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Rules vs. Targets: Climate Treaties under Uncertainty [*]

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We demonstrate the advantages of a climate treaty based solely on rules for international permit markets when there is uncertainty about abatement costs and environmental damages. Such a 'Rules Treaty' comprises a scaling factor and a refunding rule. Each signatory can freely choose the number of permits it allocates to domestic firms. For every permit so issued, an international agency is allowed to issue additional permits in accordance with the scaling factor. The agency auctions all additional permits and refunds all the revenues to the signatories according to the refunding rule. Our main finding is that for a sufficiently large scaling factor, the Rules Treaty approximates the globally optimal outcome in every state of the world. In this sense, newly arriving information is optimally processed. This is in stark contrast to treaties based on emission targets, even if countries fully comply with such targets. If countries are sufficiently homogeneous there exists, moreover, a refunding rule under which every country that abates more under the treaty than in the status quo ante can be compensated, so that all countries will participate voluntarily. If, however, countries are rather heterogeneous, some may decline to participate.

Keywords: Rules Treaties, Target Treaties, Climate Change, Uncertainty, Global Refunding Scheme, International Permit Markets

JEL: D81, H23, H41, Q54

1 Introduction

Motivation

There is now a broad consensus that the climate is changing and that this development is linked to the increasing stock of greenhouse gases (GHGs) resulting from human activity. Efforts to reverse this trend have not been successful because the international coordination necessary to realize substantial cuts in emissions is proving to be nearly impossible to achieve. This is due to a variety of reasons, but two particularly thorny problems stand out.

First, lower emissions are a global public good. The severity of climate change depends on the total stock of GHGs in the atmosphere; it is irrelevant where the emissions take place, and the benefits of abatement accrue to all countries. Thus, the voluntary abatement efforts of individual countries, if uncoordinated, will be below what is optimal from a global perspective. In fact, there is no international authority with the power to enforce desirable reductions in emissions. The result is the notorious free-rider problem: countries may not participate in a treaty, or if they do, they may not deliver on their promises.

Second, abatement costs and environmental damages are very difficult to predict. The cost-benefit analysis of any climate policy is therefore subject to much uncertainty, and finding the globally optimal level of emission reduction involves a great deal of guesswork. Any binding target[1], even an ex-ante optimal one, would turn out to be suboptimal upon the arrival of new information. Only continuous renegotiation could incorporate the steady trickle of new information, but this is impossible at an international level. This is why policy-makers hesitate to agree to binding reductions in the first place.

The interplay of incentives to free-ride and the uncertainties about costs and benefits make it particularly difficult to estimate, design, and implement a suitable climate policy. This paper proposes a blueprint for a global climate treaty that would overcome these challenges. We start from the principle that treaties should be confined to rules. Our main insight is that there exists a set of rules that will allow new information on abatement costs and environmental damages to be processed efficiently without the necessity of renegotiating the treaty.

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[1] Suppose for the moment that we could prevent free-riding and enforce any binding emission reduction target.

In the remaining part of this introduction, we first discuss briefly the various uncertainties that climate policy has to deal with. We proceed with a general description of the model and introduce our proposed climate treaty, called the 'Rules Treaty,' and several benchmark scenarios. Thereafter we show the major results. Finally, we outline the organization of the rest of the paper.

Climate Policy Architecture and the Impact of Uncertainty

In 1992 and the years that followed, all major countries signed the Framework Convention on Climate Change. All signatories acknowledge that dangerous interference with the climate system has to be prevented. Thereafter, many proposals concerning the form of a suitable policy architecture to achieve this goal have been put forward. Aldy et al. (2003) define six criteria for assessing climate policy regimes: environmental outcome, cost-effectiveness, dynamic efficiency[2], equity, flexibility in the presence of new information, and participation and compliance. They examine the performance of the Kyoto protocol, which was drawn up in 1997, and a variety of other proposals in the light of these criteria. Several results are worth highlighting. First, no proposal fulfills all criteria. Second, in very few proposals is the provision of incentives for participation and compliance considered. Third, there exists a fundamental tension between achieving efficiency and promoting participation and compliance.

International permit markets with refunding have the potential to overcome these tensions, and it gives participants the possibility of utilizing new information. In treaties that do not allow the utilization of new information, this restriction on putting learning to good use has in most cases an adverse effect on coalition formation (Dellink et al., 2007). This is especially relevant for climate change policy, since there are still tremendous uncertainties. We will now briefly show why this is the case.

The current generation has to start planning and paying for emission abatement, but most of the potential damage caused by climate change will have to be borne by future generations. It is, however, difficult to determine what sacrifices should be made today in order to reduce damage in the future, since the valuation of the damage and the abatement costs is highly controversial. Figure 1 displays the causal chain from GHG emissions to the economic valuation of global damage at each date. Each and every link involves uncertainty, and together they add up to an immense uncertainty regarding

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[2] An efficient climate policy is by definition both cost-effective and environmentally optimal.

```
                        Emissions
                     1.↓
              Atmospheric stock of GHGs
                     2.↓
              Global temperature change
                     3.↓
                   Local changes
                     4.↓
                   Local impacts
                     5.↓
            Economic valuation of local impacts
                     6.↓
            Agreggation across time and space
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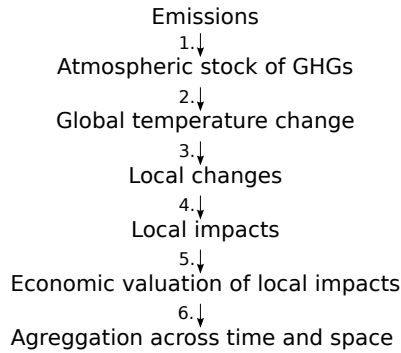
Figure 1: Multiple uncertainties in the economic valuation of environmental damage caused by GHGs emissions: every link in the causal chain is associated with uncertainties

damage. Links 1 to 3 involve uncertainties because the complexities of the earth system hinder predictions with small confidence bounds (Solomon et al., 2007). Link 4 has to do with the extent and effectiveness of changes in behavior and adaptation measures. Link 5 involves valuing these changes. Finally, aggregation across regions and generations is required, a step that involves value judgments about the distribution of real living standards at any point in time and the weights to be assigned to different generations. This is a matter on which there is much disagreement (see e.g. Stern (2007), Nordhaus (2007), Ackerman et al. (2009)). Abatement costs are somewhat less uncertain. Estimates depend mainly on the predicted speed of technological change and its impact on costs and the availability of technologies for reducing emissions or storing GHGs (Metz et al., 2007).

The Approach

The model is a variant of Gersbach and Winkler (2011) who have introduced international permit markets with refunding of revenues in permit auctions. The first modification is to allow the international agency to rescale the total number of permits; the second is to introduce uncertainty. To model the effects of uncertainty, we use the
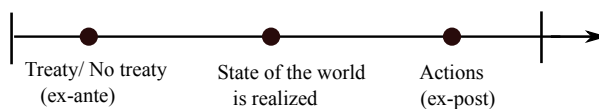
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    ├────●──────────●──────────●──────┤──────▶
      Treaty/ No treaty   State of the world   Actions
        (ex-ante)         is realized        (ex-post)
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Figure 2: General sequence of events

time-line depicted in Figure 2. During climate-treaty negotiations (ex-ante) only the probabilities and the state space of environmental damage and abatement costs in each country are known. After the negotiations, uncertainty is resolved. Actions and outcomes (distribution and trading of permits, emission choices, damage, etc.) take place under complete information (ex-post).

Benchmarks Scenarios and the Rules Treaty

In the paper we will first define the 'Global Social Optimum' as the outcome which minimizes total global costs in each state of the world (ex-post optimality).

Next, we will describe three different outcomes of international coordination (or non-coordination): the 'No-Treaty Outcome', the 'Target Treaty' and the Rules Treaty. All of them assume that an international permit market for GHG emissions is in place, and countries distribute a certain number of permits to domestic firms. Firms may, in turn, trade their permits. The initial distribution of permits is the essential difference among the three different outcomes. They can be described as follows:

The No-Treaty Outcome is the non-cooperative solution in which countries freely choose the number of permits they will distribute to domestic firms. It also serves as the outside option for all parties during treaty negotiations.

Under the (ideal) Target Treaty, signatories cooperate fully; they fix the total number of permits at a level such that, for them, it is ex-ante optimal. This total is then distributed among member countries. The total number of permits and the distribution cannot be renegotiated ex-post and countries fully comply. This (ideal) Target Treaty serves as a benchmark, as it represents the best possible treaty involving fixed targets.

The Rules Treaty works as follows: The permit market is administered by an international agency and obeys a set of rules negotiated by the participating countries. The rules comprise a scaling factor and country-specific refunding shares. After the Rules Treaty has come into force, participants can freely choose the number of emission permits they allocate to domestic firms. For every permit a country issues, the international agency is allowed to issue additional permits in accordance with the scaling factor. The agency auctions its permits to member countries' firms and reimburses all its revenues to member countries in line with the refunding shares.

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Results

We will obtain two major insights regarding the Rules Treaty.

First, it allows the processing of new information, as it fixes rules but not the number of permits. Thus, the countries will be able, and willing, to adjust their permit issuance when they receive new information on damages or abatement costs.[3] This is in stark contrast to treaties based on fixed targets, like our Target Treaty.

For large scaling factors, information processing is almost optimal under a Rules Treaty. The reason is that global costs decrease with the scaling factor, and approach the Global Social Optimum in each state of the world when scaling factors become large. This seems to be counterintuitive at first, and we will devote a large part of the paper to prove the result and to provide the intuition (see e.g. Section 5.2).

Second, the Rules Treaty can be designed to induce voluntary participation, i.e. all countries are better off ex-ante if they participate than by choosing the outside option. This can be achieved by selecting a refunding rule that compensates those countries with high expected abatement costs and low expected damages for making higher abatement efforts under the treaty than they would make under the outside option. This compensation is feasible, since expected global costs will be lower under a Rules Treaty with a strictly positive scaling factor.[4]

These two features of the Rules Treaty combine to prevent free-riding altogether if the scaling factor is sufficiently high. The first implies incentive compatibility and the second that participation is individually rational.

Against the above advantages there is the drawback that arriving at a Rules Treaty might be difficult or even impossible if countries are rather heterogeneous. Hence, Rules Treaties are more likely to come about among groupings like the EU.

Organization of the Paper

The remainder of the paper is organized as follows: In Section 2 we describe the set-up of the model. In Section 3 we define and derive closed-form solutions for the Global Social Optimum and the No-Treaty Outcome. The same is done for the Target and

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[3] Information that reveals higher marginal damages or lower marginal abatement costs, for instance, will typically induce countries to tighten permit issuance.

[4] Our outside option, the No-Treaty Outcome, corresponds to a degenerate Rules Treaty with a scaling factor of zero. In addition, global costs decrease with the scaling factor.

Rules Treaties, respectively, in Sections 4 and 5, followed by a comparison in Sections 6. Section 7 examines one way in which the formation of the Rules Treaty could be modeled. Section 8 discusses the parties' behavior under the treaties when extreme events occur. Section 9 summarizes the results, proposes some promising extensions, and draws more general conclusions.

2 The Model

2.1 Set-up

We consider a model of a global economy with n politically autonomous countries indexed by $i = 1, \ldots, n$ and run by a local planner. In each country i, GHG emissions $e_i > 0$ arise from the activity of a representative firm, which faces a country-specific abatement cost $C_i(e_i)$:

$$C_i(e_i) = \frac{1}{2\phi_i^{\omega_\phi}}(\bar{e}_i - e_i)^2, \quad e_i \in (0, \bar{e}_i), \quad \phi_i^{\omega_\phi} > 0, \quad i = 1, \ldots, n. \tag{1}$$

The abatement cost function describes the costs the firm incurs to reduce emissions. There are no abatement costs for $e_i = \bar{e}_i$, which are business-as-usual emissions. $\phi_i^{\omega_\phi}$ is the abatement cost parameter of country i. It depends on some aggregate event ω_ϕ, which can take on two values: $\omega_\phi \in \Omega_\phi := \{h_\phi, l_\phi\}$. The value $h_\phi(l_\phi)$ indicates that abatement cost are high (low) in all countries, that is $\phi_i^{h_\phi} < \phi_i^{l_\phi}$ for all i. The aggregate events h_ϕ and l_ϕ occur with probability π and $1 - \pi$, respectively. Constant parameter ratios are a special case. There, for all i, $\phi_i^{l_\phi} = \lambda^\phi \phi_i^{h_\phi}$ for some real number $\lambda^\phi > 1$.

The sum of all business-as-usual emissions across countries is denoted by $\bar{E}$ with $\bar{E} = \sum_{i=1}^n \bar{e}_i$, and the global emissions are $E = \sum_{i=1}^n e_i$. Without loss of generality we normalize the initial global stock of GHGs to zero. Global emissions cause country-specific environmental damages, $D_i(E)$, in each country, referred to as "damages", so that

$$D_i(E) = \frac{\beta_i^{\omega_\beta}}{2}E^2, \quad \beta_i^{\omega_\beta} > 0, \quad i = 1, \ldots, n . \tag{2}$$

$\beta_i^{\omega_\beta}$ is the damage parameter of country i that depends on a second aggregate event ω_β, which can also take on two values: $\omega_\beta \in \Omega_\beta := \{h_\beta, l_\beta\}$. Value $h_\beta(l_\beta)$ indicates that damages in all countries are high (low), i.e. $\beta_i^{h_\beta} > \beta_i^{l_\beta}$ for all i. The aggregate events h_β

and l_β occur with probability σ and $1 - \sigma$, respectively. Constant parameter ratios are a again a special case. There, for all i, $\beta_i^{h_\beta} = \lambda^\beta \beta_i^{l_\beta}$ for some real number $\lambda^\beta > 1$.

To describe the entire set of possible aggregate events, we define $\Omega = \Omega_\phi \times \Omega_\beta = \{(l_\phi, l_\beta), (h_\phi, l_\beta), (l_\phi, h_\beta), (h_\phi, h_\beta)\}$. We use $\omega = (\omega_\phi, \omega_\beta)$ with $\omega \in \Omega$ to denote a state of the world. $Prob(\omega)$ is the probability that ω will occur $(\sum_{\omega \in \Omega} Prob(\omega) = 1)$. A special case is stochastically independent aggregate events ω_ϕ and ω_β. The probabilities are given in Table 1:

	h_β	l_β
h_ϕ	$\pi\sigma$	$\pi(1 - \sigma)$
l_ϕ	$(1 - \pi)\sigma$	$(1 - \pi)(1 - \sigma)$

Table 1: Probabilities of states of the world if aggregate events are stochastically independent

We assume that the probabilities and the values $\beta_i^{\omega_\beta}$ and $\phi_i^{\omega_\phi}$ for all states of the world and all countries are common knowledge. Furthermore, once the aggregate events have been realized, the state of the world is also common knowledge.

To shorten the notation we will henceforth write ϕ_i^ω instead of $\phi_i^{\omega_\phi}$ and β_i^ω instead of $\beta_i^{\omega_\beta}$ if $\omega = (\omega_\phi, \omega_\beta)$ occurs. The use of the superscript ω will apply in the same way to other variables as well.

2.2 International Permit Market

As a building block for our analysis, we introduce an international permit market in the sense of Helm (2003), coupled with refunding in the sense of Gersbach and Winkler (2011), which operates after the aggregate events have occurred. In this and the next subsection, there is no administrative agency issuing additional permits, and we take the amount of emission permits each country has issued in state ω, ϵ_i^ω, as given. Later, ϵ_i^ω will be endogenized. The total number of permits $\mathcal{E}^\omega = \sum_{i=1}^n \epsilon_i^\omega$ is supplied in the market. For the functioning of the permit market, only $\mathcal{E}^\omega$ will matter. The possibility of trade guarantees that emission reductions will be cost effective.

Cost minimization behavior by the representative firms implies that in each country the marginal abatement costs will equal the permit price, p^ω, in each state ω:

$$p^\omega = -\frac{\partial C_i}{\partial e_i^\omega} = \frac{1}{\phi_i^\omega}(\bar{e}_i - e_i^\omega), \quad i = 1, \ldots, n . \tag{3}$$

Market clearing requires that the total number of permits equal global emissions:

$$\mathcal{E}^\omega = \sum_{i=1}^{n} \epsilon_i^\omega = \sum_{i=1}^{n} e_i^\omega = E^\omega. \tag{4}$$

To calculate the permit price in the permit market equilibrium for state ω, we sum equations (3) over all countries and obtain

$$p^\omega = \frac{\sum_{i=1}^{n} \bar{e}_i - \sum_{i=1}^{n} e_i^\omega}{\sum_{i=1}^{n} \phi_i^\omega} = \frac{\bar{E} - \mathcal{E}^\omega}{\Phi^\omega}, \tag{5}$$

where $\Phi^\omega := \sum_{i=1}^{n} \phi_i^\omega$ is referred to as the aggregate abatement cost parameter in state ω.

2.3 Global costs

For later use we devise expressions for global costs when international permit markets are present. For any particular state ω, we use cost minimization behavior by the firm, given by Equation 3, to rewrite optimal chosen abatement costs (Equation 1) in terms of the permit price:

$$C_i^\omega(p^\omega) = \frac{\phi_i^\omega}{2}(p^\omega)^2. \tag{6}$$

The sum of all abatement costs and damages across countries in a state ω are the global costs, denoted by K^ω. Using Equations (2) and (6), we can express global costs in terms of the permit price and the total number of permits.

$$K^\omega = \sum_{i=1}^{n} \frac{\phi_i^\omega}{2}(p^\omega)^2 + \sum_{i=1}^{n} \frac{\beta_i^\omega}{2}(\mathcal{E}^\omega)^2, \text{ and} \tag{7a}$$

$$K^\omega = \frac{\Phi^\omega}{2}(p^\omega)^2 + \frac{\mathcal{B}^\omega}{2}(\mathcal{E}^\omega)^2, \tag{7b}$$

where $\mathcal{B}^\omega := \sum_{i=1}^{n} \beta_i^\omega$ is referred to as the aggregate damage parameter.

Furthermore, the expected global cost, $\mathbb{E}[K]$, is given by

$$\mathbb{E}[K] = \sum_{\omega \in \Omega} Prob(\omega) \left[\frac{\Phi^\omega}{2} (p^\omega)^2 + \frac{\mathcal{B}^\omega}{2} (\mathcal{E}^\omega)^2 \right]. \tag{8}$$

3 Global Social Optimum and No-Treaty Outcome

In this section we derive the Global Social Optimum (SO) and the No-Treaty Outcome (NT). They serve as benchmark scenarios for the treaties we will introduce in the next section. The No-Treaty Outcome occurs when no negotiation has taken place or negotiations have failed, as countries can still operate an international permit market in such cases. It serves as the outside option, i.e. it establishes the level of costs countries have to undercut in order to be willing to participate in a treaty extending beyond the mere presence of an international permit market. Note that both the global social planner and participants in the permit market act after the state of the world has been realized.

Throughout the paper we focus on the interior solutions, in the sense that no country will choose $e_i = 0$ and reduce all emissions. Corner solutions of the type $e_i = \bar{e}_i$ cannot occur, as in such a case marginal costs of abatement are zero, while marginal damages are positive.

3.1 Global Social Optimum

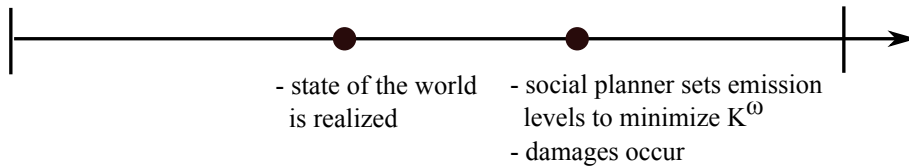The sequence of events in the Global Social Optimum is shown in Figure 3. In every state

- state of the world is realized
- social planner sets emission levels to minimize K^ω
- damages occur

Figure 3: Sequence of events in the Global Social Optimum

ω, a social planner ex-post minimizes global costs, K^ω, with respect to the emissions of countries, e_i^ω. Using Equations (1) and (2), the problem is given by

$$\min_{\{e_i^\omega\}_{i=1}^n} \sum_{i=1}^n \left[\frac{1}{2\phi_i^\omega} (\bar{e}_i - e_i^\omega)^2 + \frac{\beta_i^\omega}{2} (E^\omega)^2 \right], \tag{9}$$

9

subject to $\sum_{i=1}^{n} e_i^{\omega} = E^{\omega}$.

The necessary first-order conditions are

$$\frac{1}{\phi_i^{\omega}}(\bar{e}_i - e_i^{\omega}) = \mathcal{B}^{\omega} E^{\omega} \quad i = 1, \ldots, n. \tag{10}$$

The first-order conditions say that in the Global Social Optimum, the cost of abating an additional unit of emissions in a country i (LHS) has to equal the global damages caused by that additional unit (RHS).

Proposition 1 (Global Social Optimum)

In the Global Social Optimum, in every state of the world ω, the uniquely determined values of global emissions, $E^{SO,\omega}$, and local emissions, $e_i^{SO,\omega}$, are

$$E^{SO,\omega} = \frac{\bar{E}}{1 + \mathcal{B}^{\omega} \Phi^{\omega}}, \quad and \tag{11a}$$

$$e_i^{SO,\omega} = \bar{e}_i - \phi_i^{\omega} \frac{\mathcal{B}^{\omega} \bar{E}}{1 + \mathcal{B}^{\omega} \Phi^{\omega}}, \quad i = 1, \ldots, n. \tag{11b}$$

The proof is given in Appendix B.

We note that the Global Social Optimum can also be implemented by distributing any vector of emission permits $\{\epsilon_i^{\omega}\}_{i=1}^{n}$ to countries such that $\sum_{i=1}^{n} \epsilon_i^{\omega} = E^{SO,\omega}$ and by letting countries trade in the international permit market. In this case, the equilibrium price amounts to

$$p^{SO,\omega} = \frac{\mathcal{B}^{\omega} \bar{E}}{1 + \mathcal{B}^{\omega} \Phi^{\omega}}.$$

To rule out infeasible solutions, emissions as determined by Equation (11b) must not be negative. For the remainder of the paper, we therefore assume

Assumption 1

For all countries $i = 1, \ldots, n$ and all states ω the following condition holds:

$$\bar{e}_i \geq \phi_i^{\omega} \frac{\mathcal{B}^{\omega} \bar{E}}{1 + \mathcal{B}^{\omega} \Phi^{\omega}}.$$

3.2 No-Treaty Outcome

3.2.1 Description

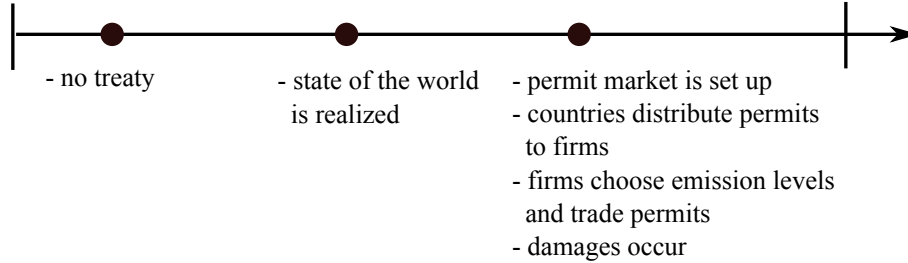The sequence of events in the No-Treaty Outcome is shown in Figure 4:

Figure 4: Sequence of events in the No-Treaty Outcome

After the state of the world has been realized, the local planner in each country is free to issue any amount of emission permits, ϵ_i^ω, which he distributes to the representative firm free of charge[5]. Given the total supply of permits, the equilibrium price is obtained via Equation (5), and the firms choose their emissions, e_i^ω, according to Equation (3). Firms buy or sell permits on the permit market such that the number of permits they hold matches their emissions.

3.2.2 Equilibrium Notation

At this stage, it is useful to differentiate the equilibrium notions used in the paper. In each state of the world, an equilibrium in the international permit market is characterized by a price of permits at which supply equals demand. An overall equilibrium for the No-Treaty case (henceforth equilibrium) is characterized by

- the Nash equilibrium of the local planners' permit choices,

- the abatement decisions by firms (taking permit prices as given), and

- market clearing.

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[5] The permits could also be auctioned by the local planner. It is well known that auctioning does not affect the cost-minimizing emission choice of firms and the total costs of a country if markets for permits are competitive. It only affects the internal distribution of costs and benefits within each country between the public and firms.

3.2.3 Equilibrium Characterization

Each country i chooses a number of permits, ϵ_i^ω, in such a way as to minimize local costs. This is the sum of the firm's abatement costs, local damage, and the cost arising from permit trading. It takes the permit choices of the other countries, $\epsilon_{j\neq i}^\omega$, as given.

$$\min_{\epsilon_i^\omega} \left[\frac{\phi_i^\omega}{2}(p^\omega)^2 + \frac{\beta_i^\omega}{2}(\mathcal{E}^\omega)^2 + p^\omega(e_i^\omega - \epsilon_i^\omega) \right], \quad i = 1, \ldots, n, \tag{12}$$

subject to (3), (4), and (5). We use the following derivatives:

$$\frac{dp^\omega}{d\epsilon_i^\omega} = \frac{\partial p^\omega}{\partial \mathcal{E}^\omega} \frac{d\mathcal{E}^\omega}{d\epsilon_i^\omega} = -\frac{1}{\Phi^\omega}(1), \text{ and}$$

$$\frac{de_i^\omega}{d\epsilon_i^\omega} = \frac{\partial e_i^\omega}{\partial p^\omega} \frac{dp^\omega}{d\epsilon_i^\omega} = -\phi_i^\omega(-\frac{1}{\Phi^\omega}) = \frac{\phi_i^\omega}{\Phi^\omega},$$

and obtain the necessary first-order conditions:

$$-\frac{\phi_i^\omega}{\Phi^\omega}p^\omega + \beta_i^\omega \mathcal{E}^\omega - \frac{1}{\Phi^\omega}(e_i^\omega - \epsilon_i^\omega) - p^\omega(1 - \frac{\phi_i^\omega}{\Phi^\omega}) = 0, \quad i = 1, \ldots, n. \tag{13}$$

The four terms represent various effects on the costs of a country i if it issues one more permit.[6]

1. $-\frac{\phi_i^\omega}{\Phi^\omega}p^\omega$: decrease of the firms' abatement costs. As there is one more permit in the market, the firm will increase its emissions by $\frac{de_i^\omega}{d\epsilon_i^\omega} = \frac{\phi_i^\omega}{\Phi^\omega}$. Its abatement costs decrease by the marginal abatement costs multiplied by the increase in emissions.

2. $+\beta_i^\omega \mathcal{E}^\omega$: increase in local damages. One more permit causes an equivalent increase of global emissions.

3. $-\frac{1}{\Phi^\omega}(e_i^\omega - \epsilon_i^\omega)$: change in trade revenues. The permit price decreases by $\frac{1}{\Phi^\omega}$. The firm has to pay less for the permits it needs to cover its emissions, e_i^ω, but it also receives less for the permits it sells, ϵ_i^ω. Depending on whether the firm is a net seller ($e_i^\omega < \epsilon_i^\omega$) or buyer ($e_i^\omega > \epsilon_i^\omega$) of permits, the revenues will decrease or increase, respectively.

4. $-p^\omega(1 - \frac{\phi_i^\omega}{\Phi^\omega})$: value of the remainder of the additional permit. From the one additional permits it receives, the firm needs the fraction $\frac{\phi_i^\omega}{\Phi^\omega}$ to cover its additional

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[6] To simplify presentation, we represent the marginal effects by the issuance of one more permit. We are aware that this is not exactly a marginal change.

emissions, and it sells the remaining fraction $(1 - \frac{\phi^\omega}{\Phi^\omega})$ at price p^ω.

Note that the first and fourth effect sum up to p^ω and the net benefit for country i from the first and the fourth effect is simply the value of the additional permit. Hence, we can rewrite the first-order conditions (13) as

$$p^\omega - \frac{1}{\Phi^\omega}(\epsilon_i^\omega - e_i^\omega) = \beta_i^\omega \mathcal{E}^\omega, \quad i = 1, \ldots, n. \tag{14}$$

The marginal benefits of issuing a permit (LHS) equal the local marginal damages (RHS).

Proposition 2 (No-Treaty Outcome)

We obtain a unique equilibrium of the No-Treaty Outcome in each state ω as a vector of permit issuance $\{\epsilon_i^{NT,\omega}\}_{i=1}^n$ and the associated total number of permits, $\mathcal{E}^{NT,\omega}$, the permit price, $p^{NT,\omega}$, a vector of country specific emissions, $\{e_i^{NT,\omega}\}_{i=1}^n$ and costs, $\{K_i^{NT,\omega}\}_{i=1}^n$, given by

$$\mathcal{E}^{NT,\omega} = \frac{\bar{E}}{1 + \bar{\beta}^\omega \Phi^\omega}, \tag{15a}$$

$$p^{NT,\omega} = \frac{\mathcal{B}^\omega \bar{E}}{n + \mathcal{B}^\omega \Phi^\omega}, \tag{15b}$$

$$e_i^{NT,\omega} = \bar{e}_i - p^{NT,\omega} \phi_i^\omega, \quad i = 1, \ldots, n, \tag{15c}$$

$$\epsilon_i^{NT,\omega} = \bar{e}_i - p^{NT,\omega} \Phi^\omega \left(\frac{\phi_i^\omega}{\Phi^\omega} + \frac{\beta_i^\omega - \bar{\beta}^\omega}{\bar{\beta}^\omega}\right), \quad i = 1, \ldots, n, \text{ and} \tag{15d}$$

$$K_i^{NT,\omega} = \frac{\phi_i^\omega}{2}(p^{NT,\omega})^2 + \frac{\beta_i^\omega}{2}(\mathcal{E}^{NT,\omega})^2 + p^{NT,\omega}[e_i^{NT,\omega} - \epsilon_i^{NT,\omega}], \quad i = 1, \ldots, n. \tag{15e}$$

$\bar{\beta}^\omega := \frac{1}{n} \sum_{i=1}^n \beta_i^\omega$ is the average damage parameter. The proof is given in Appendix B.

The difference between country-specific emissions and permit issuance in equilibrium is

$$e_i^{NT,\omega} - \epsilon_i^{NT,\omega} = p^{NT,\omega} \Phi^\omega \frac{\beta_i^\omega - \bar{\beta}^\omega}{\bar{\beta}^\omega}. \tag{16}$$

Analogous to the findings in Helm (2003), countries with damage parameters lower than the average are net sellers and vice versa.

With regard to permit issuance, two polar cases could happen in theory. First $\epsilon_i^{NT,\omega} > \bar{e}_i$, so that a country issues more permits than its business-as-usual emissions. Second, $\epsilon_i^{NT,\omega} < 0$ which means that a country destroys permits in order to reduce the global supply of permits. We allow for both polar cases.

Comparing Equations (15a) and (11a), we observe the standard result that the total emissions chosen by countries are higher than in the Global Social Optimum. The No-Treaty Outcome is inefficient, as the local planner of a country does not take into account other country's damages when choosing the number of permits.

3.2.4 Free-rider Problem

To illustrate the free-rider problem further, let us take k replica of the economy with n countries each. In this case, the total number of permits in the No-Treaty Outcome and the total emissions in the social optimum are given by

$$\mathcal{E}^{NT,\omega} = \frac{\bar{E}}{\frac{1}{k} + \bar{\beta}^\omega \Phi^\omega}, \text{ and}$$

$$E^{SO,\omega} = \frac{\bar{E}}{\frac{1}{k} + k\mathcal{B}^\omega \Phi^\omega}.$$

In the limit, as k approaches infinity, the socially optimal emissions are zero, whereas global emissions approach a constant greater than zero in the No-Treaty Outcome.

3.3 Participation Constraint

In this paper we discuss two treaties. A country signs a treaty voluntarily if the expected cost of participating is lower than in the ex-ante outside option, which we assume to be the No-Treaty Outcome. Essentially, every country is pivotal on whether a treaty is formed or not because both treaties require unanimous agreement to be put into force. This represents the most favorable scenario for achieving consensus on forming a treaty. If this participation constraint is fulfilled for all countries, we call a treaty implementable:

Definition 1 (Implementable Treaty)
A treaty is implementable if, for each country $i = 1, \ldots, n$, the expected cost of participating in the treaty is no higher than in the No-Treaty Outcome.

3.4 Example

Throughout the paper, we will illustrate the outcomes with an example. We consider three countries. Table 2 summarizes their parameters.

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Country	Business-as-usual	Abatement		Damage	
i	$\bar{e}_i$	$\phi_i^{h_\phi}$	$\phi_i^{l_\phi}$	$\beta_i^{h_\beta}$	$\beta_i^{l_\beta}$
1	60	0.2	0.7	0.6	0.3
2	70	0.4	0.8	0.7	0.35
3	80	0.6	0.9	0.7	0.45
Probabilities		50%	50%	50%	50%

Table 2: Parameters of our example

The aggregate events are assumed to be stochastically independent and occur with probability $\frac{1}{2}$. Hence, each event $\omega = (\omega_\phi, \omega_\beta)$ occurs with probability $\frac{1}{4}$.

Figure 5 shows that in the No-Treaty Outcome all firms emit more GHGs than in the Global Social Optimum. However, they issue fewer permits than their business-as-usual emissions. Country 1's damage parameter is below average and, according to Equation 16, its firm is a net seller of permits. Abatement, $\bar{e}_i - e_i^{NT,\omega}$, by the firm in country 1 is half of country 2's firm and a third of country 3's firm, which reflects the difference in the abatement cost parameters (Equation 15c).

4 Target Treaty

Fixed targets are the approach taken by the currently most important attempt to slow down climate change: the UNFCCC process, including the Kyoto Protocol. In this section, we present our model of such a type of treaty, the Target Treaty (TT). We assume that the Target Treaty minimizes expected global costs. Correspondingly, it is an 'optimal' Target Treaty. This assumption is not made to reflect the state of current target treaties such as the Kyoto Protocol. Instead, the optimal Target Treaty serves as a benchmark, as it represents the best solution that can be achieved if countries commit to emission reduction targets.

4.1 Treaty Description

The sequence of events and functions is shown in Figure 6. Target Treaty negotiations take place before the state of the world has been realized. This reflects the fact that once countries have agreed on targets, continuous renegotiation after the arrival of new
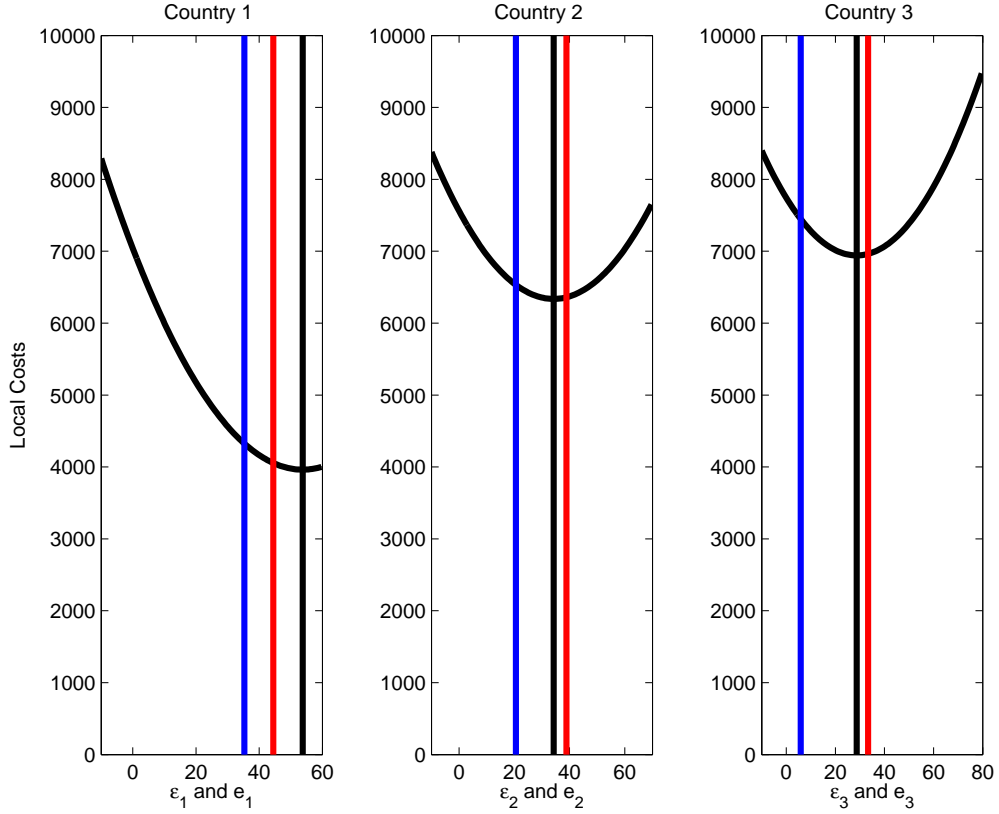
Figure 5: No-Treaty Outcome in state of the world $\omega = (h_\phi, h_\beta)$. The black curves display the local cost for a country i (see cost function (12)) in the No-Treaty Outcome, given that the other countries choose permits at the equilibrium levels. The minima of the local cost curves represent the equilibrium. Black vertical lines are the equilibrium permit issuance and red vertical lines are the equilibrium emissions. Blue vertical lines are the emissions in the Global Social Optimum. The ends of the abscissas correspond to business-as-usual emissions.

information is impossible. During the negotiation, stage an ex-ante optimal total number of permits, $\mathcal{E}^{TT}$, is determined. Countries receive permits according to an allocation, $\left\{ \epsilon_i^{TT} \right\}_{i=1}^n$, such that $\sum_{i=1}^n \epsilon_i^{TT} = \mathcal{E}^{TT}$. Formally we define the Target Treaty as follows:

Definition 2 (Target Treaty)

A Target Treaty (TT) is an aggregate number of permits $\mathcal{E}^{TT}$ and an allocation of emission permits $\left\{ \epsilon_i^{TT} \right\}_{i=1}^n$, with $\sum_{i=1}^n \epsilon_i^{TT} = \mathcal{E}^{TT}$. $\mathcal{E}^{TT}$ minimizes expected global costs.

Figure 6: Sequence of events in the Target Treaty

For the purpose of this paper, we assume that the Target Treaty is binding.

Assumption 2

The allocation of emission permits $\left\{\epsilon_i^{TT}\right\}_{i=1}^n$ is binding for all countries and in all states of the world.

This assumption implies that in any state of the world, the number of permits countries give to its firms cannot exceed or fall below ϵ_i^{TT}. The former implies complete enforceability of the permit allocation. The latter could in principle happen if, e.g., a country in a particular state is highly affected by climate change and therefore wants to lower the aggregate number of permits by not using part of its allocation. We rule out this possibility, as it only occurs in extreme cases or with few and heterogeneous countries participating in the treaty.[7] In Section 8 we will discuss cases in which targets are not exhausted by countries when an extreme event happens.

In the following, we first illustrate the ex-ante design of the Target Treaty and later derive solutions for the ex-post stage.

4.2 Ex-ante: Design of the Target Treaty

In this section we determine the ex-ante globally optimal number of permits, $\mathcal{E}^{TT}$, and discuss implementable permit allocations.

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[7] The derivation of the ex-ante optimal target becomes much more cumbersome if ex-ante the potential destruction of permits in some extreme event has to be taken into account. If countries are homogeneous, a sufficient condition that targets are binding in all events is

$$\pi\lambda^\phi + 1 - \pi \le n(\sigma + \frac{1-\sigma}{\lambda^\beta}).$$

This condition is always satisfied if the number of countries is sufficiently large.

4.2.1 Total Number of Permits

Suppose there is a designer seeking to minimize the expected global cost $\mathbb{E}[K^{TT}]$ (see Equation (8)) with respect to $\mathcal{E}$, the total number of permits:

$$\min_{\mathcal{E}} \sum_{\omega} \left\{ Prob(\omega)[\frac{\Phi^{\omega}}{2}(p^{\omega})^2 + \frac{\mathcal{B}^{\omega}}{2}\mathcal{E}^2] \right\}, \tag{17}$$

subject to Equation (5).

The necessary condition is

$$\sum_{\omega} \left\{ Prob(\omega)[-p^{\omega} + \mathcal{B}^{\omega}\mathcal{E}] \right\} = 0.$$

As the objective function (17) is strictly convex, the necessary condition is also sufficient. We use Equation (5) and the assumption that the aggregate events are independent to determine the unique $\mathcal{E}^{TT}$:

$$\mathcal{E}^{TT} = \frac{\bar{E}A}{D+A}, \tag{18}$$

where $D := \sigma \mathcal{B}^h + (1-\sigma)\mathcal{B}^l$ and $A := (\frac{\pi}{\Phi^h} + \frac{1-\pi}{\Phi^l})$.[8]

Using Equations (5) and (8), the expected global cost in the Target Treaty, $\mathbb{E}[K^{TT}]$, amounts to

$$\mathbb{E}[K^{TT}] = \frac{A}{2}\left(\frac{\bar{E}D}{D+A}\right)^2 + \frac{D}{2}\left(\frac{\bar{E}A}{D+A}\right)^2.$$

4.2.2 Allocation of Permits

Apart from the feasibility constraint $\sum_{i=1}^{n} \epsilon_i^{TT} = \mathcal{E}^{TT}$, the determination of $\mathcal{E}^{TT}$ does not pin down the country-specific allocation of permits, ϵ_i^{TT}. An example of a feasible allocation is a Target Treaty denoted by $\widetilde{TT}$, such that for each country the permit allocation equals the expected emissions in the Global Social Optimum:

$$\epsilon_i^{\widetilde{TT}} = \mathbb{E}[e_i^{SO}] = \mathbb{E}[\bar{e}_i - \frac{\bar{E}D}{(D+A)\Phi}\phi_i] = \bar{e}_i - \frac{\bar{E}D}{D+A}\left(\frac{\sigma\phi^h}{\Phi^h} + \frac{(1-\sigma)\phi^l}{\Phi^l}\right), \quad i = 1,\dots,n.$$

---

[8] If there is no uncertainty (e.g. $\pi = \sigma = 0$), Equation (18) collapses to Equation (11a).

For the purpose of this paper we do not need to model the detailed process of how permits are allocated across countries. It is nevertheless useful to show the set of implementable Target Treaties. It is characterized by the following condition:

$$\mathbb{E}[K_i^{TT}(\epsilon_i^{TT})] \leq \mathbb{E}[K_i^{NT}], \quad \forall \, i = 1, \ldots, n, \tag{19}$$

where $\mathbb{E}[K_i^{TT}(\epsilon_i^{TT})]$ and $\mathbb{E}[K_i^{NT}]$ are the expected cost for country i in a Target Treaty with permit allocation ϵ_i^{TT} and in the No-Treaty Outcome, respectively. We obtain the following lemma:

Lemma 1 (Implementable Target Treaty)
There exists an implementable Target Treaty if and only if the expected global cost of the Target Treaty is no higher than in the No-Treaty Outcome, i.e. if and only if $\mathbb{E}[K^{TT}] \leq \mathbb{E}[K^{NT}]$.

The proof of Lemma 1 is standard and given in Appendix B. The intuition is straightforward. Once a treaty is globally less costly than the outside option and the redistribution of permits (i.e. costs) across countries does not generate frictions, the treaty is ex-ante implementable.[9]

4.3 Ex-post Outcome

In what follows, we consider the solution under a Target Treaty once the state of the world has been realized. The overall equilibrium corresponds to the equilibrium in the international permit market, as the amount of permit the countries' local planner distribute to their firms is fixed according to the allocation $\left\{\epsilon_i^{TT}\right\}_{i=1}^{n}$. The allocation is independent of the state of the world. The equilibrium permit price, however, depends on this state.[10] Combining (5) and (18), we obtain

$$p^{TT,\omega} = \frac{\bar{E}D}{(D+A)\Phi^\omega}. \tag{20}$$

---

[9] We note that $\widetilde{TT}$ might not be implementable.
[10] The price only depends on the aggregate event ω_ϕ.

Given the price, firms choose the equilibrium level of emissions, $e_i^{TT,\omega}$, in accordance with Equation (3):

$$e_i^{TT,\omega} = \bar{e}_i - \frac{\bar{E}D}{(D+A)\Phi^\omega}\phi_i^\omega, \quad i = 1,\ldots,n. \tag{21}$$

Inspection of Equation (21) shows that emissions are independent of the permit allocation.

Assumption 1 does not suffice to guarantee that emissions as determined by Equation (21) do not fall below zero. To ensure $e_i^{TT,\omega}$ is non-negative, we assume for the remainder of the paper:

Assumption 3
For all countries $i = 1,\ldots,n$ and all states ω, the following condition holds:

$$\bar{e}_i \geq \frac{\bar{E}D}{(D+A)\Phi^\omega}\phi_i^\omega.$$

We sum up the properties of the Target Treaty in the following proposition:

Proposition 3 (Target Treaty)
In a Target Treaty, the total number of permits, $\mathcal{E}^{TT}$, is given by Equation (18). There exists a set of implementable permit allocations $\left\{\epsilon_i^{TT}\right\}_{i=1}^n$ if $\mathbb{E}[K^{TT}] \leq \mathbb{E}[K^{NT}]$. In equilibrium, the permit price $p^{TT,\omega}$ in each state ω is given by (20), the vector of country-specific emissions $\{e_i^{TT,\omega}\}_{i=1}^n$ is determined according to (21) and costs $\{K_i^{TT,\omega}\}_{i=1}^n$ are given by

$$K_i^{TT,\omega} = \frac{\phi_i^\omega}{2}(p^{TT,\omega})^2 + \frac{\beta_i^\omega}{2}(\mathcal{E}^{TT})^2 + p^{TT,\omega}(e_i^{TT,\omega} - \epsilon_i^{TT}), \quad i = 1,\ldots,n.$$

Finally, we observe that in general $\mathcal{E}^{TT}$ is not ex-post optimal.

Lemma 2 (Target Treaty does not Implement Global Social Optimum)
If there is uncertainty about the damage and abatement cost parameters, there exists no Target Treaty that can implement the Global Social Optimum.

The proof of Lemma 2 is given in Appendix B. The intuition is as follows: under a Target Treaty emission reductions are fixed before the state of the world is known. Hence when new information arrives, countries are not able to react but have to stick to the old,

now suboptimal, reduction targets. This is a severe disadvantage of all treaties involving fixed targets.

4.4 Example

We illustrate the properties and potential problems of the Target Treaty using the numerical example introduced in Section 3.4.[11] Figure 7 shows the ex-ante globally optimal number of permits, $\mathcal{E}^{TT}$, and the ex-post globally optimal level of emissions in each state of the world, $E^{SO,\omega}$. We observe that the Target Treaty is not ex-post optimal. In
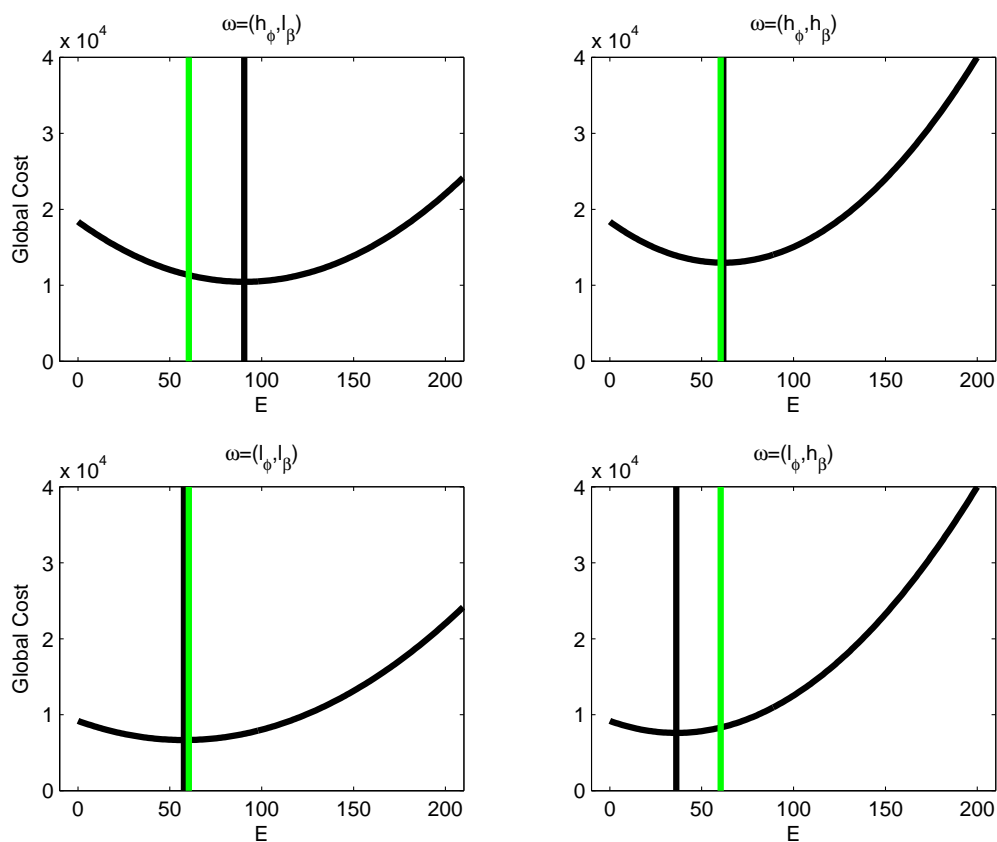
Figure 7: The black curves depict the global costs in the four different states. The vertical lines show emissions in the Global Social Optimum (black) and the Target Treaty (green).

the state $\omega = (h_\phi, l_\beta)$ abatement costs are high and damages low. Hence, the optimal

[11] The parameters fulfill Assumptions 2 and 3.

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global emissions, $E^{SO,(h_\phi,l_\beta)}$, are rather high, and $\mathcal{E}^{TT}$ is too low. By contrast in state $\omega = (l_\phi, h_\beta)$ abatement costs are low and damages high. $E^{SO,(l_\phi,h_\beta)}$ is rather low, and $\mathcal{E}^{TT}$ is too high. In the other two states, these two effects partly cancel out and $\mathcal{E}^{TT}$ is quite close to the Global Social Optimum.

5 Rules Treaty

In this section, we discuss our proposal for a climate treaty, the Rules Treaty. We start by describing how it functions, then we present the equilibrium in the ex-post stage and highlight the advantages of the Rules Treaty. Finally, we discuss treaty formation in the ex-ante stage.

5.1 Treaty Description

Figure 8 shows the functioning and sequence of events in the Rules Treaty.

Figure 8: Sequence of events in the Rules Treaty

It is similar to the No-Treaty Outcome in that countries can freely select the number of permits, ϵ_i^ω, once the state of the world is realized. However, there are two important differences.

First, there exists an international agency that is entitled to issue an additional number of permits $\mathcal{E}_A^\omega = \gamma \sum_{i=1}^n \epsilon_i^\omega = \gamma \mathcal{E}_C^\omega$, where $\gamma \in [0, \infty)$ is called the scaling factor. $\mathcal{E}_C^\omega$ denotes

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the aggregate number of permits issued by the countries. Hence, the total number of permits in the market is $\mathcal{E}_T^\omega = \mathcal{E}_A^\omega + \mathcal{E}_C^\omega = (1+\gamma)\mathcal{E}_C^\omega$.

Second, the agency sells its permits to the firms. The revenues generated are fully refunded to the countries according to a refunding rule $\{\rho_i\}_{i=1}^n$, with $\sum_{1=1}^n \rho_i = 1$. ρ_i is the share of the agency's revenues refunded to country i. The refunding rule does not depend on the abatement efforts of countries and is independent of the state of the world.

The crucial difference between the Rules Treaty and the Target Treaty is the following: During negotiations, in the Rules Treaty countries only fix the scaling factor and the refunding rule. In the Target Treaty, by contrast, emission targets are fixed.

Formally, we define the Rules Treaty as follows:

Definition 3 (Rules Treaty)

A Rules Treaty (RT) consists of the following set of parameters: $\{\gamma, \{\rho_i\}_{i=1}^n\}$. $\gamma \in [0,\infty)$ is the scaling factor, and $\{\rho_i\}_{i=1}^n$, with $\sum_{1=1}^n \rho_i = 1$, is the refunding rule.

In the following, we explore the functioning of the Rules Treaty and work backwards to solve for an implementable and optimal Rules Treaty.

1. Ex-post: Permit issuance: Upon realization of the state of the world ω, each country's local planner chooses a number of permits, given γ and $\{\rho_i\}_{i=1}^n$.

2.Ex-ante: Design of the Rules Treaty: During negotiations, countries set up a Rules Treaty $\{\gamma, \{\rho_i\}_{i=1}^n\}$ that has to be implementable and minimizes the expected global cost.

5.2 Ex-post Outcome

In this section, we describe the overall equilibrium (henceforth 'equilibrium') in each state of the world for the Rules Treaty. It is defined by the same three components (Nash equilibrium of the local planners' permit choices, abatement decisions of firms, market clearing) as those introduced in Section 3.2 for the No-Treaty Outcome.

5.2.1 Equilibrium Characterization

In every state ω, each country i receives a refund, r_i^ω, according to its share, ρ_i, of the revenues the agency generates by selling permits. It is given by

$$r_i^\omega = \rho_i p^\omega \gamma \mathcal{E}_C^\omega.$$

Each country i chooses a number of permits, ϵ_i^ω, to minimize its local costs, taking the permit choices of the other countries, $\epsilon_{j\neq i}^\omega$, as given.

$$\min_{\epsilon_i^\omega} \left[\frac{\phi_i^\omega}{2}(p^\omega)^2 + \frac{\beta_i^\omega}{2}(\mathcal{E}_T^\omega)^2 + p^\omega(e_i^\omega - \epsilon_i^\omega) - \rho_i \gamma p^\omega \mathcal{E}_C^\omega \right], \quad i = 1, \ldots, n, \tag{22}$$

subject to Equation (3), $E^\omega = \mathcal{E}_T^\omega = (1+\gamma)\sum_{i=1}^n \epsilon_i^\omega$ and $p^\omega = \frac{\bar{E} - \mathcal{E}_T^\omega}{\Phi^\omega}$. We use the following derivatives:

$$\frac{dp^\omega}{d\epsilon_i^\omega} = \frac{\partial p^\omega}{\partial \mathcal{E}_T^\omega}\frac{d\mathcal{E}_T^\omega}{d\epsilon_i^\omega} = -\frac{1}{\Phi^\omega}(1+\gamma), \text{ and}$$

$$\frac{de_i^\omega}{d\epsilon_i^\omega} = \frac{\partial e_i^\omega}{\partial p^\omega}\frac{dp^\omega}{d\epsilon_i^\omega} = -\phi_i^\omega(-\frac{1+\gamma}{\Phi^\omega}) = (1+\gamma)\frac{\phi_i^\omega}{\Phi^\omega},$$

and obtain the necessary first-order conditions for a cost minimum:

$$-(1+\gamma)\frac{\phi_i^\omega}{\Phi^\omega}p^\omega + (1+\gamma)\beta_i^\omega \mathcal{E}_T^\omega - \frac{(1+\gamma)}{\Phi^\omega}(e_i^\omega - \epsilon_i^\omega) - p^\omega\left(1 - (1+\gamma)\frac{\phi_i^\omega}{\Phi^\omega}\right)$$
$$-\rho_i \gamma p^\omega + \rho_i \frac{1+\gamma}{\Phi^\omega}\gamma \mathcal{E}_C^\omega = 0, \quad i = 1, \ldots, n. \tag{23}$$

The six terms represent various effects on the costs of a country i if it issues one more permit.[12] Effects 1-4 can be interpreted in the same way as in the No-Treaty Outcome, but they are amplified by the scaling factor. The new effects 5 and 6 are due to refunding.

1. $-(1+\gamma)\frac{\phi_i^\omega}{\Phi^\omega}p^\omega$: the decrease of the firms' abatement costs. As there are $(1+\gamma)$ more permits in the market, the firm increases its emissions by $\frac{de_i^\omega}{d\epsilon_i^\omega} = (1+\gamma)\frac{\phi_i^\omega}{\Phi^\omega}$. Its abatement costs decrease by the marginal abatement costs times the increase in emissions.

2. $+(1+\gamma)\beta_i^\omega \mathcal{E}_T^\omega$: the increase in local damages. $(1+\gamma)$ more permits cause an equivalent increase of global emissions.

---

[12] For ease of presentation and comparison to the No-Treaty Outcome, we represent the marginal effects by the issuance of one more permit. We are aware that this is not exactly a marginal change.

3. $-\frac{(1+\gamma)}{\Phi^\omega}(e_i^\omega - \epsilon_i^\omega)$: the change in trade revenues. The permit price decreases by $\frac{1+\gamma}{\Phi^\omega}$. The firm has to pay less for the permits it needs to cover its emissions, e_i^ω, but also receives less for the permits it sells, ϵ_i^ω. Depending on whether the firm is a net seller ($e_i^\omega < \epsilon_i^\omega$) or buyer ($e_i^\omega > \epsilon_i^\omega$) of permits, the revenues will decrease or increase, respectively.

4. $-p^\omega \left(1 - \frac{\phi_i^\omega}{\Phi^\omega}(1+\gamma)\right)$: the value of the remainder of the additional permit. From the one additional permits it receives, the firm needs the fraction $\frac{\phi_i^\omega}{\Phi^\omega}(1+\gamma)$ to cover its additional emissions, and it sells the remaining fraction $(1 - \frac{\phi_i^\omega}{\Phi^\omega}(1+\gamma))$ at price p^ω (if γ is sufficiently high, it has to buy the remaining fraction).

5. $-\rho_i \gamma p^\omega$: the additional refund the country receives. The agency auctions γ additional permits at price p^ω.

6. $+\rho_i \frac{1+\gamma}{\Phi^\omega} \gamma \mathcal{E}_C^\omega$: the value of the original refund decreases. As the price of the permits drops, the agency makes less revenue by selling the original permits, $\gamma \mathcal{E}_C$.

Again the first and fourth effect sum up to p^ω. Hence we rewrite the first-order conditions (23) as

$$p^\omega(1 + \rho_i \gamma) - \frac{1+\gamma}{\Phi^\omega}[\epsilon_i^\omega + \rho_i \gamma \mathcal{E}_C^\omega - e_i^\omega] = \beta_i^\omega \mathcal{E}_T^\omega(1+\gamma). \tag{24}$$

The marginal benefits of issuing a permit (LHS) equal the local marginal damages (RHS).

In the next lemma, we provide the necessary and sufficient conditions to ensure that for a country i the cost function (22) is convex.

Lemma 3 (Convexity of Cost Function)

The local cost function (22) is strictly convex in ϵ_i^ω in a state ω and for a country i if and only if

$$2\rho_i \gamma \geq \frac{\phi_i^\omega}{\Phi^\omega}(1+\gamma) - 2 - \beta_i^\omega \Phi^\omega(1+\gamma). \tag{25}$$

The proof of Lemma 3 is omitted as it is straightforward. Proposition 4 summarizes the properties of the equilibrium in the ex-post stage.

Proposition 4 (Rules Treaty)

Suppose we set up a Rules Treaty such that condition 25 holds in every state ω and for all countries i. Then there exists a unique equilibrium under a Rules Treaty with the vector of permit issuance, $\{\epsilon_i^{RT,\omega}\}_{i=1}^n$, the corresponding total number of permits issued from the countries and the agency, $\mathcal{E}_T^{RT,\omega}$, the permit price, $p^{RT,\omega}$, the vector of country specific emissions, $\{e_i^{RT,\omega}\}_{i=1}^n$, and costs, $\{K_i^{RT,\omega}\}_{i=1}^n$:

$$\mathcal{E}_T^{RT,\omega} = \frac{\bar{E}}{1 + \frac{\mathcal{B}^\omega \Phi^\omega}{1 + \frac{n-1}{1+\gamma}}}, \tag{26a}$$

$$p^{RT,\omega} = \frac{\mathcal{B}^\omega \bar{E}}{1 + \mathcal{B}^\omega \Phi^\omega + \frac{n-1}{1+\gamma}}, \tag{26b}$$

$$e_i^{RT,\omega} = \bar{e}_i - p^{RT,\omega}\phi_i^\omega, \quad i = 1,\ldots,n, \tag{26c}$$

$$\epsilon_i^{RT,\omega} = \frac{\bar{e}_i}{1+\gamma} - \frac{\gamma}{1+\gamma}\left\{\rho_i(\bar{E} - 2\Phi^\omega p^{RT,\omega}) - [\bar{e}_i - \Phi^\omega p^{RT,\omega}(\frac{\phi_i^\omega}{\Phi^\omega} + \frac{\beta_i^\omega}{\mathcal{B}^\omega})]\right\} \tag{26d}$$

$$- \frac{1}{1+\gamma}p^{RT,\omega}\Phi^\omega(\frac{\phi_i^\omega}{\Phi^\omega} + \frac{\beta_i^\omega - \bar{\beta}^\omega}{\bar{\beta}^\omega}), \quad i = 1,\ldots,n, \text{ and}$$

$$K_i^{RT,\omega} = \frac{\phi_i^{RT,\omega}}{2}(p^{RT,\omega})^2 + \frac{\beta_i^{RT,\omega}}{2}(\mathcal{E}_T^{RT,\omega})^2 \tag{26e}$$

$$+ p^{RT,\omega}(e_i^{RT,\omega} - \epsilon_i^{RT,\omega}) - \rho_i p^{RT,\omega}\gamma \mathcal{E}_C^{RT,\omega}, \quad i = 1,\ldots,n.$$

The proof of Proposition 4 is given in Appendix B. The next corollary highlights the fact that the (expected) global costs will decrease in γ. Furthermore, it characterizes the polar cases:

Corollary 1 (Properties of the Rules Treaty)

(i) *In all states ω,*

$$\frac{dK^{RT,\omega}(\gamma)}{d\gamma} < 0 \text{ for } \gamma \in [0,\infty).$$

(ii) *If $\gamma = 0$, the outcome of the Rules Treaty is the same as in the No-Treaty Outcome. Thus, $\mathcal{E}_T^{RT,\omega}(\gamma = 0) = E^{NT,\omega}$ and $K^{RT,\omega}(\gamma = 0) = K^{NT,\omega}$ for all ω.*

(iii) *If $\gamma \to \infty$, the outcome of the Rules Treaty approximates the Global Social Optimum. Thus, $\lim_{\gamma\to\infty} \mathcal{E}_T^{RT,\omega} \to E^{SO,\omega}$ and $\lim_{\gamma\to\infty} K^{RT,\omega} \to K^{SO,\omega}$.*

The proof of point (i) of Corollary 1 is given in Appendix B . Points (ii) and (iii) follow directly from Equations (26)a-(26)e by setting $\gamma = 0$ and by taking the limit $\gamma \to \infty$. [13]

The intuition of Corollary 1 is as follows: If a country issues an additional permit, changes in costs — at a global level and for the issuing country — can be separated into two parts. First, the benefits from using or selling the additional permit fully accrue to the issuing country, and thus local costs decline. The additional costs, however, have to be borne by all countries. Hence the issuing country has incentives to free-ride and overissue.[14] Second, γ additional permits are auctioned by the international agency, thus affecting global costs. As refunding shares are fixed, each country's costs move in the same direction, albeit with different weights.[15] As a consequence, local and global interests are fully aligned. The higher γ is, the more important is the latter part. This holds irrespective of how high the refunding share of the issuing country is.

To further fix ideas, suppose a Rules Treaty with a very high scaling factor is in place and the aggregate number of permits is as in the Global Social Optimum. If a country i issues an additional permit, its local costs will decrease as described above. However, there are $1 + \gamma$ additional permits in the market. Therefore, global costs, which, per definition, have been minimal at the Global Social Optimum, will increase steeply. As each country bears a fraction of the increase of total cost, the local costs of all countries including country i will increase. As γ is very high, the latter change completely dominates the former. If a country reduces permit issuance by one unit, local costs will increase both because the country can use or sell one permit less and because global costs increase steeply. Hence, neither increasing nor decreasing permit issuance is a profitable deviation for country i.

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[13] A remark is in order here. $\lim_{\gamma \to \infty} \mathcal{E}_C^{RT,\omega} \to 0$ but the individual choices of $\epsilon_i^{RT,\omega}$ will in general not be zero. It is, however, possible to set the refunding shares $\{\rho_i\}_{i=1}^n$ such that $\lim_{\gamma \to \infty} \epsilon_i^{RT,\omega} \to 0$ for all countries i in some state ω. Inspection of (26d) shows that we have to set ρ_i in such a way that the second term vanishes in state ω:

$$\rho_i^{0,\omega} = \frac{\bar{e}_i - \Phi^\omega p^{SO,\omega}(\frac{\phi_i^\omega}{\Phi^\omega} + \frac{\beta_i^\omega}{\mathcal{B}^\omega})}{\bar{E} - 2\Phi^\omega p^{SO,\omega}}. \tag{27}$$

Note that it is not possible to set ρ_i such that (27) is satisfied in all states ω.

[14] This is exactly what happens in the No-Treaty Outcome.

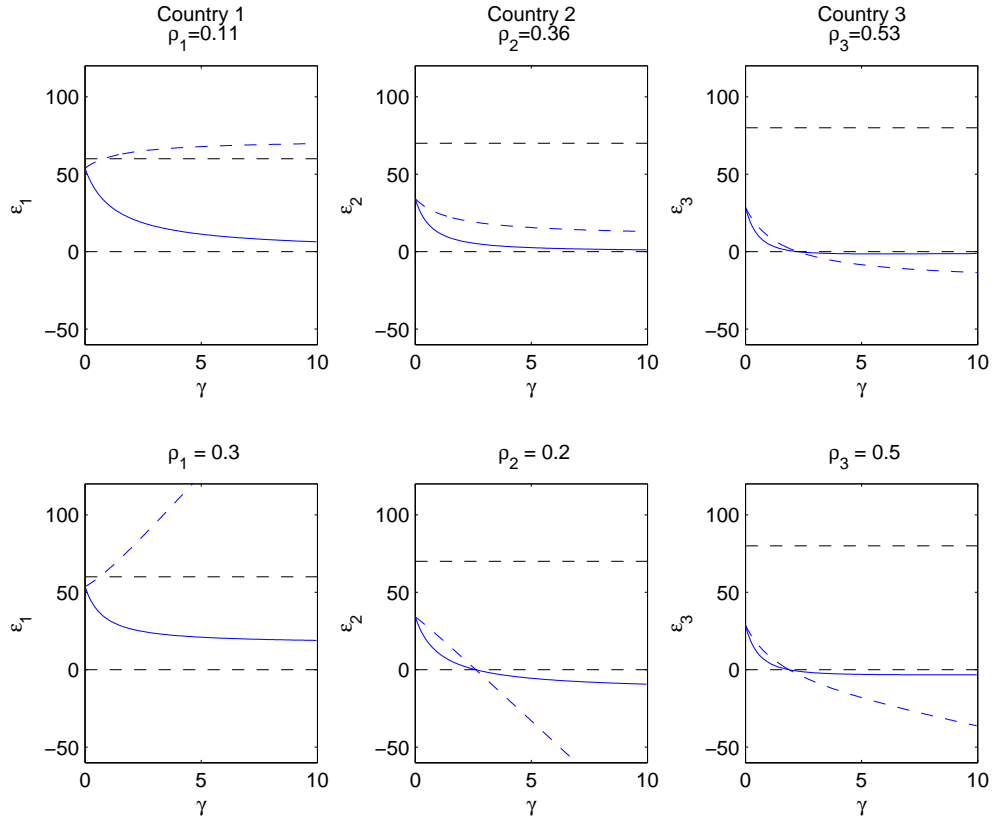[15] The Countries' costs move in the same direction as long as the refunding share is positive.

Figure 9: Equilibrium permit choices in the Rules Treaty as a function of γ for two different refunding rules (upper and lower part). In the upper part, refunding shares are given by Equation (27). The state of the world is $\omega = (h_\phi, h_\beta)$. Blue solid lines: $\epsilon_i^{RT,\omega}$. Blue dashed lines: $(1+\gamma)\epsilon_i^{RT,\omega}$. Black dashed horizontal lines are $\epsilon_i = 0$ and $\epsilon_i = \bar{e}_i$

5.2.2 Example

In the following, we use the example introduced in subsection 3.4 to show ex-post outcomes of the Rules Treaty. In accordance with Equation (26d), Figure 9 shows that every country will decrease its equilibrium issuance of permits, $\epsilon_i^{RT,\omega}$, when γ is increased. In the upper part, the refunding share for all i is $\rho_i = \rho_i^{0,\omega}$, therefore $\epsilon_i^{RT,\omega}$ approaches zero when the scaling factor becomes very large. For any other choice of the refunding rule, $\epsilon_i^{RT,\omega}$ remains bounded away from zero, as exemplified in the lower part of Figure 9.

Figure 10 depicts the difference of the equilibrium in a Rules Treaty with $\{\gamma = 1, \{\rho_i^{0,\omega}\}_{i=1}^n\}$
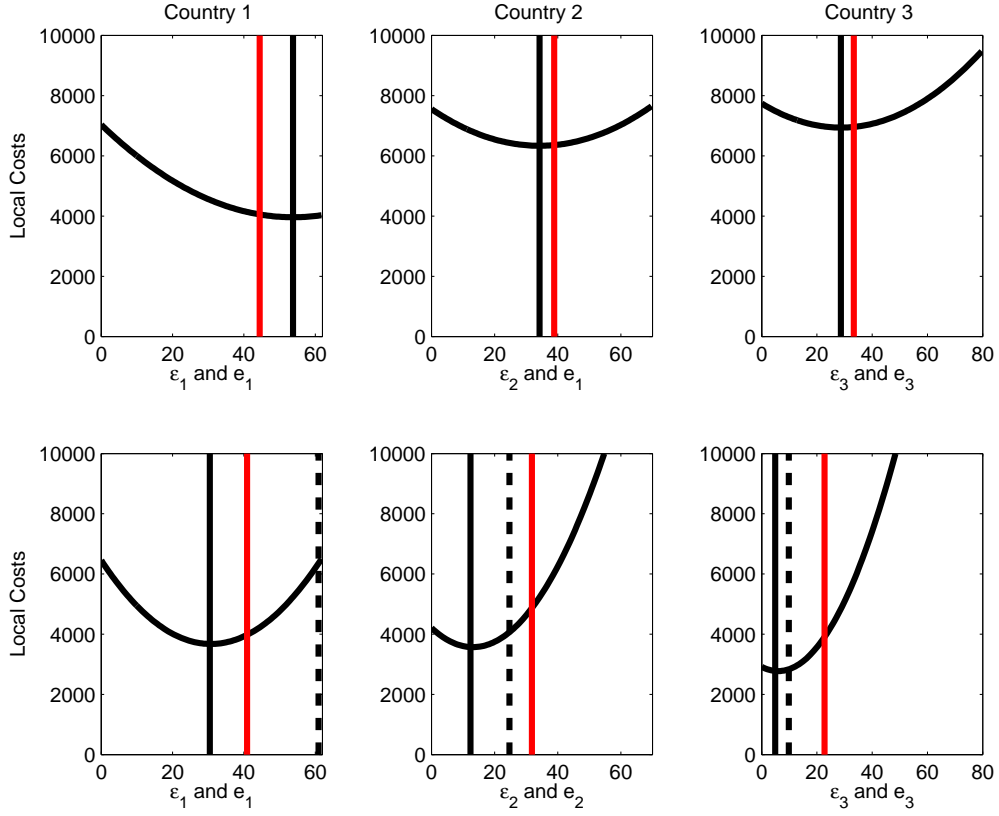
Figure 10: Upper part: No-Treaty Outcome. Lower part: Rules Treaty $\{\gamma = 1, \{\rho_i^{0,\omega}\}_{i=1}^n\}$. State of the world $\omega = (h_\phi, h_\beta)$. The black curves show the cost for a country i given that the other countries choose permits at equilibrium levels. The minima of the local cost curves represent the equilibrium. Black vertical lines are equilibrium permit issuance, and red vertical lines are equilibrium emissions. Dashed black vertical lines are $(1+\gamma)\epsilon_i^{RT,\omega}$. Ends of the abscissas correspond to business-as-usual emissions (except for country 1).

and the No-Treaty Outcome. It is apparent that in the Rules Treaty the emissions of countries are smaller than in the No-Treaty Outcome. As $\gamma = 1$, the agency will issue the same number of permits as the countries. But $\mathcal{E}_T^{RT,\omega}$ is still smaller than in the No-Treaty Outcome. As with the No-Treaty Outcome, countries with damage parameters lower than the average — here country 1 — will issue more permits (directly and indirectly via the agency) than their firms emit GHGs (compare dashed line with red line).

5.3 Ex-ante

In the following, we consider the formation of the Rules Treaty and illustrate it with an example.

5.3.1 Design of the Rules Treaty

Suppose that there is a designer aiming at minimizing the expected global cost, $\mathbb{E}[K^{RT}]$ under the constraint that the Rules Treaty is implementable.[16] Corollary 1 reduces the designer's problem to finding the largest $\gamma \in [0, \infty)$ for which an implementable refunding rule exists. For any γ, a refunding rule $\{\rho_i\}_{i=1}^n$ is implementable if and only if

$$\mathbb{E}[K_i^{RT}(\gamma, \rho_i)] \leq \mathbb{E}[K_i^{NT}] \ \forall \ i = 1, \ldots, n,$$

where $\mathbb{E}[K_i^{RT}(\gamma, \rho_i)]$ are the expected costs of country i in the Rules Treaty. We show in Proposition 5 that an implementable refunding rule exists for all $\gamma \in [0, \infty)$. As a prerequisite, we calculate the minimal refunding share, $\rho_i^{min}(\gamma)$, that guarantees the participation of a country i in the Rules Treaty for a given γ. It is defined by $\mathbb{E}[K_i^{RT}(\rho_i^{min}(\gamma), \gamma)] = \mathbb{E}[K_i^{NT}]$. This yields

$$\mathbb{E}[\frac{\phi_i}{2}(p^{RT})^2 + \frac{\beta_i}{2}(\mathcal{E}_T^{RT})^2 + p^{RT}(e_i^{RT} - \epsilon_i^{RT}(\rho_i^{min})) - \rho_i^{min}p^{RT}\gamma\mathcal{E}_C^{RT}] = \mathbb{E}[K_i^{NT}].$$

We solve for $\rho_i^{min}(\gamma)$, using $\epsilon_i^{RT}(\rho_i^{min})$ given by Equation (26d). The unique solution is:

$$\rho_i^{min}(\gamma) =$$
$$\frac{\mathbb{E}[\frac{\phi_i}{2}\left[(p^{RT})^2 - (p^{NT})^2\right] + \frac{\beta_i}{2}[(\mathcal{E}_T^{RT})^2 - (\mathcal{E}^{NT})^2] - \frac{n\beta_i\Phi}{\mathcal{B}}[(p^{NT})^2 - \frac{\gamma+n}{n(1+\gamma)}(p^{RT})^2] + (p^{NT})^2\Phi - \frac{1}{1+\gamma}(p^{RT})^2\Phi]}{\mathbb{E}[(p^{RT})^2\frac{\gamma}{1+\gamma}\Phi]}.$$

$$(28)$$

Proposition 5 (Implementable Rules Treaty)
For any $\gamma \in [0, \infty)$, suppose that Condition 25 holds for all $\rho_i \geq \rho_i^{min}(\gamma)$. Then there exists at least one implementable refunding rule, $\{\rho_i\}_{i=1}^n$. Hence the corresponding Rules Treaty is implementable.

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[16] Apart from implementability, we do not need for the purposes of this paper to model the detailed process of how permits are allocated across countries. However, for completeness, we show the Nash Bargaining Solution in Section 7.

The proof of Proposition 5 is given in Appendix B. Proposition 5 implies, notably, that a very large value of γ is implementable. Hence an implementable Rules Treaty is able to approximate the Global Social Optimum. We summarize this important result in the following corollary.

Corollary 2 (Global Social Optimum Approximately Implementable)

For very large γ, suppose that condition 25 holds for all $\rho_i \geq \rho_i^{min}(\gamma)$. Then there exists an implementable Rules Treaty that approximately establishes the Global Social Optimum.

Again the intuition for Proposition 5 and Corollary 2 is straightforward. If the scaling factor is greater than zero, global costs in the Rules Treaty will be smaller than in the No-Treaty Outcome. As we have not put any constraints on the way the refunds can be distributed, it is always possible to choose a set of refunding parameters ensuring that all countries are better off. In particular, this is true if the scaling factor is very high.

5.3.2 Example

In this section we fine-tune the intuition of Proposition 5 with the example. The upper part of Figure 11 shows the maximal γ for which the refunding rule $\{\rho_1 = 0.2, \rho_2 = 0.2, \rho_3 = 0.6\}$ is implementable in the three-country example. It demonstrates that country 1 and 2 are only better off with the Rules Treaty if approximately $\gamma < 2$. Therefore, $\gamma > 2$ cannot be implemented. However, we can increase the refund share of countries 1 and 2 at the expense of country 3, e.g. by setting $\{\rho_1 = 0.3, \rho_2 = 0.3, \rho_3 = 0.4\}$. This is shown in the lower part of Figure 11. In this case, every value $\gamma \in [0, \infty)$ is implementable. Figure 12 depicts $\rho_i^{min}(\gamma)$ as well as the Nash-bargaining (NB) solution for equal bargaining power.[17] Note that for the refunding rule $\{\rho_1 = 0.3, \rho_2 = 0.3, \rho_3 = 0.4\}$ used in the lower part of Figure 11, $\rho_i > \rho_i^{min}(\gamma)$ holds for all γ and i.

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[17] For each $\gamma \in [0, \infty)$, the Nash-bargaining solution, $\{\rho_1^{NB}, \rho_2^{NB}, \rho_3^{NB}\}$ is obtained by

$$\{\rho_1^{NB}, \rho_2^{NB}\} \in \underset{\rho_1, \rho_2}{\operatorname{argmax}} \left\{ \left(K_1^{NT} - K_1^{RT}(\gamma, \rho_1)\right)^{\frac{1}{3}} \left(K_2^{NT} - K_2^{RT}(\gamma, \rho_2)\right)^{\frac{1}{3}} \left(K_3^{NT} - K_3^{RT}(\gamma, 1 - \rho_1 - \rho_2)\right)^{\frac{1}{3}} \right\}$$

and $\rho_3^{NB} = 1 - (\rho_1^{NB} + \rho_2^{NB})$.
The refunding rule in that case is such that

$$K_1^{NT} - K_1^{RT}(\gamma, \rho_1^{NB}) = K_2^{NT} - K_2^{RT}(\gamma, \rho_2^{NB}) = K_3^{NT} - K_3^{RT}(\gamma, \rho_3^{NB}).$$
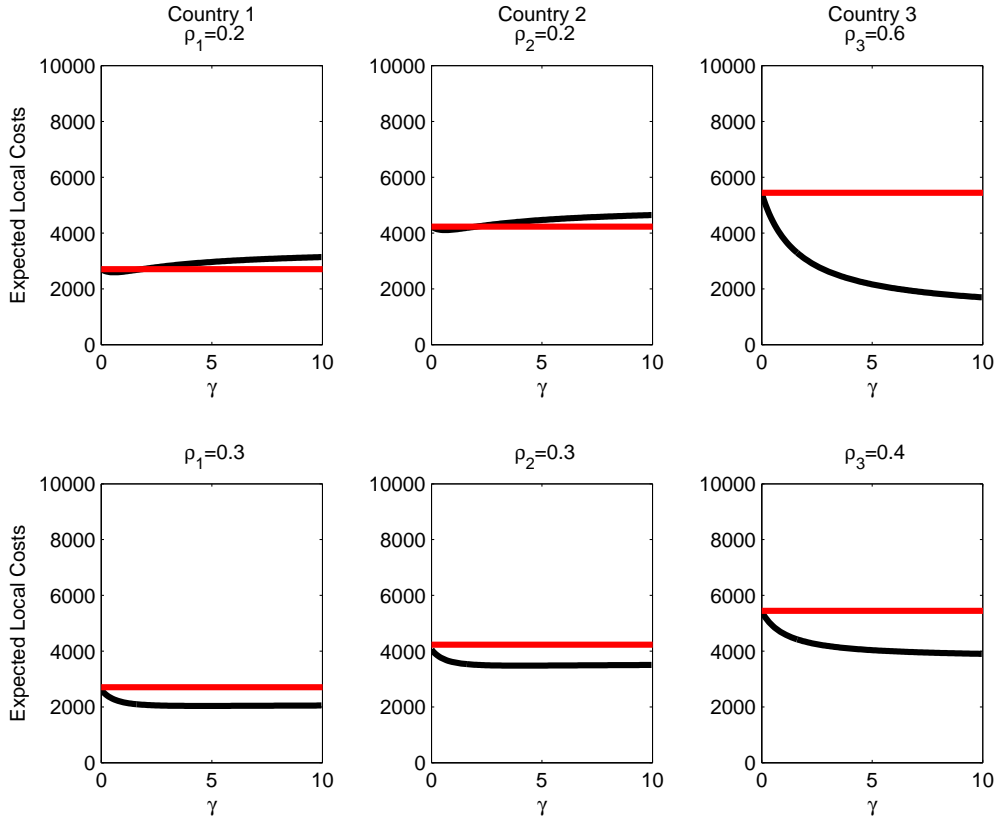
Figure 11: Illustration of the participation constraint in the Rules Treaty for two differ-
ent refunding rules (upper and lower part). The red lines are the constant
expected local costs in the No-Treaty Outcome. The black lines depict the
expected local costs in the Rules Treaty as a function of γ.

6 Rules Treaty vs. Target Treaty

In this section we compare the Rules Treaty with the Target Treaty. As we have shown
in Lemma 2 and Corollary 2, the Target Treaty is not ex-post optimal, whereas the
Rules Treaty is ex-post optimal when the scaling factor approaches infinity. However,
politicians might hesitate to agree to a very high scaling factor, one possible reason being
that is would deprive them of room to maneuver during a economic downturn. For such
cases, the comparison we state in Corollary 3 becomes relevant.
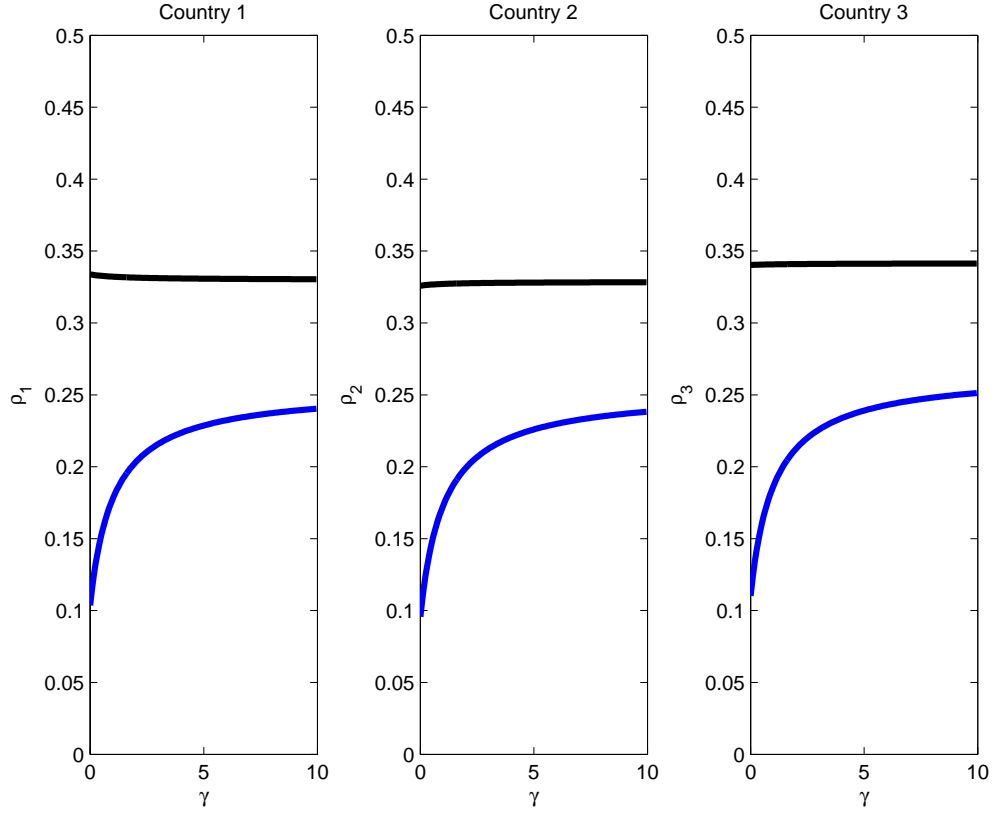
Figure 12: ρ_i^{min} (blue lines) and the Nash-bargaining solution (black lines) as a function of γ

Corollary 3 (Comparison in the General Case)

(I) *If the expected global costs of the Target Treaty are no higher than in the No-Treaty Outcome, the Rules Treaty is ex-ante superior to the Target Treaty if and only if $\gamma > \gamma^{crit}$. $\gamma^{crit} \in (0, \infty)$ is the unique critical value where the expected global costs of both treaties are the same.*

(II) *If the expected global costs of the Target Treaty are higher than in the No-Treaty Outcome, the Rules Treaty is ex-ante superior to the Target Treaty for any $\gamma \in (0, \infty)$.*

The existence of γ^{crit} in the first part of the corollary is a consequence of two results in Corollary 2. First, for γ very high, the expected global costs are globally optimal, i.e. minimal. Second, the global costs in the Rules Treaty increase when γ is decreased.

Next, we compare the treaties when countries are completely homogeneous.[18] Consider the following implementable and symmetric treaties: a Rules Treaty $RT^{Symm} = \left\{ \left\{ \frac{1}{n} \right\}_{i=1}^{n}, \gamma \text{ very large} \right\}$ and a Target Treaty $TT^{Symm} = \left\{ \frac{\mathcal{E}^{TT}}{n} \right\}_{i=1}^{n}$. We find the following corollary:

Corollary 4 (Comparison in the Homogeneous Case)

If countries are completely homogeneous, RT^{Symm} is ex-ante Pareto superior to TT^{Symm}.

The reason is that in both treaties the global costs, which are lower for RT^{Symm} than for TT^{Symm}, are shared equally among countries.

7 Treaty Formation

Up to this point, the only constraint we have imposed on the Rules Treaty during treaty formation is that it has to be implementable, i.e. every country must be ex-ante better off than in the No-Treaty Outcome. This leads to the condition that $\rho_i \geq \rho_i^{min}$ must hold for all i. In the proof of Proposition 5, we showed that $\sum_{i=1}^{n} \rho_i^{min} \leq 1$. So far we have not modeled how the remaining refunding shares, $1 - \sum_{i=1}^{n} \rho_i^{min}$, are distributed. In the following remedy this omission. We assume that a Nash-bargaining (NB) game takes place and countries have bargaining power π_i. For each $\gamma \in [0, \infty)$, the Nash Bargaining game solves

$$\min_{\{\rho_i\}_{i=1}^{n-1}} \left\{ \prod_{i=1}^{n-1} \left(\mathbb{E}[K_i^{NT}] - \mathbb{E}[K_i^{RT}(\gamma, \rho_i)] \right)^{\pi_i} \left(\mathbb{E}[K_n^{NT}] - \mathbb{E}[K_n^{RT}(\gamma, 1 - \sum_{i=1}^{n-1} \rho_i)] \right)^{\pi_n} \right\}.$$

The resulting refunding rule, $\left\{ \rho_i^{NB} \right\}_{i=1}^{n}$, is such that for all i and $\gamma \in [0, \infty)$,

$$\mathbb{E}[K_i^{NT}] - \mathbb{E}[K_i^{RT}(\gamma, \rho_i^{NB})] = \pi_i \left(\mathbb{E}[K^{NT}] - \mathbb{E}[K^{RT}(\gamma)] \right).$$

As a first example, consider the extreme case where a country j has all the bargaining power, i.e. $\pi_j = 1$. Country j's refunding share is such that all the global cost saving from the Rules Treaty, as compared to the No-Treaty Outcome, will accrue to it. All other countries $i \neq j$ receive the refunding share, ρ_i^{min}, that merely guarantees their participation. Next, consider the case where all countries are symmetric and have equal

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[18] That is, in all states ω they have the same business-as-usual emissions, damage parameters, and abatement cost parameters.

bargaining power. The resulting refunding shares after bargaining will be $\rho_i = \frac{1}{n}$ for all countries i. This corresponds to RT^{Symm} as discussed in Chapter 6, if the scaling factor is very large.

8 Extreme Events

With the current state of our knowledge, the possibility of catastrophic damages from climate change cannot be ruled out, although the probability is deemed to be small. If, against all odds, damages turn out to be extreme, both the Target Treaty and the Rules Treaty will allow countries to react to this information. However, information processing is superior in the Rules Treaty.

In a Target Treaty, the optimal ex-ante number of permits will turn out much too high, and countries may benefit from unilateral destroying permits.[19] Two cases might occur. First, all countries have more permits than they would choose in the No-Treaty Outcome. In such a case, the outcome is the same as in the No-Treaty Outcome, as all countries profit from destroying some of their permits up to the levels of the No-Treaty Outcome. Second, some countries have more and others have fewer permits than in the No-Treaty Outcome. Those who have more might choose to destroy some of their permits. At all events if information arrives indicating that marginal damages will be very high, some permits may not be used in equilibrium. In this sense, there is some reaction to new information in the Target Treaty in the case of a extreme event. However, it tends only to approximate the No-Treaty Outcome in this respect.

Countries participating in the Rules Treaty are fully able to react to information that damages are very high and correspondingly choose low numbers of permits. Because the Rules Treaty induces optimal adjustments to new information independently of its type, this holds true no matter how high damages are and how unlikely they were ex-ante.

9 Discussion

Scope of Rules Treaties
We have addressed two deficiencies in the current attempts to set up international climate

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[19] In the formal model Assumption 2 excludes such cases.

treaties on the basis of reduction targets: the underprovision of climate protection and difficulties in reacting to new information. In a simple model we have shown that the Rules Treaty has the potential to overcome both problems. First, it can be designed in such a way that countries have incentives to abate emissions up to a level that is optimal from a global perspective. Second, by construction, it induces countries to react optimally to new information.

Our simple model is sufficient to show these advantages. There are several useful extensions to it, which could provide further insights. We will discuss some of them here.

More General Functional Forms
The shapes of the damage and abatement cost functions are important as marginal costs and benefits are largely determined by this choice. Our assumption that both functions are quadratic is largely for technical convenience. The exact choice of the shape does not influence our qualitative results as long as the functional form is similar with respect to smoothness[20] and marginal abatement and damage costs are increasing. However, completely different functional forms, such as a damage function that includes a singularity – a GHG stock above which damages become extremely high –, might radically change the result of the Target Treaty. Higher abatement in the Target Treaty could be justified as an insurance against such catastrophic climate change. In our one-period model the Rules Treaty remains unaffected by these problems. However, this may no longer be true in a multi-period model.

Multi Period Framework
A sensible extension would be a multi-period framework, which would enable us to address several further points. First, we could model gradual learning about the state of the world. The main reason why this is more realistic is that due to the inertia of the climate system, true damages are slow to reveal themselves. Second, we could modify the damage function to incorporate the inertia of the climate system. Instantaneous damages might not only depend to the current level of GHG but also on the rate of change in previous periods. Third, a multi-period model enables us to consider technological constraints on abatement levels due to lock-in effects. Lock-in effects are relevant, say, in the energy system, as power plants typically have long economic lifetimes. Such constraints might imply that emission levels cannot decrease below a certain fraction of the

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[20] Smoothness means continuously differentiable abatement and damage costs.

previous period(s). Fourth, high abatement effort in a country may shift production to other countries not participating in the scheme.

Sequential Negotiation Process

Another extension is a more detailed modeling of the negotiation process. The outside option could be more realistic. Furthermore, it could include coalition formation and sequential bargaining processes with respect to the allocation of refund shares in the Rules Treaty and with respect to the permit allocation in the Target Treaty.

Geographical Scope

If countries are homogeneous, Rules Treaties are easiest to implement, so first applications of the Rules Treaties are likely to be considered for the EU. Globally, however, Rules Treaties are much harder to introduce. On average developing countries are more vulnerable to climate change because their economies rely more heavily on climate-sensitive activities than industrial countries. While Rules Treaties can be designed so that developing countries receive high compensation through high refunding shares, direct application becomes much more delicate. A small country has a negligible impact on the permit price and may overissue. To solve the problem, it might be useful to establish a hierarchical scheme: first, clusters each representing a group of countries; second, the number of permits in a cluster is allocated to individual countries.

Conclusion

Rules Treaties offer an alternative to Target Treaties. While no blueprint for a treaty is a panacea, the general idea is that focusing on rules rather than targets could be a useful principle in further attempts to solve a global public-good problem like mitigating climate change.

A Table of variables

Symbol	Meaning
i	country
n	number of countries
SO	Global Social Optimum
NT	No-Treaty Outcome
TT	Target Treaty
RT	Rules Treaty
$\bar{e}_i$	business-as-usual emissions of country i
$\bar{E}$	global business-as-usual emissions
e_i	GHG emissions of country i
ϵ_i	emission permits of country i
$\mathcal{E}$	global emission permits, $\sum_{i=1}^{n} \epsilon_i$
$\mathcal{E}_T$	In RT: total amount of emission permits, $\sum_{i=1}^{n} \epsilon_i(1+\gamma)$
$\mathcal{E}_C$	In RT: emission permits issued by countries , $\sum_{i=1}^{n} \epsilon_i$
C_i	abatement costs of country i
ϕ_i	abatement cost parameter of country i
Φ	$\sum_{i=1}^{n} \phi_i$
p	price of emission permit
E	global emission of GHG
β_i	damage parameter of country i
$\mathcal{B}$	$\sum_{i=1}^{n} \beta_i$
$\bar{\beta}$	average damage parameter
D_i	(environmental) damage of country i
K_i	local costs of country i
K	global costs, i.e. $\sum_{i=1}^{n} K_i$
$\mathbb{E}[K_i]$	expected local costs
$\mathbb{E}[K]$	expected global costs
ρ_i	refund share of country i
γ	scaling factor
r_i	refund for country i
ω_ϕ	aggregate event (abatement costs); $\omega_\phi \in \{h_\phi, l_\phi\}$
ω_β	aggregate event (environmental damages); $\omega_\beta \in \{h_\beta, l_\beta\}$
ω	state of the world $\omega = \omega_\phi \times \omega_\beta$
Ω	set of states of the world: $\omega \in \Omega$
π	probability of h_ϕ
σ	probability of h_{β_i}
D	$\pi^h \mathcal{B}^h + \pi^l \mathcal{B}^l$
A	$\left(\frac{\sigma^h}{\Phi^h} + \frac{\sigma^l}{\Phi^l}\right)$
$\rho_i^{min}(\gamma)$	given γ, refunding share where $\mathbb{E}[K_i^{RT}] = \mathbb{E}[K_i^{NT}]$
γ^{crit}	scaling factor where $\mathbb{E}[K^{RT}] = \mathbb{E}[K^{TT}]$

B Proofs

Proof of Proposition 1

Summing up the first-order conditions (10) yields

$$\bar{E} - E^\omega = \mathcal{B}^\omega E^\omega \Phi^\omega. \tag{29}$$

This can be rearranged to yield (11a).

Next, we solve Equation (10) for e_i:

$$e_i^\omega = \bar{e}_i - \mathcal{B}^\omega E^\omega \phi^\omega. \tag{30}$$

Plugging in Equation (11a) yields Equation (11b) $\qquad\square$

As Equation (9) is strictly convex, the solution is unique. Hence the necessary conditions are also sufficient.

Proof of Proposition 2

Summing up the necessary first-order conditions (14) across countries, one obtains

$$p^\omega n = \mathcal{B}^\omega \mathcal{E}^\omega. \tag{31}$$

From Equation (5) we conclude

$$\frac{\bar{E} - \mathcal{E}^\omega}{\Phi^\omega} n = \mathcal{B}^\omega \mathcal{E}^\omega. \tag{32}$$

Equation (32) can be rewritten to yield Equation (15a). Plugging Equation (15a) into Equation (31) gives Equation (15b). To derive Equation (15d), we rewrite Equation (14) as

$$\epsilon_i^\omega = \Phi^\omega \left[p^\omega - \beta_i^\omega \mathcal{E}^\omega \right] + e_i^\omega,$$

and plug in $\mathcal{E}^\omega = \frac{p^\omega}{\mathcal{B}} n$ and Equation (3). Thus we obtain

$$\epsilon_i^\omega = \Phi^\omega \left[p^\omega - \beta_i^\omega \frac{p^\omega}{\mathcal{B}} n \right] + \bar{e}_i - p^\omega \phi_i^\omega$$

$$\Leftrightarrow \epsilon_i^\omega = \bar{e}_i - p^\omega \Phi^\omega \left(\frac{\phi_i^\omega}{\Phi^\omega} + \frac{\beta_i^\omega - \bar{\beta}^\omega}{\bar{\beta}^\omega} \right).$$

As the cost function (12) is strictly convex, the solution is unique. Hence the necessary conditions are also sufficient. $\qquad\square$

Proof of Lemma 1

Suppose all countries $j \neq i$ obtain permits ϵ_j^{TT} such that $\mathbb{E}[K_j^{TT}] = \mathbb{E}[K_j^{NT}]$.

If $\mathbb{E}[K^{TT}] > \mathbb{E}[K^{NT}]$, it must be the case that $\mathbb{E}[K_i^{TT}] > \mathbb{E}[K_i^{NT}]$ for the remaining country i. Hence there cannot exist an implementable Target Treaty.

If $\mathbb{E}[K^{TT}] \leq \mathbb{E}[K^{NT}]$, it must be the case that $\mathbb{E}[K_i^{TT}] \leq \mathbb{E}[K_i^{NT}]$ for the remaining country i. Hence the Target Treaty is implementable. $\qquad\square$

Proof of Lemma 2

In order to be socially optimal for all realizations ω, $\mathcal{E}^{TT} = E^{SO,\omega}$ must hold for all ω. According to (11a) and (18), this is true if $\frac{A}{D+A} = \frac{1}{1+\mathcal{B}^\omega \Phi^\omega}$ for all ω. This is not possible, since $\frac{1}{1+\mathcal{B}^\omega \Phi^\omega}$ differs in at least two states of the world. $\qquad\square$

Proof of Proposition 4

Summing up the necessary first-order conditions (24) across countries, one obtains

$$p^\omega(n+\gamma) - \frac{1+\gamma}{\Phi^\omega}[\mathcal{E}_C + \gamma\mathcal{E}_C - \mathcal{E}_T] = \mathcal{B}^\omega \mathcal{E}_T^\omega(1+\gamma).$$

As $\mathcal{E}_T^\omega = \mathcal{E}_C^\omega(1+\gamma)$, this is

$$p^\omega(n+\gamma) = \mathcal{B}^\omega \mathcal{E}_T^\omega(1+\gamma). \tag{33}$$

From Equation (5) we conclude

$$\frac{\bar{E} - \mathcal{E}_T^\omega}{\Phi^\omega}(n+\gamma) = \mathcal{B}^\omega \mathcal{E}_T^\omega(1+\gamma). \tag{34}$$

Equation (34) can be rewritten to yield Equation (26a). Plugging Equation (26a) into Equation (33) gives Equation (26b). To derive Equation (26d), we rewrite Equation (24)

$$\epsilon_i^\omega = \frac{\Phi^\omega}{1+\gamma}\left[p^\omega(1+\rho_i\gamma) - \beta_i^\omega \mathcal{E}_T^\omega(1+\gamma)\right] - \rho_i\gamma\mathcal{E}_C^\omega + e_i^\omega,$$

and plug in $\mathcal{E}_T^\omega = \frac{p^\omega}{\mathcal{B}}^\omega \frac{n+\gamma}{1+\gamma}$, and Equations (3) and (5). Thus we obtain

$$\epsilon_i^\omega = \frac{\Phi^\omega}{1+\gamma}\left[p^\omega(1+\rho_i\gamma) - \frac{p^\omega}{\mathcal{B}^\omega}\frac{n+\gamma}{1+\gamma}(1+\gamma)\beta_i^\omega\right] - \rho_i\frac{\gamma}{1+\gamma}(\bar{E} - p^\omega\Phi^\omega) + \bar{e}_i - p^\omega\phi_i^\omega$$

$$\Leftrightarrow \epsilon_i^\omega = \frac{\Phi^\omega}{1+\gamma}p^\omega[1+\rho_i\gamma] - \frac{p^\omega}{\mathcal{B}^\omega}\frac{n+\gamma}{1+\gamma}\Phi^\omega\beta_i^\omega - \frac{\gamma}{1+\gamma}\rho_i(\bar{E}-p^\omega\Phi^\omega) + \bar{e}_i - p^\omega\phi_i^\omega$$

$$\Leftrightarrow \epsilon_i^\omega = \frac{1}{1+\gamma}p^\omega\Phi^\omega + \rho_i\frac{\gamma}{1+\gamma}p^\omega\Phi^\omega - \frac{\gamma}{1+\gamma}\frac{\beta_i^\omega}{\mathcal{B}^\omega}p^\omega\Phi^\omega - \frac{n}{1+\gamma}\frac{\beta_i^\omega}{\mathcal{B}^\omega}p^\omega\Phi^\omega$$
$$- \frac{\gamma}{1+\gamma}\rho_i(\bar{E}-p^\omega\Phi^\omega) + \bar{e}_i(\frac{1}{1+\gamma}+\frac{\gamma}{1+\gamma}) - p^\omega\phi_i^\omega(\frac{1}{1+\gamma}+\frac{\gamma}{1+\gamma})$$

$$\Leftrightarrow \epsilon_i^\omega = \frac{\bar{e}_i}{1+\gamma} + \frac{\gamma}{1+\gamma}(\rho_i p^\omega\Phi^\omega + \bar{e}_i - p^\omega\phi_i^\omega - \rho_i\bar{E} + \rho_i^\omega p^\omega\Phi^\omega - \frac{\beta_i^\omega}{\mathcal{B}^\omega}p^\omega\Phi^\omega)$$
$$+ \frac{1}{1+\gamma}(p^\omega\Phi^\omega - p^\omega\phi_i^\omega - n\frac{\beta_i^\omega}{\mathcal{B}^\omega}p^\omega\Phi^\omega)$$

$$\Leftrightarrow \epsilon_i^\omega = \frac{\bar{e}_i}{1+\gamma} + \frac{\gamma}{1+\gamma}\left\{-\rho_i(\bar{E}-2p^\omega\Phi^\omega) + \bar{e}_i - p^\omega\Phi^\omega(\frac{\phi_i^\omega}{\Phi^\omega}-\frac{\beta_i^\omega}{\mathcal{B}^\omega})\right\}$$
$$+ \frac{1}{1+\gamma}p^\omega\Phi^\omega(-\frac{\phi_i^\omega}{\Phi^\omega}+\frac{\bar{\beta}_i^\omega-\beta_i^\omega}{\bar{\beta}_i^\omega})$$

$$\Leftrightarrow \epsilon_i^\omega = \frac{\bar{e}_i}{1+\gamma} - \frac{\gamma}{1+\gamma}\left\{\rho_i(\bar{E}-2p^\omega\Phi^\omega) - [\bar{e}_i - p^\omega\Phi^\omega(\frac{\phi_i^\omega}{\Phi^\omega}-\frac{\beta_i^\omega}{\mathcal{B}^\omega})]\right\}$$
$$- \frac{1}{1+\gamma}p^\omega\Phi^\omega(\frac{\phi_i^\omega}{\Phi^\omega}+\frac{\beta_i^\omega-\bar{\beta}_i^\omega}{\bar{\beta}_i^\omega}).$$

$\square$

Proof of Corollary 1(i)

We use Equations (4) and (5) and rewrite Equation (7b) as

$$K^{RT,\omega}(E^{RT,\omega}(\gamma)) = [\frac{\Phi^\omega}{2}(\frac{\bar{E}-E^{RT,\omega}(\gamma)}{\Phi^\omega})^2 + \frac{\mathcal{B}^\omega}{2}(E^{RT,\omega}(\gamma))^2].$$

Hence,

$$\frac{d\,K^{RT,\omega}}{d\,\gamma} = \frac{\partial\,K^{RT,\omega}}{\partial\,E^{RT,\omega}}\frac{d\,E^{RT,\omega}}{d\,\gamma}.$$

For $\gamma \in [0,\infty)$, $E^{RT,\omega} \in (E^{SO,\omega}, E^{NT,\omega}]$. $K^{RT,\omega}(E^{RT,\omega})$ is by construction at its mini-

mum for $E^{RT,\omega} = E^{SO,\omega}$ and convex in $E^{RT,\omega}$. Hence, $\frac{\partial\, K^{RT}}{\partial\, E^{RT,\omega}} > 0$. Furthermore, from (26a) we obtain $\frac{d\, E^{RT,\omega}}{d\,\gamma} < 0$. $\qquad\square$

Proof of Proposition 5

Summing up $\rho_i^{min}(\gamma)$ as given by Equation (28) leads to

$$\sum_{i=1}^{n} \rho_i^{min}(\gamma) = \frac{\mathbb{E}[K^{RT}(\gamma)] - \mathbb{E}[K^{NT}]}{\mathbb{E}[(p^{RT})^2 \frac{\gamma}{1+\gamma}\Phi]} + 1. \tag{35}$$

This sum is no bigger than 1 for any $\gamma \in [0,\infty)$, as the expected global costs under the Rules Treaty are no higher than in the No-Treaty Outcome and the denominator is positive.

Hence, for any $\gamma \in [0,\infty)$, as $\sum_{i=1}^{n} \rho_i^{min}(\gamma) \leq 1$, there exists a Rules Treaty $\{\{\rho_i\}_{i=1}^{n}, \gamma\}$ with $\rho_i \geq \rho_i^{min}(\gamma)\,\forall\,i$ which, by definition of $\rho_i^{min}(\gamma)$, is implementable. $\qquad\square$

Proof of Lemma 4

RT^{Symm} approximately implements the Global Social Optimum in all states ω, and in equilibrium each country i issues the same number of permits $\epsilon_i^{RT,\omega} \approx \frac{\mathcal{E}^{SO,\omega}}{n}$. Hence, country-specific costs are distributed symmetrically in both treaties.[21] Furthermore, $\mathbb{E}[K^{RT^{Symm}}] < \mathbb{E}[K^{TT^{Symm}}]$. $\qquad\square$

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[21] If emission permits and the refunding shares are not symmetrical but chosen arbitrarily, one might imagine a case where the Rules Treaty is not a Pareto Improvement compared to the Target Treaty even though countries are symmetrical. Suppose, e.g., that $\mathbb{E}[K^{TT}] - \frac{n-1}{n}\mathbb{E}[K^{NT}] \leq \frac{\mathbb{E}[K^{SO}]}{n}$ and $\mathbb{E}[K^{TT}] \leq \mathbb{E}[K^{NT}]$. Then there exists an implementable Target Treaty TT^{Asymm} where $n-1$ countries $j \neq i$ obtain permits such that $\mathbb{E}[K_j^{TT}] = \mathbb{E}[K_j^{NT}]$, and country i receives the remaining permits. As a consequence, country i is ex-ante better off in this Target Treaty than in the Rules Treaty RT^{Symm}. Hence, RT^{Symm} is not Pareto superior to TT^{Asymm}.

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