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Semi-Lumped Model for the Calculation of Frequency Dependent Complex Permeance for U- or E-Cores

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As ferrite have simultaneously non-negligible permeability, permittivity and conductivity, the magnetic behaviour of ferrite cores is frequency dependent. This article presents a method to predict the complex permeance of ferrite E- or U-cores including electro-magnetic resonance. The cores are decomposed into basic segments in which Maxwell equations are solved analytically to obtain a network of frequency-dependent permeances or reluctances.

***Index Terms*—lumped model, complex permeance, complex reluctance, core losses**

I. Introduction

FERRITE cores are widely used to build inductors and transformers for power electronics or for EMI filters [1]. Such magnetic devices are often designed based on an optimisation procedure. Such a procedure requires to calculate different metrics about the behaviour of the component for a high number of possible configurations. Those metrics can be for example core losses or inductance as functions of the excitation amplitude and of the frequency. Nevertheless, the presence of electro-magnetic resonance inside the core could make such an assessment more difficult. Because of a non negligible permittivity of the core, both resistive and capacitive eddy currents occur in the core. Between 100 and $1000\,\mathrm{kHz}$, these make the frequency scaling of the core loss diverge from a power law and change the permeance, and hence the inductance [2].

The need for a high number of design evaluations during the optimisation process requires fast calculation which typically disqualifies Finite Element Modelling (FEM) for this task, as it is too slow despite being accurate. To solve this issue, analytical models can be used. For instance, in [2], a model of frequency dependent permeance and core losses for ring cores is presented, which uses the analytical solution to wave equations in the core from [3] and a frequency dependent material model from [4]. To make this approach available to a broader range of applications, the method needs to be adapted to core shapes other than ring cores. The material model from [4] can also be applied in the case of E- or U-cores, but the geometry of those cores is different from ring cores, so that the solution of the wave equations from [3] cannot be reused. Moreover, the geometry of these rectangular cores is not axial symmetric, unlike ring cores. Consequently, it is not possible to analyse the device by simply considering its cross section as shown in [2], [3] for ring cores. In contrast, modelling the frequency behaviour of rectangular cores demands a new method to calculate the complex permeance of the core, provided its material parameters are given.

To obtain the frequency behaviour of the core, this paper proposes to decompose the core into basic segments that can separately be reduced to their cross section. Their respective permeances are calculated based on wave equations, and finally combined as a network using frequency dependent permeances. This results in a lumped model in which the value of each lumped element is calculated with the wave equations, hence the name of "semi-lumped model". The model is finally tested against FEM and the results are discussed.

II. Splitting of an U core into basic segments

In contrast to ring cores, rectangular cores (e.g. E- or U-cores) are not axial symmetric and cannot be reduced to their 2D cross section to solve the wave equations and to get the frequency dependent permeance and core losses. Consequently, the geometries must be simplified to obtain the complex permeability of the core. In standardised low frequency permeance calculation approaches [5], the flux lines are assumed to be straight in the straight segments (leg or yoke) and rounded in the corners. This leads essentially to a decomposition of the core into segments, as shown in figure 1a) and b). The assumption of straight flux lines in the leg and yoke segment is equivalent to consider the flux distribution to be invariant along the length of the respective segment. With this assumption, the wave equations can again be reduced to a 2D case. The circular nature of flux lines in the corner [5] is analogous to ring cores. Thus, the corner segments can be replaced by equivalent ring quarters, cf. figure 1c), for

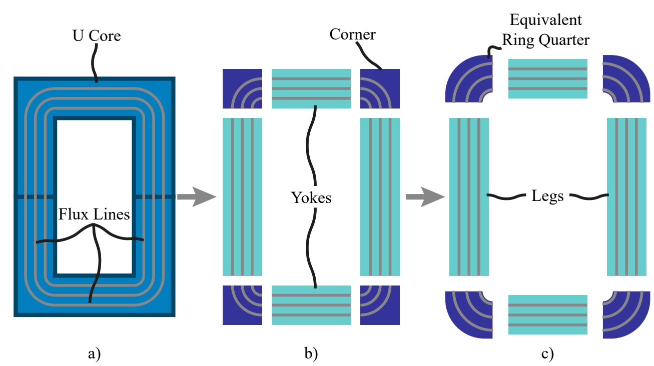

Fig. 1. Decomposition of a UU pair of cores: a) The shape of the flux lines is different in the various parts of the core (straight in the yokes and legs, bended in the corners). b) The geometry can be decomposed into several segments. c) Finally, the corners can be replaced by equivalent ring quarters.

which the frequency dependent permeance formula is already available in literature. In a final step, the permeance of the complete core can be obtained by combining the permeance of the different segments.

III. Permeance of the basic core segments

To obtain the permeance of the complete core, the permeance of the different basic segments must be calculated first. As stated before, the circular nature of flux lines in the corners enables to replace the corners with equivalent ring quarters. To define a ring quarter, three geometrical parameters are required: the height, the inner diameter and the outer diameter. The first one is identical to the one of the original corner but the two diameters must be calculated. The constraint for this computation is to have an equivalent ring quarter with the same effective cross section $A_{\mathrm{e,ring}}$ and magnetic length $l_{\mathrm{e,ring}}$ as the original corner ($A_{\mathrm{e,corner}}$ and $l_{\mathrm{e,corner}}$). These four effective core parameters are standardised [5]. A system of two equations (one for the cross section and one for the magnetic length) and two unknowns (inner and outer diameters) can be derived and solved. Besides the standardised formula for the corner cross section given in [5], an alternative expression can be used which is not based on the average of the cross sections of the neighbouring leg and yoke, but rather on the diagonal cross section of the corner. Once the equivalent ring corner has been derived, its permeance can be calculated using the solution of the wave equations given in [3]. The actual value is obtained by integrating analytically the magnetic field over the cross section [2]. The two corner cross section formulas as well as the derivation of the permeance will be discussed in the final publication. For straight segments, the basic assumption of the standardised permeance formula is that their flux lines are perfectly straight [5]. This is also assumed in the following derivation of the frequency dependent permeance of straight segments. The flux distribution analysis is then reduced to a 2D case over the cross section. Solving the wave equations results in a formula based on a combination of trigonometric functions. The various coefficients are determined with the boundary conditions. This formula is integrated over the cross section to get a permeance per unit length from which the actual permeance of the segment is calculated. The full derivation will be given in the final publication.

IV. Validation with FEM

To validate the model for the frequency dependent permeance, a comparison with results obtained by 3D FEM is performed. Several geometries are considered including a TDK-EPCOS U101x76x30 core. The frequency dependent material properties are taken as typical values from literature. The FEM models are implemented in COMSOL Multiphysics in the frequency domain. The new semi-lumped approach is implemented in MATLAB. Both formulas of the equivalent corner cross section are tested (cf. Section III). The proposed semi-lumped model predicts well the values of the real and the imaginary permeances, even above resonant frequencies, as can be seen in figure 2. It requires only 4 ms to calculate one point on a standard laptop (Intel Core i7-8665U, 16 GB of RAM) compared to 1 h required to run the FEM analysis on a computation server. As it will presented in the final paper, the standardised corner cross section formula from [5] exhibits a systematic underestimation of the low frequency permeance, what is avoided by the proposed alternative corner cross section formula. This underestimation can be explained by the fact that the corner cross section can be greater than both adjacent cross sections, meaning that taking the average underestimates the actual effective corner cross section and consequently the permeance of the corner.

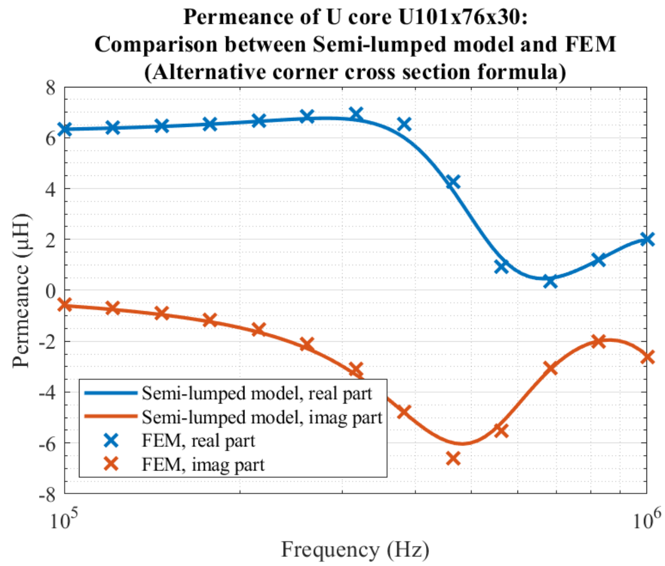

Fig. 2. Comparison between the semi-lumped model and FEM: the resonance frequencies and the associated complex permeance are satisfyingly predicted by the new model.

V. Conclusion

A new approach to model the frequency dependency of the permeance of ferrite E- or U-cores is presented. It is based on segmenting the core and describing the segments by a permeance network. There, each lumped complex permeance element is calculated including wave resonance phenomena. This approach can be used in reluctance networks of complex magnetic devices such as multi-winding topologies, even under the influence of electromagnetic resonance in the core.

Furthermore, it is shown that the standardised low frequency core permeance formula underestimates the permeance of the corners. To overcome this limitation a simple modification to the formula is proposed.

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