

DISS. ETH NO. 29065

ESSAYS ON CHECKS & BALANCES
AND CREDENCE POLICIES

A thesis submitted to attain the degree of
DOCTOR OF SCIENCES of ETH ZURICH
(Dr. sc. ETH Zurich)

presented by

Julia Wagner

M.Sc. Mathematical Modeling, University College London

born on July 20, 1996

citizen of Austria

accepted on the recommendation of

Prof. Dr. Hans Gersbach

Prof. Dr. Antoine Bommier

2023

Acknowledgments

None of the work presented in this thesis would have been possible without the great people who encouraged and supported me along the way. First and foremost, I wish to express my sincere gratitude to my supervisor Prof. Hans Gersbach, for teaching me about high standards in every aspect of being a researcher, and for giving me the freedom to choose my own research projects while offering his support and invaluable advice. I owe a special thank to my co-supervisor, Prof. Antoine Bommier, for his comments as well as for evaluating my research and to Prof. Lucas Bretschger for the time he spent chairing my PhD examination committee.

I am also indebted to Oriol Tejada, who helped me to improve my thesis and become a better researcher through many constructive and motivating discussions. I am also very grateful to Margrit Buser for having always an open door and ear, being patient with me about forgetting commata and helping me to improve my thesis significantly.

During my doctoral studies, I got to know many colleagues and academic guests at ETH Zurich and University of Zurich. A huge thank you to all of them, notably members of the Chair of Macroeconomics, for the stimulating discussions that always gave me fresh perspectives on my research problems, and Kremena, in particular, for her helpful feedback. My sincere thanks also go to Moritz, who was not only the best office room-sharing colleague imaginable, but who also motivated me everyday with his insightful feedback and many discussions about research, cycling and good food.

I am grateful to my friends and collaborators in academia: Zhaozhi, Weijun, Mako, and Seth, whose experience helped me avoid many mistakes, and whom I could always bug with my questions. To my extended family in Zurich: Aga, Ivana, Markus, Benno, Ewelina, David, Ea, Vero, Sophia, Irene, Anne, and Paul, who

constantly reminded me that there is a life outside work.

During my time as a doctoral student I got to know many people who inspired me beyond the scope of my PhD thesis. I owe special mention to Alex, Lewis, Arnout, Nhi, Liliane, Sephin, Cynthia, and Thomas. A special thank you goes to Dietrich whose humor and lighthearted spirit brought a much-needed sense of levity to the challenging times.

Finally, to those who always believed in me and were part of this journey throughout: my parents, my brother Alex, Magdalena, David, Katharina, Victoria, and many more friends in Vienna, London and around the world.

I gratefully acknowledge the financial support from the ETH Grant “Co-voting Democracy: Benefits and Cyber-risks” (project code: 2016-48) and from Chair of Macroeconomics: Innovation and Policy at ETH Zürich led by Hans Gersbach.

Zurich, December 2022

Contents

List of Figures	VI
List of Tables	VIII
Thesis Summary	X
Zusammenfassung	XII
1 Introduction	1
1.1 Motivation	1
1.2 Research Questions	6
1.3 Structure of the Thesis	7
2 Policy Reforms and Checks & Balances	10
2.1 Introduction	11
2.2 Relation to the Literature	16
2.3 Model	19
2.4 Analysis	21
2.4.1 Analysis of the political game	21
2.4.2 Analysis of the constitutional game	24
2.5 Comparative Statics	37
2.5.1 Welfare	39
2.6 Conclusion	41
3 Role of Preference Shocks	42
3.1 Introduction	42
3.2 Related Literature	44
3.3 Model	45
3.4 Analysis	47
3.4.1 The second period	47
3.4.2 The first period	49
3.5 Conclusion	56

4	Credence Policies and Experts	57
4.1	Introduction	58
4.2	Relation to the Literature	61
4.3	Model	63
4.3.1	Basic setup	63
4.3.2	Some properties	66
4.4	Voting Procedures	68
4.4.1	Benchmark: Optimal Democracy	69
4.4.2	Elections without Experts	69
4.4.3	Elections with Vote Delegation	70
4.4.4	Elections with Opinion Updating	75
4.5	Comparisons and Discussion	80
4.5.1	Comparisons with Optimal Democracy	80
4.5.2	Discussion about risk aversion	83
4.6	Special Cases	84
4.7	Extensions	86
4.8	Conclusion	86
5	Capturing Anxiety with Word Embeddings	88
5.1	Introduction	89
5.1.1	Related literature	91
5.2	Methods	93
5.2.1	Data collection and preprocessing	93
5.2.2	Anxiety index	95
5.2.3	Validation of the anxiety index	97
5.2.4	Topic modeling	100
5.3	Results	101
5.3.1	Anxiety over time	102
5.3.2	Anxiety across topics	102
5.4	Discussion and Conclusion	103
6	Conclusion and Outlook	105
A	Appendix to Chapter 2	107
A.1	List of Countries for Figure 2.3	107
A.2	Proof of Corollary 1	107
A.3	Proof of Theorem 1	108
A.4	Proof of Corollary 2	111
A.5	Proof of Corollary 3	111
A.6	Proof of Theorem 2	111
A.7	Proof of Corollary 4	114

A.8	Proof of Corollary 5	115
A.9	Proof of Theorem 3	115
A.10	Proof of Corollary 6	119
A.11	Proof of Corollary 7	119
A.12	Proof of Theorem 4	119
A.13	Proof of Theorem 5	119
A.14	Proof of Corollary 8	120
A.15	Proof of Theorem 6	120
B	Appendix to Chapter 3	122
B.1	Proof of Proposition 3	122
C	Appendix to Chapter 4	148
C.1	Proof of Proposition 4	148
C.2	Proof of the Single-Crossing Property (SPC) of Section 4.4.2	149
C.3	Proof of Proposition 5	150
C.4	Proof of Proposition 6	152
C.5	Proof of Proposition 7	159
C.6	Proof of Proposition 8	170
C.7	Proof of Proposition 9	171
C.8	Proof of Proposition 10	172
C.9	Proof of Proposition 11	176
C.10	Proof of Corollary 9	177
C.11	Proof of Corollary 10	178
C.12	Possible Extensions	180
	C.12.1 Vote Delegation: General case	180
	C.12.2 Social Planner	181
C.13	Proof of Proposition 12	183
C.14	Proof of Theorem 7	186
C.15	Proof of Proposition 13	187
C.16	Proof of Proposition 14	188
D	Appendix to Chapter 5	194
D.1	Additional information on data collection and preprocessing	194
	D.1.1 Swiss newspapers	194
	D.1.2 Austrian newspapers	195
	D.1.3 Data crawling of Austrian newspapers	195
D.2	Additional information on anxiety index	196
	D.2.1 Dictionary word list	196
	D.2.2 Extended keyword list	199
	D.2.3 Anxiety index for each country	200

D.3	Additional information on validation	200
D.3.1	Translation of Figure 5.3	200
D.3.2	Qualitative analysis	201
D.3.3	Comparison with other indices	201
D.4	Additional information on topic modeling	204
D.4.1	Number of topics	204
D.4.2	Translation of Table 5.2	205
D.4.3	Anxiety by topic over time	205
	References	218
	Curriculum Vitae	219

List of Figures

1.1	Evolution of checks and balances over 45 years in 183 countries. Source: Cruz et al. (2021).	3
2.1	Evolution of C&B over 45 years in Canada (left) and the UK (right).	14
2.2	Evolution of C&B over 45 years in Spain (left) and France (right). .	14
2.3	Comparison of 38 countries regarding checks and party polarization over 40 years. The list of all countries is in the Appendix.	16
2.4	The policy choices in periods $t = 1$ and $t = 2$	23
2.5	Schematic representation of (expected) utilities for $\Pi < \bar{\Pi}$ (left) and $\bar{\Pi} \leq \Pi$ (right).	26
2.6	Choice $(c_1)_{K,1}^*$ of agent K with peak μ_K , where $\bar{\mu}_K = \mu_L + \frac{1+p}{2}\Pi$. .	27
2.7	Choice $(c_1)_{L,2}^*$ of party L given by Equation (2.6) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.8) (right, case $\bar{\Pi} \leq \Pi$).	31
2.8	Choice $(c_1)_{R,2}^*$ of party R given by Equation (2.7) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.9) (right, case $\bar{\Pi} \leq \Pi$).	32
2.9	Choice $(c_1)_{L,3}^*$ of party L given by Equation (2.10).	36
2.10	Choice $(c_1)_{M,3}^*$ of median voter M given by Equation (2.11) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.13) (right, case $\bar{\Pi} \leq \Pi$).	36
2.11	Choice $(c_1)_{R,3}^*$ of party R given by Equation (2.12) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.14) (right, case $\bar{\Pi} \leq \Pi$).	37
3.1	Timeline of the political game.	46
3.2	The policy choices in period $t = 2$ when $0 < c < 2B$	50
3.3	The policy choices in period $t = 2$ when $2B \leq c < 2\Pi - 2B$	50
3.4	The policy choices in period $t = 2$ when $2\Pi - 2B \leq c < 2\Pi$	51
3.5	The policy choices by party R in period $t = 1$	53
3.6	The policy choices in period $t = 1$ for $B = 0$	54
3.7	The policy choices in period $t = 1$ for $B = \frac{\Pi}{2}$	55
4.1	The preferred policy p_i^* for different values of $\frac{b_i}{k_i}$ and λ^2	65
4.2	The average and median value of k_i for $0 \leq \eta \leq 1$	67

4.3	The average and median value of b_i for $\beta \leq \frac{1}{2}$ (left) and $\beta > \frac{1}{2}$ (right).	68
4.4	Policy p_{maj}^* depending on η .	71
4.5	Policy p_{del}^* for $\frac{b^E}{\mathbb{E}(k^E)} < \lambda^2$ (left) and $\lambda^2 < \frac{b^E}{\mathbb{E}(k^E)}$ (right) in Case (i).	73
4.6	Policy p_{del}^* for $\frac{b^E}{\mathbb{E}(k^E)} < \lambda^2$ (left) and for $\lambda^2 < \frac{b^E}{\mathbb{E}(k^E)}$ (right) in Case (ii).	74
4.7	Policy p_{del}^* in Case (iii) on the left side and in Case (iv) on the right side.	74
4.8	The threshold η_1 depending on β .	75
4.9	Policy p_{up}^* for $\beta < \tau < \tau + \kappa$, if $\lambda^2 < \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ (left) and if $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \lambda^2$ (right) in Case (i).	77
4.10	Policy p_{up}^* for $\tau < \beta < \tau + \kappa$, if $\lambda^2 < \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ (left) and if $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \lambda^2$ (right) with $\eta_3 = \frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2}$ in Case (ii).	77
4.11	Policy p_{up}^* for $\tau + \kappa < \beta$, if $\lambda^2 < \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ (left) and if $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \lambda^2$ (right) in Case (iii).	78
4.12	Policy p_{up}^* for $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta$ in Case (iv).	78
4.13	The threshold η_1 depending on β .	79
5.1	Methodological framework.	94
5.2	Descriptive statistics of newspaper data per source.	95
5.3	Wordcloud for anxiety pole.	98
5.4	Anxiety index for each country in the years 2020 and 2021.	101
5.5	Aggregated anxiety related to each topic (left) and each newspaper type (right).	102
C.1	A schematic representation of two utility functions that fulfill the SCP.	150
C.2	Policy p_{del}^* for $\beta > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ in Case (i).	181
C.3	Policy p_{del}^* for $\beta \leq \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, if $\lambda^2 < \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ (left) and if $\lambda^2 > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ (right) in Case (ii).	182
D.1	The anxiety index for broadsheet newspapers in the years 2020 and 2021.	200
D.2	The anxiety index for tabloids in the years 2020 and 2021.	201
D.3	The dates of the 12 waves of the survey.	204
D.4	Coherence scores for different numbers of topics.	204
D.5	Anxiety by topic over time.	206

List of Tables

- 5.1 Spearman’s rank correlation coefficients for different shifts in time. 99
- 5.2 List of topics, topic names and associated keywords. 100

- D.1 List of first sentences of articles with the highest anxiety index in Austria. 202
- D.2 List of first sentences of articles with the highest anxiety index in Switzerland. 203
- D.3 Translation of the list of topics, topic names and associated keywords. 205

Thesis Summary

This dissertation analyzes the design of political institutions in various dimensions, namely with regard to checks and balances, to the impact of preference shocks in the presence of costs of change, and to credence policies.

In an introductory chapter we motivate our research. In Chapter 2, we study a two-period model of political competition where the level of costs of change is endogenously determined and is interpreted as checks and balances. We analyze different voting rules with regard to the proposal-maker and to the majority voting rule used for approval. Our main focus are stable levels of checks and balances, in other words, levels that do not change over time. Among other findings, we show that stable checks and balances always exist, are strictly positive, lead to gridlock, and are higher if the proposal-maker is the incumbent party. Moreover, we find that higher level of checks and balances are chosen if higher majority requirements for constitutional changes are in place and if the society is more polarized.

We study an extension of this model in Chapter 3. We introduce preference shocks for all parties in the presence of costs of change and determine the policy choices. We find that when facing preference shocks, the incumbent party chooses more extreme policies. However, we can also show that policy choices become more moderate for intermediate costs of change and for a higher turnover probability.

In Chapter 4, we develop a model to analyze different decision-making processes for credence policies, i.e., for policies whose consequences are difficult to assess in the mid- and long-term. Therefore, we assume heterogeneous voters and include experts in different ways: we consider elections without experts, elections with vote delegation and elections with a published consensus expert opinion. Further, we compare these three voting processes to an optimal policy that is chosen if the fully informed electorate votes on an issue. We find that detrimental and extreme

outcomes may arise when policy decisions are delegated to experts who do not take the electorate's heterogeneity in risk aversion into account. Further, we elaborate on the positive impact of a published consensus expert opinion on the policy choice.

Finally, in Chapter 5, we assess empirically how credence policies are reflected by using the Covid-19 pandemic as a case study. Since online newspapers play a crucial role in times of high uncertainty regarding the near future and can shape a society's emotional perception of policies, we analyze online newspaper articles with natural language processing methods. We develop a new time series measure of anxiety by combining existing lexica and word embeddings. We show that the time series correlate with policy changes and infection rates. For a validation of our results, we evaluate them qualitatively and compare them with other indicators.

Zusammenfassung

Diese Dissertation befasst sich mit verschiedenen Aspekten politischer Institutionen, nämlich mit der gegenseitigen Kontrolle von politischen Parteien (Checks and Balances), den Auswirkungen von Präferenzschocks mit kostspieligen Reformen und den sogenannten “Credence Policies”, die als Politikentscheidungen, deren Konsequenzen schwer abzuschätzen sind, zu verstehen sind.

In der Einleitung begründen ausgewählte Literatur und empirische Beobachtungen die Forschungsfragen dieser Arbeit. Im zweiten Kapitel untersuchen wir ein Wählermodell mit zwei Perioden. Darin interpretieren wir irreversible Änderungskosten einer Politik als Grad der gegenseitigen Kontrolle, kurz genannt *Checks and Balances*, die endogen bestimmt werden. Wir analysieren verschiedene Abstimmungsregeln, die durch die/den AntragsstellerIn und die in Kraft gesetzte Mehrheitsregel charakterisiert werden. Unser Hauptaugenmerk liegt auf stabilen Niveaus von Checks and Balances, d.h. Niveaus, die im Laufe der Zeit unverändert bleiben. Wir zeigen unter anderem, dass stabile Niveaus von Checks and Balances immer existieren, immer positiv sind, zum politischen Stillstand führen können und höher sind, wenn sie von der amtsführenden Partei vorgeschlagen werden. Ausserdem können wir nachweisen, dass ein höheres Mass an Checks and Balances gewählt wird, wenn höhere Mehrheitsanforderungen für Verfassungsänderungen bestehen und stärkere politische Polarisierung gegeben ist.

Im dritten Kapitel präsentieren wir eine Erweiterung des Modells, in dem alle Parteien von Präferenzschocks betroffen sind und irreversible Reformkosten auftreten können. Wir zeigen, dass Präferenzschocks die amtierende Partei dahingehend beeinflussen, eine extreme Politik zu wählen. Jedoch wirken mittlere irreversible Reformkosten und eine höhere Fluktationswahrscheinlichkeit dem entgegen und führen zu moderateren politischen Entscheidungen.

In Kapitel 4 entwickeln wir ein Modell zur Analyse verschiedener Entscheidungsprozesse für Credence Policies, d.h. für Politikentscheidungen, deren Auswirkungen schwer oder unmöglich abzuschätzen sind. Dafür modellieren wir heterogene WählerInnen und binden ExpertInnenmeinungen auf unterschiedliche Weisen ein: Wir betrachten Wahlen ohne ExpertInnen, Wahlen mit Stimmrechtsübertragung und Wahlen mit einer veröffentlichten Konsens-ExpertInnenmeinung. Ausserdem vergleichen wir die Ergebnisse der drei Abstimmungsprozesse mit der idealen Politikentscheidung, die gewählt wird, wenn vollständig informierte WählerInnen abstimmen. Wir stellen fest, dass es zu extremen und nachteiligen Politikentscheidungen kommen kann, wenn die Entscheidungen an ExpertInnen delegiert werden, welche die Heterogenität der Risikoaversion der Wählerschaft in ihre Entscheidungsfindung nicht miteinbeziehen. Des Weiteren gehen wir auf die positiven Auswirkungen einer veröffentlichten konsensuellen ExpertInnenmeinung auf die politische Entscheidung ein.

Im letzten Kapitel analysieren wir empirisch, wie Credence Policies reflektiert werden, indem die Covid-19 Pandemie als Fallstudie verwendet wird. Da Online-Zeitungen in Zeiten grosser Unsicherheit für die nahe Zukunft eine wichtige Informationsquelle darstellen und die Stimmung einer Gesellschaft prägen können, analysieren wir Zeitungsartikel mit Textanalyse-Methoden. Wir entwickeln eine Zeitreihe im Bezug auf die Angstwahrnehmung, indem wir Lexica und Word-Embedding-Methoden kombinieren. Als Resultat zeigen wir, dass die Zeitreihe mit Politikänderungen und Infektionszahlen korreliert. Zur Validierung evaluieren wir die Resultate qualitativ und vergleichen sie mit anderen Indikatoren.

Chapter 1

Introduction

This thesis can be divided into two parts: *(i)* Costs of change and their interpretation as Checks and Balances; *(ii)* Credence policies, voting procedures and the role of experts in democracy. For each part, some (empirical) observations and selected literature that motivate the work presented in this thesis are discussed.

1.1 Motivation

Costs of change and their interpretation as checks and balances*

Changing a status quo policy often induces costs, so-called *costs of change* (Gersbach et al., 2019). These costs due to the policy shift might arise because additional funding might be necessary or original investments might be redundant because of a reform. Another way to interpret these reform costs, is as a summary statistics for the many institutional features of a political system and various regulations that determine checks and balances and make policy shifts costly. For instance, there might be a long legislative procedure to change the law, where legislators, either being in favor of or against the change, incur opportunity costs that lower the provision of public goods in their constituencies. Another example of costs is an increase of the vote threshold necessary in parliament to reform the status quo

*This motivation is partially based on Gersbach et al. (2022).

policy. Hence, a majority might only be obtained with the vote of some parliament members who want to obtain pork barrel policies. Recently, this was the case in the US when Republicans used the *filibuster rule* to block a bill to launch an independent commission investigating the January 6 2021 insurrection.¹ The filibuster is a procedural trick in the Senate: By prolonging debate, one or several members can delay or prevent a bill. To stop this prolonged debate, a vote with a majority of 60% is necessary. As a result, it means that nearly all major legislation requires a 60% majority to pass.

How institutional design and constitutional checks can constrain politicians' behaviour was discussed in the seminal works of Barro (1973) and Ferejohn (1986). Various forms of checks and balances and the separation of powers limit the incumbent's power to dictate policies and to implement policies for her/his own interest or for ideological agendas. The importance and role of this mechanism goes back to at least Locke (Locke, 1690) and Montesquieu (Montesquieu, 1748). How to prevent the abuse of power through constitutional checks and balances was already discussed in the federalist debate preceding the adoption of the U.S. Constitution. Founding Father James Madison captured it in *The Federalist Papers* No. 51 in the year 1788: "*In framing a government which is to be administered by men over men, the great difficulty lies in this: you must first enable the government to control the governed; and in the next place oblige it to control itself. A dependence on the people is, no doubt, the primary control on the government; but experience has taught mankind the necessity of auxiliary precautions.*" (Madison et al., 1788).

The mentioned *auxiliary precautions* include different institutional features, like the separation of powers between the executive and a bicameral legislature, as well as the indirect election of senators, among others (Acemoglu et al., 2013). Formally, Persson et al. (1997) study the influence and requirement of the separation of powers for a system of checks and balances within the structure of a government. In particular, they show that the separation leads to a reduction of the amounts of rents that political parties can extract. This suggests that voters are in favor of high checks and balances, in general. However, Acemoglu et al. (2013) elaborate on cases

¹More information can be found at <https://edition.cnn.com/2021/05/28/opinions/repulican-threat-to-democracy-filibuster-zupnick/index.html> (accessed on July 7, 2021).

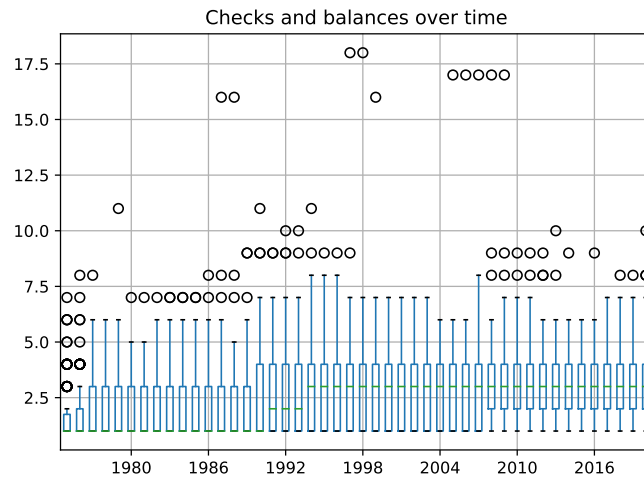


Figure 1.1: Evolution of checks and balances over 45 years in 183 countries. Source: Cruz et al. (2021).

where voters dismantle constitutional checks and balances on the executive, based on a theoretical model. Their empirical motivation includes constitutional changes increasing legislative power over the past decades throughout Latin America. Stated examples are the Constitutions of Ecuador and Peru (1979), the Constitution of Brazil (1988) and Constitution of Paraguay (1992). These variations and differences in the level of checks and balances can also be observed across different countries over the time span of 45 years, as illustrated in Figure 1.1.² One can observe that checks and balances increased overall, but vary to a large extent.

Acemoglu et al. (2013) conclude that “[...] *the extent of checks and balances in democratic political systems should be thought of as an equilibrium outcome rather than as a historically or exogenously given, immutable institutional characteristic.*”

This motivates us to develop a model in which the costs of change are interpreted as checks and balances and are determined endogenously. Changes can be proposed not only by parties but also by the citizenry. For instance, in Switzerland popular initiatives enable citizens to propose changes to the federal constitution. Considering an agent representing the citizenry also allows us to capture the pos-

²CHECKS is a discrete variable from the Database of Political Institutions [2020], available from Inter-American Development Bank, Cruz et al. (2021). We consider the variable CHECKS which is defined based on Legislative and Executive Indices of Electoral Competitiveness.

sibility that different rules are in place for approval of changes. In California, for instance, constitutional amendments must be submitted to the voters as mandatory referendum. This is not the case in other US states. We study how the role of the proposer in the government and the majority rule used for approval influence the level of checks and balances that is chosen.

It is of interest, independent of checks and balances, how costs of change affect policy choices when the political environment changes. For instance, preference shocks, due to an unexpected event like a terrorist attack or the outbreak of a war, can affect policy decisions. Montalvo (2011) empirically analyzes the effects of a terrorist attack on voters' preferences in Spain by comparing the votes cast before (certified by mail) and after the bombing. As a result, the terrorist attack led to a change in voters' decision and to a higher turnout. Therefore, preference shocks are included in an extension of our model described above. Since politicians align their interests with the voters, we assume preference shocks in parties' interests. Our goal is to study how preference shocks affect policy choices, given costs of change.

Credence policies, voting procedures and experts in democracy

Implementing a new policy is often not only connected to costs, but also to uncertainty about its consequences. So-called *credence policies* are credence in the sense that it becomes impossible, or at least only possible in a distant future, to assess whether the policy was beneficial (Gersbach, 2021). Hence, the short- and mid-term consequences of the implementation of a new policy remain unknown for quite some time. There are several potential reasons why a policy might be a credence policy. For instance, a policy could be implemented along with other measures, such that the consequences of one specific policy are difficult to assess. Another reason might be that external shocks, such as preference shocks discussed above, make it impossible to trace back the individual benefit of a policy.

Examples of such credence policies include reducing the protection of private information to fight terrorism, implementing banking regulations, slowing down climate change and managing a health crisis. Gersbach (2021) elaborates on the first two examples. Whether, and if yes, how much access to private information

of citizens has helped to fight terrorism is impossible to assess, or at least not in the short-and mid-term. The consequences of the implementation of banking regulations with regard to a potential banking crisis also remain unknown. Further, to slow down climate change or to manage a health crisis, several policies are implemented at the same time. The direct consequences of setting CO₂ targets to reduce pollution or closing schools to handle a pandemic is hard to quantify on an individual basis.

Slowing down climate change or fighting a health crisis like the Covid-19 pandemic do not only require a large variety of policies, but also expertise from different fields. In both cases, experts with diverse backgrounds advise decision-makers regarding policies. For instance, at the UN Climate Change Conference 2021, politicians were consulting experts to assess the consequences of potential policies. One example in the case of the Covid-19 pandemic is the Swiss National COVID-19 Science Task Force consisting of experts from different fields, e.g. virologists and economists, who addressed key issues regarding the Covid-19 crisis (Swiss National COVID-19 Science Task Force, 2022).

Solving complex issues that involve diverse disciplines is one of the most important challenges of modern democracies. There are different ways to approach policy decisions where some expertise is needed. A common procedure, used by the *Pirate Party* of Germany, is called *liquid democracy*. Its members can either vote themselves or delegate their voting right to proxies (Cammaerts, 2015). Further, one can also include opinion updating of voters into the voting process. In this procedure, a consensus of opinion is produced by experts and shared with voters before elections. This was implemented by the Swiss National COVID-19 Science Task Force who published its policy recommendations on its website.³ This served as a source of information for citizens before voting on initiatives, i.e., regarding the government's measures to fight the Covid-19 pandemic in November 2021.⁴

³<https://sciencetaskforce.ch/en/policy-briefs-english/> (accessed on April 5, 2022).

⁴<https://www.swissinfo.ch/ger/resultat-covid-gesetz-zertifikat/47141410> (accessed on May 29, 2022).

Hence, we want to tackle three central questions:

- How should democracies deal with such complex (credence) policy issues?
- How should experts be included in the decision-making process?
- How does communication through media shape perceptions of citizens about credence policies?

This motivates us to develop a theoretical model and conduct some empirical analysis to provide answers to these questions. First, we assess how credence policies are implemented in different voting procedures. Second, we study the influence of the inclusion of experts on the policy results. Third, we conduct an empirical analysis regarding credence policies and experts, taking the Covid-19 pandemic as a case study.

1.2 Research Questions

In this thesis, the following research questions are addressed:

- Q1 How do democracies choose their amount of checks and balances? How does the level of checks and balances depend on the proposal maker's role in government and on the majority voting rule used for the decision?
- Q2 How do preference shocks impact political parties' policy choices if policy reforms are costly?
- Q3 How should experts be included in decision-making processes in the case of credence policies?
- Q4 How are credence policies reflected in media coverage in times of high uncertainty?

1.3 Structure of the Thesis

Policy Reforms and the Amount of Checks & Balances (Chapter 2)*

In Chapter 2, we examine how democracies choose their amount of checks and balances (C&B) to answer the questions in Q1. For this purpose, we consider a model of political competition with three agents—two parties and a median voter—and costly policy reforms. We assume that the cost of a marginal reform is determined endogenously by the political parties and/or by the median voter representing the citizenry. The decision on the level of C&B is taken in the constitutional phase, i.e., before actual policies are chosen. We focus on the set of stable C&B—in other words, C&B that are not changed over time—for different constitutional rules. These rules depend on which agent has the power to propose new C&B and on the majority voting rule used for approval. Our main results show that stable C&B always exist, are never zero, lead to gridlock, and are higher if the proposal-maker is the incumbent party. We also find that depending on the constitutional rules, many C&B, if not most, are unstable. Finally, we conclude that higher majority requirements for constitutional changes and more polarized societies are conducive to greater sets of stable C&B.

Role of preference shocks in policy moderation (Chapter 3)†

In Chapter 3, we study an extension of the model discussed in Chapter 2 to tackle research question Q2. We extend the two-period model by introducing preference shocks for all political parties before the decision-making process, given costs of change. First, we examine both parties' policy choices in the second period. Second, the incumbent-party's policy choice in first period is characterized for different level of reform costs. We find that in the presence of preference shocks, policy choices are more extreme. Further, we can also show that policy choices become more moderate for intermediate marginal costs and for a higher turnover probability.

*This chapter is joint work with Hans Gersbach and Oriol Tejada. It is based on Gersbach et al. (2022).

†This chapter is single-authored work.

Credence Policies and Experts (Chapter 4)[‡]

We develop and study a theoretical model to analyze different decision-making processes for policies whose consequences are difficult, or even impossible, to assess in the mid- and long-term. To find an answer to the question Q3, we study the role of experts on such credence policies in democratic decision-making. Therefore, we assume that the electorate consists of heterogeneous voters regarding risk aversion and beliefs about policy efficiency. We compare elections without experts, elections with vote delegation to experts, and elections with published consensus expert opinion to the optimal policy when fully informed citizens vote on an issue. We find that it depends on the experts' risk aversion whether these experts have a beneficial or harmful impact on democratic decision-making. We also show that extreme and detrimental outcomes can arise when decisions are delegated to experts who act on the most accurate assessments of the consequences of policies but do not take the differences in risk aversion of the electorate into account. Publishing a consensus assessment of experts before the citizens vote can be either neutral or can improve policy-making, depending on how beliefs about the effectiveness of the policy are distributed in the electorate.

Capturing Anxiety with Word Embeddings (Chapter 5)[§]

Finally, in Chapter 5, we study how credence policies are reflected, based on an empirical analysis. We analyze online newspaper texts with natural language processing methods, by using the Covid-19 pandemic as a case study, to answer research question Q4. We argue that newspapers can play a crucial role in shaping a society's emotions in times of high uncertainty about the near future. We develop a new time series measure of anxiety perceptible in newspaper coverage. This measure is derived from 130,000 articles published in Austria and Switzerland over two years. Our findings highlight the gains from combining existing lexica and word embeddings to analyze texts. We find that the time series of both countries correlate with policy changes as well as with infection rates, and differ with regard to variance. For validation, we assess our results qualitatively and against other indicators.

[‡]This chapter is joint work with Hans Gersbach and Oriol Tejada.

[§]This chapter is single-authored work.

Chapter 6 summarizes and concludes the presented projects and elaborates on potential avenues for future research.

Chapters 2 and 4 are both joint work with Hans Gersbach and Oriol Tejada. All authors worked together on the research questions, developed the models, wrote the manuscript and finalized the paper versions of the projects. Julia Wagner was mainly responsible for most analyses and visualizations. Julia Wagner is the single author of the projects presented in Chapters 3 and 5.[‡]

[‡]Julia Wagner highly appreciated valuable input given by colleagues for both projects. In particular, from Hans Gersbach, Oriol Tejada, and Elliott Ash.

Chapter 2

Policy Reforms and the Amount of Checks & Balances*

Abstract

We examine how democracies choose their amount of checks and balances (C&B). For this purpose, we consider a simple model of political competition with costly policy reforms. The cost of a marginal reform is determined endogenously at the constitutional phase—i.e. before policies are chosen—through the choice of C&B. We characterize the set of stable C&B for different constitutional rules which vary depending on *(i)* who has the power to propose changes to C&B and on *(ii)* the qualified majority needed for approving such changes. Our main results show that stable C&B always exist, are never zero, lead to deadlock, and are higher if the proposal-maker is the party in government. We also find that higher majority requirements for constitutional changes and more polarized societies are conducive to greater sets of stable C&B.

*This chapter is joint work with Hans Gersbach (ETH Zurich and CEPR) and Oriol Tejada (Universitat de Barcelona). This chapter is based on Gersbach et al. (2022).

2.1 Introduction

Checks and balances (in short, C&B) are central elements of most democracies. As a general rule, they are implemented to keep political action by the government and/or by parliament within bounds, so that better decisions are attained collectively. One important way in which C&B affect the government's and/or the parliament majority's ability to rule is by imposing costs when policies are changed. For instance, as acknowledged in the above quote by Ferejohn and McCall Rosenbluth (2008), C&B can both increase transaction costs and cause delays in the legislative domain. Transaction costs that have to be borne by all citizens, including party members, stand out the most when C&B directly involve several rounds of bureaucratic procedures that are financed by the public budget. Delays in legislative decision-making, on the other hand, may hinder the provision of public goods by diverting political resources from the executive to the legislative arena, which is also costly for the citizenry. Such delays are most obvious when a minority can require amendments or use filibusters during the legislative process.¹ In certain occasions, C&B may also allow such small minorities to earn rents whose costs also have to be borne by the vast majority of citizens. This typically occurs when the vote threshold necessary in parliament to change the status quo is high, and thus may require the vote of some particular parliament members who, in exchange, want to obtain pork barrel projects.

In this section, we examine which level or amount of C&B is chosen in a democracy and which would be optimal. For our endeavour, we use the marginal cost of changing a policy as a summary statistics for all the constitutional provisions, laws, and regulations that govern checks and balances and ultimately determine their level, as motivated above. More specifically, we consider a two-phase game in a given political system, with two parties competing for power. In the constitutional phase, the amount of C&B is chosen by a procedure that determines *(i)* who can make proposals for a new amount of C&B and *(ii)* the majority necessary to accept a proposal for a new amount of C&B. If no new amount of C&B is approved in the

¹The filibuster is a procedural trick in the Senate: By prolonging debate, one or several members can delay or prevent a bill. To stop this prolonged debate, a vote with a majority of 60% is necessary. As a result, nearly all major legislation requires a 60% majority to pass.

constitutional phase, the given status-quo amount of C&B is maintained.

Once the constitutional phase is over, the legislative/executive phase starts. For this second phase, we consider a two-period model of political competition based on Gersbach et al. (2019). In each period, the party that is in power chooses a one-dimensional policy under uncertainty about who will hold power in the next period (if there is one). Our model can be applied both to executive offices—in which case being in power means holding office—and to legislative chambers—in which case being in power means being the median voter in the legislative chamber. Changing the first-period policy in the second period generates costs for all agents which increase (linearly) with the extent of the policy change; these are called *reform costs* and include the costs generated by the amount of C&B chosen in the constitutional phase.² The unidimensional policy therefore embodies the initiation of the provision of some good (in the first period), the supplied amount of which can later be changed (in the second period). Assuming that there are no reform costs in the first period is a simplification of the general case in which all types of policies can be reformed, but similar results would be obtained if first-period reform costs existed, as the main mechanisms leading to policy moderation would be maintained (see Gersbach et al., 2021).

To disentangle the strategic aspects present at choosing first C&B and then policies, we look for subgame perfect equilibria of the two-phase game. Then we focus on our main innovation, i.e., on how C&B are determined endogenously in the constitutional phase, before actual policies are chosen. This choice is, of course, strongly impacted by the expectations of all agents involved about what will happen in the legislative (or executive) phase. To rule out implausible behavior, we then restrict our attention to subgame perfect equilibria without stage-dominated strategies; we just call them *equilibria*. For our analysis, we also introduce the notion of stable (amounts of) C&B. C&B are *stable* if they are not changed during the constitutional phase. This allows us to study the set of stable C&B for different

²Beyond checks and balances, there may be other sources of reform costs such as physical investments to adapt policy, communication efforts to implement the new policy, or psychological costs that originate from an aversion to changes (see Gersbach et al., 2020b). All these costs can be easily integrated into our analysis by setting a lower bound of reform costs that cannot be undercut by the lowest possible level of C&B (or equivalently, the complete absence of C&B). Unless specified otherwise, we assume by the default that all reform costs are generated through the choice of the amount of C&B.

rules governing the constitutional phase.³

In the constitutional phase, parties can make a proposal and vote on it, but so can a representative voter (also called the *median voter*). Adding this third agent allows us to capture the possibility that besides the government and the legislative chambers, the citizenry can also participate in each stage of the constitutional phase, say via a popular initiative or referenda. In Switzerland, popular initiatives enable the citizens to propose changes to the federal constitution. In California, constitutional amendments must be submitted to the voters as mandatory referendum. The median voter could also be seen as a constitutional court that has a say in suggesting or interpreting constitutional rules, or it could embody the possibility that constitutional changes could be either proposed or upset at higher (national or international) levels.

Our main insights are as follows. First, no matter the constitutional rule, the set of stable C&B is non-empty and only contains strictly positive (levels of) C&B. Thus, at least in the long run, all democratic countries should display some amount of checks and balances which persist over time. These features can be observed empirically in some countries. For instance, the case of Canada and the UK is shown in Figure 2.1, which depicts C&B for the two countries over a time span of 45 years and shows that they remained relatively stable throughout this period.⁴

Second, all C&B lead to policies that are (weakly) more moderate than when no C&B are at all in place, although policy moderation does not monotonically increase with C&B for the whole range of the latter variable. Moreover, no matter the constitutional rules, the C&B chosen in equilibrium lead to deadlock, in the sense that no policy reform occurs on the equilibrium path. That is, the policy in both periods is the same regardless of how far apart the policy in the first period is from the peak of the party in office in the second period. (Yet, the C&B chosen for a given constitutional rule matters for the specific policy chosen in both periods of the legislative phase.) All this means that stable C&B can increase welfare by

³The voting rule and the proposal-making rule used in the constitutional phase can be interpreted themselves as C&B for constitutional changes. Our analysis then sheds light on how C&B at the constitutional level affect C&B on the executive and the legislative levels.

⁴CHECKS is a discrete variable from the Database of Political Institutions provided by the Inter-American Development Bank, which is defined based on Legislative and Executive Indices of Electoral Competitiveness.

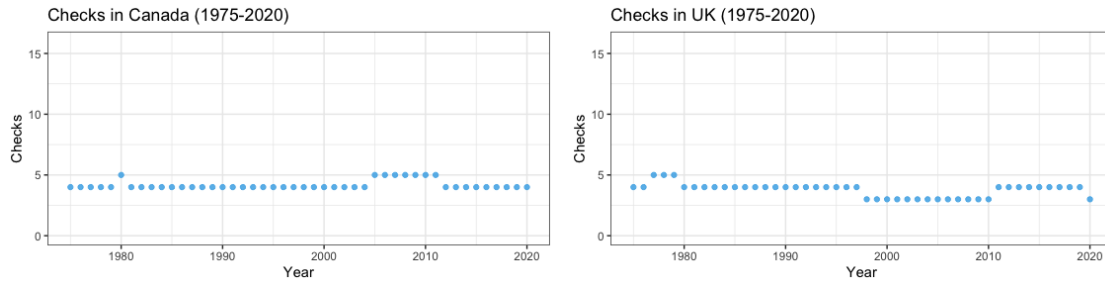


Figure 2.1: Evolution of C&B over 45 years in Canada (left) and the UK (right).

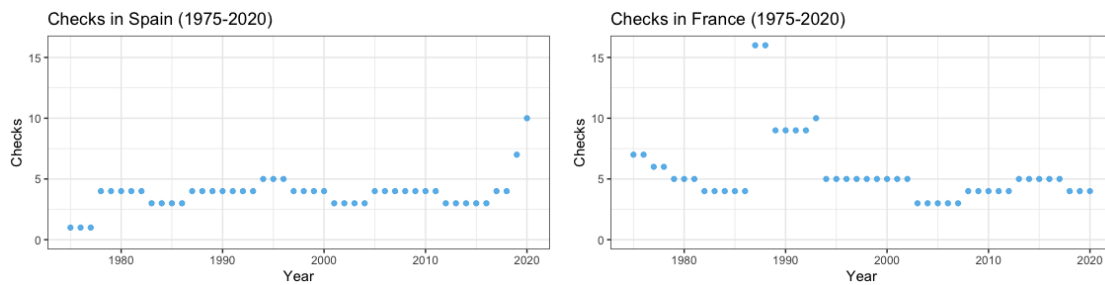


Figure 2.2: Evolution of C&B over 45 years in Spain (left) and France (right).

inducing better collective decisions: they yield moderate policies and low, or even zero, reform costs on the equilibrium path. This is in keeping with the widespread belief that checks and balances can—and *do*—improve the quality of a democracy.

Third, we also find that depending on the constitutional rules, many, if not most C&B are not stable. A broad interpretation of our theory then predicts changes in C&B in most cases, at least in the short term.⁵ For a given constitutional rule, the set of stable C&B generically depends on features of the political system, such as the degree of party rotation and the lack of symmetry of the median voter preferences with respect to the parties' preferences, which can be subject to shocks. One interpretation of this insight is that democratic societies experimenting rapid changes in different political dimensions also implement volatile C&B, and hence,

⁵More specifically, if we apply our model twice and allow some crucial parameters of the model to be different in the two games, we obtain that most C&B chosen in the first game will be changed in the second game. This can be a reasonable prediction for situations where parties heavily discount future outcomes, e.g. when parties are willing to undertake constitutional changes only once in a generation, due to the ramifications of doing so in the electoral process.

have volatile political institutions and/or regulations. From an empirical perspective, high volatility of C&B can be observed in another group of countries, as shown in Figure 2.2 for Spain and France.

Fourth, our analysis allows us to obtain comparative statics with regard to the set of stable C&B in terms of the constitutional rules by varying (i) the share of votes needed to approve a new C&B (keeping the proposal-maker fixed), and by varying (ii) the proposal-maker's identity (keeping the share of votes needed for approval of a new C&B fixed). On the one hand, an increase in the share of votes needed for approval naturally leads to a larger set of stable C&B, no matter the proposal-maker. However, more stringent vote share requirements do not necessarily lead to larger C&B, as the opposite can occur. On the other hand, it matters who can propose constitutional changes. For instance, the incumbent party proposes greater C&B—and, thus, larger reform costs—than the challenger party and the median voter. The reason is that the incumbent party wants to make it costly for the challenger party to change the policies it will enact in the first period. This captures the situations where the parties forming the government have the right to propose constitutional changes, which is common in democracies. These observations may e.g. explain the preservation of the *Filibuster Rule* in the US Senate.⁶ Even if a party could abolish such a rule at some point in time, it will probably not do it, as it would fear that its new policies would be undone as soon as the other party obtains a simple majority in the Senate.

The property that different constitutional rules and different characteristics of the political system lead to different sets of stable C&B may help to understand the differences in the amount of C&B observed across democracies and could be useful for the design of democratic institutions. For instance, our model of political competition features a positive relation between party polarization and reform costs (and, thus, amounts of C&B). Such a relationship is consistent with empirical observations, as shown in Figure 2.3, which depicts the evolution of C&B and party polarization over 40 years in 38 democratic countries chosen based on available data.⁷ Two observations are in order. First, the relation between C&B and party

⁶The majority party in the US Senate has the proposal power for both changes of C&B and legislative changes.

⁷POLARIZ is a discrete variable from the Database of Political Institutions [2020], available from Inter-American Development Bank (Cruz et al., 2021). We consider the variable POLARIZ,

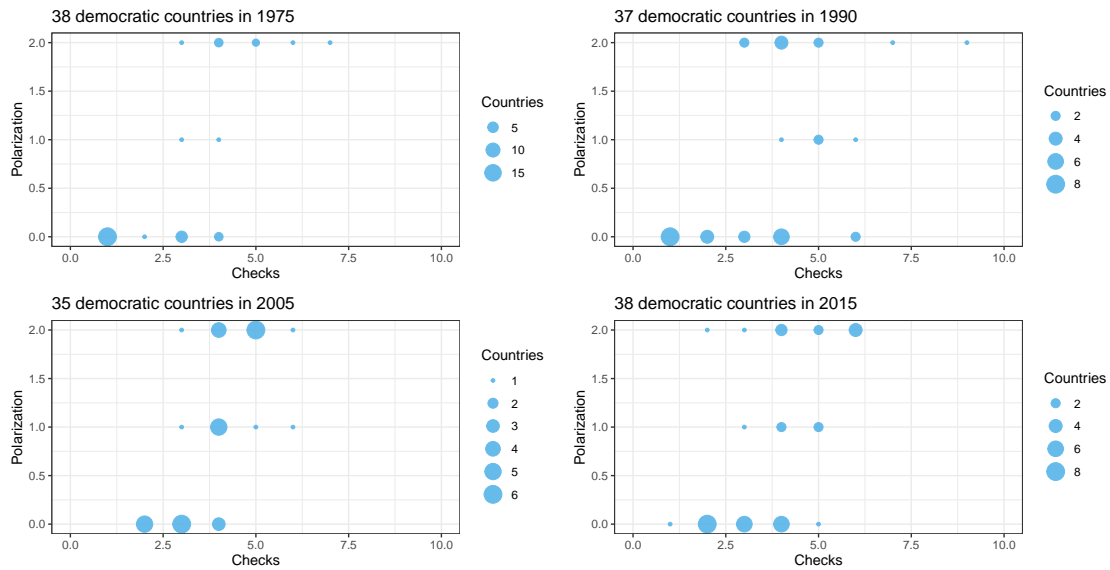


Figure 2.3: Comparison of 38 countries regarding checks and party polarization over 40 years. The list of all countries is in the Appendix.

polarization is positive throughout this time span. Second, one observes that C&B and party polarization increase over time for many countries. Since our model also predicts a positive relationship between C&B and policy polarization, the second observation might be consistent with higher party polarization leading to higher policy polarization via larger amounts of C&B.

The chapter is organized as follows: In Section 2.2 we review the papers that are most closely connected to our work. In Section 2.3 we describe the model. In Section 2.4 we first analyze the political game and, second, different constitutional rules. Section 2.5 compares different constitutional rules and discusses welfare. Section 2.6 concludes. The proofs are in Appendix A.

2.2 Relation to the Literature

We are not the first to provide a theoretical account of how political institutions arise endogenously and later may survive over time. Barberà and Jackson (2004) examine

which is defined as the maximum polarization between the executive party and the four principal parties of the legislature.

the self-stability of majority rules and, similarly to us, obtain that such rules (or constitutions) always exist but that majority rules may not survive generically. Messner and Polborn (2004) characterize voting rules that arise endogenously. Our approach is novel in that we envision the amount of C&B as costs associated with policy change and focus on its stability over time. This allows us to obtain new insights on how democratic societies choose their amount of C&B and on how this is influenced by political characteristics (such as party polarization and party rotation) and by constitutional features (such as the proposal-making rule and the majority rule governing constitutional changes).

A large literature investigates the effect of political institutions on policy-making and other aspects of elections, both from a theoretical and an empirical viewpoint (see Alesina and Rosenthal, 1996; Persson and Tabellini, 2002; Tsebelis, 2002; Persson and Tabellini, 2005; De Sinopoli and Iannantuoni, 2007; Stephenson and Nzelebe, 2010; Iaryczower and Mattozzi, 2013; Matakos et al., 2016, among many others). Our main contribution to this literature is to emphasize the off-equilibrium role of C&B in shaping policies and how the choices of C&B made in the constitutional phase lead to deadlock (i.e., to no reform) in the political stage regardless of how large or small party polarization is. This is in line with Mainwaring (1990), among many others, who argues that the division of power leads to executive deadlock. In our model, low amounts of C&B do not necessarily lead to deadlock, but it is the endogenous choice of C&B made on the equilibrium path by parties and citizens (at the constitutional stage) that exhibits this property. We also consider that reforms are neither intrinsically good nor bad. Our results are then in keeping with the idea that enacting a new fundamental policy (e.g. joining an international organization like the EU) will often not be undone, as the costs associated with such reforms will be made untenable for society.

Another strand of literature evaluates policy outcomes under different electoral systems (see e.g. Morelli, 2004; Bouton et al., 2018). We also consider different voting rules, but the novelty of our analysis is that we focus on how such constitutional rules influence policy through the choice of C&B. Our analysis identifies a link between party extremism and policy extremism, which is mediated by (the amount of) C&B, and more generally by the constitutional design. This resonates with Bordignon et al. (2016), who show that certain voting rules can moderate

party extremism. Our model assumes only one policy (or project) dimension for which one party is initially the incumbent, but our insights extend to the case where multiple project dimensions are considered in different points in time and different parties are responsible for the initial decision. It suffices to assume that parties discount such future situations sufficiently enough.

This section is also part of a growing literature devoted to analyzing the effects of reform costs—also called costs of change—in elections (see Glazer et al., 1998; Gersbach and Tejada, 2018; Gersbach et al., 2019, 2020a,b; Eraslan and Piazza, 2020; Dziuda and Loeper, 2021). We contribute to this literature by being the first to endogenize such costs, which we interpret as a reduced form of (executive and/or legislative) C&B, i.e., of all the institutions, laws, and regulations that reduce the incumbent’s ability to dictate policy. By considering different constitutional rules to determine C&B, we can use our simple stylized framework to provide new insights on the design of democracy.

Finally, C&B are the subject of substantial empirical and theoretical work, the focus of which are veto players (see e.g. Tsebelis, 1999), bicameral systems (Riker, 1992; Diermeier and Myerson, 1999; Tsebelis and Money, 1997), and/or separating the authority over different policy dimensions (see e.g. Ashworth and Bueno de Mesquita, 2017; Besley and Coate, 2003; Nakaguma, 2015; Persson et al., 1997), among others. Some papers have examined the link between C&B and policy reforms. Aghion et al. (2004) investigate how C&B in the form of the share of votes needed to block the incumbent’s policy affect the implementation of efficient reforms (see also Acemoglu et al., 2013; Forteza and Pereyra, 2019; Forteza et al., 2019; Alesina and Rosenthal, 2000). In a recent paper, Gratton and Morelli (2022) study how C&B should be set to reduce type-I errors at the expense of type-II errors in policy decision making. From an empirical perspective, Cox and Weingast (2018) show the importance of political C&B in preventing large economic declines. To the best of our knowledge, we are the first to model C&B as reform costs. Our reduced-form approach to C&B has the advantage of subsuming in one (continuous) parameter the many features, institutional and political alike, that restrict the ability of incumbents to choose policy. Our model then allows us to obtain new insights on how stable *amounts of C&B* emerge endogenously for given constitutional rules. To quote Gratton and Morelli (2022), the amount of C&B is “...at the center of the

debate over the merits of a constitution.” Our contribution can be best recognized through this premise.

2.3 Model

We consider a dynamic political game consisting of two periods ($t = 1, 2$) in which one of two parties, L and R , holds power and dictates policy. Later we extend this game by adding an initial constitutional stage, which is our main innovation. The set of policy choices is $[0, 1]$, where 0 corresponds to the leftmost policy and 1 to the rightmost policy. Without loss of generality, we assume that party R holds power in period $t = 1$. As we will see below, this implies that parties are not symmetric from a constitutional perspective. Then, in period $t = 2$, party R remains in power with probability $p \in [0, 1]$. Hence, power shifts across periods with probability $1 - p$. Besides the two parties there is also a median voter, denoted by M , whose role is visible only in the constitutional stage (we do not model elections explicitly).

In any period, agents have standard quadratic utilities over policies in $[0, 1]$, which are characterized by the agents’ peak. Moreover, reforming the policy is costly for all agents. These *reform costs* accrue only in period $t = 2$, as there is no status quo policy in place at the beginning of period $t = 1$. Specifically, given a policy $i_1 \in [0, 1]$ chosen in period $t = 1$, we assume that the policy choice $i_2 \in [0, 1]$ made in period $t = 2$ imposes utility losses on the two parties and on the median voter that amount to c_1 per unit of reform, with $c_1 \geq 0$. For simplicity, we assume that the costs are the same for the median voter and the parties. Of course, different levels of C&B can cause different costs for citizens, but in our basic model the decisive agents are alike.⁸ Therefore, if policy $i_t \in [0, 1]$ is chosen in period $t \in \{1, 2\}$, agent $K \in \{M, L, R\}$ with peak μ_K derives the following utility in this period:

$$u_K^t(i_{t-1}, i_t) := -(i_t - \mu_K)^2 - c_1 \cdot |i_{t-1} - i_t|,$$

where we write $i_0 = i_1$ for the sake of notation. Hence, unless $c_1 = 0$, the costs of policy reform across the two periods increase linearly with the absolute differ-

⁸This would also cover cases where small minorities can gain from C&B because they can earn rents in this process.

ence between the policies adopted in both periods. Assuming that costs of change are linear in the extent of the policy shift guarantees that incumbents (but not challengers) do not reform the policy in the second period they chose in the first period. Persistence in the incumbent's policy choices (but not in the challenger's) is assumed in Nunnari and Zápál (2017) and Forand (2014), among others, but we only obtain this property *in equilibrium*. Policy persistence for incumbents is a property documented in abundant theoretical and empirical research (see e.g. Miller and Schofield, 2003; Tavits, 2007).

As is standard, the agents' peaks satisfy the following condition:

$$0 \leq \mu_L < \mu_M < \mu_R \leq 1. \quad (2.1)$$

That is, the median voter has a moderate peak compared to each of the two parties.⁹ Henceforth we let $\Pi = \mu_R - \mu_L$ denote the degree of party polarization.

The game defined above, which we denote by $\mathcal{G}(c_1)$, has been studied in Gersbach et al. (2019). In the following, we extend this game by considering a constitutional phase that takes place prior to period $t = 1$, say in period $t = 0$. In this phase, the society chooses the reform costs c_1 that will be in place between the first and the second period, given the status quo reform costs $c_0 \geq 0$. The reform costs that are in place are called (*the amount of*) $C\mathcal{E}B$.

To sum up, the timeline of the constitutional game, which we generically denote by $\mathcal{G}^+(c_0)$, is the following:

- ($t = 0$) The society chooses the value of c_1 according to some procedure (to be specified later), given the value of c_0 .
- ($t = 1$) Party R chooses policy $i_{R1} \in [0, 1]$.
- ($t = 2$) Party R wins the election in $t = 2$ with probability p , where $0 < p < 1$. Otherwise, Party L wins the election. Party $K \in \{L, R\}$ who wins the election chooses $i_{K2} \in [0, 1]$.

⁹In standard models, the positions of political parties are determined by the expected position of the median voter. In the citizen-candidate model—see Besley and Coate (1997) and Osborne and Slivinski (1996)—, the two political actors locate at equidistant positions on opposite sides of the median voter's preferred position. We relax this condition and simply assume that the median voter's peak lies in between.

We consider different variants for the constitutional stage that takes place in period $t = 0$. These variants depend on who has proposal-making power and on the majority rule used. For each variant of the constitutional phase, we study the subgame perfect equilibria (or just *equilibria*) of the resulting dynamic game. As is standard, we impose the refinement that no agent uses stage weakly dominated strategies, so that in any voting stage all agents vote as if they were pivotal.¹⁰ As a non-essential tie-breaking rule we also assume that in case of indifference agents vote in favor of the proposed C&B instead of the status quo C&B.

Our main focus are the C&B that the society does not change in the constitutional phase and thus will persist. This leads to the following definition:

Definition 1 *For a given variant of the game $\mathcal{G}^+(c_0)$, the amount of C&B c_0 is stable if $c_1 = c_0$ in any equilibrium of the corresponding constitutional game.*

That is, for a given variant of the constitutional phase, C&B are stable if society does not change them and therefore remain in place. Our goal in the following section is to find the set of stable C&B for several variants of the constitutional phase. This allows useful comparative statics across variants. We generically denote such a set by \mathcal{SC} .

2.4 Analysis

Our analysis is divided into two parts. First, we focus on (sub)game $\mathcal{G}(c_1)$. Second, we focus on the different variants of game $\mathcal{G}^+(c_0)$.

2.4.1 Analysis of the political game

The following result characterizes the policies chosen on the equilibrium path in game $\mathcal{G}(c_1)$:

¹⁰This is only relevant if parties L and R as well as the median voter participate in the voting procedure during the constitutional stage.

Proposition 1 (Gersbach et al. (2019)) *In the unique equilibrium of game $\mathcal{G}(c_1)$:*

(i) *Party R 's policy choice in period $t = 1$ is*

$$i_{R1}^* = i_{R1}^*(c_1) := \begin{cases} \mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} & \text{if } c_1 \leq (1+p)\Pi, \\ \mu_L + \frac{c_1}{2} & \text{if } (1+p)\Pi < c_1 < 2\Pi, \\ \mu_R & \text{if } 2\Pi \leq c_1. \end{cases}$$

(ii) *Let $K \in \{L, R\}$ be the party that holds power in period $t = 2$. Party K 's best response to the policy $i_1 \in [0, 1]$ chosen in period $t = 1$ is*

$$i_{K2}^* = i_{K2}^*(i_1, c_1) := \min \left\{ \max \left\{ \mu_K - \frac{c_1}{2}, i_1 \right\}, \mu_K + \frac{c_1}{2} \right\}.$$

According to Proposition 1, party R never reforms policy if it keeps power. By contrast, if there is turnover, party L chooses a policy that is (weakly) closer to its own peak than the policy chosen in the previous period by party R . The extent of the policy reform carried out by party L , $|i_{L2}^* - i_{R1}^*|$, decreases with c_1 , until the policies chosen in period $t = 2$ by both parties are the same if $c_1 \geq (1+p)\Pi$. This is shown graphically in Figure 2.4, which replicates the result of Gersbach et al. (2019).

The next corollary follows immediately from Proposition 1 and states the relationship between C&B and the expected reform size defined as

$$p \cdot |i_{R1}^* - i_{R2}^*| + (1-p) \cdot |i_{R1}^* - i_{L2}^*|.$$

Corollary 1 *In the equilibrium of the game $\mathcal{G}(c_1)$, there is an inverse relation between C&B c_1 and the expected reform size. Moreover, if $c_1 \geq (1+p)\Pi$, no reform takes place regardless of who holds power, and a marginal increase of c_1 yields a more extreme policy.*

As shown in Figure 2.4, starting from $c_1 = 0$ the policy chosen by party R becomes more moderate until $c_1 = (1+p)\Pi$, and it increases monotonically thereafter until $c_1 = 2\Pi$. The effect of increasing c_1 above $(1+p)\Pi$ is then worth discussing

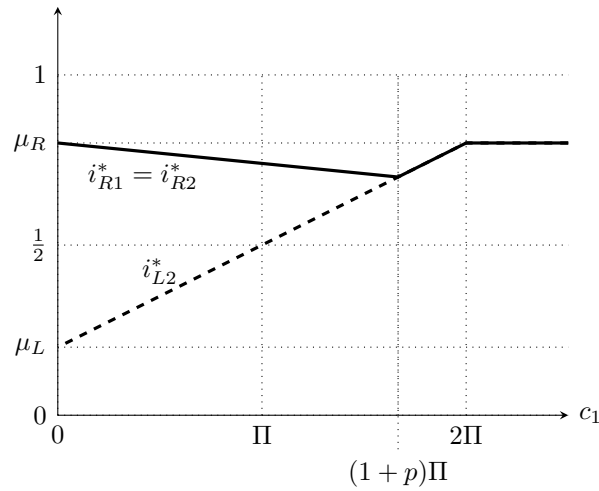


Figure 2.4: The policy choices in periods $t = 1$ and $t = 2$.

as such levels of C&B will obtain in equilibrium once we allow C&B to be determined endogenously in the constitutional phase. These larger C&B do not affect the expected reform size, but they yield more extreme policies. Although no reform is observed on the equilibrium path, the role of C&B in such cases is therefore to shape the policy via the off-equilibrium possibility of incurring reform costs if a marginal policy change is carried out.

Note that for any $c_1 \geq 2\Pi$, all policies chosen by all parties are equal to μ_R , party R 's peak. Hence, values of c_1 equal or larger than 2Π are equivalent in terms of outcomes—they lead to the same policies and to no reform costs. This yields the property that c_1 , with $c_1 \geq 2\Pi$, is stable if and only if $c_1 = 2\Pi$ is stable. Accordingly, for the sake of exposition we focus our subsequent analysis on $c_1 \in [0, 2\Pi]$.

Finally, it is worth noting that in a given political system there might be sources for costs of changes other than those that can be determined through C&B, such as economic costs associated with changes in law and structural transformation. Our model easily embodies this possibility. It suffices to assume that $c_1, c_0 \in [\underline{C}, 2\Pi]$, where $\underline{C} \geq 0$. Throughout our analysis we assume $\underline{C} = 0$, but our results can be easily accommodated to $\underline{C} > 0$. Note, in particular, that if $\underline{C} \geq 2\Pi$ then we obtain the trivial case in which μ_R is chosen in both periods no matter the party that is in office.

2.4.2 Analysis of the constitutional game

We now investigate the constitutional stage, which is the bulk of our analysis, by building on the results of the previous section. The expected lifetime utility—henceforth simply called *utility*—of agent $K \in \{M, L, R\}$ in period $t = 0$ is

$$\begin{aligned}
 H(c_1, \mu_K, p) &:= \mathbb{E}[u_K^1(i_{R1}, i_{R1}) + u_K^2(i_{R1}, i_{K2})], \\
 &= p \cdot [-(i_{R2}^*(i_{R1}^*(c_1), c_1) - \mu_K)^2 - c_1 |i_{R2}^*(i_{R1}^*(c_1), c_1) - i_{R1}^*(c)|] \\
 &+ (1 - p) \cdot [-(i_{L2}^*(i_{R1}^*(c_1), c_1) - \mu_K)^2 - c_1 |i_{L2}^*(i_{R1}^*(c_1), c_1) - i_{R1}^*(c_1)|] \\
 &- (i_{R1}^*(c_1) - \mu_K)^2,
 \end{aligned} \tag{2.2}$$

where the policy choices have been characterized in Proposition 1. If $c_1 \in [0, (1 + p)\Pi]$, Equation (2.2) can be written as

$$\begin{aligned}
 H(c_1, \mu_K, p) &= -(1 + p) \cdot \left(\mu_R - \frac{c_1}{2} \cdot \frac{1 - p}{1 + p} - \mu_K \right)^2 - (1 - p) \cdot \left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2 \\
 &- (1 - p) \cdot c_1 \cdot \Pi + \frac{1 - p}{1 + p} \cdot c_1^2.
 \end{aligned} \tag{2.3}$$

If $c_1 \in [(1 + p)\Pi, 2\Pi]$, Equation (2.2) boils down to

$$H(c_1, \mu_K, p) = -2 \cdot \left(\mu_K - \mu_L - \frac{c_1}{2} \right)^2.$$

In the following, we consider different variants for the constitutional phase in which the value of c_1 is determined. Each variant depends on the constitutional rules, i.e., (*i*) on who has the proposal-making power (incumbent party R , challenger party L , or median voter M) and (*ii*) on the majority rule used (simple majority, double majority, or unanimity). For either variant of the constitutional phase, the status quo C&B c_0 remains in place if the proposed C&B c_1 fails to gather the necessary votes.¹¹

We find that no matter the constitutional rule used, C&B are chosen such that no policy reform occurs. Moreover, there always exist stable levels of C&B. They depend generically on the agents' peak and on probability p .

¹¹In total there are nine different variants of the constitutional phase.

Simple majority rule

We start with the variant of the constitutional stage in which agent $K \in \{L, M, R\}$ can unilaterally decide on C&B c_1 . The agent with the decision-making power—called the *decision-maker*—is therefore either one of the parties or the median voter. If $K \in \{L, R\}$ is the decision-maker, this means party K 's has a parliamentary majority that suffices to approve c_1 . If party R has the majority in parliament, in particular, then both the executive power and the legislative power are in the same hands in period $t = 1$. If party L has the majority in parliament, by contrast, the executive power and the legislative power are in different hands in period $t = 1$. Assuming that the median voter is the decision-maker means that the decision about C&B c_1 can be taken by a simple majority of the electorate. In each of the three cases, the status quo C&B c_0 is immaterial for the outcomes as it can be simply overruled by the decision-maker. Alternatively, one can view this setup as equivalent to the case where agent K has the proposal-making power and only one vote from party R , party L , and the median voter M is required for approval of a new C&B. Thus the simple majority rule is in place in the constitutional phase in such a case. We denote the resulting game by $\mathcal{G}_{K,1}^+(c_0)$ for $K \in \{M, L, R\}$.

For our analysis in this and the next sections, it is convenient to define

$$\bar{\Pi} := \frac{2}{1+p} \cdot (\mu_M - \mu_L).$$

Parameter $\bar{\Pi}$ measures how polarized the median voter is compared to the challenger party L . Therefore it captures the asymmetry in preferences within the electorate. Note that

$$\frac{\bar{\Pi}}{\Pi} = \frac{2}{1+p} \cdot \frac{\mu_M - \mu_L}{\mu_R - \mu_L}. \quad (2.4)$$

The first term of the right-hand side of (2.4) increases with the turnover probability $1 - p$ and is equal to one if $p = 1$, i.e., if there is no turnover. The second term of the right-hand side of (2.4) increases in μ_M and is equal to one if $\mu_M = \mu_R$, i.e., if the median voter has the same peak as party R . To sum up, $\bar{\Pi}/\Pi$ is lower, the less likely turnover is and the less biased the median voter is in favor of the incumbent (relative to the challenger). As we show next, the relationship between $\bar{\Pi}$ and Π determines the C&B that the median voter M would choose if they were the

decision-maker in the constitutional stage.

If the simple majority rule is used in the constitutional phase, we obtain the following result:

Theorem 1 *In the unique equilibrium of game $\mathcal{G}_{K,1}^+(c_0)$, the C&B chosen is*

$$(c_1)_{K,1}^* := (c_1)_{K,1}^*(c_0) = \begin{cases} (1+p)\Pi & \text{if } K = L, \\ (1+p) \max\{\Pi, \bar{\Pi}\} & \text{if } K = M, \\ 2\Pi & \text{if } K = R. \end{cases} \quad (2.5)$$

Theorem 1 characterizes all the agents' optimal choice for the amount of C&B. To understand such choices, it is useful to plot the utilities of all agents as a function of C&B c_1 . This is done in Figure 2.5 by relying on the proof of Theorem 1.

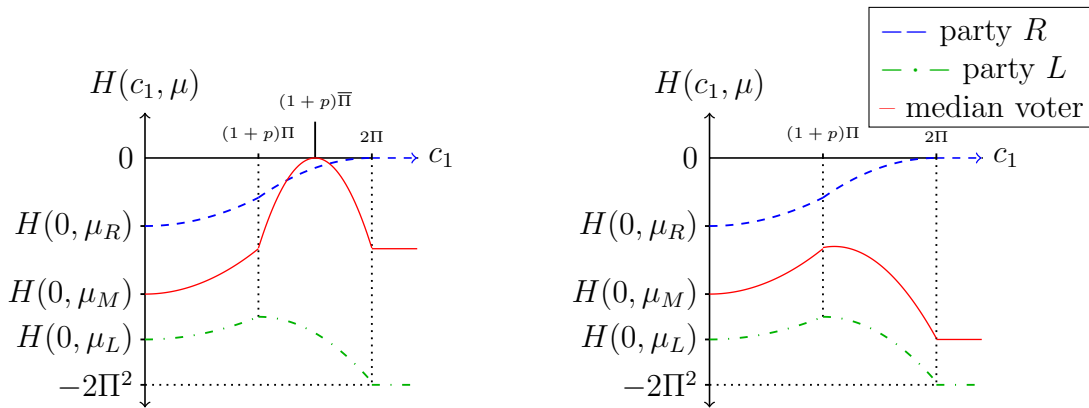


Figure 2.5: Schematic representation of (expected) utilities for $\Pi < \bar{\Pi}$ (left) and $\bar{\Pi} \leq \Pi$ (right).

The first observation is that the decision-maker's utility increases for $c_1 \in [0, (1+p)\Pi]$ no matter their peak. For party R , utility also increases for $c_1 \in [(1+p)\Pi, 2\Pi]$ and therefore reaches its maximum at 2Π . By contrast, party L 's utility decreases for values of c_1 above $(1+p)\Pi$, so its peak is at $(1+p)\Pi$. For the median voter, utility is maximal at $(1+p) \max\{\Pi, \bar{\Pi}\}$. It increases to the left of such threshold and decreases to its right. Hence, all agents have single-peak preferences regarding C&B. The relative order of their peaks is inherited from the relative order of their peaks regarding policies in $[0, 1]$ and from the fact that party R is the incumbent

and gets to choose the policy in period $t = 1$. If $\bar{\Pi} \leq \Pi$, in particular, the challenger party L and the median voter M are perfectly aligned in terms of their interests regarding C&B, despite having different peaks for the optimal policy in $[0, 1]$. Yet the median voter's peak must nonetheless be sufficiently close to that of party L relative to incumbent party R 's peak. Figure 2.6 plots the C&B c_1 chosen as a function of the decision-maker's peak, μ_K . It shows that if we start from $\mu_K = \mu_L$, then $(c_1)_{K,1}^*(\mu_K)$ does not change as we further increase μ_K until $\bar{\mu}_K = \mu_L + \frac{1+p}{2} \cdot \Pi$. From this cost threshold onward, $(c_1)_{K,1}^*(\mu_K)$ increases linearly with μ_K . The ranges above μ_R and below μ_L do not yield different results. This is because policy-making is in the hands of the two parties, which have peaks equal to μ_L and μ_R , respectively, and because parties never choose policies that are more extreme than their peaks.

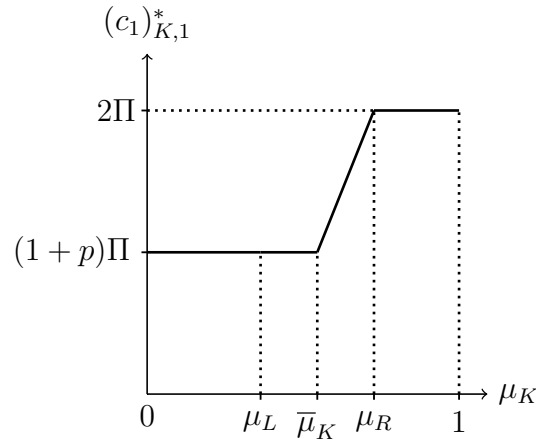


Figure 2.6: Choice $(c_1)_{K,1}^*$ of agent K with peak μ_K , where $\bar{\mu}_K = \mu_L + \frac{1+p}{2} \Pi$.

It also follows from Theorem 1 that if given the monopoly power to change C&B, party R would choose C&B that prevent any political reform and ensure that its peak is chosen in every period. This is the best outcome for party R . More interestingly, party L and median voter M also choose C&B that prevent any policy reform. This is formalized in the next corollary.

Corollary 2 *If the simple majority rule is used in the constitutional phase, no policy reform occurs.*

The above corollary follows immediately from Corollary 1, since all C&B chosen

are at least as large as $(1+p)\Pi$ if the simple majority rule is used in the constitutional phase. The property that no reforms take place on the equilibrium path is stark and leads to deadlock (policy is not changed in period $t = 2$ by neither party despite it does not generically coincide with their peaks). It obtains in our setup because costs of change are linear and losses from policies are quadratic. If costs of change were convex, albeit less convex than the quadratic utility loss function from policies, we would observe policy reforms in period $t = 2$. However, such reforms would be small, particularly for moderately convex costs of change. Later we show that the result that no reform occurs on the equilibrium path also holds for all other variants of the constitutional phase, and thus is a general result of our model of elections.

Although no reform is carried out, the implemented policy *does* vary depending on the value of C&B, at least for the range of C&B between $(1 + p)\Pi$ and 2Π , as it affects the off-equilibrium incentives to carry out a reform. Hence, the decision-maker's peak matters for policy. Indeed, conditional on choosing C&B that ensure the property that there will be no policy reform in period $t = 2$, the challenger party L chooses the smallest possible C&B, while party R chooses the largest possible C&B (up to 2Π). This is because, as stated in Corollary 1, for the range of C&B that lead to no reform there is a positive relationship between C&B and how extreme the implemented policy is. The latter is a measure of policy polarization, which in our model arises endogenously as a function of the endogenously chosen C&B.

Theorem 1 also shows that the probability of turnover, $1 - p$, has an influence on the C&B c_1 chosen. The higher p , the higher c_1 , at least weakly. This means that less policy turnover (i.e., higher p) leads to (weakly) higher C&B, which in turn leads to more extreme policies. Such a property is void for party R since the C&B chosen by the incumbent party is independent of p , but it has a bite for both the challenger party L and median voter M . For the median voter, up to a certain probability threshold $\frac{2(\mu_M - \mu_L)}{\Pi} - 1$, they propose C&B that are independent of p . Above this threshold, the C&B chosen increases linearly with p . The latter is always the case for the challenger party L . Note that the degree of instability, randomness, or rotation of the political system can be captured in our model by $1 - p$. It therefore follows that if the simple majority is used in the constitutional phase, then instability matters for C&B only if there are divided institutions. That is, only if the decision-making power at the constitutional phase is in hands different

from government. In such a case, more instability (i.e., lower p) translates into lower C&B.

Finally, the next corollary follows trivially from Theorem 1.

Corollary 3 *Suppose that agent $K \in \{L, M, R\}$ is the decision-maker in the constitutional phase and that the simple majority is used. Then*

$$\mathcal{SC}_{K,1} = \begin{cases} \{(1+p)\Pi\} & \text{if } K = L, \\ \{(1+p)\max\{\bar{\Pi}, \Pi\}\} & \text{if } K = M, \\ \{2\Pi\} & \text{if } K = R. \end{cases}$$

According to Corollary 3, most levels of C&B are not stable if an agent—the decision-maker—has the monopoly power to choose C&B in the constitutional phase. Moreover, higher party polarization translates into higher C&B.

Double majority rule

In this section we assume that agent $K \in \{L, M, R\}$ has the right to make a proposal for a new C&B that is later pitted against the status quo C&B c_0 in a (simultaneous) vote among all agents. The difference with respect to the previous section is that we now assume that two votes from the three agents (party L , party R , and median voter M) are needed to approve a new C&B in the constitutional phase instead of just one vote. If the proposal for a new C&B fails to gather at least two votes, the status quo C&B c_0 prevails. For instance, suppose that a party proposes a new C&B that is later approved by such a party and the median voter. One interpretation is that the parliamentary majority of party L or R has to approve c_1 but so must median voter M . Therefore a double majority is required, from parliament (or government) and from the electorate. In actual democracies, it is common to require that certain constitutional changes that have been approved by parliament must be additionally approved by the electorate through a referendum before they take effect. We denote the resulting game by $\mathcal{G}_{K,2}^+(c_0)$.

If the double majority rule is used in the constitutional phase, we obtain the following result, where $(c_1)_{K,2}^* := (c_1)_{K,2}^*(c_0)$ for each agent $K \in \{L, M, R\}$:

Theorem 2 *In any equilibrium of game $\mathcal{G}_{K,2}^+(c_0)$, the C&B chosen is the following:*

(i) *If $\Pi < \bar{\Pi}$,*

$$(c_1)_{L,2}^* = \begin{cases} (1+p)\Pi & \text{if } 0 \leq c_0 \leq (1+p)\Pi, \\ c_0 & \text{if } (1+p)\Pi < c_0 \leq (1+p)\bar{\Pi}, \\ \max\{(1+p)\Pi, 2(1+p)\bar{\Pi} - c_0\} & \text{if } (1+p)\bar{\Pi} < c_0 \leq 2\Pi, \end{cases} \quad (2.6)$$

$$(c_1)_{M,2}^* = (1+p)\bar{\Pi},$$

$$(c_1)_{R,2}^* = \begin{cases} \min\left\{2\Pi, \sqrt{-2\bar{H}(c_0, \mu_M, p)} + (1+p)\bar{\Pi}\right\} & \text{if } 0 \leq c_0 \leq (1+p)\Pi, \\ \min\{2\Pi, 2(1+p)\bar{\Pi} - c_0\} & \text{if } (1+p)\Pi < c_0 \leq (1+p)\bar{\Pi}, \\ c_0 & \text{if } (1+p)\bar{\Pi} < c_0 \leq 2\Pi. \end{cases} \quad (2.7)$$

(ii) *If $\bar{\Pi} \leq \Pi$,*

$$(c_1)_{L,2}^* = (1+p)\Pi, \quad (2.8)$$

$$(c_1)_{M,2}^* = (1+p)\Pi,$$

$$(c_1)_{R,2}^* = \max\{c_0, (1+p)\Pi\}. \quad (2.9)$$

Figures 2.7 and 2.8 illustrate the two parties' proposals (or choices) for C&B described in Theorem 2 depending on the status quo C&B. These proposals are voted in equilibrium by two agents and are best understood with the help of Figure 2.5. We recall that $\bar{H}(c_0, \mu_M, p)$ has been defined in Equation (2.3).

On the one hand, Figure 2.7 illustrates party L 's choice. On the left figure there is the case where $\Pi < \bar{\Pi}$, which on average yields higher C&B than when $\bar{\Pi} \leq \Pi$. This is because in the former case the interests regarding the optimal C&B diverge between the challenger party L and the median voter M , and thus the median voter M 's interests regarding the optimal C&B approaches the incumbent party R 's interests, which are to have as large C&B as possible. By contrast, if $\bar{\Pi} \leq \Pi$, a case that is illustrated by the right figure, party L 's and median voter M 's proposal

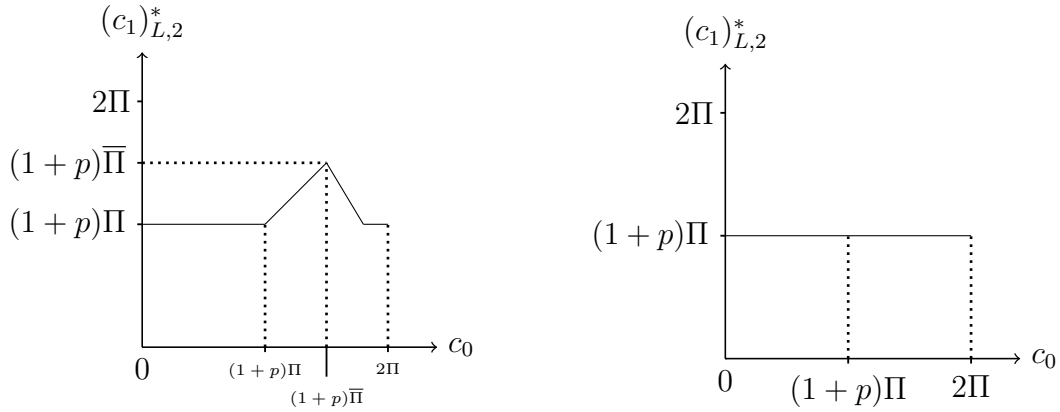


Figure 2.7: Choice $(c_1)_{L,2}^*$ of party L given by Equation (2.6) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.8) (right, case $\bar{\Pi} \leq \Pi$).

coincide. These choices for a new C&B are moreover independent of the status quo C&B c_0 , since party L and median voter M have the necessary votes to implement their desired amount of C&B.

On the other hand, Figure 2.8 illustrates party R 's choice. Similar to party L , the incumbent proposes a higher C&B on average in the case where $\Pi < \bar{\Pi}$ compared to the case where $\bar{\Pi} \leq \Pi$. In the former case, party R can rely on median voter M to push c_1 further up. Moreover, the choices for c_1 are higher if the proposal-maker is party R compared to party L .

No matter which party has proposal-making power, there is a noteworthy feature of the parties' choices for C&B if the double majority is used in the constitutional phase which we do not obtain if the single majority is used: if $\Pi < \bar{\Pi}$, so that median voter M 's and party L 's interests regarding c_1 are not aligned, then the status quo C&B c_0 matters. Similar to the logic behind Romer and Rosenthal (1978), the further away c_0 is from the median voter's preferred amount of C&B, the more leverage the party with the proposal-making power has. This implies the non-monotonicity of the choices described in Figures 2.7 and 2.8 (left cases). If the proposal-maker is the incumbent party R , its leverage increases as c_0 becomes smaller. In such a case, median voter M is willing to accept a higher C&B than their optimal one which gives them at least as much utility as the status quo c_0 . If the proposal-maker is party L , the logic is reversed and the party's leverage is

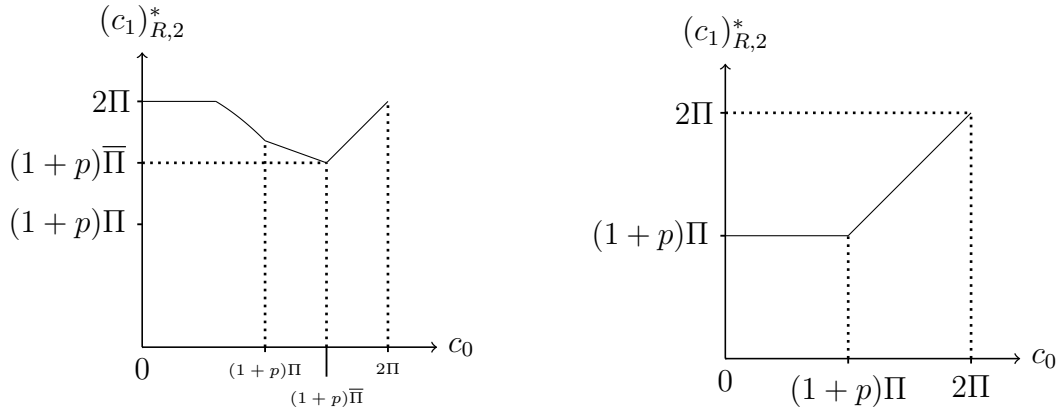


Figure 2.8: Choice $(c_1)_{R,2}^*$ of party R given by Equation (2.7) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.9) (right, case $\bar{\Pi} \leq \Pi$).

maximal if $c_0 = 2\Pi$.

As in the case of a single decision-maker analyzed in Section 2.4.2, we obtain the following corollary:

Corollary 4 *If the double majority rule is used in the constitutional phase, no policy reform occurs.*

According to Corollary 4, the proposal-maker proposes C&B that yield no reform, no matter their identity and peak. In equilibrium, such C&B must be approved by themselves and one other agent, and thus no proposals are made for C&B c_1 which are bound to be rejected. Conditional on satisfying the no-reform property, the proposal-maker chooses the C&B that maximize their own utility. Due to Equation (2.1), if either party is the proposal-maker, then they seek the vote of the median voter. If the median voter is the proposal-maker, on the other hand, then they seek the support of one of the parties (party R if the status quo c_0 is low, and party L otherwise). The median voter has a peak between those of the parties, which then implies the property that they can always rely on one of the parties to change the status quo C&B c_0 , no matter what the latter is. Then, under the double majority rule, the median voter always has their preferred amount of C&B implemented.

The next corollary also follows trivially from Theorem 2.

Corollary 5 *Suppose that agent $K \in \{L, M, R\}$ proposes c_1 and the double majority rule is used. Then*

$$\mathcal{SC}_{K,2} = \begin{cases} [(1+p)\Pi, (1+p)\max\{\Pi, \bar{\Pi}\}] & \text{if } K = L, \\ \{(1+p)\max\{\Pi, \bar{\Pi}\}\} & \text{if } K = M, \\ [(1+p)\max\{\Pi, \bar{\Pi}\}, 2\Pi] & \text{if } K = R. \end{cases}$$

Because under the double majority rule only C&B are proposed that ensure no reform in the political process, then status quo C&B below $(1+p)\Pi$ cannot be stable. Graphically, stable C&B can be easily seen in Figures 2.7 and 2.8 as they must lie on the 45°-line. Some remarks about Corollary 5 are in order.

First, consider that the challenger party L is the proposal-maker in the constitutional phase. Then the Lebesgue measure (or, just, the *size*) of the set of stable C&B is larger if party polarization, Π , is low compared to $\bar{\Pi}$. In this case, the farther apart the peaks of party L and of the median voter are, the larger the set of stable C&B becomes. By contrast, if party polarization Π is larger than $\bar{\Pi}$, the set of stable C&B is a singleton. This level of reform costs is the lowest possible C&B that guarantees no reform in the political process, and thus it is the most moderate policy conditional on no reform. The relationship between policy moderation and C&B is stated in Corollary 1.

Second, consider that the median voter M is the proposal-maker in the constitutional phase. Then, most C&B are not stable no matter the relationship between Π and $\bar{\Pi}$. As mentioned above, although the median voter never dictates policy, the fact that their peak lies between those of the parties allows them to obtain their preferred amount of C&B approved. If a social planner would like to maximize welfare measured as the median voters' utility, the social planner should then give the proposal-making power for changing C&B to the citizenry via referenda. Further details for this case can be found in Appendix B (see online material).

Third, consider that the incumbent party R has proposal-making power in the constitutional phase. Then, in contrast with the case where the challenger party L is the proposal-maker, assuming $\Pi \leq \bar{\Pi}$, the more the peaks of party L and median voter M are aligned, the larger the size of the set of stable C&B. If $\bar{\Pi} = \Pi$, then the set of stable C&B is maximal for a fixed value of Π , as it contains the whole set

of C&B that entail no policy reforms. This is shown in Figure 2.8.

Finally, it is worth noting that in general there is no monotonic relationship between the size of the set of stable C&B and the proposal-maker's identity.

Unanimity rule

In this section we assume that unanimity is required for changing the status quo C&B. This means that the proposal-maker needs the votes of the remaining two agents to have a new C&B approved. One interpretation is that such a change in the constitutional phase has to be approved by the government, by a qualified majority in parliament, and also by the electorate. This is the most stringent case. We denote the resulting game by $\mathcal{G}_{K,3}^+(c_0)$.

If the unanimity rule is used in the constitutional phase, we obtain the following result, with $(c_1)_{K,2}^* := (c_1)_{K,2}^*(c_0)$ for agent K :

Theorem 3 *In any equilibrium of game $\mathcal{G}_{K,3}^+(c_0)$, the C&B chosen is the following:*

(i) *If $\Pi < \bar{\Pi}$,*

$$(c_1)_{L,3}^* = \max \{c_0, (1+p)\Pi\}, \quad (2.10)$$

$$(c_1)_{M,3}^* = \begin{cases} \min \left\{ (1+p)\bar{\Pi}, \sqrt{-2\bar{H}(c_0, \mu_L, p)} \right\} & \text{if } 0 \leq c_0 \leq (1+p)\Pi, \\ c_0 & \text{if } (1+p)\Pi < c_0 \leq 2\Pi, \end{cases} \quad (2.11)$$

$$(c_1)_{R,3}^* = \begin{cases} \min \left\{ \sqrt{-2\bar{H}(c_0, \mu_M, p)} + (1+p)\bar{\Pi}, \sqrt{-2\bar{H}(c_0, \mu_L, p)} \right\} & \text{if } 0 \leq c_0 \leq (1+p)\Pi, \\ c_0 & \text{if } (1+p)\Pi < c_0 \leq 2\Pi. \end{cases} \quad (2.12)$$

(ii) If $\bar{\Pi} \leq \Pi$,

$$\begin{aligned} (c_1)_{L,3}^* &= \max \{c_0, (1+p)\Pi\}, \\ (c_1)_{M,3}^* &= \max \{c_0, (1+p)\Pi\}, \end{aligned} \tag{2.13}$$

$$(c_1)_{R,3}^* = \begin{cases} \min \left\{ \sqrt{-2\bar{H}(c_0, \mu_M, p)} + (1+p)\bar{\Pi}, \sqrt{-2\bar{H}(c_0, \mu_L, p)} \right\} & \text{if } 0 \leq c_0 \leq (1+p)\Pi, \\ c_0 & \text{if } (1+p)\Pi < c_0 \leq 2\Pi. \end{cases} \tag{2.14}$$

We recall that $\bar{H}(c_0, \mu_M, p)$ has been defined in Equation (2.3). If unanimity is used in the constitutional phase, every change on the status quo C&B must be approved by all agents and the proposal-making rule is then paramount to outcomes. In equilibrium, it is not generically true that proposals are approved, as it is often the case that the status quo C&B prevails. The reason is that any agent has a veto power in the voting stage over proposals that differ from such a status quo. As in the case of double majority, the status quo C&B is also a source of leverage for the proposal-maker. We therefore observe again a non-monotonic behavior for the proposals of some agents, viz. party R and median voter M . This is illustrated in Figures 2.10 and 2.11, which show voter M 's and party R 's proposal (or choice) for C&B depending on the status quo C&B, as described in Theorem 3. For instance, consider that $\Pi < \bar{\Pi}$. Then both median voter M and party R can make party L indifferent in utility terms between accepting the status quo C&B and choosing a new, larger C&B. By contrast, party L has no leverage against party R , as the latter prefers as high C&B as possible. This leads to a monotonic relationship between c_0 and c_1 , as shown in Figure 2.9.

Theorem 3 also shows that the probability of turnover, $1-p$, has an influence on the C&B c_1 chosen. As with the double majority rule, less political power turnover (i.e., higher p) leads to (weakly) higher C&B, which in turn leads to more extreme policies.

As with the previous constitutional rules, the following corollary also holds:

Corollary 6 *If unanimity is used in the constitutional phase, no policy reform occurs regardless of the proposal-maker's identity.*

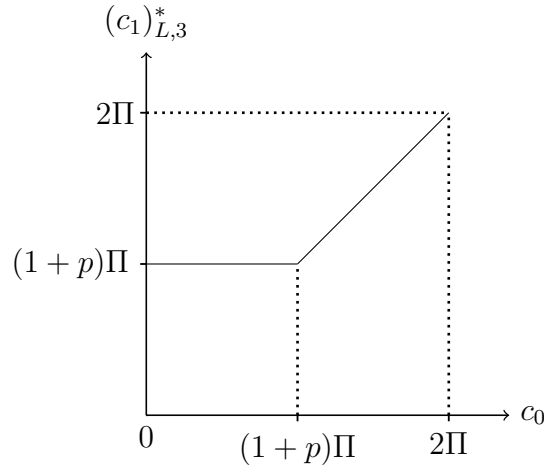


Figure 2.9: Choice $(c_1)_{L,3}^*$ of party L given by Equation (2.10).

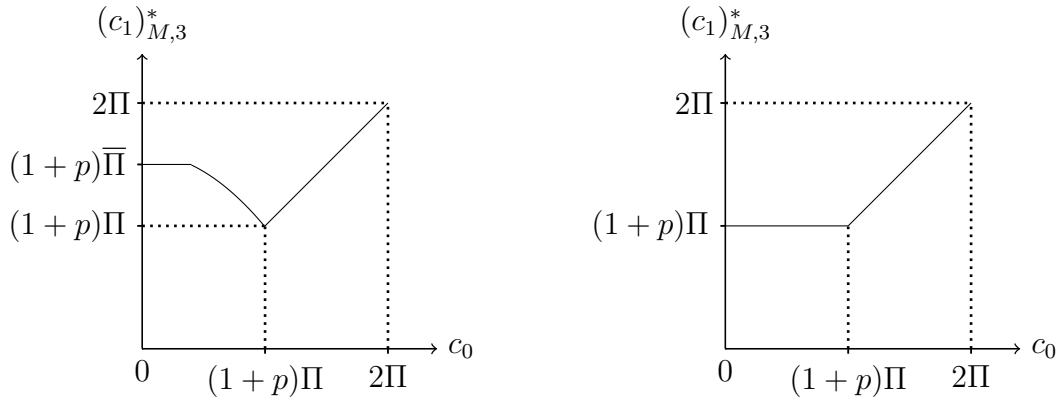


Figure 2.10: Choice $(c_1)_{M,3}^*$ of median voter M given by Equation (2.11) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.13) (right, case $\bar{\Pi} \leq \Pi$).

According to the above corollary, if the proposal-maker wants to change the status quo C&B, they must propose C&B that yield no reform which must then be approved additionally by all agents. To do so, the proposal-maker needs to choose C&B that maximize their own utility conditional on C&B satisfying the requirement that C&B must be between $(1+p)\Pi$ and 2Π . Since all agents agree that reforms should be avoided, we obtain deadlock in equilibrium. Other than this property, the three agents never agree on anything else unanimously. This is because in the

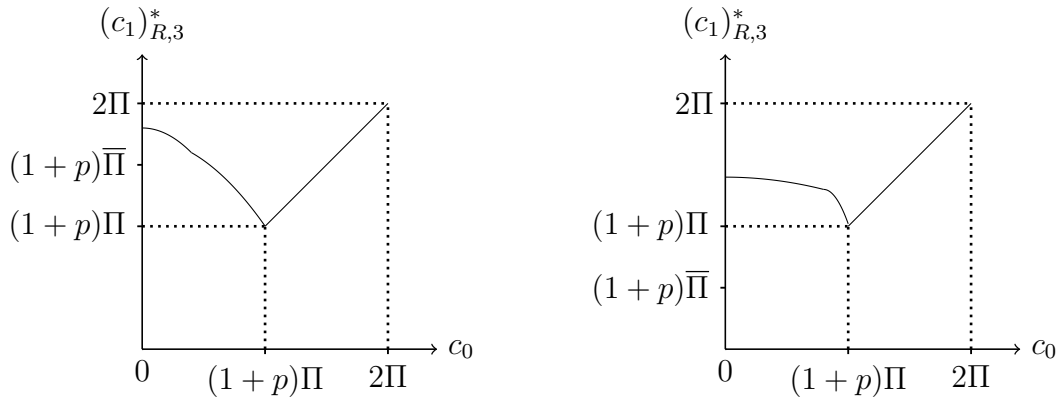


Figure 2.11: Choice $(c_1)_{R,3}^*$ of party R given by Equation (2.12) (left, case $\Pi < \bar{\Pi}$) and by Equation (2.14) (right, case $\bar{\Pi} \leq \Pi$).

range from $(1+p)\Pi$ to 2Π both parties have opposed interests regarding the amount of C&B. Hence they never agree.

Finally, the next corollary states what the set of stable C&B is if the unanimity rule is used.

Corollary 7 *Suppose that agent $K \in \{L, M, R\}$ proposes c_1 and the unanimity voting rule is used. Then*

$$\mathcal{SC}_{K,3} = [(1+p)\Pi, 2\Pi].$$

Accordingly, the set of the stable C&B is maximal if unanimity is required at the constitutional phase, and, moreover, it coincides with the set of C&B that ensure no reform in the political process. The size of either set increases with turnover probability $1 - p$ and party polarization Π . Another noteworthy property of the unanimity rule is that the proposal-maker's identity is irrelevant for the set of stable C&B.

2.5 Comparative Statics

In our setup, one can conceive of a constitution as *(i)* the amounts of C&B in place, *(ii)* a majority rule to change the status quo C&B in the constitutional phase, and

(iii) a rule that determines who has the power to make proposals for a new C&B also in the constitutional phase. In the previous section, we have analyzed endogenous levels of (i) for given combinations of (ii) and (iii). In this section, we summarize other comparative statics from the previous section regarding the *size* of the set of stable C&B. For a given constitutional phase and other fixed characteristics of the political system such as the level of party polarization, the size of the set of stable C&B can be seen as a statistic of the variance of the amounts of C&B that should be observed in democracies. It can then be relevant for empirical analyses.

The next result follows directly from Corollaries 3, 5, and 7.

Theorem 4 *The size of the set \mathcal{SC} of stable C&B*

- (i) *increases with higher majority requirements for constitutional changes on C&B,*
- (ii) *is not monotonic with respect to the proposal-maker's peak in the constitutional phase, and*
- (iii) *(weakly) increases with higher party polarization Π .*

First, consider part (i) in Theorem 4. If the simple majority rule is used in the constitutional phase, most (amounts of) C&B are not stable. Higher majority requirements in such a phase make it more difficult to obtain the necessary votes to change the status quo C&B, which naturally leads to more C&B being stable. Under the unanimity rule, in particular, not only the set of stable C&B has larger size, but it is in fact maximal among the sets of C&B that lead to deadlock.

Second, consider part (ii) in Theorem 4 and recall that party R chooses policy in period $t = 1$. Then, in general, the peak of the agent with proposal-making power in the constitutional phase matters for C&B in relation to party R 's peak but also in relation to the peak of any other agent with voting power in the constitutional phase. Consider, in particular, that double majority is used in the constitutional phase and the median voter has proposal-making power. Because the median voter has a peak that lies between the two parties' peaks, the median voter can rely on one of the parties to either change the status quo C&B or block any reform thereof. This results in the set of stable C&B being smaller (or equal) compared

to the case where the double majority rule is in place and one of the parties has proposal-making power.

Third, consider part (iii) in Theorem 4. It states that the size of the set of stable C&B increases with party polarization Π , if it changes at all. If the simple majority rule is used in the constitutional phase, the set of stable C&B is a singleton, no matter party polarization. If, by contrast, more than one agent is required to approve a change on C&B, it is more difficult to find an agreement to implement such a change, and the difficulty increases with polarization in the parties' peaks. Therefore, increased party polarization is not a threat for C&B, but implies (a potential range of) policies that diverge more from the median position.

2.5.1 Welfare

In this section we compare the different variants of the constitutional phase from a welfare perspective. We take two different approaches. First, we assume that the median voter's utility defines the welfare measure, i.e.,

$$W(c_1) = H(c_1, \mu_M) = \mathbb{E}[u_M^1(i_{R1}, i_{R1}) + u_M^2(i_{R1}, i_{K2})]. \quad (2.15)$$

We obtain

Theorem 5 *If the simple majority rule is used in the constitutional phase, the C&B proposed by the challenger party are welfare superior to the one proposed by the incumbent party if*

- (i) *party polarization is high, i.e., $\bar{\Pi} \leq \Pi \leq 1$,*
- (ii) *party polarization is low, i.e., $0 \leq \Pi < \bar{\Pi}$, and the median voter is closely aligned with the opposing party, i.e., $\mu_L \leq \mu_M < \bar{\mu}_M$, where $\bar{\mu}_M := \frac{1}{4} \cdot (3\mu_R + \mu_L + p\Pi)$.*

Theorem 5 compares the two parties as proposal-maker from a welfare perspective if the simple majority rule is in place. An interesting result is that in the presence of high polarization, the proposal by the minority party is preferred to the one by the incumbent, independent of the alignment of the median voter with the opposing party.

Notably in highly polarized societies, welfare optimizing constitutions are favorable. Hence, the following corollary states the optimal constitution from a welfare perspective, given high party polarization.

Corollary 8 *In the case of high polarization, the welfare optimizing constitutional rule is the simple majority rule if the opposing party has proposal-making power.*

Corollary 8 considers the case where the society is highly polarized. Comparing all constitutional rules leads to the result that from the median voter's perspective, the simple majority rule is the ideal rule, given that the opposing party has proposal-making power.

Next, we consider the case where the electorate is highly polarized. Then, the two parties, having approximately equal size, represent the whole electorate. Under these circumstances, the median voter plays a negligible role. Assuming that no party has a majority in parliament leads to the requirement that both parties have to approve a new C&B. In particular, the expected welfare in that case is defined as

$$\begin{aligned} W(c_1) &= H(c_1, \mu_L) + H(c_1, \mu_R) \\ &= \mathbb{E}[u_L^1(i_{R1}, i_{R1}) + u_L^2(i_{R1}, i_{K2})] + \mathbb{E}[u_R^1(i_{R1}, i_{R1}) + u_R^2(i_{R1}, i_{K2})]. \end{aligned} \quad (2.16)$$

Maximizing welfare, as described in Equation (2.16), leads to the following result:

Theorem 6 *If either the simple majority rule or the double majority rule is used in the constitutional phase, then the C&B proposed by the opposing party is welfare superior to the one proposed by the incumbent, independent of party polarization.*

As we see in Theorem 5 and 6 as well as in Corollary 8, the rule which allocates the proposal-making power to one of the agents is important. From a welfare perspective, it is better that constitutional changes can only be proposed by the minority party in parliament (non-governing party). This result is true for the simple majority rule and for the double majority rule. As already discussed in Subsection 2.4.2, the rule which allocates the proposal-making power would not influence any chosen level if the unanimity rule were in place.

2.6 Conclusion

We extended an existing model of political competition with reform costs by including a constitutional phase in which the level of such costs is endogenously determined. We argued that reform costs can be interpreted as (executive and/or legislative) C&B, which enables this chapter to yield new insights on the question which amount of C&B democratic societies choose. In our analysis, we considered different constitutional rules to determine the level of reform costs. These rules vary depending on the majority rule they use and on to whom they allocate the power to make proposals. We found that stable C&B always exist and are never zero, but that in general, many, if not most, C&B will not survive in society. We also found that endogenous C&B lead to deadlock.

Many avenues for future research can be pursued. For instance, one could add shocks to the future distribution of parties' peaks. This would induce policy reforms on the equilibrium path. One could also allow for uncertainty about the consequences of policies. In such cases, more stringent C&B may produce more information about the consequences of policies and thus may add a further rationale why C&B should be in place. These and other conceivable extensions of our model may produce further insights about which amount of C&B democracies choose or should choose.

Chapter 3

Preference Shocks, Costs of Change and Policy Moderation*

Abstract

In this chapter, we analyze the effects of preference shocks on the policy choice of political parties in the presence of costs of change. For this purpose, we study a two-period model of political competition with costly policy reforms. We find that on average, shocks in parties' preferences lead to more extreme policy choices. Moreover, we show that policy choices become more moderate for intermediate marginal costs and for a higher turnover probability.

3.1 Introduction

Sudden events like the Covid-19 pandemic in 2020 or the Russian invasion of Ukraine in 2022 can influence voters' and parties' preferences about which policies are optimal. Preference shocks of voters can be due to external factors, like an instantaneous economic crisis such as a pandemic, or be a reaction to an unexpected event, like a refugee crisis or a terrorist attack. Parties represent the preferences of groups of voters and align their positions to sudden changes in voters' preferences. Hence, we study how parties adjust policies to changes in their preferences.

*This chapter is single-authored work.

Our goal is to examine how policies are affected when simultaneously preference shocks and costs of change are present. In the absence of costs of change, the adaptation of policies would be straightforward. A shock inducing a positive (negative) shift of parties' peaks would lead to a higher (lower) policy, i.e., a policy that is more to the right (left), by the extent of the shock. What happens when there are costs of change is not obvious. Therefore, we build on a game theoretical model (Gersbach et al., 2019) and extend it by introducing preference shocks for all parties, given costly policy reforms. Gersbach et al. (2019) consider a two-period two-party model of political competition, where in each period, the incumbent party chooses a (one-dimensional) policy. The two-period model has the following characteristics. First, there are two parties that are policy-oriented and have preferences in the form of a quadratic loss function over the one-dimensional policy space, i.e., the peaks characterizing their preferences are different. Both parties experience a common preference shock before the second period. Second, changing policy of the first period in the second period imposes costs. These costs of change are also called *reform costs* and increase (linearly) with the extent of the policy change. In the first period, the policy is enacted for the first time, so reform costs occur only in the second period. Third, with a certain probability, the incumbent party in the first period remains in power in the second period when it competes with the challenging party.

The main novelty of our analysis is to introduce preference shocks in the presence of costs of change. Specifically, before the policy for the second period is chosen, a preference shock takes place which may be negative or positive. Hence, besides reform costs, preference shocks also affect the politicians' policy choices. Assessing how preference shocks and the size of reform costs impact the policies is the subject of this chapter.

In our analysis, we focus on the two parties' policy choices in the first and in the second period. In the first period, the first-period incumbent party anticipates the different policy choices for the second period and chooses a policy that maximizes its lifetime utility. We characterize the first-period incumbent party's policy choice in the presence of preference shocks and reform costs, depending on the shock size and the turnover probability.

In our main result, we find that on average, the presence of preference shocks

leads to more extreme policy choices of the first-period incumbent party, given costs of change. On the one hand, for low costs of change, the preference shocks impact policy choices more, and anticipating higher variance in the policy choices in the second period leads to more extreme policies. On the other hand, if costs of change are high, the incumbent party anticipates that changing policy in the second period is too costly and it chooses a more extreme policy that is closer to its own peak. These policy choices become more moderate for a higher turnover probability and for intermediate reform costs. Hence, if the first-period incumbent party is likely to lose power in the second period, it chooses a more moderate policy in the first period. This is also the case for intermediate reform costs, since then, changing policy becomes too expensive. Further, both parties adapt their policies in the second period for a positive or negative preference shock. If the preference shock is large relative to the reform costs, both parties move the policy more to the right, if a positive shock occurs than if a negative shock takes place.

The chapter is organized as follows. In Section 3.2 we review the scientific papers that are most connected to our work. In Section 3.3 we describe the model. In Section 3.4 we analyze the policy choices and their dependencies on parameter choices. Section 3.5 concludes. The proofs can be found in Appendix B.

3.2 Related Literature

This chapter is related to several strands of the literature.

Costs of change in policies

Our model is based on Gersbach et al. (2019), who develop a two-period model where policies impose costs on all individuals and political power might change over time. Another model associated with policy changes is by Glazer et al. (1998), who state that incumbents will choose more extreme policies in the case of large fixed costs. Another paper related to the consequences of costs of change for policy-making and elections is Gersbach and Tejada (2018). They elaborate the relation between extreme policies and efficiency in implementing policy reforms such that the politician benefits. A growing literature is devoted to analyzing the effects of

reform costs in elections (see e.g. Gersbach et al., 2020a,b; Eraslan and Piazza, 2020; Dziuda and Loeper, 2021). We add to this literature by focusing on the scenario of shocks in parties' preferences when reform costs are present.

Preference shocks

This chapter is part of a greater research agenda that investigates preference shocks from a theoretical and empirical perspective. Aragonés (2016) studies a theoretical model of political competition to analyze the effects of voters' preference shocks on parties' policy choices (see also Aragonés et al., 2019). Battaglini (2014) presents a dynamic theory of electoral competition and shows that policies can be Pareto efficient, even in the presence of preference shocks.

Empirical papers study the electoral impact of terrorist attacks to analyze if and to which extent preference shocks take place (see e.g. Montalvo, 2011, 2012; Bali, 2007; Lago and Montero, 2005). Another strand of literature argues empirically that electoral shocks can be seen as major political event and have the potential to produce political change in the short and long term (Liñeira, 2021; Fieldhouse et al., 2021). In this chapter, we argue that these shocks affect parties' preferences and we analyze how they impact policy decisions in the presence of costs of change.

3.3 Model

We consider a two-period model based on Gersbach et al. (2019). In each of the two periods ($t = 1, 2$), one of two parties, denoted by $\{L, R\}$, can choose the policy if it holds power. Without loss of generality, we assume that at the start of period $t = 1$, party R is the incumbent party. The set of policy choices is $[0, 1]$, where 0 corresponds to the left-most policy and 1 to the right-most policy. We model elections implicitly and assume for simplicity that power shifts at the start of period $t = 2$ with exogenous given probability $1 - p \in [0, 1]$. Hence, party R still holds power in the period $t = 2$ with probability p . Further, we assume that the discount factor for period $t = 2$ is one.

Both parties have quadratic utilities over policies with ideal points, given by μ_L and μ_R , where $0 \leq \mu_L \leq \mu_R \leq 1$. The degree of party polarization, i.e., the

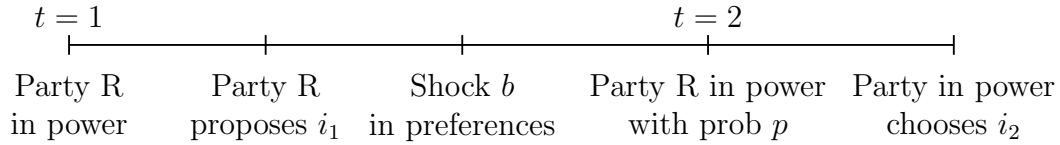


Figure 3.1: Timeline of the political game.

difference in parties' peaks, is denoted by $\Pi = \mu_R - \mu_L$. Policy changes are costly for both parties in period $t = 2$. These costs only occur in the second period, since there is no status quo policy in place before period $t = 1$. This implies that for any policy $i_1 \in [0, 1]$ chosen in period $t = 1$, the policy choice $i_2 \in [0, 1]$ in period $t = 2$ imposes marginal costs c per unit of reform additional on the two parties. These costs can be seen as utility losses for both parties. These costs of change c increase linearly with the absolute difference between the policies i_1 and i_2 .¹ The parameter c is therefore the marginal cost of policy change.

Then, if policy $i_t \in [0, 1]$ is chosen in period $t \in \{1, 2\}$, party $K \in \{L, R\}$ derives the following utility:

$$u_{tK} = -(i_t - \mu_K)^2 - c \cdot |i_1 - i_2|. \quad (3.1)$$

We extend the model in (Gersbach et al., 2019) by adding a preference shock for both parties that occurs before period $t = 2$. This preference shock takes place due to and after a sudden event, like a terrorist attack or the outbreak of a pandemic. The shock in the parties' preferences is denoted by $b \in \{-B, B\}$, such that $\mu_R + b$ and $\mu_L + b$ are the new peaks, i.e., preferences. We restrict the shock size to $B < \frac{\Pi}{2}$. Under a veil of ignorance, we assume that the shock is equally likely to be positive and negative. Different probabilities would lead to weights on the different policy choices for each scenario. For instance, if a positive shock is very likely compared to a negative shock, then parties would anticipate higher policies chosen by both parties in the second period. In the second period, party K which holds power chooses the new policy i_{K2} . Hence, each party's peak in the second period is determined stochastically and thus is unknown from the perspective of the first period. The variance of policy choices of i_1 determines the degree of volatility of

¹Following the approach of Gersbach et al. (2019), we assume that costs of change are linear. This is a first-order approximation of the general case, where costs associated with policy changes increase in the extent of the policy shift.

the political system. The bulk of the paper is devoted to analyzing the equilibrium policy decisions in such a model.

Figure 3.1 represents the timeline of the political game. In period $t = 1$, party R is in power and chooses policy i_1 . Before period $t = 2$, a shock in parties' preferences takes place, which is equally likely to be positive or negative. A positive (negative) shock induces a shift of parties' peaks to the right (left). Then, in period $t = 2$, party K that is in power chooses policy i_{K2} in period $t = 2$.

3.4 Analysis

Our focus is the analysis of the policy choices in the first and second period. First, we focus on characterizing the policy choices of both parties in period $t = 2$, depending on the size of cost of change c when preference shocks are present. Second, we calculate the first-period incumbent party's policy choices, depending on the size of the cost of change c in the first period, given preference shocks in the second period. Further, we conduct comparative statistics on other parameters, like turnover probability $1 - p$ and shock size B .

3.4.1 The second period

In this section we analyze the policy choices of party R and L in the second period. We start with the situation without preference shocks, i.e., $b = 0$, and Proposition 1 for this case in Gersbach et al. (2019). Then, the policy choices in period $t = 2$ are defined in the following proposition.

Proposition 2 (Gersbach et al. (2019)) *Let $K \in \{L, R\}$ be the party that holds power in period $t = 2$. Party K 's best response to the policy $i_1 \in [0, 1]$ chosen in period $t = 1$ is given by*

$$i_{K2}^* = i_{K2}^*(i_1, c) := \min \left\{ \max \left\{ \mu_K - \frac{c}{2}, i_1 \right\}, \mu_K + \frac{c}{2} \right\}.$$

Since we extend the model by introducing a preference shock that takes place before period $t = 2$, we have to adapt the parties' best responses as given in Proposition 2. In our model, for party $K \in \{L, R\}$ that holds power in period $t = 2$, the

best response to the policy $i_1 \in [0, 1]$ chosen in period $t = 1$ is given by

$$i_{K2}^* = i_{K2}^*(i_1, c, b) = \min \left\{ \max \left\{ \mu_K + b - \frac{c}{2}, i_1 \right\}, \mu_K + b + \frac{c}{2} \right\}. \quad (3.2)$$

As defined by Equation (3.2), the policy choices, given a policy i_1 , depend further on the value of b . If the preference shock is positive, i.e., $b = B$, resp. negative, i.e., $b = -B$, we denote the policy choice of party K by i_{K2+}^* resp. i_{K2-}^* . Hence, Equation (3.2) can be written as

$$\begin{aligned} i_{K2+}^* &= \min \left\{ \max \left\{ \mu_K + B - \frac{c}{2}, i_1 \right\}, \mu_K + B + \frac{c}{2} \right\}, \\ i_{K2-}^* &= \min \left\{ \max \left\{ \mu_K - B - \frac{c}{2}, i_1 \right\}, \mu_K - B + \frac{c}{2} \right\}. \end{aligned}$$

It follows that the incumbent party chooses the status quo policy i_1 if it lies within a certain range bounded by $\mu_K + b - \frac{c}{2}$ from below and by $\mu_K + b + \frac{c}{2}$ from above. At these thresholds, the disutility from reforming the policy is equivalent to the disutility of the incumbent party choosing a policy further away from its preferred policy, considering disruptions in its peak. This means the reform costs c need to be considered relatively to the absolute size of preference shocks B . How the costs of change c and the size of the shock B influence the policy choices of both parties in period $t = 2$ is illustrated in Figures 3.2, 3.3 and 3.4 for $B < \frac{\Pi}{4}$. One can observe some general characteristics. In case of a positive shock both parties choose a higher policy than in the case of a negative shock. Intuitively, this comes from the fact that a positive (negative) preference shock entails a higher (lower) peak of each party and hence leads to a higher preferred policy. Consider the cases where the status quo policy does not change in the second period, i.e., $i_1 = i_{K2+}$ or $i_1 = i_{K2-}$ for $K \in \{L, R\}$. One observes that the size of the intervals is independent of the preference shocks and is determined by the size of c . As the bounds described above are only binding for the case of small values of c , larger reform costs c induce the status quo in the second period. Some remarks about the figures are in order.

First, Figure 3.2 shows the policy choices of both parties for a negative and a positive preference shock, given small reform costs relatively to the size of the preference shock, i.e., $c < 2B$ for $B < \frac{\Pi}{4}$. For each party, the policies chosen for a positive and negative shock never coincide for positive reform costs c . A greater

shock size B induces a greater difference between the policies chosen in the case of a positive shock and of a negative shock for both parties. When relaxing the assumption such that the shock size B is greater, i.e., $B > \frac{\Pi}{4}$, it can even happen that the policy chosen by party R in the case of a negative shock is lower than the policy chosen by party L in the case of a positive shock for $\Pi + c < 2B$. This implies that the right-most policy that can ever be implemented by party L in the case of a positive shock is higher than the left-most policy that can ever be implemented by party R in the case of a negative shock.

Second, Figure 3.3 illustrates the policy choices, given small preference shocks relative to reform costs, i.e., $2B \leq c < 2\Pi - 2B$ for $B < \frac{\Pi}{4}$. In that case, the policy choices of both parties are weakly higher in the case of a positive shock than in the case of a negative shock. Contrary to low reform costs, party K 's policy choices coincide for positive and negative shocks if $\mu_K + B - \frac{c}{2} \leq i_1 \leq \mu_K - B + \frac{c}{2}$. If the status quo policy is in this range, both parties will not change it in period $t = 2$, no matter whether a positive or negative shock takes place.

Third, Figure 3.4 shows parties' policy choices for larger reform costs, i.e., $2\Pi - 2B \leq c < 2\Pi$. Since changing policy is very costly, the status quo policy is retained for larger intervals. Starting from $c = \mu_R - B - \frac{c}{2}$, the policy chosen by party R in the case of a negative shock is lower, i.e., more to the left, than the policy chosen by party L in the case of a positive shock.

3.4.2 The first period

In this section, we focus on party R 's policy choice in the first period. Party R is the incumbent party in the first period, and with probability p , it stays in power in the second period. Hence, party R chooses a policy i_1 such that its lifetime utility is maximized. Hence, consider the expected lifetime utility of the political game,

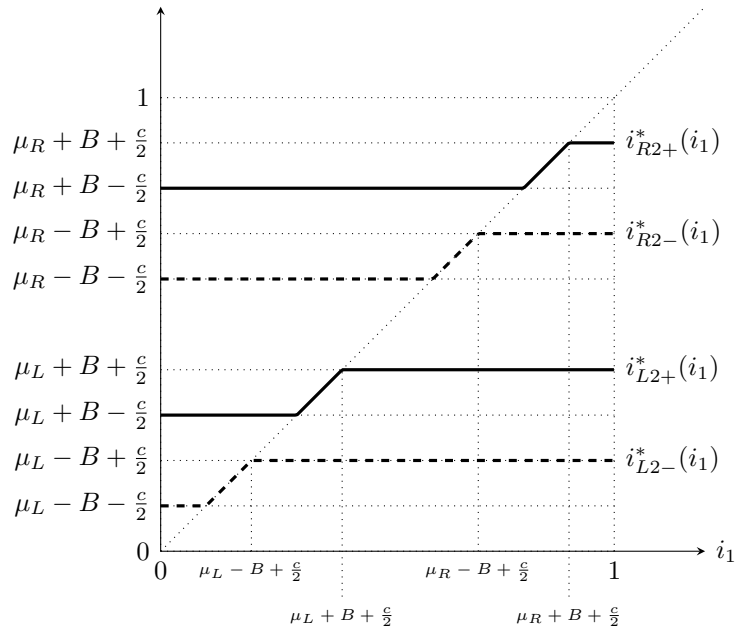


Figure 3.2: The policy choices in period $t = 2$ when $0 < c < 2B$.

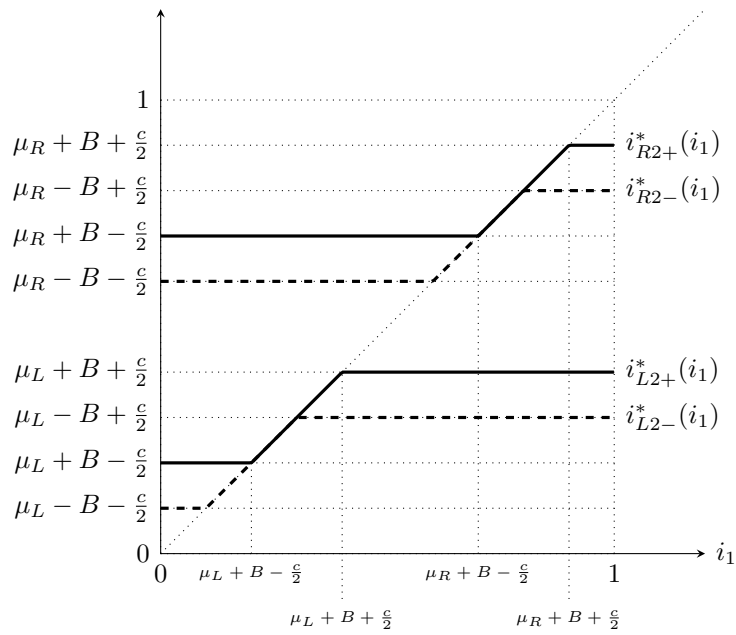


Figure 3.3: The policy choices in period $t = 2$ when $2B \leq c < 2\Pi - 2B$.

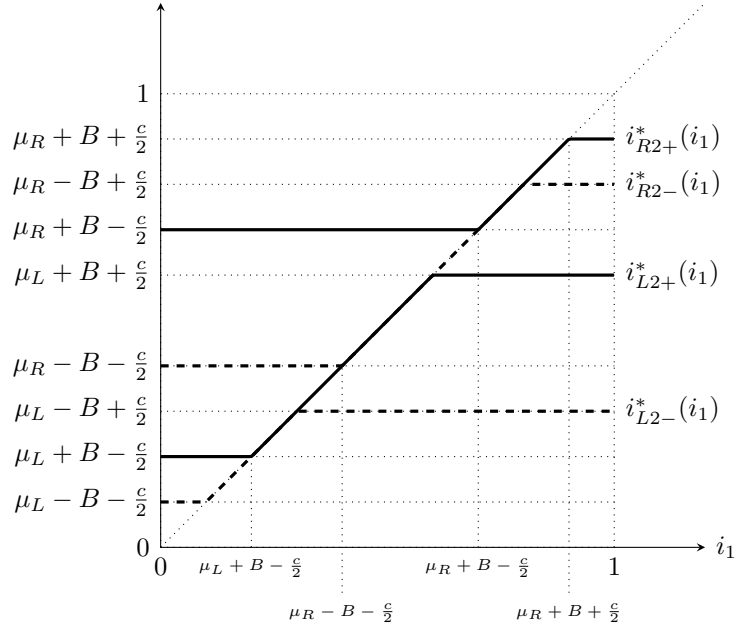


Figure 3.4: The policy choices in period $t = 2$ when $2\Pi - 2B \leq c < 2\Pi$.

called *utility*, for party R :

$$\begin{aligned}
\mathbb{E}_R(u(i_1, c)) &= -(i_1 - \mu_R)^2 + \frac{p}{2} [-(i_{R2+}^*(i_1, c) - \mu_R - B)^2 - c|i_{R2+}^*(i_1, c) - i_1|] \\
&+ \frac{p}{2} [-(i_{R2-}^*(i_1, c) - \mu_R + B)^2 - c|i_{R2-}^*(i_1, c) - i_1|] \\
&+ \frac{(1-p)}{2} [-(i_{L2+}^*(i_1, c) - \mu_R - B)^2 - c|i_{L2+}^*(i_1, c) - i_1|] \\
&+ \frac{(1-p)}{2} [-(i_{L2-}^*(i_1, c) - \mu_R + B)^2 - c|i_{L2-}^*(i_1, c) - i_1|]. \quad (3.3)
\end{aligned}$$

The first term in the utility equation, i.e., $-(i_1 - \mu_R)^2$, expresses the disutility received when choosing a policy i_1 that is further away from party R 's bliss point. Then, with probability p , party R stays in power in the second period and chooses i_{R2+}^* or i_{R2-}^* , depending on whether a negative or a positive preference shock occurs. Hence, if party R remains in power in period $t = 2$, then $-(i_{R2}^*(i_1, c) - \mu_R + b)^2$ materializes and incentivizes the incumbent party to choose a policy close to its new bliss point $\mu_R + b$. Further, expected costs of changing policy in the second period arise, captured by the term $-c|i_{R2+}^*(i_1, c) - i_1|$ resp.

by $-c|i_{R2-}^*(i_1, c) - i_1|$, if party R chooses the policy in both periods. As with party R , the last two terms capture the case where party L is in power with probability $1 - p$ in period $t = 2$ and chooses i_{L2+}^* or i_{L2-}^* , depending whether a negative or a positive preference shock occurs.

Next, we turn to our main result, which characterizes the policy choice $i_1^*(c, p, B)$ of party R , depending on the reform costs c , on the probability p that the first-period incumbent party R stays in power in period $t = 2$, and on the shock size B . The problem faced by party R to maximize its expected utility is defined by

$$i_1^* \in \arg \max_{i_1 \in [0,1]} \mathbb{E}_R(u(i_1, c)).$$

Solving the maximization problem above yields the following result:

Proposition 3 *Party R 's policy choice in the first period is*

$$i_1^* = i_{R1}^*(c, p, B) := \begin{cases} \mu_R + \frac{c(p-1)}{2} & \text{if } 0 \leq c \leq \frac{2B}{(2-p)}, \\ \mu_R - B + \frac{c}{2} & \text{if } \frac{2B}{(2-p)} < c < B(1+p), \\ \mu_R + \frac{c(p-1)}{2(1+p)} & \text{if } B(1+p) \leq c \leq (\Pi - B)(1+p), \\ \mu_L + B + \frac{c}{2} & \text{if } (\Pi - B)(1+p) < c < 2\Pi - 2B, \\ \mu_R & \text{if } 2\Pi - 2B \leq c. \end{cases}$$

According to Proposition 3, the policy choice of party R becomes more moderate as the costs c increase up to $\frac{2B}{(2-p)}$ and then becomes more extreme for values of c above this threshold, until c reaches $B(1+p)$. Starting from $B(1+p)$, the chosen policy becomes more moderate again and reaches its most moderate value $i_1^* = \mu_L + B + \frac{c}{2}$ when $c = (\Pi - B)(1+p)$. Then, the policy choice becomes more extreme again, until it reaches party R 's bliss point μ_R at $c = 2\Pi - 2B$. The policy i_1 depending on c , as described in Proposition 3, is illustrated by Figure 3.5.

Consider now the driving forces behind this result. Let us start with high costs of change, i.e., $c > 2\Pi - 2B$. Then, no party wants to change the policy, due to the high costs associated with it. As the incumbent party R chooses the policy in period $t = 1$, it implements its peak μ_R . As costs c decrease, the chosen policy becomes more moderate, due to a potential and more likely policy change

by party L . In other words, party L could threaten to change the policy in the second period, which leads to a more moderate policy chosen by party R in the first period. This is true up to a certain point, i.e., $c = (\Pi - B)(1 + p)$, where party R is indifferent as to which party chooses the policy. As costs c decrease even further, party R cares more about its peak than about the associated costs in case of a policy change and hence, chooses a more extreme policy. This is the case up to the threshold $c = B(1 + p)$. At this point, the policy again becomes more moderate, due to the high shock size B , compared to the costs of change. Party R anticipates the more varying policy choices in period $t = 2$ by both parties, which weigh more than a policy change, due to the low costs c , and chooses a more moderate policy. Then, as costs c decrease even further, if $c < \frac{2B}{(2-p)}$, the policy again becomes more extreme. In the edge case of no costs of change at all, i.e., $c = 0$, party R implements its bliss point μ_R .

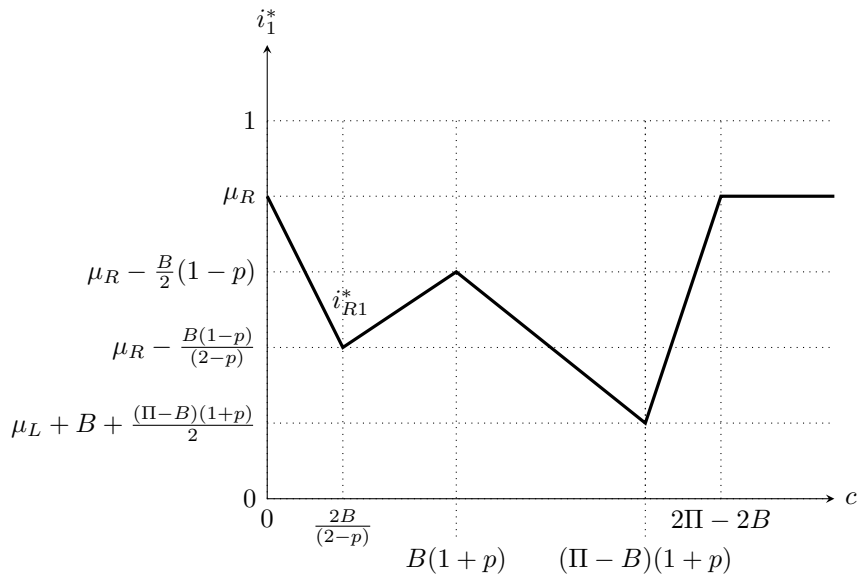


Figure 3.5: The policy choices by party R in period $t = 1$.

Consider the impact of the probability p that the incumbent party R stays in power in the second period on the policy choice i_1^* . If there is no political instability, i.e., $p = 1$, then party R can choose its own bliss point, no matter the costs of change c . At the other extreme, if party R knows that it will lose power

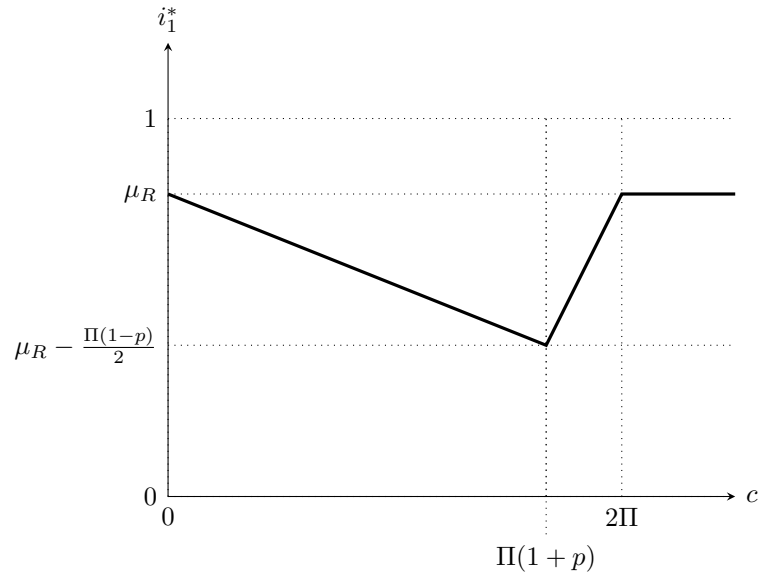


Figure 3.6: The policy choices in period $t = 1$ for $B = 0$.

in the second period, i.e., $p = 0$, then the policy i_1 does *not* become more extreme for $\frac{2B}{(2-p)} < c < B(1+p)$, since then $\frac{2B}{(2-p)} = B(1+p)$. This means that party R moderates its policy up to the point where $c = \Pi - B$. At this point, a marginal decrease in policy i_1 does not compensate for reducing of certain costs of change in period $t = 2$. Further, the higher the turnover probability $(1-p)$, the lower the extreme policy $i_1 = \mu_R - \frac{B}{2}(1-p)$ for moderate costs c . This implies that if a turnover is more likely, party R chooses more moderate policies for lower values of c . The intuition is that party R anticipates the higher likelihood that party L is in power and changes the policy accordingly to its preferences, hence party R chooses a more moderate policy.

Next, we examine how the shock size B influences party R 's policy choice in the first period. On the one hand, if there is no shock at all, i.e., $B = 0$, we obtain the same results as the ones in Gersbach et al. (2019). This case is illustrated in Figure 3.6. In the absence of a preference shock, the policy choice becomes more moderate for $0 \leq c \leq \Pi(1+p)$ and more extreme for higher values of c , up to the threshold $c = 2\Pi$. Starting from this threshold, party R will implement its bliss point μ_R . On the other hand, if the shock takes the greatest possible value, i.e., $B =$

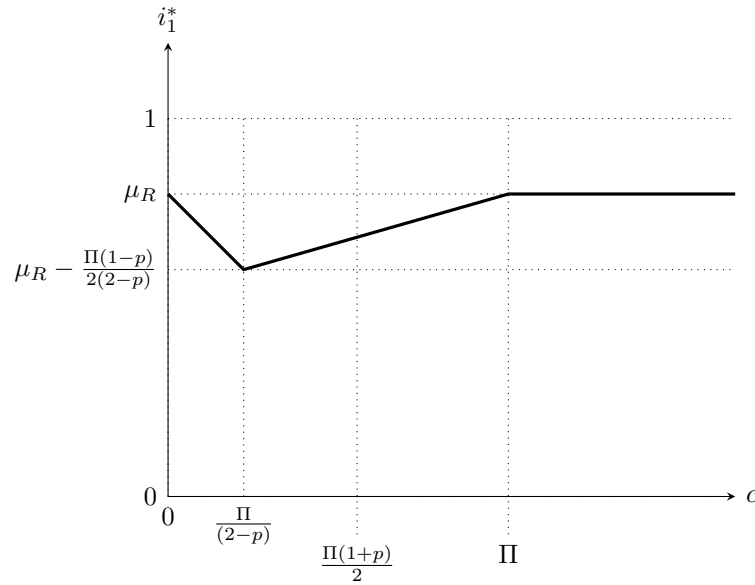


Figure 3.7: The policy choices in period $t = 1$ for $B = \frac{\Pi}{2}$.

$\frac{\Pi}{2}$, the policy choice becomes more moderate for $0 \leq c \leq \frac{\Pi}{(2-p)}$ and more extreme for values of c above this threshold, until $c = \Pi$, as depicted in Figure 3.7. There is no moderation of policy choice for higher values of c , since $B(1+p) = (\Pi - B)(1+p)$ if $B = \frac{\Pi}{2}$. Starting from $c = \Pi$, party R implements its bliss point μ_R .

One interpretation for these results is the following. If the costs of changing policy are too high, i.e., $2\Pi < c$ resp. $\Pi < c$, party R implements its peak, since it knows that a policy change in the next period would be too expensive. If the costs decrease, the policy chosen becomes more moderate, since party L can threaten to change the policy in the next period. Starting from the threshold $c = \Pi(1+p)$ resp. $c = \frac{\Pi}{(2-p)}$, the policy again becomes more extreme for decreasing costs of change. At this threshold, party R is indifferent as to which party chooses the policy in the second period. For lower costs of change, party R is more willing to take risk and cares more about the policy.

One can derive some general results from the two extreme cases of the shock size B . First, a greater shock size B implies that party R implements its bliss point for smaller sizes of c . As seen before, for the largest possible shock size $B = \frac{\Pi}{2}$, party R chooses $i_1^* = \mu_R$ for $\Pi \leq c$, whereas in the absence of a shock, i.e., $B = 0$,

party R chooses $i_1^* = \mu_R$ for $2\Pi \leq c$. The threshold $2\Pi - 2B$ for c is lower for higher values of B . This comes from the fact that for a greater shock size B , the policies chosen by both parties vary more and become higher (lower) in the presence of a positive (negative) preference shock, as seen in Subsection 3.4.1. This implies that a policy change becomes more costly for both parties and hence for a smaller value of c , party R already chooses its bliss point.

Second, in the absence of any preference shock, the policy choice is more moderate than in the case of a large preference shock, i.e., $B = \frac{\Pi}{2}$. In the general case illustrated in Figure 3.5, the greater the shock size B , the more moderate the policy choice for small costs $c \leq B(1+p)$. When the first-period incumbent party R faces small costs of change c , then deviating from the first-period policy entails relatively less disutility than choosing a policy i_1 further away from its bliss point. For a greater shock size B , the policy choices in the second period vary more and prompt party R to choose a more moderate policy. For larger costs, i.e., $c \geq (\Pi - B)(1+p)$, the opposite is true.

3.5 Conclusion

We have extended a two-period model of political competition with costs of change by including preference shocks.

Our main insight is that on average, the presence of preference shocks leads to more extreme policy choices by the incumbent party in the first period, given costs of change. Further, we have shown that a higher turnover probability and intermediate costs of change induce more moderate policy choices.

Numerous extensions could be pursued to enrich our understanding of the role of preference shocks in policy making when costs of change are present. Further areas of research could include more general shocks to parties' peaks, such as, for instance, a probability distribution of potential shocks. This would help to understand how the *likelihood* of shocks, besides the *size* of shocks, determines policy choices in the presence of costs of change. Higher likelihood of shocks might induce greater policy reforms or lead to gridlock. Possible other extensions could include endogenously-chosen costs of change set by the political actors.

Chapter 4

Credence Policies and Experts in Democracy*

Abstract

Democratic societies often decide on policies solving complex issues whose consequences are difficult, if not impossible, to predict, at least in the short-term. We examine the role of experts in democratic decision-making on such credence policies. We develop a model with heterogeneous voters regarding risk aversion and beliefs about policy efficiency. We compare *(i)* majority voting without experts, *(ii)* vote delegation to experts, and *(iii)* majority voting with published consensus expert opinion to the optimal policy if fully informed citizens voted on the issue. Experts can have a beneficial, neutral, or harmful impact on democratic decision-making. Extreme and detrimental outcomes can arise when decisions are delegated to experts who act on the most accurate assessments of the consequences of policies but do not take into account differences in risk aversion within the electorate. Publishing a consensus assessment of experts before citizens vote can be neutral or can improve policy-making, depending on how beliefs about the effectiveness of the policy are distributed in the electorate.

*This chapter is joint work with Hans Gersbach (ETH Zurich and CEPR) and Oriol Tejada (Universitat de Barcelona).

4.1 Introduction

Solving complex issues is one of the most important challenges of democratic societies. Many of these issues affect society in several dimensions, which should all be addressed by policies, ideally. In the case of *credence policies*, short- and mid-term consequences of their implementation are often difficult or even impossible to predict because they are highly uncertain. Moreover, they remain unknown for quite some time. Examples are the management of a health crisis, the slowing down of climate change, or the implementation of banking regulations. In any of these cases, experts can provide information about the consequences of policies, new or past. From a democratic perspective, the role of experts is important, as they can influence policy decisions, citizens' attitudes and hence, indirectly election outcomes.

During the COVID-19 pandemic, for instance, experts from different fields, e.g., virologists and economists, advised politicians regarding policies. Often, these experts' advice was contradictory and some of their advice could only be implemented partially. One example is the Swiss National COVID-19 Science Task Force that addressed key issues regarding the COVID-19 crisis (Swiss National COVID-19 Science Task Force, 2022) and published its policy briefs on its website.¹ These recommendations were not only relevant for policy decisions by politicians, but also served as a source of information for citizens before voting on initiatives, such as the voting on government measures to fight the pandemic in November 2021.² Another example is climate change. Notably, experts were asked to advise political leaders about it at the UN Climate Change Conference UK 2021. The question we address in this chapter is how decision-making should be designed in democracies to deal with such complex policy issues, and, in particular, how experts should be included in the decision-making process.

There are different ways for a society to take expertise into account in decision-making processes. Given that a part of the electorate is willing to listen to experts' opinions, there are two well-known procedures for the inclusion of experts in the

¹<https://sciencetaskforce.ch/en/policy-briefs-english/> (accessed on April 5, 2022).

²<https://www.swissinfo.ch/ger/resultat-covid-gesetz-zertifikat/47141410> (accessed on May 29, 2022).

decision-making process. First, one can use *Vote Delegation*: voters have the opportunity to delegate their votes to representatives.³ Second, one can rely on *Opinion Updating*: experts first reach a consensus and then make it publicly available to voters before voting day. In this chapter, we analyze and compare these two voting procedures, Vote Delegation and Opinion Updating to the simple majority vote without experts and to an optimal policy, with the goal to assess the ideal design of democracy for complex policy issues.

For this purpose, we use a theoretical model with heterogeneous voters—who form the electorate—deciding on the extent of a *credence policy*. This is a policy whose benefits cannot be observed, neither in the short run nor in the long run. The voters differ in two dimensions, risk aversion and beliefs about the efficiency of the credence policy (alternatively, the costs associated with the credence policy). While the beliefs of some voters may change in response to the experts’ inputs, the voters’ risk aversion is fixed and invariable. Yet, one part of the electorate is opinionated, meaning that it cannot be influenced by experts’ opinions. For simplicity, experts are modeled by one representative expert, whose opinion represents the experts’ consensus about the efficiency of the credence policy. We assume in this chapter, as in the case of the Swiss Covid-19 task force, that experts with different opinions discuss policy recommendations and publish one consensus.

To elaborate on the role of experts, we need to specify first what it means to be an expert. Our concept of an expert is inspired by Milgrom (1981) and Crawford and Sobel (1982), who model expertise as a single piece of information that is only possessed by an individual. Because of this knowledge advantage, we assume that the representative expert has the most accurate opinion about the efficiency of the policy. In our model, experts are also citizens but have biased opinions and risk aversion, which means that the representative expert’s risk aversion may not represent that of the whole electorate. From now on, we refer to this opinion, i.e., to the advice voiced by the expert, as the expert’s *belief*.

Overall, we consider four different voting procedures, all of which use the majority rule. In the benchmark case, the optimal policy is chosen by an “ideal” decision-making process, which we call *Optimal Democracy*. It means that the

³The Pirate parties implemented this procedure, also called *liquid democracy*, where members can either vote themselves or delegate proxies to vote for them (Cammaerts, 2015).

whole electorate has the most accurate belief, i.e., equal to the expert's belief, and nobody is opinionated. Then, we analyze three different voting procedures, which we call *Elections without Experts*, *Elections with Vote Delegation*, and *Elections with Opinion Updating*, respectively. In the first procedure, the voters decide on a credence policy without hearing any expert, based on their personal beliefs and risk aversion. In the second procedure, one part of the electorate, the non-opinionated voters, delegates its voting right to the representative expert, who takes the decision for them. This decision is based on the expert's belief and risk aversion. In the third procedure, the non-opinionated voters' beliefs are influenced by the representative expert's belief, but these voters exercise their right to vote directly and their decision is also influenced by their personal risk aversion. In each of the three procedures, the opinionated voters stick to their prior beliefs and risk aversion.

The main part of the chapter deals with the analysis of the policy outcomes of the four different voting procedures. We show that regular elections always produce executive action and the chosen policy is lower for larger shares of non-opinionated voters η . In the case of *Elections with Vote Delegation* and *Elections with Opinion Updating*, the larger the share of non-opinionated voters η , the lower or higher the chosen policy. This depends on the belief and risk aversion differences between the voters and the expert. For instance, if an expert has a belief that entails lower efficiency losses of the policy or s/he is highly risk-averse, both procedures lead to a higher policy, given the non-opinionated voters form a majority. Otherwise, the opinionated voters dictate the policy.

Next, we compare the policy outcomes to an ideal policy. There we examine two special cases. First, we consider an expert with a belief that entails higher efficiency losses of the policy. Then, in *Elections without Experts* and in *Elections with Opinion Updating* the chosen policy is always lower than the optimal policy. The latter leads to a policy which is closer to the optimal policy. From a democratic perspective, this shows that *Opinion Updating* is beneficial, since it has the advantage that voters vote themselves. Further, it can even lead to a policy closer to the optimal policy. Second, consider an expert with a belief that entails lower efficiency losses of the policy. Then, *Elections without Experts* and *Elections with Opinion Updating* can even lead to the optimal policy. This is the case for a moderate value of the belief and a large enough majority of non-opinionated voters. In general, in

Elections with Vote Delegation, the chosen policy is always lower than the optimal policy, no matter the expert's belief.

Consider now the following two special cases. First, if the representative expert has a low risk aversion and if the share of non-opinionated voters is sufficiently high, experts can have a beneficial, neutral or harmful impact on democratic decision-making. Extreme and detrimental outcomes can arise when decisions are delegated to experts who act on the most accurate assessments of the consequences of policies, but do not take into account differences in risk aversion in the electorate. Second, publishing a consensus assessment of experts before citizens vote can be neutral or can improve policy-making, depending on how beliefs about the effectiveness of the policy are distributed in the electorate.

The chapter is organized as follows. In the next section we discuss the related literature. Section 4.3 describes and analyzes the model. In Section 4.4 we elaborate the benchmark case as well as the other voting procedures, with and without experts. In Section 4.5 the outcomes are compared, followed by a discussion. In Section 4.6 we address two special cases. Section 4.7 discusses possible extensions of the model. Section 4.8 concludes.

4.2 Relation to the Literature

This chapter is related to several strands of the literature.

Proxy voting and liquid democracy

Starting in the 1960s, the literature on direct voting and proxy voting systems has grown rapidly besides the literature on traditional representation systems. Miller (1969), motivated by Tullock (1967), had a visionary proposal for direct and proxy voting, inspired by technological advantage. In this delegation system, it is possible to delegate all, some or no voting decisions to proxies. Many scholars discussed this idea for different variants of representative democracy (see e.g. Shubik, 1970; Mueller et al., 1972). Another strand of the literature elaborates on proxy voting based on axiomatic arguments as well as on proposals for practical implementations (see e.g. Dan, 2006; Green-Armytage, 2015). Liquid democracy, its potential and

risks, were assessed by Blum and Zuber (2016), Christoff and Grossi (2017), and Brill and Talmon (2018), among others. In most papers, the weight of the proxy is equal to the number of votes delegated. In our model, there is one key difference: we consider a representative expert, who not only has a belief that differs from the voters, s/he also has a different risk aversion. We add to this literature strand by analyzing the impact of the share of voters delegating their votes on the policy choice.

Government and experts

The relationship between the government and experts is the topic of a large literature. Hirschi (2018) explains the misalignment of the incentives of politicians and experts, which is illustrated by past scandals about expert committees. In Brozus et al. (2017), the history, advantages, and risks of experts' role in political processes are elaborated. Brozus et al. (2017) show that experts have had a huge influence on governmental decisions (see also Löblová, 2018). Different incentives, social or monetary in nature, can prevent experts' opinion from being integrated in the political process efficiently. Therefore, precise rules and procedures are needed to ensure that the society can benefit from the experts' advice. The design of such procedures to obtain the desired outcomes is not trivial and motivates this chapter.

Power of expert's opinions

Yet another strand of the literature questions how experts' advice influences the behavior of voters and policy makers, as well as outcomes. Recently, Callander et al. (2021) have explored the origin of experts' power by analyzing a canonical model of communication with hard information. Earlier, Callander et al. (2008) defined expertise directly over the policy process—the function that maps policies into outcomes—and showed that this generalization matters for political behavior. Kawamura and Vlaseros (2017) showed experimentally that the presence of expert information can lead to an inefficient outcome, even if it is more accurate than the one of the rest of the population. In this chapter, we analyze the impact of the experts' influence on voters' behavior and on policy outcomes.

Opinion aggregation and deliberation

A large body of literature deals with the aggregation of individual preferences or opinions through voting. The seminal work of Feddersen and Pesendorfer (1997) on voting behavior and information aggregation analyzed two-candidates elections, given noisy private information. As an example of deliberation model, Perote-Peña and Piggins (2015) developed a model combining preference transformation and aggregation. List (2018) provided an overview of democratic deliberation as well as social choice theory and discussed different models of deliberation. Dietrich and List (2017) studied probabilistic opinion pooling on general agendas and applied it to probabilistic preference aggregation. In this chapter, preference aggregation comes into effect in the scenario of Elections with Opinion Updating. Our goal is to understand better how the publicly available consensus belief influencing one part of the electorate impacts the policy outcome and whether, and if yes, how it differs from the optimal policy.

4.3 Model

4.3.1 Basic setup

There is a continuum of individuals of measure one, each indexed by $i \in [0, 1]$, who must collectively decide on the extent of a policy $p \in [0, 1]$. The policy is *credence*, i.e., the outcome of any chosen p is not informative regarding its effectiveness, and it is therefore assumed to be stochastic. There are different reasons why a policy can be credence or at least can be partly credence. The policy may be implemented along with other policy decisions or unexpected economic shocks may occur that make it difficult to establish the exact effect of the chosen policy on the observable outcomes (Gersbach, 2021). An individual voter is characterized by her/his type $(b_i, k_i) \in [0, 1] \times (0, 1]$. If policy p is chosen, each individual receives a stochastic payoff

$$\tilde{y}_i(p) \sim \mathcal{N}\left(\overbrace{-b_i \cdot p}^{:=\mu_i(p)}, \overbrace{\lambda \cdot (1-p)}^{:=\sigma(p)}\right),$$

where λ satisfies $1 < \lambda < \infty$ and is exogenously given. That is, from the perspective of voter i , $\tilde{y}_i(p)$ is drawn from a normal distribution with mean $\mu_i(p) = -b_i \cdot p$ and standard deviation $\sigma(p) = \lambda \cdot (1 - p)$. Increasing policy p therefore reduces the uncertainty of $\tilde{y}_i(p)$, but it also reduces its expected value, which can be seen as the cost of reducing the uncertainty about the outcome. The tension between uncertainty and costs is the fundamental trade-off society faces, and voters put different weights on each element of this trade-off. Increasing the absolute value of b_i (i.e., decreasing $-b_i$) means that voter i is less willing to reduce uncertainty, all else being equal, since the cost of doing so is higher. Finally, note that the higher λ , the greater the uncertainty of the payoff.⁴

All citizens value the outcome $\tilde{y}_i(p)$ through a constant absolute risk aversion utility function, albeit they differ in the extent of their risk parameter. In particular, we assume that voter i 's utility when payoff $\tilde{y}_i(p)$ is realized is

$$u_i(\tilde{y}_i(p)) := -e^{-k_i \cdot \tilde{y}_i(p)}. \quad (4.1)$$

According to Equation (4.1), voter i derives higher utility for higher values of $\tilde{y}_i(p)$. Since voter i 's coefficient of relative risk aversion is k_i and $\tilde{y}_i(p)$ is stochastic, higher values of k_i prompt voter i to prefer $\tilde{y}_i(p)$ to be less uncertain from an ex ante perspective, all else being equal. Therefore, risk-averse agents will prefer a higher policy p , since $\sigma(p)$ decreases in p . As mentioned above, higher policies reduce the mean payoff $\mu_i(p)$ and lead to the fundamental trade-off we address. This trade-off can be best illustrated by finding the policy that some voter $i \in [0, 1]$ would choose to maximize her/his expected utility, denoted by $\mathbb{E}_p[u_i(p)]$. This preferred policy solves the following problem:

$$\max_{p \in [0,1]} \mathbb{E}_p[u_i(p)]$$

The above maximization problem is equivalent to⁵

$$\max_{p \in [0,1]} \mu_i(p) - \frac{k_i \cdot \sigma(p)^2}{2} = -b_i \cdot p - \frac{k_i \cdot \lambda^2 \cdot (1 - p)^2}{2}.$$

⁴Our results do not depend qualitatively on the exact value of λ .

⁵See Sargent (1987).

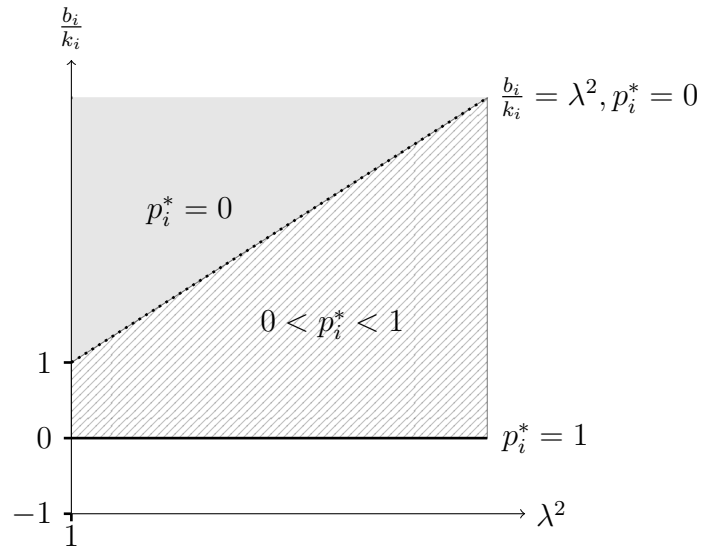


Figure 4.1: The preferred policy p_i^* for different values of $\frac{b_i}{k_i}$ and λ^2 .

One can easily verify that the policy that maximizes voter i 's expected utility is

$$p_i^* := p^*(k_i, b_i) = \max \left\{ 1 - \frac{b_i}{k_i \cdot \lambda^2}, 0 \right\}. \quad (4.2)$$

That is, voter i 's preferred policy, p_i^* , increases in k_i and decreases in b_i . On the one hand, a more risk-averse voter (who has higher k_i) prefers a higher credence policy to reduce uncertainty. On the other hand, a higher b_i implies more costly policies (for that individual) and leads to a lower choice of the credence policy. Voter i 's preferred policy is pinned down by the ratio b_i/k_i , and is thus the same for all individuals with the same ratio.

As an illustration, Figure 4.1 depicts citizen i 's preferred policy as a function of λ^2 and b_i/k_i . It shows three different areas, depending on whether $p_i^* = 0$, $0 < p_i^* < 1$, or $p_i^* = 1$. The policy is strictly positive if and only if the ratio b_i/k_i is lower than λ^2 . If the ratio b_i/k_i is zero, in particular, the preferred policy p_i^* is one for all values of λ^2 .

We further assume that the electorate consists of two distinct subsets of voters. First, there is a share η of *non-opinionated voters*, with $\eta \in [0, 1]$, whose two-dimensional type (b_i, k_i) is drawn from two independent uniform distributions,

viz. $b_i \sim U[0, 1]$ and $k_i \sim U(0, 1]$. Second, there is a share $1 - \eta$ of *opinionated voters*, who form a homogeneous group of individuals in the sense that they have a fixed ratio $b_i/k_i := \beta$, with $\beta \in [0, 1]$.⁶ From the assumption, it follows that an opinionated voter, who is risk-seeking (who has lower k_i), also has a lower value of b_i . Further, the fixed ratio implies a fixed belief of all these voters, which is a plausible real-world assumption. For instance, voters who denied that SARS-COV-2 was a real threat also had similar views on other topics.⁷ Another example is climate change denial. It has been shown empirically that there exists a strong link between the fossil fuel industry, its funders and climate denialists.^{8, 9}

Finally, there is also a group of experts, which is modeled by a representative expert, called *expert* from now on. The expert has a fixed belief b^E and risk aversion $k^E \sim \mathcal{U}[\underline{\phi}, \bar{\phi}]$. The expert's belief b^E can be seen as a consensus of all the experts' opinions about the efficiency of the policy. Therefore, it is assumed to be the most accurate belief about the costs associated with the credence policy. Due to greater knowledge on the voting issue, the expert's risk aversion differs generically from the electorate's.

4.3.2 Some properties

We focus on elections with majority voting and consider a standard Downsian model of elections in which the policy chosen is the median voter's ideal policy. As already discussed, voter i 's utility type is pinned down by the ratio b_i/k_i , so we now derive the median (and the average) voter type for each of the two (type) components: risk aversion k_i and belief b_i . For simplicity, we consider the extreme case where $b_i = \beta$ and $k_i = 1$ for any opinionated voter i .¹⁰

⁶We borrow the term “opinionated” from Caballero and Simsek (2022). That there exists only one group of opinionated voters is not a critical assumption. More groups would not change any fundamental result of the model. The results depend on the *proportion* of opinionated voters in the electorate.

⁷<https://www.thirdway.org/blog/the-politics-of-denial-from-climate-to-covid-19> (accessed on May 30, 2021).

⁸More information can be found at <https://insideclimatenews.org/news/09042020/science-denial-coronavirus-covid-climate-change/> (accessed on May 27, 2022).

⁹For more information, see <https://www.theguardian.com/environment/2020/may/21/groups-fossil-fuel-funding-urge-states-reopen-amid-pandemic> (accessed on May 27, 2022).

¹⁰As long as $b_i/k_i = \beta$ for any opinionated voter, our insights are the same.

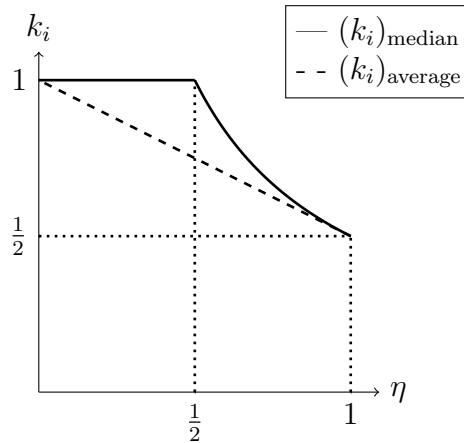


Figure 4.2: The average and median value of k_i for $0 \leq \eta \leq 1$.

First, we consider risk aversion. It can be easily verified that

$$(k_i)_{\text{average}} = \mathbb{E}[k_i] = \frac{\eta}{2} + (1 - \eta) = 1 - \frac{\eta}{2}$$

and

$$(k_i)_{\text{median}} = \begin{cases} 1 & \text{if } 0 \leq \eta \leq \frac{1}{2}, \\ \frac{1}{2\eta} & \text{if } \frac{1}{2} < \eta \leq 1. \end{cases}$$

Note that $(k_i)_{\text{median}} \geq (k_i)_{\text{average}}$ for all values of η . That is, the risk aversion distribution is negatively skewed in the electorate.¹¹ Figure 4.2 displays a graphical representation of the median and average risk aversion as a function of the share of opinionated voters.

Second, we investigate beliefs. It can be easily verified that

$$(b_i)_{\text{average}} = \mathbb{E}[b_i] = \frac{\eta}{2} + (1 - \eta)\beta = \eta \left(\frac{1}{2} - \beta \right) + \beta$$

and

$$(b_i)_{\text{median}} = \begin{cases} \frac{1}{2\eta} & \text{if } \frac{1}{2} \leq \eta\beta, \\ \beta & \text{if } \eta\beta \leq \frac{1}{2} \leq \eta\beta + (1 - \eta), \\ 1 - \frac{1}{2\eta} & \text{if } \eta\beta + (1 - \eta) \leq \frac{1}{2}. \end{cases}$$

¹¹In this case, the left tail is longer, but w.l.o.g. one can assume a positively skewed risk aversion. The intention is to include some asymmetry among the voters.

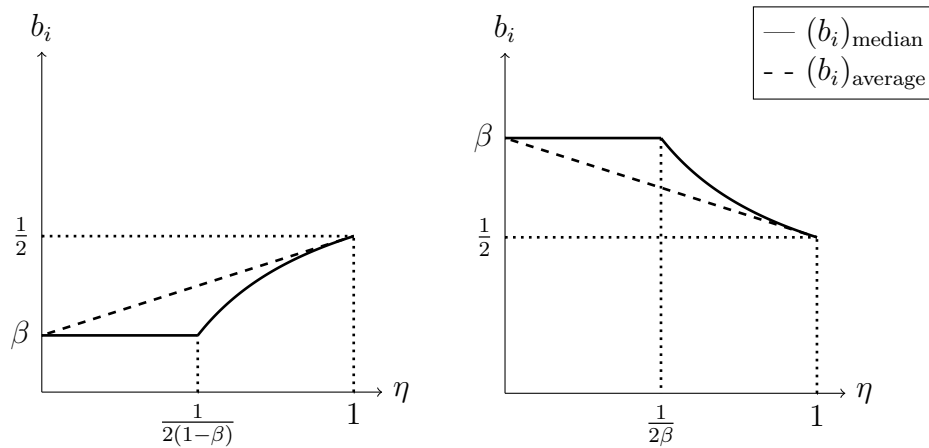


Figure 4.3: The average and median value of b_i for $\beta \leq \frac{1}{2}$ (left) and $\beta > \frac{1}{2}$ (right).

Hence, the belief distribution is positively (negatively) skewed in the electorate if $\beta > \frac{1}{2}$ ($\beta \leq \frac{1}{2}$). Figure 4.3 displays a graphical representation of the median and average belief as a function of the share of opinionated voters for both cases ($\beta > \frac{1}{2}$ and $\beta \leq \frac{1}{2}$).

4.4 Voting Procedures

In this section, we consider four different voting procedures. The first two procedures, Optimal Democracy and Elections without Experts, do not include experts. In Optimal Democracy, all voters are non-opinionated ($\eta = 1$) and have the most accurate belief (b^E), and the policy chosen (through elections with the majority rule) is therefore called *optimal*. It serves as benchmark for our analysis. In Elections without Experts, the electorate comprises non-opinionated and opinionated voters, all of whom have their own beliefs and risk aversion. We also investigate two further procedures, Elections with Vote Delegation and Elections with Opinion Updating, where the expert influences the group of non-opinionated voters. In Elections with Vote Delegation, voters can delegate their votes to the expert. In Elections with Opinion Updating, voters receive an additional signal from the expert which can be included in the decision-making process. In the following, the different procedures are described and compared to each other.

4.4.1 Benchmark: Optimal Democracy

As a benchmark case, we consider the (optimal) decision-making process in an ideal electorate. This procedure is unrealistic, but serves as a basis for comparison. In Optimal Democracy, the decision is taken by simple majority without any expert, and the whole electorate consists entirely of non-opinionated voters, i.e., $\eta = 1$. Moreover, all voters have the most accurate belief b^E , which is equal to the expert's belief. Voters only differ from each other in their risk aversion, which is represented by $k_i \sim \mathcal{U}[0, 1]$. This procedure is ideal from a democratic point of view since the decision is solely taken by the electorate and does not require any experts. Furthermore, the voters represent the characteristics of the electorate. The chosen policy, called *optimal policy* from now on, is given in the following proposition:

Proposition 4 *The optimal policy chosen in Optimal Democracy is*

$$p_{opt}^* := \max \left\{ 1 - \frac{2b^E}{\lambda^2}, 0 \right\}.$$

Hence, in Optimal Democracy, the investment in the credence policy increases with policy uncertainty, while it decreases with beliefs b_E that entail higher efficiency losses. For high values of b^E , i.e., if $b_E \geq \lambda^2/2$, the optimal policy can even be zero.

4.4.2 Elections without Experts

In this section, we consider a standard Downsian model of elections in which the policy chosen is the median voter's ideal policy. As already discussed, voter i 's utility type is pinned down by the ratio b_i/k_i . In Appendix C C, we show that this implies that the citizens' preferences satisfy the single-crossing property (SCP) considering this ratio, which ensures that the median voter theorem holds as stated in Gans and Smart (1996).

Elections without Experts differs from Optimal Democracy in two ways. First, the electorate consists of both opinionated and non-opinionated voters. Second, the non-opinionated voters draw their beliefs b_i randomly instead of sharing the expert's belief b^E . Hence, non-opinionated voters do not have the most accurate belief about the efficiency of the policy. As described in Subsection 4.3.1, a non-

opinionated voter i is characterized by $b_i \sim \mathcal{U}[0, 1]$ and $k_i \sim \mathcal{U}(0, 1]$, whereas an opinionated voter has a fixed ratio $b_i/k_i = \beta$, with $\beta \in [0, 1]$.

The policy chosen by simple majority is stated in the following proposition:

Proposition 5 *The policy chosen in Elections without Experts is*

$$p_{maj}^* := 1 - \frac{\max\left\{\beta, 2 - \frac{1}{\eta}\right\}}{\lambda^2}. \quad (4.3)$$

The above proposition shows that $p_{maj}^* > 0$, no matter the share of non-opinionated voters, η . That is, regular elections always produce executive action. (We assume that $p = 0$ means no action.) For $0 \leq \eta \leq \frac{1}{2}$, it is clear that the median value of the ratio b_i/k_i distributed in the entire electorate consisting of opinionated and non-opinionated voters is β , since the opinionated voters form the majority of the electorate. In this case, the policy chosen is $1 - \frac{\beta}{\lambda^2}$. However, this policy may also be chosen if the opinionated voters form a minority, depending on the value of β . In the polar case $\beta = 1$, the mass of opinionated voter occupies a central position in terms of the ratio b_i/k_i relative to the non-opinionated voters. In such case, the median voter is again opinionated, who then chooses policy $1 - \frac{1}{\lambda^2}$. In the polar case $\beta = 0$, by contrast, the mass of opinionated voters has the lowest ratio $\frac{b_i}{k_i}$ and the median voter is non-opinionated, who then chooses policy $1 - \frac{2 - \frac{1}{\eta}}{\lambda^2}$. In general, the larger the share of non-opinionated voters, η , the lower policy p_{maj}^* . This comes from the fact that as the non-opinionated voters, who have on average a higher median ratio b_i/k_i than opinionated voters, have more weight, the policy is lower. The policies chosen are illustrated in Figure 4.4. Finally, how close the policy chosen in Elections without Experts is to 1 depends on the scaling parameter λ . The higher λ , the higher the uncertainty and, therefore, the higher p_{maj}^* .

4.4.3 Elections with Vote Delegation

Next we consider elections where the non-opinionated voters delegate their votes to the expert. The expert, who has a greater knowledge of the voting topics and hence holds the most accurate belief b^E , votes for them together with the opinionated voters. In general, experts are themselves citizens but have biased beliefs and risk

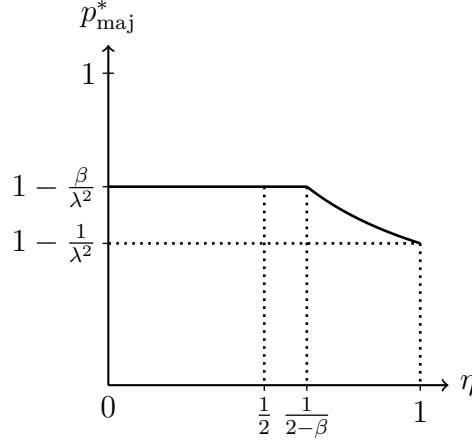


Figure 4.4: Policy p_{maj}^* depending on η .

aversion, which means that the experts' risk aversion may not represent that of the whole electorate. To account for this, we assume that the representative expert's risk aversion, k^E , is drawn from some skewed uniform distribution, i.e., $k^E \sim U[\underline{\phi}, \bar{\phi}]$. This means that for the non-opinionated part of the electorate it holds that $b_i = b^E$ and $k^E \sim U[\underline{\phi}, \bar{\phi}]$, where $0 < \underline{\phi} < \bar{\phi} \leq 1$. The preferred policy is chosen by simple majority and is given by the following proposition:

Proposition 6 *The policy chosen in Elections with Vote Delegation is*

(i) If $\beta \leq \frac{b^E}{\phi}$,

$$p_{\text{del}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})}, 0 \right\} & \text{if } \frac{1}{2} \leq \eta \leq 1. \end{cases}$$

(ii) If $\frac{b^E}{\phi} < \beta \leq \frac{b^E}{\mathbb{E}(k^E)}$,

$$p_{\text{del}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \underline{\phi})}{b^E - \beta \underline{\phi}}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})}, 0 \right\} & \text{if } \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \underline{\phi})}{b^E - \beta \underline{\phi}} \leq \eta \leq 1. \end{cases}$$

(iii) If $\frac{b^E}{\mathbb{E}(k^E)} < \beta < \frac{b^E}{\phi}$,

$$p_{del}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E}, \\ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\phi - (\phi - \phi)} & \text{if } \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E} \leq \eta \leq 1. \end{cases}$$

(iv) If $\frac{b^E}{\phi} \leq \beta$,

$$p_{del}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2}, \\ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\phi - (\phi - \phi)} & \text{if } \frac{1}{2} \leq \eta \leq 1. \end{cases}$$

The above proposition states the chosen policies depending on the fixed ratio of beliefs and risk aversion β of the opinionated voters. Similar to Elections without experts, it is clear that for $0 \leq \eta \leq \frac{1}{2}$ the median value of the ratio b_i/k_i is β as the opinionated voters form the majority of the electorate. Hence, the chosen policy is $1 - \frac{\beta}{\lambda^2}$. This policy may be also chosen, when opinionated voters form a minority depending on the value of β . This is the case for $\frac{b^E}{\phi} < \beta < \frac{b^E}{\phi}$, as stated in Cases (ii) and (iii) of the proposition. This comes from the fact that in these cases the ratio of belief and risk aversion of opinionated and of non-opinionated voters can coincide. In the polar case $\beta = \frac{b^E}{\phi}$, the mass of the opinionated voters occupies a low position compared to the non-opinionated voters. Hence, if the median voter is non-opinionated, s/he chooses a lower policy than if s/he were opinionated. In the polar case $\beta = \frac{b^E}{\phi}$, by contrast, the mass of opinionated voters has a high ratio b_i/k_i compared to non-opinionated voters, and in the case of a non-opinionated median voter, the chosen policy would be higher. For smaller (larger) values of β , i.e., $\beta < \frac{b^E}{\phi}$ ($\frac{b^E}{\phi} < \beta$), the policy changes at the threshold $\eta = \frac{1}{2}$, as stated in Case (i) (Case (iv)) of the proposition. This means that at the point where the non-opinionated voters and not the opinionated voters anymore form the majority, a clearly higher or lower policy is chosen.

Figures 4.5, 4.6, and 4.7 illustrate the policy p_{del}^* depending on the share of non-opinionated voters η as stated in Proposition 6. First, consider Cases (i) and (ii) depicted in Figures 4.5 and 4.6. Here, the ratio of belief and risk aversion of the expert, hence the share of non-opinionated voters, is higher than the ratio of opin-

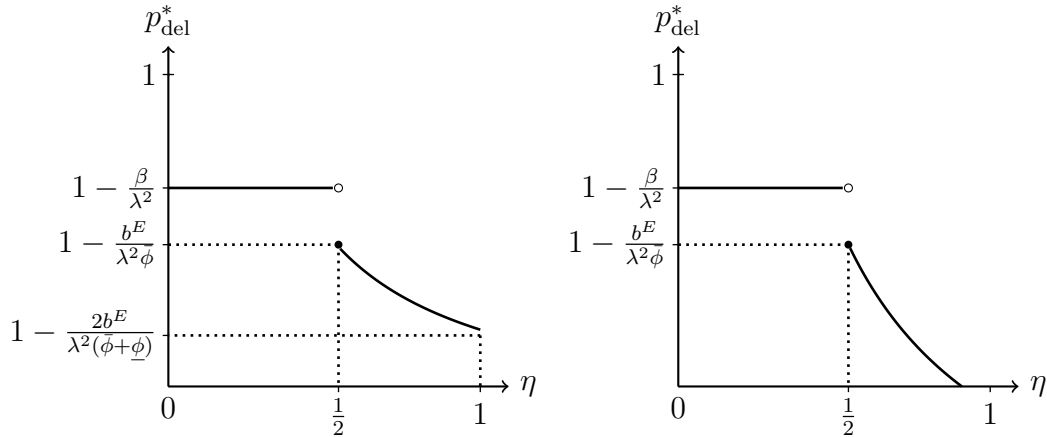


Figure 4.5: Policy p_{del}^* for $\frac{b^E}{\mathbb{E}(k^E)} < \lambda^2$ (left) and $\lambda^2 < \frac{b^E}{\mathbb{E}(k^E)}$ (right) in Case (i).

ionated voters, i.e., $\frac{b^E}{\mathbb{E}(k^E)} \geq \beta$. This implies the larger the share of non-opinionated voters, η , the lower the policy p_{del}^* . If the uncertainty parameter λ is enough low, it is possible that no action, i.e., $p_{\text{del}}^* = 0$, is chosen, given a large enough majority of non-opinionated voters. This is one difference to regular elections that always produce an executive action as shown in Proposition 8. Second, for the Cases (iii) and (iv) the opposite, i.e., $\frac{b^E}{\mathbb{E}(k^E)} < \beta$, is true. This implies that the larger the share of non-opinionated voters η , the higher the policy p_{del}^* . Further, there is always an executive action, i.e., $p_{\text{del}}^* > 0$ no matter the share of non-opinionated voters, η .

If $\beta \in \left[\frac{b^E}{\phi}, \frac{b^E}{\phi}\right]$, then the policy p_{del}^* can be seen a continuous function depending on the share of non-opinionated voters η as shown in Figures 4.6 and 4.7 (left). The threshold, denoted for now by η_1 , where the policy starts increasing, respectively decreasing, depends on the relation of the values of β , b^E and $\mathbb{E}(k_i)$. In Figure 4.8, one can see that the threshold η_1 depending on the value of β . For low and for high values of β , the threshold is constant at $\frac{1}{2}$. At the point where the ratio of expected belief to expected risk aversion is the same for opinionated and for non-opinionated voters, i.e., where $\beta = \frac{b^E}{\mathbb{E}(k^E)}$, the threshold equals 1, and therefore the policy chosen is independent of the share of non-opinionated voters η . Otherwise, the more the ratios of opinionated and non-opinionated voters are aligned, the higher the threshold η_1 .

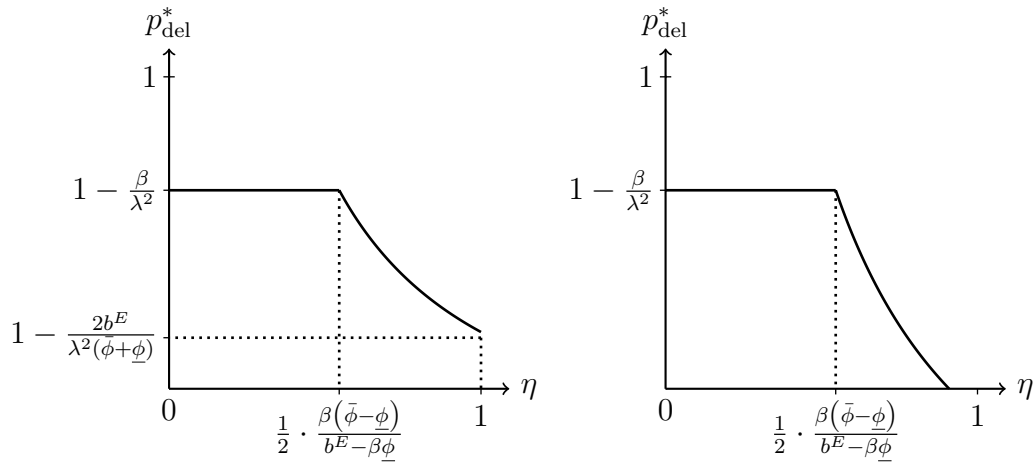


Figure 4.6: Policy p_{del}^* for $\frac{b^E}{\mathbb{E}(k^E)} < \lambda^2$ (left) and for $\lambda^2 < \frac{b^E}{\mathbb{E}(k^E)}$ (right) in Case (ii).

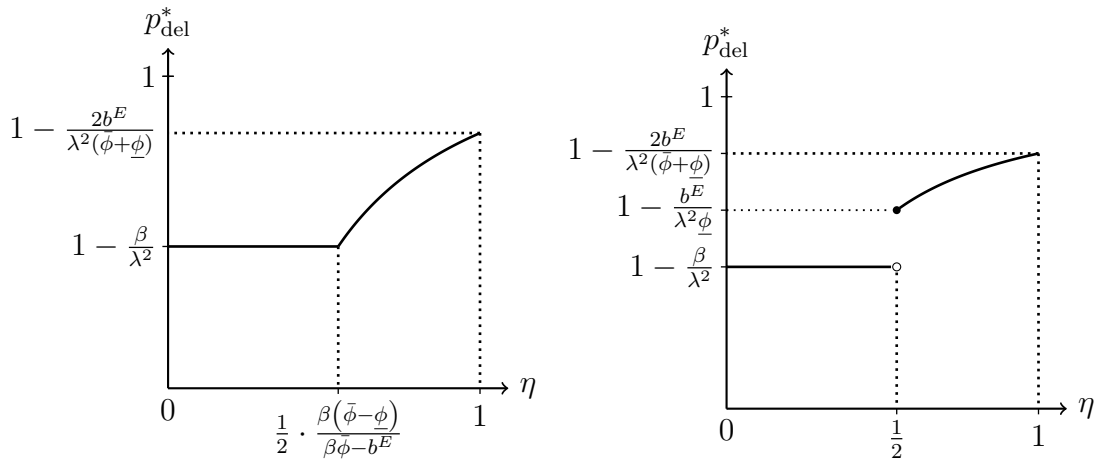


Figure 4.7: Policy p_{del}^* in Case (iii) on the left side and in Case (iv) on the right side.

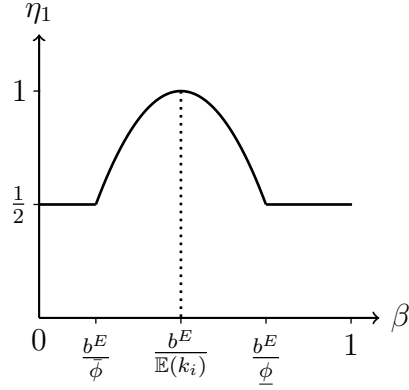


Figure 4.8: The threshold η_1 depending on β .

4.4.4 Elections with Opinion Updating

In this section we consider elections where the non-opinionated voters receive an additional signal (or piece of information) from the expert regarding the costs associated with the credence policy. As these voters receive this information, they update their beliefs by weighting their own belief by weight $\kappa \in [0, 1]$ and the expert's belief by weight $1 - \kappa$. By contrast, their risk aversion does not change. This is because the voters vote for themselves after having updated their opinion on b_i .

Hence, for the share of non-opinionated voters η share of the electorate,

$$k_i \sim \mathcal{U}[0, 1],$$

$$b'_i = \kappa b_i + (1 - \kappa)b^E \sim \mathcal{U}[(1 - \kappa)b^E, (1 - \kappa)b^E + \kappa], \text{ where } \kappa \in [0, 1].$$

To simplify notation, we let $\tau := (1 - \kappa)b^E$, so that $b'_i \sim \mathcal{U}[\tau, \tau + \kappa]$. As in the last section, the preferred policy is chosen by simple majority. Then, the chosen policy is given by the following proposition:

Proposition 7 *The policy chosen in Elections with Opinion Updating is*

(i) *If $\beta < \tau < \tau + \kappa$,*

$$p_{up}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta \leq \frac{1}{2}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \left(\tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta-1) + \kappa^2(2\eta-1)^2}}{2\eta} \right), 0 \right\} & \text{if } \frac{1}{2} < \eta \leq \eta_1, \\ \max \left\{ 1 - \frac{\eta(2\tau+\kappa)}{\lambda^2}, 0 \right\} & \text{if } \eta_1 < \eta \leq 1, \end{cases}$$

where $\eta_1 = \frac{\tau+\kappa}{2\tau+\kappa}$.

(ii) *If $\tau < \beta < \tau + \kappa$,*

$$p_{up}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta \leq \eta_2, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \left(\tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta-1) + \kappa^2(2\eta-1)^2}}{2\eta} \right), 0 \right\} & \text{if } \eta_2 < \eta \leq \eta_1, \\ \max \left\{ 1 - \frac{\eta(2\tau+\kappa)}{\lambda^2}, 0 \right\} & \text{if } \eta_1 < \eta \leq 1, \end{cases}$$

where $\eta_1 = \frac{\tau+\kappa}{2\tau+\kappa}$ and $\eta_2 = \frac{\kappa\beta}{2\kappa\beta - (\beta-\tau)^2}$.

(iii) *If $\tau < \tau + \kappa < \beta \leq 2\tau + \kappa$, i.e., $\beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$,*

$$p_{up}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{\beta}{2\tau+\kappa}, \\ \max \left\{ 1 - \frac{\eta(2\tau+\kappa)}{\lambda^2}, 0 \right\} & \text{if } \frac{\beta}{2\tau+\kappa} \leq \eta \leq 1. \end{cases}$$

(iv) *If $2\tau + \kappa < \beta$, i.e., $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta$,*

$$p_{up}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{\beta}{2\beta - (2\tau+\kappa)}, \\ 1 - \frac{\eta(2\tau+\kappa)}{(2\eta-1)\lambda^2} & \text{if } \frac{\beta}{2\beta - (2\tau+\kappa)} \leq \eta \leq 1. \end{cases}$$

The above proposition states the chosen policy for different ranges of β . In Part (i), (ii) and (iii), the fixed ratio $\frac{b_i}{k_i} = \beta$ of the opinionated voters is always lower than the ratio of beliefs and risk aversion from the non-opinionated voters. In this case, the larger the share of non-opinionated voters, the lower the chosen policy p_{up}^* . For higher values of β , i.e. $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta$, as described in Part (iv), the opposite is true.

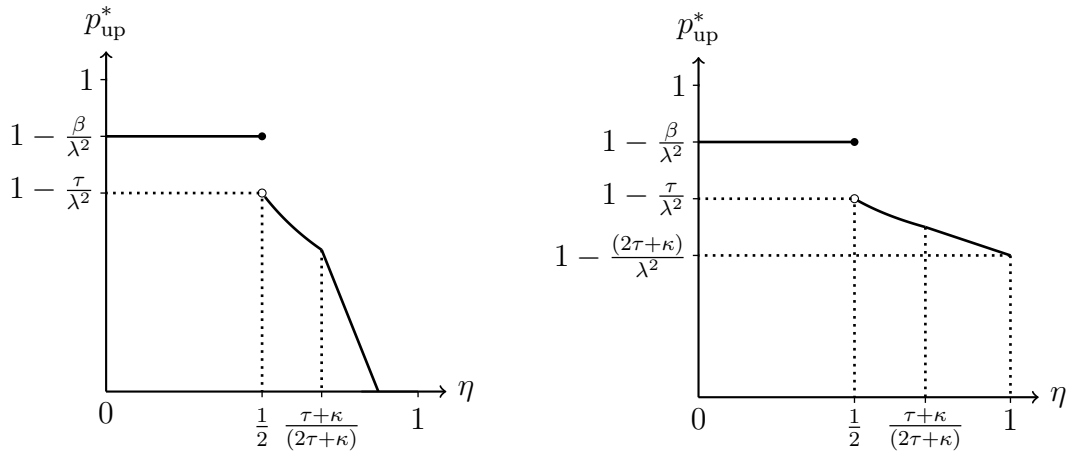


Figure 4.9: Policy p_{up}^* for $\beta < \tau < \tau + \kappa$, if $\lambda^2 < \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ (left) and if $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \lambda^2$ (right) in Case (i).

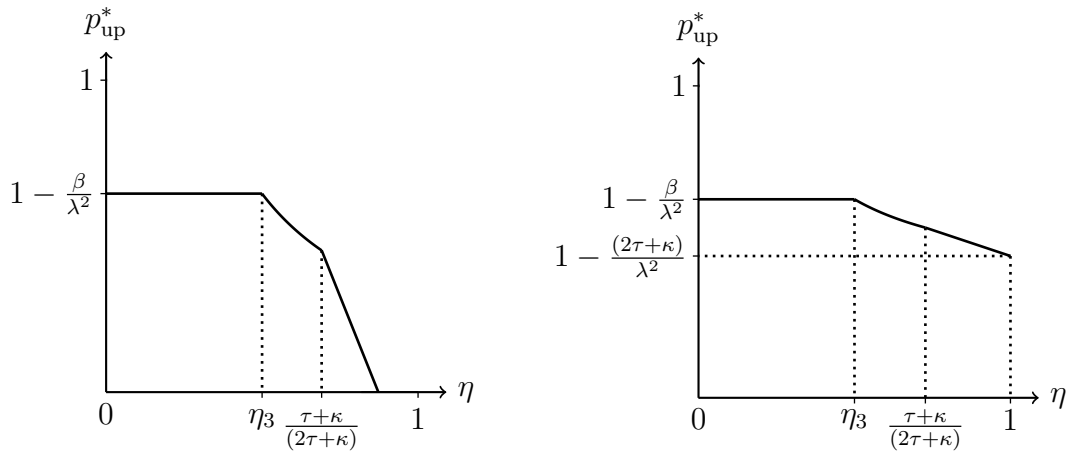


Figure 4.10: Policy p_{up}^* for $\tau < \beta < \tau + \kappa$, if $\lambda^2 < \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ (left) and if $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \lambda^2$ (right) with $\eta_3 = \frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2}$ in Case (ii).

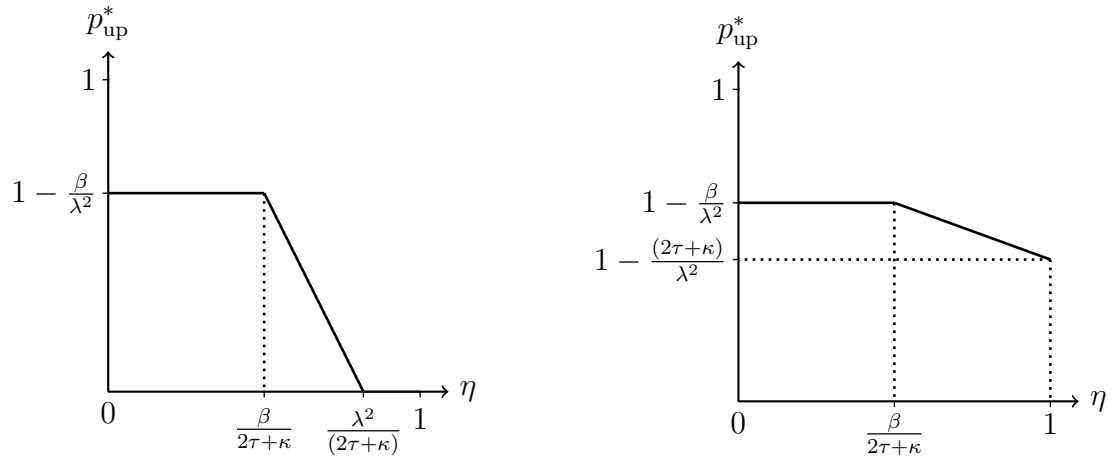


Figure 4.11: Policy p_{up}^* for $\tau + \kappa < \beta$, if $\lambda^2 < \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ (left) and if $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \lambda^2$ (right) in Case (iii).

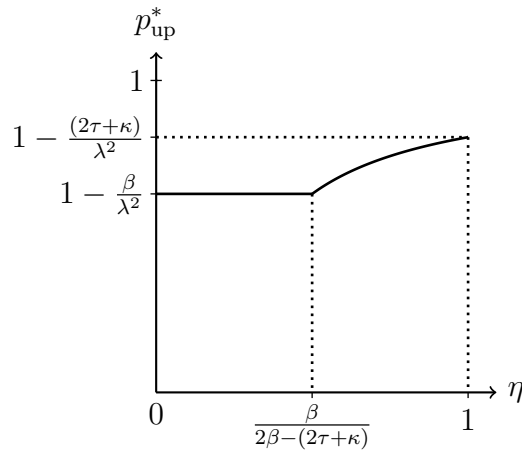


Figure 4.12: Policy p_{up}^* for $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta$ in Case (iv).

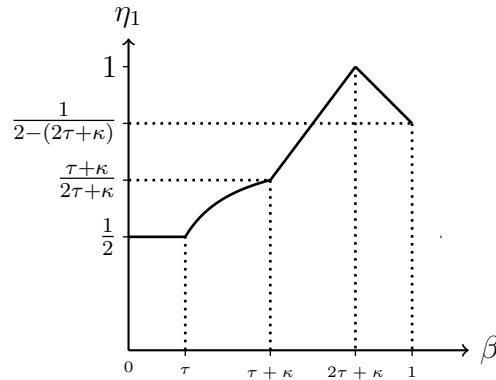


Figure 4.13: The threshold η_1 depending on β .

Figures 4.9, 4.10, 4.11 and 4.12 illustrate the policy p_{up}^* as a function of the share of non-opinionated voters η . First, consider Figures 4.9, 4.10 and 4.11 where $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} \geq \beta$. In the case of low uncertainty Opinion Updating can even lead to no action, given a large enough share of non-opinionated voters η . The necessary share of non-opinionated voters for this inaction depends on $\lambda^2 / (2\tau + \kappa)$. Hence, the lower (larger) the parameter λ (the average updated belief), the lower the threshold. This follows the intuition that lower uncertainty leads to lower policy. This might also happen on average, if non-opinionated voters have a belief that entails higher efficiency losses.

Second, Figure 4.12 illustrates the case where $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta$, as stated in Part (iv) of the proposition above. The policy is strictly positive, no matter the share of non-opinionated voters η . This means that there is always an executive action. Furthermore, the larger the share of non-opinionated voters η , the higher the policy p_{up}^* . This comes from the fact that a low ratio of expected belief and expected risk aversion either implies a belief entailing lower efficiency losses of the policy or a high risk aversion of non-opinionated voters. Hence, the more weight the non-opinionated voters have, the higher the policy chosen, to reduce uncertainty.

Similar to the case of Elections with Vote Delegation, the threshold of the share of non-opinionated voters η at which the monotonicity of the policy seen as a function depending on the share of non-opinionated voters η changes depends on the relation of β , τ and κ . Figure 4.13 shows this threshold η_1 , depending on the value

of the fixed ratio β . For a low value of β , the threshold is constantly $\frac{1}{2}$. This means that there is a discontinuity at $\eta = \frac{1}{2}$ when the policy is shown as a function of η . For higher values of β , i.e., $\beta > \tau$, the threshold increases up to the value of $2\tau + \kappa$, at which point the ratio of expected belief to expected risk aversion coincides for opinionated and non-opinionated voters. In this extreme case, the policy is independent of the share of non-opinionated voters η , which follows from the fact that all voters have the same ratio. For even higher values of β , i.e., $\beta > 2\tau + \kappa$, the threshold decreases. In general, one can observe that the more the ratios of expected belief and expected risk aversion are aligned between opinionated and non-opinionated voters, the higher the threshold η_1 .

4.5 Comparisons and Discussion

This section compares the policies chosen in the different forms of elections and discusses the influence of risk aversion on policy. The optimal policy serves as a benchmark. We focus on the case where the share of non-opinionated voters form the majority, i.e., $\eta \geq \frac{1}{2}$, as otherwise opinionated voters would dictate the policy.

4.5.1 Comparisons with Optimal Democracy

Elections without Experts and Optimal Democracy

Let us first consider Elections without Experts. The electorate consists of opinionated and non-opinionated voters. Opinionated voters shift the chosen policy and non-opinionated voters do not have the most accurate belief, either. Elections without Experts aggregate beliefs of all voters and are the best possible procedures from a democratic point of view. Comparing the policy in Elections without Experts with the optimal policy leads to the following proposition.

Proposition 8 Given $\frac{1}{2} \leq \eta \leq 1$,

(i) for $b^E < \frac{\beta}{2}$, $p_{opt}^* > p_{maj}^*$.

(ii) for $\frac{\beta}{2} \leq b^E \leq \frac{1}{2}$,

$$p_{opt}^* \begin{cases} \leq p_{maj}^* & \text{if } \frac{1}{2} \leq \eta \leq \frac{1}{2(1-b^E)}. \\ > p_{maj}^* & \text{if } \frac{1}{2(1-b^E)} < \eta \leq 1. \end{cases}$$

(iii) for $b^E > \frac{1}{2}$, $p_{opt}^* < p_{maj}^*$.

As stated in the above proposition, the policy chosen in Elections without Experts can be lower or higher than the optimal policy. First, if the expert has a belief that implies a less (more) costly policy, i.e., b_i is low (high), the policy chosen in regular elections is lower (higher) than the optimal policy. This comes from the fact that in Elections without Experts, the electorate overestimates (underestimates) the costs associated with the policy, which is better known by the expert. Second, for intermediate values of expert's belief b^E , i.e., $\frac{\beta}{2} \leq b^E \leq \frac{1}{2}$, the optimal policy can be reached. In other words, if the non-opinionated voters have a large enough majority the policy chosen coincides with the optimal policy. For a smaller (larger) share of non-opinionated voters, the optimal policy is lower (higher) than the one chosen in Elections without Experts. Non-opinionated voters shift the policy in a direction leading to a lower policy. Consider the polar case $b^E = \frac{\beta}{2}$. Then, the policies coincide for $\eta = \frac{1}{2-\beta}$. Hence, the higher the fixed ratio β implying a higher value of b^E , the higher the threshold η . In the other polar case $b^E = \frac{1}{2}$, the policies chosen in the two procedures coincide in the absence of opinionated voters, i.e., $\eta = 1$.

In general, the policy which is chosen in the absence of opinionated voters, i.e., $\eta = 1$, is closest to the optimal policy and equal to $1 - \frac{1}{\lambda^2}$.

Elections with Vote Delegation and Optimal Democracy

Next, we compare the optimal policy to the policy obtained in Elections with Vote Delegation. This form of voting allows to delegate the vote and, hence, to reach the most accurate belief about the efficiency of the policy. For the purpose of this comparison, we assume that the expert has an expected risk aversion which is lower

than the expected risk aversion from non-opinionated voters. In other words, we take $0 < \underline{\phi} < \bar{\phi} < \frac{1}{2}$. The intuition behind this assumption is that the expert has a greater knowledge on the voting topic and is better acquainted with the policy decision-making process. This leads to a more risk-seeking behavior. Comparing the policy chosen in Elections with Vote Delegation to the optimal policy leads to the following proposition.

Proposition 9 $p_{opt}^* > p_{del}^*$ for all $b^E \in [0, 1]$.

Proposition 9 states that the policy chosen in Elections with Vote Delegation is always lower than the optimal policy. This result is independent of the expert's belief. The intuition behind it is that if the expert votes instead of the non-opinionated voters, s/he chooses a lower policy due to a lower risk aversion, which follows from the assumption $0 < \underline{\phi} < \bar{\phi} < \frac{1}{2}$.

Elections with Opinion Updating and Optimal Democracy

In this section, we elaborate on the policy differences in Elections with Opinion Updating and in Optimal Democracy. In the case of Opinion Updating, the non-opinionated voters may include the expert's belief by a weight κ . This voting procedure allows the expert's belief to enter the voting decision through a change in the non-opinionated voters' beliefs. Furthermore, all voters are voting themselves and therefore represent the entire electorate. Since in this form of voting, the electorate still consists of opinionated and non-opinionated voters, the electorate does not fully adapt to the most accurate belief. A comparison of the results of Proposition 7 with the optimal policy is described in the following proposition.

Proposition 10 Given $\frac{1}{2} \leq \eta \leq 1$,

(i) If $b^E > \frac{1}{2}$, then $p_{opt}^* < p_{up}^*$ for $\frac{1}{2} \leq \eta \leq 1$.

(ii) If $b^E \leq \frac{1}{2}$ and

(a) $\beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$. First, if $b^E < \frac{\max\{\tau, \beta\}}{2}$, then

$$p_{opt}^* > p_{up}^* \text{ for } \frac{1}{2} \leq \eta \leq 1.$$

Second, if $\frac{\max\{\tau, \beta\}}{2} \leq b^E \leq \frac{1}{2}$ there exists a threshold $\eta_1 \in [\frac{1}{2}, 1]$, such that

$$p_{opt}^* \begin{cases} \leq p_{up}^*, & \text{if } \frac{1}{2} \leq \eta \leq \eta_1. \\ > p_{up}^*, & \text{if } \eta_1 < \eta \leq 1. \end{cases}$$

$$(b) \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta.$$

$$p_{opt}^* > p_{up}^*.$$

This proposition states the comparison for different values of β and b^E . First, consider the case where the expert has a belief that entails higher efficiency losses, i.e., $b^E > \frac{1}{2}$. Although in both procedures the expert's belief is included, Opinion Updating leads to a higher policy. This result is independent of the weight κ that is used by non-opinionated voters and of the share of non-opinionated voters η . Hence, the (potential) presence of opinionated voters and the voters' distribution of beliefs that differs from the expert's one lead to a higher policy than optimal.

Second, consider an expert's belief implying a less costly policy, i.e., $b^E \leq \frac{1}{2}$. In this case, the expert's belief is lower than the expected belief of the non-opinionated voters. Then, the results depend on the differences in the ratio of expected belief and expected risk aversion of the opinionated and non-opinionated voters. In the polar case $\beta = 0$, the opinionated voters have a low position compared to the non-opinionated voters. Then, the optimal policy can be reached in Elections with Opinion Updating if the expert's belief does not entail too great efficiency losses. In the polar case $\beta = 1$ the policy chosen in Elections with Opinion Updating is strictly lower than the optimal policy.

4.5.2 Discussion about risk aversion

In this section, we discuss the influence of the expert's risk aversion on the policy results. This matters only in the case of Elections with Vote Delegation, where the expert votes for the non-opinionated voters and has a different risk aversion than the voters themselves. In the first step, we establish the influence of very low risk aversion on the chosen policy p_{del}^* . Then, we elaborate on the influence of the expert's risk aversion with regard to the optimal policy.

First, consider the case where the ratio of b^E and expected risk aversion of

the expert is higher than the fixed ratio β of the opinionated voters. Then, in Elections with Vote Delegation, the larger the share of non-opinionated voters η , the lower the policy p_{del}^* . The expert's risk aversion influences the threshold η_1 , such that for all $\eta \geq \eta_1$ the chosen policy p_{del}^* is zero. The lower the expert's risk aversion, the lower the threshold η_1 . Hence, an expert with low risk aversion leads to the case where the policy already becomes zero for a small majority of non-opinionated voters. Next, consider the case where the ratio of b^E and expected risk aversion of the expert for the non-opinionated voters is lower than the ratio β of the opinionated voters. Even then, the larger the share of non-opinionated voters η , the lower the policy p_{del}^* . Further, the lower the expert's risk aversion, the lower the chosen policy. For example, in the case where the whole electorate solely consists of non-opinionated voters, i.e. $\eta = 1$, the policy chosen is $p_{\text{del}}^* = 1 - \frac{2b^E}{\lambda^2(\underline{\phi} + \bar{\phi})}$. This is the highest possible policy chosen and it is lower for lower values of $\underline{\phi}$ and $\bar{\phi}$.

Second, consider the comparison of the optimal policy with the policy p_{del}^* , where we assumed $0 < \underline{\phi} < \bar{\phi} < \frac{1}{2}$. As seen in Proposition 9, an expert with low risk aversion leads to a policy p_{del}^* , which is lower than the optimal policy, no matter the share of non-opinionated voters η or the expert's belief. In this case, the higher $\underline{\phi} + \bar{\phi}$, the more the policy p_{del}^* is aligned with the optimal policy. In the other extreme case of high risk aversion, i.e., $\frac{1}{2} < \underline{\phi} < \bar{\phi} \leq 1$, we can show that the opposite is true, as stated in the following proposition.

Proposition 11 *For high risk aversion, i.e., $\frac{1}{2} < \underline{\phi} < \bar{\phi} < 1$, it follows that $p_{\text{opt}}^* < p_{\text{del}}^*$ for all $0 \leq \eta \leq 1$ and $b^E \in [0, 1]$.*

If the expert has a high risk aversion, then the policy chosen in Elections with Vote Delegation is higher than the optimal policy. This comes from the fact that the expert votes for the non-opinionated voters, who form a majority in the electorate. Since the expert is highly risk-averse, s/he chooses a higher policy to reduce uncertainty.

4.6 Special Cases

In this section, we consider two special cases. First, we assume that the electorate does not benefit from the expert's opinion. This means that the average belief of

the non-opinionated voters coincides with the expert's belief. The policy chosen by the different voting procedures under this assumption are stated in the following corollary:

Corollary 9 *If $b^E = \frac{1}{2}$, then for a sufficiently high $\eta \gg \frac{1}{2}$, it follows that*

$$0 \approx p_{del}^* < p_{opt}^* \leq p_{maj}^* \approx p_{up}^*$$

and the expert's belief b^E does not influence the policy chosen by non-opinionated voters in Elections with Opinion Updating.

As stated in Corollary 9, Elections with Vote Delegation yield a very low policy due to the expert's low risk aversion, compared to the non-opinionated voters'. This can be detrimental for society, as the optimal policy is strictly higher. Although majority voting without experts does not always lead to the optimal policy, it can still achieve it. For a large share of non-opinionated voters, i.e., if η is close to 1, the policies chosen in Elections without Experts and in Elections with Opinion Updating coincide. These results imply that in the case where the non-opinionated voters have the most accurate belief in expectation, the expert has no additional benefit to the society. It can be even risky to include the expert in the decision-making process in the case of Vote Delegation, as the uncertainty is not taken sufficiently into account, due to a low risk aversion.

Second, consider the case where the expert's belief b^E entails higher efficiency losses of the policy. In this setting, the belief of the non-opinionated voters is lower than the experts'. The relation between the policies chosen under this assumption is stated in the following corollary:

Corollary 10 *If $b^E > \frac{1}{2}$, then*

$$0 \approx p_{del}^* < p_{opt}^* \leq p_{up}^* < p_{maj}^*.$$

As stated in Corollary 10, Elections with Vote Delegation lead to a policy that is almost equal to zero, due to the assumption that the expert's risk aversion is lower than the one of non-opinionated voters. By contrast, regular elections yield the highest policy among the four procedures. The reason is that in this voting

procedure, the expert's belief does not enter the voting decision, so that the non-opinionated voters do not take into account the (true) costs of choosing higher policies. For its part, Elections with Opinion Updating lead to a policy which is higher than the optimal policy, but still lower than the one obtained in Elections without Experts. The intuition behind this result is that including the best possible assessment, i.e., the expert's belief, can lead to a policy which is closer to the optimal policy, although it is only partially included by weight κ . Hence, Opinion Updating is beneficial to society from a welfare perspective, as it can lead to the optimal policy or at least is closer to it than the policy chosen in Elections without Experts.

The two examples stated in Corollary 9 and Corollary 10 show that the inclusion of experts must be pondered carefully before implementation, as it is not always beneficial for society.

4.7 Extensions

Many extensions and generalizations of our model and our voting procedures are possible. Examples of extensions can be found in the Appendix C. The first part deals with a generalization of elections with Opinion Updating, where the representative expert has the same risk aversion as the voters. The second part considers one approach of a welfare analysis. We consider the utilitarian welfare as well as a convex combination of the utilitarian and the maximin welfare to compare the voting procedures with regard to welfare.

4.8 Conclusion

We have presented a model where voters are heterogeneous regarding their belief about the efficiency of a policy and regarding risk aversion. The model has allowed us to include experts' beliefs, which are modeled as a consensus and are represented by a representative expert. We have compared and discussed four different voting procedures. Our main insight is that the inclusion of experts may lead to a policy which is more optimal than the policy chosen without experts. Furthermore, Opinion Updating has the same relation between the policies chosen and the share

of non-opinionated voters as voting without experts, but can even lead to a policy closer to the optimal policy. Considering two special cases has showed that including experts is not always beneficial to society.

Our analysis elaborates on the consequences of the inclusion of experts' beliefs and assesses the (dis-)advantages of the different voting procedures. The results may have broader implications for democratic policy decisions in the future.

Chapter 5

Newspaper Coverage in Uncertain Times: Capturing Anxiety through Word Embeddings*

Abstract

Newspapers can play a crucial role in shaping society's emotions in times of high uncertainty regarding the near future. This chapter develops a new time series measure of anxiety addressed perceptibly in online newspaper coverage. It is derived from articles published over two years. All articles were published in German during the Covid-19 crisis in Austria and Switzerland. The findings highlight the gains from combining existing lexica and word embeddings. To validate our methodology, we assess our results qualitatively and against other indicators of the Covid-19 crisis as well as against survey-based measures of anxiety. Intuitively, anxiety spikes at the time of the first infections and generally decreases afterwards. Comparing Austria and Switzerland, we find that the time series differ with regard to variance. The two time series correlate with infection rates in each country and are influenced significantly by a few prevailing media topics.

*This chapter is single-authored work.

5.1 Introduction

News can have a significant influence on a society's perception of the most salient current issues. Besides factual information obtained from exposure to news, readers also learn about the importance of topics from the framing in the media (McCombs and Reynolds, 2002; Sigillo and Sicafuse, 2015). How themes related to anxiety are depicted in the media can play a crucial role in times when short-term policies are highly uncertain. For example, during a pandemic, the society is affected in many dimensions, like economics or health, and policies and their implementation may vary at a very short notice. This chapter focuses on the Covid-19 pandemic as a case study analyzing the framing in the media with regard to anxiety in uncertain times. Providing information as to how the public is informed through newspapers can help to understand perceptions or behaviors of individuals and the framing of crises.

The Covid-19 pandemic has affected nearly every country in the world. Diverse countermeasures were taken, ranging from lockdowns to school closures, in the years 2020 and 2021, with the aim of reducing the spread of the virus. During this crisis, each country has been challenged with choosing a policy which is complied with the citizenry. The strategies have varied widely in Western democratic states. In the neighbouring dominantly German-speaking countries—Austria and Switzerland—the approaches have differed strongly so far, both in intensity and as to time frames. Despite a similar demographic, geographical and economical background, Austria imposed very strict restrictions during four lockdowns, whereas Switzerland chose a less restrictive approach. Comparing these two neighbouring countries and their opposing strategies can shed light on the main drivers of anxiety, such as policy changes or infection rates.

During this evolving situation, media coverage played an important role, its key goal being to inform the public. Online newspapers helped to stay up-to-date with the infection rates, with rapidly changing measures taken by the government, with restrictions implemented in other countries and with constantly updated health messages from the authorities. This led to an increased demand for reliable information (Dreisiebner et al., 2022). As key source of information and reflection of the public's sentiment towards health policies, online newspapers provided an ideal

data source for the analysis. Recently, it was shown that the media coverage of the pandemic has significant effects on people’s beliefs about its origins, on opinions about appropriate policy responses, and on overall politicization of the crisis (Hart et al., 2020; Bolsen et al., 2020). How nation-wide online newspapers have reflected the Covid-19 pandemic in Austria and Switzerland will thus be the subject of this chapter.

The chapter develops a new approach to measure anxiety in texts written in German. We extend the anxiety-related keyword lists developed by linguistic psychologists by using word embeddings. Around 130,000 Austrian and Swiss online newspaper articles were studied to adapt the list to Covid-19 related texts. The anxiety index is calculated as average share of anxiety-related keywords in articles. We validate it qualitatively and compare it to other anxiety-related indices. To assess our methodology, we include several validation techniques following the standard literature using text data (Quinn et al., 2010; Grimmer and Stewart, 2013). We show that our method only comprises anxiety-related keywords and is able to capture anxiety in Covid-19 related contexts. In addition, positive correlation with other indices reveals more clearly that the anxiety index can be seen as a good proxy for anxiety addressed perceptibly in online newspapers. A Latent Dirichlet Allocation (LDA) algorithm is used to model the prevailing topics depicted in online newspapers. Further, anxiety across different topics is calculated to analyze the main drivers for anxiety in the coverage.

As a main result, we found that the anxiety index reflects major changes in policies in the course of the pandemic. The highest peak can be observed around the time of the first infections in both countries, following a general decrease. Comparing the two countries, we notice that the time series varies more in Austria than in Switzerland, without any significant difference with regard to absolute values. Further, our analysis shows that anxiety is driven by a few main topics and that it is higher in broadsheet newspapers than in tabloids.

The chapter contributes to different strands of research. First, the methodology extends typical dictionary-based methods analyzing Covid-19-related (social) media content (Pellert et al., 2020). Second, it aligns with the literature evaluating how newspaper texts reflect certain attitudes of society, as political power (Ban et al., 2019), for instance, or sentiments (Shapiro et al., 2022). Finally, we add to a large

literature exploring word embeddings (Garg et al., 2018; Ash et al., 2021b).

5.1.1 Related literature

Our work is connected to several strands of literature.

Influence of news on society

The influence of media on society is the subject of many empirical and theoretical papers. McCombs and Reynolds (2002) show that more media exposure implies an increased consensus as to which are the most important issues within a society. Another work by Sigillo and Sicafuse (2015) evaluates how the community sentiment and the media affect policy-making. Building on these results, this chapter contributes to this strand of research by analyzing how the media reflect the Covid-19 pandemic and therefore, how they may affect society in uncertain times.

Analysis of social media content during the Covid-19 pandemic

Another strand of literature evaluates the (social) media content during the Covid-19 pandemic with regard to sentiments and topics. A recent paper by Amann et al. (2021) analyzes the Austrian, German and Swiss media coverage regarding contact tracing apps during the pandemic. They analyze the same daily newspapers as we do and discuss core issues of the perception of tracing apps in the Austrian, German and Swiss media coverage. Pellert et al. (2020) conduct a sentiment analysis of user posts on the website of derStandard.at, which is an Austrian online newspaper, on Twitter, and in a chat platform for students. Since the beginning of the pandemic in early 2020, this interactive platform shows different sentiments, based on a manually-adapted LIWC vocabulary. Another study by Eisele et al. (2022) focuses on social media and on newspaper content during the Covid-19 pandemic. By analyzing around 38,000 newspaper articles and around 1.6 million user comments with a dictionary approach, they can show an increase in emotionality during lockdowns and a shift in favor of the government. Furthermore, a large literature investigates tweets and social media posts in the course of the pandemic with regard to sentiments and topics (see e.g. Samuel et al., 2020; Kruspe et al., 2020; Mellado

et al., 2021). This chapter builds on the same data sources and dictionary methods, but focuses on the *newspaper articles* instead of social media content, and it extends the dictionary-based method.

Index measuring sentiments and emotionality

How to establish indices by accounting for specific keywords in texts is the subject of many scientific papers. One area of application is the evolution of economical factors and indicators (Handley and Li, 2020). Shapiro et al. (2022) develop a time series measure of economic sentiment derived from US newspapers. They use a sentiment-scoring model based on a new lexicon built specifically to capture the sentiment in economic news. They also show that the news sentiment index can predict movements of survey-based measures of consumer sentiment and can be used to estimate the impulse responses of macroeconomic variables. This index was also applied to the newspaper coverage during the Covid-19 pandemic¹. Baker et al. (2016) use a dictionary approach to establish an index for economic policy uncertainty.

Several papers extend diverse dictionary approaches by using word embeddings to analyze sentiments or other emotional factors in news coverage. Gennaro and Ash (2022) study emotion and reason in political language. The authors measure emotionality in six million speeches given in US Congress based on categorized dictionaries and word embeddings. A related work by Chakraborty and Bose (2020) analyzes the general sentiment of news articles during the Covid-19 pandemic with unsupervised learning based on the AFINN lexicon and a Naive Bayes-based transfer learning approach, which has been trained on a popular movie reviews data set. The chapter statistically determines how and after which delay the number of affected patients and the number of deaths due to Covid-19, impact the news sentiment in regional and world-wide news. Furthermore, the authors elaborate on other relevant factors that contribute to the rise or fall of global news sentiment related to particular countries. The anxiety index in this chapter applies methods similar to others such as word embeddings but was specifically developed for the German language and tailored to the Covid-19 pandemic.

¹<https://www.frbsf.org/economic-research/publications/economic-letter/2020/april/news-sentiment-time-of-covid-19/> (accessed on May 1, 2022).

5.2 Methods

This section gives an overview of the data and methodological framework of this chapter. Figure 5.1 illustrates the approach. First, the data was collected and preprocessed. In a next step, the anxiety index was, first, calculated based on the corpus consisting of articles of five online newspapers and, second, validated qualitatively as well as with other anxiety-related indices. Then, a topic analysis was conducted to analyze the anxiety distribution across topics.

5.2.1 Data collection and preprocessing

The database consists of two Austrian and three Swiss prominent online newspapers with nation-wide reach with differing readership. A crawling system extracted the articles from the Austrian online newspaper websites. The data of the Swiss online newspapers was received from the Swiss media database [swissdox.ch](https://www.swissdox.ch). To achieve widest possible coverage, the two keywords, “Covid-19” and “Corona”, were searched for in the title, abstract and content of the articles. Exclusively online newspaper articles published in the time frame from 1st January 2020 to 31st December 2021 and published in German were considered. The emphasis lies on official articles of these online newspapers, which means that any kind of podcast, live article (updated regularly in a given time period) or general overview article without any specific date were excluded. Under these restrictions, around 130,000 articles entered the analysis.

For each country, one tabloid and one to two broadsheet newspapers were chosen. Broadsheet newspapers produce more political news and are less accessible to the broader public due to their more in-depth, serious style (Bek, 2004; Reinemann et al., 2012; Jandura and Friedrich, 2014).

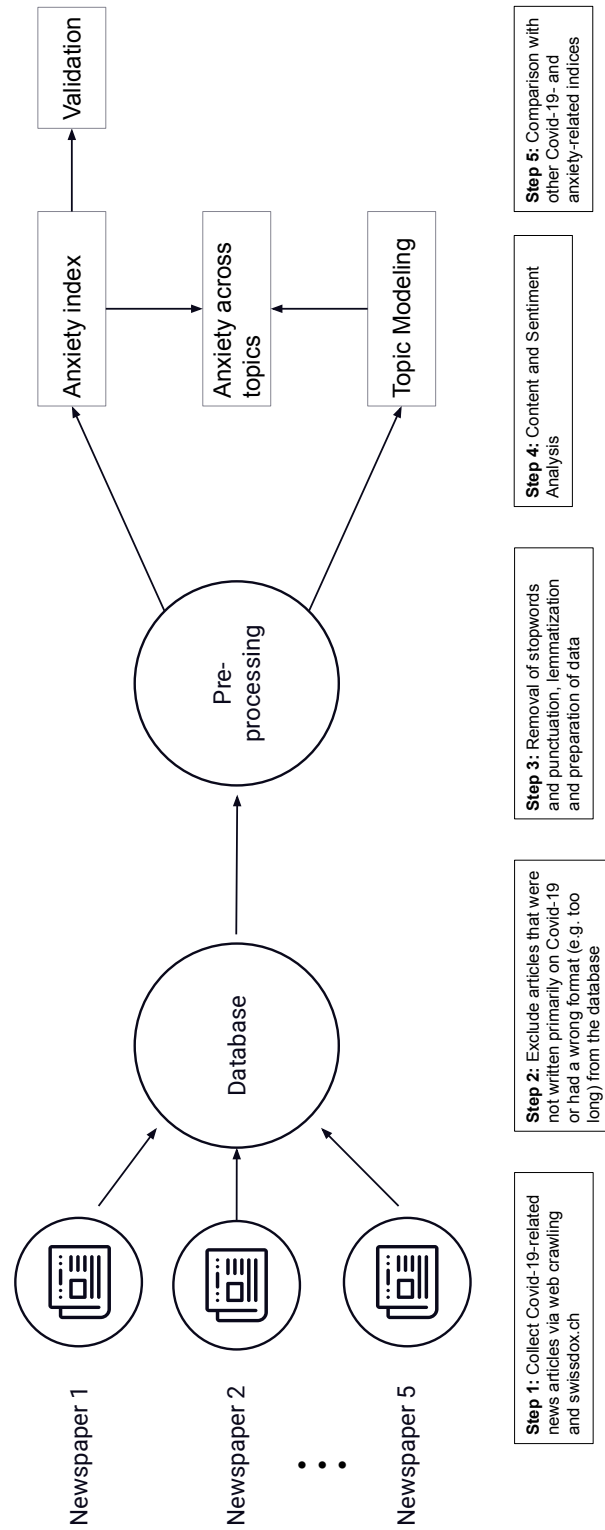


Figure 5.1: Methodological framework.

Media Overview						
Magazine	Online Reach	Number of articles	Average article length [words]	Median article length [words]	Min article length [words]	Max article length [words]
derstandard.at	2,546M	31,383	586	515	100	6,382
oe24.at	2,633M	30,727	325	267	50	3,620
nzz.ch	3M	21,324	1,210	918	100	6,947
blick.ch	2,809M	28,474	432	368	63	4,813
tagesanzeiger.ch	1,984M	17,879	772	679	80	6,304
All		129,787	618	470	50	6,947

Figure 5.2: Descriptive statistics of newspaper data per source.

In Figure 5.2, one can see an overview of the newspapers used. The Austrian online newspapers considered, *oe24.at* and *derstandard.at*, published approximately the same amount of articles together as the three Swiss online newspapers, *nzz.ch*, *bild.ch* and *der tagesanzeiger.ch*. The online reach, measured in million unique users per month², the number of articles, the average word count as well as the median, minimal and maximal article length in words are stated in Figure 5.2. The distribution of the articles in the different newspapers is not even. Around 30,000 articles were published in each of the Austrian and in one Swiss newspaper and only around 20,000 in the other two Swiss news portals, *bild.ch* and *der tagesanzeiger.ch*. Furthermore, the average length of an article published in broadsheet newspapers, *nzz.ch*, *der tagesanzeiger.ch* and *derstandard.at*, is notably higher than in the two tabloids, *bild.ch* and *oe24.at*.

For each article the title, abstract and content were merged for analysis. Then, all texts were tokenized, the punctuation and non-alphabetic expressions were removed, and all words were converted to lower case. Tokens are lemmatized using spaCy (Honnibal and Montani, 2017) and German NLTK (Bird et al., 2009) stop-words were removed.

5.2.2 Anxiety index

The anxiety index is based on word lists and word embeddings. The list of 345 anxiety-related words including 95 word stems is taken from the German Version

²More information on the online reach can be found in Appendix D.

of the Linguistic Inquiry and Word Count (LIWC) developed by linguistic psychologists (Meier et al., 2019; Pennebaker et al., 2015). One major issue in this regard is that the generic LIWC word list is not adapted to Covid-19-related texts. Hence, we extended the list of keywords based on the given corpus of 130,000 articles using word embeddings.

Word embeddings aim to represent the meaning of a word by its neighbouring words. Word embedding algorithms, like Word2Vec (Mikolov et al., 2013) and GloVe (Pennington et al., 2014), represent words by vectors. Hence, a word vector \mathbf{w} corresponds to a word w . These word vectors have a spatial interpretation, meaning that geometric distances between word vectors reflect semantic relations between words. This implies that vectors representing words that appear in similar contexts have a high cosine similarity. To be more precise, consider two word vectors \mathbf{w}_1 and \mathbf{w}_2 . Then, the more similar their semantic meaning, the higher the cosine similarity, i.e.,

$$\text{sim}(w_1, w_2) = \frac{\mathbf{w}_1 \cdot \mathbf{w}_2}{\|\mathbf{w}_1\| \cdot \|\mathbf{w}_2\|}$$

between vectors \mathbf{w}_1 and \mathbf{w}_2 . For example, the word vectors of the words *joy* and *happy* would have a higher cosine similarity than the word vectors of the words *joy* and *anger*.

The Word2Vec algorithm (Mikolov et al., 2013) is applied to the whole corpus, using the python gensim implementation, 300 dimensional vectors, an eight-word window and five epochs for training. The parameters chosen are similar to parameters used in related literature and a change would not lead to significant differences regarding the results obtained (Ash et al., 2021a; Rodriguez and Spirling, 2022; Gennaro and Ash, 2022).

Then, the list of anxiety-related keywords is extended with all words that are part of the 100 most similar words, based on the cosine similarity of the different word vectors, in at least 10% of all keywords.

In the next steps, the anxiety index i_t is established for week t . First, the relative frequency in each article p of anxiety-related keywords of the extended keyword list is calculated. We denote a keyword by k_i and the extended list of N_L keywords by L , i.e., $k_i \in L$ for all $i \in \{1, \dots, N_L\}$. Then, for each article p consisting of N_p words, where each word is denoted by w_i for $i \in \{1, \dots, N_p\}$, the relative frequency

is obtained through $f_p = \frac{\sum_i 1_{w_i \in L}}{N_p}$. Then, the average of the relative frequencies of all articles p published per week t is taken for each newspaper n . Hence, the anxiety index i_t is based on the weekly average relative frequency of anxiety-related keywords.

One difficulty with this ratio is the varying volume across newspapers and countries. This problem is solved by a standardization process similar to Baker et al. (2016). It works as follows: The time series of each newspaper is scaled by the total number of articles in the same online newspaper and month. Denote the scaled frequency counts for online newspaper n in month t by X_{nt} for one country. Then, each monthly newspaper series is standardized to unit standard deviation over the two years 2020 and 2021. To be precise, first, the time series variance σ_n^2 over the two years is calculated for each online newspaper n . Second, X_{nt} is standardized by dividing through the standard deviation σ_n for all t , i.e., $Y_{nt} := \frac{X_{nt}}{\sigma_n}$ for all t . This leads to a series Y_{nt} with unit standard deviation for each online newspaper. Then, the mean over all online newspapers of Y_{nt} in each month is calculated, yielding the series Z_t . Finally, the time series for each country is normalized to a mean of 1. This is done by calculating the mean of Z_t , denoted by M , and multiplying Z_t by $\frac{1}{M}$. This procedure leads to the normalized time series anxiety index and it is applied for each country.

5.2.3 Validation of the anxiety index

To assess whether the index captures the dynamics of the pandemic and can be used as a proxy for the anxiety level of communication through newspapers, some additional comparisons are taken into account. Several validation steps are conducted, following standard literature analyzing text as data (Grimmer and Stewart, 2013; Gennaro and Ash, 2022). First, a qualitative analysis provides evidence that the index can indeed capture anxiety-related content. Second, our results are compared to other indicators from a survey and to Covid-19-related indices. In each case, we find strong support for the validity of the anxiety index.

	t (no shift)	$t + 1$ (1-wave shift)	$t + 2$ (2-wave shift)	$t + 3$ (3-wave shift)
ρ_t	-0.013986	0.3454545	0.781818	0.633333
p -value	0.965590	0.298089	0.007547	0.067086

Table 5.1: Spearman’s rank correlation coefficients for different shifts in time.

taken from the official websites: AGES Dashboard COVID-19 for Austria, resp. Bundesamt für Gesundheit BAG for Switzerland. Since the index is a ratio of anxiety-related keywords in each article and as infection rates are given in absolute values, the relative change in infection rates is used for comparison. The anxiety index correlates moderately positively with the relative change in infection rates in Austria resp. Switzerland with a Pearson correlation coefficient of 0.27 resp. 0.36.

Second, a mental health study in Austria is considered. In 2020, a cross-sectional survey about mental well-being in twelve waves, meaning at twelve different points in time, with around 12,000 participants was conducted (Niederkrotenthaler et al., 2022). The questions addressed were suicidal ideation, depressive symptoms, anxiety and domestic violence. For anxiety the Hospital Anxiety and Depression Scale (HADS) (Zigmond and Snaith, 1983) was used, where the participants were asked to state their current anxiety sentiment compared to pre Covid-19 times on a scale from 1 (less often, weaker) to 5 (more often, stronger). The time series based on the aggregated data was standardized to unit deviation, averaged across each wave, and normalized to a mean of 1. As the assumption that the relation between the survey results and the anxiety is linear is inaccurate, Spearman’s rank correlation coefficient is taken to test for a monotonic relation. Similar to Chakraborty and Bose (2020), the correlation coefficient and the associated p -value are considered for a time lag of up to 3 periods. The intuition is that anxiety-related news can take time to affect and to be perceived by society, and hence to be reflected in the survey data. Table 5.1 states the different correlation coefficients for the different shifts in time. As one can see, there is a strong positive correlation for a shift of two waves in the time series with a p -value lower than 0.01. This means that comparing the survey results with the anxiety index from two waves earlier leads to a strong positive correlation. The finding suggests that it takes time for the

List of topics		
	Topic name	keywords
1	USA	trump, polizei, video, biden, twitter, new york, facebook, teilnehmer
2	Austria and beliefs	wiener, standard, wirklich, lesen, glauben, mutter, denken, vielleicht, angst
3	Covid in Austria	patient, variante, neuinfektionen, tirol, studie, intensivstation, infizieren, erkrankung, bezirk, sars
4	Business	milliarde, dollar, vorjahr, online, heuer, kunde, preis, handel, kurzarbeit, geschäft
5	Soccer	team, spieler, fan, fussball, saison, salzburg, league, sieg, star, trainer
6	Vaccination	impfstoff, italien, biontech, impfungen, pfizer, johnson, dose, grossbritannien, china, who
7	Policies: Decision and criticism	spö, wiener, maskenpflicht, grüner, grüne, gesundheitsminister, bundesregierung, kritik, entscheidung, partei
8	Policies: Implications	övp, schule, pcr, öffnen, negativ, schüler, gast, einreise, gastronomie, veranstaltung

Table 5.2: List of topics, topic names and associated keywords.

anxiety communicated through online newspapers to be perceived and reflected in the survey.

5.2.4 Topic modeling

To find the distribution of topics across articles, an unsupervised topic model is applied to the entire corpus. Topics models are useful to reduce the dimension of high-dimensional data and to generate topics based on word frequency. It thus allows to summarize unstructured texts. The Latent Dirichlet Allocation (LDA) (Blei, 2012), a method allowing to discover interpretative patterns or themes in texts, is used. Each topic is represented by a distribution over words and each article by a distribution over topics.

The LDA algorithm is applied to the full pre-processed corpus of all Austrian and Swiss newspaper articles. The number of topics, namely eight, is chosen based on the coherence score of the LDA Mallet model (McCallum, 2002). To evaluate the optimal number of topics, the LDA Mallet model was run with different numbers of

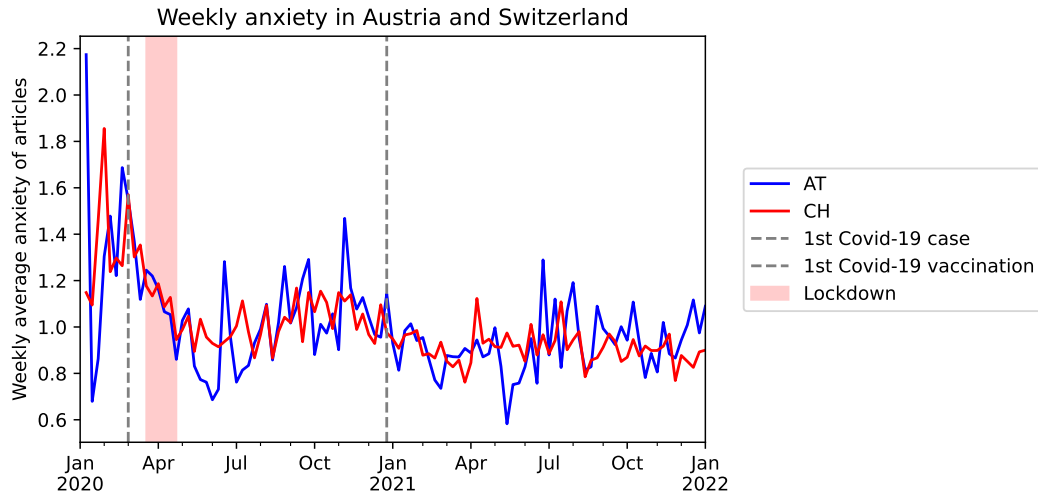


Figure 5.4: Anxiety index for each country in the years 2020 and 2021.

topics, from 2 to 18, and the model with the highest coherence score was selected. Table 5.2⁴ gives an overview of all eight topics, a chosen topic name and its associated keywords. More than half of the topics refer to Covid-19 and its implications. The first topic *USA*, covers news from the US, including the presidential elections and the Black Lives Matter movement. The remaining topics mainly stemmed from business and football.

To calculate anxiety across topics, the distribution of topics is considered for each article and used as weight for the anxiety index value. Then, for each topic, the weighted anxiety index values are aggregated.

5.3 Results

This section shows how the anxiety index varies over time in the two countries and across topics.

⁴A translation of the words in English can be found in Appendix D.

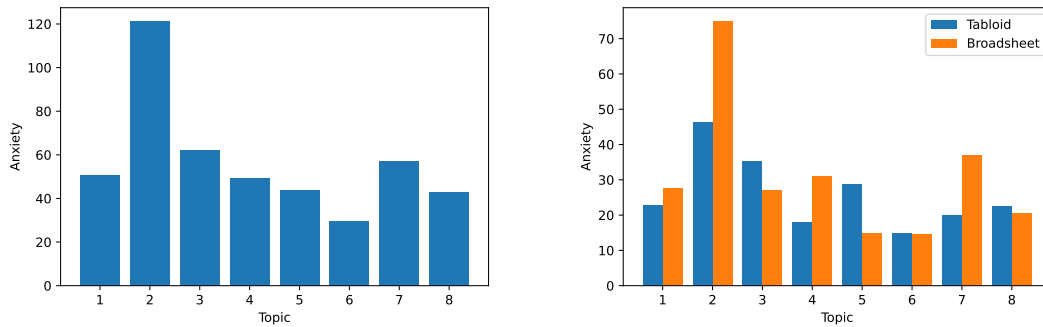


Figure 5.5: Aggregated anxiety related to each topic (left) and each newspaper type (right).

5.3.1 Anxiety over time

The main result, the anxiety index over time for each country, is shown in Figure 5.4. Besides the two indices, the plot shows the first Covid-19 case, the first Covid-19 vaccination and the (first) lockdown, which happened at the same time in both countries. For simplicity, the other three lockdowns which took place in Austria over the two years are not depicted in the graph. In general, one observes that both indices decrease over the time span, after being highest at the beginning of 2020. The intuition behind this observation is that more and more information was gained on Covid-19 and hence, more was known about its risks, which led to a decrease in anxiety. Also, both indices decrease dramatically during the first lockdown and after the first Covid-19 vaccination. Furthermore, the index for the Austrian newspapers shows a higher variation than the Swiss index. This may be due to the fact that Austria had more changes in Covid-19-related policies than Switzerland (Desson et al., 2020). For example, Switzerland had one lockdown, whereas Austria had four from 2020 to 2021.

5.3.2 Anxiety across topics

In a next step, the variation of anxiety across topics is discussed. Topics covered in newspapers may have a significant influence on the anxiety level depicted. The temporal fluctuation of Covid-19-related key themes presented in newspaper articles and its relation to the anxiety index allow more insights about the main drivers

of anxiety in the newspapers. Over the two years examined, different topics prevailed in the media. Figure 5.5 shows the aggregated anxiety per topic on the left side and per topic and type of newspaper on the right side. The topics showing the highest value of anxiety are: Austria and beliefs (Topic 2), Covid-19 in Austria (Topic 3) and Policies: Decision and criticism (Topic 7). The anxiety level is higher for emotional topics, implying that the anxiety index can be seen as a valid indicator regarding anxiety. Interestingly, as shown on the right side of Figure 5.5, broadsheet newspapers depict a higher level of anxiety than tabloids. One possible explanation for this dynamic might be that in both countries, broadsheet newspapers were mainly responsible for the communication of new policies, infection rates and politicians' statements. These also included anxiety-related information.

5.4 Discussion and Conclusion

We have developed a new measure of anxiety for the German language aimed at capturing perceptible context-based anxiety addressed in online newspaper coverage. Therefore, a dictionary-based approach was extended by using word embeddings. The contribution of this chapter is to show how newspapers reflected the course of the Covid-19 pandemic and the anxiety of society. In addition, we allow a better understanding which topics may have been the major sources of anxiety.

Validation over time, across countries and topics confirms the validity of the anxiety index. Nevertheless, this index should be complemented and extended by other methods given some of its limitations. Although dictionary-based indices to analyze texts regarding emotions or semantic content are a well-known standard in literature (see e.g. Baker et al., 2016; Ban et al., 2019), one major limitation of dictionary-based approaches is that the index heavily depends on some specific keywords. Hence, the dictionary needs to be carefully adapted to each research issue. Furthermore, dictionary-based approaches cannot take into account specifics such as negation or other words in a text stating the importance or weight of words. For example, the two phrases “feeling anxious” and “feeling a bit anxious” would have the same weight according to this method. The use of “a bit anxious” as euphemism for “extremely anxious” cannot be reflected, either.

Addressing certain limitations of this proposed framework in more detail could

include using a larger corpus and adapting the method to other languages, with the aim to test its robustness. Another direction of exploration could be using different and longer keyword lists and adding a benchmark calculated on the basis of newspaper articles published before 2020.

Yet, despite potential limitations, the validated method described in this chapter could also be applied to analyze other domains of news, like transcripts or social media channels. For example, the analysis could be applied to transcripts of political statements made during the Covid-19 pandemic to develop or blueprint for further text analysis in other, new times of crisis.

Chapter 6

Conclusion and Outlook

This thesis presents extensive answers to the key research questions on the design of political institutions stated in the introductory chapter. First, we studied stable levels of checks and balances that are chosen in political institutions. Our results suggest that higher majority requirements for constitutional changes and more polarized societies lead to more stable checks and balances. Further, we showed that democracies choose different levels of checks and balances, where most of them are not stable, which is consistent with empirical observations. Second, we found that preference shocks lead to more extreme policies in the presence of costs of change. However, intermediate costs of change and a higher turnover probability can counteract and hence, induce more moderate policies. Third, we assessed different policy decision-making procedures with experts for choosing credence policies. Among other findings, we came to the conclusion that vote delegation to experts can lead to extreme or even detrimental policy outcomes, as experts are not anticipating the electorate's risk aversion. To conclude, an empirical analysis considered a case study to elaborate on the perception of credence policies.

In the first two chapters, we extended a model of political competition with costs of change. Based on this model, many avenues for future research can be pursued. For a better comparison with chosen levels of checks and balances in different democracies worldwide, one could allow for uncertainty about the consequences of policies. This setting may benefit from more stringent rules for checks and balances and may be a rationale for greater weight allocated to checks and balances in policy

decisions. As to the extension with preference shocks in the presence of costs of change, one might combine the two extensions to analyze the policy choices with shocks when costs of change are endogenously determined.

Chapter 4 and 5 analyze credence policies from a theoretical and an empirical angle. These analysis open up other research opportunities. For instance, the theoretical model would allow to include more decision-making processes, like participatory democracy and to assess how it might improve policy-making in the presence of credence policies. To improve the theoretical approach, one could study surveys and apply different distributions of the electorate's risk aversion and beliefs. This way to continue this strand of research would complicate the analysis, but make the calculations more applicable to real-world examples. The empirical analysis could be enriched by considering more texts and more countries, as well as by using various lexica of emotions and by training the methods on texts written in pre-pandemic times.

Chapter A

Appendix to Chapter 2

A.1 List of Countries for Figure 2.3

We use the discrete variables CHECKS and POLARIZ from the Database of Political Institutions [2020], available from Inter-American Development Bank, Cruz et al. (2021). The list was chosen based on the available data. Since we want to depict a picture of a large set of democratic countries regarding the relation of C&B to polarization, we decide to use the countries with the most available data. These are: Albania, Bulgaria, Brazil, Botswana, Canada, Chile, Costa Rica, Czech Republic, Denmark, Dominican Republic, Ecuador, Spain, France, UK, Ghana, Greece, Guyana, Hungary, India, Ireland, Iceland, Israel, Republic of (South) Korea, Sri Lanka, Luxembourg, Malaysia, Netherlands, Norway, Peru, Philippines, Poland, Portugal, Paraguay, Sweden, Thailand, Taiwan, Uruguay, South Africa.

A.2 Proof of Corollary 1

Our goal is to show that the expected reform size $p \cdot |i_{R1}^* - i_{R2}^*| + (1 - p) \cdot |i_{R1}^* - i_{L2}^*|$ decreases in c_1 , if it changes at all.

First, let $0 \leq c_1 \leq (1 + p)\Pi$. From Proposition 1 we know that the expected

reform size equals

$$\begin{aligned}
(1-p) \cdot |i_{R1}^* - i_{L2}^*| &= (1-p) \cdot \left| \mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} - \left(\mu_L + \frac{c_1}{2} \right) \right| \\
&= (1-p) \cdot \left(\Pi - \frac{c_1}{2} \cdot \left(\frac{1-p}{1+p} + 1 \right) \right) = (1-p) \cdot \left(\Pi - \frac{c_1}{2} \cdot \frac{2}{1+p} \right) \\
&= (1-p) \cdot \left(\Pi - \frac{c_1}{1+p} \right),
\end{aligned}$$

and hence decreases in c_1 . Second, from Proposition 1 we also know that for $c_1 \geq (1+p)\Pi$ no reform takes place. Moreover, for $c_1 \geq (1+p)\Pi$ policies i_{R1}^* and i_{K2}^* increase in c_1 for $K \in \{L, R\}$.

A.3 Proof of Theorem 1

Our goal is to maximize utility in terms of c_1 as defined in Equation (2.2) by inserting the policy choices in the first and second period given by Proposition 1. We distinguish two ranges.

Case 1: $0 \leq c_1 \leq (1+p)\Pi$

For this range, the policies for the first and second period are $i_{R1}^*(c_1) = \mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p}$, $i_{R2}^*(c_1) = i_{R1}^*(c_1)$, and $i_{L2}^*(c_1) = \mu_L + \frac{c_1}{2}$, as stated in Proposition 1. Then, agent K 's utility is given by

$$\begin{aligned}
\bar{H}(c_1, \mu_K) &= -p \cdot \left(\mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} - \mu_K \right)^2 \\
&\quad - (1-p) \cdot \left[\left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2 + c_1 \left(\left(\mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} \right) - \left(\mu_L + \frac{c_1}{2} \right) \right) \right] \\
&\quad - \left(\mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} - \mu_K \right)^2 \\
&= -(1+p) \cdot \left(\mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} - \mu_K \right)^2 - (1-p) \cdot \left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2 \\
&\quad - (1-p) \cdot c_1 \cdot (\mu_R - \mu_L) + \frac{1-p}{1+p} \cdot c_1^2. \tag{A.1}
\end{aligned}$$

Then, the first order partial derivative of $H(c_1, \mu_K)$ w.r.t. c_1 is

$$\begin{aligned}
\frac{\partial \bar{H}(c_1, \mu_K)}{\partial c_1} &= (1-p) \cdot \left(\mu_R - \frac{c_1}{2} \cdot \frac{1-p}{1+p} - \mu_K \right) - (1-p) \cdot \left(\mu_L + \frac{c_1}{2} - \mu_K \right) \\
&\quad - (1-p) \cdot (\mu_R - \mu_L) + 2c_1 \cdot \frac{1-p}{1+p} \\
&= - (1-p) \cdot \frac{c_1}{2} - \frac{(1-p)^2}{1+p} \cdot \frac{c_1}{2} + 2c_1 \cdot \frac{1-p}{1+p} \\
&= \frac{(1-p)c_1}{1+p} \geq 0.
\end{aligned} \tag{A.2}$$

Hence, $H(c_1, \mu_K)$ is an increasing function in c_1 independent of μ_K .

Further, one can then verify that

$$H(0, \mu_M) = -(1+p)(\mu_R - \mu_M)^2 - (1-p)(\mu_L - \mu_M)^2$$

and

$$\begin{aligned}
H(\Pi(1+p), \mu_M) &= - (1+p) \left(\mu_R - \frac{\Pi(1+p)}{2} \cdot \frac{1-p}{1+p} - \mu_K \right)^2 \\
&\quad - (1-p) \left(\mu_L + \frac{\Pi(1+p)}{2} - \mu_M \right)^2 \\
&\quad - (1-p)\Pi(1+p)(\mu_R - \mu_L) + \frac{(1-p)\Pi^2(1+p)^2}{1+p} \\
&= - (1+p) \left(\mu_R - \frac{\Pi(1-p)}{2} - \mu_M \right)^2 \\
&\quad - (1-p) \left(\mu_L + \frac{\Pi(1+p)}{2} - \mu_M \right)^2 \\
&= - (1+p)(\mu_R - \mu_M)^2 - (1-p)(\mu_L - \mu_K)^2 + \frac{\Pi^2(1-p^2)}{2} \\
&= H(0, \mu_M) + \frac{\Pi^2(1-p^2)}{2}.
\end{aligned} \tag{A.3}$$

Case 2: $(1+p)\Pi < c_1 < 2\Pi$

For this range, the policies for the first and second period are $i_{R1}^*(c_1) = \mu_L + \frac{c_1}{2}$ and $i_{K2}^*(c_1) = i_{R1}^*$, as stated in Proposition 1. Then, utility as well as its first and

second order partial derivative are

$$\begin{aligned} H(c_1, \mu_K) &= -p \cdot \left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2 - (1-p) \cdot \left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2 \\ &\quad - \left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2 \\ &= -2 \left(\mu_L + \frac{c_1}{2} - \mu_K \right)^2, \end{aligned} \tag{A.4}$$

$$\frac{\partial H(c_1, \mu_K)}{\partial c_1} = -2 \left(\mu_L + \frac{c_1}{2} - \mu_K \right), \tag{A.5}$$

$$\frac{\partial^2 H(c_1, \mu_K)}{\partial c_1^2} = -1.$$

Equation (A.5) yields the unrestricted utility-maximizing C&B

$$(c_1^*)_{\text{Max}} := 2(\mu_K - \mu_L) \leq 2\Pi.$$

One can then verify that

$$(1+p)\Pi < (c_1^*)_{\text{Max}}$$

if and only if

$$\Pi < \frac{2(\mu_K - \mu_L)}{(1+p)} := \bar{\Pi}_K.$$

Hence, if $\bar{\Pi}_K \leq \Pi$, utility is decreasing for all c_1 in this range. Note that $\bar{\Pi}_L = 0$, $\bar{\Pi}_M = \bar{\Pi}$, and $\bar{\Pi}_R = \frac{2}{(1+p)}\Pi$. Note also that

$$H(2\Pi, \mu_M) = -2(\mu_L + \Pi - \mu_M)^2 = -2(\mu_R - \mu_M)^2.$$

Furthermore,

$$H(2\Pi, \mu_K) = -2(\mu_L + \Pi - \mu_K)^2 = -2(\mu_R - \mu_K)^2, \tag{A.6}$$

$$H(2\Pi, \mu_L) = -2\Pi^2,$$

$$H(2\Pi, \mu_R) = 0.$$

Finally, we determine the preferred level of C&B—i.e., the level of C&B that each agent would choose if s/he has proposal-making power—for each agent.

For party L utility increases for $0 \leq c_1 \leq (1+p)\Pi$, which can be directly

derived from Equation (A.2), and decreases for $(1+p)\Pi < c_1 < 2\Pi$ as shown in Equation (A.5). Hence, party L 's ideal C&B is $c_1^* = (1+p)\Pi$.

For the median voter M , utility increases for $0 \leq c_1 \leq (1+p)\Pi$, which can be directly derived from Equation (A.2). The exact level of party polarization impacts the shape of her/his utility for $(1+p)\Pi < c_1 < 2\Pi$. For low party polarization, i.e., if $\Pi < \bar{\Pi}$, the median voter's utility increases up to her/his utility-maximizing C&B $c_1^* = 2(\mu_M - \mu_L) = (1+p)\bar{\Pi}$ and then decreases for $c_1^* < c_1 < 2\Pi$. For high party polarization, i.e., $\bar{\Pi} \leq \Pi$, the median voter's utility decreases in the full interval $(1+p)\Pi < c_1 < 2\Pi$. Hence, the median voter's preferred C&B is $c_1^* = (1+p)\bar{\Pi}$ for $0 \leq \Pi < \bar{\Pi}$ and $c_1^* = (1+p)\Pi$ for $\bar{\Pi} \leq \Pi \leq 1$.

For party R , utility increases for $0 \leq c_1 \leq (1+p)\Pi$, which can be directly derived from Equation (A.2), and increases further for $(1+p)\Pi < c_1 < 2\Pi$, as shown in Equation (A.5), until it reaches its maximum at C&B $c_1^* = 2\Pi$. Hence, party R 's preferred C&B is $c_1^* = 2\Pi$.

A.4 Proof of Corollary 2

The corollary follows from Theorem 1. Since all chosen levels of C&B c_1 are at least $(1+p)\Pi$, no policy reform occurs.

A.5 Proof of Corollary 3

The corollary follows from Theorem 1. Since c_1 is unilaterally chosen, stable levels can only occur if the status quo level coincides with the preferred level of each agent.

A.6 Proof of Theorem 2

We distinguish two cases.

Case 1: $\Pi < \bar{\Pi}$

From the proof of Theorem 1, we know that (i) party L 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\Pi)$ and decreases for $c_1 \in ((1+p)\Pi, 2\Pi]$, that (ii)

median voter M 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\bar{\Pi}]$ and decreases for $c_1 \in ((1+p)\bar{\Pi}, 2\Pi]$, and that (iii) party R 's utility as a function of c_1 increases for $c_1 \in [0, 2\Pi]$.

First, suppose that party L is the proposal-maker. If $c_0 \in [0, (1+p)\Pi]$, then party L proposes $c_1^* = (1+p)\Pi$, since all agents prefer c_1^* to c_0 and c_1^* is party L 's optimal choice for c_1 . If $c_0 \in [(1+p)\Pi, (1+p)\bar{\Pi}]$, then party L proposes c_0 , as both the median voter M and party R dislike choices of c_1 that are further below c_0 and party L dislikes choices of c_1 that are further above c_0 . It therefore remains to consider the case where $c_0 \in [(1+p)\bar{\Pi}, 2\Pi]$. In such case, due to Equation (2.4) and Corollary 1, one can easily verify that in equilibrium, party L will propose the lowest C&B that can be accepted by median voter M . This is because party R will not accept any c_1 below c_0 . The C&B c_1 proposed by party L will then be at least $(1+p)\Pi$ and at most $(1+p)\bar{\Pi}$, due to the shape of party L 's and the median voter M 's utilities as described above, and is therefore pinned down by the following equation:

$$\begin{aligned} H(c_1, \mu_M, p) &= H(c_0, \mu_M, p) \\ \text{s.t. } (1+p)\Pi &\leq c_1 \leq (1+p)\bar{\Pi}. \end{aligned}$$

Using Equation (A.4), one can see that solving the above problem is equivalent to solving the following equation in c_1 :

$$\mu_L + \frac{c_1}{2} - \mu_M = -\mu_L - \frac{c_0}{2} + \mu_M,$$

subject to $(1+p)\Pi \leq c_1 \leq (1+p)\bar{\Pi}$. Solving the latter equation with this restriction yields the unique solution c_1^* to the above problem, namely

$$c_1^* = \max\{(1+p)\Pi, 2(1+p)\bar{\Pi} - c_0\}.$$

Second, suppose that median voter M is the proposal-maker. If $c_0 \in [0, (1+p)\bar{\Pi}]$, then the median voter M proposes $c_1^* = (1+p)\bar{\Pi}$ since both the median voter M and party R prefer c_1^* to c_0 and c_1^* is the median voter M 's optimal choice for c_1 . If $c_0 \in [(1+p)\bar{\Pi}, 2\Pi]$, then the median voter M proposes $c_1^* = (1+p)\bar{\Pi}$, as both the

median voter M and party L prefer c_1^* to c_0 and c_1^* is the median voter M 's optimal choice for c_1 . In either case, the median voter obtains her/his most preferred C&B.

Third and last, suppose that party R is the proposal-maker. Then, let us analyze the case where $c_0 \in [0, (1+p)\bar{\Pi}]$. Due to Equation (2.4) and Corollary 1, one can easily verify that in equilibrium, party R will propose the largest C&B that can be accepted by median voter M . This C&B will be at least $(1+p)\bar{\Pi}$ (and at most 2Π). The reason is that both the median voter M and party R prefer $(1+p)\bar{\Pi}$ to any c_0 lower than $(1+p)\bar{\Pi}$. The desired C&B is therefore pinned down by the following equation:

$$\begin{aligned} H(c_0, \mu_M, p) &= H(c_1, \mu_M, p) \\ \text{s.t. } (1+p)\bar{\Pi} &\leq c_1 \leq 2\Pi. \end{aligned} \tag{A.7}$$

To solve the above equation, one needs to distinguish two cases. On the one hand, suppose that $c_0 \in [0, (1+p)\bar{\Pi}]$. Then, using Equations (2.3) and (A.4), one obtains that Equation (A.7) can be rewritten as:

$$\bar{H}(c_0, \mu_M, p) = -2 \left(\mu_L + \frac{c_1}{2} - \mu_M \right)^2,$$

with the condition that the solution must lie between $(1+p)\bar{\Pi}$ and 2Π . Together with the restriction, the above equation yields the following unique solution to the above problem:

$$c_1^* = \sqrt{-2\bar{H}(c_0, \mu_M, p)} + (1+p)\bar{\Pi}.$$

It follows that the negative solution $c_1^* = -\sqrt{-2\bar{H}(c_0, \mu_M, p)} + (1+p)\bar{\Pi}$ is not feasible for $(1+p)\bar{\Pi} \leq c_1 \leq 2\Pi$. Note that as one can see from definition of the (expected) utility, as stated in Equation (2.2), the utility of all agents is negative for all $c_0 \in [0, 2\Pi]$. On the other hand, suppose that $c_0 \in [(1+p)\bar{\Pi}, (1+p)\bar{\Pi}]$. Then, using Equation (A.4), one obtains that Equation (A.7) can be rewritten as

$$\mu_L + \frac{c_1}{2} - \mu_M = \mu_M - \frac{c_0}{2} - \mu_L,$$

with the condition that the solution must lie between $(1+p)\bar{\Pi}$ and 2Π . This yields

the following unique solution:

$$c_1^* = \min\{2\Pi, 2(1+p)\bar{\Pi} - c_0\}.$$

Finally, it remains to analyze the case where $c_0 \in ((1+p)\bar{\Pi}, 2\Pi]$. In such a case, party R proposes c_0 , as it cannot have any larger C&B approved by median voter M (or party L) against c_0 . This is because both the median voter M 's and party L 's utility decreases as functions of c_1 if $c_0 \in ((1+p)\bar{\Pi}, 2\Pi]$.

Case 2: $\bar{\Pi} \leq \Pi$

From the proof of Theorem 1 we know that (i) party L 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\Pi]$ and decreases for $c_1 \in ((1+p)\Pi, 2\Pi]$, that (ii) the median voter M 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\Pi]$ and decreases for $c_1 \in ((1+p)\Pi, 2\Pi]$, and that (iii) party R 's utility as a function of c_1 increases for $c_1 \in [0, 2\Pi]$.

First, suppose that either party L or the median voter M is the proposal-maker. Since their utilities are perfectly aligned, both agents make the same proposal for C&B and approve them. Then, regardless of c_0 , either agent proposes $c_1^* = (1+p)\Pi$, since c_1^* is their optimal choice for c_1 .

Second and last, suppose that party R is the proposal-maker. Since both party L and median voter M 's utilities are perfectly aligned, in equilibrium party R will propose the largest C&B that can be accepted by either the median voter M or party L . This level of C&B will be at least as much as $(1+p)\Pi$. This is because all agents prefer $(1+p)\Pi$ to any c_0 that is lower than $(1+p)\Pi$. For $c_1 \in ((1+p)\Pi, 2\Pi]$ party R proposes c_0 , as both the median voter M and party L dislike choices of c_1 further above c_0 and party R dislikes choices of c_1 that are further below c_0 .

A.7 Proof of Corollary 4

The corollary follows from Theorem 2. Since all chosen levels of C&B c_1 are at least $(1+p)\Pi$, no policy reform occurs.

A.8 Proof of Corollary 5

The corollary follows directly from Theorem 2.

A.9 Proof of Theorem 3

We distinguish two cases.

Case 1: $\Pi < \bar{\Pi}$

From the proof of Theorem 1, we know that (i) party L 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\Pi]$ and decreases for $c_1 \in ((1+p)\Pi, 2\Pi]$, that (ii) the median voter M 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\bar{\Pi}]$ and decreases for $c_1 \in ((1+p)\bar{\Pi}, 2\Pi]$, and that (iii) party R 's utility as a function of c_1 increases for $c_1 \in [0, 2\Pi]$.

First, suppose that party L is the proposal-maker. If $c_0 \in [0, (1+p)\Pi]$, then party L proposes $c_1^* = (1+p)\Pi$, since all agents prefer c_1^* to c_0 and c_1^* is party L 's optimal choice for c_1 . If $c_0 \in [(1+p)\Pi, 2\Pi]$, then party L proposes c_0 , as party R dislikes choices of c_1 that are further below c_0 and party L dislikes choices of c_1 that are further above c_0 .

Second, suppose that the median voter M is the proposal-maker. If $c_0 \in [0, (1+p)\Pi]$, then the median voter M must propose C&B that give party L at least as much utility as under c_0 . For its part, party R simply wants to have the largest possible C&B up to 2Π . Due to the shape of the utilities of party L and of the median voter M , one can see that the proposed C&B will be in the range from $(1+p)\Pi$ to $(1+p)\bar{\Pi}$. Hence, the median voter M proposes c_1 , such that

$$\begin{aligned} H(c_0, \mu_L) &= H(c_1, \mu_L) \\ \text{s.t. } (1+p)\Pi &< c_1 \leq (1+p)\bar{\Pi}. \end{aligned}$$

Using Equations (2.3) and (A.4), one can see that solving the above problem is

equivalent to solving the following equation in c_1 :

$$\overline{H}(c_0, \mu_L, p) = -\frac{c_1^2}{2}, \quad (\text{A.8})$$

subject to $(1+p)\Pi \leq c_1 \leq (1+p)\overline{\Pi}$. The negative solution $c_1 = -\sqrt{-2\overline{H}(c_0, \mu_L, p)}$ of Equation (A.8) is not feasible for $(1+p)\Pi < c_1 \leq (1+p)\overline{\Pi}$. Solving Equation (A.8) with the restriction yields the unique solution c_1^* to the above problem, namely

$$c_1^* = \min \left\{ (1+p)\overline{\Pi}, \sqrt{-2\overline{H}(c_0, \mu_L, p)} \right\}.$$

If $c_0 \in [(1+p)\Pi, 2\Pi]$, then the median voter M will propose c_0 , as party L dislikes choices of c_1 that are further above c_0 and party R dislikes choices of c_1 that are further below c_0 .

Third and last, suppose that party R is the proposal-maker. Consider the case where $c_0 \in [0, (1+p)\Pi]$, then party R will propose the largest C&B that will be accepted by median voter M and party L . Hence, party R 's proposal must solve the following maximization problem:

$$\begin{aligned} & \max_{c_1 \in [(1+p)\Pi, 2\Pi]} c_1 \\ & \text{s.t. } H(c_0, \mu_K) \leq H(c_1, \mu_K) \text{ for all } K \in \{L, M\}. \end{aligned} \quad (\text{A.9})$$

For $K = L$, using Equation (2.3), the Constraint (A.9) can be rewritten as

$$\overline{H}(c_0, \mu_L, p) = -\frac{c_1^2}{2}.$$

The negative solution $c_1 = -\sqrt{-2\overline{H}(c_0, \mu_L, p)}$ of the equation above is not feasible for $(1+p)\Pi < c_1 \leq 2\Pi$. Together with the restriction, one obtains the following unique solution to the above problem:

$$c_1^* = \sqrt{-2\overline{H}(c_0, \mu_L, p)} \quad (\text{A.10})$$

For $K = M$, using Equation (2.3) the Constraint (A.9) can be rewritten as

$$\bar{H}(c_0, \mu_M, p) = -2 \left(\mu_L + \frac{c_1}{2} - \mu_M \right)^2.$$

The negative solution $c_1 = -\sqrt{-2\bar{H}(c_0, \mu_M, p) + (1+p)\bar{\Pi}}$ of the equation above is not feasible for $(1+p)\bar{\Pi} < c_1 \leq 2\Pi$. Together with the restriction, the above equation yields the following unique solution to the above problem:

$$c_1^* = \sqrt{-2\bar{H}(c_0, \mu_M, p) + (1+p)\bar{\Pi}}. \quad (\text{A.11})$$

Then, party R proposes the minimum of the two solutions stated in Equation (A.10) and (A.11), since then both party L and the median voter M agree.

If $c_0 \in [(1+p)\bar{\Pi}, 2\Pi]$, then the party R will propose c_0 , as party L dislikes choices of c_1 that are further above c_0 and party R dislikes choices of c_1 that are further below c_0 .

Case 2: $\bar{\Pi} \leq \Pi$

From the proof of Theorem 1, we know that (i) party L 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\bar{\Pi}]$ and decreases for $c_1 \in ((1+p)\bar{\Pi}, 2\Pi]$, that (ii) the median voter M 's utility as a function of c_1 increases for $c_1 \in [0, (1+p)\bar{\Pi}]$ and decreases for $c_2 \in ((1+p)\bar{\Pi}, 2\Pi]$, and that (iii) party R 's utility as a function of c_1 increases for $c_1 \in [0, 2\Pi]$.

First, suppose that either party L or the median voter M is the proposal-maker. Since their utilities are perfectly aligned, both agents make the same proposal for C&B. If $c_0 \in [0, (1+p)\bar{\Pi}]$, then party L and the median voter M propose $c_1^* = (1+p)\bar{\Pi}$, since all agents prefer c_1^* to c_0 and c_1^* is party L 's and the median voter M 's optimal choice for c_1 . If $c_0 \in [(1+p)\bar{\Pi}, 2\Pi]$, then party L and median voter M propose c_0 , as party R dislikes choices of c_1 that are further below c_0 and party L and the median voter dislike choices of c_1 that are further above c_0 .

Second and last, suppose that party R is the proposal-maker. Since both party L and the median voter M 's utilities are perfectly aligned, in equilibrium, party R will propose the largest C&B that can be accepted by the median voter M and

party L . If $c_0 \in [0, (1+p)\Pi]$, then party R proposes will propose the largest C&B that will be accepted by median voter M and party L . Hence, party R must solve the following maximization problem:

$$\begin{aligned} & \max_{c_1 \in [(1+p)\Pi, 2\Pi]} c_1 \\ \text{s.t. } & H(c_0, \mu_K) \leq H(c_1, \mu_K) \text{ for all } K \in \{L, M\}. \end{aligned} \quad (\text{A.12})$$

For $K = L$, using Equation (2.3), one can write Equation (A.12) as

$$\bar{H}(c_0, \mu_L, p) = -\frac{c_1^2}{2}.$$

The negative solution $c_1 = -\sqrt{-2\bar{H}(c_0, \mu_L, p)}$ of the equation above is not feasible for $(1+p)\Pi < c_1 \leq 2\Pi$. Together with the restriction, one obtains the following unique solution to the above problem:

$$c_1^* = \sqrt{-2\bar{H}(c_0, \mu_L, p)}. \quad (\text{A.13})$$

For $K = M$, using Equation (2.3), one can write Equation (A.12) as

$$\bar{H}(c_0, \mu_M, p) = -\left(\mu_L + \frac{c_1}{2} - \mu_M\right)^2.$$

Together with the restriction, the above equation yields the following unique solution to the above problem:

$$c_1^* = \sqrt{-2\bar{H}(c_0, \mu_M, p)} + (1+p)\bar{\Pi}. \quad (\text{A.14})$$

Then, party R proposes the minimum of the two solutions stated in Equation (A.13) and (A.14), since then both party L and the median voter M agree.

If $c_0 \in ((1+p)\Pi, 2\Pi]$, then the median party R proposes c_0 as the median voter M and party L dislike choices of c_1 that are above c_0 .

A.10 Proof of Corollary 6

The corollary follows from Theorem 3. Since all chosen levels of C&B c_1 are at least $(1 + p)\Pi$, no policy reform occurs.

A.11 Proof of Corollary 7

The corollary follows directly from Theorem 3.

A.12 Proof of Theorem 4

First, consider part (i). This part follows from Corollaries 3, 5, and 7. In the case of the simple majority rule, the set of stable C&B is a singleton. If the unanimity rule is in place, the size of the set of stable C&B is maximal among the set of sets of C&B that lead to no reform (i.e., to gridlock), under the assumption that $c_1 \leq 2\Pi$.

Second, consider part (ii). Then, Corollary 5 suffices to show that the size of the set of stable C&B does not change monotonically with the proposal-maker's peak when we first consider party L , then the median voter M , and finally party R .

Third and last, consider part (iii). As stated in Corollaries 3, 5, and 7, the set of stable C&B increases with party polarization Π , if it changes at all.

A.13 Proof of Theorem 5

As defined in Equation (2.15), the welfare is defined from the median voter's perspective. Since, we consider intermediate values of the status quo C&B, this means that $(1 + p)\Pi < c_0 < 2\Pi$, and welfare can be rewritten as

$$W(c_1) = H(c_1, \mu_M) = -2 \left(\mu_L + \frac{c_1}{2} - \mu_M \right)^2.$$

Consider the derivative of the welfare function w.r.t. c_1 , namely

$$\frac{\partial W(c_1)}{\partial c_1} = -2 \left(\mu_L + \frac{c_1}{2} - \mu_M \right).$$

In the case of the simple majority rule, the party with proposal-making power can decide unilaterally on C&B. As shown in Theorem 1 and Corollary 3, party L proposes the stable level $c_1 = (1 + p)\Pi$ and party R proposes the stable level $c_1 = 2\Pi$, independent of party polarization.

If $\bar{\Pi} \leq \Pi \leq 1$, then $\frac{\partial W(c_1)}{\partial c_1} < 0$ for all $(1 + p)\Pi < c_1 < 2\Pi$, implying that the C&B proposed by the opposing party is preferred from a welfare perspective.

If $0 \leq \Pi < \bar{\Pi}$, then by considering Equation (A.3) and (A.6) for μ_M , it holds that

$$\begin{aligned} H(2\Pi, \mu_M) &< H(\Pi(1 + p), \mu_M), \\ -2(\mu_R - \mu_M)^2 &< H(0, \mu_M) + \frac{\Pi^2(1 - p^2)}{2}, \\ -2(\mu_R - \mu_M)^2 &< -(1 + p)(\mu_R - \mu_M)^2 - (1 - p)(\mu_L - \mu_M)^2 + \frac{\Pi^2(1 - p^2)}{2}, \end{aligned}$$

for $\mu_L \leq \mu_M < \frac{1}{4} \cdot (3\mu_R + \mu_L + p\Pi)$, i.e., $\mu_L \leq \mu_M < \bar{\mu}_M$, and

$$H(2\Pi, \mu_M) \geq H(\Pi(1 + p), \mu_M),$$

for $\frac{1}{4} \cdot (3\mu_R + \mu_L + p\Pi) \leq \mu_M \leq \mu_R$, i.e., $\bar{\mu}_M < \mu_M < \mu_R$. This leads to the result that the C&B proposed by the opposing party is preferred from a welfare perspective, if the median voter's peak is close enough to party L 's peak.

A.14 Proof of Corollary 8

The corollary follows from the fact that the set of stable C&B of party L , as stated in Corollary 3, is the same as the median voter's ideal C&B if the simple majority rule is in place.

A.15 Proof of Theorem 6

In this case, both parties have to agree on C&B. Consider the simple majority rule and the double majority rule to compare the different stable C&B with regard to

welfare, as defined in Equation (2.16). Equation (2.16) can be rewritten as

$$\begin{aligned} W(c_1) &= H(c_1, \mu_L) + H(c_1, \mu_R) \\ &= -2 \left(\frac{c_1}{2} \right)^2 - 2 \left(\frac{c_1}{2} - \Pi \right)^2 = -2 \left(\frac{c_1^2}{2} - c_1 \Pi + \Pi^2 \right), \end{aligned} \quad (\text{A.15})$$

for $(1+p)\Pi < c_1 < 2\Pi$, independent of the constitutional rule. Then, the derivative of Equation (A.15) w.r.t. c_1 is

$$\frac{\partial W(c_1)}{\partial c_1} = -2(c_1 - \Pi) < 0,$$

since $(1+p)\Pi < c_1 < 2\Pi$.

First, consider the simple majority rule. As stated in Theorem 1 and Corollary 3, the opposing party proposes the stable level $c_1 = (1+p)\Pi$ and the incumbent party proposes $c_1 = 2\Pi$. Since welfare decreases in c_1 , the opposing party proposes a welfare superior level of c_1 .

Second, consider the double majority rule. As stated in Corollary 5, the stable C&B proposed by the opposing party is lower than the one proposed by the incumbent party, independent of party polarization. Hence, the level proposed by the opposing party is welfare superior.

Chapter B

Appendix to Chapter 3

B.1 Proof of Proposition 3

Our goal is to maximize the incumbent's expected lifetime utility stated in Equation (3.3) in terms of i_1 by inserting the policy choices of both parties in the second period. The policy choices of party $K \in \{L, R\}$ in period $t = 2$ are defined by Equation (3.2) and depend on the size of c . To determine the ideal policy i_1^* , we consider different ranges of c and analyze the policy choices i_{K2}^* of party K in the second period for each interval. We assume for simplicity that $0 < B < \Delta$, where $\Delta := \frac{\mu_R - \mu_L}{2} = \frac{\Pi}{2}$, to reduce the number of intervals. The inequalities in red indicate the changes for each interval of c with regard to the previous one.

(i) For $0 < \frac{c}{2} \leq \min\{\Delta - B, B\}$, it follows that

$$\begin{aligned} \mu_L - B - \frac{c}{2} &< \mu_L - B + \frac{c}{2} < \mu_L + B - \frac{c}{2} < \mu_L + B + \frac{c}{2} \\ &< \mu_R - B - \frac{c}{2} < \mu_R - B + \frac{c}{2} < \mu_R + B - \frac{c}{2} < \mu_R + B + \frac{c}{2}. \end{aligned}$$

(ii) For $\Delta - B < \frac{c}{2} \leq B$, given $\frac{\Delta}{2} < B$, it follows that

$$\begin{aligned} \mu_L - B - \frac{c}{2} &< \mu_L - B + \frac{c}{2} < \mu_L + B - \frac{c}{2} < \mu_R - B - \frac{c}{2} \\ &< \mu_L + B + \frac{c}{2} < \mu_R - B + \frac{c}{2} < \mu_R + B - \frac{c}{2} < \mu_R + B + \frac{c}{2}. \end{aligned}$$

(iii) For $B < \frac{c}{2} \leq \Delta - B$, given $\frac{\Delta}{2} > B$, it follows that

$$\begin{aligned} \mu_L - B - \frac{c}{2} &< \mu_L + B - \frac{c}{2} < \mu_L - B + \frac{c}{2} < \mu_L + B + \frac{c}{2} \\ &< \mu_R - B - \frac{c}{2} < \mu_R + B - \frac{c}{2} < \mu_R - B + \frac{c}{2} < \mu_R + B + \frac{c}{2}. \end{aligned}$$

(iv) For $\max\{B, \Delta - B\} < \frac{c}{2} \leq \Delta$, it follows that

$$\begin{aligned} \mu_L - B - \frac{c}{2} &< \mu_L + B - \frac{c}{2} < \mu_L - B + \frac{c}{2} < \mu_R - B - \frac{c}{2} \\ &< \mu_L + B + \frac{c}{2} < \mu_R + B - \frac{c}{2} < \mu_R - B + \frac{c}{2} < \mu_R + B + \frac{c}{2}. \end{aligned}$$

(v) For $\Delta < \frac{c}{2} \leq \Delta + B$, it follows that

$$\begin{aligned} \mu_L - B - \frac{c}{2} &< \mu_L + B - \frac{c}{2} < \mu_R - B - \frac{c}{2} < \mu_L - B + \frac{c}{2} \\ &< \mu_R + B - \frac{c}{2} < \mu_L + B + \frac{c}{2} < \mu_R - B + \frac{c}{2} < \mu_R + B + \frac{c}{2}. \end{aligned}$$

(vi) For $\Delta + B \leq \frac{c}{2}$, it follows that

$$\begin{aligned} \mu_L - B - \frac{c}{2} &< \mu_L + B - \frac{c}{2} < \mu_R - B - \frac{c}{2} < \mu_R + B - \frac{c}{2} \\ &< \mu_L - B + \frac{c}{2} < \mu_L + B + \frac{c}{2} < \mu_R - B + \frac{c}{2} < \mu_R + B + \frac{c}{2}. \end{aligned}$$

First, consider the policy choices that are independent of c , i.e., very low and very high values of i_1 . The result is the same for all values of c and hence, all regions described in (i) – (vi). If $i_1 \leq \mu_L - B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ in the second period are given by

$$\begin{aligned} i_{K2+}^* &= \min \left\{ \max \left\{ \mu_K + B - \frac{c}{2}, i_1 \right\}, \mu_K + B + \frac{c}{2} \right\} = \mu_K + B - \frac{c}{2}, \\ i_{K2-}^* &= \min \left\{ \max \left\{ \mu_K - B - \frac{c}{2}, i_1 \right\}, \mu_K - B + \frac{c}{2} \right\} = \mu_K - B - \frac{c}{2}, \end{aligned}$$

for $K \in \{L, R\}$. These policy choices lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 - c(\mu_R - B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-\left(\mu_L - \frac{c}{2} - \mu_R\right)^2 - c(\mu_L + B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-\left(\mu_L - \frac{c}{2} - \mu_R\right)^2 - c(\mu_K - B - \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} = cp + (1-p)c - 2(i_1 - \mu_R) = c - 2i_1 + 2\mu_R > 0,$$

since $\mu_R > i_1$. Then, the solution is the corner solution $i_1^*(c) = \mu_L - B - \frac{c}{2}$.

If $i_1 \geq \mu_R + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ in the second period are given by

$$\begin{aligned}i_{K2+}^* &= \min \left\{ \max \left\{ \mu_K + B - \frac{c}{2}, i_1 \right\}, \mu_K + B + \frac{c}{2} \right\} = \mu_K + B + \frac{c}{2}, \\ i_{K2-}^* &= \min \left\{ \max \left\{ \mu_K - B - \frac{c}{2}, i_1 \right\}, \mu_K - B + \frac{c}{2} \right\} = \mu_K - B + \frac{c}{2},\end{aligned}$$

for $K \in \{L, R\}$. These policy choices lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 + c(\mu_R + B + \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 + c(\mu_R - B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} = -cp - (1-p)c - 2(i_1 - \mu_R) = -c - 2i_1 + 2\mu_R < 0,$$

since $\mu_R < i_1$. Then, the solution is the corner solution $i_1^*(c) = \mu_R + B + \frac{c}{2}$.

Second, consider each interval of c separately and determine the ideal value of i_1 , depending on the value of c .

Consider (i), where $0 < \frac{c}{2} \leq \Delta - B$.

If $\mu_L - B - \frac{c}{2} < i_1 \leq \mu_L - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= \mu_L + B - \frac{c}{2}, & i_{L2-}^* &= i_1, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2}, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 - c(\mu_R - B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L - \frac{c}{2} - \mu_R)^2 - c(\mu_L + B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R + B)^2 \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R - 2i_1 + cp + \frac{(1-p)}{2}(-2B + 2\mu_R + c - 2i_1) \\ &\geq 2\mu_R - 2i_1 + cp + \frac{(1-p)}{2} \left(2\mu_R + c - 2B - 2 \left(\mu_L - B + \frac{c}{2} \right) \right) \\ &\geq 2\mu_R - 2i_1 + cp + (1-p)(\mu_R - \mu_L) > 0, \end{aligned}$$

since $\mu_R > i_1$. Then, the solution is the corner solution $i_1^*(c) = \mu_L - B + \frac{c}{2}$.

If $\mu_L - B + \frac{c}{2} < i_1 \leq \mu_L + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= \mu_L + B - \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2}, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 - c(\mu_R - B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L - \frac{c}{2} - \mu_R)^2 - c(\mu_L + B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + cp - 2i_1 \\ &\geq 2\mu_R + cp - 2\left(\mu_L + B - \frac{c}{2}\right) \\ &= 2(\mu_R - \mu_L) + cp - 2B + c \\ &= 4\Delta + cp - 2B + c > 0,\end{aligned}$$

since $B < \Delta$. Then, the solution is the corner solution $i_1^*(c) = \mu_L + B - \frac{c}{2}$.

If $\mu_L + B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= i_1, \quad i_{L2-}^* = \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, \quad i_{R2-}^* = \mu_R - B - \frac{c}{2},\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 - c(\mu_R - B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= cp - (1-p)(i_1 - \mu_R - B) - \frac{c(1-p)}{2} - 2(i_1 - \mu_R) \\ &= 2\mu_R - 2i_1 + cp + \frac{(1-p)}{2}(2B + 2\mu_R - c - 2i_1) > 0,\end{aligned}$$

since $\mu_R > i_1$ and $\frac{c}{2} < B$. Then, the solution is the corner solution $i_1^*(c) = \mu_L + B + \frac{c}{2}$.

If $\mu_L + B + \frac{c}{2} < i_1 \leq \mu_R - B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= \mu_L + B + \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2},\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 - c(\mu_R - B - \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + 2cp - c - 2i_1 \\ &\geq 2\mu_R + 2cp - c - 2\left(\mu_R - B - \frac{c}{2}\right) \\ &= 2cp + 2B > 0.\end{aligned}$$

Then, the ideal policy is the corner solution $i_1^*(c) = \mu_R - B - \frac{c}{2}$.

If $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_R - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= \mu_L + B + \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1,\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Consider the FOC and SOC of the utility with respect to i_1

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \frac{cp}{2} - p(i_1 - \mu_R + B) - c(1-p) - 2(i_1 - \mu_R) \stackrel{!}{=} 0, \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -p - 2 < 0.\end{aligned}$$

Then, a potential candidate for the globally ideal policy is given by $i_1^*(c) = \mu_R + \frac{\frac{3cp}{2} - c - Bp}{(2+p)}$. It follows that $\mu_R - B - \frac{c}{2} < \mu_R + \frac{\frac{3cp}{2} - c - Bp}{(2+p)}$. Further, it requires that $\mu_R + \frac{\frac{3cp}{2} - c - Bp}{(2+p)} \leq \mu_R - B + \frac{c}{2}$, which is only true for $\frac{c}{2} \geq \frac{B}{(2-p)}$. Otherwise, i.e., if $\frac{c}{2} < \frac{B}{(2-p)}$, it is true that $\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} > 0$. Then,

$$\begin{aligned}\mathbb{E}_R(u(i_1^*(c), c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c \left(B - \frac{c}{2} - \frac{\frac{3cp}{2} - c - Bp}{(2+p)} \right) - \left(\frac{\frac{3cp}{2} - c - Bp}{(2+p)} + B \right)^2 \right] \\ &\quad - (1-p)(\mu_L + \frac{c}{2} - \mu_R)^2 - c(1-p) \left(\mu_R + \frac{\frac{3cp}{2} - c - Bp}{(2+p)} \right) \\ &\quad + c(1-p) \left[\mu_L + \frac{c}{2} \right] - \left(\frac{\frac{3cp}{2} - c - Bp}{(2+p)} \right)^2 \\ &= (\mu_R - \mu_L)^2(p-1) + \frac{c^2(1+p^2-1.5p) - B^2p - 2Bcp^2}{(2+p)}.\end{aligned}\tag{B.1}$$

If $\mu_R - B + \frac{c}{2} < i_1 \leq \mu_R + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= \mu_L + B + \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B + \frac{c}{2},\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left(-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - \left(\frac{c}{2}\right)^2 + c(\mu_R - B + \frac{c}{2} - i_1) \right) \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Consider the FOC and SOC of the utility with respect to i_1

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= -c(1-p) - 2(i_1 - \mu_R) \stackrel{!}{=} 0, \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -2 < 0.\end{aligned}$$

Then, a potential candidate for the globally ideal policy is given by $i_1^*(c) = \mu_R + \frac{c(p-1)}{2} \in [\mu_R - \frac{c}{2}, \mu_R]$. It follows that $\mu_R + \frac{c(p-1)}{2} \leq \mu_R + B - \frac{c}{2}$. Further, it requires that $\mu_R - B + \frac{c}{2} < \mu_R + \frac{c(p-1)}{2}$, which is only true for $\frac{c}{2} < \frac{B}{(2-p)}$. Otherwise, i.e., if $\frac{c}{2} \geq \frac{B}{(2-p)}$, it is true that $\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} < 0$. Then,

$$\begin{aligned}\mathbb{E}_R(u(i_1^*(c), c)) &= \frac{p}{2} \left[-2 \left(\frac{c}{2}\right)^2 - c(2B - c) \right] - (1-p)(\mu_L + \frac{c}{2} - \mu_R)^2 \\ &\quad + c(1-p) \left(\mu_L + \frac{c}{2} - \mu_R - \frac{c(p-1)}{2} \right) - \left(\frac{c(p-1)}{2} \right)^2 \\ &= (\mu_R - \mu_L)^2(p-1) - Bcp + \frac{c^2}{2} \left(\frac{p^2}{2} - p + 1 \right).\end{aligned}\tag{B.2}$$

If $\mu_R + B - \frac{c}{2} < i_1 \leq \mu_R + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= \mu_L + B + \frac{c}{2}, \quad i_{L2-}^* = \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= i_1, \quad i_{R2-}^* = \mu_R - B + \frac{c}{2},\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-(i_1 - \mu_R - B)^2 - \left(\frac{c}{2}\right)^2 + c(\mu_R - B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + c(p-1) - 2i_1 + \frac{p}{2}(2B + 2\mu_R - c - 2i_1) \\ &= 2\mu_R + c(p-1) - 2\left(\mu_R + B - \frac{c}{2}\right) \\ &\quad + \frac{p}{2}\left(2B + 2\mu_R - c - 2\left(\mu_R + B - \frac{c}{2}\right)\right) \\ &= cp - 2B + pB + p\mu_R - \frac{cp}{2} - p\mu_R - Bp + \frac{cp}{2} \\ &= cp - 2B < 0.\end{aligned}$$

Then, the solution is the corner solution $i_1^*(c) = \mu_R + B - \frac{c}{2}$.

Consider (ii) , where $\Delta - B < \frac{c}{2} \leq B$.

The policy choices for the second period, and hence the choices of i_1 , are identical with (i) except for $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$.

If $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= i_1, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1,\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c \left(\mu_R + B - \frac{c}{2} - i_1 \right) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} [-(i_1 - \mu_R - B)^2] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c \left(\mu_L - B + \frac{c}{2} - i_1 \right) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \frac{p}{2} (c - 2(i_1 - \mu_R + B)) - (1-p)(i_1 - \mu_R - B) \\ &\quad - \frac{c(1-p)}{2} - 2(i_1 - \mu_R) \\ &= 3\mu_R + \left(B - \frac{c}{2}\right) (1 - 2p) - 3i_1 \\ &\geq 3\mu_R + B - \frac{c}{2} - 2p \left(B - \frac{c}{2}\right) - 3 \left(\mu_L + B + \frac{c}{2}\right) \\ &= 6\Delta - 2B - 2c - 2p \left(B - \frac{c}{2}\right) \geq 0\end{aligned}$$

for all values of $p \in [0, 1]$, given $c \leq 2\Delta$ and $B \leq \Delta$. Then, the ideal policy is the corner solution $i_1^*(c) = \mu_L + B + \frac{c}{2}$. Since the change in utility for party R is the same as in (i), the ideal policy i_1^* coincides with (i).

Finally, we compare the two expected utilities, given by Equations (B.1) and (B.2), of both potential globally ideal policies for $p \in (0, 1]$ if $0 < c \leq 2B$, i.e.,

$$\begin{aligned}\mathbb{E}_R \left(u \left(\mu_R + \frac{3cp - c - Bp}{(2+p)}, c \right) \right) &\leq \mathbb{E}_R \left(u \left(\mu_R + \frac{c(p-1)}{2}, c \right) \right) \quad (\text{B.3}) \\ \frac{c^2(1+p^2 - 1.5p) - B^2p - 2Bcp^2}{(2+p)} &\leq -Bcp + \frac{c^2}{2} \left(\frac{p^2}{2} - p + 1 \right) \\ 2c^2(1+p^2 - 1.5p) - B^2p - 2Bcp^2 &\leq -2Bcp(2+p) + c^2(2+p) \left(\frac{p^2}{2} - p + 1 \right) \\ 0 &\leq 2c^2 + 2B^2 - 4Bc + \frac{c^2p^2}{2} + 2Bcp - 2c^2p \\ 0 &\leq \underbrace{2(c-B)^2}_I + \underbrace{\frac{cp}{2}(cp + 4B - 4c)}_{II}.\end{aligned}$$

It is obvious that the first term I is positive. For the second term II , consider its derivative w.r.t. p , its minimum is at $p^* = \frac{2c^2 - 2cB}{c^2}$. Hence, at this point, the inequality can be written as

$$0 \leq 2(c - B)^2 + 2(c - B)(B - c) = 0.$$

Therefore, the chosen policy for this interval is defined by

$$i_1^* = \max \left\{ \mu_R - B + \frac{c}{2}, \mu_R + \frac{c(p-1)}{2} \right\}.$$

This is equivalent to

$$i_{R1}^*(c, B) := \begin{cases} \mu_R + \frac{c(p-1)}{2} & \text{if } 0 \leq c \leq \frac{2B}{(2-p)}, \\ \mu_R - B + \frac{c}{2} & \text{if } \frac{2B}{(2-p)} < c \leq B(1+p). \end{cases}$$

Note that the threshold $B(1+p)$ comes from the fact that the policy is ideal also for larger values of c as shown in the next section and coincides with the globally ideal policy $i_1 = \mu_R + \frac{c(p-1)}{2(1+p)}$ for higher values of c , that will be specified later.

Consider (iii), where $B < \frac{c}{2} \leq \Delta - B$.

The policy choices for the second period, and hence the choices of i_1 , are identical with (ii), except for $\mu_L + B - \frac{c}{2} < i_1 \leq \mu_R - B + \frac{c}{2}$.

If $\mu_L - B - \frac{c}{2} < i_1 \leq \mu_L + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= \mu_L + B - \frac{c}{2}, & i_{L2-}^* &= i_1, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2}, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - \left(\frac{c}{2}\right)^2 - c\left(\mu_R - B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L - \frac{c}{2} - \mu_R)^2 - c\left(\mu_L + B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R + B)^2 \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + cp - 2i_1 + (p-1) \left(B - \mu_R - \frac{c}{2} + i_1 \right) \\ &\geq 2\mu_R + cp - 2 \left(\mu_L + B - \frac{c}{2} \right) \\ &\quad + (p-1) \left(B - \mu_R - \frac{c}{2} + \mu_L + B - \frac{c}{2} \right) \\ &= 2(\mu_R - \mu_L) + cp - 2B + c + (p-1)(2B - c - (\mu_R - \mu_L)) \\ &= 3(\mu_R - \mu_L) - p(\mu_R - \mu_L) - 4B + 2c + 2Bp > 0,\end{aligned}$$

since $\frac{c}{2} > B$. Then, the ideal policy is defined by $i_1^*(c) = \mu_L + B - \frac{c}{2}$.

If $\mu_L + B - \frac{c}{2} < i_1 \leq \mu_L - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= i_1, \quad i_{L2-}^* = i_1, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, \quad i_{R2-}^* = \mu_R - B - \frac{c}{2},\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - \left(\frac{c}{2}\right)^2 - c\left(\mu_R - B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2 \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + cp - 2i_1 - 2(\mu_R - i_1)(p - 1) \\ &\geq 2\mu_R + cp - 2\left(\mu_L - B + \frac{c}{2}\right) + 2\left(\mu_R - \mu_L + B - \frac{c}{2}\right)(1 - p) \\ &= 8\Delta + 2cp + 4B - 2c - 4p\Delta - 2Bp > 0,\end{aligned}$$

since $B < \frac{c}{2}$ and $\frac{c}{2} < \Delta$. Then, the solution is the corner solution $i_1^*(c) = \mu_L - B + \frac{c}{2}$.

If $\mu_L - B + \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= i_1, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2},\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - \left(\frac{c}{2}\right)^2 - c\left(\mu_R - B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (\mu_L + \frac{c}{2} - \mu_R)^2 + c\left(\mu_L - B + \frac{c}{2} - i_1\right) \right] \\ &\quad - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + cp - 2i_1 - (p - 1)\left(B + \mu_R - \frac{c}{2} - i_1\right) \\ &\geq 2\mu_R + cp - 2\left(\mu_L + B + \frac{c}{2}\right) + (1 + p)\left(B + \mu_R - \frac{c}{2} - \left(\mu_L + B + \frac{c}{2}\right)\right) \\ &= 2(\mu_R - \mu_L) + cp - 2B - c + (1 + p)(\mu_R - \mu_L - c) \\ &= (3 + p)(\mu_R - \mu_L) - 2B - 2c \\ &= 6\Delta + 2p\Delta - 2B - 2c \geq 6\Delta + 2p\Delta - 3c > 0.\end{aligned}$$

Then, the solution is the corner solution $i_1^*(c) = \mu_L + B + \frac{c}{2}$.

If $\mu_L + B + \frac{c}{2} < i_1 \leq \mu_R - B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= \mu_L + B + \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2}, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - \left(\frac{c}{2}\right)^2 - c\left(\mu_R - B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c\left(\mu_L + B + \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c\left(\mu_L - B + \frac{c}{2} - i_1\right) \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + 2cp - c - 2i_1 \\ &\geq 2\mu_R + 2cp - c - 2\left(\mu_R - B - \frac{c}{2}\right) \\ &= 2cp + 2B > 0. \end{aligned}$$

Then, the solution is the corner solution $i_1^*(c) = \mu_R - B - \frac{c}{2}$.

If $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_R + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= \mu_L + B + \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c\left(\mu_L + B + \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c\left(\mu_L - B + \frac{c}{2} - i_1\right) \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \frac{cp}{2} - p(i_1 - \mu_R + B) - c(1-p) - 2(i_1 - \mu_R) \stackrel{!}{=} 0, \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -2 - p < 0.\end{aligned}$$

Then, the policy $i_1^*(c) = \mu_R + \frac{\frac{3cp}{2} - c - Bp}{(2+p)}$ is a candidate for a globally ideal policy and coincides with the policy for $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_R - B + \frac{c}{2}$ in Cases (i) and (ii). Since this candidate leads to a lower expected utility than the globally ideal policy, as shown in Inequality (B.3), the candidate can be neglected.

If $\mu_R + B - \frac{c}{2} < i_1 \leq \mu_R - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= \mu_L + B + \frac{c}{2}, \quad i_{L2-}^* = \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= i_1, \quad i_{R2-}^* = i_1,\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} [-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= -2p[i_1 - \mu_R] - c(1-p) - 2(i_1 - \mu_R) \stackrel{!}{=} 0, \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -2 - 2p < 0.\end{aligned}$$

Then, the policy $i_1^*(c) = \mu_R + \frac{c(p-1)}{2(1+p)} \in [\mu_R - \frac{c}{2}, \mu_R]$ is a potential candidate for the globally ideal policy. It follows that $\mu_R + \frac{c(p-1)}{2(1+p)} \leq \mu_R - B + \frac{c}{2}$ for $B < \frac{c}{2} \leq \Delta$. Further, it requires that $\mu_R + B - \frac{c}{2} < \mu_R + \frac{c(p-1)}{2(1+p)}$, which is only true for $\frac{c}{2} > \frac{B}{2} \left(1 + \frac{1}{p}\right)$.

Otherwise, i.e., if $\frac{c}{2} \leq \frac{B}{2} \left(1 + \frac{1}{p}\right)$, it is true that $\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} < 0$. Then,

$$\begin{aligned}
\mathbb{E}_R(u(i_1^*(c), c)) &= \frac{p}{2} \left[- \left(\frac{c(p-1)}{2(1+p)} - B \right)^2 - \left(\frac{c(p-1)}{2(1+p)} + B \right)^2 \right] \\
&\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c \left(\mu_L + B + \frac{c}{2} - \mu_R - \frac{c(p-1)}{2(1+p)} \right) \right] \\
&\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c \left(\mu_L - B + \frac{c}{2} - \mu_R - \frac{c(p-1)}{2(1+p)} \right) \right] \\
&\quad - \left(\frac{c(p-1)}{2(1+p)} \right)^2 \\
&= (\mu_R - \mu_L)^2(p-1) - B^2p + \frac{c^2(1-p)}{2(1+p)}. \tag{B.4}
\end{aligned}$$

If $\mu_R - B + \frac{c}{2} < i_1 \leq \mu_R + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}
i_{L2+}^* &= \mu_L + B + \frac{c}{2}, \quad i_{L2-}^* = \mu_L - B + \frac{c}{2}, \\
i_{R2+}^* &= i_1, \quad i_{R2-}^* = \mu_R - B + \frac{c}{2},
\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}
\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-(i_1 - \mu_R - B)^2 - \left(\frac{c}{2} \right)^2 + c(\mu_R - B + \frac{c}{2} - i_1) \right] \\
&\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\
&\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.
\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + c(p-1) - 2i_1 + p\left(\mu_R + B - \frac{c}{2} - i_1\right) \\ &\leq 2\mu_R + c(p-1) - 2\left(\mu_R - B + \frac{c}{2}\right) \\ &\quad + p\left(\mu_R + B - \frac{c}{2} - \mu_R + B - \frac{c}{2}\right) \\ &= -2c + 2B + 2Bp \\ &\leq -c + cp < 0. \end{aligned}$$

Then, the solution is the corner solution $i_1^*(c) = \mu_R - B + \frac{c}{2}$.

Consider (iv), where $\max\{B, \Delta - B\} < \frac{c}{2} \leq \Delta$.

The policy choices for the second period, and hence the choices of i_1 , are identical with (ii), except for $\mu_L - B + \frac{c}{2} < i_1 \leq \mu_R - B - \frac{c}{2}$ and $\mu_L + B + \frac{c}{2} < i_1 \leq \mu_R + B - \frac{c}{2}$.

If $\mu_L - B + \frac{c}{2} < i_1 \leq \mu_R - B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= i_1, \quad i_{L2-}^* = \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, \quad i_{R2-}^* = \mu_R - B - \frac{c}{2}, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - \left(\frac{c}{2}\right)^2 - c\left(\mu_R - B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} [-(i_1 - \mu_R - B)^2] \\ &\quad + \frac{(1-p)}{2} \left[-\left(\mu_L + \frac{c}{2} - \mu_R\right)^2 + c\left(\mu_L - B + \frac{c}{2} - i_1\right) \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R - 2i_1 + cp + \frac{(1-p)}{2}(2B + 2\mu_R - c - 2i_1) > 0, \\ &= 2\mu_R + cp - 2i_1 - (p-1)\left(B + \mu_R - \frac{c}{2} - i_1\right) \\ &\geq 2\mu_R + cp - 2\left(\mu_R - B - \frac{c}{2}\right) \\ &\quad + (1+p)\left(B + \mu_R - \frac{c}{2} - \left(\mu_R - B - \frac{c}{2}\right)\right) \\ &= cp + c + (2+p)2B > 0. \end{aligned}$$

Then, the ideal policy is defined by the corner solution $i_1^*(c) = \mu_R - B - \frac{c}{2}$.

If $\mu_L + B + \frac{c}{2} < i_1 \leq \mu_R + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= \mu_L + B + \frac{c}{2}, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-\left(\mu_L + \frac{c}{2} - \mu_R\right)^2 + c\left(\mu_L + B + \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-\left(\mu_L + \frac{c}{2} - \mu_R\right)^2 + c\left(\mu_L - B + \frac{c}{2} - i_1\right) \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \frac{cp}{2} - p(i_1 - \mu_R + B) - c(1-p) - 2(i_1 - \mu_R) \stackrel{!}{=} 0, \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -2 - p < 0. \end{aligned}$$

The policy $i_1^*(c) = \mu_R + \frac{3cp - c - Bp}{(2+p)}$ coincides with the policy for $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_R - B + \frac{c}{2}$ in Cases (i) and (ii) and for $\mu_R - B - \frac{c}{2} \leq i_1 \leq \mu_R + B - \frac{c}{2}$ in Case (iii). Since this policy leads to a lower expected utility, as shown in Inequality (B.3), it can be neglected.

Consider (v), where $\Delta < \frac{c}{2} \leq \Delta + B$.

The policy choices for the second period, and hence the choices of i_1 , are identical with (iii), except for $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$.

If $\mu_L + B - \frac{c}{2} < i_1 \leq \mu_R - B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= i_1, & i_{L2-}^* &= i_1, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= \mu_R - B - \frac{c}{2}, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - \left(\frac{c}{2}\right)^2 - c\left(\mu_R - B - \frac{c}{2} - i_1\right) \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2 \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R + cp - 2i_1 - 2(\mu_R - i_1)(p-1) \\ &\geq 2\mu_R - 2\left(\mu_R - B - \frac{c}{2}\right) + cp - 2\left(\mu_R - \mu_R + B + \frac{c}{2}\right)(p-1) \\ &\geq 2B + c + cp - (2B + c)(p-1) \\ &= 4B - 2Bp + 2c > 0. \end{aligned}$$

Then, the ideal policy is the corner solution $i_1^*(c) = \mu_R - B - \frac{c}{2}$.

If $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_L - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= i_1, & i_{L2-}^* &= i_1, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2 \right] - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \mu_R(4-p) - Bp + \frac{cp}{2} + i_1(p-4) \\ &\geq \mu_R(4-p) - Bp + \frac{cp}{2} + \left(\mu_L - B + \frac{c}{2}\right)(p-4) \\ &= (\mu_R - \mu_L)(4-p) - 2Bp + 4B + cp - 2c \\ &= 8\Delta - 2p\Delta - 2Bp + 4B + cp - 2c \\ &= \underbrace{2\Delta - 2p\Delta}_{>0} + \underbrace{6\Delta + 3B - 2c}_{=:I>0} + \underbrace{B + cp - 2Bp}_{>0} > 0,\end{aligned}$$

where $I > 0$, since $3\Delta + 1.5B > 2\Delta + 2B > c$. Then, the ideal policy is the corner solution $i_1^*(c) = \mu_L - B + \frac{c}{2}$.

If $\mu_L - B + \frac{c}{2} < i_1 \leq \mu_R + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= i_1, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1,\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c(\mu_R + B - \frac{c}{2} - i_1) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] \\ &\quad - (i_1 - \mu_R)^2.\end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 3\mu_R + B(1 - 2p) + c(p - \frac{1}{2}) - 3i_1 \\ &> 3\mu_R + B(1 - 2p) + c(p - \frac{1}{2}) - 3\left(\mu_R + B - \frac{c}{2}\right) \\ &= (1 + p)(2B - c) > 0,\end{aligned}$$

since $\frac{c}{2} > B$. Then, the ideal policy the corner solution $i_1^*(c) = \mu_R + B - \frac{c}{2}$.

If $\mu_R + B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}i_{L2+}^* &= i_1, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= i_1, & i_{R2-}^* &= i_1,\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2 \right] \\ &+ \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] \\ &- (i_1 - \mu_R)^2.\end{aligned}$$

Consider the FOC and SOC of the utility with respect to i_1

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \mu_R(3 + p) - i_1(3 + p) - (p - 1)(B - \frac{c}{2}), \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -(3 + p) < 0.\end{aligned}$$

Then, the policy $i_1^*(c) = \mu_R + \frac{(1-p)(B-\frac{c}{2})}{(3+p)} \in [\mu_R + \frac{B-\frac{c}{2}}{3}, \mu_R]$ is a candidate for the globally ideal policy. It follows that $\mu_R + B - \frac{c}{2} < \mu_R - \frac{(p-1)(B-\frac{c}{2})}{(3+p)}$ for $B < \frac{c}{2}$. Furthermore, it is necessary that $\mu_R - \frac{(p-1)(B-\frac{c}{2})}{(3+p)} < \mu_L + B + \frac{c}{2}$, i.e., $\frac{1}{2}(\Delta(3+p) - B(1+p)) < \frac{c}{2}$.

Otherwise, it follows that $\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} > 0$. Then

$$\begin{aligned}
\mathbb{E}_R(u(i_1^*(c), c^*)) &= \frac{p}{2} \left[- \left(\frac{(1-p)(B - \frac{c}{2})}{(3+p)} - B \right)^2 - \left(\frac{(1-p)(B - \frac{c}{2})}{(3+p)} + B \right)^2 \right] \\
&\quad + \frac{(1-p)}{2} \left[- \left(\frac{(1-p)(B - \frac{c}{2})}{(3+p)} - B \right)^2 - \left(\mu_L + \frac{c}{2} - \mu_R \right)^2 \right] \\
&\quad + \frac{(1-p)}{2} \left[c \left(\mu_L - B + \frac{c}{2} - \mu_R - \frac{(1-p)(B - \frac{c}{2})}{(3+p)} \right) \right] \\
&\quad - \left(\frac{(1-p)(B - \frac{c}{2})}{(3+p)} \right)^2 \\
&= \frac{1}{2}(p-1)(\mu_R - \mu_L)^2 + \frac{(-B^2(1+3p) + (1-p)c(\frac{c}{2} - 2B))}{(3+p)}.
\end{aligned} \tag{B.5}$$

If $\mu_L + B + \frac{c}{2} < i_1 \leq \mu_R - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned}
i_{L2+}^* &= \mu_L + B + \frac{c}{2}, \quad i_{L2-}^* = \mu_L - B + \frac{c}{2}, \\
i_{R2+}^* &= i_1, \quad i_{R2-}^* = i_1,
\end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned}
\mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2 \right] \\
&\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L + B + \frac{c}{2} - i_1) \right] \\
&\quad + \frac{(1-p)}{2} \left[-(\mu_L + \frac{c}{2} - \mu_R)^2 + c(\mu_L - B + \frac{c}{2} - i_1) \right] - (i_1 - \mu_R)^2.
\end{aligned}$$

Consider the FOC and SOC of the utility with respect to i_1

$$\begin{aligned}
\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= -2p(i_1 - \mu_R) - c(1-p) - 2(i_1 - \mu_R), \\
\frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -2p - 2 < 0.
\end{aligned}$$

Then, the policy $i_1^*(c) = \mu_R + \frac{c(p-1)}{2(1+p)} \in [\mu_R - \frac{c}{2}, \mu_R]$ is a candidate for the globally

ideal policy. It follows that $\mu_R + \frac{c(p-1)}{2(1+p)} < \mu_R - B + \frac{c}{2}$, since $\frac{c}{2} > B$. Furthermore, it is necessary that $\mu_L + B + \frac{c}{2} < \mu_R + \frac{c(p-1)}{2(1+p)}$, which is true for $\frac{c}{2} \leq \frac{1}{2}(2\Delta - B)(1+p)$. For $\frac{c}{2} > \frac{1}{2}(2\Delta - B)(1+p)$, it is true that

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 2\mu_R(1+p) + c(p-1) - 2i_1(1+p) \\ &\leq 2\mu_R(1+p) + c(p-1) - 2\left(\mu_L + B + \frac{c}{2}\right)(1+p) \\ &= 4\Delta(1+p) - 2c - 2B - 2Bp \\ &= 2((2\Delta - B)(1+p) - c) \\ &\leq 2(2(2\Delta - B) - c) < 0. \end{aligned}$$

Hence, for $\frac{c}{2} > \frac{1}{2}(2\Delta - B)(1+p)$, the ideal policy is the corner solution $i_1^*(c) = \mu_L + B + \frac{c}{2}$.

Next, we compare the two expected utilities, given by Equations (B.4) and (B.5), of both potential ideal policies for $p \in (0, 1]$.

$$\mathbb{E}_R \left(u \left(\mu_R + \frac{(1-p)(B - \frac{c}{2})}{(3+p)}, c \right) \right) \leq \mathbb{E}_R \left(u \left(\mu_R + \frac{c(p-1)}{2(1+p)}, c \right) \right), \quad (\text{B.6})$$

$$\begin{aligned} \frac{(-B^2(1+3p) + (1-p)c(\frac{c}{2} - 2B))}{(3+p)} &\leq \frac{1}{2}(p-1)(\mu_R - \mu_L)^2 - B^2p + \frac{c^2(1-p)}{2(1+p)}, \\ \frac{(-B^2(1+3p) + (1-p)c(\frac{c}{2} - 2B)) \cdot 2(1+p)}{(3+p)} &\leq (p^2 - 1)(\mu_R - \mu_L)^2 - 2B^2p(1+p) + c^2(1-p), \end{aligned}$$

which can be simplified to

$$\begin{aligned} (p^2 - 1)(\mu_R - \mu_L)^2 + 2B^2 + 2c^2 + c^2p + 2B^2p(1-p) - 2B^2p^3 + 4Bc - 4Bcp^2 &\geq 0, \\ \underbrace{(p^2 - 1)(\mu_R - \mu_L)^2 + c^2}_{>0} + 2B^2p(1-p) + 2B^2(1-p^3) + c^2 + c^2p + 4Bc(1-p) &\geq 0, \end{aligned}$$

since $c \geq 2\Delta$.

Therefore, the chosen policy for this interval is defined by

$$i_1 = \max \left\{ \mu_L + B + \frac{c}{2}, \mu_R + \frac{c(p-1)}{2(1+p)} \right\}.$$

This is equivalent to

$$i_{R1}^*(c, B) := \begin{cases} \mu_R + \frac{c(p-1)}{2(1+p)} & \text{if } (1+p)B \leq c \leq (2\Delta - B)(1+p), \\ \mu_L + B + \frac{c}{2} & \text{if } (2\Delta - B)(1+p) < c \leq 4\Delta - 2B. \end{cases}$$

Note that the threshold $(1+p)B$ is defined, since then $\mu_R + \frac{c(p-1)}{2(1+p)} = \mu_R - B + \frac{c}{2}$ and the threshold $4\Delta - 2B$, since then $\mu_R = \mu_L + B - \frac{c}{2}$.

Consider (vi), where $\Delta + B < \frac{c}{2}$.

The policy choices for the second period, and hence the choices of i_1 , are identical with (iv), except for $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$.

If $\mu_R - B - \frac{c}{2} < i_1 \leq \mu_R + B - \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= i_1, & i_{L2-}^* &= i_1, \\ i_{R2+}^* &= \mu_R + B - \frac{c}{2}, & i_{R2-}^* &= i_1, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} \left[-\left(\frac{c}{2}\right)^2 - c\left(\mu_R + B - \frac{c}{2} - i_1\right) - (i_1 - \mu_R + B)^2 \right] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2 \right] - (i_1 - \mu_R)^2. \end{aligned}$$

Then, the first order partial derivative of $\mathbb{E}_R(u(i_1, c))$ w.r.t. i_1 is

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \mu_R(4-p) - Bp + \frac{cp}{2} + i_1(p-4) \\ &\geq \mu_R(4-p) - Bp + \frac{cp}{2} + \left(\mu_R + B - \frac{c}{2}\right)(p-4) \\ &\geq -Bp + \frac{cp}{2} + \left(B - \frac{c}{2}\right)(p-4) \\ &\geq 2c - 4B > 0. \end{aligned}$$

Then, the ideal policy is the corner solution $i_1^*(c) = \mu_R + B - \frac{c}{2}$.

If $\mu_R + B - \frac{c}{2} < i_1 \leq \mu_L - B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= i_1, & i_{L2-}^* &= i_1, \\ i_{R2+}^* &= i_1, & i_{R2-}^* &= i_1, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} [-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2] \\ &\quad + \frac{(1-p)}{2} [-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2] - (i_1 - \mu_R)^2. \end{aligned}$$

Consider the FOC and SOC of the utility with respect to i_1

$$\begin{aligned} \frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= 4\mu_R - 4i_1, \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -4 < 0. \end{aligned}$$

Then, a candidate for the globally ideal policy is $i_1^*(c) = \mu_R$. Then,

$$\mathbb{E}_R(u(i_1^*(c), c)) = -B^2.$$

If $\mu_L - B + \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$, the policy choices of party $K \in \{L, R\}$ are

$$\begin{aligned} i_{L2+}^* &= i_1, & i_{L2-}^* &= \mu_L - B + \frac{c}{2}, \\ i_{R2+}^* &= i_1, & i_{R2-}^* &= i_1, \end{aligned}$$

which lead to the following utility of party R :

$$\begin{aligned} \mathbb{E}_R(u(i_1, c)) &= \frac{p}{2} [-(i_1 - \mu_R - B)^2 - (i_1 - \mu_R + B)^2] \\ &\quad + \frac{(1-p)}{2} \left[-(i_1 - \mu_R - B)^2 - \left(\mu_L + \frac{c}{2} - \mu_R \right)^2 + c \left(\mu_L - B + \frac{c}{2} - i_1 \right) \right] \\ &\quad - (i_1 - \mu_R)^2. \end{aligned}$$

Consider the FOC and SOC of the utility with respect to i_1

$$\begin{aligned}\frac{\partial \mathbb{E}_R(u(i_1, c))}{\partial i_1} &= \mu_R(3+p) - i_1(3+p) - (p-1)\left(B - \frac{c}{2}\right), \\ \frac{\partial^2 \mathbb{E}_R(u(i_1, c))}{\partial i_1^2} &= -(3+p) < 0.\end{aligned}$$

Then, a candidate for the globally ideal policy $i_1^*(c) = \mu_R + \frac{(1-p)(B-\frac{c}{2})}{(3+p)}$, which coincides with the policy for $\mu_R + B - \frac{c}{2} < i_1 \leq \mu_L + B + \frac{c}{2}$ in Case (v) and leads to a lower expected utility, as shown in Inequality (B.6), and hence can be neglected.

Chapter C

Appendix to Chapter 4

C.1 Proof of Proposition 4

For all voters, the beliefs are equal to the expert's, i.e., $b_i = b^E$, and the distribution of risk aversion is given by $k_i \sim \mathcal{U}[0, 1]$. Then, the cumulative distribution function for the ratio $Y := \frac{1}{k_i}$ is defined by

$$F_Y(y) = 1 - y^{-1}.$$

The median value y_{med} is the smallest value satisfying

$$F_Y(y) \geq \frac{1}{2}.$$

Hence, $y_{\text{med}} = 2$, which leads to $(k_i)_{\text{med}} = \frac{1}{2}$. This means that

$$z_{\text{med}} := \left(\frac{b_i}{k_i} \right)_{\text{med}} = \frac{b^E}{(k_i)_{\text{med}}} = 2b^E,$$

which is equivalent to

$$p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) = p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \frac{1}{\lambda^2}, 0 \right\} = \max \left\{ 1 - \frac{2b^E}{\lambda^2}, 0 \right\},$$

as stated in the proposition.

C.2 Proof of the Single-Crossing Property (SPC) of Section 4.4.2

The preferences of voters satisfy the SCP when the following statement is true:

Let $i, j \in I$ and $p, p' \in [0, 1]$ be such that $p < p'$ and $t_j > t_i$. Then, it must hold that

$$u_i(p') > u_i(p) \Rightarrow u_j(p') > u_j(p) \quad (\text{C.1})$$

and

$$u_j(p) > u_j(p') \Rightarrow u_i(p) > u_i(p'), \quad (\text{C.2})$$

where a higher type t_j is associated with a lower ratio $r_j = b_j/k_j$, such that $r_j < r_i$.

The utility function is $u_i(p) = -\exp\left(k_i b_i p + \frac{k_i^2 \lambda^2 (1-p)^2}{2}\right)$ and can be rewritten as

$$u_i(p) = -\exp\left(\frac{b_i^2}{2r_i^2} \cdot (2r_i \cdot p + \lambda^2 \cdot (1-p)^2)\right).$$

First, consider (C.1). Then, $u_i(p') > u_i(p)$ is equivalent to

$$2r_i < \lambda^2(2 - p - p').$$

Using $r_j < r_i$, we obtain

$$2r_j < \lambda^2(2 - p - p'),$$

which implies that $u_j(p') > u_j(p)$.

Second, consider (C.2). Then, $u_j(p) > u_j(p')$ is equivalent to

$$2r_j > \lambda^2(2 - p - p').$$

Using $r_j < r_i$, we obtain

$$2r_i > \lambda^2(2 - p - p'),$$

which implies that $u_i(p) > u_i(p')$.

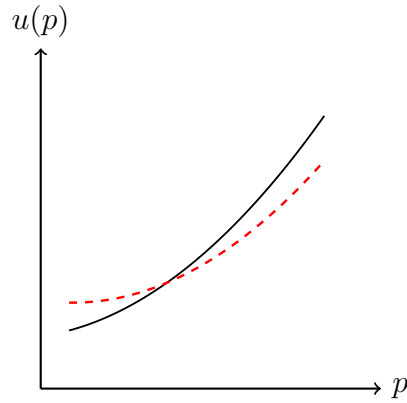


Figure C.1: A schematic representation of two utility functions that fulfill the SCP.

C.3 Proof of Proposition 5

The electorate consists of two groups of voters. For the group of non-opinionated voters, with share η , both parameters $b_i \sim \mathcal{U}[0, 1]$ and $k_i \sim \mathcal{U}[0, 1]$ are drawn independently from a uniform distribution on $[0, 1]$. Then, the cumulative distribution function for the ratio $Z := \frac{b_i}{k_i}$ for any individual i of this group of voters is

$$F_Z(z) = \mathbb{P}(Z \leq z) = \mathbb{P}(b_i \leq zk_i) = \int_0^1 \int_0^1 \mathbb{1}(b_i \leq zk_i) db_i dk_i = \int_0^1 \min\{1, zk_i\} dk_i,$$

where $\mathbb{1}(b_i \leq zk_i) = 1$ if $b_i \leq zk_i$ and $\mathbb{1}(b_i \leq zk_i) = 0$. On the one hand, if $z \leq 1$, we obtain

$$\mathbb{P}(Z \leq z) = \int_0^1 zk_i dk_i = \frac{z}{2}.$$

On the other hand, if $z > 1$, we obtain

$$\mathbb{P}(Z \leq z) = \int_0^{1/z} zk_i dk_i + \int_{1/z}^1 1 dk_i = 1 - \frac{1}{2z}.$$

For the $1 - \eta$ share of opinionated voters, the ratio has a fixed value $\frac{b_i}{k_i} = \beta$, where $0 \leq \beta \leq 1$. Then, the cumulative distribution function of the ratio $Y := \beta$

for this fraction of voters is

$$F_Y(z) = \begin{cases} 1 & , z \geq \beta. \\ 0 & , z < \beta. \end{cases}$$

Consider now the joint cumulative probability in order to find the median ratio $\left(\frac{b_i}{k_i}\right)_{\text{med}}$, which is defined as the smallest value satisfying

$$\eta F_Z(z) + (1 - \eta)F_Y(z) \geq \frac{1}{2}.$$

First, let $z < \beta$. Then, the above equation reduces to

$$\eta F_Z(z) + (1 - \eta)F_Y(z) = \eta \frac{z}{2} = \frac{1}{2},$$

or, equivalently, $z = 1/\eta$. But since $1/\eta \geq 1 > \beta$, this solution is not feasible. That is, $\eta F_Z(z) + (1 - \eta)F_Y(z) < 1/2$ for all $z < \beta$.

Second, let $\beta \leq z \leq 1$. Then, the above equation reduces to

$$\eta F_Z(z) + (1 - \eta)F_Y(z) = \eta \frac{z}{2} + (1 - \eta) = \frac{1}{2},$$

or, equivalently, $z = 2 - 1/\eta$. This requires $2 - 1/\eta \geq \beta$. If the latter condition does not hold, then $\eta F_Z(z) + (1 - \eta)F_Y(z) > 1/2$ for all $\beta \leq z \leq 1$.

Third, let $1 < z$. Then, the above equation reduces to

$$\eta F_Z(z) + (1 - \eta)F_Y(z) = \eta \left(1 - \frac{1}{2z}\right) + (1 - \eta) = \frac{1}{2},$$

or, equivalently, $z = \eta$. But since $\eta \leq 1 < z$, this solution is not feasible. That is, $\eta F_Z(z) + (1 - \eta)F_Y(z) > 1/2$ for all $z > 1$.

To sum up,

$$\left(\frac{b_i}{k_i}\right)_{\text{med}} = \max \left\{ \beta, 2 - \frac{1}{\eta} \right\},$$

which leads to the following policy, as defined in Equation (4.2):

$$p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) = p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \frac{1}{\lambda^2}, 0 \right\} = 1 - \frac{\max \left\{ \beta, 2 - \frac{1}{\eta} \right\}}{\lambda^2},$$

as stated in the proposition.

C.4 Proof of Proposition 6

For an η share of voters, i.e., the non-opinionated voters, the representative expert decides. Hence, the η share of the electorate has $b_i = b^E$, and the distribution of risk aversion is given by $k^E \sim \mathcal{U}[\underline{\phi}, \bar{\phi}]$. Then, the cumulative distribution¹ function for the ratio $Z := \frac{b^E}{k^E} \in \left[\frac{b^E}{\bar{\phi}}, \frac{b^E}{\underline{\phi}} \right]$ is given by

$$F_Z(z) = \begin{cases} 0 & z \leq \frac{b^E}{\bar{\phi}}. \\ \frac{\bar{\phi} - b^E/z}{\bar{\phi} - \underline{\phi}} & \frac{b^E}{\bar{\phi}} < z \leq \frac{b^E}{\underline{\phi}}. \\ 1 & \frac{b^E}{\underline{\phi}} < z. \end{cases}$$

For the remaining $(1 - \eta)$ share of opinionated voters, the ratio has a fixed value $\frac{b_i}{k_i} = \beta$, where $0 \leq \beta \leq 1$. Then, the cumulative distribution function of the ratio $Y := \beta$ for this fraction of voters is

$$F_Y(z) = \begin{cases} 1 & , z \geq \beta. \\ 0 & , z < \beta. \end{cases}$$

Consider now the joint cumulative probability to find the median ratio $\left(\frac{b_i}{k_i} \right)_{\text{med}}$. In general, by definition, the median x_{med} of a random variable X consists of all points x , such that

$$\text{Prob}(X \leq x) \geq \frac{1}{2} \quad \text{and} \quad \text{Prob}(X \geq x) \geq \frac{1}{2}.$$

¹The cumulative distribution function of the reciprocal $Y := \frac{1}{X}$ of a uniformly distributed variable $X \sim \mathcal{U}[a, b]$ is given by $G(y) = \frac{b-y^{-1}}{b-a}$.

For the present setting, this definition is equivalent to the following inequality:

$$\eta F_Z(z) + (1 - \eta) F_Y(z) \geq \frac{1}{2}. \quad (\text{C.3})$$

Note that for $0 < \eta \leq \frac{1}{2}$, the opinionated voters outweigh, and hence, $z_{\text{med}} = \beta$. We distinguish three different cases, depending on the value of β and $\frac{b^E}{\mathbb{E}(k^E)}$. Note that not all cases are feasible for all values of b^E . In each case, consider four different ranges of z to find the median values.

Case 1: $\beta \leq \frac{b^E}{\phi}$.

(i) For $0 < z < \beta < \frac{b^E}{\phi}$, Inequality (C.3) is equivalent to

$$0 \geq \frac{1}{2},$$

which states a contradiction. Hence, there is no feasible solution.

(ii) For $0 < \beta \leq z < \frac{b^E}{\phi}$, the median value fulfills

$$\frac{1}{2} \geq \eta.$$

In this case, $z_{\text{med}} = \beta$.

(iii) For $0 < \beta < \frac{b^E}{\phi} \leq z < \frac{b^E}{\underline{\phi}}$, Inequality (C.3) can be written as

$$\eta \cdot \frac{\bar{\phi} - \frac{b^E}{z}}{\bar{\phi} - \underline{\phi}} + (1 - \eta) \geq \frac{1}{2},$$

$$z \geq \frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})}.$$

If $z_{\text{med}} = \frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})}$, then, by assumptions of this case, $\frac{b^E}{\phi} \leq z_{\text{med}} < \frac{b^E}{\underline{\phi}}$ must

be fulfilled. Consider the first inequality

$$\frac{b^E}{\bar{\phi}} \leq \frac{2\eta b^E}{2\eta\underline{\phi} + (\bar{\phi} - \underline{\phi})},$$

$$\frac{1}{2} \leq \eta.$$

Then, the second inequality is equivalent to

$$\frac{2\eta b^E}{2\eta\underline{\phi} + (\bar{\phi} - \underline{\phi})} < \frac{b^E}{\underline{\phi}},$$

$$0 < (\bar{\phi} - \underline{\phi}),$$

which is true for all values of η . Hence, $z_{med} = \frac{2\eta b^E}{2\eta\underline{\phi} + (\bar{\phi} - \underline{\phi})}$ for $\frac{1}{2} \leq \eta \leq 1$.

(iv) For $0 < \beta < \frac{b^E}{\bar{\phi}} \leq \frac{b^E}{\underline{\phi}} \leq z$, Inequality (C.3) can be written as

$$1 \geq \frac{1}{2}.$$

Then, $z = \frac{b^E}{\bar{\phi}}$ fulfills Inequality (C.3) and it follows that this is true for all $0 \leq \eta \leq 1$. But there exist smaller values for z that fulfill the inequality.

Case 2: $\frac{b^E}{\bar{\phi}} < \beta < \frac{b^E}{\underline{\phi}}$.

Since $\bar{\phi} < 1$, it follows that $1 < \frac{\beta}{b^E}$, i.e., $b^E < \beta$.

(i) For $0 < z < \frac{b^E}{\bar{\phi}} \leq \beta$, Inequality (C.3) is equivalent to

$$0 \geq \frac{1}{2},$$

which states a contradiction. Hence, there is no feasible solution.

(ii) For $0 < \frac{b^E}{\phi} \leq z < \beta$, the median value fulfills

$$\eta \cdot \frac{\bar{\phi} - \frac{b^E}{z}}{\bar{\phi} - \underline{\phi}} \geq \frac{1}{2},$$

$$z \geq \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})}.$$

If $z_{med} = \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})}$, then, by assumptions of this case, $\frac{b^E}{\phi} \leq z_{med} < \beta$ must be fulfilled. Consider the first inequality

$$\frac{b^E}{\bar{\phi}} \leq \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})},$$

$$0 \leq (\bar{\phi} - \underline{\phi}),$$

which is true for all values of η . Next, consider the second inequality

$$\frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})} < \beta,$$

$$\eta > \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \underline{\phi})}{\beta\bar{\phi} - b^E},$$

since $\frac{b^E}{\phi} < \beta$. Then, $z_{med} = \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})}$ for $\frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \underline{\phi})}{\beta\bar{\phi} - b^E} < \eta \leq 1$.

(iii) For $\beta \leq z < \frac{b^E}{\phi}$, the median value fulfills

$$\eta \cdot \frac{\bar{\phi} - \frac{b^E}{z}}{\bar{\phi} - \underline{\phi}} + (1 - \eta) \geq \frac{1}{2},$$

$$z \geq \frac{2\eta b^E}{2\eta\underline{\phi} + (\bar{\phi} - \underline{\phi})}.$$

If $z_{med} = \frac{2\eta b^E}{2\eta\underline{\phi} + (\bar{\phi} - \underline{\phi})}$, then, by assumptions of this case, $\beta \leq z_{med} < \frac{b^E}{\phi}$ must be

fulfilled. Consider the first inequality

$$\beta \leq \frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})},$$

$$\frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \underline{\phi})}{b^E - \beta \underline{\phi}} \leq \eta,$$

where $\frac{\beta(\bar{\phi} - \underline{\phi})}{b^E - \beta \underline{\phi}} > 1$, since $\beta > \frac{b^E}{\phi}$. Consider the second inequality

$$\frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})} < \frac{b^E}{\underline{\phi}},$$

$$0 < (\bar{\phi} - \underline{\phi}),$$

which is true for all values of η . Then, $z_{med} = \frac{2\eta b^E}{2\eta \underline{\phi} + (\bar{\phi} - \underline{\phi})}$ for $\frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \underline{\phi})}{b^E - \beta \underline{\phi}} \leq \eta \leq 1$.

(iv) For $0 < \frac{b^E}{\phi} \leq \beta \leq \frac{b^E}{\underline{\phi}} \leq z$, Inequality (C.3) can be written as

$$1 \geq \frac{1}{2},$$

which is fulfilled for all $0 \leq \eta \leq 1$. Hence, $z_{med} = \frac{b^E}{\underline{\phi}}$ fulfills the inequality, but there already exist smaller values satisfying it.

Case 3: $\frac{b^E}{\phi} \leq \beta$.

(i) For $0 < z < \frac{b^E}{\phi} \leq \beta$, Inequality (C.3) is equivalent to

$$0 \geq \frac{1}{2},$$

which states a contradiction. Hence, there is no feasible solution.

(ii) For $0 < \frac{b^E}{\bar{\phi}} \leq z < \frac{b^E}{\underline{\phi}}$, the median value fulfills

$$\eta \cdot \frac{\bar{\phi} - \frac{b^E}{z}}{\bar{\phi} - \underline{\phi}} \geq \frac{1}{2},$$

$$z \geq \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})}.$$

If $z_{med} = \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})}$, then, by assumptions of this case, $\frac{b^E}{\bar{\phi}} \leq z_{med} < \frac{b^E}{\underline{\phi}}$ must be fulfilled. Consider the second inequality

$$\frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})} < \frac{b^E}{\underline{\phi}},$$

$$\frac{1}{2} \leq \eta,$$

and the first condition

$$\frac{b^E}{\bar{\phi}} \leq \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})},$$

$$0 \leq (\bar{\phi} - \underline{\phi}).$$

Then, $z_{med} = \frac{2\eta b^E}{2\eta\bar{\phi} - (\bar{\phi} - \underline{\phi})}$ for $\frac{1}{2} \leq \eta \leq 1$.

(iii) For $\frac{b^E}{\underline{\phi}} \leq z < \beta$, the median value fulfills

$$\eta \geq \frac{1}{2}.$$

Then, $z_{med} = \frac{b^E}{\underline{\phi}}$.

(iv) For $\frac{b^E}{\underline{\phi}} < \beta \leq z$, the median value fulfills

$$\eta + (1 - \eta) \geq \frac{1}{2},$$

$$1 \geq \frac{1}{2},$$

which is fulfilled for all values of η .

Then, for $\beta \leq \frac{b^E}{\phi}$, as shown in Case 1 (ii) and Case 1 (iii), the median values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{1}{2}, \\ \frac{2\eta b^E}{2\eta\phi + (\phi - \phi)} & \text{if } \frac{1}{2} \leq \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$\begin{aligned} p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) &= p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \cdot \frac{1}{\lambda^2}, 0 \right\} \\ &= \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\phi + (\phi - \phi)}, 0 \right\} & \text{if } \frac{1}{2} \leq \eta \leq 1. \end{cases} \end{aligned}$$

The result is stated in Part (i) of the Proposition. Then, for $\frac{b^E}{\phi} < \beta < \frac{b^E}{\phi}$, as shown in Case 2 (ii) and (iii), the median values of z take different values, depending on the value of b^E . First, consider Case 2 (ii). The condition $\frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E} < 1$ implies that $b^E < \frac{\beta(\bar{\phi} + \phi)}{2}$, i.e., $\frac{b^E}{\mathbb{E}(k^E)} < \beta$. Then, the median values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E}, \\ \frac{2\eta b^E}{2\eta\phi - (\phi - \phi)} & \text{if } \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E} \leq \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$\begin{aligned} p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) &= p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \cdot \frac{1}{\lambda^2}, 0 \right\} \\ &= \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\phi - (\phi - \phi)}, 0 \right\} & \text{if } \frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{\beta\phi - b^E} \leq \eta \leq 1, \end{cases} \end{aligned}$$

which are stated in Part (iii) of the proposition. Second, consider Case 2 (iii). The condition $\frac{1}{2} \cdot \frac{\beta(\bar{\phi} - \phi)}{b^E - \beta\phi} \leq 1$ implies that $b^E \geq \frac{\beta(\bar{\phi} + \phi)}{2}$, i.e., $\frac{b^E}{\mathbb{E}(k^E)} \geq \beta$. Then, the median

values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\underline{\phi})}{b^E-\beta\underline{\phi}}, \\ \frac{2\eta b^E}{2\eta\underline{\phi}+(\bar{\phi}-\underline{\phi})} & \text{if } \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\underline{\phi})}{b^E-\beta\underline{\phi}} \leq \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$\begin{aligned} p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) &= p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \cdot \frac{1}{\lambda^2}, 0 \right\} \\ &= \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\underline{\phi})}{\beta\underline{\phi}-b^E}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\underline{\phi}+(\bar{\phi}-\underline{\phi})}, 0 \right\} & \text{if } \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\underline{\phi})}{\beta\underline{\phi}-b^E} \leq \eta \leq 1. \end{cases} \end{aligned}$$

The result is stated in Part (ii) of the proposition. Then, as shown in Case 3 (ii), the median values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{1}{2}, \\ \frac{2\eta b^E}{2\eta\underline{\phi}-(\bar{\phi}-\underline{\phi})} & \text{if } \frac{1}{2} \leq \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$\begin{aligned} p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) &= p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \cdot \frac{1}{\lambda^2}, 0 \right\} \\ &= \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\underline{\phi}-(\bar{\phi}-\underline{\phi})}, 0 \right\} & \text{if } \frac{1}{2} \leq \eta \leq 1. \end{cases} \end{aligned}$$

The result is stated in Part (iv) of the proposition.

C.5 Proof of Proposition 7

In this form of voting, the electorate consists of two parts. The η share of the non-opinionated voters updates its belief, but its risk aversion is unchanged. This

means, for the non-opinionated voters,

$$k_i \sim \mathcal{U}[0, 1],$$

$$b'_i = \kappa b_i + (1 - \kappa)b^E \sim \mathcal{U}[(1 - \kappa)\gamma, (1 - \kappa)\gamma + \kappa], \text{ where } \kappa \in [0, 1] \text{ and } \gamma \in [0, 1].$$

To simplify the calculations, we set $\tau = (1 - \kappa)\gamma$, then $b'_i \sim \mathcal{U}[\tau, \tau + \kappa]$. Then, the cumulative distribution function for the ratio $Z := \frac{b'_i}{k_i}$ for this fraction of voters is

$$F_Z(z) = \mathbb{P}(Z \leq z) = \mathbb{P}(b'_i \leq zk_i) = \mathbb{E}_{k_i}(\mathbb{P}(b'_i \leq zk_i \mid k_i)) = \mathbb{E}_{k_i}(F_{b'_i}(zk_i)).$$

Clearly, $F_Z(z) = 0$ for $z \leq \tau$. Therefore, we assume $z > \tau$, then

$$\begin{aligned} F_Z(z) &= \int_0^1 \begin{cases} 0 & dk_i, & , zk_i \leq \tau \\ \frac{zk_i - \tau}{\kappa} & dk_i, & , \tau < zk_i \leq \tau + \kappa \\ 1 & dk_i, & , zk_i > \tau + \kappa \end{cases} \\ &= \begin{cases} \int_{\frac{\tau}{z}}^1 \frac{zk_i - \tau}{\kappa} dk_i & , \tau < z < \tau + \kappa \\ \int_{\frac{\tau}{z}}^{\frac{\tau + \kappa}{z}} \frac{zk_i - \tau}{\kappa} dk_i + \int_{\frac{\tau + \kappa}{z}}^1 1 dk_i & , \tau < \tau + \kappa < z \end{cases} \\ &= \begin{cases} \frac{(z - \tau)^2}{2\kappa z} & , \tau < z \leq \tau + \kappa \\ 1 - \frac{\kappa}{2z} - \frac{\tau}{z} & , z > \tau + \kappa. \end{cases} \end{aligned}$$

The second group of the voters, the $(1 - \eta)$ share of the electorate, has a fixed ratio $\frac{b_i}{k_i} = \beta$, where $0 \leq \beta \leq 1$. Then, the cumulative distribution function for the ratio $Y := \beta$ for this fraction of voters is

$$F_Y(z) = \begin{cases} 1 & , z \geq \beta. \\ 0 & , z < \beta. \end{cases}$$

The following equation states the ideal policy, which is taken by the median voter:

$$p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) = \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \left(\frac{b_i}{k_i} \right)_{\text{med}}, 0 \right\},$$

and depends on the median value of the ratio $\frac{b_i}{k_i}$. Therefore, consider now the joint cumulative probability, in order to find the median ratio, which is the smallest value

of z satisfying

$$\eta F_Z(z) + (1 - \eta)F_Y(z) \geq \frac{1}{2}. \quad (\text{C.4})$$

Note that for $0 < \eta \leq \frac{1}{2}$, the opinionated voters outweigh and hence $z_{\text{med}} = \beta$ for $0 < \eta \leq \frac{1}{2}$. We distinguish three different cases, depending on the size of β in relation to τ and $\tau + \kappa$.

Case 1: $\beta < \tau < \tau + \kappa$.

(i) $z < \beta$

For this range of z , there exists no median value for z satisfying

$$\eta F_Z(z) + (1 - \eta)F_Y(z) \geq \frac{1}{2},$$

since $F_Z(z) = 0$ and $F_Y(z) = 0$.

(ii) $\beta \leq z < \tau < \tau + \kappa$

For this range of z , Inequality (C.4) is equivalent to

$$(1 - \eta) \geq \frac{1}{2}.$$

If $0 \leq \eta \leq \frac{1}{2}$, then $z_{\text{med}} = \beta$. Otherwise, i.e., for $\frac{1}{2} < \eta \leq 1$, there exists no feasible median value for z .

(iii) $\beta < \tau < z < \tau + \kappa$

In this setting, the median value of z has to fulfill the following inequality:

$$\frac{\eta(z - \tau)^2}{2\kappa z} + (1 - \eta) \geq \frac{1}{2}. \quad (\text{C.5})$$

This can be written as

$$\underbrace{\eta(z - \tau)^2 + z\kappa(1 - 2\eta)}_{f(z)} \geq 0.$$

Consider now the equation on z defined by $f(z) = 0$. The solutions are

$$z_{1,2} = \tau + \kappa - \frac{\kappa}{2\eta} \pm \frac{1}{2\eta} \sqrt{\underbrace{(2\eta\tau + \kappa(2\eta - 1))^2 - 4\eta^2\tau^2}_{:=D}}.$$

Note that $D > 0$ for $\eta > \frac{1}{2}$. From this part onward, we only consider $\eta > \frac{1}{2}$, since otherwise, $z_{med} = \beta$. Consider the smaller solution of equation $f(z) = 0$, i.e.,

$$z_1 = \tau + \frac{1}{2\eta} \cdot \left(\kappa(2\eta - 1) - \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2} \right).$$

If $z_1 \geq z_{med}$, then, by assumptions of the case, we must have $\tau < z_1$, which implies

$$\begin{aligned} \kappa(2\eta - 1) &> \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}, \\ \kappa^2(2\eta - 1)^2 &> 4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2, \\ 0 &> 4\eta\tau\kappa(2\eta - 1). \end{aligned}$$

This states a contradiction for $\eta > \frac{1}{2}$, since then, $2\eta - 1 > 0$. Hence, z_1 cannot be z_{med} , i.e., $z_1 < z_{med}$. Note that

$$\frac{\partial f(z)}{\partial z} = 2\eta(z - \tau) + \kappa(1 - 2\eta) = 2\eta(z - (\tau + \kappa)) + \kappa$$

and

$$\frac{\partial^2 f(z)}{\partial z^2} = 2\eta > 0.$$

This means that $f(z)$ is minimized at $z_{min} := \tau + \kappa - \frac{\kappa}{2\eta}$, with $z_{min} \in (\tau, \tau + \frac{\kappa}{2}]$, since $\eta > \frac{1}{2}$. Hence, a necessary condition for equation $f(z) = 0$ to have a

solution is that $f(z_{min}) < 0$.

$$\begin{aligned}
f(z_{min}) &= \eta \cdot (z_{min} - \tau)^2 + z_{min} \cdot \kappa \cdot (1 - 2\eta) \\
&= \eta \cdot \left(\kappa - \frac{\kappa}{2\eta}\right)^2 + \left(\tau + \kappa - \frac{\kappa}{2\eta}\right) \cdot \kappa \cdot (1 - 2\eta) \\
&= \kappa \cdot \left(\eta\kappa\left(1 - \frac{1}{2\eta}\right)^2 + \left(\tau + \kappa - \frac{\kappa}{2\eta}\right)(1 - 2\eta)\right) \\
&= \kappa \cdot \left(\eta\kappa - \kappa + \frac{\kappa}{4\eta} + \tau + \kappa - \frac{\kappa}{2\eta} - 2\eta\tau - 2\eta\kappa + \kappa\right) \\
&= \kappa \cdot \left(-\frac{\kappa}{4\eta} + \tau - 2\eta\tau - \eta\kappa + \kappa\right) \\
&= \kappa \cdot \left(\kappa\left(-\frac{1}{4\eta} - \eta + 1\right) + \tau(1 - 2\eta)\right) < 0, \text{ for } \eta > \frac{1}{2}.
\end{aligned}$$

Next, we consider the larger solution of the quadratic equation $f(z) = 0$, called z_2 , and verify the conditions that must be fulfilled in this specific case. If $z_2 = z_{med}$, then, by assumptions of the case, we must have $\beta < \tau < z_2 < \tau + \kappa$, which implies

$$\tau < \tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}}{2\eta} < \tau + \kappa. \quad (\text{C.6})$$

Consider the right side of Inequality (C.6), which can be written as

$$\begin{aligned}
0 &< 2\eta\kappa - \kappa + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}, \\
0 &< \kappa(2\eta - 1) + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2},
\end{aligned}$$

which is true for $\eta > \frac{1}{2}$. Then, the left side of Inequality (C.6) is fulfilled for

$\eta < \frac{\tau + \kappa}{2\tau + \kappa}$, since the second inequality is equivalent to

$$\begin{aligned} \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2} &< \kappa, \\ 4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2 &< \kappa^2, \\ 4\eta\tau\kappa(2\eta - 1) + \kappa^2(4\eta^2 + 1 - 4\eta) &< \kappa^2, \\ 4\eta\tau(2\eta - 1) + \kappa 4\eta(\eta - 1) &< 0, \\ \tau(2\eta - 1) + \kappa(\eta - 1) &< 0, \\ \eta(2\tau + \kappa) - \tau - \kappa &< 0, \\ \eta &< \frac{\tau + \kappa}{2\tau + \kappa}. \end{aligned}$$

This means there exists a feasible solution $z_{med} = z_2$ for $\frac{1}{2} < \eta < \frac{\tau + \kappa}{2\tau + \kappa}$.

(iv) $\beta < \tau < \tau + \kappa < z$

For this range of z , the median value of z has to fulfill the following inequality:

$$\begin{aligned} \eta \cdot \left(1 - \frac{\kappa}{2z} - \frac{\tau}{z}\right) + (1 - \eta) &\geq \frac{1}{2}, \\ \eta \cdot (2\tau + \kappa) &\leq z. \end{aligned}$$

Then, the median value is $z_{med} = \eta \cdot (2\tau + \kappa)$. By assumptions of the case, we must have $z_{med} > \tau + \kappa$, i.e., the solution is feasible for $\eta > \frac{\tau + \kappa}{(2\tau + \kappa)}$.

Case 2: $\tau < \beta < \tau + \kappa$.

(i) $z < \tau$

For this range of z , there exists no median value for z satisfying

$$\eta F_Z(z) + (1 - \eta)F_Y(z) \geq \frac{1}{2},$$

since $F_Z(z) = 0$ and $F_Y(z) = 0$.

(ii) $\tau < z < \beta < \tau + \kappa$

For this range of z , the median value of z has to fulfill the following inequality:

$$\frac{\eta(z - \tau)^2}{2\kappa z} \geq \frac{1}{2}. \quad (\text{C.7})$$

Inequality (C.7) can be rewritten as

$$\eta \cdot (z - \tau) \geq \frac{\kappa}{z - \tau} \cdot z.$$

However,

$$\eta \cdot (z - \tau) \leq z \cdot \eta < z < z \cdot \frac{\kappa}{z - \tau},$$

where the first inequality holds since $z < \tau + \kappa$, the second inequality holds since $\eta < 1$, and the third inequality holds since $\tau \geq 0$. Hence, we obtain a contradiction, which means that there exists no feasible solution.

(iii) $\tau < \beta \leq z < \tau + \kappa$

In this setting, the median value of z has to fulfill the following inequality:

$$\frac{\eta(z - \tau)^2}{2\kappa z} + (1 - \eta) \geq \frac{1}{2}. \quad (\text{C.8})$$

The analysis is the same as for Case 1 (iii) of this proof, since the inequality is equivalent to Inequality (C.5). As in Case 1 (iii), Inequality (C.8) can be written as

$$\underbrace{\eta(z - \tau)^2 + z\kappa(1 - 2\eta)}_{f(z)} \geq 0.$$

Similar to Case 1 (iii), no value $z < z_2$ can be a solution. The only possible solution is

$$z_2 = \tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}}{2\eta}.$$

If $z_2 = z_{med}$, then, by assumptions of the case, we must have $\beta \leq z_2 < \tau + \kappa$,

which implies

$$\beta \leq \tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}}{2\eta} < \tau + \kappa. \quad (\text{C.9})$$

Consider the left side of Inequality (C.9). It is fulfilled for $\eta > \frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2} > \frac{1}{2}$. Then, the right side of Inequality (C.9) is fulfilled for $\eta < \frac{\tau + \kappa}{2\tau + \kappa}$, since the condition is equivalent to

$$\begin{aligned} \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2} &< \kappa, \\ 4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2 &< \kappa^2, \\ 4\eta\tau\kappa(2\eta - 1) + \kappa^2(4\eta^2 + 1 - 4\eta) &< \kappa^2, \\ 4\eta\tau(2\eta - 1) + \kappa 4\eta(\eta - 1) &< 0, \\ \tau(2\eta - 1) + \kappa(\eta - 1) &< 0, \\ \eta(2\tau + \kappa) - \tau - \kappa &< 0, \\ \eta &< \frac{\tau + \kappa}{2\tau + \kappa}. \end{aligned}$$

This means that there exists a feasible solution $z_{med} = z_2$ if and only if

$$\frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2} < \eta < \frac{\tau + \kappa}{2\tau + \kappa}.$$

(iv) $\tau < \beta < \tau + \kappa < z$

In this case, the median value of z has to fulfill the following inequality:

$$\begin{aligned} \eta \cdot \left(1 - \frac{\kappa}{2z} - \frac{\tau}{z}\right) + (1 - \eta) &\geq \frac{1}{2}, \\ \eta \cdot \left(1 - \frac{\kappa}{2z} - \frac{\tau}{z}\right) &\geq \eta - \frac{1}{2}, \\ 2z - \kappa - 2\tau &\geq 2z - \frac{z}{\eta}, \\ -\kappa - 2\tau &\geq -\frac{z}{\eta}, \\ \eta \cdot (2\tau + \kappa) &\leq z. \end{aligned}$$

Then, the median value is $z_{med} = \eta \cdot (2\tau + \kappa)$. By assumptions of the case, we must have $z_{med} > \tau + \kappa$, i.e., the solution is feasible for $\eta > \frac{\tau + \kappa}{(2\tau + \kappa)}$.

Case 3: $\tau < \tau + \kappa < \beta < 1$.

(i) $z < \tau$

For this range of z , there exists no median value for z satisfying

$$\eta F_Z(z) + (1 - \eta)F_Y(z) \geq \frac{1}{2},$$

since $F_Z(z) = 0$ and $F_Y(z) = 0$.

(ii) $\tau < z < \tau + \kappa < \beta < 1$

For this range of z , the median value of z has to fulfill the following inequality:

$$\frac{\eta(z - \tau)^2}{2\kappa z} \geq \frac{1}{2},$$

which is equivalent to Case 2 (ii) of this proof, since the inequality is equivalent, given $z < \tau + \kappa$. See Case 2 (ii) for more details. There exists no feasible solution.

(iii) $\tau < \tau + \kappa < z < \beta$

In this case, Inequality (C.4) is equivalent to

$$\eta \cdot \left(1 - \frac{\kappa}{2z} - \frac{\tau}{z}\right) \geq \frac{1}{2}. \quad (\text{C.10})$$

For $0 < \eta < \frac{1}{2}$, Inequality (C.10) can be written as

$$\frac{2\tau + \kappa}{2 - \frac{1}{\eta}} \geq z,$$

which states a contradiction, since $2 - \frac{1}{\eta} < 0$. If $\eta = \frac{1}{2}$, Inequality (C.10) is equivalent to

$$-\kappa \geq 2\tau,$$

which states a contradiction, since $\tau \geq 0$. For $\frac{1}{2} < \eta \leq 1$, Inequality (C.10) can be written as

$$\frac{2\tau + \kappa}{2 - \frac{1}{\eta}} \leq z.$$

If $z_{med} = \frac{2\tau+\kappa}{2-\frac{1}{\eta}}$, then, the condition $\tau + \kappa < z_{med} < \beta$ must be fulfilled, by assumptions of the case. This implies $\frac{\beta}{2\beta-(2\tau+\kappa)} < \eta \leq 1 < \frac{\tau+\kappa}{\kappa}$. Note that $\frac{\beta}{2\beta-(2\tau+\kappa)} < 1$ implies $\beta > (2\tau + \kappa)$.

(iv) $\tau < \tau + \kappa < \beta < z$

In this case, the median value of z has to fulfill the following inequality:

$$\begin{aligned} \eta \cdot \left(1 - \frac{\kappa}{2z} - \frac{\tau}{z}\right) + (1 - \eta) &\geq \frac{1}{2}, \\ \eta \cdot \left(1 - \frac{\kappa}{2z} - \frac{\tau}{z}\right) &\geq \eta - \frac{1}{2}, \\ 2z - \kappa - 2\tau &\geq 2z - \frac{z}{\eta}, \\ -\kappa - 2\tau &\geq -\frac{z}{\eta}, \\ \eta \cdot (2\tau + \kappa) &\leq z. \end{aligned}$$

Then, the median value is $z_{med} = \eta \cdot (2\tau + \kappa)$. By assumptions of the case, we must have $z_{med} > \beta$, i.e., the solution is feasible for $\eta > \frac{\beta}{(2\tau+\kappa)}$. Note that $\frac{\beta}{(2\tau+\kappa)} < 1$ implies $\beta < (2\tau + \kappa)$.

All feasible solutions for the different cases show the main results. The preferred policies are defined by

$$p_i^* \left(\left(\frac{b_i}{k_i} \right)_{med} \right) = p_i^* (z_{med}) = \max \left\{ 1 - z_{med} \cdot \frac{1}{\lambda^2}, 0 \right\}$$

For Case 1, i.e., $\beta < \tau < \tau + \kappa$, implying $\beta \leq (2\tau + \kappa)$, as shown in (iii) and (iv), the median values of z are defined by

$$z_{med} = \begin{cases} \beta & \text{if } 0 \leq \eta \leq \frac{1}{2}, \\ \tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta-1) + \kappa^2(2\eta-1)^2}}{2\eta} & \text{if } \frac{1}{2} < \eta \leq \frac{\tau+\kappa}{2\tau+\kappa}, \\ \eta(2\tau + \kappa) & \text{if } \frac{\tau+\kappa}{2\tau+\kappa} < \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$p_{\text{up}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta \leq \frac{1}{2}, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \left(\tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta-1) + \kappa^2(2\eta-1)^2}}{2\eta} \right), 0 \right\} & \text{if } \frac{1}{2} < \eta \leq \eta_1, \\ \max \left\{ 1 - \frac{\eta(2\tau+\kappa)}{\lambda^2}, 0 \right\} & \text{if } \eta_1 < \eta \leq 1, \end{cases}$$

where $\eta_1 = \frac{\tau+\kappa}{2\tau+\kappa}$. This policy is stated in Part (i) of the proposition.

For Case 2, i.e., $\tau < \beta < \tau + \kappa$, implying $\beta \leq (2\tau + \kappa)$, as shown in (iii) and (iv), the median values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta \leq \frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2}, \\ \tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta-1) + \kappa^2(2\eta-1)^2}}{2\eta} & \text{if } \frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2} < \eta \leq \frac{\tau+\kappa}{2\tau+\kappa}, \\ \eta(2\tau + \kappa) & \text{if } \frac{\tau+\kappa}{2\tau+\kappa} < \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$p_{\text{up}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta \leq \eta_2, \\ \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \left(\tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta-1) + \kappa^2(2\eta-1)^2}}{2\eta} \right), 0 \right\} & \text{if } \eta_2 < \eta \leq \eta_1, \\ \max \left\{ 1 - \frac{\eta(2\tau+\kappa)}{\lambda^2}, 0 \right\} & \text{if } \eta_1 < \eta \leq 1, \end{cases}$$

where $\eta_1 = \frac{\tau+\kappa}{2\tau+\kappa}$ and $\eta_2 = \frac{\kappa\beta}{2\kappa\beta - (\beta - \tau)^2}$. This policy is stated in Part (ii) of the proposition.

For Case 3, i.e., $\tau < \tau + \kappa < \beta < 1$, there are two cases, depending on the value of β . First, if $\beta > (2\tau + \kappa)$, then, as shown in (iii), the median values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{\beta}{2\beta - (2\tau + \kappa)}, \\ \frac{\eta(2\tau+\kappa)}{2\eta-1} & \text{if } \frac{\beta}{2\beta - (2\tau + \kappa)} \leq \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$p_{\text{up}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{\beta}{2\beta - (2\tau + \kappa)}, \\ 1 - \frac{\eta(2\tau + \kappa)}{(2\eta - 1)\lambda^2} & \text{if } \frac{\beta}{2\beta - (2\tau + \kappa)} \leq \eta \leq 1. \end{cases}$$

Second, if $\beta \leq (2\tau + \kappa)$, then, as shown in (iv), the median values of z are defined by

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{\beta}{2\tau + \kappa}, \\ \eta(2\tau + \kappa) & \text{if } \frac{\beta}{2\tau + \kappa} \leq \eta \leq 1, \end{cases}$$

which leads to the preferred policies

$$p_{\text{up}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{\beta}{2\tau + \kappa}, \\ \max \left\{ 1 - \frac{\eta(2\tau + \kappa)}{\lambda^2}, 0 \right\} & \text{if } \frac{\beta}{2\tau + \kappa} \leq \eta \leq 1. \end{cases}$$

Both results are stated in Part (iii) and (iv) of the proposition.

C.6 Proof of Proposition 8

Consider the policy p_{maj}^* , chosen by simple majority without experts, as stated in Equation (4.3), and compare it to policy p_{opt}^* , obtained in Optimal Democracy. First, for $\frac{1}{2} \leq \eta \leq \frac{1}{2-\beta}$, the policy is defined by $p_{\text{maj}}^* = 1 - \frac{\beta}{\lambda^2}$. From this threshold $\frac{1}{2-\beta}$ onwards, the larger the share of non-opinionated voters η , the lower the policy p_{maj}^* . It follows for $\frac{1}{2} \leq \eta \leq 1$ that

$$\begin{aligned} p_{\text{opt}}^* &> p_{\text{maj}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &> 1 - \frac{\beta}{\lambda^2}, \\ b^E &< \frac{\beta}{2}, \end{aligned}$$

which is stated in Part (i) of the proposition. Second, for $\frac{1}{2-\beta} < \eta \leq 1$, the policy is defined by $p_{\text{maj}}^* = 1 - \frac{(2-\frac{1}{\eta})}{\lambda^2}$, which takes its minimal value at $\eta = 1$. Hence, consider

the minimal value of p_{maj}^* and the optimal policy to calculate when $p_{\text{opt}}^* < p_{\text{maj}}^*$ for all $\eta \in [\frac{1}{2}, 1]$,

$$\begin{aligned} 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{1}{\lambda^2}, \\ b^E &> \frac{1}{2}, \end{aligned}$$

which is stated in Part (iii) of the proposition. For $\frac{\beta}{2} \leq b^E \leq \frac{1}{2}$, it follows that

$$\begin{aligned} p_{\text{opt}}^* &\leq p_{\text{maj}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &\leq 1 - \frac{\left(2 - \frac{1}{\eta}\right)}{\lambda^2}, \\ \eta &\leq \frac{1}{2(1 - b^E)}, \end{aligned}$$

as stated in Part (ii) of the proposition.

C.7 Proof of Proposition 9

Consider the values of p_{del}^* , as stated in Proposition 6, and compare them to the policy p_{opt}^* for $\frac{1}{2} \leq \eta \leq 1$. In each case, consider the optimal policy and the maximal value of p_{del}^* .

(i) $\beta < \frac{b^E}{\phi}$.

The policy chosen is highest at $\eta = \frac{1}{2}$, where $p_{\text{del}}^* = 1 - \frac{b^E}{\lambda^2\phi}$. Then, it follows that

$$\begin{aligned} p_{\text{opt}}^* &> p_{\text{del}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &> 1 - \frac{b^E}{\lambda^2\bar{\phi}}, \\ \frac{1}{2} &> \bar{\phi}, \end{aligned}$$

which is true given the assumption $0 < \underline{\phi} < \bar{\phi} < \frac{1}{2}$.

(ii) $\frac{b^E}{\phi} < \beta \leq \frac{b^E}{\mathbb{E}(k^E)}$.

The maximal value of p_{del}^* in this case is equal to $1 - \frac{\beta}{\lambda^2}$. Hence, it follows that

$$\begin{aligned} p_{\text{opt}}^* &> p_{\text{del}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &> 1 - \frac{\beta}{\lambda^2}, \\ \frac{\beta}{2} &> b^E, \end{aligned}$$

which is fulfilled, since according to this case where $b^E < \bar{\phi}\beta$ and $\bar{\phi} < \frac{1}{2}$.

$$(iii) \frac{b^E}{\mathbb{E}(k^E)} < \beta < \frac{b^E}{\underline{\phi}}.$$

In the absence of opinionated voters, i.e., $\eta = 1$, the highest policy is chosen, where $p_{\text{del}}^* = 1 - \frac{2b^E}{\lambda^2(\underline{\phi} + \bar{\phi})}$. Then,

$$\begin{aligned} p_{\text{opt}}^* &> p_{\text{del}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &> 1 - \frac{2b^E}{\lambda^2(\underline{\phi} + \bar{\phi})}, \\ 1 &> (\underline{\phi} + \bar{\phi}), \end{aligned}$$

which is true by assumption.

$$(iv) \frac{b^E}{\underline{\phi}} \leq \beta.$$

This case is equivalent to Part (iii), since the maximal value of p_{del}^* is identical.

Hence, we can show that in each case, the optimal policy is strictly higher than the policy chosen in Elections with Vote Delegation.

C.8 Proof of Proposition 10

Consider the values of p_{up}^* as stated in Proposition 7 and compare them to p_{opt}^* . We differentiate for $b^E > \frac{1}{2}$ and $b^E \leq \frac{1}{2}$.

- (i) If $b^E > \frac{1}{2}$, we want to show that $p_{\text{opt}}^* < p_{\text{up}}^*$ for all $\frac{1}{2} \leq \eta \leq 1$. Hence, consider for each policy result of Proposition 7 the lowest policy chosen and compare it to the optimal policy. First, for Part (i), (ii) and (iii) of Proposition 7, the

minimal value of p_{up}^* is identical and at $\eta = 1$, where $p_{\text{up}}^* = 1 - \frac{2\tau + \kappa}{\lambda^2}$. In other words, in the absence of opinionated voters, the policy chosen is the lowest. It follows that

$$\begin{aligned} p_{\text{opt}}^* &< p_{\text{up}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{2\tau + \kappa}{\lambda^2}, \\ b^E &> \frac{2\tau + \kappa}{2}, \end{aligned}$$

which is fulfilled, since $\mathbb{E}(b'_i) = \frac{2\tau + \kappa}{2}$ and

$$\mathbb{E}(b'_i) = \mathbb{E}(\kappa b_i + (1 - \kappa)b^E) = \frac{\kappa}{2} + (1 - \kappa)b^E < b^E$$

for $b^E > \frac{1}{2}$. Second, for Part (iv) of Proposition 7, the minimal value of p_{up}^* is at $\eta = \frac{1}{2}$, where $p_{\text{up}}^* = 1 - \frac{\beta}{\lambda^2}$. Then, it follows that

$$\begin{aligned} p_{\text{opt}}^* &< p_{\text{up}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{\beta}{\lambda^2}, \\ b^E &> \frac{\beta}{2}, \end{aligned}$$

which is true for $b^E > \frac{1}{2}$.

(ii) Consider now $b^E \leq \frac{1}{2}$.

(a) For $\beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ the policies p_{opt}^* are stated in Proposition 7 (i), (ii) and (iii). First, consider the maximal value of the three policies, which is equal to $p_{\text{opt}}^* = 1 - \frac{\max\{\tau, \beta\}}{\lambda^2}$. Then, for $b^E < \frac{\max\{\tau, \beta\}}{2}$ and $\frac{1}{2} \leq \eta \leq 1$, it follows that $p_{\text{opt}}^* > p_{\text{del}}^*$, since then

$$\begin{aligned} p_{\text{opt}}^* &> p_{\text{up}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &> 1 - \frac{\max\{\tau, \beta\}}{\lambda^2}, \\ b^E &< \frac{\max\{\tau, \beta\}}{2}. \end{aligned}$$

Second, consider now the different policies in more detail and establish each intersection with the optimal policy. Then, if $\beta < \tau < \tau + \kappa$ or $\tau < \beta < \tau + \kappa$, the policies are stated in Proposition 7 (i) and (ii). Consider both policies at the same time, since they are defined in the same way. For

$$\frac{\kappa \cdot \max\{\tau, \beta\}}{2\kappa \cdot \max\{\tau, \beta\} - (\max\{\tau, \beta\} - \tau)^2} \leq \eta \leq \frac{\tau + \kappa}{2\tau + \kappa}$$

the policy, if positive, is defined by

$$p_{\text{up}}^* = 1 - \frac{1}{\lambda^2} \cdot \left(\tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}}{2\eta} \right).$$

Then, it follows that

$$\begin{aligned} p_{\text{opt}}^* &\leq p_{\text{up}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &\leq 1 - \frac{1}{\lambda^2} \cdot \left(\tau + \kappa + \frac{-\kappa + \sqrt{4\eta\tau\kappa(2\eta - 1) + \kappa^2(2\eta - 1)^2}}{2\eta} \right), \\ \eta &\leq \frac{2\kappa b^E}{4\kappa b^E - (2b^E - \tau)^2}. \end{aligned}$$

This leads to the result that for $\frac{\max\{\tau, \beta\}}{2} \leq b^E \leq \frac{\tau + \kappa}{2}$:

$$p_{\text{opt}}^* \begin{cases} \leq p_{\text{up}}^*, & \text{if } \frac{1}{2} \leq \eta \leq \frac{2\kappa b^E}{4\kappa b^E - (2b^E - \tau)^2}, \\ > p_{\text{up}}^*, & \text{if } \frac{2\kappa b^E}{4\kappa b^E - (2b^E - \tau)^2} < \eta \leq 1. \end{cases}$$

If $\frac{\tau + \kappa}{2} < b^E \leq \mathbb{E}(b'_i)$, implying that $p_{\text{opt}}^* < 1 - \frac{\tau + \kappa}{\lambda^2}$, it follows that

$$p_{\text{opt}}^* \begin{cases} \leq p_{\text{up}}^*, & \text{if } \frac{1}{2} \leq \eta \leq \frac{b^E}{\mathbb{E}(b'_i)}, \\ > p_{\text{up}}^*, & \text{if } \frac{b^E}{\mathbb{E}(b'_i)} < \eta \leq 1. \end{cases}$$

This can be shown by solving for

$$\begin{aligned} p_{\text{opt}}^* &\leq p_{\text{up}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &\leq 1 - \frac{\eta(2\tau + \kappa)}{\lambda^2}, \\ \eta &\leq \frac{b^E}{\mathbb{E}(b'_i)}, \end{aligned}$$

where $\mathbb{E}(b'_i) = \frac{2\tau + \kappa}{2}$.

If $\tau < \tau + \kappa < \beta \leq 2\tau + \kappa$, as stated in Proposition 7 (iii), it follows for $\frac{\beta}{2} \leq b^E \leq \mathbb{E}(b'_i)$ that

$$p_{\text{opt}}^* \begin{cases} \leq p_{\text{up}}^*, & \text{if } \frac{1}{2} \leq \eta \leq \frac{b^E}{\mathbb{E}(b'_i)}. \\ > p_{\text{up}}^*, & \text{if } \frac{b^E}{\mathbb{E}(b'_i)} < \eta \leq 1. \end{cases}$$

This can be shown by solving for

$$\begin{aligned} p_{\text{opt}}^* &\leq p_{\text{up}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &\leq 1 - \frac{\eta(2\tau + \kappa)}{\lambda^2}, \\ \eta &\leq \frac{b^E}{\mathbb{E}(b'_i)}, \end{aligned}$$

where $\mathbb{E}(b'_i) = \frac{2\tau + \kappa}{2}$.

- (b) For $2\tau + \kappa < \beta$, i.e., $\frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)} < \beta$ the policy p_{up}^* is stated in Proposition 7 (iv). The largest value of p_{up}^* is at $\eta = 1$, where $p_{\text{up}}^* = 1 - \frac{2\tau + \kappa}{\lambda^2}$. It holds that $p_{\text{opt}}^* \geq p_{\text{up}}^*$ for $\frac{1}{2} \leq \eta \leq 1$, since

$$\begin{aligned} 1 - \frac{2b^E}{\lambda^2} &\geq 1 - \frac{2\tau + \kappa}{\lambda^2}, \\ b^E &\leq \frac{2\tau + \kappa}{\lambda^2}. \end{aligned}$$

The inequality is fulfilled, since $\mathbb{E}(b'_i) = \frac{2\tau + \kappa}{2}$ and

$$\mathbb{E}(b'_i) = \mathbb{E}(\kappa b_i + (1 - \kappa)b^E) = \frac{\kappa}{2} + (1 - \kappa)b^E \geq b^E$$

for $b^E \leq \frac{1}{2}$.

C.9 Proof of Proposition 11

Consider the values of p_{del}^* , as stated in Proposition 6, and compare them to the policy p_{opt}^* for $\frac{1}{2} \leq \eta \leq 1$. In each case, consider the optimal policy and the minimal value of p_{del}^* .

(i) $\beta < \frac{b^E}{\phi}$.

The policy chosen is lowest at $\eta = 1$, where $p_{\text{del}}^* = 1 - \frac{2b^E}{\lambda^2(\bar{\phi} + \underline{\phi})}$. Then, it follows that

$$\begin{aligned} p_{\text{opt}}^* &< p_{\text{del}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{2b^E}{\lambda^2(\bar{\phi} + \underline{\phi})}, \\ 1 &< \bar{\phi} + \underline{\phi}, \end{aligned}$$

which is true, given the assumption $\frac{1}{2} < \underline{\phi} < \bar{\phi} < 1$.

(ii) $\frac{b^E}{\phi} < \beta \leq \frac{b^E}{\mathbb{E}(k^E)}$.

This case is equivalent to Part (i), since the minimal value of p_{del}^* is identical.

(iii) $\frac{b^E}{\mathbb{E}(k^E)} < \beta < \frac{b^E}{\underline{\phi}}$.

The lowest policy chosen is at $\eta = \frac{1}{2}$, where $p_{\text{del}}^* = 1 - \frac{\beta}{\lambda^2}$. Then,

$$\begin{aligned} p_{\text{opt}}^* &< p_{\text{del}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{\beta}{\lambda^2}, \\ \frac{\beta}{2} &< b^E, \end{aligned}$$

which is fulfilled, since $\beta \underline{\phi} < b^E$. The last inequality is fulfilled by assumption of the case, i.e., $\frac{1}{2} < \underline{\phi} < \bar{\phi} \leq 1$.

(iv) $\frac{b^E}{\phi} \leq \beta$.

The policy chosen p_{del}^* takes its minimal value at $\eta = \frac{1}{2}$, where $p_{\text{del}}^* = 1 - \frac{b^E}{\lambda^2 \underline{\phi}}$.

Hence,

$$\begin{aligned} p_{\text{opt}}^* &< p_{\text{del}}^*, \\ 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{b^E}{\lambda^2 \underline{\phi}}, \\ \frac{1}{2} &< \underline{\phi}, \end{aligned}$$

which is true given the assumption $\frac{1}{2} < \underline{\phi} < \bar{\phi} < 1$.

Then, we can show that in each case, the optimal policy is strictly lower than the policy chosen in Elections with Vote Delegation.

C.10 Proof of Corollary 9

Consider the policies chosen in the different voting procedures. First, the optimal policy is equal to $p_{\text{opt}}^* = 1 - \frac{1}{\lambda^2} > 0$ for $b^E = \frac{1}{2}$. Second, in Elections without Experts, the policy chosen is equal to $p_{\text{maj}}^* = 1 - \frac{2-\eta}{\lambda^2}$ for $\eta > \frac{1}{2-\beta}$. For $\eta \leq 1$, it follows that $1 \geq 2 - \frac{1}{\eta}$ and hence, $p_{\text{maj}}^* \geq p_{\text{opt}}^*$. Third, the policy chosen in Elections with Opinion Updating for $\eta > \max \left\{ \frac{\tau+\kappa}{2\tau+\kappa}, \frac{\beta}{2\tau+\kappa}, \frac{\beta}{2\beta-(2\tau+\kappa)} \right\}$, as stated in Proposition 7, is equivalent to

$$p_{\text{up}}^* = \begin{cases} \max \left\{ 1 - \frac{\eta(2(1-\kappa)b^E + \kappa)}{\lambda^2}, 0 \right\} & \text{if } \beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}, \\ 1 - \frac{\eta(2(1-\kappa)b^E + \kappa)}{(2\eta-1)\lambda^2} & \text{if } \beta > \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}. \end{cases} \quad (\text{C.11})$$

For $b^E = \frac{1}{2}$, the policy can be simplified to

$$p_{\text{up}}^* = \begin{cases} 1 - \frac{\eta}{\lambda^2} & \text{if } \beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}, \\ 1 - \frac{\eta}{(2\eta-1)\lambda^2} & \text{if } \beta > \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}. \end{cases}$$

Next, consider the derivative of p_{up}^* , as stated in Equation (C.11), with respect to κ :

$$\frac{\partial p_{\text{up}}^*}{\partial \kappa} = \begin{cases} -\frac{\eta(-2b^E+1)}{\lambda^2} & \text{if } \beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}, \\ -\frac{\eta(-2b^E+1)}{(2\eta-1)\lambda^2} & \text{if } \beta > \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}. \end{cases}$$

Hence, it follows that $\left. \frac{\partial p_{\text{up}}^*}{\partial \kappa} \right|_{b^E=\frac{1}{2}} = 0$. This means that in this case, the expert's belief has no influence on the updated beliefs of the non-opinionated voters. Then, for $\eta \approx 1$, it follows that $p_{\text{up}}^* \approx p_{\text{maj}}^*$ for all $\beta \in [0, 1]$. Fourth, in Elections with Vote Delegation, the policy chosen for $\eta > \max \left\{ \frac{1}{2}, \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\underline{\phi})}{\beta\phi-b^E}, \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\underline{\phi})}{b^E-\beta\underline{\phi}} \right\}$ is equivalent to

$$p_{\text{del}}^* = \begin{cases} \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\bar{\phi}+(\phi-\underline{\phi})}, 0 \right\} & \text{if } \beta \leq \frac{b^E}{\mathbb{E}(k^E)}, \\ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\bar{\phi}-(\phi-\underline{\phi})} & \text{if } \beta > \frac{b^E}{\mathbb{E}(k^E)}, \end{cases}$$

which can be simplified to

$$p_{\text{del}}^* = \begin{cases} \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{\eta}{2\eta\bar{\phi}+(\phi-\underline{\phi})}, 0 \right\} & \text{if } \beta \leq \frac{b^E}{\mathbb{E}(k^E)}, \\ 1 - \frac{1}{\lambda^2} \cdot \frac{\eta}{2\eta\bar{\phi}-(\phi-\underline{\phi})} & \text{if } \beta > \frac{b^E}{\mathbb{E}(k^E)}, \end{cases}$$

for $b^E = \frac{1}{2}$. As $\underline{\phi}$ and $\bar{\phi}$ tend to zero, p_{del}^* tends to zero as well for all $\beta \in [0, 1]$.

C.11 Proof of Corollary 10

Consider the policies chosen in the different voting procedures. First, the optimal policy is equal to $p_{\text{opt}}^* = 1 - \frac{2b^E}{\lambda^2}$. Second, in Elections without Experts, the policy chosen is equal to $p_{\text{maj}}^* = 1 - \frac{2-\frac{1}{\eta}}{\lambda^2}$ for $\eta > \frac{1}{2-\beta}$. Third, the policy chosen in Elections with Opinion Updating for $\eta > \max \left\{ \frac{\tau+\kappa}{2\tau+\kappa}, \frac{\beta}{2\tau+\kappa}, \frac{\beta}{2\beta-(2\tau+\kappa)} \right\}$, as stated in

Proposition 7, is equivalent to

$$p_{\text{up}}^* = \begin{cases} \max \left\{ 1 - \frac{\eta(2(1-\kappa)b^E + \kappa)}{\lambda^2}, 0 \right\} & \text{if } \beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}, \\ 1 - \frac{\eta(2(1-\kappa)b^E + \kappa)}{(2\eta-1)\lambda^2} & \text{if } \beta > \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}. \end{cases} \quad (\text{C.12})$$

Next, consider the derivative of p_{up}^* , as stated in Equation (C.12), with respect to κ :

$$\frac{\partial p_{\text{up}}^*}{\partial \kappa} = \begin{cases} \max \left\{ -\frac{\eta(-2b^E + 1)}{\lambda^2}, 0 \right\} & \text{if } \beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}, \\ -\frac{\eta(-2b^E + 1)}{(2\eta-1)\lambda^2} & \text{if } \beta > \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}, \end{cases}$$

which is positive for all $\beta \in [0, 1]$. Since $b'_i = \kappa b_i + (1 - \kappa)b^E$, it follows that $b'_i = b_i$ for $\kappa = 1$. This implies that a decrease in κ leads to a higher weight on the expert's opinion and hence, to a lower policy p_{up}^* . It follows that $p_{\text{up}}^* < p_{\text{maj}}^*$. Next, compare p_{up}^* with p_{opt}^* . Consider first the case where $\beta \leq \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ of Equation (C.12) and $\kappa = 0$, as the policy increases in κ

$$\begin{aligned} p_{\text{opt}}^* &< p_{\text{up}}^* \Big|_{\kappa=0}, \\ 1 - \frac{2b^E}{\lambda^2} &< 1 - \frac{\eta 2b^E}{\lambda^2}, \\ 2b^E &> 2b^E \eta. \end{aligned}$$

Next, consider the second case where $\beta > \frac{\mathbb{E}(b'_i)}{\mathbb{E}(k_i)}$ of Equation (C.12) and $\kappa = 0$, as the policy increases in κ

$$\begin{aligned} p_{\text{opt}}^* &\approx p_{\text{up}}^* \Big|_{\kappa=0}, \\ 1 - \frac{2b^E}{\lambda^2} &\approx 1 - \frac{\eta 2b^E}{(2\eta-1)\lambda^2}, \\ 1 &\approx \frac{\eta}{(2\eta-1)}, \end{aligned}$$

since $\eta \approx 1$. It follows that $p_{\text{opt}}^* \leq p_{\text{up}}^*$. Fourth, in Elections with Vote Delegation, the policy chosen for $\eta > \max \left\{ \frac{1}{2}, \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\phi)}{\beta\phi-b^E}, \frac{1}{2} \cdot \frac{\beta(\bar{\phi}-\phi)}{b^E-\beta\phi} \right\}$ is equivalent to

$$p_{\text{del}}^* = \begin{cases} \max \left\{ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\phi+(\phi-\phi)}, 0 \right\} & \text{if } \beta \leq \frac{b^E}{\mathbb{E}(k^E)}, \\ 1 - \frac{1}{\lambda^2} \cdot \frac{2\eta b^E}{2\eta\phi-(\phi-\phi)} & \text{if } \beta > \frac{b^E}{\mathbb{E}(k^E)}. \end{cases}$$

As $\underline{\phi}$ and $\bar{\phi}$ tend to zero, p_{del}^* tends to zero as well for all $\beta \in [0, 1]$.

C.12 Possible Extensions

In this subsection, we sketch extensions of the basic model and a welfare approach.

C.12.1 Vote Delegation: General case

The non-opinionated voters delegate their votes to the expert. Hence, it follows for the share of non-opinionated voters η that

$$\begin{aligned} b_i &= b^E \sim \mathcal{U}[0, 1], \\ k_i &= k^E \sim \mathcal{U}[\underline{\phi}, \bar{\phi}], \end{aligned}$$

where $0 < \underline{\phi} < \bar{\phi} \leq 1$. The remaining $(1 - \eta)$ share of the electorate, i.e., the opinionated voters, sticks to its fixed ratio $\frac{b_i}{k_i} = \beta$, where $0 \leq \beta \leq 1$. The preferred policy is chosen by simple majority.

Proposition 12 *The following policies are chosen in elections with vote delegation in the general case.*

(i) *If $\beta > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, the chosen policy is defined by*

$$p_{\text{del}}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta \leq \frac{1}{\beta(\phi+\phi)}, \\ 1 - \frac{1}{\eta(\phi+\phi)\lambda^2} & \text{if } \frac{1}{\beta(\phi+\phi)} < \eta \leq 1. \end{cases} \quad (\text{C.13})$$

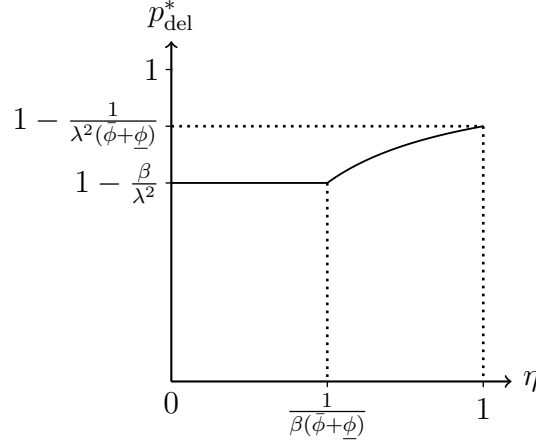


Figure C.2: Policy p_{del}^* for $\beta > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ in Case (i).

(ii) If $\beta \leq \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, the chosen policy is defined by

$$p_{del}^* = \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2-\beta(\phi+\phi)}, \\ \max \left\{ 1 - \frac{2\eta-1}{\eta(\phi+\phi)\lambda^2}, 0 \right\} & \text{if } \frac{1}{2-\beta(\phi+\phi)} \leq \eta \leq 1. \end{cases} \quad (\text{C.14})$$

The results are now compared to Elections without Experts. This leads to the following theorem.

Theorem 7

- (i) If $\mathbb{E}(b_i) < \mathbb{E}(k_i)$, Opinion Updating leads to a higher or equal ideal policy.
- (ii) If $\mathbb{E}(b_i) \geq \mathbb{E}(k_i)$, Opinion Updating leads to a lower or equal ideal policy.

C.12.2 Social Planner

In this chapter, the social planner's policy choice is discussed. The previous chapters give an overview and comparison of the policy results obtained under the different voting procedures. In this section, we are interested in the welfare optimal policy and examine if it can be replicated under each voting procedure. We consider the utilitarian social welfare function as well as the utilitarian-maximin social welfare

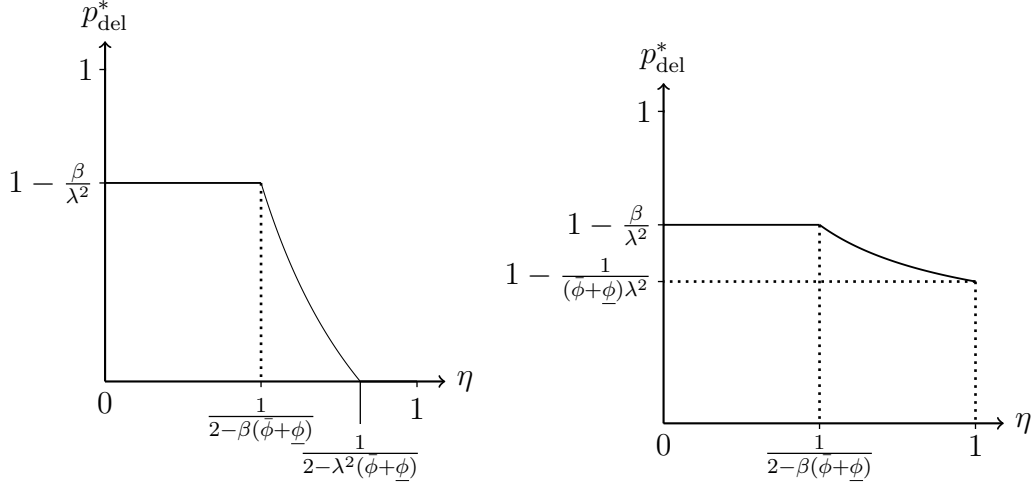


Figure C.3: Policy p_{del}^* for $\beta \leq \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, if $\lambda^2 < \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ (left) and if $\lambda^2 > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ (right) in Case (ii).

function (Bossert and Kamaga, 2020). First, the utilitarian welfare function is defined as

$$W^U(p) = \int_i \mathbb{E}[u_i(p)], \quad (\text{C.15})$$

where $\mathbb{E}[u_i(p)]$ is the expected utility. Then, we can show the following relation between the welfare function and the policy p .

Proposition 13 *The utilitarian social welfare function, as stated in Equation (C.15), increases in policy p , given $\lambda \geq \max\left\{1, \sqrt{\frac{3}{4(1-p)}}\right\}$. Hence, Vote Delegation and Opinion Updating can both lead to welfare-superior policies.*

Second, consider the utilitarian-maximin social welfare function. Given a weight $\rho \in [0, 1]$, the associated utilitarian-maximin social welfare ordering R_ρ^{UM} is a convex combination of the utilitarian and maximin welfare. R_ρ^{UM} is representable for any $\rho \in [0, 1]$ by the continuous social welfare function $W_\rho^{UM} : [0, 1] \rightarrow \mathbb{R}$ given by

$$W_\rho^{UM}(p) = \rho \int_i \mathbb{E}[u_i(p)] + (1 - \rho) \mathbb{E}[u_{i^*}(p)], \quad (\text{C.16})$$

where $\mathbb{E}[u_{i^*}(p)] = \min_{0 \leq b_i, k_i \leq 1} \mathbb{E}[u_i(p)]$. Note that the mixed utilitarian-maximin social

welfare orderings constitute a class of orderings, one for each $\rho \in [0, 1]$. The special cases of utilitarianism and of maximin are obtained for $\rho = 1$ and for $\rho = 0$. Note that given these assumptions, we claim that a policy p is replicable if and only if there is ρ , such that a social planner with ρ would choose p .

Furthermore, the expected utility of the maximin principle given by

$$\mathbb{E}[u_{i^*}(p)] = \min_{0 \leq b_i, k_i \leq 1} \mathbb{E}[u_i(p)] = \min_{0 \leq b_i, k_i \leq 1} -\exp\left(k_i b_i p + \frac{k_i^2 \lambda^2 (1-p)^2}{2}\right)$$

is minimized for $k_i = b_i = 1$. Then, for $\rho = 0$, the social welfare function is maximized at $p_{\text{maximin}}^* = 1 - \frac{1}{\lambda^2}$.

Proposition 14 *Consider the utilitarian-maximin social welfare function for a weight $\rho \in [0, 1]$. Then, there exists an interval $[1 - \frac{1}{\lambda^2}, p_u^*]$ of welfare-optimizing policies, where p_u^* is the ideal utilitarian policy, i.e., when $\rho = 1$. Furthermore, the welfare-optimizing policy increases in ρ .*

Proposition 14 states that for each weight ρ , there exists a welfare-maximizing policy $p^* > 1 - \frac{1}{\lambda^2}$. Since the ideal policy of the utilitarian principle cannot be defined explicitly, it is denoted by p_u^* .

C.13 Proof of Proposition 12

For the η share of the electorate, i.e., the non-opinionated voters, the beliefs and risk aversions are drawn independently from the following uniform distributions:

$$\begin{aligned} b_i &\sim \mathcal{U}[0, 1], \\ k_i &\sim \mathcal{U}[\underline{\phi}, \bar{\phi}], \end{aligned}$$

where $0 < \underline{\phi} < \bar{\phi} \leq 1$. Then, the cumulative distribution function for the ratio $Z := \frac{b_i}{k_i} \in [0, \frac{1}{\underline{\phi}}]$ for this fraction of voters is

$$\begin{aligned} F_Z(z) &= \mathbb{P}(Z \leq z) = \mathbb{P}\left(\frac{b_i}{k_i} \leq z\right) = \mathbb{P}(b_i \leq z k_i) \\ &= \mathbb{E}_{k_i}(\mathbb{P}(b_i \leq z k_i \mid k_i)) = \mathbb{E}_{k_i}[F_{b_i}(z k_i)] = \mathbb{E}_{k_i}[z k_i]. \end{aligned}$$

Clearly, $F_Z(z) = 0$ for $z \leq 0$. Therefore, we assume that $z > 0$. Then,

$$\begin{aligned}
F_Z(z) &= \mathbb{E}_{k_i}(\mathbb{P}(F_{b_i}(zk_i))) = \int_{\underline{\phi}}^{\bar{\phi}} \frac{1}{\bar{\phi} - \underline{\phi}} \mathbb{P}(b_i \leq zk_i) dk_i \\
&= \int_{\underline{\phi}}^{\bar{\phi}} \frac{1}{\bar{\phi} - \underline{\phi}} \min\{1, zk_i\} dk_i \\
&= \frac{1}{\bar{\phi} - \underline{\phi}} \begin{cases} \int_{\underline{\phi}}^{\bar{\phi}} zk_i dk_i & , 0 < z\bar{\phi} \leq 1 \\ \int_{\underline{\phi}}^{\frac{1}{z}} zk_i dk_i + \int_{\frac{1}{z}}^{\bar{\phi}} 1 dk_i & , 1 < z\bar{\phi} \end{cases} \\
&= \begin{cases} \frac{z}{2}(\bar{\phi} + \underline{\phi}) & , 0 < z \leq \frac{1}{\bar{\phi}}. \\ \frac{1}{\bar{\phi} - \underline{\phi}} \left(\bar{\phi} - \frac{1}{2z} - \frac{z\phi^2}{2} \right) & , \frac{1}{\bar{\phi}} < z < \frac{1}{\underline{\phi}}. \end{cases}
\end{aligned}$$

For the remaining $(1 - \eta)$ share of opinionated voters, the ratio has a fixed value $\frac{b_i}{k_i} = \beta$, where $0 \leq \beta \leq 1$. Then, the cumulative distribution function for the ratio $Y := \beta$ for this fraction of voters is

$$F_Y(z) = \begin{cases} 1 & , z \geq \beta. \\ 0 & , z < \beta. \end{cases}$$

Consider now the joint cumulative probability in order to find the median ratio, which is defined as the smallest value satisfying

$$\eta F_Z(z) + (1 - \eta) F_Y(z) \geq \frac{1}{2}. \quad (\text{C.17})$$

Consider now three cases for different values of z to find the median values.

(i) For $0 < z < \beta < \frac{1}{\bar{\phi}}$, Inequality (C.17) is equivalent to

$$\begin{aligned}
\frac{\eta z(\bar{\phi} + \underline{\phi})}{2} &\geq \frac{1}{2}, \\
z &\geq \frac{1}{\eta(\bar{\phi} + \underline{\phi})}.
\end{aligned}$$

If $z_{med} = \frac{1}{\eta(\bar{\phi} + \underline{\phi})}$, then, by assumptions of the case, we must have $z_{med} < \beta$, which implies $\beta(\bar{\phi} + \underline{\phi}) > 1/\eta > 1$, requiring $\beta(\bar{\phi} + \underline{\phi}) > 1$.

(ii) For $0 < \beta < z < \frac{1}{\phi}$, Inequality (C.17) is equivalent to

$$\begin{aligned} \frac{\eta z(\bar{\phi} + \underline{\phi})}{2} + (1 - \eta) &\geq \frac{1}{2}, \\ z &\geq \frac{2\eta - 1}{\eta(\bar{\phi} + \underline{\phi})}. \end{aligned}$$

If $z_{med} = \frac{2\eta - 1}{\eta(\bar{\phi} + \underline{\phi})}$, then, by assumptions of the case, we must have $z_{med} \geq \beta$, which implies that the solution is only feasible for $1 \geq \eta \geq \frac{1}{2 - \beta(\bar{\phi} + \underline{\phi})}$ and requires $\beta(\bar{\phi} + \underline{\phi}) \leq 1$. The second condition is $z_{med} \leq \frac{1}{\phi}$. Therefore, the solution is only feasible if $\eta \leq \frac{1}{1 - \frac{\phi}{\bar{\phi}}}$. Then, this condition is always fulfilled, since $\eta \leq 1 < \frac{1}{1 - \frac{\phi}{\bar{\phi}}}$.

(iii) For $0 < \beta < \frac{1}{\phi} \leq z$, Inequality (C.17) is equivalent to

$$\frac{\eta}{(\bar{\phi} - \underline{\phi})} \left(\bar{\phi} - \frac{1}{2z} - \frac{z\underline{\phi}^2}{2} \right) + (1 - \eta) \geq \frac{1}{2}. \quad (\text{C.18})$$

For $z = \frac{1}{\phi}$, the inequality is equivalent to

$$\eta \cdot \frac{(\underline{\phi} + \bar{\phi})}{2\bar{\phi}} + (1 - \eta) \geq \frac{1}{2}. \quad (\text{C.19})$$

This follows from algebra if we note that

$$\frac{\underline{\phi} + \bar{\phi}}{\bar{\phi}} \geq 1 \geq \frac{2\eta - 1}{\eta}.$$

Then, $z = \frac{1}{\phi}$ fulfills (C.18) and it follows from Equation (C.19) that this is true for all $0 \leq \eta \leq 1$, since $\eta \leq 1 < \frac{\bar{\phi}}{\bar{\phi} - \underline{\phi}}$. But there exist smaller values for z satisfying the inequality.

The calculations lead to two different preferred policies, depending on the relation of β and $\frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$ for $0 \leq \eta \leq 1$.

If $\beta(\bar{\phi} + \underline{\phi}) > 1$, i.e., $\beta > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, then

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta \leq \frac{1}{\beta(\bar{\phi} + \underline{\phi})}, \\ \frac{1}{\eta(\bar{\phi} + \underline{\phi})} & \text{if } \frac{1}{\beta(\bar{\phi} + \underline{\phi})} < \eta \leq 1, \end{cases} \quad (\text{C.20})$$

which leads to

$$p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) = p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \frac{1}{\lambda^2}, 0 \right\} \quad (\text{C.21})$$

$$= \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta \leq \frac{1}{\beta(\bar{\phi} + \underline{\phi})}, \\ 1 - \frac{1}{\eta(\bar{\phi} + \underline{\phi})\lambda^2} & \text{if } \frac{1}{\beta(\bar{\phi} + \underline{\phi})} < \eta \leq 1. \end{cases} \quad (\text{C.22})$$

If $\beta(\bar{\phi} + \underline{\phi}) \leq 1$, i.e., $\beta \leq \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, then

$$z_{\text{med}} = \begin{cases} \beta & \text{if } 0 \leq \eta < \frac{1}{2 - \beta(\bar{\phi} + \underline{\phi})}, \\ \frac{2\eta - 1}{\eta(\bar{\phi} + \underline{\phi})} & \text{if } \frac{1}{2 - \beta(\bar{\phi} + \underline{\phi})} \leq \eta \leq 1, \end{cases} \quad (\text{C.23})$$

which leads to

$$p_i^* \left(\left(\frac{b_i}{k_i} \right)_{\text{med}} \right) = p_i^* (z_{\text{med}}) = \max \left\{ 1 - z_{\text{med}} \frac{1}{\lambda^2}, 0 \right\} \quad (\text{C.24})$$

$$= \begin{cases} 1 - \frac{\beta}{\lambda^2} & \text{if } 0 \leq \eta < \frac{1}{2 - \beta(\bar{\phi} + \underline{\phi})}, \\ \max \left\{ 1 - \frac{2\eta - 1}{\eta(\bar{\phi} + \underline{\phi})\lambda^2}, 0 \right\} & \text{if } \frac{1}{2 - \beta(\bar{\phi} + \underline{\phi})} \leq \eta \leq 1. \end{cases} \quad (\text{C.25})$$

C.14 Proof of Theorem 7

Part (i): $\mathbb{E}(b_i) < \mathbb{E}(k_i)$

First, consider the case where $\beta > \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$. In this case, as seen in Proposition 5 and 6, the policies coincide up to the threshold $\eta = \min \left\{ \frac{1}{2 - \beta}, \frac{1}{\beta(\bar{\phi} + \underline{\phi})} \right\}$.

In the benchmark case, the chosen policy is $p_{\text{maj}}^* = 1 - \frac{(2 - \frac{1}{\eta})}{\lambda^2}$ for $\frac{1}{2 - \beta} < \eta \leq 1$. For this policy, it is true that the larger the share of non-opinionated voters η , the lower the policy, since $\frac{\partial p_{\text{maj}}^*}{\partial \eta} = -\frac{1}{\eta^2 \lambda^2} < 0$. In the case of Vote Delegation, the chosen

policy is $p_{\text{del}}^* = 1 - \frac{1}{\eta(\bar{\phi} + \underline{\phi})\lambda^2}$ for $\frac{1}{\beta(\bar{\phi} + \underline{\phi})} < \eta \leq 1$. This policy is higher for a larger share of non-opinionated voters η , since $\frac{\partial p_{\text{del}}^*}{\partial \eta} = \frac{1}{\eta^2(\bar{\phi} + \underline{\phi})\lambda^2} > 0$. The conclusion is that the policy chosen under Vote Delegation is higher than or equal to the one in the benchmark case.

Second, consider the case where $\beta \leq \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)} < 1$. In this case, as shown in Proposition 5 and 6, the policies coincide up to the threshold $\eta = \min\left\{\frac{1}{2-\beta}, \frac{1}{2-\beta(\bar{\phi} + \underline{\phi})}\right\} = \frac{1}{2-\beta}$. For a larger share of non-opinionated voters η , the outcomes differ in the two procedures. Hence, we want to show that for these outcomes, it holds that

$$\begin{aligned} p_{\text{maj}}^* &< p_{\text{del}}^*, \\ 1 - \frac{\left(2 - \frac{1}{\eta}\right)}{\lambda^2} &< 1 - \frac{2\eta - 1}{\eta(\bar{\phi} + \underline{\phi})\lambda^2}, \\ \left(2 - \frac{1}{\eta}\right) &> \frac{2\eta - 1}{\eta(\bar{\phi} + \underline{\phi})}, \\ (\bar{\phi} + \underline{\phi})(2\eta - 1) &> 2\eta - 1, \\ (\bar{\phi} + \underline{\phi}) &> 1 \end{aligned}$$

if $\mathbb{E}(k_i) > \mathbb{E}(b_i)$, given that $\eta > \frac{1}{2}$.

Part (ii): $\mathbb{E}(k_i) \leq \mathbb{E}(b_i)$

For this part, we only have to consider the case where $\beta \leq \frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)}$, since $\frac{\mathbb{E}(b_i)}{\mathbb{E}(k_i)} \geq 1$ is given. The policies coincide up to the threshold $\eta = \min\left\{\frac{1}{2-\beta}, \frac{1}{2-\beta(\bar{\phi} + \underline{\phi})}\right\} = \frac{1}{2-\beta(\bar{\phi} + \underline{\phi})}$. For a larger share of non-opinionated voters η , it follows from the proof of Part (i) that $p_{\text{maj}}^* \geq p_{\text{del}}^*$ for $(\bar{\phi} + \underline{\phi}) \leq 1$, i.e., for $\mathbb{E}(k_i) \leq \mathbb{E}(b_i)$.

C.15 Proof of Proposition 13

As stated in Equation (C.15), the utilitarian social welfare function can be rewritten as

$$W^U(p) = \int_i \mathbb{E}[u_i(p)] = \int_i -\exp\left(k_i b_i p + \frac{k_i^2 \lambda^2 (1-p)^2}{2}\right) db_i dk_i.$$

Then, the derivative with respect to p is given by

$$\frac{\partial W^U(p)}{\partial p} = \int_i \frac{\partial \mathbb{E}[u_i(p)]}{\partial p} = \int_i \mathbb{E}[u_i(p)] (k_i b_i - k_i^2 \lambda^2 (1-p)^2) db_i dk_i,$$

which can be bounded by minimizing $\mathbb{E}[u_i(p)]$. The expected utility is minimized at $k_i = b_i = 1$, which is equivalent to $\mathbb{E}[u_i(p)]|_{b_i=k_i=1} = -\exp\left(p + \frac{\lambda^2(1-p)^2}{2}\right)$. Then,

$$\begin{aligned} \frac{\partial W^U(p)}{\partial p} &\geq -\exp\left(p + \frac{\lambda^2(1-p)^2}{2}\right) \int_i (k_i b_i - k_i^2 \lambda^2 (1-p)^2) db_i dk_i \\ &= -\exp\left(p + \frac{\lambda^2(1-p)^2}{2}\right) \int_0^1 \left(\frac{k_i(2k_i \lambda^2 p - 2k_i \lambda^2 + 1)}{2}\right) dk_i \\ &= -\exp\left(p + \frac{\lambda^2(1-p)^2}{2}\right) \underbrace{\left(\frac{4\lambda^2 p - 4\lambda^2 + 3}{12}\right)}_{II} \\ &\geq 0 \end{aligned}$$

for $\forall p \in [0, 1]$ and $\lambda \geq \max\{1, \sqrt{\frac{3}{4(1-p)}}\}$. Part $I < 0$, for all values of λ and p . Part II is negative if and only if $4\lambda^2 p - 4\lambda^2 + 3 \leq 0$, which leads to the inequality $\lambda \geq \sqrt{\frac{3}{4(1-p)}}$.

Since Vote Delegation and Opinion Updating both lead to higher policies under specific conditions, both processes can be welfare-improving compared to Elections without Experts.

C.16 Proof of Proposition 14

Recall that

$$W_\rho^{UM}(p) = \rho \cdot \int_i \mathbb{E}[u_i(p)] db_i dk_i + (1 - \rho) \cdot \min_{0 \leq b_i, k_i \leq 1} \mathbb{E}[u_i(p)].$$

First, we focus on the case where $\rho = 0$. Note that

$$\begin{aligned} \min_{0 \leq b_i, k_i \leq 1} \mathbb{E}[u_i(p)] &= \min_{0 \leq b_i, k_i \leq 1} -\exp\left(k_i b_i p + \frac{k_i^2 \lambda^2 (1-p)^2}{2}\right) \\ &= -\exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right). \end{aligned} \quad (\text{C.26})$$

Hence,

$$\begin{aligned} \arg \max_{0 \leq p \leq 1} W_0^{UM}(p) &= \arg \max_{0 \leq p \leq 1} -\exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right) \\ &= \arg \min_{0 \leq p \leq 1} p + \frac{\lambda^2 (1-p)^2}{2} = \left\{1 - \frac{1}{\lambda^2}\right\}. \end{aligned}$$

Second, we investigate the case where $W_\rho^{UM}(p)$ increases/decreases differences in (p, ρ) . Therefore, we use one remark from Vives (2000):

Remark 5 (Vives, 2000)

Suppose that P and P are both convex and that f is twice continuously differentiable in (p, ρ) . Then, f increases differences in (p, ρ) if and only if

$$\frac{\partial^2 f(p, \rho)}{\partial \rho \partial p} \geq 0$$

for all $p \in P = [0, 1]$ and $\rho \in P = [0, 1)$. If the inequality is strict (except perhaps at isolated values of p or ρ), then f has strictly increasing differences.

The assumptions of Remark 5 are fulfilled for $W_\rho^{UM}(p)$. Using Equation (C.26), the welfare function can be written as

$$\begin{aligned} W_\rho^{UM}(p) &= \rho \cdot \int_i -\exp\left(k_i b_i p + \frac{k_i^2 \lambda^2 (1-p)^2}{2}\right) db_i dk_i \\ &\quad + (1-\rho) \cdot \left(-\exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right)\right). \end{aligned}$$

Then,

$$\frac{\partial W_\rho^{UM}(p)}{\partial \rho} = \int_i \overbrace{-\exp\left(k_i b_i p + \frac{k_i^2 \lambda^2 (1-p)^2}{2}\right)}{=\mathbb{E}[u_i(p)]} db_i dk_i + \exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right)$$

and

$$\begin{aligned} \frac{\partial^2 W_p^{UM}}{\partial \rho \partial p} &= \int_i \mathbb{E}[u_i(p)] (k_i b_i - k_i^2 \lambda^2 (1-p)) db_i dk_i \\ &\quad + \exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right) (1 - \lambda^2 (1-p)) \\ &=: G(p). \end{aligned}$$

Next, note that

$$G'(p) = \int_i \frac{\partial^2 \mathbb{E}[u_i(p)]}{\partial p^2} db_i dk_i + \exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right) ((1 - \lambda^2 (1-p))^2 + \lambda^2),$$

where

$$\begin{aligned} \frac{\partial^2 \mathbb{E}[u_i(p)]}{\partial p^2} &= \frac{\partial}{\partial p} \left[\mathbb{E}[u_i(p)] (k_i b_i - k_i^2 \lambda^2 (1-p)) \right] \\ &= \frac{\partial \mathbb{E}[u_i(p)]}{\partial p} \cdot (k_i b_i - k_i^2 \lambda^2 (1-p)) + \mathbb{E}[u_i(p)] \cdot k_i^2 \lambda^2 \\ &= \mathbb{E}[u_i(p)] \cdot (k_i b_i - k_i^2 \lambda^2 (1-p))^2 + \mathbb{E}[u_i(p)] \cdot k_i^2 \lambda^2 \\ &= \underbrace{\mathbb{E}[u_i(p)]}_{<0} \cdot \underbrace{((k_i b_i - k_i^2 \lambda^2 (1-p))^2 + k_i^2 \lambda^2)}_{>0} \end{aligned}$$

can be bounded by minimizing $\mathbb{E}[u_i(p)]$. The expected utility is minimized at $k_i = b_i = 1$, which is equivalent to $\mathbb{E}[u_i(p)]|_{b_i=k_i=1} = -\exp\left(p + \frac{\lambda^2 (1-p)^2}{2}\right) =: h(p, \lambda)$.

Then,

$$\begin{aligned}
\frac{\partial G}{\partial p} &\geq h(p, \lambda) \left(- \int_i ((k_i b_i - k_i^2 \lambda^2 (1-p))^2 + k_i^2 \lambda^2) + (1 - \lambda^2 (1-p))^2 + \lambda^2 \right) \\
&= h(p, \lambda) \left(\lambda^2 (1 - \int_i k_i^2) + (1 - \lambda^2 (1-p))^2 - \int_i (k_i b_i - k_i^2 \lambda^2 (1-p))^2 \right) \\
&\geq h(p, \lambda) \left(\frac{2\lambda^2}{3} + 1 - 2\lambda^2 (1-p) + \lambda^4 (1-p)^2 - \int_i (k_i b_i - k_i^2 \lambda^2 (1-p))^2 \right) \\
&\geq h(p, \lambda) \left(\frac{2\lambda^2}{3} + \frac{8}{9} - \frac{7\lambda^2 (1-p)}{4} + \frac{4\lambda^4 (1-p)^2}{5} \right) \\
&\geq \underbrace{h(p, \lambda)}_I \underbrace{\left(\frac{8}{9} - \frac{13\lambda^2}{12} + \frac{7\lambda^2 p}{4} + \frac{4\lambda^4 (1-p)^2}{5} \right)}_{II} \tag{C.27} \\
&\geq 0.
\end{aligned}$$

It is true that Part *I* is strictly positive, since it is an exponential function. Part *II* of Inequality (C.27) can be rewritten by using $\lambda^2 = t > 1$ as

$$\underbrace{t^2 \left(\frac{4(1-p)^2}{5} \right) + t \left(\frac{7p}{4} - \frac{13}{12} \right) + \frac{8}{9}}_{f(t)} \geq 0.$$

Then, the inequality is true for all values of t and for $p > \frac{13}{21}$, since then, $\frac{7p}{4} - \frac{13}{12} \geq 0$. Consider now the equation on t defined by $f(t) = 0$. The solutions are

$$t_{1,2} = \frac{\left(\frac{13}{12} - \frac{7p}{4} \right) \pm \sqrt{\left(\frac{13}{12} - \frac{7p}{4} \right)^2 - \frac{16(1-p)^2}{5} \cdot \frac{8}{9}}}{\left(\frac{8(1-p)^2}{5} \right)}.$$

Then, the discriminant D is defined by

$$D := \left(\frac{13}{12} - \frac{7p}{4} \right)^2 - \frac{16(1-p)^2}{5} \cdot \frac{8}{9},$$

which is negative for $0 \leq p \leq \frac{1}{157} (256\sqrt{10} - 683)$. Since $f(0) > 0$ for $t > 1$, it follows that $f(t) \geq 0$ for all $0 \leq p \leq \frac{1}{157} (256\sqrt{10} - 683)$. Hence, it is true for all i and $G(p)$ is an increasing function.

Consider now

$$\begin{aligned}
G(p)|_{p=p^*_{\text{maximin}}} &= \int_i \frac{\partial \mathbb{E}[u_i(p)]}{\partial p} \Big|_{p=p^*_{\text{maximin}}} \\
&+ \exp\left(p^*_{\text{maximin}} + \frac{\lambda^2(1-p^*_{\text{maximin}})^2}{2}\right) \underbrace{(1-\lambda^2(1-p^*_{\text{maximin}}))}_{=0} \\
&= \int_i \mathbb{E}[u_i(p)]|_{p=p^*_{\text{maximin}}} (k_i b_i - k_i^2 \lambda^2 (1-p^*_{\text{maximin}})) \\
&= \int_i \mathbb{E}[u_i(p)]|_{p=p^*_{\text{maximin}}} (k_i b_i - k_i^2) \\
&= - \int_i \exp\left(k_i b_i \left(1 - \frac{1}{\lambda^2}\right) + \frac{k_i^2}{2\lambda^2}\right) (k_i b_i - k_i^2) \\
&= - \int_{b_i \geq k_i} \exp\left(k_i b_i \left(1 - \frac{1}{\lambda^2}\right) + \frac{k_i^2}{2\lambda^2}\right) (k_i b_i - k_i^2) \\
&\quad - \int_{b_i \leq k_i} \exp\left(k_i b_i \left(1 - \frac{1}{\lambda^2}\right) + \frac{k_i^2}{2\lambda^2}\right) (k_i b_i - k_i^2) \\
&\geq - \int_{b_i \geq k_i} \exp\left(1 - \frac{1}{2\lambda^2}\right) (k_i b_i - k_i^2) \\
&\quad + \int_{b_i \leq k_i} \exp\left(b_i^2 \left(1 - \frac{1}{2\lambda^2}\right)\right) (k_i^2 - k_i b_i) \\
&\geq - \int_{b_i \geq k_i} \exp\left(1 - \frac{1}{2\lambda^2}\right) (k_i b_i - k_i^2) \\
&\quad + \int_{b_i \leq k_i} (k_i^2 - k_i b_i) \\
&\geq - \int_0^1 \int_{k_i}^1 \exp\left(1 - \frac{1}{2\lambda^2}\right) (k_i b_i - k_i^2) db_i dk_i \\
&\quad + \int_0^1 \int_0^{k_i} (k_i^2 - k_i b_i) db_i dk_i \\
&= - \int_0^1 \frac{(k-1)^2 k \exp\left(1 - \frac{1}{2\lambda^2}\right)}{2} dk_i + \int_0^1 \frac{k_i^3}{2} dk_i \\
&= - \frac{\exp\left(1 - \frac{1}{2\lambda^2}\right)}{24} + \frac{1}{8} > 0.
\end{aligned}$$

We just showed that $G(p)|_{p=p^*_{\text{maximin}}} > 0$ and that $G(p)$ is an increasing function in p . Hence, according to Remark 5, it follows that the welfare function $W_\rho^{UM}(p)$ has increasing differences. Consider now the following theorem to prove that the

welfare-optimizing policy is increasing in ρ .

Theorem 8 (Vives (2000)) *Suppose $f(p, \rho)$ has strictly increasing differences in (p, ρ) . Then, it follows for $\rho^L, \rho^H \in [0, 1]$, such that $\rho^H > \rho^L$ and for $p^L \in \phi(\rho^L)$ and $p^H \in \phi(\rho^H)$, we have $p^H \geq p^L$.*

If $p > p_{\text{maximin}}^*$, then $\frac{\partial^2 W_\rho^{UM}(p)}{\partial \rho \partial p} \geq 0$ and it follows from Remark 5 (Vives, 2000) that $W_\rho^{UM}(p)$ has increasing differences in (p, ρ) , i.e., it is a supermodular function, such that $p^*(\rho)$ is an increasing function in ρ according to Theorem 1 (Vives, 2000).

Chapter D

Appendix to Chapter 5

D.1 Additional information on data collection and preprocessing

D.1.1 Swiss newspapers

For Switzerland, articles of three nation-wide daily newspapers are considered: *Neue Zürcher Zeitung (NZZ)*, *Tagesanzeiger* and *Blick*. All newspapers publish their articles in print as well as online. The content that was published on the web portals *nzz.ch*, *tagesanzeiger.ch* and *blick.ch* is analyzed. The data of the online newspapers was received from the Swiss media database [swissdox.ch](https://www.swissdox.ch). Furthermore, we considered only newspaper articles with more than 100 words published on *nzz.ch* and more than 64 resp. 53 words on *blick.ch* resp. *tagesanzeiger.ch* to avoid explanatory articles that relate to a different medium, e.g. to a video or picture series. In addition, we set the maximal article length at 7,000 for *nzz.ch* to exclude the long Covid-19-related articles in the category “Wissenschaft” (i.e., research) that cover scientific discoveries rather than daily events. The online reach of the two Swiss newspapers *nzz.ch* and *blick.ch* is nearly identical, with 3 million monthly unique users¹ at *nzz.ch* and 2.809 million monthly unique users² at *blick.ch* in the

¹<https://www.nzzzone.ch/produkte/nzz-ch/> (accessed on November 24, 2021).

²<https://www.ringier-advertising.ch/portfolio/digital/blick-ch/> (accessed on January 28, 2022).

year 2021. The reach of *tagesanzeiger.ch* is around two-thirds of *nzz.ch* and *blick.ch* with around 2 million unique users³ per month in 2020. The main difference between these three media channels is the average article length, as articles published on *nzz.ch* are, on average, three times resp. twice as long as the articles published on *blick.ch* resp. *tagesanzeiger.ch*.

D.1.2 Austrian newspapers

For Austria, articles of two nation-wide daily newspapers are considered, *DerStandard* and *Oe24*, that publish their articles in print as well as online. The content was retrieved only from the web portals *derStandard.at* and *oe24.at*. All articles considered are freely available in both online newspapers, which is the vast majority of all articles published. Furthermore, any interactive or image-based article, as “Stories” or “Liveticker” (interactive information page) on *derStandard.at* and any description of video messages on *oe24.at* are excluded from the corpus. Furthermore, we considered only newspaper articles with more than 150 words published on *derStandard.at* and more than 50 words on *oe24.at* to have enough content to analyze per article. The online reach of the two Austrian online newspapers is nearly identical, with 2.546 million monthly unique users⁴ in 2020 at *derStandard.at* and 2.633 million monthly unique users⁵ at *oe24.at* in the second quarter of 2019. The average length of an article published on *derStandard.at* is about twice as great as the one of *oe24.at*. The number of articles published on *derStandard.at* analyzed is around 10 thousand more in total over the two years than on *oe24.at*.

D.1.3 Data crawling of Austrian newspapers

For the Austrian online newspaper articles, a crawling system was built to collect all articles related to Covid-19. The implementation was done with Python, using the

³<https://goldbach.com/ch/de/portfolio/print/tages-anzeiger/mediadaten> (accessed on February 9, 2022).

⁴https://sales.derstandard.at/wp-content/uploads/2021/10/DERSTANDARD_allgemeinePraesentation_211014.pdf (accessed on December 19, 2021).

⁵ For more information see <https://www.derstandard.at/story/2000109328049/online-reichweite-immer-mehr-menschen-lesen-den-standard> (accessed on November 24, 2021).

Selenium package. First, a list containing all links to the articles of each newspaper was generated through the search function of each website. Second, each article was crawled and saved in a Jason file stating the following variables: Newspaper, URL, Date, Title, Abstract, Content, Number of comments (if available).

D.2 Additional information on anxiety index

D.2.1 Dictionary word list

The German Linguistic Inquiry and Word Count (LIWC) dictionary for the sentiment “anxiety” from Meier et al. (2019) serves as a basis for the analysis. It includes 345 keywords, including 95 word stems of some keywords. These stems are denoted by an asterisk, which allows the counting of any target word that starts with this stem. The list of all used keywords is: *abgeneigt**, *abschreck**, *ängstlichste**, *angespannt**, *angst**, *anspann**, *aufgeregt**, *bang**, *beängstigendste**, *bedroh**, *befürcht**, *beschäm**, *beunruhig**, *erpress**, *erschauer**, *erschrak**, *erschrick**, *erschütter**, *feigling**, *fürchte**, *furchtbarste**, *furchterregend**, *furchtsam**, *fürchte**, *geängstigt**, *gedroht**, *gefürchtet**, *gegrübelt**, *geschämt**, *gezaudert**, *geziert**, *gezögert**, *geängstigt**, *gruseligste**, *grübel**, *horror**, *irrational**, *mied**, *missbräuch**, *missbrauch**, *missgeschick**, *nervöseste**, *panik**, *panisch**, *paranoi**, *peinlichste**, *phobi**, *quäl**, *rastlos**, *risiko**, *ruhelos**, *schäm**, *schauder**, *scheut**, *schiss**, *schlott**, *shock**, *schrecklichste**, *schuldigste**, *schutzbedürftig**, *schutzlos**, *sorgenvoll**, *spannung**, *stress**, *terror**, *übel**, *überforder**, *unbehag**, *unberechenbar**, *unbestimmt**, *ungewiss**, *unheimlich**, *unkontrolliert**, *unruhe**, *unruhigste**, *unsicherste**, *unstet**, *verärgert**, *verjag**, *verscheuch**, *verschreck**, *verstörendste**, *verstört**, *verworren**, *verzweifel**, *wehrloseste**, *widerwill**, *zaghaft**, *zauder**, *zerfähr**, *zerfahr**, *zerfuhr**, *zitter**, *zusammengezuckt**, *zusammenzuck**, *abneigen*, *abschrecken*, *ängstlich*, *ängstliche*, *ängstlichem*, *ängstlichen*, *ängstlicher*, *ängstlichere*, *ängstlicherer*, *ängstlicheres*, *ängstliches*, *beängstige*, *beängstigen*, *beängstigend*, *beängstigende*, *beängstigendem*, *beängstigenden*, *beängstigender*, *beängstigendere*, *beängstigerem*, *beängstigeren*, *beängstigeres*, *beängstigendes*, *bedenken*, *bedrohen*, *befürchten*, *beschämen*, *besorgnis*, *besorgt*, *besorgte*, *besorgtem*, *besorgten*, *besorgter*, *besorgtes*, *demütig*, *demütige*, *demütigen*, *demütigend*, *drohe*, *drohen*,

drohend, drohst, droht, drohung, drohungen, entsetzen, entsetzlich, erniedrigen, erniedrigend, erniedrigt, erniedrigte, erniedrigung, erpressen, erschauern, erschrecke, erschrecken, erschreckend, erschreckt, erschrocken, erschrockene, erschrockenem, erschrockenen, erschrockener, erschrockenes, erschüttern, erstarre, erstarren, erstarrt, erstarnte, flehe, flehen, flehend, flehende, flehenden, flehte, fürchten, furcht, furchtbar, furchtbare, furchtbarem, furchtbaren, furchtbarer, furchtbares, gefleht, gehemmt, gekillt, gemieden, gespannt, grübeln, grübel, gruselig, gruselige, gruseligem, gruseligen, gruseliger, gruseligere, gruseligerem, gruseligeren, gruseligeres, gruseliges, herzrasen, hilflos, hilflose, hilflosem, hilflosen, hilfloser, hilfloses, hilflosigkeit, kämpfen, kämpfend, kneife, kneifen, kneifst, kneift, kniff, kniffen, kniffst, meide, meiden, meidest, meidet, missbrauchen, mulmig, mutlos, mutlosigkeit, nervös, nervöse, nervösem, nervösen, nervöser, nervösere, nervöserem, nervöseren, nervöseres, nervosität, neurotisch, nöten, nöte, not, peinlich, peinliche, peinlichem, peinlichen, peinlicher, peinlicherem, peinlicheren, peinlicherer, peinliches, phobie, quälen, qual, qualen, qualvoll, schämen, scham, schamgefühl, scheu, scheue, scheuen, scheuer, scheues, scheust, schreck, schrecken, schrecklich, schreckliche, schrecklichem, schrecklichen, schrecklicher, schrecklichere, schrecklicherem, schrecklicheren, schrecklicheres, schreckliches, schüchtern, schüchterne, schüchternem, schüchternen, schüchterner, schüchternes, schüchternheit, schuld, schuldig, schuldige, schuldigem, schuldigen, schuldiger, schuldigere, schuldigerem, schuldigeren, schuldigerer, schuldigeres, schuldiges, sorge, sorgen, sorgst, sorgt, sorgte, sorgten, starr, starre, starrem, starren, starres, steif, störend, überfordern, unruhig, unruhige, unruhigem, unruhigen, unruhiger, unruhigere, unruhigerem, unruhigeren, unruhigeres, unruhiges, unsicher, unsichere, unsicherem, unsicheren, unsicherer, unsicheres, unsicherheit, verärgern, verjagen, verklemmen, verklemmt, verlegen, verlegenheit, verscheuchen, verschrecken, verstöre, verstören, verstörend, verstörende, verstörenden, verstörender, verstörendere, verstörenderem, verstörenderen, verstörenderes, verstörendes, verwirrend, verwirrt, verwirren, verwirrung, verzweifeln, verzweifeln, verzweifelt, wackelig, wackelige, wackeligem, wackeligen, wackeliger, wackeliges, wahnsinnig, wahnsinnige, wahnsinnigem, wahnsinnigen, wahnsinniger, wahnsinniges, wehrlos, wehrlose, wehrlosem, wehrlosen, wehrloser, wehrloses, zaudern, zittern, zittrig, zögern, zöger, zusammenzucken, zwängen, zwänge, zweifeln, zweifel, zweifels

For Word2Vec (Mikolov et al., 2013) calculations, 163 words of the dictionary

that are not word stems are excluded, since they do not appear in the Word2Vec dictionary, mostly due to the lemmatization of all words. The list of words excluded from the keyword list is: *ängstliche, ängstlichem, ängstlicher, ängstlichere, ängstlicherer, ängstlicheres, ängstliches, beängstige, beängstigende, beängstigendem, beängstigenden, beängstigender, beängstigendere, beängstigerem, beängstigenderen, beängstigenderes, beängstigendes, beschämen, besorgtem, besorgten, besorgter, besorgtes, demütige, drohst, drohungen, erniedrigt, erniedrigte, erschrecke, erschreckt, erschrocken, erschrockene, erschrockenem, erschrockenen, erschrockener, erschrockenes, erstarre, erstarrt, erstarnte, flehe, flehend, flehende, flehenden, flehte, furchtbare, furchtbarem, furchtbaren, furchtbarer, furchtbares, gefleht, gehemmt, gekillt, gemieden, grübel, gruselige, gruseligem, gruseligen, gruseliger, gruseligere, gruselig-erem, gruseligeren, gruseligeres, gruseliges, hilflose, hilflosem, hilflosen, hilfloser, hilfloses, kneife, kneifst, kneift, kniffen, kniffst, meide, meidest, meidet, mutlosigkeit, nervöse, nervösem, nervösen, nervöser, nervösere, nervöserem, nervöseren, nervöseres, neurotisch, nöten, nöte, peinlichem, peinlichen, peinlicher, peinlicherem, peinlicheren, peinlicherer, peinliches, qualen, schamgefühl, scheue, scheues, scheust, schrecklichem, schrecklichen, schrecklicher, schrecklichere, schrecklicherem, schrecklicheren, schrecklicheres, schüchterne, schüchternem, schüchternen, schüchterner, schüchternes, schüchternheit, schuldigem, schuldigen, schuldiger, schuldigerem, schuldigeren, schuldigerer, schuldigeres, schuldiges, sorgst, sorgte, sorgten, starrem, starres, unruhigem, unruhigen, unruhiger, unruhigere, unruhigerem, unruhigeren, unruhigeres, unruhiges, unsicherem, unsicheren, unsicherer, unsicheres, verklemmen, verklemmt, verstöre, verstören, verstörender, verstörendere, verstörenderem, verstörenderen, verstörenderes, verzweifln, verzweifl, wackeligem, wackeliger, wackeliges, wahnsinnigem, wahn-sinnigen, wahnsinniger, wahnsinniges, wehrlosem, wehrlosen, wehrloser, wehrloses, zöger, zusammenzucken, zwänge, zweifels*

Manually, the list given by LIWC is extended by the infinitive forms of the verbs, since all words in the articles are lemmatized as part of the pre-processing procedure. In total, 29 infinitives of keywords are manually added, such that lemmatized words can be matched. The list of all manually added infinitives is: *abneigen, abschrecken, bedrohen, befürchten, beschämen, erpressen, erschauern, erschüttern, fürchten, grübeln, missbrauchen, nöten, phobie, quälen, schämen, überfordern, verärgern, verjagen, verklemmen, verscheuchen, verschrecken, verzweifeln, verzweifln, zaud-*

ern, zittern, zögern, zusammenzucken, zwängen, zweifeln

Hence, the calculation for the centroid is based on a keyword list consisting of 149 words that are not stem words. The list for the Word2Vec calculations is: *abneigen, abschrecken, ängstlich, ängstlichen, angespannt, angst, aufgeregt, bang, beängstigen, beängstigend, bedenken, bedrohen, befürchten, besorgnis, besorgt, besorgte, demütig, demütigen, demütigend, drohe, drohen, drohend, droht, drohung, entsetzen, entsetzlich, erniedrigen, erniedrigend, erniedrigung, erpressen, erschauern, erschrecken, erschreckend, erschüttern, erstarren, feigling, flehen, fürchten, furcht, furchtbar, gefürchtet, gespannt, grübeln, gruselig, herzrasen, hilflos, hilflosigkeit, horror, irrational, kämpfen, kämpfend, kneifen, kniff, meiden, missbrauchen, missbrauch, missgeschick, mulmig, mutlos, nervös, nervosität, not, panik, panisch, peinlich, peinliche, phobie, quälen, qual, qualvoll, rastlos, risiko, schämen, scham, scheu, scheuen, scheuer, schiss, schock, schreck, schrecken, schrecklich, schreckliche, schreckliches, schüchtern, schuld, schuldig, schuldige, schuldiger, schutzbedürftig, schutzlos, sorge, sorgen, sorgenvoll, sorgt, spannung, starr, starre, starren, steif, störend, stress, terror, übel, überfordern, unberechenbar, unbestimmt, ungewiss, unheimlich, unkontrolliert, unruhe, unruhig, unruhige, unsicher, unsichere, unsicherheit, verärgern, verärgert, verjagen, verlegen, verlegenheit, verscheuchen, verschrecken, verstörend, verstörende, verstörenden, verstörendes, verstört, verwirrend, verwirrt, verwirren, verwirrung, verworren, verzweifeln, wackelig, wackelige, wackeligen, wahnsinnig, wahnsinnige, wehrlos, wehrlose, zaghaft, zaudern, zittern, zittrig, zögern, zwängen, zweifeln, zweifel*

D.2.2 Extended keyword list

The keyword list is extended by keywords that are most similar for at least 10% of keywords. The list of 32 additional keywords with count of keywords that are most similar to the word is: ('unerträglich', 30), ('mitleid', 24), ('unheimlich', 22), ('merkwürdig', 21), ('brutalität', 21), ('elend', 20), ('feige', 20), ('wut', 19), ('komisch', 19), ('beängstigen', 18), ('verzweiflung', 18), ('sprachlos', 18), ('verunsichern', 17), ('aggression', 17), ('unbehagen', 17), ('verstört', 17), ('ängstlich', 17), ('todesangst', 17), ('weinen', 17), ('hysterie', 16), ('bizarr', 16), ('furchtbar', 16), ('melancholie', 16), ('innerlich', 15), ('ohnmacht', 15), ('hilflos', 15), ('hilflosigkeit',

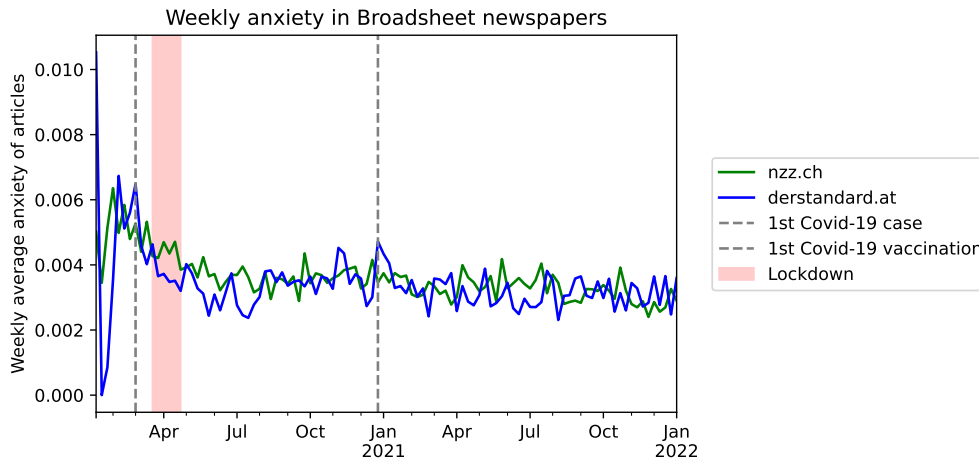


Figure D.1: The anxiety index for broadsheet newspapers in the years 2020 and 2021.

15), ('furcht', 15), ('frustration', 15), ('ungeduld', 15), ('seltsam', 15), ('grotesk', 15)

D.2.3 Anxiety index for each country

Figures D.1 and D.2 show the (unscaled) anxiety index for broadsheet newspapers resp. tabloids.

D.3 Additional information on validation

D.3.1 Translation of Figure 5.3

The words are ordered according to relevance, i.e., its distance to the centroid, starting with the smallest distance: despair, anger, unbearable, helplessness, aggression, panic, helpless, frustration, hysterics, powerlessness, discomfort, indifference, shame, irrational, anxious, frustrated, upset, anxiety about the future, horrible.

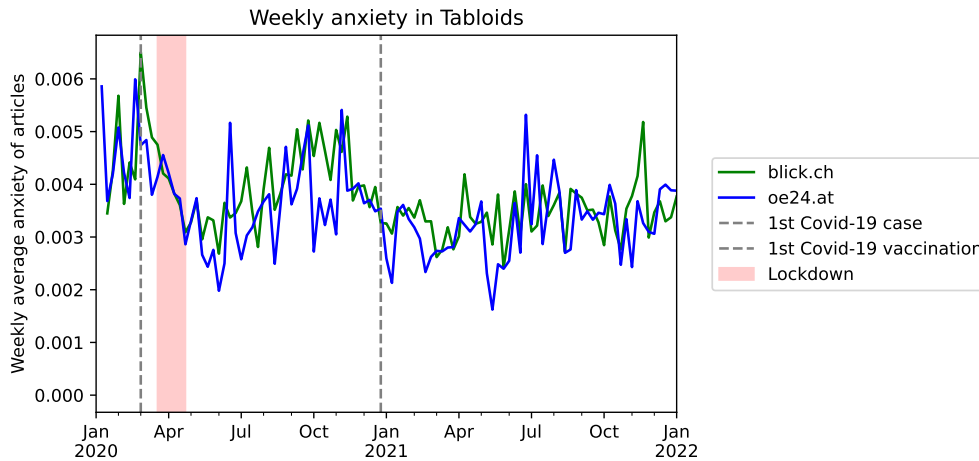


Figure D.2: The anxiety index for tabloids in the years 2020 and 2021.

D.3.2 Qualitative analysis

Tables D.1 and D.2 gives an overview of the first sentences of articles with highest anxiety index in the corpus for each country. In both countries the majority of these articles stem from the tabloid newspapers *oe24.at* and *blick.ch*. In the Swiss articles the prevailing topics are Covid-19 and related anxiety in the society, an anti-terror law, and terrorists. Austrian articles covers shortages in intensive care, Covid-19 and anxiety about related policies, and terrorists.

D.3.3 Comparison with other indices

This section provides additional information on the mental health study conducted in Austria (Niederkrotenthaler et al., 2022) and on how the comparison with the anxiety index was done. The study was run in twelve waves, which means participants were asked twelve times over one year. The exact dates of each wave, which are used to calculate the average anxiety index in each wave are stated in Figure D.3. The time series based on the aggregated survey data is standardized to unit deviation, averaged across each wave, and normalized to a mean of 1. Then, the correlation coefficient of the two time series is calculated.

Most anxiety-generating articles in Austria	
Newspaper	Sentences
ö24.at	Bundeskanzler Sebastian Kurz (ÖVP) befürchtet, dass schon in rund zwei Wochen Engpässe an den Spitälern auftreten könnten.
ö24.at	Brian May hat Corona. Corona Schock bei Queen Gitarrist Brian May testete positiv.
ö24.at	68 Prozent haben Angst um ihre Liebsten. Volle Zustimmung der Bevölkerung zu den strengen Corona Regeln der Regierung.
ö24.at	Europol Terroristen profitieren von Corona Krise: Risiko der Online Radikalisierung zugenommen.
ö24.at	Kurz befürchtet Überlastung ab Mitte April. Bereits in einigen Wochen könnte es zur Überforderung der Intensivmedizin kommen.
derstandard.at	So kann beispielsweise auch die Corona Pandemie zu Ängsten führen, sei es die Krankheit betreffend, die berufliche Existenz oder die Sorge, was politische Entscheidungen und Maßnahmen mit der Gesellschaft machen und diese dadurch womöglich verändern.
ö24.at	Trotz Lockdown: Linzer Wut-Wirtin sperrt heute wieder auf.
derstandard.at	Warum uns Ängste oft in die Irre führen. Durch die Corona Pandemie ist die Angst in unserer Gesellschaft gestiegen.
ö24.at	Kickl kritisiert in seinem Wut-Posting, dass Van der Bellen nicht auf die Gründe der Frustration und der Wut eingegangen sei.
ö24.at	Dramatische Zunahme von psychosozialen, wirtschaftlichen und politischen Spannungen im Zusammenhang mit Covid.

Table D.1: List of first sentences of articles with the highest anxiety index in Austria.

Most anxiety-generating articles in Switzerland	
Newspaper	Sentences
nzz.ch	Angst ist ein lebensnotwendiges Gefühl, weil es vor Gefahren schützt. Doch hört die Angst nicht mehr auf und dominiert sie den Alltag, kann sie zu einer Krankheit werden.
blick.ch	Die Angst vor dem Coronavirus macht auch vor Stars nicht Halt.
blick.ch	Europol: Terroristen nutzen Corona Krise, Terroristen versuchen einem Europol Bericht zufolge, die Corona Krise für ihre Zwecke auszunutzen.
blick.ch	Covid führt zu mehr Angsterkrankungen in der Bevölkerung, Wie Sie Angst erkennen und einer Chronifizierung vorbeugen erklären zwei Fachärztinnen.
blick.ch	Der Coronavirus geht um die Welt und sorgt für Panik unter den Massen.
blick.ch	Wegen Anti-Terror Gesetz: Jungparteien fürchten sich vor Polizeistaat. Jetzt beginnt der Abstimmungskampf auch ums Anti-Terror Gesetz.
blick.ch	Im US Repräsentantenhaus fliegen die Fetzen derart, dass sich einige der Abgeordneten vor den eigenen Kollegen fürchten.
blick.ch	Annett Möller, Deutsche Moderatorin, hatte live im TV Panikattacken. Die Deutsche Fernsehmoderatorin Annett Möller veröffentlichte am Oktober ihr Buch über ihre Angststörung.
nzz.ch	In Deutschland scheint die Bereitschaft zur Hysterie besonders ausgeprägt zu sein. Dabei geraten die Fakten rasch durcheinander.
nzz.ch	Die Pandemie bedeutete nicht für alle mehr Stress. Corona war für viele eine grosse Belastung.

Table D.2: List of first sentences of articles with the highest anxiety index in Switzerland.

wave	start_date	end_date
w1	2020-04-23	2020-05-05
w2	2020-05-15	2020-05-28
w3	2020-06-05	2020-06-17
w4	2020-06-26	2020-07-08
w5	2020-07-17	2020-07-30
w6	2020-08-07	2020-08-22
w7	2020-08-28	2020-09-14
w8	2020-09-18	2020-09-29
w9	2020-10-09	2020-10-21
w10	2020-10-30	2020-11-11
w11	2020-11-20	2020-11-28
w12	2020-12-11	2020-12-22

Figure D.3: The dates of the 12 waves of the survey.

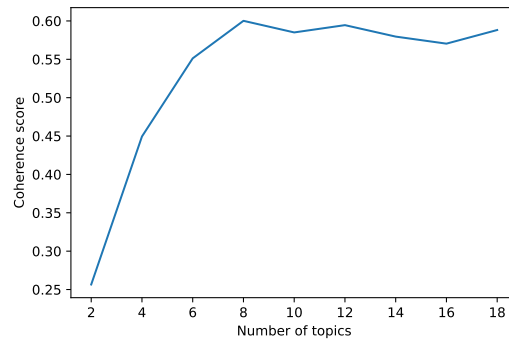


Figure D.4: Coherence scores for different numbers of topics.

D.4 Additional information on topic modeling

D.4.1 Number of topics

The number of topics, namely 8, is chosen, based on the coherence score of the LDA Mallet model (McCallum, 2002), an open source toolkit. To evaluate the optimal number of topics, the LDA Mallet model was run with different numbers of topics, from 2 to 18, and the model with the highest coherence score is selected. In Figure D.4, one can see the coherence score for a different number of topics.

List of topics		
	Topic name	keywords
1	USA	trump, police, video, biden, twitter, new york, facebook, participant
2	Austria and beliefs	Viennese, standard, reality, read, believe, mother, thinking, maybe, anxiety
3	Covid in Austria	patient, variant, new patient, tyrol, study, intensive care unit, infection, illness, region, sars
4	Business	billion, dollar, previous year, online, this year, customer, price, business, short-term working, economy
5	Soccer	team, player, fan, soccer, season, salzburg, league, win, star, trainer
6	Vaccination	vaccination, italy, biontech, pfizer, johnson, dose, Great Britain, china, who
7	Policies: Decision and criticism	spö, Viennese, mandatory masks, green party, health minister, government, critic, decision, political party
8	Policies: Implications	övp, school, pcr, open, negative, pupil, visitor, entry, gastronomy, event

Table D.3: Translation of the list of topics, topic names and associated keywords.

D.4.2 Translation of Table 5.2

A translation of the table can be found in Table D.3.

D.4.3 Anxiety by topic over time

Figure D.5 shows the average weekly anxiety per topic over time. In total, four different topics are related to Covid-19 and depict the highest level of anxiety, as shown in Figure 5.5. The topics depicted are Austria and beliefs (Topic 2), Covid-19 in Austria (Topic 3), Policies: Decision and critic (Topic 7) and Policies: Implications (Topic 8).

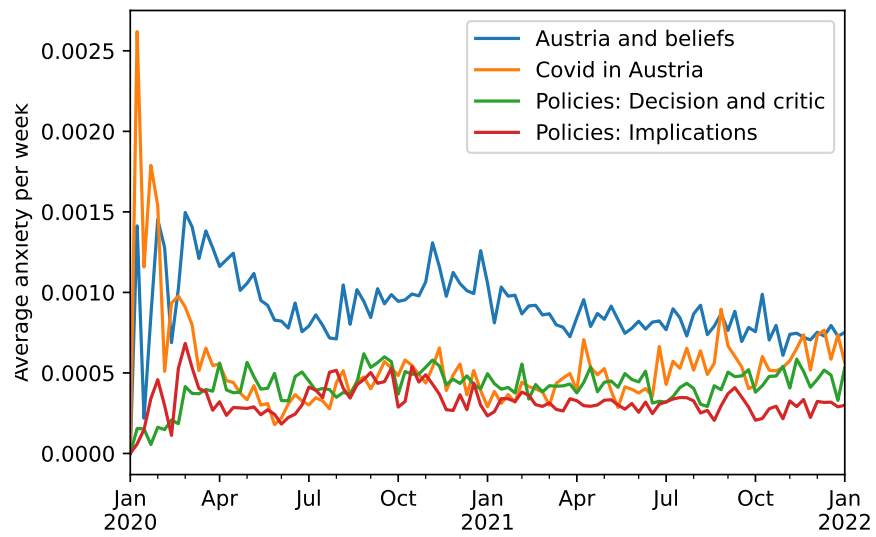


Figure D.5: Anxiety by topic over time.

References

- Acemoglu, D., Robinson, J. A., and Torvik, R. (2013). Why do voters dismantle checks and balances? *Review of Economic Studies*, 80(3):845–875.
- Aghion, P., Alesina, A., and Trebbi, F. (2004). Endogenous political institutions. *Quarterly Journal of Economics*, 119(2):565–611.
- Alesina, A. and Rosenthal, H. (1996). A theory of divided government. *Econometrica*, 64(6):1311–1341.
- Alesina, A. and Rosenthal, H. (2000). Polarized platforms and moderate policies with checks and balances. *Journal of Public Economics*, 75(1):1–20.
- Amann, J., Sleight, J., and Vayena, E. (2021). Digital contact-tracing during the Covid-19 pandemic: An analysis of newspaper coverage in Germany, Austria, and Switzerland. *Plos one*, 16(2):e0246524.
- Aragónés, E. (2016). Preference shock and party consistency. *41 Simposio de la Asociación Española de Economía (2016)*.
- Aragónés, E., Ponsatí, C., et al. (2019). Preference shocks that destroy party systems. *Barcelona Graduate School of Economics (GSE) Working Paper No. 1118*.
- Ash, E., Chen, D. L., and Ornaghi, A. (2021a). Gender attitudes in the judiciary: Evidence from US circuit courts. *Center for Law & Economics (ETH Zurich) Working Paper No. 02/2019*.
- Ash, E., Stambach, D., and Tobia, K. (2021b). Dimensions of mind in semantic space. *Available at SSRN.com 3959847*.

- Ashworth, S. and Bueno de Mesquita, E. (2017). Unified versus divided political authority. *Journal of Politics*, 79(4):1372–1385.
- Baker, S. R., Bloom, N., and Davis, S. J. (2016). Measuring economic policy uncertainty. *Quarterly Journal of Economics*, 131(4):1593–1636.
- Bali, V. A. (2007). Terror and elections: Lessons from Spain. *Electoral Studies*, 26(3):669–687.
- Ban, P., Fourinaies, A., Hall, A. B., and Snyder, J. M. (2019). How newspapers reveal political power. *Political Science Research and Methods*, 7(4):661–678.
- Barberà, S. and Jackson, M. O. (2004). Choosing how to choose: Self-stable majority rules and constitutions. *Quarterly Journal of Economics*, 119(3):1011–1048.
- Barro, R. J. (1973). The control of politicians: An economic model. *Public Choice*, 14:19–42.
- Battaglini, M. (2014). A dynamic theory of electoral competition. *Theoretical Economics*, 9(2):515–554.
- Bek, M. G. (2004). Research note. Tabloidization of news media: An analysis of television news in Turkey. *European Journal of Communication*, 19(3):371–386.
- Besley, T. and Coate, S. (1997). An economic model of representative democracy. *Quarterly Journal of Economics*, 112(1):85–114.
- Besley, T. and Coate, S. (2003). Elected versus appointed regulators: Theory and evidence. *Journal of the European Economic Association*, 1(5):1176–1206.
- Bird, S., Klein, E., and Loper, E. (2009). *Natural Language Processing with Python: Analyzing Text with the Natural Language Toolkit*. O’Reilly Media, Inc.
- Blei, D. M. (2012). Probabilistic topic models. *Communications of the Association for Computing Machinery (ACM)*, 55(4):77–84.
- Blum, C. and Zuber, C. I. (2016). Liquid democracy: Potentials, problems, and perspectives. *Journal of Political Philosophy*, 24(2):162–182.

- Bolsen, T., Palm, R., and Kingsland, J. T. (2020). Framing the origins of COVID-19. *Science Communication*, 42(5):562–585.
- Bordignon, M., Nannicini, T., and Tabellini, G. (2016). Moderating political extremism: Single round versus runoff elections under plurality rule. *American Economic Review*, 106(8):2349–70.
- Bossert, W. and Kamaga, K. (2020). An axiomatization of the mixed utilitarian-maximin social welfare orderings. *Economic Theory*, 69(2):451–473.
- Bouton, L., Llorente-Saguer, A., and Malherbe, F. (2018). Get rid of unanimity rule: The superiority of majority rules with veto power. *Journal of Political Economy*, 126(1):107–149.
- Brill, M. and Talmon, N. (2018). Pairwise liquid democracy. *International Joint Conferences on Artificial Intelligence Organization (IJCAI)*, 18:137–143.
- Brozus, L., Geden, O., Čumurović, A., Gropp, R. E., Schneidewind, U., Paqué, K.-H., and Feld, L. P. (2017). Expertenwissen im politischen Prozess: Nutzen, Grenzen und Gefahren. *Wirtschaftsdienst*, 97(4):239–255.
- Caballero, R. J. and Simsek, A. (2022). Monetary policy with opinionated markets. *American Economic Review*, 112(7):2353–92.
- Callander, S. et al. (2008). A theory of policy expertise. *Quarterly Journal of Political Science*, 3(2):123–140.
- Callander, S., Lambert, N. S., and Matouschek, N. (2021). The power of referential advice. *Journal of Political Economy*, 129(11):3073—3140.
- Cammaerts, B. (2015). Pirates on the liquid shores of liberal democracy: Movement frames of European pirate parties. *Javnost - The Public*, 22(1):19–36.
- Chakraborty, A. and Bose, S. (2020). Around the world in 60 days: An exploratory study of impact of Covid-19 on online global news sentiment. *Journal of Computational Social Science*, 3:367–400.

- Christoff, Z. and Grossi, D. (2017). Binary voting with delegable proxy: An analysis of liquid democracy. *Electronic Proceedings in Theoretical Computer Science*, 251:134–150.
- Cox, G. W. and Weingast, B. R. (2018). Executive constraint, political stability, and economic growth. *Comparative Political Studies*, 51(3):279–303.
- Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.
- Cruz, C., Keefer, P., and Scartascini, C. (2021). Database of political institutions 2020. Data retrieved from Inter-American Development Bank, <http://dx.doi.org/10.18235/0003049> (accessed on December 13, 2022).
- Dan, A. (2006). Voting by proxy. *Public Choice*, 126(1):1–26.
- De Sinopoli, F. and Iannantuoni, G. (2007). A spatial voting model where proportional rule leads to two-party equilibria. *International Journal of Game Theory*, 35(2):267–286.
- Desson, Z., Lambertz, L., Peters, J. W., Falkenbach, M., and Kauer, L. (2020). Europe’s Covid-19 outliers: German, Austrian and Swiss policy responses during the early stages of the 2020 pandemic. *Health Policy and Technology*, 9(4):405–418.
- Diermeier, D. and Myerson, R. B. (1999). Bicameralism and its consequences for the internal organization of legislatures. *American Economic Review*, 89(5):1182–1196.
- Dietrich, F. and List, C. (2017). Probabilistic opinion pooling generalized. Part one: General agendas. *Social Choice and Welfare*, 48(4):747–786.
- Dreisiebner, S., März, S., and Mandl, T. (2022). Information behavior during the Covid-19 crisis in German-speaking countries. *Journal of Documentation*, 78(7):160–175.
- Dziuda, W. and Loeper, A. (2021). Voters and the trade-off between policy stability and responsiveness. *Available at SSRN.com 3908921*.

- Eisele, O., Litvyak, O., Brändle, V. K., Balluff, P., Fischeneder, A., Sotirakou, C., Ali, P. S., and Boomgaarden, H. G. (2022). An emotional rally: Exploring commenters' responses to online news coverage of the COVID-19 crisis in Austria. *Digital Journalism*, 10(6):952–975.
- Eraslan, H. and Piazza, A. (2020). Efficiency with political power dynamics and costly policy change. *Rice University Technical Report No. 14858*.
- Feddersen, T. and Pesendorfer, W. (1997). Voting behavior and information aggregation in elections with private information. *Econometrica*, 65(5):1029–1058.
- Ferejohn, J. (1986). Incumbent performance and electoral control. *Public Choice*, 50(1/3):5–25.
- Ferejohn, J. and McCall Rosenbluth, F. (2008). Warlike democracies. *Journal of Conflict Resolution*, 52(1):3–38.
- Fieldhouse, E., Green, J., Evans, G., Mellon, J., Prosser, C., Schmitt, H., and Van der Eijk, C. (2021). *Electoral Shocks: The Volatile Voter in a Turbulent World*. Oxford University Press, Oxford.
- Forand, J. G. (2014). Two-party competition with persistent policies. *Journal of Economic Theory*, 152:64–91.
- Forteza, A., Mussio, I., and Pereyra, J. S. (2019). Does political gridlock undermine checks and balances? A lab experiment. *Documento de Trabajo. FCS-Decon; 05/19*.
- Forteza, A. and Pereyra, J. S. (2019). When do voters weaken checks and balances to facilitate economic reform? *Economica*, 86(344):775–800.
- Gans, J. S. and Smart, M. (1996). Majority voting with single-crossing preferences. *Journal of Public Economics*, 59(2):219–237.
- Garg, N., Schiebinger, L., Jurafsky, D., and Zou, J. (2018). Word embeddings quantify 100 years of gender and ethnic stereotypes. *Proceedings of the National Academy of Sciences*, 115(16):E3635–E3644.

- Gennaro, G. and Ash, E. (2022). Emotion and reason in political language. *Economic Journal*, 132(643):1037–1059.
- Gersbach, H. (2021). Elections, the curse of competence and credence policies. *Public Choice*, 186(3):491–511.
- Gersbach, H., Jackson, M., Muller, P., and Tejada, O. (2020a). Electoral competition with costly policy changes: A dynamic perspective. *CEPR Discussion Paper No. DP14858*.
- Gersbach, H., Jackson, M., and Tejada, O. (2020b). Optimal term-length and the structure of democracy. *CEPR Discussion Paper No. DP14857*.
- Gersbach, H., Muller, P., and Tejada, O. (2019). Costs of change and political polarization. *European Journal of Political Economy*, 60:101801.
- Gersbach, H., Muller, P., and Tejada, O. (2021). Using re-election thresholds to curb political polarization. *CEPR Discussion Paper No. DP16037*.
- Gersbach, H. and Tejada, O. (2018). A reform dilemma in polarized democracies. *Journal of Public Economics*, 160:148–158.
- Gersbach, H., Tejada, O., and Wagner, J. (2022). Policy reforms and the amount of checks & balances. *CER-ETH Working Paper No. 22/373*.
- Glazer, A., Gradstein, M., and Konrad, K. A. (1998). The electoral politics of extreme policies. *Economic Journal*, 108(451):1677–1685.
- Gratton, G. and Morelli, M. (2022). Optimal checks and balances under policy uncertainty. *International Economic Review*, 63(2):549–569.
- Green-Armytage, J. (2015). Direct voting and proxy voting. *Constitutional Political Economy*, 26:190–220.
- Grimmer, J. and Stewart, B. M. (2013). Text as data: The promise and pitfalls of automatic content analysis methods for political texts. *Political Analysis*, 21(3):267–297.

- Handley, K. and Li, J. F. (2020). Measuring the effects of firm uncertainty on economic activity: New evidence from one million documents. Technical Report 27896, National Bureau of Economic Research.
- Hart, P. S., Chinn, S., and Soroka, S. (2020). Politicization and polarization in COVID-19 news coverage. *Science Communication*, 42(5):679–697.
- Hirschi, C. (2018). *Skandalexperten, Expertenskandale. Zur Geschichte eines Gegenwartproblems*. MSB Matthes & Seitz, Berlin.
- Honnibal, M. and Montani, I. (2017). spaCy 2: Natural language understanding with Bloom embeddings, convolutional neural networks and incremental parsing. [open-source software library, <https://spacy.io/> (accessed on December 14, 2022)].
- Iaryczower, M. and Mattozzi, A. (2013). On the nature of competition in alternative electoral systems. *Journal of Politics*, 75(3):743–756.
- Jandura, O. and Friedrich, K. (2014). The quality of political media coverage. In Reinemann, C., editor, *Political Communication*, pages 351–374. De Gruyter Mouton, Berlin, Boston.
- Kawamura, K. and Vlaseros, V. (2017). Expert information and majority decisions. *Journal of Public Economics*, 147:77–88.
- Kruspe, A., Häberle, M., Kuhn, I., and Zhu, X. X. (2020). Cross-language sentiment analysis of European Twitter messages during the Covid-19 pandemic. *arXiv preprint arXiv:2008.12172*.
- Lago, I. and Montero, J. (2005). The mechanics of electoral change. *Claves de la Razón Práctica*, 149:36–44.
- Liñeira, R. (2021). Valence secession? Voting shocks and independence support in Scotland. *Politics and Governance*, 9(4):399–411.
- List, C. (2018). Democratic deliberation and social choice. In Bächtiger, A., Dryzek, J. S., and Warren, J. M. M. E., editors, *The Oxford Handbook of Deliberative Democracy*, pages 463–489. Oxford University Press, New York, NY.

- Löblová, O. (2018). Epistemic communities and experts in health policy-making. *European Journal of Public Health*, 28(suppl.3):7–10.
- Locke, J. (1690). [1976] *The Second Treatise of Government*. Oxford: Blackwell, 3rd edition.
- Madison, J., Hamilton, A., and Jay, J. (1788). [1961] *The Federalist Papers*. Rossiter. New York: New American Library.
- Mainwaring, S. (1990). Presidentialism in latin america. *Latin American Research Review*, 25(1):157–179.
- Matakos, K., Troumpounis, O., and Xefferis, D. (2016). Electoral rule disproportionality and platform polarization. *American Journal of Political Science*, 60(4):1026–1043.
- McCallum, A. K. (2002). Mallet: A machine learning for language toolkit. <http://mallet.cs.umass.edu>.
- McCombs, M. and Reynolds, A. (2002). News influence on our pictures of the world. In Bryant, J., Zillmann, D., Jennings Bryant, J., and Oliver, M. B., editors, *Media Effects*, pages 11–28. Routledge, New York, NY.
- Meier, T., Boyd, R. L., Pennebaker, J. W., Mehl, M. R., Martin, M., Wolf, M., and Horn, A. B. (2019). “LIWC auf Deutsch”: The development, psychometrics, and introduction of DE-LIWC2015. *PsyArXiv*, 1(a):1–42.
- Mellado, C., Hallin, D., Càrcamo, L., Alfaro, R., Jackson, D., Humanes, M. L., Màrquez-Ramàrez, M., Mick, J., Mothes, C., LIN, C. I.-H., Lee, M., Alfaro, A., Isbej, J., and Ramos, A. (2021). Sourcing pandemic news: A cross-national computational analysis of mainstream media coverage of COVID-19 on Facebook, Twitter, and Instagram. *Digital Journalism*, 9(9):1261–1285.
- Messner, M. and Polborn, M. K. (2004). Voting on majority rules. *Review of Economic Studies*, 71(1):115–132.

- Mikolov, T., Sutskever, I., Chen, K., Corrado, G. S., and Dean, J. (2013). Distributed representations of words and phrases and their compositionality. *Advances in Neural Information Processing Systems*, 26.
- Milgrom, P. R. (1981). Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 12(2):380–391.
- Miller, G. and Schofield, N. (2003). Activists and partisan realignment in the United States. *American Political Science Review*, 97(2):245–260.
- Miller, J. C. (1969). A program for direct and proxy voting in the legislative process. *Public Choice*, 7:107–113.
- Montalvo, J. G. (2011). Voting after the bombings: A natural experiment on the effect of terrorist attacks on democratic elections. *Review of Economics and Statistics*, 93(4):1146–1154.
- Montalvo, J. G. (2012). Re-examining the evidence on the electoral impact of terrorist attacks: The Spanish election of 2004. *Electoral Studies*, 31(1):96–106.
- Montesquieu, C. (1748). [1989] *De l'esprit des lois [The Spirit of the Laws]*. Cambridge University Press. [transl. and ed. by Anne M. Cohler, Basia Carolyn Miller (et. al)].
- Morelli, M. (2004). Party formation and policy outcomes under different electoral systems. *Review of Economic Studies*, 71(3):829–853.
- Mueller, D. C., Tollison, R. D., and Willett, T. D. (1972). Representative democracy via random selection. *Public Choice*, 12:57–68.
- Nakaguma, M. Y. (2015). Choosing the form of government: Theory and evidence from Brazil. *Economic Journal*, 125(589):1986–2023.
- Niederkrotenthaler, T., Laido, Z., Kirchner, S., Braun, M., Metzler, H., Waldhör, T., Strauss, M., Garcia, D., and Till, B. (2022). Mental health over nine months during the sars-cov2 pandemic: Representative cross-sectional survey in twelve waves between April and December 2020 in Austria. *Journal of Affective Disorders*, 296:49–58.

- Nunnari, S. and Zápál, J. (2017). Dynamic elections and ideological polarization. *Political Analysis*, 25(4):505–534.
- Osborne, M. J. and Slivinski, A. (1996). A model of political competition with citizen-candidates. *Quarterly Journal of Economics*, 111(1):65–96.
- Pellert, M., Lasser, J., Metzler, H., and Garcia, D. (2020). Dashboard of sentiment in Austrian social media during COVID-19. *Frontiers in Big Data*, 3:32.
- Pennebaker, J. W., Boyd, R. L., Jordan, K., and Blackburn, K. (2015). *The Development and Psychometric Properties of LIWC2015*. University of Texas at Austin, Austin, TX.
- Pennington, J., Socher, R., and Manning, C. D. (2014). Glove: Global vectors for word representation. In *[Conference] Glove: Global Vectors for Word Representation*, pages 1532–1543, Doha, Qatar. Association for Computational Linguistics.
- Perote-Peña, J. and Piggins, A. (2015). A model of deliberative and aggregative democracy. *Economics and Philosophy*, 31(1):93–121.
- Persson, T., Roland, G., and Tabellini, G. (1997). Separation of powers and political accountability. *Quarterly Journal of Economics*, 112(4):1163–1202.
- Persson, T. and Tabellini, G. (2002). *Political Economics: Explaining Economic Policy*. MIT Press, Cambridge, MA.
- Persson, T. and Tabellini, G. (2005). *The Economic Effects of Constitutions*, volume 1. MIT Press, Cambridge, MA, 1st edition.
- Quinn, K. M., Monroe, B. L., Colaresi, M., Crespín, M. H., and Radev, D. R. (2010). How to analyze political attention with minimal assumptions and costs. *American Journal of Political Science*, 54(1):209–228.
- Reinemann, C., Stanyer, J., Scherr, S., and Legnante, G. (2012). Hard and soft news: A review of concepts, operationalizations and key findings. *Journalism*, 13(2):221–239.

- Riker, W. H. (1992). The justification of bicameralism. *International Political Science Review*, 13(1):101–116.
- Rodriguez, P. L. and Spirling, A. (2022). Word embeddings: What works, what doesn't, and how to tell the difference for applied research. *Journal of Politics*, 84(1):101–115.
- Romer, T. and Rosenthal, H. (1978). Political resource allocation, controlled agendas, and the status quo. *Public choice*, 33(4):27–43.
- Samuel, J., Ali, G. G. M. N., Rahman, M. M., Esawi, E., and Samuel, Y. (2020). COVID-19 public sentiment insights and machine learning for Tweets classification. *Information*, 11(6):314.
- Sargent, T. J. (1987). *Macroeconomic Theory*. Boston: Academic Press, 2nd edition.
- Shapiro, A. H., Sudhof, M., and Wilson, D. J. (2022). Measuring news sentiment. *Journal of Econometrics*, 228(2):221–243.
- Shubik, M. (1970). On Homo Politicus and the instant referendum. *Public Choice*, 9:79–84.
- Sigillo, A. and Sicafuse, L. (2015). The influence of media and community sentiment on policy decision-making. In Miller, M. K., Blumenthal, J. A., and Chamberlain, J., editors, *Handbook of Community Sentiment*, pages 29–42. Springer, New York, NY.
- Stephenson, M. C. and Nzelibe, J. O. (2010). Political accountability under alternative institutional regimes. *Journal of Theoretical Politics*, 22(2):139–167.
- Swiss National COVID-19 Science Task Force, T. (2022). *Abschlussbericht der Swiss National COVID-19 Science Task Force (ncs-tf)*. Swiss National COVID-19 Science Task Force, Switzerland.
- Tavits, M. (2007). Principle vs. pragmatism: Policy shifts and political competition. *American Journal of Political Science*, 51(1):151–165.

- Tsebelis, G. (1999). Veto players and law production in parliamentary democracies: An empirical analysis. *American Political Science Review*, 93(3):591–608.
- Tsebelis, G. (2002). *Veto Players: How political Institutions Work*. Princeton University Press, Princeton, NJ.
- Tsebelis, G. and Money, J. (1997). *Bicameralism. Political Economy of Institutions and Decisions*. Cambridge University Press, New York, NY.
- Tullock, G. (1967). *Toward a Mathematics of Politics*. University of Michigan Press, Ann Arbor, MI.
- Vives, X. (2000). *Oligopoly Pricing: Old Ideas and New Tools*. MIT Press, Cambridge, MA.
- Zigmond, A. S. and Snaith, R. P. (1983). The hospital anxiety and depression scale. *Acta Psychiatrica Scandinavica*, 67(6):361–370.

Curriculum Vitae

Born on July 20, 1996 in Vienna, Austria

2019 – 2023 Ph.D. in Economics - ETH Zurich, Switzerland

2018 – 2019 Internship - Nationalbank, Austria

2017 – 2018 M.Sc. in Mathematical Modeling - University College London, UK

2014 – 2017 B.Sc. in Mathematics - Technische Universität Wien, Austria