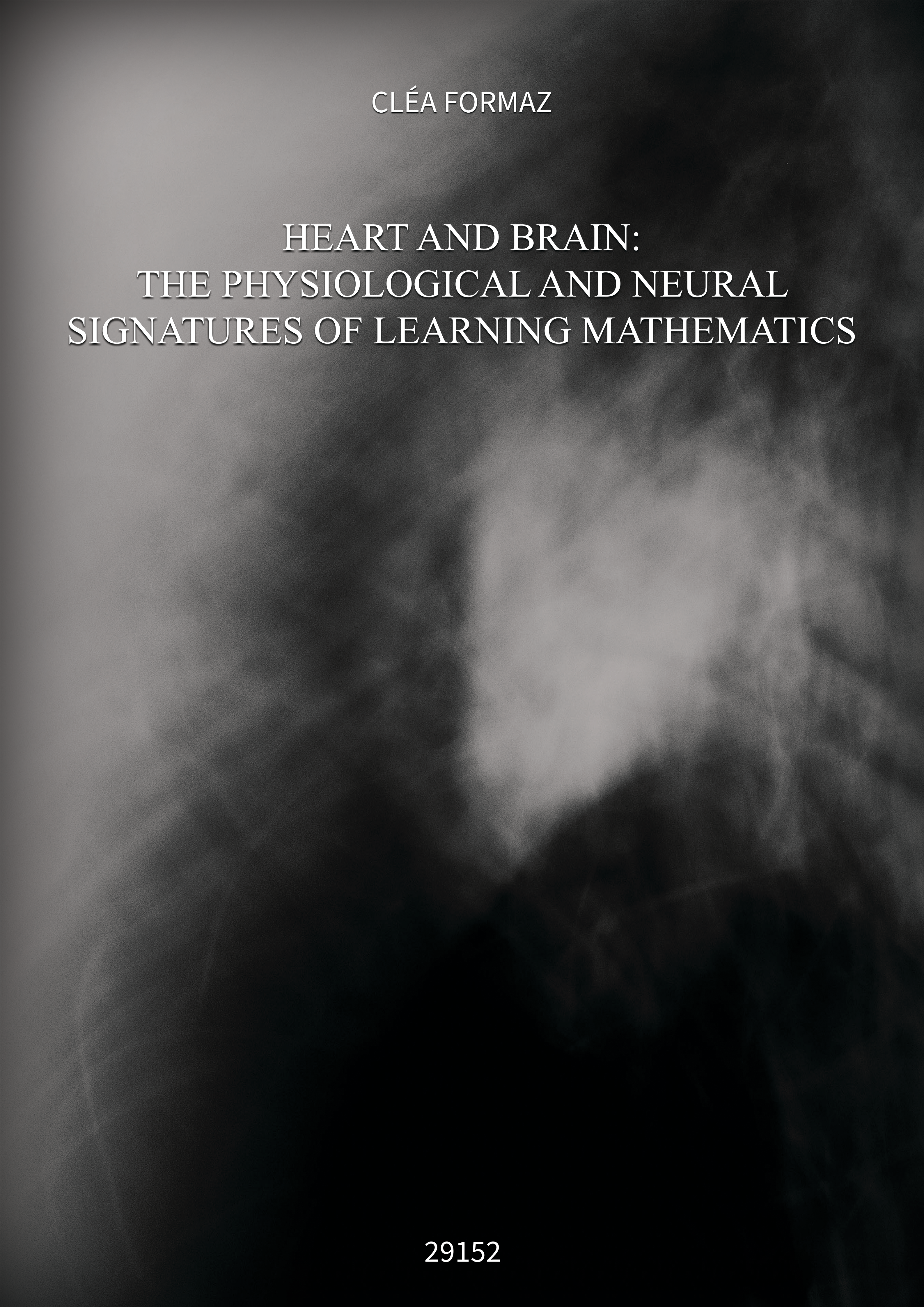

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HEART AND BRAIN: THE PHYSIOLOGICAL AND NEURAL SIGNATURES OF LEARNING MATHEMATICS

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HEART AND BRAIN: THE PHYSIOLOGICAL AND NEURAL SIGNATURES OF LEARNING MATHEMATICS

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While the old academic model addressed primarily the intellectual aspects of learning, the prevailing model suggests that we learn with our mind, heart and body. This more holistic view underscores the importance of considering all of the learner's issues.

—Eric Jensen, *Completing the Puzzle*

Abstract

This study makes three interrelated contributions pertinent to neurocognition and neurophysiology in learning. These contributions were carried out in the form of three projects that examined the neural and physiological basis of learning mathematics.

The first project is a systematic investigation of neural and physiological measurements taken while learning. A systematic review was conducted of previous literature documenting heartbeats and brain oscillations during different learning processes.

The second project analyzes neural differences between math experts and novices making sense of mathematical demonstrations. This project enabled the investigation of the brain oscillation differences between successful (expert) and unsuccessful (novice) performers. As the expert-novice literature is well-established, this study allowed for novel insights on the processes of advanced cognition through a detailed neural analysis. In addition to the neural differences, the second project also enabled the analysis of the differences in heartrates and heartrate variability between math experts and novices when exposed to mathematical demonstrations.

The third project is an investigation of the neural and physiological basis of learning through problem-solving followed by instruction (PS-I) as entailed in Productive Failure (PF). In a PS-I, learners are intentionally confronted with moments of difficulty, and even failure, as a means of preparation for improved learning from future instruction. Recent research has indicated cognitive mechanisms explaining why students learn better after encountering difficulties; however, the neural and physiological mechanisms underpinning this process have yet to be explored. This project employed an established PS-I design for the learning of a mathematical concept (standard deviation) while neural and physiological data was collected. Because moments of difficulty are not just metaphorically but literally felt in the heart, HRV may be a promising methodology for exploring the success of instructional designs that introduce desirable difficulties, as in PS-I.

Taken together, these projects allow us to build a deeper explanatory basis of advanced mathematical cognition and learning by exploring the connections between cognition, neural activity, and physiological activity.

Résumé

Cette thèse apporte trois contributions interdépendantes pertinentes pour la neurocognition et la neurophysiologie dans l'apprentissage. Ces contributions ont pris la forme de trois projets qui ont étudié les bases neuronales et physiologiques de l'apprentissage des mathématiques.

Le premier projet est une étude systématique des mesures neuronales et physiologiques lors de l'apprentissage. Une revue systématique de la littérature antérieure a été effectuée, documentant les battements cardiaques et les processus cérébraux pendant différents processus d'apprentissage.

Le deuxième projet analyse les différences neuronales entre les experts et les novices en mathématiques qui analysent des démonstrations mathématiques. Ce projet m'a permis d'étudier les différences d'oscillation cérébrale entre les sujets qui réussissent (experts) et ceux qui échouent (novices). La littérature spécialisée des études experts-novice étant bien établie, cette étude a permis, grâce à une analyse neuronale détaillée, d'apporter de nouveaux éclairages sur les processus de la cognition avancée. En plus des différences neuronales, le deuxième projet m'a également permis d'analyser les différences de battements cardiaques et de variabilité des battements cardiaques entre les experts en mathématiques et les novices pendant l'analyse de démonstrations mathématiques.

Le troisième projet est une étude de la base neuronale et physiologique de l'apprentissage par la résolution de problèmes avant l'instruction (PS-I), comme c'est le cas dans Productive Failure (PF). Dans une conception PS-I, les apprenants sont délibérément confrontés à des moments d'échec afin de se préparer à un apprentissage amélioré à partir de l'enseignement futur. Des recherches antérieures ont mis en évidence des mécanismes cognitifs qui expliquent pourquoi les élèves apprennent mieux après avoir rencontré des difficultés ; cependant, les mécanismes neuronaux et physiologiques qui sous-tendent ce processus restaient à explorer. Ce projet a utilisé une conception PS-I bien établie pour l'apprentissage d'un concept mathématique (écart-type), tout en recueillant des données neuronales et physiologiques.

En résumé, cette thèse m'a permis de construire une base d'explication plus profonde de la cognition mathématique et des processus d'apprentissage en examinant les liens entre la cognition, l'activité neuronale et physiologique.

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Chapter 1
Introduction

Neurocognition, Neurophysiology, and Mathematical Cognition

Neurocognition and neurophysiology are two disciplines that broadly seek to study, investigate, and understand, through different methodologies, the structures of the brain and its correlation with the ability of the brain to learn. Learning involves forming and strengthening neural connections and networks (e.g. Holtmaat & Caroni, 2016), from a cognitive neuroscience perspective. In addition to neural measurements, heart rate variability (HRV) analysis is emerging as a measure of learning-related cognitive functions (Forte et al., 2019). In this sense, research confirms what we intuitively sense: the palpitations of the heart are related to excitement and anxiety we sometimes associate with everyday problem solving. Yet recent findings indicate that HRV may act as an objective measure even in advanced problem solving, such as when generating solutions to complex problems. HRV is thus a promising yet underutilized measurement in the cognitive and learning sciences field. My doctoral work aims to systematically investigate heartbeats in the context of advanced mathematical cognition and learning.

In addition to heartbeat measurements, I collected measures of neural brain activity (Electroencephalography; EEG) during the learning process. Two different approaches were put in place, aiming to look at the neural signature of learning. The first study was based on math cognition with experts versus novices making sense of mathematical demonstrations. The second study evaluated the neural basis of learning through investigating the neural signature of learning through problem-solving followed by instruction (PS-I). For both studies, we were interested in the neural brain activity, which was recorded via Electroencephalography/Electroencephalogram (EEG). EEG is the record of the electric signals generated by the action of brain cells and measured by means of electrodes that are placed on the scalp (Blinowska & Durka, 2006).

The learning of mathematics was chosen as the basis of this study. The importance of mathematics is not only crucial for scientists or engineers but for every individual as it helps develop valuable skills, such as analyzing data, or describing and understanding phenomena (Niss, 1994). Because some people may have difficulties learning mathematical and abstract concepts, the present study is essential in understanding if these learning challenges have references to the brain.

The first EEG study was based on math cognition, using university students participating in a series of challenges with experts versus novices making sense of mathematical demonstrations. The study analyzed the differences between novices and experts in terms of the frontal alpha and theta brain activity. The differences in heart rates were also considered. The main goal for this first study is to see the differences in the heartbeat and the neural brain between experts and novices while exposed to mathematical proofs and whether and how stress and nervousness could affect the heartbeat of the novices and experts.

The second study evaluated the neurophysiological basis of the learning design called problem-solving followed by instruction (PS-I). Studies investigating PS-I have been suggesting cognitive mechanisms involved, but the neural signature has not yet been studied. This study focuses on the latter via EEG. PS-I is a learning design that entails conditions for learners to persist in generating and exploring representations and solution methods for solving complex problems, prior to formal instructions. Such a process may initially lead to failure (i.e., failing to generate canonical results). The heartbeat and neural differences between the different phases that students go through when being exposed to problems without having any instruction and background, compared to students who are getting the instruction, were analyzed.

Investigating physiological responses (heartbeats and brain waves) to teaching methods is indirect but might provide a less subjective measure as a proxy for cognitive engagement. The goal is to contribute to cognitive neuroscience and education research by translating the neuro-physiological mechanisms of learning to educational practice and understanding the effects of education and, more specifically, instructional designs on the learner's brain. By translating insights about the heart rate as an indirect and more objective measure of cognitive load, learning could be made more efficient by improving the student's mindfulness, inner balance, and calmness (e.g., lowering the heart rate).

Neuroscience and Education

Education is essential to any individual. Education impacts economic, political, and cultural development in today's modernizing world (Chabbott & Ramirez, 2000). Empirical studies in the field of education show that there is a positive relationship between the forms of education and an individual's economic, political, and cultural development (Chabbott & Ramirez, 2000).

Therefore, research in the field of education has become increasingly important.

Education has been referred to helping and increasing the productivity of human labor through acquiring the necessary knowledge. A general rationale constructs education as a human right. Education has been linked to the basis for human beings to improve their skills, which then allows them to participate in the economy, politics, and culture of society. Education and learning are tied to the notions of equality and human rights (Chabbott & Ramirez, 2000).

Neuroscience has become increasingly important during the past century, as many aspects of physiology, biochemistry, and the structure of the vertebrate brain have been uncovered (Goswami, 2004). Neuroscience is concerned with studying the cognitive, attentional, and emotional (among other) mechanisms, which are important when studying learning processes and education. The correlation between the brain, education, and the learning process can be studied with neuroimaging techniques, which allow us to study the human brain at work in vivo. Neuroimaging allows us to deepen our understanding of learning processes, such as solving mathematical problems, reasoning, reading, or language processing. Neuroscience makes it possible to explore educational questions with an increased understanding of the brain functions underlying learning processes (Goswami, 2004).

Investigation on brain and cognition has merged into the field of cognitive neuroscience. Cognitive neuroscience is concerned with studying the nerve cells and how neurons receive and transmit information. This study has provided a new framework, which allows us to study memory, perception, language, mathematical cognition, and other cognitive processes (Milner, Squire & Kandel, 1998). Cognitive neuroscience might therefore have an essential influence on research in learning and education, including mathematics education, as it contributes to our understanding of mathematical cognition. The study of cognitive neuroscience can generate findings about learning that cannot be uncovered solely with behavioral research. The interdisciplinary field of education and neuroscience therefore has opened a door to reveal the underlying neural – as well as physiological - mechanisms of learning (De Smedt et al., 2010).

Doctoral Project

This doctoral work aims to systematically investigate heartbeats and brain measurements in the context of advanced mathematical cognition and learning. The first project (Chapter 2) was a systematic review on heartbeat measurements and brain oscillations during the learning process. The second project (Chapter 3) was the analysis of neural differences between novices and experts making sense of mathematical proofs. Neural differences in frontal alpha and theta activity between successful (experts) and unsuccessful performers (novices) were documented and analyzed. The second project explored what an expert's neurophysiology looks like and what effects learning mathematics has on successful learners' brains. The third project (Chapter 4) of my doctoral work is the extension of EEG and ECG analysis to a more complex scenario involving learning from problem-solving followed by instruction (PS-I). Participants in this EEG and ECG study are intentionally confronted with moments of difficulty, and even failure, to prepare them to learn better from future instruction. In the third study, previous research was extended by including physiological and neural measures. During my doctoral work, I developed and documented a deeper explanatory basis of advanced mathematical cognition and learning through exploring the connections between cognition, heartbeats, and neural activity. The overall discussion (Chapter 5) highlights the main findings and possible future directions in the field of neurocognition and neurophysiology in learning.

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Chapter 2

Neural and Physiological Mechanisms During the Learning Process: A Systematic Review

Overview

Physiological signals, for example heartbeats, enable, compete with, or inhibit, information processing across psychological domains (Critchley & Garfinkel, 2018). Physiological signals have also been used as a means of measuring changes in cognition and learning (Cranford et al., 2014). However, the connection of heartbeats and brain oscillations in the learning context is unclear. This systematic review aims to analyze neural and physiological mechanisms during the learning process, focusing on different brain wave frequencies (neural mechanisms) and heart rate variability (physiological mechanisms).

The systematic review process was conducted according to the PRISMA-Guidelines. The inclusion criteria were that studies were published in English, measurement of brain oscillations along with heart rate variability (HRV) during a learning process. Learning processes are defined as tasks requiring sustained attention, working memory, memory retrieval, or problem-solving. Exclusion criteria: medical conditions, dementia, psychiatric disorders, strokes, and traumatic brain injury.

In most studies, heart rate change was positively related to the cognitive measures of learning. A higher resting state HRV has been observed to predict better cognitive performance in the considered learning process. Moreover, different brain oscillation frequencies (alpha, beta, theta, delta, gamma) are associated with different learning processes, including problem-solving, having an insight, or memory retrieval. Therefore, this review serves as an overview and a summary of the neural and physiological signature underlying learning.

The results highlight the influence of HRV on the learning process and the different brain oscillation frequencies that are responsible for different learning processes. There is a great range of interest on this topic, but there are no unifying results yet. This might be because research in neurophysiology and learning domains are still at an early stage. Despite this, HRV could be considered a promising measurement in learning processes in populations without medical disorders. From a physiological perspective, HRV has been linked to predicting performance on cognitive processes involved in learning. From a neurocognitive perspective, brain frequencies have been shown to serve as indices for specific brain activity related to learning processes.

Introduction

Heart Rate Variability

Dynamic changes in the body's physiology influence cognitive processes (Critchley & Garfinkel, 2018), but research in neurophysiology and specific learning processes, including for example memory, is still in its infancy. It is known that physiological signals, such as heartbeats, selectively facilitate, compete with, or inhibit information processing (Critchley & Garfinkel, 2018). While we know that physiological signals influence cognition, we still know little about the specific association between physiology and learning processes.

One of the physiological correlates of cognitive functioning is heart rate variability (HRV). It reflects the oscillations in the interval between consecutive heartbeats, which can be measured in milliseconds (Malik, 1996; Thayer and Lane, 2000; Reyes del Paso et al., 2013). Heart rate variability analysis can be conducted in three different domains: time, frequency, and non-linear analyses, whereas the time-domain has been seen as a preferred index of the vagal tone (e.g. Laborde et al., 2017).

For the time-domain, either the standard deviation of all R–R intervals, the time elapsed between two successive R-peaks of the heartbeats, is calculated, which reveals the components responsible for variability in the recording period (Malik, 1996); or the root mean square of successive differences (RMSSD), which would reflect the so-called vagal tone (Thayer and Lane, 2000; Kleiger et al., 2005; Shaffer et al., 2014; Laborde et al., 2017). More specifically, the RMSSD in heartbeat periods takes into consideration each successive time difference between heartbeats in milliseconds. Each of the values is squared, the result is averaged, then the square root of the total is attained. The RMSSD has been seen as a good index of the vagal cardiac control (e.g. Laborde et al., 2017).

Heart rate variability and cognitive processes related to learning processes

Higher HRV has been observed to be associated with higher emotional well-being, lower anxiety, and better emotion regulation (Mather & Thayer, 2018). Furthermore, as an emotion regulation strategy, attention allocation is related to HRV (Gross, 1998). In this sense, individuals with higher HRV have the capacity to quickly adjust cardiac influence to foster attentional engagement or disengagement with their sensory environment (Thayer & Lane, 2000). Higher emotional regulation and attentional regulation could lead to better learning processes, since the learner can focus on the to be learned subject by regulating anxiety and other feelings and giving attention to the task at hand. Previous research has investigated the association of different heartbeat measurements and cognitive processing of higher order cognitive skills such as recalling and memorizing facts (Park & Thayer, 2014; Mather & Thayer, 2018). By having a closer look on these studies, we can infer the interplay between heartbeats and the learning process.

Learning has been defined as ontogenetic adaptation, in other words as changes in the behavior of an organism that result from regularities in the environment of the organism (De Houwer et al., 2013). According to Houwer and colleagues (2013), this functional definition of learning serves to enhance research in cognitive learning. Since learning involves different cognitive processes (e.g. memory, exposure to novelty, or having an insight), it follows that we refer to it as learning processes. In learning processes, heart rate has been used as means of measuring cognitive load (Cranford et al., 2014), how different levels of challenge and difficulty may give rise to different emotional states, including boredom, engagement or anxiety (Chanel et al., 2008), and as a further indicator of the level of perceived challenge and difficulty (Hjortskov et al., 2004). These findings suggest that heartbeat measurements may correspond to different learning mechanisms. What we lack so far is an overview of the exact relationship between HRV components and the specific learning processes. When we talk about HRV components and cognition, it is important to take into consideration the cardiac vagal tone.

The cardiac vagal tone can be referred to as the indicator of the relationship between HRV and cognitive processes that are involved in learning (Hansen et al., 2003). The vagus nerve is a nerve that is composed of efferent fibers that send signals from the brain to the body. Further, it also sends sensory fibers with information from the body to the brain (Howland, 2014). The cardiac vagal tone is referred to the activity of the vagus nerve. In this sense, the cardiac vagal tone has been linked to cognitive control (e.g. Porges, 1995; Hansen et al., 2003; Duschek et al., 2009). This process is influenced by the so-called sympathetic and parasympathetic cardiac activity. While the sympathetic activity is associated with an acceleration of heart rate, the parasympathetic activity slows the heart rate (e.g. Pfeifer et al., 1983). Thus, sympathetic and parasympathetic influences are essential for us individuals to successfully adapt to changing environmental demands (Porges, 1995; Thayer and Lane, 2000, 2009). Individuals who have a reduction in vagal control could have a lack of ability to respond flexibly to changing demands. This may limit the individuals' ability to generate appropriate responses to external stimuli. Even though established theories indicate a relationship between HRV and cognition, the relationship between HRV and learning processes has yet been understudied. We therefore aimed to foster the understanding of physiological signatures and how they may correspond to specific learning processes.

Brain oscillations during the learning processes

To investigate how natural, varying behavior is guided by complex neural dynamics, researchers apply neuroimaging methods, for example electroencephalography (EEG) (Makeig et al., 2009). EEG records the integrated and synchronized activity of pyramidal neurons in the cerebral cortex, which are composed of oscillations in various frequencies (delta, theta, alpha, beta, and gamma; Klimesch et al., 2005). These frequencies serve as indices for specific brain activity related to cognitive processes, which are involved in learning (attention and memory; Basar, 1999).

EEG research shows how different changes in brain oscillations are related to different cognitive and learning processes. Changes in alpha or theta waves can give information about task difficulty, sustained attention, or cognitive load (Basar, 1999; Klimesch et al., 2005). Further EEG research shows that while problem-solving and having an insight, gamma oscillations are enhanced in right prefrontal regions (Rosen & Reiner, 2017), in right anterior superior temporal gyrus (Jung-Beeman et al., 2004), and fronto-central regions (Sheth et al., 2009). Higher gamma activity in the right prefrontal cortex has been associated with the restructuring stage of insight (Rosen & Reiner, 2017). Thus, when having insight, there seems to be differences in prefrontal cortex.

Moreover, alpha oscillations are associated with semantic information processing, with searching, accessing, and retrieving information from long-term memory (Klimesch et al., 1997). Memory codes are retrieved via longitudinal pathways linking thalamic nuclei - neuronal cell bodies that consist of the thalamus - with the cortex, whereas alpha is the predominant rhythm reflecting the activity of these pathways. Based on that, researchers found that a higher alpha frequency was correlated to better long-term memory performance (Klimesch et al., 1997). There is evidence that indicates how different brain oscillations are associated with learning processes, for example alpha oscillations are associated with semantic information processing. This systematic review provides an overview of the different brain oscillations and how they may correspond to the specific learning processes.

Altogether, the previous findings of heart rate variability and brain oscillations in cognitive functioning related to the learning process suggest that there seems to be a relationship between HRV and learning and that various brain oscillations underly certain learning processes. In this systematic review, we reveal the exact relationship between HRV and the specific learning processes, as well as the different brain oscillation frequencies that underly learning processes. Therefore, this review allows us to understand the specific connections between cognition, heart rate variability and learning and provides us with an overview of these connections.

Aims of the Systematic Review

While previous research studies successfully observed that heartbeat measurements, such as HRV are related to cognition and learning, and that different brain oscillation frequencies may underly learning processes, an overview of the above mentioned connections is missing. More specifically, there seems to be a gap in the exact relationship between HRV, or brain oscillation frequencies and the underlying learning processes. The general aims of this systematic review of the literature is to analyze the relationship between heartbeat measurements and learning processes, focusing mainly on HRV. A second aim of the systematic review is the analysis of neural brain measurements in learning, focusing on the different brain oscillations. All studies reviewed are research experiments on neurocognition, neurophysiology and learning in the absence of affective dimensions and pathological aspects.

Method

The systematic review was conducted according to the PRISMA-Guidelines (Liberati et al., 2009; Moher et al., 2009). We carried out a systematic analysis of the international literature by using PsychInfo (http://www.apa.org/psychinfo/), as well as Google Scholar. The key terms we used were as following: “heart rate variability”, “brain oscillations”, “learning”, “problem-solving”, “memory”, “cognitive load”. We also entered a combination of those terms into the databased mentioned above. Articles were selected if they were published in peer-reviewed journals. Study selections were limited to academic publications in which the full text was published in English, and the study included human populations without age, gender, or ethnicity restrictions. Eligibility criteria was the inclusion of one or more cognitive measures and one or more of the measurements of HRV. We excluded studies that included participants with medical conditions, as any medical condition could potentially show abnormal HRV and thus influence the relationship between the cognitive domain and HRV. In other words, we focused on HRV, brain oscillations, and learning studies that have been done on healthy participants.

Data Collection

For each study, we were interested in (1) author(s) and year of publication; (2) characteristics of participants (including age, years of education, gender); (3) type of HRV measures; (4) cognitive domain analyzed; and (5) nature and direction of the identified relationship. We based our interest regarding the data collection of the systematic review on the PICOS approach (Liberati et al., 2009). Please refer to the Appendix for a summary of the studies analyzed.

Studies Undertaken

We selected studies that included one of more learning processes, including memory, problem-solving, and tasks which were investigated with neuroimaging methods to study underlying neural and physiological mechanisms. Our study search was restricted and limited to academic publications in peer-reviewed journals and published in English language. We excluded studies that included participants with any sort of diagnosis of psychiatric disorders or disability. Sixteen studies were finally incorporated in this systematic review analysis.

RESULTS

Data Selection

The studies included in this review were carried out from 1986 to 2022, where the age of participants ranged from 18 years (Carroll, Turner & Hellawell, 1986; Jung-Beeman et al., 2004; Guderian, Schott, Richardson-Klavehn and Düzel, 2009) to 90 years (Alessandrini et al., 1997). The information regarding the gender of the participants was missing in some studies (Gellatly and Meyer, 1992; Sheth, Sandkühler, and Bhattacharya, 2009; Garfinkel et al., 2013; Cranford et al., 2014; Rosen and Reiner, 2017). The flowchart shows the number of studies identified in the database and examined by the authors. Please refer to the Appendix.

Heart rate variability and learning processes

We reviewed research studies to reveal a more detailed evaluation on the relationship between HRV and learning processes, focusing on studies that explicitly investigated a learning process (such as memory), and measured HRV. Six studies (Gellatly & Meyer, 1992; Alessandrini et al., 1997; Garfinkel et al., 2013; Cranford et al.,2014; Pham & Wang, 2016; Colzato et al.,

2018) found a relationship between heart rate variability (HRV) and learning processes. Conversely, one study (Hjortskov et al., 2004) did not to find any correlation between HRV and learning processes. It is also worth mentioning that a study reported in Carroll, Turner and Hellawell (1986) found that the heart rate, or the difference between resting and heart rate levels, was sensitive to variations in difficulty level of the learning process at hand. More specifically, an easier condition elicited significantly less cardiac activity than the hard and the impossible conditions (Carroll, Turner & Hellawell, 1986).

Results generated from the laboratory experiments conducted on 117 undergraduate participants emphasized on the fact that heart rate change was positively related to the cognitive and behavioral measures (Gellatly & Meyer, 1992). An important cardiovascular channel through which autonomic arousal impacts a cognitive function was established from the observations made by Garfinkel and colleagues (2013). In their study, the authors presented participants with words under limited attentional resources and time-locked to different phases of the cardiac cycle. They observed that when words were presented around systole (when the heart muscle contracts), memory was decreased compared to when the words were presented around diastole (when the heart muscle relaxes) (Garfinkel et al., 2013). While these results may highlight that there might be an important cardiovascular channel through with autonomic arousal impacts a cognitive functions, other studies have used heart rate in investigations including the cognitive load.

Heart rate usage as a means of measuring changes in the cognitive load was addressed and validated in the research work presented by Cranford et al. (2014). In their study, heart rate changes were hypothesized as a valid measurement of cognitive load. Chemistry students were compared to faculty members when solving chemistry problems. The authors found that chemistry problems of higher complexity induced a greater change in heart rate than those that have been designed to be of lesser complexity (Cranford et al., 2014). Further studies have been conducted in HRV and learning, including for example task-switching. Colzato et al. (2018) found that higher resting-state HRV predicts better task-switching. More specifically, participants with higher resting-state HRV showed smaller switch costs (i.e. greater flexibility) than individuals with lower resting-state HRV (Colzato et al., 2018). This study confirmed that after examining neurovisceral integration model, which proposes the linking of HRV and prefrontal cortex activity via vagus nerve connecting heart and brain, on all the 90 participants that higher levels of vagally mediated resting-state HRV support and promote cognitive flexibility. Altogether, this provides us with a more detailed knowledge on the connections between heart rate variability, cognition and learning. See the Appendix for a summary of the review on heart rate variability and learning processes.

Brain oscillations during learning processes

Previous research in the field has suggested that different brain oscillations are involved in specific learning processes, for example memory acquisition (e.g. Klimesch, Schack and Sauseng, 2005; Rosen & Reiner, 2017). Here, we report a detailed inspection of the studies that uncovered brain oscillations during learning processes. An association between brain oscillations and learning processes was reported in four studies (Jensen and Tesche, 2002; Jung-Beeman et al., 2004; Guderian, Schott, Richardson-Klavehn, Düzel, 2009; Rosen and Reiner, 2017). However, a notable and significant correlation between the brain oscillations and learning lacked in one study (Sheth, Sandkühler, & Bhattacharya, 2009).

The research carried out by Jensen and Tesche, 2002 emphasized on the role between brain oscillations and learning highlighting the fact that brain oscillations in the theta band, generated in the frontal brain regions play a pivotal role in memory maintenance. Experiments conducted on functional magnetic resonance imaging (fMRI) and Electroencephalogram (EEG) revealed a relationship between brain oscillations and learning processes as demonstrated in the research work by Jung-Beeman et al. (2004). In this research work, the authors observed an increased activity in the right hemisphere anterior superior temporal gyrus, when the experiment was carried out using fMRI and on the other hand, a sudden burst of high-frequency gamma-band neural activity was detected in the same area in the EEG, while the participants solved verbal problems with insight. Hence, it was concluded from this study that during comprehension the right anterior temporal area may be associated with making various connections across distantly related information. The findings using magnetoencephalographic recordings of brain activity reveal state-related aspects of memory formation in humans, and thereby brings forward approaches for improving memory through theta-related brain states (Guderian, Schott, Richardson-Klavehn, Düzel, 2009). Research further confirms that while solving spatial puzzle with insight neurological-cognitive processes and exclusive brain areas are required. It is noteworthy that specifically enhancement of right frontal gamma and beta band was observed while solving spatial puzzles with insight (Rosen and Reiner, 2017). According to Sheth, Sandkühler, & Bhattacharya, 2009, insight is represented by distinct spectral, spatial, and temporal patterns of neural activity related to cognitive processes (intrinsic to the problem itself) but not exclusively to one's subjective assessment of insight, thereby emphasizing on the fact that there is no significant relation between learning and brain oscillations. Please refer to the Appendix for a summary of the review on brain oscillations during learning processes.

Discussion

The aim of this systematic review was to analyze the relationship between heartbeat measurements (1), in addition to brain oscillations (2) involved in learning processes. We had a closer look at the literature to understand the relationship between HRV and learning. Some studies (Gellatly & Meyer, 1992; Alessandrini et al., 1997; Garfinkel et al., 2013; Cranford et al., 2014; Pham & Wang, 2016; Colzato et al., 2018), have reported that there is a relationship between HRV and learning process. Those previous studies demonstrated that assigned goal difficulty affected heart rate, cognition, and task performance.

More specifically, heart rate change was positively related to cognitive measures (such as memory), as well as behavioral measures (such as task performance) (e.g. Gellatly & Meyer, 1992). The study by Cranford and colleagues (2014) – for example - suggests that problems that are of higher cognitive load induce a greater change in heart rate than those of lesser load. The literature suggests a cognitive–affective mechanism that may mediate goal-difficulty and performance relation. The findings reinforce the idea that heartbeat measurements are associated with cognitive processes involved in learning.

The first laboratory studies have tried to identify the relationship between HRV and cognition and highlighted a positive relationship of heart rate change and cognitive measures (including memory processing, or problem-solving; e.g. Gellatly & Meyer, 1992, Cranford et al., 2014). Based on these findings, some theories have been developed to explain the relationship between HRV and cognitive functioning, including learning processes. These theories include the neurovisceral integration model, which proposes the linking of HRV and prefrontal cortex activity via vagus nerve connecting heart and brain (Thayer and Friedman, 2002). More recent studies demonstrate that higher levels of vagally mediated resting-state HRV support and promote cognitive flexibility (Colzato et al., 2018). Higher levels of resting-state HRV promote cognitive flexibility, such as the ability to switch easily from one cognitive task to another. This highlights how resting-state HRV corresponds to cognitive tasks involved in learning.

Another aim (2) of this study was to highlight whether different brain oscillation frequencies can be considered an index of generalizing different learning mechanisms. The analyzed studies found that there is an association between brain oscillations and different learning mechanisms (Jensen and Tesche, 2002; Jung-Beeman et al., 2004; Guderian, Schott, Richardson-Klavehn, Düzel, 2009; Rosen and Reiner, 2017). Brain oscillations in the theta band, generated in the frontal brain regions play a pivotal role in memory maintenance (Jensen and Tesche, 2002). Increased activity in the right hemisphere anterior superior temporal gyrus was related to a sudden burst of high-frequency gamma-band neural activity while the participants solved verbal problems with insight (Jung-Beeman et al., 2004). Brain activity reveal state-related aspects of memory formation in humans and brings forward approaches for a possible improvement of memory through theta-related brain states (Guderian, Schott, Richardson-Klavehn, Düzel, 2009). Enhancement of right frontal gamma and also beta oscillations was observed while solving spatial tasks with insight (Rosen and Reiner, 2017). In conclusion, specific brain oscillation frequencies have been observed to serve as indices for particular cognitive processes related to learning. Please refer to the table in the Appendix for chapter 2 for the study summaries of heartbeat measurements and brain oscillations.

Limitations

This systematic review of the literature aimed to analyze the scientific studies concerning the link between HRV and cognitive functioning related to learning. Some limitations should be considered. The heterogeneity of the population and measures, as well as the sample size may not allow performing a quantitative analysis (such as a meta-analysis) and affects the generalizability of the results. A systematic review, however, may be a good fit to bring together an overview of the studies in neurocognition and neurophysiology and learning.

Further research should aim to increase the studies on the relationship between HRV and learning, or other specific cognitive domains, such as memory, problem-solving, or task switching. Other essential aspects to consider in future studies are to compare the vagal aspects and the adaptability of the organ, in order to evaluate how, and especially to which extent, cardiac vagal control influences certain learning mechanisms. In this paper, we have focused on the HRV and learning in general and did not focus on vagal control. However, considering the vagal activity and the vagal recovery during different cognitive tasks more in detail could be interesting (e.g. Laborde, 2018). This type of study could allow us to better understand the relationship of HRV and learning, as the cardiac vagal tone has frequently been linked to cognitive control (e.g. Porges, 1995; Hansen et al., 2003; Duschek et al., 2009).

Future psychophysiological research should focus on choosing specific cognitive domains, such as language processing, attention factors, or visuospatial skills and investigate how those learning factors are influenced by and correlated with HRV. To the authors' knowledge, those research focuses have been disregarded by the studies until now. Of particular note is the investigation of the link to cognitive performance in language learning, attention – or other cognitive aspects – and HRV during the learning process.

In this review, we focused on the analysis of HRV and learning processes that allows us to analyze and understand their relationship. In addition, we investigated the association between brain oscillations and learning. Previous research has found a relationship between HRV and learning processes. In summary and in line with the neurovisceral integration model - which proposes the linking of HRV and prefrontal cortex activity via vagus nerve connecting heart and brain - higher levels of vagally mediated resting-state HRV support and promote cognitive flexibility and thus may be advantageous in learning. In addition, an association between brain oscillations and learning was reported, demonstrating how brain oscillations are related to different learning mechanisms, including memory retrieval, problem-solving and having an insight. In conclusion, this review highlights that the physiological and psychological aspects of learning operate in close interaction, whereas HRV influences learning processes, and different brain oscillation frequencies are related to specific learning processes mentioned in this review.

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Chapter 3
Brain, Mathematics, and Expertise: Neural Differences between Math Experts and Novices – a Study on Frontal Alpha Asymmetry and Frontal Theta

Abstract

Although differences in the cognitive abilities of math experts and novices are well documented, research on the underlying neural differences remains scarce. Especially rare are studies involving naturalistic stimuli with longer time durations. Although literature has indicated the presence of alpha and theta activity while solving mathematical problems and a pronounced asymmetry, it is unknown how these frequencies are related to the level of math expertise. We explored electroencephalography (EEG) dynamics in frontal alpha asymmetry and frontal theta activity between math experts and novices while participants watched mathematical demonstrations presented in symbolic and non-symbolic forms. Results indicated no significant difference between experts and novices in alpha asymmetry, but a significant difference in theta activity, with novices showing greater frontal theta activity than experts. This study contributes to research concerning the differences in neural processes between math experts and novices while making sense of mathematical demonstrations. In our EEG study, experts showed less frontal theta power compared to novices, indicating that they might need less cognitive control and engage more in working memory processes. Implications are discussed.

Introduction

The study of mathematical cognition, with its potential to give unique insights into the workings of the mind, constitutes a major domain of research in cognitive science, learning sciences, psychology, and neuroscience. Previous work has shown that math experts—compared to novices— when exposed to mathematical statements and judging them as meaningful, expand their math-sensitive neural network by activating a set of bilateral frontal, intra-parietal, and ventrolateral temporal regions, as shown in functional magnetic resonance images (fMRI) scans (Amalric & Dehaene, 2016). This indicates that brief mathematical statements reveal a difference between novices and experts. However, less is known about the neural differences between math experts and novices when being exposed to longer and more naturalistic math demonstrations. Even basic findings are lacking in this area, such as whether expert mathematicians show a specific activation profile when it comes to brain wave frequencies or asymmetric hemispheric activity.

Researchers have observed that neural correlates of arithmetic problem difficulty differ between individuals depending on their math ability (high or low ability) (Artemenko et al., 2018). For example, when solving a two-digit addition or subtraction, we can increase difficulty by requiring a carry or borrow operation. The carry effect (when a digit that is transferred from one column of digits to another column of more significant digits) and the borrow effect (when the same is used in subtraction) have been investigated in terms of spatial and temporal neural correlates. When comparing high and low performers in a written production paradigm, functional near-infrared spectroscopy (fNIRS) and event-related potential (ERP) research has shown that arithmetic difficulty interacted with an individual's math ability. More specifically, high math performers showed an increased activation in the frontal cortex. They claim that especially the left inferior gyrus, which is part of the frontal cortex, is associated with the carry and borrow effect, or the level of difficulty. In addition, the researchers observed differences in slow brainwaves at frontal sites (Artemenko et al., 2018). While these results show that the arithmetic processing might depend on the level of math ability, little is known about neural differences between math experts and math novices.

Previous neuroimaging (fMRI) research on mathematical expertise further revealed that bilateral areas of the fronto-parietal network were activated in experts compared to novices while exposed to Raven's Advanced Progressive Matrices and the Tower of London tasks (Desco et al., 2011). The increased activation in the frontal and parietal regions of math experts seems to be associated with enhanced skills in logical reasoning. Although it is known that during mathematical tasks, neurons in the active regions are tuned to numerical quantities (Dehaene et al., 1999; Nieder and Dehaene, 2009) and that there are differences between math experts and novices, not much is known yet about the underlying neural signature. While solving a math problem, different brain wave frequencies can be observed, depending on the phase of the problem, degree of involvement, and sensitivity of different brain regions (Lin et al., 2012). Brain wave frequency can be seen as the number of brain wave cycles within which a response may be elicited, and which is subject to variability in response time (Surwillo, 1963). Electroencephalography (EEG) records brain activity via the electroencephalogram. The electroencephalogram is the record of the electrical signals that are generated by the brain cell actions, more specifically, by the time course of the extracellular field potentials that are generated by synchronous brain cell actions (Blinowska & Durka, 2006). The following brain wave frequencies have been distinguished in EEG: delta (1–4 Hz), theta (4–8 Hz), alpha (8–13 Hz), beta (14–30 Hz), and gamma (>30 and typically <100 Hz; e.g., Pizzagalli, 2007). The frequency band oscillations are usually quantified through power spectral density (PSD), which describes the distribution of signal power at differing frequencies (Dressler et al., 2004). Frequency band types are associated with cognitive processes. For example, theta frequencies are associated with memory processes, such as retrieval and encoding (Klimesch, 1999; Nyhus & Curran, 2010), while alpha frequencies are related to visual processing prioritization (Jensen et al., 2014). Therefore, EEG can be used to investigate the association of frequency band types with cognitive processes and the expertise thereof.

EEG frequencies play a vital role in understanding brain activities while performing simple as well as complex cognitive tasks (e.g., Klimesch, 1999). Previous research has shown that alpha and theta band frequencies are associated with solving mathematical problems (e.g., Lin et al., 2015). Especially arithmetic processing has been related to theta and alpha frequency bands (Antonenko, Paas, Grabner, & van Gog, 2010; Grabner & De Smedt, 2011; Hinault & Lemaire, 2016). In the study of Lin and colleagues (2015), EEG was recorded as participants were asked to combine four single-digit numbers through basic arithmetic operators and create arithmetic expressions equaling 24. The researchers found that theta and alpha waves were related to mathematical problem-solving as well as solution latencies. In a study by Soltanlou and colleagues (2018), the researchers observed increased alpha power in EEG measurements after children worked and were trained on mathematical problems. Another study on alpha activity, the researchers found that alpha activity seems to be positively correlated in frontal regions with behavioral responses on a memory recall task. More specifically, frontal alpha activity seems to be correlated to the retrieval of learning contents including educational science concept material (Hanouneh et al., 2018). Further, the topographic distribution of spectral fluctuations was characterized by more pronounced asymmetries along the left–right and anterior–posterior axes for solutions that involved a longer search phase. However, although previous literature has indicated the presence of alpha and theta activity while solving mathematical problems and a pronounced asymmetry, it is unknown how these frequencies and the asymmetry are related to the level of math expertise.

Frontal Alpha Asymmetry

The average difference in brain activity between the left and right frontal areas, measured as hemispheric differences in alpha power, is referred to as frontal alpha asymmetry (FAA; Harmon-Jones, Gable, & Peterson, 2010). It is the power band activity of electrode F4 on the right hemisphere (RH) subtracted by electrode F3 on the left hemisphere (LH), or vice versa. In a previous study where individuals were asked to solve basic operations presented as a challenging math game, alpha asymmetry was enhanced as solution times grew longer, indicating that alpha asymmetry might be associated with higher difficulty levels, perseverance, and motivation (Lin et al., 2015). Contributing to the discussion of Lin et al.'s findings, further studies have investigated FAA in emotional and motivational processes (Quaedflieg et al., 2015), affects (Rosenfeld et al., 1996), and executive functions (Moynihan et al., 2013). The differences in frontal alpha power asymmetry between math experts and novices have not yet been studied, and our goal is to investigate this in our study. Moreover, by drawing on previous work and comparing FAA in experts and novices, we are also able to indirectly attend to differences in motivational processes, affects, and executive functions.

Frontal asymmetry has been of interest in studies investigating individual differences research on emotional and motivational processes. On the one hand, left-frontal hemisphere activity has been associated with an approach system that – in return – gets activated when participants tend to move towards a goal and experience positive emotions. On the other hand, researchers have observed a right lateralized withdrawal system, which is involved in negative affect. This seems to be activated mainly when participants encounter potentially dangerous situations (Tomarken et al., 1992; Davidson, 1998; Coan et al., 2006). While previous research has uncovered how frontal asymmetries are associated with emotional and motivational processes, we aim to elaborate on those findings and investigate the frontal alpha asymmetry differences between math experts and novices while being exposed to mathematical demonstrations.

Frontal Theta Activity

Theta activity over frontal regions is known as frontal theta. Theta oscillation (approximately 4-8 Hz) has mostly been associated with domain-general cognitive demands and the acquisition of new information (Klimesch, 1999). Also, theta appears to reflect active operations, particularly during high-level cognitive processes, such as memory encoding and retrieval, working memory retention, novelty detection, and realizing the need for top-down control

(Jacobs et al., 2016; Itthipuripat, et al., 2013; Cavanagh, et al., 2012; Rutishauser, et al., 2010). Previous research suggests that frontal theta activity might be engaged in problem-solving (Ryu et al., 2016) and particularly crucial for mathematical problem solving (Lin, et al., 2012, 2015; Pavlygina et al., 2010; Ghaderi et al., 2019).

Although there is an array of cognitive processes reflected in theta oscillation, we were interested in the memory encoding and retrieval, working memory, the need for cognitive control, and the novelty detection, as these are important factors to consider in the domain of expertise. According to Cavanagh & Frank (2014), theta oscillation may facilitate recurrent cycles of integration across inputs such as reward, or memory, to inform controlled action selection. Especially frontal theta seems to be related to emergent processes, such as cognitive control. Given that frontal theta seems to correspond to cognitive control, it should be expected that it reacts also to novelty. Cavanagh & Frank (2014) have observed that when individuals are exposed to a novelty, they share a need for increased control, and this is reflected in the frontal theta oscillations. While these mechanisms have been investigated in studies on cognition, we still know very little about neural mechanisms in mathematical cognition.

Attempting to solve a mathematical problem activates a network of various areas in the brain. These areas include parietal and posterior areas (e.g., the parietal lobe, the superior, medial, and inferior frontal gyri, the precentral, cingulate, and fusiform gyrus, the insula, parts of the cerebellum, and the basal ganglia; Arsalidou & Taylor, 2011). In a study by Grabner and colleagues (2012), the researchers observed that the theta oscillations increased after participants were exposed to arithmetic training, illustrating the sensitivity of the theta activity in arithmetic domains in the parietal regions. In addition to the parietal regions, the frontal cortex is also consistently activated in mathematical sense-making. Therefore, the activated network is referred to as the frontoparietal network (Moeller, Willmes, & Klein, 2015). Previous research has also shown that frontal areas are involved as much as parietal regions in mathematical tasks but that the frontal activation changes in accordance with the development of mathematical skills (Nieder & Dehaene, 2009). Sokolowski, Fias, Mousa, & Ansari (2017) further highlighted that besides the parietal lobe, the frontal lobe has been shown to be important to consider when investigating number processing. These studies highlight the importance of considering frontal regions as important for number processing.

Aims of the study

Depending on the format of the mathematical task at hand – symbolic (algebraic; using Arabic numerals) or non-symbolic (geometric; set or array of geometric forms or items) – two different networks within the fronto-parietal network, which are overlapping but distinct, are activated. Contrast analysis showed distinct fronto-parietal activation for symbolic and non-symbolic processing (see Sokolowski, Fias, Mousa, & Ansari, 2017 for a meta-analysis). Based on the above-mentioned neuroimaging research, there seems to be common and distinct brain regions in the frontal (and parietal) cortex that support symbolic and non-symbolic number processing. If the difference in processing symbolic versus non-symbolic mathematical demonstrations can also be distinguished when looking at FAA and theta activity and if it depends on the expertise of the person solving the task is yet to be investigated. To address this, we included symbolic and non-symbolic mathematical demonstrations in our study. We expect to see format dependent differences in the frontal cortex, as it has been shown that, especially the frontal cortex, subserves magnitude representations, rather than non-numerical cognitive processes. We explored the format dependent differences in experts versus novices, which, in the authors knowledge, has not been done before.

The main goal of the present study was to improve our understanding of neural correlates of mathematical expertise in comparison to math novices. We focused on frontal alpha and theta activity, described above, as they have been argued to be crucial for cognitive processes involved in mathematical reasoning. We are particularly interested in whether there are differences in frontal alpha and theta activity between math experts and novices when they are both exposed to the same types (symbolic and non-symbolic) of mathematical demonstrations. By focusing on the frontal cortex, which has been associated with the development of skill and expertise, and using longer mathematical demonstrations, we intend to understand the neurophysiology of mathematical expertise.

Based on the above summarized research, we hypothesize an increased frontal theta (4-8 Hz) activity in novices compared to math experts due to an enhanced need to allocate attention (Kruglanski & Gigerenzer, 2011; Brush et al., 2017) and working memory (Evans & Stanovich, 2013) to the task at hand, and due to the novice's response to novelty (Cavanagh et al., 2012). As theta oscillations have been associated with domain-general cognitive demands and the acquisition of new information and have been shown to increase with the workload (Klimesch, 1999), we expect novices to show enhanced theta oscillations. Given previous research indicating that asymmetric patterns of brain activation are related to participants' cognitive processes (Moynihan et al., 2013), and mathematics ability (e.g., Lin et al., 2015), we also explore the asymmetric alpha modulation of EEG power over frontal brain regions and hypothesize that there will be a difference in alpha asymmetry in experts versus novices while making sense of mathematical demonstrations.

Method

Participants

We recruited 23 math experts and 23 novices for the study. After EEG preprocessing, the final sample for the analysis consisted of 10 math experts (3 female, 7 male, mean age 21.00, SD = 1.83) and 18 novices (7 female, 11 male, mean age 23.7, SD = 3.94). There was no difference between the groups in terms of gender, X^2 (9, N = 28) = 8.4, p = .49. Due to signal problems, we had to measure more participants than planned, as we had to exclude participants who did not have sufficiently strong signals throughout the experiment. While being an expert was defined as having obtained a bachelor's degree in math or studying math at a master's level, a novice was defined as having no formal background in math or closely related topics, such as engineering or statistics. All participants were right-handed and reported no hearing loss or history of neurological illnesses. The experiment protocol was conducted in accordance with the Declaration of Helsinki and approved by the local Ethics Commission. All participants provided written informed consent.

Design and Task

Participants watched 16 computerized mathematical demonstrations (8 symbolic, 8 non-symbolic, pseudo-randomized) while sitting or standing. One of the 8 math demonstrations is shown in both formats in Figure 1. After each demonstration participants were asked to state their agreement on four statements using a button on a 4-button response box. The statements were the following: I had enough time to follow the math demonstration, I was familiar with the math demonstration, I understood the math demonstration, and I found this math demonstration engaging. The answer format followed a 4-point Likert scale with the options: 1 = Completely disagree, 2 = Somewhat disagree, 3 = Somewhat agree, and 4 = Completely agree.

Each math demonstration consisted of several slides, varying from 4 to 12 slides (6.9 slides on average), and 13 to 68 seconds (33.1 seconds on average). The fixed duration of each slide was the same for all the participants and was defined according to a preliminary online pilot study conducted with 25 math experts and 25 math novices. As format (symbolic versus non-symbolic) was a factor of the trials, the length of the trials in these two conditions were matched.

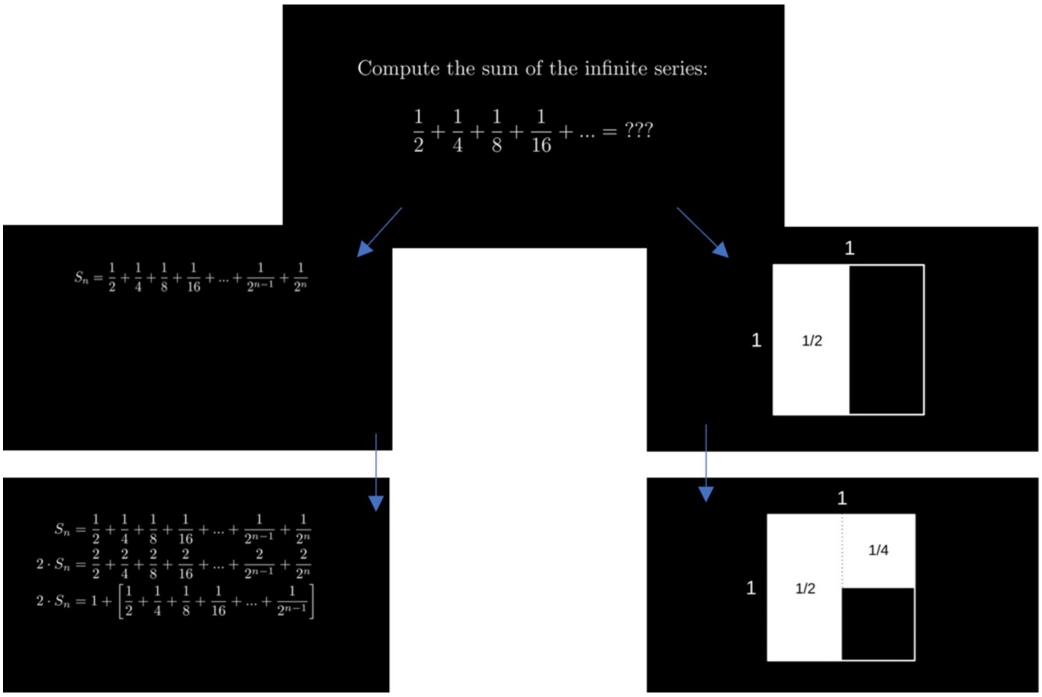


Figure 3.1: *Example of a mathematical demonstration in symbolic and non-symbolic form.*

The task was programmed in MATLAB using the PsychToolbox. After the researcher launched the program, and following instruction slides, participants could navigate through the math demonstrations by a button press. The total length of the mathematical proof demonstrations was approximately 15 minutes.

Measures

Electrical brain activity was measured with the Ant Neuro EEGO MyLab 128-channel EEG system during the presentation of the math demonstrations. Channel CPz was used as the reference electrode in the cap configuration. Data was collected with a sampling rate of 2048 Hz. The triggers were sent wirelessly via Lab Streaming Layer (LSL).

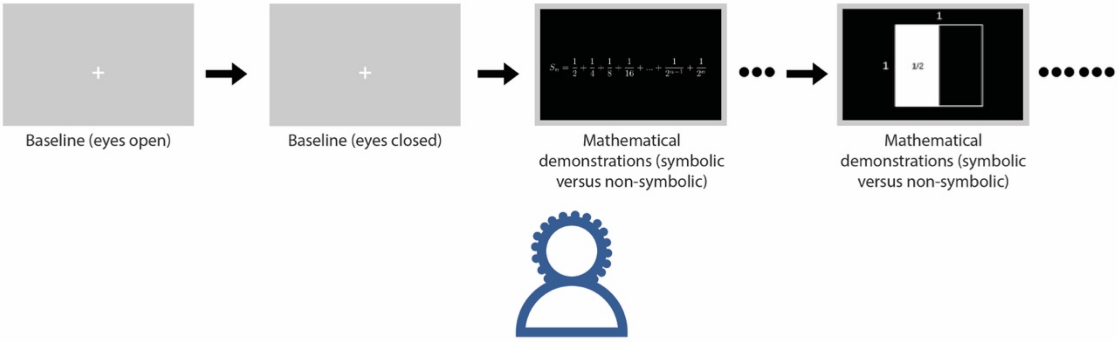


Figure 3.2: *Set-up of the study.*

Procedure and Materials

Before the experiment, an electrode cap was set on the head of the participant, with electrodes attached by an electroconductive gel. To obtain a baseline, EEG was recorded for three minutes while the participant was sitting still with open (3 minutes) and closed eyes (3 minutes). After this, the participants watched the demonstrations. They also completed other tasks which were beyond the scope of this paper.

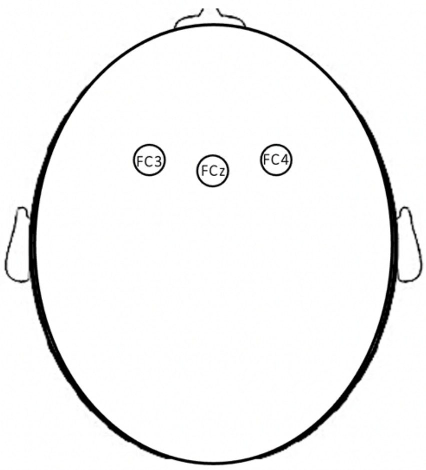


Figure 3.3: *Head Model illustrating EEG electrodes of interest.*

Data Analysis

EEG preprocessing: The EEG data were processed and analyzed using MATLAB custom scripts and the following toolbox: EEGLAB for data preprocessing (Delorme A & Makeig, 2004), including Independent Component Analysis (ICA) and artifact correction. Independent Component Analysis (ICA) was used to identify and remove eye-blink as well as eye movement artifacts (EOG; Jung et al., 2000; Hoffmann & Falkenstein, 2008). A High Pass IIR Filter (1 Hz) has been applied to obtain stationary data for ICA (Blum et al., 2019; Winkler et al., 2015) as well as for artifact subspace reconstruction (ASR; Blum et al., 2019; Chang, 2018). Initially, line noise (50 Hz) was removed using the *cleanline* plugin. Subsequently, bad channels have been removed, and the data has been corrected using ASR (parameter set to 10). Then, removed channels have been interpolated via spherical-spline interpolation (acceptable rate: 10%, excluding F3, and F4, missing those would have led to the rejection of the data set), and the data has been re-referenced to the average reference. Removing channels led to the exclusion of participants. Regarding the ICA pipeline, we generated a temporal data set of 1-second epochs for ICA, removed large artifacts in this temporal data set, and estimated the data rank for an estimation of the number of independent components to extract with ICA (*amica* algorithm; Delorme et al., 2007). The derived independent component weights were applied to the original continuous data set, and ocular and cardiac artifact components was removed via the *iclabel* plug-in (Pion-Tonachini et al., 2019). Finally, we applied a low-pass IIR filter (40Hz).

Power spectral density: PSD was calculated using Welch's method. The values for each individual were baseline corrected. The power values at each electrode for each condition were averaged over standard EEG frequency bands: theta (4–8 Hz), alpha (8–12 Hz), and subsequently log-transformed to normalize their distributions.

Averaging: A temporal data set of one-second epochs for ICA were created. The epochs have been generated according to groups and conditions. FAA was calculated by subtracting the mean alpha power at the electrode F4 from the mean alpha power at the electrode F3. Positive values indicate a left hemisphere (LH) alpha power activity dominance, whereas negative values indicate a right hemisphere (RH) alpha power dominance. Frontal theta power was calculated by the mean theta power at electrode FCz. This study was part of a larger project. While 128 channels were used to measure brain activity, the present study only focuses on the three above-mentioned channels.

Statistical analysis: We investigated the differences in FAA and theta activity using a linear mixed model with the factors group (between) and format (within). The general linear mixed model was chosen to analyze unbalanced repeated measures.

Results

The participants were asked self-evaluation reflections after each demonstration, which were evaluated in terms of their sum scores using repeated measures ANOVA. For time there was a significant difference between the groups ($F(1, 26)=17.939, p<.001$), but not in format ($F(1, 26)=1.55, p=.224$). Furthermore, for familiarity there was no significant difference between the groups ($F(1, 26)=3.151, p=.088$), but a significant difference in format ($F(1, 26)=10.482, p=.003$). In terms of understanding there was a significant difference for group ($F(1, 26)=11.259, p=.002$), and format ($F(1, 26)=28.531, p<.001$). Lastly, for engagement there was a significant difference for group ($F(1, 26)=12.294, p=.002$), and format ($F(1, 26)=8.283, p=.008$).

The results of the analysis of FAA indicate that there is no significant main effect of the group (see table 1). The results for frontal theta indicate that experts and novices differ from each other when it comes to theta activity. The results further indicate that there is a significant main effect of the format (symbolic versus non-symbolic; see table 1). Post Hoc Tests indicated that Experts (M = -7.18) show less frontal theta activity compared to Novices (M = 7.19; SE = 0.30, t = -47.5, $p_{bonferroni}$ <.001). Results further indicated that there is more theta activity in the symbolic (M = 0.125) compared to the non-symbolic (M = -.0113) math demonstrations (SE = 0.09, t = 2.72, $p_{bonferroni}$ = 0.007).

Table 3.1: *Linear Mixed Models for FAA and frontal theta.*

FAA			**95% Confidence Interval**				
Factor	**β**	**Standardized β**	**Lower**	**Upper**	**df**	**t**	**p**
Intercept	-0.212	0.173	0.551	0.127	23.8	1.225	0.232
group	0.206	0.215	0.214	0.626	154.3	0.961	0.338
math	-0.049	0.077	0.200	0.102	384.0	0.636	0.525
group x math	0.092	0.154	0.209	0.394	384.0	0.601	0.548
Theta			**95% Confidence Interval**				
Factor	**β**	**Standardized β**	**Lower**	**Upper**	**df**	**t**	**p**
Intercept	0.006	1.587	-3.104	3.116	23.7	0.004	0.997
group	14.372	0.302	13.779	14.965	389.7	47.522	< .001
math	-0.238	0.088	-0.409	-0.066	383.7	-2.720	0.007

group x math	-0.110	0.175	-0.453	0.233	383.7	-0.627	0.531

Discussion

The present study used EEG to compare FAA and frontal theta in math experts and novices making sense of longer and more naturalistic mathematical demonstrations. The math demonstrations of our study were shown in two formats, namely symbolic and non-symbolic. Based on previous research, we hypothesized that novices would show an increase in frontal theta (4-8 Hz) activity compared to math experts due to enhanced attention, working memory load, and novelty response (Kruglanski & Gigerenzer, 2011; Brush et al., 2017; Evans & Stanovich, 2013; Cavanagh et al., 2012). Our data supported the hypothesis that math experts showed less frontal theta power compared to novices. This finding indicates that novices might need more cognitive control and engage more working memory processes (thus and increase frontal theta power, e.g., Jacobs et al., 2016) while making sense of complex mathematical demonstrations. The unraveling of intuitive and analytical thinking mechanisms (Kahnemann, 2011) and their neural signatures provide insight as to how different modes of thinking might drive us based on the different neural processes that have been deciphered (Williams et al., 2019). Williams and colleagues (2019) illustrated that analytical thinking is characterized by an increase in frontal theta EEG power, indicative of the engagement of cognitive control and working memory processes. In line with previous research on mathematical cognition that indicated that, relative to baseline, theta power, as well as more pronounced asymmetries are related to mathematical problem-solving (Lin et al., 2015). At the same time, EEG research revealed that especially the frontal cortex is crucial in mathematical problem solving and math performance (e.g. Artemenko et al., 2018). Our analysis extends existing knowledge by offering new insight in terms of significantly enhanced frontal theta in novices when making sense of mathematical demonstrations. These study findings thus reaffirm the neural differences between experts and novices that seem to be present during longer and more complex math demonstrations.

Furthermore, we hypothesized that there would be a difference in alpha asymmetry over frontal brain regions in experts compared to novices while making sense of mathematical demonstrations. Previous findings have shown significant frontal asymmetries regarding different cognitive stimuli due to emotional and motivational aspects (Quaedflieg et al., 2015), affect (Rosenfeld et al., 1996), or executive functions (Moynihan et al., 2013). Our results, however, do not support the hypothesis. One might expect experts to have stronger emotional and motivational responses to mathematical tasks and enhanced executive functions during mathematical problem-solving, which would have suggested FAA (Quaedflieg et al., 2015; Rosenfeld et al., 1996; Moynihan et al., 2013) and a difference compared to novices. However, the motivation and affect might have been similar between the two groups: while math experts might feel more motivated to be exposed to familiar demonstrations, novices might be motivated to get to know them. Furthermore, a recent study on math experts and novices found that experts and novices in math do not differ significantly in aspects beyond their expertise. That is, experts and novices in math are similar in aspects such as personality and domain-general cognitive abilities (such as executive functions, Meier, Vogel, Grabner, 2021).

Another explanation for why we did not find a difference in FAA between experts and novices in math could be that lateralization (asymmetry of brain activity) does not always go hand in hand with optimal performance (Vallortigara & Rogers, 2020). This suggests that while there is a relationship between brain activation asymmetries and behavioral performance, the magnitude of the asymmetrical activation does not increase linearly with performance (Eckert, Vaden, & Iuricich, 2022). To explore the lack of significant difference, and how FAA relates to executive functions, motivation, and affect, future studies should measure these aspects objectively and subjectively at the same time.

Format, Expertise, and Interaction Effects

The results of our study further indicate that there is a significant main effect of symbolic versus non-symbolic math presentation. Though previous research has highlighted how the frontal cortex (along with the parietal cortex) supports symbolic and non-symbolic number processing in humans, there are distinguishable neural networks in fronto-parietal regions (Sokolowski, Fias, Mousa, & Ansari, 2017). We could reaffirm the effects and elaborate on those findings by observing a significant main effect in the neural signature in our research setting with longer and complex math demonstrations and in terms of frontal alpha and theta power.

Interestingly, our findings revealed a significant difference between experts and novices in terms of frontal theta activity in the symbolic versus non-symbolic format of the math demonstrations (no difference in FAA). Adding to the ongoing discussion of whether symbolic and non-symbolic processing can be differentiated on various levels (behavioral, neural timing, neural location, and neural oscillations), our data support the distinction between symbolic and non-symbolic skills. Moreover, expertise influences the differences between frontal theta activity in symbolic and non-symbolic processing, which might be in line with previous research indicating that experts show a wider and more specific neural activation when engaging in—for novices seemingly—math unrelated topics (Amalric & Dehaene, 2016). Furthermore, the effect of expertise on the format (symbolic/non-symbolic) of the tasks could be explained by recent findings of Meier, Vogel, and Grabner (2021), who compared math experts with novices in various tasks, abilities, and personality profiles. Their findings suggested that mathematicians have a more accurate representation of symbolic numbers and a stronger command of arithmetic facts. This advantage of math experts could lead to more facilitated processing of the symbolic demonstrations in our study and thus be mirrored by the frontal theta activity. Thus, our study extends previous findings by illustrating that there are significant differences in frontal theta oscillations in terms of symbolic versus non-symbolic magnitude processing.

Future research should investigate the brain-behavior relationship with self-reports or cognitive tests to develop insights into what these differences in neural activity mean. For instance, motivation and affect could be investigated with think-aloud paradigms or more specific self-reports. Such neuroscientific research associated with educational practice could bring novel insights into the expert-novice literature and a better understanding of the neural signature underlying mathematical cognition.

Conclusion

This study extends previous research examining the differences in brain processes between math experts and novices in mathematics. Although there appeared to be no difference in hemispheric dominance activity between experts and novices, frontal theta power was increased in novices compared to experts. That novices showed increased frontal theta power might be explained through their higher mental effort, greater attention, and enhanced working memory required for a subject that they are unfamiliar with. This work contributes toward understanding the neuro-dynamics of mathematical cognition and the neural differences in math experts and novices during longer and more complex math demonstrations in a more naturalistic context.

Find it in Your Heart

Heartbeat Differences between Math Experts and Novices while Making Sense of Mathematical Demonstrations: A Pilot Study

Abstract

While examining the differences in brain processes between math experts and novices in mathematics, we observed neural differences in frontal theta oscillations. Math novices showed more pronounced frontal theta activity while making sense of mathematical demonstrations, when compared to expert mathematicians. Besides looking at the neural differences between math experts and novices, we conducted a pilot study to investigate the physiological differences that may arise when mathematicians (experts) and novices in mathematics are asked to make sense of mathematical demonstrations. Previous research in neurophysiology has observed that interactions between cognitive performance and physiological measurements, including heartbeats, can provide valuable insight into intrapersonal dynamics and the measurements of learning outcomes. Heart rate and heart rate variability analysis is emerging as an objective measure of cognitive functions related to learning and can be used as a measure of cognitive load, engagement, and level of arousal during cognitive tasks. We measured experts' and novices' heart rates with electrocardiography (ECG) while they were watching mathematical demonstrations. ECG analysis showed a higher heart rate in math experts compared to novices, but no significant difference. The preliminary results of this pilot study suggest that experts might be more engaged and aroused when exposed to their material of expertise. Further studies are needed to elaborate on neurophysiological research in the learning sciences, specifically to investigate the heart rate differences between experts and novices of mathematics.

Introduction

In antiquity the heart was believed to play an essential role in cognition: Aristotle famously declared the heart to be the body's most important organ. In this account from ancient Greece, the heart was the seat of intelligence and sensation. In the modern era, we are accustomed to thinking that the ancients were misguided on this matter. However, recent research suggests that the heart may play some critical— if, as of yet, poorly understood—role in human cognition. Indeed, we now know that dynamic changes in bodily physiology influence cognitive processes (Critchley & Garfinkel, 2018). Physiological signals, such as heartbeats, selectively facilitate, compete with, or inhibit, information processing across psychological domains (Critchley & Garfinkel, 2018). The relationship between physiological signals, heartbeat measurements, and cognitive functions is trending. However, the relatively small number of studies implies that this relationship has been understudied.

There is a growing appreciation that cognition is embodied and that bodily sensations, emotions, and extracerebral changes bias cognitive processes. Therefore, learning scientists, neuroscientists, and psychologists try to characterize how and when internal (and external) bodily signals guide and support cognition (e.g., enhanced interoceptive accuracy improves memory Garfinkel et al, 2013; and decision-making Werner et al, 2013).

The measurement of cognitive activity using physiological means such as heart rate (HR) activity is a well-established research practice (Cranford et al., 2014; Darnell & Krieg, 2019; Minkley et al., 2021). Cranford and colleagues (2014) observed how the changes in heart rate are more pronounced for novices, compared to experts in a specific field when solving the same problem. However, it has been understudied how the level of engagement relates to expertise (in mathematics) and how heartbeat measurements can investigate this. There have been very few studies focusing on the differences in HR considering an individual's expertise.

Heart rate variability and learning

Monitoring physiological signals involving HR, or heart rate variability (HRV) plays a pivotal role in estimating cognitive performance. According to Alqahtani and Ramzan (2019), the usage of these physiological measurements has expanded beyond medical science. It has been implemented in diverse domains including determination of attentiveness and mental status of human subjects in modern intelligent tutoring systems and measuring human emotional responses corresponding to the surrounding environment (Alqahtani & Ramzan, 2019). Detailed literature surveys (Darnell & Krieg, 2019) confirm that in laboratory research physiological measurements are considered indirect indicators of cognitive performance; specifically, HR activity (Cranford et al., 2014; Darnell and Krieg, 2019; Minkley et al., 2021).

A recent study found that there is a strong correlation between HR and task engagement (Darnell & Krieg, 2019). Emphasizing on the fact that there is a steady decrease in HR starting from the onset to the end of the class (Darnell & Krieg, 2019). While there seems to be a remarkable rise in HR during active learning sessions, the HR immediately returned to average level after the learning session. Thus, superior learning increases HR (Scholey et al., 1999; Darnell & Krieg, 2019). Further studies supported this claim, finding that greater cognitive effort and higher order problem solving are associated with increased HR (Mulder, 1992; Sosnowski et al., 2004; Fredericks et al., 2005 and Cranford et al., 2014).

Extensive literature surveys bring to light the fact that, in earlier times, measurement of cognitive load involved post hoc collection and were subjective; however, recent studies confirm that collection of physiological data such as HR, blink rate, galvanic skin response etc. overcome the drawbacks associated with the traditional techniques (Cranford et al., 2014). At present cognitive load is measured using physiological techniques with the assumption that stress is induced on the subject's body due to cognitive overload ultimately leading to measurable physiological fluctuations in HR, which can be monitored continuously in real time while the tasks are completed by the subject (Beatty and Wagoner, 1978; Mulder, 1992; Fredericks et al., 2005; Cranford et al., 2014). It is worth mentioning that HR is used for measuring cognitive load since it can be measured in an uncomplicated and economical manner implementing any number of available commercial devices. Experimental results lead to the conclusion that greater cognitive load is associated with larger increase in HR as compared to smaller cognitive load.

In another study, conducted by Cranford and colleagues (2014), individual's HR changes were two to four times greater in case of inexperienced subjects with respect to experts for the same problem that they were asked to solve (Cranford et al., 2014). In their study, Cranford and colleagues (2014) have observed that, as expertise is developed in a discipline, the germane load of a problem can be decreased because of the learner's more robust knowledge base. Furthermore, the intrinsic load can also be lessened through the development of discipline-specific chunking strategies (Chase & Simon, 1973). Accordingly, the experienced individuals developed such strategies over the course of their studies in their field of expertise. Because of this, they could handle the higher cognitive load problems better than the inexperienced individuals (Cranford et al., 2014). This could illustrate that changes in HR are more pronounced in inexperienced participants than experts of a subject at hand.

Existing literature (Wang et al., 2018) explains that fluctuations in arousal are linked to HR. It is well documented that HR increases with arousal from sleep since it is related to autonomic reflex activation, which in turn leads to increase in HR (Horner et al., 1995; Horner, 1996; Trinder et al., 2001; Trinder et al., 2003; Azarbarzin et al., 2014). Moreover, it is found from literature that arousals associated with movement leads to more increase in HR (Sforza et al., 2000 and Azarbarzin et al., 2014). Experimental results (Azarbarzin et al., 2014) also reveal that only small changes in HR is associated with the most common arousal scale thereby leading to the conclusion that clinically an abundant number of low intensity arousals may not be crucial. There have been very few studies focusing on the differences in heart rates considering the expertise of an individual.

Since we are focusing on the domain of mathematical learning and HR, there is one important factor we must investigate, too. Some individuals experience math anxiety, which is defined as a feeling of helplessness, panic, paralysis, and mental disorganization when required to solve a mathematical problem or engage with numbers (Tobias, 1986). Anxiety can also make the heartbeat faster; in fact, in many anxiety disorders psychophysiological parameters such as HR are the most studied biomarkers (Pittig et al., 2013) and are even used to evaluate the effect of certain therapy forms (Gonçalves et al., 2015). Thus, to distinguish between the effect of potentially present math anxiety and engagement or arousal caused by expertise, we will consider and control for math anxiety in the present study.

Expertise and physiological indices of engagement and arousal

The number of heart beats per minute is referred to as heart rate (HR), and the mean value of the same is defined as mean heart rate. The fluctuations in the time intervals between adjacent heartbeats are called heart rate variability (HRV), generated by heart-brain interactions and dynamic non-linear autonomic nervous system processes. The present study relates expertise to physiological indices of engagement and arousal, and tests the following hypotheses derived from the reviewed literature mentioned above:

> **Hypothesis 1**: The mean heart rate (physiological indices of engagement) while making sense of mathematical proofs will differ in expert mathematicians compared to novices. If engaging more in a complex mathematical task is associated with lower heart rate, math experts will show lower heart rates.
>
> **Hypothesis 2:** Heart rate will be correlated with self-reported engagement and understanding of mathematical proofs. That is, the physiological indices of engagement will correlate with the self-reported indices.
>
> **Hypothesis 3**: The difference in heart rate between experts and novices is due to expertise and not math anxiety. That is the groups differ in heart rate, but not in math anxiety.

To test these hypotheses, expert mathematicians and novices were asked to make sense of a series of mathematical proofs, half of them shown in a symbolic format, and the other half in a non-symbolic format. Heart rate was assessed at baseline and during the task, and self-reports were collected after each math demonstration.

Method

Participants

In this pilot study we tested a group of N = 11 participants; five experts (3 males, 2 females, mean age = 21.1 years, defined as having obtained a bachelor in math or studying math at a master level, and six novices (4 males, 2 females, mean age = 24.7 years) with no formal background in math or closely related topics. All participants were right-handed and reported no hearing loss or history of neurological illnesses. The experiment protocol was conducted in accordance with and approved by the local Ethics Commission. All participants provided written informed consent.

Design and Task

Participants watched 16 demonstrations of mathematical arguments. After each demonstration they were asked four self-evaluation reflections, to which they answered by pressing a button on a 4-button response box. Each set of trials contained 4 excerpts of the same format (symbolic or non-symbolic) in two body positions (sitting or standing), and these sets were presented in a pseudo randomized order on a computer screen. The pseudo randomization defined the format order (symbolic first/non-symbolic first) and the order of body posture (sitting first/standing first). The body position changed once half of the demonstrations had been seen. The self- evaluation reflections were the following: I had enough time to follow the math demonstration, I was familiar with the math demonstration, I understood the math demonstration, and I found this math demonstration engaging. The answer format followed a 4-point Likert scale with the options: 1 = Completely disagree, 2 = Somewhat disagree, 3 = Somewhat agree, and 4 = Completely agree.

The task was programmed in MATLAB using the PsychToolbox. After the researcher launched the program, and following instruction slides, participants could navigate through the math demonstrations by a button press. The total length of the mathematical proof demonstrations was approximately 15 minutes. The mathematical demonstrations were the same as in the previous study with symbolic and non-symbolic formats of the demonstrations. The task was precisely the same as the one on the investigation of neural differences between experts and novices in mathematics.

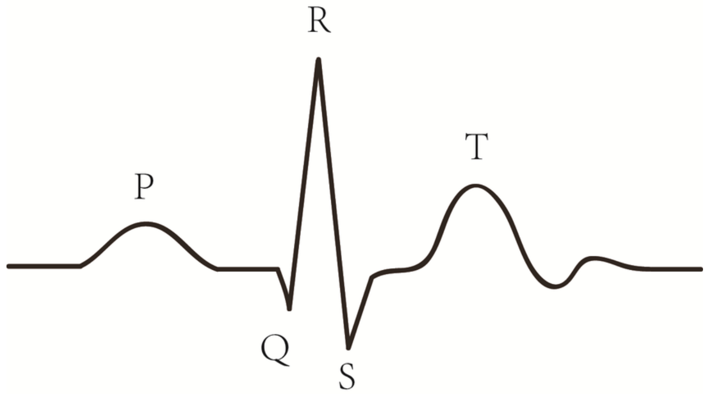
Measures

Electrical heart activity was measured with the Ant Neuro EEG/ECG MyLab system. One electrode was put at the right collarbone, another electrode below the left rib to measure heartbeat activity. The triggers were sent wirelessly via Lab Streaming Layer. Behavioral data were collected in the form of a self-report questionnaire with a 4-point Likert scale answer format.

Data Analysis

Repeated measures ANOVA was used to analyze the self-reports about time, familiarity, understanding, engagement, and anxiety (AMAS) between experts and novices.

We processed the ECG data using MATLAB based custom scripts for the analysis of heart rates (HR) according to the standards that are recommended for HR measurements. We used a real-time QRS detection algorithm to detect QRS complexes of ECG signals (Pan and Tompkins 1985). Furthermore, we used a QRS detection algorithm that is based on filter banks to identify the QRS complex. The QRS detection algorithm enables researchers to identify the QRS complex because it decomposes the ECG in sub-bands with frequency bandwidths that are implemented in MATLAB (Afonso et al., 1999). We visually inspected the ECG data to assure that the R-peaks were correctly detected. The R-peaks have been used to estimate the heart rate (HR) indexes. Repeated measures ANOVA was used to calculate differences in HR between experts and novices.



Source: Wu, J. et al. (2019).

Figure 3.4: *One typical heartbeat in ECG signal (PQRST complex).*

Results

Behavioral Results

The participants were asked self-evaluation reflections after each demonstration, which were evaluated in terms of their scores (1 = Completely disagree, 2 = Somewhat disagree, 3 = Somewhat agree, 4 = Completely agree) using repeated measures ANOVA.

Out of the total final sample of the study, the results differed significantly between experts and novices in terms of whether they had enough time for the math demonstrations ($F(1, 11)=18.631$, $p<.001$), whereas experts scored higher ($MM=27.935$) than novices ($MM=19.757$), indicating that they felt like they had enough time to observe all the demonstrations. There was also a significant difference in format ($F(1, 11)=5.040$, $p=.046$).

Furthermore, there was a marginally significant difference between the groups in terms of familiarity of the demonstration ($F(1, 11)= 6.293$, $p=.029$), whereas experts scored higher ($MM=23.012$) than novices ($MM=16.834$), indicating that they felt more familiar with the demonstrations. Also, there seems to be a significant difference in format ($F(1, 11)=6.823$, $p=.024$) in terms of familiarity.

Behavioral results also indicated a significant difference between math experts and novices in terms of their understanding of the demonstrations ($F(1, 11)=14.641$, $p=.003$), whereas experts scored significantly higher ($MM=27.035$) than novices ($MM=17.119$), indicating a better understanding of the demonstrations. In terms of the understanding, the format of the demonstrations seems to also be significantly different ($F(1, 11)=15.101$, $p=.003$).

The engagement of the demonstration was also significantly different between the groups ($F(1, 11)=10.533$, $p=.008$), whereas experts scored significantly higher ($MM=26.745$) than novices ($MM=17.947$), indicating that they were more engaged in the demonstrations. Within subjects effects of the engagement also showed a significant difference in format ($F(1, 11)=17.401$, $p=.002$).

In addition to the self-evaluation questions, we were interested in the math anxiety (AMAS). There seems to be no significant difference in terms of math anxiety between math experts and novices ($t=0.721$, $p=.487$, $df=10$).

ECG Results

We investigated the mean heart rate (meanHR) between experts and novices while making sense of mathematical demonstrations. Therefore, we conducted a repeated measure mixed ANOVA.

Table 3.2: *ANOVA – meanHR of experts versus novices.*

Cases	Sum of Squares	df	Mean Square	F	p
group	65.103	1	65.103	0.624	0.450
Residuals	938.881	9	104.320		

Table 1: *Group Descriptives.*

	Group	N	Mean	SD	SE	Coefficient of variation
meanHR	Expert	5	83.761	10.545	4.716	0.126
	Novice	6	78.875	9.940	4.058	0.126

The results of the analysis indicate that there is no significant main effect in group $F(1, 9) = 0.62$, $p=.45$.

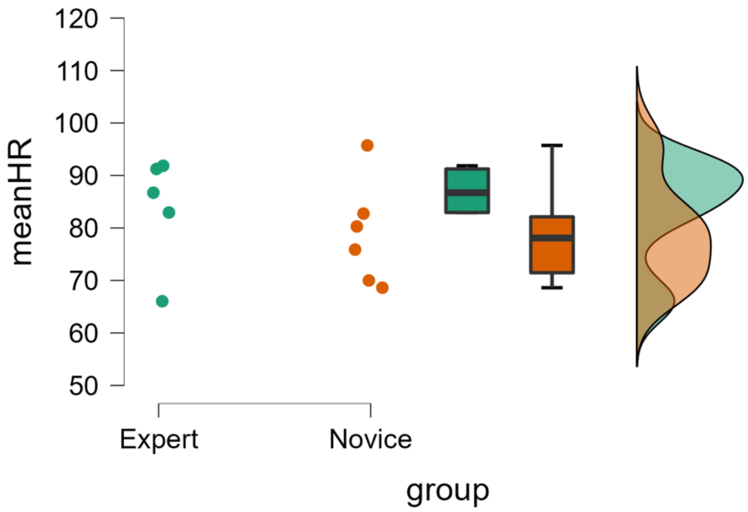
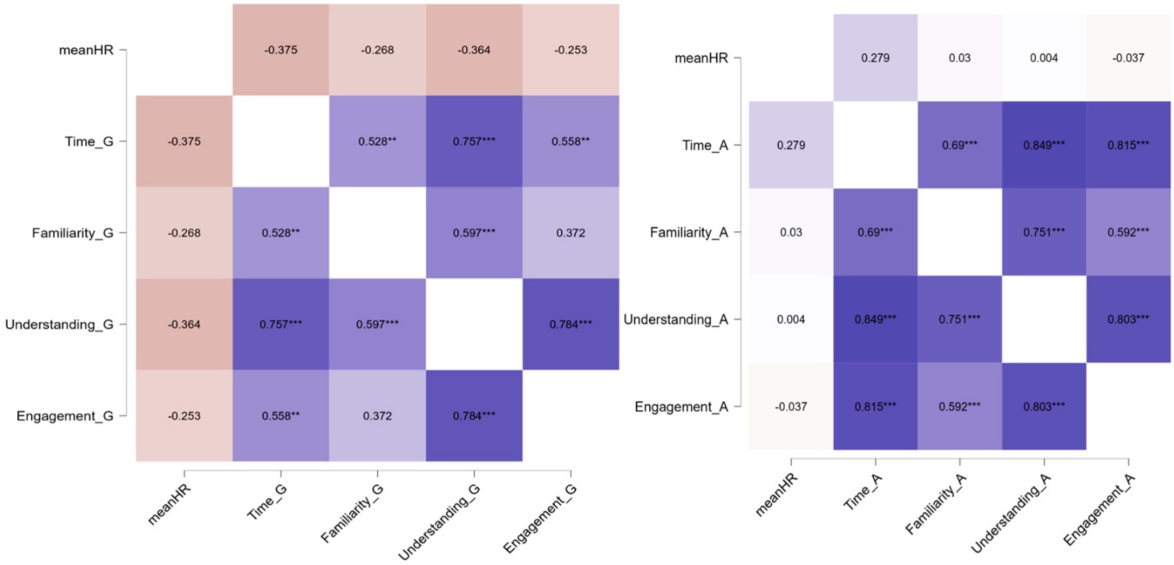


Figure 3.5: *Raincloud plot of HR (meanHR) of group Expert and Novice.*

Correlation Analysis

We further investigated whether heart rate correlates with self-reported engagement and understanding of mathematical proofs. The results of the correlation analysis indicate that there is no significant correlation between meanHR, understanding (r=0.27), and engagement (r=0.45) of the non-symbolic math demonstrations, and no significant correlation between meanHR, understanding (r=0.004) and engagement (r=0.037).



Note. * p < .05, ** p < .01, *** p < .001

Figure 3.6: *Pearson's heatmaps of meanHR and time, familiarity, understanding, and engagement of the non-symbolic math demonstrations (G), and symbolic demonstrations (A).*

Heart Rates and Math Anxiety

We finalized our analysis with an investigation of the difference in meanHR between experts and novices and math anxiety. Our results of the ANCOVA indicate no significant main effects in meanHR and math anxiety ($F(1, 7)=0.92$, $p=.369$).

Table 3.3: *ANCOVA with variable meanHR and covariate math anxiety.*

Cases	Sum of Squares	df	Mean Square	F	p	η^2
Group Expert Novice	77.032	1	77.032	0.921	0.369	0.081
Math Anxiety	287.592	1	287.592	3.439	0.106	0.303
Residuals	585.440	7	83.634			

Note. Type III Sum of Squares

Discussion

To study mathematical cognition, recent methods rely on physiological mechanisms, including heart rates, to address shortcomings associated with more traditional techniques (e.g., Cranford et al., 2014). The current research sought to validate how heart rate differs between experts and novices while making sense of long and complex mathematical demonstrations. We compared experts to novices because specific neurocognitive and neurophysiological characteristics change through experience. Our main aim was to investigate how these changes were depicted in terms of physiological indices - heart rates, and how the objective measurement of heart rates correlates with self-reported measurements.

Based on previous research, we hypothesized that the mean heart rate (physiological indices of engagement) while making sense of mathematical demonstrations differs in expert mathematicians compared to novices (*Hypothesis 1*). We expected the math experts to show lower HR as they might be more engaged in the task at hand, while exhibiting a lower cognitive load than novices (Cranford et al., 2014). Our results showed that experts had higher heart rates than novices, yet the difference was insignificant. According to Azarbarzin and colleagues (2014), an increase in heart rates is also related to arousal. Our results may therefore be explained like this: The novices and experts did not differ in heart rates because the heart rates of both groups increased due to arousal. The novices might have been aroused by the novelty of the mathematical task, whereas the experts were aroused because they enjoyed being engaged in their topic of expertise.

We further hypothesized that heart rate is correlated with self-reported engagement and understanding of mathematical demonstrations (*Hypothesis 2*). More specifically, we expected that the physiological engagement indices correlate with the self-reported ones. In other words, we expected the objective measures to correlate with the subjective ones. Our results showed that there was a significant difference between the subjective measures in math experts and novices; experts indicated that they were more engaged in the demonstrations and showed a better understanding than novices, which aligns with previous research in mathematics education (e.g., Grabner & De Smedt, 2012). However, our correlation analysis showed that heart rates were not significantly correlated to self-reported indices, such as engagement and understanding of the demonstrations. We were surprised that there was no significant correlation between heart rates and self-reported indices, as this is contradictory to previous findings (e.g., Cranford et al., 2014). However, many factors affect physiology, and it might be difficult to control participant's circumstances and the participant's current feelings within a learning or studying situation. Our study utilized self-report measures in response to mathematical sense-making of long and complex demonstrations, whereas there was no significant correlation, which might be because of the mentioned limitations in neurophysiological research, as well as the sample size of this pilot study.

Lastly, we hypothesized that the difference in heart rate between experts and novices is due to expertise and not math anxiety (*Hypothesis 3*). We expected that the groups differ in heart rate but that the heart rate difference is not due to math anxiety. We found some support for this hypothesis based on the ANCOVA; we found no significant difference in math experts compared to novices regarding math anxiety. However, we found significant differences in time, understanding, and engagement. Therefore, we can confirm that potential differences in heart rate patterns are not due to math anxiety but instead to expertise, understanding, and engagement. Since the experts and novices did not differ in their heart rates, it is difficult to conclude what the difference might have been caused by. Thus, math anxiety as a control variable should be considered in future studies. In case of a difference in heart rate, math anxiety could be excluded as the cause of the difference. Previous research established that physiological data such as heart rate differences are closely related to cognitive load, cognitive engagement, and arousal (e.g., Cranford et al., 2014). According to Darnell and Krieg (2019), there is a significant link between heart rate and cognitive task management, with heart rate increasing for higher order problem solving, greater cognitive effort, and better learning performance. It is suggested that experimental studies show an increase in heart rate with arousal (Azarbarzin et al., 2014). In this piece of work, even though the difference was not significant, a higher heart rate in math experts is observed compared to novices when making sense of mathematical demonstrations.

Limitations

There are several limitations of the current design. First, these results are based on a pilot study, limiting our findings. A small sample size based on a pilot study may make it difficult to determine if the outcome is a true finding. As this was part of a large project with EEG and ECG, we mainly collected EEG data and were only able to collect ECG data for a pilot study. While there were technical limitations on the ECG set up in our study, we put our main focus on the EEG data and the neural signature analysis. Therefore, our sample size for the ECG data part of the project was small. A higher sample size would be required for validity and to produce accuracy of the results. In the future, sample size should be increased to elaborate whether findings can be reinforced.

Heart rate is, by its nature, an individual measurement. There are many potential factors affecting heart rate recordings, such as lifestyles, biological influences, as well as mental health factors (Koch, Wilhelm, Salzmann, Rief, & Euteneuer, 2019) cognitive abilities, neural processes, and personality traits (Pham, Lau, Chen, & Makowski, 2021). While the study attempted to control for variables such as age, math anxiety and expertise, there might still be factors we should have controlled for. In future studies, self-reported measures could include various questions tapping into the factors mentioned above.

We hope this study will drive future research in this area that could readily be extended to include other disciplines. The present study could be taken as a basic idea for heartbeat investigations in the learning sciences and education. On the physiological side, a closer monitoring of the heart rate analysis during the learning activity would enable a more objective investigation of arousal, engagement, and understanding, among other factors. Correlational analysis with heartbeat measurements and self-reported measures, such as engagement and understanding can potentially increase future student cognitive engagement performance and overall student attainment. To have a closer look at heart rates during learning activity, future research could set up a study measuring heart rates via ECG, or further wearable devices, including heart rate monitor watches.

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Chapter 4
The Neural and Physiological Basis of Learning through Problem-Solving followed by Instruction (PS-I)

Abstract

Problem-solving followed by instruction (PS-I) is a learning design that includes conditions for learners to insist on generating and exploring representations and solutions to complex, novel problems before formal instruction. There is growing evidence from behavioral studies that developing solutions to novel problems before instruction can help students perform better on specific learning measures, including conceptual understanding and knowledge transfer. Recent research on PS-I has revealed cognitive mechanisms for why students learn better after encountering difficulties; however, the neurophysiological mechanisms underlying this process have not been explored. The results of the present study suggest greater alpha and theta activity and heart rate variability during PS-I compared with instruction followed by problem solving (I-PS). These results may reveal an explanatory basis for learning through PS-I and illustrate the neurocognitive benefits of this learning design.

Introduction

Problem-solving followed by instruction (PS-I), as entailed in learning through Productive Failure (Kapur, 2014), is an instructional design to advance students' conceptual understanding and transfer. This design reverses the order of more traditional teaching, in which instruction is given first, followed by a problem-solving phase (referred to as I-PS condition). In both designs, a post-test is used to assess the students' acquired knowledge. Figure 1 shows the different phases of the two conditions, I-PS versus PS-I. Within PS-I, various cognitive mechanisms have been explored (Kapur, 2014; 2015, Sinha & Kapur, 2021); however, the neural basis of this design have not yet been explored. The current research investigates the underlying neural mechanisms of PS-I using electroencephalography (EEG) to study the brain in naturalistic environments. Simultaneously, we recorded electrocardiography (ECG) to investigate heartbeat measurements during learning through PS-I.

The brain is an electrochemical organ, and current techniques, such as EEG, reveal insights into both form and function of cognitive processes, including problem-solving. Electrical activity emanating from the brain is displayed in the form of five different brainwave categories (gamma, beta, alpha, theta, and delta), ranging from the most to the least (e.g., Pizzagalli, 2007). The frequency band oscillations are commonly quantified through power spectral density. The power spectral density describes signal power distribution at differing frequencies (Dressler et al., 2004). Frequency band types are associated with cognitive processes.

EEG research has shown that alpha oscillations (8-12Hz) and theta oscillations (4-8Hz) might be of particular interest in this kind of learning paradigm, because alpha has been associated with searching, accessing, and retrieving information from long-term memory (Klimesch et al. 1997), and theta to episodic and working memory (Jensen and Tesche 2002; Kahana et al., 2001). Theta oscillations have been associated with acquiring new information (Klimesch, 1999). It appears to reflect active operations particularly during high-level cognitive processes, such as memory encoding and retrieval, and working memory retention (Itthipuripat, et al., 2013; Rutishauser, et al., 2010). Previous research suggests that frontal theta activity might be engaged in problem solving (Ryu et al., 2016), and, more specifically, mathematical problem solving (Lin, et al., 2012, 2015; Pavlygina et al., 2010; Ghaderi et al., 2019). Particularlythe frontal areas are involved in mathematical tasks (Nieder & Dehaene, 2009; Sokolowski, Fias, Mousa, & Ansari, 2017). Based on the previous EEG research, the present study focuses on frontal alpha and theta frequencies in the frontal brain regions.

The present study additionally investigates physiological measurements in the PS-I design. Intuitively, we associate fear—and fear of failure—with increased heartrate. We might, therefore, expect a link between challenging tasks, fear of failure on those tasks, and heartrate. Indeed, research has demonstrated an association between heart rate variability and cognitive performance. Different heartbeat measurements seem to be associated with both top-down and bottom-up cognitive processing (Park & Thayer, 2014; Mather & Thayer, 2018), including recalling or memorizing basic knowledge.

Higher resting-state heart rate variability appears to be related to increased activity in executive brain regions (Thayer et al., 2012), while lower resting heartrate variability seems to be related to hypoactive prefrontal regulation (Thayer & Sternberg, 2006; Park & Thayer, 2014). Consequently, higher heartrate variability is related to better cognitive performance, such as global cognition, executive functions, and attention (Forte & Casagrande, 2019). However, the interplay between heartbeats and high levels of cognition has been underexplored thus far. We use ECG to investigate different heartbeat measurements, including heart rate variability (HRV), along with differences in EEG activity related to the type of learning situation (i.e., PS-I versus I-PS).

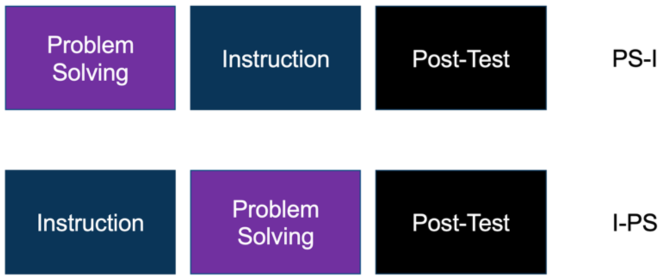

Figure 4.1: *Different Phases I-PS versus PS-I.*

In summary, we set out to study the neural and physiological mechanisms involved in the PS-I learning design. Since the setting is naturalistic in its design and participants solved problems in a manner typical of formal instruction in a university or a school, several cognitive processes, including attention, working memory, memory retrieval, creativity/search of novel ideas, were simultaneous occurring. We investigated these processes with EEG and ECG. The present study relates the PS-I design, as entailed in Productive Failure, to neural and physiological indices of information processing and tests the following hypotheses:

Research Question 1: Are there differences in brain oscillations between the phases of the conditions (problem solving vs. instruction)?

Hypothesis: We expect to see enhanced alpha activity in the problem-solving and the post-test phase compared to the instruction phase, as alpha has been associated with searching, assessing, and retrieving information (Klimesch et al. 1997). We also expect to see enhanced theta activity in the problem-solving and the post-test phase compared to the instruction phase, as theta has been associated with episodic and working memory (Jensen and Tesche 2002; Kahana et al. 2001).

Research Question 2: Are there neural differences in (brain oscillations) between the two conditions?

Hypothesis: We expect more frontal alpha and theta activity in the PS-I compared to the I-PS group, as the PS-I design has been advantageous in students' conceptual understanding and transfer, which could be reflected in frontal alpha and theta, respectively in searching, accessing, and retrieving information from long-term memory (Klimesch et al. 1997), while theta activity (4-8Hz) has been related to episodic and working memory (Jensen and Tesche 2002; Kahana et al. 2001).

Research Question 3: Are there differences in HRV between the two conditions?

Hypothesis: We expect higher HRV in the PS-I compared to the I-PS group, as the PS-I design has shown benefits in cognitive domains which could be reflected in the HRV measures (Forte & Casagrande, 2019).

We focus on the physiological and neural mechanisms individually. We are interested in the neural and physiological mechanisms of PS-I. To explore the above-mentioned research questions, we compared PS-I and I-PS conditions, using an established design (Kapur, 2014), while taking EEG and ECG measurements.

Method

Participants

We recruited 62 participants for the study (mean age 22.6 years old, 35 females, 27 males), free of any physical and mental illness, and no formal knowledge of the material taught in the experiment (standard deviation). We randomly assigned half of the participants to the PS-I group, and the other half to the I-PS group. Some participants had to be excluded from the final EEG/ECG analysis to ensure we only analyzed good EEG and ECG signals. We had 26 participants for the EEG/ECG analysis (13 females, 13 males). The Ethics Commission approved the experiment protocol. All participants gave written informed consent according to the study protocol approved by the human research ethics committee.

Procedure and Materials

We were interested in task-related changes in EEG measures, thus started with a baseline period for later comparisons. While sitting relaxed on a chair, participants were asked to keep their eyes closed. Then an EEG record was done for 3 minutes, which served as the baseline period with closed eyes. Afterwards, participants were asked to keep their eyes open for 3 minutes, which served as the baseline period with open eyes. After the baseline recordings the participants in the condition PS-I were asked to solve the standard deviation problem. A laptop was placed in front of the participants. They saw the instructions followed by the problem-solving task, the direct instruction, and the post-test. During the problem-solving phase, participants were asked to design as many different measures of consistency as possible by using data points provided to them. The average time of EEG recording from start to solution was expected to be approximately 15 minutes. Participants were asked to press a button after thinking about one possible solution, then write it down, to avoid motor artifacts. In the other condition (I-PS), the procedure started with the direct instruction, followed by the problem-solving task and the post-test. Participants started with a direct instruction which will be a theoretical input on standard deviations (same as in the other group, only the order changes), during this time EEG was recorded. This group of participants also pressed a button after solving each task during the problem-solving phase. For both groups there was a last part (posttest) in which they got a similar task to solve. We used a validated, classic Productive Failure design (Kapur, 2014), adapting it and computerizing it to match the requirements of neurophysiological research. However, we did not provide affective support to set the right expectations, but rather let the participants work independently. After the experiment, they received short questionnaires assessing math anxiety (AMAS; Hopko, Mahadevan, Bare, & Hunt, 2003), knowledge gap awareness (KGA), state curiosity (SC), germane cognitive load (GCL), learning goal orientation (LGO; Dweck, 1992) and attitude towards mistakes (ATM; Leighton, Tang, & Guo, 2015). Earlier studies used these questionnaires (Sinha & Kapur, 2021).

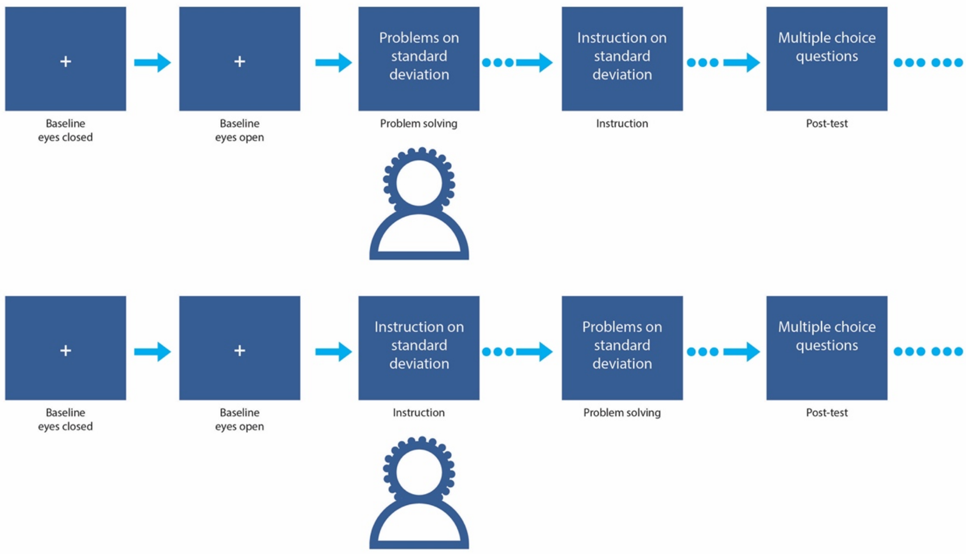

Figure 4.2: *Set-up of the study: PS-I condition (top) and I-PS condition (bottom).*

Measures

Electrical brain activity was measured with the Ant Neuro EEGO MyLab 128-channel EEG system during the study on PS-I. Channel CPz was used as the reference electrode in the cap configuration. Data were collected with a sampling rate of 2048 Hz. The triggers were sent wirelessly via Lab Streaming Layer (LSL). Simultaneously, we measured participant's heart rates by attaching an electrode to the rib and an electrode to the collarbone for electroencephalography (ECG).

Data Analysis

Behavioral data analysis: In the problem-solving phase, students were seated in front of the computer screen and asked to generate as many solutions as possible to a problem on standard deviation without any further help. Students were provided with blank A4 sheets of paper. The number of solutions generated by a student was taken as a measure of the student's prior knowledge activation and differentiation and was summarized as the *Quantity* aspect of the problem-solving phase.

We were interested in how many solutions the students generated, the tendencies they used (mean, median, and mode), and the range. Students calculated the deviation to argue that the greater the sum (or average) of the deviations, the lower the consistency. The generation of canonical solutions and using the range, dot diagrams, or line graphs were summarized as the *Quality* aspect of the problem-solving phase.

We investigated the score differences of the Post-Test questions on standard deviation as part of the behavioral data analysis. We compared the Post-Test scores between the two groups (PS-I versus I-PS). In addition, we analyzed the differences in Math Anxiety Scale (AMAS), learning goal orientation (LGO), attitude towards mistakes (ATM), knowledge gap awareness (KGA), state curiosity (SC), and germane cognitive load (GCL) between the two groups.

Table 4.1: *Variables for the behavioral and neurophysiological analysis.*

Behavioral Measure Problem-Solving Phase	Behavioral Measure Post-Test Phase
Quality: Generation of canonical solutions	Score of the Post-Test Multiple Choice Questions

Quantity: Number of solutions generated	Math Anxiety (AMAS)
	Learning Goal Orientation (LGO)
	Attitude Towards Mistakes (ATM)
	Knowledge Gap Awareness (KGA)
	State Curiosity (SC)
	Germane Cognitive Load (GCL)

EEG Measures	ECG Measures
Frontal Alpha (8-13 Hz)	Heart Rate Variability (HRV)
Frontal Theta (4-8 Hz)	

EEG data processing: Data analysis was conducted using the Brain Vision Analyser software (v1.05, Brain Products) for pre-processing and implementing the eye movement correction procedure. Matlab (The Mathworks) and EEGLAB v5.03 (Delorme A & Makeig, 2004) were used for further processing and Independent Component Analysis (ICA) and artefact correction. ICA was used to identify and remove eye-blink artifacts (EOG; Jung et al., 2000; Hoffmann & Falkenstein, 2008). A threshold algorithm was conducted to detect eyeblinks, and the data were segmented time-locked to the maximum blink excursion (−800: 1000 ms). Following this a baseline correction was made (−800:−500 ms). Finally, rare artifacts visual inspection cleaned the continuous raw data from rare non blink-related artefacts. A High Pass IIR Filter (1 Hz) has been applied to obtain stationary data for ICA (Blum et al., 2019; Winkler et al., 2015) as well as for artifact subspace reconstruction (ASR; Blum et al., 2019; Chang 2018). Initially, line noise (50 Hz) has been removed using the *cleanline* plugin. Subsequently, bad channels have been removed and the data has been corrected using ASR (parameter set to 10). Then, removed channels have been interpolated via spherical-spline interpolation (acceptable rate: 10%) and the data has been re-referenced to the average reference. Regarding the ICA pipeline, we generated a temporal data set of 1 second epochs for ICA, removed significant artifacts in this temporal data set, and estimated the data rank for an estimation of the number of independent components to extract with ICA (Amica algorithm; Delorme et al., 2007). The derived independent component weights were applied to the original continuous data set and ocular and cardiac artifact components have been removed via the iclabel plug in (Pion-Tonachini et al., 2019). Finally, we applied a low pass IIR filter (40Hz).

The data was segmented according to the experimental conditions and subsequently baseline corrected. Power was calculated concerning the different epochs/experimental conditions, i.e. according to the triggers set at each phase of the conditions and to each response to the solutions during the post-test. Subsequently, the data was averaged according to groups (PS-I, I-PS) and the different phases of the experiment (problem-solving, instruction, post-test). We extracted frontal alpha and frontal theta.

Power spectral density: Power spectral density was calculated using Welch's method. The power values at each electrode for each condition was averaged over standard EEG frequency bands. We did a planned comparison of the frontal electrodes, using standard procedures.

ECG data processing: The ECG data was processed using MATLAB based custom scripts to analyze heart rate variability (HRV) according to the recommended standards for HRV measurement. We used a QRS detection algorithm that is based on filter banks to identify the QRS complex. The algorithm enables researchers to identify the QRS complex because it decomposed the ECG in sub-bands with constant frequency bandwidths that are implemented in MATLAB (Afonso et al., 1999). The ECG data was visually inspected to assure that the R-peaks are correctly detected. Using the so-called R latencies, we obtained the inter-beat intervals (IBI). We used these values to estimate the HRV indexes. Repeated measures ANOVAs were conducted to investigate in the differences between the conditions.

Statistical analysis: Repeated measures ANOVAs were conducted to investigate the differences between the conditions and the phases.

Results

Behavioral Results for the Problem-Solving Phase

We investigated if the *Quality* of the generation of canonical solution differs in terms of the condition (Conditions: I-PS, PS-I). Thus, we conducted an independent samples test. The preliminary analysis of the Shapiro-Wilk test of normality indicated that neither of the groups satisfies the assumption of normality. Thus, we conducted Mann-Whitney U test. The results indicate that the I-PS group (M = 3.26, SD = 1.2) scored significantly higher U = 737, p < .001 than the PS-I group (M = 1.89, SD = 1.26) with a medium effect size (r = .59).

We also investigated if the *Quantity,* the number of solutions generated by a student, differs in the two groups by conducting an independent samples t-test. There was no significant difference, $t(45.65) = -1.33$, $p = .19$, while the I-PS group ($M = 2.55$, $SD = 1.02$) attained the same scores as the PS-I group ($M = 2.84$, $SD = 0.63$).

Behavioral Results for the Post-Test Phase

We investigated the differences in the post-test score between the two groups (PS-I and I-PS a). ANOVAs with the PS-I and I-PS condition as the between-subjects factors revealed no significant difference between the two conditions on the total Post-Test-Score ($F[1, 56] = 0.037$, $p = .847$), conceptual understanding ($F[1, 56] = 7.976e\text{-}4$, $p = .978$), transfer of knowledge ($F[1, 56] = 0.021$, $p = .886$), and procedural knowledge ($F[1, 56] = 0.927$, $p = .340$).

ANCOVA results revealed a significant difference in math anxiety (AMAS) ($F(1, 50) = 7.724$), $p = .008$), but no significant difference in learning goal orientation (LGO) ($F(1, 50) = 0.623$, $p = .434$), attitude towards mistakes (ATM) ($F(1, 50) = 0.162$, $p = .689$), knowledge gap awareness (KGA) ($F(1, 50)$, $p = .21$), state curiosity (SC) ($F(1, 50) = .399$, $p = .531$), and germane cognitive load (GCL) ($F(1, 50) = .069$, $p = .794$). The difference in AMAS could explain why there was no significant difference in the Post-Test score between the two conditions. Here, we are mainly interested in the different neural and physiological differences between the two conditions and the phases of the Productive Failure design. The following results were derived.

EEG Results

Frontal Alpha Activity

Table 4.2: *ANOVA for the EEG results on frontal alpha activity (baseline corrected).*

Cases	Sum of Squares	df	Mean Square	F	p	ω²
phase	0.069	2	0.035	3.266	0.039	0.005
group	1.085	1	1.085	102.202	< .001	0.115
phase ∗ group	0.002	2	0.001	0.098	0.907	0.000
Residuals	8.139	767	0.011			

Note. Type III Sum of Squares

The results indicate that the main effect of the group is significant $F(1, 767) = 102.202$, $p < .001$, $\omega^2 = 0.115$ with a medium effect size, see table 2. Tukey post hoc analysis revealed that the PS-I group (M=-.024) showed significantly more enhanced frontal alpha activity than the I-PS group (M=-.142) with a small effect size (MD = -.119, p_{tukey}< .001, d = 0.347).

Moreover, there was a significant main effect of phase $F(2, 767) = 3.266$, $p = .039$, $\omega^2 = 0.005$ with a small effect size. Tukey post-hoc analysis allowed us to derive the following results.

- The instruction (M=-.103) does not significantly differ (MD=-.019, p_{tukey}=.127) from the post-test (M=-.084).
- The instruction (M=-.103) differed significantly (MD=-.041, p_{tukey}= .047, d = 0.393) from the problem-solving phase (M=-.062) with a medium effect size.
- The post-test does not significantly differ (MD=-.021, p_{tukey}= .336) from the problem-solving phase.

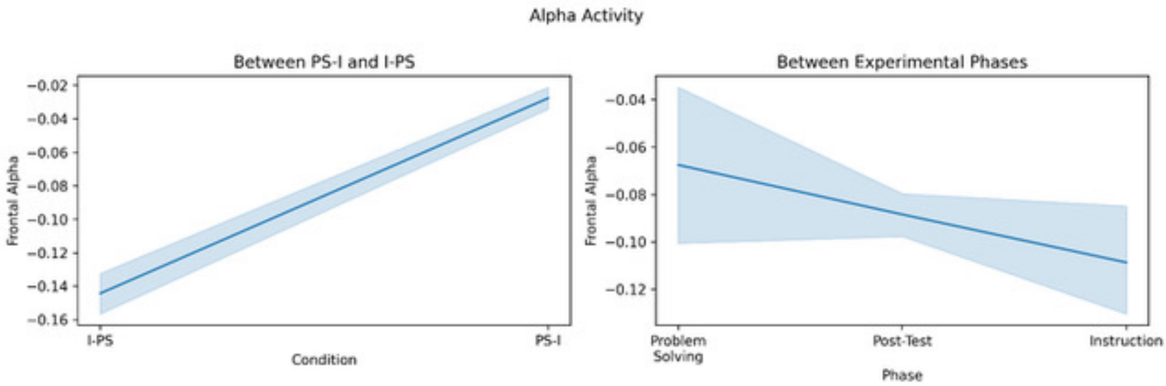

Figure 4.3: *Descriptive plot on frontal alpha activity (baseline corrected) between PS-I condition and I-PS condition, and between the different phases of the conditions (Problem solving, Instruction, and Post-Test).*

Frontal Theta Activity

Table 4.3: *ANOVA for the EEG results on frontal theta activity (baseline corrected).*

Cases	Sum of Squares	df	Mean Square	F	p	η²
group	0.241	1	0.241	9.306	0.002	0.012
phase	0.413	2	0.206	7.972	< .001	0.020
group ∗ phase	0.010	2	0.005	0.200	0.819	5.043e-4
Residuals	19.844	767	0.026			

Note. Type III Sum of Squares

The results indicate that the main effect of the group is significant $F(1, 767) = 9.306$, $p = .002$, with a medium effect size, see table 3. Thus, we conducted Tukeypost-hoc analysis and found that the PS-I group (M=.011) showed significantly enhanced frontal theta activity (MD=-.044, $p_{tukey} < .002$, $d = -0.347$) compared to the I-PS group (M=-.056) with a small effect size.

Moreover, the main effect of phase was significant $F(2, 767) = 7.972$, $p = .002$, with a small effect size. Thus, we conducted a post-hoc analysis using Tukey correction. Following results were derived.

- The instruction (*M*=-.065) does significantly differ (*MD*=-.051, p_{tukey} =.003) from the post-test (*M*=-.014).
- The instruction significantly differed (MD=-.065, p_{tukey}= .001, d = -.583) from the problem-solving phase (M=-.006) with a small effect size.
- The post-test does not significantly differ (MD=-.014, p_{tukey}= .165) from the problem-solving phase.

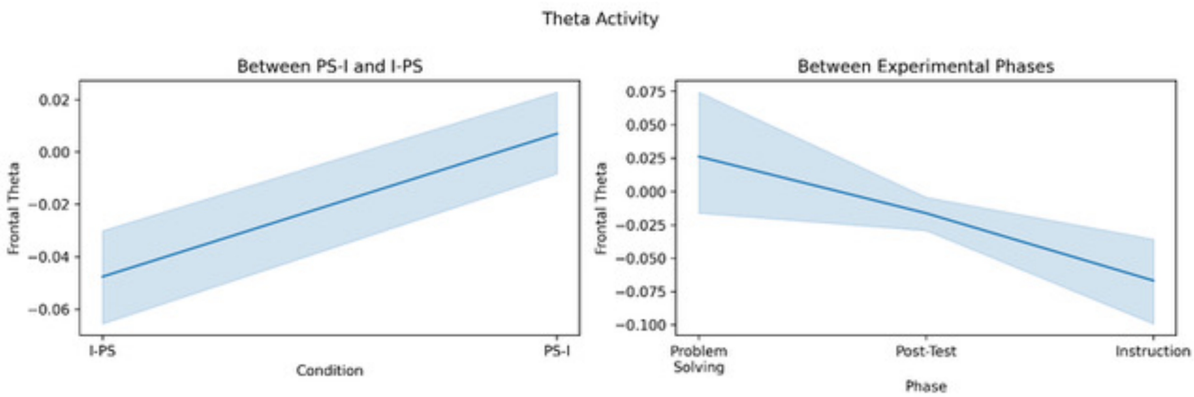

Figure 4.4: *Descriptive plot on frontal theta activity (baseline corrected) between PS-I and I-PS condition, and between the phases of the conditions (Problem solving, Instruction, Post-Test).*

ECG Results

Heart Rate Variability

The results in HRV indicate that there is a significant main effect of group $F(1, 819) = 5.506$, p=0.019, see table 4. Thus, we conducted a post-hoc analysis using Tukey correction. The I-PS group (*M*=.063) showed significantly lower HRV (*MD*=-.080, $p_{tukey} < .001$, d =-.259) than the PS-I group (*M*=-.143) with a medium effect size.

Table 4.4: *ANOVA – Heart Rate Variability (HRV).*

Cases	Sum of Squares	df	Mean Square	F	p	η²
group	0.528	1	0.528	5.506	0.019	0.007
phase	0.102	2	0.051	0.530	0.589	0.001
group ✻ phase	0.099	2	0.050	0.516	0.597	0.001
Residuals	78.584	819	0.096			

Note. Type III Sum of Squares

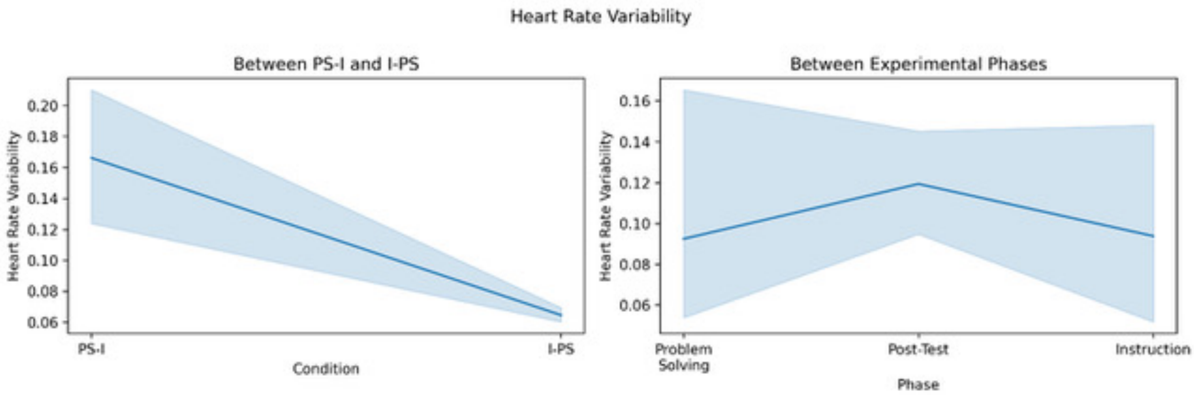

Figure 4.5: *Descriptive plot of HRV (I-PS versus PS-I).*

Discussion

We were interested in the alpha and theta oscillations between the two conditions of the PS-I design. In a naturalistic design, participants solved problems of standard deviation in front of a computer screen (two conditions; PS-I versus I-PS), while we recorded the neural and physiological signatures with EEG and ECG. By using repeated measures ANOVAs, we explored the relationship between the conditions, the phases of the conditions, and the changes in both neural oscillations and heartbeats. After discussing the initial behavioral analysis, we discuss the investigation of the neural activity (EEG), followed by the physiological activity (ECG), and end by discussing future directions in the field of neurophysiological research.

Behavioral Data

Before the EEG analysis, we analyzed the behavioral data, while the PS-I group performed a better in the post-test than the I-PS group, this difference was not statistically significant. One of the explanations could be that some of the students were showing math anxiety. Recently, there have been discussions about affect and the productive failure design (Sinha, 2021; Gashaj et al., 2022). Notably, different emotional aspects influence PF, specifically moment-by-moment determinants of affective states impact the problem-solving phase and performance in the post-test (Sinha, 2021). On the one hand, students who were exposed to failure-driven scaffolding were likely to show exclusive dynamics comprising shame had metacognitive and cognitive benefits. On the other hand, emotions such as anger and disgust have been associated with an incidence of contempt, showing a negative association (Sinha, 2021). In other words, students experiencing different emotions, including pleasurable and unpleasant emotions (Sinha, 2021), as well as math anxiety (Gashaj et al., 2022), serve as catalysts for learning. In the present behavioral results, there was a significant main effect in students' math anxiety, which may have an impact on the post-test performance.

Even though the behavioral results (no statistically significant difference between the two conditions) were contrary to expectation (that the PS-I condition outperforms the I-PS condition), we still investigated the neural and physiological differences that might appear when students are taught with different learning designs. On the one hand, we were interested in the neural and physiological signature differences between the phases of PS-I versus I-PS; on the other hand, we also investigated the differences between the two groups. The neural signatures were expressed in terms of frontal alpha and theta oscillations, whereas in the physiological signatures, we focused mainly on heart rate variability.

The Neural Basis of Learning through PS-I

Alpha oscillations are associated with semantic information processing, such as searching, accessing, and retrieving information from long-term memory (Klimesch et al., 1997). Therefore, we expected to see enhanced alpha activity in the problem-solving and the post-test phase when compared to the instruction phase. In our study, we found that, relative to the baseline, alpha oscillations were significantly lower during the instruction phase than during the problem-solving phase. During the problem-solving phase, alpha oscillations were significantly higher, suggesting greater activation of memory processes. The finding adds to the crucial issues in memory research, which is based on how the search process finds relevant information in memory. It has been suggested that alpha oscillations are related to memory performance (Klimesch et al., 1990; Klimesch et al., 1993). The present findings of higher alpha oscillations during the problem-solving phase suggest enhanced memory processes during that phase when compared to the instruction phase.

Our results further indicate that there is more alpha activity in the PS-I group compared to the I-PS group. Research on Productive Failure has indicated cognitive mechanisms for why students learn better after encountering difficulties; because the students actively search, access, and retrieve information from long-term memory. Since these cognitive activities are associated with alpha waves (Klimesch et al., 1997), .our result could reflect that the PS-I condition is advantageous in terms of memory processing and that the cognitive mechanisms that were suggested to play a role in the PS-I design might explain the better post-test performances of participants in the PS-I condition compared to the I-PS condition in previous research (Kapur, 2014).

Furthermore, our results show that theta oscillation in the frontal brain regions were significantly different between the phases of the two learning designs. Previous research has shown that theta activity is related to episodic and working memory (Jensen & Tesche 2002; Kahana et al. 2001). In our study, theta was significantly lower in the instruction phase when compared to the problem-solving phase, suggesting that in the problem-solving phase, theta activity increases, reflecting the engagement of working or episodic memory. In previous frontal theta and memory investigations, researchers observed ongoing and parametrically increasing frontal theta activity in a retention period of a task where subjects were asked to retain a list of visually presented digits (Jensen & Tesche, 2002). Our findings of the enhanced theta activity in the problem-solving phase compared to the instruction phase elaborate on those previous research results, suggesting more activity when participants were solving problems than when they were given instruction.

In line with the problem-solving phase eliciting more theta activity, we also observed a higher frontal theta activity in the PS-I group compared to the I-PS group. This could illustrate that more working memory processes might be involved in the PS-I condition – namely during the problem-solving phase, as theta activity has been related to working memory (e.g., Jensen & Tesche, 2002). This is plausible because working memory functions have been proposed to arise through the coordinated recruitment, via attention, of brain systems that have evolved to accomplish action-related functions (Postle, 2006). The PS-I condition has been proven to lead to better results in conceptual knowledge and knowledge transfer compared to the I-PS condition (Kapur, 2014; Sinha & Kapur, 2021). Enhanced working memory capacity might aid in terms of conceptual knowledge and transfer knowledge as it might maintain the knowledge better. Previous neuroscience research has shown that frontal theta activity has been related to episodic and working memory (Jensen & Tesche 2002; Kahana et al. 2001). It could be that the PS-I design is beneficial in terms of working memory, as frontal theta activity seems to be higher.

Altogether, the findings on enhanced frontal alpha and theta activity in the PS-I condition compared to the I-PS condition support – from a neuroscientific perspective - theories on working memory and long-term memory. Cognitive mechanisms have been investigated, showing that prior knowledge is critical in expanding the working memory capacity (Kirschner, Sweller & Clark, 2006). When learners are exposed to novel information, as is the case in the PS-I condition, the processing of novel information depends upon the limited working memory capacity and long-term memory. PS-I presents a situation where students first fail to solve the problem and then use this as a lesson to consolidate and assemble new knowledge, which may benefit their working and long-term memory (Kapur, 2016). From a neurocognitive perspective, enhanced alpha and theta in the frontal brain regions have been shown to enhance memory capacity (e.g. Klimesch, 1999), which supports previous findings in Productive Failure (Kapur, 2016).

The Physiological Basis of Learning through PS-I

Not only the brain activity differed in the groups, but the heart rate variability was also varying between the groups. Previous research has connected higher HRV to increased activity in executive brain regions (Thayer et al., 2012; Thayer & Sternberg, 2006; Park and Thayer, 2014). Our results suggest that since the I-PS group showed lower HRV, they might have had less activation in the executive brain regions. In other words, the PS-I designs seem to engage the executive brain regions more compared to the I-PS group. This activation might explain why PS-I as a learning design successfully enhances learning performance and outcomes.

Conclusively, there seems to be a higher HRV, higher frontal alpha, and higher frontal theta activity in the PS-I group compared to the I-PS group. Previous research investigated the effects of alpha and theta neurofeedback and HRV biofeedback on emotional and cognitive creativity. They found that alpha and theta neurofeedback may increase cognitive creativity, and HRV biofeedback may increase emotional creativity (Alaedini et al., 2018). Researchers have aimed to use stimulation-induced HRV techniques and observed that HRV increases may correspond to frequency-specific oscillatory modulation (Machetanz et al., 2021). For example, they found that HRV increases correspond to frontal elevations in the theta-band (Machetanz et al., 2021). Knowledge on the brain-heart interaction in the learning sciences is still in its infancy and current research is slowly uncovering this interaction. In our study, we observed higher HRV, which might benefit the PS-I condition with enhanced executive functions, and frontal alpha and theta activity was higher compared to the I-PS condition, which might be advantageous in memory performance.

Collectively, this study's results help us understand the brain and heart processes involved in problem-solving followed by instruction and bring valuable insights from the neuroscientific point of view to an effective learning design. Future work centered on the neural and physiological signatures of problem-solving followed by instruction may yield further insight into underlying mechanisms mediating the different phases, as well as the differences between the conditions.

Limitations and Future Directions

There are limitations to the present study. First, the fidelity of the Productive Failure design was somewhat limited. Students were showing math anxiety, and we did not control for affective support to set the right expectation, which has been done in previous studies. Furthermore, our study was in front of a computer screen, which is a different setting than the previous research in the field. Our main aim was to investigate the neural and physiological basis of the PS-I design in the absence of all aspects that relate to a high-fidelity Productive Failure design.

Second, the number of participants was also limited yet sufficient to provide evidence on the tendency of the neural and physiological differences between the phases and the groups of the PS-I and the I-PS design. In the future, we propose to increase the sample size and analyze whether the current results can be confirmed (or not). Not only the number of participants but also the level of degree was different from previous research in the field. Participants in our study were university students, whereas the design has been previously researched with high school students. This difference in age and school grade needs to be further considered when designing future studies in the field.

Third, the EEG and ECG data analysis can be elaborated with event-related synchronization (ERS) and event-related desynchronization (ERD), as well as the investigation of other brain networks that might be important to the underlying neurophysiology of PS-I. Our findings on the EEG and ECG power-spectral density analysis may provide neural and physiological mechanisms of the PS-I design, but further analysis is needed to elaborate on the current results.

Finally, a limitation of neurophysiological research is the habituation of the experimental conditions. For example, we currently cannot know yet whether the results were based on the cognitive advantages of the design or due to external factors such as math anxiety. However, neurophysiological research in the learning sciences provides valuable insights into how the nervous system works under different learning situations and during different cognitive tasks. In the future, we suggest setting up a design that tests for the different cognitive mechanisms in the PS-I learning design and set up EEG and ECG measurements to further analyze the different brain wave frequencies and thus elaborate on the neural and physiological basis of PS-I.

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Chapter 5
Overall Discussion

What are the physiological and neural signatures underlying different learning processes?

As demonstrated in Chapter 2, during the literature review, previous studies in the field of neurocognition and neurophysiology have observed a correlation between physiological and cognitive measurements. More specifically, heart rate change was positively related to the cognitive measures of learning. Previous research has observed that a higher heart rate variability (HRV) seems to predict better cognitive performance in the considered learning process or task. As shown in chapter 2, a higher HRV has been shown to be related to an increased activity in the executive brain regions, which may benefit learning and cognition. Moreover, different brain oscillation frequencies (alpha, beta, theta, delta, gamma) have been observed to be related to different learning processes, including problem-solving, insights, and memory retrieval. These results highlight the influence of heart rate variability on learning and the different brain oscillation frequencies responsible for different learning processes.

In this sense, research investigating physiological measurements underlying different learning processes confirms what we might intuitively feel: the palpitations of the heart are related to whether we are excited, or anxious, and this can influence learning outcomes. While researchers first considered HRV in medical or health domains, now we have evidence that HRV may act as an objective measure in the context of advanced problem solving. It may play an important role when generating solutions to (mathematical) problems. Collectively, HRV can be seen as a promising measurement in the field of learning and cognition. Along with HRV, brain oscillation frequencies have been uncovered to be associated with specific learning processes and allow researchers in neurocognition and neurophysiology to explore the connections between HRV, neural signatures, learning and cognition.

The neural signatures underlying math expertise

In Chapter 3, our studies concentrate on observing the neural signatures underlying math expertise. To evaluate the neural signature underlying mathematical sense-making, we tested math experts and novices with electroencephalography (EEG) recording, while they were asked to make sense of mathematical demonstrations. The study used EEG to compare frontal alpha asymmetry and frontal theta in math experts and novices, making sense of more extended and naturalistic mathematical demonstrations. Frontal alpha asymmetry is the average difference in brain activity between the left and right frontal brain areas (Harmon-Jones, Gable, & Peterson, 2010), and has been associated with emotional processes (Quaedflieg et al., 2015), affects (Rosenfeld et al., 1996), and executive functions (Moynihan et al., 2013). Frontal theta activity has been associated with active operations during high-level cognitive processes, such as memory encoding and retrieval, working memory retention, novelty detection, and other cognitive measures (e.g. Jacobs et al., 2016), and is crucial for mathematical problem solving (e.g. Lin, et al., 2012, 2015).

Based on previous research, we hypothesized that novices would show an increase in frontal theta (4-8 Hertz) activity compared to math experts due to enhanced attention, working memory load, and the novelty response (Kruglanski & Gigerenzer, 2011; Evans & Stanovich, 2013; Cavanagh et al., 2012). Our data supported the hypothesis: math novices showed more/highe frontal theta power compared to experts. This finding indicates that novices might need more cognitive control and engage more working memory processes (thus increasing frontal theta power, e.g., Jacobs et al., 2016) while making sense of complex mathematical demonstrations.

Our analysis extends existing knowledge by offering new insight into significantly more frontal theta activity in novices when making sense of mathematical demonstrations. These findings reaffirm the neural differences between experts and novices that seem to be present during longer and more complex math demonstrations. Our work contributes toward understanding the neuro-dynamics of mathematical cognition and the neural differences in math experts and novices during longer and more complex math demonstrations in a more naturalistic context.

Our research further elaborates on significant work in mathematical cognition and language by Amalric & Dehaene (2016). The researchers showed that high-level mathematical cognition is related to a different set of brain areas than language processing. While language processing and verbal semantics involve mainly left-hemisphere regions, mathematical cognition seems to recruit a bilateral network, which includes the prefrontal, parietal, and inferior temporal regions. The author’s results suggest that mathematical cognition recruits a bilateral network that is involved in number and space, and that is different from language processing (Amalric & Dehaene, 2016).

Our research elaborated on the findings by Amalric & Dehaene (2016). However, our focus was not on the differences between the brain network involved in numbers versus language processing, but rather on the underlying neural signature differences between math experts and math novices. We analyzed differences in frontal alpha asymmetry and frontal theta activity. Previous results in the field of mathematical cognition and math expertise suggested that math experts, when exposed to mathematical sense-making, activate a set of bilateral frontal areas (along with intraparietal, and temporal regions), which spared brain areas that are related to language processes. We aimed to elaborate on the evidence that there seems to be a linguistic, and a nonlinguistic brain circuit and uncover neural signatures of math expertise compared to novices. Our results did not show significant differences in frontal alpha asymmetry between math experts and novices. Thus, we were not able to add to the evidence that mathematical reasoning is related to activity amplification in corresponding brain regions that depends on individual expertise. However, we showed evidence that there seems to be a significant difference in frontal theta activity between math experts and novices, which is in line with previous research.

Along with our main finding, that math experts showed less frontal theta activity than novices when exposed to mathematical demonstrations, we also noticed behavioral differences in mathematicians when compared to novices. We were mainly interested whether participants had enough time, whether they understood the math demonstrations and how familiar and engaged they were with the math demonstrations. Math experts, unlike novices, indicated that they had enough time to solve the math demonstrations, that they understood the math demonstrations, and that they felt engaged while making sense of the math demonstrations. These findings were as expected, as previous research uncovered that math experts, compared to novices, seem to have a more accurate mental representation of mathematical knowledge, a more positive attitude, and a higher motivation towards mathematics (Meier et al., 2021). Thus, our findings show evidence that math experts differ from novices in terms of neural mechanisms, as well as in terms of behavioral characteristics.

The physiological signatures underlying math expertise

As presented as well in Chapter 3, we were interested in the differences in heart rates (HR) between math experts and novices. We compared math experts (at least a bachelor's degree in mathematics) with novices (no background in mathematics or similar field) while they were asked to observe math demonstrations in front of the computer screen. We were interested in the physiological differences of math experts and novices because specific neurophysiological characteristics change through experience. We expected to see differences in heart rates between experts and novices when exposed to mathematical demonstrations. Our pilot study found that experts had higher heart rates than novices, but the difference was not significant.

Enhanced engagement in math experts than in novices when being exposed to mathematical demonstrations might be the cause for higher heart rates in experts (Azarbarzin et al., 2014; Darnell & Krieg, 2019). Hence, based on this experimental study, it can be argued that experts are more aroused and engaged while being involved with their corresponding material of expertise than novices, leading to an increase in experts' heart rate. However, it is worth mentioning that we believe further studies are needed to investigate the heart rate differences between experts and novices of mathematics.

In line with previous research, we hypothesized that heart rate is correlated with self-reported engagement and understanding of mathematical demonstrations. In other words, we expected that the physiological indices of engagement correlate with the self-reported indices. Our results showed that math experts and novices differed significantly in terms of understanding, and engagement. Experts indicated higher levels of engagement and showed a better understanding than novices, which is in line with previous research. Contrary to our hypothesis, the correlation analysis showed that heart rates were not significantly correlated to self-reported indices, such as engagement and understanding of the demonstrations.

Lastly, we hypothesized that the difference in heart rate between experts and novices is due to expertise and not due to math anxiety. This hypothesis was supported by our findings, as we found no significant difference in math experts compared to novices in terms of math anxiety but found substantial differences in understanding, and engagement. Therefore, we can confirm that the difference in heart rate is not due to math anxiety. Heart rate differences could therefore be rather due to expertise, understanding, and engagement.

The investigation of heartbeats in the context of mathematical cognition was an extension to the previous neural (EEG) analysis described in Chapter 3. We were interested in the interaction between cognitive performance, neural signatures, and physiological measurements of math experts and novices. This is mainly because of the evidence that dynamic changes in bodily physiology are related to cognitive processes (Critchley & Garfinkel, 2018). We aimed to examine how heartbeats facilitate, or inhibit, cognitive processing involved in learning. We found significant differences in the neural signatures between experts and novices. However, we do not have enough evidence for similar results in heartbeat differences between math experts and novices while making sense of mathematical demonstrations. However, we are certain that research in how heartbeat measurements correlate with higher order cognition and learning will continue to be of high importance.

Uncovering the underlying physiological basis of learning will help us to elaborate and implement methods that improve learning.

The neural and physiological signatures underlying PS-I

Something universal happens inside of us when we experience failure. We may feel our heartbeats increasing, or feel the negative affective states followed by a failure. Many contemporary scientists believe that the quality of feeling and emotion we experience is rooted in the underlying state of our physiological processes (Damasio, 1996). Therefore, we were interested in the neural and physiological signature underlying failure. Hence, we investigated the neural and physiological basis of learning through the problem-solving followed by instruction (PS-I) design.

We investigated differences in neural signatures through electroencephalography (EEG) and physiological signatures through electroencephalography (ECG) related to the type of learning situation "Problem solving followed by Direct Instruction" (PS-I) – as entailed in the Productive Failure design (Kapur, 2014) - versus "Direct Instruction followed by Problem Solving" (I-PS). In a naturalistic design, participants solved problems of standard deviation in front of a computer screen (two states; PS-I versus I-PS) while we recorded the neural and physiological signatures. Several cognitive processes are thought to be simultaneously occurring, including semantic information processing or episodic and working memory. We investigated these processes with EEG and ECG. We focused our analysis on frontal alpha oscillations, frontal theta oscillations, and heart rate variability (HRV).

Alpha oscillations are associated with semantic information processing, such as searching, accessing, and retrieving information from long-term memory (Klimesch et al. 1997). Therefore, we expected to see enhanced alpha activity in the problem-solving and post-test phases compared to the instruction phase. Consistent with our hypothesis, we found that, relative to the baseline, the alpha oscillation was significantly lower during the instruction phase than during the problem-solving phase. During the problem-solving stage, alpha oscillations were substantially higher, suggesting greater activation of memory processes during the problem-solving phase.

This finding adds to the crucial issues in memory research, which is based on the question of how the search process finds the relevant information in memory. The cortical neural network can be considered a storage network for memory that initiates and assesses information (e.g., Klimesch, 1994). It has been suggested that alpha oscillations are related to memory performance (Klimesch et al., 1990, Klimesch et al., 1993). The present findings of higher alpha oscillations during the problem-solving phase suggest enhanced memory processes during that phase when compared to the instruction phase.

Our results further indicate that there seems to be more alpha activity in the PS-I group compared to the I-PS group. Research on Productive Failure has indicated cognitive mechanisms for why students learn better after encountering difficulties, which is the case with the PS-I group in the present study. As hypothesized, the PS-I group showed more alpha activity, potentially because the students in this group have been searching, accessing, and retrieving information from long-term memory, and those cognitive activities are associated with alpha-waves (Klimesch et al., 1997). Our findings that students in the PS-I group had higher alpha activity can therefore be interpreted to be confirming the conjectures considering cognitive mechanisms raised in previous research (Kapur, 2014).

Furthermore, our results show that theta oscillation in the frontal brain regions was significantly different between the phases (instruction, problem-solving, post-test) of the two learning conditions (PS-I versus I-PS). Previous research has shown that theta activity is related to episodic and working memory (Jensen and Tesche 2002; Kahana et al. 2001). In our study, theta was significantly lower in the instruction phase when compared to the problem-solving phase, suggesting that in the problem-solving phase, theta activity increases, reflecting the engagement of working or episodic memory. In previous frontal theta and memory investigations, researchers observed ongoing and parametrically increasing frontal theta activity in a retention period of a task where subjects were asked to retain a list of visually presented digits (Jensen & Tesche, 2002). Compared to the instruction phase, our findings of the enhanced theta activity in the problem-solving phase elaborate on those previous research results but in a novel learning setting.

In line with the problem-solving phase eliciting more theta activity, we also observed a higher frontal theta activity in the PS-I group compared to the I-PS group. This could illustrate that more working memory processes might be involved in the PS-I condition – namely during the problem-solving phase (e.g., Jensen and Tesche, 2002). This is plausible because working memory functions have been proposed to arise through the coordinated recruitment, via attention, of brain systems that have evolved to accomplish action-related functions (Postle, 2006). In the PF design, the PS-I condition has been shown to have better results in conceptual knowledge and transfer of knowledge compared to the I-PS condition (Kapur, 2014, Sinha & Kapur, 2021). Enhanced activation of working memory might aid in terms of conceptual understanding and transfer of knowledge, and previous neuroscience research has shown that frontal theta activity has been related to episodic and working memory (Jensen and Tesche 2002; Kahana et al. 2001). It could be that the PS-I condition is beneficial in terms of working memory, as frontal theta activity seems to be higher.

Not only did the brain activity differ between the groups, but the heart rate was also varying between the groups. Previous research has connected higher HRV to increased activity in executive brain regions (Thayer et al., 2012; Thayer and Sternberg, 2006; Park and Thayer, 2014). Our results suggest that since the I-PS group showed lower HRV, there might have been less activation in the executive brain regions. In other words, the PF designs may have activated the students in a way that engaged their executive brain regions more than the I-PS design. This activation might explain why PF as a learning design successfully enhances learning performance and outcomes since the students engage more actively with the material to be learned. In fact, our results suggest a higher frontal theta and alpha activity in the PS-I condition and the problem-solving phase. Thus, the higher HRV in the PS-I group is in accordance with the frontal brain activity.

Conclusively, there seems to be a higher HRV, higher frontal alpha, and higher frontal theta activity in the PS-I group compared to the I-PS group. Whereas higher HRV has been linked to cognitive engagement (e.g. Thayer et al., 2012), frontal alpha has been related to heightened working memory activation (e.g. Klimesch et al., 1997). Also, frontal theta activity has been linked to deeper conceptual understanding (e.g. Jensen and Tesche 2002). On the one hand, we observed higher HRV in the PS-I condition, which might benefit the PS-I condition with a better activation of executive functions. On the other hand, frontal alpha and theta activity was higher in the PS-I condition compared to the I-PS condition, which might be advantageous in the memory performance.

This project allowed for an extension of neural and physiological knowledge to a complex scenario involving learning from failure. Learners were intentionally confronted with moments of difficulty, or failure, before getting instruction, as a means of preparing them to learn and benefit from future instruction. We uncovered the underlying neural and physiological signatures, and with that, made a first step to indicate - from a neuroscientific perspective – why students may learn better after an initial failure. With this project, we illustrated that moments of difficulty are not just metaphorically felt in the heart and brain, but literally. We uncovered neural and physiological signatures of learning through PS-I and will continue to elaborate steps to create more effective learning scenarios.

Final Words

While, for many years, we may have had a more intellectual view of learning, it is important to consider neural and physiological signatures when investigating in learning processes and (mathematical) cognition. Taking into consideration neural and physiological signatures may provide us with a more holistic view of math learning and cognition. Human reasoning and learning cannot and should not be limited to one of the above discussed aspects alone. The understanding of processes underlying human reasoning has been investigated by Damasio over twenty years ago, when he described the so-called somatic marker hypothesis (e.g. Damasio 1996). The somatic marker hypothesis states that 'marker' signals influence processes that respond to a certain stimuli (both consciously, and unconsciously). The markers are 'somatic', because they relate to the body-state regulation (Damasio, 1996). Within the brain, these markers may be processed in the ventromedial prefrontal cortex, which is located in the frontal lobe of the brain. What's important about it is, that somatic markers may guide behavior, human reasoning, or decision-making. When we take action to those 'marker' signals, we may bias our reasoning (or decision-making) on the basis of the emotions that may arise from our body-state regulation (e.g. Damasio 1996). Heartbeats are 'somatic' markers that may be associated with emotions (such as anxiety), and thus may influence our cognition and learning. I believe that a holistic view, which takes into consideration all of the above mentioned aspects of physiological and neural signatures, such as heartbeat measurements and brain oscillation frequencies, the body-state regulation, cognition, and learning, will help us to improve educational practices, as well as emotional and physical well-being.

Thank you

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A. Appendix (Chapter 2)

Figure 2.1.1: *Flowchart of the Systematic Review.*

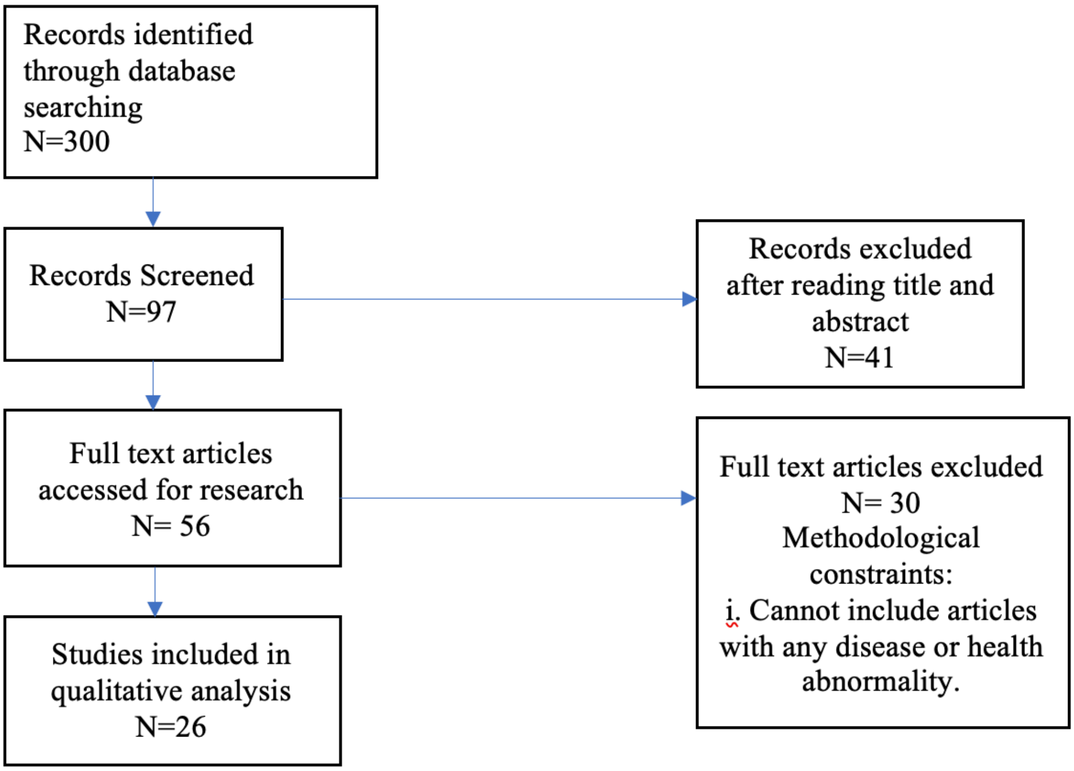


Table 2.1.1: *Study summaries of heartbeat measurements and brain oscillations.*

Heartbeat Measurements		
Study	**Relationship**	**Domain**
Pham & Wang, 2016	Participants watched three videos about different unfamiliar topics during a 24 minute learning phase. HRV data was used to determine which subject was most demanding for the subject. Subsequently, participants watched either all or none of the videos again or only the most or least demanding. Afterwards participants solved 24 multiple choice questions. Watching all or only the most demanding video(s) yielded the highest results.	Lecture on law. 24 Multiple-Choice questions in post-test. Learning/Recall. Remembering.

Gellatly & Meyer, 1992	Participants solved a letter-recognition task and were grouped depending on the difficulty of the goal they were told to reach. Subjects assigned more difficult goals on a perceptual-speed task perceived a higher performance norm, reported higher self-efficacy strength, set a higher personal goal, exhibited increased heart rate, and produced more than did subjects assigned easier goals. Perceived norm, self-efficacy strength, and personal goal were positively related with heart-rate change.	Evaluation. Self-reported Items (e.g. Arousal, Self-efficacy)
Colzato et al., 2018	Pearson's correlation coefficients were computed to test whether resting-state HRV predicted task-switching performance. Participants with higher resting-state HRV showed smaller switch costs (i.e. greater flexibility) than individuals with lower resting-state HRV.	Task-switching paradigm. Analyzing. Reaction Time.
Garfinkel et al., 2013	Physical arousal, namely the cardiac cycle, influences cognitive function. Using an emotional attentional blink (EAB) paradigm, the study showed how words detected during systole were less remembered than words during diastole. The participants' interoceptive sensitivity modulated this effect.	Word detection and memory task. Detection and remembering.
Cranford et al., 2014 Aritzeta et al., 2022	Heart rate changes were hypothesized as a valid measurement of cognitive load. Chemistry students and faculty members solved multiple chemistry problems of varying difficulty. Chemistry problems of higher complexity induced a greater change in heart rate than those that have been designed to be of lesser complexity. Changes in heart rate were less pronounced for expert chemists than less experienced chemistry students. Problem-solving on everyday problems and organizing everyday learning process in thematic spaces (according to the Amara Berri Education System; primary education).	Problem-solving task. Applying. Problem-solving.

Howell & Hamilton, 2022	73 participants completed affective questionnaires, a 5-minute baseline electrocardiogram, and the Virtual Morris Water Task. Higher baseline HRV was predictive of better performance during set-shifting. This effect was moderated by negative affect and only found in individuals with low trait negative affect, suggesting that high NA interferes with the HRV-cognition connection.	Set-shifting task. Spatial learning, memory.
Nakagawa et al., 2021	The authors propose a method to evaluate students concentration level in distance learning settings using biometric information. The change of emotion in students was evaluated by measuring EEG and HRV, while turning on and off a face-to-face video camera. Depending on the individual preferences of the subject, both EEG and HRV measurements changed after turning on the camera.	Measurements of biometric information during online class
Larmuseau et al., 2020	Participants solved exercises of probabilistic reasoning while self-reported cognitive load (CL) and physiological data was measured. CL was manipulated based on intrinsic and extraneous load. Rsults show how heart rate is significantly related to self-reported CL. Still, physiological data could not be used to detect differences in CL based on intrinsic and extraneous manipulations.	Exercises in statistics. Creating.
Scrimin et al., 2018	Participants read a science text and afterwards solved a multiple-choice test. Depending on the condition they were asked to read for themselves to gain greater understanding of the topic or to perform well on the test and gain course credit. During all phases heart-rate and HRV were measured. Results showed a general trend in which students HRV was greater in the reading phase compared to baseline. Furthermore, HR decreased during reading and increased during the test phase.	Reading comprehension. Analyzing. Applying.
Redondo-Flórez et al., 2020	82 students were divided according to their prior knowledge in laboratory practices. HRV was measured before, during and after lab practice. Both groups exhibited an anticipatory anxiety response but non-experienced students showed a lower habituation response in both subjective and objective stress records than experienced students at the end of the lab practice.	Handling of toxic materials in a supervised lab setting. Applying.

Silvennoinen et al., 2019	During ten days, self-reported qualitative and physiological data, including HRV, was collected in a sample of 14 university students. Daily differences in HRV corresponded to the students self-reports. Furthermore, alertness levels may vary between teaching methods: The data suggests how group work facilitated by the teacher may be physiologically more alerting/engaging and also more meaningful than letcures.	Comparison of physiological and qualitative data.
Zaccoletti et al., 2022	82 seventh-graders read an informational text and afterwards completed a comprehension task. Prior to reading, they were asked to either read for themselves or try to get the highest score in a ranking. Resting-state HRV was assessed. Students who were instructed to achieve a high score performed better if they had higher resting HRV. Students who were instructed to read for themselves performed worse if they exhibited a higher resting HRV.	Reading comprehension. Analyzing. Applying.
Sánchez-Conde & Clemente-Suárez, 2021	In a sample of 41 nursing degree students HRV and heart rate mean response were measured before, during and after completing a Clinical Evaluation. Results show how the stress response varying according to the difficulty of the evaluation. No clear conclusion can be drawn from the HRV data.	Analyzing. Applying. Understanding. Remembering.
Mason et al., 2018	47 school students read webpages varying for reliability and position on the topic of potential health risks associated with the use of mobile phones. Afterwards, students wrote essays on the topic which were used to analyze text comprehension. During the reading and writing session HRV and heart rate were measured. Results show no influence of the type of webpage on HRV or heart rate. However, HRV was a positive predictor of text comprehension, while higher heart rate predicted worse scores on comprehension.	Reading comprehension. Analyzing. Applying. Creating.
Kermani & Birjandi, 2017	63 university students and high schoolers received information and were provided with a biofeedback tool in order to self-generate a high psychophysiological state and optimal HRV. Results show how training on self-regulation can increase HRV scores and in turn lower the participants reading anxiety.	Reacting to Biofeedback. Applying.

Brain Oscillations		
Study	**Relationship**	**Domain**
Klimesch, Schack and Sauseng, 2005	Multiple experiments. Experiment 1 involved participants with Alzheimer's. Individual alpha frequency (IAF) was compared in multiple experiments with varying tasks. Subjects with "Good memory" had higher IAF scores than subjects with "Bad Memory".	Working memory task
Rosen & Reiner, 2017	Participants were confronted with a 10-coin puzzle and were grouped by their self-declared solving method, Insight or Incremental. Multiple EEG bands of both groups were analyzed using a 4-factor ANOVA. Significant differences between insight and incremental solvers were found in Alpha, Gamma and Theta bands.	Generating solutions. Insight and Incremental. Analyzing. Creating?
Jung-Beeman et al., 2004	Subjects solved verbal problems. They then indicated whether they solved them with or without insight. fMRI showed increased activity in the right hemisphere anterior superior temporal gyrus for insight when related to non-insight solutions. The EEG recordings revealed that there is a sudden burst of high-frequency (gamma-band) neural activity in the same area beginning 0.3 s prior to insight solutions.	Verbal problems. Creating. Applying.
Sheth, Sandkühler & Bhattacharya, 2009	Participants solved verbal puzzles and reported their level of insight. EEG recordings showed no significant differences in brain oscillations between different levels of reported insight.	Word puzzles. Evaluating. Applying.
Jensen & Tesche, 2002	Participants solved the Sternberg memory task and while whole scalp MEG data was recorded. The main result was a parametric increase in 7±8.5-Hz theta power with memory load during memory retention, and stronger theta activity in the memory task compared to a control task.	Sternberg task. Analyzing. Why understanding? Remembering.

Guderian et al., 2009	Participants performed either semantic (pleasantness-rating) or phonemic (syllable-counting) judgments on visually presented words. Whole-head magnetoencephalographic (MEG) data was recorded. Amplitudes of theta oscillations shortly preceding the onsets of words were higher for later recalled than for later-forgotten words. Furthermore, single-trial analyses revealed that recall rate in all participants tested increased as a function of increasing pre-stimulus theta amplitude. This positive correlation was independent of whether participants were preparing for semantic or phonemic stimulus processing, thus likely signifying a memory-related theta state rather than a preparatory task set.	Episodic memory encoding. Analyzing. Understanding.
Fuentes-García et al., 2019	Participants solved chess game puzzles and were classified according to their results into: high performance or low performance. HRV was assessed at baseline. During the chess problems, HRV was monitored, and immediately after chess problems, they registered subjective stress, difficulty and complexity. The Friedman test showed a significant effect of tasks in HRV indexes and perceived difficulty, stress, and complexity in both of the groups (high and low performance). A decrease in HRV was observed in both groups when chess problems difficulty increased. Interestingly, HRV was significantly higher in the high-performance group compared to the low performance group during the chess problems.	Solving chess puzzles. Analyzing.
Hjortskov et al., 2004	The researchers either added or removed computer-work-related mental stressors from a computer work session (standardized) in the laboratory. They observed a reduction in the high-frequency component of HRV. Also, an increase in the low- to high-frequency ratio was observed in the stress situation compared to the control session. They observed no significant changes in the low-frequency component of HRV, and no significant differences for subjective experience of stress.	Simple number entering task in a computer. Analyzing. Why understanding?

Kahana, Seelig & Madsen, 2001	Recent studies using implanted depth and cortical surface electrodes in humans have changed this situation by demonstrating a task-related high-amplitude activity of theta. These studies have shown that theta increases during both verbal and spatial memory tasks. Furthermore, human theta does not appear to be restricted to hippocampal sites, but rather appears over widespread regions of the neocortex.	Subjects are presented with a series of items and must indicate whether the current item matches an item that occurred n-items back in the series, thus involving simultaneous encoding, maintenance, and retrieval of information.
Başar, Başar-Eroğlu, Karakaş, Schürmann, 1999	Reviewed were multiple experiments concerning oscillatory responses to events (in the alpha, theta, and delta ranges) as possible correlates of sensory and cognitive functions. Experimental data suggests that event-related theta oscillations are related to cognitive processing and cortico-hippocampal interaction. As to delta oscillations, experimental data hint at functional correlates roughly similar to those mentioned for theta oscillations, i.e. mainly in cognitive processing.	Multiple experiments. "Cognitive processes"

Table 2.1.2: *Overview and relationship between cognitive domains of creating (CR), evaluating (EV), analyzing (AN), applying (AP), understanding (UN), and remembering (RE).*

Study	**Participants**			**Cognitive Domain**						**Domain**	**Relationship between domain and cognitive performance**
	Group	***N***	***Age***	***CR***	***EV***	***AN***	***AP***	***UN***	***RE***		
Pham & Wang, 2016	Male, Female	32	23.6 (4.2)							SDNN	Positive
Gellatly & Meyer, 1992	Undergraduate students	117	-							HR, HRV	Positive
Colzato et al, 2018	Male, Female	90	22.1 (2.5)							BPM, RMSSD, HF, LF	Positive
Garfinkel et al, 2013.	Male, Female	17	26.7 (8.6)							ECG	Positive
Rosen & Reiner, (2017)	Male, Female, Right handed	12	25.2 (6.14)							EEG	Positive
Sheth, Sandkühler, & Bhattacharya, (2009)	Male, Female, No neurological or sleep disorder	18	21.2							EEG	No significant relation
Jensen, & Tesche, (2002)	Male, Female	10	23-37							MEG, EEG	Positive

Guderian, Schott,	Male, female, right-handed	24	18-32							MEG	Positive
Carroll, Turner & Hellawell, 1986.	Young males	24	18-22							HR	No reliable period effect on HR
Alessandrini et al, 1997.	Male, female	8	71-90							-	Positive
Hjortskov et al, 2004	Female, non-experienced	12	23.7 (4.8)							HRV (ECG)	Negative
Jung-Beeman et al, (2004).	Male, female	18 (fMRI) 19 (EEG)	18-29							EEG, FMRI	Positive
Guderian, Schott, Richardson-Klavehn & Düzel, (2009).	Male, Female	24	18-32							MEG, EEG	Positive
Cranford et al, (2014)	Chemistry students, Faculty members	12 7	-							-	Positive
Aritzeta et al., 2022	Male, Female	-	7.58 (0.38)							HRV	Positive

B. Appendix (Chapter 3)

B.1 Self-Evaluation Questions

After each demonstration participants were asked self-evaluation questions on four statements using a button on a 4-button response box. The statements were the following:

a) I had enough time to follow the math demonstration.
b) I was familiar with the math demonstration.
c) I understood the math demonstration.
d) I found this math demonstration engaging.

The answer format followed a 4-point Likert scale with the options: 1 = Completely disagree, 2 = Somewhat disagree, 3 = Somewhat agree, and 4 = Completely agree.
All supplementary materials are attached.

C. Appendix (Chapter 4)

C.1 Self-Evaluation Questions

After each demonstration participants were asked self-evaluation questions on four statements using a button on a 4-button response box. The statements were the following:

a) I had enough time to follow the math demonstration.
b) I was familiar with the math demonstration.
c) I understood the math demonstration.
d) I found this math demonstration engaging.

The answer format followed a 4-point Likert scale with the options: 1 = Completely disagree, 2 = Somewhat disagree, 3 = Somewhat agree, and 4 = Completely agree.
All supplementary materials are attached.

D. Appendix (Chapter 5)

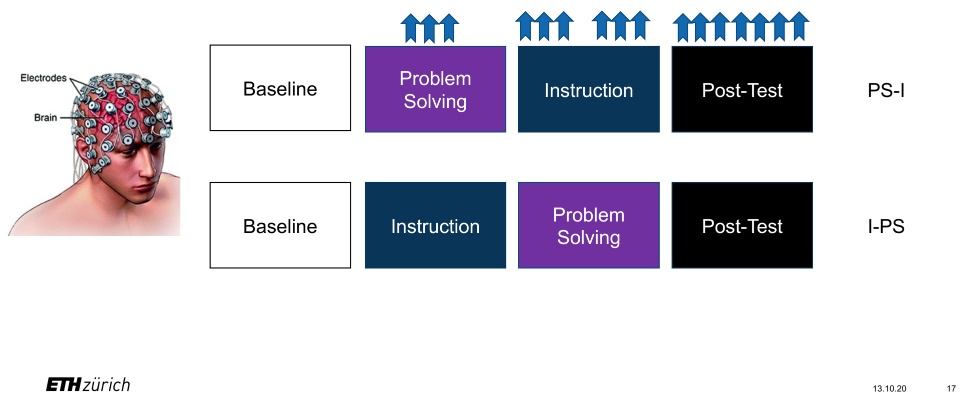


Figure 5.1.1: *Set up of the study (PS-I condition versus I-PS condition).*

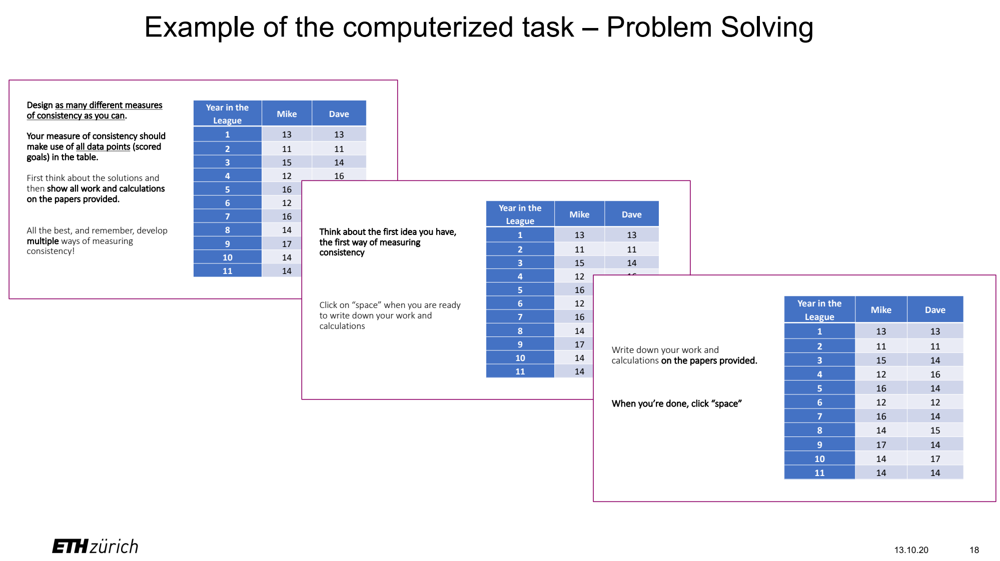


Figure 5.1.2: *Example of the computerized task – Problem Solving.*

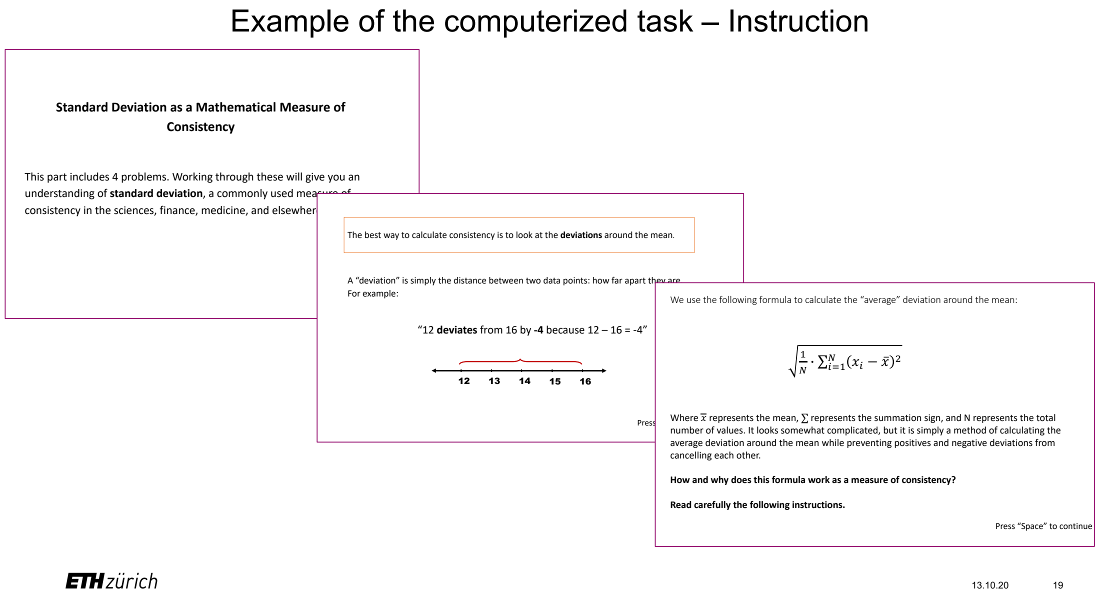


Figure 5.1.3: *Example of the computerized task - Instruction.*

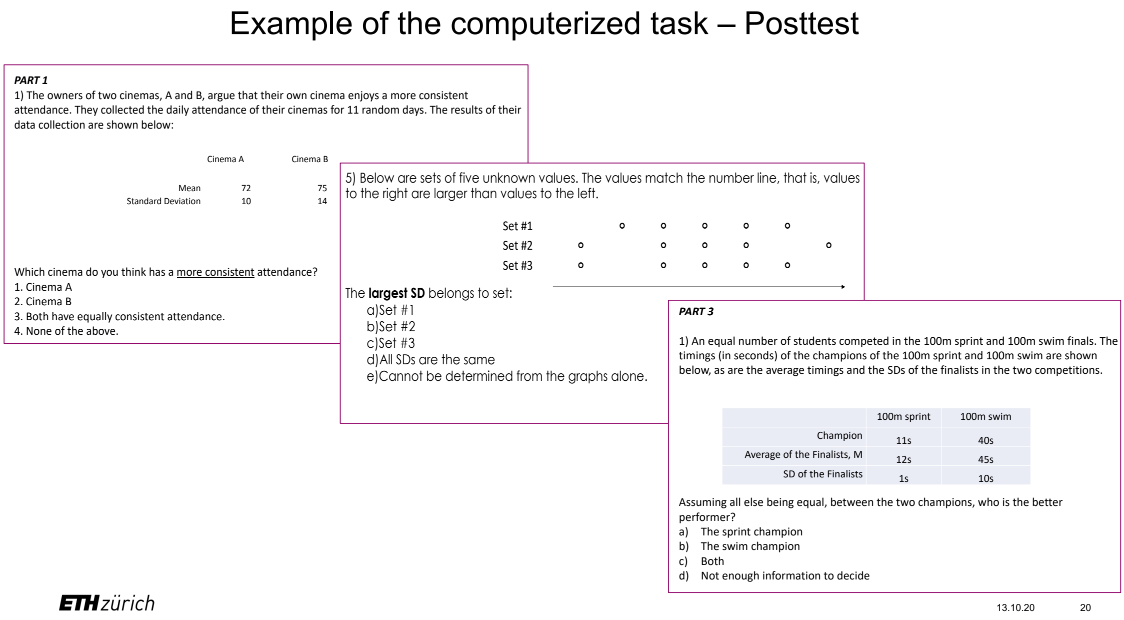


Figure 5.1.4: *Example of the computerized task - Posttest.*

D.1 Additional Forms

D.1.1 Consent form

⇒ Please read this form carefully.

⇒ Please ask the investigator or the contact person if you have any questions.

Study title: The Physiological and Neural Basis of Learning through Productive Failure
Study location: ETH Zurich Decision Science Laboratory, Haldeneggsteig 4, 8092 Zürich

Principal Investigator's Name and First Name: Formaz, Cléa
Participant's Name and First Name:

- I participate in this study on a voluntary basis and can withdraw from the study at any time without giving reasons and without any negative consequences.
- I have been informed orally and in writing about the aims and the procedures of the study, the advantages and disadvantages, as well as potential risks. My questions have been answered completely and to my satisfaction.
- I have read the written information for the volunteers. My questions related to participating in the study have been answered satisfactorily. I have been given a copy of the information for volunteers and the consent form.
- I am aware that part of the study will be video and/or voice recorded. I understand that only the researchers of the study have access to the recordings and that they will not be shared in public.
- I am aware that the EEG data will be made public in an anonymous form and that my identity cannot be recognized nor traced from these data.
- I was given sufficient time to make a decision about participating in the study.
- With my signature, I certify that I fulfill the requirements for participating in the study stated in the information for the volunteers.
- I have been informed that any possible damage to my health, which are directly related to the study and are demonstrably the fault of ETH Zurich, are covered by the general liability insurance of ETH Zurich (insurance policy no. 30/4.078.362 of the Basler Versicherung AG). However, beyond the before mentioned, my health and/or accident

insurance (e.g. for the commute to or from the study location) will be applicable.

- I agree that the responsible investigators and/or the members of the Ethics Commission have access to the original data under strictly observed rules of confidentiality.
- I am aware that during the study I have to comply with the requirements and limitations described in the information for volunteers. In the interest of my health, the investigators can, without mutual consent, exclude me from the study. I am aware of the requirements and restrictions to be observed during the study.
- I will inform the investigators regarding any medical treatment and medication (prescribed by medical doctors or self-purchased).
- I participate in this study voluntarily and consent that my personal data be used as described above. I have had enough time to decide about my participation.
- If in the course of the study, a clinical finding should occur which could lead to a diagnosis, treatment or prevention of an existing or future illness

  ☐ I want to be informed.

  ☐ I do not want to be informed.
- I would like to be informed about the results of this study

  ☐ yes ☐ no

Location, date . . . . Signature volunteer . . . .

Location, date . . . . Signature investigator . . . .

D.1.2 Information sheet

(Take a few minutes to read through and sign the consent form)

Study title: Neural Basis of Productive Failure

Principal Investigator's Name and First Name: Formaz, Cléa

Goals of the study

This study investigates the brain processes involved in productive failure in solving problems of standard deviation. By understanding the brain processes, we will have a more thorough understanding on why the productive failure design works better than the usual direct instruction in learning. This study can bring valuable insights from the neuroscientific point of view in an effective learning situation.

Research procedure

You will be sitting on a relaxed chair and an EEG/fNIRS record will be done during 2 minutes. After this baseline recording, you will be writing something with your dominant hand. You will then be asked to either directly solve the problem of standard deviation or are given instructions first. A laptop will be placed in front of you, you will see the instructions followed by the problem-solving task, after that you will receive direct instructions (or vice versa), and then complete a posttest. You will be asked to press a button after solving the task. Before the posttest starts, you will receive short questionnaires about math anxiety and general questions about feeling in learning situations, cognitive load and metacognition. Again, after solving the posttest you should press a button and the recording will stop here.

Right of withdrawal

You may withdraw your consent to use your data at any time and without any liability to you.

Data protection

The obtained data will be stored safely and reported in an anonymous form. This information will be stored electronically and anonymously, on a password-protected computer located at ETH. The data will be completely deleted 10 years after the date of the experiment. Physical copies will be securely stored.

Only the responsible investigators and/or the members of the ETH Ethics Commission will have access to the original data under strictly observed rules of confidentiality.

Contact person
Cléa Formaz
clea.formaz@gess.ethz.ch

D.1.3 Declaration of Informed Consent

During this study, you will work individually while making decisions on given situations.
There is no risk involved in participating in this study.
Your participation in the research component of this study is purely voluntary, and you may withdraw your participation or your data at any time.
Your data will be kept completely confidential.
If you have any question regarding this study, please use the contact details below.

I hereby acknowledge that I have read and understood the above information and agree to participate.

Last Name: ____________________________________

First Name: ____________________________________

Date: ____________________________________

Signature: ____________________________________

Contact person
Cléa Formaz
clea.formaz@gess.ethz.ch

D.1.4 Proof of compensation for a participant

I, __________________________ have received CHF 40 after participating the study on the physiological and neural basis of learning through Productive Failure at the Decision Sciences Lab, ETH Zürich.

Place and date: ___________________________

Signature of the participant: ___________________________

Signature of the researcher: ___________________________

D.2 Additional Questionnaires

D.2.1 Abbreviated Math Anxiety Test

Appendix Q2: Abbreviated Math Anxiety Test

Please rate each item below in terms of how anxious you would feel during the event specified.

1 Low Anxiety 2 Some Anxiety 3 Moderate Anxiety 4 Quite a bit of Anxiety 5 High Anxiety

Having to use the tables in the back of a mathematics book. ___________

Thinking about an upcoming mathematics test one day before. ___________

Watching a teacher work an algebraic equation on the blackboard. ___________

Taking an examination in a mathematics course. ___________

Being given a homework assignment of many difficult problems which is due the next class meeting. ___________

Listening to a lecture in mathematics class. ___________

Listening to another student explain a mathematics formula. ___________

Being given a "pop" quiz in a mathematics class. ___________

Starting a new chapter in a mathematics book. ___________

D.2.2. Questionnaires to assess students' incoming cognitive and motivational characteristics

Learning Goal Orientation (5-point Likert scale from "strongly disagree" to "strongly agree")

1. The opportunity to do challenging work is important to me.
2. When I fail to complete a difficult task, I plan to try harder the next time I work on it.
3. I prefer to work on tasks that force me to learn new things.
4. The opportunity to learn new things is important to me.
5. I do my best when I'm working on a fairly difficult task.
6. I try hard to improve on my past performance.
7. The opportunity to extend the range of my abilities is important to me.
8. When I have difficulty solving a problem, I enjoy trying different approaches to see which one will work.

Attitude towards Mistakes (5-point Likert scale from "strongly disagree" to "strongly agree")

1. When I make mistakes, I am afraid that others look down upon me. [AFFECT subscale]
2. If I make mistakes, I don't want others to notice them. [AFFECT subscale]
3. When I make mistakes answering classroom questions, I am overwhelmed with embarrassment. [AFFECT subscale]
4. I seldom feel bothered by the mistakes I make. [AFFECT subscale, *reversed*]
5. I believe successful students make fewer mistakes during learning than others. [COGNITION subscale]
6. I believe it is smart to avoid making mistakes during learning. [COGNITION subscale]
7. I believe making mistakes is not an efficient way to learn academic materials. [COGNITION subscale]
8. I believe making mistakes may reduce my interest in learning. [COGNITION subscale]

D.2.3. Questionnaires to assess students' task experiences

Knowledge Gap Awareness (5-point Likert scale from "strongly disagree" to "strongly agree")

1. My knowledge was insufficient to carry out these tasks.
2. These tasks made clear to me that I lack some knowledge.
3. I sometimes got stuck when trying to execute these tasks.
4. These tasks were too difficult to finish.
5. I felt that I did not manage to complete these tasks.

State Curiosity (5-point Likert scale from "strongly disagree" to "strongly agree")

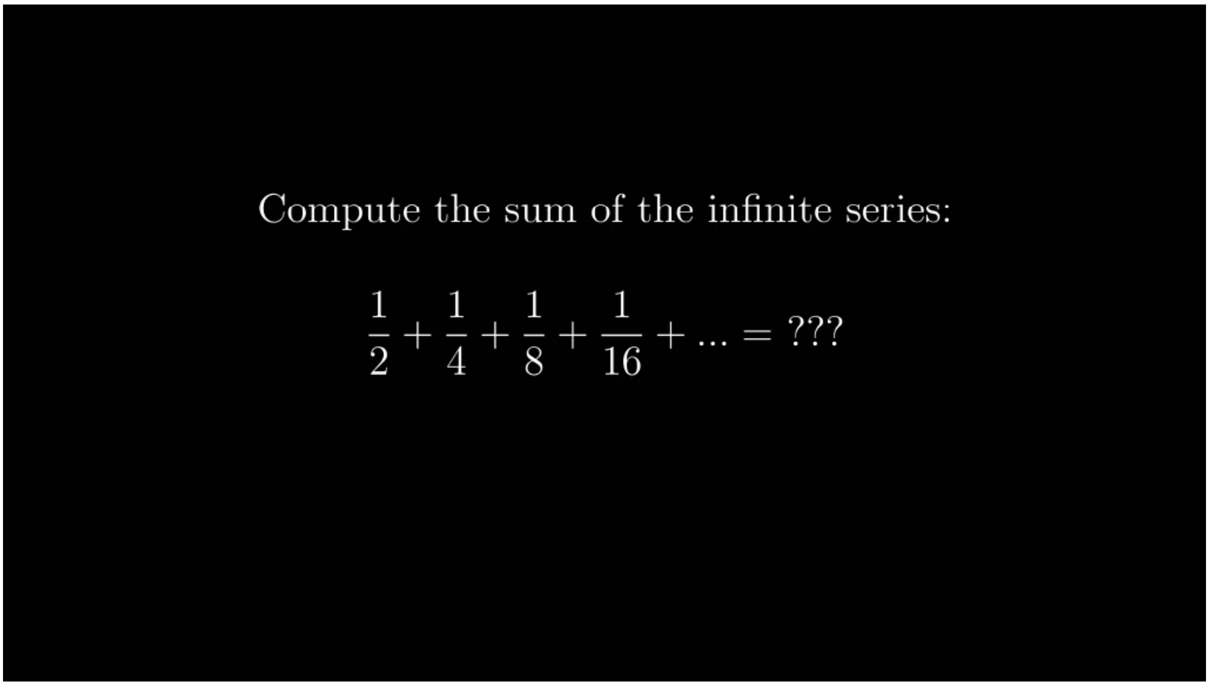
1. I want to know more.
2. I am feeling puzzled.
3. I want things to make sense.
4. I am intrigued by what is happening.
5. I feel like asking questions about what is happening.
6. Things feel incomplete.
7. I feel like searching for answers.
8. I want to explore possibilities.
9. My interest has been captured.

Germane Cognitive Load (5-point Likert scale from "strongly disagree" to "strongly agree")

1. This activity improved my understanding of the content that was covered.
2. This activity improved my understanding of the problem that was/were covered.
3. This activity improved my knowledge of the terms that were mentioned.
4. This activity improved my knowledge of how to deal with the problem/s covered.
5. This activity improved my understanding of how to deal with the problem/s covered.
6. I invested high mental effort during this activity.

All Slides from Study 1: Brain, Mathematics, and Expertise

(the same slides were used for the pilot study on the heartbeat differences between math experts and novices).



$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n}$$

$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n}$$

$$2 \cdot S_n = \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + ... + \frac{2}{2^{n-1}} + \frac{2}{2^n}$$

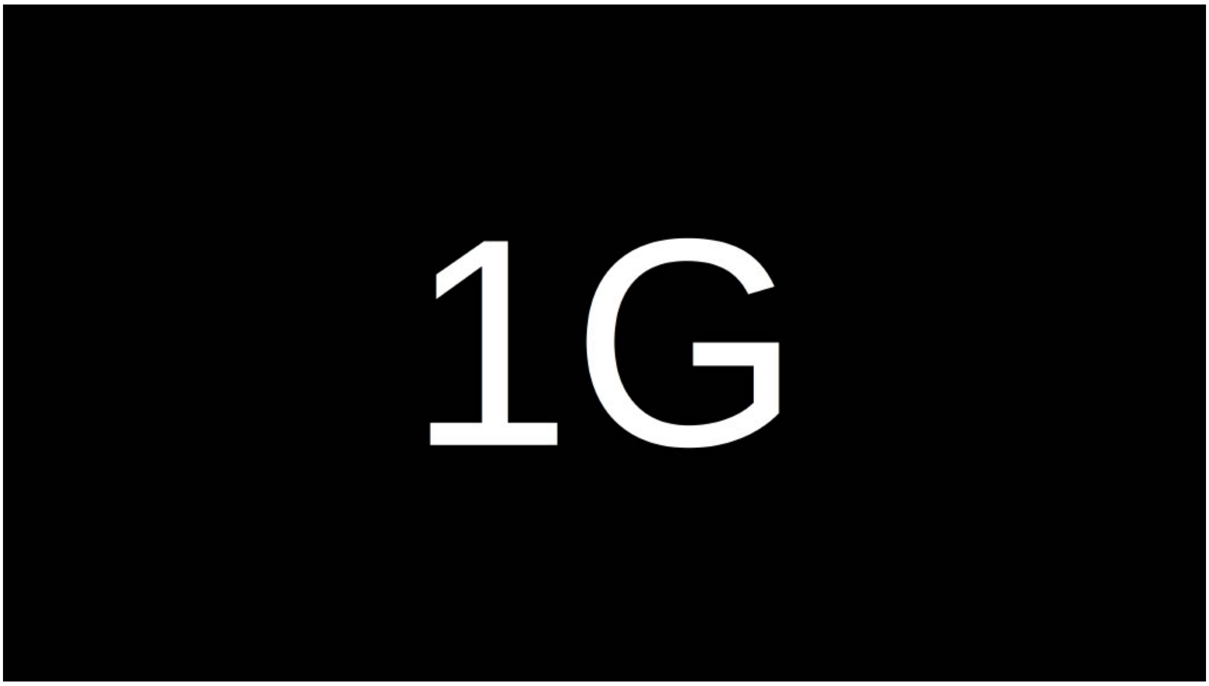
$$
\begin{aligned}
S_n &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n} \\
2 \cdot S_n &= \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + ... + \frac{2}{2^{n-1}} + \frac{2}{2^n} \\
2 \cdot S_n &= 1 + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}}\right]
\end{aligned}
$$

$$
\begin{aligned}
S_n &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n} \\
2 \cdot S_n &= \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + ... + \frac{2}{2^{n-1}} + \frac{2}{2^n} \\
2 \cdot S_n &= 1 + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}}\right] \\
&= 1 + \left[S_n - \frac{1}{2^n}\right]
\end{aligned}
$$

$$
\begin{aligned}
S_n &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n} \\
2 \cdot S_n &= \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + ... + \frac{2}{2^{n-1}} + \frac{2}{2^n} \\
2 \cdot S_n &= 1 + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}}\right] \\
&= 1 + \left[S_n - \frac{1}{2^n}\right] \\
S_n &= 1 - \frac{1}{2^n}
\end{aligned}
$$

$$
\begin{aligned}
S_n &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n} \\
2 \cdot S_n &= \frac{2}{2} + \frac{2}{4} + \frac{2}{8} + \frac{2}{16} + ... + \frac{2}{2^{n-1}} + \frac{2}{2^n} \\
2 \cdot S_n &= 1 + \left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... + \frac{1}{2^{n-1}}\right] \\
&= 1 + \left[S_n - \frac{1}{2^n}\right] \\
S_n &= 1 - \frac{1}{2^n}
\end{aligned}
$$

As $n \to \infty,\ S_n = 1 - 0 = 1$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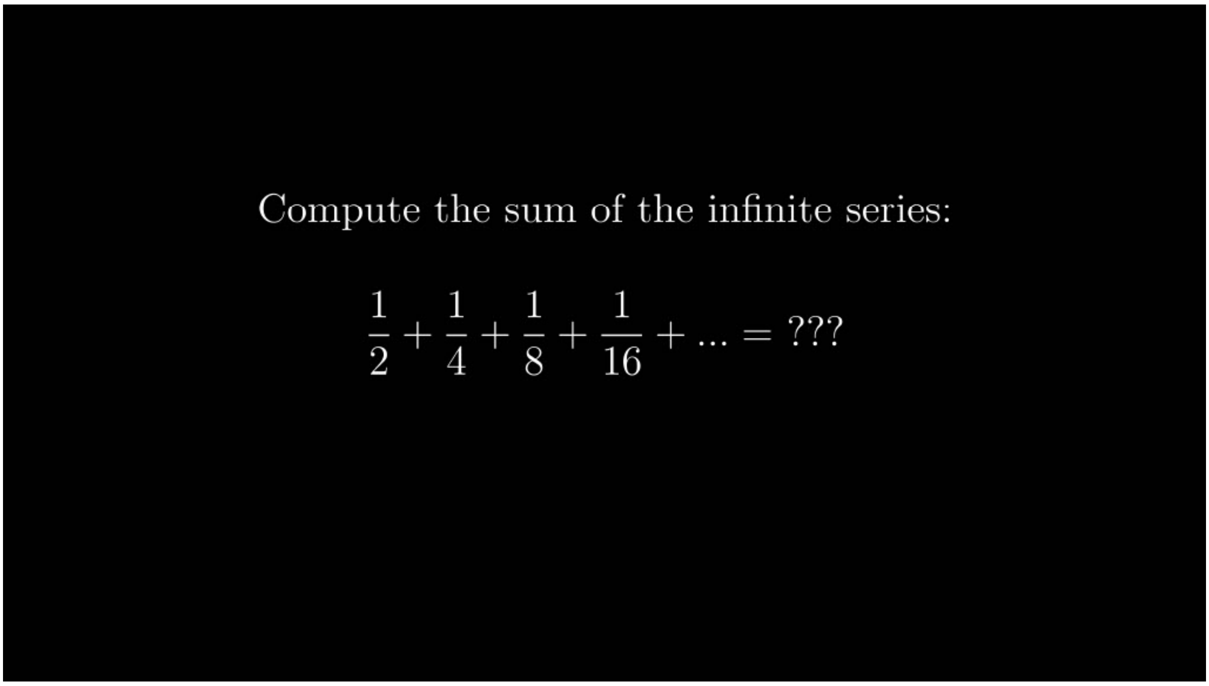
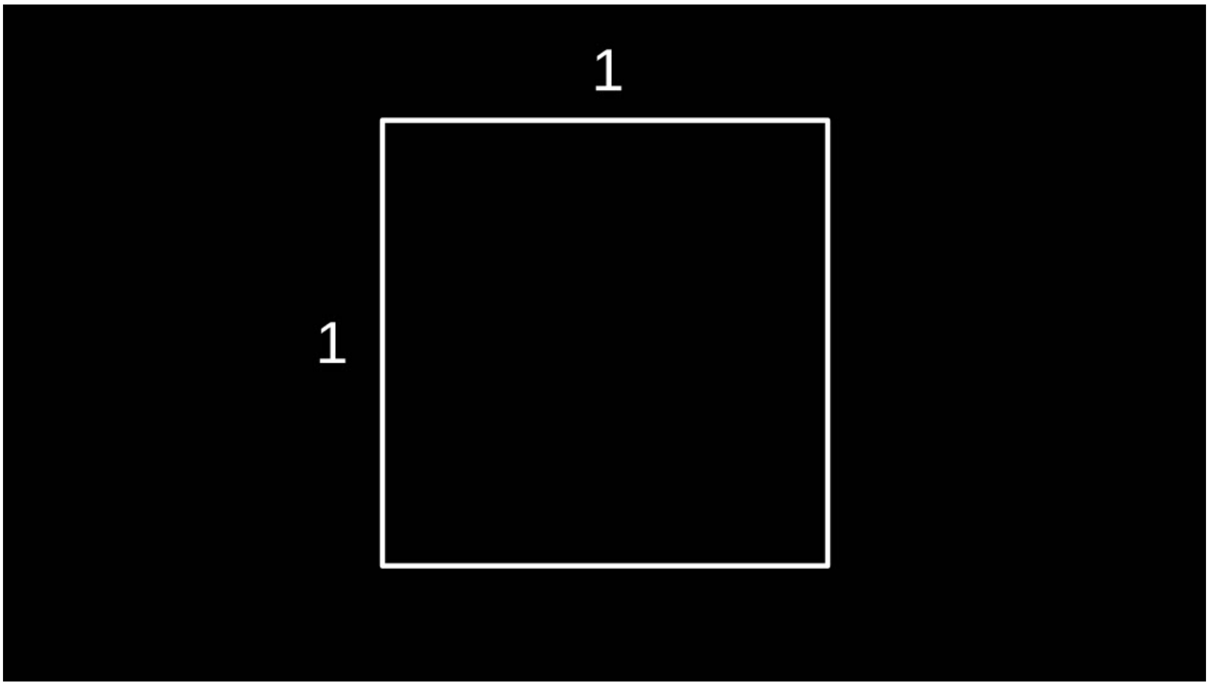

Compute the sum of the infinite series:
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ... = ???$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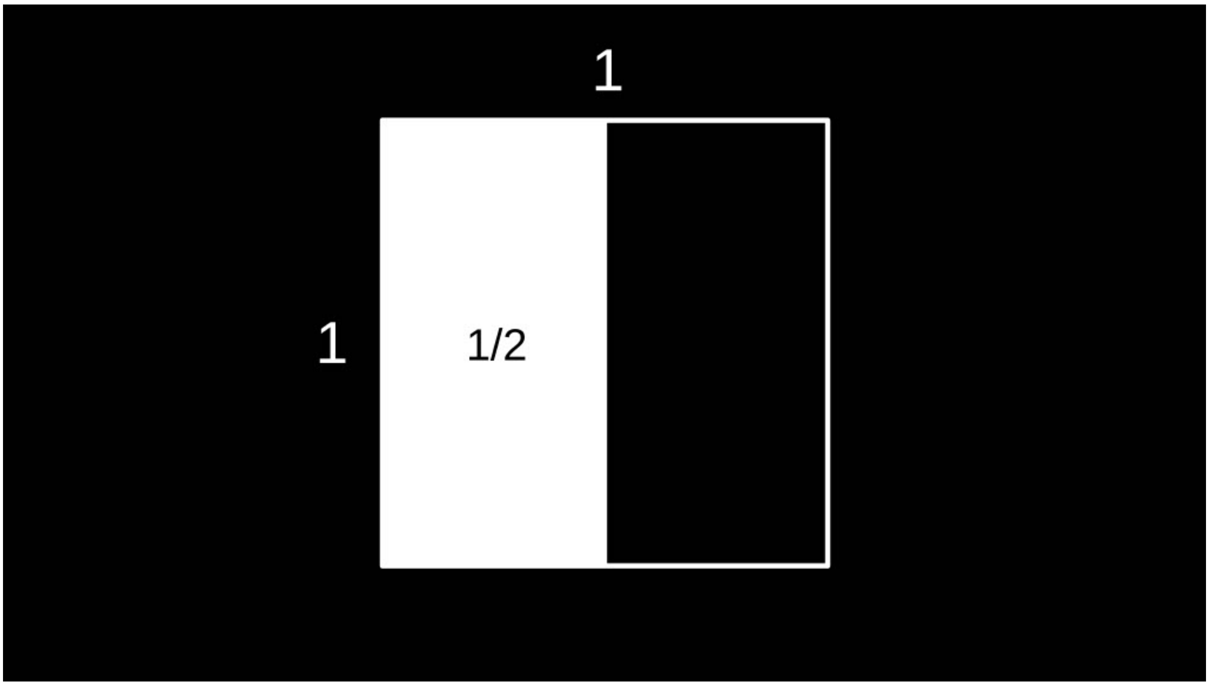
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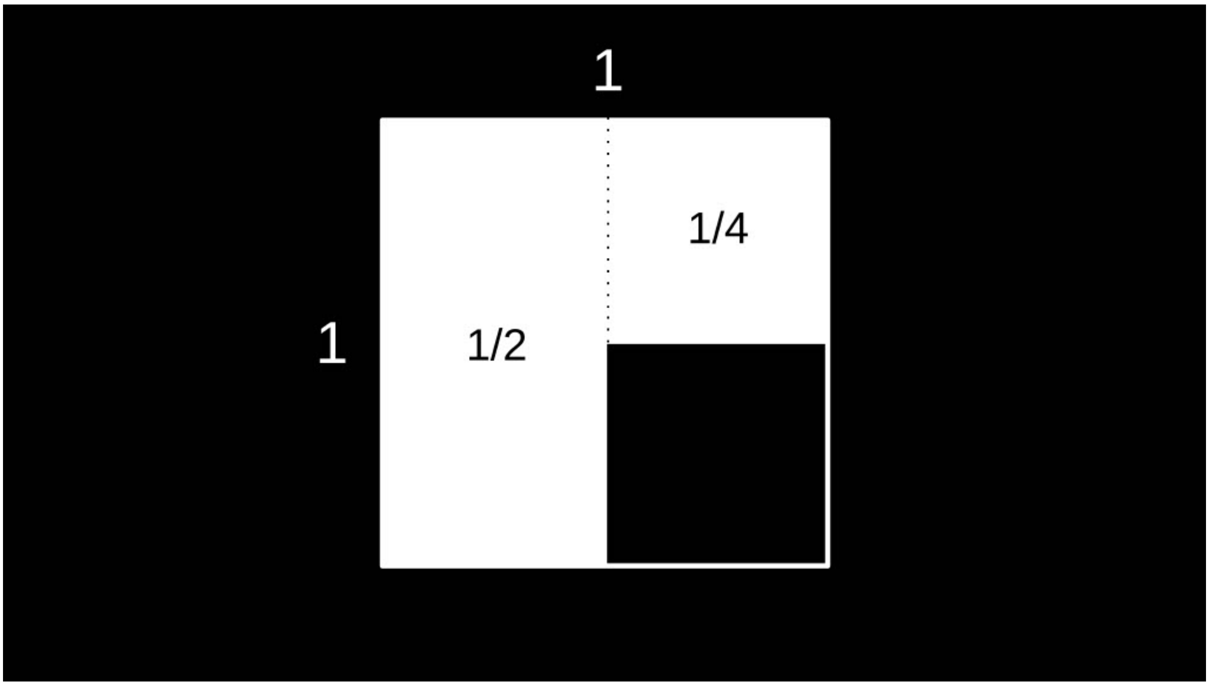
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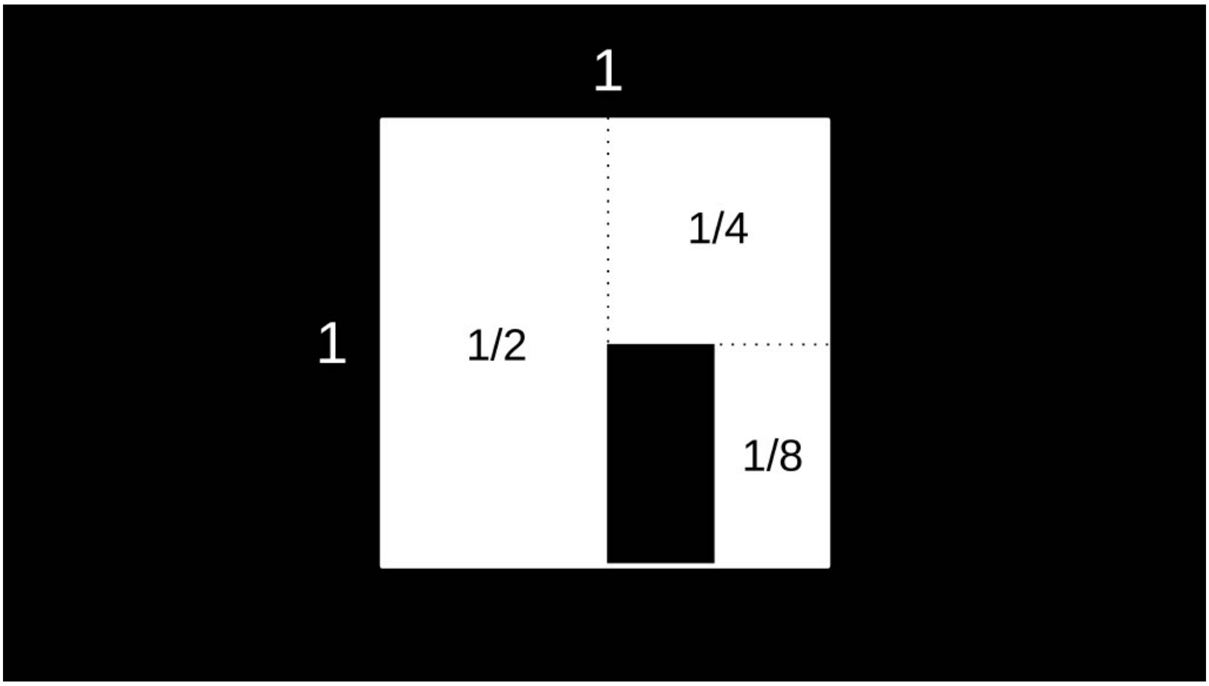
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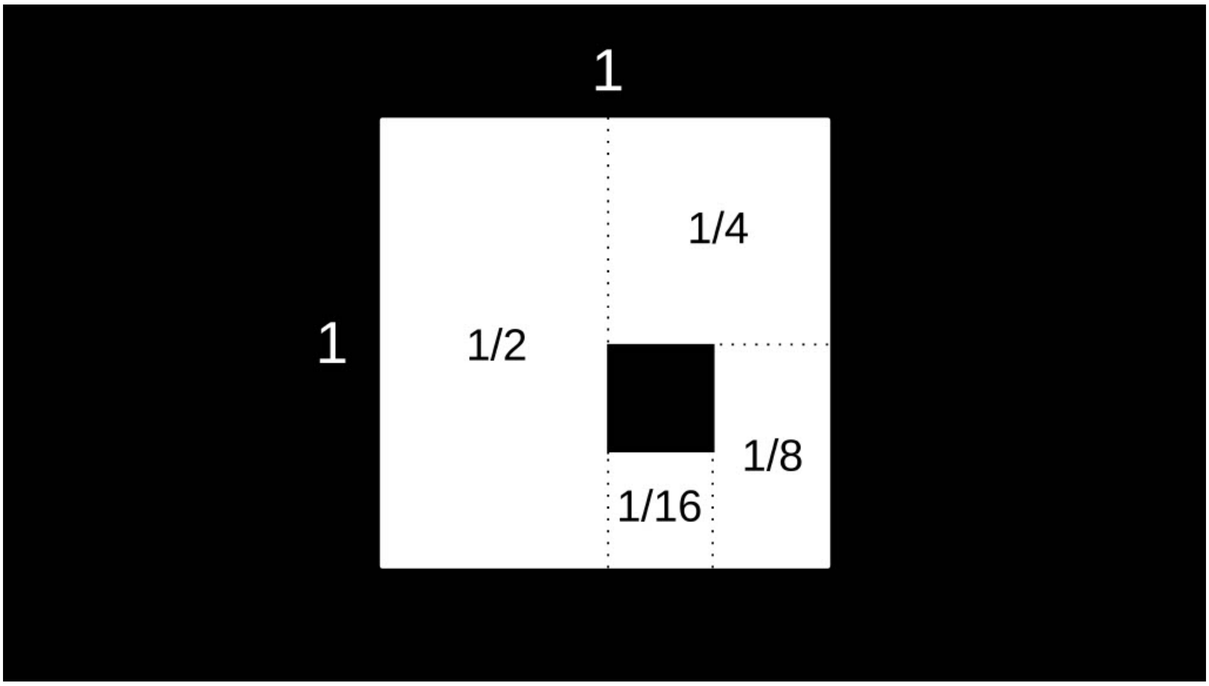
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1/2
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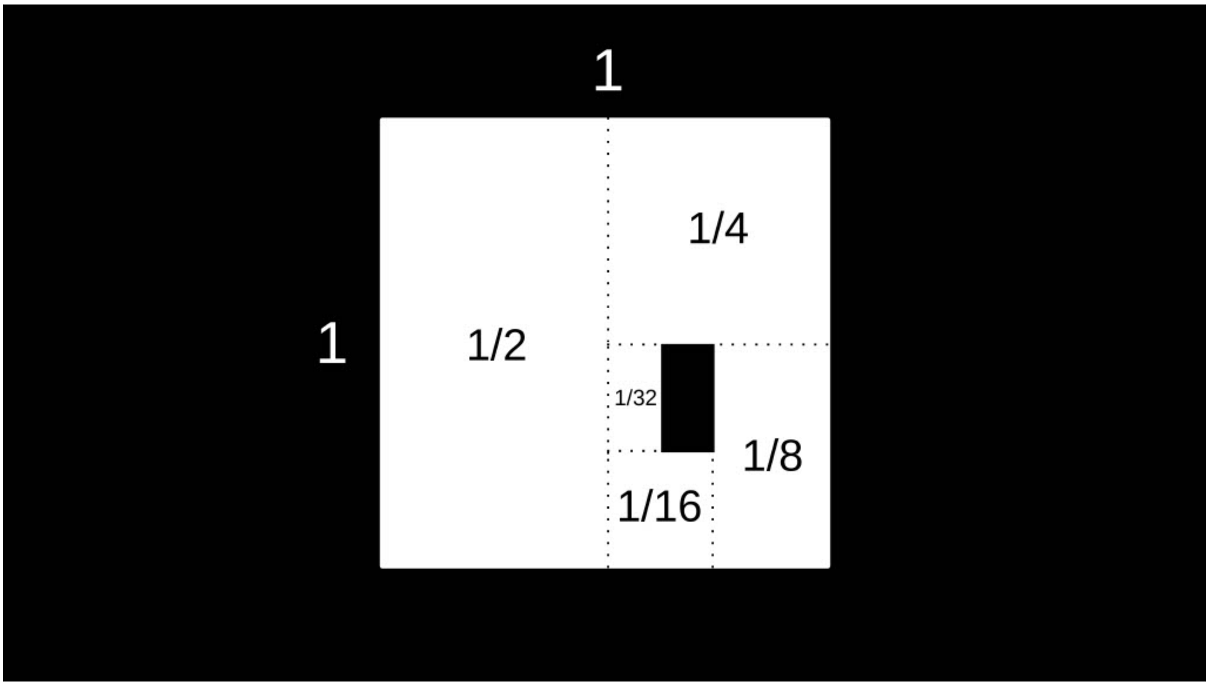
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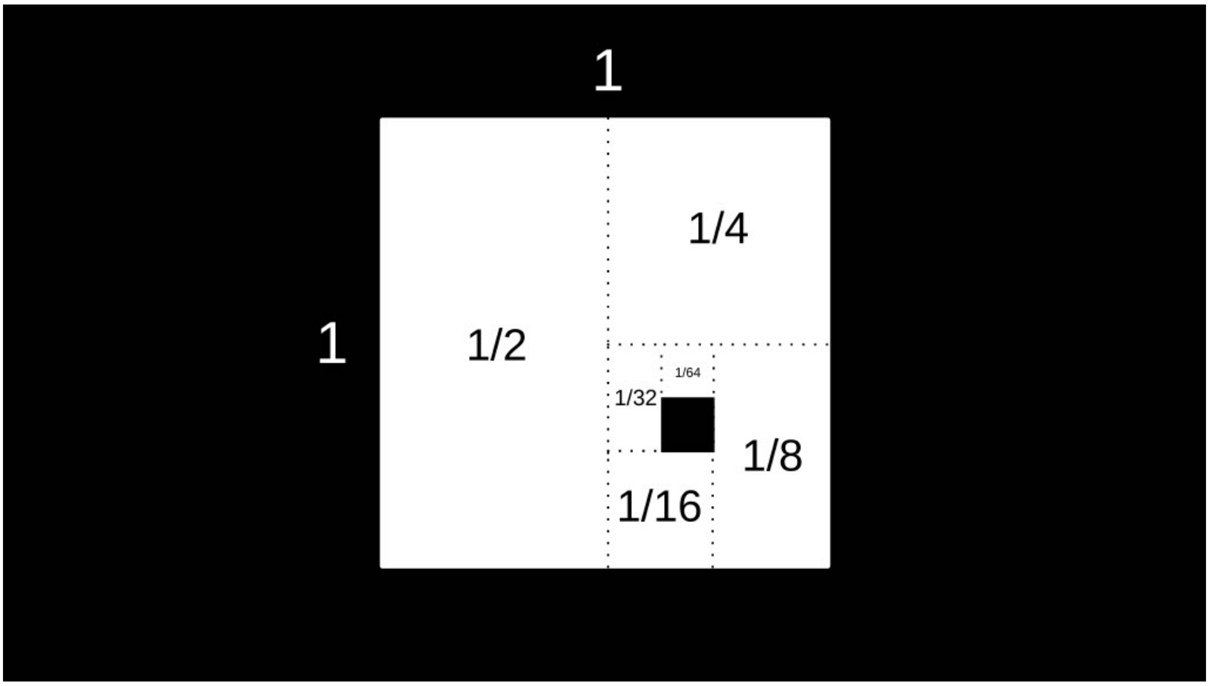
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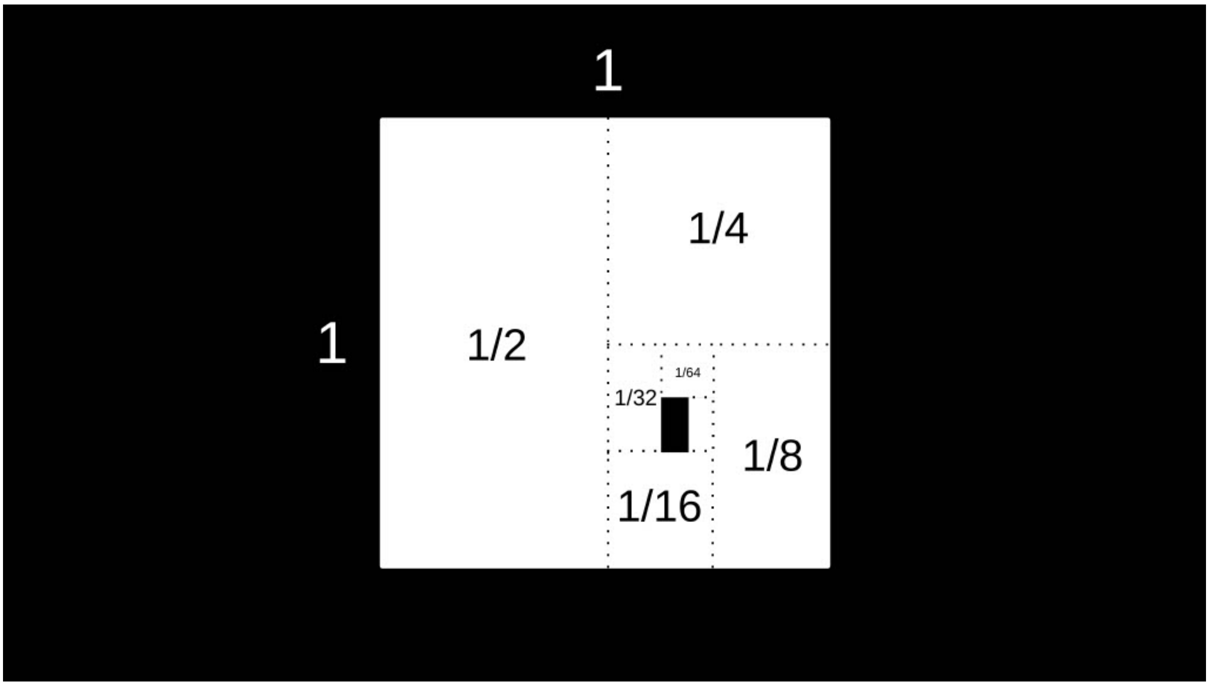
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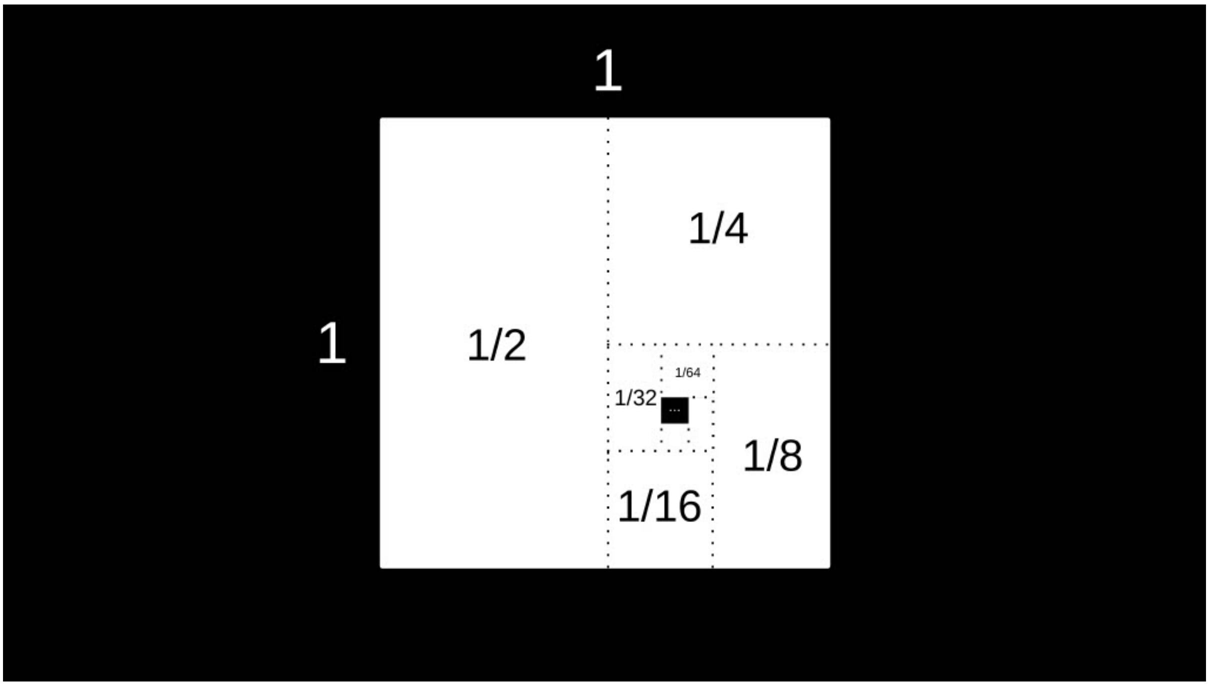
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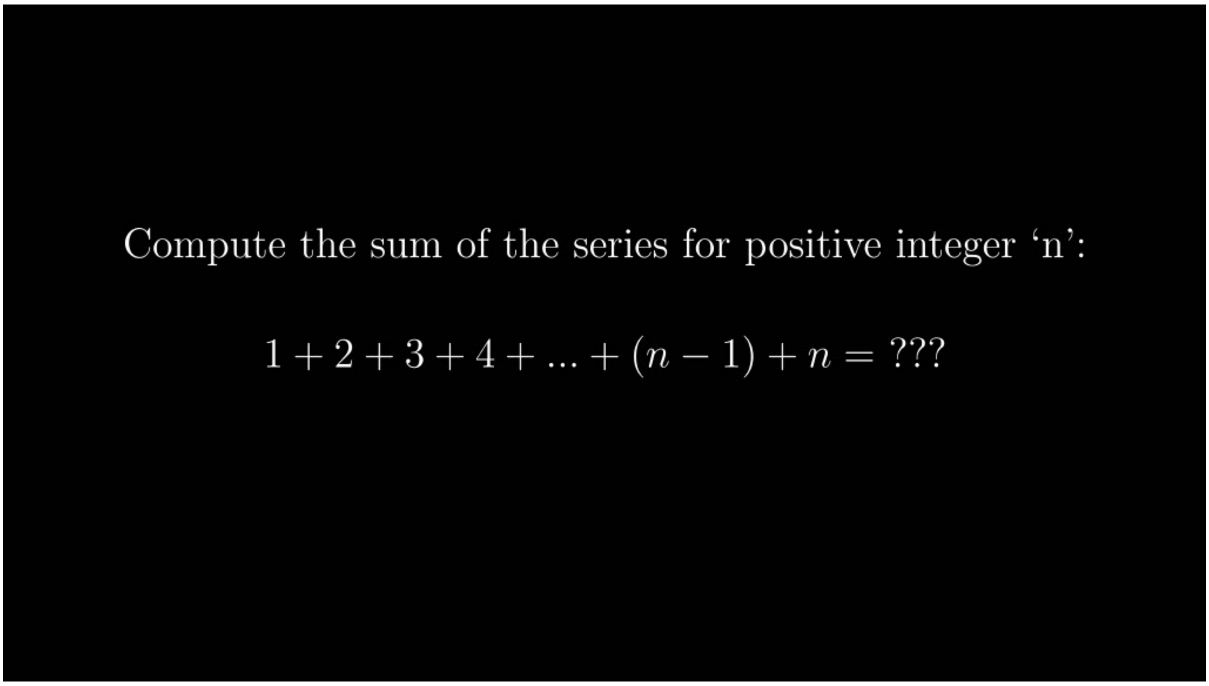
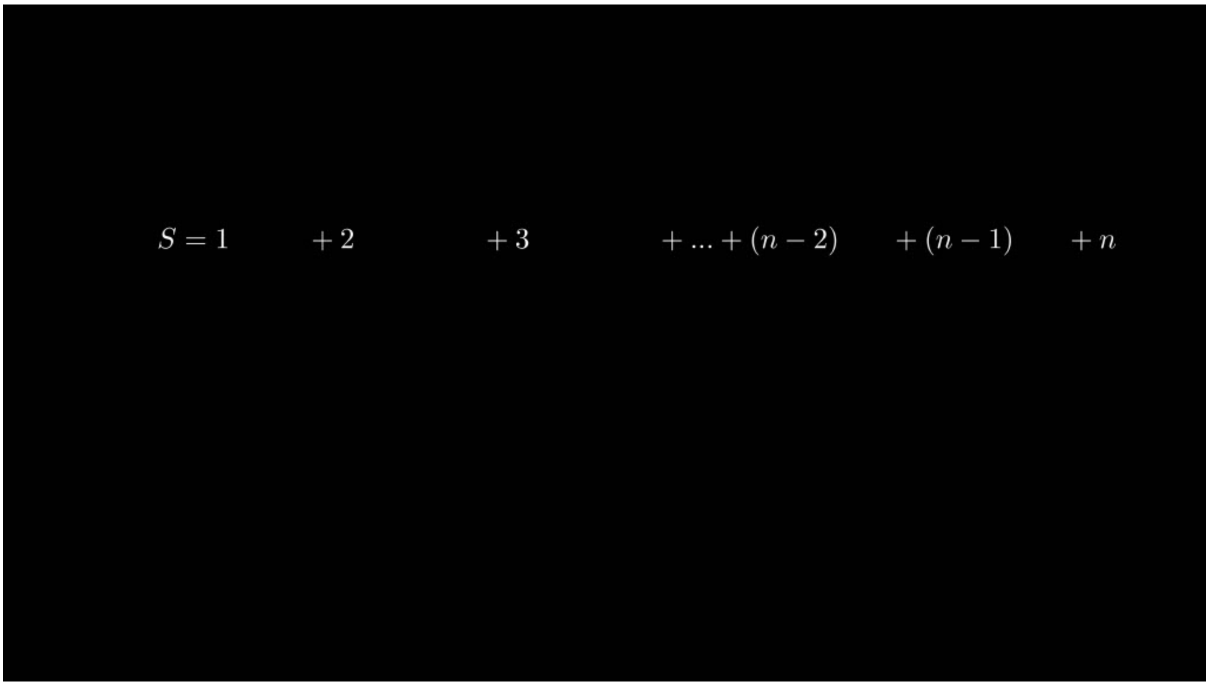
...


2A


Compute the sum of the series for positive integer ‘n’:
$1 + 2 + 3 + 4 + ... + (n-1) + n = ???$


$S = 1 \quad + 2 \quad + 3 \quad + ... + (n-2) \quad + (n-1) \quad + n$

$$
\begin{aligned}
S &= 1 &&+ 2 &&+ 3 &&+ ... + (n-2) &&+ (n-1) &&+ n \\
S &= n &&+ (n-1) &&+ (n-2) &&+ ... + 3 &&+ 2 &&+ 1
\end{aligned}
$$

$$
\begin{aligned}
S &= 1 &&+ 2 &&+ 3 &&+ ... + (n-2) &&+ (n-1) &&+ n \\
S &= n &&+ (n-1) &&+ (n-2) &&+ ... + 3 &&+ 2 &&+ 1 \\
S + S &= (n+1) &&+ (n-1+2) &&+ (n-2+3) &&+ ... + (3+n-2) &&+ (2+n-1) &&+ (1+n)
\end{aligned}
$$

$$
\begin{array}{rlllllll}
S = & 1 & +\,2 & +\,3 & +\,...+(n-2) & +\,(n-1) & +\,n \\
S = & n & +\,(n-1) & +\,(n-2) & +\,...+3 & +\,2 & +\,1 \\
S+S = & (n+1) & +\,(n-1+2) & +\,(n-2+3) & +\,...+(3+n-2) & +\,(2+n-1) & +\,(1+n) \\
2\cdot S = & (n+1) & +\,(n+1) & +\,(n+1) & +\,...+(n+1) & +\,(n+1) & +\,(n+1)
\end{array}
$$

$$
\begin{array}{rl}
S = & 1 + 2 + 3 + ... + (n-2) + (n-1) + n \\
S = & n + (n-1) + (n-2) + ... + 3 + 2 + 1 \\
S+S = & (n+1) + (n-1+2) + (n-2+3) + ... + (3+n-2) + (2+n-1) + (1+n) \\
2\cdot S = & \underbrace{(n+1) + (n+1) + (n+1) + ... + (n+1) + (n+1) + (n+1)}_{\text{n times}}
\end{array}
$$

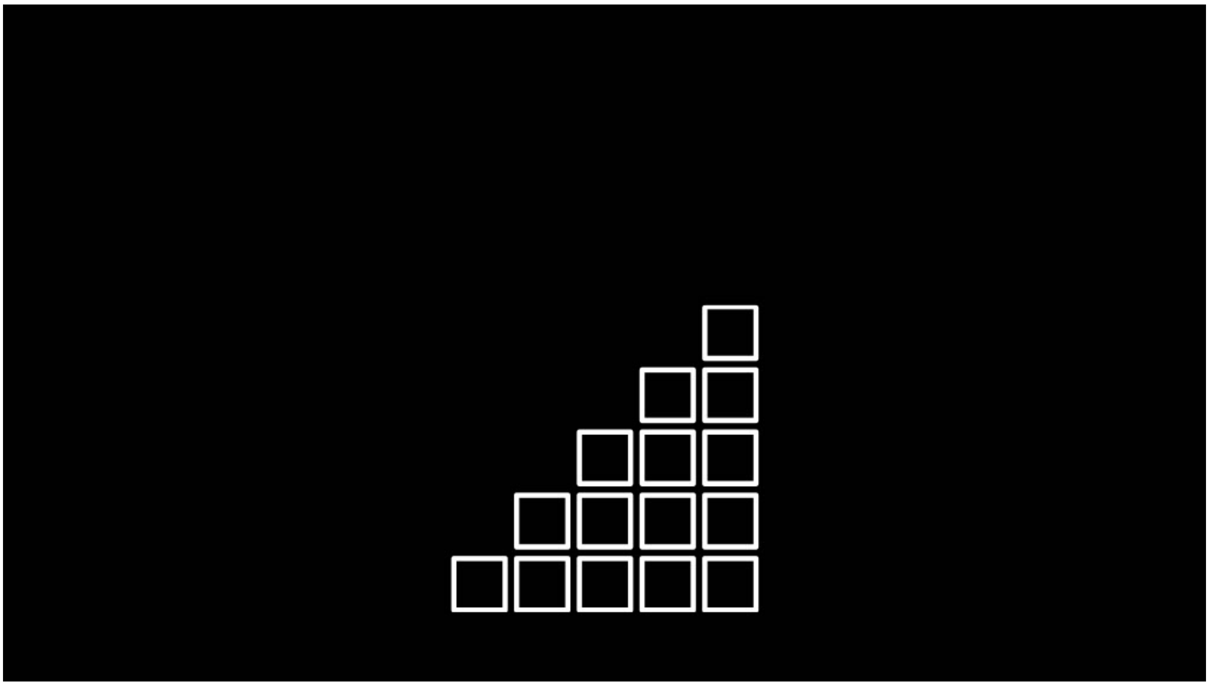
$$
\begin{aligned}
S &= 1 \quad + 2 \quad + 3 \quad + \ldots + (n-2) \quad + (n-1) \quad + n \\
S &= n \quad + (n-1) \quad + (n-2) \quad + \ldots + 3 \quad + 2 \quad + 1 \\
S + S &= (n+1) + (n-1+2) + (n-2+3) + \ldots + (3+n-2) + (2+n-1) + (1+n) \\
2 \cdot S &= \underbrace{(n+1) + (n+1) \quad + (n+1) \quad + \ldots + (n+1) \quad + (n+1) \quad + (n+1)}_{\text{n times}} \\
2 \cdot S &= n \cdot (n+1)
\end{aligned}
$$

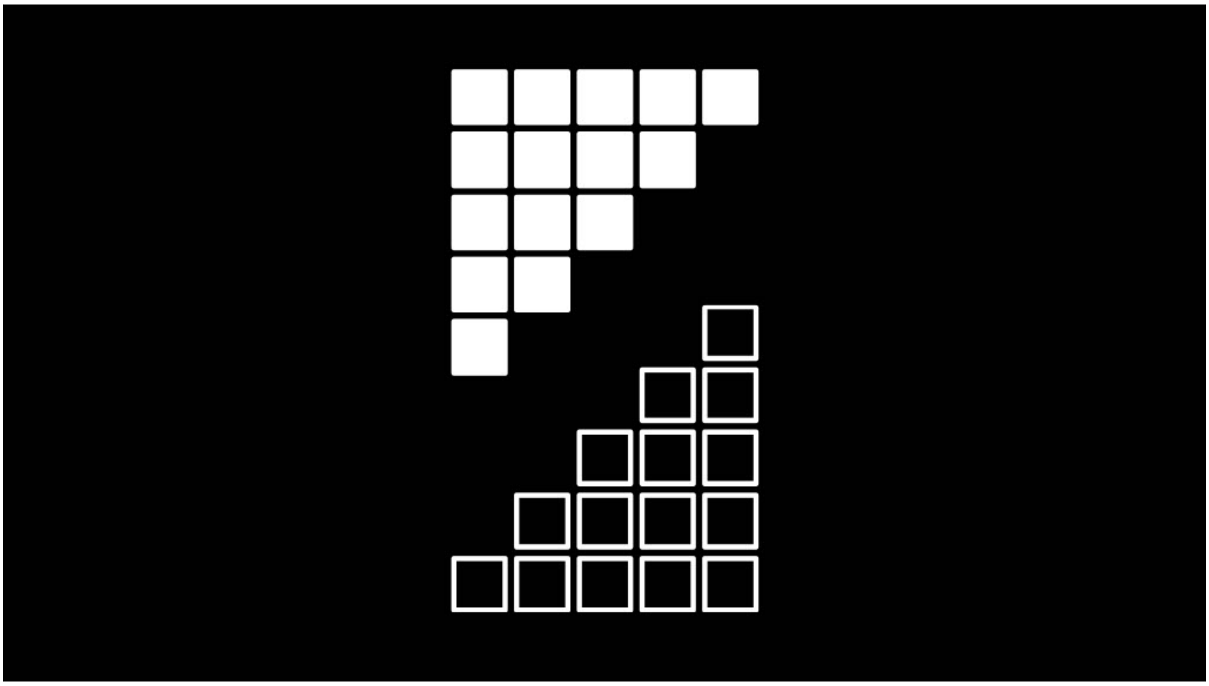
$$
\begin{aligned}
S &= 1 \quad + 2 \quad + 3 \quad + \ldots + (n-2) \quad + (n-1) \quad + n \\
S &= n \quad + (n-1) \quad + (n-2) \quad + \ldots + 3 \quad + 2 \quad + 1 \\
S + S &= (n+1) + (n-1+2) + (n-2+3) + \ldots + (3+n-2) + (2+n-1) + (1+n) \\
2 \cdot S &= \underbrace{(n+1) + (n+1) \quad + (n+1) \quad + \ldots + (n+1) \quad + (n+1) \quad + (n+1)}_{\text{n times}} \\
2 \cdot S &= n \cdot (n+1) \\
S &= \frac{n \cdot (n+1)}{2}
\end{aligned}
$$

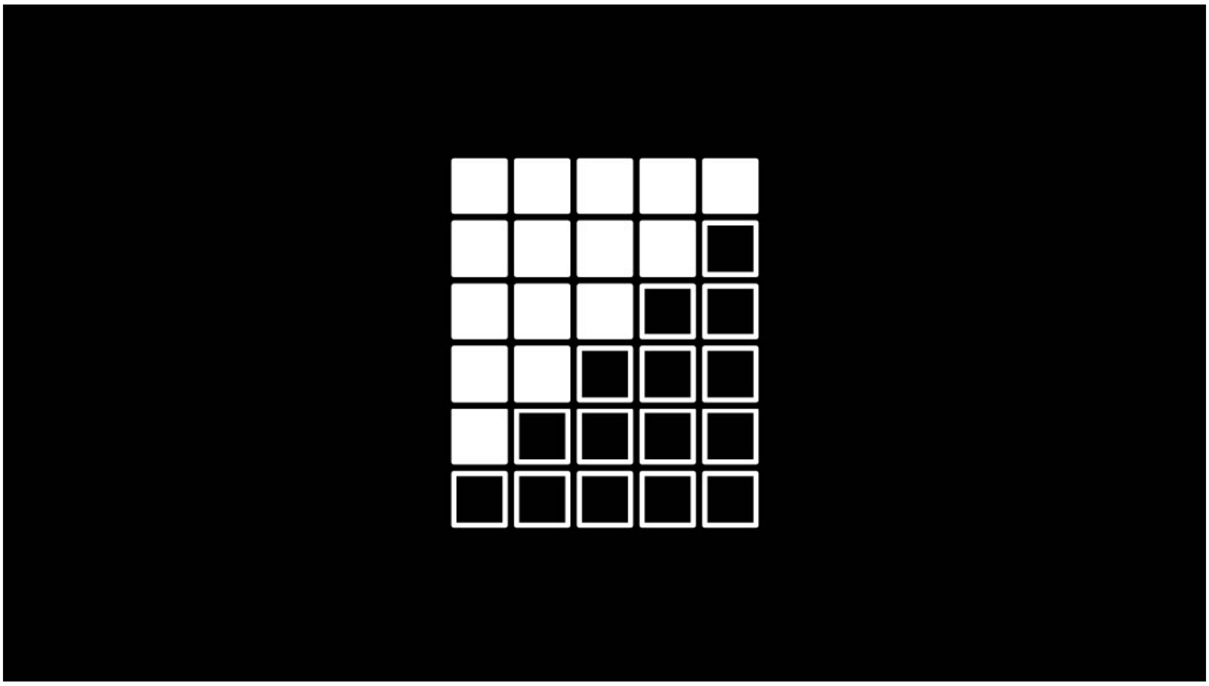
2G

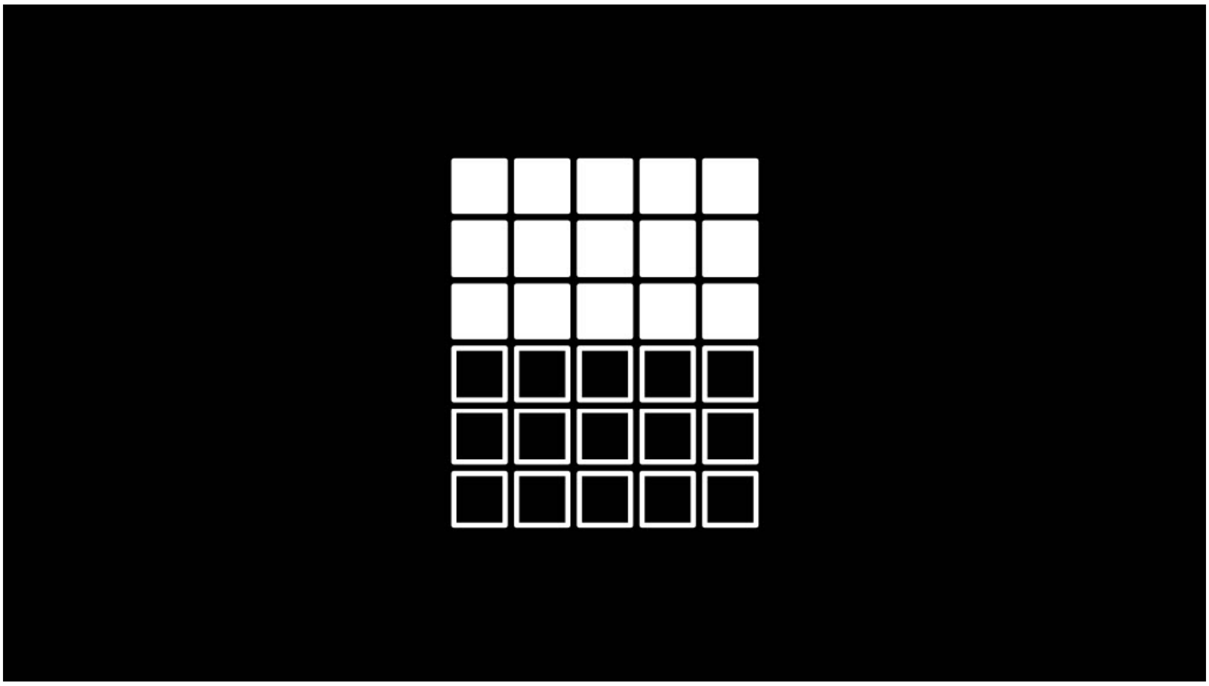
Compute the sum of the series for positive integer ‘n’:

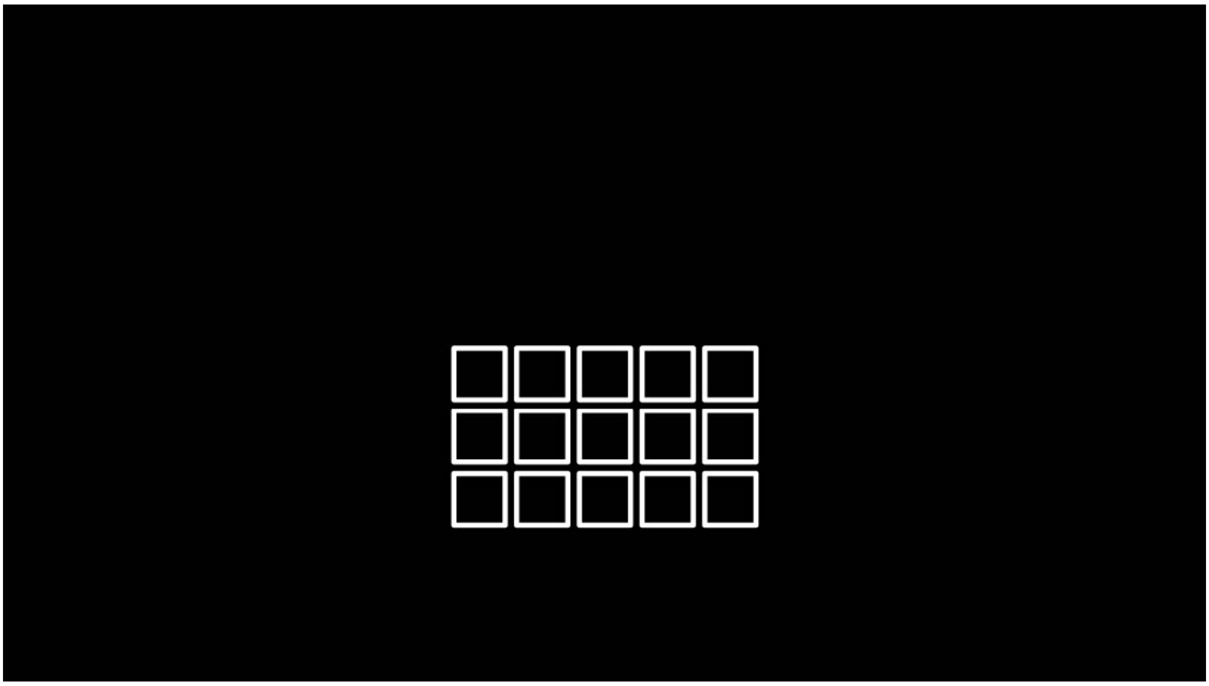
$$1 + 2 + 3 + 4 + ... + (n - 1) + n = ???$$

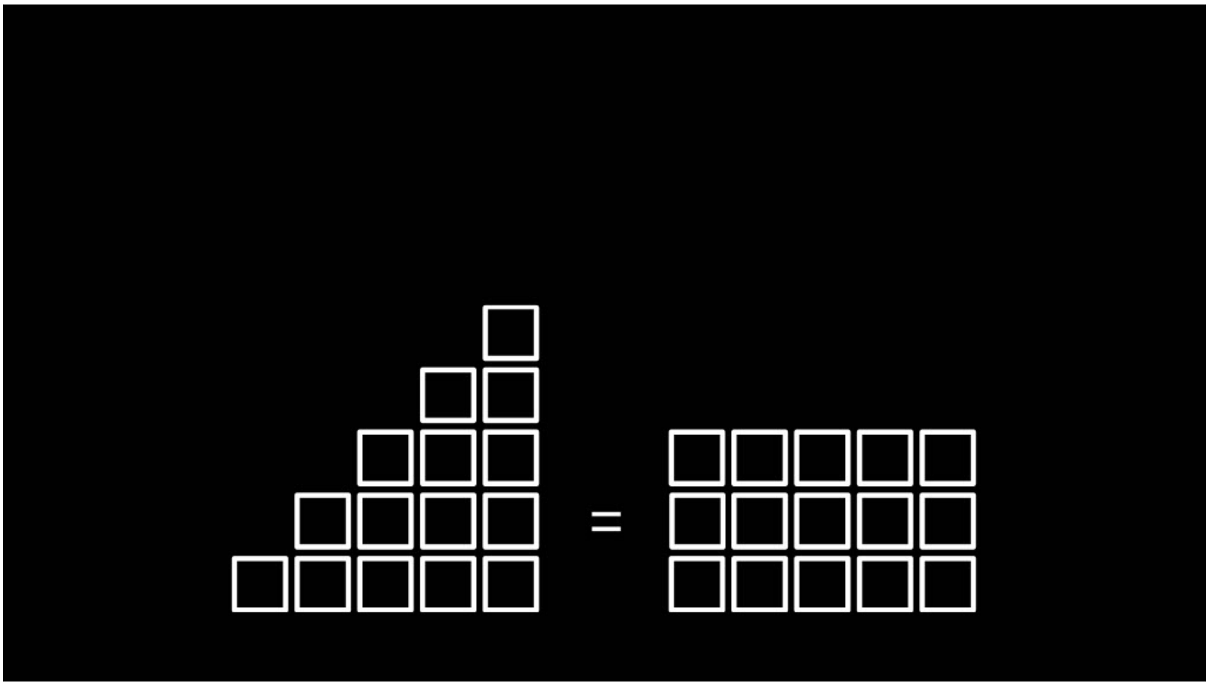












3A

Compute the sum of the infinite series:

$$\frac{1}{4}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^3+\ldots = ???$$

$$S_n = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} + \frac{1}{4^n}$$

$$S_n = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} + \frac{1}{4^n}$$

$$4 \cdot S_n = 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}}$$

$$
\begin{aligned}
S_n &= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} + \frac{1}{4^n} \\
4 \cdot S_n &= 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} \\
4 \cdot S_n &= 1 + \left(S_n - \frac{1}{4^n} \right)
\end{aligned}
$$

$$
\begin{aligned}
S_n &= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} + \frac{1}{4^n} \\
4 \cdot S_n &= 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} \\
4 \cdot S_n &= 1 + \left(S_n - \frac{1}{4^n} \right) \\
3 \cdot S_n &= 1 - \frac{1}{4^n}
\end{aligned}
$$

$$
\begin{aligned}
S_n &= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} + \frac{1}{4^n} \\
4 \cdot S_n &= 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} \\
4 \cdot S_n &= 1 + \left(S_n - \frac{1}{4^n} \right) \\
3 \cdot S_n &= 1 - \frac{1}{4^n} \\
S_n &= \frac{1}{3} \cdot \left(1 - \frac{1}{4^n} \right)
\end{aligned}
$$

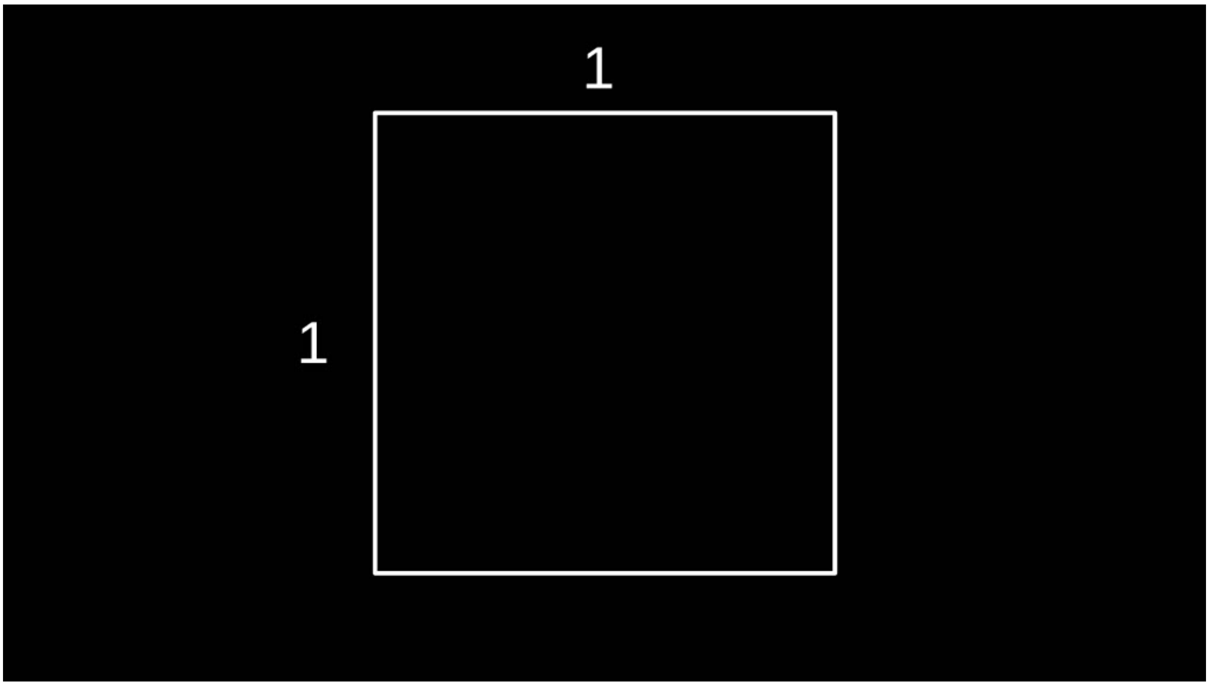
$$
\begin{aligned}
S_n &= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} + \frac{1}{4^n} \\
4 \cdot S_n &= 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + ... + \frac{1}{4^{n-1}} \\
4 \cdot S_n &= 1 + \left(S_n - \frac{1}{4^n} \right) \\
3 \cdot S_n &= 1 - \frac{1}{4^n} \\
S_n &= \frac{1}{3} \cdot \left(1 - \frac{1}{4^n} \right)
\end{aligned}
$$

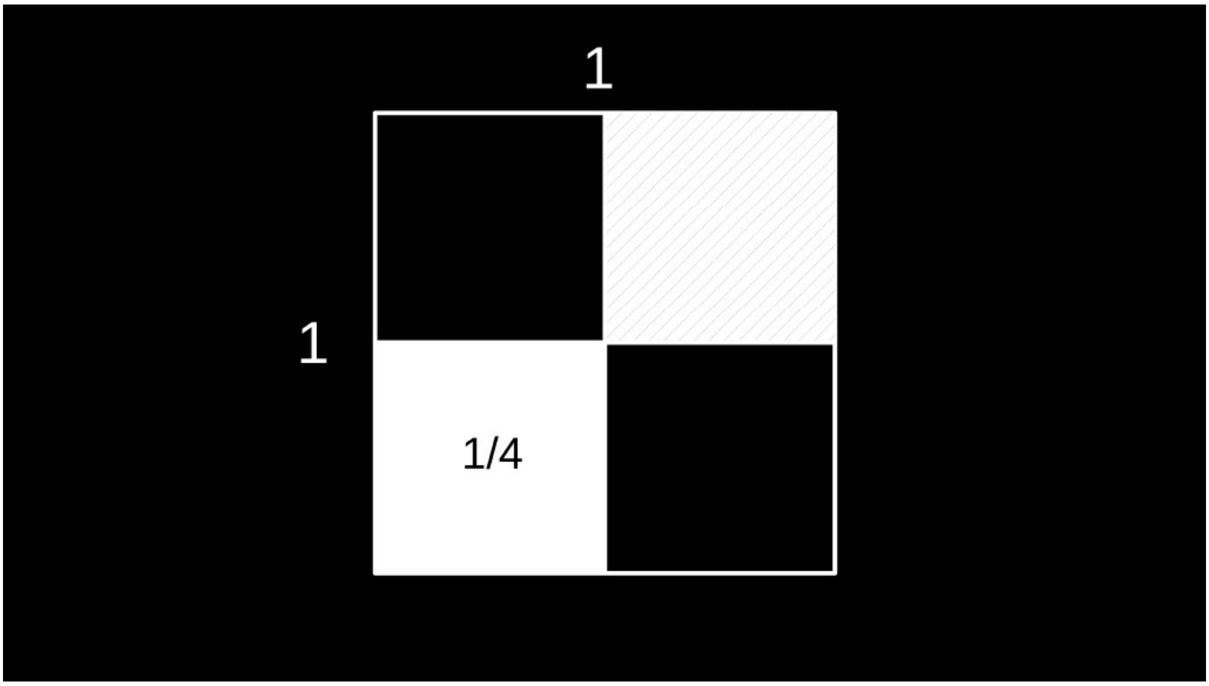
$$
\text{As } n \to \infty,\ S_n = \frac{1}{3} \cdot (1 - 0) = \frac{1}{3}
$$

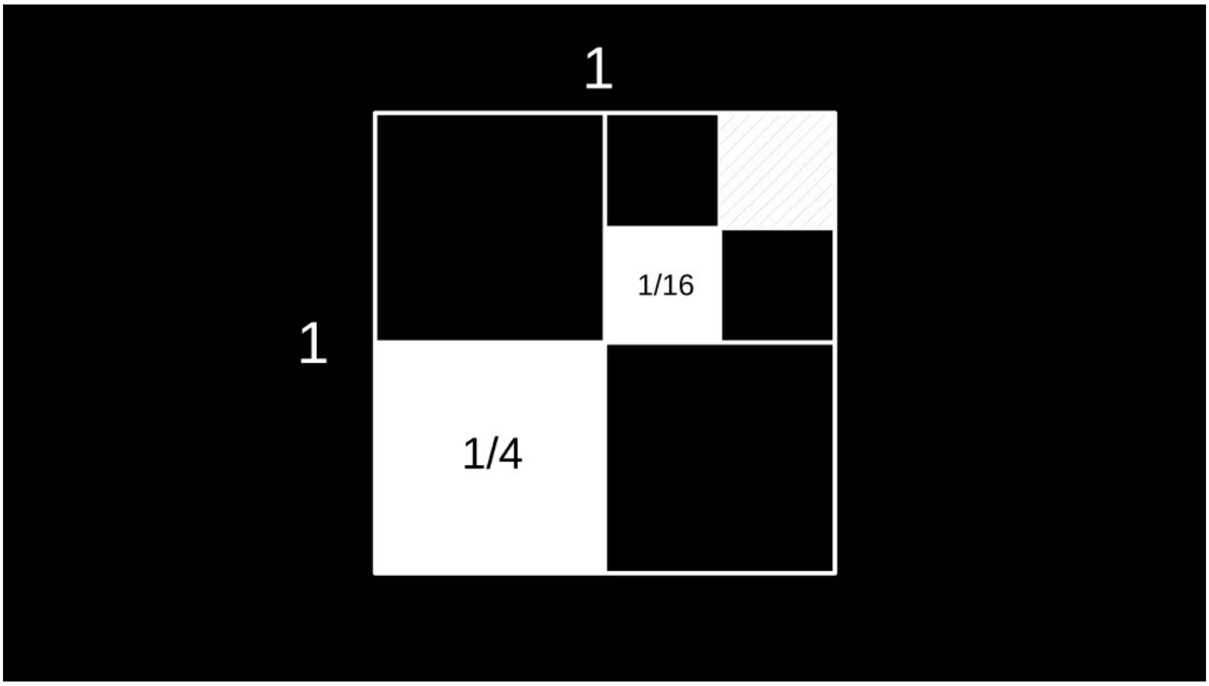
3G

Compute the sum of the infinite series:

$$\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + ... = ???$$


1
1


1
1
1/4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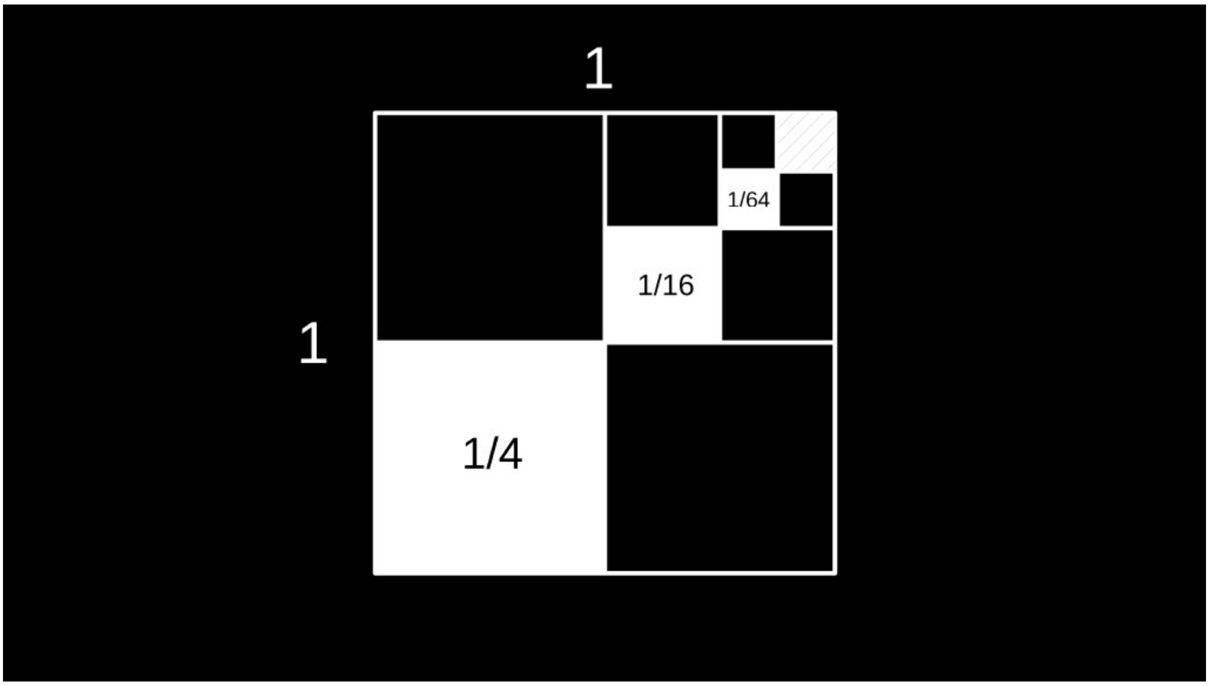
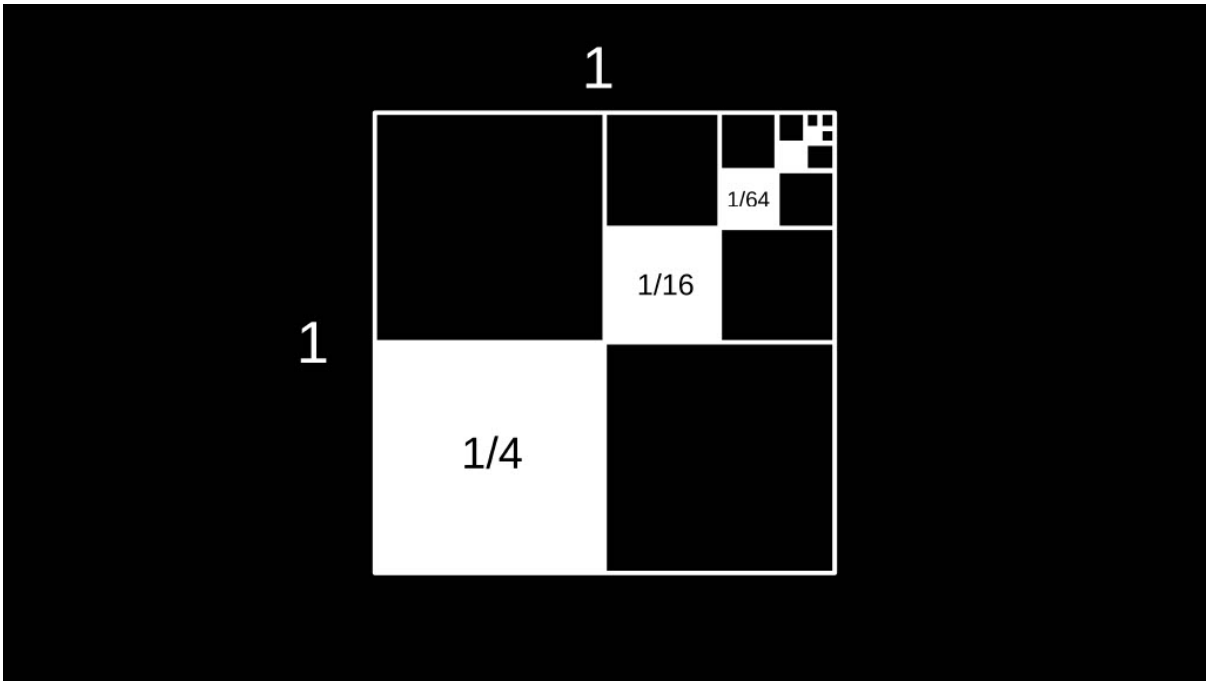
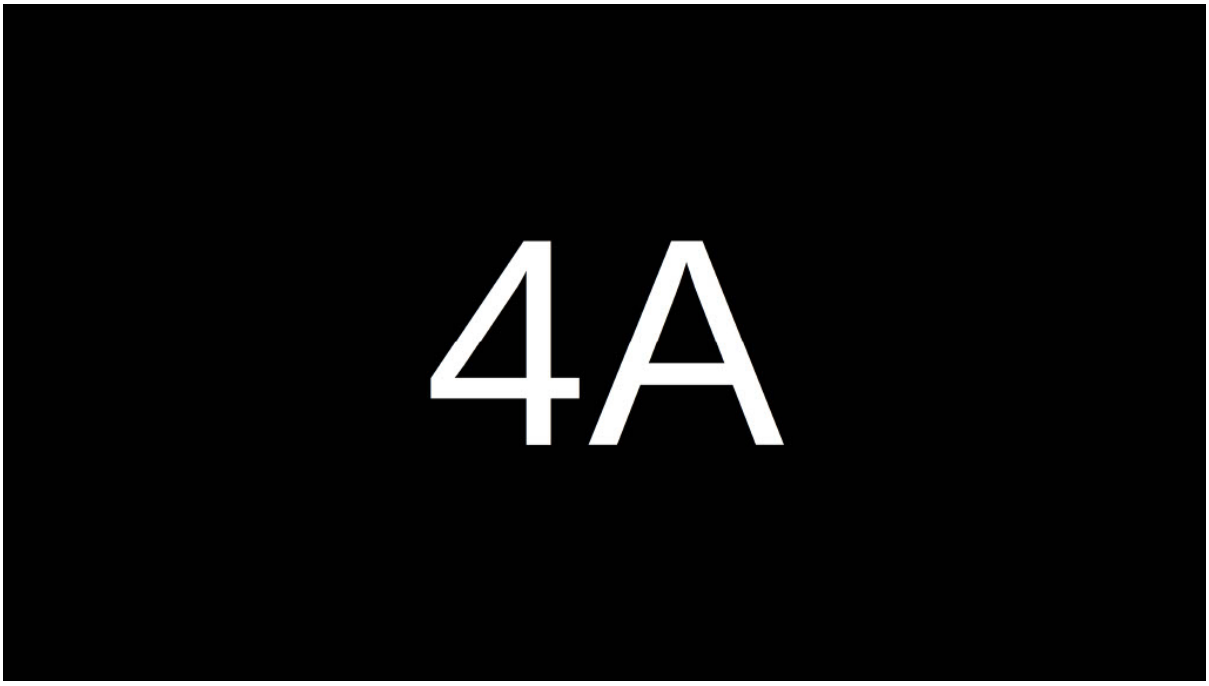
1/16
1
1/4


1
1/64
1/16
1
1/4


1
1
1/64
1/16
1/4


4A

Compute the sum of the series for positive integer ‘n’:

$$1 + 3 + 5 + ... + (2n - 1) = ???$$

$$S = 1 \qquad + 3 \qquad + 5 \qquad + ... + (2n-5) \quad + (2n-3) \quad + (2n-1)$$

$$
\begin{aligned}
S &= 1 && + 3 && + 5 && + \ldots + (2n-5) && + (2n-3) && + (2n-1)\\
S &= (2n-1) && + (2n-3) && + (2n-5) && + \ldots + 5 && + 3 && + 1
\end{aligned}
$$

$$
\begin{aligned}
S &= 1 && + 3 && + 5 && + \ldots + (2n-5) && + (2n-3) && + (2n-1)\\
S &= (2n-1) && + (2n-3) && + (2n-5) && + \ldots + 5 && + 3 && + 1\\
2S &= (2n-1+1) && + (2n-3+3) && + (2n-1+1) && + \ldots + (2n-5+5) && + (2n-3+3) && + (2n-1+1)
\end{aligned}
$$

$$\begin{aligned}
S &= 1 && + 3 && + 5 && + ... + (2n-5) && + (2n-3) && + (2n-1)\\
S &= (2n-1) && + (2n-3) && + (2n-5) && + ... + 5 && + 3 && + 1\\
2S &= (2n-1+1) && + (2n-3+3) && + (2n-1+1) && + ... + (2n-5+5) && + (2n-3+3) && + (2n-1+1)\\
&= 2n && + 2n && + 2n && + ... + 2n && + 2n && + 2n
\end{aligned}$$

$$\begin{aligned}
S &= 1 + 3 + 5 + ... + (2n-5) + (2n-3) + (2n-1)\\
S &= (2n-1) + (2n-3) + (2n-5) + ... + 5 + 3 + 1\\
2S &= (2n-1+1) + (2n-3+3) + (2n-1+1) + ... + (2n-5+5) + (2n-3+3) + (2n-1+1)\\
&= \underbrace{2n + 2n + 2n + ... + 2n + 2n + 2n}_{\text{n times}}
\end{aligned}$$

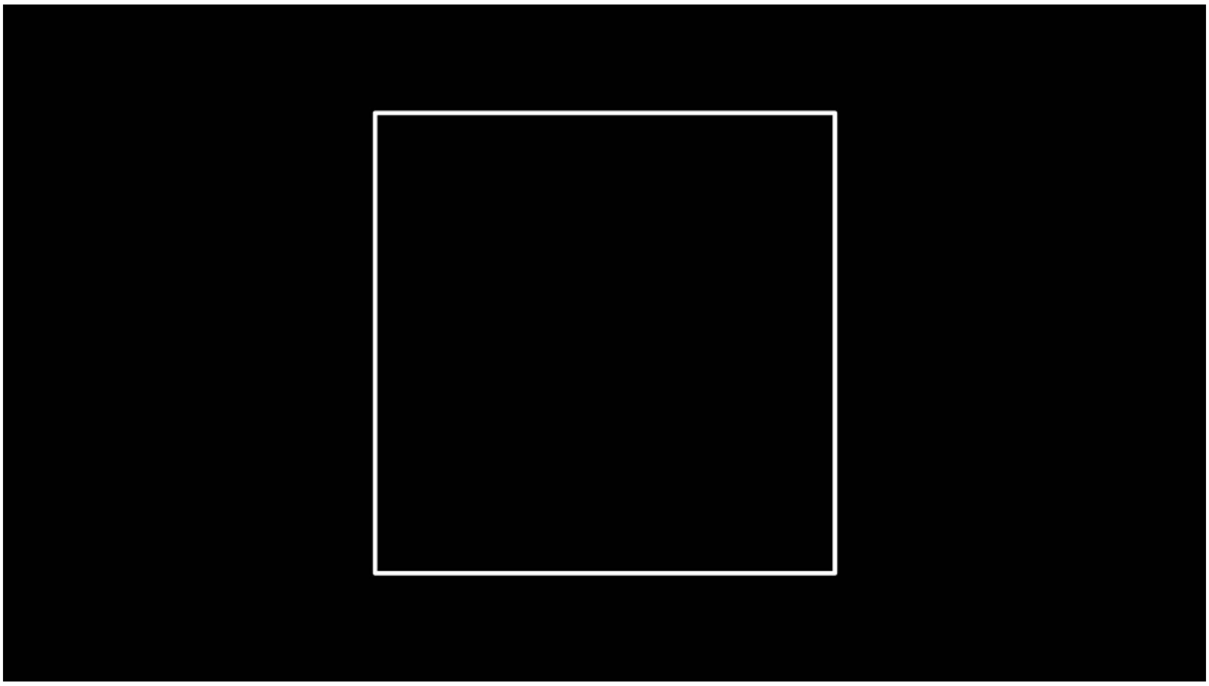
$$\begin{aligned} S &= 1 + 3 + 5 + \dots + (2n-5) + (2n-3) + (2n-1)\\ S &= (2n-1) + (2n-3) + (2n-5) + \dots + 5 + 3 + 1\\ 2S &= (2n-1+1) + (2n-3+3) + (2n-1+1) + \dots + (2n-5+5) + (2n-3+3) + (2n-1+1)\\ &= \underbrace{2n + 2n + 2n + \dots + 2n + 2n + 2n}_{\text{n times}}\\ 2S &= n \cdot (2n) \end{aligned}$$

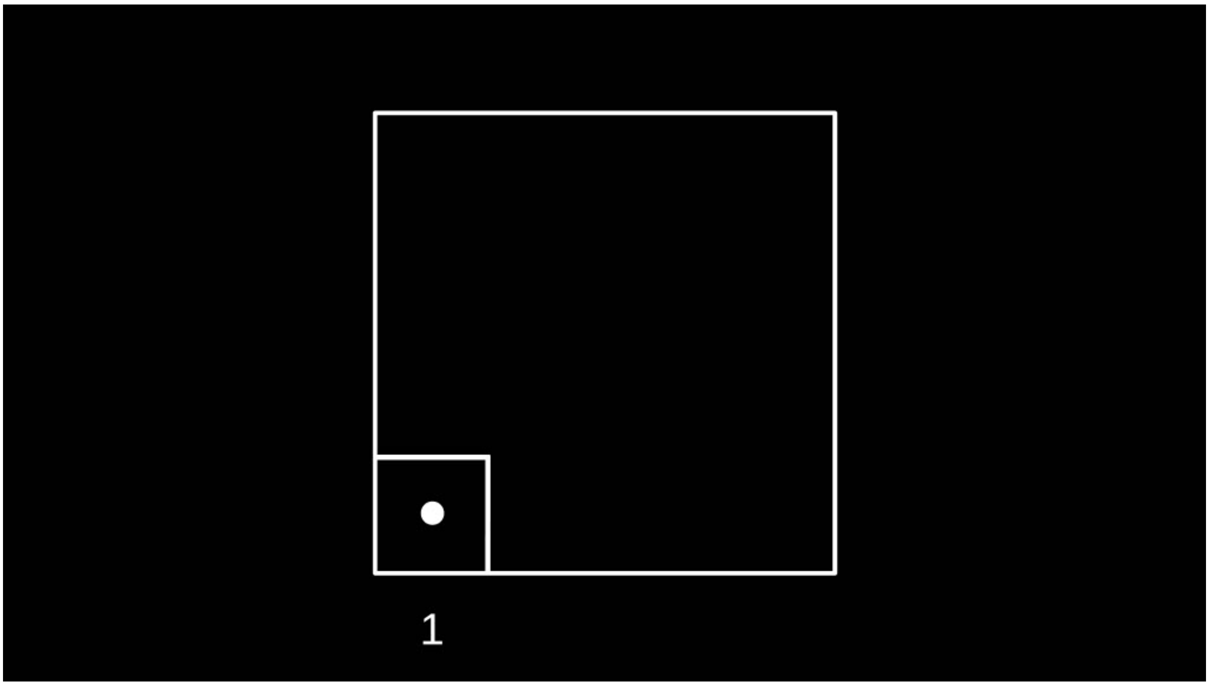
$$\begin{aligned} S &= 1 + 3 + 5 + \dots + (2n-5) + (2n-3) + (2n-1)\\ S &= (2n-1) + (2n-3) + (2n-5) + \dots + 5 + 3 + 1\\ 2S &= (2n-1+1) + (2n-3+3) + (2n-1+1) + \dots + (2n-5+5) + (2n-3+3) + (2n-1+1)\\ &= \underbrace{2n + 2n + 2n + \dots + 2n + 2n + 2n}_{\text{n times}}\\ 2S &= n \cdot (2n)\\ S &= n^2 \end{aligned}$$

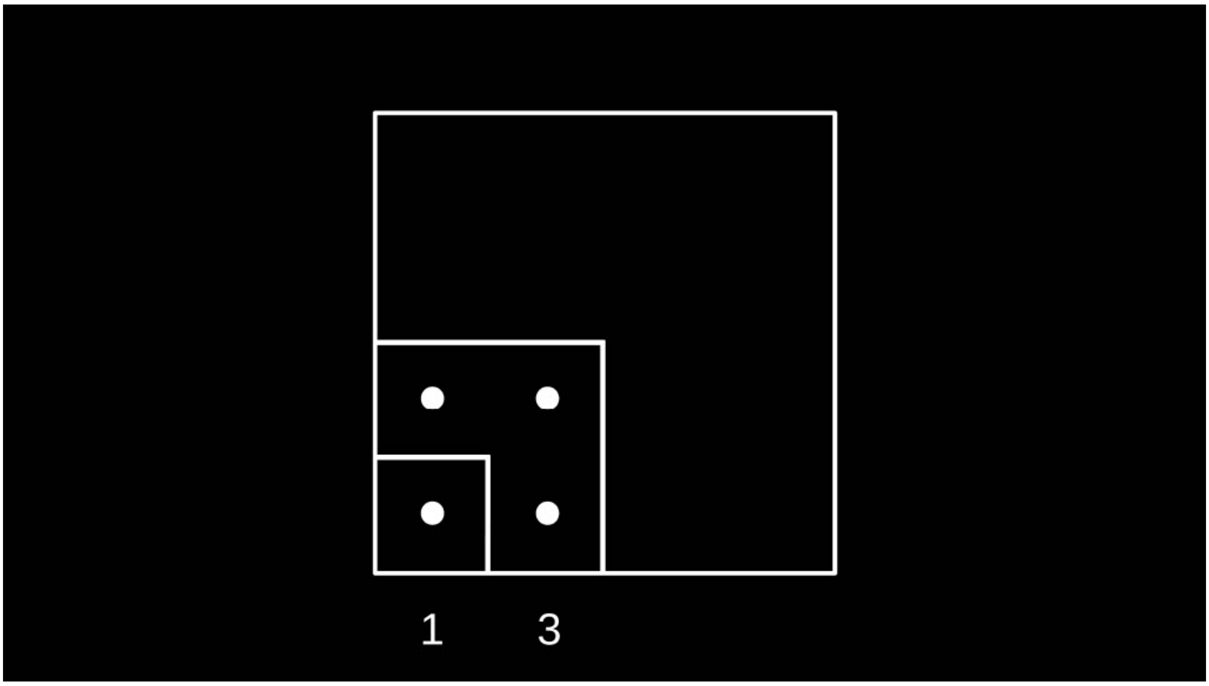
4G

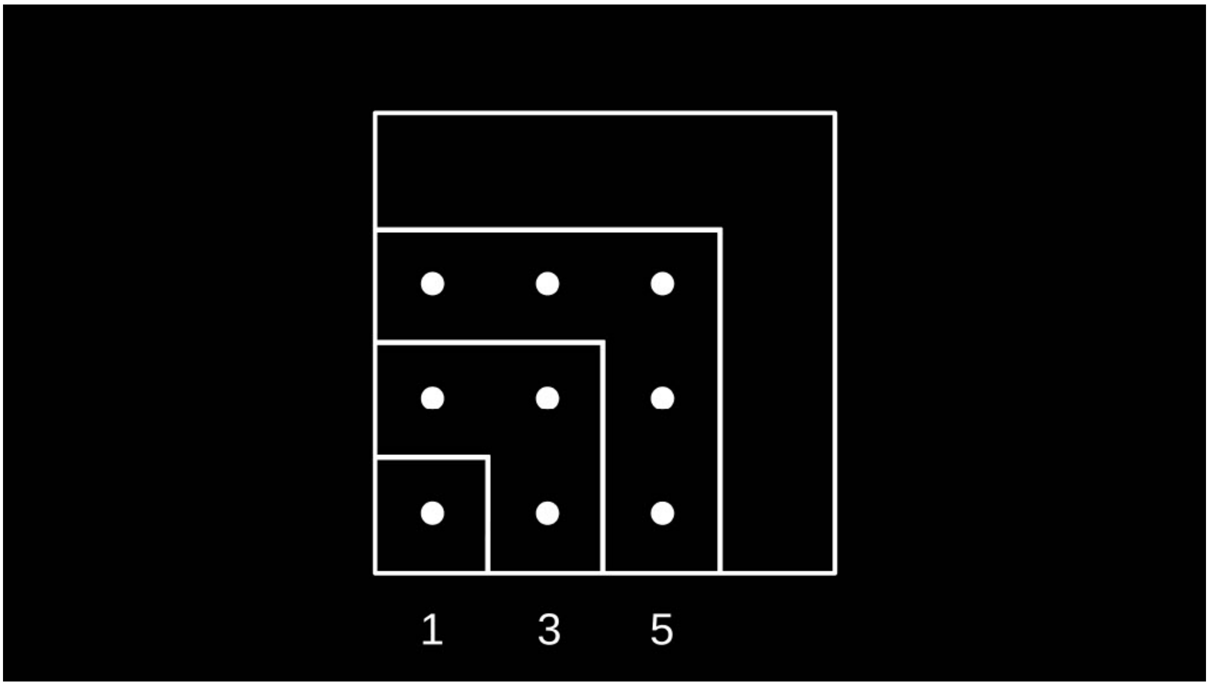
Compute the sum of the series for positive integer ‘n’:

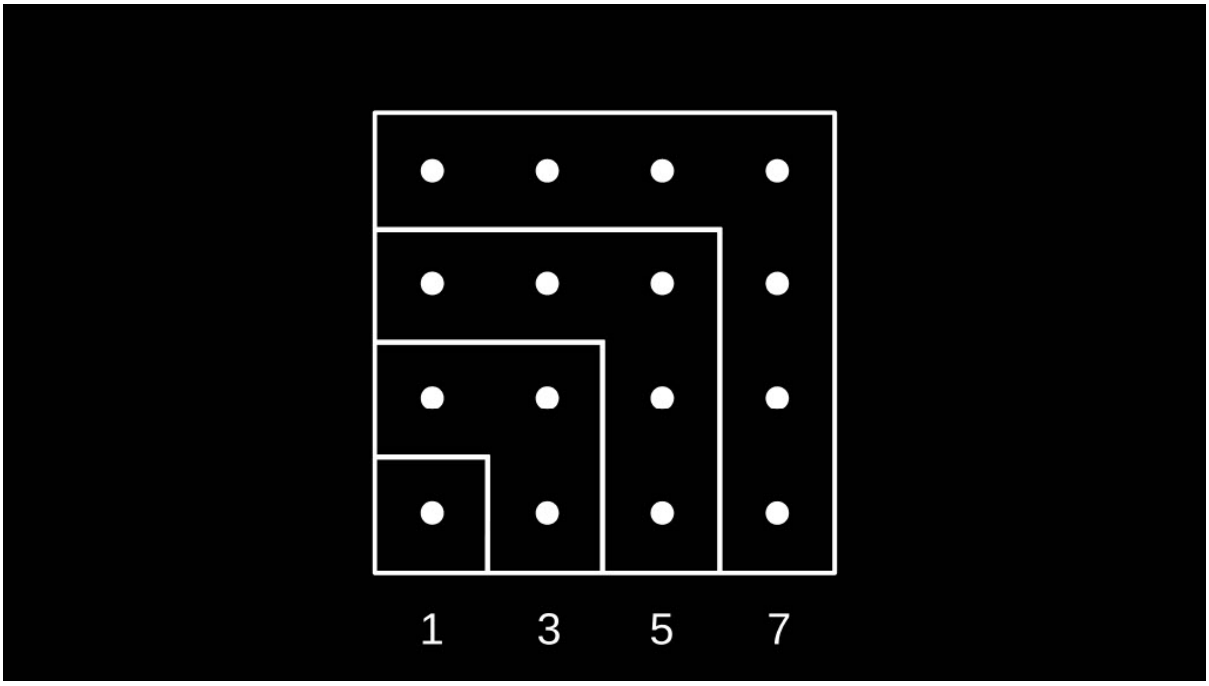
$$1 + 3 + 5 + ... + (2n - 1) = ???$$

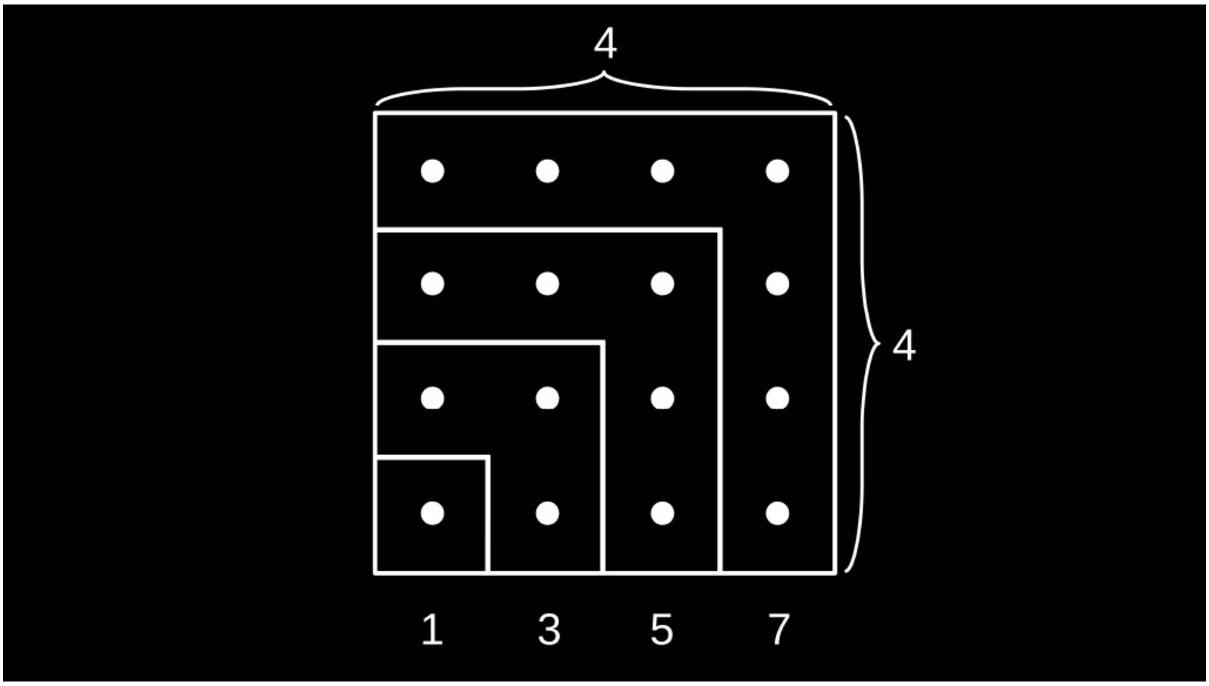



1


1 3


1 3 5

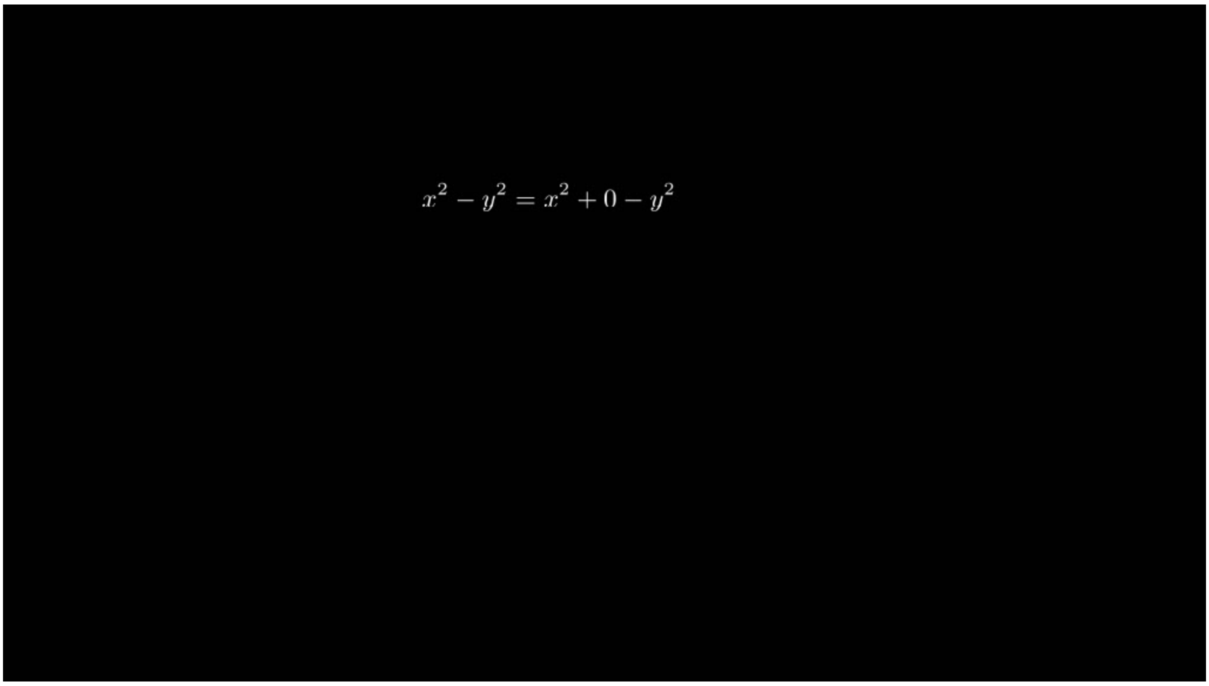

1 3 5 7

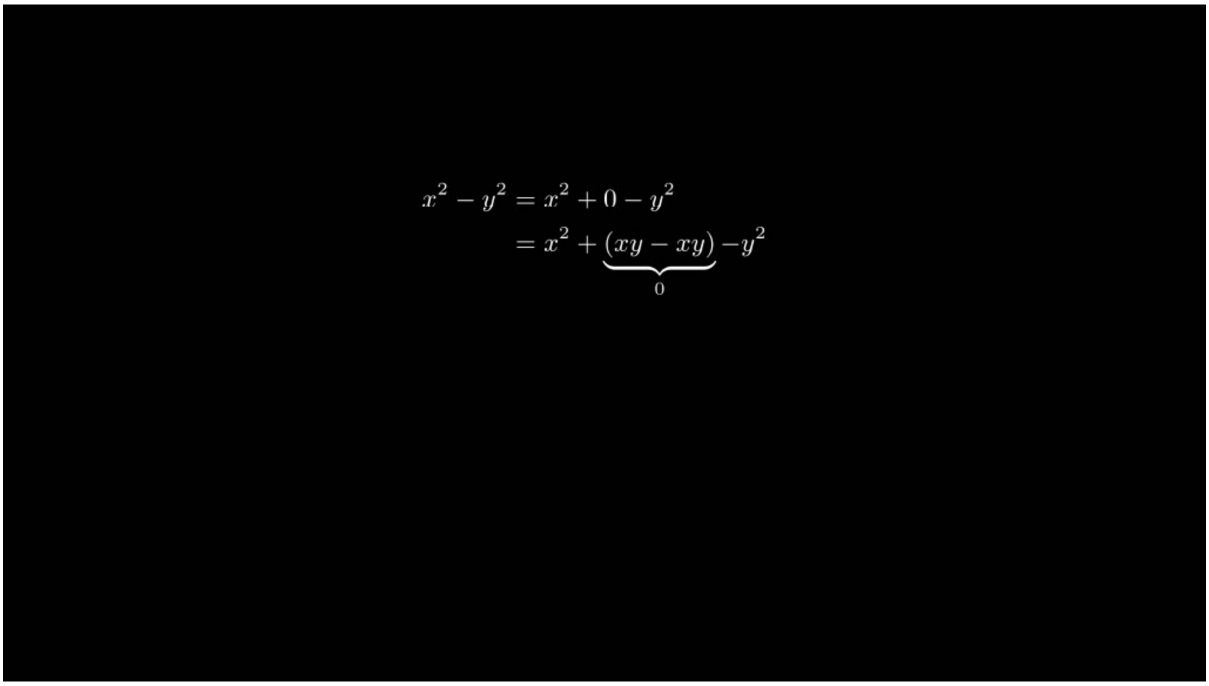

4
4
1 3 5 7

5A

The differences of squares formula:

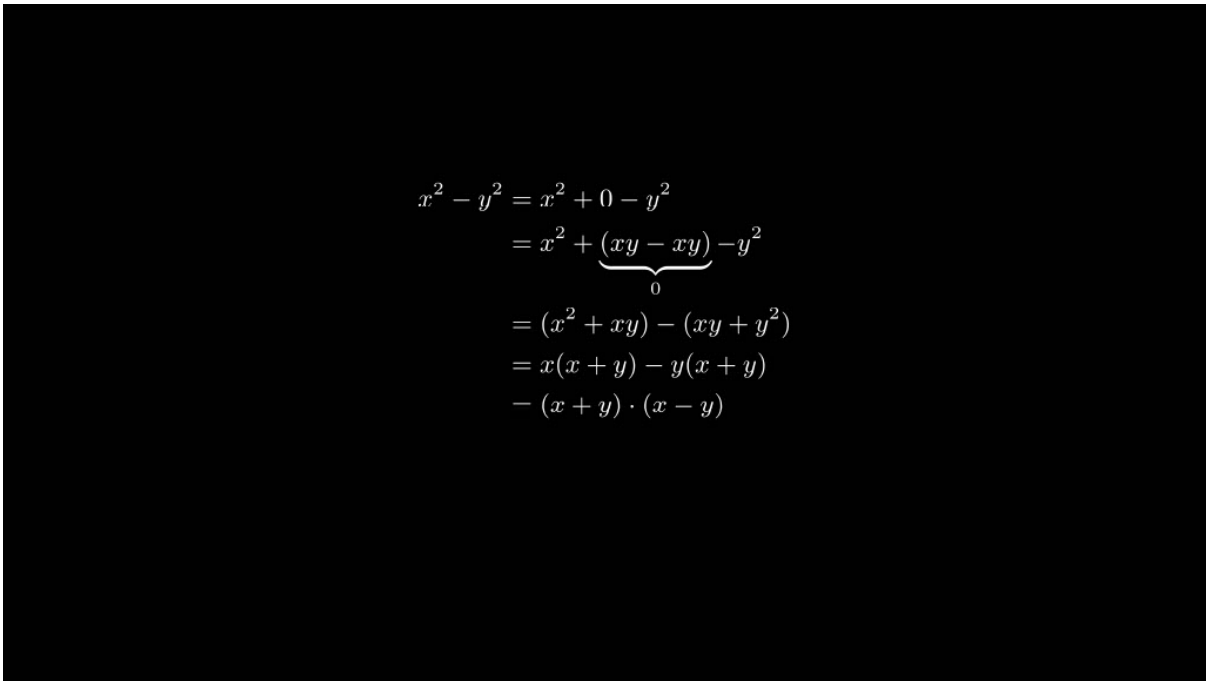
$$x^2 - y^2 = ???$$

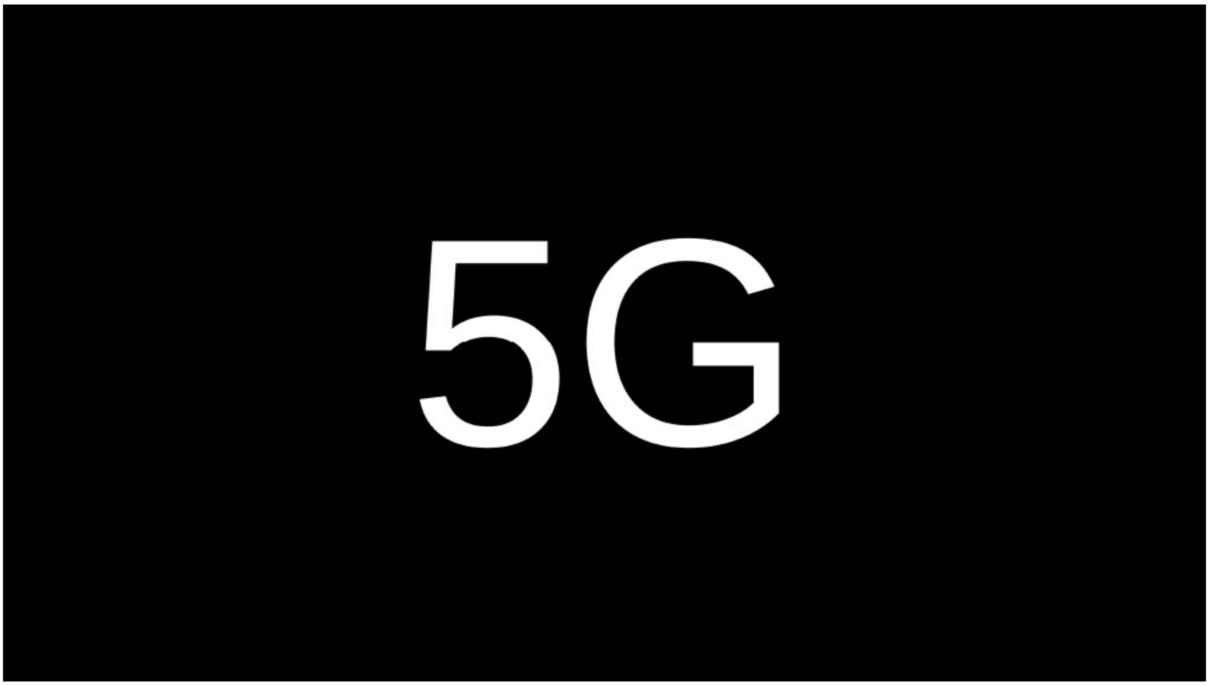

$x^2 - y^2 = x^2 + 0 - y^2$

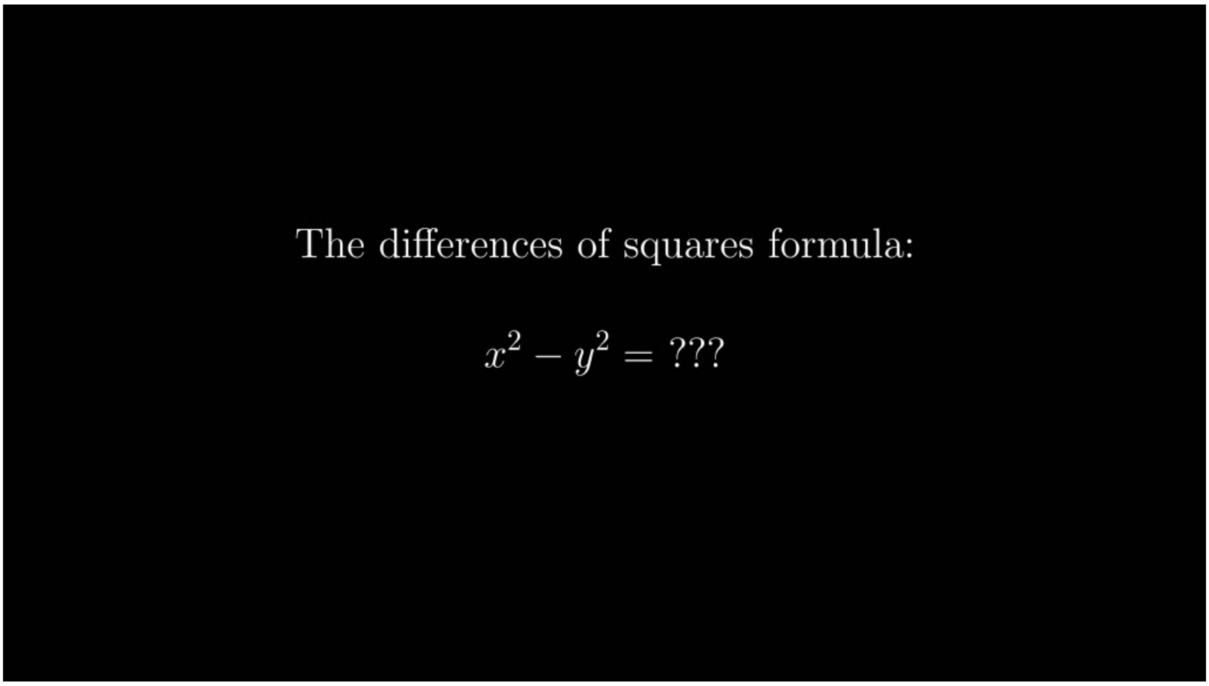

$$\begin{aligned} x^2 - y^2 &= x^2 + 0 - y^2 \\ &= x^2 + \underbrace{(xy - xy)}_{0} - y^2 \end{aligned}$$

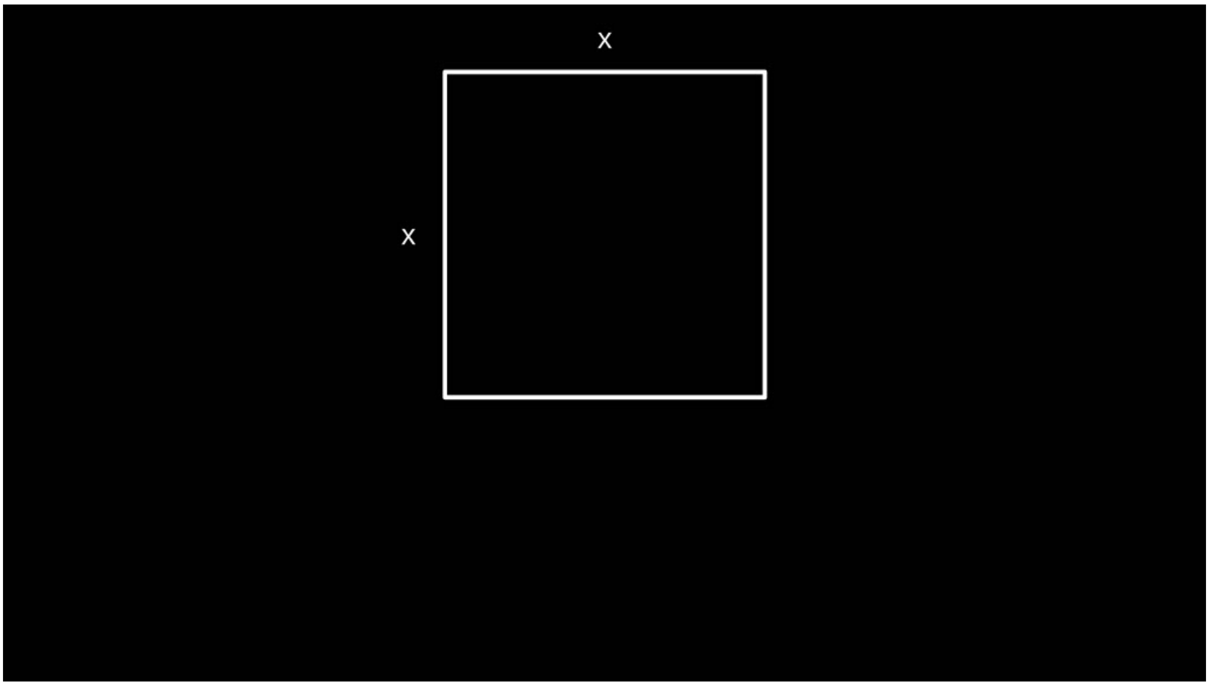
$$\begin{aligned} x^2 - y^2 &= x^2 + 0 - y^2 \\ &= x^2 + \underbrace{(xy - xy)}_{0} - y^2 \\ &= (x^2 + xy) - (xy + y^2) \end{aligned}$$

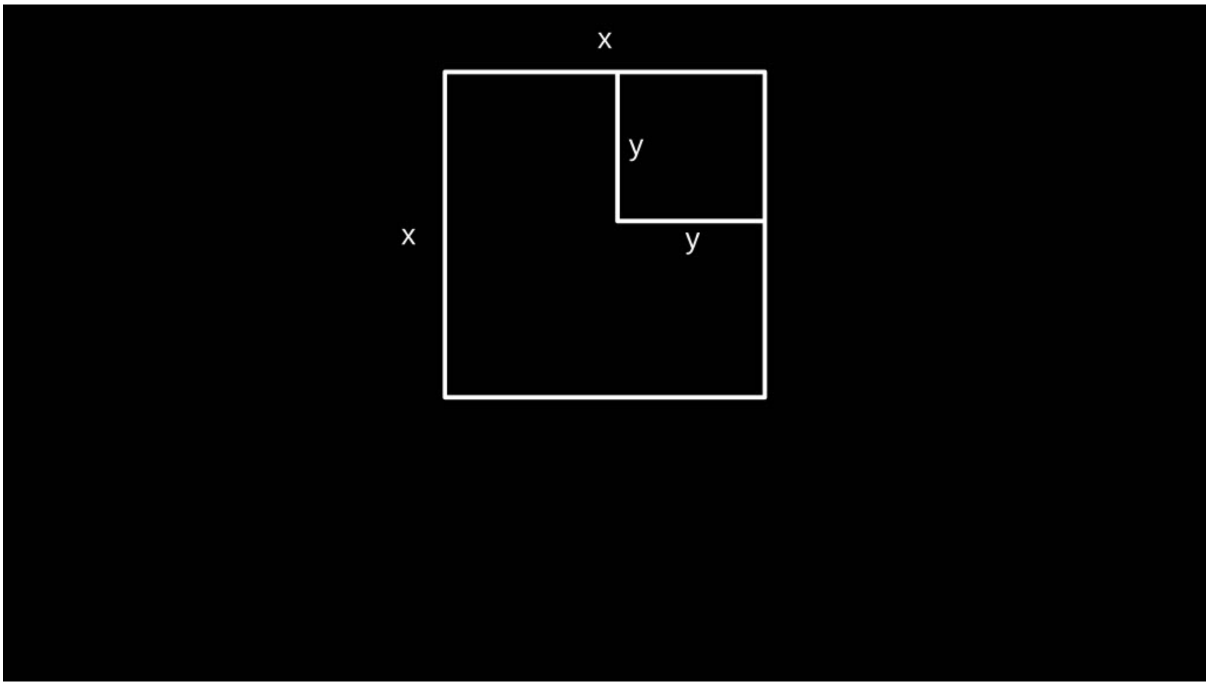
$$\begin{aligned} x^2 - y^2 &= x^2 + 0 - y^2 \\ &= x^2 + \underbrace{(xy - xy)}_{0} - y^2 \\ &= (x^2 + xy) - (xy + y^2) \\ &= x(x + y) - y(x + y) \end{aligned}$$

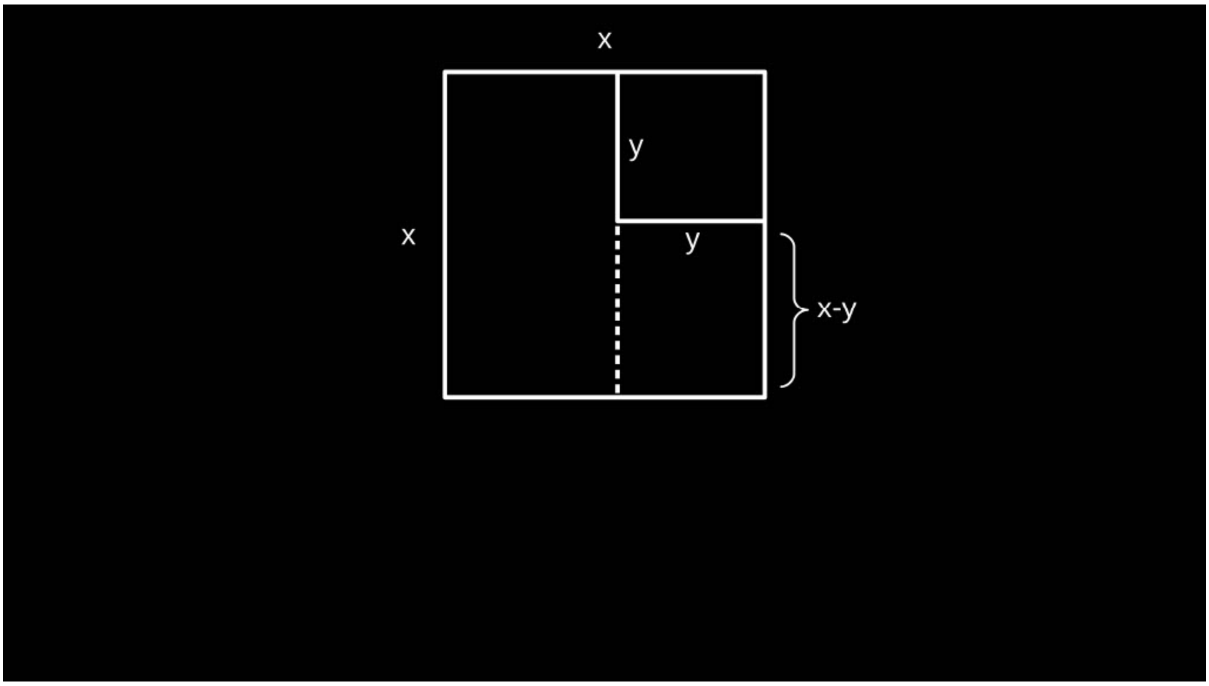

$x^2 - y^2 = x^2 + 0 - y^2$
$= x^2 + \underbrace{(xy - xy)}_{0} - y^2$
$= (x^2 + xy) - (xy + y^2)$
$= x(x + y) - y(x + y)$
$= (x + y) \cdot (x - y)$

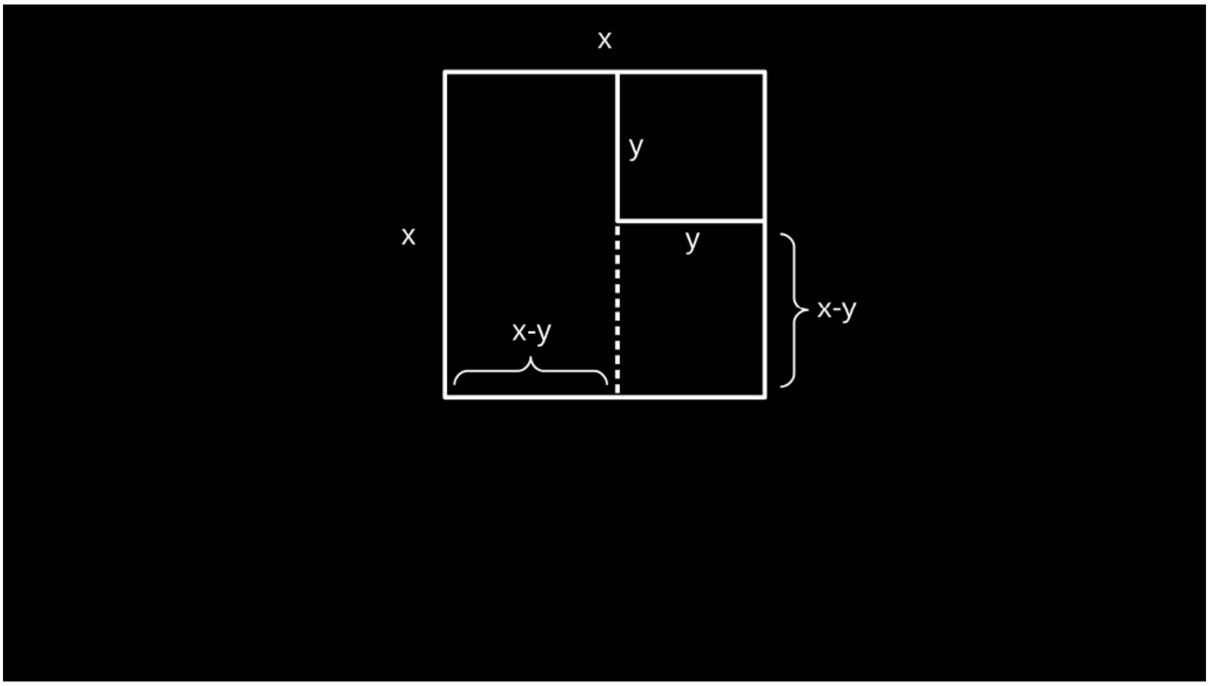

5G

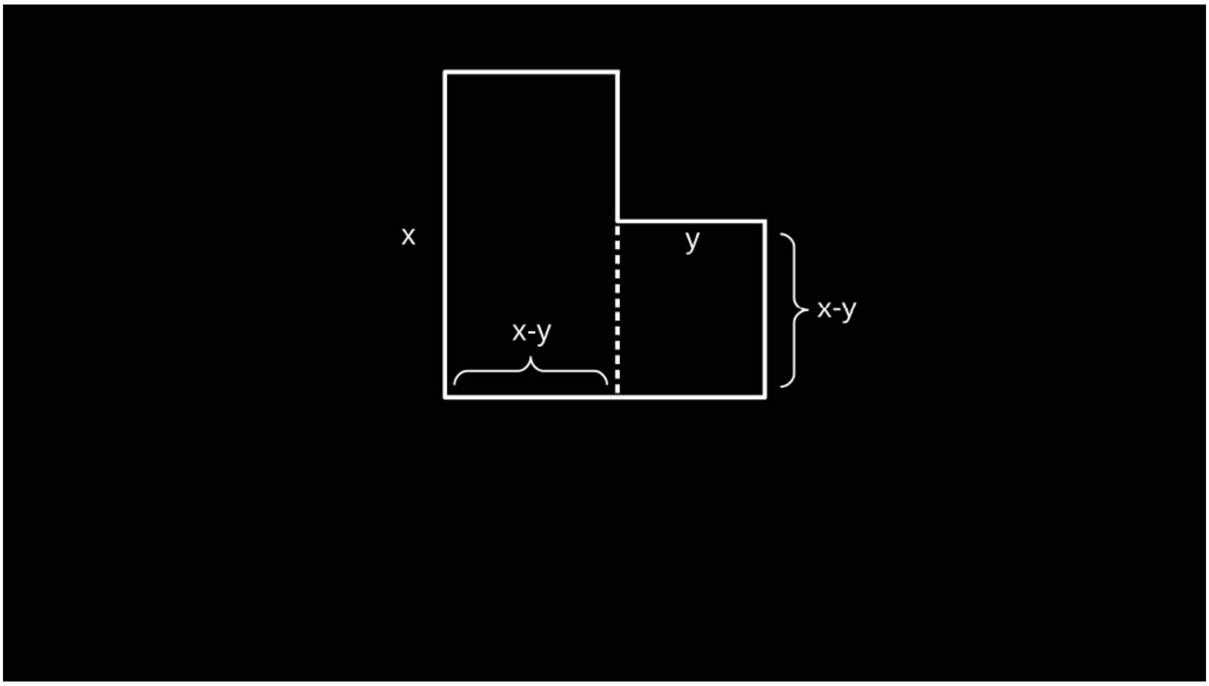

The differences of squares formula:
$x^2 - y^2 = ???$

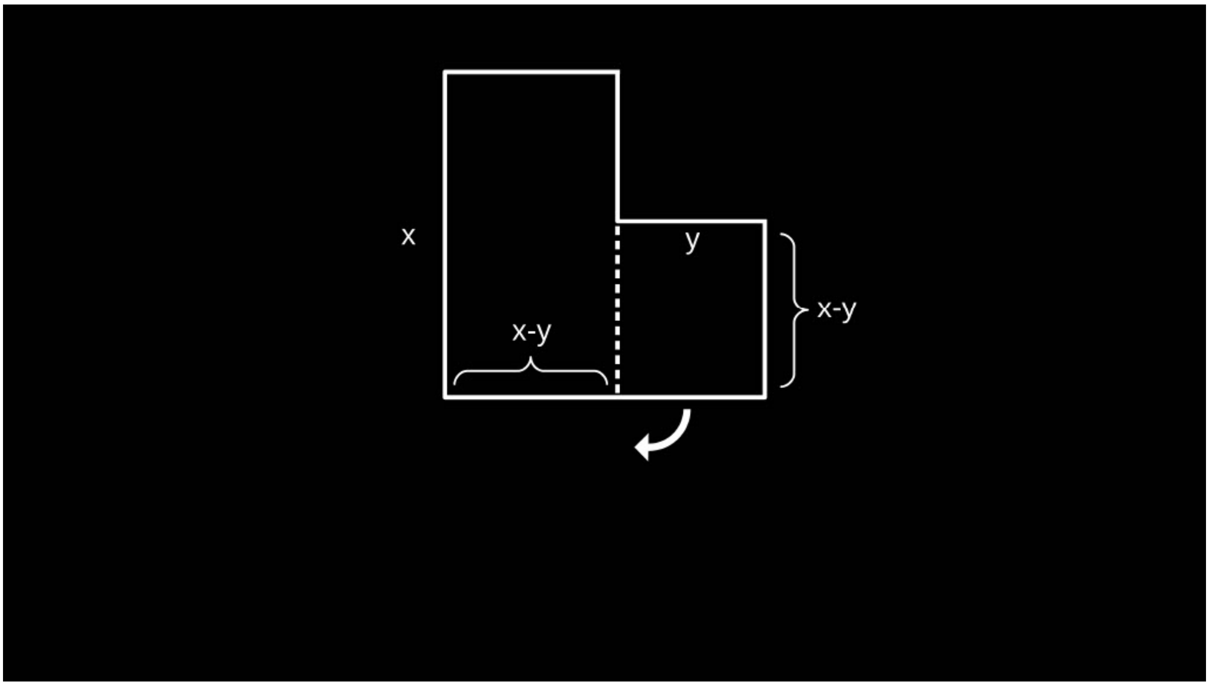

x
x

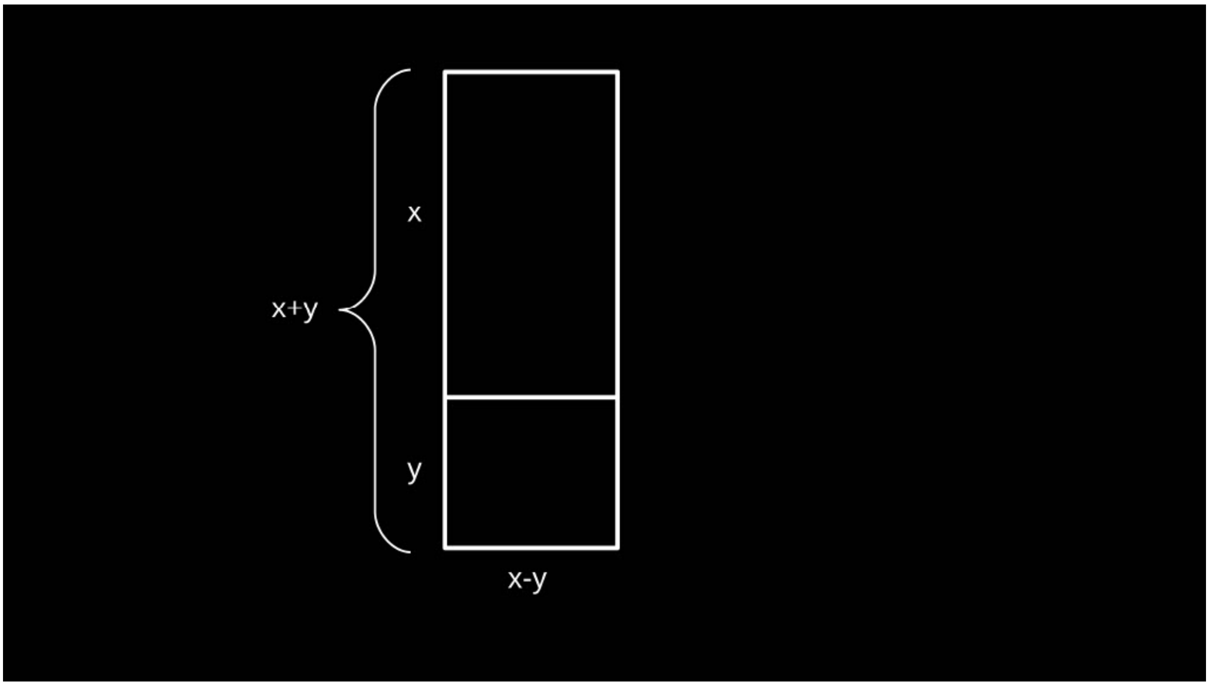

x
y
x
y


x
y
x
y
x-y


x
y
x
y
x-y
x-y


x
y
x-y
x-y


x
y
x-y
x-y


x
x+y
y
x-y

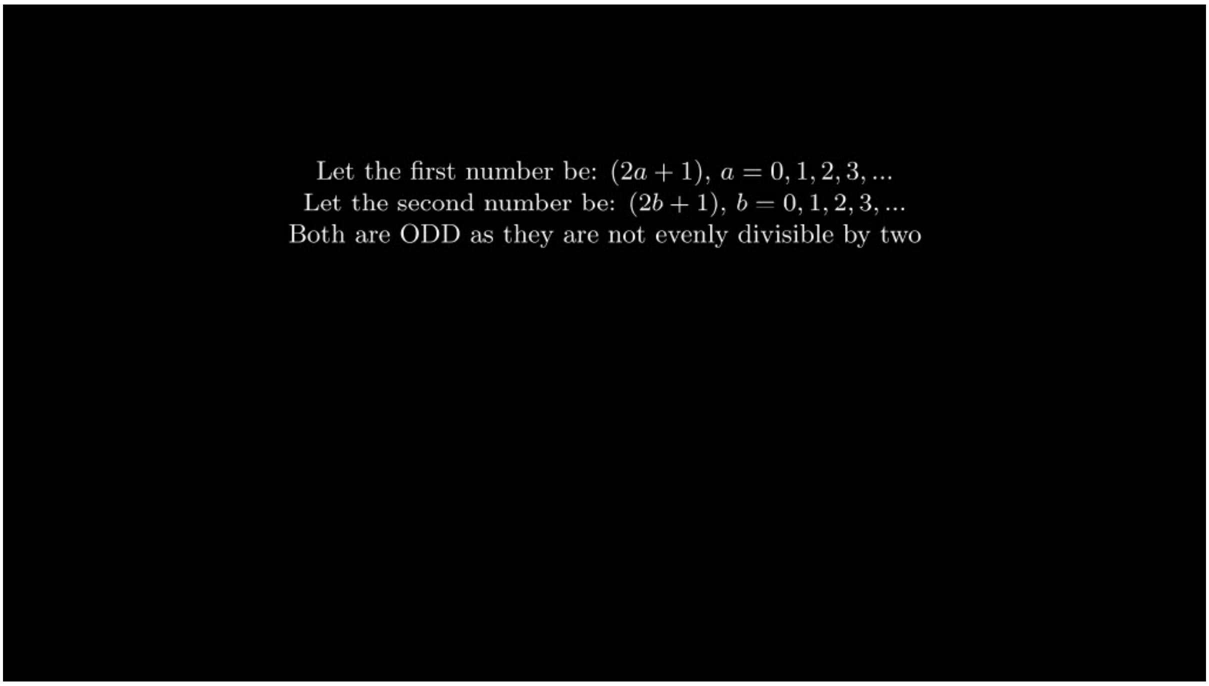
6A

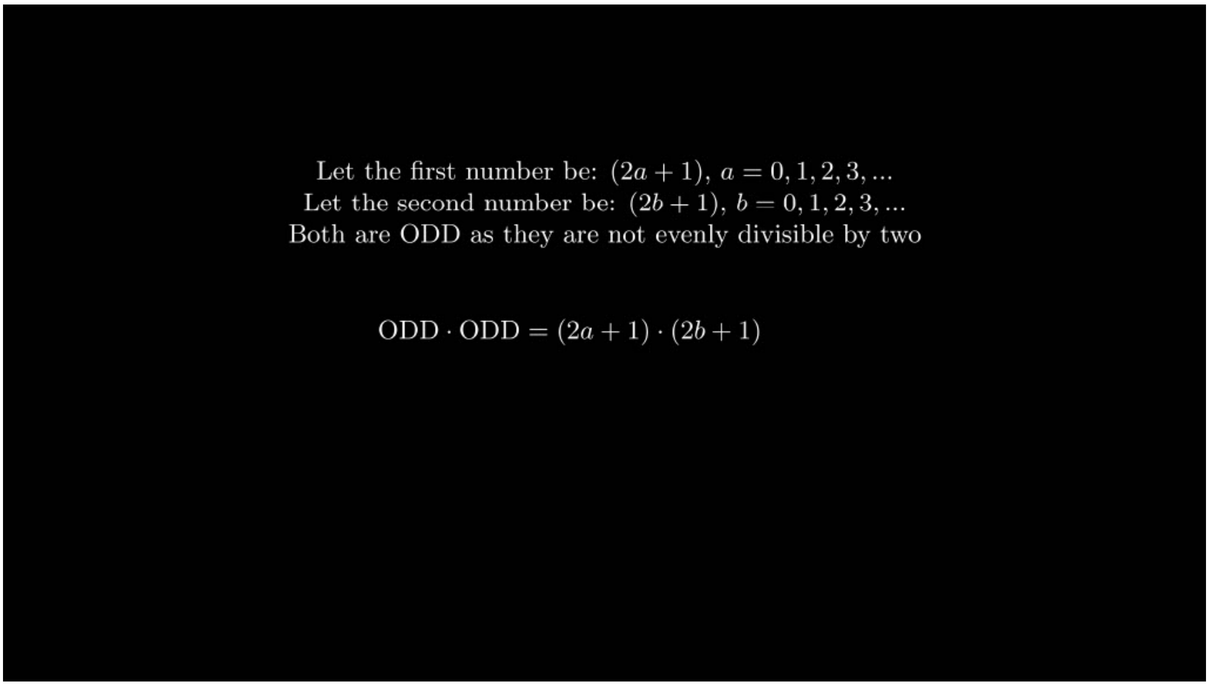
Is the product of two ODD numbers EVEN or ODD?

$$\text{ODD} \cdot \text{ODD} = ???$$

Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, ...$

Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, ...$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, ...$


Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, ...$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, ...$
Both are ODD as they are not evenly divisible by two


Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, ...$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, ...$
Both are ODD as they are not evenly divisible by two
$\text{ODD} \cdot \text{ODD} = (2a+1) \cdot (2b+1)$

Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, ...$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, ...$
Both are ODD as they are not evenly divisible by two

$$\begin{aligned}\text{ODD} \cdot \text{ODD} &= (2a+1) \cdot (2b+1) \\ &= 4ab + 2a + 2b + 1\end{aligned}$$

Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, ...$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, ...$
Both are ODD as they are not evenly divisible by two

$$\begin{aligned}\text{ODD} \cdot \text{ODD} &= (2a+1) \cdot (2b+1) \\ &= 4ab + 2a + 2b + 1 \\ &= 2 \cdot (2ab) + 2 \cdot (a+b) + 1\end{aligned}$$

Let the first number be: $(2a + 1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b + 1)$, $b = 0, 1, 2, 3, \ldots$
Both are ODD as they are not evenly divisible by two

$$\begin{aligned}
\text{ODD} \cdot \text{ODD} &= (2a + 1) \cdot (2b + 1) \\
&= 4ab + 2a + 2b + 1 \\
&= 2 \cdot (2ab) + 2 \cdot (a + b) + 1 \\
&= \text{EVEN} + \text{EVEN} + \text{ODD}
\end{aligned}$$

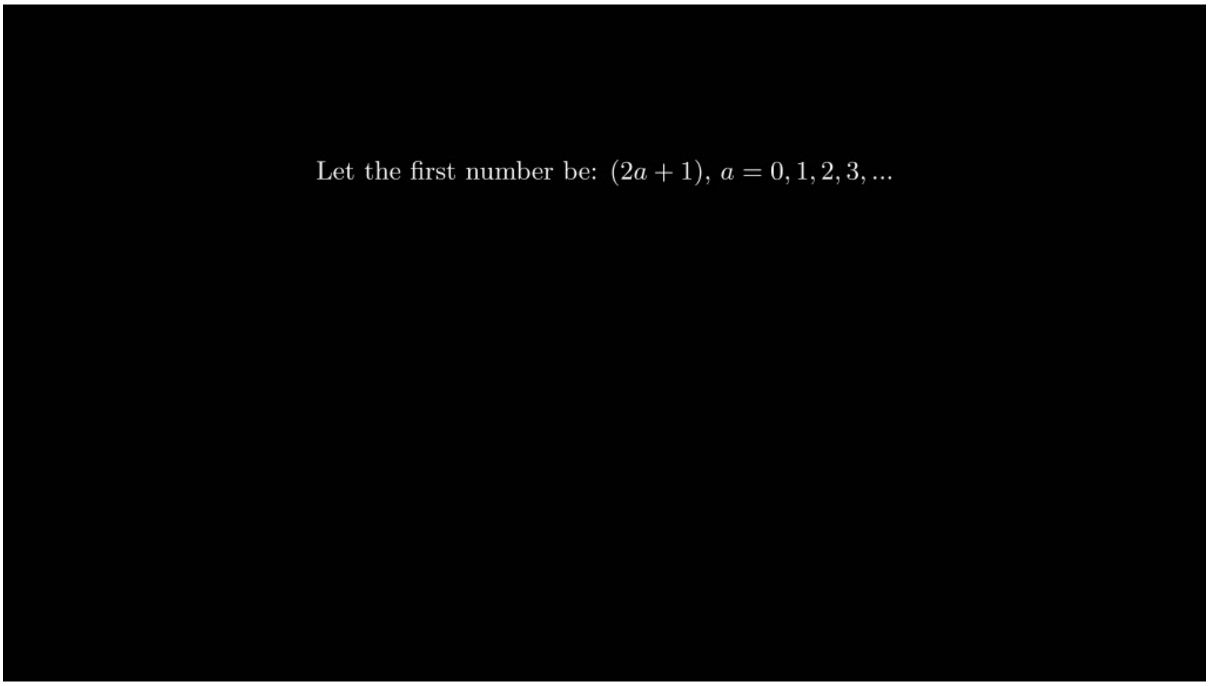
Let the first number be: $(2a + 1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b + 1)$, $b = 0, 1, 2, 3, \ldots$
Both are ODD as they are not evenly divisible by two

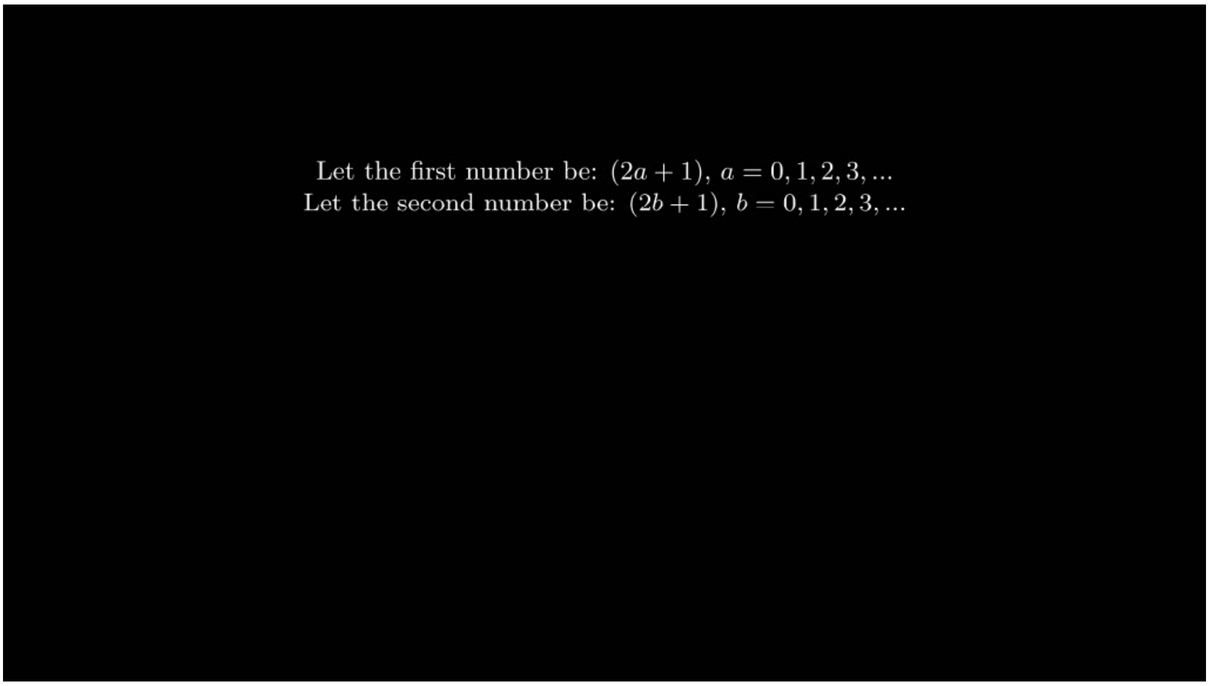
$$\begin{aligned}
\text{ODD} \cdot \text{ODD} &= (2a + 1) \cdot (2b + 1) \\
&= 4ab + 2a + 2b + 1 \\
&= 2 \cdot (2ab) + 2 \cdot (a + b) + 1 \\
&= \text{EVEN} + \text{EVEN} + \text{ODD} \\
&= \text{ODD}
\end{aligned}$$

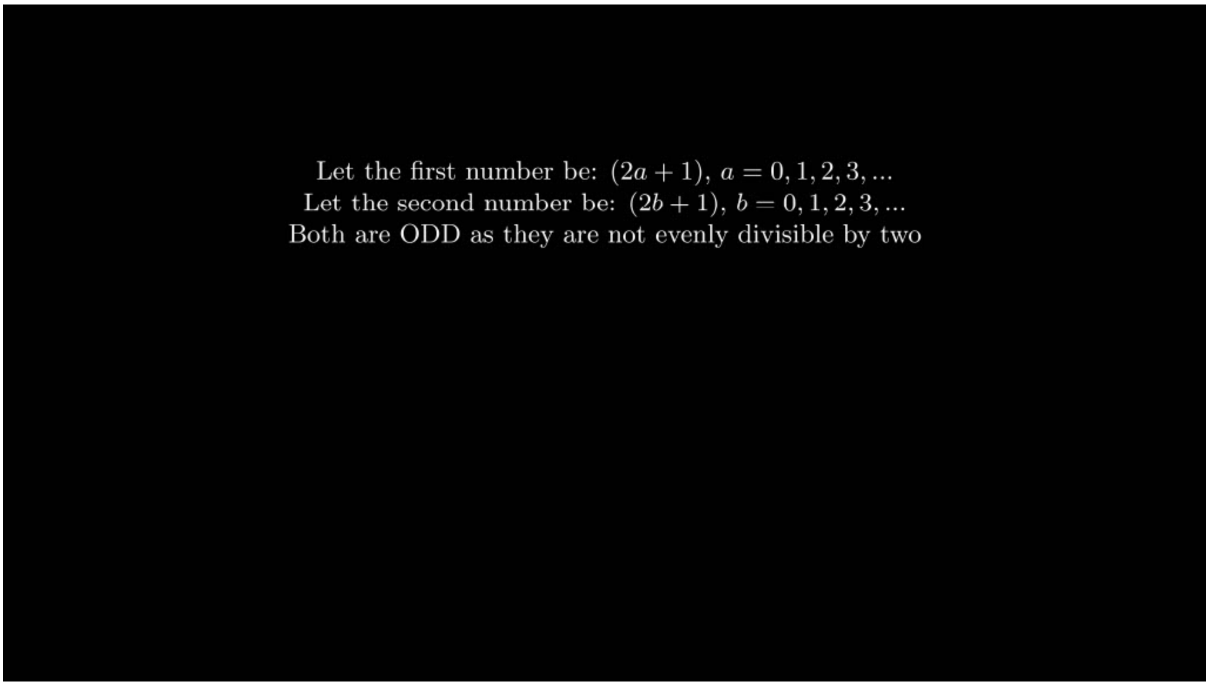
6G

Is the product of two ODD numbers EVEN or ODD?

$$\text{ODD} \cdot \text{ODD} = ???$$


Let the first number be: $(2a + 1)$, $a = 0, 1, 2, 3, \ldots$


Let the first number be: $(2a + 1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b + 1)$, $b = 0, 1, 2, 3, \ldots$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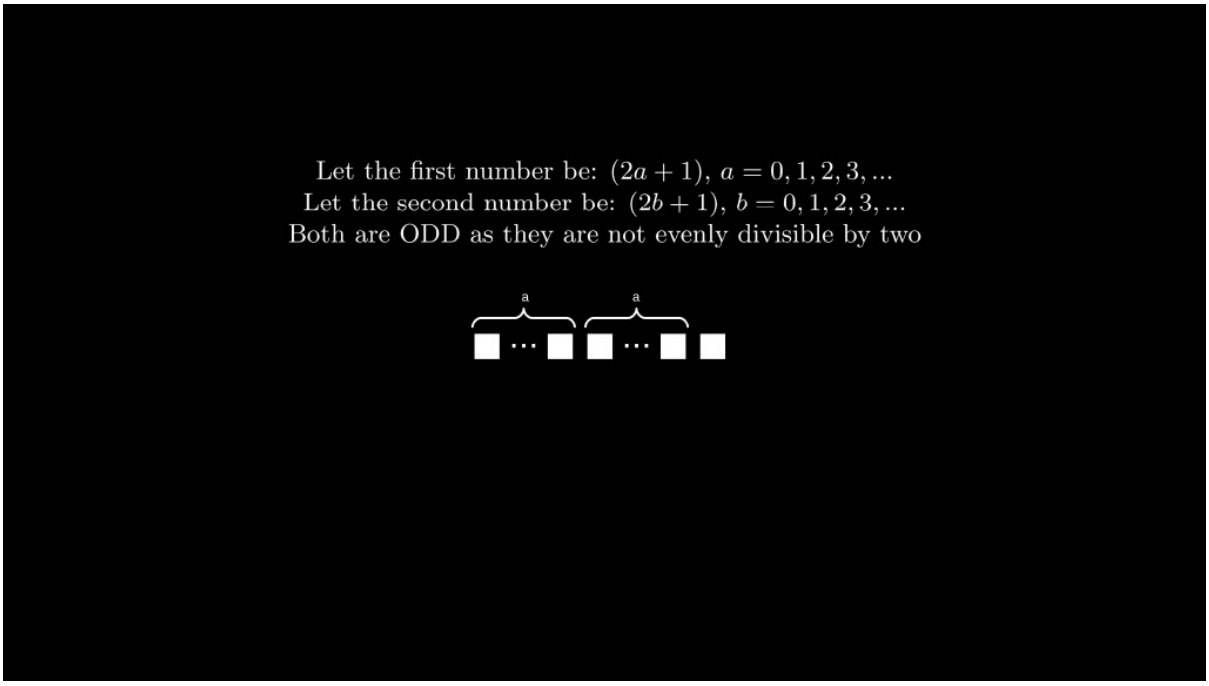
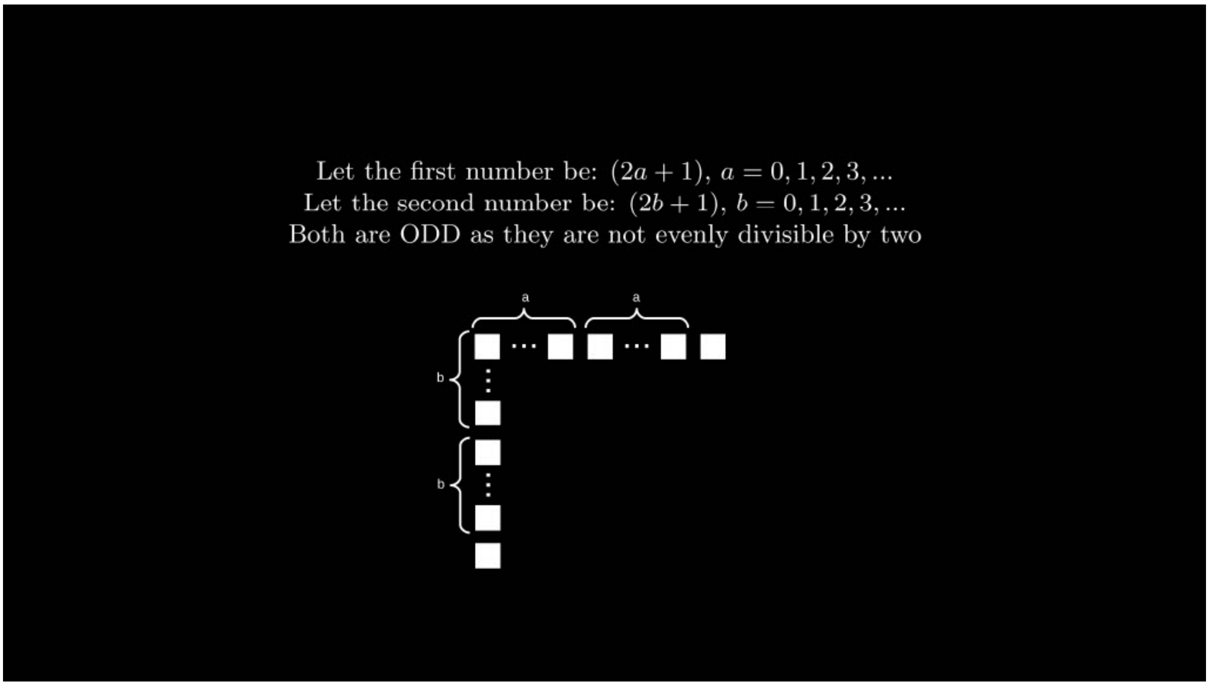
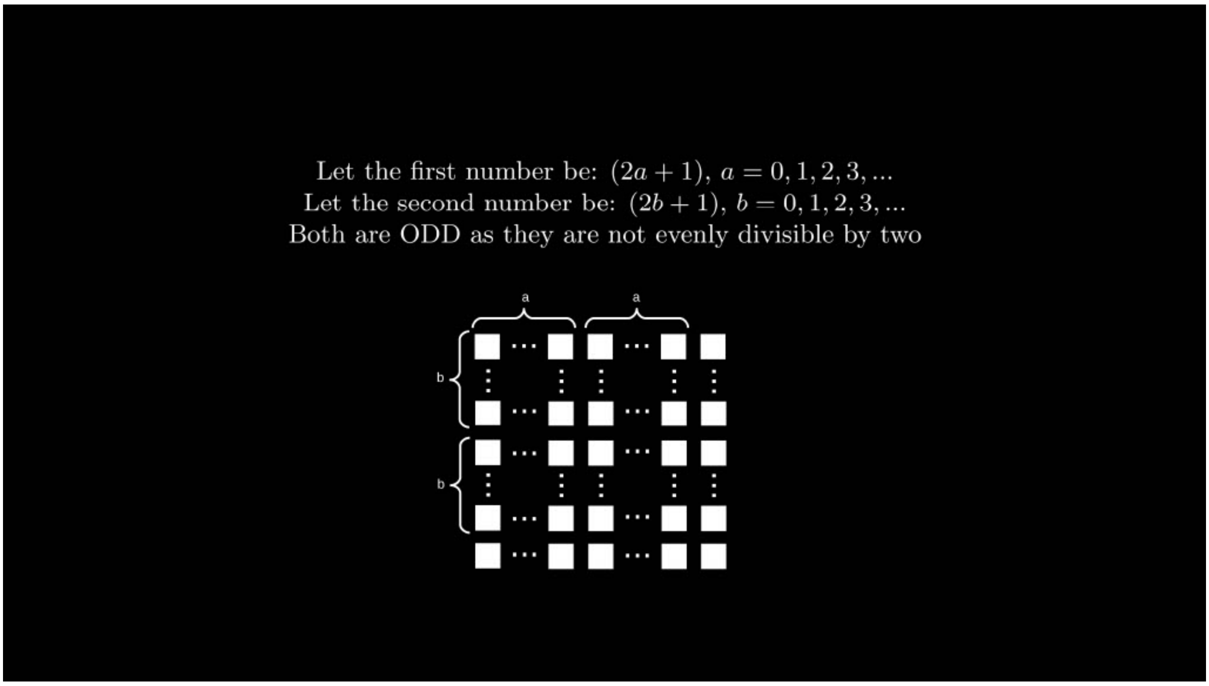
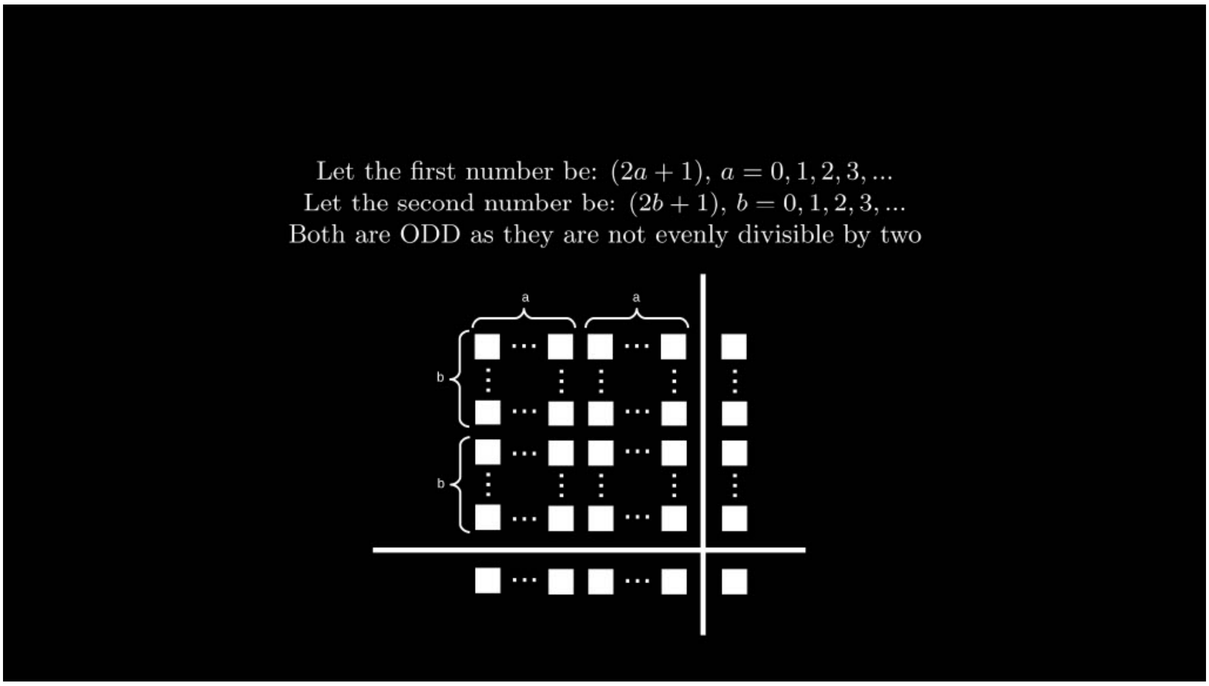
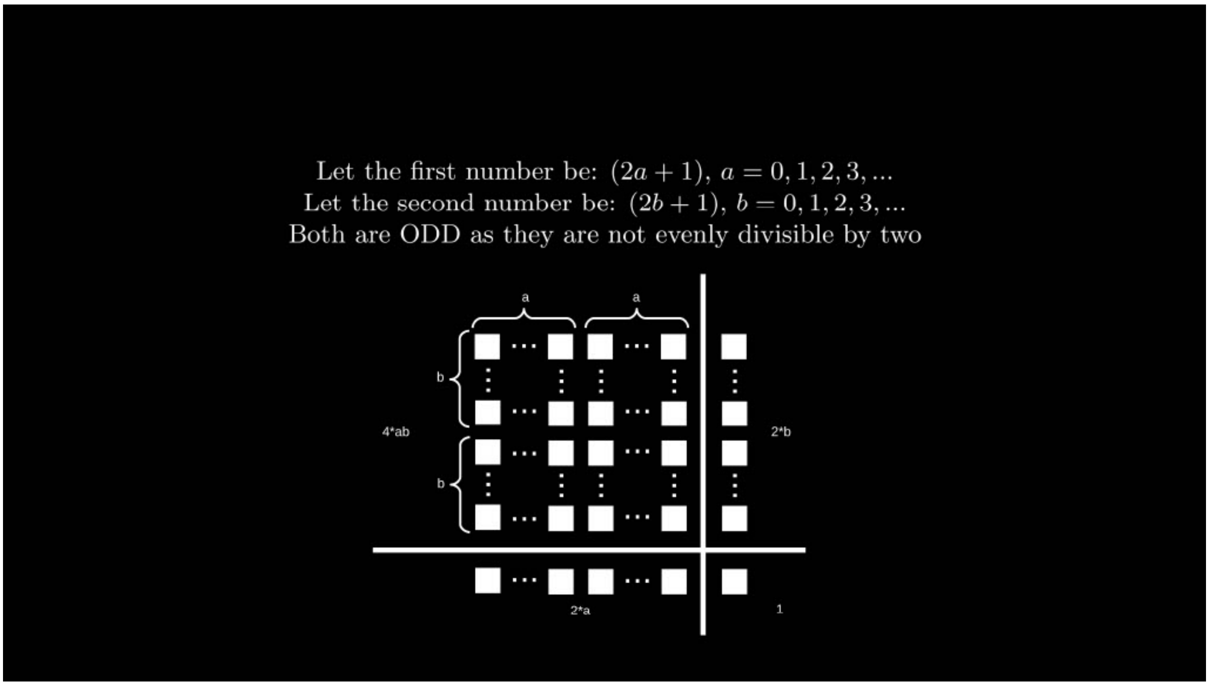
Both are ODD as they are not evenly divisible by two

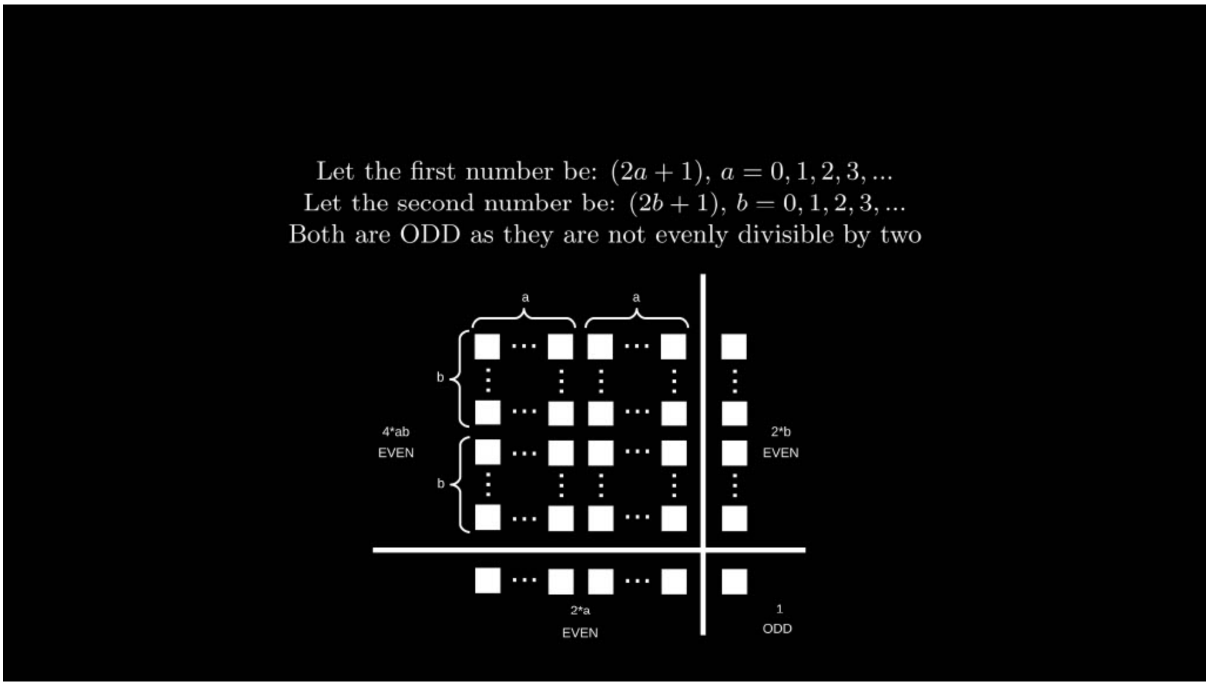

Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, \ldots$
Both are ODD as they are not evenly divisible by two
a
a


Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, \ldots$
Both are ODD as they are not evenly divisible by two
a
a
b
b


Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, \ldots$
Both are ODD as they are not evenly divisible by two
a
a
b
b


Let the first number be: $(2a+1)$, $a=0,1,2,3,...$
Let the second number be: $(2b+1)$, $b=0,1,2,3,...$
Both are ODD as they are not evenly divisible by two
a
a
b
b


Let the first number be: $(2a+1)$, $a=0,1,2,3,...$
Let the second number be: $(2b+1)$, $b=0,1,2,3,...$
Both are ODD as they are not evenly divisible by two
a
a
b
b
4*ab
2*b
2*a
1


Let the first number be: $(2a+1)$, $a = 0, 1, 2, 3, \ldots$
Let the second number be: $(2b+1)$, $b = 0, 1, 2, 3, \ldots$
Both are ODD as they are not evenly divisible by two
a
a
b
b
4*ab
EVEN
2*b
EVEN
2*a
EVEN
1
ODD


7A

Alternating Sums of Odd Numbers:

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = ???$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\text{Case n odd} = 1 - 3 + 5 \ ... - (2n-3) + (2n-1)$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\text{Case n odd} = 1 - 3 + 5 \ ... - (2n-3) + (2n-1)$$

$$= 1 + (5-3) + ... + ((2n-1) - (2n-3))$$

$$
\begin{aligned}
\sum_{k=1}^{n}(2k-1)(-1)^{n-k} &= (1)(-1)^{n-1}+(3)(-1)^{n-2}+(5)(-1)^{n-3}+...+(2n-3)(-1)^{1}+(2n-1)(-1)^{0} \\
\text{Case n odd} &= 1-3+5\ ...-(2n-3)+(2n-1) \\
&= 1+(5-3)+...+((2n-1)-(2n-3)) \\
&= 1+\underbrace{2+...+2}_{\frac{n-1}{2}\text{ times}}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=1}^{n}(2k-1)(-1)^{n-k} &= (1)(-1)^{n-1}+(3)(-1)^{n-2}+(5)(-1)^{n-3}+...+(2n-3)(-1)^{1}+(2n-1)(-1)^{0} \\
\text{Case n odd} &= 1-3+5\ ...-(2n-3)+(2n-1) \\
&= 1+(5-3)+...+((2n-1)-(2n-3)) \\
&= 1+\underbrace{2+...+2}_{\frac{n-1}{2}\text{ times}}
\end{aligned}
$$

$$
= 1+2\cdot\frac{n-1}{2}
$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\begin{aligned}
\text{Case n odd} &= 1 - 3 + 5 \; ... - (2n-3) + (2n-1) \\
&= 1 + (5-3) + ... + ((2n-1) - (2n-3)) \\
&= 1 + \underbrace{2 + ... + 2}_{\frac{n-1}{2} \text{ times}} \\
&= 1 + 2 \cdot \frac{n-1}{2}
\end{aligned}$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = n$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\text{Case n even} = -1 + 3 - 5 + 7\ ... - (2n-3) + (2n-1)$$

$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0}$$

$$\begin{aligned}\text{Case n even} &= -1 + 3 - 5 + 7\ ... - (2n-3) + (2n-1)\\ &= (3-1) + (7-5) + ... + ((2n-1) - (2n-3))\end{aligned}$$

$$
\begin{aligned}
\sum_{k=1}^{n}(2k-1)(-1)^{n-k} &= (1)(-1)^{n-1}+(3)(-1)^{n-2}+(5)(-1)^{n-3}+\ldots+(2n-3)(-1)^{1}+(2n-1)(-1)^{0}\\
\text{Case n even} &= -1+3-5+7\ \ldots-(2n-3)+(2n-1)\\
&= (3-1)+(7-5)+\ldots+((2n-1)-(2n-3))\\
&= \underbrace{2+\ldots+2}_{\frac{n}{2}\ \text{times}}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=1}^{n}(2k-1)(-1)^{n-k} &= (1)(-1)^{n-1}+(3)(-1)^{n-2}+(5)(-1)^{n-3}+\ldots+(2n-3)(-1)^{1}+(2n-1)(-1)^{0}\\
\text{Case n even} &= -1+3-5+7\ \ldots-(2n-3)+(2n-1)\\
&= (3-1)+(7-5)+\ldots+((2n-1)-(2n-3))\\
&= \underbrace{2+\ldots+2}_{\frac{n}{2}\ \text{times}}
\end{aligned}
$$

$$
= 2\cdot\frac{n}{2}
$$

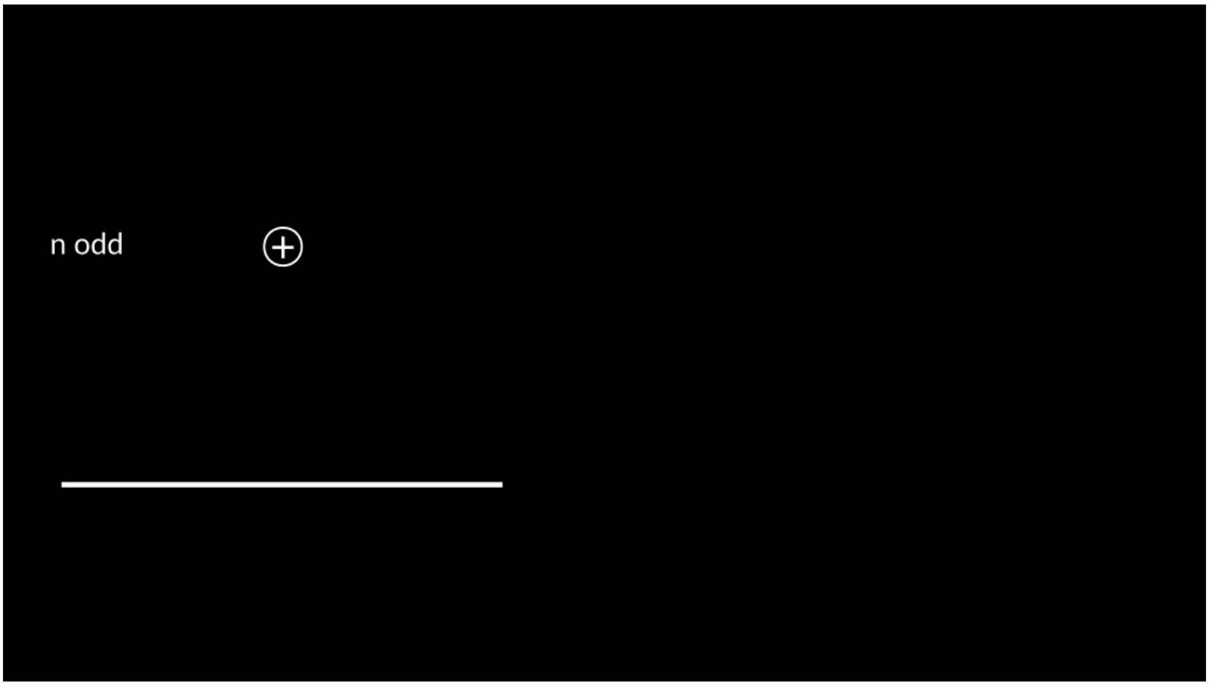
$$
\begin{aligned}
\sum_{k=1}^{n}(2k-1)(-1)^{n-k} &= (1)(-1)^{n-1} + (3)(-1)^{n-2} + (5)(-1)^{n-3} + ... + (2n-3)(-1)^{1} + (2n-1)(-1)^{0} \\
\text{Case n even} &= -1 + 3 - 5 + 7 \ ... - (2n-3) + (2n-1) \\
&= (3-1) + (7-5) + ... + ((2n-1) - (2n-3)) \\
&= \underbrace{2 + ... + 2}_{\frac{n}{2}\ \text{times}} \\
&= 2 \cdot \frac{n}{2} \\
\sum_{k=1}^{n}(2k-1)(-1)^{n-k} &= n
\end{aligned}
$$

7G

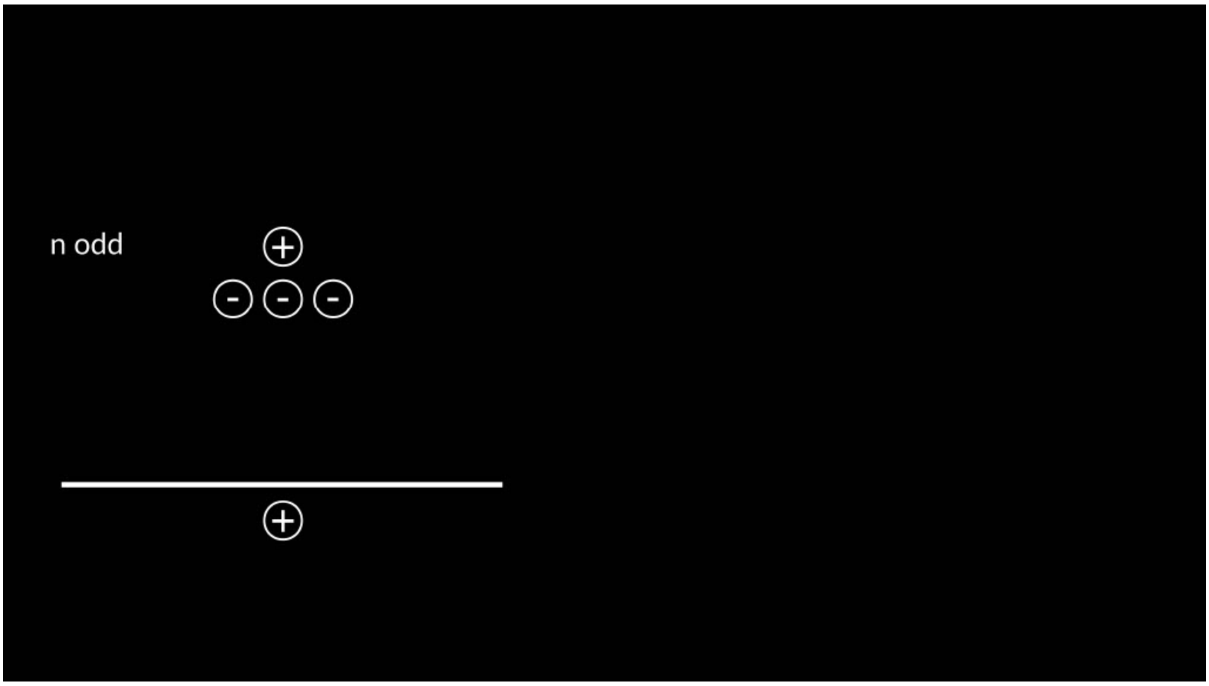
Alternating Sums of Odd Numbers:

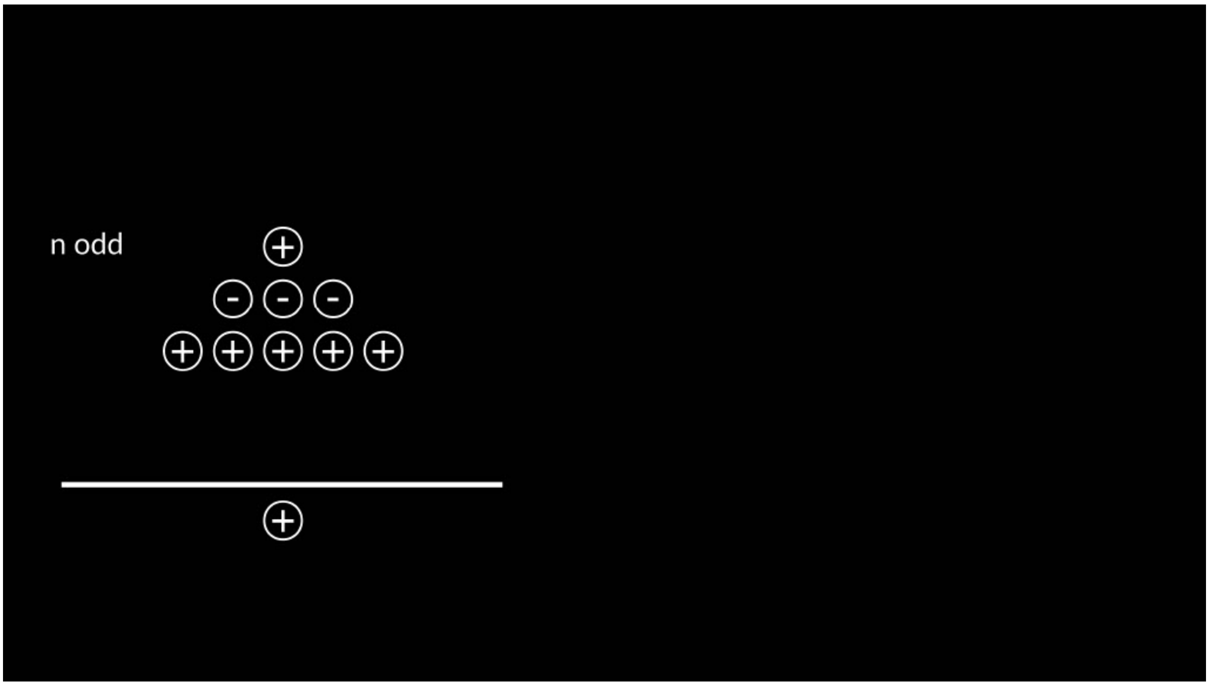
$$\sum_{k=1}^{n}(2k-1)(-1)^{n-k} = ???$$

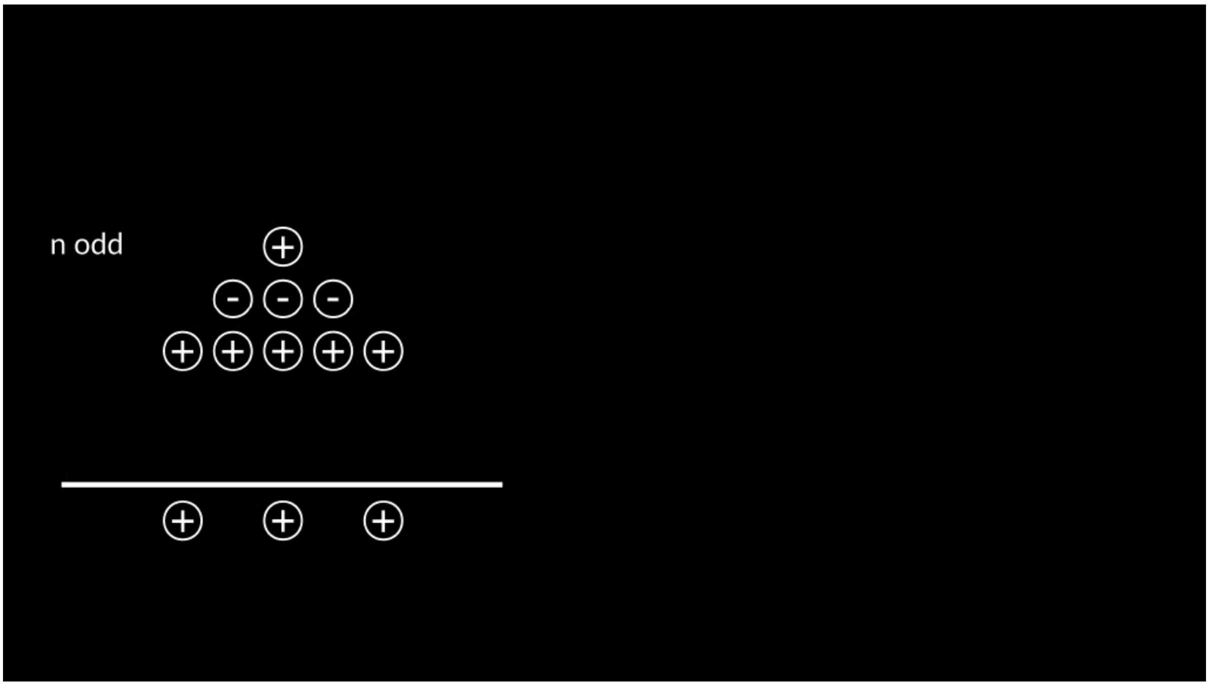
n odd

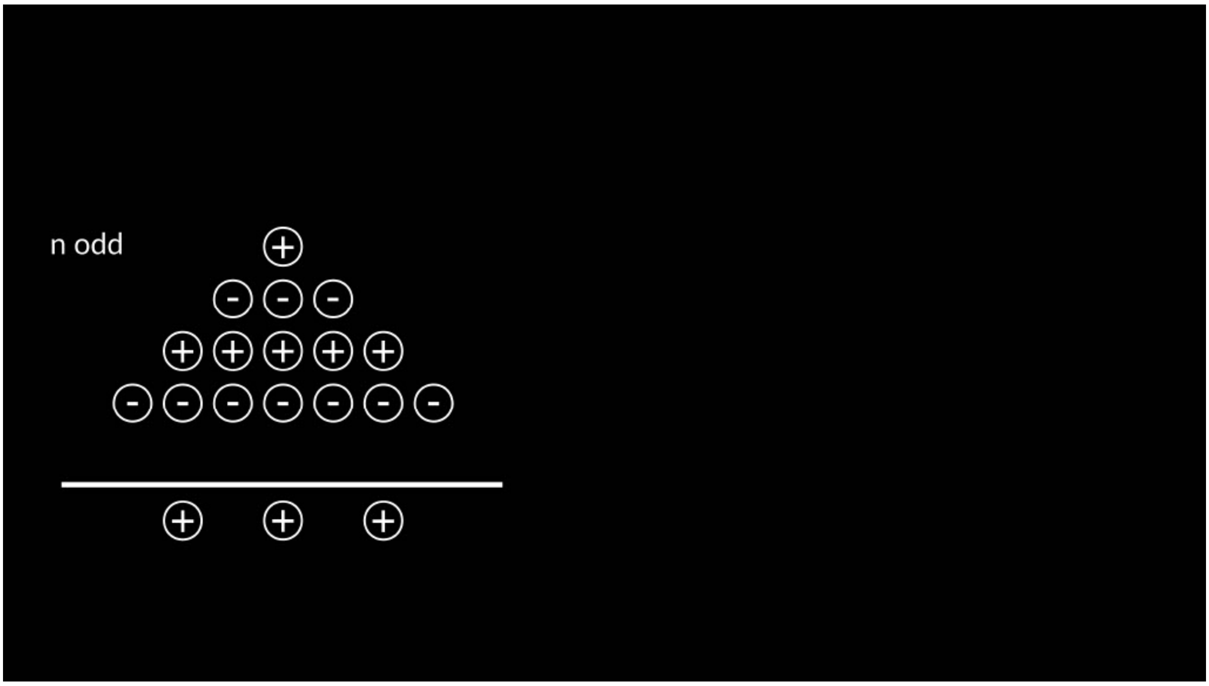

n odd

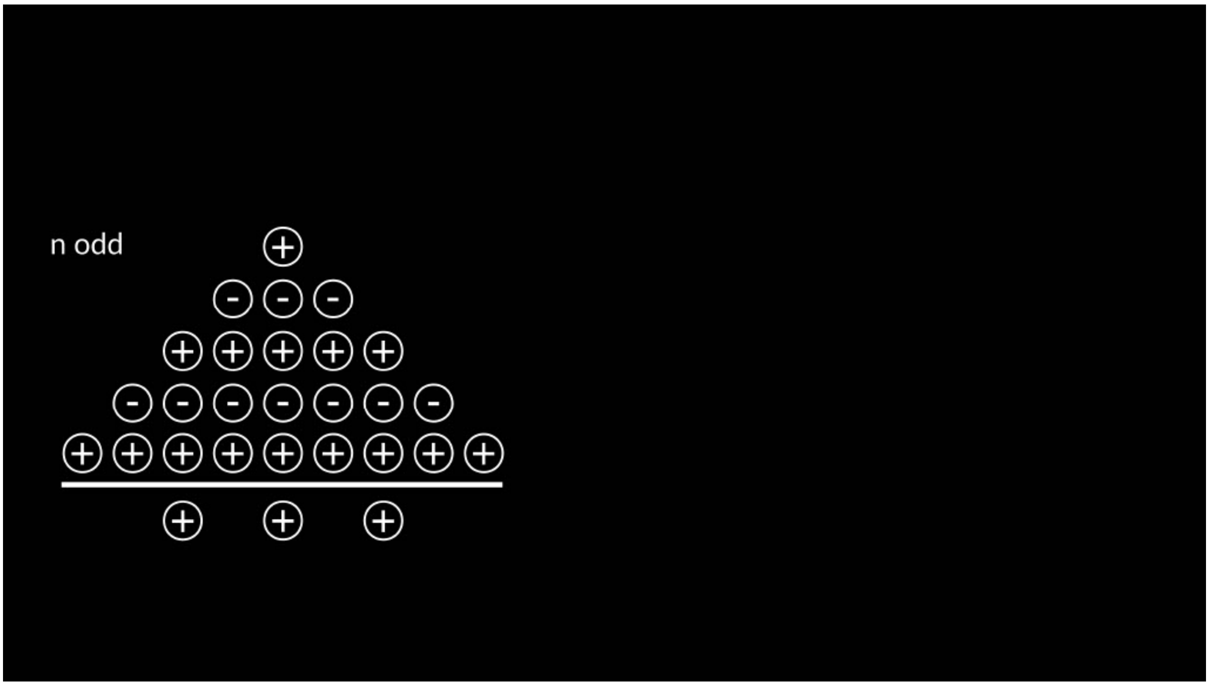

n odd

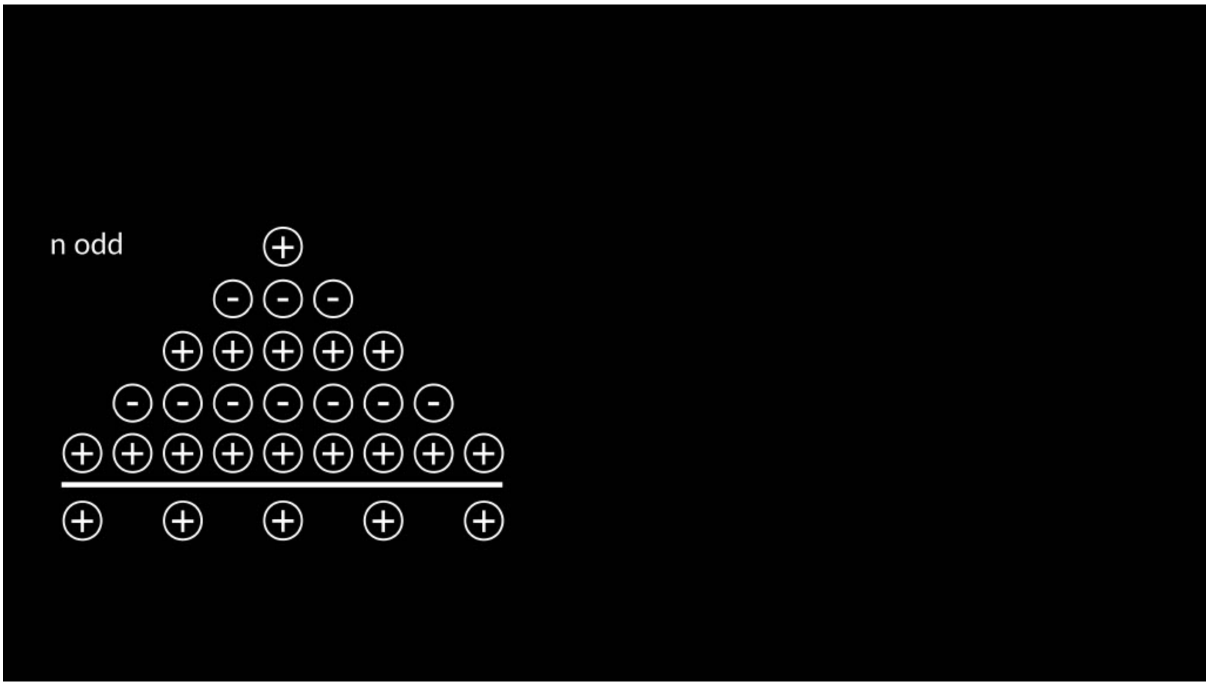

n odd

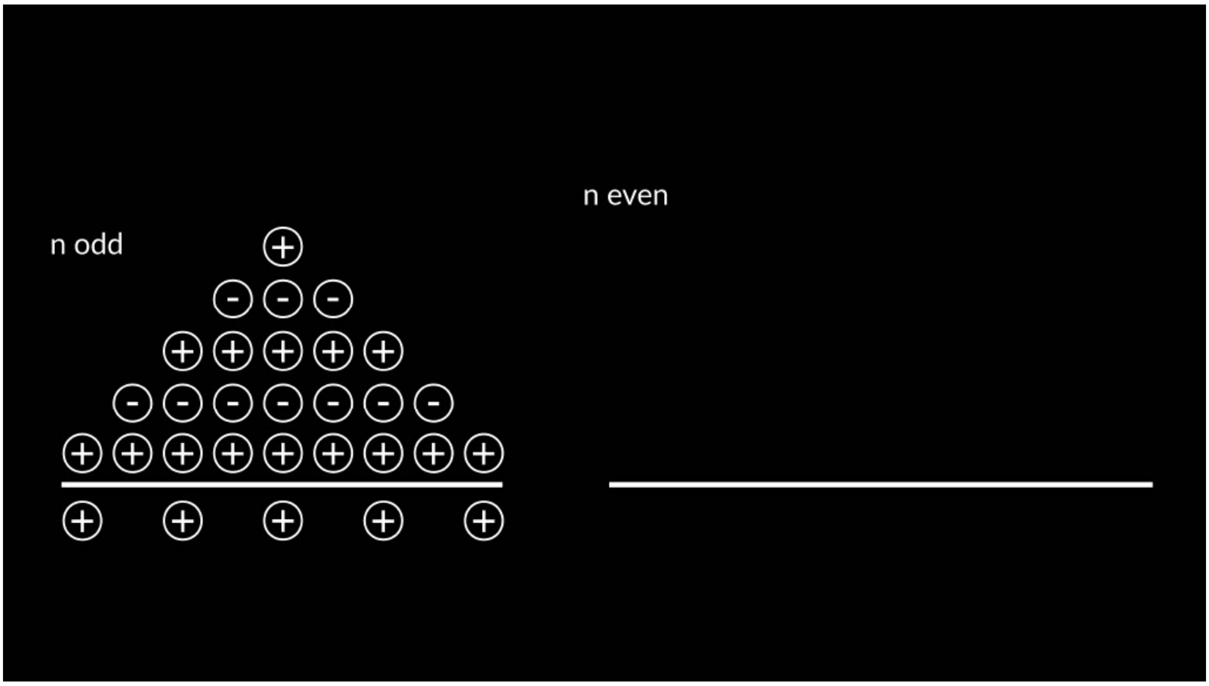

n odd

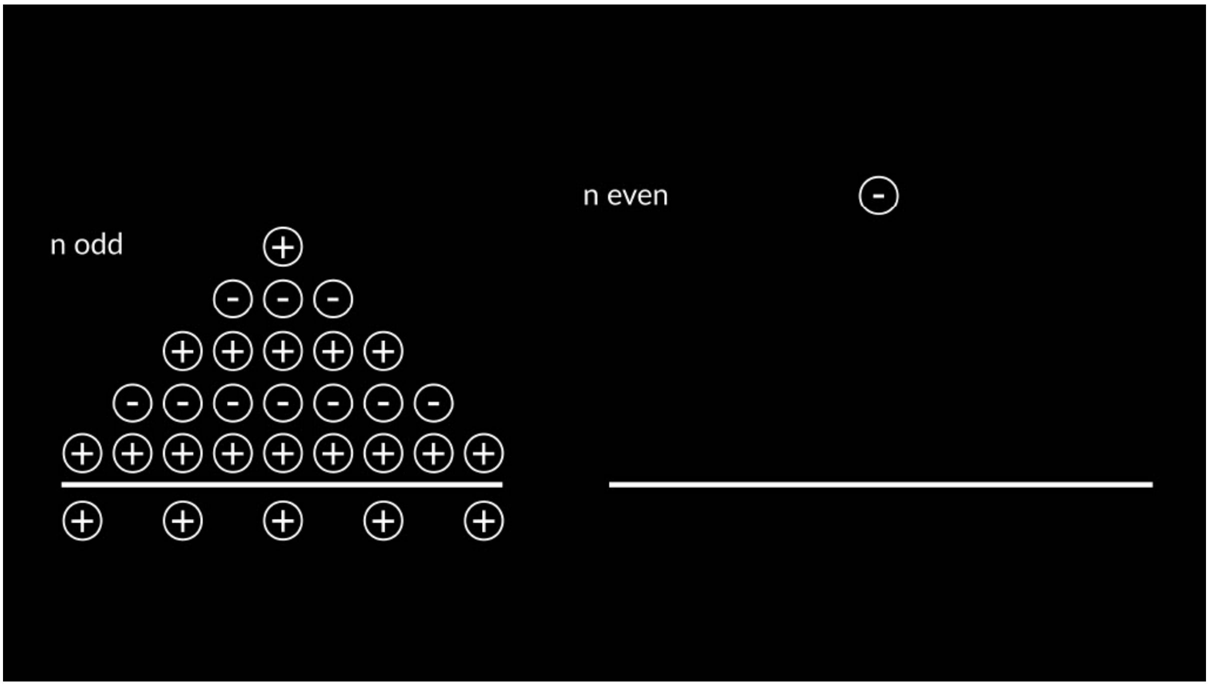

n odd

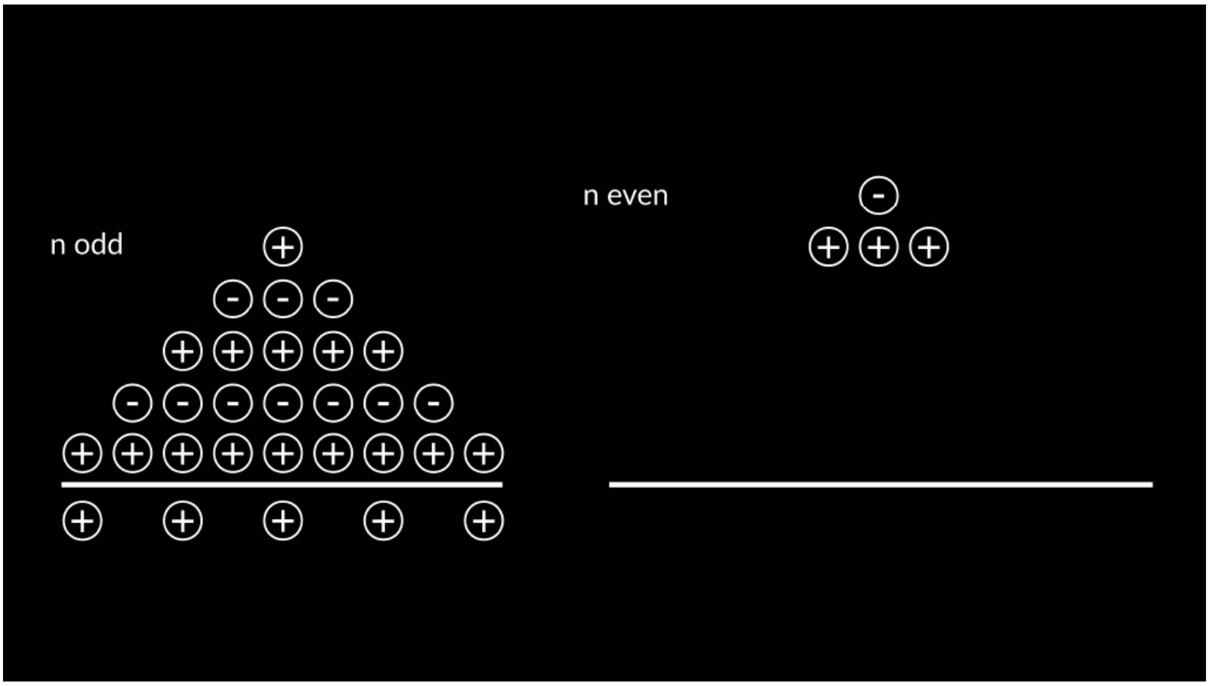

n odd

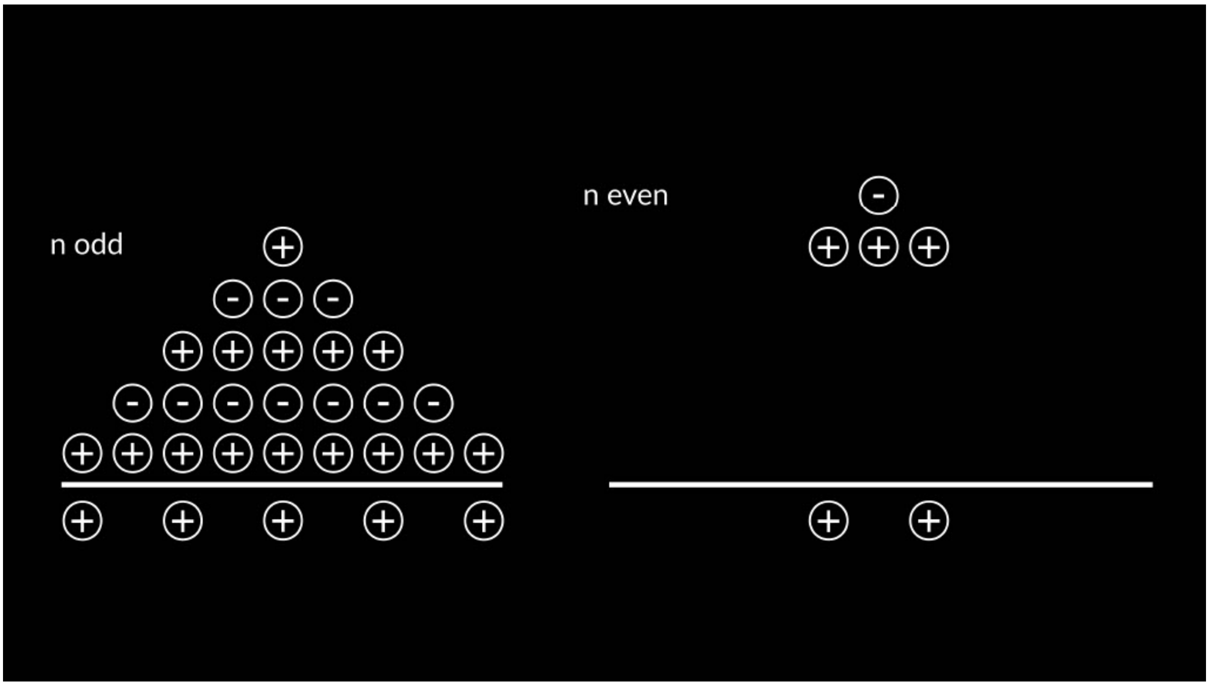

n odd

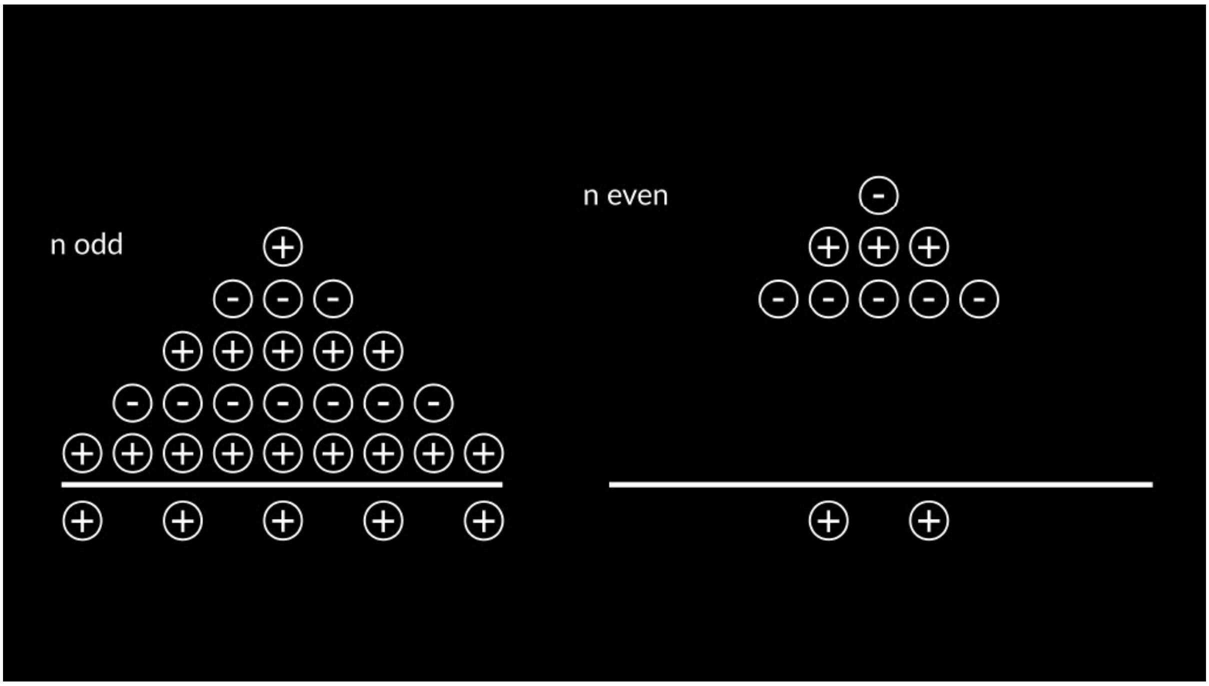

n odd

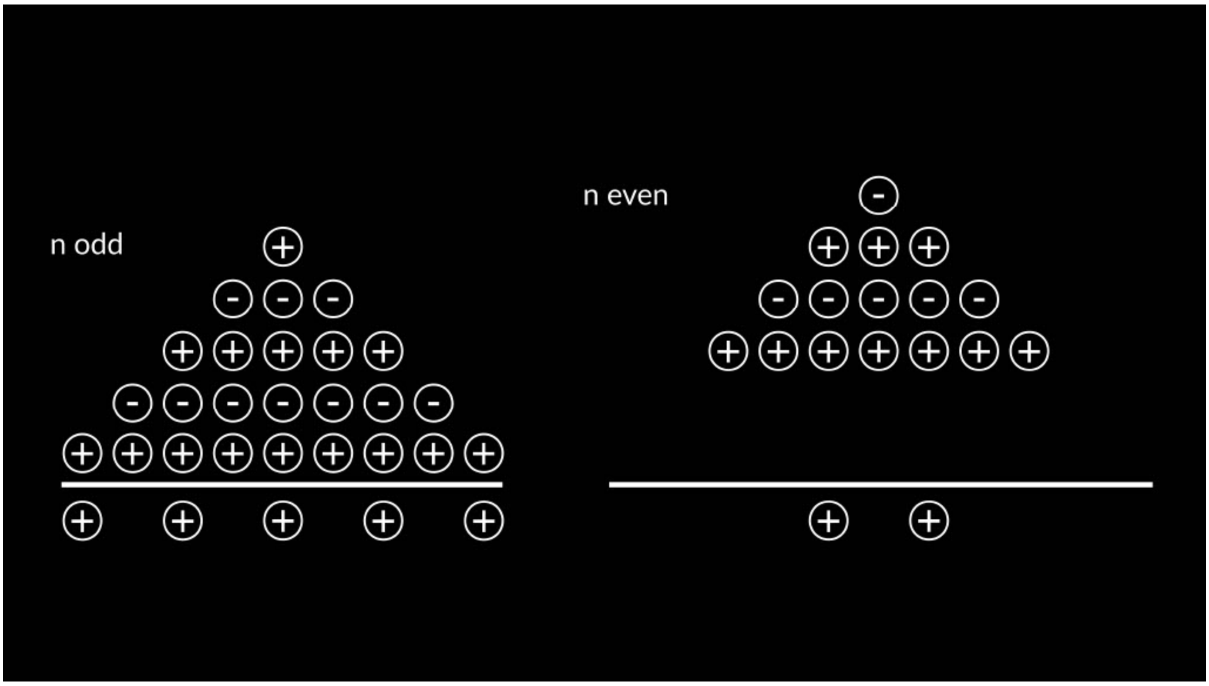

n even
n odd

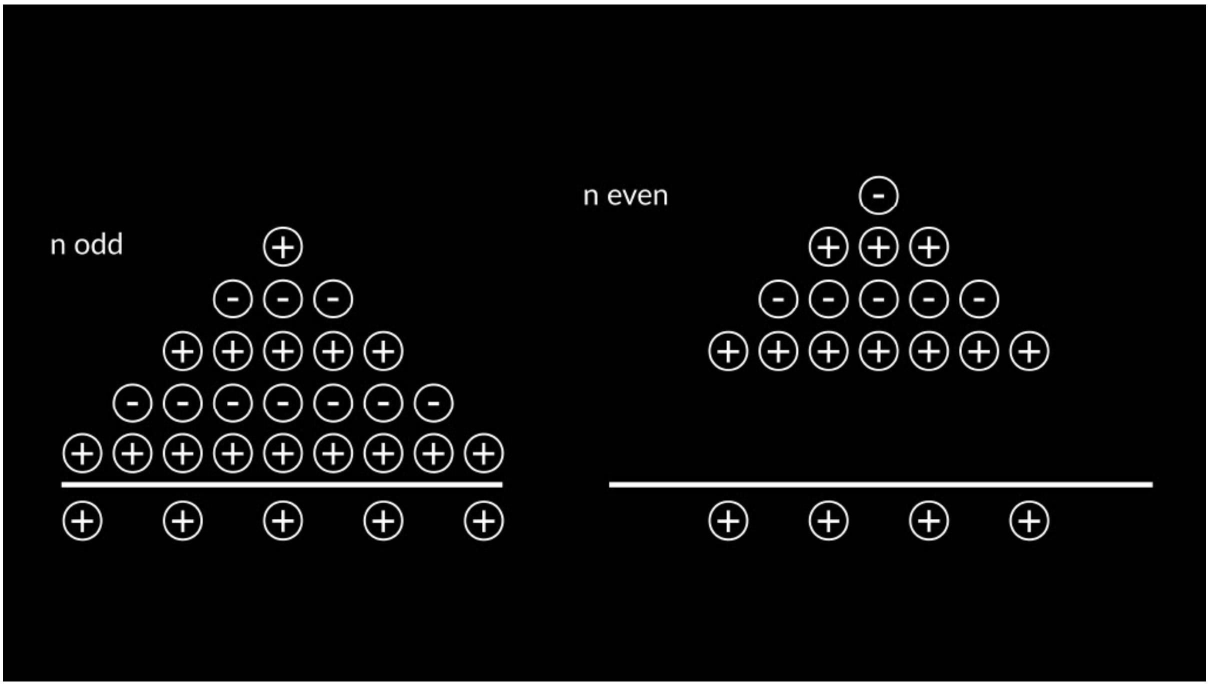

n even
n odd

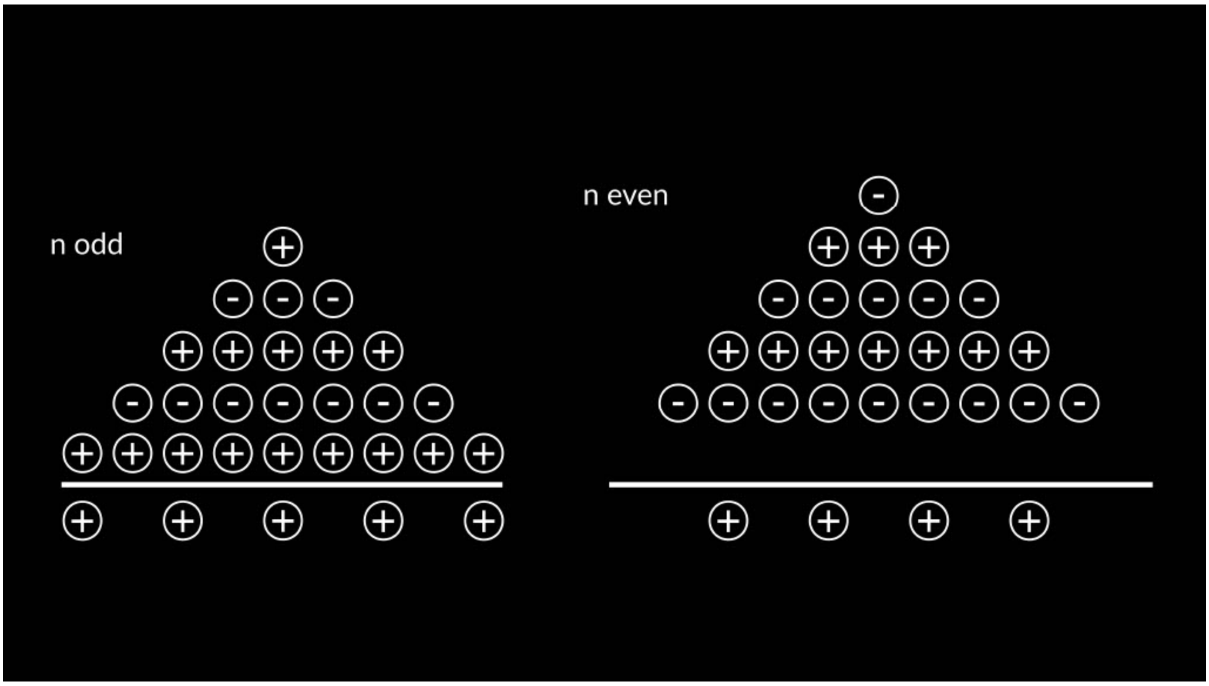

n odd
n even

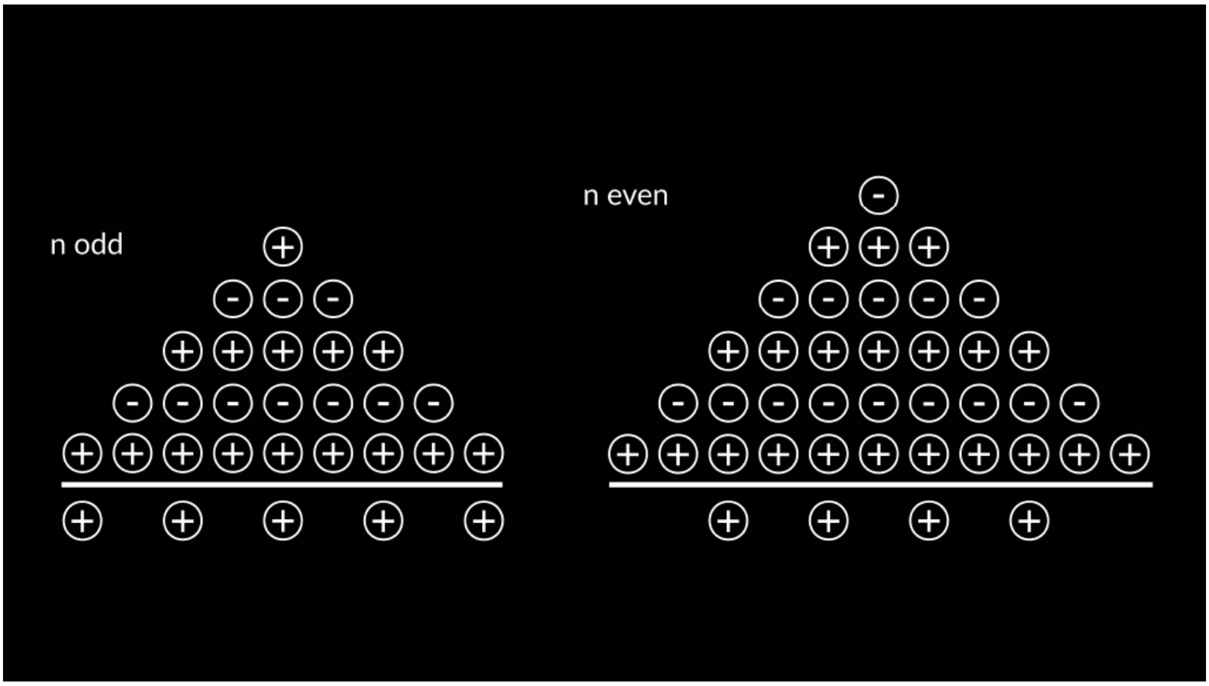

n odd
n even

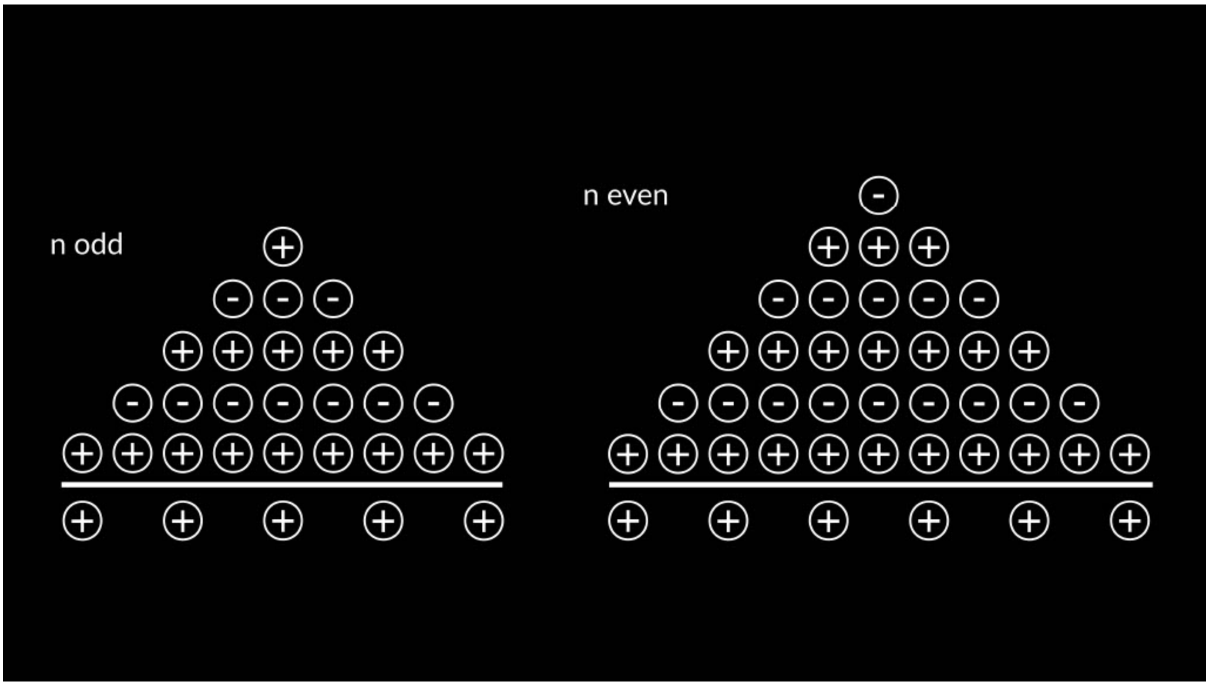

n odd
n even


n odd
n even


n odd
n even


n odd
n even

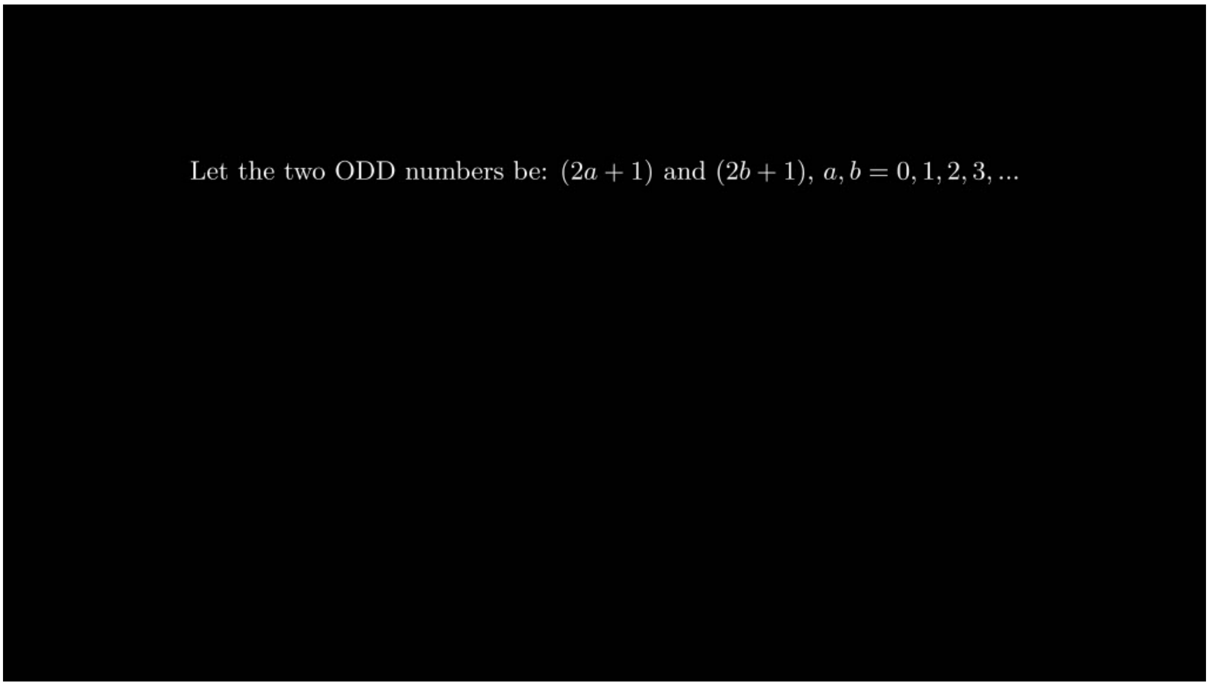

n odd
n even

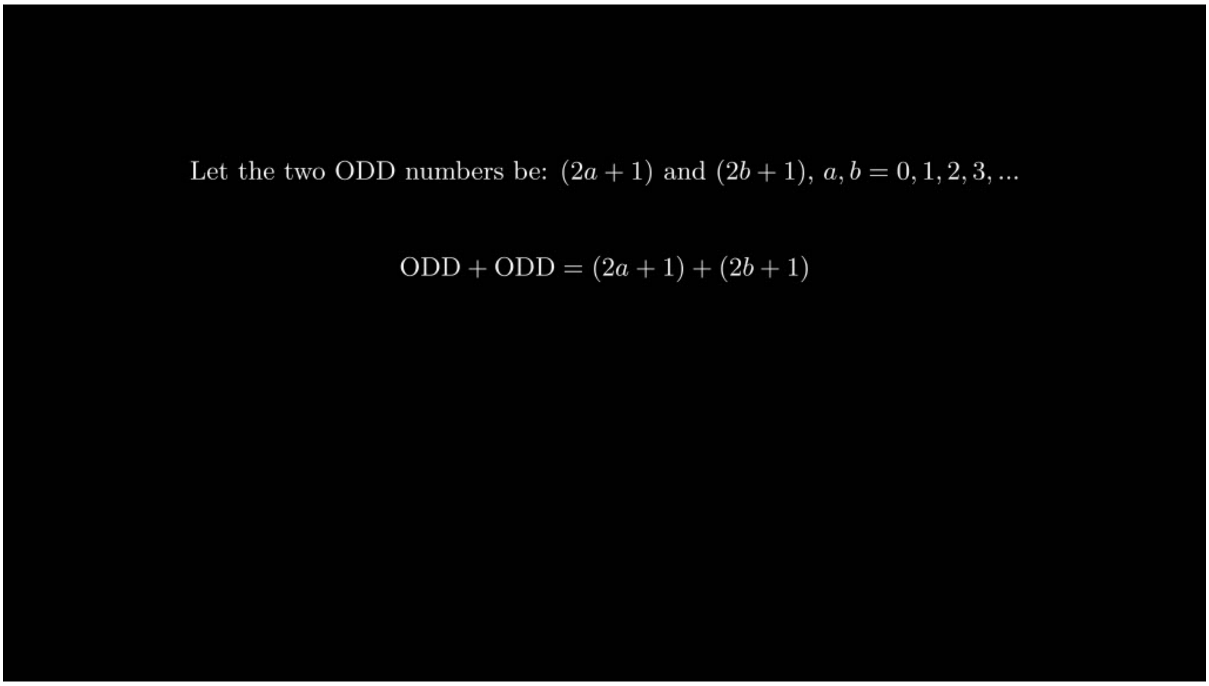

n odd
n even

8A

Is the sum of two odd numbers EVEN or ODD?

ODD + ODD = ???


Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$


Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$
$\text{ODD} + \text{ODD} = (2a+1) + (2b+1)$

Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$

$$\begin{aligned} \text{ODD} + \text{ODD} &= (2a+1) + (2b+1) \\ &= 2a + 2b + 2 \end{aligned}$$

Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$

$$\begin{aligned} \text{ODD} + \text{ODD} &= (2a+1) + (2b+1) \\ &= 2a + 2b + 2 \\ &= 2 \cdot (a+b+1) \end{aligned}$$

Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$

$$\begin{aligned}
\text{ODD} + \text{ODD} &= (2a+1) + (2b+1) \\
&= 2a + 2b + 2 \\
&= 2 \cdot (a + b + 1) \\
&= 2c, \;\; c = 1, 2, 3, \ldots
\end{aligned}$$

Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$

$$\begin{aligned}
\text{ODD} + \text{ODD} &= (2a+1) + (2b+1) \\
&= 2a + 2b + 2 \\
&= 2 \cdot (a + b + 1) \\
&= 2c, \;\; c = 1, 2, 3, \ldots \\
&= \text{EVEN}
\end{aligned}$$

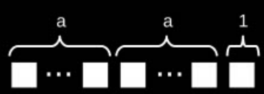
8G

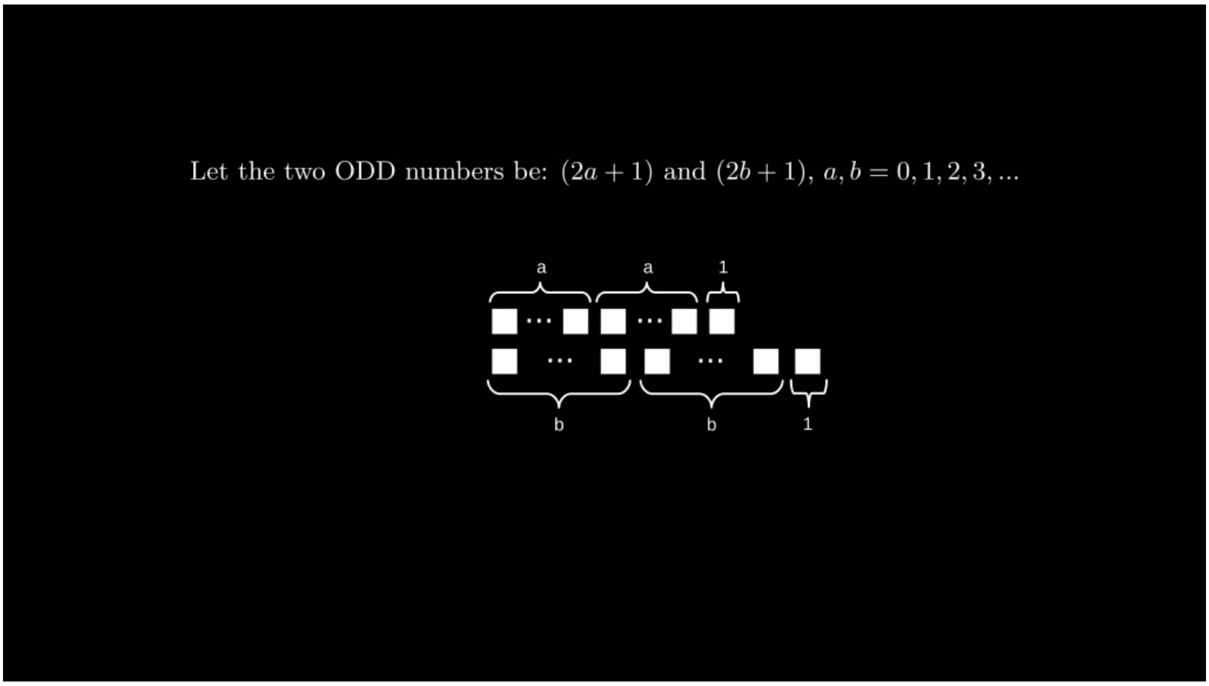
Is the sum of two odd numbers EVEN or ODD?

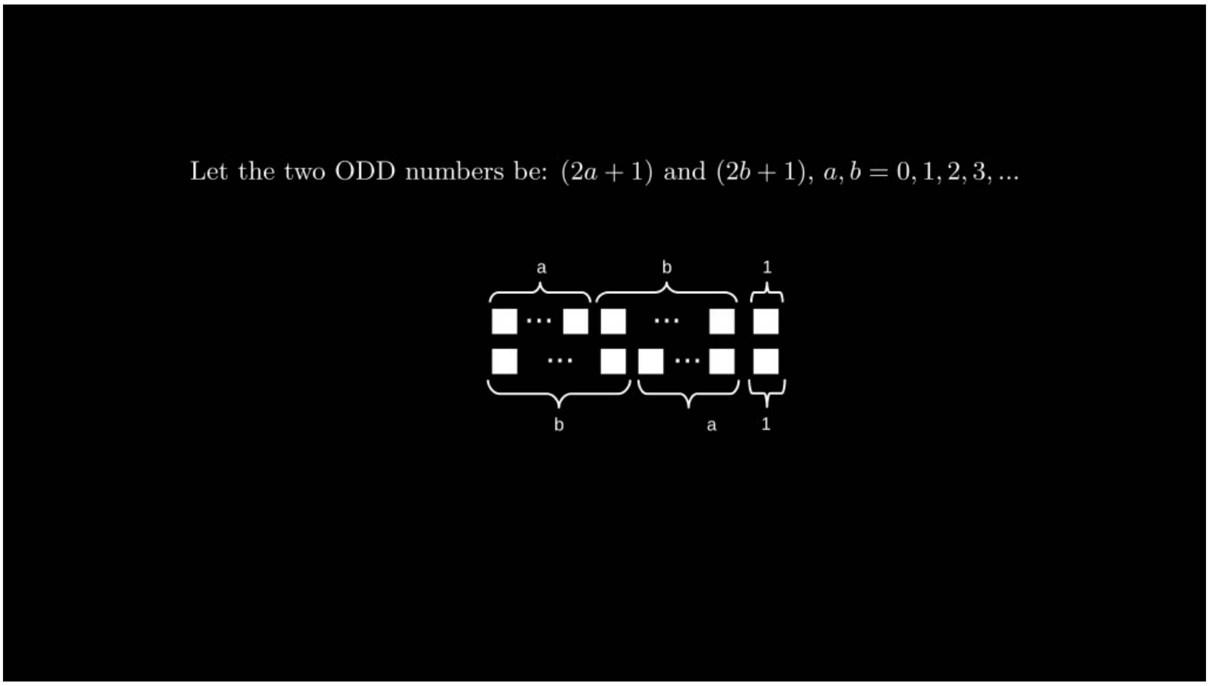
ODD + ODD = ???

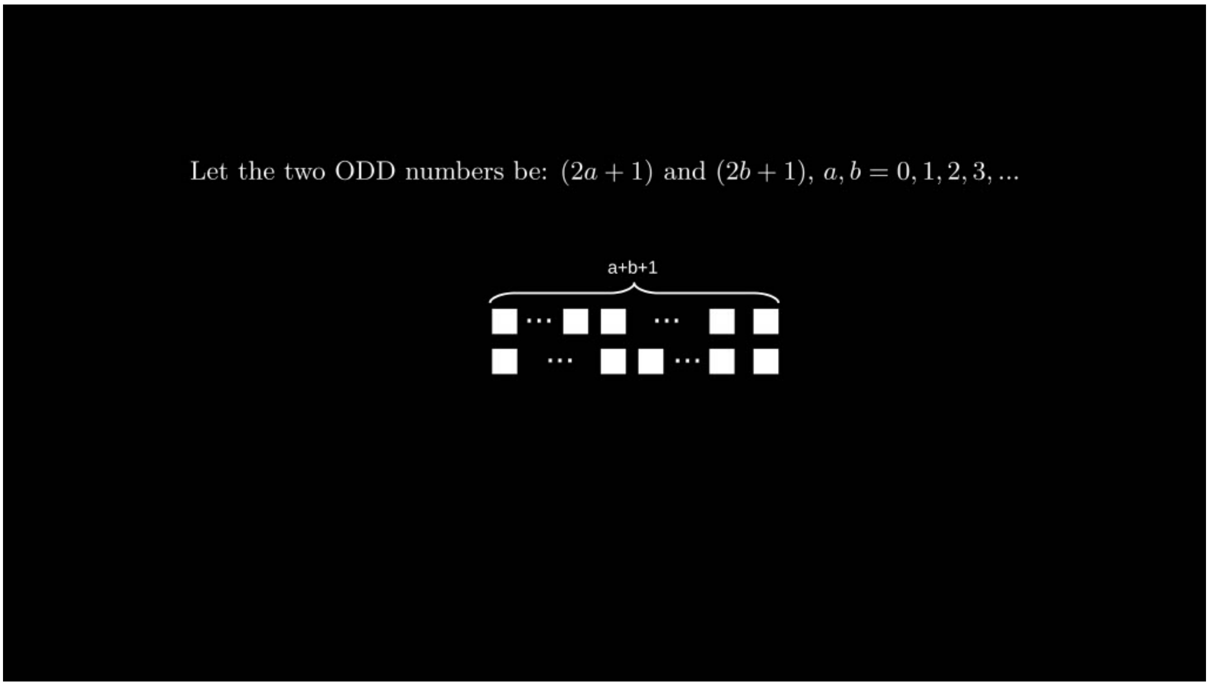
Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$

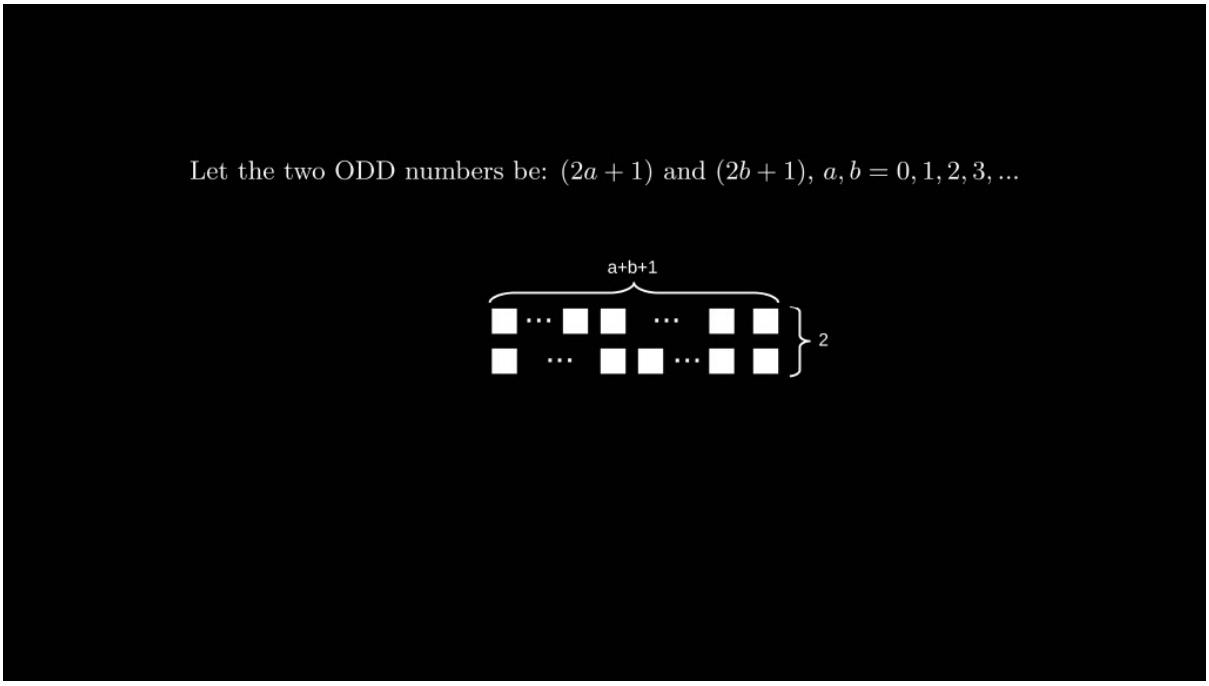
Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$

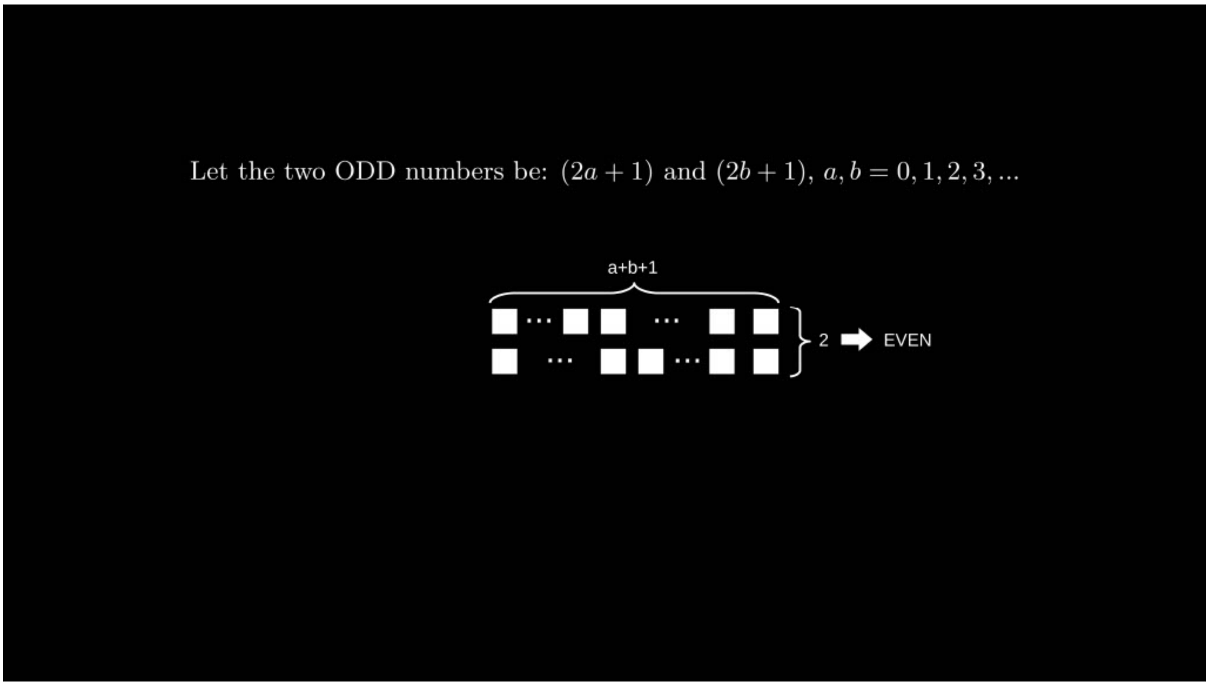



Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$
a
a
1
b
b
1


Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$
a
b
1
b
a
1


Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$
a+b+1


Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$
a+b+1
2


Let the two ODD numbers be: $(2a+1)$ and $(2b+1)$, $a, b = 0, 1, 2, 3, \ldots$
a+b+1
2
EVEN

All Slides from Study 2: The Neural and Physiological Basis of Learning through Problem-Solving followed by Instruction (PS-I).

Welcome

This experiment has 3 parts

Read the instructions carefully. To go on with the experiment click on the *space bar*.

Sometimes you might need to write down your answers, make sure you do it during the indicated time window.

To answer multiple-choice questions, please use the keys “1-6”.

Try to relax,

close your eyes and try not to think of anything

When you hear a sound you can open your eyes again.

Try to relax,

leave your eyes open and try not to think of anything

Football Fever

The organizers of the Premier League Federation have to decide which one of the two players – Mike and Dave – should receive the "The Most Consistent Player" award. The most consistent player is one who scores goals as evenly as possible.

The table on the right shows the number of
goals that each striker scored in an 11-year period.

The organizers agreed to approach this decision *mathematically* by designing a **measure** of **consistency.** They decided to get your help.

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Design <u>as many different measures of consistency as you can</u>.

Your measure of consistency should make use of <u>all data points</u> (scored goals) in the table.

First think about the solutions and then **show all work and calculations on the papers provided.**

All the best, and remember, develop **multiple** ways of measuring consistency!

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Think about the first idea you have, the first way of measuring consistency

Click on "space" when you are ready to write down your work and calculations

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Write down your work and calculations **on the papers provided.**

When you're done, click "space"

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Think about the second idea you have, the second way of measuring consistency

Click on "space" when you are ready to write down your work and calculations

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Write down your work and calculations **on the papers provided.**

When you're done, click "space"

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Think about the third idea you have, the third way of measuring consistency

Click on "space" when you are ready to write down your work and calculations

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Write down your work and calculations **on the papers provided.**

When you're done, click "space"

Year in the League	Mike	Dave
1	13	13
2	11	11
3	15	14
4	12	16
5	16	14
6	12	12
7	16	14
8	14	15
9	17	14
10	14	17
11	14	14

Welcome to the second part of this experiment :)

Here you will get some instructions and tasks with solutions, please work through them in your own pace but make sure not to take breaks until the end of this part.

Press "Space" to continue

Standard Deviation as a Mathematical Measure of Consistency

This part includes 4 problems. Working through these will give you an understanding of **standard deviation**, a commonly used measure of consistency in the sciences, finance, medicine, and elsewhere.

Press “Space” to continue

Problem 1

Math grades on 6 tests for three students: 1, 2, and 3

Note: (worst grade) F - D - C - B - A (best grade)

- **1:** A, B, A, B, A, B
- **2:** C, C, C, C, C, C
- **3:** A, F, A, F, A, F

Which of these three students is the most consistent student?
(press 1, 2, or 3 for the correct answer)

1: A, B, A, B, A, B
2: C, C, C, C, C, C
3: A, F, A, F, A, F

Of these three, 1 has the best grades, while 2 and 3 have the same overall grade on average. However, 2 is the most consistent, because their score does not go up or down.

Press “Space” to continue

Problem 2

Grades on five tests (max score: 20) for students S1 and S2

S1: 12, 13, 14, 15, 16
S2: 12, 14, 14, 14, 16

Who is the more consistent student?
(Press 1 or 2 to give an answer)

One good guess would be that S**2** is more consistent, because their scores are more "even" in the middle: 14, 14, 14. This would be a good qualitative argument. Because we have quantitative scores, we can make a stronger, quantitative argument for who is the more consistent student.

If you look at these two distributions, you can see that both start with 12 and end in 16. Both have the same midpoint, or median, of 14. How about the average, or mean?

Average or mean (*M*) for S**1** is:

$$\frac{12 + 13 + 14 + 15 + 16}{5} = \mathbf{14}$$

M for S**2** is also 14.

Press "Space" to continue

The best way to calculate consistency is to look at the **deviations** around the mean.

A "deviation" is simply the distance between two data points: how far apart they are. For example:

"12 **deviates** from 16 by **-4** because 12 – 16 = -4"

Press "Space" to continue

Problem 2, continued

Grades on five tests (max score: 20) for students S1 and S2

S1: 12, 13, 14, 15, 16

S2: 12, 14, 14, 14, 16

Calculate the deviations around the mean for **S1**. The mean is 14, so the deviations are:

- 12 deviates from the mean (14) by 12 – 14 = **-2**
- 13 deviates from the mean (14) by 13 – 14 = **-1**
- 14 deviates from the mean (14) by 14 – 14 = **0**
- 15 deviates from the mean (14) by 15 – 14 = **1**
- 16 deviates from the mean (14) by 16 – 14 = **2**

The deviations around the mean for S**1** are -2, -1, 0, 1, and 2.

Using the same method, deviations around the mean for S**2** are -2, 0, 0, 0, and 2.

Press "Space" to continue

Deviations around the mean are useful because they tell us something about how far away the data points are from the center (mean). It would be even better if we had one single value, instead of five deviations.

Ideally, we could take the average of the deviations, but this doesn't work, because negative and positive values cancel each other.

For example, the average deviation around the mean for S**1** would be (-2 + -1 + 0 + 1 + 2) / 5 = 0.

Check for S**2** yourself.

Press "Space" to continue

We use the following formula to calculate the "average" deviation around the mean:

$$\sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Where $\bar{x}$ represents the mean, $\sum$ represents the summation sign, and N represents the total number of values. It looks somewhat complicated, but it is simply a method of calculating the average deviation around the mean while preventing positives and negative deviations from cancelling each other.

How and why does this formula work as a measure of consistency?

Read carefully the following instructions.

Press "Space" to continue

Start by calculating the mean $\bar{x}$. You can think of the mean as a fixed reference point that captures some information about all the data points. In our case, the mean for S1 is 14. (Same for S2.)	$\bar{x}$
Calculate the deviation between each data point and the mean. This tells us how far away each point is from the center. For S1, this is -2, -1, 0, 1, and 2. Next, square each of these deviations. This makes the deviations positive, so positive and negative differences won't cancel out. For S1, we have 4, 1, 0, 1, and 4. (For S2, we have 4, 0, 0, 0, and 4.)	$(x_i - \bar{x})^2, i = 1, 2, \ldots, N$
Sum the squared deviation. For S1, the sum is 4 + 1 + 0 + 1 + 4 = 10. (For 2, the sum is 8.)	$\sum_{i=1}^{N}(x_i - \bar{x})^2$
Divide by the number of deviations. This gives us the average, and allows comparison of different sample sizes. For S1, the average value of squared deviations is 10/5 = 2. For S2, this value would be 8/5 = 1.6.	$\frac{1}{N} \cdot \sum_{i=1}^{N}(x_i - \bar{x})^2$
Finally, take the square root of the average of the squared deviations. This reverts units to the original size, before we squared them. This final value is called the standard deviation. For S1, the standard deviation of test scores is $\sqrt{2}$ or about 1.41. (For S2, this is $\sqrt{1.6}$ or about 1.26.)	$\sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N}(x_i - \bar{x})^2}$

The standard deviation for S**2** (1.26) is smaller than the standard deviation for S**1** (1.41).

A **smaller** standard deviation means that the data points are, on average, closer to the center (mean), and less spread out. This is a mathematical argument that S**2** is **more consistent** than S**1**.

Press “Space” to continue

Problem 3

Ice hockey teams scored the following across five games

H1: 2, 4, 6, 8, 10
H2: 2, 4, 6, 8, 12

Who is the more consistent team?
Use the standard deviation formula to calculate the answer on your own. Press *space* and calculate.

Problem 3

Ice hockey teams scored the following across five games

H1: 2, 4, 6, 8, 10
H2: 2, 4, 6, 8, 12

Who is the more consistent team? Use the standard deviation formula

$$\sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N}(x_i - \bar{x})^2}$$

to calculate the answer on your own and answer with key 1 or 2 for the correct answer.

The mean for H1 is $\frac{2+4+6+8+10}{5} = \mathbf{6}$	$\boldsymbol{\bar{x}}$
Calculate the deviation between each data point and the mean. For H1, the deviations are: -4, -2, 0, 2, and 4.	$\boldsymbol{(x_i - \bar{x})^2, i = 1, 2 \ldots N}$
Sum the squared deviation. For H1, this is: $(-4)^2 + (-2)^2 + 0^2 + 2^2 + 4^2 = 40$	$\boldsymbol{\sum_{i=1}^{N}(x_i - \bar{x})^2}$
Take the average. For H1, the average value of squared deviations is 40/5 = 8.	$\frac{1}{N} \cdot \sum_{i=1}^{N}(x_i - \bar{x})^2$
Finally, take the square root of the average of the squared deviations to reduce units to the original format. For H1, the standard deviation is $2\sqrt{2}$ or about 2.83.	$\sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N}(x_i - \bar{x})^2}$

Repeating this procedure for **H2**, the standard deviation is about 3.44.

The standard deviation for **H1** (2.83) is smaller than the standard deviation for **H2** (3.44).

A smaller standard deviation means more consistency, because the data points are on average closer to the center.

This means that **H1** is a more consistent team than **H2**.

Press "Space" to continue

Problem 4

Daily temperature over 5 days in two cities C1 and C2

C1: 0, 2, 4, 6, 8
C2: 0, 2, 4, 6, 13

Which city has a more consistent temperature during this time?

Use the standard deviation formula to calculate the answer on your own.
Press *space* and calculate.

Problem 4

Daily temperature over 5 days in two cities C1 and C2

C1: 0, 2, 4, 6, 8
C2: 0, 2, 4, 6, 13

Which city has a more consistent temperature during this time?

$$\sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

Use the standard deviation formula to calculate the answer on your own and answer with key 1 or 2 for the correct answer.

The mean for C1 is $\frac{0+2+4+6+8}{5} = \mathbf{4}$	$\bar{x}$
Calculate the deviation between each data point and the mean. For C1, the deviations are: -4, -2, 0, 2, and 4.	$(x_i - \bar{x})^2, i = 1, 2 \ldots N$
Sum the squared deviation. For C1, this is: $(-4)^2 + (-2)^2 + 0^2 + 2^2 + 4^2 = 40$	$\sum_{i=1}^{N} (x_i - \bar{x})^2$
Take the average. For C1, the average value of squared deviations is 40/5 = 8.	$\frac{1}{N} \cdot \sum_{i=1}^{N} (x_i - \bar{x})^2$
Finally, take the square root of the average of the squared deviations to reduce units to the original format. For C1, the standard deviation is $2\sqrt{2}$ or about 2.83.	$\sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} (x_i - \bar{x})^2}$

Repeating this procedure for **C2**, the standard deviation is about 4.47.
The standard deviation for **C1** (2.83) is smaller than the standard deviation for **C2** (4.47).

This means that **C1** has a more consistent temperature than **C2**.

Press “Space” to continue

Welcome to the third part of this experiment :)

Please solve the following questions. Take your time but do not take a break in between.
Use the keys 1,2,3,4, and 5 to answer the questions.

1) The owners of two cinemas, A and B, argue that their own cinema enjoys a more consistent attendance. They collected the daily attendance of their cinemas for 11 random days. The results of their data collection are shown below:

	Cinema A	Cinema B
Mean	72	75
Standard Deviation	10	14

Which cinema do you think has a <u>more consistent</u> attendance?
1. Cinema A
2. Cinema B
3. Both have equally consistent attendance.
4. None of the above.

2)
Calculate the standard deviation (SD) of the following set of marks on a statistics test:
30, 60, 50, 40, 70

1. 10
2. 10 $\sqrt{2}$
3. 20
4. 20 $\sqrt{2}$

3.) In calculating SD, explain:

i) Why is it important to square the deviations?

It is important…

1. because squaring gives each deviation its proportionate place in the result
2. in order to reduce the impact of outliers (extreme values) in the data
3. since squaring ignores small deviations and emphasizes large deviations
4. so that the values do not cancel each other out

ii) Why is it important to divide the sum of squared deviations by *n*?

It is important…

1. in order to allow comparison of samples of different sizes
2. because dividing by N means that we are capturing the complete data distribution
3. because dividing by N compensates for the overestimation in variance because of squared deviations
4. so that the values do not cancel each other out

4) Another student, Student A, used the following method to answer the question "Calculate the consistency of the following set of marks on a statistics test: 30, 60, 50, 40, 70"

$$\sqrt{\frac{(30-70)^2+(60-70)^2+(50-70)^2+(40-70)^2+(70-70)^2}{5}}=10\sqrt{6}$$

How does Student A's method compare with the **standard deviation** formula. Which one is better?

1. Student A's Formula
2. SD-Formula
3. It's the same, just written differently

5a) Consider the following six datasets:

A: {1, 5, 6, 10}

B: {4, 4, 4, 4}

C: {101, 102, 103, 104}

D: {7, 8, 9, 10}

E: {1, 2, 9, 10}

F: {1, 2, 3, 4}

Which dataset has the **smallest** SD?

5b) Consider the following six datasets:

A: {1, 5, 6, 10}

B: {4, 4, 4, 4}

C: {101, 102, 103, 104}

D: {7, 8, 9, 10}

E: {1, 2, 9, 10}

F: {1, 2, 3, 4}

Which dataset has the **largest** SD?

5c) Consider the following six datasets:

A: {1, 5, 6, 10}

B: {4, 4, 4, 4}

C: {101, 102, 103, 104}

D: {7, 8, 9, 10}

E: {1, 2, 9, 10}

F: {1, 2, 3, 4}

Which datasets have the same SD?

A. B and C

B. A, C and E

C. C, D, and F

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4. Use this information to answer Questions 4(a) to 4(e).

(a) If each of the five numbers is increased by 2, what are the new mean and SD?

1. M = 7, SD = 4
2. M = 9, SD = 4
3. M = 7, SD = 6
4. M = 9, SD = 6

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4. Use this information to answer Questions 4(a) to 4(e).

(b) If each of the five numbers is multiplied by 5, what are the new mean and SD?

1. M = 7, SD = 4
2. M = 35, SD = 4
3. M = 7, SD = 20
4. M = 35, SD = 20

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4. Use this information to answer Questions 4(a) to 4(e).

(c) If the numbers 5 and 9 are added to the dataset to make it a data set with seven numbers, how will the mean and SD change?

Mean
1. Unchanged
2. Increases
3. decreases

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4. Use this information to answer Questions 6(a) to 6(e).

(c) If the numbers 5 and 9 are added to the dataset to make it a data set with seven numbers, how will the mean and SD change?

SD:
1. Unchanged
2. Increases
3. decreases

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4

(d) If the numbers 1 and 13 are added to the dataset to make it a data set with seven numbers, how will the mean and SD change?

Mean

1. unchanged
2. Increases
3. decreases

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4

(d) If the numbers 1 and 13 are added to the dataset to make it a data set with seven numbers, how will the mean and SD change?

SD

1. Unchanged
2. Increases
3. decreases

6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4

(e) If the numbers 3 and 11 were added to the dataset to make it a dataset with seven numbers, how will the mean and SD change?

Mean

1. unchanged
2. Increases
3. decreases

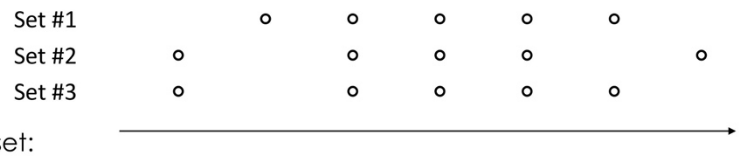
6) A data set consisting of **five** numbers has mean, M = 7, and standard deviation, SD = 4

(e) If the numbers 3 and 11 were added to the dataset to make it a dataset with seven numbers, how will the mean and SD change?

SD

1. Unchanged
2. Increases
3. decreases

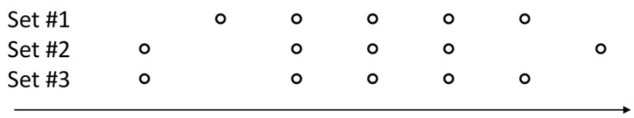
7) Below are sets of five unknown values. The values match the number line, that is, values to the right are larger than values to the left.



The **largest SD** belongs to set:

1. Set #1
2. Set #2
3. Set #3
4. All SDs are the same
5. Cannot be determined from the graphs alone.

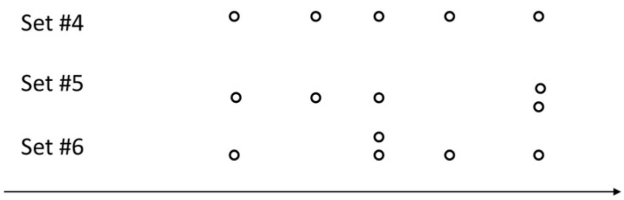
7) Below are sets of five unknown values. The values match the number line, that is, values to the right are larger than values to the left.



The **smallest SD** belongs to set:

1. Set #1
2. Set #2
3. Set #3
4. All SDs are the same
5. Cannot be determined from the graphs alone.

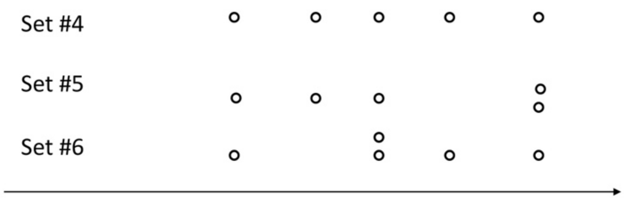
8) Below are sets of five unknown values. The values match the number line, that is, values to the right are larger than values to the left.



The **largest SD** belongs to set:

1. Set #4
2. Set #5
3. Set #6
4. All SDs are the same
5. Cannot be determined from the graphs alone.

8) Below are sets of five unknown values. The values match the number line, that is, values to the right are larger than values to the left.



The **smallest SD** belongs to set:

1. Set #4
2. Set #5
3. Set #6
4. All SDs are the same
5. Cannot be determined from the graphs alone.

9) An equal number of students competed in the 100m sprint and 100m swim finals. The timings (in seconds) of the champions of the 100m sprint and 100m swim are shown below, as are the average timings and the SDs of the finalists in the two competitions.

	100m sprint	100m swim
Champion	11s	40s
Average of the Finalists, M	12s	45s
SD of the Finalists	1s	10s

Assuming all else being equal, between the two champions, who is the better performer?

1. The sprint champion
2. The swim champion
3. Both
4. Not enough information to decide

10) David's scores for Mathematics, Physics and Chemistry in the final examinations are given below. His class's performance for the three subjects is also given below:

	Mathematics	Physics	Chemistry
David's Score	95	90	85
Class Average, M	80	80	80
Class SD	15	5	4

Relative to his class, in which subject did David perform **the best**?

1) Mathematics
2) Physics
3) Chemistry

11) David's scores for Mathematics, Physics and Chemistry in the final examinations are given below. His class's performance for the three subjects is also given below:

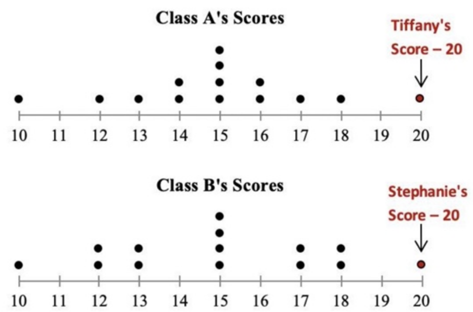
	Mathematics	Physics	Chemistry
David's Score	95	90	85
Class Average, M	80	80	80
Class SD	15	5	4

Relative to his class, in which subject did David perform **the worst**?

1) Mathematics
2) Physics
3) Chemistry

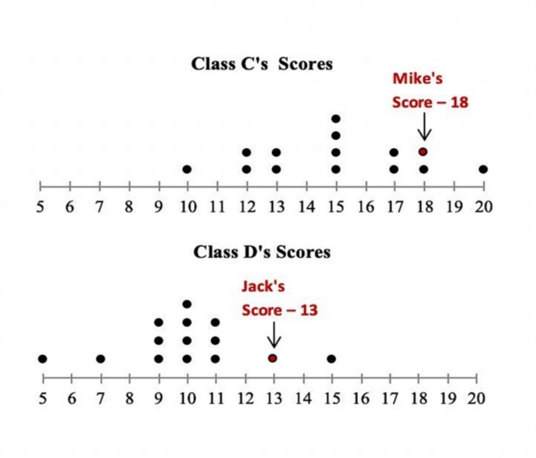
12) The diagrams below show the scores of classes A and B on a quiz. Tiffany's and Stephanie's scores are given in the diagram. Who performed relatively better in their respective classes on the quiz?

1. Tiffany
2. Stephanie
3. Both



13) The diagrams below show the scores of classes C and D on a quiz. Mike's and Jack's scores are given in the diagram. Who performed relatively better in their respective classes on the quiz?

1. Mike
2. Jack
3. Both



End

Thank you for participating!

Questionnaires