# The Ethical Charge of Articulating Mathematics

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ORIGINAL PAPER

# The Ethical Charge of Articulating Mathematics

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**Abstract**
Making mathematical statements and justifying them depend on a choice of mathematical framework(s). Such choice, this paper argues, depends on social circumstances and has social implications, rendering mathematical production ethically charged. The ethical charge of mathematics can therefore not be restricted to the impact of specific applications, research institutions, and teaching, as these may already be at least partly enabled or suppressed by the choice of mathematical frameworks.

**Keywords** Ethics of mathematics · History of mathematics · Social impact of mathematics

## 1 Motivation

Before an introduction, let me motivate this paper with a classroom exercise that I present to students. It begins with the statement:

> Pure mathematics is ethically neutral. It is only in the application and implementation of mathematical research that ethical issues might come up.

Here, "application" means scientific and industrial applications—anything from mathematical physics to technological artefacts. "Implementation" refers to the practical and institutional aspects of mathematical activities: funding, material resources, decisions on research priorities, the organization of research (e.g. flat or hierarchical), forms of teaching and communication, etc.

Then, I propose to the students to compare their evaluation of this statement to similar statements, where "mathematics" and "mathematical" are replaced by other disciplines. We start with

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> Pure physics is ethically neutral. It is only in the application and implementation of physical research that ethical issues might come up,

and go through biology, engineering, anthropology, medicine, and—just to hammer the point in, culminate with

> Pure murder science is ethically neutral. It is only in the application and implementation of murder science research that ethical issues might come up.

The general reaction (which is not universal, as we will see shortly), is that, on an intuitive level, the statement that seems convincing for mathematics, becomes less convincing as we move down the list of disciplines.

A simple explanation for these differences is that, unlike other disciplines, mathematical research is independent, or at least more easily separable, from its implementation and application. However, as the ensuing class discussion usually shows, this intuitive sentiment is far from obvious. While it is hard, for example, to separate biological or medical research from their intended applications and experimental implementation (which often raises the issue of animal suffering), there are strands of bio-medical research that are more independent. Consider, for example, basic research using simulations or modeling of real or theoretical—perhaps even counterfactual—tissues, organisms and ecosystems. Likewise, an engineer analyzing some generic technical components can sometimes be as oblivious to possible applications and as institutionally independent as a pure mathematician. And isn't "murder science" simply forensics or criminology, fields which, in themselves, to the extent that we can bracket their implementation and application, are ethically neutral?

In the opposite direction, while some mathematicians can honestly claim that they have no idea if or how their work may be applied (think, for example, of large cardinals in set theory), many others do have a good sense as to how their work is likely to be used by their colleagues in other disciplines. And the ability of mathematicians to separate their research from the way it is implemented (funding, infrastructure, research labor, teaching, communication, etc.) is not substantially different from that of researchers in other sciences, except that mathematical research tends to be cheaper than research in empirical disciplines, which may scale down the impact of its implementation. The image of the solitary mathematician, completely independent of resource struggles and institutional debates, is just as unlikely as that of the solitary biologist, who figures out the secrets of nature by solipsistic reflection and independent, non-intrusive observation. The relations between pure science, its implementation and its techno-scientific applications are indeed different in different disciplines, but these differences are not enough to establish the ethical neutrality of mathematics.

The different evaluations of the above statements do reflect some practical realities. For mathematics and physics, institutional and disciplinary divisions are more likely to explicitly uphold the division between pure, basic research and application oriented research than in the other disciplines considered above. And the same institutions find it, at least today, quite plausible that students of biology, medicine, engineering, anthropology and forensics should study the ethics of their disciplines, whereas in mathematics this is not at all taken for granted. The prevalent justifications for this disciplinary difference, however, as we saw above, are not as convincing as they may appear.

The other main reaction to the same classroom exercise is to endorse the opposite attitude. Some students assert that, in fact, in all disciplines, not just mathematics, the pure, ethically neutral knowledge should be distinguished from the ethically charged research implementation and the application of this knowledge ('ethically charged' here is a shorthand for 'having possible impact on issues with an ethical aspect'). According to this view, it is not that we mistake mathematics to be ethically neutral, but rather that we mistake other forms of pure knowledge to be ethically charged.

This too is a problematic claim. It assumes that facts (pure knowledge) are clearly distinguishable from values; that the facts in themselves are ethically independent of the material practices of their invention or discovery (e.g. medical knowledge obtained by experiments on prisoners of war); and that facts do not transform the world that they purport to neutrally describe. However, instead of following the arguments mentioned in the last sentence, I will try, in this paper, to circumvent them. I will try to argue that in a certain sense, even the mere categorization of a claim as mathematical, or the assertion of a mathematical statement *as mathematical*, are acts that may be ethically charged. A mathematician who cares about possible ethical implications should therefore be aware of this charge and take it into consideration in their mathematical practice.

## 2 Introduction and Preliminaries

In this paper I will assert that the ethical charge of mathematical practice occurs even "before" specific applications, ways of teaching and research implementations enter the discussion (where "before" concerns the logical-discursive organization of the argument, not physical chronology). I will argue that an ethical charge is potentially present in the very act of distinguishing between some claims as mathematical and others as non-mathematical. Such distinction may be implicit or explicit, and may occur either in stating a certain claim *as a mathematical claim* or in endorsing a certain view of what counts as mathematics when doing mathematical work. Of course, not every utterance of "$2+2=4$ is a mathematical statement" actually has ethical impact, and certainly not the same kind of impact when uttered by different people, so this paper will explain how and in what sense such ethical impact may come about.

I note that this is claim is not entirely new: the ethnomathematical community has documented how questionable criteria for inclusion of some practices as mathematical (modern, western, formal) as opposed to other (indigenous, practice oriented) serves to justify and perpetuate western domination ideologies (Bishop 1990;

Powell and Frankenstein 1997; Barton 2008).[1] The focus in this paper, however, is on and around what is usually considered as the mainstream of modern and historical mathematics. It also extends the discussion more broadly to our conceptions of mathematics, rather than focus on specific cultural boundaries of inclusion and exclusion.

It is now well established that, if we include contexts such as application and teaching, then mathematics is ethically charged. Some well-known examples are the appropriation of mathematics for the purpose of war (Booss-Banvnik and Høyrup 1994, 2003) and the impact of various rating algorithms on the perpetuation or even accentuation of social injustice (O'Neil 2016; Eubanks 2018). Concern with applications is also the most salient aspect of contemporary projects on ethics in mathematics, such as the Cambridge University Ethics in Mathematics Project and the Mathematics for Social Justice project (see Müller 2022 for a comparative review).

The context of teaching has also attracted attention. Ernest (2018, 2020) has done a lot to analyze the ethical aspects of mathematics teaching. He associated mainstream contemporary mathematics education with training in obedient rule following, a compulsion to quantify, decontextualization, suppression of subjectivity, ethics-free thought and more generally instrumental reason (in the Frankfurt school sense). This is further associated with an over-valuation of mathematics in the school and other social settings, where it is often distinguished as *the* model of rational knowledge and as a privileged indicator of general intelligence, rendering other forms of knowledge and skill inferior. In turn, claims Ernest, this over-valuation is bound with a distorted public image of mathematics as an absolute, univocal and universal form of knowledge. The claim is not that these characterizations of mathematics education hold universally in our society, but that they are widespread and harmful.

Note that all the above problems may be mitigated by the benefits of mathematics and could be tackled by reforming our practices in the contexts of applications, education and the public portrayal of mathematics. So the arguments quoted above are not that mathematics is simply bad, but that it is ethically charged.

The above arguments assume that we integrate applications and education into our discussion of the ethics of mathematics. But such integration requires justification. One argument in favor of such integration is established in Ernest (2021). The framework he used was MacIntyre's theory of ethics. According to this theory, one's practice is ethical only if it respects the virtues internal to the practice itself, as well as those that contribute to one's flourishing as a person and wider social virtues (MacIntyre 2007). This means that by doing mathematics strictly in line with its internal norms, neglecting the personal and social levels, one is not engaging in an ethically virtuous or even neutral practice, but in a practice that is ethically problematic. A mathematician who views ethics in such an integrative manner should link the doing of good mathematics as autonomously defined by the discipline with

[1] Refusing to acknowledge some activities by some marginalized groups as mathematical may be used to mark them as "primitive". However, imposing the title "mathematics" on such activities may also be problematic (see Larvor and François 2018, where this debate is summarized).

a consideration to personal and social good, which involves aspects of application, teaching and research implementation. Therefore, according to this view, mathematical practice is ethically charged.

My goal here is to complement this kind of argument by an argument that allows to shift the focus away from application, education and research implementation. Closer to this goal is the Wittgenstein-inspired argument by Pérez-Escobar and Sarikaya (2022), which challenges (independently of the question of ethics) the attempt to carve out a pure layer of mathematics. They argue against such attempts not only because of possible, unintended applications and of mathematically internal applications (which then link to external applications), but because even as a mere symbolic-argumentative activity, mathematics comes with potential social distinctions and meanings. In a sense, this paper pinpoints and elaborates some ways in which the distinctions and meanings associated with mathematics may be ethically charged.

For making the claim that the ethical charge of doing mathematics occurs even "before" specific applications, ways of teaching and research implementations enter the discussion, some preliminaries are in order. First, in the context of this paper, I reserve ethical judgment to actions of humans, possibly using objects or facts, rather than to objects or facts in themselves. In other words, I do not apply ethical judgment to a gun, but to producing, selling, holding or shooting a gun; similarly, I do not apply ethical judgment to a mathematical fact, but to acts of deriving, applying, or simply asserting such a fact.[2]

I further limit myself to considering ethics in the context of socially and materially embedded actions. This assumes that mathematical cognition, like other kinds of cognition, is embodied (Núñez and Freeman 1999; Freeman 2000). Mathematical actions thus bear the impact of and make an impact on social and physical reality. Indeed, even a solipsistic mathematical reflection depends on a social context of acquiring knowledge and requires some resources (at the very least, time, energy and motivation). So the kind of mathematical activity that interests me in this paper necessarily involves some kind of dissemination—at a minimum, a potentially intersubjective act of assertion. This approach excludes some platonist, realistic and soplipsistic-idealistic positions that abstract mathematics from such inter-subjectivity. While I will not engage with such positions, I will indicate, toward the end of the paper, to what extent the argument of this paper is still relevant for them.

The most basic mathematical acts that I will consider in this paper are therefore stating, justifying and disseminating mathematical claims. These actions are interdependent: the practice of stating and justifying claims is, in the context set by the previous paragraph, an intersubjective practice, and so, by definition, bound with

[2] I do not claim that this distinction necessarily holds. In fact, I believe that the boundary between human action, objects, and descriptive facts is vague, in line with philosophies that challenge the human/nonhuman distinction, broadly captured by the title "new materialisms" (e.g. Barad 2007). However, if we accept this promising, but still fringe approach, the notion of ethics would take a whole new meaning, and require a very different analysis (in particular, ethical charge would obviously "spread" into all areas of knowledge, and the claim of math neutrality would not even make sense). On similar grounds, I will tactically assume a distinction between ontology, epistemology and ethics.

dissemination. Once dissemination is taken into account, teaching and application become salient, as disseminated mathematical statements depend on the former and enable the latter.

But my goal in this paper is, as stated above, to establish the ethical charge of mathematical activities "before" specific applications, ways of teaching and research implementations enter the discussion. This may appear paradoxical, as I confine myself to socially and materially embedded mathematical practice, which, as the previous paragraph argues, is indissociable from such contexts. To clarify, then, I claim that even if we focus on asserting mathematical claims, rather than on applying them, teaching them or organizing the institutions of their research, mathematics is already potentially ethically charged.

More precisely, I argue that when asserting, justifying and disseminating mathematical statements, one is often choosing to endorse, promote or invest in certain ways or frameworks of doing mathematics. I will call such choice, endorsement or promotion "articulating mathematics". As we will see, articulations of mathematics both depend on and affect various ideological and practical concerns, which are not ethically neutral. Moreover, these articulations may enable or suppress, among other things, specific forms of application, teaching, and research implementation with obvious ethical aspects. Therefore, the goods and harms of some forms of application, teaching, and research implementation are not entirely contingent, but depend to some extent on "prior" articulations of mathematics. The ethical charge is thus potentially there already in the articulation.

The impact of an articulation depends, of course, on the context and agent. In many cases, I acknowledge, an act of articulation has hardly any impact at all. However, making a mathematical statement *as mathematical* (even implicitly), tends to associate the term "mathematics" with some articulation, and this articulation, due to its causal relations to ideological and practical concerns, is already ethically charged. To make this argument I will first explain in what way making mathematical statements involves an articulation of mathematics, and then explain the ethical charge implicated in such articulations.

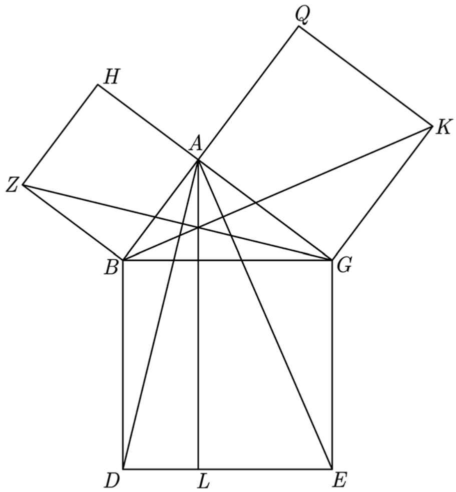
## 3 The Ways of Doing Mathematics

In itself, the statement that "the squares on the sides of a right-angled triangle equal the square on the hypotenuse" (henceforth: the right-triangle statement)[3] is not self-contained enough to serve as a basic unit of scrutiny. Indeed, it is too underdetermined. In fact, it is so underdetermined, that it fails to even have a clear truth-value. Depending on the mathematical framework in which it is stated, it may be true, false

[3] I do not call it "the Pythagorean theorem" because it was, according to contemporary near-consensus among historians, never proved by Pythagoras, who was not interested in this kind of proof (Netz 2022, ch. 1), and because it was known to Babylonians before Greek antiquity (Katz 2007, p. 100). Moreover, based on its various attested proofs, different cultures proved the statement independently of the Greeks.

or undecided. To specify this statement, we need to embed it in a mathematical system. So let us consider several possibilities.

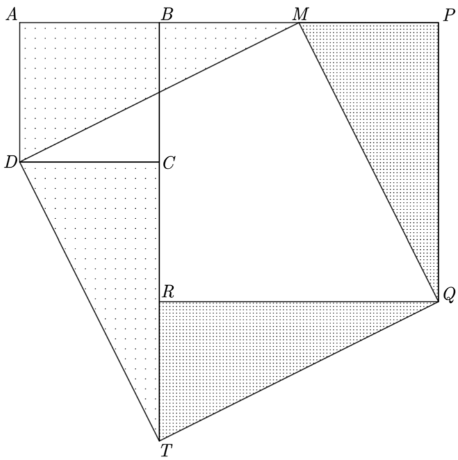
1. The above statement is famously embedded in Euclid's *Elements* as proposition I.47. There, it is determined by a system of definitions, postulates, axioms and many implicit but quite rigorous linguistic and diagrammatic inferences (for an overview of these tools and how they function—which is not at all like a modern axiomatic system—see Netz 1999, chs. 3–6). A short sketch of the proof (which, note, omits details that are crucial from the point of view of the classical Greek system of justification, and relies on previously proved theorems) depends on the following diagram:




In this diagram, the triangle ZBG equals half the square ZA, because they share the same base (ZB) and are between the same parallel lines (ZB and HG). Similarly the triangle ABD equals half the rectangle BL. But the triangles ZBG abd ABD are easily shown equal, so the rectangle BL equals the square ZA. Similarly, the rectangle GL equals the square AK. So the squares AZ and AK together equal the rectangles BL and GL together, namely the square GD.

2. Instead of such an elaborate system, the right-triangle statement may be embedded in a less codified, more intuitive cut-and-paste procedural system. But the term "intuitive" should not be taken at face value; it too depends on more or less contingent norms about geometric objects and their manipulations that depend on contextual training. By "intuitive", therefore, I mean that understanding and evaluating this argument may require less training and is closer to relatively widespread material practices (more so, at least, than the previous Euclidean argument), but not that it forms a universal, spontaneously acceptable piece of mathematical knowledge (see item 5 below). An example of an embedding of the right-triangle statement into such a system is the following argument, taken

from a treatise composed in the context of the Kerala school of mathematics, which flourished between the fourteenth and seventeenth century in India (Sarma et al. 2009, p. 180; for an earlier Arabic version by ibn Qurra, see Chemla 2005, Sect. 4):




Here BQ and BD are the squares on the sides of the right triangle, set side by side. Cutting the triangles DAM and MPQ, and pasting them, respectively, as DCT and TRQ, we get the square on the hypotenuse.

3. We could also go in the opposite direction in terms of rigorous codification. As we noted above, the Euclidean derivation of the right-triangle statement depends on many implicit norms and practices, including some that depend on observing diagrams. Toward the end of the nineteenth century, some mathematicians engaged in a project of rendering Euclidean geometry strictly axiomatic, in the sense that proofs could be formalized as sequences of statements where the derivation of each statement from the axioms and from previous statements is purely syntactically verifiable. This required comprehensive amendments and several additional axioms with respect to the Euclidean tradition (in particular, congruence axioms and topological axioms, see Hilbert and Bernays 1971), without which the statement would simply not be provable.
4. Since according to the standards of modern axiomatic systems, the Euclidean "proof" of the right-triangle statement is no proof at all, one could say that it is simply false. This applies not only to some deviant geometries, but also to geometries that obey all of Euclid's standard axioms—including the famous parallel postulate. For instance, the projective plane with the sphere induced metric and measure obeys all of Euclid's explicit axioms (in standard contemporary reconstructions of the text), but not the right-triangle statement.
5. Another approach is to claim that the right-triangle statement is true in the sense that it forms part of the practical standards of some normative professional communities, such as land surveyors or carpenters. Even as such, the observation is still embedded in some non-trivial system of mathematical practices, including

a not entirely elementary numeracy and a concept of area that is not culturally universal. For example, this notion of area requires that the magnitude of a piece of land should be dissociated from its sowing capacity, labor requirement, and embedding in three-dimensional space (i.e. ignoring whether it lies on a horizontal or an inclined plane)—all assumptions that were rejected in various times and places (Kula 1986, ch. 6; Katz et al. 2016, pp. 307–309). Note also that in such professional contexts, other mathematical conventions are often in use, which may conflict with the above statement, such as the use of the value 1+2/5 for the diagonal of a square with side 1.

6. Another kind of empirical embedding of the right-triangle statement depends on advanced practices of measurement. One would measure the sides of a right triangle, square the results, add them together, and compare the sum to the square of the measure of the hypotenuse. In this framework, it is the fact that the results are nearly the same, and that they are closer together as we improve the flatness of the surface and the precision of measurement, which justifies the statement. This framework holds the possibility that the statement turn out to be false. Indeed, it is false, if we accept contemporary physical definitions and measurement practices of cosmic-scale straight line intervals.
7. The invocations of "flatness" and "precision of measurement" above are quite problematic in themselves. According to which benchmark do we decide that a certain measurement procedure is or isn't more precise, and how do we verify that one surface is more flat than another? One might find out that we explicitly or implicitly assume the right-triangle statement in establishing the precision of a measurement practice or the flatness of a surface. In such a case, one may prefer to frame the right-triangle statement as no theorem at all, but rather as a definition, which constrains what we mean when we discuss precise measurement and flatness (e.g. Friedrichs 1965, pp. 36–37).

We see that the same statement may be true, false, undecided, or even taken as definition, and that this is not a mere sophistry, but a reflection of actual mathematical practices. This means that when we claim or prove a mathematical statement, we implicitly or explicitly invoke a framework for this statement, that is, we are implicitly or explicitly making a choice, which framework to invoke. Even if there were a platonic fact of the matter concerning the right-triangle statement, in making mathematical statements, we depend on inter-subjective frameworks of justification that exceed the purported platonic fact. We may accept one, some, all or none of the above frameworks as mathematical, but either way we are making choices. We should now figure out to what extent such choices have an ethical charge.

## 4 The Ethical Charge of Mathematical Choices

Usually, we accept some of the frameworks above as mathematical, and reject others. For example, many mathematically educated professionals would claim that the frameworks mentioned in items 3, 4 and 7 above constitute real mathematics, whereas those in items 1 and 2 are merely approximations of mathematics, and those in 5 and 6 are not mathematics per se. When we make such a choice, we implicitly or explicitly endorse or promote some frameworks at the expense of others, and may encourage directing institutional resources and symbolic capital toward them. This act may well be ethically charged. Of course the charge may be positive, rather than negative. Furthermore, a pluralist choice does not escape the charge, as it is still a choice that may affect resource distribution.

The first objection to the claim that such a choice has an ethical charge is that deciding which frameworks above are properly mathematical does not necessarily endorse or promote the associated frameworks. Naming something “mathematics” does not mean that I endorse it—it may be a mere classification. But this, I believe, is a naïve portrayal. First, mathematics is, at least in our society, a strong brand, which often commands authority and respect. Associating something with mathematics, or excluding something from mathematics, affects its status and prestige. For instance, if we say that mathematics only begins with the classical Greek geometry centered around semi-axiomatic deductive proofs and label what was done earlier in the middle east as sub-mathematical, we are contributing (whether we want to or not) to an ethically problematic hierarchization of these historical cultures.

Moreover, mathematics comes with institutions and funding. If we say that something is or is not mathematics, we imply that it should or should not be part of what is funded and culturally legitimated in university mathematics departments or in school mathematics classrooms. This means that we make an impact on the distribution of resources among potentially mathematical practices. In turn, universities and schools train and condition students to follow certain conventions, including the commitment to specific articulations of mathematics, potentially replicating and perpetuating these articulations and their impact on symbolic capital and resource distribution.

We see here how a certain articulation of mathematics has a potentially ethical impact: it leads to a self-perpetuating pattern of distribution of resources and symbolic capital, and is therefore ethically charged. I note, however, that the argument of the last two paragraphs depends on a contingent social circumstance: that mathematics enjoys a privileged (or, more generally, special) status. In a world where mathematics did not enjoy a special status, and was not associated with socially or economically privileged (or disadvantaged) institutions, classifying a statement as mathematical would escape the above implications. Under such (currently counterfactual) circumstances, the argument above would not suffice to associate an ethical charge to the articulation of some frameworks rather than others as mathematical.

If we want to make our claim more universalizable, an alternative route would be to rely on the fact that classifying some frameworks rather than others as mathematical generates a discursive domain where some things are bunched together and others are left out. This may be ethically charged, even if this domain is not a-priori privileged or disadvantaged. For example, excluding frameworks 5 and 6 above (practice oriented standards or empirical measurement) from the realm of mathematics reinforces a distinction between pure and applied mathematics. This is not an obvious distinction, as argued by Pérez-Escobar and Sarikaya (2022). Indeed, historically, this is a rather contingent distinction, which had substantial impacts, some of which are ethically charged. Associated most prominently with the classical Greek project of separating theory from practice, it did not hold much sway over mathematics in other pre-modern and early modern mathematical cultures until its re-assertion in nineteenth century Europe. To understand the potential impact of this and other articulations of mathematics, a historical overview is in order.

## 5 A Historical Narrative

It is commonplace to assert that the Platonic-Aristotelian distinction between theory and practice, which strongly favored theory, reflected and reaffirmed a widespread view among scholars in Greek antiquity and the Christian Latin west. Mathematics was an integral part of this story. Several historians (Netz 1999, ch. 7; see also the review by Latour 2008; Asper 2008) detail how Euclidean style mathematics was designed as a project of social and political distinction along a theory–practice divide. On the one hand, classical Greek geometry distinguished itself from the prevalent features of the mathematics of accountants, land surveyors and artisans at the time. Most importantly, it never measured geometrical magnitudes by numbers (although it allowed expressing *ratios* between magnitudes as *ratios* between numbers). It rejected numerical designations in diagrams, and introduced indexical letters instead. It focused on arguments rather than methods of calculation. It made a point of proving some claims that were clearly obvious (sometimes in ways that might beg the question). All this fits the preference for elaborate deductive structures over simple arguments (like the overly complex first proof of the right-triangle statement above). Classical Greek geometry further inaugurated a highly specialized, elliptic vocabulary, which was opaque without appropriate instructions. It replaced the second-person statements of practical mathematical texts by impersonal grammatical structures. It even used a passive imperative aorist (commands to let something have-been-done) in order to make mathematical diagrams appear as objectively, impersonally and primordially given.

On the other hand, classical Greek geometry distinguished itself from the devious and manipulative arguments that prevailed in political assemblies, courts of law, and debates among philosophers (even though some the latter attempted to steer clear of the sophistic style, or at least pretended to do so). It used a very tight lexicon, grammar, and rigid inferential "formulas" (that were verbal, non-symbolic) to impose a clear consensus among practitioners. This was only partially successful—the Euclidean corpus managed to generate consensus despite its sometimes imperfect structure (Mueller 1981), but as mathematical knowledge expanded beyond it, the mathematical scene became more and more adversarial (Netz 2004, p. 62).

The result was that mathematicians competent enough to write an original proof in Euclidean style were few and belonged to an elite that could, on the one hand, distance itself from practical numeracy and mensuration, and, on the other hand (despite some notable exceptions), from the rhetorical bickering of other social elites (Netz 2002, but see also Cuomo's 2019 critique of the possible overstatement of this view). This mathematical framework imposed barriers on social access to knowledge as well as a gap between theory and practice, and asserted the status of some elites as those who possess distinguished knowledge. Whether we think of this framing of mathematics as harmful, in that it gave rise to segregated, elitist, impractical knowledge, or as beneficial, in that it allowed for the autonomous development of a strand of rigorous knowledge, it is clear that it involved choices that were not ethically neutral.

The above narrative is paradigmatic in its demonstration of how the framing of mathematics reflects and affects social realities in harmful or beneficial ways. The intellectual divisions constituted by mathematics are intertwined with those ethically charged realities and are thus not ethically neutral themselves. These divisions take part in strengthening and reshaping ethically charged ideologies and practices, and, in turn, enable or suppress various ethically problematizable attitudes to applications, research and teaching.

To show that this Greek framing was indeed contingent, rather than intrinsically necessary, a comparative historiography is in order. Indeed, many mathematical cultures, like the Indian and the Chinese, with their highly evolved written mathematical argumentation cultures, never endorsed a clear theory/practice division—either in general or with respect to mathematics (Martzloff 2007, pp. 44, 47; Srinivas 2015). They did not develop the *same* mathematics as the Greeks, but were no less sophisticated. The Arabic scientific culture, which did inherit the Greek theory/practice distinction, made some genuine efforts to undermine it in the context of mathematics (Høyrup 1994), deliberately mixing Greek style theory with practical traditions. This is related by Høyrup to what he calls "fundamentalism" (the penetration of the divine into all aspects of Islamic secular life) and to the absence of a segregated *and* competent intellectual authority, which are argued to have inspired and enabled the imbrication of theory and practice. Within these non-homogeneous cultures, each community (e.g. Chinese numerate bureaucracy, Jain merchants, high caste Kerala astronomers, mosque-based "*madrasah*" schools) used its own articulation of what should pass as mathematical knowledge as a tool for identity building and attaining cultural capital. For each, the articulation of the mathematical framework itself (that is, what counts as proper mathematics, rather than only the way it was taught, researched, or applied) was entangled in religious, political, and intellectual ideologies that often related knowledge and power in ethically problematizable ways.

European early modernity also picked up on the integration of theory and practice. Zilsel's thesis (1942), which claims that early modern science depended on the *interaction* of scholastic traditions, humanistic reforms and the globalizing artisanal-practical knowledge, is now more or less consensual. This interaction meant that the separation of theory from practice no longer dominated mathematics, even as some scholars tried to hold on to it. In Hadden's account (1994, especially 135–149), Descartes is exemplary for linking the ideal and the real in his identification of primary aspects of reality with quantity (instead of endorsing, like many of his predecessors, a mathematical description, abstraction or idealization of physical reality). This articulation of mathematics also homogenizes, via a universalizable notion of quantity, various aspects of reality—possibly inspired by the universalization of exchange via monetary value in the emerging capitalist economy. This was not only part of a scientific program, but also a politically and even theologically motivated one, which was far from ethically neutral. It further enabled new applications of mathematics, such as the geometrization of space itself rather than of shapes in space (De Risi 2016), the proposed reduction of socio-political and even ethical questions to calculation ("political arithmetic", which later became statistics, Sivado 2019), and the assertion of mathematical laws governing society rather than mere mathematical regularities observed in society (Hacking 1990, chs. 13–15).

As the boundary between mathematical theory and practice was being challenged, valid mathematical reasoning became a practice ever more prone to experimentation and debate. New epistemic values, such as exploratory mathematical innovation (or, sometimes, the mere appearance of innovation) became more dominant, especially, but not only, around the emergence of calculus (Mancosu 1999; Jesseph 1999). New techniques of calculation, new notions of number and new forms of reasoning gained traction, enabling a highly adversarial and non-consensual mathematical culture (see Wagner 2022, Sect. 5), which is in itself an ethically charged development. (Even God, via his late-medieval identification with infinity and the exploratory mathematization of infinity, became somewhat mathematized!) These new articulations of mathematics—or sometimes, the reactions against these new articulations—were intertwined with new forms of mathematical communication and research and the attainment of scholarly status by new social classes. These, in turn, were correlated with the emergence of new scientific institutions (see e.g., David 2004, 2008). With all its various opportunities and problems, the shift from the old to the new framing of mathematics clearly had an ethical charge.

But a new articulation of pure mathematics did come back to the European stage (here I follow mainly Schubring 1981, 1994; see also Jahnke 1990). In nineteenth century Prussia, a combination of neo-humanism and Kantian ideas, together with the reform of university mathematics following the example of the emergent philological seminars, reframed the ethos of mathematical production around autonomy, new mathematical languages with new standards of rigor, and an imperative to research. A new articulation of mathematics emerged, which begat, among others, the proto-formal mathematics of Weierstrass and Dedekind. That this was ethically charged is demonstrated by the fact that this change was *too* successful, and was causally linked with developments that were deemed, at least from certain points of view, harmful. Disciplines and institutions that required practical mathematical training rebelled against what they perceived as the obscurantist mathematics imposed on them, and fought over who should have the right to teach mathematics, what should be taught as mathematics, and what are its disciplinary boundaries.

This resulted in a redistribution of funding and the emergence of distinct applied mathematics university chairs.

Finally, the early twentieth century foundational debates are usually considered as debates over the epistemology and ontology of mathematics. But Poincaré's rhetoric (e.g. "Formerly, when a new function was invented, it was in view of some practical end. To-day they are invented on purpose to show our ancestors' reasonings at fault, and we shall never get anything more than that out of them", 1914, p. 125) shows that the way we reframe mathematics involves an ethical attitude as well. Brouwer's arguments (e.g. 1975, pp. 417–428) also have a scope wider than one would expect, clearly manifesting the depth of his existential concerns, which turn the debate over mathematics into a debate over forms of life. These debates were therefore ethically charged even before the institutional conflicts between Hilbert and Brouwer over the latter's editorial seat in the *Mathematische Annalen* translated their debate into a struggle over the control of resources (van Dalen 2001).

Eventually, Hilbert's victory (not his dream of a formal proof of consistency, but the successful banishment of philosophical debates from mathematics in favor of a pluralism of axioms and formal systems) guaranteed a renewed consensus regarding the validity of mathematical proofs relative to well-specified systems, breathing new life into the image of mathematics as a domain of universal certainty. This was an ethically problematic achievement, as it depended, to a large extent, on exiling from the realm of mathematics all disagreements over the validity of arguments that could not be settled by Hilbert's formalizing approach (to qualify and nuance this statement, see Wagner 2022). This, in turn, had a lasting impact on the over-valuation of mathematics discussed in the introduction. This modern framing of mathematical knowledge still attracts both praise and deprecation, but is rarely perceived as neutral with respect to its impact on science and society by scholars who acknowledge its contingency (Kline 1980, chs. 12–15, is but one well known example).

Of course, this is my own concise "greatest hits" of mathematical historiography. Other historians would highlight different historical narratives and contest some of those that I presented above. Explaining why I think these narratives are the most convincing is far beyond the scope of this paper, but historical precision is anyway secondary here. Indeed, it is effectively consensual among historians that different articulations of mathematics are historically intertwined with questions of theory and practice, group identity, theology and political ideology, the scope and ontology of natural sciences, as well as economy, industry, institutional resources, authority and consensus. What such narratives establish is that the very articulation of mathematical frameworks can enable or suppress, via the values that these frameworks reflect and the divisions that they impose, beneficial or harmful social practices, including ethically problematizable forms of application, research implementation and teaching. These articulations are therefore not neutral, but ethically charged.

## 6 Back to the Main Argument

The moral of this narrative is that the way we articulate knowledge can have a social impact with clear ethical dimensions, even before the establishment of mathematics as a highly valued brand that commands privileged resources. In fact, we saw that different ways of articulating mathematics affected its status and social role. Of course, the implications of an articulation might not be understood, or known, or foreseeable for a specific person engaged in mathematical practice. However, given our historical experience, we should be able to estimate, to *some* extent, *some* of the potential repercussions, with *some* level of probability. We should be aware of probable impacts, even if we are far from certain, and this is enough to establish an ethical charge.

This ethical burden can be compared to that of a researcher making contributions to the theory of signal-processing in a modern university. Such a scientist knows, more or less, where their work is *likely* to be directly or indirectly applied, even though they cannot tell if it will actually be applied, whether the application will have positive or negative consequences, or whether it would be applied in some other unexpected domain. Discussing such a situation in the ethical framework that we all too often employ, that of personal liability, reward and punishment, is not useful. To do that, we need to plausibly link such a scientist to knowable consequences in a way that would justify punishment or reward (or at least punishment—scientists are sometimes credited and rewarded for the unexpected, indirect fruit of their work). However, it would be ethically virtuous for such a scientist to be involved in the social monitoring of technologies that might benefit from their work and alert the public to possible negative repercussions of the research in their field (Resnik and Elliott 2016). That is, I believe, also the ethical attitude mathematicians should have to the choice of frameworks that articulate their discipline.

But there's another argument against the claim that marking some frameworks rather than others as mathematical is ethically charged. One may claim that choosing the frameworks that govern mathematics is merely an epistemological issue, rather than an ethical one, and that one's epistemological goals would compel one to include or exclude some frameworks under the label "mathematical". Given such compulsion, the articulation of mathematics cannot be seen as ethical, as it fosters no real choice.

Indeed, if someone is committed to an identification of mathematics with some specific epistemic criterion or criteria (e.g. consensus, certainty, public or mechanical verifiability, applicability, intuitiveness, abstraction, constructivity, consistency, productivity, beauty, autonomy, and access to the divine—all of which are historically attested; cf. Maddy 2019), one may be compelled to articulate only some frameworks as mathematical. However, epistemological goals and criteria are not simply given in advance. Choosing which epistemic goals to identify with mathematics and which epistemic goals to promote is, again, a choice with a strong normative, ethically charged aspect. Quoting Sally Haslanger (in the context of her broader discussion of epistemology): "My suggestion is that questions of value are already implicit in traditional epistemological debates, and that the questions should be raised more explicitly" (1999, pp. 472–473; for a different argument for a "thick" epistemology of mathematics see Hunsicker and Rittberg 2022).

Other variants of this last objection are platonist, realist or idealistic stances according to which the articulation of mathematics is given a-priori, or in reality, or follows necessarily from some conditions of possibility, independently of any inter-subjective social exchange (I will refer to such articulations as "absolute"). However, even if one accepted these stances, a supposedly absolute articulation of mathematics does not imply that we need to organize our knowledge and practice around it. We are still free to organize our knowledge and practice in ways that respect or violate any absolute articulation (just like, even if we believed in some absolute distinction between physics and biology, we could still refuse to organize our scientific practice around this distinction, if we thought it would bring about better science). Therefore, a person who believes that the articulation of mathematics is absolute can consider this articulation as ethically neutral only if they do not let this supposed absolute articulation unquestionably dictate our scientific practice.

Indeed, unless one accepts obviously counterfactual, solipsistic assumptions on mathematical practice, *acting* mathematically (in the sense discussed in the preliminaries section) according to the supposedly absolute framework means that one actively invests resources in this framework, disseminates it, and usually (although not necessarily) implicitly endorses it.[4] Once it is allowed to organize our practice, even a supposedly absolute articulation of mathematics becomes an applied ideology: a system of ideas that makes a mark on the world. Therefore, even for someone who believes in an absolute articulation of mathematics, disseminating this articulation as a framework for mathematical practice would be ethically charged. The true articulation of mathematics itself would remain, for an absolutist who believed in it, neutral, but acting based on it would not.

It might be the case that, regardless of how I do or do not do mathematics, my own ethical impact as practitioner of mathematics is negligible (indeed, this is the case for most of us). This, however, only concerns the question of personal liability, to which, again, ethics should not be reduced (indeed, the fact that the effect of my personal travel plans on climate change is negligible does not imply that social transportation habits are ethically neutral). The ethically charged systemic impact of many people doing mathematics in certain frameworks may be as strong or stronger than the impact of attempting to define mathematics in terms of one framework or another. So the arguments above for the ethical charge of mathematics still hold.

[4] Of course, I may do mathematics in a certain framework because I'm good at it, not because I think it is the best or only framework for mathematics. But in doing that, I still invest in this framework, and if I am ethically minded, this implies that I believe that it is, at the very least, not more harmful to promote and disseminate this mathematical framework than to do other things I could do (mathematics in other frameworks, where I may be less proficient, or non-mathematical activities).

## 7 Conclusion

In the previous paragraphs I acknowledged that, under some circumstances, articulating mathematics in certain ways would be ethically neutral. If mathematics did not have a special status, if it were not distinguished by its institutional position and access to resources, or if the way we define mathematics did not impose itself on the organization of knowledge production and dissemination, then it might end up ethically neutral. Moreover, if doing mathematics would somehow be a solipsistic "resource neutral" endeavor, this neutrality would apply also to actually doing mathematics.

In fact, when people say that mathematics is ethically neutral, this is usually the image they have in mind: mathematics that is somehow done outside the world, or is independent of any material-social-institutional setting and impact. But this is precisely the image of mathematics that was questioned in the exercise that opened this paper, and further challenged in the subsequent sections. Actually doing mathematics commits us to doing mathematics according to certain frameworks in certain social settings, which means we necessarily advance those frameworks and disseminate them, rather than others, and commit resources to these frameworks rather than to others. In turn, these frameworks, when put to action, reflect, reinforce and reconfigure social values and realities. This impact may eventually be beneficial or harmful. Multiplying these choices onto a social scale, we bear the ethical brunt of pursuing certain ideologies of doing mathematics. This is not a merely abstract argument, but one that historical evidence and contemporary observation show to have real substance. Subjecting mathematics to different frameworks were historically, and are still today, ethically charged.

This leaves us with the following question: Why is it so common to think about mathematics, rather than other kinds of knowledge, as detached from ethically charged ideological-practical choices? What kind of social role does this characterization play? Is it about a historical ideal: our attachment to an identification with the absolutely right, supposedly a-political Greek mathematical aristocrat? Is it about pragmatic considerations: simply an indication of the relatively greater distance (rather than proper gap) between mathematics and its applications? Is it a knowledge-power play: a myth imposed by those who benefit from the prestige of mathematics, maintained by the popular alienation from, and sense of ignorance about, mathematics? Or is it a result of existential angst: our consolation for the loss of certainty with the death of God, a loss that the precarious and imperfect empirical sciences could not compensate for? Once we give up the faith in math neutrality, can we properly engage with these questions.

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