Doctoral Thesis

Wideband multi-user cooperative networks
Theory and measurements

Author(s):
Auger, Etienne

Publication Date:
2012

Permanent Link:
https://doi.org/10.3929/ethz-a-007242707

Rights / License:
In Copyright - Non-Commercial Use Permitted
Wideband Multi-user Cooperative Networks: Theory and Measurements

A dissertation submitted to
ETH ZURICH

for the degree of
Doctor of Sciences

presented by
ETIENNE AUGER
Dipl. El-Ing., ETH Zürich
born September 18, 1979
citizen of France

accepted on the recommendation of
Prof. Dr.-Ing. A. Wittneben, examiner
Univ.-Prof. Dr.-Ing. G. Bauch, co-examiner

2012
Day of Doctoral Examination : February 15, 2012
Abstract

In this thesis we study communication schemes and channel modeling for a pervasive wireless access network (PWAN). Such a network is formed by nodes equipped with heterogeneous antenna arrangements and placed up to 10 m apart from each other. Nodes can be static when they belong to the infrastructure or have limited mobility when they are carried by a person. This scenario is extremely promising for relay-based applications and fills the gap between the body area network (BAN) that has a coverage up to 1 m and the wireless local area network (WLAN) that has a coverage up to 100 m. When a PWAN is jointly considered with high node density, it allows for node cooperations, and idle or infrastructure nodes can support the communication of source-destination pairs. With an adequate signaling scheme, the PWAN can operate as a virtual antenna array (VAA) and ultimately achieve multiple-input multiple-output (MIMO) gains in a distributed way. Until now these networks have been studied using oversimplified channel models that were frequency flat and designed for cellular low node density networks. The novelty of this thesis and its main contribution is the study of a PWAN under realistic channel conditions and hardware constraints.

First we consider a PWAN in a multi-path propagation environment. Multiple user pairs communicate, supported by coherent relay nodes that have the capability of filtering with a finite impulse response (FIR) filter the incoming signal from the sources before forwarding it (filter-and-forward (FF) relays) to the destinations. The main result of this part is a multi-user zero-forcing (MUZF) relay filter gain allocation that performs a distributed MIMO channel orthogonalization independently of the sources modulation alphabet and ideally suppresses multi-user interferences (MUI). This scheme has a lower complexity than the existing orthogonal frequency-division multiplexing (OFDM) -based approach where the MUI are treated on a sub-carrier basis.

Afterwards we set our focus to a real-world implementation of a PWAN available at the Wireless Communication Group (WCG) at ETH Zurich and provide as a main result a novel non-collocated MIMO baseband channel model that takes the spatial-temporal correlations between the channels, and the nodes nomadic and behavioral mobility into account. The proposed baseband channel encloses the propagation channel, the receiver and transmitter
Abstract

antenna patterns and all analog hardware effects in the nodes. In order to design this model, the nodes that build the PWAN are characterized and their identified imperfections are discussed for their effect on the channel estimation. Finally we present the channel measurement campaign and discuss the results. The measurements outcome is a set of parameters for a non-collocated MIMO channel model that is validated by comparison with measured channels.
Résumé

Cette thèse présente des méthodes de communication et la modélisation des canaux dans un réseau local pénétrant (en anglais PWAN). Un tel réseau est constitué de nœuds équipés d’antennes avec des structures hétérogènes et séparés par des distances allant jusqu’à 10 m. Ces nœuds sont statiques lorsqu’ils sont intégrés à l’infrastructure, ou ont une mobilité réduite lorsqu’ils sont portés par une personne. Ce scénario est extrêmement prometteur pour de nouvelles applications utilisant des relais, et correspond à un réseau de taille intermédiaire situé entre le réseau local corporel (en anglais BAN) qui a une couverture jusqu’à 1 m, et le réseau local sans fil (en anglais WLAN) qui a une couverture de l’ordre de 100 m.

Si l’on considère un réseau local pénétrant avec une forte densité de nœuds, il est possible d’exploiter des coopérations entre ces derniers, les nœuds inutilisés pouvant apporter leur soutien aux paires communicantes. Lorsque les nœuds transmettent de façon adéquate, le réseau local peut opérer comme un système virtuel d’antennes et réaliser des gains similaires à ceux obtenus dans les systèmes à entrées multiples-sorties multiples (en anglais MIMO), mais de façon distribuée. Jusqu’à maintenant ces réseaux ont été étudiés pour des modèles de canaux constants en fréquence et conçus pour des réseaux de communication cellulaire à faible densité de nœuds. Le présent travail tire sa motivation de cette observation, et propose l’étude d’un réseau local pénétrant dans des conditions réalistes.

Nous considérons tout d’abord un réseau local pénétrant dans un environnement générant des canaux sélectifs en fréquence. Plusieurs paires d’utilisateurs sont engagés dans une transmission facilitée par des relais cohérents qui filtrent leur signal reçu avec un filtre à réponse impulsionnelle finie avant de le retransmettre aux nœuds destinataires. Nous proposons une méthode d’attribution multi-utilisateur des coefficients des filtres des relais à zéro-forcé (en anglais MUZF). Cette méthode supprime idéalement les interférences causées par les autres utilisateurs et orthogonalise le canal à entrées multiples-sorties multiples de manière distribuée, indépendamment du choix de l’alphabet des sources. Cet algorithme a une complexité moindre que celle d’une méthode utilisant le codage par répartition en fréquences orthogonales (en anglais OFDM) pour laquelle les interférences sont supprimées pour chaque sous-porteuse.
**Résumé**

Ensuite nous étudions un exemple de réalisation d’un réseau local pénétrant disponible au laboratoire du Wireless Communications Group (WCG) à l’ETH Zurich, et utilisons cet équipement pour obtenir le premier modèle réaliste d’un canal à entrées multiples-sorties multiples dégroupées dans la bande de base. Ce modèle respecte les corrélations spatiales et temporelles entre les canaux, ainsi que la mobilité nomade et comportementale des nœuds. Ce modèle de canal dans la bande de base contient les effets dus à la propagation, le schéma de distribution d’ondes des antennes en transmission et réception et les effets des composants électroniques analogiques des nœuds. Nous caractérisons les nœuds du système RACooN qui forment ce réseau pénétrant, et quantifions l’effet de leurs imperfections sur l’estimation du canal. Pour finir nous présentons les résultats d’une campagne de mesure. Les données récoltées sont utilisées pour calculer les paramètres du modèle du canal, qui est validé par une comparaison avec le résultat de mesures.
# Contents

Abstract i  
Résumé iii  

## 1 Introduction  
1.1 Motivation .................................................. 1  
1.2 Contribution and Outline ................................. 2  

## 2 Filter-and-Forward Relay Gain Allocation  
2.1 Introduction .................................................. 5  
2.2 State of the Art ............................................. 7  
2.3 System Model .................................................. 9  
2.4 Signal Model .................................................. 10  
2.4.1 Continuous-Time Signal Model ......................... 10  
2.4.2 Discrete-Time Signal Model .............................. 11  
2.4.3 Equivalent Model ......................................... 12  
2.5 Zero-Forcing Filter Gain Matrix .......................... 13  
2.5.1 Compound Gain ........................................... 14  
2.5.2 Zero-Forcing Condition ................................ 15  
2.5.3 Excess Cooperation ....................................... 17  
2.5.3.1 Minimum Cooperation Configuration .................. 18  
2.5.3.2 Additional Cooperations .............................. 18  
2.5.3.3 Full Cooperation ...................................... 19  
2.6 Optimization of the Nullspace Gain Vector ............... 20  
2.7 Simulation Results .......................................... 23  
2.7.1 Parameters ............................................... 23  
2.7.2 Figures of Merit .......................................... 23  
2.7.3 Performance Results ...................................... 25  
2.7.3.1 Spatial Multiplexing Gain ............................ 26
3 Imperfections of the RACooN Nodes

3.1 Introduction .................................................. 33

3.2 State of the Art ................................................. 35
   3.2.1 MIMO Sounder Architecture .............................. 35
   3.2.2 Existing Channel Sounders .............................. 36
      3.2.2.1 Existing TDMS Sounders ............................ 36
      3.2.2.2 Existing Fully-Parallel Sounders .................. 37
      3.2.2.3 Existing Semi-Sequential Sounders ................ 37
   3.2.3 Relay Network Demonstrator .............................. 37

3.3 Description of the RACooN Nodes ............................ 38
   3.3.1 Analog RFU ............................................. 38
   3.3.2 Digital STU ............................................ 40
      3.3.2.1 Hardware Description ............................... 40
      3.3.2.2 Data Structure ..................................... 40
      3.3.2.3 Time structure ..................................... 41
      3.3.2.4 Elementary Commands ............................... 42

3.4 Characterization of the Devices and Imperfections ........ 43
   3.4.1 Common Measurement Setup .............................. 43
      3.4.1.1 Choice of the Frequency ............................ 43
      3.4.1.2 Received Signal Analysis ........................... 44
   3.4.2 Carrier Frequency Offset ................................ 50
   3.4.3 System Phase Noise ..................................... 51
      3.4.3.1 Origin and Definition of Oscillator Phase Noise .... 52
      3.4.3.2 Model for Oscillator Phase Noise .................. 53
      3.4.3.3 Experimental Setup ................................. 56
   3.4.4 System Linearity ......................................... 60
      3.4.4.1 Origin of the Non-Linearities ..................... 60
      3.4.4.2 Power Gain Linearity ............................... 62
      3.4.4.3 Compression Point ................................. 62
      3.4.4.4 Intercept Points ................................. 64
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4.5</td>
<td>I-Q Imbalance</td>
<td>65</td>
</tr>
<tr>
<td>3.4.5.1</td>
<td>Origin</td>
<td>65</td>
</tr>
<tr>
<td>3.4.5.2</td>
<td>Complex Model</td>
<td>66</td>
</tr>
<tr>
<td>3.4.5.3</td>
<td>I-Q Imbalance Estimation on the RACooN Units</td>
<td>68</td>
</tr>
<tr>
<td>3.4.6</td>
<td>DC Offset</td>
<td>70</td>
</tr>
<tr>
<td>3.4.6.1</td>
<td>Definition</td>
<td>70</td>
</tr>
<tr>
<td>3.4.6.2</td>
<td>Origin</td>
<td>70</td>
</tr>
<tr>
<td>3.4.6.3</td>
<td>DC Offset Measurement</td>
<td>72</td>
</tr>
<tr>
<td>3.4.6.4</td>
<td>Model</td>
<td>73</td>
</tr>
<tr>
<td>3.5</td>
<td>Conclusion</td>
<td>78</td>
</tr>
<tr>
<td>4</td>
<td>Robust Channel Estimation with the RACooN Lab</td>
<td>79</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>79</td>
</tr>
<tr>
<td>4.2</td>
<td>State of the Art for Channel Sounding</td>
<td>80</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Impulse Sequence</td>
<td>80</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Direct Sequence</td>
<td>81</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Chirp Sequence</td>
<td>83</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Comparison Between Direct and Chirp Sequences</td>
<td>85</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Common Channel Estimation</td>
<td>86</td>
</tr>
<tr>
<td>4.2.5.1</td>
<td>Averaging</td>
<td>87</td>
</tr>
<tr>
<td>4.2.5.2</td>
<td>Pulse Compression</td>
<td>87</td>
</tr>
<tr>
<td>4.2.5.3</td>
<td>Parameter Choice</td>
<td>88</td>
</tr>
<tr>
<td>4.3</td>
<td>Gain Degradation due to Hardware Imperfections</td>
<td>89</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Summary of the Signal Processing Steps</td>
<td>89</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Averaging</td>
<td>90</td>
</tr>
<tr>
<td>4.3.2.1</td>
<td>Ideal Case</td>
<td>90</td>
</tr>
<tr>
<td>4.3.2.2</td>
<td>With Frequency Offset</td>
<td>91</td>
</tr>
<tr>
<td>4.3.2.3</td>
<td>With Phase Noise</td>
<td>93</td>
</tr>
<tr>
<td>4.3.3</td>
<td>Correlation</td>
<td>96</td>
</tr>
<tr>
<td>4.3.3.1</td>
<td>Ideal Case</td>
<td>96</td>
</tr>
<tr>
<td>4.3.3.2</td>
<td>With Frequency Offset</td>
<td>97</td>
</tr>
<tr>
<td>4.3.3.3</td>
<td>With Phase Noise</td>
<td>103</td>
</tr>
<tr>
<td>4.4</td>
<td>Robust Channel Estimation</td>
<td>105</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Observations and Motivation</td>
<td>105</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Origin of the Non-Linearities</td>
<td>106</td>
</tr>
</tbody>
</table>
## 4.4.3 Channel Estimation Robust to Non-Linearities
- **4.4.3.1 Example Using a Simulated Channel**
- **4.4.3.2 Application to Measured Channels**

## 4.4.4 For Chirp Sequence

## 4.4.5 Result Comparison

## 4.4.6 Calibration

## 4.5 Conclusion

## 5 Wideband Network Channel Measurements

### 5.1 Introduction

### 5.2 Motivation and State of the Art
- **5.2.1 Measurement Campaigns**
- **5.2.2 PWAN Channel Model**
  - **5.2.2.1 PWAN SISO Link Model**
  - **5.2.2.2 PWAN MIMO Channel Model**

### 5.3 Setup
- **5.3.1 RACooN Units**
- **5.3.2 Environment**
- **5.3.3 Behavioral Patterns**

### 5.4 Measurement Campaign
- **5.4.1 Parameter Choice**
- **5.4.2 Transmit Signal**
  - **5.4.2.1 Parameters**
  - **5.4.2.2 Multilevel Transmission**
- **5.4.3 Measurement Protocol**

### 5.5 Measurement Evaluation
- **5.5.1 Post-Processing Steps**
- **5.5.2 Unit Tracking Position**
  - **5.5.2.1 Problem Setting and State of the Art**
  - **5.5.2.2 Assumptions**
  - **5.5.2.3 Ideal Case**
  - **5.5.2.4 Robust Time-of-Arrival Estimation**
- **5.5.3 Channel Impulse Response Magnitude**
- **5.5.4 Average Power Delay Profile**
  - **5.5.4.1 Noise Floor and Non-Linearities Discussion**
  - **5.5.4.2 Sounding Sequence Discussion**
Chapter 1

Introduction

1.1 Motivation

From Marconi in 1896 and his first telegraph system to the beginning of the 1960s and the launch of the first communication satellite, the focus of scientists and industry was to develop and provide communication systems with enhanced range. In the late 1970s, satellites started offering coverage of virtually every point in the world, and focus turned to communication reliability. The Advanced Research Projects Agency Network (ARPANET) aimed at establishing reliable wired connections between remote computers. Wired meshes were introduced to create redundancy in the network and also increased the transmission rates. The concept of mesh was taken over for wireless communications in the Advanced Mobile Phone System (AMPS) system of the Bell Labs that divided in 1977 the US in geographic regions called cells. There the problematic shifted from a global coverage to a local coverage issue, the latter being the basis to achieve the former. The trend continued, narrowing down the coverage around the user while increasing the data rate support requirements. Bluetooth in 1994 defined the first pervasive wireless access network (PWAN) which is then named piconet and operating within 1 m, 10 m or 100 m depending on the transmit power requirements. It was followed by the wireless local area network (WLAN) and ZigBee from the late 1990s that all define networks with a range up to some hundred meters. More recently the multiplication of the electronic devices and the increase of digital data demand for higher speed and simple data transfer possibilities between devices. Wireless solutions based on ultra wideband (UWB) in a body area network (BAN) are a further step toward connectivity between wearable devices.

As evoked previously, local coverage was originally thought as a first step towards global coverage. In an approach to move into that direction, relaying systems have been introduced
to enable communication between nodes (or devices) that are otherwise not able to communicate because either they are too far away from each other or too simplistic to decode the received signal. These nodes can be devices, but also key nodes of shorter range networks like a BAN. Devices, key nodes and relays can form a PWAN which is the focus of this thesis.

A PWAN is a network in which nodes up to 10 m apart from each other can communicate with each other. There is a priori no hierarchy between the nodes that have the capability for self-organization. Antenna arrangements in a PWAN can be very heterogeneous, they range from collocated multiple-input multiple-output (MIMO) antennas where all antennas are organized on a regular array to a distributed antenna system (DAS) where antennas are physically separated by up to several meters and connected by a cable to the signal processing unit of the node. A PWAN is an extremely promising scenario when it is jointly considered with high node density since it allows for node cooperations. The PWAN can operate as a virtual antenna array (VAA) with an adequate signaling scheme and ultimately achieve MIMO gains.

1.2 Contribution and Outline

In chapter 2 of this thesis we consider a PWAN in a frequency selective environment. A multipath channel can occur when electromagnetic waves are reflected in the environment and impinge at the receiver with a larger delay than the line of sight (LOS) wave. Idle nodes or dedicated relays are available in the network to support the transmission of given source-destination pairs. The relays are equipped with a finite impulse response (FIR) filter used to process their incoming signal before forwarding it in a half-duplex (HDX) mode. We propose for that scenario a multi-user zero-forcing (MUZF) gain allocation scheme that orthogonalizes the equivalent two-hop source-to-destination channel. We also state a set of necessary conditions for the orthogonalization to be performed. This scheme shifts the complexity from the source-destination pairs to the relays. The user pairs require less energy for signal processing and, if they are battery-powered, extend their lifetime. In the case of dedicated relays built into a DAS, it can be assumed that the relays draw their power supply from the infrastructure and can afford complexity.

In chapter 3 we focus on a hardware implementation of PWAN nodes embodied by the Radio Access with Cooperative Nodes (RACooN) of the Wireless Communication Group (WCG) at ETH Zurich. The architecture of these nodes is analyzed. We identify five imperfections (i) carrier frequency offset (CFO) (ii) phase noise (iii) direct component (DC) offset
(iv) I-Q imbalance and (v) non-linearities that will affect the signal. We characterize them and propose a model for the nodes.

In chapter 4 we pursue our study of the RACooN nodes for using them as measurement devices to sound the channel. The imperfections identified in chapter 3 are further considered. We provide analytical and numerical expressions for the degradation of the channel estimate caused by these imperfections. We propose a signal processing method for channel estimation and assess it with respect to the imperfections. This study encloses a discussion on the sounding sequence choice.

The last chapter 5 is based on chapters 3 and 4 and presents the results of a measurement campaign performed at ETH Zurich with the RACooN nodes. We propose a measurement protocol that takes into account the discussion on the node imperfections from the previous chapters. The RACooN lab architecture allows for semi-sequential non-collocated MIMO channel estimation. Measurements are performed in two different environments and ten different setups. The nodes have a nomadic mobility and one node is subject to a behavioral pattern, that is a random movement over a finite circular area that mimics human behavior. This is the first ever-performed measurement of that kind. The measurement data are finally used to compute the parameters of a non-collocated MIMO channel model.
Chapter 2

Filter-and-Forward Relay Gain Allocation in Multi-User Wideband Networks

2.1 Introduction

In this chapter we consider a pervasive wireless access network (PWAN) with $N_u$ communicating user pairs sharing the same wireless medium. When all sources concurrently access the medium, each receiver receives $N_u$ streams, out of them $N_u - 1$ are interference streams. A rule of thumb to manage interferences at the receiver is summarized in [20] as follows: strong interference should be decoded, weak interference should be treated as noise, and if interference is as strong as the useful signal, the channel should be orthogonalized. Orthogonalization consists in processing the signals at the nodes (source, destination or relays) such that the equivalent system can be simplified to a set of $N_u$ user pairs communicating concurrently through the same medium and without multi-user interference (MUI) at the receivers. In a PWAN with high node density, two receivers, resp. two transmitters, can be close to each other. The signal transmitted by each source to its destination is interference for the other destination, and interference is assumed as strong as the signal. This motivates channel orthogonalization for MUI cancellation. In our setup, orthogonalization is achieved by using idle nodes available in the environment as relays to achieve a distributed orthogonalization.

We focus on two-hop strategies where in the first timeslot the sources transmit their data to the relays and in the second timeslot the phase and symbol synchronous relays simultaneously retransmit their processed incoming data (half-duplex (HDX) constraint) to the destinations. The relays are called

- filter-and-forward (FF) relays if they filter their incoming signal before retransmitting it,
Chapter 2 Filter-and-Forward Relay Gain Allocation

- amplify-and-forward (AF), or non-regenerative relays, as a subcategory of the FF relays that simply retransmit an amplified version of their received signal,
- decode-and-forward (DF), or regenerative relays, if they decode their incoming signal before transmitting it,
- compress-and-forward (CF), as a subcategory of the DF relays that can decode and compress their incoming signal before retransmitting it,

Due to spatial separation of the relaying terminals, relaying schemes can provide a distributed spatial diversity gain which can be used to enhance the link reliability by averaging over independent channel realizations. Relays can also be used to increase the capacity of the system by achieving a distributed spatial multiplexing (SM) gain. This essentially means that a number of source/destination pairs can communicate concurrently over the same physical channel. The system sum rate is increased while using the same bandwidth and no additional power [88]. Operating several relays leads to an increase of the average signal-to-noise ratio (SNR) at the receiver since the signal from the relays can be coherently combined. This effect is called distributed array gain. These three gains are multiple-input multiple-output (MIMO) gains and have been introduced among others in [88] in their collocated form.

In the sequel, we propose a new distributed channel orthogonalization. The proposed setup is allocating the relay gains such that (i) the user links are improved and (ii) the MUIs are suppressed by a distributed MIMO orthogonalization. The novelty of the approach is that our results enclose (i) both the frequency selective and the frequency flat case, and (ii) different cooperation patterns between the relays, offering a general framework for a practical orthogonalization scheme.

The main contributions of this chapter are
- the statement of a multi-user zero-forcing (MUZF) minimum configuration of the number of relays and relay filter taps that represents a necessary condition for channel orthogonalization,
- the zero-forcing relay gain calculation in the frequency selective case, for any number of relays and any cooperation pattern that fulfills the MUZF minimum configuration,
- the calculation of the equivalent channel spectral efficiency per stream,
- the discussion of the number of relays, relay filter taps and optimization criterion effects on the SM gain, array gain and diversity gain, based on simulation results.

The remainder of this chapter is organized as follows. We give in part 2.2 the state of the art and present the system model in part 2.3. Part 2.4 contains the signal model and reformulates it such that in part 2.5 the zero-forcing (ZF) filter gain coefficients can be derived. Part 2.6 introduces an optimization of the nullspace vector. Finally simulation results are presented in part 2.7 and we conclude our analysis in part 2.8.
2.2 State of the Art

The main results available for relaying schemes are now reviewed, with a focus on independent multi-user (MU) two-hop communication with dedicated cooperating HDX amplify-and-forward (AF) relays as support nodes. We start with a summary of theoretical results obtained for the single user pair case supported by one relay, several relays and finally several user pairs. We recall then several practical relay gain allocation schemes, for the single user pair and MU case. In the MU case, we present major relay gain allocation schemes, for frequency-flat and frequency selective channels, subject to different relay cooperation patterns.

From the first fundamental results in [26], the two-hop relay channel is a major topic of research. In this work, a single-input single-output (SISO) scenario with a single-user pair and one single relay in considered. The scenario is extended to the $N_r$ relay case in [38]. The authors show that in the $N_r$ relay case, the network capacity grows with $\log N_r$ when $N_r$ goes to infinity. Diversity gains up to the number of relays can be achieved and have been studied for one relay in [108], and several relays in [62].

For MU communication, relays are beneficial as well. A theoretical result motivating the use of single-antenna relays assisting non-cooperative single-antenna MU communication is given in [19]. The authors show that a coherent relay network can offer - asymptotically in the number of relays - in a distributed way the MIMO gains described in 2.1 without the need that users have several antennas. Further results for the finite number of relays are in [59,122] and show that relays add degrees of freedom into the network that can be exploited to increase diversity and spatial multiplexing gain.

Relay gain allocation algorithms exploit the degrees of freedom available from spatial diversity in the network and are based on AF or decode-and-forward (DF) relays. Performance comparison between AF and DF relaying is done in [77] for the single-relay case and in [9] for the multiple relay case. [77] shows that at high SNR, AF and DF relaying can extract the full system diversity. Yet DF relaying suffers from its complexity, which motivates the use of AF relaying.

For the single-user case, power allocation between the source and a single relay is investigated in [47], another work [42] treats the power allocation when several independent relays are used and [66] with relays exhibiting random behaviors of cooperation.

In the MU case, gain allocation becomes more challenging. A time-division multiple access (TDMA) scheme has the drawback that the average transmission time allocated per user (and consequently the average user rate) tends to zero when the number of users grows to
Chapter 2 Filter-and-Forward Relay Gain Allocation

infinity. This issue motivates channel orthogonalization when a large number of user pairs are in the network. The relay gain allocation algorithm must perform MUI cancellation (i.e. extract spatial multiplexing gain) and if possible extract diversity gain. Different algorithms have been proposed and evaluated: beamforming [32] with MIMO relays, ZF in [127] with cooperating SISO relays, minimum mean-square error (MMSE) in [15], QR decomposition associated with ZF in [1, 110], and in [2] for MIMO relays.

Frequency-selective channels offer a new dimension to the problem by adding temporal diversity [79] in the network. The initial idea is that a SISO system with a L-tap frequency selective fading channel offers a diversity gain up to L. In this frequency-selective context, AF relays are renamed filter-and-forward (FF) relays [24] because they can linearly filter their incoming signal with a finite impulse response (FIR) filter before forwarding it. The analysis of communication schemes in frequency selective channels can be performed by dividing the transmission band into sub-bands over which the channel can be considered as flat. This analysis translates in practical algorithms based on orthogonal frequency-division multiplexing (OFDM). This approach has been retained among others in [43] with a single relay and a two-hop transmission. Yet a per-subcarrier optimization is computationally complex since it requires (i) the calculation of a gain for each subcarrier (128 subcarriers for 802.11a) at each relay [16] and (ii) the relays to perform OFDM modulation and demodulation.

For multihop communication, so far only [78] uses a time-domain approach to study the behavior of a single-user pair in a relay assisted communication with frequency-selective channels. Yet this theoretical work proposes a network code and does not constraint the number of hops in the communication.

General results for the maximum diversity gain extractable in a frequency-selective fading environment for a MU relaying system with distributed MIMO orthogonalization are not known. Only a result for the frequency-flat case is available in [128] and shows that the system can benefit from the traditional MIMO diversity gains.

Other works for distributed MIMO orthogonalization in frequency-selective channels rely either on the use of orthogonal frequency-division multiple access (OFDMA) or of codes to separate the different user pairs. For frequency-selective channels, a specific MU scenario is studied in [31] where the MU interferences are handled using code-select code-division multiple access (CSCDMA). Performance of CSCDMA degrades rapidly when the number of users increases, limiting the applicability of the CSCDMA scheme to system with few user pairs. OFDMA is used in [115] use to deal with the MUI, but diversity is gained there by the use of a scheduler and not through the relay gain allocation.
Relaying systems can benefit from cooperations between the relays. The cooperation pattern defines the received signal exchange between the relays. We distinguish

- full cooperation when there is a single MIMO relay or a linear distributed antenna system (LDAS),
- pure relaying when relays do not exchange their received signal, like for independent SISO relays,
- asymmetric relaying when unidirectional signal exchange occurs between relays, i.e. one relay shares its received signal with another relay, but this second relay does not,
- hybrid relaying which is the association of one or several full cooperative, pure cooperative relays or asymmetric relays.

Works like [59, 61, 108] identify cooperation as a key issue for the relaying systems. In [1, 2, 21, 32] full cooperation within MIMO relays is assumed, or a fixed hybrid pattern in [66], and cannot be straightforwardly extended to any cooperation pattern. The drawback of cooperations is the increased system complexity in terms of signal exchange that grows with \( N_r \).

### 2.3 System Model

We consider \( N_u \) independent single-antenna source-destination pairs that are communicating wirelessly through a frequency-selective block fading channel. The two-hop communication is assisted by \( N_r \) HDX AF identical single- or multiple-antenna relays. The nodes are depicted in Fig. 2.1.

The relays have a hybrid cooperation pattern and global channel state information (CSI), meaning that each relay knows all channels from all sources to all relays, and from all relays to all destinations. The cooperation pattern is maintained general to cover the MIMO

![Fig. 2.1: Multi-user relaying](image)
filter-and-forward relay gain allocation. The relay case, the independent relay case and any case in-between (linear distributed antenna system (LDAS) and hybrid relaying systems consisting of combinations of SISO, single-input multiple-output (SIMO), multiple-input single-output (MISO) and MIMO relays). The sources do not have channel state information. Receivers do not cooperate and have knowledge of the equivalent source-destination channel such that they can ideally decode their received signal. The direct link is discarded by the receiver. A generalized version of the original AF (non-regenerative) relays is considered, for which the gain is not a single amplification factor but a filter. The output signal of each relay is the convolution of its input with the relay filter finite impulse response. We name this scheme filter-and-forward relaying. The relays are coherent in the sense that their local oscillators are phase synchronized to a phase reference. Sources are coherent and destinations are coherent, but they can use different phase references, i.e. one for the source tier, one for the relay tier and another one for the destination tier. Several relay configurations are possible. For the pure relaying case the relays do not cooperate, i.e. they do not exchange received signals. For \( N_u = 1 \) the relays provide diversity and improve the average SNR at the destination, leading to a distributed array gain [88]. Such a scheme increases the range and coverage of the wireless communication system. For \( N_u \geq 2 \) we may also achieve a distributed SM gain.

2.4 Signal Model

2.4.1 Continuous-Time Signal Model

\( N_u \) sources produce with rate \( W \) stochastic independent sequences taken from Gaussian codebooks. The sequences are modulated by a band-limited frequency response time-continuous pulse and stacked into the transmit source symbol vector \( \vec{s}_c(\tau) \). We assume that no CSI is available at the sources. The autocovariance matrix of \( \vec{s}_c(\tau) \) is thus

\[
\begin{bmatrix}
\vec{s}_c(\tau_1)\vec{s}_c(\tau_2)^H
\end{bmatrix} = \sigma_s^2 \delta(\Delta \tau) I_{(N_u \times N_u)},
\]

with \( \Delta \tau = \tau_1 - \tau_2 \), \( \langle \cdot \rangle \) is the ensemble average and \( \cdot^H \) is the complex conjugate transpose operator. \( \delta(\cdot) \) is the time-continuous Dirac function. All the vectors are column vectors, and \( I_{(N_u \times N_u)} \) is the \( N_u \times N_u \) identity matrix. The dimensions of the vectors and matrices are only written where useful for the clarity.

We assume all channels are band-limited to a given bandwidth \( W \). Channel matrix \( X_{c(N_r \times N_u)}(\tau) \), resp. \( Z_{c(N_u \times N_r)}(\tau) \), denotes the matrix containing the time-continuous baseband channel impulse responses from the sources to the relays, resp. from the relays to the
destinations as a function of the delay $\tau$.

The received signal vector at the relays includes the additive white Gaussian noise component $\bar{\mathbf{m}}_c(\tau)$ with zero-mean and autocovariance matrix $\sigma_m^2 \delta(\Delta \tau) \mathbf{I}_{(N_t \times N_t)}$ representing the $N_t$ noise sequences with variance $\sigma_m^2$.

The relays filter the received signal vector with the filter bank contained in matrix $\mathbf{G}_c(\tau)$ to obtain the relay transmit signal $\bar{r}_c(\tau)$. For pure relaying $\mathbf{G}_c(\tau)$ is diagonal.

The $N_u$ time-continuous stochastic local noise sequences at the destinations are stacked into vector $\bar{\mathbf{w}}_c(\tau)$ with zero-mean and autocovariance matrix $\sigma_{w}^2 \delta(\Delta \tau) \mathbf{I}_{(N_u \times N_u)}$ that denotes the local noise contribution at the destinations.

### 2.4.2 Discrete-Time Signal Model

Due to the band-limited characteristics of all signals and channels there is equivalence between the time-continuous model and the time-discrete model.

After uniform sampling of the continuous-time stochastic vector $\bar{s}_c(\tau)$ with rate $2W$, the statistical properties of the sequences $\bar{s}(k) = \bar{s}_c(\frac{k}{2W})$, $\mathbf{X}(k) = \mathbf{X}_c(\frac{k}{2W})$, $\mathbf{Z}(k) = \mathbf{Z}_c(\frac{k}{2W})$ and $\mathbf{G}(k) = \mathbf{G}_c(\frac{k}{2W})$ are similar to the properties of $\bar{s}_c(\tau)$, $\mathbf{X}_c(\tau)$, $\mathbf{Z}_c(\tau)$ and $\mathbf{G}_c(\tau)$ with the restriction that $k$ is now an integer time variable. The discrete-time stochastic noise sequences stacked in $\bar{\mathbf{m}}(k)$ and $\bar{\mathbf{w}}(k)$ are complex normal independent identically distributed zero-mean distributed as $\bar{\mathbf{m}}(k) \sim \mathcal{CN}(\mathbf{0}_{(N_t)}; \sigma_m^2 \delta[k] \mathbf{I}_{(N_t \times N_t)})$ and $\bar{\mathbf{w}}(k) \sim \mathcal{CN}(\mathbf{0}_{(N_u)}; \sigma_w^2 \delta[k] \mathbf{I}_{(N_u \times N_u)})$. $\mathbf{0}$ denotes the all-zeros vector and $\delta[\cdot]$ is the time-discrete Dirac function.

The one-sided $z$-transform is applied to the discrete sequences to obtain $\bar{s}(z^{-1}) = \mathcal{Z}\{\bar{s}\}$, $\bar{r}(z^{-1}) = \mathcal{Z}\{\bar{r}\}$, $\mathbf{X}(z^{-1}) = \mathcal{Z}\{\mathbf{X}\}$, $\mathbf{Z}(z^{-1}) = \mathcal{Z}\{\mathbf{Z}\}$, $\mathbf{G}(z^{-1}) = \mathcal{Z}\{\mathbf{G}\}$, $\bar{\mathbf{m}}(z^{-1}) = \mathcal{Z}\{\bar{\mathbf{m}}\}$ and $\bar{\mathbf{w}}(z^{-1}) = \mathcal{Z}\{\bar{\mathbf{w}}\}$.

![Fig. 2.2: Discrete-time signal model](image-url)

Fig. 2.2 depicts the discrete time signal model. For a given channel matrix $\mathbf{X}(z^{-1})$ and gain matrix $\mathbf{G}(z^{-1})$ the relay transmit signal denoted $\bar{r}(z^{-1})$ has the average sum power $\bar{P}_r$. 
As

\[ \vec{r}(z^{-1}) = G(z^{-1})X(z^{-1})\vec{s}(z^{-1}) + G(z^{-1})\vec{m}(z^{-1}), \quad (2.2) \]

we define

\[ H_1(z^{-1}) = G(z^{-1})X(z^{-1})X^H(z)G^H(z) \]

and

\[ G_1(z^{-1}) = G(z^{-1})G^H(z) \]

to express \( P_r \) as

\[ P_r = \text{tr} \left( H_1(0) \right) \sigma^2_s + \text{tr} \left( G_1(0) \right) \sigma^2_m. \quad (2.3) \]

### 2.4.3 Equivalent Model

The single-antenna destinations do not cooperate, i.e. they do not exchange their received signals. The decision vector \( \vec{d}(z^{-1}) \) follows readily in (2.4).

\[ \vec{d}(z^{-1}) = Z(z^{-1})G(z^{-1})X(z^{-1})\vec{s}(z^{-1}) + Z(z^{-1})G(z^{-1})\vec{m}(z^{-1}) + \vec{w}(z^{-1}) \quad (2.4) \]

\( H_{SD}(z^{-1}) \) is the equivalent channel matrix and \( \vec{u}(z^{-1}) \) the equivalent destination noise. The equivalent impulse response of the source/destination link \( l \) is given by \( H_{SD}[l, l](z^{-1}) \), i.e. the \( l \)-th diagonal element of the equivalent channel matrix \( H_{SD}(z^{-1}) \). The received signal autocorrelation function follows in (2.5).

\[ R_{SD}^{(l)}(z^{-1}) = \sigma^2_s H_{SD}[l, l](z^{-1}) H_{SD}^*[l, l](z) \quad (2.5) \]

The equivalent destination noise contribution \( \vec{u}(z^{-1}) \) has the covariance matrix \( \Lambda_{\vec{m}\vec{r}}(z^{-1}) \) such that

\[ \Lambda_{\vec{m}\vec{r}}(z^{-1}) = \sigma^2_m Z(z^{-1})G(z^{-1})G^H(z)Z^H(z) + \sigma^2_w \mathbf{I} \quad (2.6) \]

\( \Lambda_{\vec{m}\vec{r}}(z^{-1}) \) is in general not diagonal due to the relay noise contribution. As the destinations do not cooperate, they are not aware of the codebooks used by the other sources. Thus we consider the interstream interference as additional noise. For destination \( l \) the MUI has the autocorrelation (no source power loading) \( R_{ISI}^{(l)}(z^{-1}) \) (2.7)

\[ R_{ISI}^{(l)}(z^{-1}) = \sigma^2_s H_{SD, 0}[l, :](z^{-1}) H_{SD, 0}^*[l, k](z) \quad (2.7) \]
where $H_{SD,0}(z^{-1})$ is obtained from $H_{SD}(z^{-1})$ by setting the elements on the main diagonal to zero. The autocorrelation function of the compound noise at this destination is given by $R_{cp}(z^{-1})$ in (2.8).

$$R_{cp}(z^{-1}) = \Lambda_{\text{Int}}[l,l](z^{-1}) + R_{\text{Int}}(z^{-1})$$  \hspace{1cm} (2.8)

Thus this stream indexed $l$ supports the rate in $\text{bit} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1}$ per channel use $R^{(l)}$.

$$R^{(l)} = \frac{1}{2} \int_{0}^{1} \log_2 \left( 1 + \frac{ \sum_m R_{SD,m}^{(l)} e^{-j 2 \pi m f}}{ \sum_m R_{cp,m}^{(l)} e^{-j 2 \pi m f}} \right) df$$  \hspace{1cm} (2.9)

where $R_{SD}^{(l)}(z^{-1}) = \sum_m R_{SD,m}^{(l)} z^{-m}$ and $R_{cp}^{(l)}(z^{-1}) = \sum_m R_{cp,m}^{(l)} z^{-m}$. The pre-log factor 1/2 accounts for the two channel uses required for each transmission. This is not a capacity expression, since the $\{R_{cp,m}^{(l)}\}_m$ are not independent from the $\{R_{SD,m}^{(l)}\}_m$. Both quantities are a function of the relay gain coefficients, and also from the source signals if the MUI are not fully suppressed. Assuming that MUI are removed, the equivalent signal-to-noise ratio $\text{SNR}^{(l)}$ at destination $l$ follows in (2.10). This is the SNR of an equivalent frequency flat two-hop relay channel with no MUI and independent noise samples at the receiver that supports the same rate $R^{(l)}$.

$$\text{SNR}^{(l)} = 2^{2 R^{(l)}} - 1.$$  \hspace{1cm} (2.10)

### 2.5 Zero-Forcing Filter Gain Matrix

A simple and efficient approach to determine the filter gain matrix is the zero-forcing (ZF) criterion. For zero-forcing, we choose the filter gain matrix subject to the relay sum power constraint (2.3) such that the equivalent channel matrix $H_{SP}(z^{-1})$ is diagonal and there is no multiuser interference. Besides simplicity, an additional advantage of ZF is the transparency to the source power allocation (near-far problem). In [16, 126, 129, 130] the authors have derived the ZF gain matrix for pure relaying (no cooperation between the relays) and flat fading. In [127] these results are extended any cooperation between the relays and to the frequency selective case.
2.5.1 Compound Gain

The equivalent channel vector \( \vec{h}_{SD}(z^{-1}) = \text{vec} \left( H_{SD}(z^{-1}) \right) \) is formed by stacking in a single column the columns of the equivalent channel matrix \( H_{SD}(z^{-1}) \). The entries of vector \( \vec{h}_{SD}(z^{-1}) \) are polynomials because the entries of \( H_{SD}(z^{-1}) \) are also polynomials. The compound gain vector is accordingly defined as \( \vec{g}_0(z^{-1}) = \text{vec} (G(z^{-1})) \). Vector \( \vec{h}_{SD}(z^{-1}) \) can be expressed as the sum of polynomial products due to (2.4). We can define a compound channel matrix \( A_0(z^{-1}) \) such that

\[
\vec{h}_{SD}(z^{-1}) = A_0(z^{-1}) \vec{g}_0(z^{-1}) \tag{2.11}
\]

with \( A_0(z^{-1}) \) given by

\[
A_0(z^{-1}) = X^T(z^{-1}) \otimes Z(z^{-1}) \tag{2.12}
\]

where the symbol \( \otimes \) denotes the Kronecker product.

Cooperations Let some of the \( N_r \) relays cooperate by exchanging their received signals. A unidirectional cooperation from relay \( j \) to relay \( i \) implies that relay \( j \) communicates its received signal to relay \( i \). As a result the element \( G[i,j] \) of the gain matrix, i.e. the element \( i + (j - 1)N_r \) of the compound gain vector \( \vec{g}_0(z^{-1}) \), may be nonzero. Let \( N_{\text{coop}} \) denote the total number of cooperations between the relays (including "self-cooperations" \( i-i \)). In the pure relaying case \( N_{\text{coop}} = N_r \) and the gain matrix is diagonal. In the distributed antenna system (DAS) case (full cooperation) \( N_{\text{coop}} = N_r^2 \) and all elements of the gain matrix are nonzero.

For a given relay cooperation pattern we may drop the zero elements of the compound gain vector \( \vec{g}_0(z^{-1}) \) and the compound channel matrix \( A_0(z^{-1}) \). Vector \( \vec{g}_0(z^{-1}) \) is replaced by \( \vec{g}_{(N_{\text{coop}})}(z^{-1}) \) and \( A_0(z^{-1}) \) is replaced by \( A_{(N_0^2 \times N_{\text{coop}})}(z^{-1}) \) by removing the corresponding columns in the compound channel matrix. Thus the equivalent channel vector is given by

\[
\vec{h}_{SD}(z^{-1}) = A(z^{-1}) \vec{g}(z^{-1}). \tag{2.13}
\]

Cooperations Let some of the \( N_r \) relays cooperate by exchanging their received signals. A unidirectional cooperation from relay \( j \) to relay \( i \) implies that relay \( j \) communicates its received signal to relay \( i \). As a result the element \( G[i,j] \) of the gain matrix, i.e. the element \( i + (j - 1)N_r \) of the compound gain vector \( \vec{g}_0(z^{-1}) \), may be nonzero. Let \( N_{\text{coop}} \) denote the total number of cooperations between the relays (including "self-cooperations" \( i-i \)). In the pure relaying case \( N_{\text{coop}} = N_r \) and the gain matrix is diagonal. In the distributed antenna system (DAS) case (full cooperation) \( N_{\text{coop}} = N_r^2 \) and all elements of the gain matrix are nonzero.

For a given relay cooperation pattern we may drop the zero elements of the compound gain vector \( \vec{g}_0(z^{-1}) \) and the compound channel matrix \( A_0(z^{-1}) \). Vector \( \vec{g}_0(z^{-1}) \) is replaced by \( \vec{g}_{(N_{\text{coop}})}(z^{-1}) \) and \( A_0(z^{-1}) \) is replaced by \( A_{(N_0^2 \times N_{\text{coop}})}(z^{-1}) \) by removing the corresponding columns in the compound channel matrix. Thus the equivalent channel vector is given by

\[
\vec{h}_{SD}(z^{-1}) = A(z^{-1}) \vec{g}(z^{-1}). \tag{2.13}
\]
2.5 Zero-Forcing Filter Gain Matrix

The equivalent channel matrix $H_{SD}(z^{-1})$ has "signal elements", which are used by the respective destinations to decode the source data stream, and "interference elements", which generate MUI. In the case of non-cooperating destinations, the signal elements are the $N_u$ diagonal elements of $H_{SD}$ and the interference elements are the $N_u(N_u - 1)$ off-diagonal elements. Let $\vec{h}_{SD,s}(z^{-1})$ be the vector of signal elements of $\vec{h}_{SD}(z^{-1})$ and $\vec{h}_{SD,i}(z^{-1})$ the vector of interference elements. We define the compound signal matrix $A_s(z^{-1})$ with size $N_u \times N_{coop}$ and the compound interference matrix $A_{ZF}(z^{-1})$ with size $N_u(N_u - 1) \times N_{coop}$ such that (2.14) and (2.15) are fulfilled.

\[
\begin{align*}
\vec{h}_{SD,s}(z^{-1}) &= A_s(z^{-1})\vec{g}(z^{-1}) = \left[\vec{h}_{SD,s}[1](z^{-1}) \cdots \vec{h}_{SD,s}[N_u](z^{-1})\right]^T \quad (2.14) \\
\vec{h}_{SD,i}(z^{-1}) &= A_{ZF}(z^{-1})\vec{g}(z^{-1}) \quad (2.15)
\end{align*}
\]

with for $1 \leq p \leq N_u$,

\[
\vec{h}_{SD,s}[p](z^{-1}) = \sum_{k=0}^{L_g-1} h_{SD,s,k}^p z^{-k}. \quad (2.16)
\]

2.5.2 Zero-Forcing Condition

Let $L_G$ be the maximum number of taps of the relay causal FIR filters, i.e. each polynomial entry in $\vec{g}(z^{-1})$ has maximum degree $L_G - 1$. Multi-user zero-forcing (MUZF) relaying requires

\[
\vec{h}_{SD,i}(z^{-1}) = 0, \quad (2.17)
\]

and we look for $\vec{g}(z^{-1})$ such that (ZF condition)

\[
\vec{h}_{SD,i}(z^{-1}) = A_{ZF}(z^{-1})\vec{g}(z^{-1}) = 0. \quad (2.18)
\]

At this point we observe that a phase shift between the source, relay and destination tier phase references will introduce a phase shift in $A_{ZF}(z^{-1})$ and $\vec{g}(z^{-1})$. This will not affect the ZF condition (2.18) but will create a phase shift in the equivalent signal vector $\vec{h}_{SD,s}(z^{-1})$.

We define now vector $\vec{g}^{(p)}$ as the vector associated to polynomial

\[
\vec{g}^{(p)}(z^{-1}) = \sum_{k=0}^{L_g-1} g_{k}^{(p)} z^{-k}. \quad (2.19)
\]
such that
\[ \tilde{\mathbf{g}}^{(p)} = \left[ g_0^{(p)}, \ldots, g_{L-1}^{(p)} \right]^T. \] (2.20)

For \( 1 \leq i \leq N_u(N_u - 1) \) and \( 1 \leq j \leq N_{coop} \), we define the Toeplitz matrix \( \tilde{A}_{ZF}^{(i,j)} \), associated to polynomial \( A_{ZF}[i,j](z^{-1}) \)
\[ A_{ZF}[i,j](z^{-1}) = \sum_{k=0}^{L+L_z-2} a_k^{(i,j)} z^{-k} \] (2.21)
as the \( L_t \times L_G \) matrix such that
\[
\tilde{A}_{ZF}^{(i,j)} = \begin{bmatrix}
    a_0^{(i,j)} & 0 & \cdots & 0 \\
    a_1^{(i,j)} & a_0^{(i,j)} & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{L_t+L_z-2}^{(i,j)} & a_{L_t+L_z-3}^{(i,j)} & \cdots & a_0^{(i,j)} \\
    0 & a_{L_t+L_z-2}^{(i,j)} & \cdots & \vdots \\
    \vdots & \vdots & \ddots & \vdots 
\end{bmatrix}.
\]
(2.22)

with \( L_t = L_X + L_Z + L_G - 2 \).

Matrix \( \tilde{A}_{ZF} \) is the block matrix with size \( N_u(N_u - 1)L_t \times N_{coop}L_G \) built from the \( \tilde{A}_{ZF}^{(i,j)} \) and representing \( A_{ZF}(z^{-1}) \), and \( \tilde{g} \) is the vector with scalar entries built from the concatenation of the \( \{ \tilde{g}^{(p)} \}_{p=1,\ldots,N_{coop}} \).

The ZF condition from (2.18) can be written as
\[ \tilde{A}_{ZF} \tilde{g} = \tilde{0}. \] (2.23)

Denoting \( \operatorname{Ker}(\tilde{A}_{ZF}) \) as the kernel of matrix \( A_{ZF} \), we have [113] from the rank-nullity theorem
\[ \operatorname{rank}(\tilde{A}_{ZF}) + \operatorname{dim} \operatorname{Ker}(\tilde{A}_{ZF}) = N_{coop}L_G, \] (2.24)
where \( \operatorname{dim}(\cdot) \) is the dimension operator that returns the dimension of the image space.

The rank of a matrix is always lower or equal to the minimum between its number of rows
and columns
\[
\text{rank} \left( \tilde{A}_{ZF} \right) \leq \min(N_u(N_u - 1)L_t, N_{coop} L_G),
\] (2.25)
thus we obtain
\[
\dim \text{Ker} \left( \tilde{A}_{ZF} \right) \geq N_{coop} L_G - \min(N_u(N_u - 1)L_t, N_{coop} L_G).
\] (2.26)

If \( \min(N_u(N_u - 1)L_t, N_{coop} L_G) = N_u(N_u - 1)L_t \), we get a lower bound of the nullity (dimension of the nullspace) of \( \tilde{A}_{ZF} \) as a function of the system parameters. Otherwise only a trivial condition exists on the dimension of the subspace. Therefore a necessary condition for having the nullspace of \( \tilde{A}_{ZF} \) not reduced to \( \vec{0} \) follows
\[
N_u(N_u - 1)L_t < N_{coop} L_G
\] (2.27)
or equivalently
\[
L_G(N_{coop} - N_u(N_u - 1)) > N_u(N_u - 1)(L_X + L_Z - 2)
\] (2.28)
and we define the minimum number of relay filter taps \( L_{G,\text{gen}} \) required to remove the interferences for any number of relays \( N_t \) and \( N_{coop} \) valid for \( N_{coop} \geq N_u(N_u - 1) + 1 \) as
\[
L_{G,\text{gen}} = \begin{cases} 
\frac{N_u(N_u - 1)(L_X + L_Z - 2)}{N_{coop} - N_u(N_u - 1)} + 1, & \text{if } \frac{N_u(N_u - 1)(L_X + L_Z - 2)}{N_{coop} - N_u(N_u - 1)} \in \mathbb{N}, \\
\frac{N_u(N_u - 1)(L_X + L_Z - 2)}{N_{coop} - N_u(N_u - 1)}, & \text{otherwise.}
\end{cases}
\] (2.29)
A necessary and sufficient condition for having the nullspace of \( \tilde{A}_{ZF} \) not reduced to \( \vec{0} \) is that \( \tilde{A}_{ZF} \) has full rank.

### 2.5.3 Excess Cooperation

In this part several cases are distinguished with respect to the variable \( N_{coop} \). The case \( N_u = 1 \) is discarded since it leads to \( A_{ZF}(z^{-1}) = 0 \).

From now \( N_u > 1 \). Additionally, we require from (2.28) that \( N_{coop} > N_u(N_u - 1) \). This means that even with a very large number of relay taps, if the condition \( N_{coop} > N_u(N_u - 1) \) is not fulfilled, it is not possible to remove the MUI in the channel. A system that does not have enough relays to suppress the MUI interferences with flat fading channels is not able to suppress the MUI in the frequency selective case, independently of the number of
taps available at the relays. As a special case, a DAS with \( N_u \) antennas fulfills the condition \( N_{\text{coop}} > N_u(N_u - 1) \). This initial condition is the MUZF condition obtained for flat fading channels.

In section 2.5.3.1 the number of filter taps at the relay is discussed for the smallest number of cooperations, in 2.5.3.2 the effect of additional cooperations is discussed with respect to the number of filter taps at the relays. Finally the case where all relays fully cooperate is investigated.

### 2.5.3.1 Minimum Cooperation Configuration (Pure Relaying)

**Flat Fading Case** In the flat fading case, \( L_X = L_Z = 1 \). In that case we obtain \( N_{\text{coop}} = N_{\text{coop},\text{min}} = N_u(N_u - 1) + 1 \), which is the minimum cooperation configuration of [127].

**Frequency Selective Case** In the frequency selective case, \( L_X + L_Z - 2 > 0 \) and condition \( N_{\text{coop}} = N_{\text{coop},\text{min}} \) alone does not insure that (2.27) is fulfilled. Given \( N_{\text{coop}} = N_{\text{coop},\text{min}} \), an additional constraint on \( L_G \) given in (2.30) is required.

\[
L_G > N_u(N_u - 1)(L_X + L_Z - 2)
\]  

(2.30)

We define \( L_{G,\text{min}} \) such that

\[
L_{G,\text{min}} = N_u(N_u - 1)(L_X + L_Z - 2) + 1
\]  

(2.31)

For the minimum relay configuration \( N_{\text{coop}} = N_{\text{coop},\text{min}} \), it is necessary to have \( L_{G,\text{min}} \) taps at the relay to insure that \( \tilde{\mathbf{A}}_{ZF} \) has a nullspace not reduced to \( \mathbf{0} \). Thus the minimum filter length increases quadratically with the number \( N_u \) of source/destination pairs. Fig. 2.3 represents the length of the relay filters as a function of \( N_u \). The first three curves for \( N_{\text{coop}} = N_{\text{coop},\text{min}} \), \( N_{\text{coop},\text{min}} + 1 \) and \( N_{\text{coop},\text{min}} + 2 \) are plotted in red, blue and light green. They show the quadratic increase in the minimum number of relay filter taps required to achieve a non-empty nullspace with respect to the number of antenna pairs \( N_u \).

### 2.5.3.2 Additional Cooperations

Now the effect of additional cooperations on the minimum relay configuration case is investigated. In this paragraph \( N_{\text{coop}} > N_{\text{coop},\text{min}} \). Let \( N_{\text{exc}} = N_{\text{coop}} - N_{\text{coop},\text{min}} \) be the number
of excess cooperations. With (2.27) we obtain (2.32).

\[
\frac{N_u(N_u - 1)(L_X + L_Z - 2)}{N_{\text{exc}} + 1} < L_G
\]  

(2.32)

Thus we can trade off the relay filter length and the excess number of cooperations. An example is given in Fig. 2.3 by the curve obtained for \( N_{\text{coop}} = N_u^2 \). It shows that when the number of cooperations \( N_{\text{coop}} \) grows with \( N_u^2 \), the minimum number of taps required to achieve a non-zero nullspace increases linearly with the number of antenna pairs \( N_u \).

![Graph](image)

**Fig. 2.3:** Required minimum number of relay gain taps for a non-empty nullspace (\( L_X = L_Z = 2 \))

### 2.5.3.3 Full Cooperation

With full cooperation, \( N_{\text{coop}} = N_r^2 \). When \( N_u = N_r \), we obtain an increase linear with \( N_u \) of the minimum relay filter length (2.33).

\[
(N_u - 1)(L_X + L_Z - 2) < L_G
\]

(2.33)
2.6 Optimization of the Nullspace Gain Vector

We assume now that the nullity of $\tilde{A}_{ZF}$ is at least one. Let $K = \text{Ker}(\tilde{A}_{ZF})$ be a matrix with its columns containing the vectors generating the nullspace of $A_{ZF}$ and such that $K^H K = I$. By definition of $K$, we have $\tilde{A}_{ZF} K = 0$. Clearly any MUZF gain vector lies in this nullspace, i.e. for any vector $\tilde{y}$ we obtain a ZF gain vector $\tilde{g}_{ZF}$ such that

$$\tilde{g}_{ZF} = K \tilde{y}. \tag{2.34}$$

We refer to $\tilde{y}$ as the nullspace gain vector in the sequel. Vector $\tilde{g}_{ZF}$ is a-priori not unique and in the sequel we discuss three optimization criterions for its choice.

**max-min criterion** Following [127] we adopt in this section a heuristic approach to the optimization: (ia) determine $\tilde{y}$ such that the minimum equivalent source-to-destination link energy (i.e. the sum of the magnitude squared of the polynomial coefficients) of the diagonal elements of the equivalent channel matrix $H_{SD}(z^{-1})$ is maximized and (ii) perform this maximization subject to the constraint $\|\tilde{y}\|_F^2 = 1$ (i.e. $\text{tr}(G_1(0)) = 1$). The operator $\|\cdot\|_F$ denotes the Frobenius norm.

The max-min approach is motivated by fairness and diversity considerations. It improves the weakest link. The constraint relates to the average relay sum transmit power $\overline{P}_r$. In all simulations the nullspace gain vector is normalized such that the instantaneous sum relay transmit power satisfies $\overline{P}_r = N_d \sigma_z^2$. With this constraint the results depend neither on the number of relays nor on the number of relay filter taps.

**max-mean criterion** We compare the performance obtained with the max-min criterion to the max-mean criterion. For this max-mean criterion we determine $\tilde{y}$ such that (ib) the mean equivalent source-to-destination link energy is maximized, subject to the same constraint (ii) as before. This approach does not consider any fairness but targets the global system performance improvement, possibly by enhancing the strongest links and closing the weakest ones.

**random gain vector choice** Performance reference is provided by this random approach that choses (ic) a random nullspace gain vector subject to constraint (ii). This criterion gives a lower bound for the performance evaluation. Any criterion that performs better than the random vector choice criterion is of interest.
The dimension of the nullspace gain vector $\vec{y}$ grows with the number of cooperations $N_{\text{coop}}$ and the number of relay filter taps $L_G$. In the following we adopt the subspace approach of [127] which reduces the maximum number of dimensions without significantly compromising performance. The ZF gain vector $K\vec{y}$ captures the signal contribution such that

$$\tilde{h}_{SD,s} = \tilde{A}_sK\vec{y},$$

with

$$\tilde{h}_{SD,s} = \begin{bmatrix} h_{SD,s,0}^{(1)} & \cdots & h_{SD,s,L_{t}-1}^{(1)} \\ \vdots & & \vdots \\ h_{SD,s,0}^{(N_{u})} & \cdots & h_{SD,s,L_{t}-1}^{(N_{u})} \end{bmatrix}^T,$$

where the $\{h_{SD,s,i}^{(j)}\}_{i,j}$ are introduced in (2.16). Matrix $\tilde{A}_s$ is built similarly to $\tilde{A}_{ZF}$ in section 2.5.2.

We define the Toeplitz matrix $\tilde{A}_s^{(i,j)}$ associated to polynomial $A_s[i, j](z^{-1}) = \sum_{k=0}^{L_{X}+L_{Z}-2} b_k^{(i,j)}z^{-k}$ as the $L_{t} \times L_G$ matrix such that

$$\tilde{A}_s^{(i,j)} = \begin{bmatrix} b_0^{(i,j)} & 0 & \cdots & 0 \\ b_1^{(i,j)} & b_0^{(i,j)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{L_{X}+L_{Z}-2}^{(i,j)} & b_{L_{X}+L_{Z}-3}^{(i,j)} & \cdots & 0 \\ 0 & b_{L_{X}+L_{Z}-2}^{(i,j)} & \cdots & \vdots \end{bmatrix}$$

Matrix $\tilde{A}_s$ is the block matrix with size $N_uL_t \times N_{\text{coop}}L_G$ built from the $\tilde{A}_s^{(i,j)}$ and representing $A_s(z^{-1})$. The nullspace matrix $K$ has size $N_{\text{coop}}L_G \times N_{\text{coop}}L_G - N_u(N_u - 1)L_t$. Without loss of generality we transform the nullspace such that the elements of $\vec{y}$ have decreasing impact on the vector $\tilde{h}_{SD,s}$. The singular value decomposition of $\tilde{A}_sK$ gives

$$\tilde{A}_sK = USV^H,$$

with $U$ and $V$ unitary matrices of size $N_uL_t \times N_uL_t$, resp. $N_{\text{coop}}L_G - N_u(N_u - 1)L_t \times N_{\text{coop}}L_G - N_u(N_u - 1)L_t$, and $S$ a diagonal matrix of size $N_uL_t \times N_{\text{coop}}L_G - N_u(N_u - 1)L_t$ containing the singular values of $\tilde{A}_sK$ in decreasing order on its diagonal. We obtain thus the desired transformation, replacing $\vec{y}$ by $\vec{y}' = V\vec{y}$ such that the entries of the nullspace gain vector $\vec{y}'$
are weighted by the decreasing singular values of \( \tilde{A}_sK \). Only
\[
L_{y,s} = \text{rank} \left( \tilde{A}_sK \right) \leq \min \left( N_u L_t, N_{coop} L_G - N_u (N_u - 1) L_t \right)
\] (2.39)
elements of \( \tilde{y} \) contribute to \( \tilde{h}_{SD,s} \). The other elements do not bring signal energy at the destinations. However, they may still have impact on the performance since they influence the relay noise contribution at the destinations.

Let \( \tilde{y}_s = [\tilde{y}^T[1] \ldots \tilde{y}^T[L_{y,s}]]^T \) be the subspace gain vector and let \( K_s \) be the matrix made from the corresponding columns of the nullspace matrix \( K \). According to the considerations above, we determine the subspace gain vector as follows. With
\[
\tilde{h}_{SD,s} = \tilde{A}_sK_s\tilde{y}_s,
\] (2.40)
we solve

- max-min criterion
\[
\tilde{y}_s = \arg \max_{\tilde{y}_s} \left( \min \left( B \left( \tilde{h}_{SD,s} \odot \tilde{h}_{SD,s}^\ast \right) \right) \right)
\] (2.41)

- max-mean criterion
\[
\tilde{y}_s = \arg \max_{\tilde{y}_s} \left( \text{mean} \left( B \left( \tilde{h}_{SD,s} \odot \tilde{h}_{SD,s}^\ast \right) \right) \right)
\] (2.42)

- random vector \( \tilde{y}_s \) is a random vector.

subject to \( \| \tilde{y}_s \|_F^2 = 1 \). The symbol \( \odot \) denotes the Hadamard (element-wise) product, and \( (\cdot)^\ast \) is the complex conjugate operator. The matrix \( B \) of dimension \( N_u \times N_u L_t \) collects the energy of the equivalent channel taps of \( \tilde{h}_{SD,s}^{\text{stack}} \) for each source/destination pair, and is defined as
\[
B = \begin{bmatrix}
\tilde{v}_1^T & \tilde{v}_0^T & \cdots \\
\tilde{v}_0^T & \ddots & \\
\vdots & \ddots & \tilde{v}_1^T
\end{bmatrix}
\] (2.43)
with \( \tilde{v}_1 \) the vector of length \( L_t \) containing only 1s and \( \tilde{v}_0 \) the vector of length \( L_t \) containing only 0s. We resort to numerical optimization to solve (2.41) and (2.42).
2.7 Simulation Results

2.7.1 Parameters

For all simulation results we let the channel matrices have i.i.d. complex normal (CN) random elements. The channels are normalized such that they have each unitary energy. The source complex symbols are spatially and temporally independent, and identically CN distributed with zero mean and unit variance. Thus the source sum transmit power is proportional to the number of sources. Both sources and relays use the same sum transmit power. Relay and destination noise have the same variance \( \sigma_m^2 = \sigma_w^2 \).

The reference SNR, denoted \( \text{SNR}_{\text{ref}} \), determines the noise variance at relays and destinations

\[
\sigma_m^2 = \sigma_w^2 = 1 / \text{SNR}_{\text{ref}}. \tag{2.44}
\]

In a \((1 \times 1 \times 1)\) system, \( \text{SNR}_{\text{ref}} \) is equal to (i) the average SNR at the relay and to (ii) the average SNR at the destination if the relays were noiseless. The maximum rate supported by the link indexed \( l \) between the \( l^{th} \) source-destination pair is given in (2.9) and the equivalent SNR of this link is given in (2.10).

2.7.2 Figures of Merit

In this section we study the performance of MUZF relaying in the pure relaying case, in terms of

- distributed array gain: the average destination SNR at \( \text{SNR}_{\text{ref}} = 40 \) dB computed from (2.10) and normalized with respect to the average destination SNR of a system with \( N_u = 1, N_r = 1 \) and same number of channel taps \( L_X \) and \( L_Z \) and relay filter taps \( L_G \). This normalization allows a fair array gain estimation as it accounts for the SNR improvement offered by additional identical relays in the system.
- distributed spatial multiplexing gain: let

\[
\overline{R}(\text{SNR}_{\text{ref}}) = \frac{1}{N_u} \sum_{l=1}^{N_u} R^{(l)} \tag{2.45}
\]

be the mean rate (per user pair) as a function of \( \text{SNR}_{\text{ref}} \). We approximate this function
as:

\[
\hat{R}(\text{SNR}_{\text{ref}}) \approx \frac{b}{2} \log_2(1 + a\text{SNR}_{\text{ref}})
\]

(2.46)

and determine \(a = \hat{a}\) and \(b = \hat{b}\) such that the mean error magnitude is minimized. \(\hat{b}\) is then the estimated spatial multiplexing gain per user. It is unity when all user pairs can transmit concurrently.

- diversity gain at outage probability: this is the asymptotic slope at high SNR of the outage probability function of \(\hat{R}\) at target rate \(4 \text{ bit} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1}\) per channel use, normalized by the asymptotic slope of the outage probability function at the same target rate of the mean rate of a system with \(N_u = 1, N_r = 1\) and \(L_G = 1\), and same number of channel taps \(L_X\) and \(L_Z\). This normalization stresses the improvement offered by additional relays and their filter length on the mean rate. It measures the ability of the system to exploit the relay filters to increase the diversity gain.

![Spatial multiplexing estimation](image)

**Fig. 2.4:** Spatial multiplexing estimation (\(N_u = 2, L_X = L_Z = 2, N_r = N_{\text{coop}, \text{min}} = 3\) and \(L_G = 6\))

An example of the spatial multiplexing gain estimation is presented in Fig. 2.4 for \(N_u = 2, L_X = L_Z = 2, N_{\text{coop}} = N_r = 3\) and \(L_G = 6\). This figure displays the mean rate \(\hat{R}\) as a function of \(\text{SNR}_{\text{ref}}\) in dB. The slope of the curve is denoted \(\beta\) and is approximately 0.17. As
we estimate \( \hat{b} \approx 1.0 \).

The fit between the simulated data and the model \( \frac{b}{2} \log_2(1 + a\text{SNR}_{\text{ref}}) \) is very accurate over the range 10 dB-50 dB.

\[
\hat{b} = \frac{20}{\log_2(10)} \beta,
\]

(2.47)

Fig. 2.5: Diversity gain estimation \((N_u = 2, L_X = L_Z = 2, N_r = N_{\text{coop}, \min} = 3 \text{ and } L_G = 6)\)

Estimation of the diversity gain is illustrated in Fig. 2.5. This figure presents the outage probability at target rate 4 bit·s⁻¹·Hz⁻¹ per channel use with respect to \( \text{SNR}_{\text{ref}} \) for a system with \( N_u = 2, L_X = L_Z = 2, N_r = N_{\text{coop}} = 2 \text{ and } L_G = 6 \). The simulated outage values show a linear decay from \( \text{SNR}_{\text{ref}} = 30 \text{ dB} \) and the slope is approximately -2.2. Therefore the slope read in Fig. 2.5 is normalized by the slope of the outage probability curve estimated for a system with \( N_u=1, N_r=1, L_G=1 \) and the same number of channel taps \( L_X \) and \( L_Z \). The diversity is thus estimated with respect to a two-hop scenario with fixed relay gain.

**2.7.3 Performance Results**

Simulations are performed for \( N_u \) user pairs, \( L_X = L_Z \) channel taps and a pure relaying configuration. The number of relays \( N_r \) and relay filter taps (per relay) \( L_G \) are varied, while satisfying the condition (2.28).
2.7.3.1 Spatial Multiplexing Gain

For all investigated configurations, the estimated spatial multiplexing gain per user \( \hat{b} \) is very close to 1: MUI are perfectly removed and the sum rate of the source-to-destination links approaches at high SNR\(_{\text{ref}} \) the sum rate of \( N_u \) independent SISO channels operating concurrently. Fig. 2.6 illustrates this by displaying the system spatial multiplexing gain (i.e. \( N_u \hat{b} \)) for several node configurations and channel lengths. For \( N_u=2, N_{\text{coop,min}}=3 \) which is less than \( N_r=4 \) used in the figure (round blue markers), for \( N_u=3 \) (resp. \( N_u=4, N_u=5 \), \( N_{\text{coop,min}}=7 \) (resp. \( N_{\text{coop,min}}=13, N_{\text{coop,min}}=21 \)), which is less than \( N_r=12 \) (resp. \( N_r=24, N_r=40 \)) used in the simulation and plotted with the square green markers (triangle red, star light blue). For all these configurations, \( L_G=L_G,\text{gen} \) from (2.29). The estimated spatial multiplexing gain is nearly equal to \( N_u \) for all these configurations allowing MUI cancellation. The estimated values are slightly below the nominal value \( N_u \) due to the noise sources at the relays that degrades the fitting operation in (2.46).

![Graph showing spatial multiplexing](image)

**Fig. 2.6:** Spatial multiplexing \( (L_G = L_G,\text{min}) \)

2.7.3.2 Diversity Gain

The normalized diversity gain is plotted in Fig. 2.7. For all configurations, \( N_u=2, L_X=L_Z=2, \) and \( N_r \) and \( L_G \) are varied. The max-min optimization criterion is used to chose the relay gain vector. The minimum configuration case is identified by an arrow and shows the
2.7 Simulation Results

The smallest diversity gain estimated with respect to the case where $N_u=1$, $L_X=L_Z=2$, $N_r=1$ and $L_G=1$. In a general way, increasing the number of relays and/or relay filter taps enables the system to better exploit the channel spatial and temporal degrees of freedom and therefore improves the diversity gain estimated from the average source-to-destination equivalent link. The temporal degrees of freedom come from the multipath characteristics of each channel, and the spatial degrees of freedom arise from the $N_r$ spatial links that connect each source to its destination via the $N_r$ relays.

The case $N_{coop,min} = N_r = 3$ represents the configuration with the minimum number of relays and is plotted with the blue circle markers. The minimum number of filter taps for $N_{coop,min}$ is given by $L_{G,min}=5$ in (2.31). From this configuration, increasing the relays filter length improves the diversity gain. Configuration $N_r=4$ is plotted with the green square markers and requires according to (2.29) at least $L_{G,gen}=3$ relay filter taps. The other configuration $N_r=5$, resp. $N_r=7$, is plotted with red diamond markers, resp. magenta triangle markers, and requires at least $L_{G,gen}=2$ (resp. $L_{G,gen}=1$) relay filter taps.

For a given $N_r$, the diversity gain increases with $L_G$. The system can use the relay filter taps to spread the signal in time domain and benefits from the temporal degrees of freedom available in the channels.

For a given $L_G$, the diversity gain increases with $N_r$: the system uses the $N_{exc}$ additional relays to increase the spatial degrees of freedom in the system, since the number of two-hop source-to-destination links scales linearly with $N_r$. At the same time temporal degrees of freedom are created since each newly created channel (source-to-relay and relay-to-destination) has multiple paths. The relay filter coefficients are then responsible for the exploitation of these degrees of freedom. Our optimization criterion can improve the diversity gain when the number of degrees of freedom increases.

2.7.3.3 Optimization Criterion

We investigate now the effect of the optimization criterion proposed in section 2.6. Fig. 2.8 shows the normalized diversity gain achieved for different optimization criterions, for $N_r=4$ (green markers) and $N_r=7$ (purple markers) with respect to the number of relay filter taps $L_G$. The max-min (filled markers) and max-mean (hollow diamond and up-triangle markers) criterion are applied, and reference points are provided by a random gain vector choice (hollow square and down-triangle markers). For both $N_r=4$ and $N_r=7$, the diversity gain increase is lower than for the max-min and max-mean criterion. For all configurations, the $N_r=7$ case performs better than the max-min and max-mean criterion in the $N_r=4$ case. Therefore the complexity (generated from longer filter taps) can be balanced against additional relays with-
out system performance degradation.

From the three optimization criterion, the max-min approach achieves the best performance in terms of diversity gain. It is an heuristic approach for which we give some insights. The max-min criterion improves the lowest source-to-destination link energy and maximizes

$$\int_0^1 \sum_m R_{SD,m}^{(l)}(z^{-1}) \frac{e^{-\beta \pi m f}}{N(f)} \, df$$

for a given noise power spectrum $N(f)$ where $l$ denotes the index of the link with lowest energy. As all functions are positive,

$$\arg \max_{R_{SD}^{(l)}(z^{-1})} \int_0^1 \sum_m R_{SD,m}^{(l)}(z^{-1}) \frac{e^{-\beta \pi m f}}{N(f)} \, df = \arg \max_{R_{SD}^{(l)}(z^{-1})} \int_0^1 \log_2 \left( 1 + \sigma_s^2 \sum_m R_{SD,m}^{(l)} e^{-\beta \pi m f} \right) \, df,$$

and the max-min approach maximizes the capacity of the single-hop frequency-selective system with channel autocorrelation function $R_{SD}^{(l)}(z^{-1})$ and i.i.d. colored noise (with a given power spectrum). In that case the max-min criterion improves the outage probability at high SNR and consequently increases the diversity gain. This approach is only heuristic here since (i) $N(f)$ depends on $\sum_m R_{SD,m}^{(l)}(z^{-1})$ via the filters coefficients that affects the noise at the relays, and (ii) we propose only an achievable rate expression that is a-priori not the capacity. Nevertheless this optimization criterion shows good performance in the simulations.

The max-mean criterion performance lies between the random gain and the max-min performance. It does not specifically target the weakest link improvement and has therefore less
impact on the outage probability at high SNR.

![Graph showing normalized diversity gain vs. Lag for different node configurations: N_r = 4, random gain; N_r = 7, random gain; N_r = 4, max-min; N_r = 7, max-min; N_r = 4, max-mean; N_r = 7, max-mean.]

**Fig. 2.8:** Optimization method impact ($N_u = 2$, $L_X = L_Z = 2$)

### 2.7.3.4 Array Gain

The array gain for several node configurations is plotted in Fig. 2.9. We set $N_u = 2$ and $L_X = L_Z = 2$, and vary $N_r$ and $L_G$ using the max-min, the max-mean criterion and the reference random gain to optimize the gain vector. For all configurations, condition (2.29) is fulfilled. The points for $L_G = 4$ are marked in red and for $L_G = 10$ in green. The array gain increases when additional relays are introduced in the system, this is of interest since relays operate under power constraint given in (2.3) that only depends on $N_u$ and not on $N_r$. Increasing $N_r$ mildens the effect of a faded channel tap in the source-to-relay or relay-to-destination channels. Indeed a faded channel tap leads to small entries in $\tilde{A}_s$. Matrix $\tilde{A}_s$ has size $N_u L_t \times N_r L_G$. The $\{ h_{SD,s,j}^{(i)} \}_{i,j}$ are the weighted sum of the $N_r L_G$ entries in the $i$th line of $\tilde{A}_s$. Increasing $N_r$ mobilizes more terms into the sum, thus averaging out the effect of an eventual faded term. This is spatial averaging since the additional terms in the sum originates from the additional relays.

The points obtained for the same number of relays and different relay filter lengths are not directly comparable since the reference used to compute the array gain depends itself on $L_G$. But using the same reasoning as previously, a similar effect is anticipated.

The optimization criterion affects the results. As previously, a random gain vector provides a
reference for the two other criterions. The max-mean criterion is expected to perform better than the max-min criterion since it directly chooses the gain vector that maximizes the signal part of the SNR, whereas the max-min criterion improves the weakest link that has a lower contribution to the average SNR. This effect is visible for $L_G = 4$ and small $N_r$ where the array gain is higher for the max-mean than for the max-min criterion. But for $L_G = 10$, and $L_G = 4$ and $N_r = 7$, both optimization criterions have similar performances. This is a consequence of our channel model for which all taps are i.i.d and of the averaging effect described below that applies in a similar way on all source-to-destination links. Using longer relay filters increases the number of elements in the weighted sum that equals $h_{SD,s,j}^{(i)}$. As a result the equivalent channel taps of all source-to-destination links tends to the same value and the links SNR tends to the same value. The difference between the weakest and the strongest link vanishes and therefore both criterions perform similarly.

![Array gain estimation](image)

**Fig. 2.9:** Array gain estimation ($N_u = 2, L_X = L_Z = 2$)

### 2.8 Conclusion

We presented a two-hop multiuser cooperative relaying scheme where the coherent relays can cooperate in an arbitrary way. The channels from all sources to all relays, and from all relays to all destinations are frequency selective and perfectly known by the relays. The
receivers have access to the equivalent source-to-destination channel such that they can perfectly decode the received signal. The source, relay and destination tier are coherent, meaning that there is a phase reference for each of the three tiers. We first derived an achievable rate expression for the equivalent source-to-destination channels and the equivalent SNR. A practical method to compute the relay gain filter coefficients is then proposed, based on zero-forcing the interference terms. A distributed channel orthogonalization is achieved. Conditions on the number of relay cooperations and relay gain filter taps number are given. Three criterions for optimizing the gain vector are compared. Finally simulation results are presented, followed by a discussion on the performance of the system in terms of SM gain, diversity gain and array gain.
Chapter 3

Towards the Implementation of Relaying Schemes: Imperfections of the RACooN Nodes

In the previous chapter, we have proposed a relaying scheme for pervasive wireless access network (PWAN). We showed that under certain conditions it is possible to orthogonalize the multiple-input multiple-output (MIMO) channel and benefit from a distributed spatial multiplexing (SM) gain. The common way to evaluate the relaying scheme performances relies on simulating channel realizations using standardized channel models. Yet these models are designed for collocated MIMO systems, and are commonly assumed to model PWAN channels as well. In the next chapters we will discuss this assumption and finally propose a PWAN channel model using channel measurements performed with the Radio Access with Cooperative Nodes (RACooN) of the Wireless Communication Group (WCG) at ETH Zurich.

3.1 Introduction

The RACooN lab can be used
– as a channel sounding system for non-collocated MIMO systems,
– and as a real-world demonstrator for relaying schemes.

Although these two applications may at first appear independent, the first one is actually required to perform the second one. Indeed most of the existing relaying schemes are non-blind relaying schemes, i.e. the nodes in the relay network require some channel knowledge (that can be global, full or partial).

A channel sounding procedure will provide them with an estimation of the wireless channel.
A classification of the wireless channels is proposed in [120]. It distinguishes four channel categories

- the propagation channel (electromagnetic effects only),
- the radio channel (includes effect of transmit and receive antenna),
- the radio-frequency channel (radio channel and all analog hardware effects),
- the baseband channel (characterizing the digital baseband-to-baseband signal distortion).

In the following we will refer only to baseband channels, as they are directly accessible using the RACooN lab. Our channel measurements and channel model therefore include the propagation channel, the antenna pattern at the receiver and transmitter and analog hardware effects at radio-frequency (RF), intermediate frequency (IF) and baseband.

The motivation for this chapter is to provide an extensive understanding of the RACooN nodes and their operation. It serves as an introductory chapter for following chapters 4 and 5 that will specifically deal with channel sounding methods and their application in a measurement campaign. This chapter contributions are

- a review of the existing MIMO channel sounders with time-division multiplexed switching (TDMS), full-parallel and semi-sequential architecture,
- a review of the existing distributed MIMO demonstrators,
- an extensive analysis of the RACooN analog and digital architecture,
- a model for the node imperfections that encloses carrier frequency offset (CFO), phase noise, direct component (DC) offset,
- and a discussion of the effect of I-Q imbalance and transmitter and receiver nonlinearities on the received baseband signal.

The state of the art in section 3.2 deals with existing channel sounders that estimate baseband channels and distributed MIMO hardware demonstrators. This gives the keys to fully appreciate section 3.3 where we describe the RACooN lab architecture that allows both channel sounding and relaying scheme demonstration. Finally in section 3.4 we address the RACooN hardware imperfections and conclude in section 3.5 by providing an equivalent model for the node imperfections at the receiver side while considering the transmitter side as ideal.
3.2 State of the Art

3.2.1 MIMO Sounder Architecture

In this section existing MIMO channel sounders are presented. They are categorized according to the architecture of the receiver and the transmitter, i.e. how the sounder is accessing the medium. We distinguish

- (fully-) parallel channel sounders \[73\],
- sequential channel sounders, also named time-division multiplexed switching (TDMS) sounders \[12,76\],
- semi-sequential channel sounders \[103\].

Sounders can also be categorized according to their hardware into collocated and non-collocated sounders. Collocated sounders are such that the RF circuits of the transmitter and receiver share the same local oscillator. Non-collocated sounders have a local oscillator for each RF circuit. A major issue when operating with non-collocated sounders is to cope with the phase noise of the different oscillators, this issue concerns parallel, time-division multiplexed switching (TDMS) and semi-sequential sounders.

TDMS sounders are the most cost-effective sounders because there is only one RF circuit at the transmitter and at the receiver that is shared by all the antennas activated sequentially by a switch. This procedure gives perfect MIMO measurements only if the channel is perfectly stationary and the local oscillators are ideally stable and stay in-phase. Measurements in non-static environments or with non-static nodes are only possible for small Doppler frequencies, short pulse duration and short switching times between the antennas. There are two types of switches available:

- electromechanical switches that mechanically link the selected antenna to the RF circuit while the other antennas are disconnected,
- and microwave (or solid-state) switches that contain active directional couplers to link the antenna to the RF circuit without requiring any mechanical part.

Microwave switches operate up to 10,000 times faster than electromechanical switches, are stable and have a nearly infinite lifetime since they do not contain any moving part. As a drawback their bandwidth is limited because they contain active components that behave linearly only over a given frequency range.

On the other side, electromechanical switches offer a lower insertion loss, standing wave ratio and better isolation properties than the solid-state switch.

Parallel sounders allow the simultaneous estimation of all the channels from the transmitter to the receiver side. Yet the hardware costs are high since there are as many RF circuits
as transmit and receive antennas.

Semi-sequential sounders offer a trade-off between parallel (section 3.2.2.2) and sequential sounders (section 3.2.2.3). The transmitter side has a sequential architecture whereas the receiver side has a parallel architecture, thus creating a single-input multiple-output (SIMO) setup.

### 3.2.2 Existing Channel Sounders

We summarize in this section the literature describing existing channel sounders for MIMO wideband channel measurements operating at a center frequency between 5 GHz and 6 GHz. The description follows the architecture classification from section 3.2.1.

#### 3.2.2.1 Existing TDMS Sounders

Sequential (or TDMS) channel sounders are the most common type of channel sounders. It comprises the existing commercial channel sounder "Propsound" [3]. This device enables sequential MIMO channel measurements with collocated antennas at the transmitter and receiver side. Its bandwidth is 100 MHz and operates at center frequency 5.25 GHz in [3] using a pseudo-random binary sequence as sounding sequence.

Another TDMS channel sounder with collocated antennas is the RUSK LUND [81, 116] MIMO channel sounder. It has been designed for mobility applications with center frequency between 5 GHz and 6 GHz. Its bandwidth is 120 MHz in [116] or 240 MHz in [81]. It allows single-input single-output (SISO) channel estimation with a period between 0.8 µs and 25.6 µs. The sounding sequence is a multitone signal and the channel estimation is performed in the frequency domain. The advantage is that the channel frequency response is directly obtained by looking at the corresponding Fourier transform (FFT) bin. The drawback is that the signal envelope in time domain is not constant and the receiver and transmitter power amplifier (PA) non-linearities may be exacerbated. Synchronization is maintained by an optic fiber or by two rubidium reference clocks, depending on the application. The phase noise is kept low by a phase-locked loop (PLL).

Other authors in [55, 58] present a 5.3 GHz TDMS MIMO channel sounder with 120 MHz bandwidth. The antennas are activated by microwave switches. The probing sequence is a m-sequence and the channel estimation can occur in the analog part or the digital part of the device [54]: the receiver performs either a sliding correlation (at the IF stage in the analog domain) or a cross-correlation in the digital domain with the transmitted m-sequence.
More recently, another TDMS channel sounder has been presented in [67, 68] with a design that targets low cost. It uses microwave switches and is an open-hardware channel sounder. Open-hardware means that the device is built from conventional off-the-shelf hardware. It operates in the 2-8GHz range with 80MHz bandwidth (100MHz in [68]) and uses a multitone signal as a sounding sequence.

### 3.2.2.2 Existing Fully-Parallel Sounders

There exist only few fully-parallel MIMO channel sounders due to their high cost. A design for collocated antennas is presented in [73], existing sounders are only the ETH RACooN lab (non-collocated antennas) and the EMOS platform from the Eurecom Institute [49] (collocated antennas at the transmitter, non-collocated antennas at the receiver).

### 3.2.2.3 Existing Semi-Sequential Sounders

Semi-sequential MIMO channel sounders offer a trade-off between the costs of the parallel sounders and the performance limitation of the sequential sounders. A semi-sequential MIMO channel sounder is presented in [7]. It operates in the 5 GHz band with a bandwidth of 100MHz. There are 16 transmit antennas equipped each with a RF circuit. At the receiver side 32 antennas are grouped into 4 sets of 8 antennas each, and antennas are activated within each set by a switch.

### 3.2.3 Relay Network Demonstrator

Demonstrators for cooperative relaying schemes have been built and described in [18, 29, 49]. The MIT commodity hardware demonstrator [18] is built from off-the-shelf hardware and its purpose is to address practical issues of cooperative relaying at center frequency 916.5 MHz (i.e. demonstrate a relay gain allocation algorithm) but not to measure channels. This demonstrator has stressed the coherence issue as the major difference between real-world demonstrator and theoretical works. The Eurecom demonstrator using the Openair interface [29, 49] allows relaying demonstration and real-time MIMO channel measurements synchronously over multiple users moving at vehicular speed at a carrier frequency in the Universal Mobile Telecommunications System (UMTS) range. But its modular design makes it hard to deploy for pedestrian mobility applications as required in a PWAN.
These demonstrators suffer from imperfections that reduce the achieved performances. The phase noise issue is stated in [18] as the most severe problem. Carrier frequency offset (CFO) is another severe imperfection that affects any transmission system. We found out with the RACooN lab that direct component (DC) offset, non-linearities and I-Q imbalance have also a strong effect on the received signal. In the next sections we will focus on the RACooN lab and describe how these imperfections affect the received signal.

3.3 Description of the RACooN Nodes

The RACooN lab consists of 10 nodes (or units) operating in half-duplex mode, meaning that each node cannot transmit and receive simultaneously. Each unit is tagged with a number from 1 to 10 and is built up from three sub-units, namely
- a radio-frequency unit (RFU),
- a storage unit (STU),
- a power supply unit (PSU).

The power supply unit (PSU) provides the power to the storage unit (STU) and the radio-frequency unit (RFU) belonging to the same RACooN unit, it will not be further detailed.

3.3.1 Analog RFU

The RFU is an analog superheterodyne up- and downconverter between the 80 MHz complex baseband and radio-frequency at a user-defined frequency \( f_c \) between 5.1 GHz and 5.9 GHz. The RFU functional blocks are represented in Fig. 3.1 for the transmitter and in Fig. 3.2 for the receiver.

Each RFU is equipped with a reference clock at 10 MHz obtained through a built-in rubidium frequency normal. It is used to generate the \( f_s = 80 \) MHz baseband sampling clock and the internal RF carrier synthesizer at frequency \( f_{RF} \). The carrier frequency is set by default at \( f_c = 5.5 \) GHz but can be set by the user at any frequency between 5.1 GHz and 5.9 GHz with a 1 MHz step. This is achieved by changing the RF carrier synthesizer (named also RFsynth) at \( f_{RF} \) between 6.55 GHz and 7.35 GHz. This RFsynth signal is used in the RF mixers.

The RACooN nodes have a fast mechanical switch that enables the operation mode to switch from TX to RX and vice-versa in less than 600 ns [124]. In the transmitter, the baseband signal is converted to RF in two mixing steps. After low-pass filtering with cut-off frequency 44 MHz to remove the transmitter digital-to-analog converter (DAC) non-linear components,
the first step is a canonical quadrature modulation that shifts the baseband signal to IF at center frequency $f_{\text{IF}}$. The next bandpass IF filter is centered at $f_{\text{IF}}$ and has 100 MHz bandwidth. It removes the non-linearities produced by the mixer.

To reach radio-frequency, a second mixing step is required where the signal is upconverted from $f_{\text{IF}}$ to $f_c = 5.5$ GHz. This operation is performed by multiplying the signal with the RFsynth signal at $f_{\text{RF}} = 6.95$ GHz ($f_{\text{RF}} - f_c = f_{\text{IF}} = 1.45$ GHz), thus $w(t)$ is obtained (Fig. 3.1). A RF bandpass filter centered at $f_c = 5.5$ GHz with 1 GHz bandwidth follows the mixer and prevent RF mixer non-linearities from reaching the transmitter antenna.

We describe now the receiver. The image rejection filter removes the interferers possibly present in the image band which is the 80 MHz band around 8.40 GHz that will be shifted to the intermediate frequency at 1.45 GHz by the RF-mixer. The IF filter lets only the signal components in the range 1.4GHz-1.5GHz reach the IF mixer. The terms at frequency $f_{\text{RF}}+f_c$ are filtered out. The anti-aliasing filter with cut-off frequency 30 MHz removes the sideband frequency components of the signal at $2f_{\text{IF}}$.

We discuss at this point the frequency of the RF mixer. Another possibility is to use $f_{\text{RF}} = 4.05$ GHz for the RFsynth signal since $f_c - 4.05 = f_{\text{IF}}$ as well. Yet in that case the mirror frequency is at 9.55 GHz, which is closer to the signal frequency than the mirror frequency obtained if $f_{\text{RF}} = 6.95$ GHz. In order to relax the requirements on the slope of the filter transfer function between the stopband and the passband, one chooses $f_{\text{RF}} = 6.95$ GHz.

The receiver and the transmitter are equipped with a user-controlled analog gain. When activated, it gives a +20 dB additional amplification at the receiver and the transmitter, obtained from the variable attenuators in Fig. 3.1 and Fig. 3.2.

- At the transmitter, "Analog Gain TX" can be set to -20 dB (position L/low) or 0 dB (position H/high).
- At the receiver, "Analog Gain RX" can be set to -20 dB (position L/low) or 0 dB (position H/high).

The "Analog Gain TX" and "Analog Gain RX" are controlled by:

- the configuration file (gain determined in advance by the user),
- or the received signal strength indicator (RSSI)(automatic analog gain control),
- or the baseband power (automatic analog gain control),

and the selection of the control source is programmed by software.
3.3.2 Digital STU

3.3.2.1 Hardware Description

The STU processes the user commands and controls the RFU. It is basically a computer running under Linux and equipped with an I/O interface card. It works as an analog-to-digital converter (ADC) or DAC. It is furthermore in charge of the memory allocation, i.e. for accessing the Random-Access Memory (RAM) and the harddisk. When the unit is operating as a receiver, the I/O interface card samples at $f_s = 80$ MHz the incoming baseband signal of the I- and Q-branch and quantizes it over 14 bits for each branch before storing it in the RAM. When the unit is operating as a transmitter, the STU reads the digital complex samples from the RAM at $f_s = 80$ Msps for each dimension and converts them to time-continuous signals that are forwarded to the RFU. The STU can perform limited digital signal processing, the tasks of higher complexity are performed offline. This structure is attractive because digital signal processing does not suffer from the non-idealities of analog components but does not allow real-time processing.

3.3.2.2 Data Structure

The smallest data structure that can be accessed is the (complex) sample. The sample period $T_s$ is 12.5 ns and it occupies 32 bits. The 14 bits real and imaginary parts of the sample after quantization are coded over 16 bits each. A group of $2^{13}$ complex samples forms a buffer, i.e. a buffer is $S = 8192$ samples long and
a buffer is the shortest data structure the RAM can access. A superframe is a data structure built from the concatenation of the samples from consecutive buffers in the RAM, as shown in Fig. 3.3.

A specific RAM is available for operation, as well as a 20 GByte hard disk. The RAM contains $B=256$ buffers (each buffer contains $2^{13}$ complex samples) that can be directly addressed by the software commands receive, transmit, load, store, idle and process. The total storage capacity available in the RAM is approximately 8 MByte ($S \cdot B \cdot 32\text{ bit}$).

![Fig. 3.3: Data structure](image)

### 3.3.2.3 Time structure

The sample period $T_s$ of 12.5 ns is the shortest time structure. The different time structure defined for the RACooN (RAC) lab are presented in Fig. 3.4. A timeslot is a time structure that comprises the superframe duration, augmented by a user-defined delay $N'T_s$ that amounts a multiple of a sample duration $T_s$. In the figure, the delay between the start of the timeslot and the start of the superframe is denoted $K T_s$ and the delay between the end of the superframe and the end of the timeslot is $(N' - K)T_s$. The superframe length is user-defined. It can be one buffer, in which case the RAM contains $B$ superframes, up to $B$ buffers, in which case the RAM contains 1 superframe.
Successive timeslots define the time frame for a mission. At each timeslot the RACooN node performs one command (but a command may extend over several timeslots), as shown in Fig. 3.4.

![Timing structure](image)

**Fig. 3.4:** Timing structure

### 3.3.2.4 Elementary Commands

**Synchronization**  
Master and slave units are defined by software to set a common timeslot counter between the units. This synchronization step requires the units to be linked by USB cables during the execution of the synchronization procedure.

**Commands**  
The STU emulates a computer with an I/O interface card. Its basic set of instructions consists of six commands. A mission consists in a list of up to $N_{\text{cmd}}^{\text{max}}=512$ elementary commands written in an extensible markup language (XML) file that is executed sequentially by the RACooN nodes and repeated as many times as the user defines it. These six elementary commands are

- *receive command:* writes the superframe into the RAM,
3.4 Characterization of the Devices and Imperfections

- transmit command: transmit the superframe content,
- idle command: wait,
- load command: load data from the harddisk to the RAM,
- store command: store superframes from the RAM on the harddisk,
- process command: send data via Ethernet to another computer that will execute a specified Matlab routine.

3.4 Characterization of the Devices and Imperfections

The ideal heterodyne system described in the previous section is affected by imperfections. The main sources of imperfections in a realistic measurement system are identified in [106] and will be treated in the following sections:
- carrier frequency offset (3.4.2)
- phase noise (3.4.3)
- linearity of the analog receiver and transmitter (3.4.4)
- I-Q imbalance (3.4.5)
- DC offset (3.4.6).

3.4.1 Common Measurement Setup

The system under test consists of:
- a transmitter (RACooN unit 3) that up-modulates the signal,
- a receiver (RACooN unit 4) that down-modulates the signal,
- a RF-cable (length 2 m) and a 36 dB passive attenuator.

The units are started 2 h before measurements are performed.

In this chapter, a transmit signal has a single tone. The received signal is analyzed in order to identify qualitatively in section 3.4.1.2 and quantitatively in sections 3.4.2, 3.4.3, 3.4.4, 3.4.5 and 3.4.6 the system imperfections.

3.4.1.1 Choice of the Frequency

A baseband complex exponential test signal $s(t)$ at $f_{\text{test}}$ is transmitted

$$s(t) = Ae^{j2\pi f_{\text{test}}t}. \quad (3.1)$$
This constant magnitude signal is used to alleviate the effects of transmitter and receiver PA non-linearities. Any value for $f_{\text{test}}$ in the range $[-30 \text{ MHz}, 30 \text{ MHz}]$ is valid, we use yet a frequency that fulfills following conditions

- the frequency $f_{\text{test}}$ must be large enough such that it does not lie in the frequency range around 0 Hz that is affected by phase noise (Fig. 3.7 will later show that the band $[-2 \text{ MHz}, 2 \text{ MHz}]$ should be avoided),
- the harmonics of $f_{\text{test}}$ above 40 MHz must not lie after 80 MHz sampling at the positions $-k f_{\text{test}}$, $k \in \mathbb{N} \setminus \{0\}$ where the spectral peaks caused by I-Q imbalance are expected.

We choose $f_{\text{test}} = 13 \text{ MHz}$, the signal third and fourth harmonic at 52 MHz and 65 MHz lie after sampling at -28 MHz and -15 MHz, whereas the I-Q imbalance peaks are expected at -13 MHz and -26 MHz.

### 3.4.1.2 Received Signal Analysis

The received signal read without further processing from the harddisk of the receiver after transmission is in Fig. 3.5. The transmit signal for one measurement consists of $8 \cdot 10^5$ samples contained in one superframe of 100 buffers (c.f. section 3.3.2).

![Raw digital signal at the receiver ($f_{\text{test}}=13 \text{ MHz}$)](image)

**Fig. 3.5:** Raw digital signal at the receiver ($f_{\text{test}}=13 \text{ MHz}$)

**Clipping** The received analog signal is converted by the ADC into a discrete value in the range $[-1, 1]$. Observing the digital signal in time domain is a check for the data validity.
3.4 Characterization of the Devices and Imperfections

Indeed when the amplitude of the analog signal at the input of the ADC in the I- or Q-branch exceeds the converter maximum input value, the converter is driven to saturation, clipping occurs and its output is -1 or +1. Therefore it must be always checked that the digital values do not reach -1 or +1 in the I- and Q-branch. Fig. 3.5 shows that the digital received signal in the I and Q branch is not clipped since the real and imaginary part of the received signal are approximately in the range \([-0.5, 0.5]\).

**Initial delay** A closer look at the first 3 µs of the received samples in Fig. 3.6 shows that there is a delay between the start of the superframe and the reception of the signal at the receiver. This delay is not caused by the \(l = 2 \, \text{m}\) cable that links transmitter to receiver (the transmission through the cable causes a delay of \(d = l/c \approx 6.7 \, \text{ns}\), that is \(T_s/2\) and is below the resolution \(T_s\) of the device, therefore the propagation can be considered as instantaneous). This delay is due to the RAM access time that amounts up to several 100 ns. This delay is independent of the size of the superframe. The hardware delay is constant and amounts 775 ns or 62\(T_s\). From now the first received samples of a superframe affected by hardware delay will be removed, unless specified.

![Fig. 3.6: First 3 µs of the received signal](image)

**Received signal power spectrum** The received signal power spectrum is in Fig. 3.7. The spectrum is normalized such that the signal peak at \(f_{\text{test}}\) has 0 dB spectral magnitude. The
received data is preprocessed by a Hanning window (width 39999 samples) applied on the autocorrelation of the received signal. This is equivalent to smoothing the power spectrum curve with a convolution filter.

![Power Spectrum Graph]

**Fig. 3.7:** Received normalized signal power spectrum ($f_{test}=13$ MHz)

We comment the peaks in the power spectrum that are less than 40 dB below the test signal peak. 40 dB attenuation is a threshold commonly used by circuit designers to identify significant interferers.

The largest peak at $f_{test}=13$ MHz is the test signal. Its basis is widened due to phase noise (see section 3.4.3).

The second largest peak is at $-13$ MHz. It arises from two contributions: (i) the I-Q imbalance term (see section 3.4.5) in the IF mixer at the transmitter and receiver side (ii) baseband non-linearities at transmitter and receiver including ADC and DAC effects. Phase noise affects the contributions generated at the transmitter side.

The next largest peak is at 0 Hz and is the DC offset (see section 3.4.6). A close-up of Fig. 3.7 around 0 Hz is in Fig. 3.8. It shows that that noise around 0 Hz is due to phase noise (wide basis of the peak) and also to the presence of low power spectral lines named spurs. These spurs come from leakage current and charge pumps in the local oscillator and are located at discrete offset frequencies from the carrier [10]. They are also present around the signal peak at $f_{test}$ but are partly masked by phase noise.

Another significant peak in the spectrum is at 26 MHz = $2f_{test}$. It corresponds to the first harmonic of the test signal affected by non-linearities in baseband at the transmitter and re-
3.4 Characterization of the Devices and Imperfections

receiver and by the ADC and DAC converters. Phase noise is hardly visible since it is below the noise floor.

The last significant peak is at $-26 \text{ MHz} = -2 f_{\text{test}}$. It arises from three contributions: (i) DAC non-linearities in the I- and Q-branch, (ii) the first harmonic of the imbalance term at $-f_{\text{test}}$ and (ii) the imbalance term of the first test signal harmonic generated in the DAC and baseband circuit at the transmitter side. These contributions add and therefore the peak amplitude is nearly as high as the peak amplitudes at $f_{\text{test}}$ and $-f_{\text{test}}$.

![received signal power spectrum - zoom around 0 Hz (f_{\text{test}}=13 MHz)]

**Fig. 3.8:** Received signal power spectrum - zoom around 0 Hz ($f_{\text{test}}=13$ MHz)

**Received signal in time domain** With this knowledge of the received signal power spectrum, we plot now the magnitude of the received signal in Fig. 3.9. In the ideal case a constant magnitude is expected but measurements show that a periodic pattern adds to the expected constant magnitude. This is explained by the fact that the received signal contains a term in $f_{\text{test}}$, but also in $2 f_{\text{test}}$, $-f_{\text{test}}$, $-2 f_{\text{test}}$ and a DC.

An explanation is offered by a simple model that considers only the three largest terms ($f_{\text{test}}$, $-f_{\text{test}}$ and a DC) of the received signal.

Assume that the received baseband signal is $y(t) = e^{j\phi(t)}(ae^{j\omega_{\text{test}}t} + be^{-j\omega_{\text{test}}t} + c)$, where $a, b \in \mathbb{R}^+$, $c \in \mathbb{C}$ such that $c = \mathcal{R}\{c\} + j\mathcal{I}\{c\}$. 

47
\[ |y(t)|^2 = |ae^{i\omega_{test}t} + be^{-i\omega_{test}t} + c|^2 \]
\[ = a^2 + b^2 + |c|^2 + 2R \{c\} \cos(\omega_{test}t)(a + b) + 2I \{c\} \sin(\omega_{test}t)(a - b) + 2ab \cos(2\omega_{test}t) \]

From this model, we see that periodic terms in \( f_{test} \) and \( 2f_{test} \) add to the constant magnitude term \( a^2 \) obtained in the ideal case \((b = c = 0)\). Their presence can justify the oscillations at \( f_{test} \) and \( 2f_{test} \) observed in the magnitude of the received signal. If \( c = 0 \) and \( b \neq 0 \) (no harmonics), there is only a periodic term in \( 2f_{test} \) that adds to the constant amplitude. If \( b = 0 \) and \( c \neq 0 \) (no DC), there is only a periodic term in \( f_{test} \) that adds to the constant amplitude.

![Graph showing magnitude of the received signal](image)

**Fig. 3.9:** Magnitude of the received signal \((f_{test}=13 \text{ MHz})\)

In the next plot we show the difference between the test signal phase at the transmitter and receiver. The phase is extracted from the received signal after removing from the received signal its mean value. Fig. 3.10 shows that the phase difference is not constant with time. We use the previous model to justify this behavior. The phase difference \( \psi(t) \) is given by:

\[ \psi(t) = \arctan\left(\frac{\sin(\omega_{test}t)(a - b) + I \{c\}}{\cos(\omega_{test}t)(a + b) + R \{c\}}\right) + \phi(t) - \omega_{test}t \]
The term $\arctan\left(\frac{\sin(\omega_{test}t)(a-b)+I(e)}{\cos(\omega_{test}t)(a+b)+R(e)}\right)$ is responsible for the fast variations in Fig. 3.11. The slow changing variations are caused by a low frequency envelope term resulting from the addition of the arctan term and the phase noise $\phi(t)$ that has a power spectrum decay of $1/f^2$ around $f_{test}$ (see section 3.4.3).

The average phase offset value depends on the phase reference used for the test sequence and the constant phase offset between transmitter and receiver local oscillators. In the long term, the phase difference increases linearly with time, this is due to the carrier frequency offset between transmitter and receiver (see section 3.4.2).

![Fig. 3.10: Phase difference ($f_{test}=13$ MHz)](image)

In Fig. 3.10 the first 1 ms of the phase difference presents a large phase jump. This jump is only present at the beginning of any mission and is not influenced by the transmit sequence. When the same sequence is transmitted in two consecutive commands in the same mission, only the first occurrence is affected by this phase jump. This can attributed, according to [10], to the PLL lock time, which is the time required until the phase of the local oscillator is stabilized. Indeed the local oscillator needs to lock at the start of any mission and after each change of the switch (transmitter-receiver operation), but not within a mission that contains identical commands. This lock time is inversely proportional to the PLL bandwidth and proportional to the frequency jump, which is in our case from 0 Hz to $f_{IF}$ and $f_{RF}$. The issue is solved when an idle command is executed at the start of the mission, letting time to the local oscillators for stabilization. The idle command must be used with the parameter antennaSwitch set to receive (default value) if the unit is receiving in the next timeslot, or
set to transmit is the unit is transmitting in the next timeslot.

### 3.4.2 Carrier Frequency Offset

**Origin** Frequency offset is caused by the non-perfectly identical local oscillator nominal frequencies at transmitter and receiver. This effect adds to phase noise such that a pure tone at frequency $f_{\text{test}}$ at the transmitter lies after up- and down-conversion at frequency $f_{\text{test}} + f_{\epsilon}(t)$. We assume a simple linear model where

$$f_{\epsilon}(t) = f_{\text{off}} + f_{i}(t) \quad (3.3)$$

with

- $f_{\text{off}}$ a (time-constant) frequency offset (treated in this paragraph 3.4.2)
- and $f_{i}(t)$ a zero-mean Gaussian random process (related to phase noise and treated in paragraph 3.4.3).
The justification for this model follows in 3.4.3. The received signal instantaneous phase $\phi(t)$ and frequency $f_{\text{test}} + f_e(t)$ are related by

$$
\phi(t) = 2\pi \int_0^t (f_{\text{test}} + f_e(u)) du + \phi_0 \\
= 2\pi \int_0^t (f_{\text{test}} + f_{\text{off}} + f_1(u)) du + \phi_0 \\
= 2\pi f_{\text{test}} t + 2\pi f_{\text{off}} t + 2\pi \int_0^t f_1(u) du + \phi_0.
$$

(3.4)

The phase $\phi(t)$ of the received signal is the sum of the phase from the transmit signal, a time-proportional phase term due to frequency offset, a random term due to phase noise and the initial phase $\phi_0$ of the receiver local oscillator. As the integral of the zero-mean (Gaussian) random process $f_i(t)$ remains a zero-mean (Gaussian) random process, the frequency offset $f_{\text{off}}$ can be estimated from the slope of the phase difference between the transmit and received signal, i.e. from a curve plotting $2\pi f_{\text{off}} t + 2\pi \int_0^t f_1(u) du + \phi_0$ versus time.

**Estimation** Using the model from (3.4), a linear model is fit to the phase difference measurements. The model parameters are the slope estimating the frequency offset and the y-intersect estimating the phase constant $\phi_0$.

Fig. 3.12 shows the phase difference, the linear fit and the phase difference after removal of the estimation for $\phi_0 + 2\pi f_{\text{off}} t$. Measurements are performed for $f_{\text{test}} = 10$ MHz. The phase difference after frequency offset and phase offset compensation is the phase noise.

We estimate the frequency offset over 10 measurements in Table 3.1. In a first approximation the carrier frequency offset is independent of $f_{\text{test}}$ and $f_{\text{off}} = 2.3$ Hz (mean value) (standard deviation 0.3 Hz).

### 3.4.3 System Phase Noise

The focus of this section is

- to understand the origin of phase noise in the RACooN units,
- to model the phase noise in the RACooN transmission chain.
Chapter 3  Imperfections of the RACooN Nodes

Fig. 3.12: Frequency offset for $f_{\text{test}} = 10$ MHz

The transmitter and the receiver have a heterodyne architecture (see section 3.3.1) and use mixers to shift the frequency $f_{\text{test}}$ of the monochromatic test signal. These mixers multiply the signal input with a local oscillator signal that originates from an imperfect voltage-controlled oscillator (VCO), causing phase noise. System phase noise must be understood in this section as the baseband-to-baseband frequency spread of a monochromatic test signal sent through the system that has no carrier frequency offset. We describe first the oscillator phase noise in section 3.4.3.1, give a model in section 3.4.3.2 and finally estimate it for the RACooN units in 3.4.3.3.

3.4.3.1 Origin and Definition of Oscillator Phase Noise

In a general way, oscillator imperfections can be described in time domain by the (period) jitter [133], and in frequency domain by the phase noise.

Period jitter is defined as the time difference between a measured cycle period and the ideal cycle period. The cycle period is measured between two consecutive clock rising- (or falling) edges crossing point at a given threshold.

Phase noise is defined in [117] as “rapid, short-term, random fluctuations in the phase of a wave, caused by time-domain instabilities”. Its origins include thermal noise, shot noise and flicker noise [36].

Phase noise (or jitter) is caused by a continuous random voltage fluctuation denoted $\xi(t)$ at
3.4 Characterization of the Devices and Imperfections

<table>
<thead>
<tr>
<th>$f_{\text{test}}$</th>
<th>Frequency offset $f_{\text{off}}$ (std dev) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20 MHz</td>
<td>2.7 (0.6)</td>
</tr>
<tr>
<td>-10 MHz</td>
<td>2.3 (0.5)</td>
</tr>
<tr>
<td>-5 MHz</td>
<td>2.5 (0.6)</td>
</tr>
<tr>
<td>-3 MHz</td>
<td>2.5 (0.5)</td>
</tr>
<tr>
<td>3 MHz</td>
<td>2.5 (0.5)</td>
</tr>
<tr>
<td>5 MHz</td>
<td>2.4 (0.7)</td>
</tr>
<tr>
<td>10 MHz</td>
<td>2.1 (0.8)</td>
</tr>
<tr>
<td>20 MHz</td>
<td>1.8 (0.8)</td>
</tr>
</tbody>
</table>

Table 3.1: Frequency offset estimation

the input of the VCO. This fluctuation leads to a phase fluctuation $\phi(t)$ at the output. Writing the instantaneous phase difference $\phi(t)$ as: $\phi(t) = 2\pi f_{\text{test}} \alpha(t)$, the phase fluctuation can be interpreted as a random continuous time shift $\alpha(t)$ (jitter) in the local oscillator output signal.

3.4.3.2 Model for Oscillator Phase Noise

The output $V(t)$ of an ideal (noiseless, nondrifting) oscillator at frequency $f_{\text{test}}$ is modeled as a monochromatic signal

$$V(t) = A e^{j2\pi f_{\text{test}} t}. \quad (3.5)$$

When imperfections are considered, the output of a VCO is more generally given by [97]

$$V(t) = A(t) e^{j(2\pi f_{\text{test}} t + \phi(t))}, \quad (3.6)$$

where $A(t)$ denotes the time-varying magnitude of $V(t)$ and $\phi(t)$ denotes the phase noise.

We discuss two approximations related to

- the small magnitude noise assumption,
- and the small phase noise assumption $|\phi(t)| \ll 1$.

As the noise components related to frequency and phase are generally much above the magnitude noise [53, 104], we consider in the sequel that $A(t) = A$.

A small phase variation $\phi(t)$ leads to a multiplication of the phase noise-free received signal with $e^{j\phi(t)} \approx \theta(t) = 1 + j\phi(t)$. The power spectrum of $\theta(t)$ is denoted $S_\theta(f)$ such that
Chapter 3  Imperfections of the RACooN Nodes

\[ S_θ(f) = δ(f) + S_φ(f). \] (3.7)

Under these approximations, the power spectrum of the realistic local oscillator output is the result of the convolution of the ideal local oscillator power spectrum with \( S_θ(f) \). As the ideal local oscillator output power spectrum contains only a Dirac pulse at \( f_{\text{test}} \), the realistic local oscillator power spectrum presents around \( f_{\text{test}} \) a scaled version of \( S_θ(f) \).

We discuss in the next paragraphs two models for \( φ(t) \): the free-running oscillator and the PLL oscillator model.

**Free-running oscillator** Systems with free-running oscillators are called frequency-locked [11]. The free-running oscillator is a VCO where the output signal is not fed back into the oscillator. The output frequency of the signal is proportional to the input voltage of the VCO. As shown in 3.4.3.1, the jitter \( α(t) \) is proportional to the instantaneous phase difference and related to the voltage fluctuation \( ξ(t) \) by

\[ α(t) = k \int_0^t ξ(u)du, \]

where \( k \) is a real constant.

Assuming that the voltage perturbation modeled by the random process \( ξ(t) \) is zero-mean white Gaussian distributed [11], the instantaneous frequency \( \frac{1}{2\pi} \frac{dφ(t)}{dt} \) has also a white spectrum and is zero-mean Gaussian distributed. As \( φ(t) = 2\pi f_{\text{test}}α(t) \) and \( α(t) \) is by definition a zero-mean Wiener process, \( α(t) \) is Gaussian distributed [91] (for a fixed \( t \)) and \( φ(t) \) is also zero-mean Gaussian.

An approach with phasors in given in [104,133] to derive the phase noise power spectrum. We consider two noise components at offset \( ±f_n \) from the carrier phasor rotating at \( f_{\text{test}} \). We assume pure phase noise, i.e. the amplitude and phase of the two noise terms are the same. Since \( \frac{dφ(t)}{dt} = 2πf_1(t) \), a phase error tone \( φ(t) = 2V_n\cos(2πf_n t)/V_0 \) causes a frequency deviation \( f_1(t) = -f_n^2V_n\sin(2πf_n t)/V_0 \) with mean value \( \langle f_n^2 φ(t)^2 \rangle \). Presuming stationarity of the random process describing the phase and the frequency deviation, they are described by their power spectrum \( S_F \) and \( S_θ \) which are function of the frequency offset \( f_n \) from the carrier. It follows that \( S_F(f_n) = f_n^2S_φ(f_n) \). If the power spectrum \( S_F(f_n) \) is flat, the phase noise power spectrum decreases as \( 1/f_n^2 \). A rigorous expression for the power spectrum is obtained in [30] by non-linear perturbation analysis and the author shows that signal affected by phase noise has a Lorentzian power spectrum [11,30,104,133].
3.4 Characterization of the Devices and Imperfections

When low jitter is required, the clock is equipped with a PLL mechanism that insures that the phase noise is controlled by a lowpass filter and remains within a given bandwidth. A PLL consists in a backloop that feeds the actual VCO output and the reference oscillator signal into a phase comparator. The output of the comparator is low-pass filtered and fed at the input of the VCO as in Fig. 3.14. Thus the phase of the VCO is locked to the phase of the reference oscillator.

Phase noise power spectral density calculation is performed in [72]. Within the PLL filter bandwidth, the phase noise density is lower than for the free-running oscillator, but decreases with the same slope. Around the PLL cut-off frequency, the power spectral density is flat. Outside the PLL bandwidth, phase noise behaves like in the free-running case, it is not controlled by the PLL and the phase noise power spectrum decays as $1/ f^2$ as the result of the effect of white noise voltage sources.

[72] proves that in the PLL case, the phase noise $\phi(t)$ is an Ornstein-Uhlenbeck process. Such a process has a Lorentzian spectrum [83] and is is zero-mean Gaussian distributed with finite variance [91].

**Phase noise power spectrum** Phase noise is commonly described by the Single Sideband-to-Carrier Ratio (SSCR) which is the ratio of the phase noise power in 1-Hz bandwidth at a given offset from the carrier to the carrier power concentrated at $f_{\text{test}}$ [41, 97, 133].
Let $L(f)$ be the SSCR of the signal $V(t)$ as given in (3.6) with power spectral density $S_V(f)$. The SSCR $L(f)$ is defined as the attenuation in dB from the peak value of $S_V$ at test frequency $f_{\text{test}}$ to a value of $S_V$ at $f$ [30, 60].

$$L(f - f_{\text{test}}) = 10 \log_{10} \left( \frac{S_V(f)}{S_V(f_{\text{test}})} \right).$$

The SSCR can be understood as a frequency dependent signal-to-noise ratio (SNR) where the signal is concentrated at frequency $f_{\text{test}}$ and the noise is in the side lobes of the received signal power spectrum.

### 3.4.3.3 Experimental Setup

The power spectrum of the received signal is in the left part of Fig. 3.15 for $f_{\text{test}}=10$ MHz. The large peak at 10 MHz corresponds to the frequency of the complex transmit signal. On the right part of the figure, the received signal is filtered by windowing the autocorrelation function with a Hanning window and scaled such that the carrier peak at $f_{\text{test}}$ has unitary spectral magnitude.

![Fig. 3.15: Received signal spectrum for $f_{\text{test}}=10$ MHz](image)

The pure sinusoidal transmitted wave is spread around $f_{\text{test}}$ at the receiver, this effect is due to system phase noise.

When the frequency noise is white, the SSCR function shows $1/f^2$ sideband noise and can be fit to a Lorentzian distribution $L(f)$ that is given in (3.8).

$$L(f) = \frac{1}{\pi} \frac{\pi f_{\text{test}}^2 a}{(\pi f_{\text{test}}^2 a)^2 + f^2}$$  \hspace{1cm} (3.8)
3.4 Characterization of the Devices and Imperfections

The factor $f_{1/2} = \pi f_{\text{test}}^2 a$ represents the SSCR $-3$ dB bandwidth measured from $f_{\text{test}}$. Consequently we define the linewidth of the phase noise process as $2 f_{1/2}$ since $L(f) = L(-f)$.

The fitting operation is performed using a linear model $y = ax + b$ on a log-log representation of the phase noise (in dBc/Hz) as a function of the frequency deviation $f - f_{\text{test}}$ (in Hz). Unit dBc means that the power is normalized by the power of the carrier.

The fitting operation is illustrated in Fig. 3.16 for $f_{\text{test}}=10$ MHz. The phase noise has a $1/f^2$ decay in a good match with the model. This decay is characteristic of a random walk noise [53] (white noise frequency modulation). The flat top of the SSCR is due to the effect of the PLL. Yet due to the maximum achievable resolution in the frequency domain (90 Hz resolution for a superframe of 110 buffers) the precise behavior of the PLL cannot be observed since its bandwidth has the same order of magnitude as the measurement resolution.

Table 3.2 summarizes the parameter of the Lorentzian model (spectral linewidth) and demonstrates the match with the model (exponent, that is the slope estimated from the linear model fit).

The spectral linewidth does only slightly depends on $f_{\text{test}}$ and is therefore approximated by its mean 224 Hz. The Lorentzian model fits the measured data very accurately (for $f - f_{\text{test}}$ much larger than $f_{1/2}$).

Once the frequency offset has been removed, the phase noise presents a Gaussian distri-
### Chapter 3 Imperfections of the RACooN Nodes

#### Table 3.2: Phase noise spectral linewidth

<table>
<thead>
<tr>
<th>$f_{\text{test}}$</th>
<th>spectral line width [ Hz] (std dev)</th>
<th>exponent (std dev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20 MHz</td>
<td>202 (8)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>-10 MHz</td>
<td>220 (7)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>-5 MHz</td>
<td>225 (13)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>-3 MHz</td>
<td>266 (62)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>3 MHz</td>
<td>194 (44)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>5 MHz</td>
<td>250 (23)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>10 MHz</td>
<td>229 (8)</td>
<td>2.0 (0.0)</td>
</tr>
<tr>
<td>20 MHz</td>
<td>203 (10)</td>
<td>2.0 (0.0)</td>
</tr>
</tbody>
</table>

#### Table 3.3: Mean phase noise

<table>
<thead>
<tr>
<th>$f_{\text{test}}$</th>
<th>mean phase [rad] (std dev)</th>
<th>phase variance (std dev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20 MHz</td>
<td>$3.2 \cdot 10^{-11} \ (1.4 \cdot 10^{-10})$</td>
<td>$8.0 \cdot 10^{-3} \ (1.6 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>-10 MHz</td>
<td>$2.2 \cdot 10^{-11} \ (4.5 \cdot 10^{-11})$</td>
<td>$7.8 \cdot 10^{-3} \ (1.0 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>-5 MHz</td>
<td>$6.6 \cdot 10^{-11} \ (8.2 \cdot 10^{-11})$</td>
<td>$6.9 \cdot 10^{-3} \ (1.1 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>-3 MHz</td>
<td>$-3.3 \cdot 10^{-11} \ (8.7 \cdot 10^{-11})$</td>
<td>$7.8 \cdot 10^{-3} \ (1.0 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>3 MHz</td>
<td>$5.9 \cdot 10^{-11} \ (9.5 \cdot 10^{-11})$</td>
<td>$7.9 \cdot 10^{-3} \ (2.1 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>5 MHz</td>
<td>$5.3 \cdot 10^{-11} \ (1.5 \cdot 10^{-10})$</td>
<td>$8.1 \cdot 10^{-3} \ (1.4 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>10 MHz</td>
<td>$2.6 \cdot 10^{-11} \ (9.3 \cdot 10^{-11})$</td>
<td>$9.1 \cdot 10^{-3} \ (1.8 \cdot 10^{-3})$</td>
</tr>
<tr>
<td>20 MHz</td>
<td>$2.3 \cdot 10^{-11} \ (1.2 \cdot 10^{-10})$</td>
<td>$11.6 \cdot 10^{-3} \ (3.8 \cdot 10^{-3})$</td>
</tr>
</tbody>
</table>
3.4 Characterization of the Devices and Imperfections

...distribution, as shown for \( f_{\text{test}} = 20 \text{ MHz} \) in Fig. 3.17. The fitting curve has zero-mean and the same variance as the measured samples. We use data from Fig. 3.12 to study the phase noise distribution up to the second order in Table 3.3. This table contains the mean phase noise and its variance estimated for different \( f_{\text{test}} \). The phase noise distribution parameters up to the second order statistics are independent of \( f_{\text{test}} \). The phase noise mean value in Table 3.3 is nearly null, the variance of \( f_i \) is independent of \( f_{\text{test}} \) and is equal to the mean variance \( 8.4 \times 10^{-3} \).

![Fig. 3.17: Phase noise with Gaussian distribution for \( f_{\text{test}} = 20 \text{ MHz} \)](image)

**Relationship between phase noise variance and SSCR** Under small signal assumption, phase noise variance and SSCR are related such that

\[
\sigma_{\phi}^2 \approx 2S_V(f_{\text{test}}) \int_{0}^{\infty} L(f) df,
\]  

(3.9)

The factor 2 is needed because the integral is calculated over the positive frequency differences \( f - f_{\text{test}} \) and \( L(f) = L(-f) \).
3.4.4 System Linearity

3.4.4.1 Origin of the Non-Linearities

Non-linearities are characterized in the spectrum of a signal by the presence of new frequencies at the output of a component or block of components that were not present at the input. In time domain, if the non-linear function \( f(\cdot) \) is continuous within the dynamic range, then invoking the Weierstrass theorem, it can be uniformly approximated by a polynomial of order \( L \) with an arbitrary precision \( \epsilon > 0 \) \(^{[37, 111]} \) such that \( y(x) = \sum_{p=0}^{L} c_p x^p \), where \( L - 1 \) is the non-linearity order, \( y(x) \) is the output sample, \( x \) is the input sample and \( \{c_p\}_{p=0,...,L} \) are real coefficients. When \( L = 1 \) the input-output relationship is linear.

We consider as previously a baseband single-tone transmit signal. We investigate
- the effect of the baseband I- and Q-branch non-linearities at the transmitter side,
- the effect of the IF non-linearities at the transmitter side,
- the effect of the RF non-linearities,
- the effect of the IF non-linearities at the receiver side,
- and the effect of the baseband I- and Q-branch non-linearities at the receiver side.

### Baseband non-linearities at the transmitter

The discrete signal at the input of the I- and Q-branch is real and contains frequency components at \( f_{\text{test}} \) and \(-f_{\text{test}}\). After passing through the DAC and the baseband analog components, the signal at the input of the IF mixer in the I-branch is denoted \( s'_I(t) \) and \( s'_Q(t) \). The signals \( s'_I(t) \) and \( s'_Q(t) \) contain frequency components at \( k_1 f_{\text{test}}, k_1 \in \mathbb{Z}_1 = \{k_{1,\text{min}}, \ldots, k_{1,\text{max}}\} \subset \mathbb{Z} \) such that \( k_{1,\text{min}} f_{\text{test}} \geq -B/2 \) and \( k_{1,\text{max}} f_{\text{test}} \leq B/2 \), with \( B = 1/T_s \). These components are upmodulated to center frequency \( f_{\text{IF}} \) by the IF mixer.

### IF non-linearities at the transmitter

A realistic mixer is modeled as an ideal mixer followed by a non-linear element. The ideal IF mixer shifts the center frequency of the signal in the I- and Q-branch by \( f_{\text{IF}} \). The signal at the output of the ideal mixer contains frequency components at \( k_1 f_{\text{test}} + f_{\text{IF}}, k_1 \in \mathbb{Z}_1 \).

The IF non-linearities introduce frequency components \( l f_{\text{IF}} + l k_1 f_{\text{test}}, k_1 \in \mathbb{Z}_1, l \in \mathbb{Z} \setminus \{0\} \). The IF filter following the mixer removes the non-linearities from the ideal IF mixer.
3.4 Characterization of the Devices and Imperfections

**RF non-linearities** The frequency components from the IF band are upmodulated to RF. The RF filter removes the effect of the harmonics of the RF mixer as well as the IF non-linearities that occur after the IF filter. The RF signal at the output of the RF filter contains frequency components at $f_{RF} + k_1 f_{test}$, $k \in \mathbb{Z}_1$. The RF non-linearities that appear between the RF filter output and the RF output of the transmitter, and between the receiver RF input and the input of the image rejection filter are removed by this latter. Non-linearities between the output of the image rejection filter and the input of the RF mixer cause harmonics that are suppressed afterwards by the IF filter.

**IF non-linearities at the receiver** Similarly to the transmitter side, harmonics generated by the RF mixer and the IF circuit are removed at the IF filter.

**Baseband non-linearities at the receiver** The signal entering the baseband part of the receiver contains only the harmonics generated in baseband at the transmitter side. The frequency components outside the range $[-40 \text{ MHz}, 40 \text{ MHz}]$ and introduced in baseband are removed by the anti-aliasing filters. At the output of the anti-aliasing filter, the signal has frequency components at $k_2 f_{test}$, $k_2 \in \mathbb{Z}_2 = \{k_{2,\text{min}}, \ldots, k_{2,\text{max}}\} \subset \mathbb{Z}$ such that $k_{2,\text{min}} f_{test} \geq -B/2$ and $k_{2,\text{max}} f_{test} \leq B/2$. The ADC non-linearities affect the signal on each branch.

**Conclusion** The system non-linearities originate only from the transmitter baseband part and the receiver baseband part. At the receiver, the ADC introduces further non-linearities that are not controlled by the anti-aliasing filter of the RFU. Non-linearities from the mixers, and the IF and RF circuits are removed by IF and RF passband filters.

The system non-linearities are observed among others on Fig. 3.18. This figure shows the received power spectrum when a test signal with $f_{\text{test}} = 13$ MHz is transmitted. The harmonic of $f_{\text{test}} = 13$ MHz at 26 MHz and 39 MHz are caused by baseband non-linearities at the transmitter and receiver side and by ADC non-linearity. Harmonics at 52 MHz, 65 MHz, 78 MHz and 91 MHz are only caused by the ADC since these frequencies are blocked by the receiver anti-aliasing filter. After sampling at $f_s = 80$ MHz, these harmonics are located at -28 MHz, -15 MHz, -2 MHz and 11 MHz, and they can be identified on the figure. The I-Q peak and its first harmonic at $-2f_{\text{test}}$ are not negligible.

We investigate now the power linearity of the system in section 3.4.4.2 and characterize the level of the harmonics in 3.4.4.3 and by the 3rd order intercept point (IP$_3$) in section 3.4.4.4.
3.4.4.2 Power Gain Linearity

In order to investigate the power gain linearity of the system, we transmit a digital complex exponential signal (constant amplitude) at $f_{test}=5$ MHz and observe it after the ADC in baseband at the receiver side. Fig. 3.19 is obtained when 10 superframes of 10 buffers each are transmitted with the same transmit digital power. The mean is removed from the received digital signal and the result is projected on a phase-aligned version of the transmit digital complex exponential signal. The phase alignment is obtained by tracking the position of the correlation function maximum, computed between the baseband received and transmit signal. The result of the projection is plotted verses the projection of the transmit signal on itself. Values are normalized by the number of samples. Fig. 3.19 shows that the input-output power relationship is linear.

3.4.4.3 Compression Point

The 1dB-compression point ($CP_1$) is the point where the gain of the circuit has dropped by 1 dB from its small-signal asymptotic value. It is estimated from Fig. 3.20. This point is not reached during operation because the ADC converter is saturating before.

The peak amplitudes of the other harmonics are plotted in Fig. 3.21. They are at least 35 dB below the fundamental. The slope of the fundamental component is 1, the slope of
3.4 Characterization of the Devices and Imperfections

Fig. 3.19: Input-output power ($f_{\text{test}}=5 \text{ MHz}$)

Fig. 3.20: Input-output power of the fundamental frequency ($f_{\text{test}}=5 \text{ MHz}$)
the first harmonic is 1.7, when the polynomial model predicts a slope of 2, and the second harmonic has a slope of 2.7, when the polynomial model gives a slope of 3. Harmonic power at low transmit power is below the noise floor and is not measurable.

![Fig. 3.21: Harmonics power level](image)

### 3.4.4.4 Intercept Points

The \( n \)th order intercept point is the point where the \( n \)th order harmonic as extrapolated from small-signal conditions crosses the extrapolated power of the fundamental. The \( \text{IP}_3 \) calculated for a two-tone intermodulation distortion (IMD) has a special signification: it characterizes the system behavior when a strong interferer at frequency \( f_2 \) close to the useful signal at \( f_1 \) is present. Indeed the third order harmonic of the two-tone sum signal contains a contribution at \( 2f_1 - f_2 \) that is still in the system bandwidth. The non-linearity leading to this new spectral line originates from the baseband and the RF components. In baseband, we use complex amplitude signals with \( f_2 = 4 \text{ MHz} \), \( f_1 = 13 \text{ MHz} \) and both signals have the same amplitude. The distance between the useful signal spectral line and the interference spectral line is chosen such that it is larger than the phase noise spectral linewidth estimated in 3.4.3. The spectral component retained to compute the \( \text{IP}_3 \) is at 22 MHz and the fundamental peak amplitude is measured for \( f_1 \). The results are plotted in Fig. 3.22. The \( \text{IP}_3 \) is not reached in the operation range of the device, and in the worst case the spectral line corresponding to the third order harmonic of the two-tone IMD is 35 dB.
3.4 Characterization of the Devices and Imperfections

below the fundamental.

![Graph showing IP3 levels](image)

**Fig. 3.22**: IP3 \((f_1 = 13\, \text{MHz}, f_2 = 4\, \text{MHz})\)

3.4.5 I-Q Imbalance

We define \(\omega_{RF} = 2\pi f_{RF}, \omega_{IF} = 2\pi f_{IF}\) and \(\omega_{Synt} = 2\pi f_{Synt}\).

The I-Q imbalance is discussed in this section. Its origin is detailed in paragraph 3.4.5.1. A linear system model is then presented in paragraph 3.4.5.2 and an estimation method for the RACooN nodes is in paragraph 3.4.5.3.

3.4.5.1 Origin

The RACooN nodes operate using quadrature modulation (see 3.3.1). The receiver and transmitter circuits process the I- and Q-components of the signal in dedicated branches. Due to finite tolerances of capacitor and resistor values used in the implementation of I-Q modulators and demodulators, a perfectly balanced quadrature up- and downmodulation cannot be achieved \([118]\): the gain in the I and Q branches are different and the mixing signals in the I and Q branch are not in perfect quadrature. As a consequence, the signal from the I-branch interferes in the Q-branch and inversely.
3.4.5.2 Complex Model

The RACooN units have a superheterodyne structure. Two mixers on each branch of the transmitter and receiver circuits perform the two-step up- and downmodulation via an intermediate frequency. I-Q imbalance occurs at the IF mixer of the transmitter and receiver side where the I- and Q-branch are up- and downmodulated. The RF mixers at transmitter and receiver side have a single input and output and are not subject to I-Q imbalance. We define $m_{TX}(t)$ as the mixing signal at transmitter with frequency $f_{IF}$, phase imbalance $\phi_1$ and gain imbalance $h_1$, and $m_{RX}(t)$ as the receiver IF mixing signal with frequency $-f_{IF}$, phase imbalance $\phi_2$ in the I-branch and $\phi_3$ in the Q-branch, gain imbalance $h_2$ in the I-branch and $h_3$ in the Q-branch, where $h_1, h_2, h_3$ and $\phi_1, \phi_2, \phi_3$ are defined with respect to a unitary gain and the phase reference set by the IF mixer signal phase in the I-branch. In the ideal case $\phi_1 = \phi_2 = \phi_3 = 0$ and $h_1 = h_2 = h_3 = 1$ (no I-Q imbalance).

**I-Q imbalance spectral interpretation** The mixing signals $m_{TX}(t)$ and $m_{RX}(t)$ can be rewritten as

\[
\begin{align*}
    m_{TX}(t) &= \cos(\omega_{IF} t) + jh_1 \sin(\omega_{IF} t + \phi_1) \\
    &= A_1 e^{j\omega_{IF} t} + A_2 e^{-j\omega_{IF} t} \\
    m_{RX}(t) &= h_2 \cos(- (\omega_{IF} t + \phi_2) + jh_3 \sin(-(\omega_{IF} t + \phi_3)) \\
    &= A_3 e^{-j\omega_{IF} t} + A_4 e^{j\omega_{IF} t}
\end{align*}
\]

with

\[
\begin{align*}
    A_1 &= \frac{1 + h_1 e^{j\phi_1}}{2} \\
    A_2 &= \frac{1 - h_1 e^{-j\phi_1}}{2} \tag{3.14} \\
    A_3 &= \frac{h_2 e^{-j\phi_2} + h_3 e^{-j\phi_3}}{2} \tag{3.15} \\
    A_4 &= \frac{h_2 e^{j\phi_2} - h_3 e^{j\phi_3}}{2} \tag{3.16}
\end{align*}
\]

When there is no phase and gain imbalance, $A_1 = A_3 = 1$ and $A_2 = A_4 = 0$. I-Q imbalance leads to the appearance of a new frequency term in $-\omega_{IF}$ in baseband at the transmitter side and in $\omega_{IF}$ in baseband at the receiver side.
3.4 Characterization of the Devices and Imperfections

**Linear system model**  An equivalent representation of the system with complex signals is in Fig. 3.23.

![Diagram](image)

Fig. 3.23: Complex model

The IF mixer operation at the transmitter can be written in matrix form as

\[
\bar{u}(t) = Q_{tx} \bar{s}(t)
\]

with

\[
\bar{u}(t) = \begin{bmatrix} \mathcal{R}\{u(t)\} \\ \mathcal{I}\{u(t)\} \end{bmatrix}, \quad Q_{tx} = \begin{bmatrix} \cos(\omega_{IF} t) & -h_1 \sin(\omega_{IF} t + \phi_1) \\ h_1 \sin(\omega_{IF} + \phi_1) & \cos(\omega_{IF} t) \end{bmatrix}
\]

and

\[
\bar{s}(t) = \begin{bmatrix} \mathcal{R}\{s(t)\} \\ \mathcal{I}\{s(t)\} \end{bmatrix}
\]

The IF mixer operation at the receiver is written as

\[
\bar{r}(t) = Q_{rx} \bar{o}(t)
\]

with

\[
\bar{o}(t) = \begin{bmatrix} \mathcal{R}\{o(t)\} \\ \mathcal{I}\{o(t)\} \end{bmatrix}, \quad Q_{rx} = \begin{bmatrix} h_2 \cos(\omega_{IF} t + \phi_2) & -h_3 \sin(\omega_{IF} t + \phi_3) \\ h_3 \sin(\omega_{IF} + \phi_3) & h_2 \cos(\omega_{IF} t + \phi_2) \end{bmatrix}
\]

and

\[
\bar{r}(t) = \begin{bmatrix} \mathcal{R}\{r(t)\} \\ \mathcal{I}\{r(t)\} \end{bmatrix}
\]

A frequency flat channel is assumed. Its effect on the signal is modeled by a multiplication
Chapter 3  Imperfections of the RACooN Nodes

with a complex number $\alpha e^{j\beta}$. With matrix notation,

$$\vec{m}(t) = \alpha H \vec{y}(t),$$

with

$$\vec{y}(t) = \begin{bmatrix} \mathcal{R}\{y(t)\} \\ \mathcal{I}\{y(t)\} \end{bmatrix}$$

and

$$H = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix}$$

Finally

$$\vec{r}(t) = \alpha Q_x H Q_x \vec{s}(t)$$

$$= \alpha M_c \vec{s}(t) \quad (3.18)$$

$$= e^{j\omega_{test}t}. \quad (3.22)$$

3.4.5.3  I-Q Imbalance Estimation on the RACooN Units

We propose now a simple estimation method of the I-Q imbalance modeled by $M_c$ of a transmitter-receiver pair linked by a radio-frequency cable when

$$s(t) = s_I(t) + j s_Q(t)$$

$$= \vec{s}[1] + j \vec{s}[2]$$

$$= e^{j\omega_{test}t}. \quad (3.22)$$

To the difference with existing works based on blind I-Q imbalance estimation \cite{8,69,119}, this is a data-aided scheme based on a constant-amplitude sequence. A constant amplitude (complex) signal is transmitted, as previously, to avoid the non-linearity behavior of the PAs in the circuits.

**Estimation of $M_c$**  Functions $s_I(t) = \cos(\omega_{test}t)$ and $s_Q(t) = \sin(\omega_{test}t)$ are orthogonal. Consequently, the scalar product of $r_I(t)$ (or $r_Q(t)$) and $\cos(\omega_{test}t)$ depends solely on the coefficients of the $\cos(\omega_{test}t)$-terms, and the scalar product of $r_I(t)$ (or $r_Q(t)$) and $\sin(\omega_{test}t)$ depends solely on the coefficients of the $\sin(\omega_{test}t)$-terms.
3.4 Characterization of the Devices and Imperfections

We can compute from the received signal:

\[
\frac{1}{f_{\text{test}}} \int_{0}^{1} r(t) \cos(\omega_{\text{test}} t) \, dt = \frac{\alpha}{2} M_c[1, 1] \\
\frac{1}{f_{\text{test}}} \int_{0}^{1} r(t) \sin(\omega_{\text{test}} t) \, dt = \frac{\alpha}{2} M_c[1, 2] \\
\frac{1}{f_{\text{test}}} \int_{0}^{1} r_Q(t) \cos(\omega_{\text{test}} t) \, dt = \frac{\alpha}{2} M_c[2, 1] \\
\frac{1}{f_{\text{test}}} \int_{0}^{1} r_Q(t) \sin(\omega_{\text{test}} t) \, dt = \frac{\alpha}{2} M_c[2, 2]
\]

(3.23) \hspace{1cm} (3.24) \hspace{1cm} (3.25) \hspace{1cm} (3.26)

We define the signal-to-interference ratio (SIR), also called [119] image rejection ratio, as

\[
\text{SIR} = \frac{|M_c[1, 1] + M_c[2, 2] + \imath(M_c[2, 1] - M_c[1, 2])|^2}{|M_c[1, 1] - M_c[2, 2] + \imath(M_c[2, 1] + M_c[1, 2])|^2}
\]

(3.27)

and in dB: \(\text{SIR}_{\text{dB}} = 10 \log_{10}(\text{SIR})\). The result is independent of angle \(\beta\) because matrix \(H\) is a unitary matrix. It therefore does not affect the energy of the signal and is not visible in the figure of merit SIR.

Frequency \(f_{\text{test}}=29\) MHz is the integer frequency over the equalized baseband operating range \([-30\) MHz, \(30\) MHz\] for which the first interferer at \(-29\) MHz that is an harmonic image frequency is the 78th order harmonic of the signal. Frequency \(f_{\text{test}}=20\) MHz is the worst choice for I-Q imbalance estimation since for this frequency the image frequency at \(-20\) MHz superpose with the aliasing peaks caused by the third harmonics at 60 MHz.

The received signal power spectrum after truncation of the autocorrelation function for \(f_{\text{test}}=29\) MHz is in Fig. 3.24.

The attenuation \(\text{SIR}_{\text{dB}}\) measured at \(f_{\text{test}} = 29\)MHz and \(f_{\text{test}} = -29\)MHz is 20 dB (averaged attenuation over 10 measurements). This attenuation represents a power ratio of \(10^2\) between the two peaks, whereas a ratio of \(10^4\) with respect to the signal peak is considered as strong.
Fig. 3.24: Received signal power spectrum after truncation of the autocorrelation function \( f_{\text{test}} = 29 \text{ MHz} \)

### 3.4.6 DC Offset

#### 3.4.6.1 Definition

The DC offset is the mean value of the stationary digital signal received when the transmit digital signal is a complex exponential \( s[k] = A e^{j2\pi f_{\text{test}} k T_s} \).

#### 3.4.6.2 Origin

The DC offset observed in the digital received signal (I- and Q-branch) possibly originates from [92, 96, 132]
- self-modulation of the local oscillator at the IF mixer at the receiver side,
- local oscillator leakage at the IF mixer at the transmitter side by capacitive, substrate and bond wire coupling,
- amplifier (components B22 and B24) output DC offset at the receiver side (due to its non-linearity and input bias),
- DAC and ADC output offset,
- phase noise,
- contributions from harmonics of \( f_{\text{test}} \) that are located at \( f = 0 \text{ Hz} \) after sampling,
- phase and gain imbalance between transmitter and receiver,
3.4 Characterization of the Devices and Imperfections

- quantization noise in the ADC.

These noise sources are commented in the next paragraph.

**Remark 1:** In the following we discuss the offset values based on the polar complex representation since the I- and Q-branch present non-equal offsets. This happens for two reasons
- the local oscillator leakage in the I-branch can be significantly different from local oscillator leakage in the Q branch due to layout differences [33],
- the phase noise realizations are different in each branch.

**Remark 2:** The offset sources mentioned above are responsible for static (i.e. time-invariant) offset only. Dynamic DC offset [96, 132] is generated by self-mixing of re-radiated local oscillator leakage and interferer leakage. This occurs when the local oscillator signal leaks to the antenna, is radiated and subsequently reflected from moving objects back to the receiver. A fast fading channel will also introduce time-variant changes in the wave reflected back to the transmit antenna. This effect happens when transmission occurs via antennas and in therefore not considered in this paragraph.

**Self-modulation and leakage** The DC offset originates among others from self-mixing and leakage because the isolation between the local oscillator mixer input, the IF mixer input and the IF mixer output is not perfect [95, 96]. Due to coupling, a finite amount of feedthrough exists from the local oscillator ports to the circuits before and after the mixers. Self-modulation occurs when the induced signal at the input of the mixer is amplified and mixed with the local oscillator signal, producing a DC at the output of the mixer. Due to the heterodyne structure of the receiver, only self-mixing at the IF mixer on the receiver side will affect the baseband signal DC. Leakage is the coupling between the local oscillator and the mixer output port. It follows that the mixer signal output contains an attenuated version of the local oscillator signal. Due to the heterodyne structure, only the IF mixer leakage can affect the signal.

**Amplifier DC offset** The DC output offset of the amplifier is a well-known imperfection of this type of devices. This offset depends on the temperature.

**Phase noise** The local oscillator random phase noise modifies the complex signal DC offset phase at each mixer. A random phase component is added at the transmitter.
Chapter 3 Imperfections of the RACooN Nodes

– to any baseband DC offset component entering the IF mixer,
– to any signal component at $f_{IF}$ entering the RF mixer.

A random phase component is added at the receiver
– to any signal component at $f_{RF}$ entering the RF mixer,
– to any signal component at $f_{IF}$ entering the IF mixer.

Minor DC offset sources Quantization noise occurs in the ADC. This converter uses 14 bits to quantize the incoming signal. Its $2^{14}$ output levels extend over $[-1, 1]$, consequently one step corresponds to an amplitude output of $B = \frac{2}{2^{14} - 1} \approx 1.22 \times 10^{-4}$. Assuming a mid-tread quantization, the error made at each quantized point is between $-B/2$ and $B/2$. Thus the error due to quantization in the estimation of the mean value of a signal is upper-bounded by $B/2$.

Component non-linearity in the receiver chain introduces harmonics of $f_{test}$ in the signal. They are only partly filtered out by dedicated filters. After sampling at $f_s$=80 MHz, the harmonics multiple of $f_s$ present in the spectrum become visible in the baseband and add at the DC position. Consequently we perform measurements using $f_{test} = 29$ MHz such that only high order (low power) harmonics can interfere with the DC and disturb the measurements. I-Q imbalance can also influence the baseband DC offset since the signal in one branch is leaking in the other branch, as described in section 3.4.5.

3.4.6.3 DC Offset Measurement

The magnitude and phase of the received complex baseband DC offset are estimated. A mission consisting of
– a single transmit command for the transmitter (after an idle command as explained in 3.4.1),
– a single receive command for the receiver,
is executed 100 times. The received data containing 800,000 samples is analyzed: the samples are grouped in 400 blocks of 2000 samples, and the mean complex value of each block is calculated. The phase and magnitude of the blocks is plotted in Fig. 3.25(a) and Fig. 3.26(a) for the first 0.1 s.

We first check the stationarity of the signal mean value. The block mean phase and magnitude curves have a staircase shape where each step is 0.01 s. These represent the samples from one mission execution. Over one step, the block mean magnitude is oscillating around a time-constant mean value. At each mission repetition, this mean value is changing. The
height between two consecutive steps is random. Over one step the block mean phase is also oscillating around a mean value. The height between two consecutive steps depends on the phase offset between the transmitter and the receiver local oscillators at the beginning of the mission execution. From these two observations we conclude that the mean value of the signal is not rigorously stationary, but the mean value of the phase and argument are in first approximation stationary over the duration of one mission execution.

We will now investigate the distribution of the block mean phase and block mean magnitude within the first superframe (or mission execution, i.e. on one step). The magnitude and the phase are plotted in Fig. 3.25(b) and Fig. 3.26(b). The empirical phase distribution in Fig. 3.26(c) and magnitude distribution in Fig. 3.25(c) have a good fit with a Gaussian distribution.

For each superframe we can model the complex offset as \((A + n(t))e^{j(\phi_0 + \phi'(t))}\) where \(A\) and \(\phi_0\) are real constant (for the duration of the superframe) and \(n(t)\) and \(\phi'(t)\) are Gaussian processes with zero mean and variance \(\sigma_n^2\) and \(\sigma_{\phi'}^2\). This model can be interpreted by decomposing \(\phi'(t)\) into \(\phi'(t) = \phi(t) + \psi(t)\) where \(\phi(t)\) is phase noise. \((A + n(t))e^{j\phi(t)}\) is some transmitter noise affected by the phase noise \(e^{j\psi(t)}\). \(\phi(t)\) and \(\psi(t)\) are independent zero-mean Gaussian processes. The transmitter noise has a time-dependent amplitude and phase since it is generated by independent noise sources in the I- and Q-branch.

The validity of the model is now extended over the different timeslots. Parameter \(A\) is estimated for all the 100 timeslots and its empirical distribution is shown in Fig. 3.27 and for the noise variance \(\sigma_n^2\) in Fig. 3.28.

The empirical distribution of parameter \(\phi_0\) is plotted in Fig. 3.29. It is in first approximation uniformly distributed over \([-\pi, \pi]\). This corresponds to a random phase offset between the transmitter and the receiver for each mission. Indeed the local oscillator are reinitialized at the beginning of each mission and the phase offset can take any value between \(-\pi\) and \(\pi\).

The empirical distribution of parameter \(\sigma_{\phi'}^2\) is in Fig. 3.30 and has a \(\chi^2\) square shape, that is the distribution of a squared normally distributed random variable. From Fig. 3.29 and Fig. 3.30, we model the phase as zero-mean Gaussian.

### 3.4.6.4 Model

We model the digital DC offset by a complex random variable \((A + n(t))e^{j(\phi_0 + \phi(t))}\) with

- \(A\) is constant during one mission uniformly distributed over \([0.05, 0.08]\),
Chapter 3 Imperfections of the RACooN Nodes

(a) Mean block magnitude

(b) Zoom in the first superframe (magnitude)

(c) Empirical distribution of the magnitude, samples collected over interval [0, 0.01 s]

**Fig. 3.25:** Mean block magnitude
3.4 Characterization of the Devices and Imperfections

Fig. 3.26: Mean block phase
Fig. 3.27: Empirical distribution of parameter $A$, realizations collected over the superframes

Fig. 3.28: Empirical distribution of parameter $\sigma_n^2$, realizations collected over the superframes
3.4 Characterization of the Devices and Imperfections

Fig. 3.29: Empirical distribution of parameter $\phi_0$, realizations collected over the superframes

![Empirical distribution of parameter $\phi_0$](image)

Fig. 3.30: Empirical distribution of parameter $\sigma^2_{\phi}$, realizations collected over the superframes

![Empirical distribution of parameter $\sigma^2_{\phi}$](image)
Chapter 3 Imperfections of the RACooN Nodes

\[ y(t) = x(t) + m(t) \]

- \( n(t) \) is Gaussian distributed with zero-mean and variance \( \sigma_n^2 = 10^{-6} \).
- \( \phi_0 \) is constant during one mission and is uniformly distributed over \([\pi, \pi] \).
- \( \phi'(t) \) is Gaussian distributed with zero-mean and variance \( \sigma_{\phi'}^2 = 10^{-2} \).

3.5 Conclusion

The transmission system can be modeled in baseband as illustrated in Fig. 3.31: its output affected by the DC offset \( \epsilon(t) \), the frequency offset \( f_{\text{offset}} \), the phase noise \( \phi(t) \), a non-linear block representing I-Q imbalance and baseband non-linearities and a complex zero-mean circularly symmetric additive white Gaussian noise (AWGN) \( m(t) \).

Baseband non-linearities and I-Q imbalance effects are modeled in a worst case basis by the additional first harmonic component at \( 2f_{\text{test}} \) at 40 dB below the signal peak, by the harmonic at \( -f_{\text{test}} \) that is 20 dB below the signal peak and the harmonic at \( -2f_{\text{test}} \) that is 40 dB below the signal peak.

The phase noise is expressed as a multiplication of the signal with \( e^{j\phi(t)} \) where \( \phi(t) \) has a Lorentzian power spectral density with spectral linewidth 200 Hz around \( f_{\text{test}} \) and is Gaussian distributed with zero-mean and variance \( 8.4 \cdot 10^{-3} \).

The frequency offset is constant value \( f_{\text{offset}} = 2.3 \) Hz and disturbs the received signal by multiplying it with \( e^{j2\pi f_{\text{offset}} t} \).

The DC offset \( \epsilon(t) \) is a complex random variable \( \epsilon(t) = (A + n(t))e^{j\phi_0 + \phi(t)} \) with:

- \( A \) is constant during one mission uniformly distributed over \([0.05, 0.08] \),
- \( n(t) \) is complex Gaussian circularly symmetric distributed with zero-mean and variance \( \sigma_n^2 = 10^{-6} \),
- \( \phi_0 \) is constant during one mission and is uniformly distributed over \([\pi, \pi] \),
- \( \phi'(t) \) is Gaussian distributed with zero-mean and variance \( \sigma_{\phi'}^2 = 10^{-2} \).

---

\[ y(t) = x(t) + m(t) \]

\[ s(t) \quad \text{channel} \quad k(t) \quad \text{non-linearities I-Q} \quad x(t) \quad \times \quad \times \quad + \quad + \quad y(t) \]

Fig. 3.31: General complex model with imperfections
Chapter 4

Robust Channel Estimation with the RACooN Lab

4.1 Introduction

In the previous chapter we have described the RACooN lab and proposed a model for the nodes. We go in this chapter one step forward towards channel measurements. In this chapter we propose a robust channel estimation method, where robustness is defined with respect to the hardware imperfections that affect the channel estimate. A channel estimation method is robust when the processing gain degradation due to hardware imperfections remains below a threshold that depends on the required channel estimation accuracy. In section 4.2 we describe the state of the art for channel estimation. Then we analyze and quantify the effect of hardware imperfections on the channel estimate in section 4.3. We propose a structured definition for the processing gains relevant for channel estimation. The gain degradation caused by frequency offset, phase noise and non-linearities is evaluated in section 4.3.2 and 4.3.3. Then we propose a robust scheme for channel estimation that can cope with the imperfections in section 4.4. Furthermore, we give a novel insight for the use of direct sequence and chirp sequence for channel estimation in section 4.4.4. Finally we apply the proposed scheme on existing hardware in section 4.4.6. Our conclusions are summarized in 4.5.

The main contributions of this chapter can be summarized as follows

- we provide a review of channel sounding sequences,
- we propose a signal processing method for channel estimation with the Radio Access with Cooperative Nodes (RACooN) lab and discuss the sensitivity of the channel estimate to phase noise and carrier frequency offset (CFO),
- we provide analytical expressions for the gain degradation due to phase noise and CFO,
– we assess the robustness of our method against system non-linearities by comparing it with a dedicated non-linear system identification method,
– we experimentally compare a direct sequence to a chirp sequence and conclude for their use in the RACooN lab.

In this chapter the training (or sounding) sequence is denoted \( x[\cdot] \) and is a either a direct sequence or a chirp sequence. The signal built from repetitions of a training sequence is called a training (or sounding) signal and is denoted \( s[\cdot] \). The estimated channel in this chapter remains constant for the estimation duration. The discrete symbol duration is \( T_s \) all through this chapter.

### 4.2 State of the Art for Channel Sounding

Three categories of sounding sequences for wideband channel are available in the literature and are discussed here

– the impulse sequence, treated in 4.2.1,
– the direct sequence, treated in 4.2.2,
– and the chirp sequence, treated in 4.2.3.

State of the art channel sounders use either direct sequences or chirp sequences. Direct sequence sounders are used among others in [4, 22, 35, 56, 112], whereas chirp sequences are in use in [44, 64, 70, 99–103]. It must be noted that except for [64], all other works dealing with chirp sequences have been done by Salous. Clearly more sounders are using direct sequences than chirp sequences. For digital sounders [22, 35, 64, 70, 90] the choice of the sequence is never motivated by performance arguments but by hardware constraints (for example only one input available in [90]).

#### 4.2.1 Impulse Sequence

The impulse sequence is the most straightforward sounding sequence. It is a \( N \)-periodic signal built from a sequence that contains a very narrow time-impulse [84]. The sequence has a flat power spectrum. The frequency resolution of the estimated channel is given by \( 1/(NT_s) \) and the bandwidth of the estimated baseband channel is \( f_s = 1/T_s \). The discrete pulse duration is \( T_s \) and determines the minimum discernible path difference between echo contributions [85].

The major limitation of this sounding technique is the high peak-to-mean power ratio requirement in order to detect weak echoes. As pulsed transmitters are peak-power limited, only
moderate range coverage can be achieved \cite{101}. An alternative solution is based on pulse compression technique at the receiver \cite{35} so that more energy can be set into the sounding sequence while avoiding the peak-power limitation. Sequences that allow pulse compression techniques are the direct sequences and the chirp sequences. They allow channel estimation at a higher signal-to-noise ratio (SNR) than an impulse sequence with the same repetition rate.

\subsection{Direct Sequence}

Direct sequences are present in the literature under several names. They are called maximum-length correlation sequence, pseudo-random binary sequence (PRBS), maximum-length binary sequence, maximum-length sequence, pseudo-noise sequence, pseudo-random sequence, maximal-length linear feedback shift-register sequence, binary maximal-length linear feedback shift-register sequence or m-sequence.

These sequences have been widely used for channel sounding since the work of Cox \cite{27} because they are simple to implement in hardware and have good cyclic autocorrelation properties \cite{22}.

These sequences draw their name from the fact that they are the sequences of maximum possible period \((N = 2^m - 1)\) from an \(m\)-stage binary shift-register with linear feedback. They consist in a sequence of \((+1)\) and \((-1)\), have one more \((+1)\) than \((-1)\), and thus they have mean value \((+1)\). An example of a m-sequence for \(N = 31\) is given in Fig. 4.1.

\textbf{Correlation properties} The direct sequence has a similar autocorrelation property to white noise, therefore it is sometimes called pseudo-noise sequence.

The cross-correlation and autocorrelation properties are extensively studied in dedicated works like \cite{105} and the definition is recalled in the appendix. The periodic pattern of the cyclic autocorrelation function \(\theta_x[\cdot]\) is defined for \(\delta = 0, \ldots, N - 1\) by

\[
\theta_x[\delta] = \begin{cases} 
N & \text{for } \delta = 0, \\
-1 & \text{for } 1 \leq \delta \leq N - 1
\end{cases}
\]  

(4.1)

This is the best possible cyclic autocorrelation function for a binary sequence of period \(N\), in the sense that the cyclic autocorrelation sidepeaks are minimum (see Fig. 4.2(a)). As a side remark we note that the linear autocorrelation function has higher sidepeaks, as shown in Fig. 4.2(b).
We introduce the mainlobe-to-sidepeak ratio (MSR). It is defined as the ratio of the cyclic autocorrelation main peak to the largest sidelobe magnitude.

$$\text{MSR} = \frac{\theta_x[0]}{\max_{1 \leq \delta \leq N-1} |\theta_x[\delta]|} = N.$$  \hfill (4.2)

The MSR in this context is a measure for the comparison between the cyclic autocorrelation function and a Dirac function. An cyclic autocorrelation function with high MSR has a high peak at 0-delay and low sidelobes, and for the Dirac function, MSR = $\infty$. 

**Fig. 4.1**: m-sequence ($N=31$)

**Fig. 4.2**: m-sequence autocorrelation ($N=31$, $T_s=12.5$ ns)
4.2 State of the Art for Channel Sounding

**Power spectrum**  The power spectrum of the direct sequence is flat over the whole bandwidth except at 0 Hz (Fig. 4.3). The discrete power spectral density $S_x[\nu]$ of the periodic signal built from sequence $x[\cdot]$ is given by

$$S_x[\nu] = \begin{cases} 
\frac{1}{N^2 f_s} & \text{for } \nu = 0 \\
\frac{N+1}{N^2 f_s} & \text{for } \nu = \frac{f_s}{N}, \ldots, \frac{f_s(N-1)}{N} 
\end{cases} \quad (4.3)$$

**4.2.3 Chirp Sequence**

A chirp (or swept frequency) sequence, as defined in [99], is a sequence characterized by a linear frequency increase or decrease over a bandwidth $B$ during a period $T_R = NT_s$. The signal amplitude in time domain is represented in Fig. 4.4 for $N = 31$ and $B = 80$ MHz.

The chirp signal instantaneous frequency is plotted in Fig. 4.5(a) and is for sample $k$, $0 \leq k \leq N - 1$

$$f[k] = B \left( \frac{k}{N} - \frac{1}{2} \right). \quad (4.4)$$

Thus the instantaneous phase $\phi[k]$ of sample $x[k]$ is illustrated in Fig. 4.5(b) and is given by

$$\phi[k] = \pi \left( \frac{k^2}{N} - k \right) + \phi_0, \quad (4.5)$$
where $\phi_0$ is a constant real number representing the signal phase at $k = 0$. We further assume that $\phi_0 = 0$ without loss of generality.

The complex chirp samples are given in time domain by

$$x[k] = Ae^{j\pi(\frac{k^2}{N}-k)}, \ 0 \leq k \leq N - 1$$  \hspace{1cm} (4.6)

The constant signal amplitude makes it possible to use a non-linear power amplifier [64].

---

**Fig. 4.4:** Time-domain signal

**Fig. 4.5:** Chirp properties ($T_R = 387.5$ ns, $B = 80$ MHz)

(a) instantaneous frequency  \hspace{1cm} (b) instantaneous phase
4.2 State of the Art for Channel Sounding

**Correlation properties**  The periodic pattern of the cyclic autocorrelation function $\theta_x[\cdot]$ is given for $0 \leq \delta \leq N - 1$ by

$$\theta_x[\delta] = \begin{cases} 
N & \text{for } \delta = 0, \\
0 & \text{for } 1 \leq \delta \leq N - 1
\end{cases}$$  \hspace{1cm} (4.7)

It is represented in Fig. 4.6(a) and is the best cyclic autocorrelation function with respect to the MSR since $\text{MSR} = \infty$.

The linear autocorrelation function is represented in Fig. 4.6(b) and has higher side peaks (Fig. 4.6(a)) than the cyclic autocorrelation function.

![Graphs of cyclic and linear autocorrelation](image)

**Fig. 4.6:** Chirp autocorrelation ($T_R = 387.5$ ns, $B = 80$ MHz)

**Power spectrum**  The power spectral density of the sequence built from repetition of a chirp pattern is given by

$$S_x(\nu) = \frac{1}{N f_s} \text{ for } \nu = 0, \frac{f_s}{N}, \ldots, \frac{f_s(N - 1)}{N}$$  \hspace{1cm} (4.8)

### 4.2.4 Comparison Between Direct and Chirp Sequences

The two types of training sequences are compared in [90, 100]. Both sequences offer the same correlation gain through pulse-compression at the receiver if they have the same length. The cyclic autocorrelation function properties are invoked as the main argument to promote the use of the chirp sequences compared to direct sequences. Indeed a simple received signal
scaling after the correlator removes the effect of the chirp sequence. Yet surprisingly the majority of existing sounders employ direct sequences. Experimental comparison results are available at very high frequency (VHF) in [102] and are not directly exploitable since two different sounders are compared, one using a direct sequence, the other a chirp sequence. [90] describes a measurement campaign in the 60 GHz band and also considers both types of training sequence. The authors in this work motivate the choice of direct sequence by the experimental consideration that a direct sequence is for their channel sounder less sensitive to non-linear distortion. They underline the difficulty of predicting the severity of these effects on the signal in a general way.

### 4.2.5 Common Channel Estimation

It is assumed from now that the channel length $L_H \leq N$. We present a common channel estimation technique as it is employed in available works. A training signal $s[\cdot]$ made from periodic repetitions of a training sequence $x[\cdot]$, such that $\forall k \in \mathbb{Z}, \ s[k] = x[k \mod (N)]$, is transmitted. The periodic repetition of the sequence emulates at the receiver a cyclic convolution with the channel. The $N$-periodic pattern isolated in the received signal is the noisy output of the cyclic convolution of $x[\cdot]$ with the channel $h[\cdot]$. The received discrete-time signal $y[\cdot]$ when $x[\cdot]$ is transmitted is modeled as follows

$$\forall k \in [0, N - 1], \ y[k] = (h \odot x)[k] + n[k]$$

(4.9)

where $n[k]$ is a zero-mean complex additive white Gaussian noise (AWGN) sample with variance $N_0$ and $\odot$ denotes the cyclic convolution.
4.2 State of the Art for Channel Sounding

4.2.5.1 Averaging

If $M$ realizations of the sequence $y[\cdot]$ are available, averaging reduces the effect of the AWGN by offering an averaging gain of $10 \log_{10} (M)$ independently of the type of training sequence used.

**For impulse sequence** The periodic training signal is a repetition of impulse sequences, each containing a discrete Dirac pulse. The impulse sequence is

$$\forall k \in [0, \ldots N - 1], \ x[k] = \delta[k].$$

Therefore the received signal is directly the time-domain channel estimate with an AWGN term.

$$\hat{h}[k] = h[k] + n[k], \ 0 \leq k \leq N - 1. \quad (4.10)$$

We discuss the next signal processing steps that apply to direct and chirp sequences in section 4.2.5.2.

4.2.5.2 Pulse Compression

This section handles channel estimation when the sounding sequence is a direct sequence or a chirp sequence. Estimation relies on the cyclic autocorrelation properties of these sequences. We use two indicators to characterize the robustness of this step

- the correlation gain, defined as the dynamic range increase (i.e. SNR increase),
- the MSR of the training sequence, which is in this context a measure for the signal-to-interference ratio (SIR).

**Cyclic cross-correlation** The received signal is cyclic correlated with the transmit sounding sequence to retrieve a channel estimate. $\otimes$ denotes the cyclic correlation.

$$\forall k \in [0, N - 1],$$

$$y[k] = (h \otimes x)[k] + n[k] \quad (4.11)$$

$$y \otimes x[k] = ((h \otimes x) \otimes x)[k] + (n \otimes x)[k] \quad (4.12)$$

$$\theta_{y,x}[k] = (h \otimes \theta_x)[k] + \theta_{n,x}[k] \quad (4.13)$$
The noise sequence and the transmit sounding sequence are uncorrelated, thus \( \theta_{n,x} \approx 0 \). This technique is categorized as a compression technique since the received pulse is compressed in time-domain by the correlation operation. Indeed as the sounding sequence cyclic autocorrelation function can be approximated by a Dirac pulse (exact result for a chirp sequence), the sequence \( \theta_{y,x} \) contains \( L_H \) significant samples instead of \( N \) before correlation. The correlation operation leads to a signal dynamic range increase of \( 10 \log_{10} (N) \) expressed in dB [71] with respect to the impulse sequence.

The MSR depends on the training sequence, MSR = \( 10 \log_{10} (N) \) when a direct sequence is transmitted, and MSR = \( \infty \) when a chirp sequence is used. The total processing gain is the sum of the averaging gain and the correlation gain, and it equals \( 10 \log_{10} (MN) \). This measure does not take into account the interferences from the non-ideal training sequence autocorrelation function.

For m-sequences, an additional processing step is required. Perfect compensation of \( \theta_x \) is achieved in frequency domain by dividing term-by-term the discrete Fourier transform of \( \theta_{y,x} \) denoted \( \mathcal{F} \{ \theta_{y,x} \} \) by the training sequence discrete power spectrum \( S_x \).

### 4.2.5.3 Parameter Choice

As previously, \( N \) denotes in this section the length of the training sequence and \( M \) is the number of periodic patterns available at the receiver after transmission of the training sequence. We choose \( N \) such that \( N \geq L_H \). The sample duration is \( T_s \), which is the maximum achievable time resolution, and \( NT_s \) is the longest admissible channel excess delay.

**Total processing gain required** The choice for \( M \) and \( N \) depends on the total processing gain required, i.e., on the noise level at the receiver and the required SNR of the channel estimate. As seen in 4.2.5.2, the total processing gain is \( 10 \log_{10} (MN) \) and is a function of the product \( MN \), this product is proportional to the training signal duration \( MNT_s \) (when neglecting the first and last received samples for which the cyclic convolution is not formed).

**Frequency resolution** The frequency resolution is \( \frac{1}{NT_s} \). Large \( N \) decreases the size of the frequency bin, thus improving the frequency resolution.
4.3 Gain Degradation due to Hardware Imperfections

**Movement**  When the transmitter or the receiver are moving, channel estimation is further possible under certain conditions. The distance over which the receiver and transmitter have moved apart during $MNT_s$ must be very small with respect to the wavelength of the radio-frequency signal. We denote $v_{\text{max}}$ the largest linear speed between the receiver and the transmitter. The condition above is

$$v_{\text{max}}MNT_s \ll \frac{c}{f_c}.$$  

**Doppler speed measurement**  When the speed between the transmitter and the receiver is estimated, the Doppler sampling rate must be larger than two times the maximum Doppler frequency $f_D = \frac{v_{\text{max}}f_c}{c}$, i.e.

$$\frac{1}{MNT_s} \geq 2\frac{v_{\text{max}}f_c}{c}.$$  

Except [22], no available work specifically addresses the effect of hardware imperfections on the channel estimate.

### 4.3 Gain Degradation due to Hardware Imperfections

So far we have considered ideal hardware. In this section we discuss now the effect of the hardware imperfections on the estimation procedure. We quantify the averaging gain in section 4.3.2, and correlation gain in section 4.3.3 when the system is subject to frequency offset, phase noise and non-linearities.

#### 4.3.1 Summary of the Signal Processing Steps

A $N$-periodic sounding signal is transmitted. Channel estimation is performed as follows

1. $M$ ($N$-periodic) received patterns are isolated (using the common sample indexing if an absolute time reference is required, or using an arbitrary time reference otherwise),
2. the periodic patterns are averaged to reduce the effect of AWGN, this step offers $10 \log_{10} (M)$ averaging gain in the ideal case,
3. the average pattern is cyclic correlated with the transmit sequence, this step offers $10 \log_{10} (N)$ correlation gain,
4. (for direct sequence only) the transmit sequence power spectrum is removed from the periodic pattern at the correlator output.
Remark about the DC offset  We have seen in 3.4.6 that the signal value during one mission is not strictly stationary. The mean signal phase and amplitude are randomly changing and are described each by a Gaussian random process. Transmission of information on the baseband direct component will be performed at low SNR and should, as it is the case in the standard IEEE 802.11a [46], be avoided. In the further steps, the mean value of each pattern before averaging is removed, assuming a baseband channel blocking at direct component (DC). A mean value is added to the signal by the time-domain windowing operation. Indeed this operation is equivalent in frequency domain to a linear convolution of the channel transfer function with a filter with transfer function equal to the Fourier transform of the time-domain window. It will smooth the channel transfer function and also add an artificial mean value interpolated from the transfer function values around the DC. A filter with a wide main lobe (in frequency domain) will affect a larger frequency band around the DC component than a filter with a narrow main lobe. The width of the main lobe is coarsely inversely proportional to the width of the time-domain window. Therefore the window is subject to following trade off: a wide time-domain window let noise samples in the channel impulse response (CIR) tail and ripples in the useful band of the channel transfer function but reduces the distortion of the channel transfer function around the DC, whereas a narrow time-domain window cuts out the noise samples in the CIR tail but distorts more significantly the transfer function around DC. We decide for a time-domain narrow window, favoring the channel time-domain observation.

4.3.2 Averaging

In this section we discuss the deterioration of the averaging gain in the system. All through this chapter the averaging gain is the SNR increase obtained by averaging, all other parameters (and imperfections where it applies) remaining the same. The organization of this paragraph is sketched in Fig. 4.8. We start by defining the ideal averaging gain achieved when no imperfections affect the system in 4.3.2.1, then compute the averaging gain degradation caused by carrier frequency offset (CFO) in 4.3.2.2 and phase noise in 4.3.2.3. Non-linearities are not discussed in this section since averaging a signal affected by non-linearities gives the same averaging gain as for the ideal case.

4.3.2.1 Ideal Case

The training signal built from the repetition of a $N$-sample training sequence is transmitted. $M$ periodic patterns are isolated from the received signal. When no imperfections are
4.3 Gain Degradation due to Hardware Imperfections

![Diagram: Averaging vs Phase Noise vs CFO]

Fig. 4.8: Structure of section 4.3.2

present, each sample in the received signal is the sum of a deterministic term and a random noise term. We consider now \( y_l[k] \) which is the \( (k + 1) \)th sample of the \((l + 1)\)th pattern, with \( 0 \leq k \leq N - 1 \) and \( 0 \leq l \leq M - 1 \). We have

\[
y_l[k] = z[k] + n_l[k],
\]

where \( z[k] \) is the deterministic signal obtained from the \( (k + 1) \)th term of the cyclic convolution between the channel and the training sequence, and the \( \{n_l[k]\} \) are realizations of independent identically distributed complex discrete zero-mean AWGN random samples with variance \( N_0 \). The assumption that noise samples are independent identically distributed is enough for this paragraph. We set \( k = k_0 \) and average the samples with index \( k_0 \) over the \( M \) realizations, the result is denoted \( y[k] \).

\[
y[k_0] = \frac{1}{M} \sum_{l=0}^{M-1} y_l[k_0] = z[k_0] + \frac{1}{M} \sum_{l=0}^{M-1} n_l[k_0]
\]

As the noise terms are zero-mean and independent identically distributed, the variance of \( \frac{1}{M} \sum_{l=1}^{M} n_l[k] \) is \( \frac{N_0}{M} \). Thus averaging offers an SNR gain named averaging gain of \( 10 \log_{10} (M) \) in dB with respect to the non-averaging case.

**Numerical results for the RACooN lab** Table 4.1 contains the maximum achievable averaging gains that can be obtained when the training sequence length is \( N \) samples for different superframe sizes.

4.3.2.2 With Frequency Offset

When the transmitter and the receiver have a frequency offset \( f_{\text{off}} = \frac{\omega_{\text{off}}}{2\pi} \), the received signal is affected by a time-linear deterministic phase shift. Equation (4.14) becomes
∀k ∈ [0, N − 1], ∀l ∈ [0, M − 1],

\[ y_l[k] = z_l[k] + n_l[k] \]  

(4.16)

with

\[ z_l[k] = z[k] e^{j\omega_{\text{off}}(lN + k)T_s}. \]  

(4.17)

Compared with (4.14), \( z[k] \) is replaced by \( z_l[k] \) that depends on the pattern index \( l \). We set \( k = k_0 \). After averaging

\[
y[k_0] = \frac{1}{M} \sum_{l=0}^{M-1} y_l[k_0] 
\]

(4.18)

\[
= z[k_0] \frac{1}{M} e^{j\omega_{\text{off}}k_0T_s} \sum_{l=0}^{M-1} e^{j\omega_{\text{off}}lNT_s} + \frac{1}{M} \sum_{l=0}^{M-1} n_l[k_0] 
\]

(4.19)

\[
= z[k_0] e^{j\omega_{\text{off}}(k_0 + \frac{N}{2}(M-1))T_s} \frac{\sin c(\omega_{\text{off}}\frac{N}{2}MT_s)}{\sin c(\omega_{\text{off}}\frac{N}{2}T_s)} + \frac{1}{M} \sum_{l=0}^{M-1} n_l[k_0] 
\]

(4.20)

The SNR gain offered by averaging when the system suffers from frequency offset is

\[
10 \log_{10} \left( M \left| \frac{\sin c(\omega_{\text{off}}\frac{N}{2}MT_s)}{\sin c(\omega_{\text{off}}\frac{N}{2}T_s)} \right|^2 \right). 
\]

Therefore frequency offset degrades the averaging gain by

\[
20 \log_{10} \left( \left| \frac{\sin c(\omega_{\text{off}}\frac{N}{2}T_s)}{\sin c(\omega_{\text{off}}\frac{N}{2}MT_s)} \right| \right) \]  

when compared to the case with no frequency offset.

We discuss in the following the averaging gain term for a system with frequency offset.

If \( \forall k_1 \in \mathbb{N} \setminus \{0\} \),

\[
NT_s \neq \frac{k_1}{f_{\text{off}}}, 
\]

(4.21)
an averaging gain drop is observed when $\exists k_2 \in \mathbb{N} \setminus \{0\}$ such that

$$MNT_s = \frac{k_2}{f_{\text{off}}}. \quad (4.22)$$

In that case the averaging operation destroys the signal and no channel estimate can be calculated.

If $\exists k_1 \in \mathbb{N} \setminus \{0\}$ such that

$$NT_s = \frac{k_1}{f_{\text{off}}}, \quad (4.23)$$

then (4.24) holds and the averaging gain becomes $10 \log_{10} \left( \frac{1}{M} \right)$, which is negative. The averaging operation deteriorates the SNR and makes the channel estimate noisy.

$$\left| \frac{\text{sinc}(\omega_{\text{off}} NMT_s)}{\text{sinc}(\omega_{\text{off}} N T_s)} \right| \sim \frac{1}{M} \quad (4.24)$$

This gain is independent of the type of sounding sequence considered (chirp or direct sequence). The product $MN$ or the duration $MNT_s$ controls the sensitivity of the estimation procedure to frequency offset. When $\omega_{\text{off}} N MT_s = k\pi, \; k \in \mathbb{N} \setminus \{0\}$, the channel estimation is impossible. When $\omega_{\text{off}} N T_s = k\pi, \; k \in \mathbb{N} \setminus \{0\}$, no averaging should be performed.

**Numerical results for the RACooN lab** The SNR gain when the system is subject to frequency offset is simulated and plotted in Fig. 4.9 for $M$ up to 10,000 using the frequency offset value measured in 3.4.2, i.e. $f_{\text{off}} = 2.3$ Hz. For $N = 7$ up to 1023, the gain degradation is negligible for $M$ below 10,000. For longer sequences ($N = 2047$ and above) the frequency offset effect appears earlier and causes drastic gain drops when the term $\text{sinc}(\omega_{\text{off}} NMT_s)$ becomes null. For the RACooN lab, the maximum duration for a training signal is reached when it occupies the full Random-Access Memory (RAM), and its duration is then $B \cdot S \cdot T_s = 0.0262144$ s. The lowest frequency offset that would lead to an averaging gain drop is with these numerical values at $f_{\text{off}} = 38$ Hz, which is much larger than the frequency offset value $f_{\text{off}}$ estimated in section 3.4.2.

**4.3.2.3 With Phase Noise**

The phase noise is modeled by the addition at the receiver of a random phase offset at each sample. The phase noise terms are sampled values of a 0-mean Gaussian process with
Fig. 4.9: Averaging gain with frequency offset ($T_s = 12.5$ ns, $f_{off} = 2.3$ Hz)

variance $\sigma_0^2$ (section 3.4.3.1). The $(k+1)^{th}$ received sample in the $(l+1)^{th}$ pattern, where $0 \leq k \leq N - 1$ and $0 \leq l \leq M - 1$, is multiplied in time domain by a complex exponential $e^{j(\phi_l[k]+\phi_0)}$ where $\phi_l[k]$ is a sampled value of the phase noise process described above and $\phi_0$ is a real constant in $[-\pi, \pi]$ (constant over a mission, but uniformly distributed in $[-\pi, \pi]$ over different missions) that represents the phase offset between the local oscillator of the transmitter and the receiver at the mission start.

Equation (4.14) becomes

$$\forall k \in [0, N - 1],$$

$$\bar{y}_l[k] = z_l[k] + n_l[k]$$  \hspace{1cm} (4.25)

with

$$z_l[k] = z[k]e^{j(\phi_l[k]+\phi_0)}.$$  \hspace{1cm} (4.26)
4.3 Gain Degradation due to Hardware Imperfections

The noise remains statistically unaffected by phase noise due to its circular symmetry property. We set $k = k_0$ and average over the $M$ realizations.

$$y[k_0] = \frac{1}{M} \sum_{l=0}^{M-1} y[l]$$

$$= z[k_0] e^{j\phi_0} \frac{1}{M} \sum_{l=0}^{M-1} e^{j\phi_l[k_0]} + \frac{1}{M} \sum_{l=0}^{M-1} n_l[k_0] \quad (4.27)$$

For small phase noise (assumption justified in 3.3) and $M \geq 2$, $e^{j\phi_l[k_0]}$ is approximated as $1 + j\phi_l[k_0]$, thus

$$\frac{1}{M} \sum_{l=0}^{M-1} e^{j\phi_l[k_0]} \approx 1 + j \frac{1}{M} \sum_{l=0}^{M-1} \phi_l[k_0] \quad (4.28)$$

The variance of the term in (4.28) is $1 + \frac{\sigma^2}{M}$ if the phase noise samples $\{\phi_l[k_0]\}_{l=0,...,M-1}$ are independent.

The averaging gain, defined as the SNR gain of the system affected by phase noise before and after averaging is $10 \log_{10} \left( M + \sigma^2_\phi \right)$, $M \geq 2$, when the phase noise samples are independent.

**Numerical results for the RACooN lab**

The averaging gain with respect to the case where the signal is affected by phase noise but not averaged is given is Fig. 4.10 for independent phase noise samples. For $\sigma^2_\phi = 8.4 \cdot 10^{-3}$ the SNR gain is nearly identical to the case with no phase noise, therefore the phase noise effect on the averaging gain can be neglected in practice. The averaging gain increases with the phase noise variance. But the averaging gains of systems with different phase noise variances cannot be compared since these gains describe the behavior of two different systems. Without averaging, the signal SNR with or without phase noise is the same. Therefore the averaging gain is 0 for $M=1$. When $M$ increases, the signal subject to large phase noise variance has a poorer SNR than a second system with a lower phase noise variance, but it will benefit more from averaging than the second system, therefore the averaging gain appears larger in systems with large phase noise variance.
4.3.3 Correlation

In this section we discuss the correlation gain and MSR degradation of a system with imperfect hardware. Our systematic approach to structure the paragraph is summarized in Fig. 4.11. We summarize in 4.3.3.1 the correlation gain in the ideal case. In 4.3.2.2 we calculate the correlation gain and give an expression for the MSR in a system with frequency offset. Then in 4.3.2.3 we analyze the correlation gain and MSR when phase noise affects the system. We illustrate these theoretical results using experimental data from the RACooN lab.

Non-linearities ruin the training sequence autocorrelation properties. Analytical results require an exact model for the non-linearities, and the design of such a model is out of the scope of this work. We dedicate section 4.4 to robust channel sounding methods that do not require the estimation of the non-linearity model parameters and focus in this section on frequency offset and phase noise.

4.3.3.1 Ideal Case

After averaging, a $N$-sample sequence with samples indexed by $k$, $0 \leq k \leq N - 1$, is given by

$$y[k] = z[k] + n'[k]$$  (4.29)
4.3 Gain Degradation due to Hardware Imperfections

Correlation gain

**Fig. 4.11:** Structure of section 4.3.3

where \( n'[k] = \frac{1}{M} \sum_{l=0}^{M-1} n_l[k] \) is a zero-mean AWGN sample and \( n_l[k] \) has been introduced in (4.14). The previous averaging operation has not changed the noise distribution but only its variance.

The cyclic correlation between the sequences \( y[\cdot] \) and \( x[\cdot] \) is computed, where \( x[\cdot] \) is either a direct sequence or a chirp sequence as described by equation (4.13). In practice this is achieved by performing the linear correlation with the periodic sequence \( s[\cdot] \). The crosscorrelation was computed in section 4.2.2 and 4.2.3. This correlation operation improves the dynamic range by a factor \( 10 \log_{10} (N) \), which is the correlation gain in the system without imperfections.

4.3.3.2 With Frequency Offset

Frequency offset effect is described in [112] for a single-input single-output (SISO) channel sounder. The authors stress two detrimental effects: a reduction of the cyclic cross-correlation function peak amplitude and the MSR. We will in the sequel quantify these effects. When frequency offset is considered (at the exclusion of any other imperfection)

\[ \forall k \in [0, N - 1], \]

\[ y[k] = z''[k] + n'[k] = z'[k]e^{j\omega_{\text{off}} kT_s} + n'[k], \quad (4.30) \]
where now

\[ z''[k] = z[k] e^{j\omega \text{off}\frac{N}{2}(M-1)T_s} \frac{\text{sinc}(\frac{N}{2}M\omega \text{off} T_s)}{\text{sinc}(\frac{N}{2}\omega \text{off} T_s)} e^{j\omega \text{off} kT_s}, \]  

(4.31)

\[ z'[k] = z[k] e^{j\omega \text{off}\frac{N}{2}(M-1)T_s} \frac{\text{sinc}(\frac{N}{2}M\omega \text{off} T_s)}{\text{sinc}(\frac{N}{2}\omega \text{off} T_s)} . \]  

(4.32)

and

\[ n'[k] = \frac{1}{N} \sum_{l=0}^{N-1} n_l[k] . \]  

(4.33)

The sequence \( x'[\cdot] \) is defined such that \( \forall k \in [0, N - 1] \)

\[ x'[k] = x[k] e^{-j\omega \text{off} kT_s} , \]  

(4.34)

where \( x[\cdot] \) is defined in section 4.2.5.

The cyclic cross-correlation with \( x[\cdot] \) is computed

\[ (y \otimes x)[k] = (z'' \otimes x)[k] + (n' \otimes x)[k] \]  

(4.35)

\[ \theta_{y,x}[k] = (z' \otimes x')[k] + (n' \otimes x)[k] \]  

(4.36)

\[ \theta_{y,x}[k] = e^{j\omega \text{off}\frac{N}{2}(M-1)T_s} \frac{\text{sinc}(\frac{N}{2}M\omega \text{off} T_s)}{\text{sinc}(\frac{N}{2}\omega \text{off} T_s)} ((h \otimes x) \otimes x')[k] + (n' \otimes x)[k] \]  

(4.37)

\[ \theta_{y,x}[k] = e^{j\omega \text{off}\frac{N}{2}(M-1)T_s} \frac{\text{sinc}(\frac{N}{2}M\omega \text{off} T_s)}{\text{sinc}(\frac{N}{2}\omega \text{off} T_s)} (h \otimes (x \otimes x'))[k] + (n' \otimes x)[k] \]  

(4.38)

\[ \theta_{y,x}[k] = e^{j\omega \text{off}\frac{N}{2}(M-1)T_s} \frac{\text{sinc}(\frac{N}{2}M\omega \text{off} T_s)}{\text{sinc}(\frac{N}{2}\omega \text{off} T_s)} (h \otimes \theta_{x,x'})[k] + \theta_{n',x}[k] \]  

(4.39)

The frequency offset modifies the autocorrelation function \( \theta_{x,x'}[\cdot] \) in (4.13) by \( \theta_{x,x'}[\cdot] \). \( \theta_{n',x}[\cdot] \) has statistically the same properties as \( \theta_{n,x}[\cdot] \) because of the circular symmetry property of the noise.

We calculate now this cross-correlation term in the case of a direct sequence.

**m-sequence** The correlation gain as defined in section 4.2.5.2 can be analytically computed:

\[ \quad \]
4.3 Gain Degradation due to Hardware Imperfections

\[
\theta_{x,x'}[0] = \sum_{k=0}^{N-1} e^{j\omega_{\text{off}} k T_s} \quad (4.40)
\]

\[
= 1 - e^{j\omega_{\text{off}} NT_s} \quad (4.41)
\]

\[
= \frac{e^{j\omega_{\text{off}} \frac{N}{2} T_s} \sin(\frac{N}{2} \omega_{\text{off}} T_s)}{e^{j\omega_{\text{off}} \frac{1}{2} T_s} \sin(\frac{1}{2} \omega_{\text{off}} T_s)} \quad (4.42)
\]

\[
= N e^{j\omega_{\text{off}} \frac{N-1}{2} T_s} \frac{\sin(\frac{N}{2} \omega_{\text{off}} T_s)}{\sin(\frac{1}{2} \omega_{\text{off}} T_s)} \quad (4.43)
\]

The correlation gain is 10 \log_{10} \left( N \left| \frac{\sin(\frac{N}{2} \omega_{\text{off}} T_s)}{\sin(\frac{1}{2} \omega_{\text{off}} T_s)} \right|^2 \right) and shows a degradation of 20 \log_{10} \left( \frac{\sin(\frac{1}{2} \omega_{\text{off}} T_s)}{\sin(\frac{1}{2} \omega_{\text{off}} T_s)} \right) \geq 0 with respect to the ideal case with no frequency offset. We discuss now the correlation gain.

If \( \forall k_1 \in \mathbb{N} \setminus \{0\} \), \( T_s \neq \frac{k_1}{f_{\text{off}}} \), there is a correlation gain drop when

\[
\exists k_2 \in \mathbb{N} \setminus \{0\} \text{ such that } NT_s = \frac{k_2}{f_{\text{off}}} \quad (4.45)
\]

This condition is the same as (4.23), and no channel estimation is possible when (4.23) is fulfilled.

If \( \exists k_1 \in \mathbb{N} \setminus \{0\} \) such that \( T_s = \frac{k_1}{f_{\text{off}}} \), then the correlation gain is 10 \log_{10} \left( \frac{1}{N} \right) < 0 and the correlation gain degrades with the training sequence length. When this condition is fulfilled, (4.23) is also fulfilled, and both averaging and correlation degrade the signal SNR. An analytical expression for the MSR is not straightforward because there is no simple expression for \( x[k] \) valid for all \( N \) and all \( k \).

**Numerical results for the RACooN lab** The correlation gain is plotted in Fig. 4.12. For frequency offset up to 100 Hz, the \text{sinc} ratio is nearly 1 and the correlation gain is nearly unchanged by \( f_{\text{off}} \) and only depends on \( m \). Using Fig. 4.13 (dotted lines), the MSR degradation is extremely small and frequency offset can be neglected since the curves are nearly flat when \( f_{\text{off}} \) varies from 0 to 100 Hz (a typical value for \( f_{\text{off}} \) is 2.3 Hz). The first correlation gain drop would occur \( (N = 8191) \) with \( f_{\text{off}} = 9.767 \) kHz.
The instantaneous phase of the sample $x[k]$, $0 \leq k \leq N - 1$ is denoted $\phi_x[k]$ such that

$$\phi_x[k] = \phi_0 + \pi \left( \frac{k^2}{N} - k \right)$$

and the instantaneous phase of $x'[k]$ is $\phi_{x'}[k]$ such that

$$\phi_{x'}[k] = \phi'_0 + \pi \left( \frac{k^2}{N} - k \right) - \omega_{off}kT_s$$

The sequence $x'[\cdot]$ used for cyclic cross-correlation is therefore $
\forall k \in [0, N - 1],$

$$x'[k] = e^{j(\phi'_0 + \pi \left( \frac{k^2}{N} - k \right) - \omega_{off}kT_s)} \quad (4.46)$$

$$x[k] = e^{j(\phi_0 + \pi \left( \frac{k^2}{N} - k \right))} \quad (4.47)$$
4.3 Gain Degradation due to Hardware Imperfections

The cyclic cross-correlation function has value \( \theta_{x,x'}[0] \) such that

\[
\theta_{x,x'}[0] = \sum_{k=0}^{N-1} e^{j\phi_x[k]} e^{-j\phi_{x'}[k]}
\]

(4.48)

\[
= e^{j(\phi_0 - \phi_0')} \sum_{k=0}^{N-1} e^{j\omega_{\text{off}}kT_s}
\]

(4.49)

\[
= e^{j(\phi_0 - \phi_0')} \frac{1 - e^{j\omega_{\text{off}}NT_s}}{1 - e^{j\omega_{\text{off}}T_s}}
\]

(4.50)

\[
= e^{j(\phi_0 - \phi_0')} e^{\frac{jN\omega_{\text{off}}T_s}{2}} \sin \left( \frac{N\omega_{\text{off}}T_s}{2} \right) \sin \left( \frac{1}{2} \omega_{\text{off}}T_s \right)
\]

(4.51)

\[
= e^{j(\phi_0 - \phi_0')} e^{\frac{jN\omega_{\text{off}}T_s}{2}} \frac{1}{2} \omega_{\text{off}} T_s \sin \left( \frac{N\omega_{\text{off}}T_s}{2} \right) \sin \left( \frac{1}{2} \omega_{\text{off}}T_s \right)
\]

(4.52)

\[
\theta_{x,x'}[p] = \sum_{k=0}^{N-1-p} e^{j\phi_x[k]} e^{-j\phi_{x'}[k+p]} + \sum_{k=N-p}^{N-1} e^{j\phi_x[k]} e^{-j\phi_{x'}[k+p-N]}
\]

(4.53)

The result is the same as (4.44) with a constant phase shift of \( \phi_0 - \phi_0' \) that does not affect the gain expression. The same remarks hold as for the previous paragraph dealing with direct sequences.

In order to calculate the MSR, the cyclic cross-correlation for any delay \( p \) can be calculated

\[
\theta_{x,x'}[p] = \sum_{k=0}^{N-1-p} e^{j\phi_x[k]} e^{-j\phi_{x'}[k+p]} + \sum_{k=N-p}^{N-1} e^{j\phi_x[k]} e^{-j\phi_{x'}[k+p-N]}
\]

(4.54)

We have for \( 0 \leq k \leq (N - 1 - p) \),

\[
\phi_x[k] - \phi_{x'}[k+p] = \pi \left( \frac{k^2}{N} - k \right) - \pi \left( \frac{(k + p)^2}{N} - (k + p) \right) + \omega_{\text{off}}(k + p)T_s
\]

(4.55)

\[
= -p\pi \left( \frac{D}{N} - 1 \right) - 2p\pi \frac{k}{N} + \omega_{\text{off}}(k + p)T_s
\]

(4.56)
and, for $N - p \leq k \leq N - 1$,

$$\phi_x[k] - \phi_x'[k + p - N] = \pi\left(\frac{k^2}{N} - k\right) - \pi\left(\frac{(k + p - N)^2}{N} - (k + p - N)\right) + \omega_{\text{off}}(k + p - N)T_s$$

$$= -2k\pi\left(\frac{p}{N} - 1\right) - \pi(p - N)(\frac{p}{N} - 2) + \omega_{\text{off}}(k + p - N)T_s$$

$$= -2k\pi\left(\frac{p}{N} - 1\right) - p\pi\left(\frac{p}{N} - 1\right) + 2\pi(p - N) + \omega_{\text{off}}(k + p - N)T_s$$

Therefore

$$\theta_{x,x'}[p]$$

$$= e^{-j\pi\left(\frac{p}{N} - 1\right) - \omega_{\text{off}}(p)T_s} \sum_{k=0}^{N-1-p} e^{-j(2\pi\frac{k}{N} - \omega_{\text{off}}k)T_s}$$

$$+ e^{-j\pi\left(\frac{p}{N} - 1\right) - \omega_{\text{off}}(p-N)T_s} \sum_{k=N-p}^{N-1} e^{-j(2\pi\frac{k}{N} - \omega_{\text{off}}k)T_s}$$

$$= e^{-j\pi\left(\frac{p}{N} - 1\right) - \omega_{\text{off}}(p)T_s} \left(1 - e^{-j(N-p)(2\pi\frac{p}{N} - \omega_{\text{off}}T_s)}\right)$$

$$e^{j\pi\left(\frac{p}{N} - 1\right) - \omega_{\text{off}}(p-N)T_s} \left(1 - e^{-j(2\pi\frac{p}{N} - \omega_{\text{off}}T_s)}\right)$$

$$= e^{-j\pi\left(\frac{p}{N} - 1\right) - \omega_{\text{off}}(p)T_s} \left(1 - e^{-j(2\pi\frac{p}{N} - \omega_{\text{off}}T_s)}\right)$$

$$+ e^{j\pi\left(\frac{p}{N} - 1\right) - \omega_{\text{off}}(p-N)T_s} \left(1 - e^{-j(2\pi\frac{p}{N} - \omega_{\text{off}}T_s)}\right)$$

We resume the analysis based on Fig. 4.13 that plots the MSR using equation (4.53) and (4.60). The MSR is given by

$$\text{MSR} = \frac{||\theta_{x,x'}[0]||}{\max_{p=1,...,N-1} ||\theta_{x,x'}[p]||}.$$
than the direct sequence of the same length. But for a given frequency offset, the chirp MSR remains above the direct sequence MSR, at least for $f_{\text{off}} \leq 100$ Hz.

### 4.3.3.3 With Phase Noise

Phase noise affects the autocorrelation function properties.

$$y[k] = z[k]e^{j\phi'[k]} + n'[k].$$  \hspace{1cm} (4.62)

In this section we redefine $x'[\cdot]$ in (4.34) as (we take $\phi_0 = 0$):

$$\forall k \in [0, N - 1], \ x'[k] = x[k]e^{-j\phi'[k]},$$  \hspace{1cm} (4.63)

where $\phi'[k]$ is introduced in (4.28).

**Direct sequence**  The correlation gain is given by $\theta_{x,x'}[0]$, where $\theta_{x,x'}[\cdot]$ denotes the cyclic correlation between the two sequences $x$ and $x'$:

$$\theta_{x,x'}[0] = \sum_{k=0}^{N-1} e^{j\phi'[k]} \approx N + j \sum_{k=0}^{N-1} \phi'[k] \approx Ne^{j\phi''}$$  \hspace{1cm} (4.64)
The correlation gain is a random variable that tends to the ideal processing gain \(10 \log_{10} (N)\) for large \(N\) because the \(\{\phi[k]\}_{0 \leq k \leq N-1}\) are 0-mean.

The MSR is numerically investigated. The average MSR over 1,000 iterations is simulated with Matlab using a zero-mean Gaussian phase noise with variance between \(10^{-5}\) and 1. The results are plotted in Fig. 4.14 (dotted curves) and are commented in the next paragraph.

**Fig. 4.14: MSR with phase noise**

**Chirp sequence** The correlation gain achieved when chirp sequences are used is the same as for the direct sequence and is used to compute the MSR

\[
\theta_{x,x'}[p] = e^{-\jmath(p\pi(\frac{k}{N}-1))} \sum_{k=0}^{N-1-p} e^{-\jmath(2p\pi \frac{k}{N} + \phi'[k+p])} + e^{-\jmath(p\pi(\frac{k}{N}-1))} \sum_{k=N-p}^{N-1} e^{-\jmath(2p\pi \frac{k}{N} + \phi'[k+p-N])}
\]

The average MSR is plotted in Fig. 4.14. For small phase noise variance, the chirp MSR tends to infinity, and the direct sequence MSR curve tends to \(10 \log_{10} (N)\) that is the maximum value. The chirp MSR decreases linearly in dB with the increasing phase noise variance expressed in dB. It tends to the direct sequence MSR for large phase noise variance while still being larger than it for the same \(N\). Assuming that the MSR is a measure of the inter-symbol interference (ISI) introduced by the correlator, using a chirp sequence leads to a
4.4 Robust Channel Estimation

We have seen in the previous section that a careful choice of parameters $M$ and $N$ makes the channel estimate robust to frequency offset and phase noise. The effect of the non-linearities is discussed in this section. The case of the direct sequence and the chirp sequence is separately treated since [90] notes that the non-linear effects on the channel estimate strongly depend on the training sequence, and are difficult to predict without an exact model for the non-linearities and therefore we resort to experimental data.

We motivate the importance of the non-linearities in the channel estimation process by showing experimental data from the RACooN lab in section 4.4.1. We summarize in section 4.4.2 the origin of the non-linearities. Then in section 4.4.3 we propose a robust scheme based on direct sequence transmission. The obtained results are experimentally compared with the performance of a scheme based on a chirp sequence. We also give in this section new insights for the choice of the training sequence.

All through this section we denote $x[:]=\alpha x_0[:], \text{ where } x_0[:]$ is a direct sequence or a chirp sequence such that $\max_{k=0,...,N-1}|x[k]| = 1, \forall k \in [0, N-1], |x_0[k]| = 1$, and assume a memoryless model for the non-linearities.

4.4.1 Observations and Motivation

Without further signal processing, experimental channel estimates obtained after the steps described in 4.3.1 are dependent on the transmitted sequence level. The estimated CIR obtained for $\alpha = 0.2$ and $\alpha = 0.8$ is shown in Fig. 4.15(a) and Fig. 4.15(b) when the training
sequence is a direct sequence. A delay of 30 samples, i.e. $30T_s = 0.375 \, \mu s$ is introduced for clarity of the picture. This delay shifts the CIR by this duration in the plots. The difference between the two figures is attributed to non-linearities. In Fig. 4.15(b) one can clearly see the effect: the tail of the CIR estimated with $\alpha = 0.8$ presents magnitude peaks at $1.7 \, \mu s$ and $1.95 \, \mu s$ that are not present for $\alpha = 0.2$ in Fig. 4.15(a).

In frequency domain, the transfer function (power spectrum and phase) is plotted for $\alpha = 0.2$ in Fig. 4.16(a) and for $\alpha = 0.8$ in Fig. 4.16(b). The non-linearities affect the channel power spectrum by creating oscillations in the bandwidth. The channel transfer function phase is linear and visibly unaffected by the non-linearities. Its slope is $0.375 \, \mu s$, this is the group delay observed in Fig. 4.15. The DC point is omitted from the representation since not estimated.

### 4.4.2 Origin of the Non-Linearities

**Non-linearities from I-Q imbalance.** I-Q imbalance affects the channel estimation by adding at the receiver for any transmitted single-tone signal at $f_{\text{test}}$ an attenuated (with respect to the transmitted signal) single-tone signal at $-f_{\text{test}}$. The attenuation is independent of $f_{\text{test}}$ but depends on the amplitude of the signal. When a training sequence with flat power spectrum is transmitted, I-Q imbalance at the transmitter keeps the power spectrum flat and only I-Q imbalance at the receiver matters.

**Other non-linearities** We consider here non-linearities not generated by I-Q imbalance. We discussed previously (section 3.4.4.1) the origin of these non-linearities and located their origin in the transmitter and receiver baseband circuits including analog-to-digital converter (ADC) and digital-to-analog converter (DAC). When a direct sequence as sounding sequence is used, the transmitter baseband non-linearities do not introduce additional frequency components. We motivate this by considering a polynomial model for the baseband non-linearities as in 3.4.4. The even order non-linearities increase the transmit sequence mean value since the direct sequence are real constant amplitude sequences. The odd order non-linearities result in a scaling of the transmit direct sequence that can be removed by calibration. Therefore the transmitter baseband non-linearities do not affect the MSR. Only non-linearities at the receiver baseband matter. Chirp sequences are sensitive to transmitter non-linearities because they are complex sequences.
Fig. 4.15: Estimated CIR with direct sequence
Fig. 4.16: Estimated channel transfer function with direct sequence
4.4.3 Channel Estimation with a Direct Sequence Robust to Non-Linearities

Non-linear channel estimation has already been considered in the literature. System theory tackled the problem in [17] for a linear system in cascade with a zero-memory non-linear element followed by a second linear system. A direct sequence is transmitted and the correlation function at the output is computed between the received signal and a multi-level transmit direct sequence. The output of such a system can be decomposed as a functional series where the functionals of the decomposition are expressed in terms of Volterra Kernels [109]. The basis functions are retrieved by a multilevel transmission approach: a direct sequence with different amplitudes is transmitted. The received signal is correlated with the unscaled transmitted direct sequence. A linear system is built, where each correlation function is expressed as a weighted sum of the same functionals. The system is finally inverted to obtain the basis functions. The first basis function represents the correlation of the output of the linear elements of the system with the transmit direct sequence. This principle is applied in [37] and [138] for the estimation of wireless channels, with the non-linear element at the transmitter [138] or at the receiver [37]. But in all these works the estimation method is assessed using Matlab simulations and not with realistic systems. In the sequel we extend the method from [37] to a transmission system with I- and Q-branch, and apply the proposed method to compute the calibration functions. We also discuss extensively the relationship between the number of levels and the non-linearity order.

We consider the equivalent system of Fig. 4.17. We define the equivalent baseband channel $h_{eq}$ such that

$$h_{eq} = h_{RX} \ast h \ast h_{TX}. \quad (4.66)$$

The real baseband equivalent channel $\mathcal{R} \{h_{eq}\}$ is denoted $h_{eq,r}$ and the imaginary baseband equivalent channel $\mathcal{I} \{h_{eq}\}$ is $h_{eq,i}$, where $h_{RX}$ is the equivalent baseband receiver filter, $h_{TX}$ is the equivalent baseband transmitter filter and $h$ is the equivalent wireless baseband channel. A complex AWGN signal sample $n[k] = \mathcal{R} \{n[k]\} + j \mathcal{I} \{n[k]\}$ with $\mathcal{R} \{n[k]\}$ and $\mathcal{I} \{n[k]\}$ independent identically distributed is added to the received sample.

The baseband non-linearities at the receiver are modeled at each branch by a polynomial $P_1(\cdot)$ of order $L_1$ and $P_Q(\cdot)$ of order $L_Q$ such that:

$$P_1(x) = \sum_{p=1}^{L_1} c_{1,p} x^p \quad (4.67)$$
Fig. 4.17: Non-linear equivalent system

and:

\[ P_Q(x) = \sum_{p=1}^{L_Q} c_{Q,p} x^p. \]  (4.68)

Since the I- and Q-branch are built on the same printed board with the same architecture, we assume furthermore that \( L_I = L_Q = L_{NL} \) and, \( \forall p \in [1, L_{NL}] \)

\[ c_{I,p} = c_{Q,p} = c_p, \]  (4.69)

that is

\[ P_I(x) = P_Q(x) = P(x). \]  (4.70)

The model does not include the DC because it is not time-constant, therefore the sums above start at \( p = 1 \).

We split the received signal into its real and imaginary components \( y_I[k] \) and \( y_Q[k] \). Thus we can estimate \( h_{eq,r} \) and \( h_{eq,i} \) after transmitting \( x[\cdot] \) on each branch.

\[ y_I[k] = P ((\mathcal{R} \{h_{RX} \ast h \ast h_{TX}\} \circ x[k]) + \mathcal{R} \{n[k]\}) \]  (4.71)
\[ y_Q[k] = P ((h_{eq,r} \circ x)[k]) + \mathcal{R} \{n[k]\} \]  (4.72)

and

\[ y_Q[k] = P ((\mathcal{I} \{h_{RX} \ast h \ast h_{TX}\} \circ x)[k]) + \mathcal{I} \{n[k]\} \]  (4.73)
\[ y_Q[k] = P ((h_{eq,i} \circ x)[k]) + \mathcal{I} \{n[k]\} \]  (4.74)
where \( n[k] \) is the equivalent complex baseband noise at the receiver side and \( h_{\text{eq}} \) represents the equivalent linear channel from transmitter to receiver.

\[
P ((h_{\text{eq},x} \odot x)[k]) \text{ can also be expressed as}
\]

\[
P ((h_{\text{eq},x} \odot x)[k]) = c_1 \sum_{p_1=0}^{N-1} h_{\text{eq},1}[p_1] x[k-p_1] + c_2 \left( \sum_{p_1=0}^{N-1} h_{\text{eq},1}[p_1] x[k-p_1] \right) \left( \sum_{p_2=0}^{N-1} h_{\text{eq},1}[p_2] x[k-p_2] \right) + \ldots
\]

\[
+ c_{L_{\text{NL}}} \left( \sum_{p_1=0}^{N-1} h_{\text{eq},1}[p_1] x[k-p_1] \right) \ldots \left( \sum_{p_{L_{\text{NL}}}=0}^{N-1} h_{\text{eq},1}[p_{L_{\text{NL}}}] x[k-p_{L_{\text{NL}}}] \right)
\]

\[
= \sum_{q=1}^{L_{\text{NL}}} w_{L,q}[k]
\]

with \( \forall k \in [0, N-1], \)

\[
w_{L,q}[k] = c_q \sum_{p_1=0}^{N-1} \ldots \sum_{p_q=0}^{N-1} \prod_{j=1}^{q} (h_{\text{eq},1}[p_j] x[k-p_j])
\]

\[
= c_q ((h_{\text{eq},x} \odot x)[k])^q
\]

At this point we can notice that \( w_{L,1}[:] \) is proportional to the convolution of the noiseless linear system channel \( h_{\text{eq},1}[:] \) with \( x[:] \).

Similarly we express

\[
P ((h_{\text{eq},i} \odot x)[k]) = \sum_{q=1}^{L_{\text{NL}}} w_{Q,q}[k]
\]

with \( \forall k \in [0, N-1], \)

\[
w_{Q,q}[k] = c_q \sum_{p_1=0}^{N-1} \ldots \sum_{p_q=0}^{N-1} \prod_{j=1}^{q} (h_{\text{eq},1}[p_j] x[k-p_j])
\]

\[
= c_q ((h_{\text{eq},i} \odot x)[k])^q
\]

The cyclic cross-correlation between the received sequence \( y_i[:] \) and \( y_Q[:] \), and the transmit
sequence $x[\cdot]$ is computed.

\[ R_{y,t,x}[k] = \sum_{p=1}^{L_{NL}} R_{w,I,p,x}[k] \] (4.81)

\[ R_{y,q,x}[k] = \sum_{p=1}^{L_{NL}} R_{w,Q,p,x}[k] \] (4.82)

We need to estimate $R_{w,I,p,x}[k]$ and $R_{w,Q,p,x}[k]$ for $p = 1$ since

\[ R_{w,I,1,x}[\cdot] = c_1((h_{eq,r} \odot x) \otimes x)[\cdot] \] (4.83)

\[ = c_1(h_{eq,r} \otimes R_x)[\cdot] \] (4.84)

\[ R_{w,Q,1,x}[\cdot] = c_1((h_{eq,i} \odot x) \otimes x)[\cdot] \] (4.85)

\[ = c_1(h_{eq,i} \otimes R_x)[\cdot]. \] (4.86)

Retrieving $R_{w,I,1,x}[\cdot]$ and $R_{w,Q,1,x}[\cdot]$ is achieved by multi-level transmission. Multilevel input signals $\alpha_i x[\cdot]$ are transmitted, where $\forall i \neq j, \alpha_i \neq \alpha_j$ and $i = 1, \ldots, T$. The corresponding received sequence is then denoted $y_I(\alpha_i)$ and $y_Q(\alpha_i)$. Then $\forall i = 1, \ldots, T$,

\[ R_{y(I),x}[\cdot] = \sum_{p=1}^{L_{NL}} \alpha_i^p R_{w,I,p,x}[\cdot] \] (4.87)

\[ R_{y(Q),x}[\cdot] = \sum_{p=1}^{L_{NL}} \alpha_i^p R_{w,Q,p,x}[\cdot] \] (4.88)

It is possible to use another set of coefficients $\{\beta_i\}_{i=1,\ldots,T}$ fulfilling the same conditions as the elements $\{\alpha_i\}_{i=1,\ldots,T}$ to calculate the $\{R_{y(Q)(\beta_i),x}[\cdot]\}$. Choosing $\alpha_i = \beta_i \forall i \in [1, T]$ does not restrict the generality of the algorithm while allowing a compact notation. The equations can be written in a matrix form.
\[ \forall k \in [0, N-1], \]
\[
\begin{bmatrix}
R_y(\alpha_1),x[k] & R_q(\alpha_1),x[k] \\
R_y(\alpha_2),x[k] & R_q(\alpha_2),x[k] \\
\vdots \\
R_y(\alpha_T),x[k] & R_q(\alpha_T),x[k]
\end{bmatrix} =
\begin{bmatrix}
\alpha_1 & \alpha_1^2 & \ldots & \alpha_1^{L_N} \\
\alpha_2 & \alpha_2^2 & \ldots & \alpha_2^{L_N} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_T & \alpha_T^2 & \ldots & \alpha_T^{L_N}
\end{bmatrix}
\begin{bmatrix}
R_{w1,1},x[k] \\
R_{w1,2},x[k] \\
\vdots \\
R_{w1,L_N},x[k] \\
R_{wQ,1},x[k] \\
R_{wQ,2},x[k] \\
\vdots \\
R_{wQ,L_N},x[k]
\end{bmatrix}
\]
\[
R_y[k] = VR_w[k]
\] (4.89)

\( R_y[k] \) is a \( T \times 2 \) matrix, \( R_w[k] \) is a \( L_{NL} \times 2 \) matrix and \( V \) is a \( T \times L_{NL} \) matrix. When matrix \( V \) is square, it can be decomposed as the product of a diagonal matrix with non-zero elements and a Vandermonde matrix that has a non-zero determinant as long as \( \alpha_i \neq \alpha_j \) \( \forall i \neq j \) and \( \alpha_i \neq 0 \). Thus \( A \) is invertible. We discuss now the size of \( V \)

**\( L_{NL} \) known**
- If \( T = L_{NL} \), there is a unique solution \( R_w[k] \) to the system of equations,
- If \( T > L_{NL} \), the system is overdetermined. The rank of \( V \) is denoted \( \text{rank}(V) \) and is \( L \) because it is a Vandermonde matrix. There is a unique solution for \( R_w \) or no solution.
- If \( T < L_{NL} \), the system is underdetermined. \( \text{rank}(V) = T \) and there are infinitely many solutions for \( R_w \).

**\( L_{NL} \) unknown** In practical cases, the non-linearity order is not known. We give in the following an algorithm for its estimation. The non-linearity order can be estimated by using the case where \( V \) is square. The algorithm is run for increasing \( T = L_{hyp} \), i.e. by making an hypothesis \( L_{hyp} \) on \( L_{NL} \). At each step, an estimation \( \tilde{R}_w[k] \) for \( R_w[k] \) is obtained. The components from the line \( L_{NL} + 1 \) in the matrix \( \tilde{R}_w[k] \) are null as soon as \( T = L_{hyp} > L_{NL} \). Yet as some \( c_p, p = 1, \ldots, L \) can be a priori null, the corresponding functionals \( w_{\ell,p}[\cdot] \) and \( w_{Q,p}[\cdot] \) can also be null without that \( L_{hyp} > L_{NL} \). The algorithm must only decide for the estimate \( L_{NL} \) for \( L_{NL} \) after a large set \( R_{w1,L_{NL}+1},s[\cdot], \ldots, R_{w1,T},s[\cdot], R_{wQ,L_{NL}+1},s[\cdot], \ldots, R_{wQ,T},s[\cdot] \) is found as null.

Once \( R_{w1,1},x[k] \) and \( R_{wQ,1},x[k] \) are computed for all \( k \in [0, N-1] \) using equation (4.89), \( c_1 h_{eq}[\cdot] \) and \( c_1 h_{eq}[\cdot] \) are estimated after deconvolution of \( R_{w1,1},x[\cdot] \) and \( R_{wQ,1},x[\cdot] \) with \( R_x[\cdot] \). The value \( c_1 \) depends on the hardware and affects all measurements based on this scheme. It is compensated by using the calibration function \( c_1 (h_{eq}[\cdot] + j h_{eq}[\cdot]) \) (obtained when units are linked back-to-back by a cable) in all further channel measurements.
### 4.4.3.1 Example Using a Simulated Channel

In this example, we simulate the algorithm using Matlab. For clarity we set \( n[k] = 0 \) \( \forall k \). The channel is described by the vector \([1, 2, 3, 4]^T\), \( P(x) = x + 0.01x^2 + 0.03x^4 + 0.01x^5 \) and \([\alpha_1, \ldots, \alpha_{10}] = [0.1, \ldots, 1]\), i.e \( T = 10 \) and \( L_{\text{NL}} = 5 \). As \( T \geq L_{\text{NL}} \), it is possible to estimate \( R_{w1,1,x}[], \ldots, R_{w1,10,x}[] \). The effect of the non-linearities is clearly visible when the direct sequence is scaled by a larger coefficient: peaks appear at larger delays in the correlation function and the relative distance between the main correlation peak and the non-linear peaks decreases when \( \alpha_i \) increases. The estimated functionals in \( \tilde{R}_w[] \) are in Fig. 4.19. As expected, \( R_{w1,3,x}[] \) is nearly null because \( c_3 = 0 \), and \( R_{w3,k,x}[], k > 5 \), are also null because \( c_k = 0 \). Estimated and effective channels are plotted in Fig. 4.18. The estimation is perfect and the non-linearities are captured by the functionals \( w_{1,k}, k > 1 \). The algorithm allows a perfect estimation of the linear channel.

![Fig. 4.18: Channel estimated in a non-linear system](image)

### 4.4.3.2 Application to Measured Channels (\( L_{\text{NL}} \) a-priori unknown)

A series of measurements is performed using the RACooN lab. We use \([\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [0.2, 0.4, 0.6, 0.8]\), i.e. \( T = L_{\text{hyp}} = 4 \). The direct sequence parameter \( m = \log_2 (N + 1) = 8 \) and one buffer contains 32 direct sequences. For each \( \alpha_i \), eight direct sequences are transmitted. \( M = 7 \) full direct sequences can be exploited. The mean value is subtracted from
4.4 Robust Channel Estimation

Fig. 4.19: Functional estimation
each of the received direct sequences, and finally an averaged received direct sequence is computed. The output is circularly correlated with the unscaled transmitted direct sequence. The elements of $\mathbf{R}_y[k] \forall k$ are in Fig. 4.20(a) and Fig. 4.20(b).

As expected the main peak of the correlation functions is proportional to $\alpha_i$, i.e. to the transmit power. After resolution, the matrices $\mathbf{R}_w[k]$ are obtained for all $k$. The elements of $\mathbf{R}_w[k]$ are in Fig. 4.21(a) and Fig. 4.21(b). The correlation functions are proportional to $\alpha_i$, i.e. to the transmit power. After resolution, the matrices $\mathbf{R}_w[k]$ are obtained. The entries of $\mathbf{R}_w[k]$ are given in Fig. 4.21(a) and Fig. 4.21(b).

In contrary to the ideal case, the functionals $\mathbf{R}_{w,1,s}[\cdot]$ are not exactly null after a certain $L$. The noise (additive and phase noise) that affects the measurements has the same effect as very high order non-linearities. $L_{hyp} < L_{NL}$ and the estimated functionals in $\tilde{\mathbf{R}}_w[\cdot]$ are approximations for the $\{ R_{w,p,s}[\cdot] \}_{p=1,...,L_{hyp}}$ and $\{ R_{wQ,p,s}[\cdot] \}_{p=1,...,L_{hyp}}$. Thus expressions 4.87 and 4.88 become approximations.

The estimated real and imaginary part of the baseband channel are plotted in Fig. 4.22(a) and Fig. 4.22(b).

The estimated complex baseband channel magnitude of the CIR is plotted in Fig. 4.23(a) and the transfer function is in Fig. 4.23(b). We discuss it in the sequel.

**Limitations** Phase noise affects the received signal. Phase noise related changes are interpreted as high order non-linearities by the algorithm, leading to non-zero high-order $R_{w1,1,s}[\cdot]$ and $R_{wQ,1,s}[\cdot]$ functions. Additive noise at the receiver is also assimilated by the algorithm as high-order non-linearity.

**Quality of the estimation** We compare the CIR and transfer function of the baseband channel estimated with the multilevel approach (Fig. 4.23(a)) and without (Fig. 4.16). The multilevel approach is able to remove the non-linearities that affect the estimation, as for example in 4.15(b). Yet the cost of this processing is an increase of the noise in the tail of the estimated CIR, or in frequency domain ripples that add to the magnitude of the transfer function. This noise will be further removed by windowing the raw time-domain channel estimate obtained after multilevel transmission.

**Further improvement** A time-domain window can be applied to the raw CIR, it will flatten the tail of the raw CIR. A Tukey window is chosen for its flat top in time domain and its sides converging to 0 (in time domain), forcing the CIR to vanish at large delay. The window
4.4 Robust Channel Estimation

![Graphs showing channel estimation](image)

(a) 1st column

(b) 2nd column

Fig. 4.20: $R_y[\cdot]$
Fig. 4.21: $R_w[\cdot]$
4.4 Robust Channel Estimation

Fig. 4.22: Estimated channel
Fig. 4.23: Estimated channel
is designed such that the ratio of the taper to constant sections is equal to 0.5. The CIR after windowing is in Fig. 4.24(a) and the transfer function in Fig. 4.24(b).

![CIR and Power Spectrum](image)

**Fig. 4.24**: Estimated channel after windowing

### 4.4.4 For Chirp Sequence

In order to compare with fairness the estimate obtained with a chirp sequence to the estimate obtained using direct sequence, a scaling of the training sequences is required such that they have the same energy at all frequency bins except at DC, since the DC is not estimated (blocking channel assumption made in section 4.3.1). The amplitude of the chirp sequence is therefore multiplied by $\sqrt{\frac{N+1}{N}}$ according to 4.2.2 and 4.2.3.
When chirp sequences are considered, the transmitter non-linearities must be considered, whereas for direct sequences the focus could be limited to the receiver non-linearities as explained in section 4.4.2. The transmitter baseband non-linearities affect the complex chirp signal by adding at symbol $k$ denoted $c_ke^{j\theta[k]}$ some terms $c_p(\cos(\theta[k])^p + j\sin(\theta[k])^p)$ with $p = 2, \ldots, L_{NL}$ (assuming that the non-linearity is modeled on both branches by the same polynomial). At the receiver, the baseband non-linearities affect the signal in a similar way.

At the difference to the direct sequences, non-linear effects are not visible when the channel estimate is obtained with a chirp sequence. The raw channel estimates obtained with the same parameters and setup as in 4.4.1 are shown in Fig. 4.25(a) and Fig. 4.25(b) for $\alpha = 0.2$ and $\alpha = 0.8$. The non-linearities are not visible even for $\alpha = 0.8$ in Fig. 4.25(b). The drawback is a CIR noisy tail, leading to ripples in the power spectrum. This is shown in Fig. 4.26(a) and Fig. 4.26(b) for $\alpha = 0.2$ and $\alpha = 0.8$.

The noise in the CIR tail in Fig. 4.25 can be removed as previously by applying a time-domain Tuckey window on the raw CIR. The windowed CIR is in Fig. 4.27(a). In frequency domain, the power spectrum and the phase are in Fig. 4.27(b). The power spectrum is smoother, since applying a time-domain window is equivalent to a filtering of the frequency domain signal.

### 4.4.5 Result Comparison

We now compare the estimates obtained using a direct sequence ($\alpha = 0.2$, windowed), the multilevel windowed approach and a chirp sequence ($\alpha = 0.8$, windowed). All these estimates are robust to non-linearities. We summarize the advantages and drawbacks of the different estimate methods.

Non-linearities are not visible in the estimate obtained with a low level direct sequence since they can be below the noise level. This training sequence only excite the receiver baseband non-linearities. Insuring a low received power such that the non-linearities remain below the noise floor is in practice only possible when units are linked by a cable, as for example it is the case for determining the calibration function. When field measurements are performed, adjusting the transmit power requires a feedback link from the receiver or the use of an adaptive attenuator at the receiver. The feedback or receiver gain adaptation delay makes impossible the sounding of fast-changing channels. Furthermore in a point-to-multipoint scenario, a feedback link is not possible since the transmitter needs as many transmit signal
Fig. 4.25: Estimated CIR with a chirp sequence
levels as receivers. The feedback or receiver gain adaptation delay makes impossible the sounding of fast-changing channels. Furthermore in a point-to-multipoint scenario, a feedback link is not possible since the transmitter needs as many transmit signal levels as receivers.

The multilevel method explicitly identify and filter out the non-linearities. Yet the channel estimate obtained by the multilevel transmission approach is sensitive to noise interpreted as high-level non-linearities and setting the exit condition of the estimation algorithm is awkward without a-priori knowledge of the non-linearity order. The complexity of the scheme makes it furthermore attractive only for low-order non-linearities.

The estimate obtained with a chirp sequence is intrinsically robust to non-linearities. Yet this method exposes the signal to transmitter and receiver non-linearities, and the raw estimated CIR has a noisy tail that must be windowed.

The three CIR estimates are plotted in Fig. 4.28(a) with the same Tuckey window. The corresponding estimated transfer functions are in Fig. 4.28(b).

Due to the time resolution $T_s$, a discussion based on Fig. 4.28(a) and invoking the shortest transient time before the main peak is not appropriate. Comparing the power spectrum shows that the power spectrum obtained by the multilevel transmission approach is the most asymmetrical of the three estimates. There is no obvious reason why the equivalent baseband channel should have a flat power spectrum, therefore the choice of the sounding method is

![Fig. 4.26: Estimated transfer function with a chirp sequence](image-url)
Fig. 4.27: Estimated channel with a chirp sequence after windowing (chirp sequence, $\alpha = 0.8$)
Fig. 4.28: Comparison of estimated channels obtained with different methods
based on practical considerations. The chirp sequences are retained for the future measurements due to the simplicity of the postprocessing and their experimental immunity to the transmitter and receiver baseband non-linearities.

4.4.6 Calibration

The reciprocity principle applies to the wireless channel, i.e. from antenna to antenna. In practice the signal processing is performed in baseband, i.e. the signal is converted from baseband to radio-frequency and from radio-frequency to baseband. As the RF chain can be different at the transmitter and at the receiver, a signal transmitted from A to B passes through an equivalent filter which can be different from the filter seen by the signal transmitted from B to A. Therefore the equivalent baseband transmitter-receiver filter is estimated for all unit pairs and used as calibration filter.

The calibration function is obtained when units are linked back-to-back with a 36 dB passive attenuator component. This approach is commonly used, for example in [22, 35, 90]. [22, 35] assume a perfectly linear transmission system, only [90] discuss the non-linearities and propose to remove their effects by truncating the correlation function or neglecting them if they are below the noise floor. [40] propose a relative calibration scheme valid even without knowledge of the wireless (or wired) channel between the unit.

After testing the radio-frequency cables for a flat transfer function we stick to the common approach because of its simplicity and generate a calibration function for the linear baseband channel [120] using a chirp training sequence as described above. The units are linked by a cable and the estimated channel transfer function is therefore the product of the transmitter filter and receiver filter transfer function. This reference transfer function is removed from every measurement such that the result is simply the transfer function of the equivalent baseband physical channel.

4.5 Conclusion

In this chapter we have given an expression for the averaging gain, the correlation gain and the MSR achieved by an ideal sounding system. We have considered frequency offset and phase noise as imperfections and used the model developed in chapter 3 to analytically express the gain degradation caused by these imperfections, both for direct sequences and chirp sequences. We have proposed a measurement scheme based on direct sequence transmission that estimates the linear components of a system that contains non-linear elements.
at the receiver and illustrate it by realistic measurements using the RACooN lab. We experimentally found out that chirp sequences do not present the non-linear effects observed with direct sequences, and compare the channel estimates obtained with direct sequences and chirp sequences. Finally we discussed the choice of the sounding method and gave some insights for the choice of the training sequence.
Chapter 5

Wideband Network Channel Measurements in High Node Density Environment with Pedestrian Mobility

5.1 Introduction

This chapter presents a baseband stochastic pervasive wireless access network (PWAN) channel model for communication in environments characterized by a high node density, and node realistic mobility. This scenario is close to real-world situations and offers an interesting setup for applications that require high data rate transmission between users in a PWAN. The high node density hypothesis insures that a large number of nodes can possibly cooperate by creating a distributed multiple-input multiple-output (MIMO) system in the network. The model is realistic since it takes into account the node typical nomadic and behavioral mobility by capturing the spatial-temporal correlations between the channel taps.

After presenting the state of the art for non-collocated MIMO systems and focusing on mobility aspects in section 5.2, we present a measurement campaign performed with the ETH RACooN lab in sections 5.3 and 5.4. The outcome of this campaign is a set of parameters extensively discussed in section 5.5. Section 5.3 describes the setup, the environment and the scenario under consideration. The measurement campaign is described in section 5.4. In particular the parameters for the sounding procedure are discussed. Section 5.5 contains the evaluation of the measurement data. The post-processing steps are discussed and results are analyzed. The work comprises a robust algorithm for the unit position tracking in section 5.5.2. The data extracted from the measurements are used to parameterize a channel model proposed in sections 5.6 and 5.7. Section 5.8 validates the model by comparing the
spectral efficiency estimated for a MIMO system simulated with the channel model to the one estimated from the measured channels. The last section 5.9 summarizes our conclusions.

The contributions of this chapter are
- non-collocated MIMO channel measurements with nomadic mobility and a behavioral pattern in two different environments,
- the discussion of the channel sounder parameters,
- a relative positioning algorithm for the Radio Access with Cooperative Nodes (RACooN) nodes based on differential time-of-arrival (ToA) that is robust to carrier frequency offset (CFO),
- the experimental comparison of two sounding sequences (chirp and m-sequence),
- the estimation of the large scaling fading parameters,
- the characterization of MIMO channels fast fading parameters using the channel impulse response (CIR), power delay profile, delay spread, scattering function, Doppler spectrum and Doppler spread,
- the MIMO channel tap modeling,
- the investigation of the spatial-temporal channel tap correlation,
- a PWAN channel model for non-collocated MIMO systems.

5.2 Motivation and State of the Art

5.2.1 Measurement Campaigns

Performance of distributed MIMO systems have been so far mostly estimated using simulations, except for [16] due probably to the lack of available hardware. An evaluation of these systems in realistic conditions or at least with a realistic channel model has to be done. The focus of this chapter is to provide a channel model that can be used in distributed MIMO systems.

A realistic MIMO channel model for a PWAN must consider mobility since nodes are steadily moving [50]. Node or scatterer movements may drastically change the channel realizations.

Starting with the single-input single-output (SISO) case [88, 114], channel realizations may become uncorrelated in a dense scatterer environment under the homogeneous channel assumption\(^1\) as soon as a node or a scatterer is displaced by a fraction of \(\lambda_c\), where \(\lambda_c\) is the

---

1. Homogeneity is stationarity in the spatial dimension. It implies that signals departing/arriving from/to different directions are uncorrelated.
5.2 Motivation and State of the Art

wavelength of the carrier signal. At \( f_c = 5 \text{ GHz} \) (i.e. \( \lambda_c = 6 \text{ cm} \)), this happens after a displacement of a few centimeters. In a realistic situation the homogeneous channel assumption does not hold and the channel may remain correlated over a larger distance.

For a multiple-input single-output (MISO) channel, Wang in [123] conjectures from a ray-tracing simulation that for a given cluster of scatterers contributing to one tap, the fast fading coefficients of two links can be correlated. The author is considering a cellular scenario with two transmitters and one receiver. The receiver is surrounded by scatterers placed on one ring following the von-Mises distribution, also called circular normal distribution, which is a close approximation for the wrapped normal distribution. The scatterers cluster position follows the realistic assumptions of the urban micro-cell with non line-of-sight (NLOS) from the WinnerII channel model. The small scale fading correlation of the channel coefficients is calculated and the magnitude of the correlation coefficient is found to be not null for certain distances and angles between the two transmitters. Experimental verifications of the correlation between channels have only been performed so far for collocated antennas in [106]. In this chapter we investigate the non-collocated antenna case and estimate the time and space correlations between the channels. The data provided by this measurement campaign are used to parameterize the PWAN channel model in section 5.8.

Static collocated MIMO channel measurements have been performed in various environments: rural [63], office [74, 121], in a subway tunnel in [65] for example. Time-varying channels partly caused by pedestrian movement are studied in [28] with static collocated MIMO source and destination, and in [57] with low-mobility users.

A simple mobility scenario is introduced in [80] as nomadic mobility. In this scenario, the source and destination are only moving between successive measurement sequences but not during them. It boils down to the investigation of static scenarios that differ only by the distance between the transmitters and receivers.

Low mobility in a PWAN framework is investigated recently in [50]. Small movements of the source and destination antennas are considered, as well as human interactions. But in this work the authors limit the PWAN to one collocated MIMO source-destination pair. The authors highlight the fact that the fast fading statistics is different for each antenna element, and is not stationary. But this scenario covers imperfectly the PWAN as we define it because the network in [50] is limited to a pair of collocated antenna arrays.

Measurements and channel models specific for a high mobility scenario are proposed in the framework of vehicle-to-vehicle channel models which are a subset of the mobile-to-mobile channel models. A correlation-based vehicle-to-vehicle channel model, and measurements for collocated MIMO source and destination antennas are proposed in [82, 107].
Chapter 5  Wideband Network Channel Measurements

Some other works propose a geometric-based model, like [137] for a specific road junction geometry, and [23] for moving and fixed scatterers on the roadside. Yet the Doppler spectrum shape and the influence of the node density are strongly dependent on the environment and on the scenario considered [75, 131, 136], and these vehicle-to-vehicle geometry-based or correlation-based models cannot be used for a PWAN scenario.

In all works cited previously, antennas are collocated. A distributed antenna system (DAS) can be considered as an extension to systems where antennas can occupy different locations. A DAS consists in a single processing unit (one for the transmitter and one for the receiver) linked by cable to several physical antennas that can be positioned at different places. Measurements with a DAS have been performed in [98] using a leaky feeder for indoor measurements. The concept of DAS is extended later in [3, 48] to MIMO antennas, where for example [3] focuses on city downtown measurements without mobility. In [48], measurements are performed in a campus environment with MIMO antennas and nomadic mobility, although this concept is formally defined later in [80]. These measurements are performed for a cellular scenario with distances up to several tens of meters between the transmitter and receiver, with some nodes placed at roof level. All these cited works use cellular applications as motivation and do not apply to a PWAN scenario.

We extend in this chapter the work done for DAS to non-collocated MIMO systems. A non-collocated MIMO system consists of distributed antennas, each equipped with its own processing unit. When nodes are not collocated anymore, a framework for cooperative MIMO systems is established. An overview of existing cooperative MIMO channel models is given in [123]. The authors therein stress that existing cooperative MIMO models are based on standardized point-to-point tapped-delay line MIMO models. Yet these models cover neither the heterogeneity of the parallel links nor the parameter correlations intrinsic to cooperative MIMO channels. Standardized MIMO channel models are categorized into geometry-based and correlation-based stochastic channel models. The WinnerII cellular channel models [125] are geometry-based, and they cover relay and user mobility. But they do not explicitly contain the spatio-temporal correlation properties of the system level parameters. Indeed similarities in the environment encountered by different links create correlation between the channel model system level parameters, among other for the shadowing [89, 134]. These models also do not include correlation between small-scale fading parameters [123] that occurs even when two antennas located possibly several \( \lambda_c \) from each other receive a wave scattered by the same object. Furthermore they require a precise localization of all nodes and scatterers, which is not possible when a large number of nodes is present, and are tailored to a specific environment geometry. We will therefore consider a
correlation-based model like in [52]. The drawback of such a model is an oversimplification of the reality and an unrealistic model for the link variations, yet it is more general than geometry-based models.

### 5.2.2 PWAN Channel Model

Only few works specifically deal with a channel model for PWAN. To our knowledge the first model and measurements are performed by Karedal in [50,51]. A SISO link gain model is proposed in the first work [51], later extended [50] to a MIMO channel model.

#### 5.2.2.1 PWAN SISO Link Model

The SISO link gain model proposed in [51] identifies two origins for shadowing: environment shadowing as known from the cellular scenarios, and body shadowing caused by human movement between the antennas. Both are modeled by an additional loss in the received power, but they differ from each other by their coherence time, the environment shadowing having a longer coherence time than the body shadowing.

The amplitude of the fast fading terms is modeled by a generalized gamma distribution, this distribution is chosen because it can represent, with appropriate parameters, the Rayleigh distribution or be a good approximation for the Ricean distribution. The proposed link gain model is as follows

\[
G(d_{i,j}) = G_0 - 10\gamma \log_{10}\left(\frac{d_{i,j}}{d_0}\right) - L_e - L_b + G_r + G_{ss},
\]

where \(G(d_{i,j})\) denotes the gain in dB of the link between antenna \(i\) and antenna \(j\) at distance \(d_{i,j}\) from each other, \(G_0\) is the link gain in dB when antennas are at distance \(d_0\) from each other, \(\gamma\) is the pathloss exponent, \(L_e\) is the environment shadowing term, \(L_b\) is the body shadowing term, \(G_r\) is the relative path gain and \(\sqrt{G_{ss}}\) is the small-scale amplitude drawn from the generalized gamma distribution.

#### 5.2.2.2 PWAN MIMO Channel Model

The work in [51] is extended in [50] to give a MIMO channel model. An expression for the channel tap phase and amplitude is given, and correlation between model parameters are considered. This work is novel since it gives the first model that supports link heterogeneity
within a MIMO channel: shadowing can be different for each antenna pair, and the Ricean K-factor is link-specific. This model can be used with uniform or non-uniform antenna arrays, as long as they are collocated. The proposed model is as follows

\[ H(f; t) = \sqrt{G^\text{com}(t)} P(t) \otimes (\psi_1(K(t)) \otimes H^\text{dm}(f; t) + \psi_2(K(t)) \otimes H^\text{fd}(f; t)), \quad (5.2) \]

with

\[ P(t) = \begin{bmatrix} \sqrt{G^\text{rel}_{1,1}(t)} & \cdots & \sqrt{G^\text{rel}_{1,N_a}(t)} \\ \vdots & \ddots & \vdots \\ \sqrt{G^\text{rel}_{N_a,1}(t)} & \cdots & \sqrt{G^\text{rel}_{N_a,N_a}(t)} \end{bmatrix}, \quad (5.3) \]

and

\[ K(t) = \begin{bmatrix} K_{1,1}(t) & \cdots & K_{1,N_a}(t) \\ \vdots & \ddots & \vdots \\ K_{N_a,1}(t) & \cdots & K_{N_a,N_a}(t) \end{bmatrix}, \quad (5.4) \]

where \( \otimes \) denotes the Hadamard product of two matrices.

The Hadamard product of two \( n \times m \) matrices \( M_1 \) and \( M_2 \) is denoted \( M \) such that

\[ M = M_1 \otimes M_2 \quad (5.5) \]

and for \( 1 \leq i \leq n \) and \( 1 \leq j \leq m \),

\[ M[i, j] = M_1[i, j] \cdot M_2[i, j]. \quad (5.6) \]

Scalar \( G^\text{com}(t) \) contains the large-scale effects of distance decay and shadowing common to all links, \( \{ G^\text{rel}_{i,j}(t) \}_{1 \leq i \leq N_a, 1 \leq j \leq N_a} \) contains the individual link gains, functions \( \psi_1(\cdot) \) and \( \psi_2(\cdot) \) are defined such that:

\[ \psi_1(M) = [M \otimes [1 + M]]^{-1/2} \quad (5.7) \]

\[ \psi_2(M) = [1 + M]^{-1/2}, \quad (5.8) \]

where \([\cdot]^{1/2}, [\cdot]^{-1} \) and \([\cdot]^{-1/2} \) are element-wise operators.

The fast-fading terms are defined in the time-domain by the entries of matrices \( H^\text{dm}(f; t) \) and \( H^\text{fd}(f; t) \). The entry \( H^\text{dm}[i, j](f; t) \), resp. \( H^\text{fd}[i, j](f; t) \) is the function \( h^\text{dm}_{i,j}(f; t) \), resp.
5.2 Motivation and State of the Art

$h_{i,j}(f; t)$ such that

$$h_{i,j}^{dm}(f; t) = F_{\tau \rightarrow f} h_{i,j}^{dm}(\tau; t)$$  \hspace{1cm} (5.9)$$

and

$$h_{i,j}^{fd}(f; t) = F_{\tau \rightarrow f} h_{i,j}^{fd}(\tau; t)$$  \hspace{1cm} (5.10)$$

with

$$h_{i,j}^{dm}(\tau; t) = e^{j(\theta_{i,j} + 2\pi f_{D}^m t)} \delta(\tau)$$  \hspace{1cm} (5.11)$$

$$h_{i,j}^{fd}(\tau; t) = \lim_{Q \rightarrow \infty} \frac{1}{\sqrt{Q}} \sum_{q=1}^{Q} e^{j(\theta_{q} + 2\pi f_{D}q t)} \delta(\tau - \tau_{q}),$$  \hspace{1cm} (5.12)$$

where $\theta_q, \tau_q, f_{D}^m$ and $f_{D}q$ are random variables and $\theta_{i,j}$ is a deterministic phase delay determined by the antenna position in the array.

The distribution of the first tap is Ricean, the other taps are modeled by a sum-of-sinusoids as in [125]. This model assumes that each scatterer contributes to one tap arriving at $\tau_q$ and the tap rotates at frequency $f_{D}q$. The amplitude weights of the complex sinusoids are not a function of the delay power spectrum or the Doppler spectrum but are all equal. Their value $1/\sqrt{Q}$ is chosen such that the total average power is unitary, independent of the number of echoes. This approach using equal tap amplitude is taken from [45] and motivated by the wide sense stationarity (WSS)-uncorrelated scattering (US) channel model that is statistically completely determined by the scattering function and not primarily by the amplitudes and delays of the different paths. The cross-correlation between the entries of $K(t)$ and the entries of $P(t)$ is computed and found null. The spatial correlation between the channel parameters within $K(t)$ is also found null, as well as the correlation between the squared entries of $P(t)$.

**Limitations of the existing model** The tap model presented above has constant identical tap amplitude, conferring the channel a flat power delay profile. This channel model fulfills the WSS-US assumption but its flat power delay profile is not realistic [34]. The model from [50] is limited to collocated antennas since the same pathloss is used for all the links. The model also does not take spatial correlations between the channel taps into account.
Chapter 5 Wideband Network Channel Measurements

5.3 Setup

The setup reproduces a high node density PWAN where users are cell-phone or headset bearers standing or walking in a shop or a subway station. Six units are disposed in two different environments that differ by the number and type of scatterers and their dimension. Each set of measurements (1 to 10) features a behavioral pattern adopted by a mobile user called test-user while interacting with the other users. Section 5.3.1 describes the RACooN nodes operation. The two environments considered for the measurements are defined in section 5.3.2 and the mobility pattern of the nodes in the two environments is detailed in section 5.3.3.

5.3.1 RACooN Units

We consider the case where there are as many nodes as antennas in the system \( N_a = N_u \), the extension to collocated MIMO nodes can be done using the results of [50].

The units are equipped with identical single non-polarized antennas omnidirectional in the azimuthal plane. These antennas are placed at 1.55 m from the ground. As the antenna pattern has its main lobe concentrated in the azimuthal plane, there is no reflected wave from the ground or from the metallic racks of the piled radio-frequency unit (RFU), storage unit (STU) and power supply unit (PSU). The units are controlled by a WLAN 802.11g operating at 2.4 GHz which will not disturb the measurements performed at center frequency \( f_c = 5.5 \text{ GHz} \).

5.3.2 Environment

The environments are sketched in Fig. 5.1 and Fig. 5.2. A line of sight (LOS) is available between all units. Environment in Fig. 5.1 can represent a shop interior with tables, windows and many scattering objects. Metallic boards at the front and back walls act as reflectors for the waves, and shelves and cables on the walls scatter the signal.

The environment in Fig. 5.2 can model a subway station. The space is delimited by heterogeneous vertical structures which consist of wooden doors, metallic shields and concrete walls. The walls are smooth and there are few scatterers of the size of the wavelength. A concrete pillar stands in the middle of the space.
5.3 Setup

Fig. 5.1: Floor sketch ETF A104 (dimensions in m)

Fig. 5.2: Floor sketch (dimensions in m)
5.3.3 Behavioral Patterns

Both environments have small dimensions such that a high node density can be considered. The test-user reproduces typical small-scale human movements in ten measurement sets labeled set 1, ..., set 10. Human beings are never strictly static but move around a fixed position with a random pattern called here behavioral pattern. The random movement we are considering occurs over a disk with 25 cm radius. A similar approach was also retained in [50], but for comparing large-scale movement to small-scale movement, where large scale movements denote movements of the handheld device bearers (users) and small scale movements denote movements of the handheld devices without user movement. We combine in this chapter the concept of small scale (or behavioral) movements with nomadic mobility introduced in [80]. Nomadic mobility considers that users only move between two mission executions but are static during the measurements. We make this assumption more realistic by adding a behavioral pattern. This is a realistic assumption for a PWAN in an environment as defined in 5.3.2.

The measurement sets are depicted in Fig. 5.4 to Fig. 5.10. In measurement sets 1 and 3, the test-user is static for each position. These two sets take over the original concept of nomadic mobility introduced in [80]. In sets 2 and 4-10, the test-user is subject to the behavioral movement. Unit 7 is all through the measurements at the same location and static.

Sets 1-5 build reference measurements for sets 6-10. In sets 1-5, units have the same relative positions and are in two different environments. These sets are useful to quantify the effect of the movement and the environment on the channel statistics. They also allow for an experimental comparison of results obtained when the sounding sequence is a chirp sequence and a m-sequence.

The purpose of sets 6 to 10 is to investigate the effect of the behavioral pattern on the channel, and in particular on the spatial correlation of the taps. Set 6 models a test-user (unit 4) passing by a group constituted by units 3, 8-10. Set 7 represents a test-user (unit 4) following a group (units 3, 8-10). Set 8 represents a group breaking up, the test-user (unit 4) remaining in the center and the other users moving on diagonally opposed trajectories (units 3, 8-10). Set 9 represents the splitting of a group into two smaller groups moving in opposite directions, units 8-9 in one direction, units 3 and 10 in another direction while the test-user (unit 4) remains in the middle. Finally set 10 models a group enclosing the test-user (unit 4) in its middle that is moving together through the room.
5.3 Setup

**Set 1** emulates a mobile user joining or leaving a group. Measurements are performed in the room sketched in Fig. 5.1. The first position is when unit 4 is 1.40 m apart from unit 8. Unit 4 is shifted away from unit 8 along a line passing by unit 3 and unit 8. The static units are positioned in a cross pattern made from units 3 and 8-10 at a distance of 50 cm from each other, as depicted in Fig. 5.3.

Set 1 is purely nomadic and serves as a reference for set 2. Seven positions are investigated.

![Fig. 5.3: Antenna cross pattern](image)

At each position the mission is executed five times. 40 network channel realizations are obtained per mission execution, thus 200 network channel realizations are obtained for each position of unit 4.

**Set 2** uses the same unit placement and trajectory as set 1, but now unit 4 is subject to a behavioral pattern. The operator stands between unit 4 and the back wall.

**Set 3** uses the same scenario as set 1, i.e. static unit 4 is placed at seven different positions (see Fig. 5.5), but in the environment sketched in Fig. 5.2.

**Set 4** is based on set 3, but unit 4 is subject to a behavioral pattern.

**Set 5** is the same set as set 4 but a m-sequence is used for the channel estimation instead of a chirp sequence.

**Set 6** emulates a mobile user passing by waiting static users. Unit 4 is occupying seven positions along a line passing along a group of static units (3-8-9-10), as sketched in Fig. 5.6. The first raster point is at 50 cm from the front wall, the other raster points are separated by 1m.
Chapter 5 Wideband Network Channel Measurements

Fig. 5.4: Set 1 and 2

Fig. 5.5: Layout for sets 3, 4 and 5

Fig. 5.6: Layout for set 6
5.3 Setup

**Set 7** models a translation movement. A group constituted from units 3, 4, 8, 9 and 10 is moved according to Fig. 5.7 in the room described in Fig. 5.2. Four positions are investigated, separated by 1 m. At each position five missions are executed, thus 200 network channel estimates are available per position.

![Diagram of Set 7](image)

**Fig. 5.7:** Layout for set 7

**Set 8** models a radial movement from a common central point where unit 4 is located, as shown in Fig. 5.8. The first raster point is defined when antennas are 1 m apart from unit 4. The other raster points are on the dotted lines 50 cm apart from each other. Units 3, 8, 9 and 10 are simultaneously moved from one raster point to the next. Measurements are performed for 5 raster points, and for each raster point 5 missions are executed. Thus for each raster 200 network channel estimates are available.

![Diagram of Set 8](image)

**Fig. 5.8:** Layout for set 8
Set 9 models a group breaking up and is sketched in Fig. 5.9. Unit 4 remains in the middle of the room while units 8 and 9 move towards the front wall and units 3 and 10 move towards the back wall with 50 cm steps. Units 3, 8, 9 and 10 are static and unit 4 is moved when units are transmitting and receiving. 4 positions are measured, and for each position 5 missions are executed.

![Diagram](image.jpg)

(a) layout (dimensions in m)  
(b) picture

Fig. 5.9: Layout for set 9

Set 10 models a translation movement of unit 4 between units 3 and 10 on one side, and units 8 and 9 on the other side. It is sketched in Fig. 5.10. 7 positions are measured, and for each position 5 missions are executed. Each mission gives 24 channel matrix realizations (the estimates indexed 25 to 40 were found corrupted at some positions), giving a total of 120 network channel estimates per position. The operator is standing between unit 4 and the back wall.

After having motivated the measurement scenario, we document in the next section the parameters chosen for the measurement campaign. This section 5.4 is based on chapter 3.

5.4 Measurement Campaign

In this section the RACooN mission parameters are motivated. Section 5.4.1 presents the mission file parameters (superframe length, number of commands, order of the commands). The sounding signal is discussed in 5.4.2, more specifically we detail the multilevel transmission approach. Finally section 5.4.3 focus on the measurement protocol.
5.4 Measurement Campaign

5.4.1 Parameter Choice

We allow only one unit to transmit at a time to avoid receiver power amplifier saturation (semi-sequential operation mode detailed in 3.2.1). With \( N_u \) in the network, \( N_u - 1 \) links are simultaneously estimated and \( N_u(N_u - 1) \) estimates are gathered in total. Reciprocity is a-priori not assumed, each link is estimated in both directions. Actually reciprocity principle does apply to the radio channel but not necessarily to the filters and amplifiers at transmitter and receiver. The channel matrix is symmetric only if these filters are all strictly identical. They can be made nearly identical in a post-processing step by calibration. Calibration is a time-constant linear operation that can compensate time-constant linear signal distortions.

In order to capture fast variations of the channel we need to keep the sounding duration for each link as short as possible and we take the smallest possible superframe, i.e. one buffer. The superframe lasts \( T_{ts} \), with \( T_{ts} = (S + N')T_s = 0.1049 \text{ ms} \). Parameter \( N' \) denotes the difference between the number of samples in a superframe and in a timeslot, as defined in 3.4 (\( N' = 200 \)). Every unit is transmitting the same training signal on its turn, and is receiving the rest of the time.

The change of operation mode from transmitter to receiver and inversely requires the switch to connect alternatively the receiver or transmitter circuit to the antenna. The switching operation lasts up to 600 ns [124] and the local oscillator needs to stabilize after the switch is positioned. A delay of \( N_{sw}=10 \) timeslots, i.e. \( T_{sw} = N_{sw}T_{ts} = 1.049 \text{ ms} \) is observed, as discussed in 3.4.1.2, until the unit can again transmit or receive. Each transmit and receive command is preceded by an idle command that sets duration \( T_{sw} \) for the phase.
Chapter 5 Wideband Network Channel Measurements

to stabilize (issue discussed in 3.4.1.2).
The other units that keep receiving have to wait for the same duration to maintain the command parallelism.
The \( N_u(N_u - 1) \) channels in the network are estimated after \( T_{est} = N_u(T_{ts} + T_{sw}) \), where \( T_{est} \) is the time required to obtain one channel matrix realization. We denote by \( B_0 \) the number of buffers filled with valid data after \( T_{est} \). The time \( T_{est} \) between two channel estimations is short, leading to a high maximum estimated Doppler frequency given in (5.13). Parameter \( B_0 \) is equal to \( N_u - 1 \) because each unit is receiving the signal from the \( N_u - 1 \) other units before transmitting again. We enumerate the commands used during \( T_{est} \): transmit (1 time), receive \( (N_a - 1 \) times) and idle \( (N_a \) times), that are \( 2N_a \) commands in total.
Numerical values are in Table 5.1 for illustration.

<table>
<thead>
<tr>
<th>( N_a )</th>
<th>( T_{est} )</th>
<th>Number of buffers ( B_0 ) occupied (per unit, out of 255(^1))</th>
<th>Number of commands (out of 511(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.3078 ms</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3.4617 ms</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>4.6156 ms</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>5.7695 ms</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>6.9234 ms</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>8.0773 ms</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>9.2312 ms</td>
<td>7</td>
<td>16</td>
</tr>
<tr>
<td>9</td>
<td>10.3851 ms</td>
<td>8</td>
<td>18</td>
</tr>
</tbody>
</table>

**Table 5.1:** Network channel matrix estimation duration (one realization without storage to the harddisk)

At this point, data are stored in the Random-Access Memory (RAM). Further channel network realizations can be stored until the RAM is full or the maximum number of commands per mission is reached. With our mission structure, the limiting factor is the maximum number of commands per mission, for any value of \( N_a \) \( (N_a \geq 2) \). \( N_{rep} \) channel matrix estimates are then available.

This time-limitation of the measurement imposes an upper bound on the Doppler frequency resolution as given in (5.14). The maximum Doppler frequency that can be estimated is:

\[
 f_D^{\text{max}} = \frac{1}{2T_{est}}. \tag{5.13}
\]

1. Only 255 buffers are available for the received signal and not 256 as in section 3.3.2.2 because one buffer is used by the transmit signal.
2. Only 511 commands are available and not 512 as in section 3.3.2.4 because one command is required at the beginning of the mission to load the transmit signal from the harddisk to the Random-Access Memory (RAM).
If all channels are estimated \( N_{\text{rep}} \) times \( (N_{\text{rep}} \geq 2) \) within a mission execution, the Doppler frequency resolution is:

\[
\mathcal{f}_{\text{D}}^{\text{res}} = \frac{1}{T_{\text{est}}(N_{\text{rep}} - 1)}, \tag{5.14}
\]

**Storage operation**  The store command is time-consuming, possibly due to the harddisk access time and the bus that connects the RAM to the harddisk. Its duration is experimentally estimated in Table 5.2. \( N_{\text{sto}} \) denotes the number of timeslots required for the store command to complete. From this table obtained with experimental data we extract as rule of thumb:

\[
N_{\text{sto}} = aB_0 + b, \tag{5.15}
\]

with \( a = 3000 \), \( b = 3000 \) and \( B_0 \) the number of buffers to store, for the case where the superframe contains one buffer. Design constraints impose [124]:

\[
N_{\text{sto}} \leq N_{\text{sto}}^{\text{max}} = 65535,
\]

i.e.

\[
B_0 \leq B_0^{\text{max}} = \left\lfloor \frac{N_{\text{sto}}^{\text{max}} - b}{a} \right\rfloor = 20 \text{ buffers}.
\]

The store command execution lasts:

\[
T_{\text{st}} = N_{\text{sto}}(S + N')T_s,
\]

and some numerical values are given in Table 5.2.

<table>
<thead>
<tr>
<th>( B_0 )</th>
<th>( N_{\text{sto}} )</th>
<th>( T_{\text{st}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6000</td>
<td>0.6144025 s</td>
</tr>
<tr>
<td>2</td>
<td>9000</td>
<td>0.9216025 s</td>
</tr>
<tr>
<td>3</td>
<td>12000</td>
<td>1.2288025 s</td>
</tr>
<tr>
<td>4</td>
<td>15000</td>
<td>1.5360025 s</td>
</tr>
<tr>
<td>5</td>
<td>18000</td>
<td>1.8432025 s</td>
</tr>
<tr>
<td>6</td>
<td>21000</td>
<td>2.1504025 s</td>
</tr>
<tr>
<td>7</td>
<td>24000</td>
<td>2.4576025 s</td>
</tr>
<tr>
<td>8</td>
<td>27000</td>
<td>2.7648025 s</td>
</tr>
</tbody>
</table>

**Table 5.2:** Storage overhead for \( B_0 \) buffers (one superframe contains one buffer)
Mission structure  The maximal Doppler frequency and Doppler frequency resolution are given in Table 5.3 using formula (5.13) and (5.14). \( N_{\text{rep}} \) is the number of channel matrix estimations performed until the maximum number of commands \( N_{\text{cmd}}^{\text{max}} - 1 \) is reached. It fulfills following constraint:

\[
\left\lfloor \frac{N_{\text{rep}}B_0}{B_0^{\text{max}}} \right\rfloor + 2N_{\text{a}}N_{\text{rep}} \leq N_{\text{cmd}}^{\text{max}} - 1
\]  

(5.16)

In the previous expression, \( \left\lfloor \frac{N_{\text{rep}}B_0}{B_0^{\text{max}}} \right\rfloor \) is the number of store commands required to store all the buffers in the mission and \( 2N_{\text{a}}N_{\text{rep}} \) is the number of commands idle, transmit and receive. Once \( N_{\text{rep}} \) from (5.16) is determined, the storage total duration is:

\[
T_{\text{tot}}^{\text{st}} = T_{\text{st}} \left\lfloor \frac{N_{\text{rep}}B_0}{B_0^{\text{max}}} \right\rfloor .
\]

and two mission executions are separated by:

\[
T_{\text{miss}} = N_{\text{rep}}T_{\text{est}} + T_{\text{tot}}^{\text{st}}.
\]

The values for \( T_{\text{miss}} \) in Table 5.3 are rounded at \( 10^{-4} \) s.

<table>
<thead>
<tr>
<th>( N_{\text{a}} )</th>
<th>( f_{\text{D}}^{\text{max}} )</th>
<th>( N_{\text{rep}} )</th>
<th>( f_{\text{D}}^{\text{res}} )</th>
<th>( N_{\text{rep}}T_{\text{est}} )</th>
<th>( T_{\text{miss}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>216.66 Hz</td>
<td>126</td>
<td>3.44 Hz</td>
<td>290.7828 ms</td>
<td>48.4131 s</td>
</tr>
<tr>
<td>3</td>
<td>144.44 Hz</td>
<td>83</td>
<td>3.44 Hz</td>
<td>287.3211 ms</td>
<td>62.1589 s</td>
</tr>
<tr>
<td>4</td>
<td>108.33 Hz</td>
<td>62</td>
<td>3.49 Hz</td>
<td>286.1672 ms</td>
<td>69.0324 s</td>
</tr>
<tr>
<td>5</td>
<td>86.62 Hz</td>
<td>50</td>
<td>3.47 Hz</td>
<td>288.4750 ms</td>
<td>69.0347 s</td>
</tr>
<tr>
<td>6</td>
<td>72.22 Hz</td>
<td>41</td>
<td>3.52 Hz</td>
<td>283.8594 ms</td>
<td>75.9047 s</td>
</tr>
<tr>
<td>7</td>
<td>61.90 Hz</td>
<td>35</td>
<td>3.54 Hz</td>
<td>282.7055 ms</td>
<td>75.9035 s</td>
</tr>
<tr>
<td>8</td>
<td>54.16 Hz</td>
<td>31</td>
<td>3.61 Hz</td>
<td>286.1672 ms</td>
<td>75.9070 s</td>
</tr>
<tr>
<td>9</td>
<td>48.16 Hz</td>
<td>27</td>
<td>3.57 Hz</td>
<td>280.3977 ms</td>
<td>75.9012 s</td>
</tr>
</tbody>
</table>

Table 5.3: Mission parameters

Mission file generation  The mission files are generated dynamically. The 505 mission commands and their respective parameters are entered in an Excel sheet. This allows a visual check of the command sequence by parallelization of the instruction flow. The command flow is in Fig. A.1 until Fig. A.10.

The XML mission file of each RACooN is automatically generated. It contains a header and footer provided by the RACooN developers. A buzzer tone is integrated as a processing
command process in the mission file to signalize start and end of the store commands sequence. This actually leads to an increase of $T_{\text{miss}}$ but not of $N_{\text{rep}} T_{\text{est}}$.

### 5.4.2 Transmit Signal

With six units, the maximum repetition rate is $T_{\text{est}}=6.9234$ ms, as given in Table 5.1. This rate denotes the minimum time interval between two network channel matrix estimations. A chirp sounding sequence is chosen for its robustness to non-linearities. This choice is justified later using data from set 5 (in section 5.5.4.2).

#### 5.4.2.1 Parameters

In the following we discuss the choice for parameters $N$ and $M$ introduced in chapter 3. $N$ is the length of the training sequence and $M$ is the number of training sequence repetitions available at the receiver. From the previous paragraph, $MN \leq S$.

The delay of the largest significant contribution in the estimated channel impulse response (CIR) determines a lower bound for $N$. This requires a-priori knowledge of the environment geometry (scatterer positions) or experimental trials performed with very large $N$. For a chirp sequence, increasing $N$ increases also the sensitivity of the correlation result to $f_{\text{off}}$ (mainlobe-to-sidepeak ratio (MSR) decrease in Fig. 4.13), but reduces the sensitivity of the correlation to phase noise (MSR increases in Fig. 4.14). We use $N = 255$ as a trade-off, i.e. $m = 8$ recalling $N = 2^m - 1$, and the maximum measurable excess delay $T_d$ is:

$$T_d = NT_s = 3.1875 \mu s.$$  \hspace{1cm} (5.17)

This duration corresponds to the largest time-of-arrival difference between the first and the last CIR sample, and represents a distance difference of $cT_d = 956.25$ m between the shortest and the longest path. With this choice for $N$, the correlation gain degradation due to frequency offset is negligible, as read from Fig. 4.12.

The superframe length is one buffer, thus $M = \lceil S/N \rceil = 32$ and the averaging gain degradation due to frequency offset read from Fig. 4.9 is negligible. The maximum averaging gain is $10 \log_{10}(M) \approx 15$ dB and the maximum processing gain (averaging and correlation gain) is $10 \log_{10}(MN) \approx 39$ dB. All further processing will assume that the channel is constant over $ST_s = 0.1024$ ms.
5.4.2.2 Multilevel Transmission

The signal-to-noise ratio (SNR) at the receiver is a function of the channel and the transmit power. In a moving scenario the distance between the receiver and the transmitter changes and thus the SNR at the receiver changes too. Finding the largest transmit power without instantaneous feedback from the receiver is impossible since the receiver is not able to inform the transmitter when it is clipping. The idea behind multilevel transmission is to modify the transmit power such that the received signal power attenuation due to pathloss is partly compensated by a signal gain.

In the sequel we assume that two units never get closer than a minimum distance $d_{\text{min}}$.

The pathloss law in section 5.5.9.1 gives the large scale power loss with respect to the distance between transmitter and receiver. For any distance larger than $d_{\text{min}}$ between 2 units, the transmit signal power must be large enough such that the received SNR is high, under constraint that the analog-to-digital converter (ADC) at the receiver does not clip. This is achieved by transmission of a multilevel training signal, where the repetitions of the training sequences within the training signal have $T$ different amplitudes. The receiver selects from the received signal the sequences with the highest amplitude for which its ADC does not clip. A two-level strategy is adopted with 6 dB amplitude difference between two training sequences, respectively scaled by $\alpha_1 = 1$ and $\alpha_0 = \frac{\alpha_1}{4}$.

**Advantages:** Among the received multilevel sequences, the receiver selects the set of sequences that do not lead to clipping. The averaging gain is degraded by this selection. The trade-off between the averaging gain and the signal gain described in 5.4.2.1 is discussed now. We compare in Table 5.4 (i) the single-level strategy achieving the largest received SNR at $d_{\text{min}}$ when transmitting with power $p_0$ to (ii) the multilevel strategy with $T$ levels. For the single-level strategy, the processing gain is the sum of the averaging gain and the correlation gain, for any distance $d \geq d_{\text{min}}$. The processing gain amounts $10 \log_{10} (MN)$. For the multi-level strategy, the receiver can benefit from an additional signal gain. At $d = d_{\text{min}}$ the processing gain amounts $10 \log_{10} \left( N \left\lfloor \frac{M}{T} \right\rfloor \right)$. When the units are $d = d_k > d_{\text{min}}$ apart from each other, the receiver can exploit the sequence transmitted with power $p_k > p_0$ such that a signal gain $10 \log_{10} \left( \frac{p_k}{p_0} \right)$ adds to the processing gain achieved at $d = d_{\text{min}}$. The multilevel strategy is profitable at distance $d_k$ only if the signal gain compensates the averaging gain degradation, i.e.:

$$10 \log_{10} (M) < 10 \log_{10} \left( \left\lfloor \frac{M}{T} \right\rfloor \right) + 10 \log_{10} \left( \frac{p_k}{p_0} \right),$$

(5.18)
5.4 Measurement Campaign

<table>
<thead>
<tr>
<th>gains</th>
<th>(d_{\text{min}})</th>
<th>(d_k &gt; d_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>single-level</td>
<td>(10 \log_{10} (MN))</td>
<td>(10 \log_{10} (MN))</td>
</tr>
<tr>
<td>multilevel (T levels)</td>
<td>(10 \log_{10} \left(N \left\lfloor \frac{M}{T} \right\rfloor \right))</td>
<td>(10 \log_{10} \left(N \left\lfloor \frac{M}{T} \right\rfloor \frac{p_k}{p_0}\right))</td>
</tr>
</tbody>
</table>

Table 5.4: Processing gain with multilevel strategy

or

\[
\frac{p_k}{p_0} > \frac{M}{\left\lfloor \frac{M}{T} \right\rfloor}.
\] (5.19)

As \(\left\lfloor \frac{M}{T} \right\rfloor \leq \frac{M}{T}\), we obtain \(p_k > Tp_0\). On the other side the value \(p_k\) is upper-bounded by the receiver saturation and the maximum transmit amplitude.

The multilevel strategy is suboptimal for \(d = d_{\text{min}}\), it is only profitable when distances are changing.

**Receiver benefits:** This paragraph discusses an improvement of the multilevel strategy.

The processing gain was computed in Table 5.4 under the assumption that the receiver is considering only the training sequences with the largest transmit power. Actually, considering all non-clipping sequences increases the processing gain.

Let \(\alpha_0x[\cdot]\) be the sounding sequence, \(n[\cdot]\) the noise additive white Gaussian noise (AWGN) process with 0-mean and variance \(N_0\), and \(h[\cdot]\) the discrete channel. The sounding sequence is such that:

\[
0 < \alpha_0 \leq 1
\] (5.20)

and

\[
\forall k, \quad |x[k]|^2 = 1.
\] (5.21)

With \(z[k] = (h \odot x)[k]\), the received sequence is:

\[
y[k] = \alpha_0 z[k] + n[k],
\] (5.22)

and the received SNR of the \(k^{th}\) sample of the received periodic sequence is \(\text{SNR}_0 = \alpha_0^2 \frac{|z[k]|^2}{N_0}\).

With a multilevel strategy, a set of amplitude scaling factors \(0 < \alpha_0 < \alpha_1 < \ldots < \alpha_{T-1} \leq 1\)
Chapter 5 Wideband Network Channel Measurements

is used. After adding the received signals with different scaling factors:

\[ y[k] = \alpha_0 z[k] + n_0[k] + \sum_{p=1}^{T-1} (\alpha_p z[k] + n_p[k]) \]  \hspace{1cm} (5.23)

with \( n_p[k] \) the independent identically distributed noise for the \( k^{th} \) sample received when the transmit sequence level is \( \alpha_p \).

The received SNR of the \( k^{th} \) sample is:

\[ \text{SNR}_1 = \frac{\alpha_0^2 |z[k]|^2 (1 + \sum_{p=1}^{T-1} (\frac{\alpha_p}{\alpha_0}))^2}{TN_0} \]  \hspace{1cm} (5.24)

since \( \frac{\alpha_p}{\alpha_0} > 1 \), \( \text{SNR}_1 > \text{SNR}_0 \) and the received SNR is always improved.

5.4.3 Measurement Protocol

Five missions are executed for each position along the displacement line and up to 41 channel realizations (actually 40 because one unit did not save the last buffer of the store command without setting the corresponding IA flag in its log file to 1) are obtained for each mission execution (c.f. Table 5.3). Thus at each position in the raster 205 channel matrix realizations (snapshots) are collected with the same fast fading statistics.

Using the parameters and operation mode described in the previous sections the measurements are performed and the results are evaluated in the sequel.

5.5 Measurement Evaluation

This section contains the analysis of selected aspects from the measurement campaign. First, the post-processing steps are recalled in section 5.5.1. The strength and weakness of the methods with respect to the imperfections were intensively discussed in chapter 4. In order to exploit the nomadic aspect of the measurements, a unit tracking position algorithm is proposed in 5.5.2. The estimated channel are analyzed for selected links based on the CIR in 5.5.3, the power delay profile and delay spread in 5.5.4 and 5.5.5. The frequency offset and compensation is described in 5.5.6. Then the temporal channel variations are characterized using the Doppler spectrum and Doppler spread in 5.5.7 and 5.5.8. The large scale fading parameters are commented in 5.5.9. Finally the fast fading is characterized. The channel taps are modeled in 5.5.10 and the spatial-temporal correlation is analyzed in 5.5.11.
5.5 Measurement Evaluation

5.5.1 Post-Processing Steps

**Log files**  The log files are automatically read and checked for consistency after measurement completion. The number of executed commands is controlled, and the error flags CD (corrupted data), FATAL (fatal error), RFU (power status of the RFU), IA (interruption acknowledgment bit, set to 1 if the number of idle timeslots allocated for a store command is insufficient) and RB (Rubidium clock locked) are screened before further data processing. If any of these flags is set to 1, the measurement is repeated.

**Storage**  Data are available on the RACooN harddisks after the mission execution. Each unit has stored its buffers as specified in the XML mission file. The file name indicates the timeslot index and the buffer index. The timeslot index characterizes the mission repetition (whereas the contrary is not true since several timeslots are required to store all the buffers from one mission execution), and the buffer index characterizes the transmitter since units are transmitting in a user-defined order specified in the XML file. The files are read from the RACooN harddisk using a Matlab code. The native data are originally formatted as described in [124] and must first be converted into 16 bits complex numbers for Matlab processing.

**Clipping**  Clipping is controlled at the receiver and the possibilities offered by the multilevel transmission scheme are exploited. The buffer samples are separated in stacks of length \( N \left\lceil \frac{M}{T} \right\rceil \) according to their training sequence scaling factor, with \( N \) the sounding sequence length, \( M \) the number of sounding sequences available at the receiver and \( T \) the number of levels used by the multilevel strategy.

Clipping is checked, starting from the stack corresponding to the highest transmit sequence amplitude. The stack corresponding to highest amplitude for which no clipping is detected is retained for further processing and stored.

Two algorithms for clipping detection with similar performances are implemented. The first algorithm computes the difference between consecutive points around the points that reach the maximum digital values (+1 or -1). It decides for clipping when two or more consecutive points reach the maximum value. This method does not identify single clipped samples. The second algorithm is based on the received signal statistics. The histogram of the signal amplitude is computed and clipping is detected when the bins containing the maximum values +1 or -1 are overpopulated compared to their neighbor. The algorithm decides for clipping when the maximum values bins are at least 3 times more populated than their closest neighbor; this threshold is motivated for 0-mean signals by noting that a signal contributing to
the maximum-value bin will contribute once to the closest neighbor bin before reaching its maximum and once after reaching its maximum. The drawback of this method is that its performance depends on the signal statistics, or inversely the signal statistics determines the size of the bins and threshold; as an advantage the histogram-based method has low complexity and its execution is fast, assuming the histogram parameters are carefully chosen.

**DC removal** In a second step, the mean value of each received sequence is removed. We process each sequence individually as we found out in section 3.4.6 that the received signal mean value is not strictly stationary over one mission duration. Removing the complex mean signal value at this step is equivalent to assuming that the baseband channel is blocking at DC or that the physical channel is blocking at $f_c$. This is obviously wrong, but the channel direct component (DC) estimate is corrupted anyway by a random offset generated by the electronics that prohibits any reliable estimation of the baseband channel DC.

This operation has a later consequence. When the time-domain raw CIR is truncated by a time-domain window in order to remove the noise samples in the tail of the CIR, an artificial DC is generated. In frequency domain, this operation is equivalent to the convolution of the spectrum with the Fourier transform of the time-domain window. The convolution re-creates a DC in the spectrum, and also affects the frequency bins around the DC. The number of affected frequency bins around DC is inversely proportional to the width of the time-domain window.

**Further steps** The zero-mean sequences are averaged, the maximum processing gain achieved is $10 \log_{10} \left( \left\lceil \frac{M}{T} \right\rceil \right)$. The sequence is cyclic-correlated with the transmitted unscaled sounding sequence. Finally the signal in frequency domain is scaled to account for the flat power spectrum of the chirp sequence.

The estimated CIR can be now calibrated using calibration data obtained previously in 4.4.6. Calibration targets the compensation of the transmitter and receiver RF filters, making the channel matrix symmetric. Yet the resulting CIR showed no physically meaningful behavior and we drop this step. The reason for this effect is attributed to the samples of the calibration function located just before the upper bound and just after the lower bound of the time-window in Fig. 5.11. After inspection of the calibration function on a dB scale for the amplitude, we observe noise samples close to the physical CIR that are attenuated by the window sides but not fully removed. The window choice is crucial. As we dealt with a calibration function we opted for a window with a flat top such that the physical CIR is not
weighted by the window, and for smooth transitions outside since we did not want the spectrum to be distorted (since calibration is performed in frequency domain). Yet we assume the remaining noise samples in the calibration function destroy the calibration procedure. Nevertheless we claim that this has only a limited effect on our results since channel reciprocity is experimentally observed (section 5.5.4.2). All estimated CIR contain the same linear distortion since all nodes are built from identical components.

![Windowing of the calibration function](image)

**Fig. 5.11:** Windowing of the calibration function

### 5.5.2 Unit Tracking Position

In this section we propose a unit robust position tracking algorithm. In section 5.5.2.1 we motivate the algorithm by showing why existing methods do not apply. The system model is presented in 5.5.2.2. The tracking algorithm for ideal units is introduced in 5.5.2.3 and extended in 5.5.2.4 to cope with imperfections of the units. The advantage of this algorithm is its simplicity, robustness to frequency offset and digital hardware delay. It requires a LOS between the units and the transmission of a beacon between any two units, and the exchange of side information.
5.5.2.1 Problem Setting and State of the Art

The global positioning system (GPS) can be used for a dynamic scenario in outdoor to gain the absolute position knowledge and calculate from it the distance between the nodes. For indoor scenarios some other techniques must be used since no LOS with the GPS satellite constellation is available. Time-of-arrival (ToA), angle-of-arrival (AoA) and received signal strength (RSS) are the common principles used by state of the art positioning algorithms.

**Time-of-Arrival**  ToA methods base on an estimation of the signal propagation time since it is proportional to the distance between transmitter and receiver. It requires that all transmitters and receivers have precisely synchronized clocks and a strong LOS between the nodes. Multipaths, in particular from distant objects can cause severe errors [87]. An extension of ToA methods is the time difference-of-arrival (TDoA) method where the time difference at which the signal arrives at multiple base stations is estimated rather than the absolute arrival time.

**Angle-of-Arrival**  The AoA method is based on the measurement of the incoming signal direction with an antenna array [94,135]. It requires several nodes and computes their relative position by triangulation. This method is sensitive to the linear orientations of the antenna arrays and is not appropriate for dynamic scenario.

**Received Signal Strength**  RSS [87] is defined as the voltage or power measured by the receiver. It is related to the distance by the Friss formula (section 5.5.9.1) and is used to estimate the distance when the pathloss exponent is given. RSS is usually applied for coarse-grained localization [135]. The measurements are simple and can be performed in a dynamic scenario. But when the pathloss exponent is unknown, the RSS method cannot be used.

In the sequel we present our algorithm. It is based on a set of assumptions on the transmitter and the receiver summarized in the next section.

5.5.2.2 Assumptions

A digital system is considered where each node has (i) its own local oscillator and (ii) a sample counter that is incremented at a multiple time of the local oscillator period. The nominal local oscillator period is denoted $T_s$, and $T_s$ is chosen for simplicity as the sample duration. If $k$ is the sample counter state at a receiver, starting with $k = 0$, the signals
5.5 Measurement Evaluation

arriving at the antenna in the time range \([kT_s, (k + 1)T_s]\) contribute to the value of the digital sample indexed by \(k\). The node local oscillator is ideal in section 5.5.2.3, and non-ideal in the other sections. Jitter and frequency offset in the local oscillator signal (section 3.4.3.1) affect the pace at which the counter is incremented.

We study in the following a digital system consisting of two units labeled A and B, and describe a differential ToA method based on reciprocal ToA measurements.

5.5.2.3 Ideal Case

In the ideal case, all units have local oscillators perfectly synchronized at every instant of the measurement and they share a common time reference. Unit A (resp. B) starts transmitting when its sample counter indicates \(N_{\text{start}}^A\) (resp. \(N_{\text{start}}^B\)), i.e. at time \(N_{\text{start}}^AT_s\) for unit A and \(N_{\text{start}}^BT_s\) for unit B. For simplicity of the presentation, \(N_{\text{start}}^A \leq N_{\text{start}}^B\), i.e. unit A starts transmitting before unit B.

Signal transmission time is subject to a digital delay expressed in number of samples. This delay is caused by digital hardware (digital-to-analog converter (DAC), memory access) and represents \(N_{\text{HW,tx}}^A\) samples at transmission and \(N_{\text{HW,rx}}^A\) samples at reception at unit A. Similarly \(N_{\text{HW,tx}}^B\) and \(N_{\text{HW,rx}}^B\) denote the delays at unit B in transmission and reception. \(T_{\text{trav}}^C(N_{\text{start}})\) denotes the propagation time at speed \(c\) of the wave between unit A and B in a spatial configuration \(C\) when the transmitter sends its signal at sample \(N_{\text{start}}\). \(T_{\text{trav}}^C(N_{\text{start}})\) is the same if it is measured from A to B or from B to A at a given time.

Unit A starts transmitting at time \(N_{\text{start}}^AT_s\) and unit B receives the signal at time \(N_{\text{start}}^AT_s + N_{\text{HW,tx}}^AT_s + T_{\text{trav}}^C(N_{\text{start}}^A)\). Unit B finally stores the received signal when its counter indicates \(N_{\text{start}}^B\) such that

\[
N_{\text{start}}^B = N_{\text{start}}^A + N_{\text{HW,tx}}^A \left\lfloor \frac{T_{\text{trav}}^C(N_{\text{start}}^A)}{T_s} \right\rfloor + N_{\text{HW,rx}}^B. \tag{5.25}
\]

Unit B estimates its distance to unit A as

\[
\hat{d}_{BA} = cT_sN_{\text{start}}^B = cT_s \left( N_{\text{start}}^A + N_{\text{HW,tx}}^A + N_{\text{HW,rx}}^B \right) + cT_s \left\lfloor \frac{T_{\text{trav}}^C(N_{\text{start}}^A)}{T_s} \right\rfloor \tag{5.26}
\]

The hardware and start delays act as a constant distance offset when the distance between A and B is measured. In the next section 5.5.2.4, the node imperfections are taken into account, and more specifically the timeslot drift between the units due to the non-ideal local oscillators.
Chapter 5 Wideband Network Channel Measurements

Oversampling

Analog systems can estimate the position with theoretically infinite precision whereas digital systems have a time resolution limited by the sampling period $T_s$, leading to a native spatial localization resolution $R_s = cT_s = 3.75$ m. The resolution $R_s$ found previously is too low for considerations in a PWAN where a typical distance ranges between 1 and 10 m. Resolution enhancement is obtained by oversampling. This operation is performed offline in the digital domain. The received signal spectrum is padded with 0s until the desired spatial resolution is reached. The target resolution is $R'_s = R_s/100 = 3.75$ cm, which represents an oversampling factor of 100, i.e. a new bandwidth of $100 \cdot 80$ MHz = 8 GHz = $1/T'_s$. The unit counters are now incremented every $T'_s$.

5.5.2.4 Robust Time-of-Arrival Estimation

In this section we propose a distance estimation algorithm that takes node imperfections into account. The imperfections considered here are local oscillator phase noise, phase offset and frequency offset. We assume a strong LOS available between transmitter and receiver such that the amplitude of the LOS component at the receiver largely dominates the noise floor. We consider non-ideal units A and B in a spatial configuration $C_0$. At any time $t$, the local oscillator frequency difference between two units is modeled as in section 3.4.2 by

$$f_i(t) = f_{off} + f_i(t).$$  \hspace{1cm} (5.27)

and the phase difference at time $t$ between transmitter and receiver is $\phi_d(t)$ such that (section 3.4)

$$\phi_d(t) = 2\pi f_{off} t + 2\pi \int_0^t f_i(t) dt + \phi_0.$$  \hspace{1cm} (5.28)

When $\phi_d(t) = 2\pi$, unit counters differ by 1. The constant frequency offset is denoted $f_{off}$ and causes unit B’s timeslot counter to have a slightly different pace than unit A timeslot counter. The term $f_i(\cdot)$ is a random process related to the phase noise process $\phi_i(\cdot)$ by $f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$ and $\phi_0$ is the initial phase shift between the local oscillators. It is uniformly distributed over $[0, 2\pi]$ and constant over one mission.

The drift at time $t$ is defined as the time shift between the two units counters. The drift corrupts the value $N^B$ in (5.25) by adding a virtual $T_{\text{drift}}$ to the physical travel time $T_{\text{trav}}^C$. We propose now a scheme that allows distance estimation between the nodes even when their
local oscillators are subject to drift. Unit A sets the time reference.

**Transmission from A to B**  In the oversampled system, unit A starts transmitting when its counter indicates \(N'_{A\text{ start}}\) and unit B receives the signal when its counter indicates \(N'_{B}\) such that

\[
N'_{B} = N'_{A\text{ start}} + \frac{T^{c_0}(N'_{A\text{ start}})}{T_s} + \frac{T_s}{T_s} \int_{0}^{\frac{N'_{A\text{ start}} + N'_{A\text{ HW,tx}}}{T_s} + T_{\text{trav}}^{c_0}(N'_{A\text{ start}})} f_s(t) dt + \frac{1}{2\pi} \frac{T_s}{T_s} \phi_0 \right] + N'_{A\text{ HW,tx}} + N'_{B\text{ HW,rx}},
\]

(5.29)

where the primed variables \(N'_{B}\), \(N'_{A\text{ start}}\), \(N'_{A\text{ HW,tx}}\) and \(N'_{B\text{ HW,rx}}\) are obtained from \(N_{B}\), \(N_{A\text{ start}}\), \(N_{A\text{ HW,tx}}\) and \(N_{B\text{ HW,rx}}\) by multiplication with the oversampling factor \(\frac{T_s}{T_s}\). \(T^{c_0}(N'_{A\text{ start}})\) represents the wave travel time from unit A to unit B in configuration \(C_0\) when unit A transmits with its counter at \(N'_{A\text{ start}}\). \(\frac{1}{2\pi} \frac{T_s}{T_s} \phi_0\) is the initial local oscillator counter shift. The integral is proportional to the phase shift between the two units, and \((N'_{A\text{ start}} + N'_{A\text{ HW,tx}})/T_s + T_{\text{trav}}^{c_0}(N'_{A\text{ start}})\) is the arriving time of the signal at the analog part of unit B.

If unit B estimates its distance to unit A, it obtains \(\tilde{d}_{BA}\) such that

\[
\tilde{d}_{BA} = cT'_{s}N'_{B}
\]

\[
= cT'_{s} \left( N'_{A\text{ start}} + N'_{A\text{ HW,tx}} + N'_{B\text{ HW,rx}} \right)
\]

\[
+ cT'_{s} T_{trav}^{c_0}(N'_{A\text{ start}}) + \frac{T_s}{T_s} \int_{0}^{\frac{N'_{A\text{ start}} + N'_{A\text{ HW,tx}}}{T_s} + T_{\text{trav}}^{c_0}(N'_{A\text{ start}})} f_s(t) dt + \frac{1}{2\pi} \frac{T_s}{T_s} \phi_0 \right] .
\]

(5.30)

The estimate is corrupted by the hardware delay, the frequency offset integral and the initial phase offset difference. These terms are partially compensated in the following steps using a transmission from B to A.

**Transmission from B to A**  Unit B now transmits when its counter is at \(N'_{B\text{ start}}\). The signal arrives at unit A when the counter of unit A indicates \(N'_{A}\) such that:

\[
N'_{A} = N'_{B\text{ start}} + \frac{T^{c_0}(N'_{B\text{ start}})}{T_s} - \frac{T_s}{T_s} \int_{0}^{\frac{N'_{B\text{ start}} + N'_{B\text{ HW,tx}}}{T_s} + T_{\text{trav}}^{c_0}(N'_{B\text{ start}})} f_s(t) dt - \frac{1}{2\pi} \frac{T_s}{T_s} \phi_0 \right] + N'_{B\text{ HW,tx}} + N'_{A\text{ HW,rx}},
\]

(5.31)
where the primed variables $N^A$, $N^B_{\text{start}}$, $N^B_{\text{HW,tx}}$ and $N'^A_{\text{HW,rx}}$ are obtained from $N^A$, $N^B_{\text{start}}$, $N^B_{\text{HW,tx}}$ and $N'^A_{\text{HW,rx}}$ after multiplication by $\frac{T_s}{T'}$.

**Distance estimation within the same mission execution** Let us define

\[
\alpha^A_{C_0} = (N^A_{\text{start}} + N'^A_{\text{HW,tx}})T'_s + T'_{\text{trav}}(N^A_{\text{start}}), \quad (5.32)
\]

\[
\alpha^B_{C_0} = (N^B_{\text{start}} + N'^B_{\text{HW,tx}})T'_s + T'_{\text{trav}}(N^B_{\text{start}}). \quad (5.33)
\]

Distance $\tilde{d}_{A,B}(C_0)$ between the units in spatial configuration $C_0$ is estimated after units have shared their counter value $N^A$ and $N^B'$. Summing $N^A$ and $N^B'$ balances the effect of the frequency offset and initial phase shift.

\[
\tilde{d}_{A,B}(C_0) = \frac{1}{2} c T'_s (N^A' + N^B')
\]

\[
\tilde{d}_{A,B}(C_0) = \frac{1}{2} c T'_s \left( N^A_{\text{start}} + N^B_{\text{start}} + N'^A_{\text{HW,tx}} + N'^B_{\text{HW,tx}} + N'^A_{\text{HW,rx}} \right)
\]

\[
+ \frac{1}{2} c T'_s \left[ \frac{T'_{\text{trav}}(N^A_{\text{start}})}{T'_s} + \frac{T_s}{T'_s} \int f_\alpha(t) dt + \frac{1}{2\pi} \frac{T_s}{T'} \phi_0 \right]
\]

\[
+ \frac{1}{2} c T'_s \left[ \frac{T'_{\text{trav}}(N^B_{\text{start}})}{T'_s} - \frac{T_s}{T'_s} \int f_\alpha(t) dt - \frac{1}{2\pi} \frac{T_s}{T'} \phi_0 \right]. \quad (5.35)
\]

After a known delay $\Gamma$ multiple of $T'_s$, units occupy spatial configuration $C_1$, and assuming $\alpha^A_{C_0} \leq \alpha^A_{C_1} + \Gamma$ (unit A performs the second distance estimation after unit B has performed its first distance estimation), $\tilde{d}_{A,B}(C_1)$ is estimated.

\[
\tilde{d}_{A,B}(C_1) = \frac{1}{2} c T'_s \left( 2 \frac{\Gamma}{T'_s} + N^A_{\text{start}} + N^B_{\text{start}} + N'^A_{\text{HW,tx}} + N'^B_{\text{HW,tx}} + N'^A_{\text{HW,rx}} \right)
\]

\[
+ \frac{1}{2} c T'_s \left[ \frac{T'_{\text{trav}}(\Gamma + N^A_{\text{start}})}{T'_s} + \frac{T_s}{T'_s} \int f_\alpha(t) dt + \frac{1}{2\pi} \frac{T_s}{T'} \phi_0 \right] \quad (5.36)
\]

\[
+ \frac{1}{2} c T'_s \left[ \frac{T'_{\text{trav}}(\Gamma + N^B_{\text{start}})}{T'_s} - \frac{T_s}{T'_s} \int f_\alpha(t) dt - \frac{1}{2\pi} \frac{T_s}{T'} \phi_0 \right]
\]
The difference \( \tilde{d}_{AB} = \tilde{d}_{A,B}(C_1) - \tilde{d}_{A,B}(C_0) - c^\Gamma \) is computed in (5.37). The constant digital delays cancel

\[
\tilde{d}_{AB} = \frac{1}{2} c T_s \left[ \frac{T_{C_1}^C (\Gamma + N_{\text{start}}^A)}{T_s} + \frac{T_s}{T_s} \int_0^{\alpha_{c_1}^B + \Gamma} f_\epsilon(t) dt + \frac{1}{2\pi} \frac{T_s}{T_s} \phi_0 \right]
\]

\[
+ \left[ \frac{T_{C_0}^C (\Gamma + N_{\text{start}}^B)}{T_s} - \frac{T_s}{T_s} \int_0^{\alpha_{c_0}^B + \Gamma} f_\epsilon(t) dt - \frac{1}{2\pi} \frac{T_s}{T_s} \phi_0 \right]
\]

(5.37)

Interesting remarks can be made on (5.37) after assuming for simplification that the quantities between the brackets are integers. In that special case

\[
\tilde{d}_{AB} = \frac{1}{2} c \left( \frac{T_{C_1}^C (\Gamma + N_{\text{start}}^A)}{T_s} - \frac{T_{C_0}^C (\Gamma + N_{\text{start}}^A)}{T_s} \right)
\]

\[
+ \left[ \frac{T_{C_1}^C (\Gamma + N_{\text{start}}^B)}{T_s} - \frac{T_{C_0}^C (\Gamma + N_{\text{start}}^B)}{T_s} \right]
\]

\[
+ T_s \int f_\epsilon(t) dt - T_s \int f_\epsilon(t) dt
\]

(5.38)

In case the quantities inside the brackets are not integers, the right-hand-side term of (5.38) is an upper bound for \( \tilde{d}_{AB} \). We discuss the validity of (5.38) in the special case where \( C_1 = C_0 \).

The two spatial configurations are identical, i.e. the nodes have not moved,

\[
T_{C_1}^C (\Gamma + N_{\text{start}}^B) = T_{C_0}^C (\Gamma + N_{\text{start}}^B) = T_{C_1}^C (\Gamma + N_{\text{start}}^A) = T_{C_0}^C (\Gamma + N_{\text{start}}^A)
\]

(5.39)

and

\[
\alpha_{c_0}^C = \alpha_{c_1}^C,
\]

(5.40)

\[
\alpha_{c_0}^B = \alpha_{c_1}^B.
\]

(5.41)
Therefore (5.38) is further simplified and the only error intrinsic to the procedure remains

\[ \tilde{d}_{AB} = \frac{1}{2} c T_s \left( \int_{\alpha_A}^{\alpha_B} f_e(t) dt - \int_{\alpha_A+\Gamma}^{\alpha_B+\Gamma} f_e(t) dt \right) \]  \hspace{1cm} (5.42)

The phase noise term in the case where the local oscillator contains a phase-locked loop (PLL) is described in section 3.4.3.2. It can be modeled as a zero-mean Gaussian distributed process with finite variance, this is therefore the distribution of the random variable \( \int f_i(t) dt \). Using the model proposed in sections 3.4.2 and 3.4.3 recalled in (5.27), the frequency offset \( f_{\text{off}} \) is constant and

\[ \tilde{d}_{AB} = \frac{1}{2} c T_s \left( \int_{\alpha_A}^{\alpha_B} f_i(t) dt - \int_{\alpha_A+\Gamma}^{\alpha_B+\Gamma} f_i(t) dt \right) \]  \hspace{1cm} (5.43)

\[ = \frac{1}{4\pi} c T_s (\phi(\alpha_B) - \phi(\alpha_A) + \phi(\alpha_A + \Gamma) - \phi(\alpha_B + \Gamma)) \]  \hspace{1cm} (5.44)

The estimated distance from the reference point defined by the spatial configuration \( C_0 \) is \( \tilde{d}_{AB} \). It is not exactly zero but equals the sum of zero-mean Gaussian random variables. The phase noise model from section 3.4.3 considers the \( \{ \phi \} \) as independent identically distributed zero-mean Gaussian random variables. Thus their sum is zero-mean Gaussian distributed. This distance difference is plotted in Fig. 5.12. Three units are involved: unit 3 and unit 8 are immobile, and unit 4 is immobile during the first mission execution (samples 1-82), and is moved during the second (samples 83-165) and the third execution (samples 166-end). In the first execution, the estimated distance oscillates around a mean value and is not strictly constant. This observation confirms the previously developed model, the distance estimate is corrupted by a zero-mean Gaussian noise caused by phase noise.

In the second and third mission execution, unit 4 is moving. In the second execution, unit 3 and unit 8 are static, and unit 4 is moving away from unit 3 along a line. During the third execution unit 4 is moving closer to unit 3 along the same line. The position estimated from the link 3-8 is also plotted to validate the measurements. The frequency offset between unit 3 and unit 8 is approximately constant during each mission execution (i.e. block of 82 samples), leading to a constant (over a repetition) distance offset. The mean position error is not null for the second and third mission execution. That is caused by the offsets \( N_{HW,tx}^A, N_{HW,rx}^B, N_{HW,tx}^B \) and \( N_{HW,rx}^A \) that are not constant from one mission execution to the other. When subtracting \( \tilde{d}_{A,B}(C_0) \) from \( \tilde{d}_{A,B}(C_1) \), the constant terms cannot be compensated. Therefore the displacement within a mission can be estimated with accuracy from the initial
position at the beginning of the mission execution, but not over several executions.

![Distance tracking in a dynamic configuration](image)

**Fig. 5.12:** Distance tracking in a dynamic configuration

To summarize this section, we have presented a differential ToA tracking algorithm for the RACooN units that is robust to frequency offset and hardware delay. This algorithm has low complexity and does not require a-priori estimation of the frequency offset and other internal delays. It can estimate the displacement between two units within the same mission execution by perfectly compensating the frequency offset and hardware delay, the accuracy of the position is only limited by phase noise. The proposed localization algorithm can work with any beacon. We consider now the transmission of sounding sequences and discuss the channel estimates.

### 5.5.3 Channel Impulse Response Magnitude

The CIR magnitude of link TX4-RX3 obtained for all positions of unit 4, all mission executions and all snapshots are stacked in Fig. 5.13 such that their temporal evolution is visible. Results for set 3 are in Fig. 5.13(a) and for set 4 in Fig. 5.13(b). A delay of 30 samples (30T_s=375 ns) is introduced for visibility and shifts the LOS peak. The channel is shorter than T_d defined in (5.17) and the rays significantly contributing to the CIR energy arrive with less than 1 µs delay from the LOS. Shadowing is typically constant over a distance of 10-50λ_c, and slow fading is typically constant over a distance shorter than 100λ_c. In the following we consider that the channel is
stationary in time because slow fading and shadowing are constant over the area described by the test-user animated by a behavioral movement. For both sets under investigation, unit 3 stays at its initial position and unit 4 is changing position every 200 snapshots, that is 5 repetitions of 40 snapshots. The 7 different positions are 50 cm apart from each other. Blocks of 200 snapshots can be clearly identified in Fig. 5.13(a). Within each of these 7 blocks the channel tap magnitude is approximately constant, and is strictly constant over blocks of 40 snapshots corresponding to one mission execution. When no movement occurs, the channel is changing very slowly and appears constant over one mission repetition (0.3 s). But it has changed within 75 s such that two mission executions show two different realizations.

This block structure is not visible in Fig. 5.13(b). In set 4, unit 4 is moved with the behavioral pattern discussed in section 5.3.3. The CIR magnitude is changing faster than in set 3. The behavioral pattern of unit 4 reduces the channel coherence time and the channel cannot be considered constant during $N_{\text{rep}} T_{\text{est}}$ in a PWAN. Yet we assume that the channel remains stationary over 200 realizations (duration $N_{\text{rep}} T_{\text{est}}$).

The magnitude of the main peak (LOS peak) decreases slowly from one position to the next as the distance between unit 3 and unit 4 is increasing. The average tap amplitude decrease corresponds to pathloss and will be quantified in section 5.5.9. The tap amplitude variations around the mean value are attributed to shadowing, in particular due to the pillar in the middle of the space in Fig. 5.2. This is observed at position 5 (snapshots 801-1000), unit 4 is close to the pillar that reflects and scatters the wave at that position. These reflected and scattered waves impinge at the receiver at the same time as the LOS wave since the path length difference is below the system spatial resolution $R_s$, thus enhancing the first channel tap amplitude.

### 5.5.4 Average Power Delay Profile

The average power delay profile of a channel is the squared magnitude of the CIR averaged over a set of realizations for which the channel is stationary, in our case for each position occupied by the units.

The power delay profile of the link TX4-RX3 for set 1 and position 1 is plotted in Fig. 5.14. At this position, unit 3 and unit 4 are 1.40 m apart. The y-axis denotes the baseband channel power, i.e. it contains the effect of the transmitter and receiver filter gain and the antenna gain.

The noise floor level and the pseudo-periodic ripples in it are commented in section 5.5.4.1. A strong LOS peak is visible at 30Tₙ delay. This delay is artificial and was added for
visibility. This LOS peak corresponds to the wave that propagates without reflection from the transmitter to the receiver, and to the waves scattered by objects in the vicinity of the transmit and receive antenna such that the path difference is lower than 3.75 m.

The transition from the noise floor to the LOS peak is not steep, although no physical wave can reach the receiver before the LOS wave. This effect is attributed to frequency offset that enlarges the main peak of the cyclic autocorrelation. Indeed we observed in simulations that the MSR degradation in Fig. 4.13 is due to the widening of the cyclic autocorrelation main peak. After convolution with the physical baseband channel, as described in (4.13), this latter is spread in time domain.

The power delay profile shows a peak at a delay $6T_s$ behind the LOS peak. Such a delay corresponds to a travel path difference of 22.5 m between the wave generating the LOS peak and the wave generating the reflected peak. This peak is attributed to multiple reflections since the room dimensions are approximately $8 \times 4$ m and the additional distance traveled by a wave that is reflected one time is less than 22.5 m. As the same peak is also observed in set 2 when unit 4 is at the same position (not shown), we conclude that back-and-front wall reflections do not cause this peak, since for set 2 the operator is standing between unit 4 and the back wall, shadowing the possible reflected waves.

The observed power delay profile is similar to [98] and fits the Saleh-Valenzuela model with one cluster. A similar profile has also been obtained in [5, 6] in a large office environment at 5.8 GHz and 100 MHz bandwidth. The cluster has a linear power decay on a dB scale, or an exponential decay on a linear scale. Indeed the cluster is formed by the rays bouncing forth and back in the vicinity of the transmitter and receiver before impinging at the received antenna. At each bounce the ray undergoes a delay and its power is divided by a factor that
is a function of the reflecting surface. This explains the cluster power decay.

The power delay profile is therefore approximated with an exponential power decay \([51, 98]\). The power delay profile plotted with a dB scale is fitted with a linear curve. An example is presented in Fig. 5.15. The estimated decay is 0.15 dB/ns and is comparable with numerical values estimated in [5] for large office environment (0.18 dB/ns). This power decay is neither dependent on the unit position nor on the movement, the estimated power decays for sets 1-4 range between 0.13 dB/ns and 0.18 dB/ns. Authors like [98] observed for similar environments but larger range (1 to 100 m) that the power delay profile is not correlated with the distance between transmitter and receiver but only with the surroundings of the transmitter and receiver. We did not observe this maybe because of the shorter distance range investigated and the too large similarities between the two environments.

### 5.5.4.1 Noise Floor and Non-Linearities Discussion

In this section we discuss the noise floor level. Understanding the noise floor level is important since it will affect the delay spread computation in section 5.5.5. We analyze data from set 10 and plot in Fig. 5.16 the power delay profile of link TX4-RX3 for all the 7 positions occupied by unit 4.
The power delay profile related to the physical baseband channel estimate is concentrated in the range 0.3-0.7 $\mu$s. The physical power delay profile is scaled by the path loss coefficient as expected. Outside the 0.3-0.7 $\mu$s range a noise floor is visible.

This noise floor is surprisingly changing with the position of unit 4. It increases when the distance between unit 3 and unit 4 decreases (positions 1-3) and inversely (positions 4-7). The noise floor level change has two causes: the multilevel transmission (section 5.4.2.2) and the baseband transmitter and receiver non-linearities.

- The noise floor level is affected by the multilevel transmission. Indeed when the receiver exploits a higher level ($\alpha_k$, $1 \leq k \leq T - 1$) training sequence instead of the lowest level ($\alpha_0$) sequence, it scales the received signal (including noise) with $\frac{\alpha_0}{\alpha_k}$ and the noise floor is changed by $20 \log_{10} \left( \frac{\alpha_0}{\alpha_k} \right)$. As $\alpha_k > \alpha_0$, the noise floor is decreased when the distance is increased. We would in that case expect only two noise floor levels in Fig. 5.16. Instead, an apparently continuous shift of the noise floor is observed.

- The effect previously described superposes with a noise term caused by the non-linearities at the transmitter and receiver that scales with the transmit signal power. The non-linearities degrade the cyclic autocorrelation properties of the chirp sequence, creating a deterministic term that adds to the noise.

In order to confirm our conjectures concerning the effect of the non-linearities on the channel estimate, we simulate a simplified sounding system made from two RACooN units. In this simplified baseband model sketched in Fig. 5.17, the output of the chirp generator at
Chapter 5 Wideband Network Channel Measurements

Fig. 5.16: Noise floor level (TX4-RX3, set 10)

Fig. 5.17: Simplified simulated system
the transmitter is affected by non-linearities in the baseband. The real and imaginary parts of the complex symbols at the output of the chirp generator are multiplied by a memoryless time-invariant non-linearity modeled by polynomial $P(X)$. The complex output is then sent through a baseband channel and scaled by a pathloss factor. At the receiver, a complex zero-mean AWGN with variance $N_0$ is added to the signal. The resulting signal is then processed as described in chapter 4.3.1. Non-linearities at the receiver are not modeled here, but their effect is similar at the transmitter. In the sequel we drop the averaging gain for simplicity and keep only the correlation gain, offering a processing gain up to $10 \log_{10}(N)$.

This simplified system is implemented with Matlab and the simulated CIR magnitude estimate is plotted in Fig. 5.18. We use a real baseband channel with arbitrarily chosen discrete taps $h = [3 1 2 1 2]$, the chirp samples have magnitude 1, and $g_{PL}$ denotes the pathloss amplitude scaling factor. The reference curve is obtained for $P(X) = X$ (no non-linearity) and 0 dB pathloss ($g_{PL} = 1$). The noise floor lies $10 \log_{10}\left(\frac{N}{N_0}\right)$ below the signal level, i.e. at -54 dB for $N_0 = 10^{-3}$ and N=255. This level is observed in Fig. 5.18 (blue curve).

A non-linearity is now introduced in the baseband at the transmitter with 0 dB pathloss. The non-linearity is the same in the I and Q branch. We use an arbitrary polynomial $P(X) = X + 0.1X^2$. After correlation at the receiver, the estimated CIR magnitude is plotted in Fig. 5.18 (green curve). The red curve with the round markers represents the channel $h$ overlaps with the blue curve and the green curve for the first 5 samples. The channel is correctly estimated and not spread in time-domain because no frequency offset occurs in the

![Fig. 5.18: Output of the simulated system](image)
simulation. But the noise floor of the estimate affected by the non-linearity is now uplifted by 25 dB. The non-linearity strongly degrades the system dynamic range by destroying the cyclic autocorrelation properties of the chirp sequence.

We illustrate this observation with the following example. Using \( P(X) = X + p_1 X^2 \), the non-linearities at the transmitter are such that

\[
P(\Re \{ x[k] \}) + jP(\Im \{ x[k] \}) = x[k] + p_1(\Re \{ x[k] \})^2 + jp_1(\Im \{ x[k] \})^2 = x[k] + p_1 x_{NL}[k],
\]

with

\[
x_{NL}[k] = (\Re \{ x[k] \})^2 + j(\Im \{ x[k] \})^2.
\]

At the receiver, \( y[k] \) is received

\[
y[k] = g_{PL} (h \odot (x + p_1 x_{NL})) [k] + n[k]
\]

and after correlation

\[
(y \otimes x)[k] = g_{PL} N h[k] + n_1[k] + n_2[k]
\]

with

\[
\begin{align*}
n_1[k] &= (n \otimes x)[k], \\
n_2[k] &= g_{PL} P_1 ((h \otimes x_{NL}) \otimes x) [k]
\end{align*}
\]

The noise terms comprise now a component \( n_1[k] \) caused by the AWGN at the receiver, and a component \( n_2[k] \) caused by the non-linearities. This second component scales with \( g_{PL} \). Therefore a large path loss attenuates the non-linear effects at the transmitter, but the noise floor, caused by \( n_1[k] + n_2[k] \), does not vary linearly with the pathloss. This effect is simulated in Fig. 5.18 with a pathloss of 6 dB (magenta curve). The simulation confirms that a strong path loss decreases the noise floor.

The pseudo-periodic pattern in the noise floor can be explained as follows. Assuming \( |x[k] = 1| \ \forall k \), let us write the chirp sample \( x[k] \) as \( \cos(\phi[k]) + j \sin(\phi[k]) \). After the non-linearity, the transmit signal can be written in a general way as \( \sum_n a_n \cos(n\phi[k]) + j \sum_n b_n \sin(n\phi[k]) \). The convolution with the channel \( h[\cdot] \) and the correlation with \( x[\cdot] \) are
linear operations, their output can again be written like previously. The result has trivially
the shape of a squeezed sinusoid that is clearly visible in Fig. 5.16 for position 3.
To summarize, we have shown in this section that the noise floor variations in the power
delay profile are caused by non-linearities in the units. The noise floor splits into two contri-
butions: the signal-independent AWGN term at the receiver, and non-linear terms scaled by
the pathloss coefficient. These latter are responsible for the pseudo-periodic pattern visible
in the noise floor.

5.5.4.2 Sounding Sequence Discussion

In this section we discuss the choice of the sounding sequence based on measured power
delay profile. We use data from set 4 and set 5, both are obtained in the same environment,
with the same behavioral pattern and positions, changing only the sounding sequence.
The power delay profile of the link TX4-RX7 is plotted in Fig. 5.19 for position 7 when unit
4 is moving. In set 4 the estimation is performed with a chirp sequence, and in set 5 with a
m-sequence. The training sequences (m-sequence and chirp sequence) are scaled such that
they have the same spectral power (except at DC).
The power delay profile obtained with a m-sequence as a training sequence shows determin-
istic peaks in the tail that are not visible in the power delay profile obtained with a chirp
sequence. These peaks are located very close to the physical baseband channel samples.
To make this visible a 0.7 µs delay is applied on the power delay profile in Fig. 5.19 and
Fig. 5.20. The receiver baseband non-linearities degrade the m-sequence cyclic autocor-
relation properties, and the effects on the channel estimate are stronger than when a chirp
sequence is used in the same conditions. These peaks have the same amplitude and same
position in the estimated power delay profile for all links (not shown), they are therefore not
caused by multiple reflections and are not specific of a receiver or transmitter.

Without a-priori knowledge of the non-linearity polynomials and of the channel, it is im-
possible to separate the channel contributions from the non-linearity peaks. Calculation of
the delay spread requires to isolate the channel contributions from the noise in the power
delay profile. This can be performed by setting a level threshold for the power delay profile
or defining a time-domain window if the maximum excess delay is a-priori known to cut
out the noise samples. Both possibilities are not applicable when a m-sequence is used as a
sounding sequence.
Removing the large peaks caused by non-linearities imposes to set a high threshold that dra-
matically reduces the dynamic range. The deterministic position of the non-linear peaks
restrains the use of the time-domain windowing to cases where the maximum excess delay
is short such that the window upper bound cuts out the first non-linearity peak but leaves the channel intact. This condition strongly restricts the channel admissible maximum delay spread. For these two reasons the use of m-sequence as sounding sequence in the RACooN lab is not recommended. Data from set 5 are not further exploited and experimentally verify the conclusion of chapter 4.

![Graph showing time in microseconds and attenuation in dB for m-sequence (set 5) and chirp sequence (set 4).]

**Fig. 5.19:** Same channel estimated with chirp and m-sequence (TX4 RX7, position 4)

**Reciprocity and delay spread** The average power delay profile of links TX4-RX9 and TX9-RX4 is plotted in Fig. 5.20. The curves superimpose, the noise floor are at the same level, and the estimated effective baseband average power delay profile is the same if estimation is performed in one direction or in the other. The ripples in the noise floor were commented in 5.5.4.1. This observation is in line with the reciprocity principle that states that the wireless channel does not depend on the direction used to estimate the channel.

Based on this observation, we will present and comment the root mean square (RMS) delay spread estimated for selected sets and links. To compute the delay spread we consider that the delay spread values of a bidirectional link are realizations of the same random variable and compute the mean RMS delay spread over the realizations obtained by estimating the channel in both directions.
5.5 Measurement Evaluation

![Graph showing link reciprocity between unit 4 and unit 9 (set 1)]

**Fig. 5.20**: Link reciprocity between unit 4 and unit 9 (set 1)

### 5.5.5 Delay Spread

The delay spread is the span of the path delays. It is estimated by the RMS delay spread of the channel, which is defined as

\[
\tau_{\text{RMS}} = \sqrt{ \frac{\int_0^{\tau_{\text{max}}} (\tau - \langle \tau \rangle)^2 p(\tau) d\tau}{\int_0^{\tau_{\text{max}}} p(\tau) d\tau}} \tag{5.51}
\]

where \( p(\tau) \) is the power delay profile and

\[
\langle \tau \rangle = \frac{\int_0^{\tau_{\text{max}}} \tau p(\tau) d\tau}{\int_0^{\tau_{\text{max}}} p(\tau) d\tau} \tag{5.52}
\]

is the mean delay and \( \tau_{\text{max}} \) is the maximum excess delay.

The delay spread can be expressed in the (delay-)frequency domain by the coherence bandwidth \( B_c \). The coherence bandwidth is inversely proportional to the RMS delay spread and

\[
B_c \approx \frac{1}{\tau_{\text{RMS}}} \tag{5.53}
\]

The samples of the power delay profile that are in the noise floor do not contribute to the computation of the delay spread. They can be removed by windowing the CIR in time...
domain if the window position and width can a-priori be determined. This is not possible for a scenario with mobility since the delay of the LOS component can change as well as the excess delay. A second possibility is to select the samples contributing to the delay spread by defining a threshold for the noise floor. As we have seen in 5.5.4 that the noise floor is not constant, it is determined after visual inspection of the power delay profile of each set, each link and each position.

**Effect of the position** We collect the realizations of the RMS delay spread for the bidirectional links at position 6 for set 1 and summarize in Table 5.5 the first and second order statistics of the RMS delay spread. This table has entries in the upper diagonal only since bidirectional links are considered.

The mean RMS delay spread shows no visible correlation with distance between the units, as for example for the links 4-9 and 4-8: the mean RMS delay spread is doubled (12.8 ns for link 4-9 and 26.2 ns for link 4-8) whereas units 8 and unit 9 are close to each other. In the other side, links 8-9 and 4-9 have a similar mean RMS delay spread (14.3 ns for link 8-9, 12.8 ns for link 4-9) but the distance between unit 8 and unit 9 is much shorter than between units 4 and 9). The same remark was done in [98].

The standard deviation is approximately the same for all links, and not strictly null because the channel is slowly varying during the observation time (Fig. 5.13).

<table>
<thead>
<tr>
<th>unit</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>19.6 (0.4)</td>
<td>20.3 (1.7)</td>
<td>16.1 (1.3)</td>
<td>11.6 (0.2)</td>
<td>14.6 (1.9)</td>
</tr>
<tr>
<td>4</td>
<td>16.2 (0.9)</td>
<td>26.2 (1.3)</td>
<td>12.8 (0.5)</td>
<td>21.9 (1.8)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>16.4 (1.6)</td>
<td></td>
<td>23.4 (2.9)</td>
<td>20.3 (0.5)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>14.3 (1.6)</td>
<td>14.6 (1.3)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>16.2 (0.8)</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: Mean RMS delay spread for set 1, position 6 [ns] (standard deviation [ns])

We investigate now the evolution of the mean RMS delay spread over the different positions. The mean RMS delay spread estimated for each position and selected links involving unit 4 is plotted in Fig. 5.21(a). The RMS delay spread of these links shows no correlation with the distance between the units, for example the variation of the RMS delay spread is not monotonous when unit 4 moves from one position to the next. This observation is also in line with observations in [98].

The RMS delay spread first and second order statistics are computed from the realizations collected over the 7 positions are summarized in Table 5.6. The mean RMS delay spread
5.5 Measurement Evaluation

values are very close to the ones estimated for position 6 in Table 5.5, but the standard deviation is higher in Table 5.6 for the links involving unit 4. This is an artifact caused by the noise floor threshold that is set manually for each position. As unit 4 is moved, the noise floor is shifted and the threshold for the RMS delay spread calculation needs to be set differently. This operation is performed manually and affects $\tau_{\text{max}}$, and consequently the RMS delay spread.

To summarize our observations, in set 1 the RMS delay spread is determined by the environment and not by the unit positions. We compare now these conclusions with data obtained for set 2.

**Effect of the behavioral pattern** We compute now the RMS delay spread first and second order statistics for set 2 and position 6 and summarize these results in Table 5.7. The mean values are very close to the ones obtained for set 1 in Table 5.5. The behavioral pattern has no effect on the mean RMS delay spread. The standard deviation of the RMS delay spread for links involving unit 4 is strongly increased with respect to Table 5.5. The channel undergoes
larger changes when unit 4 has a behavioral pattern. This increases the standard deviation of the RMS delay spread evaluated at one position.

<table>
<thead>
<tr>
<th>unit</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>15.9 (2.8)</td>
<td>21.1 (1.4)</td>
<td>16.2 (1.4)</td>
<td>12.8 (1.1)</td>
<td>13.6 (1.3)</td>
</tr>
<tr>
<td>4</td>
<td>19.6 (3.7)</td>
<td>24.4 (4.7)</td>
<td>16.3 (3.3)</td>
<td>20.8 (3.2)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>16.4 (1.0)</td>
<td>19.7 (2.1)</td>
<td>19.5 (1.6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>14.4 (1.4)</td>
<td>14.1 (1.6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>15.4 (0.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7: Mean RMS delay spread for set 2, position 6 [ns] (standard deviation [ns])

We plot now in Fig. 5.21(b) the mean RMS delay spread of selected links for the seven positions. When compared with Fig. 5.21(a), the mean RMS delay spread has a lower variability in set 2. This effect is attributed to the averaging performed over a larger number of independent channel realizations generated by the behavioral pattern of unit 4. For set 1 we saw in Fig. 5.13 that the channel realizations obtained during each mission repetition were similar to each other.

**RMS delay spread distribution** The RMS delay spread cumulative distribution function of two static bidirectional links is plotted in Fig. 5.22(a) for selected links of set 3 and set 4 (all positions). Links 9-7 and 8-7 are chosen because they are static for both sets, and we are interested in the influence of moving unit 4 on the RMS delay spread of these static links. Measurements for set 3 are performed when units are all perfectly static, whereas measurements in set 4 are performed while giving unit 4 a behavioral pattern. The cumulative
distribution function shape of link 9-7 is unchanged between set 3 and set 4. The cumulative
distribution function of the two links only differ by a shift, i.e. by their mean value. The
behavioral pattern of unit 4 and the operator presence in set 4 do not affect the distribution
of the RMS delay spread of the static links in these scenarios.

We observe now in Fig. 5.22(b) the behavioral pattern effect on links 9-4, 8-4, 9-7 and
9-8. Links 9-4 and 8-4 involve unit 4, whereas link 9-7 and 8-9 are static. The cumulative
distribution function of static links 9-7 and 8-9 is plotted with dotted lines and is the same as
in Fig. 5.22(a). They have a steeper slope, i.e. a smaller second moment, than the cumulative
distribution function of links 9-4 and 8-4 which are plotted with full lines. The behavioral
movement of unit 4 leads to a higher variability of the CIR estimates and larger variations of
the RMS delay spread around its mean value.

In the next steps we will characterize the temporal variations of the channel. This requires
before all a frequency offset compensation.

5.5.6 Frequency Offset

Although the unit local oscillators are tuned with a Rubidium source before the start of
the measurement campaign, the instantaneous frequencies of any two units are never strictly
equal. The Rubidium tuning procedure will setup the local oscillators only up to a certain
precision specified in [124] as 5 Hz in the 80 MHz sampling clock.

Frequency and phase offsets are evaluated using the phase of the LOS peak for set 1 and
set 3 for each mission execution in set 1. The frequency offset is assumed constant but the
initial phase at the start of each mission is changing since the clocks are reinitialized at every
mission execution. The frequency offset is estimated from the rotation of the LOS channel
tap in the complex plane. The two units for which the frequency offset is estimated must be
immobile, otherwise Doppler frequency terms add to the frequency offset term. The average
frequency offset over all the executions of a mission is calculated, and then averaged with the
frequency offset of the link estimated in the reverse direction. The result is such that when
the frequency offset estimated from A to B is \( f_{\text{off}} \), the frequency offset estimated from B to
A is exactly \(-f_{\text{off}}\).

To illustrate the frequency offset estimation, the phase of the LOS tap of the link TX8-RX10 is plotted in Fig. 5.23 for the first mission execution. The link TX8-RX10 is static
and there is no moving scatterer in the room, thus neither Doppler shift nor Doppler spread
affect the result. The phase of the signal is linearly changing with time, the slope of the linear
Chapter 5 Wideband Network Channel Measurements

curve fitting the measured phase gives the frequency offset between the two units. The small variations of the phase around the linear approximation are caused by phase noise.

![Graph showing phase and linear approximation](image)

\[ y = -0.0042x + 0.08 \]

**Fig. 5.23:** Frequency and phase offset estimation (TX8-RX10 repetition 1)

In contrary to what we assumed previously, measurements show that the frequency offset is not strictly constant. In Fig. 5.24 the estimated frequency offset of the directional link TX8-RX10 and TX10-RX8 for set 1 and set 2 is plotted. The frequency offset changes its sign when the direction of estimation is changed, since the clock reference is changed. The frequency offset estimated per mission repetition oscillates around a time-constant value that is used later to compensate the frequency offset of all sets. Slight changes of the frequency offset may be among others caused by changes of the room temperature. Therefore an accurate frequency offset correction requires frequency offset estimation for each set. This is possible for static links, but not for links involving units with a behavioral pattern because the phase plotted in Fig. 5.23 contains additional Doppler frequency terms.

The average frequency offset for set 1 is in Table 5.8, where the entries are averaged over the bidirectional link and the 7 positions. All entries (expressed in rad.s\(^{-1}\)) are below 5 Hz as expected after the Rubidium tuning procedure. Unit 3 and unit 4 have a small frequency offset (0.14 rad.s\(^{-1}\)), therefore the frequency offsets of units 7-10 with unit 3 and unit 4 are nearly identical. Similarly units 9 and 10 have a small frequency offset (0.36 rad.s\(^{-1}\)), therefore the frequency offsets of units 9 and unit 10 with units 3-8 is nearly identical.

Using data from Table 5.8, Fig. 5.25 shows the frequency offset effect and how it is compensated for a channel tap. This figure shows the LOS tap of link TX8-RX10 in the real-
imaginary plane for the first mission execution at position 1. Due to the frequency offset, the raw data are rotating (cross markers) in this plane with the average angular frequency read in Table 5.8. This rotation is compensated by multiplying each sample with a complex exponential such that the phase of the samples is aligned to the first sample phase (circle markers). The compensation is not perfect because the frequency offset value is a mean value. We saw indeed in Fig. 5.24 that the frequency offset is not strictly constant in time but a presumed stationary random process.

The initial phase of the first tap is a random value that depends on the phase of the local oscillator at the beginning of the mission. The local oscillator are reinitialized at the start of each mission, and their initial phase values are independent. This value must be estimated for each mission if it has to be compensated. Fig. 5.25 shows (square markers) the samples after initial phase and frequency offset correction. The samples are now concentrated around

Table 5.8: Angular frequency offset estimate for set 1 [rad.s\(^{-1}\)] (standard deviation [rad.s\(^{-1}\)])

<table>
<thead>
<tr>
<th>unit</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.14 (0.76)</td>
<td>-0.79 (0.62)</td>
<td>2.28 (0.53)</td>
<td>5.67 (0.58)</td>
<td>6.06 (0.65)</td>
</tr>
<tr>
<td>4</td>
<td>-0.80 (0.56)</td>
<td>2.20 (0.89)</td>
<td>5.65 (0.70)</td>
<td>6.05 (0.81)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.03 (0.53)</td>
<td>6.54 (1.11)</td>
<td>6.78 (0.88)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3.43 (0.53)</td>
<td>3.81 (0.52)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>0.36 (0.73)</td>
<td></td>
</tr>
</tbody>
</table>
the real axis. This is due to the combined effect of imperfect frequency offset compensation and phase noise that slightly rotates the samples around the plane origin.

Fig. 5.25: Frequency and phase offset compensation

5.5.7 Doppler Spectrum

Time-varying fading due to scatterer and/or transmitter/receiver movement results in a Doppler spread, i.e. each pure tone spreads in the time domain over a finite spectral bandwidth. The Fourier transform in the time-dimension of the channel time-autocorrelation function is defined as the Doppler power spectrum which is the average power of the channel as a function of the Doppler frequency.

The frequency offset is removed from the data for each mission execution and the taps of the first channel realization set the phase reference. The normalized Doppler spectrum of the link 4-3 averaged over the 5 mission executions is plotted in Fig. 5.26 for set 4 and each position of unit 4. The spectrum has a peak at 0 Hz due to the strong LOS component. The narrow Doppler spectrum observed is typical of the case where waves with significant energy arrive from a narrow angular range [25, 136]. The main peaks of the spectra obtained for each position in Fig. 5.26 are not strictly aligned on 0Hz. This is due to the Doppler shift, and to imperfect frequency offset compensation. Indeed the value used for compensation is an average value that slightly differs from the instantaneous value. Frequency offsets
above 3.52 Hz are visible (from table 5.3) due to the finite Doppler resolution of the system, corresponding to a linear speed in the propagation direction of 0.19 m.s$^{-1}$.

The shape of the Doppler power spectrum is similar to what was observed in [136] (fig.4 e) and [13] and can be approximated above the noise floor by a zero-mean Laplacian spectrum with probability density function

$$f_{f_D}(a) = \frac{1}{\sqrt{2k_D}} e^{-\sqrt{2k_D}|a|}$$  \hspace{1cm} (5.54)

where $k_D$ is a real constant related to the Doppler spread.

The Laplacian approximation is shown in Fig. 5.27 on a normalized Doppler spectrum for set 2 (link 3-4, position 1). In the frequency range $[-30 \text{ Hz}, 30 \text{ Hz}]$ the Laplacian spectrum is a good approximation for the spectrum. In the frequency range $[-50 \text{ Hz}, -30 \text{ Hz}]$ and $[30 \text{ Hz}, 50 \text{ Hz}]$ the Doppler spectrum approximation is below the noise floor.

In the following we discuss the meaning of parameter $k_D$. High $k_D$ values are responsible for fast channel changes. Indeed parameter $k_D$ is related to the channel Doppler spread. An approximation for the channel autocorrelation function is given by

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2k_D}} e^{-\sqrt{2k_D}|f|} e^{2\pi ft} df = \frac{1}{1 + (2\pi tk_D)^2}$$  \hspace{1cm} (5.55)
The autocorrelation function is reduced by $1/2$ for $t = t_{DS}$ and

$$t_{DS} = \frac{1}{2\pi k_D}$$ (5.56)

Time $t_{DS}$ is a measure for the channel coherence time (like $t = \tau_{RMS}$ when the autocorrelation function is $1/\sqrt{2}$), and therefore $k_D$ is a measure for the channel Doppler spread.

![Fig. 5.27: Laplacian approximation of the Doppler spectrum](image)

Parameter $k_D$ of the Laplacian approximation is a random variable, it takes a new realization for each unit position and for each mission execution. We consider all realizations of $k_D$ for the bidirectional links 7-8 and 10-4, for all positions. A total of 70 samples are collected and their empirical cumulative distribution function is plotted. A good fit is found with a lognormal distribution with parameters $\mu$ and $\sigma$ from (5.57). Fig. 5.28 shows the cumulative distribution function of these two links superposed with the lognormal approximation. The fit is better for the static link in Fig. 5.28(a) than for the moving link in Fig. 5.28(b).

$$f_{k_D}(a) = \frac{1}{a\sigma \sqrt{2\pi}} e^{-\frac{(\log_e(a)-\mu)^2}{2\sigma^2}}$$ (5.57)

The parameters of the lognormal distribution that fits the empirical cumulative distribution function of $k_D$ are given in Table 5.9 for set 10. Parameter $\mu$ is the mean value of $\log_e (k_D)$ estimated over all the bidirectional link realiza-
5.5 Measurement Evaluation

Fig. 5.28: Lognormal approximation of the distribution of $k_D$

The largest values for $\mu$ are obtained for links 3-4, 4-8, 4-9 and 4-10 for which a moving unit is involved. For these links the Doppler spread has a large average $k_D$, and is changing faster than the static links 7-8 and 7-9 for example. Link 4-7 has a low $\mu$ although it involves unit 4 since LOS is for this link always insured. To the contrary, links 3-4, 4-8, 4-9 and 4-10 are partly shadowed by the operator who is standing between unit 4 and the back wall.

Parameter $\sigma$ is the standard deviation of $\log_e (k_D)$ which is Gaussian distributed, and a measure for the variability of $k_D$. Static links (3-7, 3-9 for example) have a smaller $\sigma$ than the links involving moving unit 4 (except for 4-7). This behavior was expected since links involving unit 4 are affected by the operator presence and the behavioral pattern imposed to unit 4.

<table>
<thead>
<tr>
<th>RX(↓), TX (→)</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-0.85;0.61</td>
<td>-1.81;0.17</td>
<td>-1.30;0.38</td>
<td>-1.55;0.17</td>
<td>-1.45;0.23</td>
</tr>
<tr>
<td>4</td>
<td>-1.62;0.25</td>
<td>-0.74;0.74</td>
<td>-0.82;0.48</td>
<td>-0.91;0.66</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-1.62;0.18</td>
<td>-1.81;0.17</td>
<td>-1.85;0.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-1.50;0.23</td>
<td>-1.50;0.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-1.47;0.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: Lognormal distribution parameters set 10 ($\mu; \sigma$)
5.5.8 RMS Doppler Spread

The Doppler-frequency spread is estimated by the RMS Doppler spread defined by:

$$f_{D,\text{RMS}} = \sqrt{\frac{\int_{W} (f_{D} - \langle f_{D} \rangle)^2 p(f_{D}) df_{D}}{\int_{W} p(f_{D}) df_{D}}}$$  \hspace{1cm} (5.58)$$

where $W$ is the frequency band of width $2f_{\text{D,max}}$ centered on 0, $f_{\text{D,max}}$ is given in Table 5.3, and:

$$\langle f_{D} \rangle = \frac{\int_{W} f_{D} p(f_{D}) df_{D}}{\int_{W} p(f_{D}) df_{D}}.$$  \hspace{1cm} (5.59)$$

The Doppler spread can be expressed in time domain by the coherence time $T_{c}$ in (5.60). The coherence time is inversely proportional to the RMS Doppler spread and is a measure for how fast the channel is changing in time.

$$T_{c} \approx \frac{1}{f_{D,\text{RMS}}}.$$  \hspace{1cm} (5.60)$$

The RMS Doppler spread values for all links in set 3 and set 4 are presented in Table 5.10 and Table 5.11. The RMS Doppler spread of the static links are identical in set 3 and set 4, meaning that the position of unit 4 has no influence on the Doppler spread of the links. The coherence time is much larger than the effective estimation duration $T_{\text{est}}$ and is approximately equal to $N_{\text{rep}} T_{\text{est}}$ in Table 5.3. Therefore the changes in all the channels can be neglected during one mission duration in set 3. For the links involving unit 4, the nomadic mobility investigated in set 3 does not influence the mean RMS Doppler spread, which has similar values for all the links in Table 5.10. For set 4 the same conclusion as for set 3 applies to all links that do not involve unit 4. Channels involving unit 4 are still constant during one channel matrix estimation, but the changes cannot be neglected anymore during $N_{\text{rep}} T_{\text{est}}$. The behavioral pattern has a strong effect on the mean RMS Doppler spread of links involving moving unit 4. They have a RMS Doppler spread twice as large as in set 3. This is an expected behavior, the behavioral pattern of unit 4 causes the channels involving this unit to change faster.
5.5 Measurement Evaluation

Table 5.10: RMS Doppler spread set 3 [Hz] (standard deviation [Hz])

<table>
<thead>
<tr>
<th>unit</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.22 (0.53)</td>
<td>3.27 (1.01)</td>
<td>3.51 (0.79)</td>
<td>3.09 (0.53)</td>
<td>3.36 (0.55)</td>
</tr>
<tr>
<td>4</td>
<td>3.46 (1.01)</td>
<td>3.49 (0.74)</td>
<td>3.45 (0.85)</td>
<td>3.50 (0.47)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.18 (0.44)</td>
<td>3.37 (1.58)</td>
<td>3.36 (0.56)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3.19 (0.49)</td>
<td>3.48 (0.73)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4.35 (1.72)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.11: RMS Doppler spread set 4 [Hz] (standard deviation [Hz])

<table>
<thead>
<tr>
<th>unit</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7.49 (2.24)</td>
<td>3.10 (0.49)</td>
<td>3.54 (0.85)</td>
<td>3.34 (0.80)</td>
<td>3.27 (0.49)</td>
</tr>
<tr>
<td>4</td>
<td>7.46 (2.11)</td>
<td>7.54 (2.00)</td>
<td>7.38 (2.51)</td>
<td>7.67 (2.15)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.13 (0.44)</td>
<td>3.39 (1.55)</td>
<td>3.18 (0.55)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3.26 (0.48)</td>
<td>3.47 (0.65)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4.57 (1.90)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.5.9 Pathloss Evaluation

5.5.9.1 Friis Formula

The mean path loss is the distance-dependent average signal strength attenuation. Path loss is the mean signal power attenuation caused among others by propagation, scattering, reflection and absorption. In free space this mean path loss is range-dependent and the received signal power is given by the Friis formula

\[ P_r = P_t \left( \frac{\lambda_c}{4\pi d} \right)^\gamma G_r G_t \]  

(5.61)

where
- \( P_r \) is the received power,
- \( P_t \) is the transmit power,
- \( \lambda_c = \frac{c}{f_c} \) is the wavelength,
- \( c \) is the speed of light in the medium,
- \( f_c \) is the carrier frequency
- \( G_r \) is the receive antenna gain,
- \( G_t \) is the transmit antenna gain,
- \( \gamma \) the path loss exponent
- and \( d \) is the distance between transmitter and receiver.
When all the variables enumerated above are constant except for $d$, a compact path loss model is \[ P_t = \Xi \left( \frac{d_{\text{ref}}}{d} \right)^\gamma \] (5.62)

where
- $\Xi$ is a constant
- and $d_{\text{ref}}$ is a reference distance for the antenna far-field.

In realistic propagation conditions the path loss exponent varies from 2.5 to 6 [88].

5.5.9.2 Estimation

Pathloss is a valuable figure for cellular scenario because it defines the cell bounds. For a PWAN scenario, the receiver can only move over distances of a few meters. This distance represents up to $100\lambda_c$ and over this distance pathloss is constant. Therefore in the following we will give an attenuation factor that is strictly speaking not the pathloss since it aggregates the medium and large scale fading.

Pathloss estimation is performed when the receiver is in the far field of the transmitter. The far-field region, also called Fraunhofer region, of a transmitter is the region beyond the far-field distance $d_{\text{ff}}$ [93] given by

\[ d_{\text{ff}} = \frac{2D^2}{\lambda_c}, \] (5.63)

where $D$ is the largest physical linear dimension of the antenna. Additionally,

\[ d_{\text{ff}} \gg D \]

and

\[ d_{\text{ff}} \gg \lambda_c. \]

Pathloss is specific for an environment, a propagation medium and the antennas position and height. Sets 1-2 are therefore considered separately from sets 3-10. The pathloss is estimated by the slope of the linear curve that fits the log-log-curve distance verses average CIR power. The CIR power is computed by summing the magnitude squared of the CIR taps that are above the noise floor level. The slope of the linear curve is $-10\gamma$, where $\gamma$ is the path loss exponent. The noise floor level is the same that is used to estimate the RMS delay spread in 5.5.5.
The pathloss is computed using data from set 1 and set 2. Two bidirectional links involving units located in a large distance range are selected (link 3-4 and link 8-4) and plotted in Fig. 5.29. The slope of the linear fit of the log-log curve distance versus CIR energy is computed and averaged over the two sets. The linear curves for all links have the same slope determined by the pathloss exponent and differ by a small offset. The difference between the links involving the same units (TX4-RX3, TX3-RX4 for example) is due to the noise floor threshold that is manually adjusted for each position and each link, as detailed in 5.5.4.2. The offset between links involving different units (4-3, 8-4 for example) is due to slightly different antenna gains and filter gains in the units. The antenna and filter gains are part of the baseband channel and are neither estimated nor compensated separately in this measurement campaign.

The estimated average pathloss is $\gamma = 1.7$ for set 1 and set 2 in the environment represented in Fig. 5.2. This value is below 2 which is the pathloss exponent value in free space. This can happen because the estimation for $\gamma$ is computed over a small distance range (5 m) over which shadowing cannot be separated. Pathloss values below 2 for indoor channel measurements are also reported in [93, 98] and denote a waveguide effect.

The pathloss estimated for the environment represented in Fig. 5.2 is obtained after using bidirectional links from sets 3, 4, 6 and 8. The pathloss exponent is estimated as $\gamma = 2.0$. This value is closer to the reference pathloss $\gamma = 2$ obtained in free space. A reason for this may be the larger amount of data available for the second environment that allows to average out shadowing effects.

### 5.5.10 Channel Tap

Before investigating the channel tap distribution, the frequency offset is corrected and the CIRs are normalized to isolate the channel fast fading behavior. The CIRs are normalized such that the average CIR power per link and position is unitary. This normalization removes the pathloss and shadowing effects.

#### 5.5.10.1 Ricean Distribution

The statistical characterization of short term fading relies on the central limit theorem [83]. This theorem states that the sum of $n$ independent random variables tends - when $n$ tends to infinity - to a Gaussian random variable. This approximation is good when $n$ is large, i.e. it is motivated for practical bandlimited communication systems. The bandlimited channels have a reduced temporal resolution equal to the inverse bandwidth. Each tap in the channel
model aggregates the contribution of the non-coherent rays arriving during a time window with duration equal to the system temporal resolution. For wideband systems up to a few 100 MHz this approximations is experimentally validated [106].

The Ricean distribution models the amplitude distribution of a zero-mean complex Gaussian random variable added to a constant value. The Rice probability density function is given by [88]

\[ f_x(x) = \frac{2x(K+1)}{\Omega}e^{-K\frac{(K+1)x^2}{\Omega}}I_0(2x\sqrt{\frac{K(K+1)}{\Omega}}), \quad x \geq 0. \] (5.64)

In this expression, \( x \) is the tap amplitude, \( \Omega \) is the average received power, \( K \) is the Ricean factor defined as the ratio of the power in the mean component of the channel to the power in the scattered component, and \( I_0(\cdot) \) is the 0th-order modified Bessel function of the first kind defined as

\[ I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{-x\cos(\theta)} \, d\theta \] (5.65)

When \( K = 0 \), the Ricean probability density function in (5.64) reduces to the Rayleigh probability density function.

We introduce \( s^2 \) as the power of the direct path and \( 2\sigma^2 \) as the power of the complex scattered
component. We have thus

\[ K = \frac{s^2}{2\sigma^2}, \quad \Omega = 2\sigma^2(K + 1), \]

and (5.64) can be rewritten for \( x \geq 0 \) as:

\[ f_x(x) = \frac{x}{\sigma^2} e^{-\frac{x^2 + x^2}{2\sigma^2}} I_0\left(\frac{xs}{\sigma^2}\right) \]

The distribution parameters \((K, \Omega)\) or \((s, \sigma)\) are computed as the distribution maximum likelihood estimation parameters from the measured data. The numerical fit operation is performed with the Matlab function \textit{MLE}.

5.5.10.2 First Channel Tap

**Ricean fit** We study the LOS tap amplitude in link TX8-RX10 (set 1, position 1 and mission execution 1). The cumulative distribution function of the LOS channel tap magnitude is plotted in Fig. 5.30 using the 40 taps realizations available. The empirical cumulative distribution function of the tap magnitude has a good fit with a Ricean cumulative distribution function.

**K-factor distribution** For each position, 5 realizations (as many as mission executions) of the Ricean K-factor gain are accessed for each directional link. A previous work [80] showed that the K-factor can be modeled with a lognormal distribution. Due to the small number of K-factor realizations available we cannot verify this and start from this assumption. The K-factors distribution is therefore given by the mean and the standard deviation of the Gaussian distribution random variable \(10 \log_{10}(K)\).

**Nomadic case** The K-factor estimated in set 1 and 3 is the "temporal K-factor" from [80] because units are static. Our K-factor values (around 40 dB) are larger than the ones reported in [80], a reason for this is the shorter distance between transmitter and receiver (1-3 m versus 10-30 m), providing a stronger LOS component in our setup.

**Behavioral pattern effect** We consider now a link involving moving unit 4. Link 4-8 in set 1 has a mean K-factor of 43 dB, the same link in set 2 has a mean K-factor of 23 dB. A similar
behavior is observed for all links involving unit 4. This effect is explained by two points. Firstly, the presence of the operator close to unit 4 in set 2 may affect the received signal. The human body is scattering the wave, leading to a scattering component with higher power than in set 1. The pathloss variations affecting the mean component are negligible when unit 4 moves over a few cm. Secondly, the behavioral pattern of unit 4 makes the wave addition at the receiver less coherent than for sets 1 and 3, thus the mean component power is decreased and the scattered component power increased, leading to a lower K-factor.

5.5.10.3 Other Taps

The empirical cumulative distribution function of the channel tap magnitude indexed from 2 to 10 is plotted in Fig. 5.31. The empirical distribution of these taps magnitude above the noise floor shows a good fit with a Ricean distribution, this observation is usual for low mobility channels [79]. Indeed transmitters, receivers and scatterers are static and many scattered contributions are coherent. Also the reflections of large smooth surfaces in the environments like walls, doors and windows are coherent and overlap with the diffuse scattering.
5.5 Measurement Evaluation

**Static link**  We summarize the K-factor distribution parameters for the static link TX9-RX7 (set 4, position 4) in Table 5.12. These values are estimated from the 10 realizations of the K-factor obtained by considering the 5 missions repetitions and the bidirectional links. The mean K-factor is approximately the same for all taps at a given position.

The empirical cumulative distribution function for taps 2-10 is plotted in Fig. 5.31 when unit 4 is at position 4. A Ricean distribution models appropriately taps 2 to 9, and the estimated K-factor shows no correlation with the increasing tap index. The median value of the tap magnitude decreases with the tap index, this behavior is expected since the power delay profile presents an exponential decay. The cumulative distribution function of tap 2 and 3 are crossing because tap 3 has a steeper slope than tap 2, but similar mean value.

![Rician tap distribution](image)

**Fig. 5.31:** Ricean tap distribution (TX9-RX7, set 4, position 4)

<table>
<thead>
<tr>
<th>K-factor(↓), tap (→)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>21.2</td>
<td>20.6</td>
<td>23.0</td>
<td>23.5</td>
<td>25.5</td>
<td>25.9</td>
<td>25.8</td>
<td>24.1</td>
<td>23.7</td>
</tr>
<tr>
<td>standard deviation</td>
<td>6.3</td>
<td>5.2</td>
<td>5.7</td>
<td>4.1</td>
<td>3.4</td>
<td>3.4</td>
<td>2.3</td>
<td>4.7</td>
<td>2.8</td>
</tr>
</tbody>
</table>

**Table 5.12:** Parameters of the Gaussian distribution for $10 \log_{10}(K)$ link 9-7 (set 4, position 4)

**Behavioral pattern effect**  We look now at the link TX8-RX4 for set 4 and position 4. In this example unit 4 moves with a behavioral pattern and we compare the estimated K-factors with the ones obtained for a static link. The mean K-factors of the taps are summarized in Table 5.13. In comparison to the case with no behavioral movement, the K factors drop by 20 dB. Two reasons explain this behavior. First, the operator next to unit 4 is an additional scatterer in the environment. The human body increases the scattering components...
and artificially reduces the K-factors. Secondly, the behavioral pattern of unit 4 makes the wave addition at the receiver less coherent (than without movement), thus the mean component power is decreased and the scattered component power increased, leading to a lower K-factor.

<table>
<thead>
<tr>
<th>K-factor($\downarrow$), tap ($\rightarrow$)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>6.7</td>
<td>4.2</td>
<td>-2</td>
<td>6.8</td>
<td>8.5</td>
<td>7.6</td>
<td>7.1</td>
<td>8.0</td>
<td>6.3</td>
</tr>
<tr>
<td>standard deviation</td>
<td>6.8</td>
<td>4.7</td>
<td>30</td>
<td>4.4</td>
<td>4.3</td>
<td>8.6</td>
<td>7.0</td>
<td>6.2</td>
<td>7.8</td>
</tr>
</tbody>
</table>

Table 5.13: Parameters of the Gaussian distribution for $10 \log_{10} (K)$ link 8-4 (set 4, position 4)

### 5.5.11 Correlations

**Definition** In this paragraph we summarize some properties of the correlation coefficient, also called cross-correlation coefficient. Let us consider complex vectors $\vec{x}$ and $\vec{y}$ such that $\vec{x} = [x_1, \ldots, x_M]$ and $\vec{y} = [y_1, \ldots, y_M]$. The mean values of the entries of $\vec{x}$, resp. $\vec{y}$, is $\langle \vec{x} \rangle$, resp. $\langle \vec{y} \rangle$ such that:

$$\langle \vec{x} \rangle = \frac{1}{M} \sum_{i=1}^{M} x_i$$

(5.69)

$$\langle \vec{y} \rangle = \frac{1}{M} \sum_{i=1}^{M} y_i.$$  

(5.70)

The correlation coefficient gives the quality of a linear least square fitting between the entries of $\vec{x}$ and the entries of $\vec{y}$.

For the linear fit

$$y = a + bx,$$

(5.71)

the least-square fit coefficients $b$ and $a$ are given by

$$b = \frac{\sum_{i=1}^{M} x_i^* y_i - M \langle \vec{x} \rangle^* \langle \vec{y} \rangle}{\sum_{i=1}^{M} |x_i|^2 - M |\langle \vec{x} \rangle|^2}$$

(5.72)

and

$$a = \langle \vec{y} \rangle \frac{\sum_{i=1}^{M} |x_i|^2 - \langle \vec{x} \rangle \sum_{i=1}^{M} x_i^* y_i}{\sum_{i=1}^{M} |x_i|^2 - M |\langle \vec{x} \rangle|^2}.$$  

(5.73)
If the variables are centered,

\[ \langle \bar{x} \rangle = 0 \tag{5.74} \]
\[ \langle \bar{y} \rangle = 0, \tag{5.75} \]

and (5.72) becomes

\[ b_c = \frac{\sum_{i=1}^{M} x^*_i y_i}{\sum_{i=1}^{M} |x_i|^2}. \tag{5.76} \]

Now we consider the linear fit

\[ x = a' + b'y. \tag{5.77} \]

Coefficient \( b' \) is given for centered variables by:

\[ b'_c = \frac{\sum_{i=1}^{M} y^*_i x_i}{\sum_{i=1}^{M} |y_i|^2}. \tag{5.78} \]

The squared correlation coefficient for centered variables is real-valued and defined as

\[ R^2 = b_c b'_c = \frac{\sum_{i=1}^{M} x^*_i y_i \sum_{i=1}^{M} x_i y_i^*}{\sum_{i=1}^{M} |x_i|^2 \sum_{i=1}^{M} |y_i|^2} = \frac{|\sum_{i=1}^{M} x^*_i y_i|^2}{\sum_{i=1}^{M} |x_i|^2 \sum_{i=1}^{M} |y_i|^2}. \tag{5.79} \]

If there is full correlation, the linear fit obtained for (5.71) and (5.77) coincide and

\[ y = \frac{a'}{b'_c} + \frac{1}{b'_c} x = a + b_c x. \tag{5.80} \]

Therefore

\[ b_c = \frac{1}{b'_c} \tag{5.81} \]

and

\[ R^2 = b_c b'_c = 1. \tag{5.82} \]

**Spatial-temporal correlation matrix**  The correlation matrix is in the sequel a square block hermitian matrix. There are \( N^2 \) blocks containing the correlations at time \( t \) between
selected channels. The block indexed \((i, j)\), \(1 \leq (i, j) \leq N\) contains the correlation between \(h_i(\cdot; t)\) and \(h_j(\cdot; t)\), where \(h_i\) and \(h_j\) denote two channel functions selected out of \(N\) for a given \(t\).

The entry \((p, q)\), \(1 \leq (p, q) \leq L\) of each block is the complex correlation coefficient \(b_c\) defined in (5.76) between vector \(\vec{x}\) and \(\vec{y}\), where \(\vec{x}\) contains the realizations of the \(p\)th tap of channel \(h_i(\cdot; t)\) denoted \(h_i(p; t)\), and \(\vec{y}\) contains the realizations of the \(q\)th tap of the second channel \(h_j(\cdot; t)\) denoted \(h_j(q; t)\). Taps are counted from the LOS tap, which is indexed by 1.

The correlation matrix is called "spatial-temporal" since

- a spatial aspect comes from the correlation between spatial links in the off-diagonal blocks,
- a temporal aspect comes from the off-diagonal terms of the blocks that contain the correlation between taps arriving at different delays, and from the fact that correlation is computed using realizations of the taps gathered in the time dimension.

The diagonal blocks in the matrix contain the autocorrelation matrix of the selected channels. The off-diagonal blocks contain the cross-correlation matrix between the channel taps.

All CIRs are aligned on their LOS tap. We consider \(L = 20\) taps representing the most significant CIR consecutive taps starting from the LOS tap. 40 realizations of each tap are available. The frequency offset is compensated. Indeed from (5.79) a frequency offset term modifies the correlation coefficient \(R^2\). We saw in 5.5.6 that a phase offset remains after frequency offset compensation. This phase offset is constant for a mission duration, and if the correlation matrix is computed with data originating from the same mission, these phase offsets do not affect \(R^2\). In the following we estimate the correlation matrix representing \(|R|\) for each mission repetition, and then average over the 5 mission repetitions.

We focus on
- links TX4-RX3, TX4-RX7, TX4-RX8, TX4-RX9, TX4-RX10 because (i) they share the same transmitter (ii) these links are estimated simultaneously,
- and on TX7-RX9 because (i) its transmitter and receiver are static and (ii) it shares the same receiver with link TX4-RX9.

**Correlation between static links** In this paragraph we discuss the spatial-temporal correlation matrix obtained with data from set 3 and represented in Fig. 5.32. This matrix has 1s on its main diagonal (bottom left to top right) as it is a correlation matrix.

The diagonal blocks show strong correlation values, meaning that taps within the same channel are correlated. The origin of this strong correlation is not only the physical channel
Tap correlation, but also due to an artifact. The non-ideal training sequence autocorrelation function - caused by the non-linearities in the system - is convolved with the physical CIR after the correlation step at the receiver and spreads the CIR in the delay domain. This generates a correlation between the taps of the same channel and explains the spread pattern along the diagonal elements of the diagonal blocks.

The off-diagonal blocks (except the top and right blocks) contain the cross-correlation between channels TX4-RX3, TX4-RX7, TX4-RX8, TX4-RX9 and TX4-RX10. The estimated correlation magnitude is between 0.4 and 0.7. These links share the same transmitter and are simultaneously estimated. The correlation pattern of each block shows no diagonal pattern but rather a "uniform" correlation pattern. The diagonal elements of these blocks contain the correlation between same-delay taps from different channels (pure spatial correlation). Our hypothesis is that this uniform correlation pattern comes from the common phase noise of unit 4. This explanation is motivated by the tap amplitude correlation matrix and the tap phase correlation matrix that are analyzed now in Fig. 5.33.

The taps amplitude correlation matrix is represented in Fig. 5.33(a) and shows correlation coefficients nearly null everywhere except on the main diagonal. The unitary correlation coefficient on the diagonal is typical of a correlation matrix.

Off-diagonal terms show low correlation, independently of the fact that taps are within the same link or not, or of the fact that they share the same transmitter. The phase correlation matrix in Fig. 5.33(b) contains a structured pattern. Low correlation values occur between the phase of the taps within the channels sharing the same transmitter, independently from the tap indexes considered. This is due to the noise sample independency after removal of 193
the tap mean value.

We go back now to Fig. 5.32 to complete the analysis of the correlation matrix of set 3. An interesting feature is observed in the upper right corner of each block matrix. The cross-correlation values of the last taps of the estimated CIR are high (correlation 0.7-0.8) even though the first and most significant channel taps are fully uncorrelated (correlation 0-0.2). Indeed the last channel taps are close to the noise floor. We saw in section 5.5.4 that the noise is visually not white but contains a periodic pattern attributed to the non-linearities that corrupt the autocorrelation properties of the training sequence. The non-linearity are caused by imperfections in the hardware components. As the same components are used in all the units, the non-linearities are similar. The cross-correlation between these taps gives higher values although they do not carry any information about the channel.

**Correlation and behavioral pattern** In this paragraph we discuss the correlation matrix for the same links as before, but using data from set 4. The correlation matrix is plotted in Fig. 5.34. Interpretation of the correlation matrix of sets 6-10 is done in the same way.

Link TX7-RX9 correlates like in the previous case very little with the other links. Movement does not affect the correlation of these links with static links.

We propose now a novel channel model parameterized with data extracted from our measurement campaign.
5.6 Proposed Channel Model

The proposed baseband discrete channel model is adapted from [50] and generalizes the SISO channel model to distributed MIMO systems by specifying spatial-temporal correlations between the channel taps and capturing the heterogeneity of the links. The model novelty relies on

1. an extension of the model of [50] to non-collocated antennas,
2. the support of nomadic mobility,
3. the support of link heterogeneity,
4. the consideration of correlation between taps.

The model supports non-calibrated nodes. In that case the channel matrix is non-symmetric.

The channel matrix is as follows

\[
H(\tau; t) = H_{MS}(t) \odot H_{SF}(\tau; t),
\]

where

\(- H(\tau; t)\) is a \(N_a \times N_a\) matrix containing the elements \(\{H[i, j](\tau; t)\}_{1 \leq i, j \leq N_a}\) that are the CIR from transmitter \(j\) to receiver \(i\) at time \(t\) to an impulse sent at time \(t - \tau\). The diagonal elements of \(H(\tau; t)\) are such that (see note 1)

\[
H[i, i](\cdot, \cdot) = 0, \quad 1 \leq i \leq N_a.
\]
Chapter 5 Wideband Network Channel Measurements

This is a consequence of the half-duplex hypothesis according to which a node cannot transmit and receive at the same time.

- **$\mathbf{H}_{\text{MS}}(t)$** is a $N_a \times N_a$ matrix containing the medium scale fading and the antenna transmitter and receiver gains. This matrix is deterministic and defined in (5.85) such that

\[
\mathbf{H}_{\text{MS}}[i,j](t) = \begin{cases} 
10^{\frac{1}{20}G_0[i,j]-\frac{1}{2}\gamma \log_{10}\left(\frac{d_{i,j}(t)}{d_0}\right)} & \text{if } i \neq j \\
0 & \text{if } i = j \text{ (see note 2),}
\end{cases}
\]

(5.85)

where $G_0[i,j]$ is the gain matrix expressed in dB when antennas indexed $i$ and $j$ are at distance $d_0$, $d_{i,j}(t)$ is the distance between the two antennas $i$ and $j$ at time $t$, and $\gamma$ is the pathloss exponent. Matrix $\mathbf{H}_{\text{MS}}(t)$ is symmetric if the received signals are calibrated to remove the effect of the transmitter and receiver filters. $G_0[i,j]$ depends on the transmitter filter at antenna $j$ and on the receiver filter at antenna $i$ which are a-priori all different.

- **$\mathbf{H}_{\text{SF}}(\tau; t)$** is a random $N_a \times N_a$ matrix modeling the small scale fading. It is symmetric due to the reciprocity principle and contains 0 (see note 1) on its diagonal.

The entries \(\{\mathbf{H}_{\text{SF}}[i,j](\tau; t)\}_{1 \leq i \leq N_a}^{1 \leq j \leq N_a}\) of matrix $\mathbf{H}_{\text{SF}}(\tau; t)$ are such that

\[
\mathbf{H}_{\text{SF}}[i,j](\tau; t) = \begin{cases} 
0 & \text{if } \tau < \tau_{ij}^0(t) \\
c_1(\tau, t, i, j) & \text{if } \tau \geq \tau_{ij}^0(t)
\end{cases}
\]

(5.86)

where $\tau_{ij}^0(t)$ is the time when the first signal contribution transmitted at time $t - \tau_{ij}^0(t)$ from transmitter $j$ reaches the receiver $i$.

$c_1(\tau, t, i, j)$ is the realization of a Ricean random process with parameter $K_{i,j}(t)$ depending on the antenna pair and time $t$. Elements \(\{c_1(\tau, t, i, j)\}_{\tau, i, j}\) are correlated by the spatial-temporal correlation matrix $\mathbf{K}(t)$.

**The WSS-US channel model assumption** [45, 86] We discuss now how to specify $c_1(\tau, t, i, j)$ under the WSS-US assumption. Parameters $i$ and $j$ are fixed. Let $h(\tau; t) = c_1(\tau, t, i, j)$ be the randomly time-variant CIR. $h(\tau; t)$ is linear and slowly time-varying, in other words it is the impulse response of a linear underspread system. The channel impulse response is expressed as the sum of $\tau_n$—delayed complex time-variant coefficients $\{a_n(t)\}_n$

1. Rigorously it is meant the function $0(\tau; t)$ defined such that $\forall (\tau, t) \in \mathbb{R}^2$, $0(\tau; t) = 0$. 

196
such that

\[ h(\tau; t) = \sum_n a_n(t)\delta(\tau - \tau_n). \]  

(5.87)

In the sequel we assume that \( n \geq 0 \) and \( \forall n > 0, \tau_n > \tau^0_{i,j}(t) \), which means that the channel is causal. We define the delay-Doppler-spread function \( U(\cdot; \cdot) \), also called the time Fourier transform of the channel impulse response or spreading function, as in [14] such that

\[ \forall \tau, f_D \in \mathbb{R}, \quad U(\tau; f_D) = \int_{-\infty}^{\infty} h(\tau; t)e^{-j2\pi f_D t} dt, \]  

(5.88)

Due to the complexity of describing the function \( U(\cdot; \cdot) \), it is common to restrict the description to the second order statistics of \( U(\cdot; \cdot) \), i.e. to the autocorrelation function of \( U(\cdot; \cdot) \). The autocorrelation function of \( U(\cdot; \cdot) \) is denoted \( R_U(\cdot; \cdot; \cdot; \cdot) \) such that

\[ \forall \tau_1, \tau_2, f_{D_1}, f_{D_2} \in \mathbb{R}, \quad R_U(\tau_1, \tau_2; f_{D_1}, f_{D_2}) = \langle U(\tau_1; f_{D_1})U^*(\tau_2; f_{D_2}) \rangle \]  

(5.89)

For a Gaussian channel, \( R_U \) provides an accurate description of the channel since in that case the mean and covariance provide a statistically complete description. For non-Gaussian channels it is only an approximation.

The expression of \( R_U(\cdot; \cdot; \cdot; \cdot) \) can be considerably simplified using the WSS-US assumptions [14,45]. The WSS assumption implies that the channel at different Doppler frequencies is uncorrelated. Physically it means that the scattering originates from a distinct and uncorrelated angular distribution of secondary sources. The US assumption implies that the scatterers contributing to the delay spread in the channel have independent fading. This translates that the channel echoes at different delays \( \tau_n \) are uncorrelated in amplitude. The combination of WSS and US assumptions leads to the WSS-US model which is stationary in the delay and Doppler frequency domain, or equivalently, has independent components in the Doppler frequency and delay dimensions.

With the WSS-US assumption

\[ R_U(\tau_1, \tau_2; f_{D_1}, f_{D_2}) = \delta(\tau_1 - \tau_2)\delta(f_{D_1} - f_{D_2})S(\tau_1; f_{D_1}) \]  

(5.90)

The WSS-US channel is fully characterized by the scattering function \( S(\tau; f_D) = \langle |U(\tau; f_D)|^2 \rangle \), also called delay-Doppler power spectrum and is proportional to the joint
Chapter 5 Wideband Network Channel Measurements

probability density function \( f_{\tau,f_D}(\cdot,\cdot,\cdot,\cdot) \) with the notations introduced in (A.3) [45]. When \( \tau \) and \( f_D \) are mutually independent,

\[
\begin{align*}
 f_{\tau,f_D}(\cdot,\cdot,\cdot,\cdot) &= f_{\tau}(\cdot,\cdot) f_{f_D}(\cdot,\cdot), \\
(5.91)
\end{align*}
\]

**Reciprocity** The reciprocity principle applies only to the wireless channel from antenna to antenna. For the baseband channel, the effect of the RF filters at the transmitter and receiver must be taken into account. \( H(\tau,t) \) is symmetric, i.e.

\[
\forall (i,j), \quad H[i,j] = H[j,i] \\
(5.92)
\]

if the user transmitter and receiver RF filters have the same impulse response, making the channel reciprocal. This is in general not the case but can be forced by using a calibration filter on the received data to compensate the joint effect of the transmitter and receiver RF filters. This method is limited to the case where the filter are linear and time-invariant. In a general way the baseband channels include transmitter and receiver filter effects and other imperfections, and are not reciprocal.

**Discrete channel model** Using the fact that the baseband channel is bandlimited over \([-B,B]\) in the frequency-delay dimension, we can express the model for discrete delays. The channel can be perfectly characterized in the delay dimension if it is sampled at a rate larger than the Nyquist rate \( 2B \), i.e. if the time \( T_s \) between two samples is \( T_s < \frac{1}{2B} \).

Under assumption that the channel is slowly varying, the channel is bandlimited over \([-f_{D_{\max}},f_{D_{\max}}]\) in the frequency-time (Doppler-frequency) dimension and we can characterize the model for discrete times. The channel can be perfectly characterized in the time dimension if it is sampled at a rate larger than the Nyquist rate \( 2f_{D_{\max}} \), i.e. if the time \( T_{est} \) between two estimates is \( T_{est} < \frac{1}{2f_{D_{\max}}} \).

A channel can only by characterized if it is underspread, meaning that the channel does not change during \( T_{est} \). An underspread channel fulfills condition (5.93)

\[
 f_{D_{\max}} \ll B. \\
(5.93)
\]

We propose now an implementation recipe based on the measurement evaluation.
5.7 Implementation Recipe

We propose in the sequel the implementation method for a block-fading baseband discrete channel. Links involving unit 4 have a block duration two times shorter than the other links \( N_{\text{rep}}T_{\text{est}}/2 \) vs. \( N_{\text{rep}}T_{\text{est}} \). Implementation of the model can be done as follows

1. Select an environment, a position and participating units.
2. Determine the pathloss using (5.62) and the pathloss exponent estimated in 5.5.9.
3. Determine for each link and each tab the K-factor realization using the lognormal distribution parameters explained in 5.5.10.
4. Generate independent realizations of the MIMO channel matrix using the K factor realizations computed previously. It is convenient to use (5.68) with \( 2\sigma^2 = 1 \) such that \( K = s^2 \). Thus the variance of the scattered component is the same for each tab and each link.
5. Introduce the spatial-temporal correlation between the taps. A simple method is as follows: after removing the mean value and adding a random phase to each channel coefficient, the channel tap realizations are set along columns and form \( X \). Using the eigenvalue decomposition of the correlation matrix \( R, R = UDU^{-1} \), where \( U \) is the matrix of the eigenvectors and \( D \) is a diagonal matrix containing the eigenvalues on its diagonal. We set \( W = UD^{-1/2} \), matrix \( Y = XW^H \) containing along its columns the tap realizations that have the desired correlation properties. This method is applicable here since we set the scattered component of the Ricean taps generated above to the same variance for all taps of all links. The mean component of the Ricean taps is added back.
6. Determine for each link and each position the power delay profile decay using 5.5.4.
7. Normalize the average power per tap and apply the power delay profile from 5.5.4.

5.8 Network Channel Model Validation

We validate the proposed non-collocated MIMO channel model by comparing parameters extracted from simulated and measured channels. We first validate the link modeling capability of the model by comparing the RMS delay spread cumulative distribution function estimated from measurements and from the model in 5.8.1. In 5.8.2 we validate the RMS Doppler spread model. Finally in 5.8.3 we discuss the MIMO capacity estimated from data generated by the model and from measurements.
5.8.1 RMS Delay Spread

We validate the link model performance by comparing the RMS delay spread cumulative distribution function of a static link of set 3 (9-7) and of a moving link of set 4 (8-4) estimated from measurements and from the model in Fig. 5.35.

For the static link, the RMS delay spread cumulative distribution function of the model shows a good fit with the measurement data. The mean RMS delay spread is 18.2 ns (standard deviation 1.5 ns) for the model, and 18.4 ns (standard deviation 1.3 ns) with the measurements.

For the moving link, the model has a steeper cumulative distribution function slope than the measurement data. This may be due to the modeled exponential power delay profile that oversimplifies the effective power delay profile and does not capture small variations. Yet the error committed on the mean RMS delay spread is small. For link 8-4 in set 4, the mean RMS delay spread is 16.3 ns (standard deviation 1.9 ns) with our model, and 16.6 ns (3.4 ns) for the measurements.

![RMS delay spread validation using CDF](image)

Fig. 5.35: RMS delay spread validation using CDF

5.8.2 RMS Doppler Spread

The normalized Doppler spectrum measured for a moving link TX3-RX4 (set 2, position 1, repetition 1) is plotted in Fig. 5.36 and superposed with the Doppler spectrum generated by the model. The spectra have a good match over the frequency range [-30 Hz, 30 Hz]. This
5.8 Network Channel Model Validation

Fig. 5.36: Doppler spectrum for link TX3-RX4 (set 2, position 1, repetition 1)

is the range where the spectrum is above the noise floor and for which the Laplacian approximation for the Doppler spectrum is verified. The slope in dB of the Laplacian spectrum is given by a lognormal random variable. Outside this frequency range, our Laplacian model underestimates the Doppler spectrum. The measured mean RMS Doppler spread for link 3-4 (set 2) is 7.90 Hz (standard deviation 2.54 Hz), and the model achieves a mean RMS Doppler spread of 5.18 Hz (standard deviation 2.14 Hz). The values obtained with our model are lower because the noise floor is not part of the model. Doppler frequencies higher than 30 Hz are less weighted for the RMS calculation in the model than the measurements and this leads to the lower values obtained with the model.

5.8.3 MIMO Spectral Efficiency

In this paragraph we estimate the MIMO spectral efficiency from measurements data and from data generated with the proposed channel model. We consider the 2×2 non-coltocated MIMO system made from transmitters 3 and 4, and receivers 8 and 9. First we compare the spectral efficiency of the SISO channels TX3-RX8, TX3-RX9, TX4-RX8 and TX4-RX9. The SISO link (TXj-RXi) spectral efficiency (frequency averaged and without channel state
information at the transmitter (CSIT)) in \( \text{bit/s/Hz} \) is given by

\[
C(i, j, t) = \log_2 \left( 1 + \text{SNR} |H[i, j](f; t)|^2 \right)
\]

(5.94)

\( H[i, j](f; t) = \mathcal{F}_{\tau \rightarrow f}(H[i, j](\tau; t)) \) is the Fourier transform of the channel in the delay dimension. (5.94) is estimated for \( \text{SNR}=20 \text{ dB} \). The SISO spectral efficiency cumulative distribution function of the investigated links is plotted in Fig. 5.37. It shows a good fit between the measurements and the model, for both the moving links and the static links. Links TX4-RX8 and TX4-RX9 have similar spectral efficiency since unit 8 and unit 9 are close to each other. Links TX3-RX8 and TX3-RX9 show a larger mean spectral efficiency because units 3, 8 and 9 are located at less than 1 m from each other, whereas unit 4 is at least at 1.4 m from the closest receiver. As unit 4 is moving away from units 3, 8 and 9, the spectral efficiency is decreasing and the cumulative distribution function slope is lower than for the static links. We also plot the spectral efficiency when the model is discarding the correlation between the taps, i.e. when the taps of a same link have no correlation. In that case the correlation matrix is the identity matrix. Correlation between the channel taps does not quantitatively affect the SISO spectral efficiency cumulative distribution function.

\[
\begin{array}{cccc}
\text{TX4−RX8} & \text{TX3−RX8} & \text{TX3−RX9} & \text{TX4−RX9} \\
\text{measurements} & \text{model without correlation} & \text{model with correlation} & \text{model without correlation} \\
\text{TX3−RX8} & \text{TX3−RX9} & \text{TX4−RX8} & \text{TX4−RX9} \\
\text{measurements} & \text{model without correlation} & \text{model with correlation} & \text{model without correlation} \\
\end{array}
\]

**Fig. 5.37:** SISO capacity empirical CDF

We estimate now the MIMO spectral efficiency of the non-collocated MIMO system built from the links introduced above and compare the results obtained with measurement data and simulated data obtained with the proposed model. The MIMO spectral efficiency (frequency
5.8 Network Channel Model Validation

averaged, and with no CSIT) in bit · Hz\(^{-1}\) · s\(^{-1}\) is given by (5.95).

\[
C(t) = T_s \int \log_2 \left( \det \left( I + \frac{\text{SNR}}{N_{tx}} H(f; t) H^H(f; t) \right) \right) df,
\]  

(5.95)

where \(N_{tx} = 2\) represents the number of transmitter.

The channel matrix is scaled such that for each link, the mean CIR energy is the same for the measured and the simulated links. Thus there is a per link power delay profile normalization but the 4 links have unequal energy.

The MIMO spectral efficiency cumulative distribution function is plotted in Fig. 5.38. The measurement data are represented with a blue line and the simulated data obtained with the proposed model are plotted with the red and green plain line. Obviously the model underestimates the spectral efficiency in comparison to the measurement data. For each frequency bin the model achieves a lower MIMO matrix rank. We attribute this to the phase noise that affects each link. As investigated in [12], phase noise artificially increases the rank of the MIMO matrix. In our case, we are interested in generating a baseband model that matches with the measurement data and therefore the phase noise effect must be enclosed in the model. After adding independent phase noise sources for every link of the model, the rank of the MIMO matrix is artificially increased and the results are plotted in Fig. 5.38 with the dotted lines. We see that the MIMO capacity is much better approximated by the model when it encloses the phase noise effect.

This is confirmed in Fig. 5.39 where the cumulative distribution function of the MIMO matrix eigenvalues is plotted. When no phase noise is enclosed in the model, the smallest eigenvalue estimated from the model underestimates the one obtained from the measured channel matrix. The condition of the matrix generated from the model is lower than for the measurement data. The correlation between the taps (with the same link and between different links) slightly degrades the MIMO matrix condition and therefore increases the mismatch with the measurement data. The key element is the phase noise consideration. Once included in the model, it leads to an increase of the smallest matrix eigenvalue and the MIMO matrix condition, and the distributions of the MIMO eigenvalues obtained from the model and from the measurement show a good fit.

Phase noise strongly affects the system spectral efficiency and must therefore be taken into account in a baseband channel model. A baseband channel model that discards phase noise underestimates the system spectral efficiency and does not provide a reliable model. A propagation channel model and a radio channel model do not enclose these effects.

203
Chapter 5 Wideband Network Channel Measurements

Fig. 5.38: MIMO capacity cdf

Fig. 5.39: Eigenvalues empirical CDF for all frequency bins
Spatial-temporal correlations caused by the unit behavioral patterns decrease the system spectral efficiency. This effect is opposed to the phase noise effect. In a baseband channel model, the phase noise effect largely dominates the spatial-temporal correlation effects. These latter can therefore be neglected in realistic baseband channel models. This is not the case for a propagation and a radio channel model. These channel models do not enclose phase noise and for them the spatial-temporal correlation may be relevant.

In this chapter we aimed at proposing a non-collocated MIMO baseband channel model for the RACooN lab. We showed that the tap spatial-temporal correlations do not affect the spectral efficiency of the simulated system. Including phase noise in the model is mandatory in order to increase the MIMO channel matrix rank and achieve the same spectral efficiency.

5.9 Conclusion

In this chapter we have presented a measurement campaign using the ETH-owned RACooN lab. These are the first measurements of a non-collocated nomadic MIMO system with behavioral pattern ever performed. Existing works have so far only dealt with collocated antennas and considered either a nomadic behavior or a behavioral pattern, but not both. Section 5.3 gives an overall description of the setup and operations. The RACooN nodes operation mode is detailed, the environments are described and we give a definition for the behavioral pattern and the nomadic movement. The 10 measurements sets are described and motivated.

In section 5.4 we have detailed the measurement campaign. In particular the hardware and software parameter choice is discussed. Using these results, the choice of the transmit signal is motivated, and a multilevel transmission strategy is proposed. This strategy increases the signal gain in non-collocated MIMO channel sounders and for any SISO channel sounder in a mobility scenario. Finally the measurement protocol is summarized.

Measurements are performed as described and the results are analyzed in section 5.5. The post-processing steps are discussed in 5.5.1 and choices are motivated. A unit tracking position algorithm is proposed in 5.5.2. It computes the displacement between any two units within one mission execution. The limitation of the procedure is theoretically analyzed and confirmed with experimental data. A detailed exploration of the large amount of collected data is performed in sections 5.5.3 to 5.5.11. The CIR magnitude is commented in 5.5.3. The behavioral pattern effect is clearly visible because it introduces a larger variability in the collected channel estimates. The power delay profile is analyzed in 5.5.4, and the effect of phase noise and non-linearities is discussed in details. The non-linearities in the measure-
ment system lead to a dependency of the noise floor level to the received signal power, and also to a pseudo-periodic pattern in the noise floor. The use of a chirp sequence as a sounding sequence instead of a m-sequence is also justified. In section 5.5.5 the delay spread is computed. We saw that the RMS delay spread is in our setup determined by the environment and not by the individual unit positions, and it is not affected by the behavioral pattern. The frequency offset is estimated in section 5.5.6 and is compensated in the following steps. The temporal variations of the channel are characterized and quantified in section 5.5.8 by the Doppler spectrum and the RMS delay spread. The behavioral pattern is doubling the RMS Doppler spread in comparison to the static case. The pathloss is estimated from the experimental data for both environments in 5.5.9. The channel tap distribution is investigated in 5.5.10. It was shown that all taps follow a Ricean distribution, even when a behavioral pattern is applied. But the effect of the behavioral pattern is visible on the K-factor that is lowered. In 5.5.11 the spatial-temporal correlation matrix of selected links is calculated and commented. Taps of the same link are always strongly correlated with each other. Phase noise artificially increases the correlation between links sharing the same transmitter. When a behavioral pattern affects one unit, the correlation between the links sharing this unit increases, but also the correlation with the static links increases due to the scatterer movement. Finally we use the parameters estimated from the measurement campaign to propose a baseband channel model in section 5.6. An implementation recipe is suggested in section 5.7 and the model is validated by comparing the RMS delay spread, the RMS Doppler spread and the MIMO spectral efficiency of channels generated by the model and from the measurements.
Chapter 6

Conclusion

6.1 Results

In the first chapter of this thesis we started with a pervasive wireless access network (PWAN) that includes relays which can be dedicated to support or idle nodes. We proposed a multi-user zero-forcing (MUZF) gain allocation scheme where relays assist multi-user (MU) communication by performing a distributed channel orthogonalization. This allows multiple users to concurrently access the medium without that the receivers suffer from multi-user interference (MUI). One advantage of this scheme is the complexity transfer from the receiver side to the relays, another is the access to distributed multiple-input multiple-output (MIMO) gains. The novelty of the approach lies in the signal processing performed in time-domain and the support of any cooperation pattern between the relays.

In this PWAN model, the filter-and-forward (FF) relays form a filter bank that can exploit spatial and temporal degrees of freedom in the system. We defined a minimum configuration for the number of relays and relay filter taps that is a necessary condition for channel orthogonalization. This result generalizes the MUZF minimum relay configuration case already established in the frequency flat case. We proposed an equivalent system model and an achievable rate expression for each source-to-destination link as a function of the relay filter coefficients. The relay filter gains are in general not unique and we discussed three optimization criterions. Performance are evaluated based on spatial multiplexing (SM) gain, diversity gain and array gain. Our scheme opens as many spatial streams as user pairs communicating in the network. Relays and relay filter taps that exceed the MUZF minimum configuration can enhance the equivalent source-to-destination links by offering diversity and array gain. The max-min relay gain optimization criterion targets the weakest link energy maximization and achieves a diversity gain larger than the max-mean criterion that maximizes the mean...
link energy and achieves on its turn a larger array gain than the first criterion. Both criterions overperform a random relay filter gain choice.

In the second chapter we moved into a more practical perspective by focusing on an existing prototype (the Radio Access with Cooperative Nodes (RACooN)) that models the nodes of a PWAN. This system allows for distributed MIMO channel measurements and for cooperative relaying transmissions. We reviewed existing MIMO channel sounders and concluded that they were not designed to support distributed MIMO configurations like in a PWAN. Demonstrators have been designed to support distributed antenna configurations but are either low-complexity and inappropriate for channel sounding or high-complexity and unsuitable for pedestrian mobility applications. Therefore the RACooN lab offers an equipment that trade-off the accuracy with the mobility requirements and allows for both cooperative relaying demonstrations and channel measurements. We described the node analogue and digital architecture and identified the functional blocks. In the last section we investigated the node imperfections. The carrier frequency offset (CFO) was evaluated and found in the range specified by the local oscillator calibration procedure. After CFO removal, we observed the effect of phase noise on a single-tone signal at the receiver side. We characterized the phase noise in the frequency domain and found out that it fits a Lorentzian spectrum and a zero-mean Gaussian distribution typical of a local oscillator with a phase-locked loop (PLL). The system linearity was also investigated. We found that non-linearities originate from the transmitter and receiver baseband circuits and characterized it for a single-tone signal by the harmonics power level and the $3^{rd}$ order intercept point ($\text{IP}_3$) point. I-Q imbalance is another non-linear effect that was not captured by the harmonics analysis of the previous test signal. We provided a model for I-Q imbalance in the nodes and experimentally characterized it. We found out that the I-Q peak lies 20 dB below the signal peak. Finally the direct component (DC) offset was analyzed. We explained its origin and measured it. We found out that the DC offset is approximately constant in phase and amplitude during one mission realization but is changing from one mission to the next. We analyzed its distribution over several mission realizations and came out with a model for the DC offset.

In the third chapter we discussed the possibility of performing channel estimation using the RACooN (RAC) lab. After having investigated the RACooN lab characteristics, we turned our focus on how to perform channel estimation with the available hardware, with the goal of understanding how the imperfections highlighted in the previous chapter affect the channel estimate. Three sounding sequences are described: the impulse sequence, the direct sequence and the chirp sequence. Motivated by processing gain requirements we focused on the latter two. We proposed a signal processing method to estimate the channel using these
sequences. The method has two core operations: averaging and correlation. We gave analytical expressions for the processing gain degradation that occur during these two operations, due to phase noise and CFO, for a direct sequence and a chirp sequence. Finally we studied the robustness of our method towards non-linear effects. We compared our method to a dedicated non-linear system identification method and found that this latter was not practical for our system due to the a-priori unknown non-linearity order. Using our proposed method we compared the channel estimate obtained for the two sounding sequences and found out that the chirp sequence is less sensitive to the system non-linearities even though it is affected by the transmitter non-linearities.

In the last chapter we documented a measurement campaign using the RAC lab. We used the knowledge gained from the previous chapters to discuss the RACooN lab parameters. We chose short training sequences in order to optimize the repetition rate and proposed for mobility purpose a multilevel transmission scheme that circumvents the received signal-to-noise ratio (SNR) decrease due to a larger pathloss. The measurement campaign enclosed two different environments, static nodes and nomadic mobility and the use of two different sounding sequences. We placed six RACooN nodes in ten different setups that mimic typical PWAN scenario. To the difference with existing measurement campaigns, we conferred one node a behavioral pattern, that is a random movement of the node over a disk with $25 \text{ cm}$ radius and centered at the disk center. This pattern, associated with nomadic mobility, is believed to model a realistic PWAN scenario in which human users are involved. We analyzed our measurement data and discussed the effect of the behavioral pattern on the results. Our discussion enclosed the power delay profile, the delay spread, the frequency offset estimate, the Doppler spectrum and the pathloss evaluation. We compared our numerical values with measurement results already published. Finally we used our measurement data to generate the parameters of a channel model. We validated it by comparing the statistics of channel realizations from the model and from measurements. We showed that the spatial-temporal correlation is not relevant for the model proposed, and that phase noise must be enclosed into the baseband channel model.

6.2 Outlook

In this thesis we have studied a PWAN under different perspectives. In chapter 2 we considered a PWAN as a distributed MIMO system by involving relay in the MU communication. As in a PWAN distances between nodes are short it is reasonable to consider that the
receivers also receive the direct link that includes MUI. This additional information could be exploited and be subject to future work.

The high node density assumption was used to justify the channel orthogonalization. This assumption concerned the number of communicating user pairs, yet it could apply as well to the number of relay and idle nodes. In that case, and if the system complexity is limited, all relays cannot assist the communication since channel state information (CSI) exchange (required for gain calculation) between them causes too much overhead. It would be then useful to design relay gain algorithms based on relay selection or clustering in frequency selective channels. A last extension could treat the case where the two-hop assumption is hold back. A multi-hop transmission scheme would increase the equivalent source-to-destination channel length and possibly brings more temporal degrees of freedom in the system.

In the other chapters of this thesis we considered the PWAN without underlying infrastructure and estimated therefore the channel between all nodes. We restricted ourselves to nomadic mobility because the available Random-Access Memory (RAM) only allows, for a given sampling rate, the collection of samples for a finite duration, i.e. a finite displacement of the nodes. With a hardware upgrade of the RAM size it should be possible to support pedestrian speed and extend the proposed channel model. A measurement campaign mobilizing more nodes would provide parameters for a network channel model with a larger number of users. With such a model it will be possible to simulate offline with a realistic channel model the performance of the MUZF and other relaying schemes.
A Appendix

(Linear) cross-correlation

Let $x(\cdot)$ and $y(\cdot)$ be two time-continuous functions defined for all $t \in \mathbb{R}$. The time-continuous cross-correlation function $R_{x,y}(\cdot)$ is defined as:

$$\forall \tau \in \mathbb{R}, \quad R_{x,y}(\tau) = \int_{-\infty}^{\infty} x(t)y^*(t-\tau)dt.$$  

If $x[\cdot]$ and $y[\cdot]$ are two time-discrete functions defined for all $k \in \mathbb{Z}$, the time-discrete cross-correlation function $R_{x,y}[\cdot]$ is defined as:

$$\forall \delta \in \mathbb{Z}, \quad R_{x,y}[\delta] = \sum_{k=-\infty}^{\infty} x[k]y^*[k-\delta].$$

In the special case where $x[\cdot] = y[\cdot]$, the linear cross-correlation function between $x[\cdot]$ and $y[\cdot]$ is called the linear autocorrelation function of $x[\cdot]$ and is denoted $R_{x}[\cdot] = R_{x,x}[\cdot]$.

Cyclic cross-correlation

If $x[\cdot]$ and $y[\cdot]$ are $N$-periodic discrete sequences defined for all $k \in \mathbb{Z}$, the discrete cyclic cross-correlation function is expressed as:

$$\forall \delta \in \mathbb{Z}, \quad \theta_{x,y}[\delta] = \sum_{k=0}^{N-1} x[k]y^*[k-\delta].$$

$\theta_{x,y}[\cdot]$ is also $N$-periodic.
In the special case where $x[\cdot] = y[\cdot]$, the cyclic cross-correlation function between $x[\cdot]$ and $y[\cdot]$ is called the cyclic autocorrelation function of $x[\cdot]$ and is denoted $\theta_{x}[\cdot] = \theta_{x,x}[\cdot]$. 

211
Power spectrum

Invoking the Wiener-Khinchin theorem, the Fourier transform of the discrete periodic autocorrelation function $\theta_x[\cdot]$ is the discrete power spectrum $S_x[\cdot]$ of the time sequence $x[\cdot]$, i.e.

$$\mathcal{F}\{\theta_x[\cdot]\} = S_x[\cdot]$$

Probability density function

Let $x(\cdot)$ be a complex time-continuous process defined for all $t \in \mathbb{R}$. It is described by its probability density function $f_x(\cdot; \cdot)$ such that:

$$f_x(a; t) = \frac{\partial F_x(a; t)}{\partial a}$$  \hspace{1cm} (A.1)

and

$$F_x(a; t) = P(x(t) \leq a),$$  \hspace{1cm} (A.2)

where $P(A)$ denotes the probability of event $A$.

Cross-correlation for random processes

This definition extends to random processes the definition of the correlation function given in chapter 4 for deterministic functions. Let $x(\cdot)$ and $y(\cdot)$ be two complex continuous-time random processes defined for all $t \in \mathbb{R}$ and described by their joint probability density function $f_{xy}(\cdot; \cdot; \cdot)$ such that

$$f_{xy}(a, b; t_1, t_2) = \frac{\partial^2 F_{xy}(a, b; t_1, t_2)}{\partial a \partial b}$$  \hspace{1cm} (A.3)

and

$$F_{xy}(a, b; t_1, t_2) = P(x(t_1) \leq a, y(t_2) \leq b),$$  \hspace{1cm} (A.4)

where $P(A, B)$ denotes the joint probability of events $A$ and $B$. The cross-correlation function $R_{xy}(\cdot, \cdot)$ is defined as
∀(t_1, t_2) ∈ \mathbb{R}^2, R_{x,y}(t_1, t_2) = \langle x(t_1)y^*(t_2) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t_1)y^*(t_2)f_{x,y}(a, b; t_1, t_2)da db \tag{A.5} \tag{A.6}

In the special case where \(x(\cdot) = y(\cdot)\), the cross-correlation function between \(x(\cdot)\) and \(y(\cdot)\) is called the autocorrelation function of \(x(\cdot)\) and is denoted \(R_{x,x}(\cdot, \cdot) = R_x(\cdot, \cdot)\). This definition is easily extended to discrete-time processes.

Definitions

- A **snapshot** is one realization of a network channel matrix or a channel estimate.
- A **mission** is the execution of a sequence of commands contained in a mission file that contains at most \(N^{\text{max}}_{\text{cmd}}\) commands. It controls the transmission/reception and the storage of the RAM content on the harddisk. A mission usually generates several snapshots.
- The **operator** is the person physically present in the premises where the measurements are performed.
- An **antenna** (also named **point**) is the hardware element that performs the physical signal transmission and reception. The number of antennas in the network is denoted \(N_a\).
- A **user** (also named **node**) is the hardware element that consists in one or more antennas. It can be the source or the destination of a signal, or both. The number of users in the network is denoted \(N_u\), and \(N_u \leq N_a\). If the nodes carry a single antenna, then \(N_u = N_a\).
- A **network channel matrix** is a matrix that contains all point-to-point channels measured between any two nodes in the network.
- A **link** or **channel** can be directional or bidirectional. A directional single-input single-output (SISO) wireless link between two antennas is specified as TXA-RXB and denotes the baseband channel between transmitter A and receiver B. It is commonly called a channel in this work. A bidirectional link is specified as A-B and denotes both TXA-RXB and TXB-RXA, assuming the reciprocity of the baseband channel. The collection of all possible links in a network is a network channel matrix.
Fig. A.1: Mission sequence for unit 1

<table>
<thead>
<tr>
<th>Command Sequence</th>
<th>Switch Position</th>
<th>Buffer Start</th>
<th>Buffer End</th>
<th>Duration in Timeslots</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL 10</td>
<td>RX</td>
<td>RX L 130</td>
<td>RX L 130</td>
<td>1</td>
</tr>
<tr>
<td>IDL 10</td>
<td>RX</td>
<td>RX L 140</td>
<td>RX L 140</td>
<td>1</td>
</tr>
<tr>
<td>IDL 10</td>
<td>RX</td>
<td>RX L 150</td>
<td>RX L 150</td>
<td>1</td>
</tr>
<tr>
<td>IDL 10</td>
<td>RX</td>
<td>RX L 160</td>
<td>RX L 160</td>
<td>1</td>
</tr>
<tr>
<td>IDL 10</td>
<td>RX</td>
<td>RX L 170</td>
<td>RX L 170</td>
<td>1</td>
</tr>
<tr>
<td>IDL 10</td>
<td>RX</td>
<td>RX L 180</td>
<td>RX L 180</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. A.1: Mission sequence for unit 1
<table>
<thead>
<tr>
<th>command</th>
<th>TX gain</th>
<th>RX gain</th>
<th>buffer start</th>
<th>buffer end</th>
<th>duration in timeslots</th>
<th>switch position</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL</td>
<td>RX</td>
<td>L</td>
<td>130</td>
<td>130</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>L</td>
<td>140</td>
<td>140</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>RX</td>
<td>L</td>
<td>150</td>
<td>150</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>RX</td>
<td>L</td>
<td>160</td>
<td>160</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>RX</td>
<td>L</td>
<td>170</td>
<td>170</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>RX</td>
<td>L</td>
<td>180</td>
<td>180</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>RX</td>
<td>L</td>
<td>190</td>
<td>190</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>RX</td>
<td>L</td>
<td>200</td>
<td>200</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>TX</td>
<td>L</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>TX</td>
</tr>
<tr>
<td>STO</td>
<td>RX</td>
<td>L</td>
<td>130</td>
<td>130</td>
<td>10000</td>
<td></td>
</tr>
</tbody>
</table>

Fig. A.2: Mission sequence for unit 2
<table>
<thead>
<tr>
<th>Mission name: Diss_meas_1 description: test file for 10 RACoNs</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>RACon unit</th>
<th>command ID</th>
<th>command</th>
<th>TX gain</th>
<th>RX gain</th>
<th>buffer start</th>
<th>buffer end</th>
<th>duration in timeslots</th>
<th>switch position</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>IDL</td>
<td>RX</td>
<td>L</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>TX</td>
</tr>
<tr>
<td>3</td>
<td>IDL</td>
<td>RX</td>
<td>L 140</td>
<td>140</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>RX</td>
<td>RX</td>
<td>L 150</td>
<td>150</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>IDL</td>
<td>RX</td>
<td>L 160</td>
<td>160</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>IDL</td>
<td>RX</td>
<td>L 170</td>
<td>170</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>IDL</td>
<td>RX</td>
<td>L 180</td>
<td>180</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>RX</td>
<td>RX</td>
<td>L 190</td>
<td>190</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>RX</td>
<td>RX</td>
<td>L 200</td>
<td>200</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>RX</td>
<td>RX</td>
<td>L 110</td>
<td>110</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>RX</td>
<td>RX</td>
<td>L 120</td>
<td>120</td>
<td>10</td>
<td>100</td>
<td>RX</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. A.3:** Mission sequence for unit 3.
Fig. A.4: Mission sequence for unit 4

<table>
<thead>
<tr>
<th>command</th>
<th>TX gain</th>
<th>RX gain</th>
<th>buffer start</th>
<th>buffer end</th>
<th>duration in timeslots</th>
<th>switch position</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>100</td>
<td>130</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>100</td>
<td>130</td>
<td>10</td>
<td>TX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>160</td>
<td>160</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>170</td>
<td>170</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>180</td>
<td>180</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>190</td>
<td>190</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>200</td>
<td>200</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>110</td>
<td>110</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>120</td>
<td>120</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>STO</td>
<td>130</td>
<td>150</td>
<td>160</td>
<td>170</td>
<td>180</td>
<td>190</td>
</tr>
</tbody>
</table>

10000
<table>
<thead>
<tr>
<th>command</th>
<th>TX gain</th>
<th>RX gain</th>
<th>buffer start</th>
<th>buffer end</th>
<th>duration in timeslots</th>
<th>switch position</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL</td>
<td>L</td>
<td>130</td>
<td>130</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>140</td>
<td>140</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>150</td>
<td>150</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>160</td>
<td>160</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>170</td>
<td>170</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>180</td>
<td>180</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>190</td>
<td>190</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>200</td>
<td>200</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>110</td>
<td>110</td>
<td>1</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>STO</td>
<td>L</td>
<td>130</td>
<td>140 160 170 180 190 200 110 120</td>
<td>10000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Fig. A.6:** Mission sequence for unit 6

<table>
<thead>
<tr>
<th>Command</th>
<th>TX Gain</th>
<th>RX Gain</th>
<th>Buffer Start</th>
<th>Buffer End</th>
<th>Duration (in Timeslots)</th>
<th>Switch Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>130</td>
<td>130</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>140</td>
<td>140</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>150</td>
<td>150</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>170</td>
<td>170</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>180</td>
<td>180</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>190</td>
<td>190</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>IDL</td>
<td>RX</td>
<td>RX</td>
<td>200</td>
<td>200</td>
<td>1</td>
<td>L</td>
</tr>
<tr>
<td>STO</td>
<td>130</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>L</td>
</tr>
<tr>
<td>command</td>
<td>TX gain</td>
<td>RX gain</td>
<td>buffer start</td>
<td>buffer end</td>
<td>duration in timeslots</td>
<td>switch position</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>--------------</td>
<td>------------</td>
<td>-----------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>130</td>
<td>130</td>
<td>1</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>140</td>
<td>140</td>
<td>10</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>150</td>
<td>150</td>
<td>1</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>160</td>
<td>160</td>
<td>10</td>
<td>TX</td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>180</td>
<td>180</td>
<td>1</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>190</td>
<td>190</td>
<td>10</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>200</td>
<td>200</td>
<td>1</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>110</td>
<td>110</td>
<td>1</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td>10</td>
<td>RX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>120</td>
<td>120</td>
<td>1</td>
<td>RX</td>
<td></td>
</tr>
<tr>
<td>STO</td>
<td></td>
<td></td>
<td>10000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>command</td>
<td>TX gain</td>
<td>RX gain</td>
<td>buffer start</td>
<td>buffer end</td>
<td>duration in timeslots</td>
<td>switch position</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>--------------</td>
<td>------------</td>
<td>-----------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td>L</td>
<td>130</td>
<td>130</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td></td>
<td>L</td>
<td>140</td>
<td>140</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td>L</td>
<td>150</td>
<td>150</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td></td>
<td>L</td>
<td>160</td>
<td>160</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td>L</td>
<td>170</td>
<td>170</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>TX</td>
<td></td>
<td>L</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td>L</td>
<td>190</td>
<td>190</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td></td>
<td>L</td>
<td>200</td>
<td>200</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td>L</td>
<td>110</td>
<td>110</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td></td>
<td>L</td>
<td>120</td>
<td>120</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>STO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. A.8: Mission sequence for unit 8
<table>
<thead>
<tr>
<th>command</th>
<th>TX gain</th>
<th>RX gain</th>
<th>buffer start</th>
<th>buffer end</th>
<th>duration in timeslots</th>
<th>switch position</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>130</td>
<td>130</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>140</td>
<td>140</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>150</td>
<td>150</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>160</td>
<td>160</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>170</td>
<td>170</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>180</td>
<td>180</td>
<td>10</td>
<td>TX</td>
</tr>
<tr>
<td>TX</td>
<td>L</td>
<td>RX</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>200</td>
<td>200</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td>RX</td>
<td>110</td>
<td>110</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td>L</td>
<td>RX</td>
<td>120</td>
<td>120</td>
<td>10</td>
<td>RX</td>
</tr>
<tr>
<td>STO</td>
<td>L</td>
<td>RX</td>
<td>130</td>
<td>130</td>
<td>10000</td>
<td>STO</td>
</tr>
</tbody>
</table>

Fig. A.9: Mission sequence for unit 9
<table>
<thead>
<tr>
<th>command</th>
<th>TX gain</th>
<th>RX gain</th>
<th>buffer start</th>
<th>buffer end</th>
<th>duration in timeslots</th>
<th>switch position</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>130</td>
<td>130</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>140</td>
<td>140</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>150</td>
<td>150</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>160</td>
<td>160</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>170</td>
<td>170</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>180</td>
<td>180</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>190</td>
<td>190</td>
<td>1</td>
<td>TX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>TX</td>
</tr>
<tr>
<td>TX</td>
<td>L</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>110</td>
<td>110</td>
<td>1</td>
<td>RX</td>
</tr>
<tr>
<td>IDL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RX</td>
</tr>
<tr>
<td>RX</td>
<td>L</td>
<td></td>
<td>120</td>
<td>120</td>
<td>1</td>
<td>TX</td>
</tr>
<tr>
<td>STO</td>
<td></td>
<td></td>
<td>130</td>
<td>140</td>
<td>10000</td>
<td></td>
</tr>
</tbody>
</table>

Fig. A.10: Mission sequence for unit 10
## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>analog-to-digital converter</td>
</tr>
<tr>
<td>AF</td>
<td>amplify-and-forward</td>
</tr>
<tr>
<td>AGC</td>
<td>automatic gain control</td>
</tr>
<tr>
<td>AMPS</td>
<td>Advanced Mobile Phone System</td>
</tr>
<tr>
<td>AoA</td>
<td>angle-of-arrival</td>
</tr>
<tr>
<td>ARPANET</td>
<td>Advanced Research Projects Agency Network</td>
</tr>
<tr>
<td>AWGN</td>
<td>additive white Gaussian noise</td>
</tr>
<tr>
<td>BAN</td>
<td>body area network</td>
</tr>
<tr>
<td>BER</td>
<td>bit error rate</td>
</tr>
<tr>
<td>BS</td>
<td>basis station</td>
</tr>
<tr>
<td>CC</td>
<td>cyclic convolution</td>
</tr>
<tr>
<td>CDF</td>
<td>cumulative density function</td>
</tr>
<tr>
<td>CDMA</td>
<td>code-division multiple access</td>
</tr>
<tr>
<td>CF</td>
<td>compress-and-forward</td>
</tr>
<tr>
<td>CFO</td>
<td>carrier frequency offset</td>
</tr>
<tr>
<td>CIR</td>
<td>channel impulse response</td>
</tr>
<tr>
<td>CP</td>
<td>cyclic prefix</td>
</tr>
<tr>
<td>CP₁</td>
<td>1 dB-compression point</td>
</tr>
<tr>
<td>CSCDMA</td>
<td>code-select code-division multiple access</td>
</tr>
<tr>
<td>CSI</td>
<td>channel state information</td>
</tr>
<tr>
<td>CSIR</td>
<td>channel state information at the receiver</td>
</tr>
<tr>
<td>CSIT</td>
<td>channel state information at the transmitter</td>
</tr>
<tr>
<td>DAC</td>
<td>digital-to-analog converter</td>
</tr>
<tr>
<td>DAS</td>
<td>distributed antenna system</td>
</tr>
<tr>
<td>DC</td>
<td>direct component</td>
</tr>
<tr>
<td>DF</td>
<td>decode-and-forward</td>
</tr>
<tr>
<td>DFT</td>
<td>discrete Fourier transform</td>
</tr>
<tr>
<td>DoA</td>
<td>direction-of-arrival</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>DSP</td>
<td>digital signal processor</td>
</tr>
<tr>
<td>FDMA</td>
<td>frequency-division multiple access</td>
</tr>
<tr>
<td>FDX</td>
<td>full duplex</td>
</tr>
<tr>
<td>FF</td>
<td>filter-and-forward</td>
</tr>
<tr>
<td>FIR</td>
<td>finite impulse response</td>
</tr>
<tr>
<td>GPS</td>
<td>global positioning system</td>
</tr>
<tr>
<td>HDX</td>
<td>half-duplex</td>
</tr>
<tr>
<td>HF</td>
<td>high frequency</td>
</tr>
<tr>
<td>HO</td>
<td>homogeneous channel</td>
</tr>
<tr>
<td>IC</td>
<td>integrated circuit</td>
</tr>
<tr>
<td>IDFT</td>
<td>inverse discrete Fourier transform</td>
</tr>
<tr>
<td>IF</td>
<td>intermediate frequency</td>
</tr>
<tr>
<td>IID</td>
<td>independent identically distributed</td>
</tr>
<tr>
<td>IMD</td>
<td>intermodulation distortion</td>
</tr>
<tr>
<td>IP&lt;sub&gt;1&lt;/sub&gt;</td>
<td>1&lt;sup&gt;st&lt;/sup&gt; order intercept point</td>
</tr>
<tr>
<td>IP&lt;sub&gt;3&lt;/sub&gt;</td>
<td>3&lt;sup&gt;rd&lt;/sup&gt; order intercept point</td>
</tr>
<tr>
<td>ISI</td>
<td>inter-symbol interference</td>
</tr>
<tr>
<td>LAN</td>
<td>local area network</td>
</tr>
<tr>
<td>LDAS</td>
<td>linear distributed antenna system</td>
</tr>
<tr>
<td>LF</td>
<td>low frequency</td>
</tr>
<tr>
<td>LinRel</td>
<td>linear relaying</td>
</tr>
<tr>
<td>LNA</td>
<td>low-noise amplifier</td>
</tr>
<tr>
<td>LOS</td>
<td>line of sight</td>
</tr>
<tr>
<td>LSE</td>
<td>least square error</td>
</tr>
<tr>
<td>MIMO</td>
<td>multiple-input multiple-output</td>
</tr>
<tr>
<td>MISO</td>
<td>multiple-input single-output</td>
</tr>
<tr>
<td>ML</td>
<td>maximum likelihood</td>
</tr>
<tr>
<td>MLS</td>
<td>maximum-length sequence</td>
</tr>
<tr>
<td>MLSE</td>
<td>maximum likelihood sequence estimation</td>
</tr>
<tr>
<td>MMSE</td>
<td>minimum mean-square error</td>
</tr>
<tr>
<td>MRC</td>
<td>maximum ratio combining</td>
</tr>
<tr>
<td>MSE</td>
<td>mean square error</td>
</tr>
<tr>
<td>MSI</td>
<td>multi-stream interference</td>
</tr>
<tr>
<td>MSIC</td>
<td>multi-stream interference cancellation</td>
</tr>
<tr>
<td>MSR</td>
<td>mainlobe-to-sidepeak ratio</td>
</tr>
<tr>
<td>MU</td>
<td>multi-user</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>MUI</td>
<td>multi-user interference</td>
</tr>
<tr>
<td>MUZF</td>
<td>multi-user zero-forcing</td>
</tr>
<tr>
<td>NLOS</td>
<td>non line-of-sight</td>
</tr>
<tr>
<td>OFDM</td>
<td>orthogonal frequency-division multiplexing</td>
</tr>
<tr>
<td>OFDMA</td>
<td>orthogonal frequency-division multiple access</td>
</tr>
<tr>
<td>PA</td>
<td>power amplifier</td>
</tr>
<tr>
<td>PDF</td>
<td>probability density function</td>
</tr>
<tr>
<td>PLL</td>
<td>phase-locked loop</td>
</tr>
<tr>
<td>PN</td>
<td>phase noise</td>
</tr>
<tr>
<td>PRBS</td>
<td>pseudo-random binary sequence</td>
</tr>
<tr>
<td>PSU</td>
<td>power supply unit</td>
</tr>
<tr>
<td>PWAN</td>
<td>pervasive wireless access network</td>
</tr>
<tr>
<td>RACooN</td>
<td>Radio Access with Cooperative Nodes</td>
</tr>
<tr>
<td>RAM</td>
<td>random-access memory</td>
</tr>
<tr>
<td>RF</td>
<td>radio-frequency</td>
</tr>
<tr>
<td>RFU</td>
<td>radio-frequency unit</td>
</tr>
<tr>
<td>RMS</td>
<td>root mean square</td>
</tr>
<tr>
<td>RSS</td>
<td>received signal strength</td>
</tr>
<tr>
<td>SEQ</td>
<td>sequential</td>
</tr>
<tr>
<td>SIMO</td>
<td>single-input multiple-output</td>
</tr>
<tr>
<td>SINR</td>
<td>signal-to-interference-and-noise ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>signal-to-interference ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>single-input single-output</td>
</tr>
<tr>
<td>SM</td>
<td>spatial multiplexing</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>SSCR</td>
<td>single sideband-to-carrier ratio</td>
</tr>
<tr>
<td>STU</td>
<td>storage unit</td>
</tr>
<tr>
<td>TD</td>
<td>time-domain</td>
</tr>
<tr>
<td>TDD</td>
<td>time-division duplex</td>
</tr>
<tr>
<td>TDMA</td>
<td>time-division multiple access</td>
</tr>
<tr>
<td>TDMS</td>
<td>time-division multiplexed switching</td>
</tr>
<tr>
<td>TDoA</td>
<td>time difference-of-arrival</td>
</tr>
<tr>
<td>TF</td>
<td>transfer function</td>
</tr>
<tr>
<td>ToA</td>
<td>time-of-arrival</td>
</tr>
<tr>
<td>TXD</td>
<td>transmit diversity</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
</tbody>
</table>
### Appendix

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>uncorrelated scattering</td>
</tr>
<tr>
<td>UWB</td>
<td>ultra wideband</td>
</tr>
<tr>
<td>VAA</td>
<td>virtual antenna array</td>
</tr>
<tr>
<td>VCO</td>
<td>voltage-controlled oscillator</td>
</tr>
<tr>
<td>WCG</td>
<td>Wireless Communication Group</td>
</tr>
<tr>
<td>WLAN</td>
<td>wireless local area network</td>
</tr>
<tr>
<td>WSS</td>
<td>wide sense stationarity</td>
</tr>
<tr>
<td>XML</td>
<td>extensible markup language</td>
</tr>
<tr>
<td>ZF</td>
<td>zero-forcing</td>
</tr>
</tbody>
</table>
# Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{0}$</td>
<td>all-zeros vector</td>
</tr>
<tr>
<td>$*$</td>
<td>linear convolution</td>
</tr>
<tr>
<td>$\times$</td>
<td>linear correlation</td>
</tr>
<tr>
<td>$\odot$</td>
<td>cyclic convolution</td>
</tr>
<tr>
<td>$\circ$</td>
<td>cyclic correlation</td>
</tr>
<tr>
<td>$\ominus$</td>
<td>Hadamard product</td>
</tr>
<tr>
<td>$\otimes$</td>
<td>Kronecker product</td>
</tr>
<tr>
<td>$(\cdot)^*$</td>
<td>conjugate operator</td>
</tr>
<tr>
<td>$\lfloor X \rfloor$</td>
<td>nearest integer smaller or equal to $X$</td>
</tr>
<tr>
<td>$\lceil X \rceil$</td>
<td>nearest integer larger or equal to $X$</td>
</tr>
<tr>
<td>$A_0$, $A$</td>
<td>compound channel matrix</td>
</tr>
<tr>
<td>$A_s$</td>
<td>compound signal matrix</td>
</tr>
<tr>
<td>$\tilde{A}_{s(i,j)}$</td>
<td>Toeplitz matrix associated to $A_s<a href="z%5E%7B-1%7D">i, j</a>$</td>
</tr>
<tr>
<td>$\tilde{A}_s$</td>
<td>block Toeplitz matrix built from the ${\tilde{A}_{s(i,j)}}$</td>
</tr>
<tr>
<td>$A_{ZF}$</td>
<td>compound interference matrix</td>
</tr>
<tr>
<td>$\tilde{A}_{ZF(i,j)}$</td>
<td>Toeplitz matrix associated to $A_{ZF}<a href="z%5E%7B-1%7D">i, j</a>$</td>
</tr>
<tr>
<td>$\tilde{A}_{ZF}$</td>
<td>block Toeplitz matrix built from the ${\tilde{A}_{ZF(i,j)}}$</td>
</tr>
<tr>
<td>${a_k^{(i,j)}}$</td>
<td>polynomial coefficients of $A_{ZF}<a href="z%5E%7B-1%7D">i, j</a>$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>jitter</td>
</tr>
<tr>
<td>$B$</td>
<td>system bandwidth</td>
</tr>
<tr>
<td>$B_0$</td>
<td>number of buffers</td>
</tr>
<tr>
<td>$B$</td>
<td>number of buffers in the RAM</td>
</tr>
<tr>
<td>$b$, $\hat{b}$</td>
<td>spatial multiplexing gain per user</td>
</tr>
<tr>
<td>${b_k^{(i,j)}}$</td>
<td>polynomial coefficients of $A_s<a href="z%5E%7B-1%7D">i, j</a>$</td>
</tr>
</tbody>
</table>
### A Appendix

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>speed of light in the air</td>
</tr>
<tr>
<td>$\mathbb{C}$</td>
<td>set of the complex numbers</td>
</tr>
<tr>
<td>$\mathcal{CN}$</td>
<td>complex normal</td>
</tr>
<tr>
<td>$\dim (\cdot)$</td>
<td>dimension operator</td>
</tr>
<tr>
<td>$\delta (\cdot)$</td>
<td>continuous-time Dirac function</td>
</tr>
<tr>
<td>$\delta [\cdot]$</td>
<td>discrete-time Dirac function</td>
</tr>
<tr>
<td>$\langle \cdot \rangle$</td>
<td>ensemble average</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency</td>
</tr>
<tr>
<td>$f_{1/2}$</td>
<td>SSCR $-3$ dB bandwidth</td>
</tr>
<tr>
<td>$f_c$</td>
<td>carrier frequency</td>
</tr>
<tr>
<td>$f_i$</td>
<td>random frequency</td>
</tr>
<tr>
<td>$f_{IF}$</td>
<td>intermediate frequency</td>
</tr>
<tr>
<td>$f_{off}, f_n$</td>
<td>frequency offset</td>
</tr>
<tr>
<td>$f_{RF}$</td>
<td>internal RF carrier synthesizer</td>
</tr>
<tr>
<td>$f_s$</td>
<td>sampling frequency</td>
</tr>
<tr>
<td>$f_{test}$</td>
<td>test frequency</td>
</tr>
<tr>
<td>$\mathcal{F} { \cdot }$</td>
<td>Fourier transform</td>
</tr>
<tr>
<td>$| \cdot |_F$</td>
<td>Frobenius norm</td>
</tr>
<tr>
<td>$\phi$</td>
<td>phase noise, phase</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>phase offset</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>pathloss exponent</td>
</tr>
<tr>
<td>$\mathbf{G}$</td>
<td>discrete-time relay filter matrix</td>
</tr>
<tr>
<td>$\mathbf{G}_c$</td>
<td>continuous-time relay filter matrix</td>
</tr>
<tr>
<td>$\vec{g}_0, \vec{g}$</td>
<td>compound gain vector</td>
</tr>
<tr>
<td>$\tilde{\vec{g}}_{ZF}$</td>
<td>zero-forcing (ZF) gain vector</td>
</tr>
<tr>
<td>$\tilde{\vec{g}}(p)$</td>
<td>vector associated to polynomial $\tilde{g}<a href="z%5E%7B-1%7D">p</a>$</td>
</tr>
<tr>
<td>$\tilde{\vec{g}}$</td>
<td>vector built from the ${ \tilde{g}(p) }$</td>
</tr>
<tr>
<td>$(\cdot)^H$</td>
<td>transpose conjugate operator</td>
</tr>
<tr>
<td>$\mathbf{H}_{SD}$</td>
<td>equivalent source-to-destination channel matrix</td>
</tr>
<tr>
<td>$\tilde{\mathbf{H}}_{SD}$</td>
<td>equivalent channel vector</td>
</tr>
<tr>
<td>$\tilde{\mathbf{h}}_{SD,1}$</td>
<td>equivalent signal channel vector</td>
</tr>
<tr>
<td>$\tilde{\mathbf{h}}_{SD,1}$</td>
<td>equivalent interference channel vector</td>
</tr>
<tr>
<td>$\mathbf{H}_{SD,0}$</td>
<td>$\mathbf{H}_{SD}$ with 0 on its main diagonal</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>I</td>
<td>identity matrix</td>
</tr>
<tr>
<td>( I { \cdot } )</td>
<td>imaginary part</td>
</tr>
<tr>
<td>( i, j )</td>
<td>matrix index</td>
</tr>
<tr>
<td>( j )</td>
<td>( j^2 = -1 )</td>
</tr>
<tr>
<td>( k )</td>
<td>discrete time index</td>
</tr>
<tr>
<td>( K )</td>
<td>number of samples</td>
</tr>
<tr>
<td>( \mathbf{K}, \mathbf{K}_s )</td>
<td>kernel of ( \tilde{A}_{ZF} )</td>
</tr>
<tr>
<td>( \text{Ker} \ (\cdot) )</td>
<td>Kernel operator</td>
</tr>
<tr>
<td>( \xi )</td>
<td>random voltage fluctuation</td>
</tr>
<tr>
<td>( l )</td>
<td>source-to-destination link index</td>
</tr>
<tr>
<td>( L(f) )</td>
<td>single sideband-to-carrier ratio</td>
</tr>
<tr>
<td>( L_1, L_Q, L_{NL} )</td>
<td>polynomial order</td>
</tr>
<tr>
<td>( L_G )</td>
<td>number of relay filter taps</td>
</tr>
<tr>
<td>( L_{G,\text{gen}} )</td>
<td>minimum number of relay filter taps</td>
</tr>
<tr>
<td>( L_{G,\text{min}} )</td>
<td>minimum number of relay filter taps in the minimum relay configuration</td>
</tr>
<tr>
<td>( L_t )</td>
<td>maximum number of channel taps in the source-to-destination channel matrix</td>
</tr>
<tr>
<td>( L_X )</td>
<td>maximum number of channel taps in the source-to-relay channel matrix</td>
</tr>
<tr>
<td>( L_Z )</td>
<td>maximum number of channel taps in the relay-to-destination channel matrix</td>
</tr>
<tr>
<td>( \Lambda_{\bar{m}\bar{m}} )</td>
<td>equivalent noise covariance matrix</td>
</tr>
<tr>
<td>( \lambda_c )</td>
<td>carrier wavelength</td>
</tr>
<tr>
<td>( M )</td>
<td>number of repetitions of the training sequence</td>
</tr>
<tr>
<td>( \bar{m} )</td>
<td>discrete-time noise sequence vector at the relays</td>
</tr>
<tr>
<td>( m )</td>
<td>number of stages in the shift register</td>
</tr>
<tr>
<td>( \bar{m}_c )</td>
<td>continuous-time noise sequence vector at the relays</td>
</tr>
<tr>
<td>( N )</td>
<td>length of the sounding sequence</td>
</tr>
<tr>
<td>( N' )</td>
<td>user-defined number of samples</td>
</tr>
<tr>
<td>( N_{\text{cmd}}^{\max} )</td>
<td>maximum number of commands in a mission</td>
</tr>
</tbody>
</table>
### Appendix

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{n}$</td>
<td>discrete-time equivalent noise vector at the destinations</td>
</tr>
<tr>
<td>$\mathbb{N}$</td>
<td>set of the natural numbers</td>
</tr>
<tr>
<td>$N_a$</td>
<td>number of antennas</td>
</tr>
<tr>
<td>$N_{coop}$</td>
<td>total number of cooperations</td>
</tr>
<tr>
<td>$N_{coop,\text{min}}$</td>
<td>minimum cooperation configuration</td>
</tr>
<tr>
<td>$N_{exc}$</td>
<td>excess number of cooperations</td>
</tr>
<tr>
<td>$N_r$</td>
<td>number of relays</td>
</tr>
<tr>
<td>$N_{sto}$</td>
<td>number of timeslots for one store command</td>
</tr>
<tr>
<td>$N_{sw}$</td>
<td>number of timeslot required for switching</td>
</tr>
<tr>
<td>$N_u$</td>
<td>number of users</td>
</tr>
<tr>
<td>$\nu$</td>
<td>discrete frequency</td>
</tr>
<tr>
<td>$N_{\text{HW,rx}}, N_{\text{HW,rx}}^B, N_{\text{HW,tx}}^A, N_{\text{HW,tx}}^B$</td>
<td>hardware delay at receiver</td>
</tr>
<tr>
<td>$N_{\text{HW,tx}}, N_{\text{HW,tx}}^B, N_{\text{HW,tx}}^A$</td>
<td>hardware delay at transmitter</td>
</tr>
<tr>
<td>$\mathcal{P}_r$</td>
<td>power constraint at the relays</td>
</tr>
<tr>
<td>$\psi$</td>
<td>phase</td>
</tr>
<tr>
<td>$\mathbb{R} { \cdot }$</td>
<td>real part</td>
</tr>
<tr>
<td>$\mathbf{r}$</td>
<td>discrete-time relay transmit vector</td>
</tr>
<tr>
<td>$\mathbb{R}$</td>
<td>set of the real numbers</td>
</tr>
<tr>
<td>$R(l)$</td>
<td>supported rate for link $l$</td>
</tr>
<tr>
<td>$\mathbf{r}_c^{(l)}$</td>
<td>continuous-time relay transmit vector</td>
</tr>
<tr>
<td>$\mathbf{R}_{\text{cp}}^{(l)}$</td>
<td>compound noise autocorrelation at receiver $l$</td>
</tr>
<tr>
<td>$\mathbf{R}_{\text{ISI}}^{(l)}$</td>
<td>multi-user interference autocorrelation at receiver $l$</td>
</tr>
<tr>
<td>$S$</td>
<td>number of complex samples in a buffer</td>
</tr>
<tr>
<td>$S_0(f), S_\psi(f), S_{F_i}$</td>
<td>power spectrum</td>
</tr>
<tr>
<td>$\mathbf{s}$</td>
<td>discrete-time transmit source symbol vector</td>
</tr>
<tr>
<td>$\mathbf{s}_c$</td>
<td>continuous-time transmit source symbol vector</td>
</tr>
<tr>
<td>$\text{sinc}(x)$</td>
<td></td>
</tr>
<tr>
<td>$\text{SNR}_{\text{ref}}$</td>
<td>reference SNR</td>
</tr>
<tr>
<td>$\sigma_m^2$</td>
<td>relay noise variance</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\sigma^2_s$</td>
<td>signal variance</td>
</tr>
<tr>
<td>$\sigma^2_w$</td>
<td>destination local noise variance</td>
</tr>
<tr>
<td>$\tau$</td>
<td>continuous-time index</td>
</tr>
<tr>
<td>$\tau_1, \tau_2$</td>
<td>continuous delay index</td>
</tr>
<tr>
<td>$T_R$</td>
<td>repetition rate</td>
</tr>
<tr>
<td>$T_d$</td>
<td>excess delay</td>
</tr>
<tr>
<td>$T_{ts}$</td>
<td>timeslot duration</td>
</tr>
<tr>
<td>$T_{est}$</td>
<td>estimation duration</td>
</tr>
<tr>
<td>$\text{tr} (\cdot)$</td>
<td>trace operator</td>
</tr>
<tr>
<td>$\tau$</td>
<td>transpose operator</td>
</tr>
<tr>
<td>$T_s$</td>
<td>sample period</td>
</tr>
<tr>
<td>$T_{sw}$</td>
<td>switching duration</td>
</tr>
<tr>
<td>$\theta_x [\cdot]$</td>
<td>cyclic autocorrelation of sequence $x$</td>
</tr>
<tr>
<td>$\theta_{x,y} [\cdot]$</td>
<td>cyclic crosscorrelation of sequence $x$ and $y$</td>
</tr>
<tr>
<td>$V, V_0$</td>
<td>voltage</td>
</tr>
<tr>
<td>$W$</td>
<td>rate, channel bandwidth</td>
</tr>
<tr>
<td>$\vec{w}$</td>
<td>discrete-time local noise sequence vector at the destinations</td>
</tr>
<tr>
<td>$\vec{w}_c$</td>
<td>continuous-time local noise sequence vector at the destinations</td>
</tr>
<tr>
<td>$\omega_{IF}$</td>
<td>angular IF frequency</td>
</tr>
<tr>
<td>$\omega_{RF}$</td>
<td>angular RF frequency</td>
</tr>
<tr>
<td>$\omega_{test}$</td>
<td>angular frequency test</td>
</tr>
<tr>
<td>$X$</td>
<td>discrete-time source-to-relay channel matrix</td>
</tr>
<tr>
<td>$X_c$</td>
<td>continuous-time source-to-relay channel matrix</td>
</tr>
<tr>
<td>$\vec{y}, \vec{y}'$</td>
<td>nullspace gain vector</td>
</tr>
<tr>
<td>$\vec{y}_s$</td>
<td>subspace gain vector</td>
</tr>
<tr>
<td>${ \cdot }$</td>
<td>one-sided z-transform</td>
</tr>
<tr>
<td>$Z$</td>
<td>discrete-time relay-to-destination channel matrix</td>
</tr>
<tr>
<td>$\mathbb{Z}$</td>
<td>set of the integer numbers</td>
</tr>
</tbody>
</table>
\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|}
\hline
$Z_c$ & continuous-time & relay-to-destination channel matrix \\
\hline
\end{tabular}
\end{center}
\end{table}
Bibliography


[73] W. A. Miller, B. R. Petersen, and B. G. Colpitts. Parallel MIMO channel measurement


Bibliography


[125] WINNER II. *WINNER II interim channel models*. IST, November 2006. D1.1.1 v. 1.1.


