref}} = (4\pi \cos(\theta))^2 \left[ \frac{\sigma}{\lambda} \right]^2 \quad (3.32)$$
3. Design considerations

The expression, even though only valid for smooth surfaces ($\sigma \ll \lambda$), qualitatively shows the strong influence of roughness height on the scattered power. Again, for green light and normal incidence the TIS equals approximately 6 for 100 nm and almost 60 for 300 nm roughness height.

The angular dependencies of scattered light can be described using the modified Beckmann-Kirchhoff (B-K) model which is further simplified, if one assumes that the angle of incidence corresponds approximately to the viewing direction [68]. The model is valid for rough surfaces and large angles of incidence and scatter. The qualitative dependence of the scattered intensity $I(\theta)$ on the roughness parameters $\sigma$ and $l_c$ is given in (3.33).

$$I(\theta) \propto \cos^{-1}(\theta) \exp(- (l_c \tan(\theta)/2\sigma)^2) \quad (3.33)$$

Figure 3.26 shows this relationship as a function of the incidence angle for different values of $m = \sigma/l_c$. It is obvious that the scattering behavior attains a more and more diffuse character with increasing $m$. It has to be mentioned that for $m > 0.2$ multiple scattering may occur and thus the B-K-Model has to be considered with care for moderately to very rough surfaces [69].

![Figure 3.26](image)

Figure 3.26.: Normalized scattered intensity as function of the incidence angle and for different roughness characteristics ($m = \sigma/l_c$), eq. 3.33

The original wafer surface is either polished and thus highly specular or unpolished with a certain rough texture caused by the grinding process during wafer fabrication [70]. Although unpolished its scattering property is quite poor due to a rms roughness height $\sigma$ and a correlation length $l_c$ of approximately 0.5 $\mu$m and 2.5 $\mu$m, respectively, and hence a ratio $\sigma/l_c$ of only 0.2 [71]. For such a surface topography the back scattered light is
3.8. Conclusion

If one assumes that the diaphragm surface is roughened by imposing tiny grooves into the SCS surface (\{100\}-wafer) using a wet etchant, the surface topography can be approximated as a triangular roughness profile with an angle \( \gamma \) of \( 54.74^\circ \) between the triangle edge and the surface (Fig. 3.25c). In order to maximize \( m \) the individual triangles are attached to each other. The height of a triangle \( H \) corresponds to the maximum allowable roughness height of 300 nm and the spatial surface wavelength to \( L = \sqrt{2H} \). The corresponding correlation length is estimated based on \( l_c = \sqrt{2\sigma/\Delta_q} \) [69] with \( \sigma = H/2\sqrt{3} \) and the rms slope \( \Delta_q = 2H/L \) [72]. The correlation length equals approximately \( 0.2L \) and the ratio \( \sigma/l_c \) unity. This rough estimate points out that in spite of the geometrical restrictions a highly diffuse scattering surface is theoretically feasible.

Diffraction phenomena on a surface topography with roughness elements in the sub-wavelength size range (sub-wavelength grid) should not appear because only the zero orders are transmitted or reflected into the far field [73]. Dispersion should be minimized due to randomly varying \( \sigma \) and \( l_c \) (within a certain range) over the rough surface, but a loss in scattering intensity has to be accepted.

3.8. Conclusion

Based on the present study the relevant design requirements were identified and the design of the sensor unit defined. In order to maximize the measurement performance, a resonant sensor with a diaphragm thickness of 4 \( \mu m \) (assumed thickness limit due to mechanical strength concerns) and a diaphragm size of 1.6 mm has to be aimed for. A circular diaphragm shows a slightly better sensing performance than a square diaphragm but the latter one is more advantageous in terms of fabrication concerns.

If the resonator is driven by acoustic noise only the fundamental and the second symmetric vibration mode (square diaphragm: (3,1), circular: (1,0)) are suitable to cover the whole pressure range of 14 kPa. The fundamental mode is highly damped by acoustic sound radiation and thus should not meet the required measurement performance. The higher symmetric mode shows a lower damping and a favorable damping characteristic. A decreasing attenuation with rising sensor load can at least partially compensate the loss in pressure sensitivity at higher pressure loads.

Due to an operating frequency range for mode (3,1) above 100 kHz, flow interference or interference with environmental disturbances should not occur. Drawbacks at high operating frequencies are a high directivity
of the CUT and a lower sensitivity of the SPDA (detector) due to a shorter integration time.

A reference cavity with a height of 200 µm should meet the requirements on the overall thickness of the sensor unit (i.e. foil). Due to the added stiffness effect the resonance frequency increases by approximately 10 % for the fundamental mode but only by less than 1 % for the higher mode. The temperature cross sensitivity caused by fluid loading on both sides of the diaphragm is expected to be smaller than 0.06 %/K. Viscous effects within the cavity were not quantified but should be of similar magnitude as the losses caused by acoustic radiation. A small number of microchannels with a small cross section (200 x 50 µm) should have a negligible influence on the sensing performance of the resonator.

A suitable topography of the top surface of the diaphragm should be formed by densely packed and randomly distributed pyramidal grooves with a maximum depth of 300 nm and a lateral extent of approximately 500 nm. Thus, the diffuse light scattering property is maximized and diffraction, dispersion and the impact on the mechanical strength of the diaphragm are minimized.

The first prototype of the plate resonator, whose fabrication is described in the next chapter and on which all experimental tests shown in the present report are based, has a square diaphragm with a thickness of 7 µm and different sizes between 1.4 and 2 mm. Its cavity is 400 µm high. These sensor dimensions do not provide the highest measurement performance since they are mainly motivated by a simpler fabrication using 400 µm thick wafers and a mechanically more robust diaphragm.
4. Sensor unit fabrication

The main components of a sensor unit are the plate resonator, its carrier (laser structured Kapton foil) which also provides a pneumatic link between the sensor's cavity and the pressure controller. The resonator is fabricated in MEMS technology on a Silicon on Insulator (SOI) wafer (batch-fabrication) whereas for the polymer carrier a fabrication method based on laser ablation is applied. The present chapter discusses the basic fabrication techniques used for the manufacture of the first prototype of the sensor unit and also includes a detailed description of the entire fabrication process.

4.1. Plate resonator

In order to obtain a pressure sensor with a precise, uniform and reproducible thickness of the diaphragm two MEMS fabrication processes using standard photolithography and wet or dry anisotropic etching are usually employed. The first one is based on wet etching of a single crystal silicon (SCS) wafer in combination with an electrochemical etch stop method and the other on wet or dry etching of a SOI wafer [70, 60].

A SOI wafer is a composite of two silicon layers separated by a thin Buried Oxide Layer (BOX). The thinner silicon layer is called the Device Layer (DL) and corresponds to the diaphragm whereas the support of the diaphragm is provided by the Handle Layer (HL), the second silicon layer of the SOI. During the release of the diaphragm the HL layer is removed within predefined regions (photolithography) using anisotropic wet or Deep Reactive Ion Etching (DRIE). The etch stop is precisely defined by the BOX layer. During the DRIE process a cavity is formed with almost vertical walls whereas wet etching usually (e.g. KOH etchant) produces walls angled 54.74° to the substrate ({100}-wafer) surface and thus leads to a larger overall chip size.

Due to the high complexity of a precise and repeatable electrochemical etch stop method and its more difficult integration into the entire fabrication process the SOI-based fabrication method was chosen in the present work. The drawback of higher costs due to the expensive substrate is negligible during the preliminary phase.

As described in Section 3.7 the most suitable surface topography of the
4. Sensor unit fabrication

diaphragm’s top surface features a densely packed and random arrangement of grooves with a depth and width of approximately 300 and 500 nm. In order to roughen the whole diaphragm top surface of the resonator (up to 4 mm²) one would need to implement up to 16 millions tiny grooves per sensor into the silicon surface. This creates stringent constraints on the fabrication process.

Wet etching of square grooves requires a predefined opening within the oxide mask which corresponds to a square window with the size of a groove’s footprint or an arbitrary geometry with a prolongation (in or perpendicular to the wafer flat direction) similar to the grooves’s width (approx. 500 nm) [60]. With common optical photolithography processes (UV light) the limiting feature size is around one micron [74]. Furthermore, the fabrication of an appropriate chromium mask would be very time-consuming or even unfeasible. Using (electron)-beam lithography features far smaller than one micron can be transferred directly to the oxide mask, but the writing process is sequential and thus the writing too time consuming for the present application [74].

Due to the mentioned difficulties, a novel fabrication process for an efficient roughening of a silicon surface was developed in the present work. The process is based on thin film deposition of poly-silicon (poly-Si) and wet etching in TMAH (Tetramethylammonium Hydroxide) and does not require any photolithographic masking steps.

\[ \text{(a)} \]

\[ \text{(b)} \]

**Figure 4.1.** a) Schematic representation of HSG formation [75] b) Transmission electron microscopy of a HSG on a-Si after annealing during 20 min [76]

The morphology of poly-Si thin films deposited on a Si/SiO₂ surface by low-pressure chemical vapour deposition (LPCVD) using silane (SiH₄) gas depends on different process parameters such as deposition and annealing temperature, silane pressure, etc.. For a silane pressure of 20 Pa and a deposition temperature above 590 °C poly-Si is formed whereas below 560 °C amorphous silicon (a-Si) is deposited which can be crystallized in a
subsequent annealing process [75]. If \textit{in-situ} annealing under high vacuum is performed the formation of crystalline silicon hemispheres (HSG) occurs.

During annealing first the nuclei are formed which then grow by capturing migrating Si atoms on the a-Si surface (cf. Fig. 4.1a). As shown by Watanabe et al. [77] the size and density of the HSGs depend on the annealing time and temperature. At low temperatures large and low density whereas at high temperatures small and high density grains are formed. The depletion of a-Si around a grain (cf. Fig. 4.1b) forms a pattern which was used as a mask required for structuring the actual passivation layer (SiO\textsubscript{2} film).

The fabrication of the prototype was carried out at the Center of Micronanotechnology (CMI) at the EPFL (École Polytechnique Fédérale de Lausanne) in Switzerland. The detailed process is described next.

\subsection*{4.1.1. Fabrication process}

![Diagram of sensor design and geometrical parameters](image)

<table>
<thead>
<tr>
<th>Value [mm]</th>
<th>$L_S$</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_S$</td>
<td>$\approx 0.4$</td>
<td></td>
</tr>
<tr>
<td>$l_M$</td>
<td>1.4, 1.6, 1.8, 2</td>
<td></td>
</tr>
<tr>
<td>$\Delta l$</td>
<td>0.1, 0.2, 0.3, 0.4</td>
<td></td>
</tr>
</tbody>
</table>

\textbf{Figure 4.2.:} Sensor design and geometrical parameters of the 1\textsuperscript{st} prototype (CAD drawing)

Figure 4.2 shows the front and back side of the sensor and the relevant dimensions of the current prototype. The prototype does not feature the required overall height $H_S$ of 200 $\mu$m but is 400 $\mu$m high due to better handling capabilities during fabrication. The overall chip size $L_S$ of 4.5 mm also accounts for a larger safety margin to insure a strong support of the diaphragm. The minimum allowable chip size has to be evaluated in a separate study. Four different diaphragm sizes $l_M$ between 1.4 and 2 mm with a diaphragm thickness of 7 $\mu$m were manufactured. To account for a potential failure of the diaphragm due to the reduced fracture strength caused by the roughness, a margin $\Delta l$ of 0.1 to 0.4 mm between the rough area and the diaphragm’s edges was considered in the current design\textsuperscript{1}. The

\textsuperscript{1}For a deflected clamped diaphragm the maximum stresses emerge close to the edges.
4. Sensor unit fabrication

The rear side of the sensor is structured (10 µm wide and 100 µm deep cavities) to improve the sealing and attachment between sensor and Kapton carrier.

**Table 4.1.:** Schematic representation of the fabrication process and the relevant process parameters

<table>
<thead>
<tr>
<th>Step</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong></td>
<td>Wet oxidation (LPCVD) 500 nm, passivation layer (PL)</td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
<td>Photolitho DL, AZ9260_2µm, mask 1 Oxide dry etch (DRIE) 500 nm Resist removal O₂-plasma</td>
</tr>
<tr>
<td><strong>Step 3</strong></td>
<td>Dry oxidation (LPCVD) 10 nm</td>
</tr>
<tr>
<td><strong>Step 4</strong></td>
<td>a-Si deposition (LPCVD) ≈10 nm, T = 500 °C, t = 25 min in-situ annealing (UHV) T = 510 °C, t = 60 min</td>
</tr>
<tr>
<td><strong>Step 5</strong></td>
<td>Oxide wet etch (BHF) T = 22 °C, t = 30 sec</td>
</tr>
</tbody>
</table>
4.1. Plate resonator

<table>
<thead>
<tr>
<th>Step</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 6</td>
<td>Dip in HF(1%) ( T = 22 , ^\circ C, t = 30 , \text{sec} )</td>
</tr>
<tr>
<td>Silicon wet etch (5wt % TMAH + 0.5wt % surfactant)</td>
<td>( T = 60 , ^\circ C, t = 5 , \text{min} )</td>
</tr>
<tr>
<td>Step 7</td>
<td>Backside photolitho ( \text{AZ1512.2}\mu m, ) mask 2, front to backside align.</td>
</tr>
<tr>
<td>Oxide dry etch (DRIE)</td>
<td>100 nm</td>
</tr>
<tr>
<td>Resist removal</td>
<td>O(_2)-plasma</td>
</tr>
<tr>
<td>Step 8</td>
<td>Backside photolitho ( \text{S1805 G2.1}\mu m, ) mask 3, align. on mask 2</td>
</tr>
<tr>
<td>Oxide dry etch (DRIE)</td>
<td>1500 nm</td>
</tr>
<tr>
<td>Step 9</td>
<td>Silicon dry etch (DRIE) ( \text{Si-Bosch process [78]} )</td>
</tr>
<tr>
<td>Step 10</td>
<td>Oxide removal ( \text{(BHF)} ) ( T = 22 , ^\circ C, t = 15 , \text{min} )</td>
</tr>
</tbody>
</table>
4. Sensor unit fabrication

Table 4.1 shows a schematic illustration of the fabrication process flow together with the most important process parameters. As substrate a 4" \{100\}-SOI wafer was used. In a first step, a passivation layer (PL) of 500 nm wet silicon oxide (SiO$_2$) is deposited on the substrate. Accordingly, the already existing SiO$_2$-layer on the back side was thickened to 1.5 µm. With standard photolithography (mask 1) and Deep Reactive Ion Etching (DRIE) of SiO$_2$ square windows are defined on the device layer where rough surface has to be created (step 2).

After resist removal, a 10 nm thick dry oxide film (DOX) is deposited on the wafer to get a mask layer for the roughening process (step 3). During silicon deposition in a LPCVD furnace an approximately 10 nm thick amorphous silicon layer (a-Si) is formed (step 4). The HSGs are formed during \textit{in-situ} annealing under ultra high vacuum (UHV) based on the procedure described before. As shown in Figure 4.3a the grains are randomly distributed on the a-Si surface and show a size between 20 and 100 nm. The darker area adjacent to a grain corresponds to the depletion zone (consumed a-Si). Within this zone the DOX layer is released and can be subsequently removed in Buffered Hydrofluoric Acid (BHF) (step 5).

The individual roughness elements are etched in a surfactant (NCW-601A) added Tetra Methyl Ammonium Hydroxide (TMAH) bath (step 6). TMAH is highly selective to SiO$_2$ (>10000:1) and therefore mainly suitable for silicon etching where thin masks (PL) are used [79] (cf. Fig 4.3c). By adding surfactant in TMAH the etch rate is reduced and the controllability of the etching process improved.

The backside is structured based on two lithography processes. During the first step the square pattern (on sensor’s rear face) is defined and transferred into the oxide mask up to an etch depth of 100 nm using DRIE (step 7). The pattern is aligned on the rear substrate surface with respect to the structure on the front side (front to backside alignment). After resist removal, the mask windows corresponding to the sensor cavities are added to the oxide mask during the second lithography and DRIE (etch depth: 1.5 µm)(step 8).

For the alignment of mask 3 alignment marks applied during the previous lithography step are used. The complete structure on the backside of the substrate is dry etched (DRIE) in one step. To obtain vertical cavity walls the Si-Bosch process [78] is applied (step 9). The etch process stops as soon as the BOX layer is reached (after approx. 400 nm). The resist (1 µm thick) and the remaining oxide delay the silicon etch of the backside pattern relative to the etch of the sensor’s cavity. Therefore, as well as

\[\begin{array}{l}
\text{Device Layer: } 7\pm0.5 \mu m, \text{ Doping: pB, Resistivity: } 1-3 \Omega cm, \text{ BOX: } 1 \mu m, \text{ Handle} \\
\text{Layer: } 400\pm15 \mu m \\
\text{pyramidal grooves with } \{111\}-\text{walls (cf. Fig. 4.3d)}
\end{array}\]
due to a decreasing etch rate with increasing aspect ratio of the structure (height/width) the corresponding grooves become approximately only 100 \( \mu \text{m} \) deep.

In the last step all remaining oxide is removed in BHF during 15 minutes (step 10). Sawing of the wafer was carried out without any protective coating of the front side.

**Remark** During the second photolithography step (step 7) an alignment error occurred (front to backside alignment) which could not be reversed at that late fabrication stage. The misalignment caused a shift between the rough area on the top surface of the diaphragm and the diaphragm...
4. Sensor unit fabrication

itself. The extent of this shift depends on the chip location on the wafer and reaches values up to 200 µm. The shift may cause an asymmetric stress distribution within the deflected diaphragm and thus may influence the vibrating behavior.

4.1.2. Optical characterization of the surface roughness

A quantitative optical characterization of the surface roughness using an appropriate measurement device such as a light scattering goniometer was not yet performed. However, the light scattering property of the surface was estimated based on two statistical roughness parameters, the rms roughness height $\sigma$ and the correlation length $l_c$, and the theoretical model for light scattering on rough surfaces described in Section 3.7 (cf. eq. 3.33). The roughness parameters were determined based on surface profile data obtained from measurements using an atomic force microscope (AFM).

The dispersion and diffraction property of the surface were estimated purely by a visual inspection. As is apparent in Figure 4.6a the rough surface appears nearly white if illuminated with white light. Hence, dispersion of light due to wavelength dependent scattering properties of the surface should be small. During illumination with monochromatic coherent light no diffraction effects were observed.

The roughness profile measurements were carried out using a commer-

![Figure 4.4:](image)

**Figure 4.4:** a) AFM image of the rough diaphragm surface (scan area 10 x 10 µm) b) Autocorrelation function $G$ based on the profile data (sample 1 to 4) and Gaussian fit of $G$ for sample 1
cial AFM with a MFP-3DTM controller (Asylum Research, Santa Barbara, CA) operated in tapping mode in air. Ultrasharp Silicon cantilevers (NSC15/Cr-Au, Mikromasch, Tallinn, Estonia) were used with a nominal tip radius of 50 nm. The acquired surface profiles represent a scan area of 10 x 10 µm and exhibit a resolution of 512 x 512 pixels. Four images were obtained from separate locations across the sample to ensure reproducibility. A good agreement between the surface topography acquired using the scanning electron microscope (SEM) (Fig. 4.3d) and the AFM (Fig. 4.4a) can be noticed.

Table 4.2.: Statistical roughness parameters used to characterize the light scattering property of the rough diaphragm surface

<table>
<thead>
<tr>
<th>Sample</th>
<th>σ [nm]</th>
<th>σₜ [nm]</th>
<th>lₖ [nm]</th>
<th>m [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61.3±4.7</td>
<td>272.2±24</td>
<td>156.7</td>
<td>0.39</td>
</tr>
<tr>
<td>2</td>
<td>63.4±4.9</td>
<td>263±23</td>
<td>153.3</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>63.5±4.3</td>
<td>259.2±22</td>
<td>151.8</td>
<td>0.42</td>
</tr>
<tr>
<td>4</td>
<td>65.8±6.1</td>
<td>277±34</td>
<td>162.3</td>
<td>0.41</td>
</tr>
</tbody>
</table>

The analysis of the surface profile data was performed using the software Gwyddion. The rms and the maximum roughness heights σ and σₜ, respectively shown in Table 4.2 represent the mean value and the standard deviation of all 512 1D profiles (column of an image) of an AFM image. The correlation length lₖ can be determined based on the autocorrelation function G of the profile data and corresponds to the translation τ required to drop from the peak value of G by a factor of e⁻¹ [72]. Figure 4.4b shows the autocorrelation function of all 4 acquired images determined using the aforementioned software. Only the translation range between 0 and 1 µm is shown. The definition used for lₖ is only valid if the G characteristic is Gaussian. The Gaussian fit of the data corresponding to sample 1 (red solid lined) confirms the validity of lₖ determination based on the present roughness data.

All samples show similar roughness parameters. The rms and the maximum roughness height is approximately 63 nm and 270 nm, respectively and lₖ corresponds to approximately 155 nm. The corresponding ratio \( m = \sigma / \tau \) of approximately 0.4 indicates a surface with a highly diffuse scattering property (cf. Fig. 3.26). If one assumes that the tip of the probe cannot fully reproduce the V-shape of the roughness elements a higher effective σ and thus even a higher parameter m should result.

The obtained σ is far below the critical value of approximately 200 nm
4. Sensor unit fabrication

where a degradation of the mechanical fracture strength has to be expected. Hence, a further improvement of the scattering surface property is conceiv-able if $\sigma$ is further increased and $l_c$ decreased. This would require a better understanding of the roughening process and a detailed study of the relevant process parameters in order to obtain an optimum surface roughness.

4.2. Sensor Carrier

For experimental sensor characterization purposes acrylic holders were used which enable a fast and simple adaptation of the pneumatic tubing design without the need for sensor replacement. The microchannels (200 $\mu$m wide and 50 $\mu$m deep) were milled using a mechanical CNC-milling machine with a cutter diameter of 0.2 mm. The design of those sample holders is briefly described in Section 5.2.

In order to verify the measurement system under real measurement conditions (i.e. wind tunnel) a NACA 0012 airfoil was equipped with five plate resonators along its chord of 300 mm (cf. Sec. 6.1). They are embedded in a thin Kapton™ foil which also acts as a carrier of microchannels used for setting and controlling the pressure in the individual sensor cavities. Below, the sensor foil design and the fabrication procedure are described.

![Sensor foil design](image)

**Figure 4.5.** Sensor foil design: a) Sensor placement on the foil and layout of the pneumatic system, through-holes for pneumatic access of the foil (zoomed view) b) Assembly of the Kapton™ foil (4-layer composite) and layout of the sensor carrier (layer 2)

The sensor stripe is 240 mm long and 40 mm wide. Figure 4.5a shows the placement of the individual sensing devices on the foil. Due to the
occurrence of higher static pressure gradients within the front section of the wing the first three sensors are closely spaced at 30 mm followed by the fourth and fifth with a separation distance of 40 mm and 61 mm. The foil forms a composite of four Kapton™ layers with a thickness of 125 µm each. The adhesive film (25 µm thick) on one side of each layer provides a sufficiently strong bond between the layers.

The microtubing system is implemented in the bottom layer (layer 1). As it is shown in Sections 3.6.3 and 5.3.4 a tubing system containing only a small number of channels with a cross section of 200 µm x 50 µm has a negligible influence on the sensing characteristic of the plate resonator. Hence, each sensor is supplied by eight radially oriented channels (approx. 8 mm long) which end in a square cavity on the one end and in three circumferential channels on the other end. The square structure in the center forms the ending of the sensor’s cavity. Eight channels per sensor were chosen in order to fulfill the demand on symmetry and redundancy (channel obstruction). Five microchannels of same size as mentioned before interconnect the individual sensing devices. A through-hole (Ø1 mm) at the end of each longitudinal channel forms the pneumatic link between foil and wind tunnel model (cf. Fig. 4.5a, zoomed view).

Layer 2 covers the structure of the bottom layer and also provides the pneumatic access to the sensor’s cavity. In order to minimize the overall foil thickness the sensors are recessed by 75 µm into layer 2. The bottom floor of such a recess is only 50 µm thick and thus reinforced with ribs to improve its stability within the remaining gap outside of the chip (cf. Fig. 4.5b). The dies (diaphragm size: 2mm, thickness: 7 µm) are glued on layer 2 using an elastic silicon adhesive (RTV110) (cf. Fig. 4.6a). The same adhesive is used to fill the remaining gap between sensor and the openings in layers 3 and 4.

Critical dimensions of the Kapton layer such as the small thickness and the large area which has to be structured were the crucial factors for choosing laser micromachining as a fabrication tool instead of mechanical micromilling. Surface structuring by material removal based on laser ablation is a cost efficient technology which usually offers a precision in the submicron range. For polymer structuring where high precision and surface quality are required it is important that the removal of material mainly occurs by direct bond breaking of molecular chains due to the absorbed photon energy (i.e. photochemical ablation) [80]. Thermal ablation, i.e. breaking the bonds by heat (melting and vaporization) leads to surface damages and has to be minimized by choosing the right process parameters. For long wavelength lasers (upper visible and infrared range) the thermal ablation dominates the material removal, because of photon energies which are significantly lower than the dissociation energies of common molecular bonds.
4. Sensor unit fabrication

Figure 4.6.: a) Sensor attached to the carrier foil (layer 2). Layer 3 and 4 are not added yet b) Surface and edge quality after laser ablation (channel depth 50 µm and width 200 µm), optical profilometer (zoomed view)

High photon energies of UV lasers and a strongly increased absorption behavior for $\lambda$ smaller than 550 nm of polyimides [81, 82] justify the laser ablation of polymers in the UV wavelength range. Thermal ablation can also be reduced if the laser pulses are sufficiently short to minimize heat diffusion and therefore energy losses into the substrate. Hence, nano- or picosecond lasers are used for a precise structuring of polymers [83].

Figure 4.7.: Laser (ablation) structuring facility at IPPE (FHNW)

In the present work a laser micromachining (research) facility at the Institute of Product and Production Engineering IPPE at FHNW\(^4\) was

\(^4\)University of Applied Sciences Northwestern Switzerland (FHNW), Windisch (AG)
4.2. Sensor Carrier

Table 4.3.: Process parameters for Kapton<sup>TM</sup> structuring

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>350 nm</td>
<td>Translation speed</td>
<td>0.9 m/s</td>
</tr>
<tr>
<td>Output power</td>
<td>0.52 W</td>
<td>Ablation rate</td>
<td>4 µm/pulse</td>
</tr>
<tr>
<td>Energy per pulse</td>
<td>0.52 µJ</td>
<td>Lens (focal length)</td>
<td>100 mm</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>1 MHz</td>
<td>Laser spot (diameter)</td>
<td>≈10 µm</td>
</tr>
<tr>
<td>Pulse width</td>
<td>≈10 ps</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

used (cf. Fig. 4.7). The system is based on a picosecond UV (355 nm) laser source (Time-Bandwidth Duetto) and a fast and precise galvanometer scanner (Scanlab). The laser beam is focused down to a spot diameter of approximately 10 µm using a lens with a focal length of 100 mm and a aperture of 1.5 mm.

The sample was mounted on a micropositioning stage which enables a precise adjustment of the sample and laser beam in space. For fine adjustment of the sample position with respect to the focal length (z-position) the flame front was observed during laser ablation using an ocular, searching for the maximum flame intensity. Due to the limited scan area of about 70 mm the particular foil structure was divided in 5 individual designs. After each writing step the sample was shifted by a certain amount using the positioning table (x-direction). In order to cope with possible misalignment between the writing steps, the microchannels are broadened by a spot of 1 mm in diameter at each joint.

Figure 4.6b shows the laser structured microchannels based on the process parameters listed in Table 4.3. The images recorded with an optical microscope and profile measurements using an optical profilometer (zoomed view) demonstrate the high surface and edge quality which is attainable by a suitable laser ablation process.
5. Experimental verification of the resonator behavior

In order to verify the pressure sensing behavior of the newly fabricated resonant sensors experimental tests were carried out in a temperature and pressure controlled test cell.

5.1. Measurement setup and procedure

The test cell is made out of acrylic and forms a cylindrical pressure vessel with a inner diameter of 170 mm, a length of 200 mm and a wall thickness of 4 mm (cf. Fig. 5.1). The rear closure (acrylic) includes the pneumatic and electrical interfacing and is screwed tightly to the flange of the cylindrical vessel. The front cover is formed by an aluminum ring which serves as a supporting frame for the optical access, a round glass window with a diameter of 170 mm and a thickness of 6 mm. A screw-locking system which is based on a fine thread on the outer and inner side of the aluminum flange and ring, respectively, enables a fast access to the cell. For heat insulation
and absorption of sound waves the inner wall of the cell is covered with a synthetic felt. The test cell is mounted on a 3-axis tilt stage in order to simplify the alignment between sensor and the interrogation and excitation unit located outside the cell (cf. Fig. 5.1).

The temperature was set and controlled using a Peltier heater and two heat exchangers between 20 and 30 °C (control accuracy < 0.02 °C ). An electrical fan at the inlet of the air heat exchanger forces the flow to pass the exchanger and the air circulation in the cell (cf. Fig. 5.2). The heat on the other side of the Peltier element is dissipated using a water heat exchanger driven by a water pump. The heater is supplied with a DC voltage between ±10 Volts. The corresponding power supply (control unit [4]) is controlled by an analog output port of the data acquisition unit DAQ I (NI PCI-6036E) and a control algorithm implemented in Matlab. The gas temperature is monitored near the resonator using a thermocouple. The heater and the two heat exchangers form a unit which also serves as a carrier for the sample holder. The whole unit is suspended by one bracket which is mounted in a guide rail on the inner wall of the test cell and thus is freely movable in the longitudinal direction.

Two pressure control systems are implemented in the present measurement setup (cf. Fig. 5.2). One system is used to set and control the pressure in the reference cavity coupled to the sensor’s diaphragm (back pressure). It is based on a fast and highly accurate (< 10 Pa) pressure controller (PACE 5000, GE Sensing). The controller is linked with a high pressure source (< 50 kPa) formed by a pressure vessel containing compressed dry air. As a low pressure source a tank connected to a vacuum pump is used. The pressure in the vacuum tank is electronically monitored and the pump switched on using an electronic switch if the pressure exceed -0.3 bar (controller protection). A serial interface (RS-232) between controller and PC allows a remote, full control of the device using Matlab.

The pressure in the test cell is set and controlled using a custom built control system. The system is based on 6 pneumatic valves driven by the digital output ports of the data acquisition card DAQ I and a control algorithm implemented in Matlab. For each channel (high, low and ambient pressure) two valves are used, one for a fast and rough adjustment and the second, throttled valve for the fine control of the cell pressure. The same high and low pressure sources are applied for both control systems. The cell pressure is measured using a pressure transducer (Honeywell 26PC Series) with a measurement range of ±35 kPa and a measurement uncertainty smaller than 70 Pa. The accuracy of the sensor is further improved after a re-calibration using the PACE 5000 controller.

$1^{50 \text{ kPa} \equiv \text{maximum input pressure!!}}$


5.1. Measurement setup and procedure

Figure 5.2.: Measurement setup as used for optical excitation in combination with a control of the cell pressure and temperature and the back pressure (2 control systems). If acoustic excitation was applied, the test cell was kept open (no temperature and pressure control) and only the back pressure was controlled.

The pressure and temperature in the test cell are acquired using the DAQ I system and are used as actual value input signals for the control algorithm. During the resonator readout both pressure control systems were switched off after the nominal value was reached. As soon as the actual cell pressure
deviated by more than ±20 Pa (back pressure ±10 Pa) from the nominal value, the controller was switched on again and the actual measurement was aborted.

For the majority of the tests the pressure in the test cell was kept at ambient conditions. The sensor was loaded by setting and controlling the back pressure as it is the case during in-situ calibration\(^\text{2}\). This measurement configuration differs from the actual one\(^\text{3}\) due to the different pressure boundary conditions on both sides of the diaphragm, but it simplifies the measurement procedure considerably because only one pressure level has to be controlled. However, the different pressure boundary conditions alter the mass loading of the diaphragm and hence its vibrating behavior. This effect was experimentally verified and is treated in Section 5.3.1.

Due to the limited space inside the test cell acoustic excitation could not be applied for investigations where temperature adjustment and control were required. Instead, an external pulsed (< 1 MHz) laser diode (FP-64 635 nm, 10 mW, Laser Components GmbH) was used as an excitation source. The diaphragm is locally heated by the laser beam and the induced thermal stresses are driving the resonant structure with the particular excitation frequency [84]. The diode is supplied with 5 V\(_{\text{DC}}\) and modulated using a TTL signal delivered by a programmable waveform generator (WW1071, Tabor Electronics). In order to maximize the excitation strength the laser beam was focused (laser spot < 50 µm at a working distance of approx. 100 mm) and positioned in the center area or at the edge of the diaphragm where maximum mechanical stresses arise during a transverse diaphragm vibration.

The vibration amplitude was then measured using a single-point laser Doppler vibrometer (LDV) (CLV-2534-3, Polytec GmbH) from outside of the test cell (working distance approx. 200 mm). Similar to the laser diode the laser beam was focused (spot diameter approx. 25 µm) and positioned on the diaphragm so as to maximize the interrogation signal using a linear micro-positioning stage.

The LDV output signal was sampled with 500 kHz during 0.2 s using a fast data acquisition card DAQ II (NI PCI-6110). Its amplitude was then determined based on a dual phase lock-in demodulation algorithm\(^\text{4}\) (5.1). The acquired LDV signal \(u(t)\) is described as a sine wave with an amplitude \(U\), a frequency \(\omega\) and an arbitrary phase \(\phi\). If the signal is multiplied once by a sine and once by a cosine with equal frequency, and a time averaging over a sufficiently long time is carried out, the amplitude is determined from the two time averaged signals. The corresponding diaphragm displacement

\(^2\)cf. Sec. 2.3
\(^3\)back pressure is kept constant at a certain level and the ambient pressure varies
\(^4\)cf. Sec. 2.2.2, [85]
is evaluated if $u(t)$ (representing a velocity) is divided by $\omega$.

$$u(t) = U \sin(\omega t + \phi)$$

$$c_o = \frac{1}{T} \int_0^T u(t) \sin(\omega t) dt = \frac{U}{2} \cos(\phi)$$

$$c_1 = \frac{1}{T} \int_0^T u(t) \cos(\omega t) dt = \frac{U}{2} \sin(\phi)$$

$$U = 2 \sqrt{c_o^2 + c_1^2}$$

During a test run the excitation frequency was swept across the range of interest with steps of 20 Hz in the neighborhood of the resonance peak. The resonance frequency was found by locating the peak value of the displacement amplitude spectrum. Due to the constant excitation power delivered by the laser diode over the frequency range of interest very smooth amplitude spectra are obtained (no averaging necessary). An accurate evaluation

![Figure 5.3.: Displacement amplitude spectrum of a clamped square diaphragm (size: 1.8 mm, thickness: 7 µm) for a pressure load of 5.2 and 6 kPa (obtained with optical excitation) a) mode (1,1) and principle of $Q$-factor determination based on the amplitude spectrum b) (3,1) mode](image)

of the resonance frequency is enabled in this way (cf. Fig. 5.3).

Compared to acoustic noise excitation the monochromatic optical excitation provides a better analysis of the vibrating behavior regarding the damping behavior ($Q$-factor) and interference phenomena caused by fluid-structure coupling. The quality factor was determined from the amplitude spectrum based on the definition $Q = f_{res}/\Delta f_{3dB}$ where $f_{res}$ and $\Delta f_{3dB}$
5. Experimental verification of the resonator behavior
denote the resonance frequency and the band width of the amplitude spec-
trum, respectively [23] (cf. Fig. 5.3a).

During the tests with acoustic excitation the test cell was kept open. The capacitive ultrasound transducer (CUT) and the sensor were 1 m apart and were aligned in order to maximize the interrogation signal. The power supply of the CUT\textsuperscript{5} was fed with narrow band electrical noise generated in Matlab using a white noise generator and an elliptic filter algorithm. The signal was then delivered by the aforementioned wave generator to the power supply. The resonant diaphragm vibration induced by the acoustic excitation was recorded using the LDV.

For demodulation of the vibrometer output signal an ordinary FFT and a more robust FFT approach, the Welch method, which averages multiple Fourier spectra, was applied [86]. A rectangular window, a segment length of 500 and an overlap of 50\% were used for the Welch algorithm as implemented in Matlab. In total 100 ksamples acquired with 500 kHz sample rate were used for processing. Averaging over 20 subsequent measurements was performed.

5.2. Sample holder

The sensor holder is designed with the objective to enable a simple pneu-
matic access to the sensor’s cavity (back pressure control) and a simple modification of the pneumatic tubing design without the need for sensor replacement.

It is made up of two micro mechanically milled parts out of acrylic (cf. Fig. 5.4a). They are fused together using an adhesive film. The upper part exhibits an enlarged recess into which the sensor is inserted using a silicon filler material. Due to the recessed mount only a thin (approx. 0.1 mm) supporting structure remains between sensor and the bottom part of the holder (cf. Fig. 5.6). Hence, the sensor’s cavity is not considerably enlarged and the stability of the holder is sufficiently high to enable the separation of the two holder parts without sensor damage.

The specific geometry of the recess prevents any interference effects be-
tween the plate resonator and potential acoustic resonances within the open cavity. Such a phenomenon is obvious in Figure 5.5a where the damping behavior of a 1.4 mm large resonator recessed in a 3 mm deep and 5.5 mm wide cavity with vertical walls (old sensor holder) is shown\textsuperscript{6}. The interaction between cavity and diaphragm causes destructive and constructive interference effects, leading to a strongly varying damping behavior.

\textsuperscript{5}cf. Sec. 2.1
\textsuperscript{6}Old holder geometry is indicated by the dashed outline in Fig. 5.6
5.2. Sample holder

Figure 5.4: Sample holder designs: a) Holder design with an optimized recess geometry in the upper part b) Bottom part of the sensor holder (microchannel layout) c) Laser structured Kapton foil as the upper part of the sensor holder d) Rear side geometry of the front cover of the test cell used for tests with an open reference cavity of the sensor. The sensor and its holder on the front side are not shown.

with very high damping at certain frequencies (or pressure loads). Using an improved holder design (new holder) with an enlarged cavity, a relatively uniform damping behavior is achieved over the operating range of the sensor (dashed line in Fig. 5.5a). If compared with a holder configuration where the sensor is flush mounted with the holder surface (embedded in a 0.15 mm thick Kapton foil, cf. Fig. 5.4c) no particular variations in the damping behavior are obvious (cf. Fig. 5.5b). Furthermore, Figure 5.5b

\[ \text{\textsuperscript{7}cf. closed and open test cell} \]
5. Experimental verification of the resonator behavior

Figure 5.5: Verification of the sensor holder design and sensor positioning in the test cell: a) Q-factor versus pressure load of a 1.4 mm sensor attached to the old (chip 6) and the improved (chip 33) holder design. b) Q-factor vs. frequency for mode (1,1) and (3,1) of a 1.8 mm large sensor (chip 42) mounted on a Kapton carrier (open & closed cell) and on the newly designed holder.

Figure 5.6: Sketch of the sensor holder and geometry of the pneumatic tubing system (lateral view). Dashed outline indicates the recess geometry of the old holder design.

Verifies a negligible boundary effect caused by the front cover of the test cell, if a separation distance between the sensor and wall of approximately 80 mm is maintained.

The bottom part of the sensor holder includes the microchannel layout (radially oriented channels), a circular ring channel as settling chamber and a pneumatic access (cf. Fig. 5.4b). In order to verify the influence of the pneumatic tubing system on the resonator’s sensing behavior different microchannel layouts with varying channel number, length and depth were
manufactured and tested (cf. Table 5.1). The minimum channel width of 200 \(\mu\text{m}\) and a depth of 50 \(\mu\text{m}\) were limited by the available mechanical milling process. The width and the depth of the settling chamber are the same for all holder configurations but its volume changes dependent on the channel length (cf. Fig. 5.6).

**Table 5.1.** Considered microchannel layouts: the channels are all radially oriented, exhibit a square cross section and are all 200 \(\mu\text{m}\) wide.

<table>
<thead>
<tr>
<th>Config.</th>
<th>Chip No.</th>
<th>No. of channels</th>
<th>Height [(\mu\text{m})]</th>
<th>Width [(\mu\text{m})]</th>
<th>Length [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63</td>
<td>1</td>
<td>50</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>61</td>
<td>4</td>
<td>50</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
<td>8</td>
<td>50</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>62</td>
<td>4</td>
<td>50</td>
<td>200</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>59</td>
<td>4</td>
<td>50</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>57</td>
<td>4</td>
<td>100</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>58</td>
<td>4</td>
<td>200</td>
<td>200</td>
<td>10</td>
</tr>
</tbody>
</table>

To identify the effect of a closed reference cavity on the sensing behavior of a plate resonator, tests with an open cavity were performed. For these tests the actual front cover of the test cell was replaced by a 10 mm thick plate of acrylic. The afore-described sensor holder (upper part) was mounted centrally on the outer side of that plate and aligned with a through hole in the plate. To preserve approximately the original depth of the sensor’s cavity and thus to prevent any acoustic resonance effects, the rear side of the plate is thinned down to a wall thickness of 3 mm on an area of 70 mm x 50 mm. The area in the vicinity of the through hole was additionally shaped similar to the recess of the actual sensor holder (cf. Fig. 5.4d).

Due to a thermal expansion coefficient mismatch the sensor carrier and mount (i.e. silicon filler) may have a considerable influence on the temperature sensitivity of the sensor unit. Hence, the experimental verification of the temperature cross sensitivity was carried out using the final Kapton foil containing 5 sensing elements and applied for wind tunnel tests on a NACA 0012 wing (cf. Chapter 6). The foil design and dimensions are described in Section 4.2. In order to embed the 240 mm long foil in the test cell, the pressure vessel of the cell was extended by an additional cylindrical element.
5. Experimental verification of the resonator behavior

Figure 5.7.: Installed pressure sensor foil in the test cell (left). During handling, acrylic covers protect the sensing elements which are not under investigation. Test cell extension required for the placement of the foil inside the test cell (right) with a length of 200 mm (cf. Fig. 5.7). The foil was clamped at one single point to minimize mechanical cross influences by the mount itself.
5.3. Results

5.3.1. Influence of reversed pressure boundary conditions

The discrepancy between the sensor loading procedure applied during calibration and the loading principle valid during the actual measurements was quantified by tests where both types of loading were verified. As shown in Section 3.6.1 the shift in resonance frequency caused by fluid loading is small (< 2%) and thus the effect of different pressure boundary conditions in a range between ±10 kPa even smaller.

Being able to properly capture the mentioned effect experimentally, the temperature in the test cell was not controlled so as to avoid any pressure fluctuations induced by the controller. To minimize the measurement error caused by a varying gas temperature, the two tests with an equally loaded sensor but using a different loading procedure were performed successively. Additionally, the pressure transducer used for cell pressure measurements was calibrated using the pressure controller and calibrator PACE 5000. All results shown in the present section are based on tests carried out on 4 sensing elements of the pressure sensor stripe described in Section 4.2.

Figures 5.8 and 5.9 show the deviation in resonance frequency between the two types of sensor loading for a symmetric (3,1) and an asymmetric vibration mode (1,2). For mode (3,1) a pressure load range between 0.6

\[\text{Figure 5.8:} \ \text{Discrepancy in resonance frequency for mode (3,1) between the calibration procedure } f_{\text{calib}} \text{ and the measurement procedure } f_{\text{meas}} \text{ for sensor 1 to 4 on the Kapton stripe. Markers: experiment, solid line: data fit, dashed line: theory}\]
5. Experimental verification of the resonator behavior

and 12.6 kPa and for mode (1,2) a range between 0.6 and 4.6 kPa were considered. Dependent on the back pressure $p_b$ setting, the pressure in the test cell $p_m$ varied between $\pm 10$ kPa for mode (3,1) and between $\pm 5$ kPa for the asymmetric mode. For mode (3,1) and a $p_b$ of -3 kPa the corresponding cell pressure ranges between -2.4 and 9.6 kPa and for a $p_b$ of -10.2 kPa between -9.6 and 2.4 kPa. For mode (1,2) the corresponding values are: 0 to 4 kPa at $p_b = -0.6$ kPa and -4 to -0.4 kPa at $p_b = -5$ kPa. The error bars indicate the measurement uncertainty if optical excitation and LDV interrogation are applied. The dashed lines represent the theory where fluid loading (cf. Sec. 3.6.1) is assumed on both sides of the diaphragm and the solid line denotes a polynomial of 2nd degree which fits the data using the least square method (cf. Table 5.2).

![Figure 5.9](image)

**Figure 5.9.** Discrepancy in resonance frequency for mode (1,2) between the calibration procedure $f_{\text{calib}}$ and the measurement procedure $f_{\text{meas}}$. Markers: experiment, solid line: data fit, dashed line: theory

The discrepancy between the two loading principles increases with increasing $p_m$ but does not exceed $\pm 200$ Hz at a $p_m$ of $\pm 10$ kPa for mode (3,1). At zero cell pressure both pressure boundary conditions are the same and the deviation remains within or close to the measurement uncertainty.

For mode (1,2) a similar qualitative behavior is obvious but the effect is more pronounced than for the symmetric mode. Deviations in resonance frequency of $\pm 150$ Hz are attained already at a $p_m$ of $\pm 5$ kPa. Interesting is also the disagreement between theory and experiment for the asymmetric mode whereas a fair agreement can be noticed for the symmetric mode.

If one assumes an average pressure sensitivity of the resonator of 3 Hz/Pa

\(^9\) pressure range where acoustic excitation is feasible

\(^{10}\) standard deviation based on 20 subsequent measurements
Table 5.2: Coefficients of a polynomial of 2\textsuperscript{nd} degree that fits the experimental data in Figures 5.8 and 5.9

<table>
<thead>
<tr>
<th></th>
<th>(c_1) [Hz/Pa(^2)]</th>
<th>(c_2) [Hz/Pa]</th>
<th>(c_3) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mode (3,1)</td>
<td>-44.48 (\cdot) 10(^{-8})</td>
<td>-17.35 (\cdot) 10(^{-3})</td>
<td>19.5</td>
</tr>
<tr>
<td>mode (1,2)</td>
<td>-11.58 (\cdot) 10(^{-7})</td>
<td>-28.7 (\cdot) 10(^{-3})</td>
<td>28.76</td>
</tr>
</tbody>
</table>

and a measurement uncertainty of 20 Pa an \textit{in-situ} calibration without a correction is valid within a pressure measurement range of \(\pm 2.5\) kPa. Beyond this range a correction of the data as described in Section 2.3 has to be performed using the polynomial defined in Table 5.2.

The influence of the different types of loading on the damping behavior of mode (3,1) and (1,2) is shown in Figure 5.10. For both modes of vibration a decreasing attenuation with a decreasing pressure on both sides of the diaphragm is obvious but not very pronounced (dashed line)\textsuperscript{11}. It is assumed that the deviation is mainly caused by a reduced acoustic radiation on the outer side of the diaphragm at lower gas densities. The opposite effect of higher damping with increasing cell pressure (\(p_b = -3\) kPa for (3,1) and \(p_b = -0.6\) kPa for (1,2)) cannot be identified by the experiment.

\textsuperscript{11}At \(p_b = -10.2\) or -5 kPa (1,2) and \(p_{\text{load}} < |p_b|\) the cell pressure and thus the gas density is reduced.
5. Experimental verification of the resonator behavior

However, in spite of the small discrepancy in the damping behavior which may appear between the two sensor loading principles, the qualitative behavior remains the same and thus the sensor loading procedure by means of back pressure variation applied for tests in the test cell and for in-situ calibration is considered to be valid.

5.3.2. Measurement performance

Figure 5.11 shows the operating lines corresponding to a 1.8 mm large sensor (chip 59)\textsuperscript{12} and to all structural modes of vibration in a frequency range between 30 and 150 kHz. The solid lines represent the data determined using the optical excitation and the markers the results based on acoustic noise excitation. The pressure load was adjusted by setting and controlling the back pressure in a range between 0.8 and 13.6 kPa with 0.8 kPa steps. An unloaded diaphragm was not considered because the efficiency of optical excitation drops significantly for unloaded sensor operation. It seems that a certain level of mechanical stress has to be present within the diaphragm in order to allow for optical excitation.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.11}
\caption{Optical (solid lines) versus acoustic noise (markers) excitation: Operating lines for all vibration modes in a frequency range between 30 and 150 kHz (chip 59). Frequency bands of the noise: $f_{\text{band},(11)}$: 25-80 kHz, $f_{\text{band},(12)}$: 50-90 kHz, $f_{\text{band},(22)}$: 75-110 kHz, $f_{\text{band},(31)}$: 95-145 kHz}
\end{figure}

The agreement between optical and acoustic excitation is satisfactory for the fundamental (1,1) and the (3,1) mode. The discrepancy obvious for the asymmetric modes can be explained by the effect of a mode dependent

\textsuperscript{12}All details regarding the corresponding sensor holder and measurement settings are listed in Appendix C
5.3. Results

receptivity for acoustic sound of a plate resonator described in Section 3.5.1. There, the symmetric modes were classified as efficient sound radiators and therefore also good receivers for sound. Hence, they dominate over their neighboring modes and can be acoustically driven over the whole considered operating range. The asymmetric modes can only be partially or driven not at all by narrow band acoustic noise. As soon as the neighboring operating lines overlap with the operating frequency band of the CUT for a particular mode the more receptive mode for sound is excited.

Figure 5.12 compares the mode specific damping of a 1.8 mm large plate resonator\textsuperscript{13} determined by experimental and theoretical means (dashed lines). The experiment represents the total occurring losses (solid lines) whereas the theory considers only losses caused by acoustic radiation on the outer side of the diaphragm\textsuperscript{14}. A qualitative agreement between the radiation and the total losses is obvious even though the latter are considerably higher for certain modes. Hence, the effect of mode specific receptivity for sound can also be identified based on the experimental data. For further discussion regarding the mode specific damping of a plate resonator, refer to Section 3.5.

![Figure 5.12: Total versus acoustic radiation losses: Q-factor vs. resonance frequency for all vibration modes in a frequency range between 30 and 150 kHz (chip 63). Total losses: experiment is based on optical excitation. Radiation losses: theory](image)

The total residual losses $Q_r$ could include damping sources such as damping caused by viscous, internal material losses, losses by the support, etc.. In order to determine experimentally the total residual losses one would

\textsuperscript{13}cf. chip 63

\textsuperscript{14}cf. Sec. 3.5
need to perform the tests in vacuum where no acoustic damping occurs. Such tests would require a suitable test environment which was not available in the present work. Furthermore, to maintain the required pressure load on the sensor, the pressure conditions in the reference cavity would differ from the actual one and hence, probably also the viscous losses caused by the cavity. However, if one assumes that the acoustic damping $Q_a$ is accurately determined by the theory, the residual losses can be estimated based on the relationship $1/Q_{tot} = 1/Q_a + 1/Q_r$ [23].

Figure 5.13: Radiation versus residual losses: Damping coefficient $\eta = 1/Q$ [%] as function of the resonance frequency (chip 63), $\eta_a$: theory, $\eta_r = \eta_{tot} - \eta_a$.

Figure 5.13 shows the two fractions of damping based on the data of Figure 5.12 for all considered modes. They are expressed as a damping coefficient $\eta = 1/Q$. The damping behavior of mode (1,1) and (3,1) is clearly dominated by acoustic radiation losses whereas for the asymmetric modes this source of damping is smaller and comparable to $\eta_r$.

The reason why mode (1,1) exhibits the highest $\eta_r$ is not fully understood. Viscous losses caused by the gas-filled cavity should be lower for the symmetric than for the asymmetric modes$^{15}$. A resonator operating in its fundamental mode induces forces and moments resulting from the vibration and thus is called an unbalanced resonator in contrast to an balanced one represented by an asymmetric mode [23]. The high $\eta_r$ of mode (1,1) could therefore be caused by energy losses dissipated at sensor’s mount. Although the (3,1) mode is also classified as a symmetric mode, the pronounced center peak is balanced by the antiphase vibration of the 4 peripheral bumps$^{16}$.

$^{15}$ cf. Sec. 3.6.2  
$^{16}$ cf. Fig. 3.6
5.3. Results

Therefore, and due to an operation at high frequencies the lowest residual losses could result.

Mode (1,2) shows an abnormal rise of residual losses for frequencies higher than 70 kHz. A possible reason for this behavior is an interference between acoustic and structural modes of vibration discussed in Sections 3.6.2 and 5.3.5.

![Figure 5.14](image)

**Figure 5.14.** Pressure sensitivity versus pressure load of a 1.8 mm large plate resonator (chip 59) and all vibration modes in a frequency range between 30 and 150 kHz & comparison between experiment (markers) and FEM (solid lines)

As is shown in Figure 5.14 the experiment (markers) confirms the numerically\(^\text{17}\) (ANSYS) predicted higher pressure sensitivity for symmetric compared to the asymmetric modes. A fair quantitative agreement between experiment and FEM is obvious as well. The abnormal decay in pressure sensitivity for mode (1,2) is probably associated with the high residual losses within the mentioned operating range (cf. Fig. 5.13).

A resonator with a high $Q$-factor (i.e. low damping) exhibits a high measurement performance due to its ability to better reject external noise sources and therefore facilitates the identification of the resonance frequency. The influence of damping on the measurement performance of the plate resonator is quantified by considering the uncertainty of resonance frequency determination (cf. Fig. 5.15). The uncertainty is expressed as the standard deviation of 20 subsequent measurements based on acoustic noise excitation and an ordinary Fast Fourier Transform as demodulation algorithm. If one assumes an average pressure sensitivity of the resonator of 3 Hz/Pa and takes into account the required system sensitivity of 20 Pa a maximum uncertainty of 60 Hz is allowed (i.e. 30 Hz standard deviation).

\(^{17}\)cf. Appendix A.1
5. Experimental verification of the resonator behavior

The highly damped fundamental mode is too far above this limit, while the (3,1) mode shows a considerably better measurement performance. Only mode (1,2) fulfills the required measurement performance, but it can only be utilized for sensing within a limited pressure range (< 4 kPa, cf. Fig. 5.12).

![Figure 5.15.](image)

Figure 5.15.: Measurement uncertainty versus Q-factor for chip 59 and modes (1,1), (1,2), (2,2) and (3,1). Measurement uncertainty represents the standard deviation of 20 subsequent measurements based on acoustic noise excitation

As already mentioned before, the cross-sensitivity of the sensor to temperature changes may considerably be influenced by the sensor carrier.

![Figure 5.16.](image)

Figure 5.16.: Temperature cross sensitivity versus pressure load for all sensing elements on the Kapton foil and mode (3,1). Hollow markers: temperature range between 20 and 24 °C, solid markers: 24 to 28 °C
Hence, the corresponding experimental tests were carried out on the final sensor carrier, the Kapton stripe. The tests were conducted at 20, 24 and 28 °C on all 5 sensing elements of the sensor stripe. Figure 5.16 shows the temperature cross sensitivity for mode (3,1) plotted over a pressure range of 13 kPa. The hollow markers represent the corresponding value for a temperature range between 20 and 24 °C and the solid markers the range between 24 and 28 °C. Except for a slightly loaded sensor the cross sensitivity is well below 30 Hz/K across the whole considered pressure range or smaller than 0.1 %/K if a frequency range of 30 kHz is assumed.

If compared with an estimated temperature sensitivity caused by the fluid loading on both sides of the diaphragm of $<0.03 \text{ %/K}$ (valid for mode (3,1)) a considerable effect of the sensor or foil mount on the sensitivity can be assumed. The fact, that all sensors show a temperature sensitivity with similar magnitude but different sign, points more to a cross influence by the mount of the foil in the test cell. Mode (1,2), considered in Figure 5.17, shows a similar behavior as mode (3,1).

However, even though more investigations regarding the influence of sensor packaging on the temperature sensitivity have to be carried out, the present results already show a low temperature sensitivity which decreases with increasing pressure load, i.e pre-stressing of the diaphragm.

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18 cf. Fig. 3.17b and 3.19b
19 Sensor stripe position in the test cell adjusted for each sensor.
5. Experimental verification of the resonator behavior

5.3.3. Influence of the sensor size

The influence of the sensor size on the sensing behavior was studied on a clamped square diaphragm with an edge length of 1.4, 1.6, 1.8 and 2 mm.

![Figure 5.18](image)

*Figure 5.18.*: a) Influence of the diaphragm size on the operating line of the sensor and b) on the pressure sensitivity, mode (1,1), diaphragm size: 1.4, 1.6, 1.8, 2 mm: chip 46, 47, 44, 42

![Figure 5.19](image)

*Figure 5.19.*: a) Influence of the diaphragm size on the operating line of the sensor and b) on the pressure sensitivity, mode (1,2)
5.3. Results

Figure 5.20.: a) Influence of the diaphragm size on the operating line of the sensor and b) on the pressure sensitivity, mode (3,1)

all with a diaphragm thickness of 7 µm\(^{20}\). Figures 5.18 to 5.20 show the effect on the operating lines and the pressure sensitivity for the fundamental mode (1,1), the asymmetric mode (1,2) and the second symmetric mode (3,1).

In general, the diaphragm becomes stiffer with decreasing size and consequently shows higher resonance frequencies. This effect is smallest for the fundamental mode and gets more pronounced for higher modes. The pressure sensitivity can be increased by increasing the sensor size but only for a slightly loaded sensor. The higher the pressure load is the smaller the gain in sensitivity becomes. For higher loading the pressure sensitivity is about the same for all considered sensors or even higher for smaller diaphragms. This effect is obvious for both symmetric modes but is quite pronounced for the fundamental mode with approximately 10 % higher sensitivity for a 1.4 mm sensor compared with the next larger one (cf. Fig. 5.18b).

The influence of the sensor size on the damping behavior is shown in Figure 5.21. The \(Q\)-factor is plotted versus the pressure load except in Figure 5.21a, where all 3 modes are shown as a function of frequency. If the damping behavior is considered as function of the pressure load, all modes show an increased damping with diaphragm size. Only the 1.4 mm large sensor operating in mode (1,2) and (3,1) behaves differently. If it is operating in mode (1,2), it exhibits the highest damping over the whole considered pressure range whereas for mode (3,1) this behavior is only

\(^{20}\) cf. chip 46, 47, 44 and 42 in Appendix C
Experimental verification of the resonator behavior

Figure 5.21: Influence of the sensor size on the damping behavior, a) mode (1,1), (1,2) and (3,1) plotted vs. frequency, b) mode (1,1) c) mode (1,2) and d) mode (3,1) as function of the pressure load

apparent for pressure loads above 6 kPa. This abnormal behavior should not be related to the damping caused by acoustic radiation. As is shown in Figure 3.15b the $Q$-factor averaged over the operating range increases with decreasing sensor size.

Basically, a more pronounced influence of the sensor size on the damping behavior for mode (1,1) and (3,1) than for the asymmetric mode can be noticed.

Figure 5.22 points out the validity of the theoretical prediction of the
Figure 5.22: Experimental validation of the semi-empirical approach (contour plots in a $a-t$ diagram ($a$: edge length, $t$: thickness) a) resonance frequency for mode (1,1) and an unloaded sensor, b) resonance frequency for mode (3,1) and maximum pressure load of approx. 14 kPa, c) pressure sensitivity averaged over the operating range for mode (1,1) and d) for mode (3,1)

sensing behavior based on the semi-empirical approach described in Section 3.5.2. The figures show the considered values expressed as a contour plot in a $a-t$ diagram where $a$ denotes the edge length of the diaphragm and $t$ its thickness. Figure 5.22a shows the resonance frequency for an unloaded sensor operating in mode (1,1) and Figure 5.22b points out the frequency for mode (3,1) and maximum pressure load. The two values limit the operating frequency range of the resonator and therefore are important
5. Experimental verification of the resonator behavior

design parameters. A very good agreement between theory and experiment is obvious for a 1.6 and 1.8 mm large sensor. The smallest and largest sensor show lower frequencies than predicted by the theoretical approach except the 1.4 mm large sensor and mode (1,1), where the theory underestimates the resonance frequency.

A further important design parameter is the average pressure sensitivity which is shown in Figure 5.22c for mode (1,1) and in Figure 5.22d for mode (3,1). The experimentally determined values deviate more from the theory as is the case for the resonance frequency. The experiment shows a less pronounced influence of the diaphragm size on the mean sensitivity as predicted by the theory. For mode (3,1) the theory predicts lower sensitivities than the experiment but the opposite is obvious if the fundamental mode is considered (except the 1.4 mm sensor).

5.3.4. Influence of the microchannel layout

In the present study the sensing behavior of a 1.8 mm large sensor combined with different microchannel layouts according to Table 5.1 was verified. Even though an identical sensor was used for all tests, the sample holder was reassembled for each holder configuration and thus small deviations between the individual tests have to be expected.

Figure 5.23.: Influence of different number of microchannels on the sensing behavior: a) operating lines b) damping behavior plotted vs. the resonance frequency

\[^{21}\text{cf. also Appendix A.1.2}\]
\[^{22}\text{averaged over the operating range of the sensor}\]
Figure 5.23 shows the operating lines (left) and the damping behavior (right) for the aforementioned plate resonator coupled with a tubing system.

**Figure 5.24.**: Influence of the channel length on the sensing behavior

**Figure 5.25.**: Influence of the channel size (cross section) on the sensing behavior

having 1, 4 or 8 radially outwards directed channels\(^{23}\). The three considered vibration modes (1,1), (1,2) and (3,1) account for almost all frequencies.

\(^{23}\)config. 1 to 3 in Table 5.1
5. Experimental verification of the resonator behavior

within a range between 30 and 140 kHz. Hence, all acoustic resonance effects caused by the tubing system should be identified if present.

As is obvious there is no significant deviation between the three holder configurations. The higher damping apparent for the 4 channel configuration and mode (3,1) at higher sensor loading is not interpreted as an interference effect between the channels and the resonator because no consistency with the other configurations exists. A similar statement is valid for the comparison of microchannel layouts with different channel lengths\textsuperscript{24} (cf. Fig. 5.24) and different channel cross sections\textsuperscript{25} (cf. Fig. 5.25).

The operating line for the resonator operating in mode (1,2) and combined with 2.5 mm long channels slightly deviates from the other configurations (cf. Fig. 5.24a). The shift is caused by a diaphragm geometry which is not perfectly symmetric and thus the (1,2) and (2,1) modes appear not at exactly the same frequency. As mentioned in Section 4.1.1 the asymmetry is caused by a displacement between the rough surface and the diaphragm as a consequence of an incorrect mask alignment during sensor fabrication. Apart from slightly shifted operating lines the damping behavior remains the same for both asymmetric modes.

5.3.5. Influence of the reference cavity

Two questions were addressed during the experimental study regarding the influence of the reference cavity on the sensing behavior of the resonator. First, it was attempted to verify the significance of the viscous losses caused by the gas-filled cavity and second, the abnormal damping behavior for mode (1,2) at high sensor loading was investigated.

The size of the reference cavity was enlarged by a circular hole with a diameter of 1.8 mm within the bottom part of the sample holder. Downsizing of the actual sensor cavity was not feasible because an inlet inserted into the cavity would cause high viscous losses due to squeezed film damping within the remaining tiny gap between inlet and cavity side walls.

Again, the operating lines and the $Q$-factor of a 1.8 mm plate resonator were considered during the tests. The original cavity height of 0.5 mm adds to the sensor cavity (0.4 mm high) and the 0.1 mm thick sensor support belonging to the top part of the sensor holder\textsuperscript{26}. The cavity height was enlarged to 0.8 mm, 1 mm and 1.3 mm by the aforementioned circular cavity in the bottom part of the sensor holder\textsuperscript{27}.

If one considers the fundamental mode in Figure 5.26b a damping be-

\textsuperscript{24}config. 5, 4 and 2 in Table 5.1
\textsuperscript{25}config. 2, 6 and 7 in Table 5.1
\textsuperscript{26}cf. Fig. 5.6
\textsuperscript{27}chip 66: $H_c = 0.5$ mm, chip 67: 0.8 mm, chip 68: 1 mm and chip 69: 1.3 mm
behavior, which could be related to viscous effects within the cavity, is only apparent at low sensor loading or low operating frequencies. The missing effect at higher frequencies could be interpreted as a consequence of higher shear wave numbers and thus, a reduced influence of the viscous effect (cf. Sec. 3.6.2). Mode (1,2) and (3,1) show a higher damping for the smallest considered cavity across almost the whole operating range of the sensor but an effect of shear wave number as obvious for mode (1,1) cannot be identified. Between the larger cavities no significant variation in the damping behavior is obvious.

The operating line for mode (1,2) exhibits a discontinuity between 10 and 17 kPa (cf. Fig. 5.26a) which coincides with an abnormally high damping between 85 and 100 kHz (cf. Fig. 5.26b). At frequencies higher than 100 kHz the quality factor starts to increase again. Both observations indicate a destructive interference between the structural and the acoustic fundamental cavity mode \( f_{100} = 95 \text{ kHz} \). This assumption is further confirmed if the coincidence between the frequency range where strong damping occurs and acoustic cavity modes (cf. Table 3.2) for smaller diaphragms and consequently smaller cavities is considered (cf. Fig. 5.21a).

Mode (3,1) shows similar interference effects but only for a cavity height of 1 mm and 1.3 mm. The destructive interference for the largest cavity appears in a frequency range which includes the acoustic mode defined by the cavity height \( f_{001} \approx 131 \text{ kHz} \)^{28}. The resonator coupled with a 1

\[^{28}\text{cf. eq. 3.28}\]
mm high cavity seems to interfere with the second acoustic mode \( f_{110} \approx 135 \text{ kHz} \) which is similar to the fundamental mode defined by the lateral dimensions of the cavity or sensor size. Why this interference effect does not appear for the two smaller cavities is not clarified yet.

Figure 5.27.: Comparison between an open and a closed sensor cavity: a) operating lines b) damping behavior plotted vs. the resonance frequency for a 1.8 mm large sensor and for all modes within a frequency range between 30 and 150 kHz

The sensing behavior of a plate resonator with a closed and an open cavity is compared in Figure 5.27. The sample holder used for tests with an open cavity is described in Section 5.2. For both cavity configurations a 1.8 mm large plate resonator\textsuperscript{29} was used. The added stiffness effect caused by a closed cavity for the fundamental mode is clearly apparent (cf. Fig. 5.27a). The higher modes are less influenced by the non-viscous fluid loading effect (added mass or stiffness\textsuperscript{30}) and thus no significant deviation between the operating lines corresponding to an open and a closed cavity is obvious. A similar finding arises if the damping behavior for the asymmetric modes is considered. This points to a negligible effect of viscous losses in a closed cavity, especially if one considers the more pronounced viscous effect in a closed cavity for asymmetric compared with symmetric modes\textsuperscript{31}.

Interesting is also the considerably higher damping for the symmetric modes (1,1) and (3,1) in an open cavity configuration. The corresponding

\textsuperscript{29}closed cavity: chip 63, open cavity: chip 64
\textsuperscript{30}cf. Sec. 3.6.2
\textsuperscript{31}cf. Sec. 3.6.2
mode shape causes a displacement of the fluid trapped in the cavity due to a change in cavity volume. The emerging alternating and highly dissipative flow at the cavity outlet (vortex generation) could induce the high losses which are observed.
6. Wind tunnel tests

The wind tunnel tests were carried out in order to verify the resonator behavior in a flow and a highly vibrating environment. For reference purposes, the established laser Doppler vibrometer (LDV) instead of the full-field interferometer was employed as interrogation system. A pressure tap system installed on a NACA 0012 wing was used as a reference pressure measurement technique.

6.1. Measurement setup and procedure

The tests were carried out in the boundary layer wind tunnel of the Institute of Fluid Dynamics at ETH. The wind tunnel is an open circuit facility of the blower type originally used for boundary layer investigations [87] (Fig. 6.1, sketch). A large settling chamber equipped with 4 screens and an entry cone with a high contraction ratio of 15 deliver a flow of high quality. The flow is driven by a 75 kW DC motor up to flow rates of 12 m$^3$/s corresponding to flow speeds in the test section of approximately 60 m/s. The flow velocity is determined by measuring the static drop over the contraction cone and the air temperature using a thermocouple in the test section. The calibration of the corresponding measurement chain is carried out using a Pitot-Prandtl tube and a water manometer (scale resolution 0.1 mmH$_2$O). The probe is positioned in proximity of the cone exit and outside the boundary layer. The test section, equipped with a removable ceiling panel, is 0.4 m wide, 0.5 m high and 0.7 m long. An open ceiling allows for an optical and acoustic access to the wind tunnel model (cf. Fig. 6.2b). In order to minimize the leakage flow through the open ceiling an adjustable, 2.5 m long diffusor is attached to the outlet of the test section (cf. Fig. 6.1). A diffusor angle of approximately 0.39° (top plate) maintains parallel flow conditions over almost the entire height of the test section. A rounded upper edge at the exit of the contraction cone reduces pulsations induced by acoustic pipe resonances.

As the wind tunnel model a NACA 0012 airfoil with a chord $c$ of 0.3 m and a span of 0.4 m was chosen. The wing is mounted horizontally between the two side walls of the test section (cf. Fig. 6.2a). Using an electronic inclinometer (Jewell Sensors) the angle of attack $\alpha$ is manually adjusted with an accuracy of approximately 0.05° (cf. Fig. 6.1). For reference pressure measurements 29 pressure taps ($\varnothing$ 0.8 mm) are installed adjacent
6. Wind tunnel tests

![Diagram of wind tunnel facility](image)

**Figure 6.1.:** Sketch of the wind tunnel facility (modified from [87]). Main dimensions of the test section and installed test equipment (left bottom). Variable diffusor (2.5 m long) used for flow adjustments in the test section (right bottom).

The resonant sensors (lateral displacement 30 mm) and spaced 10 mm apart along the chord. A technical drawing with exact positions of the individual taps is provided in Appendix D.2.

The read-out is achieved using a mechanical pressure scanner providing 48 ports\(^1\) (Scanivalve Corp., model 48J9) and a pressure transducer of type PDCR 22 (GE Druck Limited) with a measurement range of 7 kPa and a measurement uncertainty of 10 Pa. All measured pressure data, temperature and the angle of attack are acquired using a data acquisition card NI USB-6009 (National Instruments) with 300 Hz sampling rate. The scanner is remotely controlled using a TTL signal delivered by the analog output port of the USB DAQ card and Matlab.

The sensor stripe is 240 mm long, 40 mm wide and 0.6 mm thick. It

\(^1\) channel assignment cf. Appendix D.2
6.1. Measurement setup and procedure

is equipped with 5 sensing elements (diaphragm size: 2 mm, thickness: 7 µm) distributed along its length according to Table 6.1. Its design and manufacture is described in Section 4.2. The sensing behavior of all sensors was verified in the test cell making use of the optical excitation principle (cf. Sec. 5.3.2 and Appendix D.1). Due to the small model dimensions and the comparatively large sensor height of the prototype, the sensor stripe is recessed in a milled slot to prevent flow disturbances by the sensor unit. A 20 mm long and 2 mm wide slit in the wall of the recess allows for a pneumatic access to the sensor unit (cf. Fig. 6.3b). An aluminum cap covering the orifice forms a pressure port which is connected to the external pressure controller (PACE 5000) by means of a silicon tube (cf. Fig. 6.3a).

Table 6.1.: Position of the individual MEMS sensing elements on the wing (distance from the leading edge normalized with the wing chord \(c\))

<table>
<thead>
<tr>
<th>MEMS Nr.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position (x/c)</td>
<td>0.100</td>
<td>0.201</td>
<td>0.300</td>
<td>0.433</td>
<td>0.636</td>
</tr>
</tbody>
</table>

The head of the laser Doppler vibrometer is located approximately 0.5 m above the model (cf. Fig. 6.1 and 6.2b). For a precise alignment between sensor and detector the head is mounted on a 2-axis linear micro-positioning stage and a 3-axis tilt stage (tripod head). A CCD camera with a macro
6. Wind tunnel tests

Figure 6.3.: Pneumatic access to the sensor stripe: a) Pneumatic feedthrough (slit) in the model wall covered by an aluminum cap as a tube connector b) CAD sketch of the pneumatic access on the sensor stripe side

lens facilitates the positioning of the laser beam in the diaphragm center (cf. Fig. 6.2b). Tests with a focussed (spot diameter approx. 70 µm) and a defocussed laser beam with a spot diameter similar to the diaphragm size were carried out. Using the latter configuration the full-field optical interrogation was approximated and hence, its feasibility to identify higher vibration modes verified.

The ultrasound transducer (UT) is also adjustable in its direction of radiation and is positioned approximately 1.5 m away from the model. For each sensor and angle of attack the LDV and the UT are adjusted so as to maximize the interrogation signal.

The measurement procedure and data postprocessing related to the resonant sensors comply with the procedure applied during the investigations in the test cell using the acoustic excitation (cf. Sec. 5.1). All tests shown here are based on resonator operation in mode (3,1). For the majority of the tests the reference pressure in the sensors’ cavity was set to -3 kPa using the pressure controller PACE 5000. The pressure tap data and the flow velocity were acquired simultaneously with the LDV output signal and the readout of the pressure controller (back pressure control and monitoring).

The reference wall pressure at the resonators’ location is determined by a linear interpolation between the data of neighboring taps. The sensor calibration was carried out on the model (in-situ) and was regularly repeated during the measurement campaign. According to Section 2.3 the final pressure data are determined based on the calibration curve (spline interpolation) and a correction for the differing pressure conditions (i.e.
mass loading) present on the two sides of the diaphragm during calibration and measurement\(^2\).

### 6.2. Results

The sensing behavior of the newly developed system was verified on two sensing elements S1 and S2 located at \(x/c\) of 0.1 and 0.2. During the tests free stream velocities \(U_\infty\) between 12.5 and 35 m/s \((Re_c = 2.5 \cdot 10^5 - 7 \cdot 10^5)\) and an angle of attack \(\alpha\) between 0 and 16° were investigated. A maximum negative wall pressure \(p_{wall}\) of \(-1.2\) kPa at S1 location, 12° angle of attack and at maximum flow speed can be achieved.

The impact of environmental vibrations on the measurement performance was assessed by monitoring the standard deviation (i.e. measurement un-

![Figure 6.4](a) and (b)](a) and (b)

**Figure 6.4.** Impact of environmental vibrations on the sensing behavior expressed as the standard deviation of 20 subsequent measurements versus the chord Reynolds number. Signal demodulation is based on the Welch algorithm. a) S1 b) S2

uncertainty) of the pressure data obtained from 20 subsequent measurements and using the Welch algorithm\(^3\) for signal demodulation (cf. Fig. 6.4). Although the environmental vibrations are considerably larger at higher flow speeds (or \(Re_c\)) the measurement uncertainty remains smaller than 20 Pa over the whole considered operating range of the airfoil. The rising uncertainty at 16° angle of attack and higher free stream velocities is caused by

\(^2\)cf. Sec. 5.3.1

\(^3\)cf. Sec. 5.1
6. Wind tunnel tests

high pressure fluctuations within the separated flow on the wing.

![Graphs showing pressure coefficient vs. Re_c](image1)

![Graphs showing discrepancy in percent between two measurement systems](image2)

**Figure 6.5.** Comparison of the MEMS resonator and the pressure tap system, S1: left Fig., S2: right Fig., a) and b) $c_p$ versus $Re_c$, markers: MEMS resonator, markers & solid line: pressure taps, c) and d) Discrepancy in percent between the two measurement systems in reference to the pressure tap data vs. $Re_c$.

The pressure data collected at the S1 and S2 locations by the MEMS resonator and the pressure tap system are compared in Figure 6.5. The wall pressure is expressed dimensionless, i.e. as a pressure coefficient $c_p = (p_{wall} - p_\infty)/(1/2\rho U^2)$ where $p_\infty$ denotes the static pressure of the free stream (cf. Fig. 6.5a and b). The error bars represent the measurement uncertainty as defined above. The corresponding pressure tap values are
well below 10 Pa and are not illustrated. The deviation in percent between the two measurement systems in reference to the tap data is shown in Figure 6.5c and d.

If one considers the $c_p$ characteristics obtained by the pressure taps for $Re_c$ smaller than $3 \times 10^5$ and higher angle of attack, an airfoil performance is apparent which shows a pronounced dependency on $Re_c$. Such a behavior is usually caused by low $Re$ boundary layer (BL) phenomena such as separation of the laminar BL, bubble formation, etc. [88]. For higher $Re_c$ and dependent on $\alpha$ the $c_p$ slightly in- or decreases with $Re_c$ but mostly converges towards a constant value.

Sensors S1 and S2 show neither a quantitative nor a qualitative agreement with the reference data. Due to a measurement uncertainty of approximately 20 Pa (cf. Fig. 6.4) and pressure values smaller than -300 Pa at low $Re_c$ or zero angle of attack both sensors deviate by up to ±40 % from the reference data. For higher $Re_c$ and consequently higher pressure values the systematic measurement error drops below 20 %. Furthermore, a discrepancy of approximately 10 % for S1 and even well below 10 % for S2 can be stated for $\alpha$ larger than 12°.

However, the sources of interference which cause that deviant measurement behavior are not yet conclusively identified. The findings gained so far are discussed as follows.

![Figure 6.6](image)

**Figure 6.6.** Discrepancy in [Pa] between MEMS resonator (S1) and pressure tap after readjustment of the sensor unit prior to each run. a) Readjustment of the LDV head b) Focussed (beam diam. approx. 75 µm) and defocussed (beam diam. approx. 2 mm) LDV laser beam, $\alpha = 0°$
6. Wind tunnel tests

Measurement errors which may arise on the pressure tap side should not cause a measurement discrepancy of such an extent. Typical sources of error for pressure tap systems such as developing eddies in the cavity formed by the tap or flow turbulence should be smaller than 1% of the dynamic pressure, i.e. less than 10 Pa [89, 90].

As discussed before the environmental vibrations (i.e. machine vibrations) seem to have no influence on the sensing performance of the resonant sensors. In order to verify the system’s sensitivity to measurement adjustments, repeatability tests were carried out with readjustment of the LDV head alignment and the beam position on the target prior to each run (cf. Fig. 6.6a). Furthermore, tests based on a focussed and defocussed interrogation laser beam (spot diameter approx. 75 µm vs. 2 mm) were performed (cf. Fig. 6.6b). A focussed laser beam can be precisely positioned in the diaphragm center where maximum vibration amplitude for mode (3,1) occurs. Hence, the interrogation signal is maximized and cross coupling between neighboring vibration modes prevented. On the other hand, a varying beam position during or between individual runs may induce a measurement error which ought to be quantified by these tests.

Figure 6.6a and b show the discrepancy in Pascal between the MEMS resonator and the reference system obtained from the repeatability tests on S1. The data are plotted as function of $Re_c$ and $\alpha$ and the marker’s color highlights the different repeatability runs. All data are within the 20 Pa uncertainty range obtained from measurements without system readjustment. An influence of measurement adjustments on the sensing accuracy

![Figure 6.7](image_url)
of the MEMS resonator can therefore be neglected. The change in sign between the data obtained for zero and $8^\circ$ angle of attack in Figure 6.6a is discussed later in this section. The tests based on a defocussed laser beam illuminating the whole diaphragm surface also demonstrate the feasibility of higher mode detection by means of full-field interrogation of the resonator.

The Welch algorithm applied for signal demodulation is more robust than an ordinary FFT but due to averaging and smoothing steps an alteration of the power spectrum caused by high frequency environmental noise is conceivable. A comparison of both algorithms, again expressed as the discrepancy in Pascal between the tap and the MEMS resonator data, does not confirm this concern (cf. Fig. 6.7). The corresponding data are all positioned within the 20 Pa uncertainty range. The higher robustness of the Welch algorithm compared to the FFT is clearly demonstrated by a smaller deviation between the individual runs.

![Diagram](image)

**Figure 6.8.:** Interference mechanism between flow and resonator: Asymmetric diaphragm deformation due to a) pressure gradient b) wall shear stress

A flow past a curved body induces a pressure gradient in the streamwise direction $dp/dx$ which can reach values up to 20 Pa/mm for the considered wing, sensor location and operating conditions. If related to the current size of the resonating element of 2 mm a non-uniform load distribution of up to 40 Pa could induce a vibrating behavior which deviates from the sensor calibration. The deviation could originate from an averaging by the diaphragm and from an asymmetric stress distribution within the prestressed diaphragm (cf. Fig. 6.8a).

A similar interference effect may result from the wall shear stress $\tau_w$ caused by viscous effects in the flow (cf. Fig. 6.8b). If one assumes a $\tau_w < 5 \text{ N/m}^2$ for $Re_c < 7 \cdot 10^5$ and its inherent diaphragm loading by shear (tangential force) the pressure gradient should play the dominant role.\(^4\)

Figure 6.9 shows the measurement discrepancy in Pascal between the

\(^4\)The estimation is based on turbulent flow over a 0.3 m long flat plate [91]
6. Wind tunnel tests

Figure 6.9.: a) and b) Measurement discrepancy in [Pa] between MEMS resonator and pressure tap for different $\alpha$ and $Re_c$, c) and d) Same data as above but plotted vs. the tap pressure, e) and f) Same data as above but plotted vs. the pressure gradient in streamwise direction, S1: left Fig., S2: right Fig.
plate resonator and the reference technique as a function of $Re_c$, $p_{\text{wall}}$ and $dp/dx$, again for S1 (left) and S2 (right). The measurement discrepancy increases with increasing $Re_c$ but shows also a pronounced dependence on the angle of attack (cf. Fig. 6.9a and b). A similar behavior can be identified if the data are plotted over the corresponding pressure gradient (cf. Fig. 6.9e and f).

The pronounced dependency on $\alpha$ strongly indicates an impact of $\tau_w$ and $dp/dx$ on the resonator’s behavior. This assumption is further confirmed if the low measurement discrepancy for a separated flow regime at $\alpha = 16^\circ$ is considered where both flow parameters vanish (cf. Fig. 6.9e and f). However, if one considers the comparatively high discrepancy at zero angle of attack where only a small pressure gradient exists the wall shear stress and not the pressure gradient appears to be the dominant parameter.

Unfortunately it is not feasible to quantify experimentally the error contribution of the two flow parameters independently. The influence of the wall shear stress could be further investigated on a flat plate where no pressure gradient exists. The pressure gradient on the other hand cannot easily be separated from the shear forces. The magnitude of the contribution of the two flow parameters can only be effectively clarified if numerical simulations (FEM) are performed.

![Figure 6.10](image)

**Figure 6.10.** Measurement discrepancy in [Pa] between MEMS resonator and pressure tap as function of $Re_c$ and different pressure settings in the reference cavity of the sensor (initial pre-stressing of the diaphragm), S1: empty markers, S2: solid markers, $\alpha = 12^\circ$

The extent of the interference between flow and the resonator should be related to the level of the stress within the diaphragm. With increasing mechanical stresses the impact of the flow should decrease. In fact, tests
6. Wind tunnel tests

where the back pressure and thus the initial pre-stressing of the diaphragm was increased from -3 to -9 kPa show a tendency towards a reduced measurement discrepancy but surprisingly not very pronounced (cf. Fig. 6.10). The data represent S1 (empty markers) and S2 (solid markers) at 12° angle of attack and different $Re_c$.

A further finding obtained from the wind tunnel tests is the negative measurement discrepancy for S1 and $\alpha$ of zero and 4° whereas for a higher $\alpha$ the error becomes positive (cf. Fig. 6.9). The different sign of the discrepancy should not be related to the sign of the pressure gradient because at zero angle of attack a favorable and at 4° an adverse pressure gradient is present at S1 location (cf. Fig. 6.9e). In order to verify if the boundary layer (BL) characteristics has the mentioned influence on the measurement discrepancy tests were performed with a laminar and a turbulent BL state present at the resonator and tap locations (cf. Fig. 6.11). The study was conducted on S1 and at zero angle of attack in order to maintain a laminar BL over the whole considered $Re_c$ range. Transition between a laminar and a turbulent BL was triggered using a 0.5 mm high transition strip attached at $x/c$ of approximately 0.03. The data for a laminar BL are indicated by round and for a turbulent BL by square markers. In fact, the sign of the error seems to depend on the BL state. Accordingly, the negative sign of the error for $\alpha$ of zero and 4° (cf. 6.9a) can be associated with a laminar BL at the S1 location whereas for higher $\alpha$ the BL becomes turbulent and thus $\Delta p$ positive. The reason for that behavior is not clarified yet and requires further investigations.

**Figure 6.11.** Measurement discrepancy in [Pa] between MEMS resonator and pressure tap as function of $Re_c$ for a laminar BL and a tripped, turbulent BL at S1 location and $\alpha = 0^\circ$
Finally, tests with a sensor recessed in a shallow cavity additionally confirmed the afore-made assumption of an impact of the flow on the sensor’s vibration. The flow in the cavity is fully separated and thus neither the pressure gradient nor the wall shear stress should influence the sensing behavior. For the tests the sensor S3 and the neighboring pressure taps were covered by a 0.5 mm thick Kapton layer with a square opening of 3 mm$^2$ exposing the resonator (cf. Fig. 6.12). The pressure taps are recessed in circular cavities with a diameter of 2 mm each.

The pressure data obtained from the pressure tap and the MEMS resonator are expressed as a pressure coefficient and plotted versus the Reynolds number for $\alpha$ of 0, 4 and 8° (cf. Fig. 6.13a). The deviation from the pressure tap data in reference to the corresponding wall pressure is given in

\[ \frac{p_{\text{MEMS}} - p_{\text{tap}}}{p_{\text{tap}}} \% \]

These results are plotted in Fig. 6.13b.

\[ c_{p} - \text{Re}_c \text{ characteristics obtained with the pressure tap system and the MEMS resonator} \]

\[ \text{Discrepancy between the two measurement systems in reference to the corresponding wall pressure} \]
6. Wind tunnel tests

Figure 6.13b. The measurement discrepancy shows a different characteristic as without the cavity. At low wall pressure values or \( Re_c \) the discrepancy reaches values of up to 50\% but decreases notably with increasing \( Re_c \). At maximum \( Re_c \) a discrepancy of approximately 2\% is obtained. It is conceivable that at low \( U_\infty \) an intense recirculation arises within the cavity which could cause the variation in static pressure as it is observed. At higher flow speeds a separated flow region with a more turbulent character and thus lower impact on the static pressure may appear. However, further investigations are necessary in order to understand the observed phenomena and to define the minimum depth and a suitable geometry for the sensor’s outer cavity.

In summary, the wind tunnel tests revealed a robust but biased sensing behavior using the surface mounted pressure sensors. The observed systematic measurement error seems to be caused by an interference between the resonator and the flow. However, the responsible mechanism remains unidentified. Further investigations in a simple flow configuration such as on a flat plate are necessary in order to enable an adaptation of the sensor design.
7. Feasibility test: Full-Field Optical Interrogation

The experimental verification of the full-field optical interrogation system (2 cameras and self-referencing interferometer setup) as described in Section 2.2 could not be completed for the present report. However, preliminary tests based on a single camera system and carried out without considering the flow environment have shown the feasibility of sensor interrogation using the full-field interferometer system.

Figure 7.1.: Setup of the single camera full-field interrogation system: 1 Fresnel beam expander, 2 articulated laser arm, 3 fiber optics delivery system, 4 beam splitter, 5 beam expander, 6 SPDA, 7 lens, 8 ultrasound transducer, 9 laser, 10 sensor

The measurement setup as shown in Figure 7.1 is based on a 1.4 mm plate resonator\textsuperscript{1} driven in its fundamental mode by acoustic noise excitation. It is interrogated using an ordinary interferometer setup based on one SPDA (pOCTii detector, Heliotis) which was operating in the lock-in detection mode (i.e. demodulation frequency sweep). The SPDA, equipped with a standard lens (focal length 85 mm) and positioned at a distance of approximately 1 m from the sensor, registered the optically rough diaphragm surface on 4 pixels of the detector array (144 x 90 pixels).

\textsuperscript{1}chip 23 cf. Appendix C
7. Feasibility test: Full-Field Optical Interrogation

Figure 7.2.: Demodulated image (lock-in amplitude) obtained from the SPDA at resonant operation of a 1.4 mm resonator (chip 23) and a pressure load of 4.5 kPa (only a cutout of the detector array is shown)

The ultrasound transducer (CUT) was placed at a distance of 1.5 m from the sensor and was fed with narrow band electrical noise with a frequency band between 40 and 100 kHz\(^2\).

The source beam of a CW laser (Coherent Verdi V5, \(\lambda = 532\) nm, output power 1 W) is split into two parts using a Fresnel beam sampler. The object beam containing 95 % of the power is expanded to cover a target area of around 0.4 m x 0.4 m. The reference beam is redirected towards the camera using a fiber optic delivery system. A collimator expands the reference beam to cover the detector area. A beam splitter is used to mix the light from the scene and the light from the reference beam. Due to interference the result of this superposition is a harmonic, intensity-modulated signal superimposed on a constant offset\(^3\).

The demodulation of the alternating part of the signal is based on a dual phase lock-in detection on the pixel level as described in Section 2.2.2. The demodulation frequency of the SPDA is swept from 50 to 80 kHz in 100 steps (300 Hz per step) and thereby the amplitude spectrum of the interrogation signal is recorded. The resonance frequency is found at the peak value of the spectrum (cf. Fig. 7.3). A total averaging over 200 measurements was performed per run (i.e. constant operating point of the resonator).

The sensor was loaded by setting and controlling the reference pressure

\(^2\)power amplifier settings cf. Sec. 2.1

\(^3\)cf. eq. 2.1, Sec. 2.2.1
in the sensor’s cavity (back pressure) between 0.3 and 11.5 kPa (0.5 kPa steps) below the ambient value using the pressure controller (PACE 5000, cf. Sec. 5.3.2).

During post processing the raw data are first analyzed to locate the pixel cluster which corresponds to the diaphragm (cf. Fig. 7.2). After that an estimate of the resonance frequency is determined based on the obtained signal amplitude spectrum (peak detection). Finally the raw data within an interval of 8 kHz around the resonance frequency is fitted using the least-squares method with a Gaussian profile (cf. Fig. 7.3). The final resonance frequency is extracted using a peak detection based on the Gaussian fit.

Figure 7.4 compares the resonance frequencies obtained for different pressure loads in the considered range (operating lines) with data from tests where optical excitation and the laser vibrometer (LDV) were applied on the same sensor and carrier. The two methods show a very good agreement in the pressure load range between 3 and 8 kPa but diverge between 8 and 11 kPa. The deviation goes along with the drop in $Q$-factor below 50 within the mentioned pressure load range. Due to this strongly damped oscillation behavior the amplitude at resonance is too small to generate sufficiently large signals on the detector. It has to be mentioned that both tests (SPDA and LDV) were carried out using the first generation sample holder where acoustic resonance effects on the upper side of the holder caused this abnormal damping behavior for the fundamental mode$^4$.

$^4$cf. Fig. 5.5a, Sec. 5.2
Figure 7.4.: Operating line of the resonator (chip 23) acquired using the single point (LDV) and the full-field (SPDA) interrogation method (mode (1,1)). The $Q$-factor characteristic (red) is based on LDV measurements and optical excitation.

However, the tests have verified the feasibility of optical full-field interrogation using the SPDA and a plate resonator having appropriate optical surface properties (rough surface). In order to reach the required measurement performance (accuracy and measurement uncertainty) a resonator with a $Q$-factor larger than 50 is required. For a plate resonator with dimensions and operating conditions (e.g. frequency) as considered in the present work only the higher vibration modes can satisfy this requirement.
8. Conclusions and outlook

The feasibility of static pressure sensing based on a plate resonator operating at ambient pressure conditions, being remotely interrogated by optical means and driven by acoustic noise excitation was verified. In order to obtain a system performance which is required to allow for wall pressure measurements in low-speed wind tunnel surveys several constraints on the system design are mandatory:

1. The resonator has to be operated as a differential pressure sensor with a predefined and controlled reference pressure on the back side of the diaphragm in order to maximize the pressure sensitivity and minimize cross sensitivities.

2. The second symmetric mode of vibration\(^1\) has to be utilized for pressure sensing in order to reduce the damping of the resonator.

3. The diaphragm has to be recessed in a shallow cavity in order to prevent any impact on the resonator by the flow (e.g. wall shear stress and pressure gradient)

4. The top surface of the diaphragm requires a sufficiently rough topography in order to obtain an optically diffuse surface reflection property.

**Constraint 1** is mainly motivated by the demand for maximizing the pressure sensitivity of the resonator. Except for small sensor loading, the pressure sensitivity of the resonator decreases with increasing pressure load. Hence, the initial loading should preferably be adapted to the considered measurement range with the aim to be minimized. Control of the reference pressure is necessary in order to prevent any impact on the sensing performance by the gas behavior in a closed cavity, i.e. change in reference pressure due to thermal gas expansion and due to a change in cavity volume (static diaphragm displacement). A further benefit of reference pressure adjustment and control is the feasibility of *in-situ* sensor calibration and thus the compensation of packaging induced stresses during installation of the sensor unit. And at last, for signed pressure readings the reference pressure has to be adjusted below the ambient value (initial loading) during measurements.

\(^1\)circular diaphragm: (0,1) and square: (3,1)
A suitable design of the sensor unit which allows for the afore-described sensor operation requires a sensor carrier with integrated microchannels. They interconnect the individual sensor cavities and also allow for a pneumatic access to the sensor array. The carrier, a thin Kapton foil, and the sensing elements form a unit which may complicate or constrain the installation of the system. Although only one single small tube is required as a link between sensor unit and controller, its implementation may also restrict the system installation. The current overall thickness increase of the sensor unit due to the pneumatic tubing system is approximately 60 µm (cf. sensor height of 200 µm) but a channel height smaller 50 µm (current value) seems to be feasible in terms of fabrication and operating constraints.

Experimental tests revealed an overall temperature cross-sensitivity of approximately 0.1 %/K (cf. the estimated value of approx. 0.03 %/K caused by gas loading effects) with a varying sign depending on the considered resonant element on the sensor stripe. That points to a significant influence on the temperature sensitivity by sensor packaging and by the fixation of the sensor stripe on the wind tunnel model. Further experimental investigations and numerical simulations (FEM) have to be performed in this regard, to identify existing limitations and to further optimize the design.

The interaction between the plate resonator and the gas-filled cavity and tubing system can be of different types. Basically, no significant impact on the sensing behavior by the microchannels was observed. The added stiffness effect imposed by the cavity arises only for the symmetric modes of vibrations and is negligible for the higher symmetric mode (3,1) which is best suited for pressure sensing in the present application. Viscous effects caused by the fluid in the cavity seem to have no significant influence on the sensing behavior for the mode of interest. Interference between acoustic cavity and structural plate modes seems to degrade the sensing behavior due to destructive interference, but only if the resonator is operating in the asymmetric mode (1,2) and at higher pressure loading. Such sensor operation is not relevant for the present application.

**Constraint 2** The fundamental vibration mode of a plate resonator is highly damped by acoustic sound radiation. A resonator with a quality factor well below 100 does not provide the required measurement performance. Asymmetric higher modes show much lower damping than the fundamental mode but are also less receptive to acoustic sound and thus cannot or can only be partially (i.e. in a limited pressure range) driven by acoustic noise excitation. The second symmetric mode (3,1) shows Q-factors well above 100 and can also be driven over the whole considered pressure range of 14 kPa.
The mode shows a similar pressure sensitivity characteristic as the fundamental one with an average sensitivity of approximately 3 Hz/Pa, valid for a diaphragm size between 1.4 and 2 mm and a thickness of 7 µm (current prototype). A measurement uncertainty of approximately 0.1 % of FS (approx. 10 Pa) was attained using acoustic excitation and LDV measurements. In contrast to the fundamental mode the damping decreases with increasing sensor loading and thus may compensate for the loss in pressure sensitivity with increasing pressure load. The corresponding mode shape shows side lobes vibrating with an opposite phase to the center peak. This counterbalances arising forces and moments and hence, reduces losses induced by the plate or sensor clamping. Reactive fluid loading effects such as added mass on the outer side of the diaphragm and added stiffness caused by the gas-filled cavity are significantly smaller than predicted for the fundamental mode.

The drawback of operating the sensor in mode (3,1) is related to an existing minimum diaphragm size limitation. The limitation is imposed by the maximum allowed operating frequency of 150 kHz which is mainly defined by the operating range of the ultrasound transducer. The minimum diaphragm size becomes approximately 1.6 mm if the maximum pressure measurement range of 14 kPa is required and approximately 1.4 mm for a measurement range of 7 kPa. The goal of minimizing the diaphragm size is justified by concerns regarding the spatial resolution, installation on small models with a high surface curvature, and a simpler handling of the flow induced impact on the sensing behavior of the plate resonator (cf. constraint 3). The fundamental mode operating at much lower frequencies compared to the (3,1) mode does not impose this restriction on the sensor design.

**Constraint 3** Characteristic flow parameters for wall-bounded flows such as the wall shear stress and the flow-induced pressure gradient seem to influence the sensing behavior of the plate resonator and impose a systematic measurement error. A maximum absolute discrepancy between resonator and reference measurement technique (pressure tap system) of approximately 150 Pa for a wall pressure of approximately 800 Pa was observed. Tests with a recessed sensor in a shallow cavity confirmed the impact of the flow on pressure sensing. Here, separated flow conditions are present and thus the pressure gradient and wall shear stress vanish. Measurement discrepancies smaller than 5 Pa at maximum chord Reynolds number of \(7 \cdot 10^5\) were achieved. On the other hand, significant discrepancies at low Reynolds numbers were observed which are probably caused by flow recirculation within the recess at low flow velocities. A further effect caused by the flow which is not understood yet is a measurement error which seems
to change its sign dependent on the boundary layer characteristics (laminar or turbulent). Further experimental and numerical investigations are required in order to clarify the flow phenomena which seem to influence the sensing behavior of the resonator.

In addition, a suitable geometry of the sensor’s top side has to be evaluated with the objective to minimize the impact of the flow on the sensor and vice versa. In this regard it is conceivable to cover the recess by a thin, perforated and transparent membrane. Viscous losses caused by squeezed film damping between the plate and the resonant diaphragm can be minimized by an appropriate design of the perforation (e.g., open area, hole geometry, etc.) [92]. However, experimental tests would have to be performed to verify the influence of such a protective plate on the vibration behavior, the optical interrogation and acoustic excitation.

**Constraint 4** The stringent requirements on the surface roughness topography were basically fulfilled. A novel process was developed which allows an efficient wet etching of tiny grooves directly into the silicon surface. No mechanical failure was observed during extensive experimental resonator testing. Hence, no significant degradation of the diaphragm’s fracture strength caused by the notching effect is assumed. The surface appears white if illuminated with white light. Consequently, dispersion of light due to wavelength dependent light scattering on the rough surface should be small. As expected for a rough surface containing roughness elements in the subwavelength range of visible light no diffraction phenomena occur. Experimental tests using the full-field interferometer system verified a sufficiently high diffuse scattering property of the surface. The plate resonator was clearly identified by the detector. The rough surface was further characterized by surface profilometry using an atomic force microscope. Based on the profile data a rms roughness height $\sigma$ of approximately 65 nm and a correlation length $\tau$ of approximately 150 nm were obtained. The ratio between the two parameters $m = \sigma/\tau$ represents a characteristic parameter in a theoretical model applied for light scattering on rough surfaces. A value larger than 0.4 already indicates a highly diffuse scattering property.

However, the optical property of the diaphragm surface could still be improved if the height and the density of the roughness elements are further increased. The limiting rms roughness height of approximately 200 nm due to mechanical strength concerns is not yet reached. A defined adaptation of the roughness structure would require a detailed study of the relevant fabrication process parameters in order to evaluate and be able to obtain an optimum density and size of the hemispherical silicon grains which define the porosity of the passivation layer.
8.1. Outlook

The attempt of sensor miniaturization should be the main objective during the further sensor development. A small sensor increases the spatial resolution of the system and facilitates the installation on smaller wind tunnel models with large surface curvature. The smaller the sensor is the smaller the cavity on its outer side becomes and potential flow disturbances by the recess are minimized. The sensor size also defines the necessary depth of the recess in order to maintain separated flow conditions over the whole diaphragm surface. A smaller sensor leads to a shallower cavity and consequently to a reduced overall sensor height.

To preserve or even increase the pressure sensitivity of the resonator during diaphragm size reduction, its thickness has to be decreased. The limiting diaphragm thickness depends on mechanical strength concerns and would have to be determined in experimental tests and by taking into account an appropriate surface roughness topography. As already mentioned in constraint 2 the minimum size of a resonator operating in mode (3,1) is limited by the maximum operating frequency of the ultrasound transducer. A transducer which can operate at higher frequencies would extend this limit towards smaller sensor size. The need for a sufficiently strong excitation and an interrogation signal further limits the minimum diaphragm size and would also be a subject for further experimental tests.

As already discussed before, the further development of the sensor unit design should additionally address the outer geometry of the sensor (flow interaction), the optimization of the diaphragm’s top surface topography (roughness) and the packaging issue (temperature cross-sensitivity).

The feasibility of full-field interrogation using an ordinary interferometer setup based on one smart pixel detector array (SPDA) which was operating in the lock-in detection mode (i.e. demodulation frequency sweep) was verified. The verification of the actual interrogation system which is based on a self-referencing interferometer setup, a dual SPDA camera system and the pulse pair statistics demodulation approach has not yet been carried out and should be performed both outside and within a flow environment (i.e. wind tunnel).

Due to the high directivity of an ultrasound transducer (UT) operating at frequencies higher than 50 kHz a transducer array would be required in order to excite all sensing elements distributed within a larger target area. All relevant design parameters of an UT array, such as the number of required transducers, their arrangement and adjustment have to be defined in an appropriate study in terms of a high and uniformly distributed
8. Conclusions and outlook

excitation power within the target area. In order to optimize the excitation performance and increase the upper limit of the operating frequency range up to 200 kHz a custom-made UT might be required and thus would have to be designed and fabricated.

Spatially resolved measurements of flow induced wall shear stress is still a challenging task during flow investigations. It is conceivable that both, the wall pressure and the shear stress might be acquired simultaneously using the resonant sensing principle. That would require two closely spaced plate resonators, one which is mounted flush with the model surface and the other one which is recessed in a cavity. The former one is sensitive to both flow parameters whereas the recessed one senses only the static pressure. The wall shear stress might then be evaluated based on the measured frequency deviation between the two sensing elements. Such a technique becomes feasible only if the size of the plate resonator can be reduced considerably. Furthermore, the modulation of the resonance frequency of a plate resonator by shear stresses would have to be investigated to identify the relevant design parameters and thus to maximize the shear stress sensitivity of the corresponding sensor.
A. FEM Implementation

A.1. ANSYS

In order to evaluate the change in resonance frequency due to a transverse center displacement of a clamped, planar plate of different shape vibrating in vacuum the finite element software ANSYS 12.1 was used. The diaphragm displacement is caused by a pressure load of up to 14 kPa. The fundamental and higher structural vibration modes of a circular, square and a rectangular diaphragm with an aspect ratio of 1.5 and 2.5 were considered and their mode shapes determined. The simulation is based on a large-deflection static analysis (NLGEOM, ON and PSTRES, ON) followed by a prestress modal analysis (PSTRES, ON).

A.1.1. Mesh study

A numerical model which is suitable to reproduce a MEMS plate resonator operating as a pressure sensor has to deal with some difficulties caused by the geometry, material and operating conditions. An appropriate computational grid for a thin structure which is based on solid elements is usually expensive in computing because the thickness of the diaphragm has to be sufficiently resolved and a high aspect ratio of the elements (size/thickness) prevented. Therefore, shell elements are usually used for such geometries but care has to be taken if large deflections of the structure (deflection larger than the plate thickness) are considered. Single crystal silicon (SCS) exhibits an anisotropic material behavior which can have a non-negligible influence on the vibration behavior of the resonator and has to be verified.

To evaluate a suitable FEM model, different elements such as the SOLID45, SOLID185, SOLID186 (circular diaphragm) and the SHELL63 element were considered and verified using experimental data. A study regarding the mesh density was performed as well. The SOLID185 element allows to implement an anisotropic material model which introduces a linear relationship between stress and strain and is formulated as a tensor. The matrix elements (stiffness coefficients) required for SCS material are: $c_{11} = 166$ GPa, $c_{12} = 64$ GPa and $c_{44} = 80$ GPa [93]. In addition, besides dealing with material anisotropy directly also tests based on an isotropic material model but with a Young’s modulus representing a particular orientation of
the crystal lattice of SCS\(^1\) were carried out. In Table A.1 all important parameters and details regarding the FEM model are listed.

\[ E_{(100)} = 130 \text{ GPa}, \quad E_{(110)} = 170 \text{ GPa} \quad \text{and} \quad E_{(111)} = 185 \text{ GPa} \]

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**Figure A.1.** Operating lines of a square (1.4 mm and 6.7 µm thick) resonator, mode (1,1): a) Element study based on an isotropic material model \( E = 130 \) GPa: IS I: SOLID185, IS II: SOLID45, IS IV: SHELL63, b) Ani- vs. isotropic material model: AS: anisotropic SOLID185, IS III: SOLID45 & \( E = 168 \) GPa, IS II: SOLID45 & \( E = 130 \) GPa, experiment: chip 46 cf. Appendix C.

Figure A.1 shows the resonance frequency as function of the pressures load for a 1.4 mm large and 6.7 µm thick, square diaphragm operating in the fundamental mode. The left figure compares the aforementioned elements using an isotropic material model and \( E_{(100)} = 130 \) GPa with experimental data (chip 46 cf. Appendix C). The SOLID45 element shows a good agreement with the experiment whereas the shell element overestimates the frequency shift for higher pressure loads. The SOLID185 element represents a sensor which is too stiff inside the low pressure range (\( \lesssim 6 \) kPa). If the anisotropic material model is used this effect becomes more pronounced (cf. Fig. A.1b). Using the SOLID45 element and isotropic material with \( E_{(110)} = 168 \) GPa the resonance frequency is overestimated over the whole pressure range.

The SOLID45 element seems to provide accurate results already with a low resolution grid containing 80 elements per edge length and 4 elements in the direction of the diaphragm thickness (cf. Fig. A.2). Using the SOLID185 element (isotropic and anisotropic model) the solution converges towards a constant solution (resonance frequency) first for a grid with

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\(^1\)E\((100)\) = 130 GPa, \(E_{(110)} = 170\) GPa and \(E_{(111)} = 185\) GPa \cite{94}
### Table A.1: Tests carried out during the mesh study and the corresponding model parameters

<table>
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<th>Test</th>
<th>AS</th>
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<th>IS II</th>
<th>IS III</th>
<th>IS IV</th>
<th>IS V</th>
<th>IS VI</th>
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<td>SOLID45</td>
<td>SHELL63</td>
<td>SHELL63</td>
<td>SOLID186</td>
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<td>8200</td>
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more than 130 elements per edge length. This convergence is considerably delayed if the transverse center deflection instead of the resonance frequency is considered. As is shown in Figure A.3 3 elements (SOLID45) are sufficient to discretize the diaphragm’s thickness.

![Figure A.2.](image)

**Figure A.2.**: Mesh density and element study for a square diaphragm (1.4 mm and 6.7 μm thick) at $p_{\text{load}} = 10$ kPa: Mesh density with respect to the thickness: 4 elements, AS: SOLID185, IS: SOLID45 and 185 & $E = 130$ GPa a) Resonance frequency b) transverse center deflection

Similar to the rectangular diaphragm a large discrepancy in resonance frequency prediction is also apparent for a circular diaphragm using SHELL63 and SOLID186 elements (cf. Fig. A.4). The disagreement gets much more pronounced if higher vibration modes are considered (cf. Fig. A.4b). No experimental data were available to validate the results obtained for a circular diaphragm. However, the SOLID186 elements which allow to deal with irregular geometries were used as a reference case. This assumption is justified if one considers the similar behavior as apparent for a square diaphragm and a very good agreement with a semi-empirical approach shown in Appendix A.1.2. A grid with 2200 elements (SOLID186) and just one element representing the diaphragm thickness still provides a good accuracy (cf. Fig. A.5). However, especially for higher modes and to maximize the accuracy of the simulation a finer grid should be used (No. of elements: 8800 and thickness division: 2, cf. Table A.2).

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2 all model parameters are listed in Table A.1
Figure A.3.: Mesh density study for a square diaphragm (1.4 mm and 6.7 µm thick) at \( p_{\text{load}} = 10 \text{ kPa} \): SOLID45 & \( E = 130 \text{ GPa} \), a) Resonance frequency and b) transverse center deflection vs. mesh density with respect to the diaphragm edge and thickness.

Figure A.4.: FEM model verification for a circular diaphragm with \( D = 2.25 \text{ mm} \) and \( t = 7 \text{ µm} \) (model parameters cf. Table A.1): Element study (Shell63 vs. SOLID186) a) Operating lines for the fundamental mode and b) for mode (0,1) (cf. Fig. 3.6)
A. FEM Implementation

Figure A.5.: FEM model verification for a circular diaphragm ($D = 2.25 \text{ mm}$, $t = 7 \mu\text{m}$) at $p_{\text{load}} = 14 \text{ kPa}$: Mesh density (total number of elements) for SOLID186 (cf. Table A.2) a) Resonance frequency for the fundamental mode and b) for mode (0,1)

Table A.2.: Parameters defining the mesh size (circular diaphragm: $D = 2.25 \text{ mm}$, $t = 7 \mu\text{m}$)

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<th>No. of elements</th>
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<th>8200</th>
<th>18800</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of nodes</td>
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<td>Max. element size [$\mu\text{m}$]</td>
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<td>30</td>
<td>23</td>
</tr>
<tr>
<td>Thickness division</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

A.1.2. Validation

Figure A.6 compares the FEM model with experimentally determined data (full markers, cf. Chapter 5). The data represent operating lines (i.e. resonance frequency as function of the pressure load) for a square, clamped diaphragm. Four different devices with a diaphragm size between 1.4 and 2 mm and a thickness of 7 $\mu\text{m}$ are considered. The pressure load varies between 0 and 14 kPa. The left figure shows the behavior for the fundamental (1,1) and the right one for the (3,1) mode. The FEM model is based on SOLID45 elements (isotropic & $E = 130 \text{ GPa}$) with a maximum element size of 20 $\mu\text{m}$ (lateral extent). The diaphragm thickness is represented by 3 elements. The experimental data are based on tests shown in Section 5.3.3 (cf. Fig. 5.18a and 5.20a).

A fair agreement between experiment and FEM is obvious especially
Figure A.6.: Validation of the FEM model (SOLID45) using experimental data (full markers, cf. Sec. 5.3.3). The data represent the resonance frequency as function of the pressure load for a clamped, square diaphragm with a size between 1.4 and 2 mm and a thickness of 7 µm a) fundamental mode (1,1) b) (3,1) mode for large diaphragms and the fundamental mode. The smallest considered sensor with a diaphragm size of 1.4 mm exhibits a higher sensitivity within the low pressure load range compared with FEM. This behavior is mainly obvious if the (3,1) mode is considered. Apart from that the FEM model seems to overestimate the pressure sensitivity for the fundamental mode but predicts a lower sensitivity for the higher mode.

Figures A.7 and A.8 show the validation of the semi-empirical approach described in Section 3.5.2. Again, the operating lines for a clamped, square\(^3\) and a circular\(^4\) diaphragm of different size and thickness operating in the fundamental and the (3,1) mode are considered. The solid lines represent the theoretical approach and the markers the FEM model. Based on the FEM data the constant \(c_{ij}\) in equation 3.17 is determined considering the best obvious fit between the data (cf. Table 3.1). The blue dashed line in Figure A.7a points out the suitability of the semi-empirical approach if the constant \(c_{ij}\) stated in [47] is employed. Considered is only the largest diaphragm with a thickness of 7 µm operating in the fundamental mode.

---

\(^3\)FEM: SOLID45 isotro. & \(E = 130\) GPa, max. element size: 20 µm, thick. div.: 3
\(^4\)FEM: SOLID186 isotro. & \(E = 130\) GPa, max. element size: 30 µm, thick. div.: 2
A. FEM Implementation

Figure A.7.: Validation of the semi-empirical approach (solid lines) using data determined by ANSYS (markers). The data represent the resonance frequency as function of the pressure load for a clamped, square diaphragm with a size between 1.4 and 2 mm and a thickness of 7 and 6 µm: a) fundamental mode (1,1) b) (3,1) mode

Figure A.8.: Validation of the semi-empirical approach (solid lines) using data determined by ANSYS. The data represent the resonance frequency as function of the pressure load for clamped, circular diaphragm with a diameter between 1.8 and 2.25 mm and a thickness between 5 and 7 µm: a) fundamental mode (0,0) b) (0,1) mode
A.2. COMSOL

A.2.1. Plate-cavity interaction

In order to quantify the added stiffness and mass effect caused by the gas-filled cavity coupled to the resonator’s diaphragm numerical simulations were performed using the FEM package COMSOL 3.5a. The clamped, square and unloaded diaphragm was modeled using shell elements and was coupled to the acoustic model (rectangular cavity) by imposing pressure boundary conditions on the structural side and acceleration boundary conditions on the acoustic side. On the remaining cavity walls hard wall boundary conditions were imposed. The resonance frequency and structural mode shapes were determined based on a time-harmonic analysis without including any damping. As an excitation source a point force at the position $x = a/4$ and $y = b/4$ was applied.

![Figure A.9.](image)

**Figure A.9.:** Validation of the numerical model (COMSOL) based on data published by Beltman et al. [53]: a) Comparison of the present model with experimental data b) Comparison of the present model with the reference model

The numerical model was validated based on numerical and experimental data published by Beltman et al. [53]. The experiments were performed on an airtight box covered with a flexible, clamped plate both out of aluminum. The plate was 0.49 x 0.245 m large and 1 mm thick. The gap beneath the plate was varied between 1 and 50 mm. The numerical model was based on two types of calculations, a standard acoustic finite element method used for gap width from 300 mm down to 50 mm and a viscous acoustic finite element method (B2000) for gap widths between 1 and 50 mm. For the
A. FEM Implementation

Table A.3: Model parameters (COMSOL 3.5a): Recalculation of the data published by Beltman et al. [53]

<table>
<thead>
<tr>
<th>Fluid properties (air):</th>
<th>$R_i = 287$ J/kgK</th>
<th>$T_c = 293$ K</th>
<th>$p_{atm} = 1$ bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate properties:</td>
<td>$\rho_{al} = 2710$ kg/m$^3$</td>
<td>$E = 70e9$ N/m$^2$</td>
<td>$\nu = 0.3$</td>
</tr>
<tr>
<td>Mesh settings:</td>
<td>hauto: 3</td>
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<td></td>
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<tr>
<td></td>
<td>hmaxfac: top face 0.015, side 0.04, bottom 0.1</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>methodfac: triaf</td>
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</tr>
<tr>
<td></td>
<td>hmaxsub: [1,0.08]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excitation force:</td>
<td>$f = p_0S_d$ with $p_0 = 20$ Pa and $S_d = 0.49 \cdot 0.245 \cdot 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis:</td>
<td>time-harmonic, $f_{range} = 30 - 350$ Hz with 0.5 Hz steps</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Present numerical simulations only gap widths between 2 and 300 mm were considered. In Table A.3 all important model parameters are listed.

Figure A.9b shows a good agreement between the two numerical models only for the two first structural vibration modes. For higher modes the present model predicts lower resonance frequencies than the reference model. If the present model is compared with the experimental data stated by Beltman et al. a good agreement for all considered modes is obvious (cf. Fig. A.9a). It seems that the present model provides a higher overall accuracy than the reference model.

Table A.4: Model parameters (COMSOL 3.5a): Plate resonator (clamped, square diaphragm, size: 1.4 mm, thickness: 7 µm) coupled to an air-filled cavity

<table>
<thead>
<tr>
<th>Fluid properties (air):</th>
<th>$R_i = 287$ J/kgK</th>
<th>$T_c = 293$ K</th>
<th>$p_{atm} = 1$ bar</th>
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</thead>
<tbody>
<tr>
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<td>$E = 130e9$ N/m$^2$</td>
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<td></td>
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<td>methodfac: triaf</td>
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<td></td>
<td>hmaxsub: [1,0.08]</td>
<td></td>
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</tr>
<tr>
<td>Excitation force:</td>
<td>$f = p_0S_d$ with $p_0 = 20$ Pa and $S_d = (1.4 \cdot 10^{-5})^2$</td>
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<td></td>
</tr>
<tr>
<td>Analysis:</td>
<td>time-harmonic, 2 refinement steps: 100 and 20 Hz steps</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table A.4 all important parameters are listed regarding the numerical model employed to quantify the nonviscous fluid loading of the silicon plate resonator (size: 1.4 mm, thickness: 7 µm) caused by the air-filled reference cavity of the sensor. The results are shown in Section 3.6.1.
B. Numerical implementation of the Rayleigh integral approach

The numerical implementation of the Rayleigh integral approach and thus the computation of the radiation efficiency\(^1\) and the reactive part of fluid loading of the resonant structure\(^2\) is based on the work of Lemmen et al. [42].

![Schematic representation of the computational domain and parameter definition](image)

**Figure B.1:** Schematic representation of the computational domain and parameter definition

B.1. Radiation efficiency

The computational domain corresponding to the vibrating surface (i.e. diaphragm) is divided in \(N\) quadrilateral 4-node elements (cf. Fig. B.1). Assuming a linear velocity field within the elements, the double integral in (B.2) is solved using a 1-point Gauss quadrature (B.3).

\[
\sigma = \frac{P_s}{P_p} \tag{B.1}
\]

\[
P_s = \frac{\omega \rho}{4\pi} \int_{A'} \int_A v(\vec{r}_s) \frac{\sin(kR)}{R} v(\vec{r}_s') dA dA' \tag{B.2}
\]

\(^1\)cf. Sec. 3.5
\(^2\)cf. Sec. 3.6.1
B. Numerical implementation of the Rayleigh integral approach

The velocities in the center of an element $v_{0,i}$ correspond to the average of the corner nodes $v_k$ (valid for a linear velocity field within an element). They are determined if the transverse surface displacement characterizing a specific modal shape is multiplied by the corresponding frequency $\omega$

$$P_s = 16 \frac{\omega \rho}{4\pi} \sum_{i=1}^{N} \sum_{j=1}^{N} v_{0,i} \frac{\sin(kR_{ij})}{R_{ij}} v_{0,j} |D_{0,i}| |D_{0,j}| \quad (B.3)$$

$$v_{0,i} = \frac{1}{4} \sum_{k=1}^{4} v_k \quad D_{0,i} = \frac{1}{4} A_i$$

$D_{0,i}$ is a scale factor (Jacobian) which accounts for an arbitrary size of an element and equals a quarter of the element’s surface area $A$.

$$P_p = \rho c \int_A \frac{1}{2} v^2(r_s) \, dA = 4 \frac{\rho c}{2} \sum_{i=1}^{N} v_{0,i}^2 |D_{0,i}| \quad (B.4)$$

The computations were carried out on a grid with more than 2000 elements using Matlab. This grid resolution is well above the required $N = 9mn$ elements to obtain accurate results [42]. $m, n$ are indices defining the mode number.

B.2. Fluid loading (reactive part)

The AVMI factor $\beta$ is computed by numerically solving equation B.6 and B.7. The numerical implementation is analogous to the approach described above.

$$\beta = \frac{W_f}{W_s} \quad (B.5)$$

$$W_f = 16 \frac{\rho}{4\pi} \sum_{i=1}^{N} \sum_{j=1}^{N} v_{0,i} \frac{\cos(kR_{ij})}{R_{ij}} v_{0,j} |D_{0,i}| |D_{0,j}| \quad (B.6)$$

$$W_p = 4 \frac{\rho s t}{2} \sum_{i=1}^{N} v_{0,i}^2 |D_{0,i}| \quad (B.7)$$
C. Sensor module description
### C. Sensor module description

<table>
<thead>
<tr>
<th>ID</th>
<th>Sensor (top)</th>
<th>Excitation</th>
<th>Mode</th>
<th>Chip</th>
<th>Size</th>
<th>Type</th>
<th>Depth</th>
<th>Width</th>
<th>Height</th>
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<tbody>
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<td>new</td>
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<td>new</td>
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<td>1.8</td>
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<td>1.8</td>
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<td>200</td>
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<td>1.8</td>
<td>10</td>
<td>200</td>
</tr>
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<td>39</td>
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<td>new</td>
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<td>1.8</td>
<td>10</td>
<td>1.8</td>
<td>10</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table C.1: Sensor module description**

All sensors exhibit a diaphragm thickness of 7 \( \mu m \).
D. Wind tunnel tests

D.1. Sensing characteristics of the sensor strip

**Figure D.1.** Operating lines for all 5 sensing elements on the sensor stripe. Measurements are based on optical excitation a) mode (1,2) b) mode (3,1)

Only mode (1,2) and (3,1) are considered during the verification tests carried out in the test cell, using the LDV system and optical excitation. For mode (1,2) a pressure range between 0.6 and 4.6 kPa was considered. Beyond this range the fundamental mode (1,1) dominates the vibration of the plate resonator if driven by acoustic noise excitation (cf. Sec. 5.3.2). The fundamental mode is strongly damped by acoustic radiation and thus not considered for the present wind tunnel tests.
D. Wind tunnel tests

Figure D.2.: Damping behavior (quality factor) of all 5 sensing elements on the sensor stripe. Measurements are based on optical excitation a) mode (1,2) b) mode (3,1)

D.2. Pressure tap system

Figure D.3.: Pressure tap position on the NACA 0012 wing (c = 300 mm)
### Table D.1.: Mechanical pressure scanner: Channel assignment

<table>
<thead>
<tr>
<th>Channel</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 28</td>
<td>pressure taps</td>
</tr>
<tr>
<td>29</td>
<td>MEMS pressure sensor stripe</td>
</tr>
<tr>
<td>30 to 35</td>
<td>free</td>
</tr>
<tr>
<td>36</td>
<td>Prandtl-tube, static pressure</td>
</tr>
<tr>
<td>37</td>
<td>Prandtl-tube, total pressure</td>
</tr>
<tr>
<td>39</td>
<td>static pressure in the settling chamber</td>
</tr>
<tr>
<td>40</td>
<td>static pressure at contraction cone exit</td>
</tr>
</tbody>
</table>
Bibliography


Bibliography


Publications

Reviewed Articles


Conference Proceedings


Conference Talks


