Doctoral Thesis

Time reverse modeling of acoustic emissions in structural concrete

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TIME REVERSE MODELING OF
ACOUSTIC EMISSIONS IN STRUCTURAL CONCRETE

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Doctor of Sciences

presented by

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Abstract

Acoustic emission (AE) is caused by the release of stored strain energy due to an irreversible deformation process or internal friction. The released energy induces elastic waves that propagate inside the structure. The emitted waves can be recorded at the surface of the structure by piezoelectric sensors and converted into acoustic signals. A convenient application of AE analysis in structural engineering is the localization of concrete cracking and reinforcement ruptures. However, for progressed crack patterns, common localization procedures fail, because the waves that are recorded by the sensors have either been reflected repeatedly, experienced interference with other waves or are very weak. Therefore, in this dissertation a new method of AE analysis is developed, which is based on a different physical principle from that of existing methods and which takes advantage of the wave field interferences.

The present work aims at establishing a novel localization method, which is based on wave propagation and is called time reverse modeling (TRM). TRM has recently been used in the area of exploration geophysics. In this dissertation, it is applied for signal-based AE analysis of reinforced concrete (RC) specimens. TRM uses signals obtained from physical experiments as input, such as those recorded by piezoelectric sensors. The signals are re-emitted numerically into a structure in a time-reversed manner. In the so-called inverse simulation, the wavefronts interfere and appear as dominant concentrations of energy at their origins, which marks the source of the AE. A numerical concrete model with constituents randomly distributed in space is established to represent the propagation behavior of concrete in a realistic manner. Numerical and physical experiments on concrete cuboids (120 × 118 × 160 mm), using Ricker wavelets and pencil-lead breaks as AE sources, were carried out to validate TRM for signal-based AE analysis of concrete. A slender RC beam (120 × 200 × 1700 mm) was subjected to four-point bending. The results confirm that TRM is suitable for studying RC structures and can also be used on large-scale specimens. The influence of cracks on the accuracy of localization is investigated. A parameter- and signal-based analysis is performed. The energy concentrations are spatially imaged using TRM. The accuracy of the TRM results is corroborated by three-dimensional crack distributions that were obtained from X-ray CT images of the tested specimens. Further, TRM is compared to a proved localization method.

In this dissertation it is shown that TRM is suitable to localize AE in both, unreinforced and reinforced concrete. AE sources due to concrete cracking can be localized successfully with TRM, regardless of the presence of previous cracks in the concrete specimen. Using numerical and physical experiments, it is demonstrated that multiple AE sources can be imaged by simply using one set of AE waveforms only.
Kurzfassung


Chapter 1

Introduction

1.1 Motivation and goals

At present, the preservation of existing structures is of equal significance as the construction of new ones. Assessment of the health condition of existing structures is becoming more and more important. The employment and advancement of innovative ultrasonic non-destructive testing (NDT) methods such as acoustic emission (AE) analysis hence seem like logical steps. AE stands for the release of strain energy due to a deformation process or internal friction. The released energy causes the excitation of elastic waves that propagate inside the structure. The emitted waves are recorded at the surface of the structure by piezoelectric sensors and converted into acoustic signals. The gained information is of practical interest for NDT. AE is a passive phenomenon mostly associated with a destruction process. In structural engineering concrete cracking and reinforcement rupture can be detected by analyzing AE events that occur during loading. The AE events can be analyzed either qualitatively or quantitatively. Qualitative AE analysis methods make use of basic parameters of recorded signals and aim at identifying the load history and the stage of degradation of the material. In quantitative AE analysis methods all characteristics of an AE source are analyzed, and therefore the wave propagation between source and sensors need to be considered. In previous studies in the field of geophysics, important results have been achieved with respect to determination of arrival times of the primary waves (picking) [54], source localization and moment tensor analysis [5]. Further progress depends on how wave propagation is modeled in cracking and cracked concrete, and how other elements of structural concrete such as reinforcement, post-tensioning tendons or inserts are taken into account.

This dissertation is related to research on AE carried out at the Institute of Structural Engineering of ETH Zurich since 1997. In [46] Köppel investigates the principal possibilities and limitations of AE analysis as a NDT method for reinforced concrete (RC) structures from a civil engineering perspective. Tests to failure were carried out on RC beams with spans of up to 19.70 m. In particular, the AE released by damage processes such as cracking of concrete, bond deterioration or friction are studied. As approximate locations and types of damage processes were known in these experiments, this information is used to assess whether AE analysis yields
accurate results. The results of the AE source localization are further used to investigate the
effectiveness of different measures for increasing accuracy and reliability. With the results of
pull-out tests an imaging technique for displaying the densities of AE sources is developed in
[99]. For quantitative analysis of selected AE sources, the relative moment tensor inversion is
used, which so far has only been applied to RC in [33]. In Schechinger [78] and [79] further
research in a geophysical context is presented. The focus is on elastic wave propagation in RC,
on the improvement of the traditional analytical methods and their application to laboratory
experiments. Automation of arrival time picking and quantification of localization accuracy are
discussed. Using two-dimensional numerical simulations it is demonstrated how tendons disturb
wave propagation in concrete [84]. Automation of data picking is used to analyze in detail the
vast amount of data collected in the experiments. A linearized algorithm is presented, with which
localization errors are estimated for each AE source and visualized using three-dimensional error
ellipsoids. Further, tension and bending tests of RC beams are described, in which the external
loading caused intense cracking with high AE activity. The crack topography is accurately
localized by AE analysis. Increasing damage of the test specimen prevented the author from
following the deterioration process in the later phases of the experiment. In a field trial described
in [24] the commercially used system “SoundPrint” is applied to a post-tensioned bridge built in
1952, which showed severe signs of corrosion of the prestressing steel due to improper grouting
of the ducts. Sensors located along the lateral faces of the bridge recorded several spontaneous
wire breaks, some of which could subsequently be confirmed by carrying out destructive tests
on the bridge. The accuracy of the test had previously been verified by the supplier with blind
tests, during which artificial wire breaks were successfully identified and localized by the system
[25].

The numerical simulation of elastic wave propagation is a powerful tool to visualize the elastic
energy flow and distribution in infinite and semi-infinite media. With increasing acceptance of
numerical methods and rapidly growing performance of high-performance computer clusters
the elastodynamic equations of motion can be solved quickly, making it possible for numerical
simulations of the elastic wave propagation to be carried out in a qualitative manner. For a better
understanding of AE due to concrete cracking, it is necessary to carry out three-dimensional
numerical simulations with complex models, for example representing heterogeneous, uncracked
or cracked media. The aim of this dissertation is to establish a novel localization method for NDT
applications, which is based on wave propagation and is called time reverse modeling (TRM)
[92]. In particular, TRM is transferred from exploration geophysics to ultrasonic NDT and
applied in signal-based AE analysis for the first time. TRM uses signals obtained from physical
experiments such as AE recorded by piezoelectric sensors, which are numerically re-emitted as
sources into a specimen in a time-reversed manner. In the inverse simulation the wavefronts
interfere and focus on their origin. The beauty of TRM is that it bypasses the picking procedure
and requires less user interaction. Cracking is a very complex process. When multiple cracks
form simultaneously in a structure usually at the $\mu$s scale, the released AE interfere with each
other. In that case, localization based on arrival time of the first motion as presented in [78] will
yield some results, but it is uncertain whether the obtained location has a physical origin or not.
1.2. Outline

For a material with progressed deterioration the procedure will be inaccurate or fail altogether. However, the recorded waveforms contain important information about their origin (i.e. the point of crack nucleation), propagation path and medium, and should not be pre-selected and thus reduced to the primary wave part (onset time) only. Further to TRM, a representative numerical model of concrete for wave propagation simulations is developed in this dissertation. To model concrete numerically in three dimensions (3D), a numerical concrete model (NCM) is established with concrete constituents randomly distributed in space. The NCM is informed by the model used by Schechinger [78]. To optimize TRM on concrete, physical experiments on a small concrete cuboid (120 × 118 × 160 mm) were carried out. Using an adjusted setup, a RC specimen (120 × 200 × 1700 mm) was tested to failure in a four-point bending test, during which AE measurements were taken. The cracked specimens (both cuboid and beam) were scanned with X-rays to determine their inner three-dimensional crack patterns, which are subsequently used to verify the TRM results. Finally, the influence of cracking progress on the localization capability of TRM is discussed and compared to the linearized localization algorithm mentioned above.

In this dissertation, it is attempted to answer the following questions: (1) Are the advantages of TRM greater than those of the linearized localizations algorithm? (2) Does a progressed cracking pattern in a RC specimen significantly affect the TRM localization? (3) In what areas and on what scale can TRM be applied in the future?

1.2 Outline

Chapter 2 “Instrumentation and analysis tools for acoustic emissions” contains an introduction to ultrasonic NDT in general and to AE analysis in particular. For a better understanding of the difference between what is measured physically and what is processed digitally, some background information about the measurement instrumentation is given, focusing on digital signal processing. The two core areas in AE analysis, parameter-based (qualitative) and signal-based (quantitative) analysis as well as their essential parameters are described and the state-of-the-art applications relevant to this dissertation are summarized.

In Chapter 3 “Elastic wave propagation in structural concrete”, the governing equations of motion are derived assuming an infinite and homogeneous continuum so as to establish a link to the compression wave and shear wave velocities, assuming an infinite and homogeneous continuum. Source representations of AE are briefly described. The interaction of wavefronts with a boundary, the occurrence of surface waves as well as the frequency-dependent behavior of waves in concrete are discussed. An approach for modeling concrete numerically as a spatial randomly-distributed medium in 3D, referred to as the NCM, is presented. X-ray computed tomography (CT) scans on small concrete specimens (120 × 120 × [160, 180] mm) were carried out. The obtained X-ray slice data is post-processed to obtain a numerical X-ray CT model. Because the numerical modeling of the NCM of large specimens requires a significant amount of computational resources, a homogenized medium with effective elastic properties (EEP) is established, which allows the idealization of the material as homogeneous but also to retain
the wave propagation properties. Simulations of elastic wave propagation in the NCM and the post-processed X-ray CT model are performed on uncracked and cracked concrete.

In Chapter 4 “Time reverse modeling in ultrasonic non-destructive testing”, a novel localization technique based on wave propagation, time reverse modeling (TRM), is described. TRM is applied for the first time in the area of ultrasonic NDT. TRM uses numerical algorithms for elastic wave propagation in combination with an imaging condition. Elastic waves emitted from a real AE source such as a pencil-lead break are recorded by piezoelectric sensors at the specimens’ surfaces. Alternatively, a synthetic excitation source generated numerically and the corresponding calculated displacements at the original sensor positions are used. The medium is modeled using the EEP of the NCM. Please note that the EEP are a major part of TRM as well. The inverse computation is carried out for both the measured AE waveforms and the numerically computed displacements. In both cases, the complete time-reversed waves are fed into the medium as sources located at the original sensor positions using the previously determined velocity model. The induced wavefronts propagate and subsequently concentrate at the source location. To verify the accuracy of TRM, numerical tests on cuboids ($120 \times 118 \times 160$ mm) are carried out, using excitation sources that are either obtained numerically (Ricker wavelet) or experimentally (pencil-lead break).

The Chapter 5 “Experimental investigations” is concerned with three topics: sensor calibration, experiments on a concrete cuboid (preliminary experiments) and a four-point bending test (main experiment). A calibration method based on the Hertzian impact of small steel spheres (at the mm scale) on concrete and aluminum is used to determine the instrument response of the piezoelectric sensors. Applying the deconvolution theorem, the waveforms recorded at the specimens’ surfaces are deconvolved (unfolded) to eliminate the disturbance, i.e. the sensors’ physical response to a stimulus. A physical experiment is carried out on a cuboid, which has the same characteristics as that used for the numerical studies in Chapter 4. AE waveforms obtained from a double punch test on a concrete specimen ($120 \times 118 \times 160$ mm) are used as input for the inverse computation. The results of TRM (in 3D) are compared to crack distributions obtained from X-ray CT scans. They are further used to optimize the configuration parameters for the four-point bending test of a larger RC specimen ($120 \times 200 \times 1700$ mm). TRM is applied to a slender RC beam containing two tensile reinforcing bars. The beam is loaded in five stages until close to failure of the compression zone. The beam is scanned with X-rays in its initial and final state (stabilized) to obtain inner snapshots of its starting and final condition. The three-dimensional crack patterns in the final state are used for comparison with the TRM results. Parameter- and signal-based analyses are carried out on classical AE parameters as well as on the waveforms recorded during the entire experiment. The focus is on the signal-based AE analysis, and TRM is compared to an established localization method, the so-called AIC-based linearized localization algorithm [79]. For the localization, AE selected because of their low signal-to-noise (S/N) ratios are used. The TRM results are compared to the three-dimensional crack patterns obtained from X-ray CT and from the AIC-based linearized localization.
1.2. **Outline**

In Chapter 6 “Discussion, conclusions and outlook”, the results from Chapter 5 are summarized and conclusions are drawn using the information presented in the previous chapters. The features, advantages and disadvantages of TRM are formulated and discussed critically. An outlook touching on possible future applications of TRM and recommendations for further research on structural concrete are given.
Chapter 2

Measurement and application of acoustic emissions

2.1 Principle of acoustic emissions

Acoustic emission in NDT is physically similar to an earthquake in seismology [32]. The principle can best be explained by pointing out analogies of both phenomena. An earthquake is a sudden movement of the Earth’s crust. The dislocation generates elastic waves, known as seismic waves. These waves propagate spherically (3D) in the Earth’s interior and cylindrically (2D) along the surface of the Earth. The wave motion is detected by seismographs that are arranged on the Earth’s surface. AE can be understood as a kind of microseismity that is generated due to a failure process. AE is defined as a spontaneous release of strain energy caused by processes such as cracking or internal friction, which result in elastic waves propagating in a structure. The emitted waves are recorded as normal displacements on the specimens’ surfaces using piezoelectric sensors. They are converted to voltage signals, which are referred to as acoustic emission (see for example [32]). The recorded voltage signals (in NDT) are identical to seismograms with the difference that both occur in different frequency ranges. Seismographs are designed to record waves at the kilometer scale (Hz, low frequency waves), whereas piezoelectric sensors are sensitive at the meter to millimeter scale (Hz to MHz, high frequency waves).

2.2 Measurement and storage instrumentation

2.2.1 Measurement chain and recording system

Acoustic emission measurement is influenced by each of the elements in the measurement chain such as the piezoelectric sensors, pre-amplifier and data acquisition system (see Fig. 2.1), each of which exhibits a different degree of sensitivity towards external stimuli.

Elastic waves are released in the test specimen (e.g. due to cracking) and propagate inside the body. When the wave particles arrive at the surface they are detected by the piezoelectric sensors. These mechanical waves are converted into voltage signals, pre-amplified and transferred
to the acquisition system. Depending on the S/N ratio the stored signals can be either high- or low-pass filtered [68]. The analog-to-digital (A/D) converter converts continuous-time signals into discrete-time signals, which are then stored. Note that data acquisition equipment capable of storing continuous-time signals is also available on the market but was not used in the present experimental work. The discrete-time (AE) signals can be analyzed either in a parameter-based (qualitative) or signal-based (quantitative) manner.

For the description of the single elements of the measurement chain the so-called transfer function is introduced. The transfer function (instrument response) is characterized by the response of a physical system to an arbitrary stimulus. The journey of the initial signal from the AE source to the data acquisition system is influenced by the instrument response of each of the chain elements. Finally, the signal received by the data acquisition system is convolved into the time domain by the transfer functions of each of the measurement chain elements such as the sensor, the pre-amplifier and the data acquisition system.

**Sensor**

A very important element in the measurement chain is the sensor or transducer. The sensor converts mechanical waves that arrive at the surface into voltage signals. In the AE community piezoelectric sensors are commonly employed to measure the wave motion at the surface. In piezoelectric sensors, the so-called piezoelectric effect, an interaction between the mechanical and the electrical state of lead-zirconate-titanate (PZT) materials, is utilized. Contraction of the PZT material due to an applied pressure yields a proportional voltage increment. The piezoelectric effect is reversible [90]. The PZT sensors employed in the current study are dominantly responding to mechanical stimuli in the normal direction. Note that all physical experiments are carried out using KSB250 sensors (Ziegler Instruments); therefore, only the description of this type of sensor is given in this section. No reliable information concerning the inner life of the sensor is provided by the manufacturers; the following is a qualitative description only. The sensor is encased in a protective housing of steel, which is mounted on a wear plate and attached
2.2. Measurement and storage instrumentation

(a) Steel bail
Adjustable screw
Vacuum grease
Glue
Sensor
Piece of cork

(b)

![Graph showing normalized instrument response on a logarithmic scale of two KSB250 sensors.](image)

Fig. 2.2: (a) Schematic of sensor mounted on a concrete surface. (b) Normalized instrument response on a logarithmic scale of two KSB250 sensors (#09.04 and #6.04) of identical structure and configuration.

The sensor is positioned by a steel bail glued to the surface, and pressed to the surface by an adjustable screw as well as a protective piece of cork (see Fig. 2.2, a). A thin layer of vacuum grease is applied to the sensor to ensure unbroken contact with the surface (coupling). A pencil-lead break [15] located close to the sensor is used to ensure that coupling has been achieved. Good coupling is important, because mistakenly included air bubbles disturb the energy transmission between medium and sensor. A non-linear sensor response cannot be ruled out, but in this study a purely linear response is assumed. To study the non-linear behavior of the sensors, refined experimental and three-dimensional finite-element considerations are needed, which are beyond the scope of this work.

The instrument response of two KSB250 sensors of the same type, determined using the Vallen Sensor Tester at EMPA Dübendorf, is shown in Fig. 2.2 (b). The sensors were coupled opposite each other on the testing equipment. A frequency generator induced a broadband impulse to cover the entire bandwidth of the sensors. In Fig. 2.2 (b) it can be seen that the KSB250 sensors are sensitive in the range 50-250 kHz. Although both sensors (with sensor identifications #09.04 and #6.04) are of the same structure and configuration, their calibration curves vary. The sensor responses are not smooth, rather they look discontinuous. More information on the calibration of the KSB250 sensors used in this study can be found in Section 5.1.

Pre-amplifier

The signals detected on the surface are relatively weak; therefore they need to be pre-amplified for further processing in the voltage range 0.01 mV-100 V. The voltage amplification (in dB) is expressed in terms of the gain $G_{AE}$. The gain describes a logarithmic relationship between an output voltage signal $v(t)_{out}$ and input voltage signal $v(t)_{in}$ [15]. The conversion of the signals from mV to dB can be written as
Chapter 2. Measurement and application of acoustic emissions

(a)

(b)

Fig. 2.3: (a) AEP4 pre-amplifier (Vallen Systeme) and (b) corresponding normalized instrument response.

\[ v(t) \, [\text{dB}] = 20 \log (v(t) \, [\text{mV}]) + 60 \, [\text{dB}] \, . \]  

(2.1)

For AE measurements the gain is formulated as

\[ G_{\text{AE}} \, [\text{dB}] = v(t)_{\text{out}} \, [\text{dB}] - v(t)_{\text{in}} \, [\text{dB}] = 20 \log \left( \frac{v(t)_{\text{out}} \, [\text{mV}]}{v(t)_{\text{in}} \, [\text{mV}]} \right) \, . \]  

(2.2)

Note that all physical experiments for this study were carried out using the AEP4 pre-amplifier (Vallen Systeme) displayed in Fig. 2.3 (a). The gain can be adjusted to either 34 dB or 40 dB. In all experiments (see Chapter 5) a gain of 40 dB was used. The pre-amplifiers exhibit a characteristic instrument response in a similar way the PZT sensors do. To ensure a linear amplification of the input voltage signal, the instrument response of the pre-amplifier has to be constant over the frequency range of the employed sensor type. The instrument response of an AEP4 pre-amplifier provided by the manufacturer Vallen Systeme is displayed in Fig. 2.3 (b). It can be seen that the instrument response is relatively flat over the range \([0; 600]\) kHz.

Data acquisition

Data acquisition of the pre-amplified signals can be performed either analogously or digitally. The biggest disadvantage of analogous acquisition systems is that data recording is limited to the length of the magnetic tape. For recording high frequencies the tape revolves at high speed, and large portions of the available tape length are used up. For the purpose of recording thousands of AE signals, the advantages of digital acquisition systems are unrivaled. Digital data acquisition systems are fast and can store large amounts of data (in the gigabyte range). Data in digital format is ready to be processed without any further preparation. All physical experiments except
2.2. Measurement and storage instrumentation

(a) Continuous

(b) Discrete

Fig. 2.4: (a) A 100 μs long segment of the continuous-time AE signal $v(t)$ and (b) the corresponding discrete-time sequence $v(t_n)$ of 100 samples ($\Delta t = 1.0 \mu s$). The amplitudes of both (a) and (b) are normalized.

the sensor calibration in Section 5.1 were carried out with a commercial AE recording system, the AMSY5 from Vallen Systeme (see Chapter 5). The AMSY5 that was used for this research is equipped with eight parallel channels for recording AE. Four external channels are available for measurement of other physical variables such as the force or the deflections of the investigated structure. Measuring AE parameters such as hits at the same time as external parameters such as the force is a good way to observe the response of the structure in realtime and to synchronize the equipment after the experiments.

Signal processing

“The term signal is generally applied to something that conveys information. Signals may, for example, convey information about the state or behavior of a physical system” [68]. Signals are typically synthesized for the communication between a physical phenomenon and a machine. The signals used in this work contain information about the acoustic emission source. “Continuous-time signals are defined along a continuum of time and are thus represented by a continuous independent variable. Continuous signals are often referred to as analogous signals. Discrete-time signals are defined at discrete times, and thus, the independent variable has discrete values; that is, discrete-time signals are represented as sequences of numbers” [68]. The continuous-time signal displayed in Fig. 2.4 (a) cannot directly be processed; it needs to be converted into a digital signal by the A/D converter first. The corresponding discrete-time signal is displayed in Fig. 2.4 (b). The continuous-time signal of length $T = 100 \mu s$ is discretized with $N = 100$ samples, time increments $\Delta t = 1.0 \mu s$ and with end time $T = N \Delta t$. For simplicity’s sake the $n^{th}$ time step will hereafter be denoted as $t_n = n \Delta t$. Note that in the following chapters the discrete-time AE signals $v(t_n)$ will be referred to as $v(t)$. 
The discretization of continuous-time signals is an error-prone procedure. Sampling plays a key role in avoiding disturbing effects such as aliasing and leakage. It is important to remember that a continuous-time signal has to be reconstructible from the discrete-time signal. The sampling is governed by the size of time increments $\Delta t$ yielding the sampling frequency $f_s = \frac{1}{\Delta t}$. A discrete frequency $f_m = m\Delta f = \frac{n}{N\Delta t}$ can be calculated. The frequency increments $\Delta f$ correspond to a multiple of the overall frequency $f = \frac{1}{T}$. Therefore, for smaller time increments higher frequencies become dominant. However, to avoid aliasing the sampling frequency is restricted to $f_s = 2f_N$, where $f_N$ is the so-called Nyquist frequency [63]. The relationship $f_N = 0.5f_s$ is known as the Nyquist-Shannon sampling theorem [86]. Leakage occurs in the frequency analysis of a finite-length discrete-time signal such as that displayed in Fig. 2.4 (b). Where the leakage effect is present, it appears as if some energy had leaked out of the original signal spectrum [68]. Representative are a series of lobes in the frequency spectrum (see also Fig. 5.7, $d_0 = 2$ mm, middle row). Leakage can be reduced by applying a window function with different spectral characteristics.

2.2.2 Detection of signals and events

Triggering

Triggering is an important topic in the detection of discrete-time AE signals. AE signals occur at different S/N ratios, which means that no unique triggering procedure can be used. The criteria for such a triggering can vary [32]. Typically, the acquisition equipment is configured to start recording as soon as a limit amplitude, called the threshold $v_T$ (see Fig. 2.5, a), is reached. The threshold can be a static (fixed), or a dynamic value that adapts to the strength of the amplitudes. If multiple channels are involved, selecting the first channel that detects the arrival of the wave motion as the trigger is advantageous. AE signals that do not meet the triggering criterion are simply excluded. Thus, triggering is also a pre-selection criterion to record a specific range of AE for further analysis.

Storage of AE parameters

The classic AE analysis uses certain AE parameters such as maximum peak amplitude $v_{\text{max}}$, rise time $t_R$, duration $t_D$, starting time $t_s$ and end time $t_e$ (see Fig. 2.5, a). The storage of these parameters reduces the total amount of stored data significantly. The AE parameters can be stored instantaneously, whereas the waveforms (see Fig. 2.5, b) cannot be processed as quickly. Note that AE parameters recorded with respect to a pre-defined threshold cannot be reanalyzed with respect to a different one.

Storage of waveforms

The storage of waveforms is a complex process. The sampling frequency controls the accuracy of the wave reconstruction (see Section 2.2.1) and the quantity of data that needs to be stored.
2.3. Acoustic emission analysis

The duration $T$, for which the waveform is recorded, is defined before the measurement and is an important parameter. If the amplitude of an initial AE signal exceeds the threshold, the waveform is stored for $T = |t_s - t_{pre}| + |t_N - t_s|$, where $t_s$ is the starting time, $t_{pre}$ is the pre-trigger time and $t_N$ is the end time (see Fig. 2.5, b). The pre-trigger time implies that the waveforms are buffered on a longer time frame than $T$ before storage. For example an AE signal of duration $T = 409.1 \, \mu s$ and $t_{pre} = 100 \, \mu s$ will be cut after an additional time period of $309.1 \, \mu s$ has elapsed. A modification of this example is used in all physical experiments in Chapter 5.

2.3 Acoustic emission analysis

The data stored in the acquisition system, which forms the basis for any further analysis, is influenced by the geometry and material properties of the investigated specimen as well as by the characteristics of the measurement chain elements. It is important that these influences be removed from the parameters associated with the AE by the filtering process. To meet this demand the data can be analyzed either by ignoring the disturbances (qualitative approach) or by taking influences such as wave propagation behavior and instrument response into account (quantitative approach).

The qualitative (parameter-based) AE analysis methods are time and resource efficient, because only characteristic AE parameters (see Tab. 2.1) found in a waveform are stored and analyzed. Quantitative (signal-based) AE methods consider the complete waveform, the dynamical material properties and the wave propagation path (Green’s function) of the investigated specimen. Quantitative AE methods are more complex and require considerable computational effort.
Chapter 2. Measurement and application of acoustic emissions

### AE parameter Signal Definition

<table>
<thead>
<tr>
<th>AE parameter</th>
<th>Signal</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hit $H_t$ [-]</td>
<td><img src="signal-hit.png" alt="Signal" /></td>
<td>Sum of accumulated counts of one AE signal. Number of times the AE signal crosses the threshold $v_T$ in one polarity.</td>
</tr>
<tr>
<td>Count $C_t$ [-]</td>
<td><img src="signal-count.png" alt="Signal" /></td>
<td></td>
</tr>
<tr>
<td>Signal duration $t_D$ [μs]</td>
<td><img src="signal-duration.png" alt="Signal" /></td>
<td>Difference between the first ($t_s$) and the last time ($t_e$) an AE signal crosses the threshold $v_T$.</td>
</tr>
<tr>
<td>Peak amplitude $v_{(max)}, v_{(min)}$ [V, mV]</td>
<td><img src="signal-peak.png" alt="Signal" /></td>
<td>Maximum/minimum voltage excursion of the AE signal within duration $t_D$.</td>
</tr>
<tr>
<td>Rise time $t_{R_{(max)}}, t_{R_{(min)}}$ [μs]</td>
<td><img src="signal-rise.png" alt="Signal" /></td>
<td>Time difference between the first threshold crossing $t_s$ and the peak amplitude of the AE signal.</td>
</tr>
</tbody>
</table>

**Tab. 2.1:** Definitions of characteristic AE parameters according to [15].

### 2.3.1 Parameter-based acoustic emission analysis

**Characteristic AE parameters**

Parameter-based AE techniques identify AE wave packets by searching for particular parameters. Essential parameters for interpretation include hit $H_t$, count $C_t$, signal duration $t_D$, peak amplitude $v_{max,min}$ and rise time $t_R$, and their occurrence rate or accumulated trend in the time domain can be used [30]. In the present work, peak amplitudes exceeding a defined threshold value are considered as AE events; signals below the threshold value are considered as noise. In Tab. 2.1 the definitions of the most important AE parameters are listed.

**Occurrence of AE signals**

The evaluation of AE parameters is mostly focused on counting occurrences and deriving trends. Only qualitative information about the AE source can be obtained. The hits and counts depend
2.3. Acoustic emission analysis

strongly on the pre-defined threshold. Setting the threshold too low may result in too much noise being recorded. Without knowledge of the transient waveforms it cannot be positively concluded whether an AE occurs, for example due to cracking or due to a material inhomogeneity along the propagation path of the elastic wave [78]. The maximum peak amplitude $v_{\text{max}}$ is related to the magnitude of a corresponding AE event. Note that the peak amplitude is influenced by the radiation pattern of the source, interference with other waveforms, the geometry of the specimen, the location of the sensor relative to the source, the Green’s function [5] of the medium and the instrument response of the employed PZT sensor [32]. The parameter-based AE analysis is being applied successfully in many fields (see Section 2.3.3). When interpreting results, it is important to keep in mind what type of informations AE parameters can provide.

Kaiser effect

“The Kaiser effect is defined as the absence of detectable acoustic emission until the previous maximum load\(^1\) of the applied load level has been exceeded [15]. The Kaiser effect is considered to be the most important phenomenon in parameter-based AE analysis, found by Kaiser by studying AE from tensile loading of metallic materials [39]. A relationship between the generating AE signals, the volumetric change in the respective materials and their ability to absorb ultrasonic waves was established. In tension and bending, the Kaiser effect first occurs after the cracking load has been reached and can be used to differentiate between initial loading and reloading” [44].

2.3.2 Signal-based acoustic emission analysis

Waveforms in the time domain

A stored discrete-time waveform (see Fig. 2.6, a) is by far richer in information than stored AE parameters (see Tab. 2.1). The visual evaluation of a waveform provides preliminary information concerning the propagation path and the frequency content of the complete signal. Such evaluation criteria can be the shape of the wave packet, the regularity of zero crossings and the onset times of the P-, S- and Rayleigh waves (see Section 3.1). Using the onset times of the P-waves of different waveforms, relative distances between the source of an AE and the sensor positions can be estimated. The relative distances can be used either for localization of AE sources or for approximation of the P-wave velocity. Waves propagating in a heterogeneous medium can be distorted, as opposed to waves that propagate in a homogeneous medium (see Fig. A.4). The waveforms in signal-based AE analysis are the counterparts to the seismograms in seismology [5]. The frequency content of a signal can be calculated from a spectral analysis. The qualitative classification to a high-frequency or low-frequency signal can be made visually. Waveforms in the time domain are essential input values for the localization approach using time reverse modeling, which is presented in Chapter 4.

\(^1\)Note that in concrete the Kaiser effect can be observed up to around 75% of the failure load.
Chapter 2. Measurement and application of acoustic emissions

Fourier transformation

The Fourier transformation (FT) is a mathematical tool, which can be used to reduce partial differential equations to ordinary differential equations [100]. Spectral analysis of an AE signal \( v(t) \) is possible, if the signal is transferred from the time into the frequency domain.

In using FT, it is assumed that a continuous-time signal \( h(t) \) can be represented as the superimposition of harmonic waves of the form

\[
h(t) = A_h e^{i(\omega t + \varphi_0)}, \tag{2.3}
\]

where \( \omega = 2\pi f \) is the angular frequency, \( A_h \) is the amplitude and \( \varphi_0 \) is the phase. \( A_h \) and \( \varphi_0 \) are varied with respect to \( \omega \), to yield \( h(t) \). If \( h(t) \) is absolutely integrable on \( -\infty < t < \infty \), then the continuous Fourier transform \( H(\omega) \) is defined as

\[
H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-i\omega t} dt. \tag{2.4}
\]

Reciprocally, the inverse FT of \( H(\omega) \) is defined as

\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{i\omega t} d\omega. \tag{2.5}
\]

The FT is a complex function of the form

\[
H(\omega) = A(\omega) e^{i\varphi(\omega)} = \Re(\omega) + i\Im(\omega), \tag{2.6}
\]

where \( A(\omega) \) is the frequency-dependent amplitude of the function \( H(\omega) \). Hence, Eq. (2.6) can be decomposed into a real and an imaginary frequency-dependent part \( \Re(\omega) \) and \( \Im(\omega) \), respectively.

Given a discrete-time signal of finite length such as the AE signal \( v(t_n) \) (see Fig. 2.6, a), the Fourier integral in Eq. (2.4) turns into a finite Fourier series of length \( T \). For \( T = N\Delta t \) the discrete Fourier transform (DFT) can be derived as

\[
V(\omega_m) = \Delta t \sum_{\omega_m=0}^{N-1} v(t_n) e^{-i\omega_m n \Delta t}, \tag{2.7}
\]

where \( \omega_m \) is the discrete angular frequency with \( m = 0, \cdots, N - 1 \). Fig. 2.6 (b) displays the normalized power spectral density \( V(f_m) \) in terms of the discrete frequency \( f_m \). The inverse DFT

\[
v(t_n) = \frac{1}{N\Delta t} \sum_{n=0}^{N-1} V(\omega_m) e^{i\omega_m n \Delta t} \tag{2.8}
\]

is obtained analogously to Eq. (2.5). Note that in the following chapters the discrete frequency response \( V(f_m) \) will simply be referred to as \( V(f) \).
2.3. Acoustic emission analysis

2.3.1 Time domain

\[ t_n = n\Delta t [\mu s] \]

\[ v(t_n) = \frac{v(t_n)}{\max[v(t_n)]} \]

\[ \Delta t \]

In time domain analysis, the signal is sampled at regular intervals, and the amplitudes are normalized to the maximum value. The time difference \( \Delta t \) is used to represent the sampling interval.

2.3.2 Frequency domain

\[ f_m = m\Delta f \text{ [kHz]} \]

\[ V(f_m) = \frac{V(f_m)}{\max[V(f_m)]} \]

\[ \Delta f \]

In frequency domain analysis, the signal is transformed into the frequency domain using the Fourier transform. The frequency difference \( \Delta f \) is used to represent the frequency resolution.

Fig. 2.6: (a) Discrete-time signal \( v(t_n) \) of 409.6 \( \mu s \) length, zoomed to the first 100 \( \mu s \), with time increments \( \Delta t = 1.0 \times 10^{-7} s \). (b) Corresponding power spectral density \( V(f_m) \) with frequency increments \( \Delta f \approx 2.5 \text{ kHz} \) and a Nyquist frequency \( f_N = 5.0 \text{ MHz} \). The amplitudes of both (a) and (b) are normalized.

The calculation of the coefficients of the Fourier series (2.7) or (2.8) involves \( O(N^2) \) computational operations. For large \( N \) the computational time increases quadratically. Cooley and Tukey [9] published an algorithm, the so-called fast Fourier transform, that reduces the overall computational operations to \( O(N \log N) \). However, in the present work only the DFT algorithm is applied to Fourier transformations.

2.3.3 Applications of acoustic emission analysis to structural concrete

Parameter-based AE methods

In the last decade a broad spectrum of research on the analysis of AE parameters has been carried out. In this section, the most important literature concerning parameter-based AE analysis on plain and reinforced concrete is briefly reviewed. Successful applications using the Kaiser effect for the analysis of reinforced concrete beams are presented in [67]. To quantify the Kaiser effect, Ohtsu et al. [67] define the so-called load ratio and supplement it with the calm ratio to define four cases that are related to the severity of the concrete damage. With this method, it can be distinguished whether acoustic emissions occur during loading or during unloading cycles.

Researchers recognized a long time ago that there exists an analogy between AE and earthquakes, with the main differences being the different frequencies and different scale. Therefore, a method from seismology, the so-called \( b \)-value analysis, is adopted in [81] (rock) for the analysis of AE peak amplitudes. The \( b \)-value defines the slope of the amplitude distribution at a double logarithmic scale, with smaller and larger \( b \)-values indicating macroscopic and microscopic fracture, respectively. Kurz et al. [48] and Colombo et al. [8] apply the seismic \( b \)-value analysis
Chapter 2. Measurement and application of acoustic emissions to unreinforced and reinforced concrete. For a better convergence of the method, an improved $I_b$-value analysis is proposed in [89].

In recent studies [44], mathematical formulations for the definitions of the load ratio and the calm ratio are defined. In these studies, AE events are characterized with respect to cracking modes (bending and shear), and AE parameters are applied to estimate pre-loads of already damaged slabs. In [4], parameter-based approaches such as the analysis of the cumulative number of AE events and the $I_b$-value are combined with wave velocity measurements to monitor the deterioration progress of RC beams. In [93], damage evaluation of structural concrete is performed by combining the AE rate with damage mechanics.

Signal-based AE methods

Signal-based AE techniques consider the complete waveform. Typical characteristics are the compression wave (P-wave) peak, the shear wave (S-wave) peak and the Rayleigh wave peak. In the recent years computer-based AE analysis techniques have found increasing acceptance and have led to more in-depth investigations on how to recognize the characteristics of complete waveforms. In the following paragraph, the signal-based AE techniques that constitute the basis for this research are briefly reviewed.

Grosse et al. [31] present a variety of innovative AE techniques such as wavelet decomposition, moment tensor inversion and evaluation of the magnitudes of coherence functions. In [5], the onset times of P-waves of AE events are used to localize the sources, similarly to how it is done in seismology. In [49], two picking approaches are discussed: one is based on the Akaike Information Criterion (AIC), the so-called AIC picker, and the other is based on the Hinkley criterion. Both approaches analyze the energy envelope of an AE signal with respect to extremal values that represent the onset time of the P-wave motion. An interesting field application is presented in [25], where the quality of prestressing tendons is controlled using continuous acoustic monitoring. One of the first prestressed bridges in Switzerland, the Ponte Moesa in Roveredo, was inspected with a commercial monitoring system (SoundPrint). Waveforms of blind tests with provoked artificial wire breaks are compared to real wire breaks that occurred spontaneously during the monitoring. Another field application is introduced in [58], where a beamforming tool developed for passive sonar and seismological applications is applied to AE signals. The method uses the fact that the Rayleigh wave carries energy over long distances and can be detected by an array of sensors. AE signals that arrive at the sensors at different times are stacked together to form a ray, which directs itself towards the AE source.

Initially, investigations on the source characteristics of AE were dominated by the works of Ohtsu et al. [65], [66] and [64] on moment tensor analysis. The eigenvalue analysis of the moment tensor provides information concerning the source kinematics of AE sources that can be connected to the fracture modes. In [51], the moment tensors are utilized for signal-based AE analysis of micro-cracking in plain mortar beams. Later publications take the investigations of concrete fracture from the micro to the macro level [50]. The study addresses micro-mechanical phenomena and how phenomena such as micro-cracking affect the mechanical properties and
2.3. Acoustic emission analysis

Fracture formation on the macro-scale. In pull-out tests [33] and splitting tests [19] on concrete, AE sources in three-dimensional space were localized successfully and moment tensor inversion is performed to investigate crack kinematics. Hypocenters (earthquake foci) displaying the pull directions (“beach balls”) are determined and validated with characteristic stress trajectories. A simplified moment tensor analysis tool for crack identification and classification of concrete materials, called SiGMA procedure, is presented in [88].

Increasingly faster computer processors and high-performance computer clusters allow researchers to solve the elastodynamic wave equation and to simulate elastic wave propagation in a time- and resource-efficient manner. The simulation of elastic wave propagation as demonstrated in [17], [75] and [83] is an effective way to visualize the elastic energy flow and distribution in infinite and semi-infinite continua, and to further our understanding of the physical fundamentals of acoustic emissions. In these references, localization techniques based on the numerical simulations of ultrasonic fields are presented. The so-called time-reversal mirror, presented by Fink [20], provides the basis for the main localization technique of the present work, time reverse modeling. The principle of TRM is described in detail in Chapter 4. In [52], TRM is used in the area of NDT, where a defect (notch) on the surface of a cylindrical rod is localized using the time reversal of the wave motion and measured using laser interferometry. These days, TRM is an established tool for the localization of hydrocarbon reservoirs [92] and is successfully being used for the localization of acoustic emission sources [76]. Wave propagation approaches can also be applied qualitatively to characterize concrete [3] and damage in concrete [4] but are not further discussed in this document.

Signal-based AE combined with other established NDT approaches

Combining signal-based AE approaches with proved linearized, probabilistic or high-resolution X-ray CT approaches, is becoming more and more popular for the verification of localization results. Such a symbiosis is presented in [79], where the localization of AE due to concrete cracking in a reinforced and prestressed four-point bending experiment using the AIC-picker is supplemented by error analysis. Eight channels are used, leading to an overdetermined system of equations (more than four equations), which allows for error estimation. Additional wave propagation simulations are performed to study the influence of reinforcement bars and a prestressing tendon on the wave propagation behavior. A Bayesian AE localization algorithm is developed in [85]. Deterministic methods involving model parameters such as the sensor locations, the P-wave velocity and the onset time use mean values. The Bayesian network is described with probability density functions obtained from calibration measurements. The probabilistic approach is flexible, since additional data from observation can be incorporated to update the network at any time. High-resolution X-ray CT is commonly applied in the NDT community to obtain images of the crack distribution in a material, which is then used to verify the localization of AE sources obtained with the picking algorithm [42] or time reverse modeling [45]. X-ray CT images, which are available in digital format, can be pre-processed and used to study the wave propagation behavior in realistic uncracked and cracked concrete models [43].
Chapter 3

Elastic wave propagation in structural concrete

3.1 Fundamentals

A wave is a mechanical disturbance in the state of equilibrium propagating from its source to other positions [1]. On an atomic scale, a wave propagates by interaction between particles such as atoms and molecules. Important to note is the time domain of interest. The waves become relevant when the time increments of propagation are on a similar scale as the time the wave needs to travel through a medium. Waves are induced by excitations such as mechanical or piezoelectric impacts, explosions, spontaneous cracking processes and earthquakes. In an infinite continuum, there exist two types of wave motions: longitudinal and transversal. The longitudinal (compressional or pressure) waves propagate parallel to the direction of travel of the particle. The transversal (shear) waves propagate with particle motion orthogonal to the propagation direction. In a finite medium with a plane boundary, the longitudinal and transversal waves have to interact with this discontinuity. The incoming waves are reflected and refracted. For certain incidence angles, total reflection occurs and surface waves develop. Fig. 3.1 displays some snapshots of the numerical simulation of elastic wave propagation in two-dimensional (ij-)space. The snapshots illustrate the complexity of the wave interactions with the boundaries and the interference between wavefronts. If waves are reflected repeatedly, and the reflected wavefronts interfere with each other, it becomes impossible to positively identify either longitudinal or transversal wave. Note that in ideal fluids (i.e. liquids, gases and plasmas) no shear forces are transmitted between the particles, hence only longitudinal waves exist.

\footnote{For a continuum “it is assumed that properties averaged over a very small element, for example, the mean mass density, the mean displacement, the mean interaction force, etc., vary continuously with position in the medium, so that we may speak about the mass density, the displacement and the stress, as functions of position and time” [1].}
3.1.1 Equations of motion

To explain elastic wave propagation, the governing equations in elastodynamics are reviewed. The equations of motion in the time domain are derived as in [1]. The kinematic relations

$$
\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})
$$

(3.1)
describe the correlation between the strain tensor $\varepsilon_{ij} = \varepsilon(x,t)$ and the displacement field $u_i = u(x,t)$ on $\mathbb{R}^3$. Using Einstein’s summation convention [14], the coordinate axes are defined by $x_j$ and the base vectors $i_j$ ($j = 1, 2, 3$). Assuming an isotropic medium, a linear stress-strain relationship and applying Hooke’s law

$$
\sigma_{ij} = C_{ijkl} \varepsilon_{kl},
$$

(3.2)

the constitutive equations can be formulated for a linear elastic continuum. In Eq. (3.2), $\sigma_{ij} = \sigma(x,t)$ and $C_{ijkl} = C(x)$ represent the stress tensor and a symmetric fourth-order elastic tensor, respectively. The elastic tensor provides information about the material properties and can be expressed in terms of either Young’s modulus $E$ and shear modulus $G$, or $E$ and Poisson’s ratio $\nu$. For an isotropic and linearly elastic material the Poisson’s ratio must assume values $\nu \in [0; 0.5]$. In elasticity theory it is common to express Hooke’s law using Lamé’s constants $\lambda$ and $\mu$ and Kronecker-Delta $\delta_{ij}$ [72]. Hence, Eq. (3.2) can be rewritten as

$$
\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij},
$$

(3.3)

which are referred to as the constitutive equations. The resulting relationships are summarized in Tab. 3.1. Applying the linear momentum theorem to stresses acting on an infinitesimal element the dynamic equilibrium equations

\footnotesize
\begin{align*}
\text{Kronecker-Delta: } \delta_{ij} &= \begin{cases} 
1 & i = j \\
0 & i \neq j
\end{cases}
\end{align*}

\normalsize
are obtained, where $\rho = \rho(x)$ is a space-dependent density and $f_i = f(x,t)$ is a space- and time-dependent body force. Note that $\sigma_{ij,j} = \frac{\partial \sigma_{ij}}{\partial x_j}$ is the first partial derivative of the stress tensor with respect to the $x_j$-direction, and $u_{i,tt} = \frac{\partial^2 u_i}{\partial t^2}$ is the second partial derivative of the displacement field with respect to time $t$. Plugging the constitutive equations (3.3) and the kinematic relations (3.1) into the dynamic equilibrium equations (3.4), a system of three coupled partial differential equations

$$\rho u_{i,tt} = (\lambda + \mu) u_{k,ki} + \mu u_{i,kk} + f_i \quad (3.5)$$

is obtained, also known as the equations of motion. Eq. (3.5) can also be expressed in vector notation

$$\rho \ddot{\mathbf{u}} = (\lambda + \mu) \nabla \nabla \cdot \mathbf{u} + \mu \nabla^2 \mathbf{u} + \mathbf{f}, \quad (3.6)$$

which is a more suitable form for the separation of the displacement field.

### 3.1.2 Decomposition of the displacement field

The system of equations (3.5) or (3.6) exhibits a disadvantageous feature in that it couples the displacement components $\mathbf{u} = [u_i, u_j, u_k]$. The components of the displacement vector $\mathbf{u}$ can be uncoupled by involving the derivatives of the Helmholtz potentials $\Phi$ and $\Psi$ [1]. The displacement field can be written as

$$\mathbf{u} = \nabla \Phi + \nabla \times \Psi + \mathbf{f}, \quad (3.7)$$

where $\nabla \Phi$ is the gradient of $\Phi$, and $\nabla \times \Psi$ is the rotation of $\Psi$. In the absence of body forces ($f = 0$), Eq. (3.7) is substituted into Eq. (3.6) to separate the displacement field into a dilatational (rotation-free) and an isochore (volume constant) part. Rewriting the substituted Eq. (3.6) as

$$\rho \frac{\partial^2}{\partial t^2} [\nabla \Phi + \nabla \times \Psi] = (\lambda + \mu) \nabla \nabla \cdot [\nabla \Phi + \nabla \times \Psi] + \mu \nabla^2 [\nabla \Phi + \nabla \times \Psi], \quad (3.8)$$

and prescribing $\nabla \cdot \nabla \Phi = \nabla^2 \Phi$ and $\nabla \cdot \nabla \times \Psi = 0$, the equivalent term
Chapter 3. Elastic wave propagation in structural concrete

\[ \nabla \left[ (\lambda + \mu) \nabla^2 \Phi - \rho \ddot{\Phi} \right] + \nabla \times \left[ \mu \nabla^2 \Psi - \rho \ddot{\Psi} \right] = 0 \]  

(3.9)
is obtained. The two uncoupled scalar differential equations

\[ \nabla^2 \Phi = \frac{1}{c^2} \ddot{\Phi} \quad \text{and} \quad \nabla^2 \Psi = \frac{1}{c^2} \ddot{\Psi} \]  

(3.10)
are separated in a way that each of them satisfies the equations of motion. Solving Eqs. (3.10), the particle velocities of the dilatational and isochore waves

\[ c_p = c_\Phi = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad \text{and} \quad c_s = c_\Psi = \sqrt{\frac{\mu}{\rho}} \]  

(3.11)
are obtained. Because the Poisson’s ratio in solids must be \(0 \leq \nu \leq 0.5\) (\(\nu_{\text{concrete}} \approx 0.2\)), the particles of the potential \(\Phi\) propagate faster than those of the potential \(\Psi\). Hence, the velocity of the faster-moving particles \(c_\Phi\) is more commonly referred to as the P-wave (primary wave) velocity \(c_p\). Analogously, the velocity of the slower-moving particles \(c_\Psi\) is referred to as the S-wave (secondary wave) velocity \(c_s\). Both velocities are related to each other by the dynamic material constant

\[ \kappa = \frac{c_p}{c_s} = \sqrt{\frac{\lambda + 2\mu}{\mu}} = \sqrt{\frac{2(1 - \nu)}{1 - 2\nu}} \quad \text{(for concrete \(\kappa = 1.63\))}. \]  

(3.12)
The P- and S-wave are also called body waves because they propagate spherically through the interior of bodies.

### 3.1.3 Source representation of acoustic emissions

**Seismic sources**

Seismology recognizes two different source categories of internal sources: faulting sources and volume sources. “A faulting source is an event associated with an internal surface, such as slip across a fracture plane. A volume source is an event associated with an internal volume, such as a sudden (explosive) expansion throughout a volumetric source region” [5]. The two source categories can be expressed using the so-called moment tensor

\[ M_{ij} = C_{ijkl} e_k e_l \Delta V, \]  

(3.13)
where \(\Delta V\) is the crack volume, \(C_{ijkl}\) is the elastic tensor, and \(e_k\) and \(e_l\) are the unit vectors parallel and normal to the crack surface, respectively. Physically, the moment tensor provides information about the source kinematics (of acoustic emissions) [32]. Given the \(3 \times 3\) components of \(M_{ij} = M(x, t)\) on \(\mathbb{R}^3\), the source mechanism can be identified by inversion and connected to either shear or tensile mode, or a combination of both [88]. In this study, the moment tensor is used to simulate different excitations such as explosion or double-couple sources. The body force \(f\) can be represented by the moment tensor

\[ f(x, t) = \frac{\partial M_{ij}}{\partial x_j}, \]  

(3.14)
3.1. Fundamentals

a partial derivative of the moment tensor $\partial M_{ij}$ with respect to the $x_j$-direction. The elements of the tensor are either Ricker wavelets or else they vanish:

(a) explosion source: $M_{11} = M_{22} = M_{33} = w_i(x, t)$, with the remaining components set to zero

(b) double-couple source: $-M_{12} = -M_{21} = w_i(x, t)$, with the remaining components set to zero

The wavelet $w_i(x, t)$ can be any arbitrary function.

Point dislocation source

For many purposes such as AE analysis, a point force can represent the AE source adequately. An AE source can be approximated by a source point source, which can be derived assuming pairs of local dislocation sources. In seismology, Aki and Richards [5] formulate the solution for a point dislocation source. For displacement $u_i = u(x, t)$ at point $x$ and time $t$ in direction $x_i$, caused by a point force $f$ acting at an arbitrary point 0 in direction $x_j$ at time $\tau$ with a general time-varying impulse $X_0(t)$ (in this case the coordinate origin), the explicit formula

$$ u(x, t) = \frac{\hat{r}}{4\pi \rho} (3\hat{r} \gamma_j - \delta_{ij}) \frac{1}{r^3} \int_{r/c_p}^{r/c_s} \tau X_0(t - \tau) \, d\tau $$

is given. In Eq. (3.15) the following notations are used:

- $r := |x|$ denotes the distance of point $x$ to the location of the point force $f$
- $\gamma_i := \frac{\hat{r}}{r} = \frac{\partial r}{\partial x_i} \ (i = 1, 2, 3)$ denotes the cosine of direction
- NF stands for the near-field part of the solution
- FFP stands for the far field of the P-wave part of the solution
- FFS stands for the far field of the S-wave part of the solution

The terms FFP and FFS decline for small distances and become dominant for long distances due to $r^{-1}$. Vice versa, NF becomes dominant for small distances, and it declines for long distances due to $r^{-3}$.

No exact source functions of AE sources have been identified as yet. The Ricker wavelet $w_i(x, t)$ is only an ordinary approximation to perform the numerical simulations. In the numerical studies either the Ricker or Ricker2 wavelet, the first or second derivative of the Gaussian function, respectively, are used (see Fig. 3.2). The wavelet is centered on a dominant frequency $f_{\text{dom}}$. 
3.1.4 Plane waves in a semi-infinite continuum

In three-dimensional space, body waves propagate spherically in all directions at the same speed. The wave equations (3.5) or (3.6) can be expressed in terms of either Cartesian or radial coordinates. In the present work the Cartesian coordinate system is used. To discuss the interaction of wavefronts with a boundary, an unbounded infinite continuum is truncated into two semi-infinite parts with one boundary each. At the time when a wavefront approaches the boundary \( y \geq 0 \), it is sufficiently accurate to consider the interaction in the \( xy \)-plane instead of considering three-dimensional space. A plane wave propagating with phase velocity \( c \) from the position of vector \( x \)

\[
\mathbf{u} = q (\mathbf{x} \cdot \mathbf{p} - ct) \mathbf{d}
\]

is defined by a function \( q \), with the unit vectors \( \mathbf{p} \) and \( \mathbf{d} \) describing the directions of propagation and particle motion, respectively. The longitudinal and transversal waves are correlated in the following way: \( \mathbf{d} = \pm \mathbf{p} \) and \( \mathbf{d} \cdot \mathbf{p} = 0 \). The unit vector \( \mathbf{p} \) is normal to plane \( \mathbf{x} \cdot \mathbf{p} (= ct) \). The plane wave can be expressed using the harmonic solution

\[
\mathbf{u}^{(n)} = A_h \mathbf{d} e^{ik_w(x \cdot \mathbf{p} - ct)}
\]

with a varying amplitude \( A_h \) for \( n = 0, 1, 2 \) and wave number \( k_w = \frac{\omega}{c} \), where \( \omega \) is the angular frequency.

### Wave interaction with a free boundary

Waves originating from the interior of the domain are called incident waves. The incident waves are reflected and refracted at the boundary, see Fig. 3.3 (a). The interaction with the boundary couples the P- and the S-wave. An incoming P-wave with amplitude \( A_0 \) and angle of incidence
3.1. Fundamentals

\[ (a) \quad \mu_0 \mu_1 \mu_2 \]

\[ (b) \]

\[ (c) \]

**Fig. 3.3:** (a) Schematic of an incident and reflected wave-system, (b) reflection of the incident P-wave off a free boundary and (c) reflection of a SV-wave off a free boundary (adapted from [29]).

\theta_0, as illustrated in Fig. 3.3 (b), is reflected as a P-wave and refracted into an SV-wave\(^3\) with amplitudes \(A_1\) and \(A_2\) and angles of reflection \(\theta_1\) and refraction \(\theta_2\), respectively. Analogously, an incoming SV-wave is reflected off the boundary as an SV-wave and refracted into a P-wave (see Fig. 3.3, c). In three-dimensional space an SH-wave\(^4\) exists. Its interaction with a boundary is decoupled resulting in a reflected SH-wave with the same angle of incidence. The SH-wave is not further discussed here. In a material with \(\nu < 0.26\) and for certain angles of incidence, both incident P- and SV-waves are transformed into SV- or P-waves, respectively. This phenomenon is called mode conversion.

The interactions of the waves with the boundary depend on the angle of incidence \(\theta_0\) and Poisson’s ratio \(\nu\) only and are independent of the wavelength \(\Lambda_0\). Snellius [12] formulated a relationship between the angles of incidence \(\theta_0\) and refraction \(\theta_1\) and the corresponding phase velocities \(c_{(0)}\) and \(c_{(1)}\) (Fig. 3.3, a)

\[
\frac{\sin \theta_0}{\sin \theta_1} = \frac{c_{(0)}}{c_{(1)}}. \quad (3.18)
\]

This relationship is known as Snellius’ law of refraction. Note that for the case of an incident SV-wave the angle of refraction \(\theta_1\) will always be larger than the angle of reflection \(\theta_2\). Assuming \(c_p > c_s\) and \(\sin(\theta_1) < 1\), the so-called critical angle is

\[
\theta_{cr} = \sin^{-1}\left(\frac{c_s}{c_p}\right). \quad (3.19)
\]

---

\(^3\)SV-waves are S-waves with vertically polarized particles. The particles move in the plane of observation and normal to the direction of wave propagation.

\(^4\)SH-waves are S-waves with horizontally polarized particles. The particle motion is perpendicular to the plane of observation and is normal to the direction of wave propagation [100].
Fig. 3.4: Relative amplitude ratios of the reflected P-wave (a) and SV-wave (b) for various values of Poisson’s ratio $\nu$ at the solid-air interface (adapted from [6]).

For example an incident SV-wave with amplitude $A_0$ (denoted as SV($A_0$)) exhibits a critical angle $\theta_{cr} = 32.1^\circ$ in a material with $\nu = 0.2$. In cases where $\theta_{cr} > \theta_1$, neither the reflected P-wave with amplitude $A_1$ (denoted as SV($A_1$)) nor the corresponding angle $\theta_1$ are defined. If $\theta_i = \theta_{cr} = 90^\circ$, the reflected P-wave travels parallel to the surface. In this case, a surface wave, called inhomogeneous plane wave, is created. This wave does not carry any energy and exists only to fulfill the boundary conditions. Figs. 3.4 (a) and (b) display the behavior of the reflected amplitude ratios of Eq. (3.17) with respect to $\theta_0$, derived from the geometric relation in Figs. 3.3 (b) and (c) and Snellius’ law (3.18) for various values of $\nu$ [6].

Rayleigh surface waves

Besides body waves, waves exist that propagate along the free surface of a body, called Rayleigh surface waves [71]. The Rayleigh wave particles move retrograde on elliptic trajectories. The particle displacements decay exponentially with respect to the depth below the Earth’s surface. In Fig. 3.5 (b) the normalized relative displacement components $\frac{u_y}{u_x(z=0)}$ (normal motion) and $\frac{u_y}{u_x(z=0)}$ (tangential motion) are plotted for a body with $\nu = 0.2$. At a depth of $z \approx 0.6$ m, only vertical displacement exists. Beyond that depth the particle motion reverses from prograde to retrograde.

Rayleigh waves occur during earthquakes, where high-frequency waves travel along the surface of the Earth over long distances and are responsible for grave destruction. In several references, e.g. [1], [29] and [5], a non-trivial solution to calculate the Rayleigh wave velocity $c_R$ iteratively is given as

$$\left[ 2 - \left( \frac{c_R}{c_s} \right)^2 \right]^2 = 4 \sqrt{1 - \left( \frac{c_R}{c_s} \right)^2} \sqrt{1 - \left( \frac{c_R}{c_p} \right)^2}$$ (3.20)

and can be approximated as
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Fig. 3.5: (a) Overview of particle velocities $c_p$, $c_s$ and $c_R$ of P-wave, S-wave and Rayleigh wave, respectively, calculated iteratively using Eq. (3.20) for a material with density $\rho = 2200$ kg/m$^3$, Young’s modulus $E = 36730$ N/mm$^2$ and variable Poisson’s ratio $\nu$. (b) Normalized tangential and normal particle displacements $\frac{u_x}{u_x(z=0)}$ and $\frac{u_y}{u_y(z=0)}$ of the Rayleigh wave with respect to the depth $z$ below the surface.

$$c_R = \frac{0.862 + 1.14\nu}{1 + \nu} c_s. \quad (3.21)$$

As can be seen in Fig. 3.5 (a), the velocity of the Rayleigh wave is slower than the velocities of either P- or S-wave ($c_R < c_s < c_p$). The peak of the Rayleigh wave always follows after the peaks of both the P-wave and the S-wave. More detailed information on Rayleigh waves and other surface phenomena can be found in [96].

3.1.5 Dispersion and distortion

In a continuum, the phase velocity $c$ is independent of the frequency $f = \frac{\omega}{2\pi}$, which means that the wave is non-dispersive. Concrete is a strongly heterogeneous material due to its composition. For some frequencies, the aggregate and especially the air voids in the concrete can act as scatterers. The phase velocity is hence dependent on the frequency $c \mapsto c(\omega)$; concrete is thus a dispersive material. In particular, the large differences in acoustic impedance between the cement matrix and the air voids cause reflection and refraction of the wavefront. The acoustic impedance is a material parameter that represents the ratio of the wave stress and the particle velocity. With air (free boundary) the highest acoustic impedance contrast can be achieved. In a dispersive system, the pulse of a propagating wave always becomes distorted, i.e. the shape of the pulse changes during propagation. Dispersion can be due to geometrical properties or strongly visco-elastic material behavior, for example where $c(\omega) = \sqrt{\frac{E(\omega)}{\rho}}$. In the case of concrete, dispersion is mainly due to the scattering effect of air voids and inclusions rather than aggregate grains. This dispersion causes relatively high damping of the wave amplitudes. In physical experiments Köppel [46] determined a damping coefficient of 45 dB/m of the compressional
wave in concrete. The snapshots in Figs. 3.6 (a)-(c) show wave dispersion in a homogeneous cuboid. Typical behavior for dispersion is that (1) the shape of the wavefront changes with time and (2) that the zero-crossing distances become irregular, an indication that new frequencies occur.

### 3.2 Numerical concrete model

Concrete is the most commonly used materials in the structural engineering industry. This material is made up of different constituents such as aggregate grains of arbitrary shape, air voids and cement matrix and hence is a strongly heterogeneous material. Every concrete mix is unique due to the arbitrary distribution of grains in the casting form and an unknown content of air inclusions that cannot be eliminated completely by compaction. The size distribution as well as the packing density of the aggregate depend on the grading curve used. All these aspects make the numerical modeling of concrete a challenging task. Realistic assumptions need to be made to reflect the physical composition of the concrete (i.e. geometry and distribution of the constituents) and to approximate the elastic properties such as P-wave velocity $c_p$, S-wave velocity $c_s$ and density $\rho$. It is realistic to assume a random spatial distribution of concrete constituents, and hence this assumption is used for the creation of the numerical model [82].

#### 3.2.1 Coordinate transformation

In a Cartesian coordinate system, the coordinate transformation of an arbitrary point $P(x, y, z)$ to point $P'(x', y', z')$ is carried out. Three cases are distinguished: translation (Fig. 3.7, a), rotation (Fig. 3.7, b) and a combination of both.
3.2. Numerical concrete model

In Fig. 3.7 (a), the position vector \( \mathbf{r} = [x, y, z] \) is translated by the position vector \( \mathbf{a} = [a_1, a_2, a_3] \) with respect to the coordinate origin \( \mathbf{0} = [0, 0, 0] \). Its transformed elements are

\[
\begin{align*}
x &= x' + a_1 \\
y &= y' + a_2 \\
z &= z' + a_3.
\end{align*}
\]  

(3.22)

In plane rotation, the rotated vector \( \mathbf{r}' = [x', y'] \) of vector \( \mathbf{r} = [x, y] \) is expressed as

\[
\begin{align*}
x' &= x \cos \varphi - y \sin \varphi \\
y' &= x \sin \varphi + y \cos \varphi
\end{align*}
\]  

(3.23)

or in matrix formulation as

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
\cos \varphi & -\sin \varphi \\
\sin \varphi & \cos \varphi
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}.
\]  

(3.24)

It can be seen that the angle of rotation \( \varphi \) is the only argument in (3.24). In three-dimensional space, rotation with respect to axes \( x, y \) and \( z \) is possible. Therefore, a coefficient matrix, the so-called rotation matrix \( \mathbf{R} \)

\[
\begin{bmatrix}
x' \\
y' \\
z'
\end{bmatrix} =
\begin{bmatrix}
\xi_1 & \eta_1 & \zeta_1 \\
\xi_2 & \eta_2 & \zeta_2 \\
\xi_3 & \eta_3 & \zeta_3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}.
\]  

(3.25)

is introduced. In Eq. (3.25) the vector \( \mathbf{r}' \) is transformed into matrix \( \mathbf{R} \) with respect to the three Euler angles \( \vartheta, \psi \) and \( \varphi \). The position of the transformed coordinate system is described by the three Euler angles (see Fig. 3.7, b). The angle of nutation \( \vartheta \) is defined as the positive angle between the \( y \)-axis and the \( y' \)-axis and is limited to \( 0 \leq \vartheta < \pi \). The angle of precision \( \psi \) is defined as the positive angle between the \( x \)-axis and the intersection line of the \( xy \)-plane and the \( x'y' \)-plane. \( \psi \) is valid in the range \( 0 \leq \psi < \pi \). The angle of rotation \( \varphi \) is formulated between
the positive \(x'\)- and \(y'\)-direction in the \(x'y'\)-plane and is limited to \(0 \leq \varphi < 2\pi\). Rotation with respect to the coordinate axes and combination of coefficients yields the coefficient vectors

\[
\begin{bmatrix}
\cos \psi \cos \varphi - \cos \vartheta \sin \psi \sin \varphi \\
\cos \psi \sin \varphi + \cos \vartheta \sin \psi \cos \varphi \\
\sin \psi \sin \vartheta
\end{bmatrix}
\quad \eta =
\begin{bmatrix}
- \sin \psi \cos \varphi - \cos \vartheta \cos \psi \sin \varphi \\
- \sin \psi \sin \varphi + \cos \vartheta \cos \psi \cos \varphi \\
\cos \psi \sin \vartheta
\end{bmatrix}
\quad \zeta =
\begin{bmatrix}
\sin \vartheta \sin \psi \\
- \sin \vartheta \cos \psi \\
\cos \vartheta
\end{bmatrix}
\]

of the rotation matrix

\[
R =
\begin{bmatrix}
\cos \psi \cos \varphi - \cos \vartheta \sin \psi \sin \varphi & - \sin \psi \cos \varphi - \cos \vartheta \cos \psi \sin \varphi & \sin \vartheta \sin \psi \\
\cos \psi \sin \varphi + \cos \vartheta \sin \psi \cos \varphi & - \sin \psi \sin \varphi + \cos \vartheta \cos \psi \cos \varphi & - \sin \vartheta \cos \psi \\
\sin \psi \sin \vartheta & \cos \psi \sin \vartheta & \cos \vartheta
\end{bmatrix}.
\tag{3.26}
\]

The vectors \( \xi = [\xi_1, \xi_2, \xi_3], \eta = [\eta_1, \eta_2, \eta_3] \) and \( \zeta = [\zeta_1, \zeta_2, \zeta_3] \) are used in the numerical implementation of the NCM.

### 3.2.2 Medium with randomly distributed constituents

The geometry of the concrete constituents is simplified by using basic shapes such as ellipsoids (aggregate grains) and spheres (air voids), which are randomly distributed in space as suggested in [82]. More complex approximations for particle shapes such as those presented in [34] are not deemed necessary. The refinement proposed in [34] provides little additional information for the simulation of wave propagation for a governing wavelength of \(\Lambda_{AE} \approx 40\) mm (\(c_p \approx 4000\) m/s and \(f_{dom} = 100\) kHz).

**Spheres and ellipsoids**

A sphere is defined in the global coordinate system according to Figs. 3.7 (a) and (b). The sphere fulfills the equation

\[
x^2 + y^2 + z^2 = r^2,
\tag{3.27}
\]

where the radius \(r\) is valid for \(x, y\) and \(z\). Applying a translation by \(a = [a_1, a_2, a_3]\) yields

\[
(x - a_1)^2 + (y - a_3)^2 + (z - a_3)^2 = r^2.
\tag{3.28}
\]

For an ellipsoid of the form

\[
\left(\frac{x}{r_a}\right)^2 + \left(\frac{y}{r_b}\right)^2 + \left(\frac{z}{r_c}\right)^2 = 1,
\tag{3.29}
\]

the radii \(r_a, r_b\) and \(r_c\) are measured on the \(x\)-, \(y\)- and \(z\)-axis, respectively. Applying translation and rotation to the local coordinates of the ellipsoid, the equation

\[
\left[\frac{\xi_1(x-a_1)+\eta_1(y-a_2)+\zeta_1(z-a_3)}{r_a}\right]^2 + \left[\frac{\xi_2(x-a_1)+\eta_2(y-a_2)+\zeta_2(z-a_3)}{r_b}\right]^2 + \left[\frac{\xi_3(x-a_1)+\eta_3(y-a_3)+\zeta_3(z-a_3)}{r_c}\right]^2 = 1
\tag{3.30}
\]

is obtained.
3.2. Numerical concrete model

Fig. 3.8: Two-dimensional representation of the numerical concrete model with aggregate grains (gray) of maximum diameter \(d_{\text{max}} = 16\,\text{mm}\) and air voids (white) randomly distributed in space, translated by vector \(\mathbf{a}\), rotated by angle \(\varphi_i\) and surrounded by a homogeneous cement matrix (light gray). The shades of gray indicate the random density values allocated to each of the grains.

Random positions of aggregate grains and air voids

The translation vector \(\mathbf{a} = [a_1, a_2, a_3]\) and the Euler angles \(\vartheta, \psi\) and \(\varphi\) are multiplied with \(X_i^{[0;1]}\) (a random number between 0 and 1) and rewritten as

\[
\begin{align*}
  a_{1,i} &= n_x X_i^{[0;1]} \\
a_{2,i} &= n_y X_i^{[0;1]} \\
a_{3,i} &= n_z X_i^{[0;1]}
\end{align*}
\]

\[
\begin{align*}
  \vartheta_i &= \pi X_i^{[0;1]} \\
  \psi_i &= \pi X_i^{[0;1]} \\
  \varphi_i &= 2\pi X_i^{[0;1]}.
\end{align*}
\] (3.31)

This random number can be generated by the Fortran subroutine \texttt{random number()} [2]. Eqs. (3.31) allocates random positions and angles to the grains and the air voids in the NCM (see Fig. 3.8).

Randomly varying wave velocities and density

To ensure random elastic material properties for each grain, the wave velocities \(c_p\) and \(c_s\) as well as the density \(\rho\) are varied according to Tab. 3.2. The variation in density can be seen in Fig. 3.8 as different shades of gray of the aggregate grains. The ratios of the minimum and maximum variance of the elastic parameters are similar. Hence, the same random number \(X_i^{[0;1]}\) is applied to each parameter. The randomly distributed elastic properties are defined as

\[
\begin{align*}
  c_{p,i} &= c_p X_i^{[0;1]} \\
  c_{s,i} &= c_s X_i^{[0;1]} \quad \text{and} \quad \rho_i = \rho X_i^{[0;1]}.
\end{align*}
\] (3.32)

To control the accuracy of the developed NCM, cross-sections displaying the spatial distribution of the elastic material parameters are shown in Figs. 3.9 (a)-(c).
3.2.3 Distribution curves of the aggregate grains

Two practical approaches to control the size distribution and packing density of the aggregate mix in concrete are presented. The traditional way is to apply grading curves developed empirically, such as the Fuller Curve [26] or the EMPA Curve [102]. These idealized curves are rarely achieved in practice, because generally only a few sieving ranges of a fabricated aggregate mix are controlled. The remaining sieving ranges do not necessarily fit the theoretical curves. The alternative way is to adopt the attributes of a mixture fabricated in practice. Such a concrete mix generally meets all the required quality criteria.

An ideal grading curve as presented by Fuller [26] yields the volumetric part \( V_{0/d_i} \) of the grading range \([0; d_i]\) with

\[
V_{0/d_i} = 100 \left( \frac{d_i}{d_{\text{max}}} \right)^n
\]  

(3.33)

<table>
<thead>
<tr>
<th>Property [Unit]</th>
<th>Cement</th>
<th>Aggregate*</th>
<th>Air voids</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_p, c_p^* ) [m/s]</td>
<td>3950</td>
<td>4180 ± 210</td>
<td>0</td>
</tr>
<tr>
<td>( c_s, c_s^* ) [m/s]</td>
<td>2250</td>
<td>2475 ± 125</td>
<td>0</td>
</tr>
<tr>
<td>( \rho, \rho^*_i ) [kg/m³]</td>
<td>2050</td>
<td>2610 ± 130</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Tab. 3.2: A list of material properties used for numerical simulations according to [82] is shown. The cement matrix is modeled as a homogeneous material. The aggregate (gravel sand) is modeled with properties scattered between the minimum and the maximum value. Because hardly any energy is transmitted by air, the wave velocities for the air voids are set to zero.
3.2. Numerical concrete model

where \(d_i\) represents a variable aggregate size and \(d_{\text{max}}\) represents the maximum aggregate size. For a discrete model Eq. (3.33) can be written as

\[
V_{d_{\text{min}}/d_i} = \frac{100}{1 - \left(\frac{d_{\text{min}}}{d_{\text{max}}}\right)^n} \left[\left(\frac{d_i}{d_{\text{max}}}\right)^n - \left(\frac{d_{\text{min}}}{d_{\text{max}}}\right)^n\right].
\]  

In Eq. (3.34), the ratio \(\frac{d_{\text{min}}}{d_{\text{max}}}\) describes the grading range of the minimal aggregate size \(d_{\text{min}}\) over \(d_{\text{max}}\). \(n\) represents the exponent of the grading curve, which depends on the grain shape. As an example an elliptic grain \(n = 0.5\) is chosen. The minimal aggregate size is \(d_{\text{min}} \approx 0.1\) mm, and a gravel sand with almost no fine grains below 0.125 mm is assumed after dressing. The resulting percentage of \(d_{\text{min}} < 0.125\) mm is not of interest for numerical modeling due to the expected wavelengths of \(\Lambda \approx 40\) mm [78].

The volumetric part of the EMPA Curve \(V_{\text{EMPA}}\) [102] is formulated similar to Eq. (3.33) as

\[
V_{\text{EMPA}} = 50 \left(\frac{d_i}{d_{\text{max}}} + \sqrt{\frac{d_i}{d_{\text{max}}}}\right).
\]

The Fuller Curve is a theoretical description of the required aggregate composition to achieve the highest packing density of the aggregate. The EMPA Curve does not achieve the same optimal packing density but ensures a better fabrication to be a good compromise. The volumetric percentages of the Fuller Curve (red) and the EMPA Curve (blue) are displayed in Fig. 3.10.

3.2.4 Concrete mixture

The grading curve with a maximum aggregate size \(d_{\text{max}} = 16\) mm of an actual concrete mixture is considered to be the most representative approximation of the grain distribution. The concrete mix is assumed to consist only of aggregate and a cement matrix (no additives). The total mass of the assumed mix \(m_{\text{tot}} = m_{\text{cem}} + m_{\text{aggr}} = 44.93\) kg, mass of the cement of \(m_{\text{cem}} = 7.00\) kg and 1.0% air voids results in a mass of aggregate \(m_{\text{aggr}} = 37.93\) kg. The mass fractions \(m_{\text{aggr,}i}\) are
related to the percentages and volumes of the respective ellipsoids. The volume per ellipsoid \( V_{e,i} \) is determined using the semi-major axis \( r_a \) and the semi-minor axis \( r_c = 0.4r_a \). \( r_b \) is assumed to be equal to \( r_c \). Note that the volume percentage of the subsieve fraction \( V'_{e,i} \) is given with respect to the aggregate mass, and \( V_{e,i} \) is related to the total mass \( m_{tot} \). In total, 420,563,631 grains are placed in the NCM of size 100 × 100 × 100 mm.

### 3.2.5 Numerical implementation

In the numerical concrete model four phases are superimposed: air continuum, cement matrix, aggregate and air voids. The NCM described in the implementation scheme in Fig. A.1, and its edges are parallel to the \( x- \), \( z- \) and \( y- \) axes. Note that the \( y- \) direction denotes the third dimension. Some notations in the scheme follow a Fortran syntax. First, the air continuum (Phase 1) of size \( n_x \times n_z \times n_y \) is generated, and the material parameters \( (c_p, c_s \text{ and } \rho) \) are stored in the model tensor \( \text{model}(i=1,nz;j=1,nx;k=1,ny) \). Afterwards, the cement matrix (Phase 2) of size \((n_x-4) \times (n_z-4) \times (n_y-4) \) is generated and stored as \( \text{model}(i=3,nz-2;j=3,nx-2;k=3,ny-2) \). The superimposition of Phase 1 with Phase 2 creates a homogeneous model of the cement matrix surrounded by a 2 gp thick layer of air (all surfaces are free surfaces). Phase 3 (grains) and Phase 4 (air voids) cannot be generated in a similarly trivial manner. The biggest challenge here is to create an algorithm that places all grains and air voids at unique spatial locations, so that the shapes do not overlap. Therefore, some additional variables are introduced:

- **structure** specifies if a grid point is occupied (1) or not (0 or 2)
- **porecount** stores the number of positioned grains/air voids in agreement with Section 3.2.2
- **intersec** regulates the intersection query, where \( \neq 1 \) stands for non-intersection
- **popey** specifies if Eq. (3.30) or (3.28) is true (1) or not (0)
- **zaehler** is an additional counter for the **porecount** variable

Any variables postfixed with the character 0 pertain to the grains. The algorithm works in a similar way for both grains and air voids, but for the grains an additional rotation in space is required. For this reason the following description of the algorithm will focus on a single loop of the algorithm to place a single grain. At the start of the algorithm, the variables **structure0** and **porecount0** are set to zero. The Euler angles (\( \theta, \psi \text{ and } \phi \)) and the components of the translation vector \( (a_1, a_2 \text{ and } a_3) \) are calculated according to Eqs. (3.31), with the Fortran subroutine **random_number()** called for each of the parameters. Next, if the ellipsoid is created successfully \( \text{(popey0=1)} \) it is placed in the model. The radius \( r_a \) is prescribed by a grading curve (see Section 3.2.3), and for the sake of convenience \( r_b \) is assumed to be equal to \( r_c \). The ratio of the axes of the ellipsoid is controlled by the ratio of \( r_c \) and \( r_a \) which is limited to \( 0.4 < r_c/r_a < 1.0 \). The intersection query determines whether points of the new ellipsoid share grid points with an ellipsoid placed in a previous loop. If this is the case \( \text{(intersec0=1)} \),
3.3 X-ray computed tomography of structural concrete

X-ray CT is an imaging method that was originally developed for medical tasks, where CT images are used as a diagnostic tool to evaluate the condition of organs hidden within the human body [18]. In this case, the patient is a concrete block. This technology offers structural engineers an innovative and non-destructive way of visualizing the interior of a structure on a microscale or macroscale level. Even though concrete has a higher density than the human body, a concrete specimen can be CT scanned in slices and visualized in a volumetric representation after post-processing of the CT images. The digital format facilitates further investigations such as simulations of elastic wave propagation.

For this research, X-ray CT scans of two plain concrete specimens of sizes $l \times d \times h = 120 \times 120 \times [160, 180]$ mm were carried out. The obtained X-ray slice data of the plain concrete specimens is post-processed to get a numerical X-ray CT model for verification of the developed NCM.

---


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Fig. 3.11: The three phases of concrete are used in the three-dimensional NCM to model the material behavior of a $100 \times 100 \times 100$ mm concrete cube, discretized with $400 \times 400 \times 400$ gp: (a) cement matrix, (b) aggregate and (c) air voids [45].
3.3.1 Obtaining X-ray computed tomography scans of concrete

The X-ray CT scans were carried out with a Dual-Source CT scanner provided by the Institute of Diagnostic Radiology of the University Hospital of Zurich. In this scanner, two tubes, each containing an X-ray source and an X-ray recording system and positioned opposite to each other, rotate around an object and generate oversampled X-ray slice data of 0.6mm thickness. The inner structure of the concrete specimen is visualized in CT images (see Fig. 3.12, a). Pixels (2D) and voxels (3D) are displayed in different shades of gray that correspond to their relative radio densities. Radio density is a property of relative attenuation of X-rays through a material [62]. The X-ray CT parameters used for the scans are 140kV and 350mAs energy, yielding a voxel size of $254 \, \mu m^3$ ($6.3 \times 6.3 \times 6.3 \, \mu m$). The X-ray slice data is acquired in the DICOM (Digital Imaging in Communications and Medicine) format. A useful feature of scanning concrete with X-rays is that contrasts in radio density are amplified and displayed over an image data range. This information is used to locate the boundaries between the different phases, or to separate concrete constituents in the post-processing step (see Fig. 3.12, b).

3.3.2 Threshold segmentation of X-ray slice data of concrete cuboids

The post-processing (threshold segmenting) of a sequence of X-ray slice data is referred to as image segmentation [87]. Image segmentation is a process of separating subregions of X-ray slice data using threshold values, e.g. of phases defined over an image data range (see Fig. 3.13). The histogram in Fig. 3.13 represents a distribution of the number of gray scale values in the X-ray slice data of the concrete specimen. The image data range displays the mean attenuation of the material in the range $[-1024; 3071]$, the Hounsfield scale. Two-dimensional pixels are generated
3.3. X-ray computed tomography of structural concrete

Gray scale value $[0; 256]$

Phase window

Hounsfield scale [HU]

-1024 0 3071

Fig. 3.13: Schematic representation of the image segmentation for an arbitrarily defined phase window. The window sets a lower and an upper threshold limit within an image data range $[-1024; 3071]$ in Hounsfield units (HU). Values of the X-ray slice data, or the gray area of the histogram, are assigned to gray scale values within $[0; 256]$.

in a matrix with respect to the viewing direction. Voxels (3D) are X-ray slices multiplied by their thicknesses. Pixels and voxels that are below a manually or automatically defined threshold value can be assigned to a particular material. In order to obtain CT images, the CT volume data can be manipulated in a process called windowing using the calculated Hounsfield units (HU). The minimum and maximum image intensity is specified by the phase window (see Fig. 3.13). Typically, the maximum resolution differs from 256 to 1024 shades of gray over the window height. In the present research 256 is used as the maximum gray scale value. The edges that emerge during segmentation are smoothed and islands inside the grains are filled using an automatic image filter. This process is applied in all the segmentations referred to in the present work. A comparison of the CT image of the concrete specimen in Fig. 3.12 (a) and the visualization of the post-processed X-ray data in Fig. 3.12 (b) clearly demonstrates the potential of the threshold segmentation of concrete specimens.

Application to uncracked concrete specimens

Based on the sequence of X-ray slice data of the concrete specimen ($120 \times 120 \times 180$ mm) displayed in Fig. 3.12 (a), the three phases of interest cement matrix, grains and air voids are threshold-segmented. Fig. 3.14 (a) shows the segmented constituents. The three post-processed phases displayed in Fig. 3.14 (b) are considered to be successfully segmented, but if one takes a closer look at the specimen’s edges, it can be seen that few grains were segmented there. Possible explanations for this deficiency include insufficient resolution of the CT scanner or an improperly working segmentation algorithm. Applying the threshold segmentation with respect to air inclusions, a larger air bubble, mistakenly generated, is also identified. The fraction of each concrete constituent in the specimen can be quantified precisely after segmentation.
Chapter 3. Elastic wave propagation in structural concrete

Fig. 3.14: Visualization of the threshold segmentation of the three concrete phases aggregate grains, cement matrix and air inclusions. (a) The cement matrix [745; 1480] HU is displayed as transparent. (b) The aggregate grains [1480; 3071] HU are clearly visible. (c) Air inclusions [−1024; 745] HU, air voids and an air bubble approx. 40 mm in length are displayed in black. Air is most easily identified in CT images due to its high radio density contrast.

Application to a cracked concrete specimen

The highest contrast in acoustic impedance in concrete is always related to air inclusions and cracks. Excluding free boundaries, air inclusions and cracks are considered to be the most significant influences on wave propagation. Hardly any energy is transmitted across a crack that splits a specimen into discrete parts or semi-separate regions, and what little is transmitted cannot be readily quantified. A cracked specimen (see Figs. 3.15, a-b) from a test similar to the double-punch loading test is used for preliminary studies. More information on the experimental setup can be found in Section 5.2. Note that crack growth cannot be observed with the medical CT scanner used in the present work. Fig. 3.15 (a) shows the crack pattern splitting a concrete block into two parts. Applying threshold segmentation to visualize the enclosed air, a separating crack can be identified (see Fig. 3.15, b). It is interesting to see that totally separated and half-separated areas can be identified within the separating crack. Similarly, reinforcing bars in RC can be threshold-segmented using the range [3000; 3071] HU, also see Chapter 5.

3.3.3 Comparison of threshold-segmented X-ray slice data and the NCM

For a specimen scanned with X-ray tomography each of a sequence of 400 X-ray slices is used to calculate the percentage of air voids using a coarse and a fine segmentation, respectively. A maximum value of 1.43% (fine segmentation) and a minimum value of 0.35% (coarse segmentation) are calculated. The results show that a percentage of air voids in the range [0.5; 1.0] per cent is realistic and can be considered a conservative assumption (an upper bound) for two-dimensional numerical simulations. From Tab. 3.3 it can be seen that an assumption of 1% of air voids
3.3. X-ray computed tomography of structural concrete

Fig. 3.15: (a) Visualization of X-ray slice data of a cracked concrete cuboid (120 × 120 × 160 mm). Cropped CT images in the xy-, xz- and yz-planes are projected onto the lateral and lower surfaces of the bounding box. A separating crack is visible on the specimen surface (experimental details see Section 5.2). (b) Threshold-segmented air inclusions with emphasis on the crack volume, which forms a separating crack. Some semi-separated areas can be observed close to the top and the bottom surface.

is shown to be a conservative approximation for future two-dimensional and three-dimensional wave propagation simulations in concrete.

To demonstrate the differences between the threshold-segmented X-ray CT model and the NCM, the determined fractions of aggregate are compared for both models. In Tab. 3.3 it can clearly be seen that the percentages of aggregate and cement matrix differ considerably. The accuracy of the threshold segmentation is lower, because it depends on a manually defined threshold. Due to the CT resolution some fine aggregate fractions cannot be visually identified and segmented. A more detailed analysis shows that most grains above the sieving range of 2 mm could be segmented successfully. Any remaining unidentified fractions are allocated to the cement matrix. Note that this is the highest accuracy that can be achieved in concrete with the Dual-Source CT scanner that was used.

<table>
<thead>
<tr>
<th></th>
<th>Cement matrix</th>
<th>Aggregate</th>
<th>Air voids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segmented X-ray CT [%]</td>
<td>46.83</td>
<td>52.20</td>
<td>0.96</td>
</tr>
<tr>
<td>Computed with NCM [%]</td>
<td>15.43</td>
<td>83.57</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Tab. 3.3: Based on segmented X-ray slice data of an uncracked concrete block of size 120 × 120 × 160 mm without any noteworthy air inclusions, the percentages of the fractions are calculated and listed in the first row (segmented X-ray CT). The NCM is used to determine an ideal concrete mix, containing fractions of grains following Fuller’s curve [26], cement paste and 1% of air voids, and is listed in the second row (computed with NCM).
3.4 Numerical modeling of wave propagation with the finite-difference method

The elastic wave equations (3.5) can be solved using a staggered-grid finite-difference (FD) technique in the time domain, see e.g. [97], [98], [53] and [70]. In particular, a rotated staggered-grid scheme [75] is used. One of the attractive features of the staggered-grid approach is that the various difference operators are all naturally centered at the same point in space and time. Thus, the grid positions are staggered not only spatially but also temporally, so that the velocities are updated independently of the stresses. This allows for a very efficient and concise implementation scheme. The benefit of the rotated staggered-grid formulation is that the components of the elastic tensor \( c_{11} = \lambda + 2\mu \), \( c_{12} = \lambda \) and \( c_{44} = \mu \) are placed at the same position at the center of the node grid. Details of the formulation with respect to its accuracy as well as a stability analysis of the method can be found in the literature cited above. Note that in all numerical simulations performed in this work the Neumann stability criterion \( \frac{\Delta t c_p}{\Delta x} \leq 1 \) is satisfied.

3.4.1 Numerical implementation

The finite-difference algorithm described below is adapted from the abovementioned references to suit the present research. The algorithm is based on a stress-velocity formulation. Therefore, the dynamic equilibrium (3.4) and the constitutive equations (3.3) are expressed in terms of the velocity \( v_i \) and stress \( \sigma_{ij} \). Rewriting Eq. (3.4) as

\[
\frac{\partial^2 u_i}{\partial t^2} = \frac{1}{\rho} \frac{\partial \sigma_{ij}}{\partial x_j},
\]

it can be transformed to an equation in dependence of the velocity \( v_i = \frac{\partial u_i}{\partial t} \)

\[
\frac{\partial v_i}{\partial t} = \frac{1}{\rho} \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j}.
\]

By replacing the sum notation with the index \( k \) to yield

\[
\frac{\partial v_i}{\partial t} = \frac{1}{\rho} \frac{\partial \sigma_{ik}}{\partial x_k}
\]

it can be expressed in discrete form as a function of stress

\[
\Delta v_i = \Delta t \frac{\Delta \sigma_{ik}}{\rho \Delta x_k}.
\]

Eq. (3.39) is rearranged to yield the discrete derivative of the velocity \( v_i \), where the partial derivatives \( \partial \) are discretized to discrete derivatives \( \Delta \). Expanding Eq. (3.3) to

\[
\sigma_{ij} = \lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),
\]

the stress \( \sigma_{ij} \) can be expressed as a function of the velocity
The discrete form of this equation is obtained using similar steps as previously employed to find \( \Delta v_i \). Hence, the discrete stress is written as

\[
\Delta \sigma_{ij} = \Delta t \left[ \lambda \frac{\Delta v_k}{\Delta x_k} \delta_{ij} + \mu \left( \frac{\Delta v_i}{\Delta x_j} + \frac{\Delta v_j}{\Delta x_i} \right) \right].
\]  

### 3.4.2 Discretization on a two-dimensional spatial grid

The staggered grid contains a system of unit cells in the space (and subsequently time) domain. The circles in the unit cells in Figs. 3.16 (a) and (b) show the relative positions at which the following field variables are evaluated: velocities \((v_x \text{ and } v_z)\) and stresses \((\sigma_{xx}, \sigma_{zz} \text{ and } \sigma_{xz})\).

To avoid having to carry out additional transformations of the equations (3.39) and (3.42), two numerical grids are created: the node grid and the centered grid [75]. Both grids are rectangular. To express that a variable (e.g., \(v_i\)) is evaluated at \(x = x_i, z = z_j\) and \(t = t_n\), also written as \((x_i, z_j, t_n)\), it is followed by \(|_{i,j}^{n}\), e.g. \((v_i)|_{i,j}^{n}\). The grid spacings in the \(x\)- and the \(z\)-direction are \(x_i = i \Delta x\) and \(z_j = j \Delta y\), respectively, and \(t_n = n \Delta t\) is the time step. Central differencing is used for both the spatial and the temporal derivatives. In central differencing, the derivative is placed by the difference operator at the center points of the unit cells. The center points of the unit cells of one grid are at the same positions as the node points of the unit cells of the other grid. For example, the derivative of the velocity \(v_i\) is calculated at the same location as the stress \(\sigma_{ij}\) and vice versa.

The derivated, discrete velocity components \(\frac{\Delta v_i}{\Delta x_i}\) are obtained by averaging the velocities of two neighboring node points as...
On a three-dimensional spatial grid the derivated velocity components are averaged and yield the explicit formulas

\[
\frac{\Delta v_i}{\Delta x} = \frac{1}{2\Delta x} \left[ \left( v_i|_{i+1,j} - v_i|_{i,j} \right) + \left( v_i|_{i+1,j+1} - v_i|_{i,j+1} \right) \right]
\]

and

\[
\frac{\Delta v_i}{\Delta z} = \frac{1}{2\Delta z} \left[ \left( v_i|_{i,j+1} - v_i|_{i,j} \right) + \left( v_i|_{i+1,j+1} - v_i|_{i+1,j} \right) \right]
\]

where \(\Delta x = x_{i+1} - x_i\) and \(\Delta z = z_{j+1} - z_j\), see also Fig. 3.16 (a).

The differences of neighboring centered-grid points are averaged and yield the explicit formulas

\[
\frac{\Delta \sigma_{ij}}{\Delta x} = \frac{1}{2\Delta x} \left[ \left( \sigma_{ij}|_{i+1,j+1} - \sigma_{ij}|_{i,j} \right) + \left( \sigma_{ij}|_{i+1,j+1} - \sigma_{ij}|_{i+1,j+1} \right) \right]
\]

and

\[
\frac{\Delta \sigma_{ij}}{\Delta z} = \frac{1}{2\Delta z} \left[ \left( \sigma_{ij}|_{i,j+1} - \sigma_{ij}|_{i,j} \right) + \left( \sigma_{ij}|_{i+1,j+1} - \sigma_{ij}|_{i+1,j+1} \right) \right]
\]

where \(\Delta x = x_{i+1/2} - x_i\) and \(\Delta z = z_{j+1/2} - z_j\).

### 3.4.3 Discretization on a three-dimensional spatial grid

On a three-dimensional spatial grid the derivated velocity components \(\frac{\Delta v_i}{\Delta x}\) are calculated by averaging the differences of four neighboring node points:

\[
\frac{\Delta v_i}{\Delta x} = \frac{1}{4\Delta x} \left[ \left( v_i|_{i+1,j,k} - v_i|_{i,j,k} \right) + \left( v_i|_{i+1,j,k+1} - v_i|_{i,j,k+1} \right) + \cdots \left( v_i|_{i+1,j,k+1} - v_i|_{i,j,k+1} \right) \right],
\]

\[
\frac{\Delta v_i}{\Delta z} = \frac{1}{4\Delta z} \left[ \left( v_i|_{i,j+1,k} - v_i|_{i,j,k} \right) + \left( v_i|_{i+1,j,k} - v_i|_{i+1,j,k} \right) + \cdots \left( v_i|_{i+1,j,k} - v_i|_{i+1,j,k} \right) \right]
\]

and

\[
\frac{\Delta v_i}{\Delta y} = \frac{1}{4\Delta y} \left[ \left( v_i|_{i,j+1,k} - v_i|_{i,j,k} \right) + \left( v_i|_{i+1,j,k} - v_i|_{i+1,j,k} \right) + \cdots \left( v_i|_{i+1,j,k} - v_i|_{i+1,j,k} \right) \right]
\]

with \(\Delta y = y_{k+1} - y_k\). Note that because of the cubic grid geometry the grid spacing is set to \(\Delta h = \Delta x = \Delta z = \Delta y\).
The derived, discrete stress components $\frac{\Delta \sigma_{ij}}{\Delta x}$ are calculated in a similar manner as the derived discrete velocity components but the evaluation is carried out with respect to the centered grid rather than the node grid. The explicit formulas are

$$\frac{\Delta \sigma_{ij}}{\Delta x} = \frac{1}{4\Delta x} \left[ \left( \sigma_{ij} |_{i, j, k+1/2}^{n+1} - \sigma_{ij} |_{i, j, k+1/2}^{n} \right) \right] + \left( \sigma_{ij} |_{i, j, k+1/2}^{n+1} - \sigma_{ij} |_{i, j, k+1/2}^{n} \right) + \ldots$$

and

$$\frac{\Delta \sigma_{ij}}{\Delta y} = \frac{1}{4\Delta y} \left[ \left( \sigma_{ij} |_{i, j, k+1/2}^{n+1} - \sigma_{ij} |_{i, j, k+1/2}^{n} \right) \right] + \left( \sigma_{ij} |_{i, j, k+1/2}^{n+1} - \sigma_{ij} |_{i, j, k+1/2}^{n} \right) + \ldots$$

and

$$\frac{\Delta \sigma_{ij}}{\Delta z} = \frac{1}{4\Delta z} \left[ \left( \sigma_{ij} |_{i, j, k+1/2}^{n+1} - \sigma_{ij} |_{i, j, k+1/2}^{n} \right) \right] + \left( \sigma_{ij} |_{i, j, k+1/2}^{n+1} - \sigma_{ij} |_{i, j, k+1/2}^{n} \right) + \ldots$$

and

3.4.4 Temporal discretization

To complete the finite-difference algorithm, the spatially discretized equations (3.47)-(3.52) also need to be discretized temporally. The temporal discretization is displayed schematically in two dimensions in Fig. 3.16 (b). The velocity $v_i$ in

$$v_i |^{n+1} = v_i |^{n} + \Delta v_i |^{n-1}$$

is first updated temporally at $t_n+\Delta t = \Delta t(n+1)$. As can be seen in Fig. 3.16 (b), the centered grid used to determine stress $\sigma_{ij}$ is spatially and temporally shifted by $[x_i+\frac{1}{2}, y_j+\frac{1}{2}, z_k+\frac{1}{2}]$ (in 3D) and $t_n+\frac{1}{2}$, respectively. The stress at $t_n+\frac{1}{2} = \Delta t(n + \frac{1}{2})$

$$\sigma_{ij} |^{n+\frac{1}{2}} = \sigma_{ij} |^{n+\frac{1}{2}} + \Delta \sigma_{ij} |^{n+\frac{1}{2}}$$

$$= \sigma_{ij} |^{n+\frac{1}{2}} + \Delta t \left[ \lambda \frac{\Delta v_k |^{n+1}}{\Delta x_k} \delta_{ij} + \mu \left( \frac{\Delta v_i |^{n+1}}{\Delta x_j} + \frac{\Delta v_j |^{n+1}}{\Delta x_i} \right) \right]$$

(3.54)
is calculated with the updated velocity \( v_{i}^{n+1} \) of Eq. (3.53) to complete one time step of the numerical wave propagation simulation.

### 3.4.5 Boundary conditions

Boundary conditions need to be prescribed for the outer points of both the node grid and the centered grid. The boundaries of the model are generally modeled as free surfaces. This is achieved by setting \( \sigma_{11} = \sigma_{12} = \sigma_{13} = 0 \), where index 1 represents the direction normal to the surface, and indices 2 and 3 represent the directions parallel to the surface of the boundary.

### 3.5 Effective elastic properties

Where a particular problem requires a large numerical model, the complexity of the NCM may use up a large amount of computational resources. For such a case it is possible to apply homogenized material properties \( c_{p,\text{eff}}, c_{s,\text{eff}} \) and \( \rho_{\text{eff}} \) to the NCM and to the X-ray slice data. These properties are called the effective elastic properties and can be determined using different numerical approaches. A summary of the most important methods used to determine the EEP for various types of heterogeneous materials is presented in [73]. A dynamic wave propagation method is used to determine the EEP for both a numerical concrete model and a segmented X-ray CT model. In this method, a representative volume element (RVE) is embedded between two layers of a homogeneous medium, one at the top and one at the bottom. The layers are modeled as having periodic boundary conditions for motion in the two horizontal directions. In addition, the layer at the bottom is modeled as having absorbing boundary conditions for motion in the vertical direction. A plane respectively line excitation source (for 3D respectively 2D modeling) is placed at the top layer. P-waves and S-waves are induced separately and calculated at the positions of two horizontal planes respectively lines of receivers located at the top and the bottom of the RVE. The emerging time delay of the mean peak amplitude between the top and the bottom receivers is used to determine the EEP [77].

### 3.5.1 Two-dimensional numerical examples

**Numerical concrete model**

In the following numerical example, a NCM with \( 400 \times 400 \) gp is used. The model is surrounded by a \( 200 \) gp thick homogeneous layer (\( c_{p} = 4180 \) m/s, \( c_{s} = 2475 \) m/s and \( \rho = 2610 \) kg/m\(^3\)) at the top and at the bottom. The grid spacing is set to \( \Delta h = 0.00025 \) m. The excitation signal is a low frequency line wavelet (Ricker) with a dominant frequency \( f_{\text{dom}} = 100 \) kHz and a time increment \( \Delta t = 0.5 \times 10^{-8} \) s.

The scattering material properties in Tab. 3.2 are used for the aggregate grains. Ten NCMs are generated for each of the following percentages of air voids: 0.5\%, 1.0\%, 2.0\%, 3.0\%, 4.0\%.

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3.5. Effective elastic properties

and 5.0%. It is found that random material properties affect the computed EEP in a proportional way. In Figs. 3.17 (a)-(c), the calculated EEP are plotted with respect to the percentages of air voids in the NCM. With increasing percentages of air inclusion in the model the velocities and density decrease almost linearly. Applying excitation signals of different dominant frequencies (50 kHz, 200 kHz and 400 kHz) to the NCMs, it is found that the correlation between the percentage of air voids and the velocities is independent of the dominant frequency (see Figs. A.2, a-f). The slopes of the regression lines of the P-wave ($y_p$) respectively S-wave ($y_s$) velocities differ. The slopes of the S-wave regression lines are smaller than those of the P-wave velocity (compare Figs. 3.17, a and b, and Figs. A.2, a-f). This makes sense, because the S-wave peak exhibits an amplitude that is larger and not affected as strongly by air inclusions and aggregate as that of the P-wave. The S-wave velocities decrease linearly and with similar slopes for different frequencies, whereas the slopes of the regression lines of the P-wave velocities vary slightly for different frequencies. At 50 kHz, the S-wave velocities show strong scattering. The mean effective elastic parameters for the two-dimensional NCM with 1% air voids and an excitation signal with a dominant frequency of $f_{dom} = 100$ kHz are $\bar{c}_{p,\text{eff}} = 3894$ m/s, $\bar{c}_{s,\text{eff}} = 2282$ m/s and $\bar{\rho}_{\text{eff}} = 2457$ kg/m$^3$.

**Threshold-segmented X-ray slice data**

The EEP for X-ray slice data are determined in a similar manner as those for an NCM. The only difference between the two approaches is that the model from the X-ray scans is available a priori and does not need to be generated numerically.

The grid points used for the finite-difference algorithm are assigned elastic material properties of either grains, air voids or cement matrix. Note that the used material parameters are identical with the parameters shown in Tab. 3.2 but without the scattering of the aggregate density. The EEP are determined for every 25$^{th}$ of the 400 available X-ray slices, and the results are shown in Figs. 3.18 (a) and (b). It can be seen that the EEP are similar for most of the slices. The

![Fig. 3.17:](image-url) Effective elastic properties for NCM models of 100 × 100 mm, discretized on 400 × 400 gp, with different percentages of air voids: (a) P-wave velocity $c_{p,\text{eff}}$ [m/s], (b) S-wave velocity $c_{s,\text{eff}}$ [m/s] and (c) density $\rho_{\text{eff}}$ [kg/m$^3$].
first value of the series in Fig. 3.18 (b) is very small compared to the others. The reason for this is that at the bottom of the specimen, where the 25th slice is located, the concrete is not as dense as in the interior of the specimen because of insufficient compaction of the material. In this location, the main constituent is the cement matrix, which has a lower density than the aggregate (see Tab. 3.2). The mean values of the EEP are $\bar{c}_{p,\text{eff}} = 4271 \text{ m/s}$, $\bar{c}_{s,\text{eff}} = 2541 \text{ m/s}$ and $\bar{\rho}_{\text{eff}} = 2322 \text{ kg/m}^3$. Results obtained from the segmented X-ray CT models compare favorable with the parameters obtained from the NCM.

### 3.5.2 Three-dimensional EEP of concrete

The EEP are calculated for the three-dimensional NCM of size $400 \times 400 \times 400 \text{ gp}$ ($4 \text{ gp} \equiv 1 \text{ mm}$) shown in Figs. 3.11 (a)-(c) and for the threshold-segmented X-ray CT model displayed in Figs. 3.14 (a)-(c). Note that the X-ray CT model is trimmed to a cube of $100 \times 100 \times 100 \text{ mm}$. The numerical setup for calculating the EEP of both models are identical to those of the two-dimensional numerical examples in Section 3.5.1.

To verify the numerical (NCM) and semi-numerical (segmented X-ray CT model) calculations of $c_p$ and $c_s$, the P-wave velocity was measured physically on a concrete plate ($50 \times 500 \times 500 \text{ mm}$). Wave propagation is assumed in the longitudinal directions of the plate. Six piezoelectric sensors (KSB250, Ziegler Instruments) were arranged diagonally on the upper concrete surface. The sensors were mounted at irregular intervals to avoid systematical errors. The excitation signal was delivered by dropping three steel balls with diameters $d_\phi = 4 \text{ mm}$, $3 \text{ mm}$ and $2 \text{ mm}$, respectively, onto the surface. The sensor closest to the location of impact acted as a trigger. The onset times of the first wave motion were recorded. The relative sensor distances $|r_i - r_j|$ with respect to the relative arrival times $|t_i - t_j|$ are plotted in Figs. 3.19 (a)-(c) for the three steel balls.
3.6. Wave propagation in cracked structural concrete

3.6.1 Plain uncracked concrete

In the following paragraph, the performances of the NCM and the segmented X-ray CT model for numerical simulations of elastic wave propagation are discussed. The NCM discretized on $400 \times 400$ gp is generated with the numerical setup as described in Sections 3.5.1 and 3.5.2. An explosion source (a Ricker wavelet $w_2(x, t)$) is excited at $[n_x, n_z] = [100, 100]$ gp. The generated

<table>
<thead>
<tr>
<th>Property [Unit]</th>
<th>NCM</th>
<th>X-ray CT</th>
<th>Experimental$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{p, \text{eff}}, c_{p}^*$ [m/s]</td>
<td>3987</td>
<td>4050</td>
<td>4129$^*$</td>
</tr>
<tr>
<td>$c_{s, \text{eff}}, c_{s}^{**}$ [m/s]</td>
<td>2328</td>
<td>2541</td>
<td>2529$^{**}$</td>
</tr>
<tr>
<td>$\rho_{\text{eff}}, \rho^{*}$ [kg/m$^3$]</td>
<td>2200</td>
<td>2322</td>
<td>2690$^*$</td>
</tr>
</tbody>
</table>

Tab. 3.4: Effective elastic properties determined numerically (NCM) and semi-numerically (X-ray CT), and experimentally determined elastic properties. $^*$Measured ($d_{\alpha} = 2$ mm) property and $^{**}$calculated property from measurement using $\kappa = 1.63$.  

Fig. 3.19: Evaluation of velocity measurements on a concrete surface. Shown is the relative sensor distance $|r_i - r_j|$ [mm] versus the relative arrival time $|t_i - t_j|$ [$\mu$s] for three steel balls: (a) $d_{\alpha} = 4$ mm ($f = 43$ kHz), (b) $d_{\alpha} = 3$ mm ($f = 57$ kHz) and (c) $d_{\alpha} = 2$ mm ($f = 87$ kHz). The standard deviation $s$ and the standard deviation of the mean P-wave velocity $s_{c_{p}}$ are noted in the graphs.

Although the three ball impacts produce different frequency contents, the variation in the average velocities $c_{p}$ is small (Figs. 3.19, a and b) and not significant (Fig. 3.19, c). The EEP from the numerical calculations and the corresponding properties determined in physical experiments are shown in Tab. 3.4. The X-ray CT-based calculations match the experimental data well. The calculated NCM-based wave velocities are approximately 200 m/s below the experimental values but are still acceptable. The NCM properties are adopted for all numerical simulations performed in this work.
NCM is shown in Fig. 3.20 (a) with the location of the excitation source marked with a red circle. The snapshot in Fig. 3.20 (b) shows a propagating wave field with a dominant frequency $f_{\text{dom}} = 50$ kHz. The waves are reflected and scattered due to interactions with the boundaries and air voids. The aggregate grains appear to have no influence on the wave propagation itself but only on the effective particle velocities depicted in Figs. 3.17 (a) and (b). The amplitudes of the vertical displacement field displayed in Figs. 3.20 (b) and 3.21 (b) have a similar magnitude.

The segmented X-ray slice data is discretized on $400 \times 400$ gp in the numerical model (see Fig. 3.21, a). Again, the numerical setup as described in in Sections 3.5.1 and 3.5.2 is used. An explosion source is excited at $[n_x, n_z] = [100, 100]$ gp. The snapshot of a wavefront of the X-ray CT model in Fig. 3.21 (b) looks similar to that of the NCM in Fig. 3.20 (b), and it can be seen that the scattering in both images is due to the air inclusions in the concrete. Since the wave propagation behavior is very similar in both models in that the wavefronts are scattered equally and the vertical displacements are in a similar range, it can be concluded that it is not necessary to carry out simulations in three dimensions. However, because the spatial spreading of the wave field in the third dimension causes a decay in the amplitudes, the wave velocities in the two-dimensional model need to be decreased.

### 3.6.2 Cracked concrete cuboid

Exciting a three-dimensional cracked specimen, the elastic energy emitted from the source at $[n_x, n_z, n_y] = [174, 176, 158]$ mm is transmitted partially across the semi-separating barrier (the crack). The model used for the numerical simulations consists of two phases: Phase 1 is homogeneous with the equivalent elastic properties of the NCM (see Tab. 3.4), and Phase 2...
3.6. Wave propagation in cracked structural concrete

Fig. 3.21: (a) Visualization of the two-dimensional segmented X-ray CT model (400 × 400 gp, gp ≡ mm) in terms of the density distribution (a): cement matrix (light gray), grains (dark gray) and air voids (white). The position of the excitation source is marked with a red circle. The shades of gray of the color bar correspond to the density $\rho$ [kg/m$^3$]. (b) Vertical displacement wave field [nm] at time $t = 6.6 \mu$s of the numerical simulation.

The wavefront remains intact, although the amplitudes decrease significantly. The simulation is carried out for a cracked specimen (Phase 1 plus Phase 2) and for a homogeneous specimen consisting only of Phase 1. Displacements normal to the surface at sensor positions #1 to #4 (coordinates see Tab. 3.5) are calculated. The displacements are plotted in Fig. A.4 for the cracked (blue) and homogeneous (red) specimen. It can be seen that the shape of the first wave motion is preserved at all sensor positions except for position #2.

The crack has a damping effect on the amplitudes (see Figs. A.3, a-c). One would expect identical displacements at sensor positions #1 and #4, because they are at the same distance from the source. The reasons for the different values are the non-symmetric shape of the Ricker2 wavelet $w_3(x, t)$.

<table>
<thead>
<tr>
<th>Pos.</th>
<th>$n_x$ [gp]</th>
<th>$n_z$ [gp]</th>
<th>$n_y$ [gp]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>174</td>
<td>0</td>
<td>158</td>
</tr>
<tr>
<td>#2</td>
<td>58</td>
<td>0</td>
<td>158</td>
</tr>
<tr>
<td>#3</td>
<td>0</td>
<td>58</td>
<td>79</td>
</tr>
<tr>
<td>#4</td>
<td>0</td>
<td>176</td>
<td>158</td>
</tr>
</tbody>
</table>

Tab. 3.5: Coordinates of the sensor positions.
Fig. 3.22: Snapshots of the wave propagation simulation in a cracked cuboid of size $n_x \times n_z \times n_y = 232 \times 232 \times 316 \text{gp}$ (2 gp $\equiv 1\text{ mm}$). The snapshots are taken at intervals of $\Delta t = 125\,\mu\text{s}$. The dominant frequency of the excitation source is $f_{\text{dom}} = 25\,\text{kHz}$. In snapshot (a) the simulation of the wave propagation is superimposed with the threshold-segmented air voids and crack. The displacement wave field in (a)-(c) is normalized.

wavelet as well as the presence of two air voids along the propagation path in $n_x$-direction (see Fig. A.3, a). In Fig. A.3 (b) it can be seen that the displacement field at sensor position #4 is missing the main peaks of the first wave motion. In Figs. A.5 (a)-(l) orthogonal and oblique X-ray CT slices are displayed. The orthogonal slices are oriented normal to the $n_xn_z$-plane. The oblique slices are oriented normal to the $n_xn_z$-plane and rotated about the $n_y$-axis with the source coordinates set to be the origin of rotation. The information in the displacement plots (see Fig. A.4) is enhanced by the snapshots displayed in Figs. A.5 (a)-(l). A difference between the displacement field in the orthogonal slices at sensor position #1 (see Figs. A.5, a-c) and sensor position #4 (see Figs. A.5, j-l) can also be observed. In Figs. A.5 (d)-(f) it can again be seen that the wavefront is scattered and damped due to its interaction with the crack.

3.6.3 Reinforced concrete

When reinforcing bars and prestressing tendons are placed into a concrete specimen, an anisotropic medium is obtained. In this dissertation, no numerical simulations are performed to study the wave propagation behavior of such specimens. In [78] and [79], the influence of reinforcing bars and prestressing tendons on wave propagation behavior is discussed. Single reinforcing bars act as scatterers of the wave field. A grouted prestressing tendon with a diameter of 100 mm has a strongly disturbing effect on the wavefront. The prestressing wires in the conduit behave as a waveguide. As wave particles propagate faster in steel ($c_{p,\text{steel}} = 5900\,\text{m/s}$, $c_{s,\text{steel}} = 3200\,\text{m/s}$ and $\rho_{\text{steel}} = 7820\,\text{kg/m}^3$ [78]) than in the surrounding concrete, the wave propagates in an anisotropic manner. A badly grouted duct may cause total reflection of the wavefront because of the presence of air inclusions.
Chapter 4

Time reverse modeling in ultrasonic non-destructive testing

4.1 Time reverse modeling of waves

Time reverse modeling (TRM) is applied in several fields of science such as medical and earth sciences [21]. Prior time reverse studies focus on filtering single events from recordings of low signal-to-noise ratio [41] or on spatial and temporal accuracy of single event localization [27]. To apply TRM in ultrasonic non-destructive testing, an approach from exploration physics is adopted. The main challenge in doing so is to solve the elastodynamic wave equation with the limited data recorded at the boundaries of the modeling domain.

4.1.1 Principle

TRM is based on ultrasonic wave fields that are reversed in time and re-emitted numerically into the specimen, where they concentrate on the location of the original source [22]. The procedure consists of two steps: the forward procedure and the inverse simulation, which also includes imaging (see Fig. 4.1). The forward procedure provides the input signal, either numerically with full control of its origin, or experimentally, obtained for example from acoustic emission measurements. In the numerical forward simulation, a time-dependent excitation is applied at a spatial point in the model. Displacements at sensor positions at the model’s boundaries are calculated over time. The resulting time-dependent displacements at the sensor positions, determined either numerically or experimentally, serve as input signals for subsequent TRM simulations.


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4.1.2 Forward simulation in concrete

The spatial domain of interest is denoted by $\Omega$ with positions $\mathbf{x} \in \Omega$. The time $t$ is defined in $[0; T]$ with end time $T$. The displacement field $u_i = \mathbf{u}(\mathbf{x}, t) \in \mathbb{R}^3$ fulfills the wave equations (3.5) for the linear-elastic continuum $\in \Omega \times [0; T]$. Eq. (3.5) is used in the forward (numerical) simulation to calculate the displacements at the sensor positions on the domain’s boundary $\partial \Omega$. The wave equation is discretized as described in Section 3.4. The forward simulation provides a time series of the displacements (Fig. 4.2)

$$\mathbf{u}^{(k)}(t) = \mathbf{u}(\mathbf{x}^{(k)}, t)$$  \hspace{1cm} (4.1)

at positions $\mathbf{x}^{(k)}$ of the sensor locations

$$S = \left\{ \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, ... \mathbf{x}^{(N)} \right\} \subset \partial \Omega.$$  \hspace{1cm} (4.2)

$N$ denotes the number of sensors that are used for the simulation.
4.1. Time reverse modeling of waves

4.1.3 Inverse simulation

The inverse simulation is performed for the effective elastic properties (see Section 3.5), the velocities $c_{p,\text{eff}}$ and $c_{s,\text{eff}}$ and the density $\rho_{\text{eff}}$, which are determined in the numerical forward simulation. Note that in the inverse simulation experimentally measured material parameters can be used instead of the EEP. The body force $\mathbf{f}$ in Eq. (3.5) is set to zero. The recorded displacements

$$
\mathbf{u}(\mathbf{x}(^k), t) = [u_x(\mathbf{x}(^k), t), u_y(\mathbf{x}(^k), t), u_z(\mathbf{x}(^k), t)]
$$

on the boundary $\partial \Omega$ are fed into the domain as excitation sources.

Full time reverse modeling

Due to time invariance an initial condition for a homogeneous wave equation can be recovered in a time-reversed computation. The time reversal requires the time series of all boundary values for times $t \in [0; T]$ as well as the displacement field and its velocity at time $T$. An approximation of the initial conditions can be found even if only the displacement field $\mathbf{u}(\mathbf{x}, T)$ and its velocity $\partial \mathbf{u}(\mathbf{x}, T) / \partial t$ at time $T$ can be determined from a forward simulation, but no knowledge of the boundary values is available. In that case the final displacement fields are used as the initial conditions

$$
\mathbf{u}(\mathbf{x}, t = 0) = \mathbf{u}(\mathbf{x}, T) \quad \text{in } \Omega
$$

$$
\frac{\partial \mathbf{u}(\mathbf{x}, t = 0)}{\partial t} = -\frac{\partial \mathbf{u}(\mathbf{x}, T)}{\partial t} \quad \text{in } \Omega,
$$

where the velocity is taken as negative because of the time reversal. Since the numerical method is based on a two-step time integrator, see [75], this reversed computation is realized from the last two displacement fields at times $T$ and $T - \Delta t$. Both fields are used in reverse order as initial fields for the two-step integrator in a reversed order. The boundary conditions are the same as in the forward computation.
Chapter 4. Time reverse modeling in NDT

The result of a full TRM simulation such as this can be considered a benchmark for a source TRM, since much more information is used and the localization of the initial excitation is generally much better (see the example below). However, Full TRM can seldom be used in practice due to the lack of knowledge of the displacement field at all boundary points in realistic situations.

Source time reverse modeling

This TRM simulation is based on wave equations (3.5) with \( x \in \Omega \) and \( t \in [0; T] \). The coefficients from the forward computation are used. No body force is present throughout the simulation \( (f = 0) \). Initial conditions for \( u(x, t) \) and \( \frac{\partial u(x,t)}{\partial t} \) vanish, so that the equation is governed by the boundary conditions. On boundary \( \partial \Omega \), recorded signals \( u(x(k), t) \) are fed as sources into the domain. Formally, the inverse displacement field is written as

\[
\begin{align*}
  u(x^{(k)}, t) &= u(x^{(k)}, T - t) \quad \text{for } x \in S \subset \partial \Omega \quad (4.6) \\
  u(x^{(k)}, t) &= 0 \quad \text{for } x \in \partial \Omega \setminus S \quad (4.7)
\end{align*}
\]

so that inhomogeneous Dirichlet data [61] is found exclusively in the source locations \( S \). Note that the time series is fed into the computation backwards in time; the TRM simulation hence reverses the forward computation. The term “Source TRM” emphasizes the fact that the time signals are implemented as sources of wave excitations. In Source TRM only receiver signals from selected points are used. This method is incomplete in comparison with Full TRM, where the equation is provided with time-reversed receiver signals at every boundary point. It is found that time-reversed signals are needed only at a few boundary points to achieve very good results [74]. Note that Source TRM has also been applied successfully to real models, i.e. recorded receiver data (representing the forward simulation) has been used for the numerical time reverse simulation in order to locate a physical excitation source. An example in the field of exploration geophysics can be found in [92].

In the numerical method, the actual domain \( \Omega \) is supplemented by a layer of almost vacuum with zero Dirichlet conditions at the outer computational boundary, as described above. Hence, the time signals \( u(x^{(k)}, t) \) are introduced into the numerical grid at the boundary grid points \( \partial \Omega \) of the medium. To avoid scattering, the signals are superimposed on any existing displacement values at these grid points that are the results of interior and surface waves. By using this technique the signals are interpreted as time series of localized initial conditions, whose evolutions are superimposed in a time-delayed manner.

Imaging the maximum particle displacement

To display the displacement field, Steiner [92] introduced the idea of taking only the maximum absolute value of the particle displacement into account, which is defined as

\[
u_{\text{max}}(x) := \max_{t \in [0; T]} \| u(x, T - t) \|.
\] (4.8)
4.1. Time reverse modeling of waves

Fig. 4.3: (a) Snapshot of the two-dimensional vertical displacement wave field within area $n_x \times n_z$ [gp] at one time step during the forward simulation, using a homogeneous velocity. The original source position is marked with a black circle. (b) The Full TRM calculation yields the best possible inverse source localization (marked by the maximum displacement wave field). (c) The Source TRM approach uses the recorded displacements at 12 receiver positions (marked with 12 green crosses in image (a)), located in the $n_x, n_z$-plane, as input [76]. The vertical displacement wave field in (a) and the maximum particle displacement in (b) and (c) are normalized.

The maximum particle displacement $u_{\text{max}}$ for every point $x \in \Omega$ of the inverse wave field is stored for all times $t \in [0; T]$ of modeling. The global maximum of the TRM field makes it possible to localize the original source location, i.e. the source location of the forward simulation.

Comparison of Full TRM and Source TRM

A generic example is used to illustrate the differences between Full TRM and Source TRM. A homogeneous 2D model surrounded by a thin vacuum layer is considered. The model ($\Delta h = 0.0001 \text{m}$) is of size $100 \times 100 \text{mm} \ (1 \text{mm} \equiv 10 \text{gp})$ with the compressional and shear wave velocity set to $c_{p,\text{eff}} = 3894 \text{m/s}$ and $c_{s,\text{eff}} = 2282 \text{m/s}$, respectively, and the density set to $\rho_{\text{eff}} = 2457 \text{kg/m}^3$ (see Section 3.5.1). An initial body force source (Ricker2 wavelet with $f_{\text{dom}} = 100 \text{kHz}$) acting horizontally is placed at $[n_x, n_z] = [400, 300] \text{gp}$ and marked with a black circle in Fig. 4.3 (a). The modeling is done with a second order time update and a second order spatial differentiation operator as described in [75], using a time increment $\Delta t = 1.8 \times 10^{-8} \text{s}$. The complete wave field is stored at two consecutive time steps at the simulation end, and this information is used as the input data for Full TRM. The full wave field (i.e. the vertical and horizontal displacement field) is also recorded at 12 sensor positions (marked with green crosses in Fig. 4.3 (a) over the duration of the simulation. This information is used as the input data for Source TRM.

Fig. 4.3 (b) shows the TRM field obtained from the Full TRM simulation. It can be seen that the largest displacement values are found around the location of the initial source. The TRM field of the Source TRM simulation is displayed in Fig. 4.3 (c). In this image, the location of the initial source is identified accurately. Note that hereafter Source TRM will simply be referred to as TRM.
Chapter 4. Time reverse modeling in NDT

(a) Horizontal force  (b) Explosion  (c) Double couple

Fig. 4.4: Examples of governing source excitations in geophysics and NDT with characteristic radiation patterns detected by the TRM field [76]. The maximum particle displacement in (a)-(c) is normalized.

TRM source patterns

In signal-based AE analysis, it is important not only to locate the initial source of an AE but also to identify the excitation (radiation) pattern. To simulate different excitation patterns, the external force $f$ of the forward computation (3.5) is derived from a given moment tensor $M_{ij}$. It is possible to extract the specific form of $M_{ij}$ from a time reverse computation using only a few (twelve) source data on the boundary. This is shown in the following brief study of three different source types.

In Figs. 4.3 (a)-(c) a body force source acting horizontally is used. This force is modeled by a moment tensor with vanishing entries except for the $M_{11}$ component, which is given by a Ricker2 wavelet localized around the source location $x_s$. In addition, an explosion source and a double-couple source are investigated. The elements of the moment tensor for the considered source types are as follows:

(a) horizontal force: $M_{11} = w_i(x, t), M_{22} = M_{12} = M_{21} = 0$

(b) explosion: $M_{11} = M_{22} = w_i(x, t), M_{12} = M_{21} = 0$

(c) double couple: $-M_{12} = -M_{21} = w_i(x, t), M_{11} = M_{22} = 0$

The TRM field for the horizontal force is shown in Fig. 4.3 (b). An enlarged section of this image is shown in Fig. 4.4 (a). Different initial excitation types result in different source patterns. The radiation patterns for an explosion and a double-couple source are shown in Figs. 4.4 (b) and (c), respectively. For both sources the same source wavelet is used. In Figs. 4.4 (a)-(c) it can also be seen that different types of excitation sources can be distinguished using the resulting TRM field. A catalog of typical TRM fields from generic excitations can be an useful tool to identify the excitations in realistic TRM simulations.
4.2 Inverse simulation with acoustic emission

4.2.1 Forward simulation in the numerical concrete model

In order to create a synthetic but realistic data set, a double-couple source with a source time function as described in Section 3.1.3 is used. The discretization details as described in Section 4.1.2 are adopted. In the snapshot shown in Fig. 4.5 (a) a wave field excited at \([n_x, n_z] = [400, 300]\) \(\text{gp}\) is shown. A double-couple source (Ricker2 wavelet, \(f_{\text{dom}} = 200\text{kHz}\) and \(\Delta t = 1.6 \times 10^{-8}\text{s}\)) is used. Because a heterogeneous NCM (see Section 3.2) is used, more scattering occurs than in the wave field in Fig. 4.3 (a). Two components of the emitted waves (horizontal and vertical displacement) are recorded at 12 sensor positions (marked by green crosses in Fig. 4.5, a).

Fig. 4.5: (a) Forward and inverse simulation in the two-dimensional heterogeneous NCM (real) for two sensor modifications: (b) all (two) components (allcomp) and (c) one component (onecomp) of the calculated displacements [76]. The vertical displacement wave field in (a) and the maximum particle displacement in (b) and (c) are normalized.

4.2.2 Inverse simulation with the NCM velocity model

The time reverse computation is executed for two sensor modifications, recording either two components (displacement normal and parallel to the surface) or one component (displacement normal to the surface). Steiner et al. [92] applied TRM successfully to data measured with three-components seismometers. For most AE applications (i.e. for piezoelectric sensors) one component (displacement normal to the surface) is recorded at the sensor positions. The performance of the inverse simulation using an exact velocity model with a two-component sensor modification is analyzed and discussed. By using both displacement components the time-reversed propagating waves focus on the source coordinates they originated from. An excellent result is obtained (Fig. 4.5, b) in that the source type can be visually identified by means of the characteristic radiation patterns shown in Fig. 4.4 (c). If only the displacement component normal to the concrete surface is used, good focus can also be achieved (Fig. 4.5, c). The radiation
4.2.3 Inverse simulation with effective elastic properties for concrete

Using the effective elastic properties $c_{p,\text{eff}}$ and $c_{s,\text{eff}}$ of the NCM, a reverse computation similar to that in Section 4.2.2 is carried out. The heterogeneous NCM as described in Section 3.2 is used for the forward simulation (see Fig. 4.6, a), but for the time reverse modeling the previously determined EEP are adopted. When both sensor components are used, a very good result is achieved (Fig. 4.6, b). In the backward propagation direction no scattering occurs, and as opposed to the inverse simulation with the exact velocity model displayed in Fig. 4.5 (b), a convergence of elastic energy can be observed. In Fig. 4.6 (b), the double-couple source pattern is identifiable. Using only the displacement component normal to the concrete surface a blurred result is obtained (Fig. 4.6, c). Significant artifacts due to surface waves are visible at the boundaries, and only a rudimentary source pattern can be seen. Possible solutions to improve the localization include the placement of more sensors on the surface, magnification of the energy induced in the model and interpolation of the zero values between sensor positions on the boundary with respect to the incoming waves. An irregular sensor arrangement may be useful to improve the performance of the method and to clarify the influence of the sensors on the wave focus.

4.2.4 Further imaging conditions

The re-emitted wave fields interfere with each other at all times $t \in [0; T]$. In some cases the interferences are difficult to identify, when only imaging of the maximum particle displacement $u_{\text{max}}(x)$, such as that shown in Fig. 4.6 (c), is used. Therefore, additional imaging conditions are
4.2. Inverse simulation with acoustic emission

Rotation-free waves (dilatation)
\[ \nabla \varepsilon(x, T-t) = \lambda \nabla \Phi(x, T-t) + 2 \mu \nabla \Phi(x, T-t) \]

Isochore waves
\[ \nabla \times \Psi(x, T-t) = 0 \]

Evaluation of max. displacement field on \( t \in [0; T] \)
\[ u_{\text{max}}(x) \]

Maximum particle displacement

Maximum energy density (P-wave)
\[ E_p(x) \]

Evaluation of max. energy-flow density on \( t \in [0; T] \)
\[ E_s(x) \]

Maximum energy density (S-wave)

Maximum total energy density
\[ E_{\text{tot}}(x) \]

**Fig. 4.7:** Imaging conditions according to [61] for the far-field term of the displacement field in a homogeneous and unbounded medium [45].

required. In total, four imaging conditions are derived to handle this problem (see Fig. 4.7): the maximum particle displacement \( u_{\text{max}}(x) \), the two maximum densities for the separated energy parts of P- and S-wave \( E_p(x) \) and \( E_s(x) \), respectively, and the maximum total energy density \( E_{\text{tot}}(x) \). Note that the relation \( E_{\text{tot}}(x) = E_p(x) + E_s(x) \) is derived assuming wave propagation in a homogeneous and unbounded medium in the far field.

**Maximum energy density**

If the inverse wave field \( u(x, T-t) \) is separated into rotation-free \( (\nabla \Phi(x, T-t) = 0) \) and isochore parts \( (\nabla \times \Psi(x, T-t) = 0) \), then two further imaging conditions can be derived as shown in [61] and [92]: the maximum energy density of the P-wave

\[ E_p(x) := \max_{t \in [0; T]} (\lambda + 2\mu) [\nabla \cdot u(x, T-t)]^2 \tag{4.9} \]

and the maximum energy density of the S-wave

\[ E_s(x) := \max_{t \in [0; T]} \mu [-(\nabla \times u(x, T-t))]^2 . \tag{4.10} \]

Due to some limitations of these conditions (addressed in [13] and [38]), a more promising imaging condition of maximum total energy density is proposed [76]:

\[ E_{\text{tot}}(x) := \max_{t \in [0; T]} \{\sigma(x, T-t) \cdot \varepsilon(x, T-t)\} . \tag{4.11} \]

The maximum total energy density \( E_{\text{tot}} \) at any point \( x \) in domain \( \Omega \) is the maximum of the scalar product of the stress tensor \( \sigma_{ij} = \sigma(x, T-t) \) and the strain tensor \( \varepsilon_{ij} = \varepsilon(x, T-t) \) for the entire modeling time \( t \in [0; T] \).
Fig. 4.8: Performance of three applied imaging conditions: (a) maximum particle displacement $$u_{max}(x) [m]$$, (b) maximum energy density of P-wave $$E_p(x) [J/m^3]$$ and (c) maximum energy density of S-wave $$E_s(x) [J/m^3]$$ (courtesy of E. H. Saenger [74]). The sensor arrangement is identical to that displayed in Fig. 4.6 (a).

In Figs. 4.8 (a)-(c) and Fig. 4.9 (b), the results of the four applied imaging conditions (4.8), (4.9), (4.10) and (4.11) are presented. Two explosion sources (Ricker2 wavelet, $$f_{dom} = 200\, kHz$$ and $$\Delta t = 1.6 \times 10^{-8} \, s$$) of similar strengths, located at $$[n_x, n_z] = [400, 300] \, \text{gp}$$ and $$[400, 500] \, \text{gp}$$ are applied simultaneously. In Figs. 4.8 (a) and (b) it can be seen that the scalar fields still appear blurred. In Fig 4.8 (c) only a single source location at $$[n_x, n_z] = [400, 300] \, \text{gp}$$ can be identified. Strong artifacts are present due to the high wave amplitudes at all boundaries. The performance of the total maximum energy density imaging condition, displayed in Fig. 4.9 (b), is without doubt the most promising one. Both source locations are readily identifiable as dominant concentrations of energy.

In this example two source locations are successfully localized by using only one set of twelve wave signals. This is the biggest advantage of TRM compared to picking-based localization algorithms, in which the number of sets of the provided wave signals must match that of the sources that need to be localized.

4.2.5 Influence of imprecise effective elastic properties

To study the robustness of the maximum total energy density imaging condition, Saenger et al. [76] vary the effective velocities within TRM. The EEP of the NCM (Tab. 3.4) are used for the forward simulation (see Fig. 4.9, b). Two additional inverse simulations are performed with lower and higher velocity values $$c_p = c_{p,eff} \mp 100 \, \text{m/s}$$ and $$c_s = c_{s,eff} \mp 100 \, \text{m/s}$$, respectively. The dominant energy concentrations are easily identified in both cases, see Figs. 4.9 (a) and (c), but a decrease in radiation intensity of the dominant energy concentrations can be observed for both cases. For sensors arranged on a single surface, a shift in the TRM focus can occur. Therefore, the chosen sensor arrangement ensures that the source locations can be localized accurately.
4.3 Application of TRM to a real NDT example

4.3.1 Feasibility study: purely numerical excitation

In this subsection, a feasibility study of TRM applied to real AE signals is carried out. The employed AE equipment consists of eight piezoelectric sensors capable of recording displacements normal to the surface. The numerical forward simulation involves \( N = 8 \) sensor locations \( x^{(1,...,N)} \), at which the calculated displacements \( u(x^{(1,...,N)}, t) \) normal to \( \partial \Omega \) are recorded (see Fig. 4.2). The excitation signal is a Ricker2 wavelet. The numerical forward simulation is performed with a sampling frequency \( f_s = 10 \text{MHz} \) for a duration of \( t = 409.6 \mu \text{s} \). The Ricker2 wavelet is induced at \( [n_x, n_z, n_y] = [120, 30, 135] \text{gp} \), at 15 grid points distance from sensor #4. The NCM (\( \Delta h = 0.001 \text{m} \)) is used as the numerical model for the simulation. A dominant frequency \( f_{\text{dom}} = 100 \text{kHz} \) and a time increment \( \Delta t = 5.0 \times 10^{-9} \text{s} \) are used. The model with \( 120 \times 118 \times 164 \text{gp} \) (gp \( \equiv \text{mm} \)) is implemented using the rotated staggered-grid FD scheme [75]. The inverse simulation is performed for a duration of \( t = 600 \mu \text{s} \), which allows for sufficient interference of the entire wave fields. After applying the imaging condition from Eq. (4.11) and cutting a slice of the scalar energy field at \( n_y = 135 \text{gp} \), two dominant energy concentrations at the position of sensor #4 (Fig. 4.10, a) can be seen.

In general, the localization with TRM works well for the purely numerical example. The location of the source can easily be identified in Fig. 4.10 (a). On a critical note, the energy radiated by the sensors themselves may interfere with the focus of the source, regardless of which imaging conditions in Eq. (4.8)-(4.11) are applied. For instance, this can happen if the source is located too closely to a sensor. To eliminate the sensor’s energy radiation, in [101] an intermediate step is suggested. The stored displacements \( u(x^{(k)}, t) \) are numerically re-emitted into the system without being reversed in time. It is important to remember that only the components normal to the surface are recorded, and that the correct orientation of the sensors at the sample surface is therefore crucial. The corresponding scalar energy field,
(a) $E_{\text{tot}}(u(x, T - t)) \ [\text{J/m}^3]$  
(b) $E_{\text{tot}}(u(x, t)) \ [\text{J/m}^3]$  
(c) $\hat{E}_{\text{tot}}(x) \ [-]$

![Specimen cross-sections at $n_y = 135\ \text{gp}$ showing maximum total energy densities for (a) $E_{\text{tot}}(u(x, T - t)) \ [\text{J/m}^3]$, (b) $E_{\text{tot}}(u(x, t)) \ [\text{J/m}^3]$ and (c) $\hat{E}_{\text{tot}}(x) = \frac{E_{\text{tot}}(u(x, T - t))}{E_{\text{tot}}(u(x, t))} \ [-]$. The blue circles indicate the numerical Ricker2 excitation sources located at $[n_x, n_z, n_y] = [120, 45, 135] \ \text{gp}$, at 15\ gp distance from sensor #4. The green crosses represent the position of sensor #4 [45].](image)

4.3.2 Real acoustic emission: excitation by pencil-lead break

Having successfully demonstrated the TRM localization in concrete for a numerical excitation source, in this section real AE waveforms from pencil-lead breaks are used instead of numerical excitations. Three physical experiments were carried out (experimental setup see Section 5.2). The coupling of all sensors was tested by breaking a pencil lead, also known as the Hsu-Nielsen source [32], at a distance of approx. 10 mm from the respective sensor. Each break provided a set of transient signals, which is used as input for the inverse simulation. The normal displacements recorded by the piezoelectric sensors at the sample surface were stored, time-reversed and numerically re-emitted into the system. The previously determined EEP of the NCM are used for the numerical model of the inverse simulation. The durations of the recorded AE and of the inverse simulation are the same as those used in the purely numerical example. The adopted numerical setup is identical to that described in Section 4.3.1.
4.3. Application of TRM to a real NDT example

Fig. 4.11: Specimen cross-sections showing the related maximum total energy density $\hat{E}_{\text{tot}}(x)$ [–] at (a) $y = 68\,\text{mm}$, (b) $y = 95\,\text{mm}$ and (c) $y = 71\,\text{mm}$. Dominant energy concentrations due to the three pencil-lead breaks located close to sensors #3, #6 and #7 (coordinates see Tab. 5.4) can be seen. The blue dashed circles indicate the positions of the AE sources. The green crosses represent the sensors close to the pencil-lead breaks [45]. The color bar of $\hat{E}_{\text{tot}}(x)$ in (a)-(c) is identical to that displayed in Fig. 4.10 (c).

The resulting related maximum total energy densities can be seen in Figs. 4.11 (a)-(c). The AE sources (dashed circles) close to the respective sensors are easily identifiable, although some artifacts are still visible. In Fig. 4.11 (a), the dominant energy concentration appears at $z \approx 10\,\text{mm}$ rather than at the surface. The reason for this is the intermediate step in Eq. (4.12). When the scalar field $E_{\text{tot}}(\mathbf{u}(x, T-t))$ is divided by $E_{\text{tot}}(\mathbf{u}(x, t))$, part of the source energy is eliminated along the boundary. A localization error cannot be excluded but is not quantified in this study. These satisfactory results indicate that TRM can be used for localizing AE sources due to concrete cracking of larger specimen e.g. beams, which is the main aim of this research.
Chapter 5

Experimental investigations

5.1 Sensor calibration and signal deconvolution

5.1.1 Problem formulation and Hertz theory

In the calibration experiments, solid spheres of defined masses are dropped from a specific height on to what is assumed to be an elastic half-space. The phenomenon that occurs when the sphere impacts on the solid is frequently discussed in contact mechanics and referred to as the contact problem [37], [28]. Hertz [35] developed a quasi-static solution to solve this problem. In his theory Hertz assumes that the frictionless compression process at the contact area between the solid sphere and the elastic half-space (see Fig. 5.1, a) happens extremely slowly compared to the induced elastic wave, so that the local indentation during the time of contact can be considered as static. The second assumption is that the duration of the impact is long compared to the time the wave needs to travel from the epicenter to the edges of the solid and back. Hence, the equilibrium can be formulated for the instant during the impact when the indentation reaches its maximum (see Fig. 5.1, b).

**Hertzian impact of a solid sphere on an elastic half-space**

Using Newton’s second law of motion [55], a force impulse $F(t)$ is defined as the product of the mass $m$ of a body and its acceleration $\frac{dv}{dt}$

$$F(t) = -m\frac{dv}{dt},$$

(5.1)

where the minus sign denotes compression. Substituting the derivative of the indentation $\delta_z$ for the velocity $v$ in the time domain yields

$$F(t) = -m\frac{d^2\delta_z}{dt^2}.\quad (5.2)$$

According to Hertz theory [37], a force acting during the impact of a sphere with radius $R_1$ on a plane surface can be defined as

$$F = \frac{4}{3} R_1^{1/2} E^* \delta_z^{3/2} = K \delta_z^{3/2},$$

(5.3)
Chapter 5. Experimental investigations

(a) $z = 0$

(b) $m F_{\text{max}} \pm z_{\text{max}}$

Fig. 5.1: (a) Contact between a solid sphere and an elastic half-space at time $t = 0$. (b) Enlarged detail of the maximum indentation of the solid sphere $\delta z_{\text{max}}$ at the point of impact.

where $1/E^* = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$ with Young’s moduli $E_1$, $E_2$ and Poisson’s ratios $\nu_1$, $\nu_2$ for the ball and the surface, respectively. The constant $K$ depends on geometries and elastic constants of the two bodies. Note that the following equations are derived unsigned. Assuming that $v$ acts in the $z$-direction only, the force impulse $F(t)$ can be obtained from Eqs. (5.2) and (5.3) and be written as

$$F(t) = K \delta_z^{3/2}.$$  

(5.4)

Integrating with respect to $\delta_z$ and considering that $\frac{d\delta_z}{dt} = 0$ when the velocity reaches zero, the maximum indentation is obtained as

$$\delta_{z,\text{max}} = \left[ \frac{5 m_1 v_0^2}{4K} \right]^{2/5}.$$  

(5.5)

Note that $v$ at time $t = 0$ is denoted as $v_0$ and referred to as the incoming or the impact velocity of the sphere. Substituting $m_1 = \frac{4}{3} \pi \rho_1 R_1^3$, where $\rho_1$ is the density of the sphere in Eq. (5.5), the total impact or contact duration $T_c$ is calculated as

$$T_c = 2.94 \frac{\delta_{z,\text{max}}}{v_0} = 5.0824 \left( \frac{\rho_1}{E^*} \right)^{2/5} R_1 v_0^{-1/5}.$$  

(5.6)

One solution for $\delta_z(t)$ is the maximum indentation $\delta_{z,\text{max}}$ multiplied by an arbitrary function of time $q$:

$$\delta_z(t) = \delta_{z,\text{max}} q \left( \frac{vt}{\delta_{z,\text{max}}} \right).$$  

(5.7)

Hunter [36] showed that the approximation

$$q \left( \frac{vt}{\delta_{z,\text{max}}} \right) \cong \sin \left( \frac{\pi t}{T_c} \right)$$  

(5.8)

is valid for $0 \leq t \leq T_c = 2.94 \frac{\delta_{z,\text{max}}}{v}$. Therefore, the function of the indentation $\delta_z(t)$ that will be used in the numerical simulations is written as
5.1 Sensor calibration and signal deconvolution

\[ R_1 = \frac{d}{2} \text{[mm]} \]
\[ Re [-] \]
\[ C_D [-] \]
\[ F_D [N] \]
\[ |v_0 - v_{0,D}| \text{[m/s]} \]

| \( R_1 \) | \( d \text{[mm]} \) | \( Re \) | \( C_D \) | \( F_D \) | \( |v_0 - v_{0,D}| \) |
|---|---|---|---|---|---|
| 1.0 | 279 | 0.7 | \( 5.20 \times 10^{-6} \) | \( 6.95 \times 10^{-6} \) |
| 1.5 | 208 | 0.8 | \( 3.34 \times 10^{-6} \) | \( 6.93 \times 10^{-6} \) |
| 2.0 | 139 | 1.0 | \( 1.86 \times 10^{-6} \) | \( 6.90 \times 10^{-6} \) |

Tab. 5.1: List of calculated drag forces \( F_D \) and influence of air drag in the Hertzian impact velocity \( |v_0 - v_{0,D}| \) for balls of different radii \( R_1 \) impacting on concrete. \( Re \) is the Reynolds number and \( C_D \) is the drag coefficient.

\[ \delta_z(t) = \delta_{z,\text{max}} \sin \left( \frac{\pi t}{T_c} \right) \quad \text{for} \quad 0 \leq t \leq T_c, \quad (5.9) \]
and analogously the force impulse is

\[ F(t) = F_{\text{max}} \sin \left( \frac{\pi t}{T_c} \right)^{3/2}, \quad (5.10) \]

with the maximum force

\[ F_{\text{max}} = K\delta_{z,\text{max}}^{3/2} = 3.0295R_1^2E^*2/5\rho_1^{3/5}v_0^{6/5}. \quad (5.11) \]

Assuming free fall of the sphere, the impact velocity becomes

\[ v_0 = \sqrt{2gH}, \quad (5.12) \]

with the gravitational acceleration \( g = 9.81 \text{ m/s}^2 \) and falling height \( H \). In all following examples, a uniform falling height of the steel ball of \( H = 50 \text{ mm} \) is used, resulting in a maximum velocity of \( v_0 \approx 0.9905 \text{ m/s} \).

**Relevance of air drag**

The drag force can be calculated according to the Newtonian friction law

\[ F_D = \frac{1}{2} C_D \rho_F A v_0^2, \quad (5.13) \]

where \( C_D \) is the drag coefficient, \( \rho_F \) is the density of the surrounding fluid and \( A \) is the reference area [10]. The Reynolds number \( Re = \frac{\rho v_0 R_1}{\eta} \) is calculated with \( \rho_F = 1.2041 \text{ kg/m}^3 \) and the characteristic dynamic viscosity \( \eta = 17.1 \times 10^{-6} \text{ Ns/m}^2 \) at 20\(^\circ\)C. Parameter \( v_{0,D} \) represents the impact velocity of the ball including the air drag and can be calculated using Eq. (5.11). The parameters listed in Tab. 5.1 are calculated using experimental \( Re \)-dependent \( C_D \) values adopted from [80]. It can clearly be seen that the forces due to air drag are very small and hence will not be taken into account in further calculations.
Chapter 5. Experimental investigations

Energy losses during impact

Energy losses during impact depend on the elasticity of the slab, the ball material, slab and ball roughnesses as well as heat dissipation [37]. The coefficient of restitution $e$, defined as the ratio of rebound velocity to impact velocity, indicates the degree of plasticity of the collision between two bodies [28]. Values of $e = 1$ and $e = 0$ represent perfectly elastic and perfectly plastic impact, respectively. The coefficient of restitution can also be expressed as $e = \sqrt{2gH'}/\sqrt{2gH} = \sqrt{H'/H}$, where $H'$ is the rebound height and $H$ is the initial dropping height. The ball velocity after impact can be expressed as $v' = ev_0$. Consequently, the maximum rebound force in Eq. (5.11) is redefined as $F_{\text{max}}' = 3.0295R_2^3E_{\text{elastic}}(H')^{3/5}v_0^{6/5}$. Fig. 5.2 (a) illustrates the absolute values of the time-force curves (with and without losses) for a 4 mm steel ball impacting on the two materials of interest, concrete and aluminum. The significant difference between the initial and the rebound force on aluminum indicates strong plastic effects during impact. Nevertheless, for simplicity’s sake in the following numerical examples the initial impacts are assumed to be elastic. More details on inelastic effects can be found in [103] and [16].

5.1.2 Impact experiments on small-scale slabs

Experimental setup

The experimental setup sketched in Fig. 5.3 consists of a planar and smooth concrete slab with dimensions $50 \times 500 \times 500$ mm. The slab is four-point-supported on a flexible material to limit the amount of energy transmitted to the environment. Further experiments carried out on a highly polished aluminum slab were performed with an identical experimental setup. Three steel balls of different diameters ($d_\phi = 4$ mm, 3 mm and 2 mm, respectively), displayed in Fig. 5.4 (a), were trigger-released from an electromagnet at height $H = 50$ mm to impact at the central spot, indicated in Fig. 5.3 by the black cross located at the top surface of the slab. Two PZT sensors
5.1. Sensor calibration and signal deconvolution

Fig. 5.3: The experimental setup consists of a smooth, four-point-supported concrete slab (50 × 500 × 500 mm). Three steel balls with diameters $d_o = 4$ mm, $3$ mm and $2$ mm, respectively, are trigger-released from an electromagnet at height $H = 50$ mm to meet the point of impact indicated by the black cross. On the bottom surface of the slab two PZT sensors are located.

were mounted at the bottom of the slab at distances of 35 mm from the epicentral axis (white crosses in Fig. 5.3). They recorded the wave motion radiated from the epicenter of the impact as voltage signals at time $T = 400 \mu s$. The wave motion was recorded with eight PZT sensors of type KSB250 (Ziegler Instruments).

The typical sensitivity bandwidth of two KSB250 sensors of identical structure and sensor configuration is displayed in Fig. 2.2 (b). No reliable information concerning the inner life of the sensors is available from the manufacturers; the following is a qualitative description only. The KSB250 sensor, which is mounted on a wear-plate, is encased in a protective housing of steel. The possibility of a non-linear sensor response cannot be excluded but in this study a purely linear response is assumed. Transient data was acquired using a digital oscilloscope (WaveSurfer, LeCroy) at a sampling rate of 1.0 GHz and was 3-bit filtered in realtime to minimize the noise. A thin layer of grease was applied to the sensors to ensure unbroken contact.

Steel ball impact on concrete and aluminum

To estimate the coefficient of restitution $e$, photographs with long exposure times, capturing the bouncing curves of a steel ball on a smooth concrete slab and a highly polished aluminum slab (see Fig. 5.4, b, top and bottom), are evaluated. The experiments were carried out in a dark room, and the ball was illuminated by a light ray. The light trace was photographed using a digital camera and a AF-S NIKKOR DX 18-70 mm lens (object lens 1:3.5-4.5, aperture 10, ISO 500) with a shutter speed of 2 s. The slab is slightly inclined to increase the parabolic trajectories of the bouncing ball. Fig. 5.4 (b, top) shows that the first rebound height is approximately 4/5 of the falling height, which leads to the conclusion that there is little plastic deformation at the point of impact.
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Fig. 5.4: (a) Three steel balls of diameters $d_\circ = 4\text{mm}$, $3\text{mm}$ and $2\text{mm}$, respectively. (b) Photographs showing the light trace of the bouncing trajectory of a 4mm steel ball dropped onto a smooth concrete surface (top picture) and a highly polished aluminum surface (bottom picture) from a height of $H = 50\text{mm}$.

A coefficient of restitution of $e \approx 0.88$ is determined for concrete (see Tab. 5.2). Depending on whether the ball hits cement matrix, aggregate or air void (roughness on surface) in concrete, the rebound height may vary. This difference can clearly be seen between the third and fourth parabolic bounce trajectory. The tests were repeated several times, and only the twenty best runs are taken into account. Similar pictures were taken of steel balls bouncing on a highly polished aluminum slab (Fig. 5.4, b, bottom). Aluminum has a lower strength than concrete, and considerably lower rebound heights were observed due to the high plastic losses during impact. This is reflected in the relatively low coefficient of restitution of $e \approx 0.61$ as shown in Tab. 5.3. Plastic indentations in the aluminum were visible to the naked eye. Environmental Scanning Electron Microscopy (ESEM) pictures were also taken of the indentation area of the first impact on concrete (Fig. 5.5, a) and aluminum (Fig. 5.5, b). For further details on ESEM technology see [11]. The indentation displayed in Fig. 5.5 (a) looks like a brittle crater, whereas the contour of indentation displayed in Fig. 5.5 (b) appears very sharp, indicating distinct plastic deformation. These observations call for a more detailed investigation of the plasticity aspects of the impact in the future work.

Generally, the rebound height of the ball on aluminum declines more continuously than that on concrete. Good repeatability of rebound heights and measured amplitudes was achieved for both materials. In fact for the reproducibility of experiments on concrete, the diameter of the steel ball is the decisive parameter. To avoid inaccurate results due to the unevenness of the concrete surface, the steel ball diameter should not be less than 2mm. The impact location on an aluminum slab needs to be changed after every impact, because the plastic indentations on the slab surface can distort the results.
5.1. Sensor calibration and signal deconvolution

5.1.3 Numerical examples

Impact of steel ball on a concrete plate

A common concrete mix (C30/37) with a maximum aggregate size of 16 mm was used to fabricate the 50 × 500 × 500 mm slab. A Young’s modulus of \( E_2 = 3.40 \times 10^4 \text{ N/mm}^2 \) and a Poisson’s ratio of \( \nu_2 = 0.24 \) are assumed. The ball consisted of hardened steel (Material 1.3505, 100Cr) with a Young’s modulus of \( E_1 = 2.04 \times 10^5 \text{ N/mm}^2 \), a density of \( \rho_1 = 7835 \text{ kg/m}^3 \) and a Poisson’s ratio of \( \nu_1 = 0.30 \). From the material parameters the maximum force \( F_{\text{max}} \), maximum indentation \( \delta_{z,\text{max}} \) and contact duration \( T_c \) are calculated according to Hertz theory (see Tab. 5.2). The maximum force is substituted into Eq. (5.10) to obtain the excitation (force impulse) for the numerical simulations of elastic wave propagation and to calculate the displacements at the sensor positions.

Impact of steel ball on an aluminum plate

The aluminum plate with dimensions 50 × 500 × 500 mm was made of commercially available aluminum (AlMg4.5 Mn0.7 / 5083, Material 3.3547) with a Young’s modulus of \( E_2 = 7.0 \times 10^4 \text{ N/mm}^2 \) and a Poisson’s ratio of \( \nu_2 = 0.34 \). The steel ball was identical to that

<table>
<thead>
<tr>
<th>( R_1 = \frac{d^2}{2} ) [mm]</th>
<th>( F_{\text{max}} ) [N]</th>
<th>( \delta_{z,\text{max}} ) [mm]</th>
<th>( T_c ) [( \mu \text{s} )]</th>
<th>( f_{\text{dom}} = \frac{1}{T_c} ) [kHz]</th>
<th>( e ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10.23</td>
<td>3.94×10^{-3}</td>
<td>11.68</td>
<td>85.6</td>
<td>0.878</td>
</tr>
<tr>
<td>1.5</td>
<td>23.01</td>
<td>5.90×10^{-3}</td>
<td>17.52</td>
<td>57.1</td>
<td>0.884</td>
</tr>
<tr>
<td>2.0</td>
<td>40.91</td>
<td>7.87×10^{-3}</td>
<td>23.36</td>
<td>42.8</td>
<td>0.874</td>
</tr>
</tbody>
</table>

**Tab. 5.2:** Maximum force \( F_{\text{max}} \), maximum indentation \( \delta_{z,\text{max}} \), contact duration \( T_c \) and dominant frequency \( f_{\text{dom}} \) for impact of steel balls of varying radii \( R_1 \) on concrete, calculated with Hertz theory.
used on concrete. Again, the resulting parameters, summarized in Tab. 5.3, are used as input for calculation of the displacements at the sensor positions.

### 5.1.4 Instrument response function and sensor calibration

The instrument response function $i(t)$, which is derived assuming linearity between the input and output quantities $v(t) = i[u_n(t)]u_n(t)$, is sometimes referred to as the transfer function. An input quantity, in this case a wave motion $u_n(t)$ at the surface, is transformed into an output quantity by the transfer function. In the present case the output quantity is a voltage signal $v(t)$. The aim is to determine the instrument response by solving a linear convolution problem with respect to $i(t)$, with a known $v(t)$ from the physical experiments and a known $u_n(t)$ from a reference measurement.

#### Derivation of the instrument response function

In [57], an approach involving Green’s function $G_{in}$ is suggested to solve the linear convolution problem. Green’s function $G_{in}(x, t; \xi, \tau)$ represents the response of a linear system, for example the displacement at position $x = [x, y, z]$ at time $t$ due to an unit impulse applied at position $\xi = [x_\xi, y_\xi, z_\xi]$ at time $\tau$ in the direction normal to the surface [5]. The space- and time-dependent displacement $u_i(x, t)$ can be expressed as the convolution product

$$u_i(x, t) = G_{in}(x, t; \xi, \tau) \otimes f_n(\xi, \tau),$$  \hspace{1cm} (5.14)

where $f_n(\xi, \tau)$ is the source function acting at position $\xi$ at time $\tau$. Assuming the motion occurs and the force acts in the normal direction only ($i = n = 3$), Eq. (5.14) can be simplified to a scalar equation

$$u_3(x, t) = g_{33}(x, t; \xi, \tau) \otimes f_3(\xi, \tau),$$  \hspace{1cm} (5.15)

where index 3 refers to the normal direction. In the present work, the equivalent displacement from the numerical simulation of elastic wave propagation, calculated at sensor positions $u_3(t) \equiv u_3(x, t)$, are used instead of calculating the convolution product between the Green’s function $G_{in}$ and the unit impulse $f_n$. The linear relationship can hence be rewritten as

$$v(x, t) = i(x, t) \otimes u_n(x, t),$$  \hspace{1cm} (5.16)
where \( v(x,t) \) is a space- and time-dependent voltage signal, and \( u_n(x,t) \) is the calculated space- and time-dependent displacement at the sensor position. Analogously, Eq. (5.16) can be simplified to

\[
v(t) = i(t) \otimes u_3(t).
\] (5.17)

Having obtained \( v(t) \) from physical experiments and \( u_3(t) \) from numerical simulations, the instrument response \( i(t) \) is found by deconvolution [60], in this case by inversion in the frequency domain

\[
I(\omega) = V(\omega) \cdot U_3(\omega)^{-1},
\] (5.18)

expressed in terms of the angular frequency \( \omega = 2\pi f \), where \( \omega \in \mathbb{C} \). The reference measurement can also be performed by using other devices such as a laser interferometer. Note that the device should be capable of measuring surface particle motion at the nano-scale.

**Calibration procedure**

In the frequency domain, the instrument response \( I(\omega) \) is determined by solving Eq. (5.18) according to the schematic in Fig. 5.6. Due to noise that is introduced by the physical measurements, the voltage signals \( v(t) \) are windowed (Blackmann-Harris). The window of time length \( t_{\text{max},\text{C}} = t_{w,\text{C}} = 250 \mu s \) (concrete) and \( t_{\text{max},\text{A}} = t_{w,\text{A}} = 150 \mu s \) (aluminum) is centered on the arrival time of the first wave motion. The displacements \( u_3(t) \) at \( t_{\text{max}} \) are used without applying any windowing function. The voltage signal \( v(t) \) and the displacement \( u_3(t) \) are transferred from the time domain into the frequency domain by DFT: \( v(t) \rightarrow V(\omega) \) and \( u_3(t) \rightarrow U_3(\omega) \). For deconvolution purposes the voltage signal \( v(t) \) is downsampled to a sampling rate of 10.0 MHz. After calculation of \( V(\omega) \) and \( U_3(\omega)^{-1} \), \( I(\omega) \) is obtained by multiplication. Note that the spectra presented in the results are expressed with respect to the frequency \( f \in \mathbb{R} \).

**Instrument response of the KSB250 sensors**

The KSB250 sensors were primarily used for AE measurements in a controlled laboratory environment. The sensors had previously been used in mid-scale experiments on RC [46], [79] and [44]. Their sensitivity range is suitable for AE measurements at the level of the cracking frequency of concrete (approx. 100-150 kHz). The main objective of the calibration experiments was to determine the instrument response of the employed PZT sensors (KSB250) for deconvolution of recorded signals, which is then used for AE analysis using TRM.

Comparing the results (see Fig. 5.7) obtained from experiments on concrete (blue) and on aluminum (red), it can be seen that the waveforms are distorted. The amplitudes of the recorded waves \( v(t) \), shown in Fig. 5.7 are damped. In the normalized spectra of the Fourier transforms \( |V(f)| \) for the three ball diameters, damping of the amplitudes can be observed for concrete (C, blue). Some high-frequency components become dominant in concrete due to the scattering effect of its air voids, which were introduced during fabrication. The spectra of 3 mm steel balls
Fig. 5.6: Schematic of the calibration procedure for a known voltage signal $v(t)$ and displacement $u_3(t)$.

impacting on concrete respectively aluminum are similar, see Fig. 5.7 (middle row and column). The instrument response shows some discontinuities in the spectrum, also known as spectral leakage, and several dominant frequencies in both materials. Similar behavior can be observed in the spectra obtained during the calibration of the sensors displayed in Figs. C.1 (a)-(f). The sensor with Id. 9.08 (see Fig. C.1, g) differed by responding dominantly to lower frequencies. The sensor with Id. 9.01 malfunctioned during the experiment described in Section 5.3, but for completeness’ sake the instrument response of this sensor is also included in this dissertation (Fig. C.1, h). Note that the following explanations only relate to the remaining sensors.

5.1.5 Signal deconvolution

Deconvolution is the inverse of a convolution operation such as Eq. (5.17). The instrument response $I(\omega)$ (in the frequency domain) of the respective sensor is used as a customized filter to reconstruct the original signal $u_3(t)$ (in the time domain) and to filter out the sensor’s disturbance.

Deconvolution theorem

Rewriting Eq. (5.17) with respect to the displacement normal to the surface yields

$$U_3(\omega) = V(\omega) \cdot I(\omega)^{-1}$$

and in the time domain

$$u_3(t) = v(t) \otimes i(t)^{-1}. \quad (5.20)$$
In the frequency domain the inverse of the instrument response $I(\omega)^{-1}$ is multiplied by the voltage signal $V(\omega)$ to obtain the original signal $U_3(\omega)$. The original signal can be transformed into the time domain by inverse DFT: $U_3(\omega) \rightarrow u_3(t)$. A set of six signals, for example AE recorded by sensors #1 to #6, are deconvolved. Note that each of the sensors exhibits a unique instrument response (see Figs. C.1, a-h). In Fig. 5.8 it can be seen that a phase shift occurs in all plots, although the original signals (blue) can be reconstructed successfully from the recorded signals (red). To eliminate this effect the recorded signals need to be zero-phase filtered, a process which is also known as forward-backward filtering. In zero-phase filtering, a recursive filter\(^1\) is used twice so that the phase shift is mathematically canceled.

\(^1\)“Recursive filters are also called Infinite Impulse Response (IIR) filters, since their impulses are composed of decaying exponentials” [91]. According to [91], no linear-phase recursive filters are available.
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Fig. 5.8: A set of six signals (sensors #1-#6) in the time domain $v(t) \text{ [mV]}$ (red) is deconvolved without applying any filtering criterion to yield a phase-shifted set of signals $u_3(t) \text{ [nm]}$ (blue).

Zero-phase filtering

The input voltage signal $v(t)$ is filtered forward and backward in time by applying a recursive filter twice. Note that in doing so the amplitudes of the response are squared. To derive the zero-phase filter, the artificial variables $\tilde{u}_3(t)$ and $w(t)$ are introduced in the linear convolution problem in Eq. (5.17) as

$$\tilde{u}_3(t) = v(t) \otimes i(t)^{-1}$$  \hspace{1cm} (5.21)

and

$$w(t) = \tilde{u}_3(-t) \otimes i(t)^{-1},$$  \hspace{1cm} (5.22)

where $\tilde{u}_3(-t)$ is the time reversal of $\tilde{u}_3(t)$. In the time domain, the displacement with a zero-phase shift $u_0^3(t)$ is defined as

$$u_0^3(t) = w(-t) = \left[ \tilde{u}_3(-t) \otimes i(t)^{-1} \right] (-t),$$  \hspace{1cm} (5.23)

where the time reversal of Eq. (5.21) into Eq. (5.22), which is time reversed in turn. The deconvolution is performed in the frequency domain and is written as

$$U_3^0(\omega) = \tilde{U}_3(\omega^{-1}) \cdot I(\omega^{-1}).$$  \hspace{1cm} (5.24)

Finally, the deconvolved signal is transferred into the time domain by inverse DFT; $U_3^0(\omega) \rightarrow u_0^3(t)$. The deconvolved set of signals with a zero-phase shift are displayed in Fig. 5.9 for sensors #1 to #6 (blue).
5.2. Experiments on a concrete cuboid

For the physical experiment, the test setup of the classic double punch test according to [7] was adopted. Because the sensors can be mounted more easily on a plane surface, a cuboid was used instead of a cylinder. Originally, the test was developed for determining the concrete tensile strength $f_{ct}$, which can be approximated by the formula $F_u/[\pi(1.2dh - p^2)]$; more details can be found in [7] and [56].

5.2.1 Experimental setup

A concrete cylinder of height $h$ (160mm) and diameter $2d$ (120mm) was compressed by two steel punches of diameter $2p$ (30mm), located concentrically on its top and bottom surfaces (see dotted line in Fig. 5.10, a). The greatest amount of fracture energy was released due to the separation failure. Acoustic emissions were recorded by eight PZT sensors, which were positioned on the specimen’s surfaces at the coordinates shown in Tab. 5.4. The PZT sensors were clamped to the surfaces, and a thin layer of grease was applied to the sensors to ensure ideal coupling. Before measuring the AE, the sensors were relatively calibrated with pencil-lead breaks.

A commercial AE recording system (AMSY5, Vallen Systeme) with transient PZT sensors (KSB250, Ziegler Instruments), sensitive at a bandwidth of 50-250kHz (see also Fig. 2.2, b), was used. The sensors respond dominantly to wave motion normal to the surface. The signals were preamplified (AEP4, Vallen Systeme) with a gain of 40dB. During the entire loading history, complete waveforms were recorded with a sampling frequency $f_s = 10\text{MHz}$ for a duration $T = 409.6\mu s$. The load was applied in displacement-controlled increments of 0.004mm/s until separation failure occurred. The corresponding load-deformation curve can be seen in Fig. 5.11.

Fig. 5.9: The same set of six signals (sensors #1-#6) as shown in Fig. 5.8 (red) is deconvolved applying a zero-phase filter to yield the set of signals without phase shift $u_0^3(t) [\text{nm}]$ (blue).
When the ultimate load $F_u$ (106.4 kN) was reached, the cylinder split radially. Sliding took place at the cone surfaces, whilst separation occurred at the radial crack surfaces between the outer segments (see dashed line in Fig. 5.10, b). The predictable crack pattern that developed is used in this dissertation to check the accuracy of the TRM localization procedure. After the test, the failed cuboid was stabilized and scanned with X-rays to determine its inner crack pattern. Four AE sets (TR$_{75}$, TR$_{82}$, TR$_{84}$ and TR$_{180}$), selected for their good S/N ratios, were used for AE analysis. AE sets TR$_{75}$ to TR$_{84}$ occurred consecutively at loads between 48.3 and 51.2 kN, whereas TR$_{180}$ occurred at a load of 63.3 kN. No correlation between the selected AE sets and the decays in the load could be observed in Fig. 5.11.

### 5.2.2 X-ray computed tomography scan of the specimen after failure

X-ray CT scans were carried out with a Dual-Source CT scanner provided by the Institute of Diagnostic Radiology of the University of Zurich (see also Section 3.3). A sequence of X-ray slice data was obtained and threshold-segmented. During the segmentation process, the crack volume is separated using a lower and upper threshold limit of the mean radio density of air [-1024;745], specified in Hounsfield units (HU). Details of the post-processing procedure for similar specimens can be found in Section 3.3.2. The separating crack, which split the concrete specimen into two parts, can be seen in Fig. 5.10 (c).

<table>
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</tbody>
</table>

**Tab. 5.4:** Coordinates of the sensor positions on the concrete cuboid.
5.2. Experiments on a concrete cuboid

5.2.3 TRM of acoustic emissions due to concrete cracking

Acoustic emissions that were recorded during the double punch test are used as input signals for the inverse simulation. The adopted numerical setup is the same as that for TRM of AE due to pencil-lead breaks (see Section 4.3). The results are presented for the four selected AE sets with comparable input signals: TR,75, TR,82, TR,84 and TR,180 (see Figs. C.2, a-d). Fig. 5.12 shows the related maximum total energy density \( \hat{E}_{\text{tot}}(x) \) resulting from AE set TR,180 for one cross-section located at \( y = 41 \text{mm} \), which is superposed with the threshold-segmented three-dimensional crack volume of the failed specimen. Additionally, the voxels representing \( \hat{E}_{\text{tot}}(x) \) are displayed in the entire cuboid. In a strict sense Eq. (4.12) performs a normalization, hence \( \hat{E}_{\text{tot}}(x) \) is dimensionless. The voxels display the maximum of the multiple of \( \hat{E}_{\text{tot}}(u(x, T - t)) \) on the dimensionless scale \([2.5;3.5]\), where 0 and 3.5 are the minimum and maximum of magnitude on the scale, respectively. The four localized AE occurred during crack formation and not during the final stage of the experiment. Please note that hardly any energy can be transmitted across the separating crack in the specimen. Hence, after development of this crack TRM may no longer be effective.

The cross-sections presented in Figs. 5.13 (a)-(c) exhibit energy concentrations of similar quality. The energy concentrations are displayed on the dimensionless scale \([1.0;3.5]\), where 0 and 3.5 are the minimum and maximum of magnitude on the scale, respectively. It is interesting to note that in Fig. 5.13 (a) two sources can be identified. This is not necessarily a conflict, as it is possible that two AE are released almost simultaneously. Hence, the information of two sources may be contained in one set of recorded displacements. As demonstrated numerically in Section 4.2, two random sources excited at the same time in a three-dimensional, heterogeneous medium can be localized successfully. Note that the results presented in Fig. 5.12 and Figs. 5.13 (a)-(c) are obtained from AE signals without applying deconvolution. The clear energy concentration displayed in Fig. 5.13 (b) is obtained using only four waveforms. In Fig. 5.12 a few...
Fig. 5.12: The related maximum total energy density $\hat{E}_{\text{tot}}(x)$ [-] at $y = 41\text{ mm}$ is superposed with the threshold-segmented crack volume of the test specimen ($x \times z \times y = 120 \times 118 \times 160\text{ mm}$) after failure. The dominant energy concentrations at the intersection of cross-section and threshold-segmented crack indicate the TRM-localized AE due to concrete cracking. The color bars show the energy concentration in the cuboid (left-hand side, checkerboard pattern) and at the shown cross-section (right-hand side), respectively.

less dominant energy concentrations (voxels) are visible. These represent either simultaneously released AE or artifacts. Therefore, all AE waveforms recorded during the four-point bending test are deconvolved to improve TRM performance (see Section 5.3).

5.3 Four-point bending test

5.3.1 Test specimen and experimental setup

A four-point bending test was carried out on a slender RC beam of size $b \times h \times l = 120 \times 200 \times 1700\text{ mm}$ (see Fig. 5.14). The beam contained two tensile reinforcing bars (diameter 8 mm). The bars were bent up on the left-hand side of the beam as displayed schematically in Fig. 5.14. The beam with a span of $l_k = 1500\text{ mm}$ was supported on two steel plates ($100 \times 120 \times 20\text{ mm}$). A PTFE (Polytetrafluorethylene) layer of 5 mm thickness was placed above the right-hand side steel plate to allow horizontal movement of the beam along the longitudinal beam axis (see Fig. 5.15). Below the steel plates load cells were located to measure the applied load. The beam was loaded to failure using two concentrated vertical loads $F$ with shear arms $a = 450\text{ mm}$. The load was induced by two low-height cylinders, which were resting on a stiff girder and were located below the loading cells. The load was regulated by a hand pump. An auxiliary construction was connected to the stiff girder by tension rods to ensure correct load application at the supports. The two point loads were introduced via steel roller bearings.
5.3. Four-point bending test

Fig. 5.13: Specimen cross-sections illustrating the related maximum total energy density $\hat{E}_{\text{tot}}(x)\,[-]$ for acoustic emission sets (a) TR\_75 at $y = 43$ mm, (b) TR\_82 at $y = 110$ mm and (c) TR\_84 at $y = 25$ mm. The scalar energy field is superposed with the threshold-segmented crack of the test specimen after failure. The location of the sources in (a)-(c), visible as energy concentrations, agree well with the crack location. The color bar of $\hat{E}_{\text{tot}}(x)\,[-]$ is identical to that displayed in Fig. 5.12 (right-hand side).

The load was increased manually to the following five load levels: 13.4 kN (load level 1, LS1), 17.4 kN (load level 2, LS2), 22.1 kN (load level 3, LS3), 27.0 kN (load level 4, LS4) and 30.0 kN (load level 5, LS5). After reaching each of the load levels the beam was completely unloaded. Characteristic AE parameters (see Tab. 2.1) as well as the complete waveforms ($T = 409.6$ $\mu$s) were detected by eight PZT sensors (KSB250, Ziegler Instruments), pre-amplified with the AEP4 pre-amplifier (Vallen Systeme) and recorded using AMSY5 equipment. The PZT sensors were clamped to the surfaces, and a thin layer of grease was applied to the sensors to ensure unbroken contact. Before measurement of the AE, the sensors were relatively calibrated with pencil-lead breaks. The sensors were arranged at irregular distances at the faces and on top of the beam (see Tab. 5.5).

### Tab. 5.5: Coordinates of the sensor positions on the RC beam in its original (untested) state and corresponding sensor identifications.

<table>
<thead>
<tr>
<th>Sensor Id.</th>
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<th>#3</th>
<th>#4</th>
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</table>
Fig. 5.14: Elevation and cross-section of the RC beam \((b \times h \times l = 120 \times 200 \times 1700\,\text{mm})\) with a span \(l_k = 1500\,\text{mm}\). The shear arms are \(a = 450\,\text{mm}\).

The prediction and the observation is mainly due to the fact that only one reinforcing bar was tested to obtain its effective yield strength \(f_y\) (519.3 N/mm²). Therefore, no statistical evaluation could be performed to obtain the variance in material properties. Because the deformed beam was to be scanned using X-rays, the experiment was stopped at load level LS5 (30.0 kN) to avoid structural collapse. Generally, the beam appeared very ductile, with failure initiating in the compression zone above crack 13 (see Figs. 5.18, a-e).

At load level 1 (see Fig. 5.18, a), cracks 1, 2 and 3 started to form at midspan. At load level 2 (see Fig. 5.18, b), the cracks had increased in width and length, thereby reducing the extent of the compressive zone. New flexural cracks (cracks 4-9) appeared visually as a comb-like structure, with the compressive zone acting as the backbone of the comb and the tensile zone as its concrete “teeth” [40]. A more complex flexural-cracking pattern could be observed after cracks 10 and 11 had formed at load level 3 (see Fig. 5.18, c). Approaching load level 4 (see Fig. 5.18, d), minor flexural cracks formed beside crack 12. A diagonal crack (crack 13) formed at the left load application point. At that time a large deflection at midspan could be observed by the naked eye. Increasing the load to 30.0 kN (load level 5) caused no further cracks (see Fig. 5.18, e). Existing cracks 1 to 13 increased in width and length, and branching cracks formed. After complete unloading the irreversibly deformed beam was scanned with X-rays.

5.3.3 Parameter-based acoustic emission analysis

Counting the accumulated AE counts \(C_t\) and hits \(H_t\) is a simple way to assess in realtime the damage of a structure during an experiment. It is also helpful to establish whether termination of an experiment is necessary, and it can be used for analytical purposes after completion of the test. In Fig. 5.16 the AE counts for the four-point bending experiment are displayed with respect to global time \(t_{\text{global}}\) (red). Further, the load-time curve for the entire load history is shown (blue). The counts occur as dominant peaks (primary AE activity), which represent nucleation of new cracks, and as considerably smaller peaks, which are concentrated around the dominant peaks and indicate the closing of cracks associated with internal friction (secondary AE activity).
5.3. Four-point bending test

The number of dominant $C_t$-peaks generally matches that of the nucleated new cracks in Figs. 5.18 (a)-(e) at each load level. The notable exception occurs at load level 1, where two cracks formed simultaneously, indicated by the presence of two dominant peaks (Fig. 5.16). Between load levels 1 and 2, the maximum number of counts but only a small number of hits was detected, which is in agreement with the nucleation of six new flexural cracks (see Fig. 5.18, b). Between load levels 4 and 6 secondary AE activity was dominant. In Fig. 5.17 (a) it can be seen that sensor #8 (Id. 9.01) recorded hardly any results over the duration of the measurement, which indicates sensor malfunction. The greatest number of hits was recorded at load level 5, with a comparatively low amount of counts – the opposite of the behavior observed at load level 2. This is typical for opening of existing cracks and for internal friction of the crack edges.

5.3.4 Localization with time reverse modeling

A number of sets of AE signals (denoted with TR$_\#$), selected because of their good S/N ratios, are deconvolved as described in Section 5.1.4. The deconvolved and normalized AE sets are used as input signals for the inverse simulation. The waveforms are of length $T = 395 \mu s$. Considering the dimensions of the beam ($120 \times 200 \times 1700$ mm) and a P-wave velocity of $c_{p,\text{eff}} = 3987$ m/s (NCM, see Tab. 3.4), it takes one waveform $431 \mu s$ to travel across the beam. Adding the length of the waveform to the travel time yields $826 \mu s$ of minimum simulation time. The inverse simulation is performed for a duration of $T = 1000 \mu s$, which allows for sufficient interference of the entire wave fields. Further details of the numerical setup are adopted from Section 4.3.

The entire analysis is performed for each of the load levels. A load-time curve (blue) with the corresponding AE counts (red) is displayed together with the AE sets (TR$_\#$) used for TRM localization (see for example Fig. 5.19). The load $F$ and global time $t_{\text{global}}$ of the selected AE sets are documented with respect to their load levels (see Tabs. C.1-C.5). Note that the normalized recorded (red) and deconvolved (blue) waveforms of the AE sets (marked in bold) are displayed in Figs. C.5-C.9. The TRM results as well as the voxels representing the related maximum total energy density $\hat{E}_{\text{tot}}(x)$ are visualized in a three-dimensional view (see for example Fig. 5.24, a).
Fig. 5.16: Load-time curve of the four-point bending test (blue) and corresponding accumulated AE counts \( C_t \) [–] (red) for the eight sensors.

In the background a CT image in the \( xy \)-plane at \( z = 118 \) mm is displayed. The CT image of the beam was taken after failure of the structure so that the open concrete cracks became visible in images taken by the medical X-ray CT scanner. In Section 5.2.3 it was demonstrated that the three-dimensional crack distribution is well suited to indicate where the AE respectively the related maximum total energy density \( \hat{E}_{\text{tot}}(x) \) should be located and what results may be excluded as artifacts. For illustration purposes, threshold-segmented reinforcing bars are also shown inside the specimen. The black bounding box shows the extent of the X-ray slice data of the deformed specimen. Note that due to the irreversible deformation of the beam after failure the X-ray slice data is of size \( z \times y \times x = 140 \times 247 \times 1745 \) mm (\( \text{mm} \equiv \text{gp} \)) and no longer identical with the original X-ray data. In its original state the numerical model of the TRM simulations is of size \( n_z \times n_y \times n_x = 124 \times 204 \times 1704 \) gp, including a vacuum layer of 2 gp that envelopes the entire numerical model. Note that all inverse simulations were performed with a numerical model representing the undamaged beam.

For visualization of both models, the X-ray CT model (deformed, \( z \times y \times x \)) and the voxels of \( \hat{E}_{\text{tot}}(x) \) (original state, \( n_z \times n_y \times n_x \)) are superposed at the coordinates \( [z, y, x] = [15, 13, 34] \) mm and \( [n_z, n_y, n_x] = [0, 0, 0] \) gp, respectively. The supports and the load application point are indicated by green and red boxes, respectively. The blue dots represent the sensor positions (coordinates see Tab. 5.5). To compare the locations of the voxels of \( \hat{E}_{\text{tot}}(x) \) with those of the cracks, the crack volume of selected cracks was threshold-segmented (see also Section 3.3.2). The threshold-segmented crack volumes of selected cracks superimposed with \( \hat{E}_{\text{tot}}(x) \) are displayed in enlarged perspective views (see for example Figs. 5.24, b-e). Note that the related maximum total energy density \( \hat{E}_{\text{tot}}(x) \) is referred to as energy concentration or energy pattern in the following descriptions. The scale of the color bar of \( \hat{E}_{\text{tot}}(x) \) is identical to that one displayed in Fig. 5.12 (left-hand side, checkerboard pattern).
5.3. Four-point bending test

Fig. 5.17: (a) AE hits \( H_t [\%] \) detected by sensors #1 to #8. (b) Percentage of AE hits detected at load levels 1 to 5.

**Load level 1**

At load level 1, three cracks formed (see Fig. 5.18, a). The AE were released at about the same time as \( F_{cr} \) (7.6 kN) was reached (Kaiser effect). It appears that two cracks formed simultaneously, indicated by the fact that only two dominant peaks of accumulated counts were observed up to LS1 (see Fig. 5.16, red). The AE sets marked in Fig. 5.19 (TR\(_s\)#) are located just before the dominant peaks. In Fig. 5.24 (a) and Fig. C.3 (a), a perspective view and side view of the CT image (at \( z = 118 \) mm) at the end of the experiment are superimposed with voxels of \( \hat{E}_{tot}(x) \) of the selected AE sets. Note that the inverse simulations \( (E_{tot}(u(x, T - t)), E_{tot}(u(x, t))) \) and \( \hat{E}_{tot}(x) \) are performed for every single AE set separately and are subsequently superimposed in the visualization process. In Fig. C.3 (a), some isolated energy concentrations at midspan as well as the accumulation of energy along the left compression diagonal can be seen. The energy concentrations near the right-hand side load application point and support are located at the bottom of the beam, where AE are also released due to friction at the support. The accumulation of energy around crack 13 (see Fig. 5.24, b), caused by AE sets TR\(_2\), TR\(_{12}\), TR\(_{26}\) and TR\(_{50}\), which became visible only at LS4 is primarily related to fracture. In some cases, the locations of energy concentrations are found outside the crack volume. This inaccuracy can be explained by the relative movement of the sensor positions during the deformation process and the deformation of the left-hand side of the beam. This effect is more pronounced at higher load levels, where deformation increases significantly. In Fig. 5.24 (c) a few less dominant energy concentrations can be observed, which match crack 1. TR\(_{49}\) is less dominant but is located exactly on the crack surface, whereas TR\(_{12}\) is located in the compression zone. A dominant energy pattern, which is caused by friction at the load application point, is located close to the lateral surface of the beam. In Fig. 5.24 (d), a significant energy concentration at the bottom
Fig. 5.18: (a)-(e) Crack patterns in beam at load levels 1 to 5. (a)-(d) At load levels 1 to 4 the new cracks are numbered consecutively (circled numbers). (e) At load level 5 the numbers of all cracks of the experiment are shown.
5.3. Four-point bending test

![Load-time curve of load level 1 of the four-point bending test](image)

**Fig. 5.19:** Load-time curve of load level 1 of the four-point bending test (blue) with selected AE sets (TR. #) marked and the corresponding AE counts (red) normalized with respect to global time $t_{global}$ [s]. The normalized recorded (red) and deconvolved (blue) waveforms of AE sets marked in bold (TR.2, TR.12, TR.26, TR.47 and TR.49) are displayed in Figs. C.5 (a)-(e).

![Load-time curve of load level 2 of the four-point bending test](image)

**Load level 2**

At load level 2, the largest number of new cracks formed (see Fig. 5.18, b), which is reflected by a low amount of hits (see Figs. 5.17, a and b) and the highest number of counts (see Fig. 5.20) recorded at any load level. The six dominant $C_t$-peaks (marked in red in Fig. 5.20) exactly match the number of new cracks that formed at LS2. In Fig. 5.20, each dominant peak is followed by a decrease in load (blue). In Fig. 5.25 (a) and Fig. C.3 (b), it can be seen that the isolated energy concentrations at midspan that occurred during LS1 (see Fig. 5.24, a) are also present in LS2. It can further be seen that the accumulation of $E_{tot}(x)$ along the compression diagonal on the left-hand side of the load application point keeps increasing. Around crack 13 (see Fig. 5.25, b) several dominant energy concentrations can be observed. One strong energy pattern can be identified (TR.221). Note that in the enlarged detail in Fig. 5.25 (b), crack 4 is present, but unfortunately it could not be threshold-segmented from the X-ray slice data.

Some energy concentrations located in the fifth and eighth octant\(^2\) are most likely related to

\[^2\]An octant is one of the eight divisions of a three-dimensional coordinate system. It is similar to the two-dimensional quadrant. Note that the origin of the coordinate system is defined in the center of each enlarged detail.
Chapter 5. Experimental investigations

Fig. 5.20: Load-time curve of load level 2 of the four-point bending test (blue) with selected AE sets (TR_#) marked and the corresponding AE counts (red) normalized with respect to global time $t_{\text{global}}$ [s]. The normalized recorded (red) and deconvolved (blue) waveforms of AE sets marked in bold (TR_162, TR_210, TR_221 and TR_228) are displayed in Figs. C.6 (a)-(d).

the formation of crack 4. In Fig. 5.25 (c) very dominant concentrations of $E_{\text{tot}}(x)$ (TR_126, TR_161, TR_210 and TR_228) can be observed, which are associated with the formation of crack 1. A few artifacts can also be observed in the second and third octant. One clear focus of energy can be observed in Fig. 5.25 (d), where AE set TR_210 is located close to the crack surface. A dominant energy pattern on the front face of the beam, where branching of crack 11 occurs, can be seen in the detail shown in Fig. 5.25 (e). The corresponding AE set (TR_221) was released at a lower load level than that at which crack 11 initially formed. Similarly to AE sets TR_2 and TR_12 at LS1, AE set TR_210 appears to contribute simultaneously to the imaging of $E_{\text{tot}}(x)$ of cracks 1, 5 and 13 at load level 2.

Load level 3

At load level 2 a considerable number of cracks is already present in the beam (see Fig. 5.18, b). In Fig. 5.18 (c) two new cracks, crack 10 and 11, are visible. One of the cracks (it is not possible to positively correlate the cracks to specific peaks) formed at a high level of accumulated counts, and the second crack formed at approximately 30% of magnitude level of the dominant $C_l$-peak (see Fig. 5.21). Crack 10 propagated in the near field of sensor #7. The energy concentrations are located mainly at the left-hand side of the beam (see Fig. 5.26, a and Fig. C.3, c). The accumulated energy concentrations are located around crack 13 (see Fig. 5.26, b), but compared to the previous load level the discrepancy between the voxels of $E_{\text{tot}}(x)$ (TR_279, TR_287 and TR_305) and the crack surface increases. It seems that some energy concentrations (TR_297 and TR_305) approximately match the area where crack 4 is located (in the fifth and eighth octant). Some less dominant concentrations can be observed around the surface of crack 1 (see Fig. 5.26, c). TR_305 and TR_293 are located exactly on the crack surface. TR_293 is hidden...
Fig. 5.21: Load-time curve of load level 3 of the four-point bending test (blue) with selected AE sets (TR.#) marked and the corresponding AE counts (red) normalized with respect to global time $t_{\text{local}}$ [s]. The normalized recorded (red) and deconvolved (blue) waveforms of AE sets marked in bold (TR_279, TR_286, TR_293, TR_305 and TR_360) are displayed in Figs. C.7 (a)-(e).

behind crack 1 in the figure and is related to swaging of the compression zone. Two dominant energy concentrations (TR_286 and TR_360) can be identified around crack 10 (see Fig. 5.26, d). The concentration of energy at the bottom of the enlarged detail (TR_286) can be related to the separation of crack 10, similar to TR_2 that can be related to the formation of crack 10 (TR_2) in Fig. 5.24 (d). The radiation pattern of TR_286 (at LS3) is similar to the pattern of TR_2 (at LS1). The locations of the energy concentrations presented in Fig. 5.26 (e) are inconclusive and are more related to microcracking than to the formation of crack 7, with the exception of TR_360, which can be related to fracture. Again, AE sets TR_279, TR_287 and TR_305 each contributed to several energy concentrations simultaneously.

Load level 4

In Fig. 5.22, the two dominant $C_t$-peaks (red) accurately match the observed number of cracks visible in Fig. 5.18 (d). Both peaks indicate the formation of new cracks 12 and 13. The first peak is related to diagonal crack 13 and associated with strong secondary AE activity after crack formation. Formation of crack 12 was accompanied by a very low number of counts. Simultaneously, existing cracks 1 to 10 reopened and increased in width and length. Most of the selected AE sets, clearly marked in Figs. 5.27 (b)-(e), are located before the first dominant $C_t$-peak on the load-time curve (crack 13) and at loads below the predicted ultimate load $F_u$ (7.6 kN). Similarly to LS1 to LS3, isolated energy concentrations still occur at midspan, and strong accumulations of energy at the left-hand side of the beam can be observed (see Fig. 5.27, a). Four out of five dominant energy concentrations (TR_388, TR_390 and TR_420) are located outside the crack (see Fig. 5.27, b). In the superposition of $\hat{E}_{\text{tot}}$ (LS4) and the principal stress trajectories calculated at $F_{cr}$ (7.6 kN), displayed in side view (see Fig. C.4, d), it can be seen that
these concentrations are located along the hyperbolic trajectories in the compression zone (red) on the left-hand side of the beam. No associated tensile components (blue) can be observed, and it is interesting to observe that such strong patterns can occur due to compression only. The remaining AE set (TR_374) generates two energy concentrations at the same time, one of which matches the crack surface. In contrast to previous load levels almost all energy concentrations at this load level exhibit remarkable volumes. This can be explained by the formation of diagonal crack 13, which was accompanied by a significant release of friction energy. In Fig. 5.27 (c), two dominant energy concentrations can be identified. TR_374 is located in the compression zone and TR_390 at the tip of crack 10. The isolated energy concentration (TR_390) displayed in the detail of crack 5 (see Fig. 5.27, d) can be seen clearly without any noticeable artifacts. One dominant energy concentration (TR_423) can be observed on the face of the bounding box of Fig. 5.27 (e), indicating formation of crack 7. A less dominant concentration (TR_388) is located in the compression zone. AE sets TR_374, TR_388 and TR_390 can be seen in several details.

Load level 5

At the final load level (LS5) no new cracks except a few branching cracks formed (Fig. 5.18, e). This fact is reflected qualitatively by the low number of accumulated counts (see Fig. 5.23, red) but the highest number of accumulated hits (see Figs. 5.17, a and b) of all load levels. On the load-time curve, one collection of AE sets is located close to the load plateau at the maximum load. The other collection is found after the second decrease of load (see Fig. 5.23, blue). Although the beam exhibited a significant amount of cracks at this stage, some satisfactory results could be obtained. AE sets TR_467, TR_513, TR_527 and TR_534 are concentrated around crack 13 (see Fig. 5.28, b). TR_537 can neither be related to the fracture process nor
5.3. Four-point bending test

Fig. 5.23: Load-time curve of load level 5 of the four-point bending test (blue) with selected AE sets (TR.#) marked and the corresponding AE counts (red) normalized with respect to global time $t_{\text{global}}$ [s]. The normalized recorded (red) and deconvolved (blue) waveforms of AE sets marked in bold (TR.467, TR.484, TR.513, TR.527, TR.534, TR.559 and TR.562) are displayed in Figs. C.9 (a)-(g).

to friction. Interestingly, TR.527 appears a second time in the second and third octant and is most likely an artifact. In Fig. 5.28 (c), two energy concentrations (TR.484 and TR.523) can be related to fracture. Two unidentifiable artifacts can be observed in the edges of the detail. In the detail showing crack 10 (see Fig. 5.28, d) one dominant (TR.464) and one less dominant energy concentration (TR.562) can be seen. Some dominant energy concentrations, which are not located on principal stress trajectories, can be observed in Fig. C.4 (e). In Fig. 5.28 (e), a dominant energy concentration can be observed close to crack 6 (branching of crack 2).
(a) $\hat{E}_{\text{tot}}$ (LS1)

CT image ($z = 118$ mm) at end of experiment
Bounding box (X-Ray slice data)
Load application point
Support

Threshold-segmented reinforcing bar
Sensor position

(b) Crack 13
(c) Crack 1, rear side

(d) Crack 10
(e) Crack 7

Fig. 5.24: (a) $\hat{E}_{\text{tot}}(x)$ [–] (AE sets of LS1) are superimposed with a CT image (LS5) in the $xy$-plane and the threshold-segmented reinforcing bars. Enlarged details of crack surfaces superimposed with $\hat{E}_{\text{tot}}(x)$ [–] of AE sets (b) TR$_2$, 12, 26 and 50, (c) TR$_{12}$, 47, 49 and 50, (d) TR$_2$, 12 and 26, and (e) TR$_{32}$, 47, 49 and 50.
5.3. Four-point bending test

(a) $\hat{E}_{\text{tot}}$ (LS2)

(b) Crack 13

(c) Crack 1

(d) Crack 5

(e) Crack 11

Fig. 5.25: (a) $\hat{E}_{\text{tot}}(x)$ [-] (AE sets of LS2) are superimposed with a CT image (LS5) in the $xy$-plane and the threshold-segmented reinforcing bars. Enlarged details of crack surfaces superimposed with $\hat{E}_{\text{tot}}(x)$ [-] of AE sets (b) TR$_{95}$, 117, 119, 126, 162, 173, 210, 217, 221 and 228, (c) TR$_{126}$, 161, 210 and 228, (d) TR$_{210}$ and 214, and (e) TR$_{162}$, 173 and 221.
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(a) $\hat{E}_{\text{tot}}$ (LS3)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.26a.png}
\caption{(a) $\hat{E}_{\text{tot}} (x)$ \text{[-]} (AE sets of LS3) are superimposed with a CT image (LS5) in the \textit{xy}-plane and the threshold-segmented reinforcing bars. Enlarged details of crack surfaces superimposed with $\hat{E}_{\text{tot}} (x)$ \text{[-]} of AE sets (b) TR\_279, 287, 293 and 305, (c) TR\_293 and 305, and (d) TR\_279, 286 and 360, (e) TR\_279, 286, 287 and 360.}
\end{figure}
5.3. Four-point bending test

(a) $\hat{\varepsilon}_{\text{tot}}$ (LS4)

(b) Crack 13

(c) Crack 10

(d) Crack 5, rear side

(e) Crack 7

Fig. 5.27: (a) $\hat{\varepsilon}_{\text{tot}} (x) [-]$ (AE sets of LS4) are superimposed with a CT image (LS5) in the $xy$-plane and the threshold-segmented reinforcing bars. Enlarged details of crack surfaces superimposed with $\hat{\varepsilon}_{\text{tot}} (x) [-]$ of AE sets (b) TR, 374, 388, 390 and 420, (c) TR, 374 and 390, and (d) TR, 390, (e) TR, 388, 411, 422 and 423.
(a) $\hat{E}_{\text{tot}}$ (LS5)

Fig. 5.28: (a) $\hat{E}_{\text{tot}}(x)$ [–] (AE sets of LS5) are superimposed with a CT image (LS5) in the $xy$-plane and the threshold-segmented reinforcing bars. Enlarged details of crack surfaces superimposed with $\hat{E}_{\text{tot}}(x)$ [–] of AE sets (b) TR$_{467}$, 513, 523, 527, 534 and 537, (c) TR$_{484}$ and 523, and (d) TR$_{464}$ and 562, (e) TR$_{559}$. 
5.3.5 Localization with an AIC-based linearized algorithm

Having demonstrated that TRM works for AE signals (see Section 5.3.4), a comparison is performed with a localization method successfully applied in previous works at ETH Zurich. The AIC-based linearized localization algorithm involves an automatic onset-time detection and is based on the arrival time of the first wave motion (P-wave) and a homogeneous velocity model [79]. The main focus is on determining the correct onset time for AE waveforms that occur at different S/N ratios. The picking procedure relies on the AIC values [49], which are determined by applying a Hilbert envelope to single waveforms in order to determine their partial energy content. In particular the Hilbert envelope also defines a time frame, wherein the onset point can be found by searching the extreme values of the AIC function with respect to time. Once the onset times of a set of AE waveforms are determined, the locations of the respective sources can be calculated iteratively. More details on the implementation of the AIC-based linearized localization algorithm can be found in [78] and [49].

For the localization presented in Figs. 5.29 (a)-(e), all recorded AE sets are used (TR_1-TR_573) without any pre-selection and without applying deconvolution. The majority of the onset times was picked manually. An accuracy criterion is set by limiting the maximum length of the semi-major axis of the error ellipsoids to 100 mm (or half the beam height, \( h = 200 \) mm). The threshold of the AIC-picker is set to floated so it can adapt to the different magnitudes of the Hilbert envelope. For optimal convergence of the localization method, the P-wave velocity is varied in the range \([3300;3550]\) m/s. The results are displayed in Figs. 5.29 (a)-(e). In the background of these figures, cracks corresponding to the cracking patterns in Figs. 5.18 (a)-(e) for each of the load levels are shown. The best results of each load level are assembled in Figs. 5.29 (a)-(e). The numbering system for the cracks is the same as that in Figs. 5.18 (a)-(e) and is used to highlight the new cracks at each load level. The only exception is LS5, where no new cracks formed (Fig. 5.29, e). The results are presented in side view with sensor positions (blue crosses), supports (green triangles) and load application points (red triangles) clearly marked. The error ellipsoids (gray) are projected either onto the front face of the beam, with the AE sources marked by black dots, or they are displayed in the beam interior, with the AE sources marked by gray dots.

At load level 1, some successfully localized AE events can be observed in Fig. 5.29 (a). The P-wave velocity was \( c_p = 3550 \) m/s. Six out of ten error ellipsoids of the AE events exhibit small semi-axes, which indicates that the localization accuracy is high. At this load level the specimen is still uncracked so that reasonable results can be expected. One AE event located on the left-hand side (at \( x \approx 600 \) mm) and three AE events on the right-hand side (at \( x \approx 1100 \) mm) of the beam correspond to crack 5 and 8, respectively. Interesting to note is that those cracks become visible only at load level 2. The associated AE sets may have been released due to micro-cracking and initiation of cracks 5 and 8. The large majority of cracks formed at load level 2, which is shown by the multitude of dominant peaks of the accumulated AE counts displayed in Fig. 5.16 (red). For \( c_p = 3360 \) m/s many AE events are localized (Fig. 5.29, b).
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(a) LS1, \( c_p = 3550 \text{ m/s} \) (73 AE sets)

(b) LS2, \( c_p = 3360 \text{ m/s} \) (200 AE sets)

(c) LS3, \( c_p = 3300 \text{ m/s} \) (100 AE sets)

(d) LS4, \( c_p = 3300 \text{ m/s} \) (100 AE sets)

(e) LS5, \( c_p = 3300 \text{ m/s} \) (100 AE sets)

Fig. 5.29: AIC-based linearized localization results for load levels 1 to 5 at selected velocities. The new cracks that formed at each load level are numbered according to Figs. 5.18 (a)-(e). The sensor positions are marked with blue crosses, the supports with green triangles and the load application points with red triangles. The gray ellipses are projections of the error ellipsoids either projected on the front with AE source (black dot) face, or in the beam interior with AE source (gray dot).

For load level 2, the accuracy of the localization decreases (larger error ellipsoids) due to the presence of distinct flexural cracking pattern. For load level 3 (Fig. 5.29, c) to load level 5 (Fig. 5.29, e) the P-wave velocity was \( c_p = 3300 \text{ m/s} \). The convergence of the method decreases
due to the progressed cracking pattern. This is emphasized by the lower getting amount of AE events on one hand and the higher volume of the error ellipsoids on the other. The localization method converges for approximately 10-15% of AE sets. This is a satisfactory result, although a better performance could be achieved with the AIC-based linearized localization method by Schechinger [78]. The main reason for the suboptimal performance might be corruption of the waveforms during measurement with the acquisition system.
Chapter 6

Discussion, conclusions and outlook

6.1 Discussion

In this section the performance of the NCM presented in Section 3.2 is described, the main findings of the experimental investigations from Chapter 5 are discussed, and a critical assessment of the core topics addressed in this dissertation is given.

The three-dimensional NCM (see Section 3.2) could be established as a representative numerical model for the numerical simulations of elastic wave propagation in concrete. In order to develop a realistic concrete model the sieving-range distributions were modeled after the portions of an actually fabricated concrete mixture. Simplification of the shapes of the concrete constituents as ellipsoids (grains) and spheres (air voids) did not noticeably influence the wave propagation behavior. The NCM was verified numerically by comparing the EEP ($c_{p,eff}$, $c_{s,eff}$ and $\rho_{eff}$) of the NCM to those of the threshold-segmented CT model of a real concrete specimen. In physical experiments, only the P-wave velocity could be measured and compared for concrete specimens. The NCM provides the basis for all numerical considerations concerning structural concrete.

Hertz theory, used for the calibration of the employed PZT sensors (KSB250, Ziegler Instruments), is shown to perform well for the deconvolution of the recorded AE signals (see Section 5.1). The combination of Hertz theory and physical steel ball impact works well for obtaining a reproducible impulse. Wave motion due to Hertzian impact and steel ball impact is calculated numerically (as displacement) and was also recorded physically by the PZT sensors (as voltage signal) during experiments, respectively. Comparing both quantities yields the instrument response for deconvolution of the recorded AE signals. However, as illustrated in ESEM pictures, the first impact on concrete (see Fig. 5.5, a) and on aluminum (see Fig. 5.5, b) is not elastic. In steel ball experiments, distinct indentations could be observed on both materials; on aluminum they were even visible by the naked eye. The indentations influence the form and frequency of the impulse and cannot be neglected in the calibration approach. Furthermore, non-linear behavior of the KSB250 sensor cannot be excluded. However, the deconvolution based on the Hertzian (elastic) impact was applied to all of the recorded AE waveforms. As can be seen in Figs. C.5-C.9, the disturbing frequencies can be filtered out in most of the cases.
Without being deconvolved, the waveforms displayed in Fig. C.5 (b, sensor #6) and Fig. C.5 (d, sensor #8) cannot be used for localization at all. The instrument responses of the sensors employed in the experiments differed (see Figs. C.1, a-h), and some of the sensors were at the end of their life spans. The pre-amplifier (AEP4, Vallen Systeme) performed without any noteworthy limitations. The employed acquisition system (AMSY5, Vallen Systeme) needs to be discussed more critically. The AMSY5 is a high-quality system that is used amongst other things for on-site monitoring of bridges [25], [24]. For the fundamental research of this dissertation the limitations of this system quickly became apparent. In the author’s opinion the system’s maximum sampling rate of 10 MHz is sufficient for on-site applications. However, for fundamental research with state-of-the-art sensors that are sensitive in the MHz range, such as the Glaser-type conical PZT sensors [59], a higher rate is required. For sensor calibration, a digital oscilloscope (WaveSurfer, LeCroy) with a sampling rate of 1.0 GHz was used. The analysis software (VisualAE) that was pre-installed on AMSY5 provided no user-friendly way to export the recorded AE sets of waveforms automatically. It was only possible to manually export single waveforms, and considering that up to several thousand waveforms are recorded during a single experiment, this represents a serious handicap. To solve this problem, self-developed Excel macros were used to export all \((8 \times 573)\) waveforms automatically.

In Chapter 4, TRM is successfully transferred from exploration geophysics to a NDT application, signal-based AE analysis, using small-scale experiments on concrete cuboids \((120 \times 118 \times 160 \text{mm})\). Its applicability is corroborated by several numerical (see Section 4.2) and some physical (see Sections 4.3 and 5.2) examples. In the numerical studies in Section 4.3.1 it is interesting to see that even for imprecise velocity assumptions two dominant energy concentrations are present (see Fig. 4.9, b), even though only one set of (eight) waveforms is used as input for the inverse simulation. Similar results (see Figs. 4.9, a and e) could be observed in the physical experiment on a concrete cuboid (see Fig. 5.13, a). Further, radiation patterns of different source types such as a horizontal force (see Fig. 4.4, a), an explosion (see Fig. 4.4, b) and a double couple (see Fig. 4.4, c) are identified after inverse simulation and imaging within TRM. The stability of TRM is influenced by several parameters such as discretization of the numerical grid, complexity of the velocity model, number of employed PZT sensors, imaging conditions, and heterogeneity and stage of degradation of the investigated medium. The inverse simulations in Sections 5.2 and 5.3 are performed assuming that one grid point is equivalent to one millimeter \((1 \text{gp} \equiv 1 \text{mm})\), which is a minimum requirement in concrete applications at the centimeter- to meter-scale. It is unnecessary to model the porosity in the interfacial transition zone between aggregate grain and cement matrix for an anticipated wavelength of \(\Lambda_{AE} \approx 40 \text{mm}\). A finer discretization \((e.g. \ 10 \gp \equiv 1 \text{mm})\), as shown in Section 4.2, consumes additional computational resources but provides better resolution. The essential parameter for choosing the appropriate numerical grid size is the frequency of acoustic emission \((\approx 100 \text{kHz})\). Note that these recommendations are based on measurements carried out with PZT sensors limited to a sensitivity range of up to 1.0 MHz. For frequencies above 1.0 MHz and realistic dimensions of the specimen (meter scale), the high damping of concrete \((45 \text{dB/m} [46])\) makes a reliable measurement of AE extremely difficult. However, on the centimeter scale, the NCM should
be further refined with respect to the porosity in the interfacial transition zone and use more realistic aggregate shapes. For any mesh size the Neumann stability criterion \( \left( \frac{\Delta t_{cp}}{2\Delta x} \right) \) has to be satisfied. What numerical setup leads to failure of TRM was not examined in this dissertation. The inverse simulations were performed on the NCM (see Section 4.2.2) and on a representative medium with EEP of the NCM (see Section 4.2.3). Although the wave fields are scattered by the air voids included inside the NCM, a clear focus of energy and visible radiation pattern can be observed in Figs. 4.5 (b) and (c) for both sensor specifications (allcomp and onecomp). Using a homogeneous velocity model a blurred focus of energy is obtained (see Figs. 4.6, b and c). The radiation pattern of the double-couple source can be identified accurately for both sensor specifications (allcomp and onecomp). Imprecise velocity assumptions have a damping effect on the interferences (compare Figs. 4.9, a-c) but still yield to satisfactory results. Comparing Full TRM with Source TRM in Section 4.2 using a numerical example, demonstrates how the TRM performs when the full wave field available at every point of domain \( \Omega \) is reduced to single points on the boundary \( \partial \Omega \) in the inverse simulation. In the physical experiments, eight KSB250 sensors were employed; hence eight input signals were available for the inverse simulation. The author considers this the minimum required number of sensors to perform successful imaging with TRM. Surprisingly, in the case of AE set TR82, which was caused by concrete cracking (see Fig. C.2, b and Section 5.2.3), it is possible to identify a clear concentration of energy located near the crack surface by using only four undeconvolved waveforms (see Fig. C.2, b). Note that sensors may fail during experiments (see Section 5.3) or that waveforms may not be recorded properly by all channels, as can be seen in Figs. C.2 (a)-(d). In such cases it is useful to have additional waveforms to work with. Of the four applied imaging conditions Eqs. (4.8)-(4.11), the maximum total energy density \( \mathbf{E}_{tot}(x) \) yields the clearest focii of energy at effective velocities \( c_{p,eff} = 3987 \text{ m/s}, c_{s,eff} = 2328 \text{ m/s} \) and density \( \rho_{eff} = 2200 \text{ kg/m}^3 \) (compare Figs. 4.8, a-c and Fig. 4.9, b). To prevent artifacts from appearing at the boundaries (see Fig. 4.9, b) and especially in the sensors’ near field (see Fig. 4.10, a), the maximum total energy density \( \mathbf{E}_{tot}(x) := \mathbf{E}_{tot}(x, T - t) \) needs to be normalized by the non-reversed \( \mathbf{E}_{tot}(x, t) \) to obtain the related maximum total energy density \( \hat{\mathbf{E}}_{tot}(x) \) (Eq. (4.12)). This improves the results (see Fig. 4.10, c), but not all artifacts can be eliminated (see Figs. 4.11, a-c). The main reasons for this are the heterogeneity of the concrete as well as the fact that the information of the TRM field was available at a limited number of boundary points (at 8 sensor positions) only. Hypothetically, multiple AE could have been released simultaneously, some of which may have been mistakenly identified as artifacts. All numerical simulations were performed in an untested, undeformed medium without cracks. As shown in Section 3.6.2, a progressed cracking pattern does affect the wave propagation behavior. In the TRM results presented in Sections 5.2 and 5.3.4, it appears that the cracking progress has no significant influence on the TRM performance. This behavior is addressed in the following section.

Correctly assigning an energy concentration of \( \hat{\mathbf{E}}_{tot}(x) \) to a real AE event can be a difficult task. To verify the results, it is suggested that the actual crack distributions be compared with the locations of \( \hat{\mathbf{E}}_{tot}(x) \). X-ray CT scans and the visualization of the post-processed (threshold-segmented) CT models of uncracked and cracked concrete specimens (see Section 3.3) are useful
in doing so. An additional advantage of the digitalized specimens is that the different concrete phases (aggregate, cement paste and air inclusions) can be displayed in 3D. In this dissertation, the threshold-segmented phases are used to verify and to improve the development of the NCM. Further, comparisons of the three-dimensional crack distributions with voxels of $\hat{\mathbf{E}}_{\text{tot}}(\mathbf{x})$ are used to eliminate non-plausible results. The only noticeable disadvantage of the employed X-ray CT is that cracks below a crack mouth opening distance of approx. 0.5 mm could not be visualized at all.

The preliminary experiments on concrete cuboids provide helpful information for the optimization of the numerical setup details such as the duration of the simulation $T$, discretization on a spatial grid ($\text{gp} \equiv \text{mm}$) and for finding the right imaging conditions for optimal TRM performance. Setup and methodology for the application of TRM to a four-point bending experiment on a slender RC beam ($120 \times 200 \times 1700 \text{mm}$), including the use of X-ray CT, are presented. On this larger scale TRM performed in a similar way as in the small-scale experiment. The computation, running on the high-performance cluster Brutus of ETH Zurich on 298 processors in parallel, took approximately 7 hours per single AE set of eight waveforms and twice as long to calculate $\mathbf{E}_{\text{tot}}(\mathbf{x}, t)$ and $\mathbf{E}_{\text{tot}}(\mathbf{x}, T - t)$. Only 53 (9%) of 573 AE sets, each with a good S/N ratio, were selected and analyzed. The overall computational time was about one month in total. The number of selected AE sets varied between seven and thirteen per load level (see Figs. 5.19-5.23). A brief parameter-based AE analysis described in Section 5.3.3 (see Fig. 5.16) provides the first information about the temporal and qualitative evolution of AE events, hits and counts. The knowledge of the temporal distribution of accumulated counts, provided by the VallenAE software in realtime, was helpful during the experiment. If required the measurement could have been interrupted promptly. The selected AE sets denoted with TR$_{\#}$ are marked on the load-time versus counts-time curve and are displayed in Figs. 5.19-5.23 for each load level. This is helpful in comparing the time of occurrence of the energy concentrations with the times at which cracks first appeared at each load level (Figs. 5.18, a-e). The voxels of $\hat{\mathbf{E}}_{\text{tot}}(\mathbf{x})$ displayed in the details of LS1 and LS2 (see Figs. 5.24, b-e and Figs. 5.25, b-e) are the energy concentrations that best match the crack surfaces. In general, the major global maxima of $\hat{\mathbf{E}}_{\text{tot}}(\mathbf{x})$ are visible without any noteworthy artifacts, which is due to the fact that when the maxima occurred the medium was in relatively good condition and still uncracked. No dominant reflections can be observed in the waveforms displayed in Figs. C.5 (a)-(e) and Figs. C.6 (a)-(d). In some cases the normalized undeconvolved waveforms (red) contain reflections. However, the good condition of the beam allowed energy transmission for optimal interference of the wavefields. At LS3 (see Fig. 5.26, a) and LS4 (see Fig. 5.27, a), comparatively strong energy concentrations appeared. Several clear energy concentrations could be observed (see Figs. 5.26, c and d, and Figs. 5.27, d and e), although many cracks had already formed. The waveforms displayed in Figs. C.7 (a)-(d) and Figs. C.8 (a)-(d) do not differ qualitatively from those selected at LS1 and LS2. A small number of reflections appear in some of the waveforms. At the final load level LS5, clear observations of $\hat{\mathbf{E}}_{\text{tot}}(\mathbf{x})$ could be made as well (see Figs. 5.28, c-e). Special attention should be paid to crack 13, which was very active throughout all load levels. The AE activity around crack 13 governed the entire experiment although crack 13 only became visible at LS4.
Looking at the principal stress trajectories during cracking (see Figs. C.4, a-e), it can be seen that tensile components (blue) appear hyperbolically between supports (green triangles) and load application points (red triangles). It is possible that micro-cracks nucleated around the surface of crack 13 and along the tensile trajectories (blue) when the cracking load $F_u$ (7.6 kN) was reached. A further increase in the load resulted in the formation of macro-cracks to yield a visible crack 13 at LS4. The energy concentrations on the left-hand side boundaries displayed in Figs. C.4 (a)-(e) can neither be associated with a physical event nor excluded as artifacts. Note that when the reinforcement was activated, cracks and associated AE events also occurred due to friction and debonding along the reinforcing bars. Because the bars were bent up on the left-hand side, an accumulation of AE in this area is also plausible. It can be observed that from LS3 onwards the energy concentrations do match the crack surfaces and do not only appear around crack 13 as observed at LS1 and LS2. One possible reason for this is the distinct deformation of the beam that changes the relative distances between the sensor positions. The deflection at midspan and dominant crack 13 were visible by the naked eye after failure at LS5 (see Fig. 5.18, e). The inverse simulation is performed on an uncitled numerical model in its original (uncracked) state ($n_x \times n_y \times n_z$) and does not consider any deformation. For that reason, the mismatch between the voxels of $\hat{E}_{tot}(x)$ and the definite crack distributions from X-ray CT images increases at higher load levels. Theoretically, if the waves are reflected at the crack surfaces, the reflections are contained in the waveforms that are recorded at the surface. While performing the inverse simulation the interferences should cancel each other. What remains are the reflections of the crack topography. The reflected surfaces should theoretically become visible as something like negative images (similar to ultrasound images), provided that the inverse simulation is performed in an homogeneous and uncracked medium and the crack formation releases elastic energy on a similar level of magnitude as the reflections. This idea, which is only briefly introduced in this dissertation, is worthy of future investigation. In the physical experiments, the relatively low number of PZT sensors (eight) and in particular failure of sensor #8 (Id. 9.01) affected TRM performance the most. However, it could be demonstrated that even under suboptimal circumstances TRM performs well.

To compare TRM with a proven localization method, the TRM results are compared to those of the AIC-based linearized localization algorithm presented in [79]. All (573) originally recorded AE sets are used as input for the localization procedure. In a preliminary investigation, a few AE events could be localized using the effective velocity $c_{p, eff} = 3987 \text{ m/s}$. To learn whether other P-wave velocities resulted in better convergence, $c_p$ is varied within the range $[3300; 3550] \text{ m/s}$. The obtained results are of varying quality (see Section 5.3.5). The low applied velocities ($c_p < 3500 \text{ m/s}$) are not necessarily unrealistic. It is known that the effective P-wave velocity decreases in a cracked medium or a medium with significant air inclusions (see also Sections 3.5 and 3.6.1). Cracks partially or completely intersect the propagation path of a propagating wave field (see Figs. A.5, a-l). The P-wave velocity that is recorded in a cracked medium can be that one of a significantly delayed P-wave or that one of the interfering reflections. However, velocities below 3300 m/s would not make any physical sense and are therefore not used.
Chapter 6. Discussion, Conclusions and Outlook

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<th>Time reverse modeling</th>
<th>AIC-based linearized localization</th>
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<tr>
<td><strong>Velocity model</strong></td>
<td>Complex (heterogeneous) or trivial (homogeneous) velocity model</td>
<td>Trivial velocity model, complex model possible but has not been used so far</td>
</tr>
<tr>
<td><strong>User interaction</strong></td>
<td>Marginal, numerical setup needs to be adjusted once; significant if using the NCM</td>
<td>Noticable user interaction required</td>
</tr>
<tr>
<td><strong>Pre-processing of waveforms</strong></td>
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<td>Not required</td>
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<td><strong>Computational time</strong></td>
<td>Significant for medium and large specimens</td>
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<td><strong>Simplicity of physical principle</strong></td>
<td>Straightforward principle, time-reversed mirror [20]</td>
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<td><strong>Special features</strong></td>
<td>Multiple energy concentrations can be imaged with one set of AE waveforms; radiation patterns of different source types can be visualized</td>
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<td><strong>Area and scale of application</strong></td>
<td>Fundamental research, small (to middle) scale</td>
<td>Applicable on-site (executable in realtime), small to large scale</td>
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<td><strong>Assessment and recommendation</strong></td>
<td>Displays what really happens, suitable for deeper study of crack mechanism, can be applied in a second run</td>
<td>Fast method, calculated 3D coordinates represent first AE activity with unclear physical meaning, can be applied in a first run</td>
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Tab. 6.1: Time reverse modeling versus AIC-based linearized localization for selected criteria.

In the case of the four-point bending test, the presence of a progressed cracking pattern affected the AIC-based linearized localization more significantly than TRM. The former localization method and in particular the localization algorithm assume an undisturbed propagation path of the wave, while the cracks acting as reflectors and scatterers intersect this straight path. TRM uses the entire waveform as input. The waveform contains information on the propagation path and on interferences, which is helpful for focusing on the source location. If a specimen exhibits a large amount of cracks so that the communication between the sensors is disrupted, also TRM becomes ineffective.
6.2 Conclusions

TRM can be used in certain cases in signal-based AE analysis of concrete and RC specimen. The possible scale of investigation depends on the number and the scale of the sensors. For optimal communication between the sensors at least four sensors should cover a maximum relative distance of 1.0 m. The performance of TRM can be improved significantly if state-of-the-art sensors (for example Glaser-type conical PZT sensors [59]) are employed. A disadvantage of TRM in AE analysis is that only a single set of waveforms can be re-emitted into the medium in the inverse simulation. Considering that the inverse simulations needs to be performed twice for large-scale specimens, once to obtain $E_{tot}(x, T - t)$ and once to obtain $E_{tot}(x, t)$, the simulation can only be carried out for a few AE sets in a reasonable amount of time. It would be more efficient to “superimpose” AE sets by merging them into a single AE set. Note that using the classic way of superimposing all waveforms of the AE sets would corrupt the TRM procedure and not yield meaningful results. Provided that continuous recording equipment is available, a set of “super waves”, i.e. waveforms recorded in the millisecond range (i.e. for $f_s = 1.0\,\text{MHz}$) could be used as input. However, due to its long computational time TRM is not an ideal application for on-site use in realtime. In the author’s opinion its potential lies in applications in fundamental research on selected small- (to middle-)scale problems.

In the following paragraphs, an attempt is made to answer the three research questions posed in the introduction of this dissertation: (1) Are the advantages of TRM greater than those of the linearized localization algorithm? (2) Does a progressed cracking pattern in a RC specimen significantly affect the TRM localization? (3) In what areas and on what scale can TRM be applied in the future?

(1) To assess the presented methods, TRM and the AIC-based linearized localization algorithm, the advantages and disadvantages of both methods are reviewed. In particular, aspects such as the complexity of the velocity model, user interaction, manipulation of waveforms, computational time, simplicity of the physical principle and special features are addressed (see Tab. 6.1). TRM is based on a complex (heterogeneous) velocity model, but it is also possible to use a trivial (homogeneous) velocity distribution. Besides a significant effort for generating the NCM, little user interaction is required. Initially, the setup of the numerical simulations needs to be adjusted to the specific structure, but subsequently the inverse simulation runs without requiring any further user intervention. The implementation and physical experiments that are needed for signal deconvolution are time consuming, but are not required for the AIC-based linearized localization algorithm. The sensor response needs to be determined once and can subsequently be applied to each measurement. Within TRM the waveforms are used without any simplification, besides deconvolution where appropriate. The handicap of TRM is that for large specimens computational time increases significantly. TRM is based on the so-called time reversed mirror [20], where the wave fields reversed in time concentrate at the location from where they were emitted. The special feature is that several energy concentrations or localized AE events can be imaged by using only one set of AE waveforms as input.
It is an original concept that radiation patterns of different source types can be identified after inverse simulation and imaging. A major deficit of TRM is that no error estimation has been performed so far. The AIC-based linearized localization algorithm uses a trivial velocity model assuming a homogeneous velocity distribution. Theoretically, complex velocity models can be applied as well, but so far this has not yet been done. This method requires some user interaction as the threshold level needs to be adjusted to the different S/N ratios of the AE signals. Evaluation of the Hilbert envelope and the AIC-function to determine the onset time works efficiently, once implemented, but requires a significant amount of user interaction while running. The localization calculation, which is based on the relative time differences, the velocity model and the sensor coordinates, is very time efficient. Although the procedure is nested, the required computational time is small and independent of the specimen’s dimensions. It is an important feature of the method that error estimation including quantification of error ellipsoids is possible, if more than four sensors are involved. The physical principle of the method relies on picking of the first wave motion, after which the first AE activity is localized in three-dimensional space.

(2) After comparing both methods presented in this work with respect to their capability of localizing AE events for a progressed cracking pattern, it appears that TRM is capable of displaying more results than the AIC-based linearized localization method. Even though the energy concentrations at LS3 are more intense than those at LS1 and LS2 and do not match the crack surfaces as well, in many cases clear foci of energy can be observed (see Section 5.3.4). Note that only 53 AE sets of the entire experiment were used for TRM, and no convergence problems have been encountered. It is important to note that it is still unclear what is being displayed by imaging $\hat{\mathbf{E}}_{\text{tot}}(\mathbf{x})$. The amount of successfully localized AE events with the AIC-based linearized localization algorithm is smaller (approx. 10-15% of the 573 available AE sets converge) but general information concerning the accuracy of localization can be provided by this method.

(3) Assuming that a large number of sensors and the numerical setup and hardware specifications as described in this dissertation are used, the large computational time of the inverse simulation limits the applicability of TRM to a model size of approx. $5.0 \times 10^7 \text{gp}^3$. The AIC-based linearized localization algorithm does not depend on computational resources, hence it can be applied on a large scale without any limitations. It can be said that the methods complement each other on different scales of application. The author recommends that for any size specimen an AIC-based linearized localization be performed initially and deconvolution be carried out. To investigate structural details, especially progressed crack patterns, TRM should be used. Finally, the results of both methods can be superimposed for visualization. TRM (implemented in Fortran) can be used at the centimeter scale and is carried out only after the experiment due to its long computational time. The AIC-based linearized localization algorithm (implemented in Matlab) works almost in realtime and can be applied to on-site structures at the meter scale.
6.3 Outlook

Future works of the author will focus on steel ball impacts on different materials. In particular, plasticity effects will be quantified so that the force impulse can be modified accurately. Physical experiments to measure the force impulse would be useful to investigate the plastic influences of a Hertzian impact. Further, non-linear FEM calculations could be carried out to complement the physical experiments.

Small- and large-scale physical experiments using state-of-the-art PZT sensors (for example Glaser-type PZT sensors) and new acquisition equipment (for example Elsys Transient Recorder System) are scheduled to be carried out in the near future. It will be attempted to relate the TRM results from the small-scale experiments to fracture mechanics. In particular, radiation patterns can be verified using fracture models, which can be supplemented with moment tensor analysis. There is further potential to develop new imaging conditions to improve the accuracy of TRM; such conditions could for example be developed from probabilistic considerations. Micro CT scans could be used to visualize cracks at a smaller scale than is possible with the medical X-ray scanner employed in this dissertation. It is the author’s aim to perform TRM in the threshold-segmented CT medium of a cracked and deformed specimen itself. On a large scale, experiments on RC slabs or girders are conceivable, provided that a great number of sensors is available.

It is strongly recommended that a new acquisition software be developed in future research. It is suggested to numerically implement a LabView environment that can store the desired AE parameters as well as the waveforms in a binary format at a higher sampling rate (at least 1.0 GHz). The binary format is best suited to be incorporated into Matlab or Fortran subroutines for post-processing purposes. An analysis tool could be developed, which can display the evolution of classic AE parameters in realtime and perform preliminary analysis during experiments. The benefit of a self-developed acquisition software is that the user can set the pre-selection criteria and that the processes and subroutines can be reviewed by the scientific community.

Finally, please allow for an excursus beyond the area of structural concrete, a “destructive” application of the time reversed mirror. Reversing the principle itself so that the sensors act as active sources, the induced wave fields will concentrate at a certain spatial point. Provided that the sensors can induce a sufficient amount of energy into the medium, it is theoretically possible that the focus can provoke a local destruction at that certain spatial point. Such an application is of interest for medical science, for example to detect [23] or to destroy [69] noxious organic material hidden inside a human or animal body.
Nomenclature

Abbreviations

2D Two dimensions
3D Three dimensions
A/D Analog-to-digital converter
AE Acoustic emission
AIC Akaike Information Criterion
allcomp All components
CT Computed tomography
DFT Discrete Fourier transform
DICOM Digital Imaging in Communications and Medicine
EEP Effective elastic properties
EMPA Swiss Federal Laboratories for Materials Science and Technology
ESEM Environmental Scanning Electron Microscope
FD Finite differences
FFP Far field of the P-wave part of the solution
FFS Far field of the S-wave part of the solution
FT Fourier transformation
IIR Infinite Impulse Response
ISO International Organization for Standardization
LS Load level
NCM Numerical concrete model
NDT Non-destructive testing
NF Near-field part of the solution
### Nomenclature

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>onecomp</td>
<td>One component</td>
</tr>
<tr>
<td>P</td>
<td>Primary, longitudinal (wave)</td>
</tr>
<tr>
<td>PTFE</td>
<td>Polytetrafluorethylen</td>
</tr>
<tr>
<td>PZT</td>
<td>Piezoelectric (sensor)</td>
</tr>
<tr>
<td>RC</td>
<td>Reinforced concrete</td>
</tr>
<tr>
<td>RVE</td>
<td>Representative volume element</td>
</tr>
<tr>
<td>S/N</td>
<td>Signal-to-noise</td>
</tr>
<tr>
<td>SH</td>
<td>Horizontally polarized secondary, transversal (wave)</td>
</tr>
<tr>
<td>SiGMA</td>
<td>Simplified Green’s functions for moment tensor analysis</td>
</tr>
<tr>
<td>SV</td>
<td>Vertically polarized secondary, transversal (wave)</td>
</tr>
<tr>
<td>TR</td>
<td>Transient set of waveforms</td>
</tr>
<tr>
<td>TRM</td>
<td>Time reverse modeling</td>
</tr>
</tbody>
</table>

### Upper-case roman letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>Reference area</td>
</tr>
<tr>
<td>(A(\omega))</td>
<td>Arbitrary frequency-dependent amplitude</td>
</tr>
<tr>
<td>(A_0)</td>
<td>Amplitude of the incoming P- or SV-wave</td>
</tr>
<tr>
<td>(A_1)</td>
<td>Amplitude of the reflected or refracted P-wave</td>
</tr>
<tr>
<td>(A_2)</td>
<td>Amplitude of the reflected or refracted SV-wave</td>
</tr>
<tr>
<td>(A_h)</td>
<td>Amplitude of the harmonic solution</td>
</tr>
<tr>
<td>(C(x), C_{ijkl})</td>
<td>Elastic tensor, vector and index notation</td>
</tr>
<tr>
<td>(C_D)</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>(C_t)</td>
<td>Count</td>
</tr>
<tr>
<td>(E, E^*, E_1, E_2)</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>(E(\omega))</td>
<td>Frequency-dependent Young’s modulus</td>
</tr>
<tr>
<td>(E_p(x))</td>
<td>Maximum energy density of the P-wave</td>
</tr>
<tr>
<td>(E_s(x))</td>
<td>Maximum energy density of the S-wave</td>
</tr>
<tr>
<td>(E_{tot}(x))</td>
<td>Maximum total energy density</td>
</tr>
<tr>
<td>(\hat{E}_{tot}(x))</td>
<td>Related maximum total energy density</td>
</tr>
<tr>
<td>(F)</td>
<td>Force</td>
</tr>
<tr>
<td>(F(t))</td>
<td>Force impulse</td>
</tr>
</tbody>
</table>
Nomenclature

$F'_{\text{max}}$ Maximum rebound force

$F_{\text{cr}}$ Cracking load

$F_D$ Drag force

$F_{\text{max}}$ Maximum force

$F_u$ Ultimate load

$G$ Shear modulus

$G_{\text{AE}}$ Gain for AE measuring

$G_{in}, G_{in}(x, t; \xi, \tau)$ Green’s function

$H$ Falling height

$H(\omega)$ Fourier transform of an arbitrary continuous-time signal $h(t)$

$H'$ Rebound height

$H_t$ Hit

$I(f), I(\omega)$ Fourier transform of the instrument response $i(t)$

$I(\omega^{-1})$ Time reversal of the instrument response $I(\omega)$

$K$ Material constant influenced by geometry and elastic constants $C_{ijkl}$

$M_{11}, \cdots, M_{33}$ Components of the moment tensor

$M(x, t), M_{ij}$ Moment tensor (vector resp. index notation)

$N$ Absolute number, number of sensors, number of samples

$O$ Computational operation

$P$ Arbitrary spatial point

$P'$ Transformed arbitrary spatial point $P$

$R$ Rotation matrix

$R_1$ Radius of the sphere

$S$ Sensor locations

$T$ End time

$T_c$ Contact duration

$U_{3}^{0}(\omega)$ Fourier transform of the normal displacement with zero-phase shift $u_3^0(t)$

$U_3(f), U_3(\omega)$ Fourier transform of the normal displacement $u_3(t)$

$\tilde{U}_3(\omega)$ Fourier transform of artificial variable $\tilde{u}_3(t)$

$\tilde{U}_3(\omega^{-1})$ Time reversal of $\tilde{U}_3(\omega)$

$V(f), V(\omega)$ Fourier transform of the continuous-time voltage signal $v(t)$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(f_m)$, $V(\omega_m)$</td>
<td>Fourier transform of the discrete-time voltage signal $v(t_n)$</td>
</tr>
<tr>
<td>$V_{e,i}$</td>
<td>Volume percentage of the subsieve fraction</td>
</tr>
<tr>
<td>$V_0/d_i$</td>
<td>Volumetric part of the Fuller Curve</td>
</tr>
<tr>
<td>$V_{\text{EMPA}}$</td>
<td>Volumetric part of the EMPA Curve</td>
</tr>
<tr>
<td>$V_{e,i}$</td>
<td>Volume per ellipsoid</td>
</tr>
<tr>
<td>$V_{d_{\text{min}}/d_i}$</td>
<td>Discrete model of the volumetric part of the Fuller Curve</td>
</tr>
<tr>
<td>$X_0(t)$</td>
<td>Time-varying impulse acting at an arbitrary point 0</td>
</tr>
<tr>
<td>$X_i$, $X_i^{[0;1]}$</td>
<td>Random number in the range [0; 1]</td>
</tr>
</tbody>
</table>

**Lower-case roman letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Shear arm</td>
</tr>
<tr>
<td>$\mathbf{a}$</td>
<td>Translation vector</td>
</tr>
<tr>
<td>$a_1$, $a_2$, $a_3$</td>
<td>Components of translation vector $\mathbf{a}$</td>
</tr>
<tr>
<td>$a_{1,i}$, $a_{2,i}$, $a_{3,i}$</td>
<td>Randomly varying components of translation vector $\mathbf{a}$</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of specimen</td>
</tr>
<tr>
<td>$c$</td>
<td>Phase velocity, particle velocity</td>
</tr>
<tr>
<td>$c(\omega)$</td>
<td>Frequency-dependent velocity</td>
</tr>
<tr>
<td>$c_{11}$, $\ldots$, $c_{44}$</td>
<td>Components of the elastic tensor</td>
</tr>
<tr>
<td>$c_{\text{eff}}$</td>
<td>Effective wave velocity</td>
</tr>
<tr>
<td>$c_p$, $c_\Psi$</td>
<td>P-wave velocity</td>
</tr>
<tr>
<td>$c_p,\text{eff}$</td>
<td>Effective P-wave velocity</td>
</tr>
<tr>
<td>$c_{p,i}$</td>
<td>Randomly varying P-wave velocity</td>
</tr>
<tr>
<td>$c_{p,\text{steel}}$</td>
<td>P-wave velocity of steel</td>
</tr>
<tr>
<td>$c_R$</td>
<td>Rayleigh wave velocity</td>
</tr>
<tr>
<td>$c_s$, $c_\Phi$</td>
<td>S-wave velocity</td>
</tr>
<tr>
<td>$c_{s,\text{eff}}$</td>
<td>Effective S-wave velocity</td>
</tr>
<tr>
<td>$c_{s,i}$</td>
<td>Randomly varying S-wave velocity</td>
</tr>
<tr>
<td>$c_{s,\text{steel}}$</td>
<td>S-wave velocity of steel</td>
</tr>
<tr>
<td>$\bar{c}_p$</td>
<td>Mean measured P-wave velocity</td>
</tr>
<tr>
<td>$\bar{c}_p,\text{eff}$</td>
<td>Mean effective P-wave velocity</td>
</tr>
<tr>
<td>$\bar{c}_s$</td>
<td>Mean S-wave velocity (calculated from $\bar{c}_p$)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$\bar{c}_{s,\text{eff}}$</td>
<td>Mean effective S-wave velocity</td>
</tr>
<tr>
<td>$d$</td>
<td>Radius of concrete cylinder (double punch test), depth of specimen</td>
</tr>
<tr>
<td>$\mathbf{d}$</td>
<td>Unit vector describing the direction of particle motion</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Variable grain size</td>
</tr>
<tr>
<td>$d_{\text{max}}$</td>
<td>Maximum aggregate size, maximum diameter of aggregate</td>
</tr>
<tr>
<td>$d_{\text{min}}$</td>
<td>Minimum aggregate size</td>
</tr>
<tr>
<td>$d_o$</td>
<td>Ball diameter</td>
</tr>
<tr>
<td>$e$</td>
<td>Coefficient of restitution (Hertz theory), Euler number</td>
</tr>
<tr>
<td>$e_k, e_l$</td>
<td>Unit vectors (index notation)</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$\mathbf{f}, f(\mathbf{x},t)$, $f_i$</td>
<td>Space- and time-dependent body force (vector resp. index notation)</td>
</tr>
<tr>
<td>$f_3(\xi,\tau)$</td>
<td>Source function acting at position $\xi$ and time $\tau$ in the normal direction</td>
</tr>
<tr>
<td>$f_{ct}$</td>
<td>Concrete tensile strength</td>
</tr>
<tr>
<td>$f_{\text{dom}}$</td>
<td>Dominant frequency</td>
</tr>
<tr>
<td>$f_N$</td>
<td>Nyquist frequency</td>
</tr>
<tr>
<td>$f_n, f_n(\xi,\tau)$</td>
<td>Source function acting at position $\xi$ and time $\tau$</td>
</tr>
<tr>
<td>$f_m$</td>
<td>Discrete frequency</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Sampling rate, sampling frequency</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Effective yield strength of reinforcement steel</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$g_{33}(\mathbf{x},t;\xi,\tau)$</td>
<td>Component ($i = n = 3$) of Green’s function</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of specimen</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>Arbitrary continuous-time signal</td>
</tr>
<tr>
<td>$i, j, k$</td>
<td>Spatial coordinates</td>
</tr>
<tr>
<td>$i(t)$</td>
<td>Instrument response</td>
</tr>
<tr>
<td>$i(\mathbf{x},t)$</td>
<td>Space- and time-dependent instrument response</td>
</tr>
<tr>
<td>$i_j$</td>
<td>Base vector (index notation)</td>
</tr>
<tr>
<td>$k_w$</td>
<td>Wave number</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of specimen</td>
</tr>
<tr>
<td>$l_k$</td>
<td>Span of beam (four-point bending test)</td>
</tr>
<tr>
<td>$m$</td>
<td>Arbitrary variable</td>
</tr>
</tbody>
</table>
\( m, m_1 \) \hspace{1cm} \text{Body mass (Hertz theory)}

\( m_{\text{aggr}} \) \hspace{1cm} \text{Mass of aggregates}

\( m_{\text{aggr},i} \) \hspace{1cm} \text{Mass of aggregate fractions}

\( m_{\text{cem}} \) \hspace{1cm} \text{Mass of cement}

\( m_{\text{tot}} \) \hspace{1cm} \text{Total mass}

\( n \) \hspace{1cm} \text{Arbitrary variable}

\( n_x, n_y, n_z \) \hspace{1cm} \text{Numerical coordinates}

\( p \) \hspace{1cm} \text{Radius of steel punch (double punch test)}

\( \mathbf{p} \) \hspace{1cm} \text{Unit vector describing the direction of propagation}

\( q \) \hspace{1cm} \text{Arbitrary function}

\( r \) \hspace{1cm} \text{Radial coordinate, radius}

\( \mathbf{r} \) \hspace{1cm} \text{Position vector}

\( \mathbf{r}' \) \hspace{1cm} \text{Rotated position vector}

\( r_a, r_b, r_c \) \hspace{1cm} \text{Radii of the ellipsoids corresponding to } x-, y- \text{ and } z\text{-direction, resp.}

\( r_i, r_j \) \hspace{1cm} \text{Relative sensor positions (Cartesian coordinates)}

\( s \) \hspace{1cm} \text{Standard deviation}

\( s_{\text{cp}} \) \hspace{1cm} \text{Standard deviation of the mean P-wave velocity}

\( t \) \hspace{1cm} \text{Continuous time variable}

\( t_D \) \hspace{1cm} \text{Duration}

\( t_{\text{global}} \) \hspace{1cm} \text{Global time}

\( t_i, t_j \) \hspace{1cm} \text{Relative arrival times}

\( t_{\text{max},A} \) \hspace{1cm} \text{Maximum window size for aluminum}

\( t_{\text{max},C} \) \hspace{1cm} \text{Maximum window size for concrete}

\( t_N, t_e \) \hspace{1cm} \text{End time}

\( t_n \) \hspace{1cm} \text{Discrete time variable}

\( t_{\text{pre}} \) \hspace{1cm} \text{Pre-trigger time}

\( t_R \) \hspace{1cm} \text{Rise time}

\( t_s \) \hspace{1cm} \text{Starting time}

\( t_{w,A} \) \hspace{1cm} \text{Arbitrary window size for aluminum}

\( t_{w,C} \) \hspace{1cm} \text{Arbitrary window size for concrete}

\( u(t) \) \hspace{1cm} \text{Displacement, wave motion}
\( \mathbf{u}, \mathbf{u}(\mathbf{x}, t), u_i, u_i(\mathbf{x}, t) \) Displacement field (vector resp. index notation)

\( \mathbf{u}(t) \) Displacement at sensor position \( \mathbf{x}_s \)

\( \mathbf{u}(\mathbf{x}, T) \) Displacement field at time \( T \)

\( \mathbf{u}(\mathbf{x}^{(k)}, T - t) \) Inverse displacement field

\( u_3^0(t) \) Normal displacement with zero-phase shift

\( \mathbf{u}^{(k)}(t), \mathbf{u}(\mathbf{x}^{(k)}, t) \) Time series of displacements at sensor positions \( \mathbf{x}^{(k)} \)

\( u_3(t), u_3(\mathbf{x}, t) \) Displacement normal to the surface

\( \tilde{u}_3(t), w(t) \) Artificial variables

\( \tilde{u}_3(-t), w(-t) \) Time reversal of artificial variables \( \tilde{u}_3(t), w(t) \)

\( u_i, u_j, u_k \) Displacement components

\( u_n(t) \) Displacement, wave motion at the surface

\( u_n(\mathbf{x}, t) \) Calculated space- and time-dependent displacement at sensor position

\( u_x, u_y, u_z \) Displacement components

\( u_x(\mathbf{x}^{(k)}, t) \) \( x \)-component of displacements at sensor positions \( \mathbf{x}^{(k)} \)

\( u_y(\mathbf{x}^{(k)}, t) \) \( y \)-component of displacements at sensor positions \( \mathbf{x}^{(k)} \)

\( u_z(\mathbf{x}^{(k)}, t) \) \( z \)-component of displacements at sensor positions \( \mathbf{x}^{(k)} \)

\( \mathbf{u}_{\text{max}}(\mathbf{x}) \) Maximum particle displacement

\( v \) Velocity

\( v' \) Ball velocity after impact

\( v(t) \) Continuous-time voltage signal

\( v(\mathbf{x}, t) \) Space- and time-dependent voltage signal

\( v(t_n) \) Discrete-time voltage signal

\( v(t)_{\text{in}} \) Input voltage signal

\( v(t)_{\text{out}} \) Output voltage signal

\( v_0 \) Velocity at time 0, impact velocity

\( v_{0,D} \) Impact velocity including air drag

\( v_i \) Velocity, index notation

\( v_i, v_j, v_k \) Velocity components

\( v_{\text{max}} \) Maximum peak amplitude

\( v_{\text{min}} \) Minimum peak amplitude

\( v_T \) Threshold
Velocity components in Cartesian coordinate system

Wavelet function \((i = 1, 2, 3)\)

Cartesian coordinates

Arbitrary position vector, point in space

Transformed Cartesian coordinates

Sensor positions

Coordinate directions (index notation)

Components of arbitrary position vector \(\xi\)

Source location

Regression line of the P-wave velocity

Regression line of the S-wave velocity

Regression line of the density

Discrete derivative

Wavelength

Typical wavelength of acoustic emissions

Helmholtz potentials

Helmholtz potential of the inverse displacement field (dilatational part)

Helmholtz potential of the inverse displacement field (isochore part)

Spatial TRM domain

Cosine of \(x_i, x_j\)-direction

Kronecker-Delta

Indentation in \(z\)-direction

Time dependent function of the indentation

Maximum indentation

Strain tensor (vector resp. index notation)

Strain tensor of the inverse displacement field

Coefficient vectors
Nomenclature

$\zeta_1, \zeta_2, \zeta_3$ Transformation coefficients

$\eta$ Characteristic dynamic viscosity

$\eta_1, \eta_2, \eta_3$ Transformation coefficients

$\vartheta$ Angle of mutation

$\vartheta_i$ Randomly varying angle of mutation

$\theta_0$ Angle of incidence of the P- or SV-wave

$\theta_1$ Angle of reflection or refraction of the P-wave

$\theta_2$ Angle of refraction or reflection of the SV-wave

$\theta_{cr}$ Critical angle

$\theta_i$ Arbitrary angle

$\kappa$ Dynamic material constant

$\lambda, \mu$ Lamé’s constants

$\nu, \nu_1, \nu_2$ Poisson’s ratio

$\nu_{\text{concrete}}$ Poisson’s ratio of concrete

$\xi$ Arbitrary position vector (sensor calibration)

$\xi_1, \xi_2, \xi_3$ Transformation coefficients

$\rho, \rho(x)$ Space-dependent density

$\rho_1$ Density of the sphere (Hertz theory)

$\rho_F$ Density of surrounding fluid

$\rho_{\text{eff}}$ Effective density

$\rho_i$ Randomly varying density

$\rho_{\text{steel}}$ Density of steel

$\bar{\rho}_{\text{eff}}$ Mean effective density

$\sigma(x, t), \sigma_{ij} (\sigma_{ik})$ Stress tensor (vector resp. index notation)

$\sigma(x, T - t)$ Stress tensor of the inverse displacement field

$\sigma_1, \sigma_2, \sigma_3$ Stress tensor components in directions 1, 2 and 3

$\sigma_{xx}, \sigma_{xz}, \sigma_{zz}$ Stress tensor components in Cartesian coordinate system

$\tau$ Time variable of the body source

$\varphi$ Angle of rotation

$\varphi_0$ Phase

$\varphi_0(\omega)$ Frequency-dependent phase
### Nomenclature

<table>
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<tr>
<td>$\phi_i$</td>
<td>Randomly varying angle of rotation</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Angle of precision</td>
</tr>
<tr>
<td>$\psi_i$</td>
<td>Randomly varying angle of precision</td>
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<td>$\omega$</td>
<td>Angular frequency</td>
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<td>$\omega_m$</td>
<td>Discrete angular frequency</td>
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### Other symbols

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<td>Crack volume</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>Frequency increment</td>
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<tr>
<td>$\Delta h$</td>
<td>Grid spacing</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time increment</td>
</tr>
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<td>$\Delta x, \Delta y, \Delta z$</td>
<td>Grid spacing in $x$-, $y$- and $z$-direction</td>
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<td>$\Im(\omega)$</td>
<td>Imaginary frequency-dependent part of the complex function $H(\omega)$</td>
</tr>
<tr>
<td>$\Re(\omega)$</td>
<td>Real frequency-dependent part of the complex function $H(\omega)$</td>
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<td>$\phi$</td>
<td>Diameter of reinforcing bar</td>
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<tr>
<td>$\partial \Omega$</td>
<td>Boundary of the TRM domain</td>
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<tr>
<td>$\rightarrow$</td>
<td>Discrete Fourier transformation operator</td>
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Appendix A

Elastic wave propagation in structural concrete

A.1 Implementation schematic for numerical concrete model
Fig. A.1: Flow chart of numerical implementation scheme of the NCM.
A.2 Frequency dependency of the EEP
Fig. A.2: Effective velocities $c_{\text{eff}}$ [m/s] plotted for different frequencies.
A.3 Simulation of wave propagation in cracked concrete
Fig. A.3: Snapshots of the wave propagation simulation ($f_{dom} = 25$ kHz) in a cracked cuboid displaying the top view of the cross-section at $n_y = 158$ gp (sensor positions #1, #2 and #4 are indicated by green crosses). Coordinates of the positions of sensors #1, #2 and #4 can be found in Tab. 3.5. The normalized displacement wave fields are plotted at intervals of $\Delta t = 125 \mu s$.

Fig. A.4: Displacements [fm] calculated at sensor positions #1 to #4 for a cracked (blue) and a homogeneous (red) specimen.
A.3. Simulation of wave propagation in cracked concrete

Fig. A.5: Snapshots of the wave propagation simulation taken at intervals of $\Delta t = 125 \mu s$. The positions of sensors #1 to #4 are located within the respective planes of wave propagation. The displacement wave fields are normalized.
Appendix B

Time reverse modeling in NDT

B.1 Application of TRM to a real NDT example

Fig. B.1: (a) Displacements [fm] calculated at sensor positions #1 to #8 due to a (numerical) Ricker2 wavelet (see Fig. 3.2) excitation located near sensor #4. (b)-(d) Three AE sets of waveforms [mV] (voltage signals) recorded by KSB250 sensors #1 to #8 due to a (physical) pencil-lead break applied near the positions of sensors #3, #6 and #7, respectively.
B.1. Application of TRM to a real NDT example

Fig. B.1: (a) Ricker2 wavelet (numerical) near sensor #4

Fig. B.1: (b) Pencil-lead break (physical) near sensor #3
Fig. B.1: (c) Pencil-lead break (physical) near sensor #6

Fig. B.1: (d) Pencil-lead break (physical) near sensor #7
Appendix C

Experimental investigations

C.1 Sensor calibration and signal deconvolution

Fig. C.1: (a)-(h) Voltage-time history (left column) and normalized spectrum on a logarithmic $y$-scale of the Fourier transform $|V(f)|$ at time $t_{\text{max}}$ (middle column) for KSB250 sensors #1 to #8. The normalized instrument response $|I(f)|$ is determined in the frequency domain at time $t_w$ zoomed to the range of significant energy (right column). The results are displayed for three steel balls dropped onto concrete (blue) and aluminum (red), having diameters of $d_o = 4$ mm (top row), $d_o = 3$ mm (middle row) and $d_o = 2$ mm (bottom row). The instrument response and the convolution process are described in Section 5.1.4 for sensor #8 with Id. 09.03 (f).
C. Experimental investigations

Fig. C.1: (a) Sensor #1

Fig. C.1: (b) Sensor #2
C.1. Sensor calibration and signal deconvolution

Fig. C.1: (c) Sensor #3
Id. 6.04, \( d_a = 3 \text{ mm} \)

\[ v(t) \rightarrow V(f) \]

\[ V(f) \cdot U(f)^{-1} \]

Fig. C.1: (d) Sensor #4
Id. 09.04, \( d_a = 4 \text{ mm} \)

\[ v(t) \rightarrow V(f) \]

\[ V(f) \cdot U(f)^{-1} \]
C. Experimental investigations

Fig. C.1: (e) Sensor #5

Id. 09.03, $d_a = 4\, \text{mm}$

Id. 09.03, $d_a = 3\, \text{mm}$

Id. 09.03, $d_a = 2\, \text{mm}$

Fig. C.1: (f) Sensor #6

Id. 09.03, $d_a = 4\, \text{mm}$

Id. 09.03, $d_a = 3\, \text{mm}$

Id. 09.03, $d_a = 2\, \text{mm}$
C.1. Sensor calibration and signal deconvolution

Fig. C.1: (g) Sensor #7

- Id. 9.08, \( d_a = 4 \text{ mm} \)
- Id. 9.08, \( d_a = 3 \text{ mm} \)
- Id. 9.08, \( d_a = 2 \text{ mm} \)

Fig. C.1: (h) Sensor #8 (damaged)

- Id. 9.01, \( d_a = 4 \text{ mm} \)
- Id. 9.01, \( d_a = 3 \text{ mm} \)
- Id. 9.08, \( d_a = 2 \text{ mm} \)
C.2 Experiments on concrete cuboid

Fig. C.2: Four AE sets of waveforms [mV] (a) TR_75, (b) TR_82, (c) TR_84 and (d) TR_180, which were recorded by KSB250 sensors #1 to #8. AE sets (a)-(d) were used as input for the inverse simulation in TRM of the concrete cuboid, presented in Section 5.2.
C.2. Experiments on concrete cuboid

Fig. C.2: (a) TR.75

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.2: (b) TR.82

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C. Experimental investigations

Fig. C.2: (c) TR_84

Fig. C.2: (d) TR_180
C.3 Four-point bending test
Fig. C.3: (a)-(e) Side views of the RC beam showing the related maximum total energy density $\hat{E}_{\text{tot}}(x)$ [–] of the selected AE sets at load levels 1 to 5, respectively. $\hat{E}_{\text{tot}}(x)$ is superimposed with a CT image in side view (at $z = 118$ mm) of the cracked RC beam and a threshold-segmented reinforcing bar in the final state (at LS5). The color bar of $\hat{E}_{\text{tot}}(x)$ is identical to that displayed in the three-dimensional counterpart in Figs. 5.24-5.28 (a). The green and red rectangles represent the supports and load application point, respectively.
C.3. Four-point bending test

(a) $\hat{E}_{\text{tot}}$ (LS1) and principal stress trajectories at $F_{\text{cr}}$

(b) $\hat{E}_{\text{tot}}$ (LS2) and principal stress trajectories at $F_{\text{cr}}$

(c) $\hat{E}_{\text{tot}}$ (LS3) and principal stress trajectories at $F_{\text{cr}}$

(d) $\hat{E}_{\text{tot}}$ (LS4) and principal stress trajectories at $F_{\text{cr}}$

(e) $\hat{E}_{\text{tot}}$ (LS5) and principal stress trajectories at $F_{\text{cr}}$

Fig. C.4: (a)-(e) Side views of the RC beam showing the related maximum total energy density $\hat{E}_{\text{tot}} (x)$ [–] of the selected AE sets at load levels 1 to 5, respectively. $\hat{E}_{\text{tot}} (x)$ is superimposed with the principal stress trajectories at $F_{\text{cr}}$ (7.6 kN), and tensile and compressive stress components are marked in blue and red, respectively. The color bar of $\hat{E}_{\text{tot}} (x)$ is identical to that displayed in Figs. 5.24-5.28. The green and red triangles represent the supports and load application point, respectively.
C. Experimental investigations

**Fig. C.5:** (a)-(e) Normalized waveforms of the selected AE sets (red) recorded during LS1 by KSB250 sensors #1 to #8 and corresponding deconvolved and normalized AE sets \( u(t) \equiv u_0^3(t) \), blue) that were used as input for the inverse simulation in TRM of reinforced concrete (Section 5.3).

**Fig. C.6:** (a)-(d) Normalized waveforms of the selected AE sets (red) recorded during LS2 by KSB250 sensors #1 to #8 and corresponding deconvolved and normalized AE sets \( u(t) \equiv u_0^3(t) \), blue) that were used as input for the inverse simulation in TRM of reinforced concrete (Section 5.3).

**Fig. C.7:** (a)-(e) Normalized waveforms of the selected AE sets (red) recorded during LS3 by KSB250 sensors #1 to #8 and corresponding deconvolved and normalized AE sets \( u(t) \equiv u_0^3(t) \), blue) that were used as input for the inverse simulation in TRM of reinforced concrete (Section 5.3).

**Fig. C.8:** (a)-(d) Normalized waveforms of the selected AE sets (red) recorded during LS4 by KSB250 sensors #1 to #8 and corresponding deconvolved and normalized AE sets \( u(t) \equiv u_0^3(t) \), blue) that were used as input for the inverse simulation in TRM of reinforced concrete (Section 5.3).

**Fig. C.9:** (a)-(g) Normalized waveforms of the selected AE sets (red) recorded during LS5 by KSB250 sensors #1 to #8 and corresponding deconvolved and normalized AE sets \( u(t) \equiv u_0^3(t) \), blue) that were used as input for the inverse simulation in TRM of reinforced concrete (Section 5.3).
C.3. Four-point bending test

Fig. C.5: (a) LS1 – TR_2

Fig. C.5: (b) LS1 – TR_12
C. Experimental investigations

Fig. C.5: (c) LS1 – TR.26

Sensor #1

Recorded

Deconvolved

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.5: (d) LS1 – TR.47

Sensor #1

Recorded

Deconvolved

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C.3. Four-point bending test

Fig. C.5: (e) LS1 – TR.49

![Graph showing data for Sensor #1, Sensor #2, Sensor #3, Sensor #4, Sensor #5, Sensor #6, Sensor #7, Sensor #8.]

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<th>$F_{cr}^*$</th>
<th>TR.2</th>
<th>TR.9</th>
<th>TR.12</th>
<th>TR.26</th>
<th>TR.32</th>
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<td>$F$ [kN]</td>
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<td>8.8</td>
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<td>308</td>
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Tab. C.1: Collection of AE sets selected at LS1 with corresponding load $F$ versus global time $t_{global}$. The predicted cracking load $F_{cr}$ is denoted with $^*$. 
Fig. C.6: (a) LS2 – TR

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.6: (b) LS2 – TR

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C.3. Four-point bending test

Fig. C.6: (c) LS2 – TR_221

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.6: (d) LS2 – TR_228

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C. Experimental investigations

Fig. C.7: (a) LS3 – TR.279

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.7: (b) LS3 – TR.286

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C.3. Four-point bending test

Fig. C.7: (c) LS3 – TR_293

Sensor #1

![Sensor #1](image1)

Sensor #2

![Sensor #2](image2)

Sensor #3

![Sensor #3](image3)

Sensor #4

![Sensor #4](image4)

Sensor #5

![Sensor #5](image5)

Sensor #6

![Sensor #6](image6)

Sensor #7

![Sensor #7](image7)

Sensor #8

![Sensor #8](image8)

Fig. C.7: (d) LS3 – TR_305

Sensor #1

![Sensor #1](image9)

Sensor #2

![Sensor #2](image10)

Sensor #3

![Sensor #3](image11)

Sensor #4

![Sensor #4](image12)

Sensor #5

![Sensor #5](image13)

Sensor #6

![Sensor #6](image14)

Sensor #7

![Sensor #7](image15)

Sensor #8

![Sensor #8](image16)
Fig. C.7: (e) LS3 – TR 360

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C.3. Four-point bending test

**Fig. C.8: (a) LS4 – TR.374**

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

**Fig. C.8: (b) LS4 – TR.390**

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
Fig. C.8: (c) LS4 – TR_420

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.8: (d) LS4 – TR_423

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C.3. Four-point bending test

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<th>TR_126</th>
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<td>16.6</td>
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Tab. C.2: Collection of AE sets selected at LS2 with corresponding load $F$ versus global time $t_{\text{global}}$.

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Tab. C.3: Collection of AE sets selected at LS3 with corresponding load $F$ versus global time $t_{\text{global}}$.

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Tab. C.4: Collection of AE sets selected at LS4 with corresponding load $F$ versus global time $t_{\text{global}}$. 
C. Experimental investigations

Fig. C.9: (a) LS5 – TR_467

Fig. C.9: (b) LS5 – TR_484
C.3. Four-point bending test

Fig. C.9: (c) LS5 – TR

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.9: (d) LS5 – TR

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C. Experimental investigations

Fig. C.9: (e) LS5 – TR_534

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8

Fig. C.9: (f) LS5 – TR_559

Sensor #1

Sensor #2

Sensor #3

Sensor #4

Sensor #5

Sensor #6

Sensor #7

Sensor #8
C.3. Four-point bending test

Fig. C.9: (g) LS5 – TR_{562}

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<th>TR_{467}</th>
<th>TR_{484}</th>
<th>TR_{500}</th>
<th>TR_{513}</th>
<th>TR_{523}</th>
<th>TR_{527}</th>
<th>TR_{532}</th>
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Tab. C.5: Collection of AE sets selected at LS1 with corresponding load $F$ versus global time $t_{global}$. The predicted ultimate load $F_u$ is marked with "*".
C. Experimental investigations
Bibliography


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