Trajectory Optimization Framework for Rehabilitation Robots - Supplementary Material

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Trajectory Optimization Framework for Rehabilitation Robots with Multi-Workspace Objectives and Constraints Supplementary Material

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I. Hybrid Differentiation

To calculate the higher-order derivatives of the parameterization, a linear combination between forward and backward finite difference coefficients c of precision 2 was applied [?]. The forward coefficients are:

$$\begin{aligned}\boldsymbol{c}_{F0} &= \{1\}\\ \boldsymbol{c}_{F1} &= \{-1.5, 2, -0.5\}\\ \boldsymbol{c}_{F2} &= \{2, -5, 4, -1\}\\ \boldsymbol{c}_{F3} &= \{-2.5, 9, -12, 7, -1.5\}\end{aligned} \tag{1}$$

And the backward coefficients:

$$\begin{aligned}\boldsymbol{c}_{B0} &= \{1\}\\ \boldsymbol{c}_{B1} &= \{0.5, -2, 1.5\}\\ \boldsymbol{c}_{B2} &= \{-1, 4, -5, 2\}\\ \boldsymbol{c}_{B3} &= \{1.5, -7, 12, -9, 2.5\}\end{aligned} \tag{2}$$

The hybrid differentiation function results in:

$$\begin{aligned}\left(\frac{\delta^j \boldsymbol{q}}{\delta^j t}\right)_k &= f_j(\boldsymbol{q}_{k-j-1}, \dots, \boldsymbol{q}_{k+j+1})\\ &= \begin{cases} A, & \text{if } k < j+1 \\ B, & \text{if } k > N-j-1 \\ \alpha A + \beta B, & \text{else} \end{cases} \qquad (3)\\ \beta &= \frac{k-j-1}{N-2j-2}, \quad \alpha = 1-\beta\\ A &= \boldsymbol{c}_{Fj}[\boldsymbol{q}_k, \dots, \boldsymbol{q}_{k+j+1}]^T\\ B &= \boldsymbol{c}_{Bj}[\boldsymbol{q}_{k-j-1}, \dots, \boldsymbol{q}_k]^T\end{aligned}$$

The parameterization derivative can also be expressed in dense form with matrix D_j, containing the corresponding forward and backward coefficients as elements.

II. CJC to HND Transformation ($\chi_{\text{CJC}}^{\text{HND}}$)

The transformation from CJC and HND can be calculated via forward kinematics. In the following, these abbreviations are used for rotations in 3D-space:

$$\begin{aligned}\boldsymbol{R}_x(\theta) &:= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}\\ \boldsymbol{R}_y(\theta) &:= \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}\\ \boldsymbol{R}_z(\theta) &:= \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}\end{aligned} \tag{4}$$

The rotation from the torso fixed system to the hand can be calculated as a series of consecutive rotations:

$$\begin{aligned}\boldsymbol{R}_{\text{POE}} &= \boldsymbol{R}_z(q_{\text{POE}})\\ \boldsymbol{R}_{\text{AOE}} &= \boldsymbol{R}_{\text{POE}}\boldsymbol{R}_y(-\frac{\pi}{2})\boldsymbol{R}_z(q_{\text{AOE}})\\ \boldsymbol{R}_{\text{IER}} &= \boldsymbol{R}_{\text{AOE}}\boldsymbol{R}_y(-\frac{\pi}{2})\boldsymbol{R}_z(q_{\text{IER}})\\ \boldsymbol{R}_{\text{EFE}} &= \boldsymbol{R}_{\text{IER}}\boldsymbol{R}_y(-\frac{\pi}{2})\boldsymbol{R}_x(-\frac{\pi}{2})\boldsymbol{R}_z(q_{\text{EFE}})\\ \boldsymbol{R}_{\text{WPS}} &= \boldsymbol{R}_{\text{EFE}}\boldsymbol{R}_y(\frac{\pi}{2})\boldsymbol{R}_z(q_{\text{WPS}})\\ \boldsymbol{R}_{\text{WFE}} &= \boldsymbol{R}_{\text{WPS}}\boldsymbol{R}_x(\frac{\pi}{2})\boldsymbol{R}_z(q_{\text{WFE}})\\ \boldsymbol{R}_{\text{WUR}} &= \boldsymbol{R}_{\text{WFE}}\boldsymbol{R}_y(-\frac{\pi}{2})\boldsymbol{R}_z(q_{\text{WUR}})\\ \boldsymbol{R}_{\text{HND}} &= \boldsymbol{R}_{\text{WUR}}\boldsymbol{R}_x(\pi)\boldsymbol{R}_z(-\frac{\pi}{2})\end{aligned} \tag{5}$$

And the corresponding hand coordinates w.r.t. the glenohumeral joint in world coordinates:

$$\begin{aligned}\boldsymbol{p}_{\text{UA}} &= \boldsymbol{R}_{\text{IER}} \begin{pmatrix} 0 \\ 0 \\ l_{\text{UA}} \end{pmatrix}\\ \boldsymbol{p}_{\text{FA}} &= \boldsymbol{p}_{\text{UA}} + \boldsymbol{R}_{\text{EFE}} \begin{pmatrix} l_{\text{FA}} \\ 0 \\ 0 \end{pmatrix}\\ \boldsymbol{p}_{\text{HND}} &= \boldsymbol{p}_{\text{FA}} + \boldsymbol{R}_{\text{WUR}} \begin{pmatrix} 0 \\ \frac{l_{\text{WR}}}{2} \\ 0 \end{pmatrix}\end{aligned} \tag{6}$$

Here, l_{UA} is the upper arm length, l_{FA} is the forearm length and l_{WR} is the wrist length of the patient. Further, the elbow

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swivel angle ϕ is calculated as follows:

$$
\begin{aligned}
r &= \frac{1}{1+e^{-1000(\alpha-0.01)}}, \quad \alpha = 1-\frac{|\boldsymbol{u}_z\cdot\boldsymbol{p}_{\text{HND}}|}{||\boldsymbol{p}_{\text{HND}}||}\\
\boldsymbol{n} &= r\left(\boldsymbol{u}_z\times\boldsymbol{p}_{\text{HND}}\right)+(1-r)\,\boldsymbol{u}_x\\
\phi &= \text{acos}\left(\frac{\boldsymbol{n}\cdot(\boldsymbol{R}_{\text{EFE}}\boldsymbol{u}_z)}{||\boldsymbol{n}||\cdot||\boldsymbol{R}_{\text{EFE}}\boldsymbol{u}_z||}\right),
\end{aligned} \tag{7}
$$

where r estimates the elbow condition to prevent singularities when the arm is fully extended. $\boldsymbol{u}_x$ and $\boldsymbol{u}_z$ are unit vectors in x and z direction respectively. The final HND coordinates result in:

$$
\begin{pmatrix}
\boldsymbol{p}_{\text{HND},x}\\
\boldsymbol{p}_{\text{HND},y}\\
\boldsymbol{p}_{\text{HND},z}\\
\phi\\
Q(\boldsymbol{R}_{\text{HND}})_w\\
Q(\boldsymbol{R}_{\text{HND}})_x\\
Q(\boldsymbol{R}_{\text{HND}})_y\\
Q(\boldsymbol{R}_{\text{HND}})_z
\end{pmatrix} \tag{8}
$$

with Q being the conversion from rotation matrix to quaternion.

III. HND to CJC Transformation ($\chi^{\text{CJC}}_{\text{HND}}$)

The normal vector of the elbow triangle $\boldsymbol{n}$ can be determined by the following system of equations

$$
\begin{cases}
\boldsymbol{n}\cdot\boldsymbol{z}_n = cos(\theta),\\
\boldsymbol{n}\cdot\boldsymbol{p}_{\text{FA}} = 0,\\
||\boldsymbol{n}|| = 1
\end{cases} \tag{9}
$$

which, when solving analytically, yields two solutions $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ for a possible positive and negative θ:

$$
\begin{aligned}
a &= \frac{\boldsymbol{p}_{\text{FA,y}}\boldsymbol{z}_{n,z}-\boldsymbol{p}_{\text{FA,z}}\boldsymbol{z}_{n,y}}{\boldsymbol{p}_{\text{FA,x}}\boldsymbol{z}_{n,y}-\boldsymbol{p}_{\text{FA,y}}\boldsymbol{z}_{n,x}}\\
b &= \frac{-cos(\theta)\boldsymbol{p}_{\text{FA,y}}}{\boldsymbol{p}_{\text{FA,x}}\boldsymbol{z}_{n,y}-\boldsymbol{p}_{\text{FA,y}}\boldsymbol{z}_{n,x}}\\
c &= \frac{\boldsymbol{p}_{\text{FA,z}}\boldsymbol{z}_{n,x}-\boldsymbol{p}_{\text{FA,x}}\boldsymbol{z}_{n,z}}{\boldsymbol{p}_{\text{FA,x}}\boldsymbol{z}_{n,y}-\boldsymbol{p}_{\text{FA,y}}\boldsymbol{z}_{n,x}}\\
d &= \frac{cos(\theta)\boldsymbol{p}_{\text{FA,x}}}{\boldsymbol{p}_{\text{FA,x}}\boldsymbol{z}_{n,y}-\boldsymbol{p}_{\text{FA,y}}\boldsymbol{z}_{n,x}}\\
e &= \sqrt{c^2\left(1+c^2-d^2-b^2\left(c^2+1\right)\right)}\\
\boldsymbol{n}_{1,x} &= b, \quad \boldsymbol{n}_{1,y} = \frac{d-e}{c^2+1}, \quad \boldsymbol{n}_{1,z} = -\frac{e+c^2d}{c^3+c}\\
\boldsymbol{n}_{2,x} &= b, \quad \boldsymbol{n}_{2,y} = \frac{d+e}{c^2+1}, \quad \boldsymbol{n}_{2,z} = \frac{e-c^2d}{c^3+c}
\end{aligned} \tag{10}
$$

Further, the upper arm position $\boldsymbol{p}_{\text{UA}}$ can be found:

$$
\begin{aligned}
r &= \frac{\boldsymbol{n}\times\boldsymbol{p}_{\text{FA}}}{||\boldsymbol{n}\times\boldsymbol{p}_{\text{FA}}||}\cdot\frac{\sqrt{s\left(s-l_{\text{FA}}\right)\left(s-l_{\text{UA}}\right)\left(s-||\boldsymbol{p}_{\text{FA}}||\right)}}{2||\boldsymbol{p}_{\text{FA}}||},\\
h &= \frac{\boldsymbol{p}_{\text{FA}}}{||\boldsymbol{p}_{\text{FA}}||}\cdot\frac{\sqrt{s\left(s-l_{\text{FA}}\right)\left(s-l_{\text{UA}}\right)\left(s-||\boldsymbol{p}_{\text{FA}}||\right)}}{2||\boldsymbol{p}_{\text{FA}}||\cdot\tan\left(\text{acos}\left(\frac{l_{\text{UA}}^2+||\boldsymbol{p}_{\text{FA}}||^2-l_{\text{FA}}^2}{2l_{\text{UA}}||\boldsymbol{p}_{\text{FA}}||}\right)\right)},\\
\boldsymbol{p}_{\text{UA}} &= r+h
\end{aligned} \tag{11}
$$

Having the elbow position, the first two clinical joints can be determined:

$$
\begin{aligned}
q_{\text{POE}} &= \text{atan2}\left(\left(\frac{\boldsymbol{p}_{\text{UA}}}{l_{\text{UA}}}\right)_y, \left(\frac{\boldsymbol{p}_{\text{UA}}}{l_{\text{UA}}}\right)_x\right)\\
q_{\text{AOE}} &= \text{acos}\left(-\frac{\boldsymbol{p}_{\text{UA}}}{l_{\text{UA}}}\cdot\boldsymbol{e}_z\right)
\end{aligned} \tag{12}
$$

The rotation from the torso fixed system to the AOE frame $\boldsymbol{R}_{\text{AOE}}$ can be calculated as seen in eq. (5). Next, the third and fourth clinical joints can be determined:

$$
\begin{aligned}
\boldsymbol{u} &= \boldsymbol{R}_{\text{AOE}}^T\boldsymbol{p}_{\text{FA}}\\
q_{\text{IER}} &= \text{atan2}\left(\boldsymbol{u}_z, \boldsymbol{u}_y\right)\\
q_{\text{EFE}} &= \text{acos}\left(\frac{l_{\text{FA}}^2+l_{\text{UA}}^2-||\mathbf{p}_{\text{FA}}||^2}{2l_{\text{FA}}l_{\text{UA}}}\right)-\pi
\end{aligned} \tag{13}
$$

The wrist joints can be determined by finding the Euler ZYX coordinates from the resulting rotation matrix:

$$
R_D = \mathbf{R}_{\text{EFE}}^T\mathbf{R}_{\text{WUR}} \tag{14}
$$