Use Less, Pay More
Can Climate Policy Address the Unfortunate Event for Being Poor?

Author(s):
Bretschger, Lucas; Supaphiphat, Nujin

Publication Date:
2012-08

Permanent Link:
https://doi.org/10.3929/ethz-a-007351770

Rights / License:
In Copyright - Non-Commercial Use Permitted
Use Less, Pay More: Can Climate Policy Address the Unfortunate Event for Being Poor?

L. Bretschger and N. Suphaphiphat

Working Paper 12/166
August 2012

Economics Working Paper Series
Use Less, Pay More: Can Climate Policy Address the Unfortunate Event for Being Poor?

Lucas Bretschger¹ and Nujin Suphaphiphat²

June 2012

Abstract

The paper develops a two-region endogenous growth model with climate change affecting the countries’ capital stocks negatively. We compare two different policies aimed at supporting less developed countries: climate mitigation by rich countries, which diminishes the increase in stock pollution and hence capital depreciation, and income transfers in the tradition of development aid. Under a mild set of assumptions we find that active climate policies are more efficient for rich economies and also, remarkably, better for poor countries than additional development aid. The main reason is the difference between the two policies with respect to their effects on economic growth. The results are robust with respect to possible model extensions.

Keywords: Climate policy, development aid, endogenous growth, stock pollution

JEL Classification: O10, Q52, Q54

¹CER-ETH Center of Economic Research at ETH Zurich, ZUE F7, CH-8092 Zurich, Tel. +41 44 632 21 92, Fax +41 44 632 13 62, email: lbretschger@ethz.ch.
²CER-ETH Center of Economic Research at ETH Zurich and International Monetary Fund, 700 19th Street N.W., Washington, D.C., 20431, email: nsuphaphiphat@imf.org.

Acknowledgements: will be added
1 Introduction

One of the most intriguing insights of recent climate research is that the (negative) effects of climate change are severely biased against the poor economies, see World Bank (2010). Main reasons are the internationally unequal availability of capital and knowledge for adaptation as well as differences in climate vulnerability. The situation is aggravated by the finding that climate change not only affects income levels but also development prospects, see Dell et al. (2009), Collier et al. (2010), and Bretschger and Valente (2011). If poorer countries have to use an ever-growing share of their scarce capital for the protection of the economy against rising temperatures their prospects for positive economic development are severely hampered. Using less of world environmental services but paying more when these services deteriorate, climate change becomes another unfortunate event for being poor on the planet.

Active climate policies of advanced countries may improve economic conditions of the poor but the more traditional direct income transfers are also effective in this respect. It has recently been argued that the direct financial support of poor countries, i.e. development aid, might be equally or even more efficient to foster development and to facilitate climate adaptation in poor countries, see Schelling (1995, p. 400) who writes: "Abatement expenditures should have to compete with alternative ways of raising consumption utility in the developing world." The contribution of the present paper is to provide a theoretical framework to evaluate the different policy alternatives.

The paper develops a novel two-region endogenous growth model with global stock of pollution and mitigation expenditures to derive the impact of different policies, specifically policies aimed at supporting less developed countries to cope with the effects of climate change. Climate change is assumed to cause higher capital depreciation because buildings and machines are suitable for a given temperature range, but are damaged with higher temperatures and the associated higher frequencies of environmental problems such as floods and droughts. The same argument can be made for the knowledge stock because climate change requires new forms of technological know-how, partly replacing existing knowledge. The model reflects that global warming is caused by a variety of economic activities such as energy use, deforestation, land use changes, and farming methods, all of which are assumed to be linked with the use of capital. The first policy option is active climate mitigation...
by rich countries, which diminishes stock pollution and hence capital depreciation on a worldwide level. The second consists of steady income transfers from North to South in the tradition of development aid. We provide a full characterization of the solutions for the different scenarios.

Under a mild set of assumptions we find no evidence that climate policies of the North are not efficient for poor countries in comparison to direct income transfers, because they affect not only consumption levels but the growth rate of the economies. The direct benefit for the rich countries and the absence of efficiency losses when transferring income between different economies are also in favor of climate policy. This is not to say that development aid is not a desirable instruments for many other e.g. humanitarian reasons. But the paper derives that it appears useful to complement the traditional policies with active climate policies to strengthen the support for poorer countries. The model results continue to hold when we introduce international trade and polluting resources into the model.

The paper starts from the original idea of Schelling (1995), who discusses climate abatement costs and different policies to support developing countries. The framework is based on the literature on pollution and growth, in particular Withagen (1995), Michel and Rotillon (1995), Bovenberg and Smulders (1995), Smulders and Gradus (1996), Stokey (1998), Brock and Taylor (2005, 2010), and on the papers on climate change and growth, see Grimaud et al. (2007), and Bretschger and Valente (2011). It is also related to the literature on natural exhaustible resources and growth, especially on Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974), Barbier (1999), Scholz and Ziemes (1999), Xepapadeas (2006), Bretschger and Smulders (2007), and Peretto (2009), and in particular with the two-country resource models provided in Daubanes and Grimaud (2010) and Berlinschi and Daubanes (2012).

The remainder of the paper is organized as follows. Section 2 develops the basic two-country model and balanced growth. Section 3 introduces mitigation policy, while section 4 analyzes development aid. Section 5 discusses our findings and presents a calibration of the model. Section 6 introduces model extensions. Finally, section 7 concludes.
2 Two-Country Growth Framework

2.1 Overview

We consider a model of a world with global stock pollution and two different endogenously growing regions called "countries". The "rich" Country 1 and the "poor" Country 2 produce final goods by employing two types of capital; physical capital and knowledge capital, respectively. In the basic model, capital use, which is proportional to output, increases the stock of global pollution, which entails global temperature increase. Increasing temperatures raise capital depreciation rates and thus negatively affect capital accumulation in both countries. The rich country accumulates both types of capital while the poor country accumulates physical capital and receives a part of knowledge from the rich country through international knowledge diffusion. This asymmetry realistically reflects the fact that knowledge is created in the industrialized countries (the vast majority of patents is still developed and granted in the rich countries) and part of it is disseminated to the rest of the world via international knowledge transmission. We first set up the basic model and derive the conditions for balanced growth. We then evaluate two types of policies: climate mitigation policy and development aid. In a later section, we extend the basic model to international trade and polluting exhaustible resources.

In order to understand the effects of global pollution on long-run growth most clearly, the baseline model is exempt from goods trade. The general results still hold when we extend the model with international trade, see section 6. In addition, we will treat the cases of polluting natural resources and of credit constraints in the poor economy separately in the end.

2.2 Rich country and pollution

This section describes the basic framework of the model and derives the conditions for balanced growth path in the baseline case in which both countries are affected by global pollution but do not impose any policy measure.

Country 1 produces final goods \(Y_1\) with physical capital \(K\) and knowledge capital \(B\) using the production technology

\[ Y_1(t) = F_1 [B_1(t), K_1(t)] = AB(t)^\alpha K(t)_1^{1-\alpha} \]  

(1)

where \(A\) and \(0 < \alpha < 1\) are given parameters. The production function exhibits
constant returns to scale with respect to the inputs. Assuming that an increase in
global pollution $\dot{P}$ harms the existing capital stock and that the associated depreciation
rate $\delta$ is the same for both capital types, $\delta_A = \delta_K = \delta$, capital accumulation for
Country 1 follows the law of motion

$$\dot{K}_1(t) + \dot{B}_1(t) = Y_1(t) - C_1(t) - \delta \cdot \dot{P}(t)$$

(2)

where $C_1 > 0$ is the consumption level; $\delta > 0$ serves as a capital exposure parameter
that captures the effect of the change in global pollution on the depreciation of
the country’s capital stock. Eq. (2) is the resource constraint for the economy: the
change in overall capital stock in the country equals output less consumption and
depreciation generated by the change in global pollution. The Appendix shows that
in the optimum we have $B_1/K_1 = \alpha/(1 - \alpha)$, see (A.5), which represents equalized
(net) marginal productivities of $B_1$ and $K_1$.\(^3\) We define "broad" capital as $\bar{K}_1 = \bar{B}_1 + K_1 = K_1/(1 - \alpha) = Y_1/A$. With the two countries labelled $i = 1, 2$, the change
in global pollution is assumed to be given by

$$\dot{P}(t) = \psi \sum_i \bar{K}_i(t), \quad i = 1, 2$$

(3)

where $\psi > 0$ is a parameter representing the impact of broad capital on the change in
global pollution stock. Note that pollution stock increase depends on both countries’
capital stocks $\bar{K}_i$, which are proportional to the respective output levels $Y_i$.\(^4\)

A representative household in Country 1 wishes to maximize overall utility, $U_1$, as given by

$$U_1 = \int_0^\infty e^{-\rho t} \log C_1(t) \, dt$$

(4)

where $C(t)$ is the total consumption at time $t$ and $\rho > 0$ is the rate of time preference.

**Proposition 1** On a balanced growth path, consumption growth increases in pro-
ductivity $A$ and is low when the capital exposure parameter $\delta$, the pollution impact
parameter $\psi$, and the discount rate $\rho$ are high.

\(^3\)Provided that historical capital stocks deviate from the given ratio, i.e. when $B_0/K_0 \neq \alpha/(1 - \alpha)$,
equal marginal productivities are the result of an adjustment process, in which the economy invests
in one capital type only until the returns become equal, which is then maintained forever.

\(^4\)Relating pollution to capital or output is identical to assuming pollution stemming from a
resource with abundant supply like coal; the case of resources with limited supply like oil is presented
separately below. Because the main model results do not change we stick with the more convenient
assumption in the main part of the paper.
Proof. Consumption growth is given by

\[ \dot{C}_1 = (1 - \alpha)\tilde{A} - \delta\psi - \rho \]  

where \( \tilde{A} \equiv A \left( \frac{\alpha}{1 - \alpha} \right)^{\alpha} \) see the Appendix for the derivation.

The steady state consumption growth rate in Country 1 is sustainable if and only if

\[ (1 - \alpha)\tilde{A} > \delta\psi + \rho \]  

Hence, according to proposition 1, the product of the capital exposure parameter \( \delta \) and the pollution impact parameter \( \psi \) plus the discount rate \( \rho \) must not exceed the constant \((1 - \alpha)\tilde{A}\) for consumption growth to remain positive.

2.3 Poor country

The poor Country 2 has a similar setup as Country 1 in terms of consumption, final production, and the effect of global pollution. The only exceptions are that (i) the size of the physical capital stocks differs, i.e. we have \( K_1 > K_2 \), and that (ii) Country 2 does not produce any knowledge capital, but profits from international knowledge diffusion. Following Daubanes and Grimaud (2010), a diffusion process with a constant time-lag between an innovation in the rich country and its availability in the poor country results in the relationship\(^5\)

\[ B_2(t) = \theta B_1(t), \quad 0 < \theta < 1. \]  

Country 2 accumulates physical capital \((K_2)\); the law of motion for the physical capital stock becomes

\[ \dot{K}_2(t) = Y_2(t) - C_2(t) - \delta \dot{P}(t) \]  

where \( Y_2 = AB_2^\alpha K_2^{1-\alpha} \). A representative household in Country 2 wishes to maximize the discounted value of lifetime utility

\[ U_2 = \int_0^\infty e^{-\rho t} \log C_2(t) \, dt \]  

subject to Eq.(8). The growth of consumption in Country 2 becomes, see the Appendix.

\(^5\)A similar treatment of technical progress in an international context is also used in Berlinski and Daubanes (2012), where (labor) productivity in the "South" is a constant fraction of that in the "North."
Interestingly, consumption growth depends on the capital gap between the countries \((K_1/K_2)\), but the steady-state analysis below shows how this aspect of the model can be conveniently accommodated.

### 2.4 Balanced Growth Paths

With regard to the growth rates in both countries we state

**Proposition 2** In equilibrium, the consumption growth rates are equal between the two countries; they equal the capital growth rates according to

\[
\dot{C}_i = \dot{B}_i = \dot{K}_i = \dot{Y}_i = (1 - \alpha)\bar{A} - \delta\psi - \rho \quad (i = 1, 2)
\]

**Proof.** See the Appendix. ■

The economic intuition is that, due to constant returns to broad capital in the rich country and proportional international knowledge diffusion, we obtain balanced growth in the world economy. The ratio of the two (physical) capital stocks turns out to be \(K_1/K_2 = 1/\theta\), see the Appendix. As usual in linear growth models, the system jumps immediately to the balanced growth path, without transitional dynamics.

Before proceeding it is important to note that this model result is realistic, despite a number of growth miracles and disasters of poor countries receiving high attention in the public. The dispersion of of world income has not changed much in the last decades, see e.g. Acemoglu and Ventura (2002); accordingly, when taking averages of rich and poor economies the growth rates were quite similar.

The steady state levels of consumption-to-capital ratios in both countries \(\chi_i\) can be written as

\[
\begin{align*}
\chi^{ss}_1 &= \frac{\rho - \delta\psi\theta}{1 - \alpha} \\
\chi^{ss}_2 &= \alpha\bar{A} - \delta\psi \frac{1 + \theta\alpha}{\theta(1 - \alpha)} + \rho
\end{align*}
\]

which exhibits the fundamental asymmetry between the two countries.\(^6\)

\(^6\)Because country 2 only accumulates physical capital, the consumption-to-capital ratio is higher than country 1. To ensure that the consumption level in country 1 is always higher than in country 2 we impose the restriction \(\frac{\chi^{ss}_1}{\chi^{ss}_2} > \theta\).
3 Mitigation Policy

This section introduces the first policy option, which is a mitigation policy of the rich country to reduce global pollution increase. Specifically, we aim at analyzing the effect of a climate mitigation policy on Country 1 and, especially, the poor country’s growth and income. The rich Country 1 is assumed to collect a tax on consumption with a proportional rate $\tau$ and use revenues for mitigation measures, reducing depreciation from lower global pollution increase. We assume that tax revenue reduces $0 < \eta < 1$ units of increase in global pollution.

With this, the effective increase in pollution stock (which we label $\Delta$) becomes equal to $(1 - m)$ times the potential pollution stock increase $\dot{P}$. In particular,

$$\dot{M}(t) = (1 - m)\dot{P}(t) \equiv (1 - m)\psi \sum_i \dot{K}_{m_i}(t)$$

(14)

where $m(\tau)$ represents the mitigation rate corresponding to the level of the tax rate chosen by the government, see below.

3.1 Rich country

Let the subscript "m" represent our model variables with the mitigation policy. With a tax on consumption, Country 1’s resource constraint in Eq.(2) becomes

$$\dot{K}_{m_1}(t) + \dot{B}_{m_1}(t) = \hat{K}_{m_1}(t) = Y_{m_1}(t) - (1 + \tau)C_{m_1}(t) - \delta \dot{M}(t)$$

(15)

The change in overall capital in Country 1 equals total output less consumption, tax induced by consumption, and the depreciation rate incurred by the change in global pollution after mitigation policy. The introduction of the tax has two effects. In the short run, it decreases consumption, but in the long run, the effect might be reversed through the effect of lower capital depreciation. For simplicity, we will take the tax rate as given in order to compare with development policy. Later on, we will allow tax rates and corresponding mitigation rates to vary. Note that, an alternative approach is to set mitigation rate ($m$) as an objective rate by the social planner, which relates to the amount of tax rate imposed by the rich Country 1. When $m$ is close to zero, the policy makers aim at low mitigation, and thus does not slow down the depreciation rate significantly. When $m$ is close to unity, the mitigation policy can eliminate the change in global pollution almost fully. In this case, the policy allows
to minimize the change in pollution, which results in minimum (climate-induced) depreciation of capital.

**Proposition 3** A consumption tax with the revenues used for climate mitigation reduces the instantaneous household consumption in the rich country but increases its consumption growth rate.

**Proof.** Following the procedure of the previous section but now including mitigation policy, the growth rate of consumption becomes

\[ \dot{C}_{m1} = (1 - \alpha)A - (\delta(1 - m)\psi + \rho) \]  

see the Appendix for the derivation.

The mitigation policy incurred by a tax on consumption increases the growth rate of consumption because the policy reduces the depreciation rate through a slow down in the change of global pollution. However, the tax does not distort the growth rate, it only decreases the level of (initial) consumption. Interestingly, the term \(1 - m\) directly appears in the growth equation, so that an increase in abatement efficiency \(m\) has a direct dynamic impact on the economy.

From the economy’s resource constraint, Eq.(15), the growth rate of physical capital after tax on consumption and mitigation policy becomes

\[ \dot{K}_{m1} = (1 - \alpha)A_1 - \delta(1 - m)\psi \left( \frac{1 + \theta} {1 - \alpha} \right) - (1 + \tau_m) \frac{C_{m1}} {K_{m1}} - \delta(1 - m)\psi \frac{K_{m2}} {K_{m1}} \]

which is quite a complicated expression. However, since the ratio between two types of capital is the same as the baseline model, the production for the final output is, again, an "AK"-type, in which, all variables grow at the same rate, so that we can conveniently write

\[ Y_{m1}(t) = \tilde{A}K_{m1}(t) \]

\[ \dot{Y}_{m1} = \dot{K}_{m1} = \dot{B}_{m1} = \dot{C}_{m1} \]
3.2 Poor country

The mitigation policy is conducted by collecting a consumption tax in Country 1. However, it creates a positive externality to Country 2 such that the mitigation policy reduces the change in *global* pollution. As a result, the depreciation rate in Country 2 will also fall in the same proportion as Country 1. The growth rate of physical capital in Country 2 is derived in the Appendix.

**Proposition 4** Mitigation policy by the rich country unambiguously increases both growth rate and consumption level of the poor country.

**Proof.** With the same properties as the baseline model, the balanced growth paths after the mitigation policy are

\[
\hat{C}_{mi} = \hat{K}_{mi} = \hat{B}_{mi} \quad (i = 1, 2)
\]

Consequently, the ratio of physical capital between two countries are the same as before. Specifically,

\[
\frac{K_{m2}(t)}{K_{m1}(t)} = \theta
\]

As a result of the policy \((m > 0)\), \(\hat{C}_{m2}\) increases (parallel to \(\hat{C}_{m1}\)) while the output gap between the two countries is unchanged.

\[
\hat{C}_{m1} = \hat{C}_{m2} = (1 - \alpha)\bar{A} - (\delta(1 - m)\psi + \rho)
\]

\[
Y_{m2}(t) = \theta Y_{m1}(t)
\]

so that the level and growth effects are established.

The steady state level of for consumption-to-physical-capital ratios \(\chi\) can be derived as

\[
\chi_{m1}^{ss} = \frac{\rho - \delta(1 - m)\psi\theta}{(1 + \tau_m)(1 - \alpha)}
\]

\[
\chi_{m2}^{ss} = \alpha\bar{A} - \delta(1 - m)\psi\left(\frac{1 + \alpha\theta}{(1 - \alpha)\theta}\right) + \rho
\]

The effect of mitigation policy supported by the tax revenue from consumption in Country 1 generates a positive growth effect not only on the own country but also on Country 2. The underlying reason is that the mitigation policy aims to reduce the change in global pollution. Consequently, it equally decreases depreciation rates in both countries.
3.3 Balanced Budget

To ensure a balanced budget of the government, tax revenue from consumption must be entirely spent for mitigation policy, that is we have

\[ \tau C_{m1} = m \delta \dot{P} \]  

(25)

The rate of mitigation effectiveness can then be found as

\[ \eta = \frac{\tau (1 - \alpha) \chi_{m1}^{qs}}{\delta (1 + \theta)} \]  

(26)

Substitute Eq.(23), the relationship between a tax rate and mitigation rate can be found as

\[ m(\tau) = \frac{(\rho - \delta \psi \theta) \tau}{(1 + \theta + \tau) \delta \psi} \]  

(27)

The result establishes that the mitigation rate is monotonically increasing in the tax rate.

4 Development aid

As a second policy option, the rich Country 1 can collect a tax on consumption with a proportional rate \( \tau_d \) and use revenues to raise output in Country 2. For the purpose of comparison, we will apply the same tax rate as in the mitigation policy case.

\[ \tau_m = \tau_d \]  

(28)

The subscript \( d \) is introduced to label our model variables in case of the development aid policy.

4.1 Rich country

Country 1 collects tax revenue from consumption to give to Country 2. As a result, the first order conditions for Country 1 become

\[ \frac{1}{C_{d1}(t)} = (1 + \tau) v_{d1} \]  

(29)

Eq.(29) states that a marginal utility of consumption equals to the shadow value of overall capitals included tax. Since the tax revenue does not generate any additional
benefits to Country 1 in terms of global pollution reduction, the shadow price for physical and knowledge capitals are the same as the baseline case, Eq.(A.3), and Eq.(A.4), respectively. Consequently, the growth rate of consumption in Country 1, the ratio of physical to knowledge capital are the same. Specifically,

\[
\frac{K_{d1}(t)}{B_{d1}(t)} = \frac{1 - \alpha}{\alpha} \tag{30}
\]

\[
\hat{C}_{d1} = (1 - \alpha)\bar{A} - \delta\psi - \rho \tag{31}
\]

In addition, with the tax on consumption, consumption in Country 1 is now decreased by the amount of the tax rate, without any offsetting gain from lower depreciation and higher growth as in the mitigation case. For capital growth we get

\[
\hat{K}_{d1}(t) = (1 - \alpha)\bar{A} - \delta\psi \left(\frac{1 + \theta\alpha}{1 - \alpha}\right) - (1 + \tau)\frac{C_1(t)}{K_1(t)} - \delta\psi\frac{K_2(t)}{K_1(t)} \tag{32}
\]

### 4.2 Poor country

Country 2 receives the lump-sum revenue \((D_1)\) from Country 1. The resource constraint for Country 2 then becomes

\[
\hat{K}_{d2}(t) = Y_{d2}(t) + D_1(t) - C_{d2}(t) - \delta \cdot \hat{P}(t) \tag{33}
\]

**Proposition 5** Development aid raises the income level of the poor economy but does not increase its growth rate.

**Proof.** As a result of the policy, the growth rate of consumption in the poor country 2 remains unchanged while the output gap between the two countries is diminished, according to

\[
\hat{C}_{d1} = \hat{C}_{d2} = (1 - \alpha)\bar{A} - (\delta\psi + \rho) \tag{34}
\]

\[
Y_{d2}(t) + D_1(t) = (1 + \gamma)Y_2(t) = (1 + \gamma)\theta Y_{d1}(t) \tag{35}
\]

where \(\gamma \equiv \frac{\bar{A}(\mu - \delta\phi)}{\theta (1 - \alpha)(1 + \tau\phi)}\) (see the Appendix) \(\blacksquare\)

The effect of development policy in terms of a transfer from Country 1 to Country 2 has no growth effect because the policy does not affect the growth equation. However, the policy changes the level of output such that the output gap between two countries shrink proportionally. The consumption-capital ratios \(\chi\) can be written as

\[
\chi = \frac{(1 - \alpha)\bar{A} - (\delta\psi + \rho)}{(1 + \gamma)\theta Y_{d1}(t) - \delta\psi\frac{K_2(t)}{K_1(t)}}
\]
\[ \chi^{ss}_{d1} = \frac{(\rho - \delta \psi \theta)}{(1 - \alpha)(1 + \tau_d)} \]  

\[ \chi^{ss}_{d2} = \alpha \tilde{A} - \delta \psi \frac{1 + \alpha \theta}{(1 - \alpha) \theta} + \rho + \frac{\tau}{\theta} \chi^{ss}_{d1} \]  

According to (37) it turns out that the steady state consumption-capital ratio in country 2 depends on the constant transfer of the rich country 1, which is highly plausible.

5 Discussion and Calibration

In our model, global pollution has a negative effect biased against poor economies because the rich country generates more pollution but both of them face the effect of pollution at the equal rate \( \delta \). Specifically, using Eq.(7), and Eq.(A.14), one can see that

\[ \tilde{K}_2(t) = \theta \tilde{K}_1(t) \]  

Eq.(38) implies the contribution of global pollution incurred by both countries. Applying Eq.(3), Eq.(A.11) and Eq.(A.12) then say that - even when the poor country emits a lower share of pollution \( \theta < 1 \) - it faces the same negative effect towards capital accumulation through deteriorating depreciation.\(^7\)

5.1 Policy Comparison

Figure 1 and 2 display the effect of the two competing policies on consumption for Country 1, and Country 2, respectively. \( Cm1 \) and \( Cm2 \) represent the effect of mitigation policy on consumption in Country 1 and Country 2. \( Cd1 \) and \( Cd2 \) show the effect of development policy on consumption in both countries, respectively. And finally, \( Cb1 \) and \( Cb1 \) are the consumption levels when there is no policy (baseline case). The policy takes place at time \( t = 0 \).

\[ ****\text{Figures 1 and 2****} \]

A higher vulnerability of the poor country could be additionally included by assuming an internationally different \( \delta \). But the effect on the results would be easily seen in the model so that this aspect is omitted for the sake of briefness.
As can be seen, the mitigation policy has a positive impact on the growth rate of output in both countries. This is because the mitigation policy in Country 1 decreases the change in global pollution, resulting in the deceleration of depreciation. However, the policy instantaneously decreases the level of consumption-capital ratio in the rich country after tax because the consumption tax decreases country 1’s consumption level even though there is a positive growth effect of mitigation measures. The policy provides benefits to country 2 in both instantaneous level effect and growth effect.

On the other hand, the development policy does not have an impact on growth but affects the level of output in Country 2 such that the output gap between the two countries is now smaller. This is because the amount of transfer is proportional to the output in Country 2.

Comparing between two policies, the rich country 1 is always better off from the mitigation policy because even though the policy reduces the ratio of consumption-capital, the magnitude of a drop in mitigation policy is always less than the one in development policy. This is because there exists some positive effect from the mitigation policy through a slowdown in depreciation via a change in global pollution. Moreover, mitigation policy creates higher growth effect. However, the effect of the policy becomes ambiguous for country 2. because both policies increases the ratio of consumption-to-capital, the magnitude of the jump varies depending on various parameters. Fortunately, the model allows us to solve the problem analytically.

5.2 Welfare

Given an AK-like production function with log utility, welfare of both countries can be solved in a closed-form manner. We define welfare as the sum of the present value $PV$ of the log of consumption over an infinite time horizon. In particular,

$$U_j^{ssi} = \frac{1}{\rho} \left( \log C_{ssi}(0) + \frac{g_j}{\rho} \right)$$  \hspace{1cm} (39)

where $j = b, m, d$ stands for the baseline model, mitigation policy and development policy, respectively. The determination of welfare can be divided into two parts. The first one is the initial value of consumption, called "instantaneous level", and the second one is the growth part, called "growth".

It is obvious for country 1 that welfare is higher under mitigation policy as a result of an increase in both instantaneous level and growth factors. For country 2, on the
other hand, it is possible that the instantaneous level of consumption might be lower in the mitigation policy at the beginning. However, as mitigation policy generates higher growth effect, the accumulative effect might be higher in term of welfare.

5.3 Calibration

So far, we have derived an important distinction between the two policies: growth versus level effects. To evaluate the relevance for practical use, we consider some "realistic" values for the parameters in the model. Obviously, in the case of the parameters for climate impact and exposure $\psi$ and $\delta$, it is difficult to directly apply the information from international climate studies to our framework. In the following, we compare the growth effect from a mitigation policy against the level effect from the development aid, assuming identical cost in terms of present welfare.

The used parameter values are summarized in Table 1. Assume country 1 sets tax as its objective at 15 percent.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.05</td>
<td>$m(\tau)$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.70</td>
<td>$A$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.10</td>
<td>$\psi$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.70</td>
<td>$\tau$</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Assume that initial capital for country 1, $K_1(0)$, is 100 units. Variables of interest are then calculated and presented in Table 2.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Value</th>
<th>Variables</th>
<th>Value</th>
<th>Variables</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^b$</td>
<td>0.0386</td>
<td>$U^b_{ss1}$</td>
<td>65.13</td>
<td>$U^b_{ss2}$</td>
<td>63.93</td>
</tr>
<tr>
<td>$g^m$</td>
<td>0.0415</td>
<td>$U^m_{ss1}$</td>
<td>64.61</td>
<td>$U^m_{ss2}$</td>
<td>67.51</td>
</tr>
<tr>
<td>$g^d$</td>
<td>0.0386</td>
<td>$U^d_{ss1}$</td>
<td>62.33</td>
<td>$U^d_{ss2}$</td>
<td>62.33</td>
</tr>
</tbody>
</table>

The calibrated results show that the steady state growth rates of two economies in the baseline and development policy are the same while the growth rate in mitigation policy is 0.3 percent higher due to the deceleration in depreciation.

When we consider welfare between two policies, calibration shows several interesting results. Firstly, mitigation policy is always better than development policy for country 1 due to the growth effect. Secondly, both development and mitigation policies increase welfare in country 2 comparing to the baseline. The size of the increase
depends on the amount of income transfer in development policy, or a positive change in the growth rate in mitigation policy. In this particular case, mitigation policy is also better than development policy.

Moreover, when we allow a tax rate to vary, the results shown in Table 3 are interesting. Firstly, a higher tax rate leads to higher growth through higher mitigation rate. However, the mitigation rate is always less than tax rate. As a result, a higher tax rate leads to lower level of welfare for country 1 regardless of policies. However, country 1’s welfare from mitigation policy is still higher than the one in development policy. From country 2’s perspective, the higher tax rate increases welfare in both policies.

<table>
<thead>
<tr>
<th>τ(%)</th>
<th>m(%)</th>
<th>g^m(%)</th>
<th>U^m_{ss1}</th>
<th>U^d_{ss1}</th>
<th>U^m_{ss2}</th>
<th>U^d_{ss2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.00</td>
<td>18.95</td>
<td>4.24</td>
<td>64.42</td>
<td>61.48</td>
<td>68.52</td>
<td>67.19</td>
</tr>
<tr>
<td>25.00</td>
<td>23.08</td>
<td>4.32</td>
<td>64.23</td>
<td>60.67</td>
<td>69.45</td>
<td>67.78</td>
</tr>
<tr>
<td>30.00</td>
<td>27.00</td>
<td>4.40</td>
<td>64.04</td>
<td>59.88</td>
<td>70.35</td>
<td>68.31</td>
</tr>
<tr>
<td>35.00</td>
<td>30.73</td>
<td>4.47</td>
<td>63.84</td>
<td>59.13</td>
<td>71.17</td>
<td>68.79</td>
</tr>
</tbody>
</table>

We conclude that if climate mitigation policy is indeed able to have an (non-negligible) impact on the growth rate, as suggested by the model, it is not only favorable for the welfare of the poor economy but also good for the rich economy, because country 1 gets a compensation for lower current consumption (due to the tax) in the form of a higher consumption growth rate.

Focusing on country 2, higher taxes increase welfare monotonically. However, the one with mitigation policy is strictly higher. This is because tax increases both instantaneous level effect and growth effects. Hence, we can conclude that

**Proposition 6** Under reasonable parameters, welfare in both countries are higher in mitigation policy than the one in development policy. Hence, mitigation policy is more efficient than development policy in both countries.

Let us also discuss the robustness of the difference between growth and level effects in the model. As regards the effects of climate change, we can be reasonably certain that existing capital will be partly destroyed if the effects are as predicted by science results; hence, the result of growth effects of climate change appears to be robust. That a transfer to poor economies has no impact on their savings decision is also fair.
to assume. However, if clearly designed development aids are able to increase the
growth rate of the poor economy, the results for the second policy option are more
favorable. It would involve improving the conditions of capital build-up, in particular
the ability of the lagging economy to accumulate knowledge at its own. In such a
case, poor countries would start to behave like the rich countries in the model and
probably catch-up in terms of income and consumption level. In such a more equal
world, the effects of climate change would no longer be biased against a group of
countries, but still have negative growth effects. Another interesting case to study
are credit constraints in the poor country, which we discuss in the next section.

6 Model extensions

6.1 International trade

An interesting issue is whether the results continue to hold when we introduce goods
trade between the countries. One might suspect that trade has a moderating effect
when policies are introduced, because an increase in a country’s growth rate may
affect the terms of trade and thus income. On the other hand, because the model is
linear, we may also expect the terms of trade effects coming out of goods trade to
reinforce our results, not changing the main model conclusions.

In this section, we allow the two countries to trade final goods ($Y$). Thus, the
countries produce final output for both countries and consume both domestic and
imported final goods. Denote subscript “$ij$” as a variable produced in country $i$ and
consumed by country $j$. The superscript “$T$” stands for trade. Utility functions of
both economies are represented by

$$U_i^T = \int_0^\infty e^{-\rho t} u_i^T(t) dt$$

$$u_i^T = \log C_{ii}(t) C_{ji}^{1-\beta}(t)$$

The household’s utility for each country depends on the consumption levels of
domestic and imported final goods. Let $\beta$ denote the domestic bias, where $\beta \in
[0.5, 1)$. A country gets utility from consuming imported goods, but the higher is the
bias, the less a representative household prefers imported goods.

$^8 \beta \neq 1$ because otherwise we would not have international trade.
The resource constraints for both countries are different from the baseline model. Specifically,

\[
\begin{align*}
\dot{K}_1^T(t) + \dot{B}_1^T(t) &= Y_1^T(t) - (C_{11}(t) + C_{12}(t)) - \delta \hat{P}(t) \\
\dot{K}_2^T(t) &= Y_2^T(t) - (C_{22}(t) + C_{21}(t)) - \delta \hat{P}(t)
\end{align*}
\]  

(42)

The law of motion for capital accumulation depends on both domestic and export consumption goods for both countries. Production functions and evolution of global pollution remain the same as in the baseline model.

In equilibrium, the value of export has to equal to the value of import. Using $P_{C1}$ as a numeraire, trade balance condition becomes

\[
C_{12}(t) = P_2(t)C_{21}(t)
\]  

(43)

**Proposition 7** With international good trade, the balanced growth path exists if and only if

\[
\hat{C}_{ii} = \hat{C}_{ij} = \hat{B}_i^T = \hat{K}_i^T = \hat{Y}_i^T = (1 - \alpha)\hat{A} - \delta \hat{\psi} - \rho
\]  

(44)

When countries open to trade, the growth rates of the economy do not change compared to autarky. The only change happens in the consumption-to-capital ratios.

**Proof.** see the Appendix. ■

We conclude that our previous results about the two policy options are not changed when introducing trade into our model.

### 6.2 Polluting exhaustible resources

Another possible model variant is to posit that the source of pollution is non-renewable resource use. Denoting resource extraction per unit of time by $R$ and labelling the environmental impact by $\zeta$ the change in global pollution is then given by

\[
\dot{P}(t) = \zeta \sum_i R_i(t), \quad i = 1, 2
\]  

(45)

From this and given Eq.(2), it becomes clear that the negative effect of climate change on the capital stock stops as soon as the resource is fully depleted. Provided that the resource is essential, this state will only be approximated in the (very) long run, see Bretschger and Valente (2011). During (the more relevant) transition, it is safe to assume that the rich Country 1 will always use relatively more resources, i.e.
we have $R_1(t) > R_2(t)$ ($\forall t < \infty$).\(^9\) With this, it is still true that the rich country pollutes more but both countries suffer equally through capital depreciation, which was our first main result. Secondly, like in the case of polluting capital, mitigation policy\(^10\) has again positive growth effects for both countries as in our main model, because Eqs. (2) and (8) still hold. Put differently, independent of the factor causing pollution, any policy reducing global warming has a growth effect, which was the next finding of our main model. Moreover, by virtue of (7) the productivity gap between the two economies is still given. The only difference between this case and our main model is the now-emerging nonlinearity in pollution increase, which is due to the nonlinear resource profile (resulting of intertemporal optimization), see Bretschger and Valente (2011). Consequently, also with the model variant, development aid in the form of a pure income transfer cannot have a growth effect, it increases the consumption level as in the previous case. To conclude, the model results continue to hold with polluting non-renewable resources.

\section{Credit Constraints}

Credit constraints are often seen as major obstacles that prevent underdeveloped countries from prospering (see e.g. Zeller et al. 1997). One might suspect that our results need to be adjusted in the case that country 2 is originally credit constrained but gets access to lending by development aid. Indeed, even though the poor country receives knowledge transfers from the rich economy, development needs to be supported by an optimum growth of home capital.

Let us assume that, prior to the development policy, the poor country 2 faces a credit constraint such that the capital/knowledge ratio is not optimum; specifically we then have $\theta B_1 / K_2 < \alpha/(1 - \alpha)$, given $B_2 = \theta B_1$. Denote $\gamma < 1$ as a fraction of optimum capital $\bar{K}_2$ (where $\theta B_1 / \bar{K}_2 = \alpha/(1 - \alpha)$) which gives actually installed capital according to

$$K_2 = \begin{cases} \gamma \bar{K}_2 & \text{if } K < \bar{K}_2 \\ \frac{K_2}{\bar{K}_2} & \text{if } K_2 = \bar{K}_2 \end{cases}$$

Suppose development aid provided by the rich country 1 eases up credits such that the capital stock is equal (or beyond) the optimum level. As a result, the poor

\footnote{\(\text{We might, for example, use a production function like } Y_i(t) = F_i[B_i(t), K_i(t), R_i(t)] = AB_i(t)\tilde{K}(t)^{1-\alpha}R_i(t)\alpha \text{ and assume an internationally equal resource price.}\)}
economy now can optimally utilize the knowledge capital diffused from country 1. Since capital stock cannot jump, it will adjust gradually to the optimum level $K_2$, so that the aid has a positive (transitional) growth effect. But as returns to (home build) capital are decreasing in country 2 like in a neo-classical growth setting, this growth impact will peter out over time. Importantly, on the balanced growth path with aid overcoming the credit constraints, the economy has the same growth rate as in the previous section, i.e. it grows at the rate $(1 - \alpha)\bar{A} - \delta\psi - \rho$. Thus, the growth effect is only transitional. This confirms the basic distinction between aid and mitigation policy (having a permanent growth effect as derived above).

7 Conclusions

The paper develops a two-region endogenous growth model with capital use increasing the stock of greenhouse gases and climate change affecting the existing capital stock negatively. We compare two different policies aimed at supporting less developed countries: climate mitigation by rich countries, which diminishes the increase in stock pollution and hence capital depreciation, and income transfers in the tradition of development aid.

Under a mild set of assumptions we find that active climate policies are more efficient for supporting the development of poor countries compared to additional development aid. The main reason is the positive impact of climate policies on the countries’ growth rates.

The model results are robust when including goods trade and polluting resources. Growth-enhancing development aid could moderate the difference between the policy outcomes in our model. It would be interesting to study the issue in the context of climate change. This is left for future research.

References


Appendix

8.1 Proof of Proposition 1

In order to find optimal choices of $C_1$, $K_1$, and $B_1$, the Hamiltonian is written as

$$H_1 = \log C_1 + v_1 \left[ AB_1^\alpha K_1^{1-\alpha} - C_1 - \delta P \right]$$

(A.1)

where $v_1$ is the shadow price of overall capital. We substitute Eq.(3) into Eq.(A.1) and differentiate with respect to consumption, physical capital, and knowledge capital. Then, the first-order conditions for the maximization of the Hamiltonian are

$$\frac{1}{C_1} = v_1$$

(A.2)

$$-\dot{v}_1 + \rho v_1 = v_1 \left[ (1 - \alpha)A \left( \frac{K_1}{B_1} \right)^{-\alpha} - \delta \psi \right]$$

(A.3)

$$-\dot{v}_1 + \rho v_1 = v_1 \left[ \alpha A \left( \frac{K_1}{B_1} \right)^{1-\alpha} - \delta \psi \right]$$

(A.4)

Equating Eq.(A.3) and Eq.(A.4), the ratio of physical to knowledge capital becomes constant

$$\frac{K_1}{B_1} = \frac{1 - \alpha}{\alpha}$$

(A.5)

Substituting Eq.(A.5) into Eq.(1), the final output can be expressed in term of physical capital as

$$Y_1 = \tilde{A} K_1, \quad \tilde{A} \equiv A(\frac{\alpha}{1 - \alpha})^\alpha > 0$$

(A.6)

Finally, log differentiating Eq.(A.2) and dividing Eq.(A.3) by $v_1$, the growth of consumption, denoted by $^\wedge$, can be expressed as

$$\dot{C}_1 = -\dot{v}_1 = (1 - \alpha)\tilde{A} - \delta \psi - \rho$$

(A.7)
8.2 Consumption growth in Country 2

The Hamiltonian for the optimization problem in Country 2 can be written as

\[ H_2 = \log C_2 + v_2 \left[ AB_2^\alpha K_2^{1-\alpha} - C_2 - \delta \hat{P} \right] \]  
\[ \text{(A.8)} \]

where \( v_2 \) is the shadow price of physical stock in Country 2.

Following the previous analysis, the growth rate of consumption in Country 2 is

\[ \dot{C}_2 = -\dot{v}_2 = (1 - \alpha)AB_2^\alpha K_2^{-\alpha} - (\delta \psi + \rho) \]  
\[ \text{(A.9)} \]

Substituting Eq.(7) and Eq.(A.5) into Eq.(A.9), the growth of consumption in Country 2 becomes

\[ \dot{C}_2 = (1 - \alpha)A \left( \left( \frac{\theta \alpha}{1 - \alpha} \right) \left( \frac{K_1}{K_2} \right) \right)^\alpha - (\delta \psi + \rho) \]  
\[ \text{(A.10)} \]

which directly yields the expression in the main text.

8.3 Proof of Proposition 2

When substituting Eq.(A.5) into Eq.(2), the growth rate of physical capital in country 1 can be written as\(^{11}\)

\[ \dot{K}_1 = (1 - \alpha)\hat{A}_1 - \delta \psi \left( 1 + \frac{K_2}{K_1} \right) - (1 - \alpha) \frac{C_1}{K_1} \]  
\[ \text{(A.11)} \]

The growth rate of physical capital in Country 1 does not only depend on its own economic activities, but also depends on the pollution generated by Country 2. By substituting Eq.(7), Eq.(A.5), and Eq.(3) into Eq.(8) and dividing by \( K_2 \), the growth rate of physical capital in Country 2 is

\[ \dot{K}_2 = \hat{A} \left( \frac{\theta K_1}{K_2} \right)^\alpha - \delta \psi \left( \left( \frac{1 + \theta \alpha}{1 - \alpha} \right) \left( \frac{K_1}{K_2} \right) + 1 \right) - \frac{C_2}{K_2} \]  
\[ \text{(A.12)} \]

Note that if global pollution is exempt from the model, the ratio of physical capitals in two countries will disappear from (A.11), and Eq.(A.12) which leaves us with the traditional model of capital accumulation.

\(^{11}\)Note that since \( K_1 \) is proportional to \( B_1 \), the growth rate of physical capital equals to the growth rate of knowledge capital and equals to the growth rate of overall capitals. Specifically, \( \dot{K}_1 = \hat{B}_1 = (\dot{K}_1 + \dot{B}_1) \)
Following Eq.(7), Eq.(A.5), balanced growth path in the poor country 2 can only exist if and only if

\[ \hat{B}_2 = \hat{K}_2 \]  

(A.13)

That implies the same growth rates of all capitals in both countries which result in a constant ratio of physical capital in country 1 to country 2. Denote \( \chi_i \equiv \frac{C_i}{K_i} \), the ratio of physical capital in country 1 to country 2 can be derived by applying Eq.(A.7), Eq.(10), Eq.(A.11), and Eq.(A.12). Specifically,

\[ \frac{K_1}{K_2} = \frac{1}{\bar{\theta}} \]  

(A.14)

Substitute Eq.(A.14) into Eq.(10), the growth rates of consumption between two countries are equal to each other. With Eq.(A.14), consumption, knowledge capital, physical capital, and output in both countries must grow at the same rates, i.e.

\[ \hat{C}_i = \hat{B}_i = \hat{K}_i = \hat{Y}_i = (1 - \alpha)\bar{A} - \delta\psi - \rho \quad (i = 1, 2) \]  

(A.15)

### 8.4 Proof of Proposition 3

Following the procedure of the previous section, Country 1 wishes to maximize the present value of lifetime utility Eq.(4) subject to Eq.(14), and Eq.(15). The Hamiltonian can be written as

\[ H_{m1} = \log C_{m1} + v_{m1} \left[ AB_{m1}^{\alpha} K_{m1}^{1-\alpha} - (1 + \tau_m)C_{m1} - \delta(1 - m)\psi \sum_i (K_{mi} + B_{mi}) \right] \]  

(A.16)

Differentiate Eq.(A.16) with respect to consumption \( C_{m1} \), and both types of capital \( (K_{m1}, B_{m1}) \), the first-order conditions become

\[ \frac{1}{C_{m1}} = (1 + \tau_m)v_{m1} \]  

(A.17)

\[ -\dot{v}_{m1} + \rho v_{m1} = v_{m1} \left[ (1 - \alpha)A \left( \frac{K_{m1}}{B_{m1}} \right)^{-\alpha} - (\delta(1 - m)\psi + \rho) \right] \]  

(A.18)

\[ -\dot{v}_{m1} + \rho v_{m1} = v_{m1} \left[ \alpha A \left( \frac{K_{m1}}{B_{m1}} \right)^{1-\alpha} - (\delta(1 - m)\psi + \rho) \right] \]  

(A.19)

Comparing to the baseline case, Eq.(A.17) includes the tax component. It reduces the consumption level, given the shadow price of capital. However, the tax does not
affect the growth rate of consumption. Specifically, it can be shown by log differentiating Eq. (A.17) with respect to time and dividing by $C_{m1}$ that the growth rate of consumption is negatively related to the growth rate of shadow price of capital.

$$\dot{C}_{m1} = -\dot{v}_{m1}$$  \hspace{1cm} (A.20)

Equate Eq. (A.18) to Eq. (A.19), the ratio between two capitals is the same as before. This is because the mitigation policy affects the depreciation rate in both types of capital symmetrically.

$$\frac{K_{m1}}{B_{m1}} = \frac{1 - \alpha}{\alpha}$$  \hspace{1cm} (A.21)

Even though the ratio between physical and knowledge capitals is the same, the mitigation policy creates an impact on the accumulation of capital stocks. In particular, the evolutions of both types of capital stock, Eq. (A.18) to Eq. (A.19), are positively affected by mitigation policy, $\eta$, in which it slows down the depreciation rate. As a result, the growth rate of shadow price of capital stock can be derived by dividing Eq. (A.18), and Eq. (A.19) by $v_{m1}$

$$-\dot{v}_{m1} = (1 - \alpha)A\left(\frac{K_{m1}}{B_{m1}}\right)^{-\alpha} - (\delta(1 - \eta)\psi + \rho) = \alpha A\left(\frac{K_{m1}}{B_{m1}}\right)^{1-\alpha} - (\delta(1 - m)\psi + \rho)$$  \hspace{1cm} (A.22)

Finally, substitute Eq. (A.21) into Eq. (A.22), and put it into Eq. (A.20), so that the growth rate of consumption becomes

$$\dot{C}_{m1} = (1 - \alpha)A\left(\frac{1 - \alpha}{\alpha}\right)^{-\alpha} - (\delta(1 - m)\psi + \rho)$$  \hspace{1cm} (A.23)

which gives directly the expression in the main text.

### 8.5 Poor country with mitigation policy

The new resource constraint and the Hamiltonian for Country 2 after the mitigation policy from Country 1 are

$$\dot{K}_{m2} = AB_{m2}^{\alpha}K_{m2}^{1-\alpha} - C_{m2} - \delta M$$  \hspace{1cm} (A.24)

where $\dot{M} = (1 - m)\dot{P} \equiv (1 - m)\psi(K_1 + \dot{K}_2)$

$$H_{m2} = \log C_{m2} + v_{m2} \left[AB_{m2}^{\alpha}K_{m2}^{1-\alpha} - C_{m2} - \delta M\right]$$  \hspace{1cm} (A.25)
Notice that, Country 2 does not have to pay tax, hence, there is no consumption distortion in this case and we get

\[ \frac{1}{C_{m2}} = v_{m2} \]  
(A.27)

\[ \dot{C}_{m2} = -\dot{v}_{m2} = (1 - \alpha)A \left( \frac{K_{m2}}{B_{m2}} \right)^{-\alpha} - (\delta(1 - m)\psi + \rho) \]  
(A.28)

Substituting Eq.(7), and Eq.(A.21) into Eq.(A.28), the growth rate of consumption in Country 2 depends on the relative stocks of physical capital between 2 countries. Specifically,

\[ \dot{C}_{m2} = (1 - \alpha)\tilde{A} \left( \frac{\theta K_{m1}}{K_{m2}} \right)^{\alpha} - (\delta(1 - m)\psi + \rho) \]  
(A.29)

Dividing Eq.(A.24) by \( K_{m2} \), the growth rate of physical capital in Country 2 is

\[ \dot{K}_{m2} = \tilde{A}\theta^{\alpha} \left( \frac{K_{m1}}{K_{m2}} \right)^{\alpha} - \frac{C_{m2}}{K_{m2}} - \delta(1 - m)\psi \left( \frac{1 + \theta^{\alpha}}{1 - \alpha} \right) \left( \frac{K_{m1}}{K_{m2}} + 1 \right) \]  
(A.30)

### 8.6 Proof of Proposition 5

After setting up the Hamiltonian and first-order condition, it is apparent to see that a lump-sum transfer does not have any impact on growth rate. The underlying reason is that the growth rate of consumption depends only on the growth rate of shadow price of physical capital which does not depend upon the amount of lump-sum transfer. As a result, the growth rate of consumption after the development policy is

\[ \dot{C}_{d2} = (1 - \alpha)\tilde{A} \left( \frac{\theta K_{d1}}{K_{d2}} \right)^{\alpha} - \delta\psi - \rho \]  
(A.31)

Assuming an equivalent of one income unit of rich country 1 increases output of Country 2 by \( \gamma \), i.e., \( D_1 = \gamma Y_2 \). Consequently, the growth rate of physical capital can be derived as

\[ \dot{K}_{d2} = (1 + \gamma)\tilde{A} \left( \frac{\theta K_{d1}}{K_{d2}} \right)^{\alpha} - \frac{C_{d2}}{K_{d2}} - \delta\psi \left( \frac{1 + \theta^{\alpha}}{1 - \alpha} \right) \left( \frac{K_{d1}}{K_{d2}} + 1 \right) \]  
(A.32)
The growth rate of physical capital in Country 2 has now increased by the amount of lump-sum transfer. As the development policy has no effect on the knowledge spillover and returns on capitals, the ratio of physical capital in Country 1 to Country 2 is the same as before

\[ \frac{K_{d2}}{K_{d1}} = \theta \] (A.33)

As a result of the policy, \( \dot{C}_2 \) remains unchanged (parallel to \( \dot{C}_1 \)) while the output gap between the two countries is diminished, according to:

\[ \dot{C}_{d1} = \dot{C}_{d2} = (1 - \alpha)\bar{A} - (\delta\psi + \rho) \] (A.34)

\[ Y_{d2} = (1 + \gamma)Y_2 = (1 + \gamma)\theta Y_{d1} \] (A.35)

### 8.7 Model solution with trade

Setting up the Hamiltonain for both countries and use Eq.(43), we have

\[ P_2 = \frac{C_{11}}{C_{22}} \] (A.36)

\[ \frac{\beta}{1 - \beta} = \frac{C_{11}}{P_2 C_{21}} = \frac{C_{22}}{C_{12}} \] (A.37)

Eq.(A.36) measures relative price of final goods between two countries. The Cobb-Douglas utility function implies that intratemporal allocation between the value of domestic and imported goods is constant in Eq.(A.37). In addition, the growth rate of domestic consumption in country 1 and country 2 can be written as

\[ \dot{C}_{11} = (1 - \alpha)\bar{A} - \delta\psi - \rho \] (A.38)

\[ \dot{C}_{22} = (1 - \alpha)A \left( \frac{\theta\alpha}{1 - \alpha} \left( \frac{K_{1T}}{K_{2T}} \right) \right)^\alpha - (\delta\psi + \rho) \equiv (1 - \alpha)\bar{A} \left( \frac{\theta K_{1T}}{K_{2T}} \right)^\alpha - (\delta\psi + \rho) \] (A.39)

Log differentiate Eq.(A.37) and apply Eq.(A.36), the growth rates of domestic and imported goods in both countries can be written as

\[ \dot{C}_{11} = \dot{C}_{12}, \dot{C}_{22} = \dot{C}_{21} \] (A.40)
Capital accumulation in both countries can be written as

\[ \dot{K}_1^T = (1 - \alpha) \dot{A} - (1 + \theta \alpha) - (1 - \alpha) \beta^{-1} C_{11} K_1^T - (1 - \alpha) \delta \psi \frac{K_2^T}{K_1^T} \]  
\[ \dot{K}_2^T = \dot{A} \left( \frac{\theta K_1^T}{K_2^T} \right)^\alpha - \delta \psi \left( \left( 1 + \theta \alpha \right) \left( \frac{K_1^T}{K_2^T} \right) + 1 \right) - \beta^{-1} C_{22} \left( \frac{K_2^T}{K_1^T} \right) \]  

(A.41)

Following the same analysis as above, the ratio of capital between country 1 and country 2 is constant. Specifically,

\[ \frac{K_2^T}{K_1^T} = \theta \]  

(A.42)

Log-differentiating Eq.(40) with respect to time and applying the balanced growth path conditions, the growth of welfare after trade becomes

\[ \dot{u}_i^T = \dot{C}_i = (1 - \alpha) \dot{A} - \delta \psi - \rho \]  

(A.43)

The growth rate of welfare from trade is the same as the one without trade and equal across countries. The underlying reason is that knowledge spillover allows the countries to grow at the same rate.

To find the steady state, subtract Eq.(44) and Eq.(A.41) and set it equal to zero. It can be written as

\[ \frac{C_{11}}{K_1^T} = \beta \left( \frac{\rho - \delta \psi \theta}{1 - \alpha} \right) \]  
\[ \frac{C_{22}}{K_2^T} = \beta \left( \alpha \dot{A} - \delta \psi \frac{1 + \theta \alpha}{\theta(1 - \alpha)} + \rho \right) \]  

(A.44)

Using Eq.(A.37), the ratios between total consumption to capital in country 1 and country 2 are the combination of production in both countries that reflects the share of domestic and imported goods. In particular, the steady state levels of consumption-capital ratio between two countries can be written as

\[ \frac{C_{11} + C_{21}}{K_1^T} = \beta \left( \frac{\rho - \delta \psi \theta}{1 - \alpha} \right) + \theta (1 - \beta) \left( \alpha \dot{A} - \delta \psi \frac{1 + \theta \alpha}{\theta(1 - \alpha)} + \rho \right) \]  
\[ \frac{C_{22} + C_{12}}{K_2^T} = \beta \left( \alpha \dot{A} - \delta \psi \frac{1 + \theta \alpha}{\theta(1 - \alpha)} + \rho \right) + (1 - \beta) \left( \frac{\rho - \delta \psi \theta}{1 - \alpha} \right) \]
Figure 1
Figure 2
Working Papers of the Center of Economic Research at ETH Zurich

(PDF-files of the Working Papers can be downloaded at www.cer.ethz.ch/research).

12/166  L. Bretschger and N. Supaphiphat
Use Less, Pay More: Can Climate Policy Address the Unfortunate Event for Being Poor?

12/165  P. S. Schmidt and T. Werner
Verified emissions and stock prices: Is there a link? - An empirical analysis of the European Emission Trading Scheme

12/164  H. Gersbach, S. Imhof and O. Tejada
Channeling the Final Say in Politics

12/163  J. Daubanes and L. Leinert
Optimum Tariffs and Exhaustible Resources: Theory and Evidence for Gasoline

12/162  C. N. Brunnschweiler, C. Jennings and I. A. MacKenzie
Rebellion against Reason? A Study of Expressive Choice and Strikes

12/161  A. Lange and A. Ziegler
Offsetting versus Mitigation Activities to Reduce CO2 Emissions: A Theoretical and Empirical Analysis for the U.S. and Germany

12/160  L. Bretschger and F. Lechthaler
Common Risk Factors and the Macroeconomy: New Evidence from the Japanese Stock Market

12/159  H. Gersbach and Q. Oberpriller
Rules vs. Targets: Climate Treaties under Uncertainty

12/158  H. Gersbach and O. Ponta
Unraveling Short- and Farsightedness in Politics

12/157  A. Bommier and F. Le Grand
Too Risk Averse to Purchase Insurance? A Theoretical Glance at the Annuity Puzzle

12/156  I. A. MacKenzie and M. Ohndorf
Restricted Coasean Bargaining

11/155  A. Bommier
Life-Cycle Preferences Revisited

11/154  C. N. Brunnschweiler and S. Valente
International Partnerships, Foreign Control and Income Levels: Theory and Evidence
11/153 R. Ramer
Dynamic Effects and Structural Change under Environmental Regulation in a CGE Model with Endogenous Growth

11/152 I. A. MacKenzie and M. Ohndorf
Optimal Monitoring of Credit-based Emissions Trading under Asymmetric Information

11/151 J. Daubanes and P. Lasserre
Optimum Commodity Taxation with a Non-Renewable Resource

11/150 A. Schäfer and M. T. Schneider
Endogenous Enforcement of Intellectual Property, North-South Trade, and Growth

11/149 H. Gersbach and V. Hahn
Inflation Forecast Contracts

11/148 D. Schiess and R. Wehrli
Long-Term Growth Driven by a Sequence of General Purpose Technologies

11/147 P. F. Peretto and S. Valente
Growth on a Finite Planet: Resources, Technology and Population in the Long Run

11/146 H. Gersbach, N. Hummel and R. Winkler
Sustainable Climate Treaties

11/145 H. Gersbach and H. Haller
A Human Relations Paradox

11/144 L. Bretschger and S. Valente
International Trade and Net Investment: Theory and Evidence

11/143 H. Gersbach
Campaigns, Political Mobility, and Communication

11/142 J. G. Becker
On the Number of α-Pivotal Players

On the Construction of Common Size, Value and Momentum Factors in International Stock Markets: A Guide with Applications

10/140 L. Leinert
How do unanticipated discoveries of oil fields affect the oil price?

10/139 H. Gersbach, M. T. Schneider and O. Schneller
Basic Research, Openness, and Convergence
10/138 L. Bretschger and V. Kappel  
Market concentration and the likelihood of financial crises

10/137 M. T. Schneider and R. Winkler  
Growth and Welfare under Endogenous Lifetime

10/136 V. Hahn  
Sequential Aggregation of Verifiable Information

10/135 A. Bommier, M.-L. Leroux and J.-M. Lozachmeur  
On the Public Economics of Annuities with Differential Mortality

10/134 A. Bommier, A. Chassagnon and F. Le Grand  
Comparative Risk Aversion: A Formal Approach with Applications to Saving Behaviors

10/133 A. Bommier and B. Villeneuve  
Risk Aversion and the Value of Risk to Life

10/132 L. Bretschger and S. Valente  
Endogenous Growth, Asymmetric Trade and Resource Taxation

10/131 H. Gersbach and N. Surulescu  
Default Risk in Stochastic Volatility Models

10/130 F. Schwark  
Economics of Endogenous Technical Change in CGE Models - The Role of Gains from Specialization

10/129 L. Bretschger, R. Ramer and F. Schwark  
Long-Run Effects of Post-Kyoto Policies: Applying a Fully Dynamic CGE model with Heterogeneous Capital

10/128 M. T. Schneider, C. Traeger and R. Winkler  
Trading Off Generations: Infinitely-Lived Agent Versus OLG

10/127 V. Kappel  
The Effects of Financial Development on Income Inequality and Poverty

10/126 M. T. Schneider  
The Larger the Better? The Role of Interest-Group Size in Legislative Lobbying

10/125 A. Ziegler  
Individual Characteristics and Stated Preferences for Alternative Energy Sources and Propulsion Technologies in Vehicles: A Discrete Choice Analysis

10/124 P. F. Peretto and S. Valente  
Resource Wealth, Innovation and Growth in the Global Economy