SERVICEABILITY ANALYSIS OF REINFORCED CONCRETE
BASED ON THE TENSION CHORD MODEL

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presented by
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Preface

"Structural Engineering is the art of moulding materials we do not wholly understand into shapes we cannot precisely analyse, so as to withstand forces we cannot really assess, in such a way that the community at large has no reason to suspect the extent of our ignorance."

Dykes (1978)

This quotation refers to the complex and hardly predictable load sequences and the highly variable concrete properties, which limit the accuracy of serviceability limit state deformation predictions in particular. Nevertheless, practical applications require reasonably reliable estimates of both integral and local deformations under service loads.

The present thesis illustrates how the Tension Chord Model can be used to estimate the serviceability limit state behaviour of tensile and bending members. The Tension Chord Model is an analytical model that was developed within Prof. Marti's group at the Institute of Structural Engineering (IBK) of ETH Zurich. During the past twenty years it has been used to model the deformation behaviour of tensile ties and beams. Even though most previous applications were focused on ultimate limit state solutions, the model was also found to give good crack width and deflection predictions under service loads. This is why the present PhD thesis work was initiated with the aim of systematically applying the Tension Chord Model for serviceability design and verifying its prediction quality.

The scope of the theoretical work summarized in Chapter 4 was guided by a number of questions, which arose during the course of the work. At the beginning, a discussion (Burns et al. 2008) written in response to a technical note on tension stiffening (Gilbert 2007) initiated the work on bending members and led to the transformed section property-dependent tension stiffening and cracking equations. In a next step, the tension stiffening equations were combined with Ghali’s creep and shrinkage curvature coefficients (Burns 2008). However, as a result of subsequent parameter studies, the much simpler Effective Modulus Method was used for further long-term deflection calculations. At this point, the topic of low shrinkage concrete directed the work towards shrinkage cracking, leading to the discussion on restraint-induced cracking under different boundary conditions. Finally, the calculations for the ConCrack prediction contest led to the end-restraint cracking approach being extended to account for elastic restraint conditions and the simultaneous presence of shrinkage and cooling-induced strains.

This research work could not have been carried out had it not been for my supervisor Prof. Dr. Peter Marti, who gave me the opportunity to be part of his research group at the Institute of Structural Engineering of ETH Zurich. I am also much obliged to Prof. Dr. György L. Balázs for being my co-supervisor and inviting me to join the fib Serviceability Models Task Group 4.1. My greatest thanks are reserved for my wonderful colleagues, who shared my time at IBK. I would like to particularly thank Stephan Etter, Hans Seelhofer and Patrick Fehlmann for their much appreciated technical advice, Andreas Galmarini for making sure this work got written and Almut Pohl for helping me prepare the presentations. Last but not least, I would like to express my gratitude to my parents and my husband Rodrigo Pérez for their support and understanding.

Zurich, September 2011

Clare Burns
Abstract

In the present thesis the Tension Chord Model is applied to describe the deformation behaviour of reinforced concrete (RC) tensile and bending members in the serviceability limit state.

After an introduction (Chapter 1), the most frequently used crack width and deflection approaches are described in Chapter 2. The two different concepts of transfer length adopted by the slipping-bond and concrete-cover crack width approaches are compared in the first part of the chapter. The second part of the chapter discusses the tension stiffening models of Branson and Rao, which differ especially for low reinforcement ratios. Further, global deflection approaches are compared.

Chapter 3 contains the basic principles required for the crack width and deflection approaches presented in Chapter 4. The first two parts of the chapter summarise the concrete and steel material properties used for the serviceability limit state. The third part of the chapter deals with approaches for determining the State I and II section deformations under sustained and variable loads. It is shown that the Age-Adjusted Effective Modulus Method (AAEMM) and the Effective Modulus Method (EMM) provide similar deformations for ordinary RC sections under sustained loads. In the fourth part of the chapter the Tension Chord Model is presented and its two main assumptions, which are the rigid-perfectly plastic bond stress-slip relation and the constant tensile strength, are discussed.

The first part of Chapter 4 provides expressions for considering tension stiffening in bending members under short-term and sustained loads as well as during unloading. The moment-induced crack width equations given in the second part of the chapter are functions of the transformed section properties. They are followed by a discussion on restraint-induced cracking due to uniform cooling- and shrinkage-induced strains. Besides span-to-depth ratios, the third section of the chapter illustrates system-dependent interpolation coefficients for global deflection calculations.

In Chapter 5 the approaches discussed in Chapter 4 are verified with a selection of test series from various research institutions. The test data comparisons show that the State II section approach (cracked elastic state neglecting tension stiffening) leads to consistent upper limit predictions for the local and integral deformations of members with constant moment distributions. In combination with the Tension Chord Model the State II section approach provides good lower limit deformation predictions. In most cases, the Tension Chord Model gives a good estimate of the maximum crack spacing. However, both the deformation and crack width comparisons show that the effective deformations vary arbitrarily between the lower (considering tension stiffening) and upper (no tension stiffening) limits. Further, the support moments predicted for the two-span beams are all larger than the measured values.

Chapter 6 discusses the approaches illustrated in Chapter 4. It is shown that lightly (strongly) reinforced end-restrained tensile members are characterized by widely (closely) spaced cracks with large (small) crack openings. The cracking behaviour of bending members is strongly dependent on the effective reinforcement ratio of the tension chord. Moment redistributions have a large influence on beam deflections, while the relative influence of tension stiffening decreases with increasing load level and time.

The overall conclusions are summarized in Chapter 7.
Kurzfassung


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1 Introduction

1.1 Background

The Tension Chord Model (TCM) was developed within the framework of the research project Deformation Capacity of Structural Concrete that was carried out between 1992 and 2002 at the Institute of Structural Engineering (IBK, ETH Zurich) within the group led by Prof. Marti. Sigrist (1995) developed the Tension Chord Model for the purpose of determining the deformation capacity of plastic hinges in reinforced concrete (RC) bending members. Alvarez (1998) formulated the complete stress-strain relationship for a reinforced and prestressed tension tie under load and restraint. He applied the model to determine the amount of minimal reinforcement necessary for limiting crack widths and preventing brittle failure at first cracking. Kaufmann (1998) developed the Cracked Membrane Model based on the basic principles of the Tension Chord Model. Fürst (2001) used the Tension Chord Model to discuss the behaviour of prestressed tension ties subjected to loading cycles, drawing conclusions concerning fatigue. Kenel (2002) implemented fracture mechanics aspects into the model. Pfyl (2003) combined the Tension Chord Model with the Fibre Effectiveness Model in order to describe the deformation behaviour of tensile and bending members made of steel fibre reinforced concrete and steel reinforcing bars. Thoma (2004) developed a stochastic model for prestressed tensile ties based on the Tension Chord Model. Heinzmann (Heinzmann 2006) incorporated the influence of self-equilibrating stresses due to shrinkage. The steel strains and integral deformations predicted by the Tension Chord Model show a good agreement with the measured values obtained in large scale tests (Sigrist 1995; Alvarez 1998; Kenel 2002).

1.2 Aims

Following up on the previous work outlined above, the present thesis was initiated with the aim of verifying the applicability of the Tension Chord Model for serviceability design. The work was carried out with the following goals:

1. to discuss and extend the Tension Chord Model crack width and deformation approach for tensile and bending members under short-term and sustained service loads.
2. to validate the resulting approach with test data from short- and long-term tensile and bending tests from various research institutes.
3. to compare the resulting approach with other crack width and deflection equations.
4. to evaluate the decisive input parameters for serviceability limit state predictions.

1.3 Research Significance

Due to the requirements for modern buildings concerning aesthetics, functionality and durability a number of deformation-sensitive solutions such as high-tech cladding, exposed concrete and white-
tank waterproofing (see Fig. 1.1) are becoming increasingly popular. At the same time, demanding cost and construction time limitations, paired with a constant increase in material strengths, favour slender, monolithically connected members with rapid construction schedules, which all together result in less favourable conditions for serviceability.

As a consequence, serviceability design is becoming an increasingly important aspect of building construction. Regardless of this tendency most codes still treat serviceability as a secondary issue (Beeby 1999) and their prediction models differ considerably (see Chapter 2). The Tension Chord Model represents a consistent tool for modelling both the cracking and deformation behaviour of tension ties and one-way bending members.

**Fig. 1.1** Significance of Serviceability Design.

### 1.4 Overview

Chapter 2 of this work summarizes the predominant crack width and deflection prediction approaches. The first part of the chapter discusses the basic principles behind the slipping-bond and concrete-cover crack width approaches. The second part of the chapter provides an overview of the most well-known tension stiffening and deflection prediction approaches for bending members.

Chapter 3 contains the basic principles used for the crack width and deflection approaches provided in Chapter 4. The chapter is divided into four parts. The first part discusses the concrete material parameters relevant for serviceability. The second part on reinforcement behaviour is very short, because elastic steel behaviour is assumed under service loads. The third part deals with different approaches for quantifying the State I and II section behaviour under sustained moments. Finally, the fourth part of the chapter illustrates the basic principles and assumptions behind the Tension Chord Model.

Chapter 4 is one of the two core chapters of this work. The first part of the chapter deals with tension stiffening in bending members, under sustained loads and during unloading. The second and
thrid parts contain a selection of consistent crack width and deflection prediction equations, derived based on the State I and II deformation behaviour combined with the Tension Chord Model.

Chapter 5 is the second core chapter of this work. A selection of test series from various research institutions is used to validate the serviceability approaches described in Chapter 4. The first part of the chapter contains three series of small tensile elements with sustained axial loads, eight shrinkage specimens and a selection of deformation-controlled tensile tests. The second part contains three series of four-point bending tests (two of them with sustained loads), a series of simple beams under sustained uniformly distributed loads, the two ConCrack four-point bending tests and three series of two-span beams. The last part of the chapter contains four experimental campaigns with simply supported slabs as well as corner, edge and internal flat-slab panels under sustained loads.

Chapter 6 contains a discussion involving parameter studies carried out for the Tension Chord Model crack and deflection equations (Chapter 4), as well as comparisons with the models discussed in Chapter 2.

Overall conclusions are provided in Chapter 7.

1.5 Limitations

The present work is limited to steel reinforced concrete tension ties and tension chords in one-way bending members with uniaxial stress states. The cross-sections are rectangular and the material behaviour is assumed elastic. Second order effects, shear deformations, non-linear creep and differential shrinkage/temperature effects are not considered. When not stated otherwise, average material properties are used. Cyclic loading is not considered.
2 State of the Art

The two main concerns of serviceability design of reinforced concrete (RC) structures consist in limiting crack widths and deflections. This chapter summarizes the most well-known approaches currently used for these purposes. Section 2.1 discusses the basic principles behind the crack width prediction equations based on the slipping-bond approach and the concrete-cover approach. Section 2.2 summarizes the most used approaches for considering tension stiffening at section level for the purpose of determining deflections at member level.

2.1 Crack Width

In a cracked tension chord the strain difference between the ductile steel (with service strains up to 2 ‰) and the brittle concrete (with tensile strains below 0.1 ‰) is absorbed by the cracks. The transfer length \( l_0 \) (also referred to as transmission length) defines the strain-difference gathering length belonging to each crack, which corresponds to the length on each side of a crack, which is affected by the crack. Since the concrete surface stresses are reduced within the transfer length, new cracks form outside the transfer length, leading to a final crack spacing \( s_r \) between once and twice the transfer length (Base et al. 1966; Broms 1965). The corresponding crack widths are

\[
w = \int_{s_r} \left( \varepsilon_s - \varepsilon_c \right) dx = s_r \left( \varepsilon_{su} - \varepsilon_{cm} \right)
\]

Under short-term loads the strain in the concrete is small and often ignored. There are different approaches for determining the transfer length \( l_0 \) and the development of the steel (and concrete) strains on either side of the crack. Both depend on the model used to describe the load transfer between the steel and the concrete through mechanical interlock.

Besides the purely empirical crack width equations (e.g. Gergely and Lutz 1968) there are two main approaches for modelling the load transfer between the steel and concrete and accommodating the strain incompatibility. The first approach based on Saliger (1936), Kuuskoski (1950) and Rehm (1961) assumes a slipping bond at the interface between the two materials. In this case the transfer length is defined as the length necessary to introduce the force \( F_c = f_{ct} A_c \) into the concrete and depends on the bond stress, the reinforcement ratio and the bar diameter. The second approach based on Base et al. (1966) and Broms (1965) assumes a transfer length defined by the concrete cover. The traditional (Continental) European code equations are based on the former, while the bar spacing limit in ACI 318 (2008) is derived from the latter approach. A comprehensive summary of the individual crack width equations is given in Borosnyói and Balázs (2005).

2.1.1 Slipping-Bond Approach

The slipping-bond approach models the load transfer between the steel and the concrete with fictitious bond stresses at the interface between the two materials. The strain incompatibility is accommodated by slip (relative displacement between steel and concrete). The distribution of bond stresses on either side of the cracks was initially assumed to be a function of the ultimate bond stress (Con-
sidère 1902; Saliger 1936) and later determined from the differential equation of slipping bond (Kuuskoski 1950; Rehm 1961). In the first case (Saliger 1936) the transfer length

\[ l_0 = k \frac{f_{\text{ct}}}{\rho \tau_{\text{b,max}}} \]  

(2.2)
is defined by the ratio of tensile and bond strength and the assumed bond stress distribution factor \( k \).

In the second case, the slip \( \delta(x) \) and shear stress \( \tau_b(x) \) distributions on either side of a crack are obtained by solving the differential equation of slipping bond

\[ \frac{d^2 \delta}{dx^2} = \frac{4\tau_b[1 + \rho(n-1)]}{\varnothing E_s(1-\rho)} \]  

(2.3)

for a given bond stress - slip relationship, e.g.

\[ \tau_b = C\delta^u \]  

(2.4)

Eq. (2.4) leads to analytical solutions for initial cracking \((0 < \alpha < 1)\) (Noakowski 1985; Balázs 1993) and the limiting cases of \( \alpha = 0 \) and \( 1 \) (e.g. Rehm 1961). More complex bond stress – slip relationships usually require numerical solutions. The reinforcement ratio \( \rho \) refers to \( A_s/(bh) \) for uniformly reinforced tension ties and \( A_s/(bh_{ef}) \) for tension chords in bending elements. The tensile concrete stresses are assumed to be uniformly distributed over the tension chord, implying plane concrete sections and leading to equal crack widths from the bar to the concrete surface.

Analytical crack width approaches based on Eq. (2.3) usually adopt Eq. (2.4) with \( \alpha = 0 \) and an average bond stress \( C = \tau_b \) (Watstein and Parsons 1943; Bachmann 1967; König and Fehling 1988; Sigrist 1995; Marti et al. 1998). This solution also provides the basis for the Model Code 1990 (Comité Euro-International du Béton 1993), Model Code 2010 (Fédération Internationale du Béton 2010b) and Eurocode 2 (Part 1) (European Committee for Standardization 2004) transfer length

\[ l_0 = \alpha_1 \frac{C}{2} + \alpha_2 \frac{f_{\text{ct}} \varnothing}{2\tau_b \rho} \]  

(2.5)

and load crack width

\[ w_{\text{max}} = 2l_0 \left( \frac{\sigma_{s,\text{ll}}}{E_s} - \alpha_3 \frac{f_{\text{ct}}(1 + np)}{\rho E_s} - \alpha_4 E_{\text{s,ef}} \right) \]  

(2.6)
as well as for the Eurocode 2 (Part 3) (European Committee for Standardization 2011) end restraint crack width equation

\[ w_{\text{max}} = 2l_0 \left( k_1 k_2 \frac{f_{\text{ct}}(1 + np)}{2\rho E_s} \right) \]  

(2.7)

The code equations differ from the strictly analytical solution \((\alpha_1 = 0, \alpha_2 = \alpha_3 = 0.5, \alpha_4 = 0/1, k_1 = k = 1)\) in an attempt to incorporate the influence of cover, curvature, different bond conditions and shrinkage strains. Eurocode 2 (EC 2) suggests \( \alpha_1 = 3.4, \alpha_2 = 0.61/0.30 \) (tension/bending for \( \tau_b = 1.8 f_{\text{ct}} \)), \( \alpha_3 = 0.6/0.4 \) (short-/long-term), \( \alpha_4 = 0, k = 1/0.65 \) \((h \leq 300 \text{ mm}/h \geq 800 \text{ mm})\) and \( k_1 = 0.5...1 \) (1 for pure tension). Model Code 2010 (MC 2010) adopts \( \alpha_1 = 0, \alpha_2 = 0.5, \alpha_3 = 0.6/0.4 \) (short-/long-term) and \( \alpha_4 = 0/1 \) and additionally provides a multiplier for large concrete covers. Except for long-term loading during the crack formation phase, the bond stress \( \tau_b = 1.8 f_{\text{ct}} \).
Both codes use Schiessl's (1989) effective reinforcement ratio

$$\rho_{ef} = \frac{A_t}{2.5b(h-d)} \geq \frac{3A_s}{b(h-h_{fl})}$$

(2.8)

for RC bending elements.

### 2.1.2 Concrete-Cover Approach

In this case the transfer length is defined by the controlling concrete cover. Broms (1965) distinguishes between internal and visible cracks and uses load spreading circles inscribed between two neighbouring cracks to check if a new intermediate crack in a one-bar tensile tie (Fig. 2.1 (a–c)) reaches the concrete surface or not. He considers a new crack short enough to remain invisible if it forms between cracks spaced closer than $s_r = 2(c + \Phi/2)$ (Fig. 2.1 (b)), making

$$l_0 = c + \frac{\Phi}{2}$$

(2.9)

the transfer length for visible cracks in circular single-bar tension ties. Similarly, for their so called "non-slip" approach, Base et al. (1966) use an elastic theory solution to show that a length approximately equal to the concrete cover $c$ is necessary to distribute concentrated tensile stresses to the concrete surface in a one-bar tensile tie. They consider the sum of the micro-crack widths at the steel-concrete interface to amount to similar values as the sum of the surface crack widths. Neither approach considers if the concrete tensile stresses that reach the member surface are large enough to cause cracking.

![Fig. 2.1](image_url)

Circular single-bar tensile tie between two cracks illustrating Broms' (1965) load spreading circles and the corresponding strut and tie models.

For a multiple-bar tension chord Broms assumes that the local cracks at each reinforcement bar join and reach the surface if they occur between two existing cracks, which are spaced at a distance larger than the bar spacing and larger than twice the transfer length according to Eq. (2.9). He suggests a
visible crack spacing between once and twice the transfer length of a single bar and an average (not maximum) surface crack width equal to

$$w_{\text{avg}} = 2 \left( c + \frac{\varnothing}{2} \right) \varepsilon_{s,\text{II}}$$  \hspace{1cm} (2.10)$$

Based on their theoretical consideration and on test data comparisons Base et al. suggest a maximal crack width

$$w_{\text{max}} = 3.3 \, c \, \varepsilon_{s,\text{II}} \frac{h - x_{\text{II}}}{d - x_{\text{II}}}$$  \hspace{1cm} (2.11)$$
corresponding to a maximal crack spacing of 3.3 $c$.

![Fig. 2.2](image)

Fig. 2.2 Maximum crack widths according to Frosch’s Eqs. (2.12) and (2.13) and the EC 2 and MC 2010 versions of Eqs. (2.6) and (2.5) with $\alpha_3 = 0.6$, $\alpha_4 = 0$ and $\tau_b = 1.8 \, f_{\text{ct}}$: (a) $\varnothing$ 16 mm @ 250 mm ($\rho = 0.4 \%$); (b) $\varnothing$ 10 mm @ 100 mm ($\rho = 0.5 \%$). Note that $f_{\text{ct}} = 2$ MPa, $n = 6$, $E_s = 205$ GPa, $c = 30$ mm. The dimensions are in [mm].

The current (since the 2005 edition) ACI 318 (ACI Committee 318 2008) maximal bar spacing for crack width limitation is derived from Frosch’s (1999) bottom face maximum crack width equation

$$w_{\text{max}} = 2 l_0 \frac{\sigma_{s,\text{II}}}{E_s} \frac{h - x_{\text{II}}}{d - x_{\text{II}}}$$  \hspace{1cm} (2.12)$$

which is directly based on Eq. (2.10) with a bottom face transfer length

$$l_0 = \sqrt{\left( c_b + \frac{\varnothing}{2} \right)^2 + \left( \frac{8}{2} \right)^2}$$  \hspace{1cm} (2.13)$$

This bottom face transfer length corresponds to the maximum distance between a point on the bottom beam face and the centre of the nearest reinforcing bar (controlling concrete cover), which increases with increasing bar spacing $s$. For bottom fibre side crack widths Frosch (1999) suggests a transfer length of

$$l_0 = \sqrt{\left( c_b + \frac{\varnothing}{2} \right)^2 + \left( c_s + \frac{\varnothing}{2} \right)^2}$$  \hspace{1cm} (2.14)$$
Fig. 2.2 illustrates Frosch’s Eqs. (2.12) and (2.13) together with the EC 2 and MC 2010 versions of Eqs. (2.6) and (2.5). Despite the additional cover term, the EC 2 version ($\alpha_1 = 3.4$, $\alpha_2 = 0.3$) predicts lower maximum crack widths than the original MC 2010 version ($\alpha_1 = 0$, $\alpha_2 = 0.5$).

2.2 Deflections

The general way to determine deflections of beams and one-way slab strips is to integrate the curvatures determined at each section $\chi(x)$, by applying the principle of virtual work

$$a = \int_0^l \chi(x) M(x) \, dx$$

(2.15)

By this means axial forces and variable stiffnesses (e.g. due to cracking or curtailed reinforcement) can be considered and long-term effects and tension stiffening can be dealt with at section level.

![Fig. 2.3 Schematic illustration of the moment and curvature distributions in a partially cracked beam.](image)

2.2.1 Section Level

The two most used models for incorporating tension stiffening at section level in bending elements were developed in the 1960's by Branson and Rao. Branson (see Branson 1977) empirically derived an effective moment of inertia

$$I_{ef} = \left( \frac{M}{M} \right)^{m} I_1 + \left[ 1 - \left( \frac{M}{M} \right)^{m} \right] I_{II}$$

(2.16)

used with $m = 4$ for considering tension stiffening at section level. Reference is given to his book (Branson 1977) because the author does not hold the original paper (Branson 1963). Eq. (2.16) can be rearranged to

$$\Delta \chi_{II} = \left( 1 - \frac{I_{II}}{I_{II} + \left( \frac{M}{M} \right)^{4} \left( I_1 - I_{II} \right)} \right) \frac{M}{E_{cm} I_{II}}$$

(2.17)
Rao (1966) derived the semi-empirical expression

$$\Delta e_s = \left( \frac{\sigma \tau \mu \alpha}{\sigma_{s,II}} \right) 0.18 \frac{f_{ct}}{\rho E_s}$$

(2.18)

for the average steel strain reduction due to tension stiffening in cracked flexural elements subjected to pure bending. According to Mourachev (Mourachev et al. 1971), the quotient in the brackets corresponds to the empirical tension stiffening factor found by Nemirovsky (1949) [not cited, as the original publication is not available]. The second part of the equation, as shown by Rostásy et al. (1976), who extended the equation to other load cases, corresponds to the sudden increase in steel strains upon cracking.

Rao's tension stiffening factor is implicitly included in the EC 2 (2004) and the MC (2010b) interpolation equation

$$\chi = \beta \left( \frac{\sigma_{s,II,r}}{\sigma_{s,II}} \right)^2 \chi_1 + \left( 1 - \beta \left( \frac{\sigma_{s,II,r}}{\sigma_{s,II}} \right)^2 \right) \chi_{II}$$

(2.19)

For pure bending elements ($\sigma_{s,II,r}/\sigma_{s,II}$) can be replaced with ($M_r/M$) and Eq. (2.19) can be rearranged to

$$\Delta \chi_{es} = \beta \left( \frac{M_r}{M} \right) \left[ \frac{M_r}{E_{cm}I_{II}} - \frac{M_r}{E_{cm}I_1} \right]$$

(2.20)

Factor $\beta$ is equal to 1 and 0.5 for short-term and long-term/cyclic loading, respectively. Fig. 2.4 compares Branson's and Rao's equations. The agreement between the models depends on the load level and reinforcement ratio.

---

**Fig. 2.4** Related tension stiffening curvature reductions according to Branson with Eq. (2.17) and Rao with Eq. (2.20): (a) $\rho = 0.2 \%$; (b) $\rho = 1.0 \%$. Note that $\rho^* = 0$, $d/h = 0.85$ and $n = 6$. The curves are cut off at $M = M_y$. 

---
2.2.2 Member Level

In order to avoid curvature integrations as in Eq. (2.15), global stiffness

\[ I_{ef} = \zeta_i I_i + (1 - \zeta_i) I_{II} \]  

(2.21)

or global deflection

\[ a = \zeta_a a_i + (1 - \zeta_a) a_{II} \]  

(2.22)

interpolation coefficients \( \zeta_i \) or \( \zeta_a \) can be used. Besides tension stiffening, these have to account for a non-uniform stiffness (cracked and uncracked regions).

Branson (1977) suggested the same interpolation coefficient he derived at section level (Eq. (2.16)) to be used at member level, but with a power \( m = 3 \). This equation is given in ACI 318 (2008) together with a global multiplier (Branson 1971) for long-term effects, leading to

\[ a = \left(1 + \frac{\xi}{1 + 50g^2}\right) \frac{\xi M d^2}{I_{ef} E_c} \]  

(2.23)

where \( \xi \) depends on the load duration and \( k \) is the integration factor.

EC 2 and MC 2010 both refer to the direct interpolation of deflections with the same interpolation coefficient

\[ \zeta_a = \beta \left( \frac{M_r}{M_a} \right)^2 \]  

(2.24)

used for curvatures (Eq. (2.19)) as the simplest method of deflection prediction. For sustained loads they use the effective modulus method (see Eq. (3.65)) with a shrinkage curvature equation (Trost and Mainz 1968).

<table>
<thead>
<tr>
<th>method</th>
<th>variable stiffness</th>
<th>tension stiffening</th>
<th>creep &amp; shrinkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-08</td>
<td>( \zeta_i = \left( \frac{M_i}{M_a} \right)^3 )</td>
<td>implicit</td>
<td>( \Delta u_i = \frac{\xi}{1 + 50g^2} a_i )</td>
</tr>
<tr>
<td>EC 2 MC 2010</td>
<td>( \zeta_a = \beta \left( \frac{M_r}{M_a} \right)^2 )</td>
<td>implicit</td>
<td>( \chi_c = \frac{M}{I_{ef} E_{cr}} + \frac{v_{cr} S_{cr}}{I_{ef}} )</td>
</tr>
<tr>
<td>Bilinear Method</td>
<td>( \zeta_a = \beta \left( \frac{M_r}{M_a} \right) )</td>
<td>implicit</td>
<td>( \Delta \chi_c = \chi_c k \delta + k_n \frac{E_n}{d} )</td>
</tr>
<tr>
<td>MC 90 simplified</td>
<td>( a = a_i \left( \frac{h}{d} \right)^\eta (1 - 20g^2) )</td>
<td>implicit</td>
<td>creep implicit</td>
</tr>
</tbody>
</table>

Table 2.1 Global deflection prediction approaches for pure bending.

The bilinear method (Favre et al. 1981) interpolates the deflections with a lower power version

\[ \zeta_a = \beta \left( \frac{M_r}{M_a} \right) \]  

(2.25)

of Rao's section level interpolation coefficient (due to the integration) and uses Trost and Mainz' (1968) interpolation coefficients (see Chapter 3.3.2) to account for long-term effects. The MC 90
State of the Art

(Comité Euro-International du Béton 1993) deflection equation is a simplified version of the bilinear method. Table 2.1 gives an overview of the global deflection approaches at member level.

2.3 Summary

This chapter gives an overview of the most-used crack width and deflection equations.

- For determining the crack spacing Model Code and Eurocode suggest Eq. (2.5), which is a modified version of the slipping-bond transfer length. The ACI Building Code Requirements for Structural Concrete implicitly use Eq. (2.13), which is based on Broms's load spreading circles.

- For considering the influence of tension stiffening at section level Model Code and Eurocode suggest Rao's Eq. (2.20), while the ACI Building Code Requirements for Structural Concrete suggest Branson's Eq. (2.17). At cracking both approaches provide curvature reductions equal to the full difference between the State I and II curvature, which are then reduced towards zero with increasing load levels. However, between these two points, especially for low reinforcement ratios, the two approaches differ (see Fig. 2.4).

- For determining the global deflections of partially cracked beams Model Code and Eurocode interpolate the deflections with Eq. (2.22), while the ACI Building Code Requirements for Structural Concrete interpolate the stiffnesses with Eq. (2.21). The interpolation coefficients are given in Table 2.1. Tension stiffening is considered implicitly within the interpolation.
3 Basic Principles

The first two sections of this chapter summarize the concrete (Section 3.1) and steel (Section 3.2) material behaviour required for short- and long-term serviceability design. The next two sections contain the two basic approaches combined in Chapter 4 to form the short- and long-term crack width and deflection equations. The first approach (Section 3.3) describes the short- and long-term section behaviour in bending of uncracked (State I) and cracked (State II, \( f_{cr} = 0 \)) sections. The second approach refers to the Tension Chord Model used to model the interaction between steel and concrete between the cracks in cracked tension chords (Section 3.4). A summary of the chapter is given in Section 3.5.

3.1 Concrete Material Behaviour

Traditionally, the average 28-day concrete compressive strength is used to describe all other concrete properties. It is usually obtained from tests on 300 mm tall, 150 mm diameter cylinders (\( f_{cm} \)) or cubes with 150 mm side lengths. Mainly due to the smaller aspect ratio, the latter is slightly larger (\( f_{cm,cube} \approx 1.25 \ f_{cm} \)). The current European code equations use the characteristic (5% fractile) cylindrical compressive strength

\[
f_{ck} = f_{cm} - 8 \ [\text{N/mm}^2] \tag{3.1}
\]

while the ACI codes refer to the specified cylindrical compressive strength \( f'_c \), which is an average value.

The cement classes S, N and R refer to slow, normal and rapid strength development rates. According to the EN 197 (European Committee for Standardization 2000) classification, S includes CEM 32.5N (\( f_{c,7d} \geq 16 \text{ MPa} \)), N includes CEM 32.5R and 42.5N (\( f_{c,2d} \geq 10 \text{ MPa} \)) and R includes CEM 42.5R and 52.5N (\( f_{c,2d} \geq 20 \text{ MPa} \) and 30 MPa) as well as 52.5R (\( f_{c,2d} \geq 30 \text{ MPa} \)).

3.1.1 Short-Term

Under service conditions it is common praxis to assume linear elastic material behaviour to describe the instantaneous deformations

\[
e_c(t_0) = \frac{\varepsilon_c(t_0)}{E_c(t_0)} \tag{3.2}
\]

in tension and compression. Alternatively, Sargin's (1971) stress-strain relationship

\[
\sigma_c = -f_{cm} \frac{k_\alpha \xi - \xi^2}{1 + (k_\alpha - 2)\xi} \quad \text{with} \quad \xi = -\varepsilon_c \frac{E_c}{f_{cu}} \quad \text{and} \quad k_\alpha = \frac{E_c \varepsilon_{cu}}{f_{cm}} \tag{3.3}
\]

in [‰ and MPa] describes the short-term behaviour of concrete in compression below the compressive strength, where the failure strains are approximately...
Basic Principles

\[ \varepsilon_{cu} \approx 0.6 \left( f_{cm} \right)^{1/3} \text{ [in } \% \text{ and MPa]} \] (3.4)

The concrete tensile strength is a decisive parameter for serviceability design. Besides defining the cracking loads, the concrete tensile strength also influences the deformation behaviour of cracked tension chords. Most current (European) cracking models are based on the average direct tensile strength, which requires a demanding test setup. This is why the tensile strength is often obtained from splitting tensile tests, where

\[ f_{ct} \approx (0.7...1.0) f_{ct,sp} \] (3.5)

or flexural tensile tests, where

\[ f_{ct} \approx \frac{1.5 (h/100 \text{ mm})^{0.7}}{1 + (h/100 \text{ mm})^{0.7}} f_{ct,fl} \] (3.6)

As indicated in Eq. (3.2), the stresses, strains and material properties in Eqs. (3.2) to (3.6) apply to the concrete age \( t_0 \) when the stress is applied. In the absence of material tests, the average 28-day elastic modulus (Fédération Internationale du Béton 2010a)

\[ E_c = \alpha_E E_0 \left( \frac{f_{cm}}{10} \right)^{1/3} \text{ [in MPa]} \] (3.7)

and the average direct tensile strength

\[ f_{ct} = 0.3 \left( f_{ck} \right)^{2/3} \text{ [in MPa]} \] (3.8)

can be determined from the 28-day compressive strength. \( E_0 = 21.5 \text{ GPa and } \alpha_E = 0.7 \) to 1.2 (1 for quartzite aggregates). The compressive strength

\[ f_{cm}(t) = \beta(t) f_{cm} \] (3.9)

and elastic modulus

\[ E_c(t) = \left[ \beta(t) \right]^m E_c \] (3.10)

at loading ages other than 28 days can be estimated with the time factor

\[ \beta(t) = \exp \left\{ \alpha \left( 1 - \sqrt{\frac{28}{t}} \right) \right\} \] (3.11)

where \( \alpha = [0.38/0.25/0.20] \) for cement types [S/N/R] and \( m \) is between 0.5 (Fédération Internationale du Béton 2010a) and 1 (European Committee for Standardization 2004). Eq. (3.11) is not usually used for the concrete tensile strength because its development is sensitive to many hard-to-quantify parameters.

3.1.2 Long-Term

The concrete strains under sustained loads increase due to creep, while the concrete stresses under sustained deformations decrease due to relaxation. Further concrete shrinks due to loss of water and cement hydration. For service compressive stresses below approximately \( 0.4 f_{cm} \) it is common practice to treat concrete as an aging, linear visco-elastic material where the strain at time \( t \)
Concrete Material Behaviour

\[ \varepsilon(t) = \sigma_c(t_0) J(t, t_0) \]  (3.12)
due to a sustained stress applied at \( t_0 \) is defined by the compliance function \( J(t, t_0) \). Accordingly the stress at time \( t \)

\[ \sigma_c(t) = \varepsilon_c(t_0) R(t, t_0) \]  (3.13)
due to a sustained strain applied at \( t_0 \) is defined by the relaxation function \( R(t, t_0) \). The compliance function is either determined experimentally or with empirical expressions for \( \varphi(t, t_0) \) (see Section "Creep Coefficient" below) according to

\[ J(t, t_0) = \frac{1 + \varphi(t, t_0)}{E_c(t_0)} \quad \text{or} \quad \frac{1}{E_c(t_0)} + \frac{\varphi(t, t_0)}{E_c} \]  (3.14)
depending on whether \( \varphi(t, t_0) \) is related to the elastic modulus at the age of loading or at 28 d. As relaxation tests are not feasible, the relaxation function is derived from the compliance function, making use of the fact that the two functions are related mathematically (see Section "Aging Coefficient" below).

According to Boltzmann’s (1874) principle of superposition for visco-elastic materials, strains due to stresses applied either suddenly at \( t_0 \) or gradually over a period of time between \( t_0 \) and \( t \), as well as shrinkage- or temperature-induced strains \( \varepsilon_{ci} \) can be determined individually and added together. Assuming the first definition of \( \varphi \) given in Eq. (3.14)

\[ \varepsilon_c = \sigma_c(t_0) \frac{1 + \varphi(t, t_0)}{E_c(t_0)} + \int_{t_0}^{t} \frac{1 + \varphi(t, \tau)}{E_c(\tau)} d\tau + \varepsilon_{ci}(t, t_0) \]  (3.15)

In practice gradual concrete stress variations occur e.g. due to internal stress redistributions between the steel and concrete caused by creep and shrinkage. By introducing the aging coefficient \( X \) (Trost 1967) and assuming \( E_c(\tau) = E_c(t_0) \), the integral term in Eq. (3.15) can be simplified according to

\[ \varepsilon_c = \sigma_c(t_0) \frac{1 + \varphi(t, t_0)}{E_c(t_0)} + \Delta\sigma_c(t) \frac{1 + X\varphi(t, t_0)}{E_c(t_0)} + \varepsilon_{ci}(t, t_0) \]  (3.16)
The aging coefficient \( X \) takes into account that \( \Delta\sigma_c(t) = \sigma_c(t) - \sigma_c(t_0) \) is applied gradually to an aging concrete.

Eq. (3.16) leads to

\[ \sigma_c(t) = \sigma_c(t_0) + E_{c,a} \left[ \varepsilon_c - \sigma_c(t_0) \frac{1 + \varphi(t, t_0)}{E_c(t_0)} - \varepsilon_{ci}(t, t_0) \right] \]  (3.17)

where

\[ E_{c,a} = \frac{E_c(t_0)}{1 + X\varphi(t, t_0)} \]  (3.18)
denotes the age-adjusted effective modulus. Eqs. (3.17) and (3.18) belong to the age-adjusted effective modulus method (AAEMM), named this way by Bažant (1972), because he found the aging coefficient \( X \) to be strongly dependent on the age of the concrete when the load is applied. The input parameters of Eqs. (3.17) and (3.18) are the temperature- and shrinkage-induced strains \( \varepsilon_{ci} = \varepsilon_{ci}(t, t_0) \), the creep coefficient \( \varphi = \varphi(t, t_0) \) and the aging coefficient \( X \). The free shrinkage strains \( \varepsilon_{sh} \), the creep coefficient and the aging coefficient are discussed in the following three sections.
Free Shrinkage Strains

It is usually assumed that the total free shrinkage strains are composed of drying $\varepsilon_{sh,d}$ (loss of water) and autogenous $\varepsilon_{sh,a}$ (cement hydration) shrinkage strains. The total and autogenous shrinkage strains can be determined experimentally. If the shrinkage specimens are not the same size as the corresponding concrete member, the size dependent development rate of the drying shrinkage strains needs to be taken into account, for example with Eq. (3.21) given below.

There are a wide range of empirical shrinkage equations for estimating the shrinkage strains. They usually contain a drying shrinkage term composed of a final value and a time function (first term) and in some cases an autogenous shrinkage term (second term). The shrinkage equations are based on the concrete compressive strength even though shrinkage is only indirectly related to the compressive strength over the water-to-cement ratio, which directly influences both the drying (water) and autogenous (cement) shrinkage. A selection of shrinkage equations is given here and illustrated in Fig. 3.1. When not stated otherwise, the input variables are to be inserted in the following units: the relative humidity RH as a percentage, the notional size of the member $h_0 = 2 \cdot \frac{V}{S} = 2 \frac{A_c}{u}$ in [mm], the concrete properties $f_{cm}$ and $E_c$ in [MPa], the shrinkage duration $t-t_s$ in [d] and the end of curing (= beginning of shrinkage) $t_s$ in [d]. The equation versions given here apply to a relative humidity (RH) between approximately 0.4 and 0.9. V/S stands for the volume to surface ratio.

CEB Model:

The original CEB shrinkage approach (version MC 90) was developed by Task Group 9 (Müller and Hilsdorf 1990) especially for Model Code 90 (Comité Euro-International du Béton 1993). The approach contains a final drying shrinkage value of

$$
\varepsilon_{sh,d,\infty} = -1.55 \cdot 10^{-6} \left(1 - RH^3\right) \left[160 + 10\alpha \left(9 - 0.1f_{cm}\right)\right]
$$

with $\alpha = [4/5/8]$ for [S/N/R] cement and a time factor

$$
\beta(t, t_s) = \left[\frac{(t-t_s)}{0.035 \cdot h_0^2 + (t-t_s)}\right]^{0.5}
$$

For the current fib (Fédération Internationale du Béton 1999; 2010a) approach (version MC 2010), an autogenous shrinkage term

$$
\varepsilon_{sh,a}(t) = -\alpha_{as} 10^{-6} \left(\frac{0.1f_{cm}}{6 + 0.1f_{cm}}\right)^{2.5} \left(1 - e^{-0.2\varepsilon}\right)
$$

and additional factors for considering the cement type in the drying shrinkage term

$$
\varepsilon_{sh,d,\infty} = -1.55 \cdot 10^{-6} \left(1 - RH^3\right) \left[(220 + 110 \cdot \alpha_{ds1}) \cdot e^{-f_{cm}\alpha_{ds2}}\right]
$$

were added. Factors $[\alpha_{as}/\alpha_{ds1}/\alpha_{ds2}]$ are equal to [800/3/0.013] for S type cement, [700/4/0.012] for N type cement and [600/6/0.012] for R type cement.

The EC 2 (European Committee for Standardization 2004) shrinkage equation includes a reduction factor 0.85 $k_h$ for the final drying shrinkage strains

$$
\varepsilon_{sh,d,\infty} = -1.55 \cdot 10^{-6} \left(1 - RH^3\right) \left[(220 + 110 \cdot \alpha_{ds1}) \cdot e^{-f_{cm}\alpha_{ds2}}\right] 0.85k_h
$$
and uses a different power of \( h_0 \) in the time factor

\[
\beta(t,t_s) = \left[ \frac{(t-t_s)}{0.04 \cdot h_0^{1.5} + (t-t_s)} \right]^{0.5}
\]  

(3.25)

as well as a slightly simpler expression for the autogenous shrinkage strains

\[
e_{\text{ah},s}(t) = -2.5 \cdot 10^{-6} (f_{\text{ah}} - 10)(1 - e^{-0.22})
\]  

(3.26)

The type of cement is considered with the coefficients \( \alpha_{d1} \) and \( \alpha_{d2} \) defined above. Factor \( k_h = [1/0.85/0.75/0.7] \) considers the notional sizes \( h_0 = [100/200/300/\geq 500 \text{ mm}] \).

**ACI 209:**

The ACI (ACI Committee 209 1992) shrinkage strains consider the experimentally determined final shrinkage value \( \varepsilon_{\text{sh},\infty} = -780 \cdot 10^{-6} \) and time factor (Branson and Christiason 1971)

\[
\beta(t,t_s) = \frac{(t-t_s)}{(t-t_s) + 35}
\]  

(3.27)

for standard conditions (RH = 0.4 and \( h_{\text{avg}} = 150 \text{ mm} \)). For non-standard conditions \( 150 \text{ mm} \leq h_{\text{avg}} \leq 300 \text{ mm} \) empirical correction factors apply, leading to

\[
e_{\text{sh}}(t, t_s) = \varepsilon_{\text{ah},s}(t) \beta(t,t_s) \left\{ \begin{array}{ll}
1.23 - 0.00150 \cdot h_{\text{avg}} & \text{for} \quad t - t_s \leq 360 \text{ d} \\
1.17 - 0.00114 \cdot h_{\text{avg}} & \text{for} \quad t - t_s \geq 360 \text{ d}
\end{array} \right.
\]  

(3.28)

According to Branson and Christiason (1971), the empirical correction factors were derived by Christiason in his MS thesis in 1970.

**GL 2000 Model:**

Gardner and Lockman's (2001) model considers final shrinkage strains

\[
e_{\text{sh},\infty} = 1000 \cdot 10^{-6} \sqrt[30]{1.18 \cdot f_{\text{cm}}} (1 - 1.18 \cdot \text{RH}^4)
\]  

(3.29)

and the CEB time factor

\[
\beta(t,t_s) = \left[ \frac{(t-t_s)}{0.15 \cdot (V/S)^{0.5} + (t-t_s)} \right]^{0.5}
\]  

(3.30)

with \( V/S \) instead of \( h_0 \). The model was derived with test specimens with \( h_0 < 38 \text{ mm} \). Factor \( \alpha \) is equal to \([1/0.7/1.15]\) for ASTM C150 (American Society for Testing and Materials 2011) Portland cement types [I/II/III], where type I is a general purpose cement with \( f_{c,3d} = 12 \text{ MPa} \), type II features a moderate sulphate resistance with \( f_{c,3d} = 10 \text{ MPa} \) and type III is characterized by a relatively high early strength with \( f_{c,3d} = 24 \text{ MPa} \).
**Basic Principles**

**Full B3 Model:**

The most complex shrinkage equation (Bažant and Baweja 1995) does not explicitly consider autogenous shrinkage. It considers final shrinkage strains

\[
\varepsilon_{sh,0} = -\alpha_1 \alpha_2 \cdot 10^{-6} \left[ 1.9 \cdot 10^{-2} w^{0.28} + 270 \right] \left[ 607 \left( 4 + 0.85 (t_s + \tau_{sh}) \right) \right]^{0.5} \left( 1 - RH^3 \right)
\]

and the time factor

\[
\beta(t, t_s) = \tanh \left( \frac{t - t_s}{\tau_{sh}} \right)
\]

with

\[
\tau_{sh} = 8.5 t_s^{-0.08} f_c^{0.25} \left( k_s h_0 \right)^2
\]

Factor \( \alpha_1 = [1/0.85/1.1] \) for cement types [I/II/III], factor \( \alpha_2 = [0.75/1.2/1] \) for curing conditions steam/normal/RH 100\% and factor \( k_s = [1; 1.15; 1.25; 1.3; 1.55] \) for an [infinite slab; infinite cylinder; infinite square; prism; cube]. \( w \) is the water content in \([\text{kg/m}^3]\) and \( h_0 \) in \([\text{cm}]\).

**Fig. 3.1** Predicted free shrinkage strains for a 320 mm thick slab strip \( (h_{avg} = h_0 = 2 \cdot \text{V/S} = 320 \text{ mm}) \) made of C25/35 concrete with \( f_{cm} = 33 \text{ MPa} \) \( (f_{ck} = 25 \text{ MPa}) \). The slab strip is located in a RH = 40\% ambient and moist cured for \( t_s = 7 \text{ days} \). Further a class N, type I cement and a water content of 150 kg/m\(^3\) are assumed.

Fig. 3.1 illustrates the shrinkage equations for a 320 mm thick slab strip made from normal strength concrete. The difference between the MC 90 and MC 2010 curves illustrates the influence of the autogenous shrinkage term. Extensive test data comparisons carried out by Gardner (2004) for all four models show that the CEB (version MC 2010), B3 and GL2000 models underestimate on average the measured values, while the ACI 209 model underestimates again at early-ages, is neutral or overestimates at durations below 1000 days and underestimates at later durations. For shrinkage predictions based solely on the concrete tensile strength, Gardner (2004) found variation coefficients between 25\% (GL 2000) and 34\% (ACI 209). For shrinkage durations above 3000 days the variation coefficients dropped to values between 15\% (GL 2000) and 30\% (ACI 209). This coincides with Müller and Hilsdorf (1990) who evaluated a variance coefficient for the CEB model equal to 33\% for general and 19\% for final shrinkage predictions.
Creep Coefficient

If the compliance function is determined experimentally from length measurements on creep specimens, the creep coefficient results from

$$\varphi = E_c(t_0)J(t,t_0) - 1$$

(3.34)

or

$$\varphi = E_cJ(t,t_0) - \frac{E_c}{E_c(t_0)}$$

(3.35)

depending on its definition. ACI 209 (1992) and the B3 model (Bažant and Baweja 1995) adopt Eq. (3.34), while the CEB (Comité Euro-International du Béton 1993; Fédération Internationale du Béton 1999; European Committee for Standardization 2004; Fédération Internationale du Béton 2010a) and GL 2000 (Gardner and Lockman 2001) models adopt Eq. (3.35).

Similar to the shrinkage strains, there is a wide range of empirical creep prediction equations. The most well-known expressions are summarized here and illustrated in Fig. 3.2. When not stated otherwise, the input variables are to be inserted in the following units: the relative humidity RH as a percentage, the notional size of the member $h_0 = 2 \cdot V/S = 2A_v/u$ in [mm], the concrete properties $f_{cm}$ and $E_c$ in [MPa] and $t$ and $t_0$ in [d].

CEB model:

The CEB creep coefficient

$$\varphi = \varphi_\infty \beta(t,t_0)$$

(3.36)

was developed by Task Group 9 (Müller and Hildsdorf 1990) especially for MC 90 (Comité Euro-International du Béton 1993) and consists of a final creep coefficient

$$\varphi_\infty = \left(1 + \alpha_1 \frac{1 - \text{RH}}{0.1 \cdot \sqrt{h_0}} \right) \frac{16.8 \cdot \alpha_2}{\sqrt{f_{cm}} \left(0.1 + t_0^{0.2}\right)}$$

(3.37)

and a time factor

$$\beta(t,t_0) = \left[ \frac{(t-t_0)}{\beta_H + (t-t_0)} \right]^{0.3}$$

(3.38)

where

$$\beta_H = 1.5 \cdot h_0 \left[ 1 + (1.2 \cdot \text{RH})^{18} \right] + 250 \cdot \alpha_4 \leq 1500 \cdot \alpha_3$$

(3.39)

The factors

$$\alpha_4 = (35/f_{cm})^{0.7}; \quad \alpha_2 = (35/f_{cm})^{0.2}; \quad \alpha_3 = (35/f_{cm})^{0.5}$$

(3.40)

considering the concrete strength were added for the current fib (Fédération Internationale du Béton 2010a; Fédération Internationale du Béton 1999) and EC 2 (European Committee for Standardization 2004) versions (they are equal to 1 in the MC 90 version).

ACI 209 Model:

The ACI (ACI Committee 209 1992) creep coefficient (Eq. (3.34)) is based on an experimentally determined (Branson and Christiason 1971) final creep coefficient $\varphi_\infty = 2.35$ and a time factor
Basic Principles

\[
\beta(t,t_0) = \frac{(t-t_0)^{0.6}}{10 + (t-t_0)^{0.6}} \tag{3.41}
\]

for standard conditions \((t_0 = 7d, \text{RH} = 0.4 \text{ and } h_{\text{avg}} = 150 \text{ mm})\). For non-standard conditions empirical correction factors apply, leading to

\[
\varphi = \varphi_c \beta(t,t_0) 1.25 t_0^{0.118} (1.27 - 0.67 \text{ RH}) \cdot \begin{cases} 
1.14 - 0.00092 \cdot h_{\text{avg}} & \text{for } t - t_0 \leq 360 \text{ d} \\
1.10 - 0.00067 \cdot h_{\text{avg}} & \text{for } t - t_0 \geq 360 \text{ d}
\end{cases} \tag{3.42}
\]

According to Branson and Christiason (1971) the empirical correction factors were derived by Christiason in his MS thesis. The \(h_{\text{avg}}\) correction factor is valid for \(h_{\text{avg}}\) between 150 and 380 mm.

**GL 2000 Model:**

Gardner and Lockman’s (2001) creep coefficient considers basic and drying creep

\[
\varphi = \Phi \left\{ \frac{2(t-t_0)^{0.3}}{(t-t_0)^{0.3} + 14} + \left( \frac{7(t-t_0)}{t_0(t-t_0) + 7t_0} \right)^{0.5} + 2.5 \left( 1 - 1.086 \cdot \text{RH}^2 \right) \left[ \frac{(t-t_0)}{(t-t_0) + 0.15 \cdot (V/S)^2} \right]^{0.5} \right\} \tag{3.43}
\]

and accounts for drying before loading \((if t_0 \geq t_s)\) with the factor

\[
\Phi = \left[ 1 - \left( \frac{t_0 - t_s}{t_0 - t_s + 0.15 \cdot (V/S)^2} \right)^{0.5} \right]^{0.5} \tag{3.44}
\]

**Full B3 Model:**

Bažant and Baweja’s (1995) creep coefficient results with Eq. (3.34) from the compliance function

\[
J(t,t_0) = J_{\text{basic}}(t,t_0) + J_{\text{drying}}(t,t_0,t_s) \tag{3.45}
\]

The compliance function considers instantaneous strain \(\alpha_i\) due to a unit stress (determined with the asymptotic modulus), the theoretical basic creep compliance

\[
J_{\text{basic}} = \alpha_2 Q + \alpha_1 \ln \left[ 1 + \left( \frac{t-t_0}{0.1} \right)^{0.1} \right] + \alpha_4 \ln \left( \frac{t}{t_0} \right) \tag{3.46}
\]

and the drying creep compliance (if \(t_0 \geq t_s\))

\[
J_{\text{drying}} = 7.57 \frac{E_{\text{sh,c}}}{f_{\text{cm}}} \left( e^{-8f(t)} - e^{-8f(t_0)} \right)^{0.5} \tag{3.47}
\]

\(Q\) stands for a binomial integral, approximated by

\[
Q = \left( 0.086 t_0^{2/9} + 1.21 t_0^{4/9} \right)^{-1} \left[ 1 + \left( \frac{0.086 t_0^{2/9} + 1.21 t_0^{4/9}}{t_0^{0.5} \ln \left[ 1 + \left( t - t_0 \right)^{0.1} \right]} \right)^{-1} \right] \tag{3.48}
\]
The additional empirical factors are

\[
\alpha_1 = \frac{0.6 \cdot 10^6}{E_c} \quad \alpha_2 = 185.4 \sqrt{c} f'_{cm}^{0.9} \quad \alpha_3 = 0.29 \frac{w}{c} \quad \alpha_4 = 20.3 \left( \frac{a}{c} \right)^{-0.7}
\]  

(3.49)

and

\[
H(t) = 1 - (1 - RH) \tanh \left[ \frac{t - t_s}{8.5t_s} - 0.08 f'_{cm} - 0.25 \left( k, h_0 \right)^2 \right]
\]  

(3.50)

The input variables are inserted in the following units: \(h_0\) in \([cm]\), water and cement content \(w\) and \(c\) in \([kg/m^3]\), water to cement ratio \(w/c\) in [-] and aggregate to cement (weight) ratio \(a/c\) (e.g. 7) in [-]. Time \(t_0\) in [d] is the major of \(t_0\) (concrete age at loading) and \(t_s\) (concrete age at end of curing). The shrinkage parameters \(\varepsilon_{sh,\infty}\) (Eq. (3.31)) and \(k_s\) are defined in the shrinkage section above.

Fig. 3.2 illustrates the different creep equations. So they can be compared, the creep coefficients based on Eq. (3.35) are multiplied with \(E_c(t_0)/E_c\). The kink in the ACI 209 curve is caused by the change in Eq. (3.42) at \(t - t_0 = 360\) days. The same test comparison referred to above for shrinkage (Gardner (2004)) contains compliance comparisons illustrating variation coefficients between 26 % (GL 2000) and 37 % (CEB) for test data comparisons based solely on the measured concrete compression strength. All four models have a tendency to underestimate the creep compliance (especially the ACI 209 model). Müller and Hilsdorf (Müller and Hilsdorf 1990) found a 20 % variation coefficient for the CEB model.

![Creep coefficients for a 320 mm thick slab strip (h_{avg} = h_0 = 2 \cdot V/S = 320 mm) made of C25/35 concrete with f'_{cm} = 33 MPa and E_{cm} = 32 GPa. The slab strip is located in a RH = 40 % ambient and loaded at t_0 = 7 days. For the B3 model a type I cement with w = 150 kg/m³, w/c = 0.5 and a/c = 7 is assumed.](image)

**Aging Coefficient**

Inserting \(\sigma_c(t_0) = \varepsilon_c E_c(t_0)\) (Eq. (3.2)) and \(\sigma_c(t) = \varepsilon_c R(t, t_0)\) (Eq. (3.13)) into Eq. (3.16) with \(\Delta \sigma_c(t) = \sigma_c(t) - \sigma_c(t_0)\) and solving for \(X\) (Ghali and Favre 1986) leads to

\[
X = \frac{E_c(t_0)}{E_c(t_0) - R(t, t_0)} - \frac{1}{\phi(t, t_0)}
\]  

(3.51)
where $R(t, t_0)$ is the relaxation function, defined as the stress at age $t$ due to a unit strain introduced at age $t_0$. This means that the aging coefficient can be determined from the relaxation function. The relaxation function is obtained by numerically solving the Volterra Integral

$$E_c(t_0)J(t, t_0) + \int_{t_0}^{t} J(t, t_0) \frac{\partial R(t, t_0)}{\partial t} \, dt = 1$$

(3.52)

which mathematically links the relaxation and compliance functions and is derived by inserting $\sigma_c(t) = \varepsilon_c R(t, t_0)$ into Eq. (3.15) and dividing by $\varepsilon_c$.

Ghali & Favre (1986) provide a step-by-step analysis to numerically determine $R(t, t_0)$ according to Eq. (3.52). They divide the time period between $t_0$ to $t$ of a given time-stress function into $i$ intervals and apply the stress increments ($\Delta \sigma_c$) at the middle of the intervals. In this case, Eq. (3.15) can be written as

$$E_c \left( t_{i,\text{end}} \right) = \sum_{j=1}^{i} \left[ \frac{1 + \varphi(t_{j,\text{end}}, t_j)}{E_c \left( t_j \right)} \left( \Delta \sigma_c \right)_j \right] + \left( \Delta \sigma_c \right)_i \frac{1 + \varphi \left( t_{i,\text{end}}, t_i \right)}{E_c \left( t_i \right)}$$

(3.53)

if the last term of the summation is written separately and zero shrinkage is assumed. While $t_i$ refers to the middle of the interval, $t_{i,\text{end}}$ refers to the end of the interval. Assuming $\varepsilon_c \left( t_{i,\text{end}} \right) = \varepsilon_c = \text{constant}$ and equal to 1 during all $i$ intervals, solving Eq. (3.53) for

$$\left( \Delta \sigma_c \right)_i = \frac{E_c \left( t_i \right) \left[ \varepsilon_c \left( t_{i,\text{end}} \right) - \sum_{j=1}^{i} \left[ \frac{1 + \varphi \left( t_{j,\text{end}}, t_j \right)}{E_c \left( t_j \right)} \left( \Delta \sigma_c \right)_j \right] \right]}{1 + \varphi \left( t_{i,\text{end}}, t_i \right)}$$

(3.54)

leads to the $(\Delta \sigma_c)$ values for $i = 1, 2, \ldots$, which can be added up to give the relaxation function

$$R \left( t_{i,\text{end}}, t_0 \right) = \frac{1}{\varepsilon_c} \sum_{j=1}^{i} \left( \Delta \sigma_c \right)_j$$

(3.55)

at the end of the $i$th interval.

**Example 1**

This example illustrates the step-by-step analysis for determining the aging coefficient. Thereto the linear time-stress curve illustrated in Fig. 3.3 is divided into 6 intervals with $i = 1$ to 6 (the first interval contains the sudden stress application at $t_i = 28$ d).

![Adopted time-stress diagram for Example 1.](image)
Eqs. (3.10) and (3.11) are used to determine the elastic modulus

$$E_i(t) = E \left[ \exp \left( 0.25 \left( 1 - \frac{28}{\sqrt{t}} \right) \right) \right]^{0.5}$$

with $E = 30$ GPa, while Eq. (3.38) is used to determine the creep coefficient

$$\varphi(t_i, t) = \left[ 1 + \frac{1 - 0.4}{0.12/35} \sqrt[3]{0.12} \right]^{1.12} \left( \frac{t - t_i}{302.5 + (t - t_i)} \right)^{1.12}$$

with $h_0 = 35$ mm, $f_{cm} = 35$ MPa, RH = 0.4 and $\beta_H = 302.5$.

The stress increments belonging to a constant unit strain are determined with Eq. (3.54) in the units [GPa and days]

$$\Delta \sigma_i = \frac{E_i \left( 28 \right)}{1 + \varphi(28, 28)} = 30$$

$$\Delta \sigma_2 = \frac{E_i \left( 56 \right)}{1 + \varphi(44, 56)} \left[ 1 - \Delta \sigma_1 \left( 1 + \varphi(28, 28) \right) \right] = -26.526$$

$$\Delta \sigma_3 = \frac{E_i \left( 112 \right)}{1 + \varphi(140, 112)} \left[ 1 - \Delta \sigma_2 \left( 1 + \varphi(44, 56) \right) \right] = 0.718$$

$$\Delta \sigma_4 = \frac{E_i \left( 168 \right)}{1 + \varphi(196, 168)} \left[ 1 - \Delta \sigma_3 \left( 1 + \varphi(140, 56) \right) \right] = -0.286$$

$$\Delta \sigma_5 = \frac{E_i \left( 224 \right)}{1 + \varphi(252, 224)} \left[ 1 - \Delta \sigma_4 \left( 1 + \varphi(196, 56) \right) \right] = -0.245$$

$$\Delta \sigma_6 = \frac{E_i \left( 280 \right)}{1 + \varphi(308, 280)} \left[ 1 - \Delta \sigma_5 \left( 1 + \varphi(252, 56) \right) \right] = -0.196$$

Adding up the above values according to Eq. (3.55) leads to $R = 3.465$ GPa. Thus, Eq. (3.51) with $\varphi(308d, 28d) = 3.16$ and $E_i(28d) = 30$ GPa leads to

$$X = \frac{30 \text{ GPa}}{30 \text{ GPa} - 3.46 \text{ GPa}} - \frac{1}{3.16} = 0.81$$

According to Eq. (3.16), this means that $\varphi = 3.16$ applies to the 10 MPa introduced suddenly at 28 days, while $X\varphi = 2.56$ applies to the 5 MPa introduced gradually.

Trost (1967) found the practical range of $X$ to be between 0.8 and 0.9, so that often a constant value of 0.8 is used.

### 3.2 Steel Material Behaviour

All of the service design models and approaches given in this thesis are based on the assumption of elastic steel behaviour

$$\sigma_s = \sigma_{s0} + E_s \left( e - e_{si} \right)$$

where $\sigma_{s0}$ stands for initial stresses and $e_{si}$ stands for induced steel strains (e.g. temperature-induced). When not stated otherwise, the reinforcement steel is assumed to have an elastic modulus of 205 GPa up until the yield limit.
3.3 Un-cracked and Cracked Elastic State Section Behaviour

![Diagram of section behaviour](image)

**Fig. 3.4** Example of a State II section with $M = 150$ kNm: (a) $\varphi = \varepsilon_{th} = 0$; (b) Eqs. (3.58) and (3.59) with $\varphi = 3, \varepsilon_{th} = 0, X = 0.8$; (c) Eqs. (3.58) and (3.59) with $\varphi = 3, \varepsilon_{th} = -0.6 \%, X = 0.8$; (d) EMM (Eq. (3.66)) with $\varphi = 3, \varepsilon_{th} = 0$; (e) shrinkage alone ($M = 0$) with $\varepsilon_{th} = -0.6 \%$. Note that $f_{ct} = 0$ MPa, $E_s = 205$ GPa, $n = 6$, lengths in [mm] and curvatures in [mrad/m].

It is common practice to determine the in-service section deformations of RC cross-sections (notation see Fig. 3.5) assuming plane sections

$$\varepsilon = \varepsilon_0 + \chi z$$

(3.57)

elastic steel and concrete behaviour and Mörsch's (1908) simplified design concept for cracked sections. According to Mörsch's simplified design concept, once the concrete tensile strength is reached in the extreme fibres, the section is assumed to reach the cracked elastic state, where $f_{ct} = 0$ is adopted. This is equivalent to assuming an infinitely small crack spacing. In this work the uncracked elastic state is referred to as State I, while the cracked elastic state is referred to as State II. With these assumptions

$$N = \int_{-h/2}^{h/2} \sigma b dz + A_s (\sigma_s - \sigma_e)_{z=d-0.5h} + A_s' (\sigma_s - \sigma_e)_{z=d'-0.5h}$$

(3.58)
and

\[ M = \int_{-h/2}^{h/2} \sigma_z z b dz + \left( d - \frac{h}{2} \right) A \left( \sigma_x - \sigma_z \right)_{z=d-0.5h} + \left( d' - \frac{h}{2} \right) A' \left( \sigma_x - \sigma_z \right)_{z=d'-0.5h} \]  \tag{3.59}

can be solved for State I and State II sections.

Fig. 3.5  Notation for Eqs. (3.57) to (3.59): (a) cross-section; (b) stress resultants (c) State I strains; (d) State II strains.

3.3.1 Short-Term Loads

As a consequence of setting \( f_{ct} = 0 \) after cracking, the State II moment-curvature relation of bending elements is linear (like the State I relation), as long as the depth of the compression zone remains constant. This is the case for short-term bending without axial force.

Thus, with \( N = 0 \) and the short-term elastic concrete behaviour (Eq. (3.2)), the equilibrium Eqs. (3.58) and (3.59) lead to the well known load-independent transformed State I and II section properties used to determine the deformations of uncracked and cracked sections, respectively. For a rectangular section the section properties are summarized in Table 3.1 and their quotient is illustrated in Fig. 3.6. The figure illustrates the importance of correctly locating the cracked regions in lightly reinforced elements (e.g. \( \rho < 0.5 \% \)).

<table>
<thead>
<tr>
<th>State I</th>
<th>State II</th>
</tr>
</thead>
</table>
| \( x \) | \( d' \left[ \sqrt{b_0^2 + 2b_0 - b} \right] = d' f_t \)  
\[ x = \frac{0.5 + \left( \alpha_1 \rho + \alpha_2 \rho' \right) (n-1)}{1 + \alpha_1 (p + p') (n-1)} = h \cdot f_t \] | \( b_0 = np + (n-1) \rho' \) and \( b_0 = np + \alpha_1 (n-1) \rho' \) |
| \( A \) | \( bd \left[ f_u + b_0 \right] \)  
\[ A = bh \left[ 1 + \alpha_1 (p + p') (n-1) \right] \] |  |
| \( I \) | \( bd \left[ \frac{f_u}{3} + np \left( 1 - f_t \right) + (n-1) \rho' (f_t - \alpha_3) \right] = bd' g_1 \)  
\[ I = bh^3 \left( 1/2 + 0.5 - f_t \right) + \alpha_1 (n-1) \rho (\alpha_t - f_t) + \rho' \left( f_t - \alpha_3 \right) \] | \( bd' g_1 \)  
\[ I = bh^3 \left( 1/2 + 0.5 - f_t \right) + \alpha_1 (n-1) \rho (\alpha_t - f_t) + \rho' \left( f_t - \alpha_3 \right) \] |

Table 3.1 State I and II section properties for rectangular cross-sections with \( \rho = A_i / (bd) \), \( \rho' = A_i' / (bd) \)  
\( \alpha_1 = d/h \), \( \alpha_2 = d'/h \) and \( \alpha_3 = d'/d \).
Example 2

In this example the State II curvatures and steel stresses illustrated in Fig. 3.4 (a) are calculated. Although not needed here, the State I section properties are also determined, as they are used in Example 7. With \( n = 6 \), \( \rho = 2\% \), \( \rho' = 0 \) and \( d/h = 0.85 \), the equations in Table 3.1 lead to the short-term section parameters

\[
f_I = \frac{0.5 + (21 - 1)0.85^2 \cdot 2 \%}{1 + 0.85 \cdot 2 \% \cdot (21 - 1)} = 0.527
\]

\[
f_{II} = \sqrt{(2 - 2 \%)^2 + 2 \cdot 2 \% - 24 \%} = 0.384
\]

and

\[
r_{II} = \frac{1}{12} \left( \frac{1}{2} - \frac{0.527}{2} \right)^2 + 0.85 \cdot (6 - 1) \left[ 2 \% \cdot (0.85 - 0.527)^2 \right] = 0.0929
\]

\[
r_{II} = \frac{(0.384)^2}{3} + 6 \cdot 2 \% \cdot (1 - 0.384)^2 = 0.0644
\]

As a consequence for \( h = 0.5 \) m the section properties

\[
x = 0.5 \text{ m} \cdot 0.527 = 264 \text{ mm}
\]

\[
x_{II} = 0.85 \cdot 0.5 \text{ m} \cdot 0.384 = 163 \text{ mm}
\]

and

\[
E_{II} = 34 \text{ GPa} \cdot 1 \text{ m} \cdot 0.5^2 \text{ m}^2 \cdot 0.0929 = 79378 \text{ kN m}^2
\]

\[
E_{II} = 34 \text{ GPa} \cdot 1 \text{ m} \cdot (0.85 - 0.2)^2 \text{ m}^2 \cdot 0.0644 = 33787 \text{ kN m}^2
\]

are obtained. They define the cracked section curvature

\[
\chi_{II} = \frac{150 \text{ kNm}}{33787 \text{ kN m}} = 4.4 \text{ mrad/m}
\]

and the steel stress

\[
\sigma_{II} = 205 \text{ GPa} \cdot \frac{150 \text{ kNm}}{33787 \text{ kN m}^2} \cdot (0.425 \text{ m} - 0.163 \text{ m}) = 238 \text{ MPa}
\]

illustrated in Fig. 3.4 (a).

Fig. 3.6   Stiffness ratio of a rectangular cross-section.

3.3.2    Sustained Loads

For sustained loads Trost's (1967) Eq. (3.17) expresses the concrete behaviour. With Eq. (3.17), Eqs. (3.58) and (3.59) lead to the section deformations illustrated in Fig. 3.4 (b) and (c) for the example section used in Example 2.
Age-Adjusted Effective Modulus Method (AAEMM)

Once long-term effects such as creep and shrinkage are involved, the depth of the compression concrete becomes time-dependent, leading to a non-linear State II moment-curvature relation. In order to avoid this nonlinearity and allow the superposition of short- and long-term curvatures in cracked sections, both Trost & Mainz (1968) and Ghali & Favre (1986) adopt the simplifying assumption of a time-independent effective concrete area for State II sections. Thereby any concrete not in the compression zone at time $t_0$ is neglected, while any concrete no longer in compression at time $t$ due to shrinkage is considered. With this assumption they derived curvature correction coefficients $\kappa_{\varphi}$ and $\kappa_{sh}$ for determining the additional creep

$$\Delta \chi_{\varphi} = \kappa_{\varphi} \cdot \varphi \cdot \chi_{t_0}$$  \hspace{1cm} (3.60)

and shrinkage

$$\Delta \chi_{sh} = \kappa_{sh} \cdot \frac{-\varepsilon_{sh}}{d}$$  \hspace{1cm} (3.61)

curvatures by solving Eqs. (3.58) and (3.59) with Trost's (1967) concrete material behaviour (Eq. (3.17)).

Fig. 3.7  Ghali & Favre's (1986) curvature coefficients according to Table 3.2: (a) creep; (b) shrinkage. Note that $\rho' = 0$, $n = 6$, $d/h = 0.85$ and $X = 0.8$.

Ghali & Favre's (1986) coefficients

$$\kappa_{\varphi} = \frac{A_{x} \left( x_{a} - x \right) \left( x_{c} - x_{a} \right) + I_{c}}{I_{a}}$$  \hspace{1cm} (3.62)

and

$$\kappa_{sh} = \frac{-A_{a} d \left( x_{c} - x_{a} \right)}{I_{a}}$$  \hspace{1cm} (3.63)

are stated here because, due to the convenient location of the axis origin in the centroid of the age-adjusted section, they are more practical. The so-called age-adjusted transformed section properties...
(x_a, I_a) used in Eqs. (3.62) and (3.63) and the curvature coefficients themselves are given in Table 3.2 for rectangular sections. While for uncracked sections the age-adjusted and short-term transformed section properties are identical except for n = E_a/E_c being replaced by n_a = E_a/E_{c,a}, entirely different equations are required for cracked sections. This is considered the main disadvantage of this method. The age-adjusted elastic modulus is given by Eq. (3.18).

Fig. 3.7 illustrates the curvature coefficients for ρ' = 0, n = 6 and d/h = 0.85. For ρ = 0.2 % a creep coefficient of φ = 2 leads to total curvatures and deflections around 3 and 1.2 times as large as the initial values for an uncracked or fully cracked beam, respectively. In contrast, the additional shrinkage curvatures would be negligible in the uncracked beam and approximately –c_{sh}/d in the fully cracked beam.

<table>
<thead>
<tr>
<th>State I aged-adjusted transformed section</th>
<th>State II aged-adjusted transformed section</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_a [ h \left( \frac{0.5 + (\alpha_3 \rho + \alpha_4 \rho')}{1 + \alpha_1 (\rho + \rho')} \right) (n_a - 1) ] = h \cdot f_{x_a}</td>
<td>d \left[ \frac{0.5 f_{x_a}^3 + b_{x_a}}{f_{n_a} + b_{x_a}} \right] = d \cdot f_{x_a}</td>
</tr>
<tr>
<td>A_a bh[1 + \alpha_1 (\rho + \rho') (n_a - 1)]</td>
<td>bd[f_{n_a} + b_{x_a}]</td>
</tr>
<tr>
<td>I_a bh \left[ \frac{1}{12} \left( 0.5 - f_{x_a} \right)^3 + \alpha_1 (n_a - 1) \right] \cdot \left[ \rho (f_{x_a} - f_{x_a}) + \rho' (f_{x_a} - f_{x_a}) \right] = bh^2 \cdot g_{x_a}</td>
<td>bd \left[ \frac{1}{3} \left( f_{x_a} - f_{x_a} \right) + \rho (f_{x_a} - f_{x_a}) + \rho' (f_{x_a} - f_{x_a}) \right] = bd \cdot g_{x_a}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State I net concrete section</th>
<th>State II net concrete section</th>
</tr>
</thead>
<tbody>
<tr>
<td>with centroid of the age-adjusted section</td>
<td>with centroid of the age-adjusted section</td>
</tr>
<tr>
<td>x_c h \left[ \frac{1}{10 - \alpha_1 (\rho + \rho')} \right]</td>
<td>d \left[ \frac{f_{x_a}^3 + \rho' \left( f_{x_a}^3 - f_{x_a} \right) }{f_{n_a} + \rho' \left( f_{x_a}^3 - f_{x_a} \right) } \right]</td>
</tr>
<tr>
<td>A_c bh[1 - \alpha_1 (\rho + \rho')]</td>
<td>bd[f_{n_a} - \rho']</td>
</tr>
<tr>
<td>I_c bh \left[ \frac{1}{12} \left( \frac{1}{2} - f_{x_a} \right)^3 \right] - \alpha_1 \left[ \rho (f_{x_a} - f_{x_a}) + \rho' (f_{x_a} - f_{x_a}) \right] = bh^2 \cdot g_{x_a}</td>
<td>bd \left[ \frac{1}{3} \left( f_{x_a}^3 - f_{x_a} \right) - \rho (f_{x_a} - f_{x_a}) - \rho' (f_{x_a} - f_{x_a}) \right] = bd \cdot g_{x_a}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State I curvature coefficients</th>
<th>State II curvature coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kp \left[ g_{x_a} + (1 - \alpha_1 (\rho + \rho')) \right] \left[ \frac{0.5 - (\alpha_3 \rho + \alpha_4 \rho')}{1 - \alpha_1 (\rho + \rho')} \right] (f_{x_a} - f_1)</td>
<td>g_{x_a} + (f_{x_a} - \rho') \left[ \frac{f_{x_a}^3 + \rho' \left( f_{x_a}^3 - f_{x_a} \right) }{f_{n_a} + \rho' \left( f_{x_a}^3 - f_{x_a} \right) } \right]</td>
</tr>
<tr>
<td>Ksh \left[ \frac{\alpha_1 \left( \frac{1 - \alpha_1 (\rho + \rho')}{1 - \alpha_1 (\rho + \rho')} \right) - f_{x_a}}{g_{x_a}} \right]</td>
<td>\left[ \frac{f_{x_a}^3 + \rho' \left( f_{x_a}^3 - f_{x_a} \right) }{f_{n_a} + \rho' \left( f_{x_a}^3 - f_{x_a} \right) } \right]</td>
</tr>
</tbody>
</table>

**Table 3.2** Ghali & Favre’s (1986) State I and II curvature coefficients for rectangular cross-sections together with the necessary age-adjusted section properties. Note that n_a = E_a / E_{c,a}, α_1 = d/h, α_2 = d’/h, α_3 = d’/d and f_{x_a} according to Table 3.1.
Effective Modulus Method (EMM)

If shrinkage is neglected and X is set equal to 1, Eq. (3.17) is reduced to

\[ \sigma_c(t) = e_c E_{c,ef} \]  

with the effective modulus

\[ E_{c,ef} = \frac{E_c(t_0)}{1 + \varphi(t,t_0)} \]  

For \( N = 0 \), Eqs. (3.58) and (3.59) with Eq. (3.64) yield

\[ \chi = \frac{M}{E_{c,ef} I_{ef}} \]  

with the advantage that both the State I and II effective transformed section properties and deformations can be determined with the classic section equations (e.g. Table 3.1) using \( n = E_s/E_{c,ef} \), Eqs. (3.64) and (3.65) are referred to as Effective Modulus Method (EMM). The EMM is the oldest (e.g. McMillan 1916) and simplest method for considering creep deformations of RC members (Bazant and Najjar 1973). For a non-aging material (e.g. very old concrete) with a bounded final creep coefficient the EMM gives the same results as the AAEMM for \( t \to \infty \). Both EC 2 (European Committee for Standardization 2004) and the simplified method in ACI 209 (ACI Committee 209 1992) refer to the effective modulus for considering the influence of creep in RC members.

With \( X = 1 \) the statically determinate shrinkage curvature is

\[ \Delta \chi_{sh} = -e_c \frac{A_{x_I}(x_{y,II} - x_c)}{I_{ef}} \]  

In uncracked sections \( x_{I,c} \) and \( A_{I,c} \) result with \( n = 0 \) from the \( x_I \) and \( A_I \) equations in Table 3.1 and the extreme fibre uncracked shrinkage stresses are

\[ \Delta \sigma_{sh} = -E_{c,ef} \left[ e_c \left( 1 - \frac{A_{y,II}}{A_{y,II}} \right) - \Delta \chi_{sh,II} \left( h - x_{y,II} \right) \right] \]  

In cracked sections \( x_{II,c} \) and \( A_{II,c} \) result from the corresponding equations in Table 3.2, where \( f_{II} \) is determined with \( n_{s,y} \) according to Table 3.1. For cracked rectangular sections without compression reinforcement with \( A_{II,c} = b x_{III,II} \) and \( x_{III,II} = 0.5 x_{III,II} \) Eq. (3.67) can be simplified to

\[ \Delta \chi_{sh,II} = -e_c \frac{A_{y,II}}{d - \frac{1}{2} x_{III,II}} \]  

and for \( M = 0 \) (\( x_{III,II} = 0 \)) further to

\[ \Delta \chi_{sh,II} = -e_c \frac{A_{y,II}}{d} \]  

Example 3

This example illustrates how the State II curvatures and steel stresses in Fig. 3.4 (d) and (e) are calculated according to the EMM. Although not needed here, the State I section properties are also determined. With \( n_s = 6 \) \((1+3) = 24\), \( \rho = 2\% \) and \( d/h = 0.85 \), the equations in Table 3.1 lead to the long-term section parameters
Basic Principles

\[
\begin{align*}
  f_{\text{ef}} &= \frac{0.5 + (24 - 1)0.85^2 \cdot 2 \%}{1 + 0.85 \cdot 2 \% \cdot (24 - 1)} = 0.598 \\
  f_{\text{I,ef}} &= \sqrt{(24 - 2 \%)^2 + 2 \cdot 24 \cdot 2 \% - 24 \cdot 2 \%} = 0.611 \\
  R_{\text{ef}} &= \frac{1}{12} \left( \frac{1}{2} \cdot 0.598 \right) + 0.85 \cdot (24 - 1) \left( 2 \% \cdot (0.85 - 0.598)^2 \right) = 0.1178 \\
  R_{\text{II,ef}} &= \frac{(0.611)^2}{3} + 24 \cdot 2 \% \cdot (1 - 0.611)^2 = 0.1487
\end{align*}
\]

Therefore, the compression zone heights are

\[
\begin{align*}
  x_{\text{ef}} &= 0.5 \text{ m} \cdot 0.598 = 299 \text{ mm} \\
  x_{\text{II,ef}} &= 0.85 \cdot 0.5 \text{ m} \cdot 0.611 = 260 \text{ mm}
\end{align*}
\]

and the stiffnesses are

\[
\begin{align*}
  E_{\text{ef}}I_{\text{ef}} &= \frac{34 \text{ GPa}}{(1 + 3)} \cdot 1 \text{ m} \cdot 0.5^4 \text{ m}^4 \cdot 0.1178 = 25148 \text{ kN m}^2 \\
  E_{\text{II,ef}}I_{\text{II,ef}} &= \frac{34 \text{ GPa}}{(1 + 3)} \cdot 1 \text{ m} \cdot (0.85 - 0.2)^4 \text{ m}^4 \cdot 0.1487 = 19496 \text{ kN m}^2
\end{align*}
\]

For Fig. 3.4 (a) the section curvature

\[
\chi_I = \frac{150 \text{ kNm}}{19496 \text{ kN m}^2} = 7.7 \text{ mrad/m}
\]

is defined by (Eq. (3.66)), while the steel stress is

\[
\sigma_{\text{II,ef}} = \frac{205 \text{ GPa} \cdot 150 \text{ kNm}}{19496 \text{ kN m}^2} \cdot (0.425 \text{ m} - 0.260 \text{ m}) = 261 \text{ MPa}
\]

The State II shrinkage strains at $M = 0$ (Fig. 3.4 (e)) are estimated with

\[
\Delta \chi_{\text{IIIII}} = \frac{-0.6 \%}{0.425 \text{ m}} = 1.4 \text{ mrad/m}
\]

**Fig. 3.8** Comparison of AAEMM (same continuous lines as in Fig. 3.7) and EMM (dashed lines) curvature coefficients: (a) creep; (b) shrinkage. Note that $\rho' = 0$, $n = 6$ and $d/h = 0.85$.

Fig. 3.8 illustrates the small difference between Ghali & Favre's (1986) AAEMM curvature coefficients (Fig. 3.2) and the EMM coefficients

\[
\kappa_\phi = \frac{1}{\phi} \left( I \left( \frac{1 + \phi}{I_{\text{ef}}} - 1 \right) \right)
\]

(3.71)
and \( \kappa_\varphi = -d \Delta \chi_{ab}/c_{ij} \) derived from Eqs. (3.66) and (3.67). In comparison to the AAEMM, the EMM yields slightly smaller creep curvature coefficients and slightly larger shrinkage coefficients. The creep curvature coefficient is smaller because the entire creep coefficient \( \varphi \) is applied to the full stress reduction caused by internal stress redistributions from the concrete to the steel.

### 3.3.3 Sustained and Variable Loads

If Trost's time-dependent material behaviour equation (Eq. (3.17))

\[
\varepsilon = \frac{\sigma_{c,0}\cdot g}{E_c} (1 + \varphi) + \frac{\sigma_{c,0}\cdot g}{E_c} (1 + X\varphi) + \varepsilon_{c,0} + \frac{\sigma_{c,0}\cdot g + q - \sigma_{c,0}\cdot g}{E_c}
\]

(3.72)

is extended by the elastic strains due to variable loads \( (q) \) applied at time \( t \) (last term in Eq. (3.72)) the concrete compression stresses due to permanent and variable loads at time \( t \) are equal to

\[
\sigma_{c,i,g+q} = E_c \left[ \varepsilon - \varphi \frac{\sigma_{c,0}\cdot g}{E_c} - X\varphi - \frac{\sigma_{c,0}\cdot g - \sigma_{c,0}\cdot g}{E_c} \right]
\]

(3.73)

![Graphs](a,b,c,d,e)

**Fig. 3.9**  Unloading after sustained loading. Note that \( f_{ct} = 3 \text{ MPa} \) before initial cracking and 0 thereafter, \( E_s = 205 \text{ GPa}, n = 6, \varepsilon_{ch} = -0.6 \%_o, \varphi = 3, X = 0.8, \) lengths in [mm] and curvatures in [mrad/m].
With a concrete compression behaviour according to Eq. (3.73), the section equilibrium equations (3.58) and (3.59) lead to the State II section deformations due to sustained and variable loads. This is shown in Fig. 3.9 for an example slab section, which is unloaded after sustained loading. As can be seen from the figure, the height and location of the compression zone is variable, indicating a strongly non-linear deformation behaviour. Even for $X = 1$ and $\varepsilon_{sh} = 0$, the deformation behaviour remains non-linear, as the compression zone depth depends on the ratio of the sustained and variable load-portions. For hand-calculations involving additive sustained and variable loads it is suggested that the section deformations be determined assuming the effective section properties in the case of dominating sustained loads and elastic section properties in the case of dominating variable loads.

### 3.4 Tension Chord Model

The Tension Chord Model (TCM) models the interaction between the reinforcement and concrete between the cracks in cracked tension chords. This can be regarded as a refinement to the State II section behaviour, which neglects the effect that is referred to as tension stiffening. The model applies to uniaxial stress states (e.g. tension ties or Bernoulli beams) and was initially derived to analyse the deformation capacity of plastic hinges (Sigrist 1995; Alvarez 1998) in the ultimate limit state, but is also a practical tool for serviceability limit state calculations.

#### 3.4.1 Basic Assumptions

![Rigid-perfectly plastic bond stress-slip relations adopted by the Tension Chord Model: (a) Sigrist (1995); (b) Marti, Sigrist, et al. (1997).](image)

The Tension Chord Model is based on Kuuskoski's and Rehm's differential equation of slipping bond and adopts two principal assumptions. The first assumption is the strongly simplifying bond stress – slip relationship (Fig. 3.10 (a)) at the interface between the steel and concrete (Sigrist 1995). Before steel yielding the bond stress transmitted between the reinforcing steel and the concrete is assumed to be constant and equal to $\tau_b = \tau_{b0}$. Based on pull out tests carried out by Engström and Shima and theoretical parameter studies, Sigrist suggested $\tau_{b0} = 2 f_{ct}$ for ordinary ribbed bars (relative rib areas around 0.05 to 0.07), with $f_{ct}$ according to Eq. (3.8). Sigrist's suggestion coincides with the findings of test data based parameter studies (250 tests) carried out at TU Darmstadt and reported by König and Fehling (1988). These parameter studies started with a linear bond stress-slip relationship with $\tau_b = \tau_0 + \delta \tau'$ and concluded that average crack widths predicted with $\tau_b = 2 f_{ct}$ ($\tau' = 0$) and an integration coefficient of 0.6 (belonging to the linear bond stress-slip relationship) give the best agreement.
with measured crack widths. On this basis, Chapter 7 of MC 90 (1993) and MC 2010 (fib 2010b) suggest $\tau_b = 1.8 f_{ct}$ and an integration coefficient of 0.6 for short-term loads.

Marti, Sigrist, et al. (1997) suggested the bond stress – slip diagram illustrated in Fig. 3.10 (b). It is assumed that as soon as a positive slip occurs bond stresses equal to $\tau_{b0}$ are activated. These remain constant unless the slip is released and bond stresses in the opposite direction $-\tau_{b0}$ are activated. Positive slip and bond stress values apply to a bar being pulled out of the concrete, while negative slip and bond stress values apply to a bar being pushed into the concrete. Later, based on his Bragg grating sensor measurements Kenel (2002) suggested negative bond stress values equal to $-0.5 \tau_{b0}$.

The second main assumption behind the Tension Chord Model consists in assuming a constant tensile strength $f_{ct}$ for the entire tensile tie or chord.

The two assumptions lead to an upper and lower limit value for the final crack spacing (stabilized cracking) defined by parameter $\lambda$ [0.5...1] (Marti et al. 1998). The lower limit ($\lambda = 0.5$) is established if a crack forms a transfer length away from a previous crack, while the upper limit occurs ($\lambda = 1$) if a crack forms twice the transfer length away from a previous crack. The transfer length (or alternatively referred to as transmission length) is defined as the length on either side of a crack where the bond is disturbed (see Fig. 3.11). No primary cracks will form within the transfer length because in this region the concrete stresses are smaller than the concrete tensile strength. If a crack forms further away than twice the transfer length, then there is an undisturbed region between the two cracks where a new crack can form. Depending on the crack spacing, the average concrete tensile stresses between two cracks (see Fig. 3.11) are

$$\sigma_{c,t,avg} = \lambda \frac{f_{ct}}{2}$$

As a consequence the average steel strains are smaller than they would be without the concrete between the cracks. This effect is referred to as tension stiffening.
### 3.4.2 Tensile Members

![Fig. 3.12](image)

**Fig. 3.12** Tensile tie with a length of 8 \( I_0 \) and \( \lambda = 1 \) under (a) an imposed axial load \( N \); (b) an imposed axial deformation \( \varepsilon_e \) or restrained cooling strains \( \varepsilon_e = -\varepsilon_{\Delta T} \); (c) restrained shrinkage-induced strains \( \varepsilon_i = -\varepsilon_{sh} \) according to Marti, Alvarez, et al. (1997).

**Axial Load**

If a constant concrete tensile strength is assumed, upon reaching

\[
N = bh f_{ct} (1 - \rho + np)
\]

(3.76)

all cracks form simultaneously in a force controlled RC tension tie, which immediately changes from State I to State II (Fig. 3.12(a)). For the cracking load the steel stresses in the cracked sections are

\[
\sigma_{s,II} = \frac{f_{ct}}{\rho} \left( 1 + \rho n - \rho \right)
\]

(3.77)

With average concrete stresses according Eq. (3.75), the equilibrium equation \( E_s \rho h \varepsilon_{s,II} = E_s \rho h (\varepsilon_{s,II} - \Delta \varepsilon_{s,II}) + bh (1 - \rho) \lambda/2 f_{ct} \) (Fig. 3.11) yields the average steel strain reduction due to tension stiffening

\[
\Delta \varepsilon_{s,II} = \frac{\lambda f_{ct} (1 - \rho)}{2 \rho E_s}
\]

(3.78)

in tensile ties with evenly distributed reinforcement \( \rho = A_s/bh \). In tensile elements with concentrated reinforcement at the edges \( \rho \) refers to the effective reinforcement ratio. The crack widths are

\[
W = \frac{2 \lambda I_0}{E_s} \left( \sigma_{s,II} - \frac{\lambda}{2} \sigma_{s,II,II} \right)
\]

(3.79)

Factor \( \lambda \) is set equal to 1 and 0.5 to consider the maximum and minimum crack spacing, respectively.

** Restrained and Imposed Deformations**

Marti et al. (Marti, Sigrist, et al. 1997; Marti, Alvarez, et al. 1997; Marti et al. 1998; Alvarez 1998) use the Tension Chord Model for a restrained tensile tie. They distinguish between internal and external restraint, referring to internal restraint for fully restrained shrinkage-induced strains and referring to external restraint for fully restrained temperature–induced (cooling) strains or imposed defor-
mations. In addition to the two basic Tension Chord Model assumptions, the shrinkage- and temperature- induced strains are assumed to be uniformly distributed over the concrete and entire section, respectively.

The **crack formation phase** starts when the restrained or imposed strains reach \( f_{ct}/E_s \). In contrast to the axially loaded tension tie, there is an extensive crack formation phase (see Fig. 3.12(b) and (c)) as each crack causes a global stress relief in the member. The axial force in the member is determined by imposing equilibrium and strain compatibility in the uncracked regions as well as enforcing the boundary conditions (constant member length or the imposed deformation length). The equations given here (Alvarez 1998) apply to a tension tie with cracks spaced further apart than twice the transfer length (\( \lambda = 1 \)), where \( j \) denotes the crack number. For imposed deformations (\( \varepsilon_i = \varepsilon_{imposed} \)) or restrained cooling-induced strains (\( \varepsilon_i = \varepsilon_{sh} \)) (see Fig. 3.12(b)) the crack steel stresses

\[
\sigma_{s,II} = \frac{2\ln \rho \tau_{s0} \left[ 1 + \rho n - \rho \right]}{j \rho \left( 1 - \rho \right)^2} \left[ \frac{1 + j \rho \tau_{s0} \left( 1 - \rho \right)^2 \varepsilon_i}{\ln \rho \tau_{s0}} - 1 \right] - \frac{E_s \varepsilon_i}{2
\]

(3.80)

fluctuate below \( \sigma_{s,II} \) (Eq. (3.77)), while the corresponding crack widths

\[
w = \frac{2 \rho n \tau_{s0} \left[ 1 + \rho n - \rho \right]}{j \rho \left( 1 - \rho \right)^2} \left( 1 + \frac{\alpha}{2 - \sqrt{1 + \alpha}} \right)
\]

(3.81)

with

\[
\alpha = \frac{j \rho \tau_{s0} \left( 1 - \rho \right)^2}{\ln \rho \tau_{s0}}
\]

(3.82)

fluctuate below \( \sigma_{s,II} l_0 / E_s \). For shrinkage-induced strains (\( \varepsilon_i = \varepsilon_{sh} \)) (see Fig. 3.12(c)), which don’t affect the reinforcement, the crack steel stresses

\[
\sigma_{s,II} = \frac{2\ln \rho \tau_{s0} \left[ 1 + \rho n - \rho \right]}{j \rho \left( 1 - \rho \right)^2} \left[ \frac{1 + j \rho \tau_{s0} \left( 1 - \rho \right)^2 \varepsilon_i}{\ln \rho \tau_{s0}} - 1 \right] - E_s \varepsilon_i
\]

(3.83)

fluctuate below \( \sigma_{s,II} - E_s \varepsilon_i \). Their maximum value is (point B in Fig. 3.12(c))

\[
\sigma_{s,II,max} = f_{ct} \left[ \frac{1}{\rho} - 1 - \frac{\rho \tau_{s0} \left( 1 - \rho \right)^2}{4 \rho \tau_{s0}^2} \right]
\]

(3.84)

The **stabilized cracking** phase starts at

\[
\varepsilon_i = \frac{f_{ct}}{E_s} + \Delta \varepsilon_{s,II}
\]

(3.85)

After this point the steel stresses increase

\[
\sigma_{s,II} = E_s \left( \varepsilon_i + \Delta \varepsilon_{s,II} \right)
\]

(3.86)

in the case of imposed or restrained cooling-induced strains or remain constant

\[
\sigma_{s,II} = E_s \Delta \varepsilon_{s,II}
\]

(3.87)

for shrinkage-induced strains. In both cases the crack widths increase. Further information on end-restrained and deformation-induced cracking can be found in Chapter 4.2, where the equations for
shrinkage- and temperature-induced cracking are combined and extended to partially end-restrained tension ties.

### 3.5 Summary

The State I and II deformations of general RC beam sections result from solving the equilibrium Eqs. (3.58) and (3.59). Thereto plane sections according to Eq. (3.57), elastic steel material behaviour according to Eq. (3.56) and Trost’s concrete material behaviour according to Eq. (3.17) can be assumed.

- The creep coefficient and free shrinkage strains can be estimated e.g. with the MC 90 Eqs. (3.20), (3.21) and (3.36). The aging coefficient $X$ is between 0.8 and 1; for ordinary RC sections the influence of $X$ is usually negligible and $X = 1$ can be adopted.

- In the absence of an axial force ($N = 0$) the equilibrium equations (3.58) and (3.59) provide the transformed section properties for determining the short-term moment-induced section deformations. Table 3.1 contains the transformed section properties for rectangular sections.

- The transformed section properties can also be used to determine the long-term moment-induced section deformations ($N = 0$, $X = 1$) if the modular ratio $n$ is determined with the effective modulus according to Eq. (3.65).

- The cross-section shrinkage curvatures (no external restraint) are provided by Eq. (3.67).

- The Tension Chord Model complements the State II section behaviour by providing the transfer length (Eq. (3.74)) for determining the crack spacing as well as the average concrete stresses (Eq. (3.75)) in the tension chord. These concrete tensile stresses reduce the tensile steel strains according to Eq. (3.78).
4 Tension Chord Model for Serviceability Problems

The complex and hardly known load history and the highly variable concrete properties (see Chapter 3.1) represent the main difficulties for predicting the deformations of reinforced concrete (RC) structural members. For reasons of simplicity, deformation prediction approaches (see Chapter 2) usually neglect the unknown load history (e.g. pre-cracking due to construction overloads) and the corresponding initial deformations. For these reasons, integral (e.g. midspan deflections) and in particular local deformation predictions (e.g. strains, curvatures, crack widths) will never achieve the same accuracy as ultimate load predictions (Marti 1983). Still, practical applications require reasonably reliable estimates of the magnitude of both integral and local deformations.

The Tension Chord Model (see Chapter 3.4) represents a consistent tool for modelling the in-service deformation behaviour of tensile members with uniaxial stress states, such as tension ties and tension chords in one-way bending members. So far (see Chapter 3.4), the model has been used to describe the short-term load- and restraint-induced cracking behaviour of tensile elements (Marti, Sigrist, et al. 1997; 1998; Alvarez 1998) as well as the load-induced cracking and deflection behaviour of statically determinate bending elements (Kenel 2002; Kenel et al. 2005).

Based on this previous work, this chapter aims at illustrating the wide range of serviceability prediction equations for designing tensile and one-way bending members that can be derived based on the Tension Chord Model. Thereto, in Section 4.1 the Tension Chord Model is extended in order to account for the effective concrete area in bending elements, the loss of tension stiffening due to bond creep and the influence of unloading. The Tension Chord Model is then combined with the long-term section deformation approaches for bending elements discussed in Chapter 3.3 to obtain hand-calculation equations for predicting short- and long-term crack widths (Section 4.2) and deflections (Section 4.3). The crack width predictions are focused on moment-, restraint-, and deformation-induced cracking with different boundary conditions. Further, the influence of the reinforcement detailing on the deflections of statically indeterminate members is discussed and limiting span-to-depth ratios are derived. In the whole chapter, the stress in the reinforcement steel is assumed to remain below the yield limit and shear deformations are neglected.

4.1 Tension Stiffening

4.1.1 Short-Term

The Tension Chord Model considers the contribution of the tensile concrete by assuming a linear distribution of concrete tensile stresses between the cracks (Fig. 4.1 (a)). In bending members the concrete stresses are further assumed to be uniformly distributed over the net concrete area $A_{c,ef}$ of a tension chord that has the same centroid as the tensile steel (Fig. 4.1 (c)). This leads to average concrete strains in the tension chord equal to

$$\varepsilon_{c,cr} = \frac{\sigma_{c,cr}}{E_c} = \frac{\lambda f_{ct}}{2E_c} \quad (4.1)$$

$$\lambda = \frac{A_{c,ef}}{A_{cr}}$$

$$\lambda = \frac{A_{c,ef}}{A_{cr}}$$

37
Factor $\lambda$ stands for the crack spacing (see Chapter 3.4.1) and varies between 0.5 and 1 and $f_{ct}$ is defined by Eq. (3.8). The most general way to consider Eq. (4.1) is by inserting the average concrete tensile force ($\sigma_{ct,Ac,ef}$) into the section equilibrium equations (Eqs. (3.58) and (3.59)). For tension chords with constant section properties and tension forces (that is $V = 0$ in bending elements) this yields the average steel strain reduction

$$\Delta \varepsilon_{s,ts} = \frac{\lambda f_{ct} (1 - \rho_{ef})}{2 \rho_{ef} E_c}$$

and the corresponding average curvature reduction

$$\Delta \chi_{ts} = \frac{\Delta \varepsilon_{s,ts}}{d - x_{II}}$$

for tension chords in bending elements (Fig. 4.1 (d)). The latter equation assumes a constant compression zone height ($x_{II}$), which allows its superimposition with the State II curvature. The main disadvantage of Eqs. (4.2) and (4.3) is their dependency on the effective reinforcement ratio of the tension chord $\rho_{ef} = A_s / (A_{c,ef} + A_s)$.

Marti (2004) suggested determining the effective reinforcement ratio by equating the State II steel stresses in the actual bending element and the corresponding steel stresses in the tension chord at cracking. In bending members without axial forces ($N = 0$), the State II steel stresses at $M = M_r$ are equal to $\sigma_{s,II,r} = (M_r n (d - x_{II})/I_{II})$, while the corresponding steel stresses in the fictitious tension chord are equal to $\sigma_{s,II,r} = f_{ct} (1/\rho_{ef} + n - 1)$. Solving for $\rho_{ef}$ leads to

$$\rho_{ef} = \left[ \frac{n I_1 (d - x_{II})}{I_{II} (h - x_I)} - n + 1 \right]^{-1}$$

and a net concrete area

$$A_{c,ef} = n A_s \left[ \frac{I_1 (d - x_{II})}{I_{II} (h - x_I)} - 1 \right]$$

In contrast to Schiessl’s effective reinforcement ratio (Eq. (2.8)), Eq. (4.4) is compatible with the Bernoulli bending theory. Inserting Eq. (4.4) into Eq. (4.2) leads to

$$\Delta \varepsilon_{s,II} = \frac{\lambda}{2} \left[ \varepsilon_{s,II,r} - \frac{f_{ct}}{E_c} \right]$$

while Eq. (4.4) in Eq. (4.3) produces

$$\Delta \chi_{II} = \frac{\lambda}{2} \left[ \frac{M_r}{E_c I_{II}} - \frac{f_{ct}}{E_c (d - x_{II})} \right]$$

The first terms in the brackets of Eqs. (4.6) and (4.7) correspond to the State II steel strain and curvature, respectively at $M = M_r$. The $2^{nd}$ term in Eq. (4.6) represents the State I steel strains at $M = M_r$, while the second term in Eq. (4.7) represents those State I steel strains divided by the State II steel lever arm. Both equations are load-independent. The fact that they could have been derived directly from Fig. 4.1 (a) for $M = M_r$ proves the consistency of the approach.

Eqs. (4.1) and (4.6) form the basis for crack width predictions, while Eq. (4.7) is a very practical equation for considering tension stiffening in bending elements, as the effective reinforcement ratio is implicitly considered. Eqs. (4.6) and (4.7) are derived for $V = 0$, but represent good approxi-
Tension stiffening also when $V \neq 0$, as the tension force gradient due to shear within the cracked elements is usually small (see Seelhofer (2009)).

\[ \chi_{\Delta} = \chi_{\bar{\chi}} - \Delta \chi_{\bar{\chi}}. \]

\[ \lambda = 1 \]

\[ \Delta \chi_{\bar{\chi}} = \frac{\lambda}{2} \left[ \chi_{\bar{\chi}} \left( d - c_{\bar{\chi}} \right) - \frac{f_{ct}}{E_c \left( d - c_{\bar{\chi}} \right)} \right] \]

Note that Eq. (4.10) is load-dependent. $\chi_{\bar{\chi}}$ and $c_{\bar{\chi}}$ denote the State II curvature and height of the compression zone at cracking.

4.1.2 Long-Term

Under sustained loads, creep relaxes the concrete stresses involved in the mechanical interlock between the steel and the concrete (Fig. 4.2 (a)). As a consequence, the effective bond stresses are reduced and the slip is increased. In order to consider the loss of bond stress due to creep, factor
is introduced. This factor (Fig. 4.2 (b)) is determined according to the Effective Modulus Method (Eq. (3.65)) and used to reduce the concrete strains

\[
e_{c,ls} = k\frac{\lambda f_{ct}}{2E_c} \tag{4.12}
\]

and the steel strain reduction

\[
\Delta e_{s,ls} = k\frac{\lambda}{2} \left[ e_{s,ls} - \frac{f_{st}}{E_c} \right] \tag{4.13}
\]

The crack spacing of existing cracks is not affected by bond creep and \(\lambda\) remains between 0.5 and 1. Besides bond creep, the average curvature reduction

\[
\Delta \chi_{ls} = k\frac{\lambda}{2} \left[ \frac{M_e}{E_c I_{ll}} \left( d - x_{ll,ct} \right) - \frac{f_{st}}{E_c \left( d - x_{ll,ct} \right)} \right] \tag{4.14}
\]

is further affected by the drop in the neutral axis due to compression zone creep. The height of the compression zone \(x_{ll,ct}\) is determined with the effective elastic modulus (Eq. (3.65)).

Neglecting the shrinkage restraint between the steel and the cracked concrete, shrinkage further increases the slip without influencing the bond stresses. The assumption of a constant bond stress, which is reduced in time due to bond creep, impedes the modelling of secondary cracks caused by restrained shrinkage stresses in the tensile concrete between the cracks. If at all, secondary cracks are most likely to form in \(\lambda = 1\) crack elements and are therefore included within the range of \(0.5 \leq \lambda \leq 1\).

The strongly simplified relationship between \(k_{\varphi}\) and \(\varphi\) (Fig. 4.2 (b)) is suggested in view of the highly complex nature of the stresses involved in the mechanical interlock and the large uncertainties involved in determining the long-term concrete parameters. Note that while \(\lambda \in [0.5...1]\) takes into account the crack spacing, \(k_{\varphi} \in [0...1]\) considers the contribution of the tensile concrete between the cracks, where \(k_{\varphi} = 1\) implies \(\tau_b = 2 f_{ct}\) and \(k_{\varphi} = 0\) implies \(\tau_b = 0\). In this sense, \(k_{\varphi}\) can be seen as a general factor for expressing the contribution of tension stiffening for a given crack spacing. \(k_{\varphi}\) is used in
this way in the test data comparisons illustrated in Chapter 5.2.1 to show the limiting cases of full ($k_\rho = 1$) and no ($k_\rho = 0$) tension stiffening.

### 4.1.3 Unloading

Again based on the load-transfer model illustrated in Fig. 4.2 (a) the strongly simplified bond stress – slip relationship illustrated in Fig. 4.3 (a) is adopted. The relation implies that the bond stress is lost if the slip is reduced. In other words, pulling the steel bar out of the concrete is resisted by bond stresses, but no bond stresses have to be overcome for it to retrace its way back in again. Before unloading (Fig. 4.3 (b)) the bond stresses are activated along the entire tension chord. Reducing the State II steel stresses in the crack (Fig. 4.3 (c)), reduces the slip along the length $x_c$, where the bond stresses disappear. It is assumed that the slip along the remaining sections stays constant (no relative movement).

Fig. 4.3 Half of a tension cord during unloading (a) assumed bond stress – slip relationship; (b) concrete and steel stresses before unloading; (c) concrete and steel stresses during unloading.

During unloading (Fig. 4.3 (c)) the tension stiffening steel strain reduction is

$$\Delta \varepsilon_{s, II}^u = \frac{\left(\sigma_{s, II} - \sigma_{s, m}\right)(\lambda I_0 - x_c)}{2\lambda I_0 E_s} \tag{4.15}$$

with

$$\left(\sigma_{s, II} - \sigma_{s, m}\right) = \frac{f_{\alpha} \left(1 - \rho\right)(\lambda I_0 - x_c)}{\rho^l} \tag{4.16}$$

determined by formulating equilibrium for $N$ at the crack and at the symmetry axis leads and

$$x_c = \frac{\rho^l \Delta \sigma_{s, II}}{f_{\alpha} \left(n \rho + 1 - \rho\right)} \leq \lambda I_0 \tag{4.17}$$

from formulating equilibrium for $\Delta N$ at the crack and at a distance $x_c$ away from the crack. Introducing Eqs. (4.16) and (4.17) in (4.15) leads to
\[ \Delta \epsilon_{s,\text{ts}} = \frac{f_{ct}(1-\rho)}{2pE_c} \left( \lambda - \frac{\rho \Delta \sigma_{s,\text{II}}}{f_{ct}(1-\rho + np)} \right) \geq 0 \] (4.18)

which is valid for \( \Delta \sigma_{s,\text{II}} \) between zero and

\[ \Delta \sigma_{s,\text{II}} = \frac{\lambda f_{ct}}{\rho} (1 - \rho + np) = \lambda \sigma_{s,\text{II},r} \] (4.19)

where \( x_t = \lambda l_0 \) is reached. Replacing \( \Delta \sigma_{s,\text{II}} \) with \( E_s (\epsilon_{s,\text{avg,max}} + \Delta \epsilon_{s,\text{ts}} - \epsilon_{s,\text{avg}} - \Delta \epsilon_{u_{s,\text{ts}}}) \) leads to

\[ k_u = \frac{\Delta \epsilon_{s,\text{ts}}}{\Delta \epsilon_{s,\text{II}}} = \frac{\Delta \epsilon_{s,\text{avg}} + \Delta \epsilon_{s,\text{ts}}}{\Delta \epsilon_{s,\text{II}}} + \frac{2pn(pn - \rho + 1) - 2(pn - \rho + 1)}{(1-\rho)^2} \frac{\Delta \epsilon_{s,\text{avg}} + \rho^2 n^2}{\Delta \epsilon_{s,\text{II}}(1-\rho)^2} \] (4.20)

\( k_u \) is between 1 and 0 and \( \Delta \epsilon_{s,\text{avg}} = \epsilon_{s,\text{avg,max}} - \epsilon_{s,\text{avg}} \).

### 4.2 Crack Width

The Tension Chord Model is based on the slipping-bond approach (Chapter 2.1.1) to crack width prediction, which assumes that the crack spacing and concrete strains remain constant over the depth of the tension chord. In this case, the crack width

\[ w = 2\lambda l_0 (\epsilon_{s,\text{avg}} - \epsilon_{c,\text{avg}}) \] (4.21)

is constant over the entire depth of the member and is defined as the integral of the difference between the steel and concrete strains within the crack influence region, which according to the Tension Chord Model is equal to \( 2\lambda l_0 \) (\( l_0 \) and \( \lambda \) are defined in Chapter 3.4.1). Inserting the average steel strains

\[ \epsilon_{s,\text{avg}} = \epsilon_{s,\text{II}} - \frac{k_u \lambda}{2} \left( \epsilon_{s,\text{II},r} - \frac{f_{ct}}{E_c} \right) \] (4.22)

determined with Eq. (4.13) and the average concrete strains

\[ \epsilon_{c,\text{avg}} = \frac{k_u \lambda f_{ct}}{2E_c} + \epsilon_{c,\text{II},r} \] (4.23)

determined with Eq. (4.12) into Eq. (4.21) results in

\[ w = 2\lambda l_0 \left( \epsilon_{s,\text{II}} - k_u \frac{\lambda}{2} \epsilon_{s,\text{II},r} - \epsilon_{c,\text{II},r} \right) \] (4.24)

Eq. (4.24) forms the basis for determining the widths of moment-, restraint- and deformation-induced cracks in tension ties and chords. The decisive components of this equation are the crack influence length \( 2\lambda l_0 \) and the load-case dependent State II steel strains \( \epsilon_{s,\text{II}} \) at the time when the crack width is predicted. The concrete tensile strains \( k_u \lambda/2 \epsilon_{s,\text{II},r} \) due to tension stiffening reduce the crack width, while shrinkage- or temperature-induced concrete contraction strains \( (\epsilon_{c,\text{II}}) \) increase the crack width. The State II steel strains at cracking \( \epsilon_{s,\text{II},r} \) are a function of the stress resultant at cracking, determined at the time of loading.

At the beginning of the crack formation phase, the crack influence region \( 2\lambda l_0 \) extends over twice the transfer length \( l_0 \) (Eq. (3.74)) and \( \lambda = 1 \). At the end of the crack formation phase the extent of the crack influence region corresponds to the stabilized crack spacing.
Crack Width

\[ s_r = 2\lambda l_0 \]  
(4.25)

which according to the Tension Chord Model (Chapter 3.4.1) is assumed to be between once \( (\lambda = 0.5) \) and twice \( (\lambda = 1) \) the transfer length. In many practical cases the crack spacing corresponds to the stirrup spacing; in this case \( \lambda \) is defined. In all cases, \( \lambda = 1 \) is decisive for predicting maximum crack widths.

Both the crack widths at the concrete surface and the bar surface are relevant for crack width control. The former are of interest for aesthetical considerations, while the latter may be of interest for durability and, in the absence of a compression zone, liquid tightness. The secondary Goto cracks (Goto 1971), which form at the bar ribs, considerably reduce the crack width at the bar surface, as the integral strain difference between the steel and concrete is absorbed by a larger amount of cracks. With increasing distance from the bar surface, the Goto cracks disappear until at the concrete surface only the primary cracks are visible. If the shear lag effect is neglected, the theoretical crack widths determined with the Tension Chord Model correspond to theoretical surface crack widths, as they are based on the surface crack spacing.

4.2.1 Bending

For bending members with \( N = 0 \), the State II steel strains at \( M = M_r \) correspond to

\[ \varepsilon_{s, II} = \frac{M_r (d - x_{II})}{E_c I_{II}} = \frac{f_c l (d - x_{II})}{E_c l (h - x_{I})} \]  
(4.26)

They are a function of the concrete material parameters \( (f_c, E_c) \) and the section properties \( (\rho, \rho', d/h, d'/h) \). Neglecting the influence of \( V \) on tension stiffening, Eq. (4.26) and Eq. (4.24) lead to

\[ w = 2\lambda l_0 \left[ \varepsilon_{s, II} - k_o \frac{\lambda f_c l (d - x_{II})}{2E_c l (h - x_{I})} - \varepsilon_{cs} \right] \]  
(4.27)

The transfer length

\[ l_0 = \frac{n f_c \varnothing}{4\tau_b} \left( \frac{I_{II} (d - x_{II})}{I_{II} (h - x_{I})} - 1 \right) = \frac{\varnothing n}{8} \left( \frac{I_{II} (d - x_{II})}{I_{II} (h - x_{I})} - 1 \right) \]  
(4.28)

results from inserting Eq. (4.4) into Eq. (3.74).

The first term in the brackets of Eq. (4.27) corresponds to the State II steel strains. In bending members they are usually determined at the centroid of the tensile reinforcement. However, in deep beams with multiple layers of tensile reinforcement, the stress gradient between the different steel layers causes an increasing crack opening with increasing distance from the neutral axis. This can be considered by determining the State II steel strains at the height of the specific reinforcement layer. For short-term loads the State II steel strains correspond to

\[ \varepsilon_{s, II} = \frac{M (d - x_{II})}{E_c I_{II}} \]  
(4.29)

For sustained loads the steel strains can either be determined with the section equilibrium equations (Eqs. (3.58) and (3.59)) and Trost's concrete material behaviour (Eq. (3.17)) or estimated with the effective concrete properties (EMM, neglecting shrinkage). In many cases they can also be assumed time-independent, as the influence of long-term effects on the steel strains is often negli-
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Because the increase in curvature is compensated by the increase in compression zone height (see e.g. Mari et al. 2010).

The second term represents the average contribution of the tension concrete, assumed to be constant within the whole effective concrete area (Fig. 5.10 (c)). For maximum crack widths \( \lambda = 1 \). For short-term predictions \( k_{\phi} = 1 \), while for sustained loads \( k_{\phi} \) can be reduced according to Fig. 4.2 (c). If the tensile concrete is neglected \( k_{\phi} = 0 \) and

\[
w = 2\lambda I_0 \left[ \varepsilon_{s,II} - \varepsilon_{ci} \right]
\]

(4.30)

The third term reflects any shrinkage- or temperature-induced concrete strains. The assumption of a constant bond stress between the steel and concrete implies that imposed concrete contraction strains are not restrained in cracked sections.

For short-term crack width predictions with \( \varepsilon_{ct} = 0 \) and \( k_{\phi} = 1 \), Eq. (4.27) can be simplified to

\[
w = 2\lambda I_0 \frac{M (d - x_{II})}{E I_{II}} \left[ 1 - \frac{\lambda}{2} \left( \frac{M}{M_t} \right) \right]
\]

(4.31)

At cracking Eq. (4.27) with \( \varepsilon_{s,II} = \varepsilon_{s,II} \) leads to

\[
w' = \frac{\lambda M c}{4} \left( \frac{I_1 (d - x_{II})}{I_{II} (h - x_1)} - 1 \right) \left[ 1 - \frac{\lambda}{2} \frac{f_{ct} (d - x_{II})}{E I_{II} (h - x_1)} \right]
\]

(4.32)

**Example 4**

This example illustrates the maximal crack width caused by a moment \( M = 70.5 \) kNm in a slab strip with sections and material properties according to Fig. 3.9. It is assumed that the slab strip is reinforced with \( \varnothing 12 \) mm bars (spaced at 150 mm). Table 3.1 provides the section parameters: \( f_I = 0.505 \), \( f_{II} = 0.185 \), \( g_I = 0.0851 \) and \( g_{II} = 0.0161 \).

A transfer length (Eq. (4.28)) of

\[
l_c = \frac{6 \cdot \varnothing}{8} \left( \frac{0.0851 \cdot (1 - 0.185)}{0.0161 \cdot 0.85 \cdot (1 - 0.505)} - 1 \right) = 8.3 \varnothing
\]

leads to a maximal (\( \lambda = 1 \)) crack spacing of 16.6 \( \cdot 12 \) mm = 200 mm. With \( f_{ct} = 3 \) MPa, the State II steel strains at cracking (Eq. (4.26)) equal

\[
\varepsilon_{s,II} = \frac{6 \cdot 3 \text{ MPa}}{205 \text{ GPa}} \left( \frac{0.0851 \cdot (1 - 0.185)}{0.0161 \cdot 0.85 \cdot (1 - 0.505)} \right) = 1.06 \%
\]

while the State II steel strains at \( M = 70.5 \) kNm equal \( \varepsilon_{s,II} = 475 \) MPa / 205 GPa = 2.32 \% (Stage A in Fig. 3.9). On this basis Eq. (4.27) provides a crack with

\[
w = 2 \cdot 8.3 \cdot 12 \text{ mm} \left( 2.32 \% - \frac{1.06 \%}{2} \right) = 0.36 \text{ mm}
\]

Shrinkage strains of \( \varepsilon_{sh} = -0.6 \% \) and a reduced \( k_{\phi} \) value of 2/3 increase the crack width to

\[
w = 2 \cdot 8.3 \cdot 12 \text{ mm} \left( 2.32 \% - \frac{1.06 \%}{3} + 0.6 \% \right) = 0.51 \text{ mm}
\]

if the steel strain increment due to long-term effects is neglected or

\[
w = 2 \cdot 8.3 \cdot 12 \text{ mm} \left( 2.44 \% - \frac{1.06 \%}{3} + 0.6 \% \right) = 0.54 \text{ mm}
\]

for \( \varepsilon_{s,II} = 500 \) MPa / 205 GPa = 2.44 \% (Stage B in Fig. 3.9).
4.2.2 Restrained Deformations

Restrained cracking is caused by shrinkage- or temperature-induced concrete strains $\varepsilon_{ci}$ (contractive strains), which are restrained by neighboring section parts (internal restraint) or boundary conditions (external restraint). The resulting tensile stresses can cause surface cracks, if they are concentrated at the surface (e.g. mass concrete), or separation cracks, if they extend over the entire section (e.g. flat plates in low-moment regions). In the second case the whole section requires a minimum reinforcement and in absence of a compression zone the cracks can cause leakage.

This paragraph discusses the two principal external restraint cases illustrated in Fig. 4.4, neglecting load-induced stress resultants. It is assumed that the shrinkage- and temperature-induced concrete strains $\varepsilon_{ci} = \varepsilon_{sh} + \varepsilon_{T}$ are uniformly distributed over the section. In the case of temperature, the concrete strains are accompanied by steel strains $\varepsilon_{si} = \varepsilon_{T} = -\alpha \Delta T$. Shrinkage-induced concrete strains range between 0 and $-1 \%$. Assuming a thermal expansion coefficient $\alpha = 0.01 \%/K$, a temperature drop of 100 °C causes a strain increase of $-1 \%$.

Prior to cracking, the concrete tensile stresses

$$\sigma_c = -RE_c \varepsilon_{ci} \tag{4.33}$$

increase until at

$$\varepsilon_{ci,t} = -\frac{f_{ct}}{E_c R} \tag{4.34}$$

the crack formation process starts. The degree of restraint $R$ ranges between 0 and 1. Creep releases a part of the restraint tensile stresses and can be considered for example by replacing $E_c$ with $E_{c,ef}$ (Eq. (3.65)) in Eqs. (4.33) and (4.34). Eq. (4.24) with

$$\varepsilon_{s,II} = \frac{f_{ct}}{\rho E_s} \left(1 + np - p\right) \tag{4.35}$$

defines the crack widths

$$w = 2l_0 \left(\varepsilon_{s,II} - \varepsilon_{ci} - \frac{k_s \lambda f_{ct} \left(1 + np - p\right)}{2\rho E_s}\right) \tag{4.36}$$

where

$$\varepsilon_{s,II} = \frac{\sigma_c}{E_s} + \varepsilon_{ci} \tag{4.37}$$

Unless stated otherwise the transfer length $l_0$ is defined by Eq. (3.74) and $k_s = 1$ is assumed.
The two basic concepts of crack control consist in either trying to prevent cracks from forming (Eq. (4.33)) or reducing their widths (Eq. (4.36)). If restraint-induced axial forces are combined with load-induced bending moments, the $M-N$ interaction has to be considered for determining the State II steel stresses (see Specimen RG8 in Chapter 5.2.1).

**End-Restraint**

The member deformations $\Delta l = -N/(khb)$ (Fig. 4.4 (a)) are defined by the axial stiffness of the support $k$ [N/mm$^2$] and the axial restraint force $N$, which is assumed to be constant within the member. Two dimensionless coefficients are defined as

$$\kappa = \frac{lk}{E_s} \quad (4.38)$$

and

$$a_i = \frac{A_i}{bh} = n\rho - \rho + 1 \quad (4.39)$$

**Before cracking**

With $\sigma_{s,1} = E_s (\epsilon_i - \epsilon_{si})$ and $\sigma_{c,1} = E_c (\epsilon_i - \epsilon_{ci})$, $\Delta l = l \epsilon_i$ leads to

$$\frac{\Delta l}{l} = \frac{n\rho \epsilon_{si} + (1 - \rho) \epsilon_{ci}}{nk + a_i} \quad (4.40)$$

the concrete stresses

$$\sigma_{c,i} = nE_c \frac{\rho \epsilon_{si} - (\rho + \kappa) \epsilon_{ci}}{nk + a_i} \quad (4.41)$$

the degree of restraint

$$R = \frac{n\rho \left(1 - \frac{\epsilon_{ci}}{\epsilon_{si}}\right) + nk}{nk + a_i} \quad (4.42)$$

and the axial force

$$N_i = bh \left[a_i \sigma_{s,i} + \rho E_s (\epsilon_{si} - \epsilon_{ci})\right] \quad (4.43)$$

**Crack Formation Phase**

The crack formation phase starts at

$$\epsilon_{ci,i,1} = -f_{\sigma,i} \left(\kappa + a_i/n\right) + E_s p \epsilon_{si}$$

when $\sigma_{s,i}$ first reaches the concrete tensile strength. If $k > 0$ each new crack causes a global stress relief in the member. This leads to an extensive crack formation phase, during which the axial restraint force $N$ decreases and increases between the development of the cracks.

The steel stresses in the cracks peak just before a new crack forms, when the concrete stresses in the uncracked sections reach the concrete tensile strength (Fig. 4.5 (b)). For $\sigma_{c,1} = f_{\sigma,i}$, equilibrium ($\rho \sigma_{s,1} = \rho \sigma_{s,i} + (1 - \rho) \sigma_{c,1}$) and strain compatibility ($\rho \sigma_{s,1} = n \sigma_{s,i} + E_s (\epsilon_{ci} - \epsilon_{si})$) lead to
Crack Width

\[ \sigma_{s,II,r} = \frac{f_c d_1}{\rho} - E_s (\varepsilon_{sI} - \varepsilon_{cI}) \]  

(4.45)

with the corresponding axial force \( N_r = bh \rho \sigma_{s,II,r} \), deformations \( \Delta l = -N_r/(k b h) \) and crack widths

\[ w_r = \frac{\lambda (2 - \lambda) l_0 f_{ct} a_1}{\rho E_s} \]  

(4.46)

This crack width \( w_r \) corresponds to the crack width obtained if the member is loaded with its cracking load. Both \( \sigma_{s,II,r} \) and \( w_r \) are independent of the support stiffness \( k \). They are illustrated with dashed lines in Fig. 4.6 for a lightly reinforced example member.

Fig. 4.5  End-restraint with \( \lambda = 1 \) and \( \kappa = \infty \): Stresses and strains due to shrinkage- and temperature-induced compressive strains: (a) just after crack \( j \); (b) just before crack \( j + 1 \); (c) stabilized cracking.

According to the Tension Chord Model (see also 3.4.2), the member deformations in-between two cracks \( j \) and \( j + 1 \) (Fig. 4.5 (a)) correspond to

\[ \Delta l = (l - j l_{\text{crack}}) \frac{\sigma_{s,I}}{E_s} + j l_{\text{crack}} \left( \frac{\sigma_{s,II} - \sigma_{s,I}}{E_s} - \frac{l_{\text{crack}} f_{ct} (1 - \rho)}{4 \rho b_0 E_s} \right) = -\frac{\rho \sigma_{s,II}}{k} \]  

(4.47)

Cracks spaced no closer than twice the transfer length (\( \lambda = 1 \)) have an influence region \( l_{\text{crack}} \) equal to twice the reduced transfer length (Fig. 4.5 (a))

\[ l_b = \frac{(\sigma_{s,II} - \sigma_{s,I}) b_0}{f_{ct} (1 - \rho)} \]  

(4.48)

where

\[ \sigma_{s,I} = \frac{n \rho \sigma_s (1 - \rho) E_s (\varepsilon_{sI} - \varepsilon_{cI})}{a_1} \]  

(4.49)
is the steel stress in the uncracked sections. Assuming that no cracks are closer twice the transfer length, inserting Eqs. (4.48) and (4.49) into (4.47) leads to

\[
\sigma_{sl,II} = \frac{\mu(\kappa n + a_t)}{\kappa \rho} \sqrt{1 - \frac{2E_s \kappa n (\kappa e_{si} - \rho(e_{si} - e_{ci}))}{\mu'_{\sigma} (\kappa n + a_t)^2}} - 1 - E_s (e_{si} - e_{ci}) \tag{4.50}
\]

where

\[
\mu = \frac{np}{2l_0 f(1 - \rho)} \tag{4.51}
\]

\[N = \sigma_{sl,II} bh \rho \] and \[\Delta l = -N/kbh\]. Equating Eqs. (4.45) and (4.50) and setting \(j = 1\) leads to the induced concrete strains at point B

\[
e_{ci,r} = \frac{\mu f_{\sigma} (\kappa n + a_t)^2}{2nE_s (\kappa + \rho)} \left[ 1 - \frac{\kappa n}{\mu (\kappa n + a_t) + 1} \right] + \frac{\rho e_{si}}{\kappa + \rho} \tag{4.52}
\]

where the steel stresses (Eq. (4.45) with \(e_{ci} \neq e_{si}\)) are maximal (see Fig. 4.6 (b)).

---

**Fig. 4.6** End-restrained tension chord with \(l = 40 l_0 \approx 10\) m and \(k = \infty\); (a) State II steel stresses for \(e_{si} = e_{ci}\); (b) State II steel stresses for \(e_{si} = 0\); (c) crack widths for \(e_{si} = e_{ci}\); (d) crack widths for \(e_{si} = 0\). Note that \(\rho = 0.6\%\), \(\varnothing = 12\) mm, \(f_{\sigma} = 2\) MPa, \(n = 6\), \(E_s = 205\) GPa, \(\tau_b = 2 f_{\sigma}\) and \(\lambda = 1\).
With Eq. (4.50), a crack influence region equal to 2 \( l_b \) and a concrete contribution reduced by the factor \( l_b / l_0 \), the crack width Eq. (4.36) yields

\[
w = \frac{l_b f_{cm} \mu^2 (\kappa n + a_l)^2 a_l}{(\kappa n)^2 \rho E_s} \left[ \sqrt{1 - \frac{2E_s \kappa n (\kappa e_{cl} - \rho (e_{sl} - e_{st}))}{\mu f_{cm} (\kappa n + a_l)^2}} - 1 \right]^2
\]  

(4.53)

Cracks that form closer than twice the transfer length away from an existing crack cause a smaller global stress relief than according to Eq. (4.50). For the limiting case of all cracks following the first crack forming a transfer length away from the previous crack \( (\lambda = 0.5) \), \( l_{\text{crack}} \) in Eq. (4.47) equals \( l_0 \).

For \( k = \infty \) (Fig. 4.6), Eqs. (4.50) and (4.53) can be simplified to

\[
\sigma_{r,\Pi} = \frac{a_l \mu f_{cm}}{\rho} \left( \sqrt{1 - \frac{2E_s e_{cl}}{\mu f_{cm}}} - 1 \right) - E_s (e_{sl} - e_{st})
\]  

(4.54)

and

\[
w = \frac{l_b f_{cm} a_l \mu^2}{\rho E_s} \left( \sqrt{1 - \frac{2E_s e_{cl}}{\mu f_{cm}}} - 1 \right)^2
\]  

(4.55)

Stabilized Cracking

After the crack formation phase is concluded (Fig. 4.5 (c)), \( j_{l_{\text{crack}}} = l \) and \( l_{\text{crack}} = 2\lambda l_0 \) so that Eq. (4.47) can be simplified to

\[
\Delta l = l \left( \frac{\sigma_{s,\Pi}}{E_s} + e_{sl} - \frac{\lambda f_{cm} (1 - \rho)}{2\rho E_s} \right) = -\frac{\rho \sigma_{s,\Pi}}{k}
\]  

(4.56)

Inserting \( \sigma_{s,\Pi,r} \) according to Eq. (4.45) into Eq. (4.56) yields the induced concrete strains

\[
e_{cl,r,\Pi} = \frac{f_{cm}}{2\rho E_s} \left( \frac{\lambda (1 - \rho) \kappa}{\kappa + \rho} - 2a_l \right) + \frac{e_{sl} \rho}{\kappa + \rho}
\]  

(4.57)

necessary to finalize the crack formation phase with

\[
j_{un} = \frac{l}{2\lambda l_0}
\]  

(4.58)

Cracks. Further, solving Eq. (4.56) leads to the stabilized cracking steel stresses

\[
\sigma_{s,\Pi} = \frac{\kappa}{\kappa + \rho} \left( \frac{\lambda f_{cm} (1 - \rho)}{2\rho} - E_s e_{cl} \right)
\]  

(4.59)

with the corresponding crack widths

\[
w = 2\lambda l_0 \left[ \frac{\lambda f_{cm} n \kappa + a_l}{2E_s} + e_{sl} \frac{\rho}{\kappa + \rho} - e_{st} \right]
\]  

(4.60)

as well as axial force \( N = \sigma_{s,\Pi} bh \rho \) and deformations \( \Delta l = -N/(kbh) \).
For $k = \infty$ (Fig. 4.6), Eqs. (4.57), (4.59) and (4.60) can be simplified to

$$\varepsilon_{c_{i,r,jn}} = \frac{f_{ct}}{2\rho E_s} \left[ \lambda (1 - \rho) - 2a_i \right]$$  \hspace{1cm} (4.61)

and

$$\sigma_{s,II} = \frac{\lambda f_{ct}}{2\rho} (1 - \rho) - E_s \varepsilon_{si}$$  \hspace{1cm} (4.62)

as well as

$$w = 2\lambda I_0 \left[ -\varepsilon_{cj} - \frac{\lambda n f_{ct}}{2E_s} \right]$$  \hspace{1cm} (4.63)

For $k = 0$ both $\varepsilon_{c_{i,r,1}}$ (Eq. (4.44)) and $\varepsilon_{c_{i,r,jn}}$ (Eq. (4.57)) can be simplified to

$$\varepsilon_{cj,r} = \frac{f_{ct}}{\rho E_s} a_0 + \varepsilon_{si}$$  \hspace{1cm} (4.64)

as there is no global stress relief in the member due to cracking and all cracks form simultaneously if a constant concrete tensile strength is assumed. Eq. (4.59) yields $\sigma_{s,II} = 0$, while Eq. (4.60) yields

$$w = 2\lambda I_0 \left[ -\frac{\lambda f_{ct} (1 + np - \rho)}{2\rho E_s} - \varepsilon_{cj} + \varepsilon_{si} \right]$$  \hspace{1cm} (4.65)

Example 5

In this example the State II steel stresses and crack widths of the approximately 10 m long (40 $l_0$) end-restrained tension chord illustrated in Fig. 4.6 are determined. The tension chord has a reinforcement ratio of $\rho = 0.6\%$ established by $\varnothing = 12$ mm bars. The material properties are $f_{ct} = 2$ MPa, $n = 6$ and $E_s = 205$ GPa. The tension chord is submitted to a) uniform cooling induced strains (e.g. due to loss of hydration heat) with $\varepsilon_{si} = \varepsilon_{ci}$ and b) uniform shrinkage strains with $\varepsilon_{si} = 0$. A restraint stiffness of $k = \infty$ is assumed.

The length of the tension chord is chosen to be 20 times the double transfer length (Eq. (3.74))

$$l_0 = \frac{12 \text{ mm} \cdot (1 - 0.6\%)}{-8 - 0.6\%} = 249 \text{ mm}$$

so that 20 cracks lead to the maximal final crack spacing ($\lambda = 1$). Further $a_0$ (Eq. (4.39)) = $6 \cdot 0.6\% - 0.6\% + 1 = 1.03$ and

$$\mu = \frac{6 - 0.6\% \cdot 40 \cdot 249 \text{ mm}}{2 \cdot 249 \text{ mm} \cdot (1 - 0.6\%)} = \frac{0.724}{j}$$

With $R = 1$, cracking starts when the imposed concrete strains reach (Eq. ((4.44)))

$$\varepsilon_{c_{i,j}} = \frac{-6 \cdot 2 \text{ MPa}}{205 \text{ GPa}} = -0.06\%$$

Uniform cooling - induced strains

During the crack formation phase the cracks $j = 1$ to 20 occur. Assuming that the cracks are not spaced closer than twice the transfer length $\lambda$ is set equal to 1. After a crack $j$ has formed, the steel stresses increase according to (Eq. (4.54))

$$\sigma_{s,II} = 1.03 \cdot 0.724 \cdot 2 \text{ MPa} \left( \frac{1 - 2 \cdot 205 \text{ GPa} \cdot \varepsilon_{c_{i,j}}}{0.724 - 0.6\%} \right)$$

until (Eq. (4.45))

$$\sigma_{s,II} = \frac{1.03 \cdot 2 \text{ MPa}}{0.6\%} = 343 \text{ MPa}$$

is reached (dashed horizontal line in Fig. 4.6 (a))) and the crack $j + 1$ forms. Accordingly, the crack widths of cracks spaced further apart than twice the transfer length increase with (Eq. (4.55))
Crack Width

\[ w = \frac{249 \text{ mm} \cdot 2 \text{ MPa} \cdot 1.03 \cdot 0.724^2}{j^2 \cdot 205 \text{ GPa} \cdot 0.6 \%} \left( \frac{1 - 2 \cdot 205 \text{ GPa} \cdot \varepsilon_{cr}}{0.724 \cdot 6 \cdot 2 \text{ MPa} - 1} \right)^2 \]

and remain below (Eq. (4.46) and dashed horizontal line in Fig. 4.6 (c))

\[ w_r = \frac{249 \text{ mm} \cdot 2 \text{ MPa} \cdot 1.03}{0.6 \% \cdot 205 \text{ GPa}} = 0.4 \text{ mm} \]

Note that for a concrete tensile strength of 3 MPa instead of 2 MPa the maximal steel stresses would be 515 MPa instead of 343 MPa and the maximal crack width 0.6 mm instead of 0.4 mm. The crack formation phase is concluded with 20 cracks when (Eq. (4.61))

\[ \varepsilon_{cr,ik} = \frac{2 \text{ MPa}(1 - 0.6 \% + 2 \cdot 6 \cdot 0.6 \%)}{2 \cdot 0.6 \% \cdot 205 \text{ GPa}} = -0.87 \% \]

is reached. With \( \alpha = 0.01\%/\text{K} \) this requires a temperature drop of 87 °C.

Uniform shrinkage-induced strains

In this case (Eq. (4.54))

\[ \sigma_{cr} = \frac{1.03 \cdot 0.724 \cdot 2 \text{ MPa}}{j \cdot 0.6 \%} \left( \frac{1 - 2 \cdot 205 \text{ GPa} \cdot \varepsilon_{cr}}{0.724 \cdot 6 \cdot 2 \text{ MPa} - 1} \right) + 205 \text{ GPa} \cdot \varepsilon_{cr} \]

and (Eq. (4.45))

\[ \sigma_{cr,cr} = \frac{1.03 \cdot 2 \text{ MPa}}{0.6 \%} + 205 \text{ GPa} \cdot \varepsilon_{cr} = 343 \text{ MPa} + 205 \text{ GPa} \cdot \varepsilon_{cr} \]

include the terms \( E \cdot \varepsilon_{cr} \), while the crack width equations remain the same. Free shrinkage strains equal to –0.87 % are necessary to conclude the crack formation phase. For \( \rho = 2 \% \) instead of 0.6 % free shrinkage strains of –0.30 % are sufficient to conclude the crack formation phase.

Continuous Edge-Restraint

The edge-restrained cracking behaviour is influenced by the section thickness \( h \) and the bond stresses \( \tau_c \) along the continuous edge Fig. 4.4 (b). For small thicknesses it makes sense to assume plain sections, while for large \( \tau_c \) values load-introduction lengths are negligible. Therefore, three cases are regarded here: Case (a) \( \tau_c \approx f_{ct} \) and \( h \) small (e.g. concrete overlays); Case (b) \( \tau_c \approx f_{ct}/100 \) and \( h \) small (e.g. flat ground slabs); and Case 3(c) \( \tau_c \approx f_{ct} \) and \( h \) large (e.g. base restrained walls). In line with the Tension Chord Model, a rigid-perfectly plastic bond-slip behaviour is assumed at the restrained edge, leading to constant bond stresses \( \tau_c \).

Case (a):

Before cracking \( R = 1 \). The stress-relief per crack is purely local and there is no crack formation phase if a constant tensile strength is assumed. The build-up of the concrete stresses on either side of a crack

\[ l_0 = \frac{f_{ct} \left(1 - \rho \right)}{\tau_c + \frac{4\rho \tau_b}{h}} \]

is established by the bond stresses \( \tau_b \) between the cracked concrete and the reinforcement as well as by the bond stresses \( \tau_c \) between the cracked concrete and the base. With \( \rho = 0 \), Eq. (4.66) is also valid for non-reinforced sections where \( \varepsilon_r = 2 \lambda_h f_{ct} / \tau_c \). Assuming average steel strains equal to zero, results in crack widths equal to Eq. (4.63). Values for \( \tau_c \) between different layers of concrete can be found for example in Randl & Wicke (2000).

Example 6

This example discusses the influence of shrinkage on a 10 cm thick, 1 m wide strip of unreinforced overlay concrete. Both the tensile strength of the overlay concrete and the bond stress \( \tau_c \) between the overlay and base concrete are assumed to be 2 MPa. Further \( n = 6 \) is adopted. According to Eq. (4.34) with \( R = 1 \) all cracks are formed at
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\[ \varepsilon_{ci} = \frac{-6.2 \text{ MPa}}{205 \text{ GPa}} = -0.06 \% \]

with a crack spacing (Eq. (4.66))

\[ s_c = 2k \cdot \frac{f_{c}}{\tau_c} \]

between once \((\lambda = 0.5)\) and twice \((\lambda = 1)\) the overlay thickness, corresponding to 100 to 200 mm. Free shrinkage strains equal to \(\varepsilon_s = -0.6 \%\) lead to a maximal crack width Eq. (4.63) of

\[ w = 200 \text{ mm} \left[ -\varepsilon_s - \frac{6.2 \text{ MPa}}{2 \cdot 205 \text{ GPa}} \right] = 200 \text{ mm} \left[ 0.6 \% - 0.03 \% \right] = 0.1 \text{ mm} \]

Note that the dominant factor in the above equations is the crack spacing. If the overlay thickness is doubled to 20 cm or the bond stress is halved to 1 MPa, crack widths up to 0.2 mm have to be expected.

Case (b):

Before cracking, the axial force in the member

\[ N_1 = \tau_c h x \] (4.67)

is built up from the free end \((x = 0)\) towards the centre of the member \((x = l/2)\) (Fig. 4.4 (b)). If the length of the member is larger than twice the load introduction length

\[ l_a = \frac{hE_c}{\tau_c} \left[ (1 - \rho) \varepsilon_{ci} + n \rho \varepsilon_{si} \right] \]

(4.68)

its central sections are fully restrained with \(N_1 = -E_c h [ (1 - \rho) \varepsilon_{ci} + n \rho \varepsilon_{si}] \). If \(l_a = l/2\) is reached before cracking starts, then the bond stresses \(\tau_c\) are activated over the whole member. In this case the degree of restraint

\[ R = \frac{E_c (\varepsilon_{ci} - \varepsilon_{si}) n \rho - x \tau_c / h}{(1 + n \rho - \rho) E_c \varepsilon_{ci}} \leq 1 \]

(4.69)

increases with \(x\) and the crack formation process occurs gradually from the middle outwards. The axial force \(N\) and the corresponding State II steel strains decrease with increasing distance from the member centre. Assuming \(\sigma_{s,II} = x \tau_c / \rho h\), Eq. (4.36) leads to

\[ w = 2\lambda l_0 \left[ \frac{\tau_c x}{h \rho E_s} - \frac{\lambda f_{ct}}{2 \rho E_s} \left( 1 + n \rho - \rho \right) + \varepsilon_{si} - \varepsilon_{ci} \right] \]

(4.70)

The influence of \(\tau_c\) on the transfer length is negligible. For ground slabs under self-weight the bond stresses are equal to \(\tau_c = \mu h \gamma\) and Eqs. (4.68) and (4.70) are independent of the slab thickness.

Case (c)

The degree of restraint in uncracked base restrained walls varies over the height and width of the wall. The sections at the bottom of the wall are fully restrained by the base and the cracks only provide local stress relief, so that at the base the cracking behaviour corresponds to Case (a). Not all of the base cracks extend up to the top of the element; the ones that do are denominated primary cracks. The regions around the top wall corners are hardly influenced by the base restraint and they do not crack unless the horizontal reinforcement ratio is very large (see end-restraint for \(\kappa = 0\)). The wall regions outside the edge and base regions are restrained by the neighbouring wall sections and according to Marti, Sigrist et al. (1997) their cracking behaviour can be modelled with end-restrained tension chords.
Crack Width

The restraint in the assumed tension chords is reduced by the formation of each crack, meaning that the tension chord length corresponds to the influence region of a crack. In long \((l \to \infty)\) walls (Fig. 4.7 (a)) the horizontal extent of the stress relieved region belonging to a crack (= tension chord length) is zero at the wall base and increases with increasing distance from the base. In shorter walls (Fig. 4.7 (b)) the tension chords are also limited by the low restraint regions. For long walls \((l \to \infty)\), Marti, Sigrist et al. (1997) suggest tension chord lengths equal to \(2z\), with \(z\) being the distance from the base. This assumption is in agreement with the observed primary crack spacing ranging between \(h\) and \(2h\) (ACI Committee 207 1990) in unreinforced walls.

4.2.3 Imposed Deformations

Deformation-induced cracking is caused by tension strains \(\varepsilon_c = \Delta l/l\), which are imposed for example through lateral support movements or shrinkage of stiffer neighbouring members. Once

\[
\varepsilon_{c,r,1} = \frac{f_{ct}}{E_c} \quad (4.71)
\]

is reached the crack formation phase starts. The cracking process caused by imposed deformations closely resembles the cracking process initiated by restrained deformations for \(k \to \infty\) and \(\varepsilon_{ci} = \varepsilon_{si}\). During the crack formation phase each crack causes a global stiffness relief defined by

\[
\Delta l_s = l \frac{\sigma_{s,1}}{E_s} + j l_b \left( \frac{\sigma_{s,II}}{E_s} - \frac{\sigma_{s,1}}{E_s} \right) = l \varepsilon_c 
\quad (4.72)
\]

Eq. (4.48) defines the reduced transfer length, with

\[
\sigma_{s,1} = \frac{np}{1 + np - \rho} \sigma_{s,II} \quad (4.73)
\]

leading to

\[
\sigma_{s,II} = \frac{\mu f_{ct}}{\rho} \left( 1 + np - \rho \right) \left( \frac{2E_c \varepsilon_c}{\mu f_{ct}} - 1 \right) \quad (4.74)
\]

and

\[
w = \frac{l_b f_{ct} \mu^2}{\rho E_s} \left( \frac{2E_c \varepsilon_c}{\mu f_{ct}} - 1 \right)^2 \quad (4.75)
\]
Eqs. (4.74) and (4.75) are valid for \( \lambda = 1 \) and correspond to Eqs. (4.54) and (4.55) for \( \varepsilon_{ci} = \varepsilon_{si} = -\varepsilon_{e} \). Accordingly, during crack formation the steel stresses and crack widths remain below

\[
\sigma_{s,ii} = \frac{f}{\rho}(1 + n\rho - \rho) \tag{4.76}
\]

and

\[
w' = \frac{\lambda(2-\lambda)l_0f_{ct}(1 + n\rho - \rho)}{\rho E_s} \tag{4.77}
\]

and start to increase with

\[
\sigma_{s,ii} = \frac{\lambda f_{ct}}{2 \rho}(1 - \rho) + E_s \varepsilon_e \tag{4.78}
\]

and

\[
w = 2\lambda \varepsilon_0 \left[ \varepsilon_e - \frac{\lambda f_{ct}}{2E_s} \right] \tag{4.79}
\]

for stabilized cracking once

\[
\varepsilon_{s,e,je} \geq \frac{f_{ct}}{2pE_s}(2\varepsilon_0 - \lambda(1 - \rho)) \tag{4.80}
\]

### 4.3 Deflections

![Basic load-deformation behaviour of a flexural member.](image)

Fig. 4.8 Basic load-deformation behaviour of a flexural member.

Together with the cracking load and the yield load, the deflections

\[
a = kl^2 \chi_a \tag{4.81}
\]

determined with a constant uncracked (State I) or cracked (State II) elastic stiffness provide a good basis for deflection estimates (Fig. 4.8). However, in many cases, the actual deflections have to be located somewhere between these two limiting cases, as most members have both uncracked and cracked regions. Further, depending on the load case, tension stiffening, long-term effects and variable section properties, as well as statically indeterminate moment redistributions due to non-uniform stiffness distributions (e.g. due to cracking or curtailed reinforcement) may need considering. With
Eqs. (4.7) and (4.14), the Tension Chord Model provides the section equations for considering tension stiffening.

For general section properties, the integral member deformations (e.g. deflection or rotation) of one-way bending members

\[ \delta = \int \chi M l d\xi \]  

result from integrating the section curvatures, for example with an equivalent statical system featuring a unit load/moment located at the section and in the direction of the required deformation. If the moment distribution is stiffness-dependent Eq. (4.82) can also be used to determine the moment redistribution coefficient

\[ \gamma = 1 - \frac{M_a}{M_{a,el}} \]  

by enforcing a kinematic boundary condition (e.g. zero rotation at a clamped end or zero deflections at a supported end). Table 4.1 summarizes the section moments and integration parameters for the basic structural systems illustrated in Fig. 4.9.

<table>
<thead>
<tr>
<th>Static system</th>
<th>( M(\xi) )</th>
<th>( M_{a,el} )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( \xi_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
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</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
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<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
<tr>
<td>[ \delta ]</td>
<td>( M_{a,el} )</td>
<td>( \frac{Ql}{4} )</td>
<td>( \frac{1}{12} )</td>
<td>( \frac{1}{8} )</td>
<td>( \frac{1}{8} )</td>
</tr>
</tbody>
</table>

Table 4.1  Moment distribution, elastic moment at critical section, midspan (free end) deflection integration factors \( k_1 \) for a constant member stiffness and \( k_2 \) for a constant curvature distribution (\( -\Delta \chi \) and \( \Delta \chi \) in negative and positive moment regions, respectively) and location where moment is zero. Note that \( \xi = x/l \) according to Fig. 4.9.
4.3.1 Interpolation Equations

Interpolation equations are a well-known tool for determining the moment-induced deflections

\[ a = \zeta_1 a_1 + (1 - \zeta_1) a_{II} \]  

(4.84)

of partially cracked flexural members with constant State I and State II stiffnesses. Effectively the State II stiffness is only strictly constant in symmetrically reinforced members, or, if the transition zones are neglected, in members with equally reinforced span \((M > 0)\) and support \((M < 0)\) sections (i.e. \(\rho_{span} = \rho_{sup}\) and \(\rho'_{span} = \rho'_{sup}\)). In contrast to most interpolation coefficients (see Chapter 2.2.2), the interpolation coefficients suggested in Table 4.2 are system-dependent (see Fig. 4.10). They were directly derived from Eq. (4.82) and are expressed as a function of

\[ \psi = \frac{M}{M_{a,el}} \]  

(4.85)

Eq. (4.81) with \(k_1\) (Table 4.1) and \(\chi_a\) equal to the absolute value of the critical section curvature (Fig. 4.8) provides the State I and State II deflections. If creep needs to be considered \(\chi_a\) can be determined with the effective stiffness according to Eq. (3.65).

![Fig. 4.9](image)

Moment and curvature distributions for the four structural systems regarded in this chapter. The regions where \(|M| > Mr\) are marked in gray. The tensile and compression reinforcement is illustrated with continuous and dashed lines, respectively. \(\xi = x/l = 0...0.5\) in Figs. (a) and \(\xi = x/l = 0...1\) in Figs. (b) to (d).

It is also possible to use interpolation equations for determining the tension stiffening deflection reduction

\[ \Delta a_{ts} = (1 - \zeta_2) \Delta a_{ts,II} \]  

(4.86)

and the shrinkage deflections
\[
\Delta a_{sh} = \zeta_2 a_{sh,\text{I}} + (1 - \zeta_2) a_{sh,\text{II}} \tag{4.87}
\]

for partially cracked members with constant State I and State II stiffnesses. Table 4.2 provides the interpolation coefficient \(\zeta_2\). The absolute curvatures \(|\Delta \chi|\) are constant and Eq. (4.81) with \(k_2\) (Table 4.1) yields \(\Delta a_{sh,\text{I}}\), \(a_{sh,\text{II}}\) and \(a_{sh,\text{sh}}\). Thereto, the Tension Chord Model provides \(\Delta \chi_{sh}\) (Eq. (4.7) or (4.14)), while Eqs. (3.67), (3.69) and (3.70) provide the shrinkage curvatures. Independent of the method chosen for determining the State II shrinkage curvature the superposition of Eqs. (4.84) and (4.87) represents an estimation, as the interaction of creep and shrinkage on the height of the compression zone is not considered.

By reducing the cracking moment
\[
M_r = \frac{(f_{ct} - \Delta \sigma_{sh}) J_{\text{ref}}}{h - x_{\text{ref}}} \tag{4.88}
\]

with \(\Delta \sigma_{sh}\) according to Eq. (3.68) shrinkage-induced cracking can be accounted for in factor \(\psi\).

<table>
<thead>
<tr>
<th>Static system</th>
<th>(\xi_1)</th>
<th>(\xi_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\underline{q})</td>
<td>(\psi)</td>
<td>(\psi)</td>
</tr>
<tr>
<td>(\underline{Q})</td>
<td>(\psi)</td>
<td>(\psi)</td>
</tr>
<tr>
<td>(\underline{Q} \quad \underline{q})</td>
<td>(\psi)</td>
<td>(\psi)</td>
</tr>
<tr>
<td>(\underline{q} \quad \underline{q})</td>
<td>(\psi)</td>
<td>(\psi)</td>
</tr>
</tbody>
</table>

\(\psi = M_r / M_{\text{ef}}\) ranges between 1 and 0 unless stated otherwise. *For the clamped-ended beam and the propped cantilever, cracked span sections are assumed.

Provided that a constant cracked stiffness can be assumed, the interpolation Eqs. (4.84) to (4.87) consider cracking, tension stiffening and long-term effects. This makes them particularly suitable for thin lightly reinforced flexural members with similar, non-curtailed support and span reinforcement, as in this case the contribution of the uncracked sections and tension stiffening are important factors.
(see e.g. Fig. 6.14). Even in members with constant tension and compression reinforcement ratios there are still statically indeterminate moment redistributions ($\gamma \neq 0$) due to cracking and shrinkage. These moment redistributions can either be estimated numerically with Eq. (4.82) or neglected ($\gamma = 0$), taking into account that compatibility is not fulfilled.

![Graph](image)

**Fig. 4.10** Contribution of the uncracked member regions $\zeta_1$ (Table 4.2) for partially cracked members with constant section properties. For the clamped-ended beam and the propped cantilever it is assumed that the support and span sections are cracked and $\gamma = 0$.

**Example 7**

This example illustrates how the interpolation coefficients given in Table 4.2 are used to determine the long-term deflections of a 4 m ($l/h = 8$) cantilever with a sustained uniform load (inclusive self-weight) of 30 kN/m corresponding to 75 % of the yield load. The cantilever sections correspond to the section illustrated in Fig. 3.4 (upside down) with the section properties determined in Example 2 and Example 3.

According to Table 4.2 the short-term elastic cantilever deflections at the free end are

$$a_1 = \frac{30 \text{ kN/m} \cdot 4^4 \text{ m}^4}{8 \cdot 79378 \text{ kN m}^2} = 12 \text{ mm}$$

and

$$a_2 = \frac{30 \text{ kN/m} \cdot 4^4 \text{ m}^4}{8 \cdot 33787 \text{ kN m}^2} = 28 \text{ mm}$$

With $f_y = 3 \text{ MPa}$, $M_a = 29.5 \text{ kNm}$ and $\lambda = 1$, Eq. (4.7) leads to

$$\Delta \Delta _{s} = \frac{1}{2} \left[ \frac{29.5 \text{ kN m} }{33787 \text{ kN m}^2} - \frac{6 \cdot 3 \text{ MPa}}{205 \text{ GPa} (0.85 - 0.5 m - 0.163 m)} \right] = 0.27 \text{ mrad/m}$$

and the tension stiffening deflections

$$\Delta \Delta _{s.a} = \frac{4^2 \text{ m}^2}{2} \cdot 0.27 \text{ mrad/m} = 2.2 \text{ mm}$$

Interpolating with

$$\psi = \frac{29.5 \text{ kNm}}{0.5 \cdot 30 \text{ kN/m} \cdot 4^4 \text{ m}^4} = 0.123$$

and the coefficients (Table 4.2)

$$\zeta_1 = 0.123^2 = 0.015$$

$$\zeta_2 = 0.123$$

leads to

$$a = 0.015 \cdot 12 \text{ mm} + (1 - 0.015) \cdot 28 \text{ mm} - (1 - 0.123) \cdot 2.2 \text{ mm} = 25 \text{ mm}$$
As shrinkage strains equal to -0.6 ‰ cause tensile stresses $\Delta \sigma_{sh} = 3.3 \text{ MPa}$ (Eq. (3.68)) in the extreme concrete fibres, the whole cantilever is assumed to be cracked for the long-term deflection predictions. The cracked elastic deflections

$$\Delta a_{e} = \frac{30 \text{ kN/m}}{819496 \text{ kN/m}^2} \times 49 \text{ mm}$$

and with Eq. (4.14) and $k_s = 1/4$

$$\Delta a_{ts} = \frac{4^2}{2} \frac{m^2}{m} \times 0.14 \frac{\text{ mrad}}{m} = 0.9 \text{ mm}$$

the tension stiffening deflection reduction

$$\Delta a_{ts} = \frac{4^2}{2} \frac{m^2}{m} \times 0.14 \frac{\text{ mrad}}{m} = 0.9 \text{ mm}$$

and finally with $\Delta a_{sh} = 1.77 \text{ mrad/m}$ (Eq. (3.69) the State II shrinkage deflections

$$\Delta a_{sh} = \frac{4^2}{2} \frac{m^2}{m} \times 1.77 \frac{\text{ mrad}}{m} = 14 \text{ mm}$$

lead to deflections

$$a = 49 \text{ mm} - 0.9 \text{ mm} + 14 \text{ mm} = 62 \text{ mm}$$

EC 2 and MC 2010 (European Committee for Standardization 2004; Fédération Internationale du Béton 2010b) both recommend a limit of span/250 for the sag of a beam under the quasi-permanent combination of loads (for the cantilever $2 \times 4 \text{ m} / 250 = 32 \text{ mm}$). The sag is assessed relative to the supports and span refers to the distance between the supports. A limit of span/500 (= 16 mm) is suggested for the deflections, which occur after any deformation-sensitive adjacent members (e.g. claddings) have been fitted. Further, precamber can be provided to compensate maximal span/250 = 32 mm. For the example cantilever this means that with a camber of 30 mm the first limit can be fulfilled. However, the second limit cannot be fulfilled, unless half of the long-term deflections are awaited before any deformation-sensitive adjacent members are fitted.

### 4.3.2 Influence of Reinforcement Detailing

In most practical cases the flexural reinforcement is curtailed and the cracked stiffness is not constant. Fig. 4.11 illustrates the influence of the reinforcement detailing on the moment redistribution coefficient $\gamma$ (Eq. (4.83)) and related deflections

$$\alpha = \frac{a_{ii}}{a_{ii,avg}}$$

for a fully cracked clamped-ended beam and a propped cantilever as a function of the stiffness ratio. Positive $\gamma$ values indicate larger span moments, while negative $\gamma$-values indicate larger support moments relative to the elastic moment distribution. The stiffness ratio

$$\kappa = \frac{E_c I_{II,\text{sup}}}{E_c I_{II,\text{span}}}$$

is defined by the stiffnesses in the critical support and span sections. For both structural systems the figure illustrates two limiting situations: A perfectly curtailed reinforcement with a cracked stiffness affine to the moment distribution (crosses) and a non-curtailed reinforcement distribution with $E_c I_{II,\text{sup}}$ and $E_c I_{II,\text{span}}$ in the positive and negative moment sections (continuous lines). The average stiffness deflections $a_{ii,avg}$ were determined with Eq. (4.81) and the average cracked stiffness 0.5 ($E_c I_{II,\text{sup}} + E_c I_{II,\text{span}}$), while $a_{ii}$ and $\gamma$ were determined with Eq. (4.82). The affine stiffness distribution leads to support moments that are 12.5% and 17.2% larger than according to the elastic moment distribution. Tension stiffening and shrinkage are neglected, while creep is implicitly considered within $\kappa$. As can be seen from Fig. 4.11, the stiffness ratio $\kappa$ strongly influences the moment redistribution coefficient.
\( \gamma \) (Fig. 4.11 (a)), while the extent of the reinforcement curtailing has a considerable influence on the size of the State II deflections (Fig. 4.11 (b)).

\[
\Delta a = (1 - \zeta) a_{II}
\]  

(4.91)

The State II deflections \( a_{II} \) result from Eq. (4.81) with \( k_2 \). In this case the tension stiffening curvature reduction is variable and the corresponding deflection reduction has to be determined by integrating the individual curvatures. However, in beams with large reinforcement ratios, where curtailing is most likely necessary, the load-levels are usually high and thus tension stiffening is less important (see e.g. Fig. 6.14). Note that in statically indeterminate systems \( \zeta_2 \) is dependent on \( \gamma \). For initially restraint free \( (e_{ci} = e_{si} = 0) \) cracked \( (f_{cr} = 0) \) members with affine reinforcement distributions \( \gamma \) is equal to -0.125 if both ends are clamped \( (\zeta_0 = 0.5) \) or -0.172 if one end is clamped and one end supported \( (\zeta_0 = 1 - 20.5/2) \). Other values for \( \gamma \) have to be estimated or determined with Eq. (4.82).
4.3.3 Span-to-Depth Ratios

Fig. 4.12 Span-to-depth ratios (Eq. (4.94)) for maintaining $a/l \leq 1/240$ (1/320) at midspan (or cantilever end) up until reaching 80% (60%) of the characteristic yield moment in the critical section: (a) cantilever; (b) simple beam; (c) propped cantilever; (d) clamped-ended beam. Note: $E_i = 205$ GPa, $f_{sy} = 500$ MPa, $f_{ct} = 2$ MPa, $d/h = 0.85$, $d'/h = 0.15$, $\rho = 0.25$, and $m_r = f_{ct}/6$.

Combining Eqs. (4.81) and (4.84) and neglecting the contribution of the curvatures in the uncracked member regions as well as shrinkage, leads to the midspan (or cantilever end) deflection equation

$$a = \frac{k(1-\zeta)l^2M_{a,el}}{(EI)_{ll,a}}$$

(4.92)

where the subscript $a$ denotes the section with the largest moment $M_{a,el}$. This equation can be used with $k = k_1$ or $k_2$ and $\zeta = \zeta_1$ or $\zeta_2$ (Table 4.1 and Table 4.2) in order to capture the two limiting detailing options of a constant cracked stiffness (constant $EI$) and an affine cracked stiffness (constant $\chi$). With $M_{a,el} = \phi M_i$, $M_i = m_y bd^2$, $M_r = m_r bh^2$ and $I_{ll,a} = bd^3 g_{ll,a}$ (Table 3.1), Eq. (4.92) is rearranged to form the limiting span to depth ratios

$$(l/d)_{lim} = \frac{(a/l)_{lim} g_{ll} E_i}{\phi m_y nk(1-\zeta)}$$

(4.93)
where the interpolation coefficient $\zeta$ is a function of the moment ratio (Eq. (4.85))

$$\psi = \frac{m_y}{m_y \phi(d/h)} \quad (4.94)$$

If creep needs to be considered, $g_{II}$ and $n$ in Eq. (4.93) are determined with the effective modulus (Eq. (3.65)).

Fig. 4.12 illustrates the limiting span-to-depth ratios for rectangular sections for $(a/l)_{lim} = 1/240$ at $\phi = 0.8$. This limit is equivalent to $(a/l)_{lim} = 1/320$ at $\phi = 0.6$ or $(a/l)_{lim} = 1/192$ at $\phi = 1$. ACI 318 (ACI Committee 318 2008) recommends $(a/l)_{lim} = 1/240$ for the deflections occurring after the attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) for floor constructions supporting or attached to non-structural elements not likely to be damaged by large deflections.

### 4.4 Summary

**Crack widths** in members with uniaxial stress states are defined by Eq. (4.24). For maximal crack widths $\lambda$ is set equal to 1, while $k_\psi$ can be chosen between 1 (full tension stiffening according to the Tension Chord Model) and zero (no tension stiffening). The State II steel strains are determined as described in Chapter 3. For specific cases simplified equations can be provided:

- For bending members with axial force, Eq. (3.74) with an effective reinforcement ratio according to Eq. (4.9) defines the transfer length. The required section properties and deformations at cracking have to be determined with the section equilibrium equations according to Chapter 3.
- For bending members without axial force Eq. (4.27) provides the crack width, while Eq. (4.28) provides the transfer length.
- For axially loaded tensile members Eq. (4.36) provides the crack width, while Eq. (3.74) provides the transfer length.
- For elastically end-restrained tensile members Eqs. (4.45) and (4.46) (with Eq. (3.74)) provide an upper limit for the steel stresses and crack widths during the crack formation phase. The crack formation phase is delimited by Eqs. (4.44) and (4.57).
- For continuous edge-restrained concrete overlays Eq. (4.66) provides the transfer length, while Eq. (4.63) provides the crack width.
- For continuous edge-restrained walls the lengths of the fictitious tension chords are estimated according to Fig. 4.7. The fictitious tension chords can then be approximately treated as end-restrained tensile members.
- For tensile ties under imposed deformations Eqs. (4.76) and (4.77) (with Eq. (3.74)) provide an upper limit for the steel stresses and crack widths during the crack formation phase. The crack formation phase is delimited by Eqs. (4.71) and (4.80).

**Beam deformations** are defined by Eq. (4.82), which can also be used to determine the moment distribution in statically indeterminate members. The State I and II section curvatures are determined according to Chapter 3. Tension stiffening is accounted for by including the additional force provided by the average concrete tensile stresses (see Eq. (4.1)) and the effective concrete area (Eq. (4.8)). For specific cases simplified equations can be provided:
• For beams \((N = 0)\) with uniform stiffnesses (State I or State II), Eq. (4.81) with Table 4.1 provides the midspan (or free end) load-induced deflections, as well as additional shrinkage deflections (with Eq. (3.67)) and deflection reductions due to tension stiffening (with Eq. (4.7)).

• For partially cracked beams \((N = 0)\) with uniform State I and II stiffnesses, the State I and II deflections can be interpolated according to Eqs. (4.84), (4.86) and (4.87). Thereto Table 4.2 provides the system dependent interpolation coefficients. For statically indeterminate systems the moment redistributions due to cracking have to be neglected or determined with Eq. (4.82).

• For extensively cracked beams (with \(N = 0\)) with strongly curtailed tensile reinforcements, the load-induced deflections can be estimated with Eq. (4.91). For statically indeterminate systems the moment redistributions due to cracking have to be determined with Eq. (4.82).
5 Verification

So far, in terms of serviceability behaviour, mainly short-term test data validations at section and member level have been carried out for the Tension Chord Model. Sigrist (1995) found a good agreement between the deflection measurements from his test specimens (Sigrist and Marti 1993) and the corresponding calculated deflection curves. With Bragg grating sensor measurements Kenel and Marti (2002) showed that a linear steel strain distribution between the cracks is a reasonable assumption. Kenel also carried out test data comparisons for load deformation curves with a selection of short-term test series (Polak and Killen 1998; Pfyl and Marti 2001; Kenel and Marti 2002) with lightly reinforced specimens.

This chapter is focused on long-term serviceability behaviour and aims at validating the prediction approaches discussed in Chapter 4. Thereto a selection of test series from various research institutions is used. Appendix A contains an overview of the test data. An example calculation is provided for most test series. Some of the test data comparisons also include predictions carried out with the code equations discussed in Chapter 2.

5.1 Tensile Members

5.1.1 Loss of Tension Stiffening

![Fig. 5.1 Long-term tensile tests: (a) Günther and Mehlihorn (1989); (b) Scott & Gill (1990); (c) Beeby and Scott (2006). The dimensions are in [mm].](image)

As long-term effects influence the deformation behaviour of both the compression and tension chord, it is not convenient to experimentally determine the long-term loss of tension stiffening from bending tests. Therefore, the long-term tensile tests illustrated in Fig. 5.1 (see also Table A. 1 in Appendix A) are used to experimentally determine the quotient of the long-term and immediate bond stress $k_0$ (Eq. 65).
(4.11). \( k_\phi = \Delta \varepsilon_{\text{II}} / \Delta \varepsilon_{\text{II}} = (\varepsilon_{\text{II}} - \varepsilon_{\text{II,avg}}) / (\varepsilon_{\text{II}} - \varepsilon_{\text{II,avg}}) \) is determined from the calculated State II strains and the average strain measurements after immediate and sustained loading. The bond stress quotient is plotted versus time (Fig. 5.2 (a)) and versus the MC 90 creep coefficient (Eq. (3.36)) (Fig. 5.2 (b)), assuming RH = 0.4. Fig. 5.2 (b) also illustrates the bond stress quotient according to Eq. (4.11) as well as according to Model Code and Eurocode 2 (\( k_\phi = 0.4/0.6 = 0.67 \), see Eq. (2.6)).

![Fig. 5.2](image)

**Fig. 5.2** Loss of tension stiffening in the tensile tests illustrated in Fig. 5.1: (a) vs. time; (b) vs. \( \varphi \), where \( \varphi \) is estimated according to Eq. (3.36) assuming RH = 0.4 and \( \alpha_{\text{II,avg}} = 1 \).

The short specimen lengths and the large reinforcement ratios do not facilitate the delicate task of estimating the tension stiffening steel strain reductions from the measured average steel strains and the calculated State II steel strains. Further, the calculated State II steel strains are slightly underestimated as the shrinkage-induced compression steel strains, which are released at cracking (see specimen RL1 in Chapter 5.1.2) are neglected. This means that if the restrained shrinkage strains released at cracking are larger than the steel strain reduction, then the tension stiffening values obtained will appear negative (see Heinzmann 2006). This is the case for Specimens D6 and D7 (not plotted in Fig. 5.2).

### 5.1.2 Restrained Deformations

**End-Restrained Shrinkage Specimens Tested at the University of New South Wales**

Nejadi and Gilbert (2003) tested eight (S1/a/b to S4/a/b), very lightly reinforced end-restrained specimens with 2 m long test lengths and variable reinforcement ratios (Fig. 5.3 and Table A. 2 in Appendix A). A section reduction at mid-length defined the location of the first crack. Fig. 5.4 compares the predicted and experimental State II steel stresses and crack widths. The experimental State II steel stresses are determined from the average of nine strain gauge measurements taken within 50 mm of the first crack. The experimental crack widths represent the maximum crack opening observed at \( t = 122 \) d or \( t = 150 \) d (S2-b). The free shrinkage strains (see Fig. 5.5) were measured on companion specimens (600/600/100 mm) with the same section dimensions as the test specimens.

All predictions are carried out according to Chapter 4.2.2 (see example below for specimen S1) with the 28-day concrete properties of batch I unless mentioned otherwise. Any strains caused by the loss of hydration heat are neglected (i.e. \( \varepsilon_{\text{ci}} = \varepsilon_{\text{sh}} \) and \( \varepsilon_{\text{sh}} = 0 \)), the axial restraint stiffness \( k \) is as-
sumed to be $\infty$ and creep is neglected, leading to $R = 1$. The numbered vertical lines in Fig. 5.4 indicate that a new crack has formed.

![Diagram](image)

**Fig. 5.3** Overview of Nejadi and Gilbert's (2003) shrinkage tests: (a) plan view; (b) elevation; (c) sections of specimens S1 to S4. The specimens were made in pairs, e.g. S1-a and S1-b. The dimensions are in [mm].

**Predictions test specimen S1**

With $R = 1$ and the 7-day concrete properties (N.B. also with the 28-day properties), the crack formation phase starts at (Eq. (4.44))

$$
\varepsilon_{sl} = -\frac{1.55 \text{ MPa} \cdot 12}{205 \text{ GPa}} = -0.09 \%.
$$

The transfer length (Eq. (3.74))

$$
l_0 = \frac{12 \text{ mm} \cdot (1 - 0.565 \%)}{8 \cdot 0.565 \%} = 264 \text{ mm}
$$

leads to initial crack influence regions equal to 528 mm ($\lambda = 1$) and a final crack spacing between 264 and 528 mm ($\lambda = 0.5$ to 1). $\lambda = 1$ is used for the following calculations. The end of the crack formation phase at (Eq. (4.61)) with $a_i = 1.045$

$$
\varepsilon_{sl} = -\frac{1.97 \text{ MPa} \cdot 1.045}{2 \cdot 0.565 \% \cdot 205 \text{ GPa}} (2 \cdot 1.045 - 1 + 0.565 \%) = 0.93 \%.
$$

with at total of (Eq. (4.58))

$$
j = \frac{2 \text{ m}}{528 \text{ mm}} + 1 = 5
$$

cracks is not reached. The additional crack is added to account for the first edge crack at the beginning of the instrumented length.

During the crack formation phase the ascending steel stress curves (Eqs. (4.54) and (4.45) with $a_i = 1.045$) are defined by

$$
\sigma_{sl,II} = \frac{1.045 \cdot 1.97 \text{ MPa} \cdot \mu}{0.565 \%} \left( \frac{2 \cdot 205 \text{ GPa} \cdot \varepsilon_{sl}}{9 \cdot 1.97 \text{ MPa} - 1} \right) + 205 \text{ GPa} \cdot \varepsilon_{sl} \leq \frac{1.045 \cdot 1.97 \text{ MPa}}{0.565 \%} + 205 \text{ GPa} \cdot \varepsilon_{sl}
$$

where (Eq. (4.51))

$$
\mu = \frac{9 \cdot 0.565 \% \cdot 5}{(1 - 0.565 \%)}
$$

The crack number is defined by $j$ (1 to $j_{cr}$). The corresponding crack widths (Eqs. (4.55), (4.46))

$$
w = \frac{264 \text{ mm} \cdot 1.97 \text{ MPa} \cdot 1.045 \cdot \mu}{0.565 \% \cdot 205 \text{ GPa}} \left( \frac{2 \cdot 205 \text{ GPa} \cdot \varepsilon_{sl}}{9 \cdot 1.97 \text{ MPa} - 1} \right) \leq \frac{264 \text{ mm} \cdot 1.97 \text{ MPa} \cdot 1.045}{0.565 \% \cdot 205 \text{ GPa}} = 0.47 \text{ mm}
$$

are expected to remain below 0.47 mm. In Fig. 5.4 the upper limits on the right hand side of the above equations for $\sigma_{sl,II}$ and $w$ are illustrated with dashed lines.
Fig. 5.4  Nejadi and Gilbert's (2003) end-restrained shrinkage specimens. The predicted curves use the 28-day concrete properties of batch I and $\lambda = 1$. 
According to Fig. 5.5 the predicted start of the crack formation phase $\varepsilon_{sh} = -0.09 \%$ is reached at 8 days. By this time all specimens except S1-b have cracked (see experimental steel stresses in Fig. 5.4). Even though early-age creep is neglected, the predictions do not underestimate the cracking age.

The end of the crack formation phase is not reached by any of the specimens. At 122 days (-0.46 \% free shrinkage strains) Specimen S3-a has one large crack, while Specimen S3-b has two cracks (one crack is predicted). The short specimen length (2 m), which is responsible for the large stiffness loss at first cracking, is the only reason the experimental steel stresses in S3-a/b remain below the yield limit. In a longer member, such a low reinforcement ratio would lead to steel yielding (see Fig. 5.6). Specimens S2-a/b and S4-a/b end up with three cracks (2 are predicted for S2 and three are predicted for S4), while four cracks can be observed on the S1 specimens. All of the observed cracks penetrate through the full depth of the specimen cross-sections.

Fig. 5.5 Shrinkage strains measured on Nejadi and Gilbert's (2003) 600/600/100 mm shrinkage specimens for Batch I and Batch II concrete. The predicted start of the crack formation phase -0.09 \% is reached at 8 days (dashed line).

Fig. 5.6 Influence of the member length on the steel stress development of an end-restrained tension chord: (a) specimen S3 with its original length of 2 m; (b) Specimen S3 with an assumed alternative length of 10 m.
**Continuous edge-restrained walls**

Al Rawi and Kheder tested 14 (Al Rawi and Kheder 1990) and 8 (Kheder 1997) small scale base-restrained mortar walls and observed 37 (Kheder 1997) full size walls under shrinkage. Unfortunately, the material properties given in the papers are insufficient for recalculation; however the qualitative results are summarized here.

Kheder's (1997) mortar walls have $l/h$ ratios of 2 (4 walls) and 3 (4 walls) and horizontal reinforcement ratios of 0.2 % and 0.8 %. All eight walls have crack free top corner regions as illustrated in Fig. 4.7 (b). This observation illustrates the influence region of an unrestrained edge or crack and agrees with the model assumption of a crack influence region (fictitious tension chord length), which increases from zero at the base to $2h$ at the wall top (see continuous edge-restraint Case (c) Chapter 4.2.2).

Further, the walls feature closely spaced base cracks that are slightly further apart and wider in the walls with lower horizontal reinforcement ratios at the base. The largest crack widths observed on Kheder's mortar walls occur in the bottom quarter of the walls. This coincides with Al Rawi and Kheder's (1990) findings. They observed maximal crack widths somewhere at mid-height of short walls as opposed to at the top of long walls. These findings support the model assumption of end-restrained tension chords with crack widths that fluctuate between a maximum and minimum value (see Fig. 4.6 (b)). While the former is primarily defined by the horizontal reinforcement ratio, the latter increases with increasing tension chord length (see Fig. 5.6). As the longest fictitious tension chords are located at the top of long walls and somewhere in the middle of short walls (see Fig. 4.7), the chances of finding the largest crack widths at these locations are high. All tests support the notion that it is practically impossible to prevent base cracks, which even for joints spaced at $2h$ were observed (Kheder 1997) until part way up the wall.

### 5.1.3 Imposed Deformations

**Deformation Controlled Tensile Tests carried out at EPF Lausanne**

![Diagram of deformation controlled tensile tests](image)

**Fig. 5.7** Overview of four deformation controlled tensile tests carried out by Jaccoud et al. (1984): (a) plan view; (b) sections of specimens B-11 to B-14. The imposed deformations were controlled with inductive displacement transmitters over a length of 1 m. The dimensions are in [mm].
Fig. 5.8  Jaccoud et al.'s (1984) deformation controlled tensile members. The vertical dashed lines illustrate the predicted ($\lambda = 1$) transition between the crack formation phase and stabilized cracking phase.
Jaccoud et al. (1984) performed 12 deformation-controlled tests on lightly reinforced concrete tensile elements (see Table A. 3 in Appendix A). Four of them (all from the B1-Series) are illustrated in Fig. 5.7 and used for the test data comparisons in Fig. 5.8. The imposed deformations were controlled with inductive displacement transmitters over a length of 1 m. The corresponding average strains are illustrated together with the experimental State II steel stresses and the opening of the largest crack in Fig. 5.8. The figure also illustrates the corresponding predictions based on Chapter 4.2.3 (see example B14). The predicted curves are based on a tension chord length equal to 1 m, corresponding to the length of the inductive displacement transmitters used to control the deformations. While the length of the tension chord has no influence on the strains necessary to start and end the crack formation phase, it does define the final amount of cracks and therefore the strain difference between two consecutive cracks. The tension chord length also has no influence on the maximal steel stresses and crack widths during or after the crack formation phase.

The positive influence of the reinforcement ratio on the State II steel stresses and crack widths during the crack formation phase is clearly visible in Fig. 5.8. However, the specimens with higher reinforcement ratios have a shorter crack formation phase, so that in the case of large imposed deformations, the positive influence of the additional reinforcement is at least partially lost once the crack formation phase is concluded (i.e. compare Fig. 5.8 (f) and (h)).

Predictions test specimen B14

The crack formation phase starts at

\[ \varepsilon_c = \frac{2.43 \text{ MPa} \cdot 6.4 \cdot 0.08 \%}{205 \text{ GPa}} = 0.08 \% \]

and ends at (Eq. (4.80) with Eq. (4.39))

\[ \varepsilon_e = \frac{2.43 \text{ MPa}}{2 \cdot 0.93 \% \cdot 205 \text{ GPa}} \left(1 + 2 \cdot 6.4 \cdot 0.93 \% - 0.93 \% \right) = 0.71 \% \]

With a transfer length (Eq. (3.74))

\[ l_0 = \frac{10 \text{ mm} \cdot (1 - 0.93 \%)}{8 \cdot 0.93 \%} = 133 \text{ mm} \]

and a maximal crack spacing of 266 mm (\(\lambda = 1\)), a total of (Eq. (4.58))

\[ j_c = \frac{1 \text{ m}}{266 \text{ mm}} = 4 \]

cracks is necessary to conclude the crack formation phase. With \(j\) equal to the crack number, factor

\[ \mu = \frac{6.4 \cdot 0.93 \% \cdot j_c}{j(1 - 0.93 \%)} \]

During the crack formation phase the steel stresses (Eqs. (4.74) and (4.76) with \(\alpha_i = 1.050\))

\[ \sigma_{e,II} = \frac{1.050 \cdot \mu \cdot 2.43 \text{ MPa}}{0.93 \%} \left[ \frac{1 - 2 \cdot 205 \text{ GPa} \cdot \varepsilon_c}{\mu \cdot 6.4 \cdot 2.43 \text{ MPa} - 1} - 1 \right] \approx \frac{2.43 \text{ MPa} \cdot 1.050}{0.93 \%} = 274 \text{ MPa} \]

remain below 274 MPa, while the crack widths (Eqs. (4.75) and (4.77))

\[ w = \frac{133 \text{ mm} \cdot 2.43 \text{ MPa} \cdot 1.050 \cdot \mu \cdot 0.93 \% \cdot 205 \text{ GPa}}{\mu \cdot 6.4 \cdot 2.43 \text{ MPa}} \left[ 1 + \frac{2 \cdot 205 \text{ GPa} \cdot \varepsilon_c}{\mu \cdot 6.4 \cdot 2.43 \text{ MPa} - 1} \right] \leq \frac{133 \text{ mm} \cdot 2.43 \text{ MPa} \cdot 1.050}{0.93 \% \cdot 205 \text{ GPa}} \approx 0.18 \text{ mm} \]

remain below 0.18 mm. In Fig. 5.8 the two upper limits on the right hand side of the equations for \(\sigma_{e,II}\) and \(w\) are illustrated with dashed lines. Once the crack formation phase is concluded the steel strains (Eq. (4.78))

\[ \sigma_{e,II} = \frac{2.43 \text{ MPa}(1 - 0.93 \%)}{2 \cdot 0.93 \%} + \varepsilon_c \cdot 205 \text{ GPa} = 129 \text{ MPa} + \varepsilon_c \cdot 205 \text{ GPa} \]

and crack widths (Eq. (4.79))

\[ w = 266 \text{ mm} \left( \varepsilon_c \cdot \frac{6.4 \cdot 2.43 \text{ MPa}}{2 \cdot 205 \text{ GPa}} \right) \]

start to increase.
5.2 Beams

5.2.1 Statically Determinate Beams

Espion et al. (Espion 1988; Espion and Halleux 1990) provided a detailed summary of long-term one-way bending tests carried out between 1907 and 1988. A selection of recent test series with complete sets of measured concrete properties is used for the test data comparisons carried out in this chapter. Further, the predictions of two large beam specimens, belonging to the French benchmark program ConCrack (NECS 2010) are included.

Unless mentioned otherwise, the Tension Chord Model (TCM) deflection predictions are determined with the interpolation equations given in Chapter 4.3.1, using the interpolation coefficient corresponding to the predominant load. The TCM crack widths are determined with the equations given in Chapter 4.2.1 and unless mentioned otherwise represent the crack openings at the height of the centroid of the tensile reinforcement. The deflections are assumed to be zero at the beginning of loading. Most of the predictions assume \( \lambda = 1 \) (maximum crack spacing). Factor \( k_{\Phi} = 0 \ldots 1 \) is used according to Chapter 4.1.2 to indicate the contribution of the tensile concrete between the cracks.

For comparison, code crack width (see Chapter 2.1) and deflection (see Chapter 2.2) predictions are included in some of the figures. The deformation predictions referred to as ACI use Brandon's effective moment of inertia according to Eq. (2.16) with \( m = 3 \) or 4. The deformation predictions referred to as EC use Rao's interpolation coefficient according to Eq. (2.17) or (2.24). The Frosch crack widths refer to Eq. (2.12) with either the bottom (Eq. (2.13)) or side (Eq. (2.14)) transfer length, where there is no distinction between the short/long-term predictions. The crack widths denoted MC refer to Eq. (2.6) with Eq. (2.8) with parameters \( \alpha_1 = 0, \alpha_2 = 0.5 \) as well as \( \alpha_3 = 0.6/0.4 \) and \( \alpha_4 = 0/1 \) for short/long-term predictions.

Short-Term 4-Point-Bending Tests carried out at ETH Zurich

Kenel and Marti (2002) tested five thin, lightly reinforced slab strips (B1 to B5) with two different reinforcement ratios and concrete types under short-term four-point-bending. Fig. 5.10 illustrates the section and midspan member deformations of specimens B1, B3 and B4 (the prestressed specimen B2 is not considered). The experimental section curvatures (marked by crosses) are taken from Fig. 4.16 in Kenel (2002), while the measured midspan deflections (labelled test) are obtained from Figs. 5.2 (b), 5.14 (b) and 5.21 (b) in Kenel and Marti (2002). The measured crack widths (marked by crosses) in Fig. 5.11 are maximal crack widths observed on the specimen side face in the constant
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moment zone (see Figs. 5.4, 5.19, 5.25 and 5.32 in Kenel and Marti (2002)), while the corresponding crack steel stresses are determined from the average support moment maintained during the load step (average vertical load \( Q \)) considering the slab self-weight and the test equipment \( (M_a = 1.2 \text{ m } (Q + 1.1 \text{ kN}) + 0.5 \cdot 1.42 \text{ m}^2 4.72 \text{ kN/m}) \).

\[
M_a = 1.2 \text{ m } (Q + 1.1 \text{ kN}) + 0.5 \cdot 1.42 \text{ m}^2 4.72 \text{ kN/m})
\]

**Fig. 5.10** Deformations of Kenel and Marti's (2002) 4-point bending tests. The crosses and lines labelled test mark the measured curvatures and deflections, respectively. Illustrated are predictions according to the Tension Chord Model (TCM with \( \lambda = 1 \)), ACI 318 and EC 2. The dimensions are in [mm].
Deflection and crack width predictions test specimen B1

For a rectangular section without compression reinforcement (Table 3.1)

\[ f_t = \frac{0.5 + (6.1-1)0.84 \cdot 0.38 \%}{1 + 0.84 \cdot 0.38 \% \cdot (6.1-1)} = 0.505 \]

and

\[ g_t = \frac{1}{12} \left( \frac{1}{2} - 0.505 \right)^2 + 0.84 \cdot (6.1-1) \left[ 0.38 \% \left( 0.84 - 0.505 \right)^2 \right] = 0.0852 \]

as well as

\[ f_{nt} = \sqrt{(6.1-0.38 \%)^2 + 2 \cdot 6.1 \cdot 0.38 \% \cdot 6.1 - 0.38 \%} = 0.193 \]

and

\[ g_{nt} = \frac{(0.193)^3}{3} + 6.1 \cdot 0.38 \% \cdot (1 - 0.193)^3 = 0.0175 \]

lead to compression zone heights

\[ x_1 = 0.2 \text{ m} \cdot 0.505 = 101 \text{ mm} \]
\[ x_2 = 0.84 \cdot 0.2 \text{ m} \cdot 0.193 = 32 \text{ mm} \]

the stiffnesses

\[ E_I = 34 \text{ GPa} \cdot 1 \text{ m} \cdot 0.2^3 \text{ m}^2 \cdot 0.0852 = 23237 \text{ kN m}^2 \]
\[ E_I = 34 \text{ GPa} \cdot 1 \text{ m} \cdot (0.84 - 0.2)^3 \text{ m}^2 \cdot 0.0175 = 2828 \text{ kN m}^2 \]

and the cracking moment

\[ M_t = \frac{3.2 \text{ MPa} \cdot (1 \text{ m} \cdot 0.2^3 \text{ m}^2 \cdot 0.085)}{0.2 \text{ m} - 0.101 \text{ m}} = 22.0 \text{ kN m} \]

which together with (Eq.4.14)

\[ \Delta x_m = \frac{k_x}{2} \left[ \frac{22.0 \text{ kN m}}{2828 \text{ kN m}^2} - \frac{3.2 \text{ MPa}}{34 \text{ GPa} (0.84 - 0.2 \text{ m} - 0.032 \text{ m})} \right] = k_x \cdot 3.55 \text{ mrad} \]

define the section deformation behaviour illustrated in Fig. 5.10 (a, c & e).

The plotted midspan deflections in Fig. 5.10 (b, d & f) represent the distance measured between the slab at midspan and under the point loads (equivalent to the midspan deflections in the classic 4-point-bending setup). The self-weight of the jacks is 1.1 kN. The slab self-weight (4.72 kN/m) is neglected for the deflection calculations but considered for \(\psi\). With (see Table 4.2) \(\xi_1 = 1.2 \text{ m} / 3.6 \text{ m} = 1/3\) the State I deflections

\[ a_i = \frac{23}{9 \cdot 24} \frac{1.2 \text{ m} \cdot (Q + 1.1 \text{ kN}) \cdot 3.6^2 \text{ m}^2}{23237 \text{ kN m}^2} = 0.07 \text{ mm kN} \cdot Q + 0.1 \text{ mm} \]

and the State II deflections

\[ a_i = \frac{23}{9 \cdot 24} \frac{1.2 \text{ m} \cdot (Q + 1.1 \text{ kN}) \cdot 3.6^2 \text{ m}^2}{2828 \text{ kN m}^2} = 0.59 \text{ mm kN} \cdot Q + 0.7 \text{ mm} \]

can be determined. The tension stiffening deflections \((\lambda = 1)\) are

\[ \Delta a_{x,x} = \frac{3.6^2 \text{ m}^2 \cdot k_x \cdot 3.55 \text{ mrad}}{8} = k_x \cdot 5.8 \text{ mm} \]

The deflections of the partially cracked beam

\[ a = \zeta_i \cdot a_i + (1 - \zeta_i) \cdot a_i - (1 - \zeta_i) \cdot \Delta a_{x,x} \]

are interpolated with (Eq.4.85)

\[ \psi = \frac{22.0 \text{ kN m}}{1.2 \text{ m} \cdot (Q + 1.1 \text{ kN}) + 0.5 \cdot 4.72 \text{ kN/m} \cdot 1.4^2 \text{ m}^2} \]

and (Table 4.2)

\[ \zeta_1 = \frac{8}{23} \psi \] and \[ \zeta_2 = \frac{4}{9} \psi \]

The maximal crack widths in the constant moment zone are determined with (Eq. 4.28))
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\[ \frac{l_p}{\varepsilon} = \frac{6.1}{8} \left( \frac{0.0852 \cdot (1-0.193)}{0.0175 \cdot 0.84^2 \cdot (1-0.505)} - 1 \right) = 7.8 \]

and (Eq.(4.26))

\[ \varepsilon_{\text{u},\text{II}} = \frac{6.1 \cdot 3.2 \, \text{MPa} \cdot 208 \, \text{Ga}}{0.0852 \cdot (1-0.193) \cdot 0.0175 \cdot 0.84^2 \cdot (1-0.505)} = 1.056 \% \]

leading to (Eq. (4.27) with \( \lambda = 1 \))

\[ \frac{w}{\varepsilon} = 2 \cdot 7.8 \left( \frac{\sigma_{\text{II},w} - k_p \cdot 1.056 \%}{E_i} \right) \]

illustrated in Fig. 5.11. Factor \( k_p \) is set equal to 0 for neglecting the tensile stresses in the concrete and equal to 1 for considering average concrete stresses equal to half the concrete tensile strength (for \( \lambda = 1 \)).

![Fig. 5.11](image_url)

**Fig. 5.11** Maximal side face crack widths in the constant moment zone of Kenel and Marti's (2002) 4-point bending tests (marked as crosses): (a) specimen B1; (b) specimen B3; (c) specimen B5; (d) specimen B4. Illustrated are predictions according to the Tension Chord Model (\( \lambda = 1 \)), MC 2010 and Frosch (slab face).

Both the section and member deformations (Fig. 5.10) are underestimated to a greater or lesser extent by all approaches if tension stiffening is considered. The direct tensile strength leads to reasonable cracking loads if the self-weight of the slab is considered. In these slab strips without stirrups, most of the observed maximal crack widths are located between the Tension Chord Model predictions with and without (\( k_p = 0 \ldots 1 \)) considering the contribution of the concrete between the cracks. The crack spacing on either side of the relevant crack in B1 is larger than predicted with \( \lambda = 1 \).
Long-Term 4-Point-Bending Tests carried out at EPF Lausanne

Jaccoud and Favre (1982) tested seven identical slab strips (Series C) at different sustained load levels (see Fig. 5.13). The experimental section deformations, midspan deflections and maximal crack widths are taken from Tables 14 to 18 in Jaccoud and Favre (1982). The State II steel stresses illustrated vs. the measured crack widths are calculated short-term State II steel stresses.

Fig. 5.12  Overview of Jaccoud and Favre's (1982) bending tests Series C: (a) structural system; (b) section. The dimensions are in [mm].

Fig. 5.13  Measured (crosses) and predicted 28-day and 388-day section deformations and midspan deflections for Jaccoud and Favre's (1982) 4-point bending tests. Illustrated are predictions according to the Tension Chord Model (TCM, $\lambda = 1$), ACI 318 and EC 2.

The 28-day deformation predictions are carried out as described above for the Kenel and Marti (2002) tests. The 388-day predictions are illustrated in the example below. The measured free shrink-
age strains at 388 days are reduced based on Eq. (3.21) from -0.35 ‰ to -0.25 ‰ in order to account for the size difference between the shrinkage specimens and the test specimens. As stated in the report, the crack spacing is defined by the stirrup spacing (200 mm). This can be seen in Fig. 5.14 (c) and (d).

### 388-day deformation predictions for the C specimens

With \( n = 6.9, f_i = 0.508, f_a = 0.245, g_i = 0.0865, \) and \( g_a = 0.0277 \) (Table 3.1), the 28-day section properties are \( x_1 = 81 \text{ mm}, x_2 = 32 \text{ mm}, E_1 = 7892 \text{ kNm}^2 \) and \( E_2 = 1395 \text{ kNm}^2 \), while \( M_t = 9.4 \text{ kNmm} \) and \( \Delta m = 2.91 \text{ mrad/m} (\lambda = 1) \). Accordingly with \( n = 6.1 (1 + 2.31) = 22.8 \) the effective 388-day section properties are \( x_{1,ef} = 84 \text{ mm}, x_{2,ef} = 52 \text{ mm}, E_{1,ef} = 2602 \text{ kNm}^2 \) and \( E_{2,ef} = 1057 \text{ kNm}^2 \). With \( k_a = 0.3 \) (Eq. (4.11) with \( p = 2.31 \)) and \( \lambda = 1 \) the 388-day tension stiffening curvature reduction is (Eq. (4.14))

\[
\Delta \gamma_{ts} = k_a \left( \frac{2.8 \text{ MPa}}{30 \text{ GPa} (0.131 \text{ m} - 0.052 \text{ m})} \right) = k_a 3.6 \text{ mrad/m}
\]

The 388-day free shrinkage strains are reduced based on Eq. (3.21) from \( \varepsilon_{sh} = -0.35 \text{ ‰} \) measured on the shrinkage specimen \( (h_0 = 60 \text{ mm}) \) to \(-0.25 \text{ ‰} \) for the test specimen \( (h_0 = 132 \text{ mm}) \) in order to account for the size difference. With \( k_{ef} = 0.290 10^{-3} \text{ m}^4 \) and the net uncracked concrete properties \( A_c = 0.119 \text{ m}^2 \) and \( x_c = 80 \text{ mm} \), this leads to shrinkage curvatures (Eq. (3.67))

\[
\Delta \gamma_{a,1} = -0.25 \% \left( \frac{0.119 \text{ m}^2 (0.080 \text{ m} - 0.084 \text{ m})}{0.290 10^{-3} \text{ m}^4} \right) = 0.5 \text{ mrad/m}
\]

and (Eq. (3.69))

\[
\Delta \gamma_{a,2} = -0.25 \% \left( \frac{3}{(3 - 0.131 \text{ m} - 0.052 \text{ m})} \right) = 2.2 \text{ mrad/m}
\]

The concrete tensile stresses in the extreme fibres (Eq. (3.68))

\[
\Delta \sigma_{a,1} = \frac{205 \text{ GPa}}{22.8} \left( -0.25 \% \left( 1 - 0.119 \text{ m}^2 \right) \left( 0.134 \text{ m} - 0.084 \text{ m} \right) - 0.2 \text{ mrad/m} \right) = 0.6 \text{ MPa}
\]

reduce the cracking moment (Eq. (4.88)) to

\[
M_r = \frac{(2.8 \text{ MPa} - 0.6 \text{ MPa}) \cdot 0.290 10^{-3} \text{ m}^4}{160 \text{ mm} - 84 \text{ mm}} = 8.6 \text{ kN} \text{ m}
\]

With \( g = 2.9 \text{ kN/m} \) and \( \xi_Q = 1 \text{ m} / 3.1 \text{ m} = 0.323 \), the midspan deflections in Fig. 5.13 (d) are determined according to Chapter 4.3.1. The State I deflections \( a_i = \left( \frac{1 - 4 \cdot 0.323^2}{24} \right) \frac{1 \text{ m} \cdot Q \cdot 3.1^2 \text{ m}^2}{2602 \text{ kNm}^2} + \frac{5 \cdot 2.9 \text{ kN/m} \cdot 3.1^4 \text{ m}^4}{384 - 2602 \text{ kNm}^2} = 0.40 \text{ mm/kN} \cdot Q + 1.4 \text{ mm} \)

the State II deflections

\[
a_i = \left( \frac{3 - 4 \cdot 0.323^2}{24} \right) \frac{1 \text{ m} \cdot Q \cdot 3.1^2 \text{ m}^2}{1057 \text{ kNm}^2} + \frac{5 \cdot 2.9 \text{ kN/m} \cdot 3.1^4 \text{ m}^4}{384 - 1057 \text{ kNm}^2} = 0.98 \text{ mm/kN} \cdot Q + 3.3 \text{ mm}
\]

the tension stiffening deflections \( (\lambda = 1) \)

\[
\Delta \sigma_{a,2} = \frac{3 \cdot 2 \text{ m}^2 \cdot k_a \cdot 3.6 \text{ mrad}}{8} \text{ m} = k_a 4.3 \text{ mm}
\]

and the shrinkage deflections

\[
\Delta \sigma_{a,1} = \frac{3 \cdot 2 \text{ m}^2 \cdot 0.2 \text{ mrad}}{8} \text{ m} = 0.6 \text{ mm}
\]

\[
\Delta \sigma_{a,2} = \frac{3 \cdot 2 \text{ m}^2 \cdot 2 \text{ mrad}}{8} \text{ m} = 2.6 \text{ mm}
\]

are interpolated (Eqs. (4.84), (4.86) and (4.87))

\[
a = \zeta_1 a_1 + (1 - \zeta_1) a_2 - (1 - \zeta_2) \Delta \sigma_{a,1} + \zeta_2 \Delta \sigma_{a,1} + (1 - \zeta_2) \Delta \sigma_{a,2}
\]

with the 4-point bending interpolation coefficients (Table 4.2)

\[
\zeta_1 = 8 \cdot q^2 \cdot 0.323^2 = 0.32 \cdot q^2
\]

\[
\zeta_2 = 4 \cdot q^2 \cdot 0.323^2 = 0.42 \cdot q^2
\]

and
Verification

\[ \psi = \frac{8.6 \text{kNm}}{1.1 \text{m} \cdot Q + 0.125 \cdot 2.9 \text{kN/m} \cdot 3.1 \text{m}^2} \]

corresponding to the predominant load.

**Fig. 5.14** Maximum crack widths in the constant moment zone of Jaccoud and Favre's test Series C (1982) with stirrups spaced at 200 mm: (a) at 28 days; (b) at 388 days; (c) at 28 days assuming max crack spacing = stirrup spacing; (d) at 388 days assuming max crack spacing = stirrup spacing. Illustrated are predictions according to the Tension Chord Model (\( \lambda = 1 \)), MC 2010 and Frosch (bottom side).

The related transfer length (Eq. (4.28)) equals

\[
l_\text{trans} = \frac{6.9}{8} \left( \frac{0.0865 \cdot (1 - 0.245)}{0.0277 \cdot 0.82^2 \cdot (1 - 0.508)} - 1 \right) = 5.3
\]

and the State II steel strains at cracking (Eq. (4.26)) equal

\[
\varepsilon_{s,II} = \frac{6.9 \cdot 2.8 \text{ MPa}}{205 \text{ Ga}} \left( \frac{0.0865 \cdot (1 - 0.245)}{0.0277 \cdot 0.82^2 \cdot (1 - 0.508)} \right) = 0.67 \%
\]

The related maximal crack widths in the constant moment zone (Eq. (4.27) with \( \lambda = 1 \)) are determined for both a 127 mm crack spacing equal to twice the transfer length

\[
w = 2.5 \left( \frac{\sigma_{s,II} - k_\psi + 0.67 \%}{2} + 0.25 \% \right)
\]

and a 200 mm crack spacing equal to the stirrup spacing. Factor \( k_\psi \) is set equal to 0.3 (Eq. (4.11) with \( \varphi = 2.31 \)) and 0.
Gilbert and Nejadi (2004) tested 6 beams (B1-a/b to B3-a/b) and 6 slabs (S1-a/b to S3-a/b) with reinforcement ratios between 0.4 and 0.8 % under loads sustained for 400 days. The free shrinkage strains $\varepsilon_{sh} = -0.825 \text{‰}$ measured on 600/160/600 mm shrinkage specimens are corrected with Eq. (3.21) to account for the specimen size (see example calculation), leading to $\varepsilon_{sh} = -0.76 \text{‰}$ for the beams and $\varepsilon_{sh} = -0.87 \text{‰}$ for the slabs.

Fig. 5.16 compares the measured and predicted midspan deflections, while Fig. 5.17 and Fig. 5.18 illustrate the maximum crack widths observed at the bottom of the specimen side faces. The steel stresses plotted vs. the measured crack widths correspond to the calculated short-term State II steel stresses given in the test report.
Fig. 5.16  Measured (crosses) and predicted 14- and 400-day midspan deflections for Gilbert and Nejadi's (2004) flexural members: (a) beams B1/2-a/b; (b) beams B3-a/b; (c) slabs S1-a/b; (d) slabs S2-a/b; (e) slabs S3-a/b. Illustrated are predictions according to the Tension Chord Model ($\lambda = 1$), ACI 318 and EC 2 (line types and parameters according to figure (a)).
Fig. 5.17 Maximum side face crack widths for Gilbert and Nejadi’s (2004) test beams: (a) B1-a & b at loading; (b) B1-a & b 386 days later; (c) B2-a & b at loading; (d) B2-a & b 386 days later; (e) B3-a & b at loading; (f) B3-a & b 386 days later. Illustrated are predictions according to the Tension Chord Model ($\lambda = 1$), MC 2010 and Frosch (side face).
400-day predictions Specimens S1

With \( n = 9, f_1 = 0.508, f_0 = 0.240, g_0 = 0.0859 \) and \( g_0 = 0.0265 \) (Table 3.1), the 14-day section properties are \( x_1 = 82 \text{ mm}, x_2 = 31 \text{ mm}, E_{cI} = 3266 \text{ kNm}^2 \) and \( E_{cII} = 536 \text{ kNm}^2 \), while \( M_1 = 3.3 \text{ kNm} \) and \( \Delta_{x_1} = 2.64 \text{ mrad/m} \) (Eq. (4.7) with \( \lambda = 1 \)). Accordingly, with \( n = 9 \), the effective 400-day section properties are \( x_1,_{ef} = 84 \text{ mm}, x_2,_{ef} = 47 \text{ mm}, E_{cI,ef} = 1269 \text{ kNm}^2 \) and \( E_{cII,ef} = 430 \text{ kNm}^2 \), while \( M_{ef} = 0.6 \text{ kNm} \) and \( \Delta_{x_1} = 1.2 \text{ mrad/m} \) (Eq. (4.14) with \( \lambda = 1 \) and \( k_\phi = 0.4 \)). Free shrinkage strains equal to \( \varepsilon_{sh} = -0.87 \% \) lead to \( \Delta_{x_1} = 1.43 \text{ mrad/m} \) (Eq. (3.67)) and \( \Delta_{x_2,sh} = 7.60 \text{ mrad/m} \) (Eq. (3.69)).

The 400-day State I and State II deflections

are interpolated with (Table 4.2)

while the 400-day tension stiffening deflections

and the shrinkage deflections

are interpolated with (Table 4.2)

where

The maximal crack widths (in the middle third of the beam) are determined with (Eq. (4.28))

and (Eq. (4.26))

leading to maximal crack widths (Eq. (4.27) with \( \lambda = 1 \))

With a crack spacing of \( 2 \cdot 7.45 \cdot 12 \text{ mm} = 180 \text{ mm} \), shrinkage alone causes a crack opening of 0.16 mm.

As there are no stirrups, the reinforcement ratio primarily defines the related crack spacing, which together with the State II steel stresses and the free shrinkage strains defines the size of the crack widths (Fig. 5.17 and Fig. 5.18). According to the slab crack width predictions, slab S1 with the lowest reinforcement ratio \( (s_{r,max} = 180 \text{ mm}) \) should have the largest cracks followed by slabs S2 \( (s_{r,max} = 116 \text{ mm}) \) and S3 \( (s_{r,max} = 84 \text{ mm}) \). In reality, the order is the other way round (see Fig. 5.18). It must, however, be taken into account that at the low steel stress levels found in the slabs, the predicted crack widths are close and generally reflect the observed values. A look at the crack patterns shows that the largest crack influence region is just over 250 mm in S1, around 200 mm in S2 and S3-a, but over 250 mm in S3-b, where the observed crack spacings are considerably larger than predicted.
Fig. 5.18 Maximum side face crack widths (bottom fibre) for Gilbert and Nejadi’s (2004) test slabs: (a) S1-a & b at loading; (b) S1-a & b 386 days later; (c) S2-a & b at loading; (d) S2-a & b 386 days later; (e) S3-a & b at loading; (f) S3-a & b 386 days later. Illustrated are predictions according to the Tension Chord Model ($\gamma = 1$), MC 2010 and Frosch.
ConCrack Benchmark Specimens RL1 and RG8

The ConCrack Specimens RL1 and RG8 (NECS 2010; Buffo-Lacarriere et al. 2011) belong to the Benchmark tests carried out within the national French project CEOS (Comportement et Evaluation des Ouvrages Speciaux vis-à-vis de la fissuration et du retrait). The Beam Specimens RL1 and RG8 were submitted to 4-point bending tests after either 2 months of free shrinkage and cooling (RL1) or 1 month of restrained shrinkage and cooling (RG8). The axial deformations of RG8 were restrained by two tube struts.

Apart from some minor changes, the predictions illustrated here represent a part of the author's results (Buffo-Lacarriere et al. 2011) submitted for the blind stage of the prediction contest carried out for the Benchmark tests.

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Table 5.1 Measured concrete properties C50/60 for RL1 and RG8: 1) splitting tensile strength from three 160 x 320 mm prisms (NF EN 12390-6); 2) refractometer with 70 x 70 x 280 mm specimens ($h_0 = 17.5$ mm, $t_s = 1$ d); 3) prismatic 110 x 220 mm specimens loaded at 40% of their compressive strength (12.2MPa); 4) 22°C for RL1 and 17°C for RG8; 5) measured under adiabatic conditions in a thermally isolated 300 mm cubic container.

Test Specimen RL1

Specimen RL1 was cast on the 12th of October 2009 ($t = t_0$). After the formwork was removed on the 14th of October ($t = 2$ d) the specimen remained supported on wooden log elements (Fig. 5.19, left) under outdoor environmental conditions ($\approx 10$ °C) until it was moved to the testing bench (Fig. 5.19, right and Fig. 5.20 (a)) on the 27th of November ($t = 46$ d). The four point bending test took place on 14th of December ($t = 63$ d). The specimen was loaded up to $Q = 2250$ kN and unloaded back to $Q = 0$. The predictions are carried out with the structural system illustrated in Fig. 5.20 (c) and the 28-day concrete properties (Table 5.1). The specimen is $b = 1.6$ m wide, $h = 0.8$ m thick (with exception of the end regions) and 6.1 m long. 16 $\varnothing$ 32 mm bars provide the tensile reinforcement with $\rho = A_s/\pi b d = 1.11$ %, while 8 $\varnothing$ 25 mm bars provide the compression reinforcement with $\rho' = 0.34$ % (Fig. 5.20 (b)). The distribution reinforcement (0.14 % from 8 $\varnothing$ 16 mm bars) is not taken into account for the predictions. There are stirrups ($\varnothing = 16$ mm) spaced at 350 mm in the constant moment zone and at 310 mm in the rest of the beam.
Fig. 5.19 Specimen RL1 (NECS 2010): (a) during 63 days of free shrinkage and cooling; (b) on testing bench.

Fig. 5.20 Specimen RL1: (a) schematic illustration of the specimen on the testing bench; (b) section; (c) static system used for predictions. The dimensions are in [mm]. The measurement locations in brackets are located on the other half of the symmetrical beam.
Fig. 5.21 Specimen RL1: Concrete temperature measured at points SU, C and SL during 30 days after casting.

Predictions 4-point bending test RL1

The static bending test takes place at $t = 56$ d. Cooling after hydration is assumed to cause evenly distributed strains equal to 0.012 %$\cdot^\circ$C$ \cdot 30^\circ$C = 0.36 %, which due to the lack of external restraint cause an even shortening and do not affect the bending behaviour. Uniformly distributed autogenous and drying shrinkage strains equal to -0.12 ‰ (measured) and 0.16 · -0.37 ‰ = -0.08 ‰ (reduced according to Eq. (3.21) to account for $h_0 = 267$ mm and $t_s = 2$ d) are adopted. The internal restraint due to -0.2 % shrinkage is not large enough to cause separation cracks, so that RL1 is modelled as uncracked at the beginning of the static bending test.

The RL1 deformation predictions are carried out with the structural system illustrated in Fig. 5.20 (c) and the moment distribution

$$ M = \begin{cases} -\frac{E}{2}(\Delta l + x) & \text{if } -\Delta l \leq x \leq 0 \\ Qx + \frac{E}{2}(x - \Delta l^2 - x^2) & \text{if } 0 \leq x \leq a \\ Qa + \frac{E}{2}(x - \Delta l^2 - x^2) & \text{if } a \leq x \leq \frac{l}{2} \end{cases} $$

where $g = 32$ kN/m, $\Delta l = 0.5$ m, $a = 1.75$ m and $l = 5.1$ m (for $x$ and $z$ see Fig. 5.20). The average section deformations $\varepsilon_{avg} = \varepsilon_{avg} + z \varepsilon_{avg}$ are defined by the equilibrium equations

$$ 0 = b \int_{-\lambda}^{\lambda} \sigma_z dz - A_\sigma c - A_\sigma c (\varepsilon_{avg}) E_c + A (\varepsilon_{avg} + k \Delta e_{cr}) E_c \\
M = \int_{-\lambda}^{\lambda} \sigma_z dz - \frac{\varepsilon}{A_\sigma c} - z A_\sigma c - z A_\sigma c (\varepsilon_{avg}) E_c + z A (\varepsilon_{avg} + k \Delta e_{cr}) E_c $$

The 28-day concrete properties with $n = 5.1$ and therefore $E_1 = 2954 \times 10^3$ kNm$^2$, $E_2 = 881 \times 10^3$ kNm$^2$, $x_1 = 409$ mm, $x_2 = 200$ mm as well as $E_s = 200$ GPa are used. The concrete stresses $\sigma_i = (\varepsilon_{avg} - \varepsilon_i) E_c$ are determined with $\varepsilon_i = -0.2$ % and assumed zero in the tension zone of cracked sections.

Tension stiffening is considered with the effective reinforcement ratio (Eq. (4.4))

$$ \rho_{eff} = \left( \frac{5.1}{2954 \times 10^3 \text{kNm}^2 (0.722 \text{m} - 0.200 \text{m}) / 881 \times 10^3 \text{kNm}^2 (0.8 \text{m} - 0.409 \text{m})} - 5.1 + 1 \right)^{-1} = 5.33\% $$

Factor $\lambda$ is set equal to 1 because the transfer length (Eq. (4.28) with $d = 0.722$ m and $\Theta = 32$)

$$ l_b = \frac{5.1 - 32 \text{mm}}{8} \left( \frac{2954 \times 10^3 \text{kNm}^2 (0.722 \text{m} - 0.200 \text{m}) / 881 \times 10^3 \text{kNm}^2 (0.8 \text{m} - 0.409 \text{m})}{-1} \right) = 71 \text{mm} $$

implies a maximum crack spacing equal to 142 mm, which is less than half the stirrup spacing. With $f_{ct} = 4.65$ MPa this leads to (Eq. (4.2))

$$ \Delta e_{cr, i} = k_s \frac{4.65 \text{MPa} (1 - 5.33\%) - 200 \text{GPa}}{2 \times 5.33\%} = k_s 0.21\% $$

The vertical displacements (relative to supports)
Verification

\[ u = \int_a^b \sqrt{2} \int_{x_0}^{x} \sqrt{2} \int_{x_0}^{x} dx \]

at \( x = x_a \) are determined with

\[ \sigma_i = \begin{cases} \left(1 - \frac{x}{T}\right)x & \text{if } 0 \leq x \leq x_a \\ x_a - \frac{x}{T}x & \text{if } x_a \leq x \leq \frac{4}{2} \end{cases} \]

and

\[ \sigma_z = \frac{x}{T} \]

For comparison, the elastic midspan deflections at \( Q = 2250 \text{ kN} \) are estimated with the elastic deflection equation

\[ a = \frac{3 - 4}{24} \frac{1.75m^2}{5.1m} = \frac{1.75m \cdot 2250 \text{ kN}(5.1m)}{881 \cdot 10^3 \text{ kNm}} = 12.2 \text{ mm} \]

for 4-point bending according to Table 4.1.

The maximum crack widths (Eq. (4.21))

\[ w_{\text{max}} = 142 \text{ mm} \left( \varepsilon_{\text{avg}} + 0.2 \%e - k_0 4.65 \text{ MPa} \right) = 142 \text{ mm} \left( \varepsilon_{\text{avg}} + 0.2 \%e - k_0 0.06 \%e \right) \]

result from the average section strains (= average steel strains) and average concrete strains. For reasons of simplicity tension stiffening is neglected for all predictions during unloading \( (k_0 = 0) \).

Judging from Fig. 5.21 the assumption of a uniform concrete (and steel) temperature distribution is not unreasonable, but the assumed 30 °C temperature drop after hydration is too small. The predicted and measured steel strains at points (see Fig. 5.20 (a)) P14 \((z = -0.34 \text{ m} ; x = 1.63 \text{ m})\), P16 \((z = -0.34 \text{ m} ; x = 1.880 \text{ m})\) and P17 \((z = 0.35 \text{ m} ; x = 1.610 \text{ m})\) as well as the concrete compressive strains at P2 \((z = 0.383 \text{ m} ; x = 2.360 \text{ m})\) and P4 \((z = 0.393 \text{ m} ; x = 1.775 \text{ m})\) are compared in Fig. 5.22. The predicted and measured strains are set equal to zero at the beginning of static loading \( (\varepsilon = 0 \text{ at } Q = 0) \). The unloading strains are not illustrated because the difference between loading and unloading is small on the compression side, while the steel strain measurements on the tension side are only available up to \( Q = 1 \text{ MN} \). The dashed lines on the tension side illustrate predictions neglecting shrinkage (no difference is visible on the compression side). It can be seen that, the predicted steel strain reduction due to tension stiffening is about the same size as the shrinkage strains released at cracking.

Fig. 5.22 Specimen RL1: Predicted average strains during the 4-point bending test with \( \lambda = 1 \): P14, P16 and P17 are steel strains; while P2 and P4 are concrete strains (see Fig. 5.20).
Judging from the cracking load and the tensile steel strains, it seems that the predictions either over-estimate shrinkage or underestimate the concrete tensile strength.

Fig. 5.23 Specimen RL1: Predicted and measured vertical displacements during the 4-point bending test at locations (see Fig. 5.20 (a)) P9, P7, P10 and P8.

Fig. 5.23 compares the vertical displacements at points P9 (\(x = 2.38\) m), P7 (\(x = 1.565\) m), P10 (\(x = 1.245\) m) and P8 (\(x = 0.17\) m) (see Fig. 5.20) relative to the supports (\(x = 0\)). Thereto, the potentiometric displacement sensor measurements are corrected with the displacements measured at P8 and P11 according to

\[
\begin{align*}
\mu_{x,\text{corr}} &= \mu_z + \frac{x_{P11}U_x,P8 - x_{P8}U_x,P11}{x_{P8} - x_{P11}} \tag{5.1}
\end{align*}
\]

in order to eliminate the displacements attributed to the elongation of the macalloy bars. Fig. 5.24 illustrates the predicted maximum crack widths at the concrete surface (\(z = -0.400\) m) as well as the maximum crack widths observed on the specimen.
Verification

Fig. 5.24  RL1 predicted and observed maximal crack widths. It is assumed that the observed values refer to the maximum crack opening found on the entire specimen.

Test Specimen RG8

Test specimen RG8 is illustrated in Fig. 5.25 and Fig. 5.26. The central part of the specimen is $b = 0.5$ m wide, $h = 0.8$ m thick, 5.1 m long and symmetrically reinforced with $10 \varnothing 32$ mm bars (see Fig. 5.26 (b)) providing $\rho_{TC} = (A_s + A_s')/(bh) = 2.01\%$ or $\rho = \rho' = A_s/(bd) = A_s'/(bd) = 1.10\%$. The distribution reinforcement ($4 \varnothing 12$ mm) is not taken into account for the predictions. 16 mm diameter stirrups are spaced at 220 mm in the middle of the beam (over a length 1100 mm) and at 200 mm elsewhere. Two steel tubes ($A_{\text{tube}} = 35563$ mm$^2$) on either side of the specimen (Fig. 5.26 (c)) restrain its axial deformations. The tubes are braced against the eccentrically prestressed heads of the specimen.

Fig. 5.25  Specimen RG8 (NECS 2010).

RG8 was cast on the 7th of April 2010 in isolating formwork (on all four sides), which was removed 2 days later ($t = 2$ d). The THM (Thermo-Hydro-Mechanic) test ($t = 2d$ to $30d$) consisted in letting the axially restrained member shrink and cool down under environmental conditions ($\approx 15$ °C). During this time the specimen was supported on wooden log elements. At $t = 30$ d the 4-point bending test
Verification

took place. The specimen was placed on the testing bench (Fig. 5.26 (a)) and loaded in 50 kN (20 minutes) steps up to 800 kN (per point load).

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<td>4.5</td>
<td>-0.060/-0.016</td>
<td>-0.08</td>
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<td>-0.30</td>
<td>-0.38</td>
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<td>4.5</td>
<td>-0.08</td>
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<td>-0.09</td>
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<td>-0.57</td>
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<td>4.5</td>
<td>-0.10</td>
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<td>-0.54</td>
<td>-0.637</td>
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<td>4.5</td>
<td>-0.10</td>
<td>25</td>
<td>-0.60</td>
<td>-0.70</td>
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<td>4.5</td>
<td>-0.11</td>
<td>20</td>
<td>-0.66</td>
<td>-0.77</td>
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<td>4.5</td>
<td>-0.080/-0.033</td>
<td>-0.11</td>
<td>15</td>
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<td>4.6</td>
<td>-0.100/-0.050</td>
<td>-0.15</td>
<td>15</td>
<td>-0.72</td>
<td>-0.87</td>
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<td>4.7</td>
<td>-0.113/-0.066</td>
<td>-0.18</td>
<td>15</td>
<td>-0.72</td>
<td>-0.90</td>
<td>-0.72</td>
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<tr>
<td>30</td>
<td>39.1</td>
<td>4.7</td>
<td>-0.18</td>
<td>15</td>
<td>-0.72</td>
<td>-0.90</td>
<td>-0.72</td>
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</table>

**Table 5.2** Assumed concrete properties for the RG8 blind stage predictions with 1) linear interpolation of values in-between measurements (coloured in grey); 2) autogenous shrinkage as measured (Table 5.1); 3) drying shrinkage reduced according to Eq. (3.21); 4) the cooling rate of 5 °C per day is assumed based on Lohmeyer & Ebeling (2009) pg. 117; 5) with the measured concrete dilatation coefficient of 12 μm/m°C.

**Predictions THM-test RG8**

During the THM-test the central part of RG8 ($l = 5.1$ m and $\rho = \rho_{nc} = 2.01$ %) is modelled according to 4.2.2 as an end-restrained beam with a restraint stiffness

$$k = \frac{2A_{w0}E_i}{lhh} = \frac{2 \times 35.63 \text{ mm}^2 \times 200 \text{ GPa}}{5.1 \text{ m} \times 0.5 \text{ m} \times 0.8 \text{ m}} = 7.0 \text{ N/mm}^3$$

and (Eq. (4.38))

$$\kappa = \frac{kl}{E_i} \frac{5.1 \text{ m} \times 7.0 \text{ N/mm}^3}{200 \text{ GPa}} = 0.178$$

For comparison the case of no external ($\kappa = 0$) restraint is also regarded.

Before cracking the compression force in the steel struts (Eq. (4.43))

$$R = -\frac{bh}{2} \left[(1 + np - \rho)\sigma_{ci} + pE_i(\varepsilon_{ci} - \varepsilon_a)\right]$$

is determined with (Eq. (4.41))

$$\sigma_{ci} = E_i \frac{bh\varepsilon_a - (\rho + \kappa)\varepsilon_{ci}}{nk + a_i}$$

During the crack formation phase an upper limit for

$$R = -\frac{bh}{2} \left[(1 + np - \rho)f_{ct} + pE_i\varepsilon_a\right]$$
is estimated with $\sigma_{z} = f_{c}$. The force fluctuations between the cracks are not considered.

**Fig. 5.26** Specimen RG8: (a) schematic illustration of specimen on testing bench; (b) regular section; (c) schematic plan view illustrating steel struts (d) static system used for predictions. The dimensions are in [mm]. Point D in brackets is on the other half of the symmetrical specimen.

The relative displacement between points C and D

$$\Delta l_{CD} = l_{CD} \frac{2R}{E_{s}}$$

results from $R$ for $\kappa \geq 0$ or is equal to

$$\Delta l_{CD} = l_{CD} \left[ \varepsilon_{u} + \frac{(\varepsilon_{u} - \varepsilon_{a})(1 - p)}{1 + np - p} \right]$$

for $\kappa = 0$. The cracking behaviour during the crack formation phase is modelled assuming a transfer length equal to

$$l_{0} = \frac{\varnothing (1 - p)}{8p} = \frac{32 \text{ mm} (1 - 2.01 \%)}{8 \cdot 2.01 \%} = 195 \text{ mm}$$
With \( \lambda \) between 1 and 0.5 this leads to a final amount of cracks between (Eq. (4.58))

\[
J = \frac{5.1 \text{ m}}{2 \cdot \lambda \cdot 195 \text{ mm}} = 13 ... 26
\]

and with \( \lambda = 1 \) a maximum crack width (Eq. (4.46) equal to

\[
w = \frac{195 \text{ mm} \cdot 4.65 \text{ MPa} \left( 1 + 2.01\% \cdot (5.1 - 1) \right)}{2.01\% \cdot 200 \text{ GPa}} = 0.25 \text{ mm}
\]

The cooling and shrinkage induced steel \( \varepsilon_{si} \) and concrete \( \varepsilon_{ci} \) strains adopted for the blind stage predictions are given in Table 5.2 and Fig. 5.27 (a). As there are 26 stirrups \( \lambda = 0.5 \) is assumed. According to \( \varepsilon_{c1} \) (Fig. 5.27 (b)) the crack formation phase starts at \( t = 5 \text{ d} \) with \( \varepsilon_{c1} = -0.25 \% \) (Table 5.2) and ends (Eq. (4.57) with \( \varepsilon_{c1} = -0.72 \% \) and \( a_1 = 1.083 \)) at

\[
\varepsilon_{c26} = \frac{4.65 \text{ MPa}}{2 \cdot 2.01\% \cdot 200 \text{ GPa}} \left( 0.5(1-2.01\%) \cdot 0.178 - 2 \cdot 1.083 \right) \left( -0.72\% + 2.01\% \right) = -1.068 \%
\]

This implies an average concrete strain increase of

\[
\Delta \varepsilon_{ci} = \frac{-1.068 \% + 0.25\%}{26 - 1} = -0.032 \%
\]

per crack, resulting in 20 cracks at \( t = 30 \text{ d} \).

With the simplified temperature evolution adopted based on the measured concrete temperatures (\( T_1 \) in Fig. 5.27 (a)) and \( \lambda = 0.5 \), the crack formation phase starts at (Fig. 5.27 (b)) \( t = 3 \text{ d} \) with \( \varepsilon_{c1} = -0.10 \% \) (Table 5.2) and ends (Eq. (4.57) with \( \varepsilon_{c1} = -0.42 \% \) and \( a_1 = 1.083 \)) at

\[
\varepsilon_{c13} = \frac{4.65 \text{ MPa}}{2 \cdot 2.01\% \cdot 200 \text{ GPa}} \left( 0.5(1-2.01\%) \cdot 0.178 - 2 \cdot 1.083 \right) \left( -0.42\% + 2.01\% \right) = -1.041 \%
\]

This implies an average concrete strain increase of

\[
\Delta \varepsilon_{ci} = \frac{-1.041 \% + 0.10\%}{25} = -0.038 \%
\]

per crack, resulting in \((-0.60 \% + 0.10 \%)/-0.038 = 13 \) cracks at \( t = 30 \text{ d} \).

Fig. 5.27 illustrates the predicted and measured behaviour of Specimen RG8 during the THM test. The concrete temperature evolutions denominated \( T_1 \) and \( T_2 \) correspond to the temperature evolution predicted for the blind stage and an evolution approximating the measured concrete temperatures, respectively. The concrete temperatures measured in RL1 (Fig. 5.20) and RG8 (Fig. 5.27 (a)) are similar. They indicate concrete hardening at a temperature of around 50 °C before the maximum temperature of 55 °C is reached after 24 hours. After this point the concrete starts to cool down to ambient temperature, reaching its hardening temperature (\( \Delta l \approx 0 \)) at around 2 days and ambient temperature at 4 days. It is cooling below the hardening temperature, which leads to tensile stresses in an axially restrained member. The present predictions assume \( \varepsilon_{si} = 0 \) and \( \varepsilon_{ci} = -0.03 \% \) at \( t = 2 \text{ days} \).

As concrete shrinkage strain and tensile strength measurements were available for the blind stage predictions and the concrete temperature evolutions were provided afterwards (Fig. 5.27 (a)), the main assumption behind the refined THM predictions is the restraint stiffness. This is why the predictions in Fig. 5.27 were carried out with a restraint stiffness corresponding to no restraint (\( \kappa = 0 \)) and the stiffness of the two 5.1 m long steel struts (\( \kappa = 0.178 \)). The latter stiffness represents an upper limit, as neither the coupling elements at the end of the steel struts nor the flexibility of the end beams is taken into account. For \( \kappa = 0 \) there should not be any cracking, while for \( \kappa = 0.178 \) cracking is predicted to start at 5 or 3 days (Fig. 5.27 (b)), depending on the rate of cooling. For \( \kappa > 0 \), the maximal tensile force (= –2 \( R \)) in the member during the crack formation phase (Fig. 5.27 (c)), which is reached each time before a crack forms, is independent of the cooling strains and restraint stiffness.

The restraint stiffness, however, defines the relative axial displacement between C and D shown in Fig. 5.27 (d). The figure illustrates the good agreement between the displacements calculated with \( \kappa = 0.178 \) and the displacements measured with the 2.5 m long optic fibre sensor located between the points C and D. In this context it remains unclear why the predicted amount of 20 (for \( T_1 \)) or 13 (for \( T_2 \)) cracks strongly overestimates the 3 cracks (at 3, 7 and 10 days) observed during the
THM test. Unfortunately experimental strut forces \((R)\), which could disclose the axial force in the member, are not available.

**Fig. 5.27** THM test Specimen RG8: (a) concrete temperature during cooling; (b) development of concrete stresses before cracking (neglecting creep); (c) axial force per steel strut; (d) relative displacement between points C and D.

**Predictions static bending test RG8**

Based on the THM predictions and for reasons of simplicity RG8 is assumed to be fully cracked at the start of the 4-point bending test. With \(\kappa = 0.178\), \(\lambda = 0.5\) and Eq. (4.59)

\[
N = -2R = 2.01 \% \cdot 500 \cdot 800 \text{ mm} \cdot 0.178 \cdot 2.01\% \cdot \left( \frac{0.5 \cdot 4.65 \text{ MPa} (1 - 2.01\%) - \varepsilon_{cr} \cdot 200 \text{ GPa}}{2 \cdot 2.01\%} \right)
\]

equals 1451 kN for \(\varepsilon_{cr} = -0.72 \%\) (T1, see Fig. 5.27 (a)) and 1071 kN for \(\varepsilon_{cr} = -0.42 \%\) (T2). The corresponding average member strains are -0.11 \% in the first case and -0.07 \% (Fig. 5.28 (b)) in the second case.

Within an iterative procedure, a midspan moment \(M_a\) is assumed, leading to a moment distribution (self-weight neglected)

\[
M = \begin{cases} 
\frac{M_a x}{l} & \text{if } 0 \leq x \leq 1.75 \text{ m} \\
M_a & \text{if } 1.75 \text{ m} \leq 0.5l 
\end{cases}
\]

On this basis, the equilibrium equations

\[
N = \int_A \sigma_c dA + \int_A E_c \left( \varepsilon_{avg} - \varepsilon_{cr} + \varepsilon_{cr} \Delta \varepsilon_{avg} \right) + \int_A E_a \left( \varepsilon_{avg} - \varepsilon_a + \varepsilon_a \Delta \varepsilon_{avg} \right)
\]

and
Verification

\[ M = \int_{z} \sigma_z z A + z' A E_z (\varepsilon_{avg} - \varepsilon_{avg}) + z A E_z (\varepsilon_{avg} - \varepsilon_{avg}) + z A E_z (\varepsilon_{avg} - \varepsilon_{avg}) \]

are used to provide the average section strains

\[ \varepsilon_{avg}(z) = \varepsilon_{avg} + \Delta \varepsilon_{avg} \]

as a function of the assumed midspan moment \( M_0 \) and the unknown axial force \( N \), which then results from

\[ N = -2kbb \int_{z} \varepsilon_{avg} \, dx \]

The bottom steel layer is defined by \( \rho' = 1.1\% \) and \( d' = 75 \text{ mm} \), while the top steel layer is defined by \( \rho = 1.1\% \) and \( d = 725 \text{ mm} \).

Tension stiffening (top and bottom reinforcement) is implicitly accounted for within the steel term according with

\[ \begin{cases} 1 & \text{if } \varepsilon_{steel} > -0.07 \%_0 \\ k_s & \text{if } -0.07 \%_0 \geq \varepsilon_{steel} \geq -0.35 \%_0 \\ 0 & \text{if } \varepsilon_{steel} \leq -0.35 \%_0 \end{cases} \]

assuming the tension force carried by the concrete to have the same centroid as the steel. The first line in the equation applies to most of the top reinforcement (Fig. 5.28 (c)) where the average steel strains increase under bending. The other two lines apply to most of the bottom reinforcement (Fig. 5.28 (d, e)) where the steel strains decrease under bending. Eq. (4.2) with \( \lambda = 0.5 \), \( n = 5.1 \) and \( \rho = 2.01\% \) provides \( \Delta \varepsilon_{steel} = 0.29 \%_0 \), while Eq. (4.20) with \( a_0 = 1.083 \) provides

\[ k_s = \frac{0.07 \%_0 \varepsilon_{steel}}{0.29 \%_0} + 1.231 - 2.210 \left( \frac{0.07 \%_0 \varepsilon_{steel}}{0.29 \%_0} + 0.011 \right) \]

Fig. 5.28  Assumed steel strains during the 4-point bending test for \( \kappa = 0.178 \) and \( \varepsilon_{steel} = -0.42 \%_0 \): (a) assumed bond stress-slip relation; (b) at the start of the 4-point bending test; (c) in most of the top reinforcement during the bending test; (d) & (e) in most of the bottom reinforcement during the static bending test (tensile force changes into compression force). When \( \delta \) reaches zero the cracks close.

The compression concrete becomes load-bearing once \( \varepsilon_{avg} \) reaches \( \varepsilon_i \) (and cracks close) and

\[ \sigma_z = -f_c \frac{\varepsilon\xi - \varepsilon^2}{1 + (\varepsilon - 2)\xi} \]

according to Sargin (1971). In this case \( f_c = 60 \text{ MPa}, \varepsilon_{avg} = 2.3 \%_0, E_z = 39.1 \text{ GPa}, \xi = (\varepsilon + \varepsilon_i)/\varepsilon_{avg} \) and \( k_s = \varepsilon_i E_z/f_c \).

Once \( N \) belonging to \( M_0 \) is determined, the reaction force in each steel strut \( R = -N/2 \), the relative displacement between C and D
Verification

\[ \Delta l = 2 \int_{x_a}^{x_b} \epsilon_{avg} \, dx \]

the vertical displacement at \( x = x_a \) (for \( \vec{M}_1 \) and \( \vec{M}_2 \) see RL1)

\[ u_v = \int_0^{x_a} \vec{M}_1 \chi_{avg} \, dx + \int_0^{x_b} \vec{M}_2 \chi_{avg} \, dx \]

as well as the average crack widths in the constant moment zone

\[ w_{avg} = 220 \text{ mm} \left( \epsilon_{avg} - \epsilon_0 - \frac{f_y}{2E_y} \right) \]

can be determined as a function of

\[ Q = \frac{M_a + N a_m}{1.75 \text{ m}} \]

where \( a_m \) is the absolute value to the midspan deflections relative to the supports.

The average section deformations at three different load levels are shown in Fig. 5.29. Fig. 5.30 illustrates the tensile steel strains at \( x = 1.9 \text{ m} \) and \( 2.35 \text{ m} \), the vertical displacements at \( x = 2.1 \text{ m} \) and \( 1.45 \text{ m} \), the force per steel strut and the relative displacement between points C and D. All four figures show predictions carried out with the full stiffness of the steel struts (\( \kappa = 0.178 \)) and the two temperature evolutions \( T_1 \) and \( T_2 \) (see Fig. 5.27 (a)). Independent of the temperature evolution chosen, the predicted steel strains (Fig. 5.30 (a)) reflect the measured values up until the vertical load-range where the compression concrete is activated. This could mean that the compression concrete is activated later than predicted (e.g. because the shrinkage strains are underestimated) or that there is steel yielding. In view of the THM predictions and the fact that \( \epsilon_{avg} = -0.42 \%_0 \) does not reduce the yield strains below 2.1 \% even for \( f_y = 500 \text{ MPa} \), both explanations seem unlikely. As the measured deflections and axial displacements start increasingly exceeding the predicted values above \( Q = 400 \text{ kN} \), it is likely that (as found for RL1) there are additional effects (e.g. rigid body translations) influencing the global deformation behaviour during the 4-point bending test.
Fig. 5.30  Four-point bending test RG8: (a) tensile steel strains; (b) vertical displacements; (c) force per steel strut; (d) displacement between C and D. The predictions are carried out assuming $\kappa = 0.178$ and either $\epsilon_{ci} = -0.90 \% \& \epsilon_{sj} = -0.72 \%$ ($T_1$) or $\epsilon_{ci} = -0.60 \% \& \epsilon_{sj} = -0.42 \%$ ($T_2$).

5.2.2  Statically Indeterminate Beams

In this section three test series of two-span beams are used to discuss the influence of the reinforcement detailing and the corresponding moment redistributions on the deflections of statically indeterminate beams.

**Two-span beams tested at Gent State University**

Taerwe (1981) tested four two-span beams (C1 to C4) with 5 m spans and approximately the same ultimate loads but different reinforcement detailing. The experimentally determined moment distribution coefficients (determined from support moments given in Figs 17 and 18 of Taerwe (1981)) and the measured deflection curves are compared to the predictions carried out according to Chapter 4.3.
Fig. 5.31  Overview of Taerwe’s (1981) beams: (a) structural system; (b) beam sections. The dimensions are in [mm].

Predictions Specimen CB 3

With $n = 6.8$ the section properties in the negative moment regions are $E_{I,\text{sup}} = 33937$ kNm², $E_{II,\text{sup}} = 4674$ kNm², $M_{\text{sup}} = 13.5$ kNm and $\Delta L_{\text{sup}} = 1.31$ mrad/m ($\lambda = 1$). Accordingly the section properties in the positive moment regions are $E_{I,\text{span}} = 36037$ kNm², $E_{II,\text{span}} = 11904$ kNm², $M_{\text{span}} = 14.7$ kNm and $\Delta L_{\text{span}} = 0.46$ mrad/m ($\lambda = 1$). The average cracked stiffness is $E_{I,\text{avg}} = 8289$ kNm² and the ratio of the cracked stiffnesses $\kappa = E_{I,\text{sup}} / E_{I,\text{span}} = 0.4$.

With a middle support moment ($x = 0$) equal to

$$M_{x,\text{sup}} = \begin{cases} \frac{Q \cdot 3 \cdot m^2 \cdot (5 \cdot m + 2 \cdot m)}{2 \cdot 5^2 \cdot m^2} + \frac{2 \cdot \text{kNm}^2 \cdot 5^2 \cdot m^2}{8} & \text{if } 0 \leq x \leq 3 \text{ m} \\ \end{cases}$$

the moment distribution is

$$M(x) = \begin{cases} (1-\gamma) M_{x,\text{sup}} \left( 1 - \frac{x}{5 \text{ m}} \right) + \frac{Q \cdot 2 \cdot m}{5 \cdot m} x + 2 \cdot \text{kNm}^2 \left( \frac{5 \cdot m^2 - x^2}{2} \right) & \text{if } 0 \leq x \leq 3 \text{ m} \\ (1-\gamma) M_{x,\text{sup}} \left( 1 - \frac{x}{5 \text{ m}} \right) + \frac{Q \cdot 2 \cdot m}{5 \cdot m} x + 2 \cdot \text{kNm}^2 \left( \frac{5 \cdot m^2 - x^2}{2} \right) - Q(x - 3 \text{ m}) & \text{if } 3 \text{ m} \leq x \leq 5 \text{ m} \end{cases}$$

This leads to the curvature distribution

$$\chi = \begin{cases} \frac{M}{4674 \text{ kNm}^2} + k_1 1.31 \text{ mrad/m} & \text{if } M < -13.5 \text{ kNm}^2 \\ \frac{M}{33937 \text{ kNm}^2} & \text{if } -13.5 \text{ kNm} \leq M \leq 0 \\ \frac{M}{36037 \text{ kNm}^2} & \text{if } 0 \leq M \leq 14.7 \text{ kNm} \\ \frac{M}{11904 \text{ kNm}^2} - k_0 0.46 \text{ mrad/m} & \text{if } 14.7 \text{ kNm} \leq M \end{cases}$$

The compatibility equation

$$0 = \int_0^5 \chi \cdot M_1 \, dx$$

with $M_1 = x - 5 \text{ m}$ defines the moment distribution with factor $\gamma$. With Fig. 4.11 the State II distribution factor $\gamma$ for $\kappa = 0.4$ is estimated (as Q in CB3 is not at midspan) to be 0.7. The deflections under the load are determined with

$$a = \int_0^5 \chi \cdot M_2 \, dx$$

and $M_2 = x - 3 \text{ m}$. The average State II deflections are determined with $\gamma = 0$ and

$$a = \int_0^5 \frac{M}{8289 \text{ kNm}^2} \cdot M_2 \, dx$$

Fig. 5.32 compares the predicted and experimentally determined moment redistribution coefficients (Eq. (4.83)) and deflections of specimens 1 to 3. Also illustrated are the State II redistribution coefficients (dashed horizontal lines) estimated with Fig. 4.11 (a) and the State II average stiffness deflec-
tions (dashed lines). The deflections are set equal to zero at \( Q = 0 \). Due to the nature of the static system, the support and span cracking loads are similar.

For all three specimens illustrated in Fig. 5.32, the measured midspan support moments are smaller than predicted and the measured deflection curves are initially steeper and then flatter than the predicted curves. The measured deflections are larger for \( \kappa > 1 \) and smaller for \( \kappa < 1 \) than the average stiffness deflections, confirming the influence of the stiffness ratio illustrated in Fig. 4.11 (b). Fig. 5.33 compares the measured deflection curves of all four specimens, which are very similar despite

Fig. 5.32  Moment redistribution coefficients (Eq. (4.83)) and deflections for Taerwe's (1981) specimens CB1 to CB3, with \( \kappa = E_i I_{1,\text{sup}} / E_i I_{1,\text{span}} \) and \( \lambda = 1 \).
the different stiffness ratios and average cracked stiffnesses. Elastically reinforced CB1 ($\kappa = 1, E_c I_{II, CB1}$) has the smallest deflections, followed by CB4 ($\kappa = 0.9, 0.92 E_c I_{II, CB1}$) and CB2 ($\kappa = 2.1, 1.13 E_c I_{II, CB1}$) and finally CB3 ($\kappa = 0.4, 0.81 E_c I_{II, CB1}$). It can be seen that both the average stiffness and the stiffness distribution influence the deflections. Further, Fig. 5.33 shows the influence of the reinforcement detailing on the yield load and therefore the service load range of the members.

![Comparison of the measured deflection curves for Taerwe's (1981) specimens CB1 to CB4](image)

**Two-span slab strips tested at ETH Zurich**

![Overview of Alvarez and Marti's (1996) slab strips: (a) structural system; (b) slab section. The dimensions are in [mm].](image)

Alvarez and Marti (1996) tested 3 slab strips (ZP1 to ZP3), two with the same support reinforcement and two with the same span reinforcement. Fig. 5.35 illustrates the predicted and experimentally determined moment distribution coefficients and deflections. In all cases the moment redistributions between support and span cracking are overestimated. This is also reflected by the measured deflection curves between the support and span cracking loads; they are less stiff than predicted. The predicted State II (fully cracked) support-to-span moment redistributions are experimentally reached by ZP2 and ZP3, while the predicted State II span-to-support moment redistributions are not reached by...
ZP1. Nevertheless, after span cracking, the measured deflections are between the curves predicted with $\lambda = 1$ and $\lambda = 0$.

Fig. 5.36 compares the measured deflection curves for specimens ZP1 ($\kappa = 1.38, E_{I_{II,avg,ZP1}}$), ZP2 ($\kappa = 0.79, 1.3 E_{I_{II,avg,ZP1}}$) and ZP3 ($\kappa = 0.66, 0.7 E_{I_{II,avg,ZP1}}$). It can be seen that the deformation behaviour of these three specimens is dominated by the average cracked stiffness.

**Fig. 5.35** Moment distribution coefficients and deflection curves for Alvarez and Marti’s (1996) specimens ZP1 to ZP3 with $\kappa = E_{s,II,avg} / E_{s,II,span}$ and $\lambda = 1$. The curves end at nominal steel yielding.
Predictions Specimen ZP1

With $n = 6.7$ the section properties in the negative moment regions are $E_I^{\text{spec}} = 40055$ kNm$^2$, $E_{II}^{\text{spec}} = 7679$ kNm$^2$, $M_r^{\text{spec}} = 40.5$ kNm and $\Delta \chi_{ts}^{\text{spec}} = 2.26$ mrad/m ($\lambda = 1$). Accordingly the section properties in the positive moment regions are $E_I^{\text{spec}} = 38452$ kNm$^2$, $E_{II}^{\text{spec}} = 5559$ kNm$^2$, $M_r^{\text{spec}} = 39.1$ kNm and $\Delta \chi_{ts}^{\text{spec}} = 3.16$ mrad/m ($\lambda = 1$). The average cracked stiffness is $E_r^{\text{avg}} = 6619$ kNm$^2$ and the ratio of the cracked stiffnesses $\kappa$ is 1.4. With a middle support moment equal to

$$M_{x_{sup}} = \left[ (1.65 \text{ m})Q + \frac{6 \text{ kNm}^{-1} \cdot 6 \text{ m}^2}{8} \right]$$

and a midspan support reaction ($x = 0$) equal to

$$A = Q \left( 4.44 \text{ m} \cdot \frac{6}{6} \text{ m} + \frac{6 \text{ kNm}^{-1} \cdot 6 \text{ m}}{2} \right)$$

the moment distribution is

$$M(x) = \begin{cases} (1 - \gamma)M_{x_{sup}} + A \cdot (x - x^2 \cdot 6 \text{ kNm}^{-1}) \quad & \text{if } 0 \leq x \leq 2.58 \text{ m} \\ (1 - \gamma)M_{x_{sup}} + A \cdot (x - 2.58 \text{ m}) - \frac{x^2 \cdot 6 \text{ kNm}^{-1}}{2} \quad & \text{if } 2.58 \text{ m} \leq x \leq 4.98 \text{ m} \\ (1 - \gamma)M_{x_{sup}} + A \cdot (x - Q(2 \cdot x - 7.56 \text{ m})) - \frac{x^2 \cdot 6 \text{ kNm}^{-1}}{2} \quad & \text{if } 4.98 \text{ m} \leq x \leq 6 \text{ m} \end{cases}$$

With $M_1 = x - 6 \text{ m}$ and $M_2 = x - 3.6 \text{ m}$ the rest of the calculation occurs as illustrated for CB 3.

![Fig. 5.36](image)

Comparison of the measured deflection curves for Alvarez and Marti’s (1996) specimens ZP1 to ZP3.

Two-span beams tested during 400 days at the University of New South Wales

![Fig. 5.37](image)

Overview of Bakoss et al.’s (1982b; 1982a) beams: (a) structural system; (b) slab section. The dimensions are in [mm].
Bakoss et al. (1982b; 1982a) tested two 2-span beams (2B1 and 2B2) with midspan point loads sustained for 400 days. The two identical beams have an elastic, not curtailed reinforcement detailing with the same support and span reinforcement ratios ($\rho_{\text{span}} = \rho_{\text{sup}} = 1.7\%$). The support and span cracking loads are similar. The beams were tested horizontally in order to avoid the influence of self-weight. Therefore, the predictions are carried out with the interpolation equations given in Table 4.2, neglecting any moment redistributions ($\gamma = 0$).

![Fig. 5.38](image)

**Fig. 5.38** 23-day and 423-day deflection curves of the Bakoss et al. (1982b; 1982a) beams 2B1 and 2B2 ($\lambda = 1$). The average measured deflections (from three readings taken at the mid-points of three identical spans) are illustrated with crosses.

**423-day deflection predictions**

With $n = 7.5$ the 23-day stiffnesses are $E_c I_I = 880$ kNm$^2$ and $E_c I_{II} = 408$ kNm$^2$ and the compression zone heights are $x_I = 80$ mm and $x_{II} = 51$ mm. Accordingly, with $n = 24.8$ the effective 423-day stiffnesses are $E_c, e_I I = 332$ kNm$^2$ and $E_c, e_{II} I = 256$ kNm$^2$ and the compression zone heights are $x_{e_I} = 89$ mm and $x_{e_{II}} = 77$ mm. The State I net concrete section properties ($n = 0$) are $A_c = 0.0148$ m$^2$ and $x_c = 74$ mm. As the support and span stiffnesses are equal, if the splice sections are neglected, $\kappa$ is 1. The free shrinkage strains were measured on specimens with the same sections as the beams, so no conversions are necessary. The measured flexural tensile strength is transformed to

$$f_{ctf} = 4.9 \text{ MPa}$$

according to Eq. (3.6). The 23-day cracking moment equals 1.5 kNm.

The 423-day tension stiffening curvature reduction (Eq. (4.14) with $\lambda = 1$ and $\kappa = 0.3$) is

$$\Delta \chi_{e} = \kappa \varphi \left[ \frac{1.5 \text{kN m}(0.131 \text{ m} - 0.051 \text{ m})}{2(408 \text{kN m}^2(0.131 \text{ m} - 0.077 \text{ m}))} \right] = 0.5 \text{ mrad/m}$$

while the shrinkage curvatures are (Eq. (3.67))

$$\Delta \chi_{sh} = -0.64 \% \left[ \frac{0.0148 \text{ m}^3(0.074 \text{ m} - 0.089 \text{ m})}{0.0401 \text{ 10}^{-5} \text{ m}^2} \right] = 3.6 \text{ mrad/m}$$

and (Eq. (3.69))

$$\Delta \chi_{sh,k} = 0.64 \% \left[ \frac{3}{3 - 0.131 \text{ m} - 0.077 \text{ m}} \right] = 6.1 \text{ mrad/m}$$

The maximum concrete shrinkage tensile stresses at 423 days (Eq. (3.68))

$$\Delta \sigma_{sh,k} = -8.28 \text{ GPa} \left[ -0.64 \% \left[ 1 - \frac{0.01478 \text{ m}^3}{0.02027 \text{ m}^2} \right] \left( 0.15 \text{ m} - 0.089 \text{ m} \right) - 3.6 \text{ mrad/m} \right] = 3.2 \text{ MPa}$$
indicate that the entire beam is cracked by the end of the test, so that there is no need for interpolations. The total deflections of the fully cracked beam are estimated by adding the State II deflections

\[ a_n = \frac{7 \cdot 3 \cdot Q \cdot 3.5^4 \text{ m}^4}{144 \cdot 16 \cdot 256 \text{ kN m}^2} = 1.5 \text{ mm kN}^{-1} \]

the tension stiffening deflections \((\lambda = 1, k_\phi = 2/3, \zeta_\Phi = 3/11)\)

\[ \Delta a_{n,II} = \left( \frac{3}{11} - \frac{1}{8} - \frac{9}{121} \right) \cdot 3.5^2 \text{ m}^2 \cdot 0.5 \text{ mrad} = 0.4 \text{ mm} \]

and the State II shrinkage deflections

\[ \Delta a_{s,II} = \left( \frac{3}{11} - \frac{1}{8} - \frac{9}{121} \right) \cdot 3.5^2 \text{ m}^2 \cdot 6.1 \text{ mrad} = 5.5 \text{ mm} \]

### 5.3 Slabs

Tabulated elastic solutions and linear finite element programs provide the deflections of uncracked flat plates. However, once cracking starts, the stiffness matrix becomes dependent on the stress-resultants and, on principle, non-linear finite element calculations are required in order to consider the influence of cracking on the bending and twisting stiffnesses. This means that flat-slab deflection predictions must rely on either non-linear finite element calculations or approximate methods.

This Chapter aims at illustrating the influence of cracking, creep and moment redistributions on flat plate deflections with three test series of large-scale flat plates under sustained uniform loads. As a reference, the observed deflections are compared with the corresponding deflections \(a = k_a q l^4 / D\) estimated with the uncracked elastic stiffness

\[ D^I = \frac{E_t h^3}{12(1 - \nu^2)} \]  

as well as a reduced stiffness

\[ D^{II} \approx D^I \frac{I_{II,avg}}{I_g} \]  

in an attempt to account for cracking. The latter is a gross simplification neglecting both the anisotropic nature of a cracked slab and the fact that the twisting stiffness is more affected by cracking than the bending stiffness. Creep is considered with the effective elastic modulus (Eq. (3.65)). The cracking loads \(q_c = m_r / (k_M \bar{f})\) are estimated with

\[ m_r = \frac{(f_{cr} - \Delta \sigma_{sh,1}) h^2}{6} \]  

neglecting any moment redistributions due to cracking.

**Simply supported rectangular slabs tested at EPF Lausanne**

Jaccoud and Favre (1982) (Fig. 5.39 (a)) tested three 4 m by 4 m, 120 mm thick slabs (B1 to B3), which are simply supported at the edges and have clamped corners. Tellenbach (1984) tested three 2.7 m by 2.7 m, 110 mm thick slabs. D1 and D2 (Fig. 5.39 (b)) have simply supported edges and clamped corners, while D3 (Fig. 5.39 (c)) is only supported by edge columns. In contrast to the other two slabs, D2 is skewly reinforced. All slabs were loaded with sustained uniform loads and the measured deflections are illustrated in Fig. 5.40.
In Fig. 5.40 the measured deflections are marked with crosses. According to the test reports, all slabs except B1 featured advanced crack patterns on the bottom surface after initial loading. B1 was uncracked at initial loading, but partially cracked by the end of the test. The deflection estimates determined with $D^I$ (Eq. (5.2)) and $D^{II}$ (Eq. (5.3)) do not include shrinkage, which in the absence of span compression reinforcement, is considered responsible for the difference between the observed and estimated $D^{II}$ deflections.

373-day deflection predictions for B1

The 373-day stiffnesses $D^I = 1139$ kNm and $D^{II} = 492$ kNm are estimated with $v = 0, n = 25.9, \rho_{avg} = 0.55\%$ and $d_{avg} = 95$ mm. With a midspan deflection factor $k_a = 0.00406$ according to e.g. Bareš (1971) the deflections are

$$a_i = 0.00406 \frac{q \cdot 4^4 m^4}{1139 \text{ kNm}} = 0.91 \text{ mm/kNm}$$

and

$$a_a = 0.00406 \frac{q \cdot 4^4 m^4}{492 \text{ kNm}} = 2.11 \text{ mm/kNm}$$

For a simply supported unit width slab strip with $A_c = 0.119$ m$^2$, $x_c = 58.9$ mm, $I_{ef} = 0.1584 \times 10^{-3}$ m$^4$, $x_{ef} = 63.4$ mm and $x_{ef} = 39.0$ mm (average stiffness) the reduced free shrinkage strains $\varepsilon_{sh} = -0.33 \%$ lead to $\Delta \chi_{sh, I} = 0.9$ mrad/m, $\Delta \chi_{sh, II} = 4$ mrad/m and $\Delta \sigma_{sh} = 0.7$ MPa. As a consequence

$$\Delta a_{\chi, I} = \frac{1}{8} 4^4 m^4 \cdot 0.9 \text{ mrad/m} = 1.8 \text{ mm}$$

$$\Delta a_{\chi, II} = \frac{1}{8} 4^4 m^4 \cdot 4 \cdot 0.9 \text{ mrad/m} = 7.9 \text{ mm}$$

as well as

$$M_c = \left(3.05 \text{ MPa} - 0.7 \text{ MPa}\right) \left(1 \text{ m} \cdot 0.12^2 \text{m}^2\right) = 5.7 \text{ kNm}$$

The cracking load $q_c$ is determined assuming a maximum span moment equal to 0.0368 $q_L^2$ (e.g. Bareš (1971)) for $v = 0$. The shrinkage deflections of 7.9 mm (not illustrated in Fig. 5.40) correspond to the difference between the observed and predicted $D^{II}$ deflections at $t = 373$ days.
Fig. 5.40 Mid-panel slab deflections for slabs B1 to B3 (Jaccoud and Favre 1982) and D1 to D3 (Tellenbach 1984). Slabs B1, B2, B3, D1 and D2 are simply supported at their edges (with clamped corners), while slab D3 is only supported at its corners.
Flat-slab corner panels under sustained loads tested at the University of New South Wales

Guo and Gilbert (2002) tested seven 7.2 m by 6.2 m flat slabs supported on columns, which were pinned (S6) or fixed (S2, S4, S5 and S7) at the base. Each slab has four 3 m by 3 m corner panels of the type illustrated in Fig. 5.41 (a). Slabs S2, S4, S6 and S7 were uniformly loaded at 14 days and then maintained loaded for 1 to 2 years depending on the specimen. Slab S5 was loaded with 6.23 kN/m² at 14 days unloaded again at 15 days and then left for 732 days under self-weight (2.16 kN/m²).

Fig. 5.41  Overview of Guo and Gilbert's (2002) uniformly loaded flat plates. They are supported by 9 columns and consist of four identical corner panels: (a) one corner panel; (b) reinforcement in section B-B. The dimensions are in [mm]. The two bottom and the two top layers have the same bar spacing. The upper two layers are limited to an area of 1.8 m by 1.8 m over the columns.

Fig. 5.42 shows the measured deflections (crosses) of the slabs (average of four panels). The figure also contains elastic deflection estimates determined with $D^I$ (Eq. (5.2)) and $D^II$ (Eq. (5.3)) (continuous lines) as well as estimated support and span cracking loads (dashed lines). They were determined with the elastic factors $k_s = 0.01326$, $k_M = -0.256$ (support, $v = 0$) and $k_M = 0.0533$ (span, $v = 0$) given in the CEB Bulletin No. 158 (Comité Euro-International du Béton 1985) for the corner panel of a twelve-panel example slab. Note that the support-to-span moment ratio of Guo and Gilbert's slabs is 5. The stiffnesses $D^I$ and $D^II$ ($\rho_{avg}$ and $d_{avg}$ of the bottom reinforcement) are determined with the 14-day and corresponding long-term effective elastic modulus (for $\varphi$ see Table A. 6 in Appendix A) assuming $v = 0$. The additional shrinkage stresses are determined as shown above for B1.

As can be seen in Fig. 5.42, the applied load levels are between the 14-day support and span cracking loads. Accordingly, after loading all slabs feature cracks on the top surface above the columns. Despite the presence of these support cracks, the observed 14-day deflections correspond to the values estimated with the uncracked stiffness $D^I$. Support-to-span moment redistributions due to cracking are considered responsible for maintaining the uncracked stiffness. After sustained loading (for 1 to 2 years depending on the specimen) all specimens feature cracks on the bottom surface. Nevertheless, even the deflections of slab S4 with the most advanced crack pattern still correspond to the uncracked $D^I$ deflections (determined with the effective modulus ratio).
Fig. 5.42  Mid-panel deflections of Guo and Gilbert's (2002) uniformly loaded panels. The predicted deflections (continuous lines) are determined with $D^I$ (Eq. (5.2)) and $D^{II}$ (Eq. (5.3)).
Cardington in-situ concrete frame building

Fig. 5.43  Plan view of the Cardington in-situ building. The flat plates are 250 mm thick and consist of twelve 7.5 m by 7.5 m column-supported panels. The dimensions are in [mm].

At Cardington a seven-story in-situ concrete frame (Vollum et al. (2002)) was built and tested under service loads. Each floor slab has twelve 7.5m by 7.5m floor panels (Fig. 5.43), which were tested under sustained loads. The flat plates are 250 mm thick and column supported. The columns are fixed at both ends.

<table>
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<tr>
<th>floor</th>
<th>design</th>
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<th>striking</th>
<th>concrete type</th>
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<td>6</td>
<td>FE</td>
<td>blanket cover two-way mats 25.5</td>
<td>1</td>
<td>3</td>
<td>pump 40 % GGBS</td>
<td>56</td>
</tr>
</tbody>
</table>

Table 5.3  Cardington flat plates. Notes a) No additional reinforcement for serviceability; b) north and south panels; c) Ground Granulated Blastfurnace Slag.

The floors have different reinforcement detailings, back-prop conditions and concrete properties (see Table 5.3). The concrete properties are given in Table A.6 of Appendix A. The flat plates were stripped of their formwork 1 to 7 days after loading (see Table 5.3), left for approximately one year under self-weight and then loaded with 3 kN/m² for another year.

The mid-panel deflections of floors 1 to 6 are illustrated in Fig. 5.44 (average of two internal panels) and Fig. 5.45 (average of four edge panels). The Cardington floor plan corresponds to the example flat plate given in the CEB report 158 (Comité Euro-International du Béton 1985). The elastic deflection and moment coefficients provided by the report are used to estimate the deflections and cracking loads. The cracking moment (Eq. (5.4)) is estimated assuming shrinkage strains of -0.3 ‰ (reduced from -0.5 ‰ to account for the specimen size).
The deflections after striking correspond to the elastic short-term deflections even though self-weight alone leads to cracking at the supports. According to the assumed moment distribution (neglecting moment redistributions), the construction overloads cause cracking on the bottom surface of the edge slabs and between the internal columns. This agrees with the visual inspection, where the slabs were found to be extensively cracked. Despite the extensive cracking, particularly the mid-panel deflections of the edge panels remain close to the corresponding elastic deflections determined with the 600-day creep coefficient. While all floors have similar deflections after striking, there is a considerable scatter after 800 days. In both cases the floors 1 and 4 with the highest 28-day concrete strength show the smallest deflections.
Fig. 5.45  Mid-panel deflections edge panels: $\nu = 0$, $E_c = 27 \text{ GPa}$, $\varphi = 2.2$, $\epsilon_{sh} = -300 \times 10^{-6}$, $k_a = 0.00917$, $k_M = -0.256$ (top face over internal columns), $k_M = 0.0536$ (bottom face between internal columns) and $k_M = 0.0576$ (bottom face span).

5.4 Summary

This chapter verifies the approaches discussed in Chapter 4 with a selection of test series from various research institutions. These test data comparisons show that:

- A good upper limit can be provided for the steel stresses, crack widths and integral deformations of end-restrained tensile ties during the crack formation phase (see Fig. 5.4 and Fig. 5.27 (d)). However, the theoretical saw teeth-like steel stress, crack width and deformation fluctuations (see Fig. 4.6) caused by the discrete cracks must be regarded as guidelines.

- The short-term State I and II section and corresponding member behaviour of beams is well understood (see e.g. Fig. 5.10, Fig. 5.13, Fig. 5.16, Fig. 5.22, Fig. 5.23 and Fig. 5.38) as long as there are no moment redistributions. The stiffness-dependent moment distributions of statically indeterminate beams are a challenge to predict (see Fig. 5.32 and Fig. 5.35). For the two-span beams illustrated in this chapter the predicted support moments altogether overestimate the experimentally determined support moments.
• In combination with the shrinkage Eqs. (3.67) and (4.87), the Effective Modulus Method (EMM) enables straightforward predictions for the State I and II section and corresponding member deformations of beams under sustained loads (see e.g. Fig. 5.13, Fig. 5.16 and Fig. 5.38).

• With the effective reinforcement ratio according to Eq. (4.4) the Tension Chord Model mostly provides good estimates for the maximum crack spacings ($\lambda = 1$) in bending members. Together with the State II steel and concrete shrinkage strains they lead to reasonable upper ($k_\nu = 0$) crack width limits (see e.g. Fig. 5.11, Fig. 5.17, Fig. 5.18). Especially for low reinforcement ratios Schiessl's effective reinforcement ratio (Eq. (2.8)) leads to less consistent crack width predictions because, as pointed out in Chapter 4.1.1, it is incompatible with the Bernoulli bending theory (see MC 2010 curve in Fig. 5.11). The prediction quality of Frosch's crack width equation is highly variable and there can be an unrealistically large difference between the predicted side face and bottom crack widths (see e.g. Fig. 5.18).

• The Tension Chord Model provides good lower limits for the average steel strains, average section curvatures, crack widths and integral deformations, which are determined assuming the maximum influence of tension stiffening ($\lambda = 1$ and $k_\nu$ according to Eq. (4.11)). However, it is not possible to locate the effective influence of tension stiffening between the upper ($k_\nu = 0$) and lower deformation limits (see Fig. 5.2, Fig. 5.10, Fig. 5.11, Fig. 5.13, Fig. 5.14, Fig. 5.16, Fig. 5.17, Fig. 5.18, Fig. 5.22, Fig. 5.23 and Fig. 5.38). For the statically determinate beams regarded here, the EC 2 (2004) load – deformation curves are mostly located between the two limits provided with the Tension Chord Model.

• The predicted initial cracking loads determined with the average tensile strength both over- and underestimate the observed cracking loads (see Fig. 5.4, Fig. 5.10, Fig. 5.22 and Fig. 5.23). Further, the span cracking loads in statically indeterminate beams are strongly influenced by the moment redistributions (see Fig. 5.32 and Fig. 5.35).

• The uncracked elastic (effective modulus) slab panel deformations (corner, edge and centre panels) represent reasonable estimates even for extensively cracked slabs (see Fig. 5.42, Fig. 5.44 and Fig. 5.45). Possibly moment redistributions and membrane action are responsible for preserving the stiffness.
6 Discussion

6.1 Crack Width

Crack control can imply preventing cracks or limiting their widths. The concrete tensile stresses that lead to cracking are defined by the nature of the imposed or restrained deformations and the boundary conditions, while the crack widths are defined by the State II steel stresses, the crack spacing and the strains in the concrete between the cracks. This chapter discusses the influence of these parameters.

6.1.1 Tensile Members

Fig. 6.1 to Fig. 6.4 illustrate the cracking behaviour of a tensile tie according to Chapters 4.2.2 and 4.2.3 for reinforcement ratios above $\rho = A_s/(bh) = 0.5 \%$. The discussion includes axial loads as well as imposed and restrained deformations. The restrained deformations consist of uniform concrete shrinkage strains as well as uniform cooling-induced steel and concrete strains. Restraint stiffnesses $\kappa$ (Eq. (4.38)) between the limiting cases of $\kappa = \infty$ and 0 are considered. The discussion also applies to the fictitious tension chords used to model base restrained walls (see Case (c) in Section Continuous Edge-Restraint of Chapter 4.2.2). The dashed vertical lines apply to the example member discussed below.

In order to prevent cracks from forming, the load-, deformation- or restrained concrete strains must not reach the values that start the crack formation phase (see Fig. 6.1). These limiting concrete strains increase with increasing concrete tensile strength, but unfortunately so do the corresponding steel stresses (Fig. 6.2) and crack widths (Fig. 6.4) during the crack formation phase. The start of restraint-induced cracking (Fig. 6.1 (c, d)) can further be delayed by reducing the shrinkage- or cooling-induced strains or the degree of restraint. The influence of the reinforcement ratio is small, except for very low restraint stiffnesses.

For limiting crack widths it is convenient to distinguish between the crack formation phase and stabilized cracking. Fig. 6.1 illustrates the concrete strains delimiting the crack formation phase. In contrast to the strains at the start of the crack formation phase, the strains at the end of the crack formation phase are strongly dependent on the reinforcement ratio. For deformation- or restraint-induced cracking, increasing the reinforcement ratio reduces the length of the crack formation phase and therefore increases the rate at which the cracks form. As a constant tensile strength is assumed, axially loaded tensile ties (Fig. 6.1 (a)) and members with internally restrained shrinkage strains ($\kappa = 0$) (Fig. 6.1 (d)) do not feature a theoretical crack formation phase, because cracking does not globally relieve the stress level. Under axial load all cracks form simultaneously upon reaching $N = N_r$, causing the strain increase illustrated in Fig. 6.1 (a).
The most important measure for crack width limitation is to ensure that the reinforcing steel does not yield under service loads. Fig. 6.2 illustrates the maximum steel stresses during the crack formation phase. They correspond to the steel stresses reached at \( N = N_r \) for axial loading and each time before a new crack forms during the entire crack formation phase for deformation- or cooling- induced cracking (Fig. 6.2 (a)). For shrinkage-induced cracking (Fig. 6.2 (b)), the illustrated steel stresses correspond to the steel stresses reached in the first crack just before a second crack occurs (point B in Fig. 4.6 (b)). In both cases the steel stress is primarily dependent on the concrete tensile strength and the reinforcement ratio. For serviceability concerns, yielding must also be prevented during stabilized cracking. Chapters 4.2.2 and 4.2.3 contain equations for determining the steel stresses during stabilized cracking.

The second most important parameter is the transfer length (Fig. 6.3 (a)), which is directly proportional to the crack width. According to the slipping-bond approach, increasing the reinforcement ratio or reducing the bar diameter decreases the transfer length.
Discussion

Fig. 6.2 Maximum State II steel stresses during the crack formation phase (Eq. (4.45)): (a) load, deformation or restraint-induced cracking with $\varepsilon_{si} - \varepsilon_{ci} = 0$ (e.g. due to cooling); (b) restraint-induced cracking with $\varepsilon_{si} - \varepsilon_{ci} \neq 0$ (e.g. due to shrinkage) and $\varepsilon_{ci,r,b}$ acc. to Eq. (4.52)). Note that $n = 6$.

Fig. 6.3 Tensile tie: (a) transfer length (Eq. (3.74)) related to bar diameter; (b) immediate contribution of the tensile concrete (Eq. (6.1) with $\lambda = k_0 = 1$). Note that for $f_{ct} = 2$ MPa and $E_s = 200$ GPa the quotient $10^5 f_{ct}/E_s$ equals 1.

Fig. 6.3 (b) illustrates the average reduction of the steel and concrete strain difference (third term in Eq. (4.36))

$$\Delta\varepsilon_{ts} = \frac{k_0 k_{f_{ct}} (1 + n\rho - \rho)}{2\rho E_s}$$

(6.1)

due to tension stiffening upon initial ($k_0 = 1$) loading. For $\rho = 0.5 \%$, $f_{ct} = 2$ MPa and $E_s = 200$ GPa the strain difference responsible for the crack opening is reduced by 1 % if $\lambda = k_0 = 1$. However, both bond creep (Fig. 5.2) and the release of restrained shrinkage strains (see Fig. 5.22) reduce the influence of tension stiffening.
Fig. 6.4 Crack widths: (a) axial load (Eq. (3.79)); (b) imposed deformations (Eqs. (4.77) and (4.79)); end-restrained deformations (Eqs. (4.46)) and (4.60)) due to (c) shrinkage/cooling with $\kappa = \infty$; (d) cooling with $\kappa = 0.05$; (e) shrinkage with $\kappa = 0.05$; (f) shrinkage with $\kappa = 0$. Note that $n = 6$ and $\lambda = 1$. The diagrams are valid as long as there is no steel yielding. Note that for $f_{ci} = 2$ MPa, $E_s = 200$ GPa and $\varnothing = 10$ mm the factor multiplying $w$ equals 1.

Fig. 6.4 illustrates the maximum ($\lambda = 1$) crack widths during the crack formation phase and stabilized cracking for imposed loads, imposed deformations and restrained deformations. The curves are valid for steel strains below the yield limit. During the crack formation phase the crack widths are identical in all illustrated cases and strongly dependent on the concrete tensile strength. The higher the concrete tensile strength, the larger is the crack width. This makes the concrete tensile strength an important parameter for tensile ties with extensive crack formation phases. After conclusion of the crack
formation phase the crack widths increase (see also Fig. 5.8). During stabilized cracking, the influence of the concrete tensile strength decreases.

Fig. 6.5 illustrates minimum reinforcement ratios for limiting the crack widths illustrated in Fig. 6.4 (a) and Fig. 6.4 (c). According to Fig. 6.5 (b), a reinforcement ratio of 1.2 % ensures a related crack width $w_{\text{lim}} 10^{-4} E_s / (\varnothing f_{\text{ct}})$ below 0.1 (e.g. 0.2 mm for $\varnothing = 10$ mm and $f_{\text{ct}} = 2$ MPa or 0.4 mm for $\varnothing = 10$ mm and $f_{\text{ct}} = 4$ MPa) as long as the shrinkage- and cooling-induced concrete strains remain below -0.5 ‰. 

![Diagram](image)

**Fig. 6.5** Minimum reinforcement ratios for crack width control: (a) axial load (Eq. (3.79)); (b) $\kappa = \infty$ end-restrained shrinkage and cooling strains (Eqs. (4.46) and (4.63)). Note that $n = 6$ and $\lambda = 1$. For $f_{\text{ct}} = 2$ MPa and $E_s = 200$ GPa the quotient $10^5 f_{\text{ct}}/E_s$ equals 1.

**Example Member**

The dashed lines in Fig. 6.1 to Fig. 6.5 belong to an example member with $\rho = 1 \%, f_{\text{ct}} = 2$ MPa, $\varnothing = 10$ mm, $n = 6$ and $E_s = 200$ GPa. Independent of the load case, the example member has a transfer length of 124 mm (Fig. 6.3 (a)) and a final crack spacing between 124 mm ($\lambda = 0.5$) and 248 mm ($\lambda = 1$). The concrete between the cracks reduces the steel and concrete strain difference responsible for the crack width by 0.53 ‰ for $\lambda = 1$ (Fig. 6.3 (b)).

Under an axial load the example member cracks at $N = N_c$, causing the strains in the member to increase from 0.06 ‰ to 0.56 ‰ (Fig. 6.1 (a)). At $N = N$, the State II steel stresses equal 210 MPa (Fig. 6.2 (a)) with a maximal crack width of 0.13 mm (Fig. 6.4 (a)). Further loading up to a 400 MPa steel stress leads to maximum crack widths of 0.37 mm (Fig. 6.4 (a)).

Imposed deformations (Fig. 6.1 (b)) and fully end-restrained ($\kappa = \infty$) (Fig. 6.1(c, d)) cooling-/shrinkage-strains with absolute values between 0.06 ‰ and 0.56 ‰ (crack formation phase) also lead to maximum crack widths of 0.13 mm (Fig. 6.4 (a)). After the crack formation phase is concluded the crack widths increase, e.g. to 0.24 mm at 1 ‰.

Reducing the restraint stiffness $\kappa$ shifts the crack formation phase, e.g. for shrinkage-induced strains to the limits of $\varepsilon_{ci} = -0.23 \%$ and -0.64 ‰ for $\kappa = 0.05$ or to $\varepsilon_{ci} = -1.05 \%$ for $\kappa = 0$ (Fig. 6.1 (d)). This does not affect the crack widths during the crack formation phase (Fig. 6.4 (d to f)), but the crack widths during stabilized cracking decrease with decreasing restraint stiffness. Independent of the restraint stiffness ($\kappa > 0$), for restrained cooling strains the maximum steel stress during the crack formation phase (Fig. 6.2 (a)) is 210 MPa. In contrast, for restrained shrinkage strains (Fig. 6.2 (b)) the maximal steel stress during the crack formation phase depends on the length of the tensile tie and the restraint stiffness.
### 6.1.2 Bending Members

Fig. 6.6 illustrates the basic parameters of moment-induced cracking according to Chapter 4.2.1. The State II steel stresses at cracking (Fig. 6.6 (a)) are determined with the State II section properties.

<table>
<thead>
<tr>
<th>$\sigma_{II}$, $\sigma_{I}$</th>
<th>$\rho_{ef}$</th>
<th>$\rho$</th>
</tr>
</thead>
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<tr>
<td>250</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>200</td>
<td>0.50</td>
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<td>1.00</td>
<td>1.50</td>
</tr>
<tr>
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<td>1.50</td>
<td>2.00</td>
</tr>
<tr>
<td>50</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

The effective reinforcement ratio (Fig. 6.6 (b)) is more controversial. However, regardless of whether Schiessl's (Eq. (2.8)) or the Tension Chord Model (Eq. (4.4)) expression is chosen, the effective reinforcement ratio soon reaches values, at which the influence of the tensile concrete is small (Fig. 6.6 (c)). As can also be seen in Fig. 5.14 and Fig. 5.17, for $\rho = A_v/(bd)$ above approximately 0.5% it does not make any difference if the contribution of the tensile concrete

$$\Delta \varepsilon_{ct} = k_0 \frac{M_f d_i (d-x_i)}{2E_i I_{ii} (h-x_i)}$$  \hspace{1cm} (6.2)

is considered ($k_0 = 1$) or not ($k_0 = 0$). Consequently, Frosch's (1999) crack width Eq. (2.12) does not consider the influence of the tensile concrete. However, the apparently small difference between effective reinforcement ratios has a large influence on the tensile concrete contribution (Fig. 6.6 (c)) at low reinforcement ratios. This can also be seen in Fig. 5.11 (d). The difference between the two effective reinforcement ratios most significantly influences the transfer length (Fig. 6.6 (d)). For the $d$, $c$ and $\varnothing$ values assumed in Fig. 6.6 (d), Frosch's bottom face transfer length and the Tension Chord Model are shown.
Model transfer length are similar. In contrast, Schiessl’s effective reinforcement ratio used for the MC 2010 and EC 2 equations leads to larger transfer lengths. As a consequence, the MC 2010 steel stress vs. crack width relations in Figs. 5.14, 5.17, 5.18 are much steeper than the tension chord model relations. If an additional cover term is considered (EC 2), then an even larger transfer length results.

Fig. 6.7 illustrates related moment-induced crack widths determined with Eq. (4.27). For the short-term crack widths \( \varepsilon_{ci} = 0 \) with \( k_0 = 1 \) are adopted, while the long-term crack widths assume shrinkage strains equal to \(-50 \frac{f_{ct}}{E_s}\) as well as \( k_0 = 0 \) and 1. Again, comparing the \( k_0 = 0 \) and 1 curves in Fig. 6.7 (b) shows that for reinforcement ratios above approximately 0.5 % and certainly above 1 %, the contribution of the tensile concrete becomes insignificant. In contrast, the influence of shrinkage is clearly visible. This can also be seen in the crack width measurements illustrated in Fig. 5.14, Fig. 5.17 and Fig. 5.18.

**Example Member**

The dashed lines in Fig. 6.7 illustrate an example slab strip with \( \rho = \frac{A_s}{bd} = 0.2 \% \), \( f_{ct} = 2 \text{ MPa} \), \( \phi = 10 \text{ mm} \) and \( E_s = 200 \text{ GPa} \). In this case the factor multiplying \( w \) equals 1. For the example member, the steel stresses at cracking (Fig. 6.6 (a)) are 100 · 2 MPa = 200 MPa and the transfer length (Fig. 6.6 (d)) is 12 · 10 mm = 120 mm, implying a maximal crack spacing of 240 mm. This leads to an initial crack width (Fig. 6.7 (a)) of 0.12 mm. Increasing the steel stresses up to 150 · 2 MPa = 300 MPa leads to 0.25 mm cracks (Fig. 6.7 (a)). Alternatively (Fig. 6.7 (b)), if the cracking moment is maintained, shrinkage strains equal to -0.50 \( \frac{\varepsilon_{ci}}{E_s} \) also lead to 0.25 mm cracks or even 0.35 mm cracks, if tension stiffening is completely lost (\( k_0 = 0 \)).

**Fig. 6.7**  Maximum moment-induced crack widths (Eq. (4.27)) with \( n = 6, \rho' = 0, \lambda = 1 \) and \( d/h = 0.9 \): (a) \( k_0 = 1 \) and \( \varepsilon_{ci} = 0 \); (b) \( k_0 = 0 \)…1 and \( \varepsilon_{ci} \phi f_{ct} = -50 \). With \( f_{ct} = 2 \text{ MPa} \), \( E_s = 200 \text{ GPa} \), \( \phi = 10 \text{ mm} \) the factor multiplying \( w \) equals 1.

Fig. 6.8 illustrates the minimum reinforcement ratios necessary for limiting the moment-induced crack widths in Fig. 6.7. For example, in order to limit bending crack widths below 0.15 \( \phi f_{ct} / (E_s 10^{-4}) \) (e.g. 0.3 mm for \( \phi = 10 \text{ mm} \) and \( f_{ct} = 2 \text{ MPa} \) or 0.6 mm for \( \phi = 20 \text{ mm} \) and \( f_{ct} = 2 \text{ MPa} \)) at steel stresses equal to 150 \( f_{ct} \) (300 MPa for \( f_{ct} = 2 \text{ MPa} \)) a minimal reinforcement ratio \( \rho = A_s/(bd) \) of 0.4 % is necessary for short-term loads (Fig. 6.8 (a)) or 0.7 % for sustained loads (Fig. 6.8 (b)).
6.2 Deflections

There are a wide range of parameters influencing the deflections of RC bending members, such as the boundary conditions of the structural system, the load-history, the geometry and detailing of the individual sections as well as material parameters. The task of obtaining all of these input parameters at an accuracy level in line with our analytical models is already a challenge for uniformly reinforced simple beams tested under laboratory conditions, but hardly possible for the statically indeterminate members with curtailed reinforcement and unknown load histories used in practice. This means that strong simplifications are necessary. This chapter aims at identifying and discussing the decisive parameters for practical deflection estimates based on the equations presented in Chapter 4.3 and the test data validations discussed in Chapters 5.2 and 5.3.

6.2.1 Section Level

Influence of Cracking

Fig. 6.9 illustrates the ratio of the State I and II curvature for rectangular sections (Table 3.1), using the EMM (see Chapter 3.3.2) to consider creep. Both creep and additional tensile reinforcement reduce the difference between the uncracked and cracked section stiffness. If a lightly reinforced uncracked section is falsely modelled as a cracked section, then the immediate curvature is overestimated by over 90 %, while for a section with $\rho = 3 \%$ and $\varphi = 3$, the curvature would only be overestimated by approximately 20 %. It is worth noting that the stiffness loss at cracking is larger for $\rho' = \rho$ than for $\rho' = 0$. 
Curvature ratio for rectangular sections with $\rho \geq 0.15\%$: (a) without creep; (b) with creep according to the EMM. Note that $n = 6$ and $d' = h - d$.

Influence of Tension Stiffening

The reinforcement ratio has a much larger influence on the absolute tension stiffening curvature reduction (Fig. 6.10) than on the relative tension stiffening curvature reduction (Fig. 6.11). According to the Tension Chord Model (Eq. (4.14)), tension stiffening initially reduces the State II curvature by 50 to 20\% and by less than 20\% under sustained loads (Fig. 6.11). In comparison, Rao’s semi-empirical tension stiffening approach (Eq. (2.20)) features a larger tension stiffening influence at moment ratios below 2. It is likely that the average strain measurements from the constant moment zone of the 4-point bending tests Rao used to derive his equation contained uncracked strains at low load levels due to the scatter in the concrete tensile strength.

Influence of tension stiffening at section level according to Eq. (4.14). Note that $\rho' = 0$, $n = 6$ and $\lambda = 1$. 
Influence of Creep

According to the EMM, a creep coefficient of three increases the immediate uncracked section curvature by 100 to 300 % (Fig. 6.12 (a)). The creep curvature of uncracked sections can be reduced by increasing the tensile and compression reinforcement ratios or the effective depth. In a cracked section (Fig. 6.12 (b)) the creep curvature is much less sensitive to the tensile reinforcement ratio. However, in the presence of high tensile reinforcement ratios, the positive influence of the compression reinforcement is notable. For $\rho < 3 \%$ the State II curvature is increased by up to 100 % in the case of $\varphi = 3$. For comparison, the dashed lines illustrate the curvature ratios determined with Ghali and Favre's (1986) curvature coefficients (Eq. (3.60)).

![Graphs illustrating the influence of tension stiffening at section level.](image)

**Fig. 6.11** Relative influence of tension stiffening at section level according to Eq. (4.14). Rao's tension stiffening Eq. (2.20) is illustrated with dashed lines. Note that $\rho' = 0$, $n = 6$, $\lambda = 1$ und $d = 0.85 \, h$.

Influence of Shrinkage

The shrinkage curvatures (Eq. (3.67)) illustrated in Fig. 6.13 are related to the basic shrinkage curvature $-\varepsilon_{sh}/d$. For a given shrinkage strain, the basic shrinkage curvature decreases linearly with increasing effective section depth. In an uncracked section (Fig. 6.13 (a, c)), the shrinkage curvature increases with increasing tensile reinforcement ratio, unless compression steel is provided. Under the given assumptions, shrinkage does not cause additional curvatures in uncracked symmetrically reinforced sections. In the absence of compression reinforcement, the shrinkage curvature of cracked sections (Fig. 6.13 (b, d)) approximately corresponds to the basic shrinkage curvature $-\varepsilon_{sh}/d$. Compression reinforcement reduces the shrinkage curvature particularly in the case of large tensile rein-
Discussion

For comparison the curvature ratios determined with Ghali and Favre's (1986) curvature coefficients (Eq. (3.61)) are illustrated with dashed lines.

\[
\chi = \frac{\varphi}{\varphi' + 1}
\]

\[
\rho = 0, d = 0.8 h = 0.9 h
\]

\[
\rho = \rho, d = 0.8 h = 0.9 h
\]

**Fig. 6.12** Influence of creep at section level: (a) uncracked section; (b) cracked section. The reinforcement and effective depth parameters apply to the EMM (continuous lines) and Ghali and Favre's (1986) curvature coefficients (dashed lines according to Eq. (3.60)). Note that \( n = 6 \) and \( d' = h - d \).

**Comparison of Effects**

For an uncracked rectangular cross section with a given moment \( M \) the initial curvature \( \chi_I \) is primarily a function of the section height \( h \) and the concrete elastic modulus \( E_c \). If \( M \) is sustained until \( \varphi \) reaches 3, the resulting curvature is between 2 and 4 times the initial value. In non-symmetrically reinforced sections shrinkage will further increase the curvature. The shrinkage curvature increases with increasing \( \rho \) up to around \(-\varepsilon_{sh}/d\) at \( \rho = 3\% \). An increasing section height has a positive influence on the initial curvature and the basic shrinkage curvature. Both tensile and compression reinforcement have a positive influence on the creep curvature. Asymmetric reinforcement distributions have a negative influence on the shrinkage curvature.

If the same section is cracked, the initial curvature \( \chi_{II} \) is between 2 and 10 times larger depending on the reinforcement ratio. Creep maximally doubles \( \chi_{II} \) in absence of compression reinforcement. For \( \varepsilon_{sh} = -1\% \), \( E_s = 200 \) GPa and \( f_{ct} = 2 \) MPa the multiplier of \( \Delta \chi_{II} \) in Fig. 6.10 and of \( \Delta \chi_{sh} \) in Fig. 6.13 are equal and the influence of tension stiffening and shrinkage can be compared. In the absence of compression reinforcement, the initial tension stiffening curvature reduction at low reinforcement ratios (Fig. 6.10 (a)) is comparable with the shrinkage curvature for shrinkage strains \( \varepsilon_{sh} = -1\% \) (Fig. 6.13 (d)), while the influence of tension stiffening under sustained loads (Fig. 6.10 (b)) is much lower. An increasing section height has a positive influence on the basic shrinkage strains, while an increasing reinforcement ratio reduces the stiffness loss at cracking, the influence of tension stiffening and shrinkage.
Fig. 6.13 Influence of shrinkage at section level according to Eq. (3.67): (a) uncracked section with \( \varphi = 3 \); (b) cracked section with \( \varphi = 3 \); (c) uncracked section with \( \varphi = 1 \); (d) cracked section with \( \varphi = 1 \). The dashed lines illustrate Ghali and Favre’s (1986) shrinkage curvature coefficient (Eq. (3.61)). Note that \( n = 6 \) and \( d' = h - d \).

### 6.2.2 Member Level

#### Constant Stiffness

Fig. 6.14 shows the related deflections of a simple beam with uniformly reinforced rectangular sections. The illustrated deflections consider State I and II regions and optionally tension stiffening and creep. They are related to the corresponding State II regions determined with either \( E_c (\varphi = 0) \) or \( E_{c,ef} (\varphi = 3) \). Before cracking \( (M_c/M_l \leq 1) \) the ratio of the State I and II deflections corresponds to the curvature ratio \( (\kappa_{II}/\kappa_I) \) (Fig. 6.9). After initial cracking, the uncracked member regions are considered with the interpolation coefficients given in Table 4.2. For moment ratios above 1.5 the midspan deflections are reduced in comparison to the corresponding fully cracked deflections by up to 10 % due to remaining uncracked sections and by up to 30 % due to tension stiffening (upper limit with \( \lambda = 1 \)). The load-independency of the absolute tension stiffening term has the advantage that the interpolation coefficients can be combined with the moment ratio of the critical load case. The critical moment ratio occurs for example during construction if large loads are applied at an early age, making \( M_c \) large and \( M_l \) small.
The uniformly loaded cantilever interpolation coefficient (simplified method) suggested by EC 2 (Eq. 2.20) for all structural systems, leads to a stiffer deflection behaviour just after initial cracking and predictions between the Tension Chord Model limits \( \lambda = 1 \) and \( \lambda = 0 \) above \( M_s / M_r = 1.25 \). Also under sustained loads the EC 2 long-term factor \( \beta = 0.5 \) leads to predictions between the Tension Chord Model limits. The 4-point bending tests illustrated in Chapter 5.2.1 show that the measured deflections are arbitrarily located between the limits of \( \lambda = 0 \) and \( 1 \).

Fig. 6.14  Deflections of a simple beam with \( n = 6, \rho' = 0, \lambda = 1 \) and \( d = 0.85 \) for \( M_s \leq M_r \). The reference State II deflections are determined with \( E_c (\varphi = 0) \) or \( E_{c,ef} (\varphi = 3) \).

Fig. 6.15 illustrates the related deflections of a symmetrically reinforced clamped-ended beam. The load-level is expressed by the ratio of the elastic support moment and the initial cracking moment. The related deflections consider cracking and arbitrarily creep and tension stiffening. They are determined with Eq. (4.82) in order to capture the moment redistributions that occur between support and span cracking. These moment redistributions are dependent on the ratio of the State I and II stiffness as well as the ratio of the support and span moment. They delay the extension of the cracked regions at the supports and preserve the overall stiffness until span cracking starts. In the two examples illustrated in Fig. 6.15, the span cracking load is reduced by approximately 25 %. The universal EC 2 interpolation coefficient does not consider moment redistributions and consequently overestimates the deflections between the cracking loads. The experimental load-deflection curves of Alvarez and Martí's (1996) 2-span slab strips feature an increased stiffness between support and span cracking, which is not quite as pronounced as predicted. Also the flat plates discussed in Chapter 5.3
Discussion

with elastic support-to-span cracking load ratios between 4 (edge panels) and 14 (internal panels) feature an uncracked deflection behaviour despite the presence of cracked regions.

![Graphs showing deflections](image)

**Fig. 6.15** Deflections of a symmetrically reinforced clamped-ended beam for $M_a \leq M_f$ with $n = 6$, $\rho' = \rho$, $\lambda = 1$ and $d = 0.85 h$.

**Variable Stiffness**

Besides cracking also a non-uniform reinforcement detailing can lead to moment redistributions (see Fig. 4.11), which affect the deflections as well as the extent of the serviceability range (load range below steel yielding). Fig. 4.11 illustrates the influence of the support-to-span stiffness ratio $\kappa$ on the State II deflections of a clamped-ended beam and a propped cantilever.

For a non-curtailed support and span tensile reinforcement, the State II deflections (Fig. 4.11 (b)) of the end-restrained example beam are approximately equal to the State II deflections determined with the average stiffness ratio, as long as the support stiffness is larger than the span stiffness ($\kappa > 1$). In contrast a larger span stiffness can imply up to 2/3 larger State II deflections. The State II deflections of the propped cantilever exceed the average stiffness State II deflections for positive $\kappa$ values, while a slightly larger span stiffness tends to have a favourable influence on the State II deflections. In both cases a reinforcement distribution, which is perfectly affine to the moment, increases the State II deflections by up to 50%. The influence of the stiffness ratio can be appreciated for test specimens CB 1 to CB 3 shown in Fig. 5.32. Further, Fig. 5.33 shows that despite the significantly larger cracked stiffness, CB2 has the same deflections as CB4. This is most likely because of its unfavourable stiffness distribution.
Fig. 6.16 compares the span to support stiffness and the moment ratios for the clamped-ended example member in Fig. 4.11. The dashed diagonal line stands for the case where the stiffness ratio coincides with the moment ratio. It can be seen that in general the provided support stiffness is too low for the corresponding moment. E.g. a uniform reinforcement distribution ($\kappa = 1$) causes a moment ratio of 2, while the elastic reinforcement distribution ($\kappa = 2$) causes a moment ratio of 3. The theoretical stiffness ratio of 5.6 corresponds to a moment ratio of 5.6 in a fully cracked member.

![Fig. 6.16 Support to span moment ratio vs. stiffness ratio](image)

### 6.3 Summary

The *cracking behaviour of tensile ties* under axial loads as well as imposed or end-restrained deformations is strongly influenced by the extension of the crack formation phase:

- In contrast to tensile ties with imposed or end-restrained deformations, axially loaded tensile ties have an instantaneous crack formation phase if a constant concrete tensile strength is assumed along the member.

- During the crack formation phase the maximum crack widths are primarily defined by the reinforcement ratio and the concrete tensile strength (see Fig. 6.4). They are neither dependent on the magnitude of the imposed or restrained deformations (e.g. shrinkage or cooling strains) nor on the restraint stiffness.

- Once the crack formation phase is concluded the crack widths increase with increasing load or deformation (see Fig. 6.4).

- Lightly reinforced members are characterized by an extended crack formation phase (Fig. 6.1) and largely spaced cracks (Fig. 6.3 (a)) that are accordingly wide (Fig. 6.4). By increasing the reinforcement ratio the extension of the crack formation phase is reduced, causing a larger amount of thinner, closer spaced cracks.

The *cracking behaviour of bending members* is strongly dependent on the effective reinforcement ratio of the tension chord:

- The parameter study carried out in this chapter (Fig. 6.6 (b)) shows that the bending-theory-conform effective reinforcement ratio used by the Tension Chord Model is larger than the effec-
Discussion

tive reinforcement ratio suggested by Schiessl (Eq. (2.8)). This leads to notably shorter transfer lengths (Fig. 6.6 (d)) and maximum crack spacings. This can also be seen in the test data comparisons in Chapter 5.2.1, where the gradient of the MC 2010 crack width vs. steel stress curves is larger than the Tension Chord Model gradient. Within these test data comparisons the predictions carried out with the Tension Chord Model effective reinforcement ratio are more consistent.

- The large effective reinforcement ratios found in bending members make the influence of tension stiffening negligible for predicting moment-induced crack widths in members with reinforcement ratios over 0.5 % (Fig. 6.6 (c)). This is also clearly shown in the test data comparisons in Chapter 5, where, unless the reinforcement ratio is very small, the predicted crack width curves with $k_\psi = 0$ and 1 are close.

- In terms of crack width, -0.25 ‰ shrinkage strains are approximately equivalent to 50 MPa additional State II steel stresses.

Section deformations are strongly influenced by cracking:

- In lightly reinforced sections cracking reduces the section stiffness by up to 90 % (see Fig. 6.9 (a)). Creep e vents out the State I and II stiffness difference.

- For $f_{ct} = 2$ MPa and $\lambda = k_\psi = 1$ the tension stiffening curvature reduction reaches 1 ‰ /d at low reinforcement ratios (see Fig. 6.10 (a)). This is in the range of the curvature caused by shrinkage strains $\varepsilon_{sh} = -1$ ‰ ($\varphi = 3$) in strongly reinforced uncracked sections or singularly reinforced cracked sections (Fig. 6.13 (a)).

Moment redistributions have a strong influence on beam deflections:

- The deflections of partially cracked statically determinate beams approximate the State II deflections with increasing reinforcement ratio and creep coefficient (Fig. 6.14).

- The deflections of statically indeterminate beams are strongly influenced by the moment redistributions. Support-to-span moment redistributions due to cracking (Fig. 6.15) have a positive influence on the global stiffness between the support and span cracking loads, but reduce the span cracking load considerably.

- The optimum support-to-span stiffness ratio for deflections (e.g. Fig. 4.11 (b)) does not necessarily coincide with the optimum support-to-span stiffness ratio for preventing early yielding (e.g. Fig. 6.16).
7 Conclusions and Outlook

7.1 Conclusions

- Serviceability design is a topic of growing importance. This is mainly due to the increasing slenderness of our structures, which is driven by aesthetical and economical concerns and made possible by increasing material strengths. The service life spans of most structures require long-term serviceability approaches.

- In most practical applications the accuracy of serviceability limit state predictions is limited by the complex and hardly known load history and the highly variable concrete properties. Nevertheless, reasonably reliable estimates of the magnitude of both integral and local deformations are required.

- In contrast to the semi-empirical cracking and deflection approaches illustrated in Chapter 2, the Tension Chord Model (Chapter 3.4) is an analytical model, which is suitable for both cracking and deflection predictions. The only empirical parameter in the model is the magnitude of the constant bond stress adopted between the steel and concrete. This makes the model applicable for a broad range of boundary conditions. The assumption of a constant bond stress is reasonable in view of the large scatter (e.g. concrete tensile strength) or the unknown components (e.g. load history) of the input parameters. The magnitude of the bond stress is in agreement with the findings of a parameter study involving 250 tests reported by König and Fehling.

- Within the scope of this work the cracking and deformation behaviour of tensile ties and beams is modelled using the Tension Chord Model:
  - new expressions are provided for determining the tension stiffening curvature reduction and the transfer length in bending members. They are based on an effective bending-theory-conform reinforcement ratio (see Chapter 4.1.1).
  - a factor $k_e [1...0]$ is introduced (Eq. (4.11)) to express the effective contribution of the tensile concrete for a given crack spacing defined according to Eq. (4.25) by factor $\lambda [1...0.5]$.
  - the approach for describing the cracking behaviour of end-restrained tensile ties is extended to account for the combined presence of shrinkage and cooling-induced strains as well as elastic end-restraint conditions. This approach can also be used for estimating the cracking behaviour of base-restrained walls.
  - deflection interpolation coefficients and span-to-depth ratios are given for basic static systems. The interpolation coefficients can also be used to determine deflection reductions due to tension stiffening as well as additional shrinkage deflections. The universal interpolation coefficient provided in EC 2 (2004) represents a uniformly loaded cantilever.

- The State I and II section behaviour forms the basis for all crack width and deformation predictions. Reliable predictions can be provided for the State I and II section deformations due to short-term or sustained moments and/or axial forces. For standardly reinforced concrete sections creep can be considered with the Effective Modulus Method ($X = 1$).
• The State I and II section deformations lead to good upper limit deflection predictions for partially cracked statically determinate beams. However, for statically indeterminate beams the prediction quality for the deformations decreases due to the challenge of predicting the moment distribution. The influence of moment redistributions can be significant for statically indeterminate beams and slabs. The latter are further affected by membrane action.

• Besides the State II steel strains, the maximum crack spacing is the crucial parameter for determining crack widths. With the bending-theory-conform effective reinforcement ratio introduced in Chapter 4, the Tension Chord Model provides reasonable maximum crack spacings for bending members. This applies to beams without stirrups or with stirrups, which are spaced further apart than the theoretical maximum crack spacing.

• With the Tension Chord Model ($\lambda = k_o = 1$) good lower limits for the average steel strains and section curvatures as well as crack widths and integral deformations are obtained. However, depending on a number of parameters, such as load duration, crack spacing and section detailing, the effective influence of tension stiffening can be anywhere between this upper limit ($\lambda = k_o = 1$) and zero ($k_o = 0$). Fortunately, crack width predictions in bending members with reinforcement ratios above 0.5 % are not sensitive to $k_o$. Also the deflections of heavily reinforced beams (which feature service load levels well above the cracking load level) and the deflections under sustained loads (where bond creep reduces the influence of tension stiffening) are insensitive to $k_o$. Further, it must be taken into account that tension stiffening can be present but this is not necessarily the case.

• The Tension Chord Model represents an excellent tool for understanding and describing the shrinkage- and cooling-induced cracking and deformation behaviour of elastically end-restrained tension ties. Good upper limits are predicted for the steel stress, deformation and crack widths during the crack formation phase. They are primarily defined by the concrete tensile strength and the reinforcement ratio and do not depend on the magnitude of the induced strains or the restraint stiffness.

7.2 Outlook

• A better understanding of the moment redistributions under service loads is necessary to improve the prediction quality of statically indeterminate beam deflections. This would also provide valuable information for slab deflections.

• The application of the end-restraint cracking approach described in Chapter 4.2.2 for base-restrained walls needs to be verified with large-scale tests.

• In this work Eqs. (4.66) & (4.63) and Eqs. (4.67) to (4.70) are suggested for modelling the cracking behaviour of concrete overlays and ground slabs. These equations need to be verified with test data and possibly further developed.
Notation

Alphabetic Characters

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<td>m</td>
<td>related bending moment</td>
</tr>
<tr>
<td>$M$</td>
<td>bending moment</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>bending moment of equivalent statical system</td>
</tr>
<tr>
<td>n</td>
<td>modular ratio ($= E_s/E_c$)</td>
</tr>
<tr>
<td>N</td>
<td>axial force</td>
</tr>
<tr>
<td>q</td>
<td>variable load (live load), uniformly distributed load</td>
</tr>
<tr>
<td>Q</td>
<td>single load</td>
</tr>
<tr>
<td>$R$</td>
<td>degree of restraint, axial force</td>
</tr>
<tr>
<td>s</td>
<td>crack spacing, bar spacing</td>
</tr>
<tr>
<td>t</td>
<td>time in days</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>u</td>
<td>displacement</td>
</tr>
<tr>
<td>V</td>
<td>shear force</td>
</tr>
<tr>
<td>w</td>
<td>crack width, water content</td>
</tr>
<tr>
<td>x</td>
<td>(horizontal) coordinate; depth of centroid of transformed section</td>
</tr>
<tr>
<td>y</td>
<td>coordinate</td>
</tr>
<tr>
<td>z</td>
<td>(vertical) coordinate</td>
</tr>
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</table>

Greek Letters

<table>
<thead>
<tr>
<th>Character</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>related deflections</td>
</tr>
<tr>
<td>$\beta$</td>
<td>fib and EC 2 tension stiffening factor, time factor creep/shrinkage</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>moment redistribution coefficient</td>
</tr>
</tbody>
</table>
Notation

\( \delta \) = integral member deformation (e.g. deflection or rotation), bond slip
\( \Delta \) = difference
\( \varepsilon \) = strain (compression negative, tension positive)
\( \varepsilon_{sh} \) = free shrinkage strains \( \varepsilon_{sh} (t, t_{sh}) \)
\( \zeta \) = deflection interpolation coefficient
\( \eta \) = factor
\( \kappa \) = end-restraint stiffness factor, cracked stiffness ratio, curvature coefficient
\( \lambda \) = tension stiffening coefficient
\( \nu \) = Poisson's ratio
\( \xi \) = related (horizontal) coordinate, ACI long-term coefficient
\( \rho \) = tensile reinforcement ratio \( A_s/(bd) \) of a bending member,
      reinforcement ratio \( A_s/(bh) \) of a tensile member
\( \rho' \) = compression reinforcement ratio \( A_s'/(bd) \) of a bending member
\( \rho_{TC} \) = total reinforcement ratio \( (A_s + A_s')/(bh) \) of a bending member
\( \sigma \) = stress (compression negative, tension positive)
\( \tau_b \) = bond stress
\( \varphi \) = creep coefficient \( \varphi (t, t_0) \)
\( \chi \) = curvature
\( \psi \) = moment ratio \( M_r/M_{u,el} \)
I, II = uncracked state, cracked state
\( \emptyset \) = nominal diameter

Subscripts

a = age-adjusted, maximum (e.g. midspan), load-introduction length
b = bond between steel and concrete
c = concrete, at restrained edge
c_t = tensile concrete
e = imposed deformation
ef = effective
g = gross area of concrete section
i = induced strains
M = moment-induced
r = cracking
s = reinforcing steel
sh = shrinkage
t = time
t_s = tension stiffening
u = unloading
y = yielding
\( \Phi \) = creep
\( \chi \) = curvature
0 = at loading, short-term, construction, at origin of coordinates \( (z = 0) \)
I = surcharge
I, II = uncracked state, cracked state

Abbreviations

ACI = American Concrete Institute
AAEMM = Age-Adjusted Elastic Modulus Method
EC = Eurocode
EMM = Elastic Modulus Method
fib = Fédération Internationale du Béton
MC = Model Code
RC = Reinforced Concrete
TCM = Tension Chord Model
Literature


Frosch, R., 1999, “Another Look at Cracking and Crack Control in Reinforced Concrete”, ACI Structural Journal, V. 96, No. 3, pp. 437–442.


Kaufmann, W., 1998, Strength and Deformations of Structural Concrete Subjected to In-Plane Shear and Normal Forces, IBK-Bericht Nr. 234, ETH, Zürich, 147 pp.


Marti, P., 2004, Reinforced Concrete I Lecture Notes, Department of Civil, Environmental and Geomatic Engineering, ETH, Zürich, 92 pp.


# Appendix A – Test Data Used in Chapter 5

## Tensile Tests

<table>
<thead>
<tr>
<th>Spec.</th>
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<th>$A_s; A_t$</th>
<th>$\rho = A_s/bh$</th>
<th>$f_{cm}$</th>
<th>$f_{cm}$</th>
<th>$E_s$</th>
<th>$f_y$</th>
<th>$t_0$</th>
<th>$t_0$</th>
<th>$N$</th>
<th>$\varepsilon_{cm}$</th>
<th>$t$</th>
<th>$s_{cm}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit</td>
<td>mm</td>
<td>%</td>
<td>MPa</td>
<td>MPa</td>
<td>GPa</td>
<td>d</td>
<td>kN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### Günther & Mehlhorn (1989)

| D1    | 1200; 70; 70 1010; - | 1.63 | 21'' $n/a$ | 202 | 29 | 146 | 16.0 | 0.58 $e$ | 0.74 $e$ | 142 | 142 |
| D1    | 1200; 70; 70 1010; - | 1.63 | 21'' $n/a$ | 202 | 29 | 144 | 16.0 | 0.73 $e$ | 0.93 $e$ | 113 | 113 |
| D2    | 22'' $n/a$ | 203 | 25 | 148 | 18.8 | 0.99 $e$ | 1.16 $e$ | 111 | 111 |
| D5    | 33'' $n/a$ | 203 | 157 | 180 | 18.8 | 0.96 $e$ | 1.16 $e$ | 141 | 141 |
| D3    | 28'' $n/a$ | 203 | 275 | 175 | 22.0 | 1.16 $e$ | 1.36 $e$ | 129 | 129 |
| D8    | 32'' $n/a$ | 198 | 158 | 179 | 22.0 | 1.09 $e$ | 1.29 $e$ | 129 | 129 |
| D6    | 1200; 90; 90 1010; - | 0.98 | 20'' $n/a$ | 203 | 49 | 176 | 21.8 | 1.16 $e$ | 1.41 $e$ | 114 | 114 |
| D7    | 20'' $n/a$ | 203 | 49 | 179 | 21.8 | 1.17 $e$ | 1.42 $e$ | 106 | 106 |

### Scott & Gill (1990)

| ST1   | 885; 100; 100 1012; - | 1.13 | 42'' 2.7'' | n/a | 40 | 73 | 26.0 | 0.696 $d$ | 0.883 $d$ | single crack |
| ST2   | * | 42'' 3.4'' | n/a | 116 | 27 | 32.5 | 0.970 $d$ | 1.095 $d$ | single crack |
| ST3   | * | 43'' 2.5'' | n/a | 35 | 36 | 15.0 | 0.509 $d$ | 0.612 $d$ | single crack |

### Beeby & Scott (2006)

| T16B1 | 1200; 120; 120 1016; - | 1.4 | 19'' 1.7'' | n/a | 28'' 127 | 74.0 | 1.507 | 1.550 |
| T16B2 | * | 55'' 3.2'' | n/a | 28'' 50 | 71.5 | 1.492 | 1.590 |
| T16B3 | * | 99'' 5.0'' | n/a | 28'' 119 | 78.0 | 1.442 | 1.607 |
| T20B1 | 220; 120; 120 1020; - | 2.18 | 27'' 2.3'' | n/a | 28'' 133 | 74.0 | 1.050 | 1.097 |
| T20B2 | * | 71'' 3.1'' | n/a | 28'' 56 | 72.8 | 1.020 | 1.109 |
| T20B3 | * | 94'' 3.9'' | n/a | 28'' 48 | 70.5 | 0.901 | 0.976 |

Note that a) 80% of concrete cube strength; b) 90 % of splitting tensile strength; c) measured with inductive displacement transducers; d) strain-gauged reinforcement bar; and (e) assumed. When not given $E_s$ is assumed to be 205 GPa.
### Table A. 2: End-restrained shrinkage tests (Nejadi and Gilbert 2003).

<table>
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<tr>
<th>Spec.</th>
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<th>$\rho = A_s/\rho$</th>
<th>$f_{cm} = f_{cm}$</th>
<th>$E_{cm}$</th>
<th>$E_s$</th>
<th>$t_0$</th>
<th>$\varepsilon_{sh}$</th>
<th>$L_s$</th>
<th>$\sigma_{II}$</th>
<th>$w_{max}$</th>
<th>$s_{cm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>%</td>
<td>GPa</td>
<td>GPa</td>
<td>MPa</td>
<td>d</td>
<td>%</td>
<td></td>
<td>MPa</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>S1a*)</td>
<td>2000; 600; 100 3012; -</td>
<td>0.565</td>
<td>24.3</td>
<td>197</td>
<td>22.9</td>
<td>n/a</td>
<td>3</td>
<td>150</td>
<td>0.98</td>
<td>1</td>
<td>AIII#)</td>
</tr>
<tr>
<td>S1b*)</td>
<td>2000; 600; 100 3010; -</td>
<td>0.393</td>
<td>28.4</td>
<td>210</td>
<td>23.2</td>
<td>8.8</td>
<td>3</td>
<td>150</td>
<td>0.98</td>
<td>1</td>
<td>AIII#)</td>
</tr>
<tr>
<td>S2a*)</td>
<td>2000; 600; 100 3012; -</td>
<td>0.262</td>
<td>24.3</td>
<td>197</td>
<td>22.8</td>
<td>n/a</td>
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<td>0.98</td>
<td>1</td>
<td>AIII#)</td>
</tr>
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<td>S2b*)</td>
<td>2000; 600; 100 3010; -</td>
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<td>197</td>
<td>22.8</td>
<td>n/a</td>
<td>3</td>
<td>150</td>
<td>0.98</td>
<td>1</td>
<td>AIII#)</td>
</tr>
</tbody>
</table>

a) concrete properties at 28 d batch I; b) concrete properties at 28 d batch II; c) tested length d) indirect tensile strength (brazil test); e) strain gauge measurements Nejadi & Gilbert (2003) Appendix III.

### Table A. 3: Deformation controlled tensile tests (Jaccoud et al. 1984). The deformations were controlled over a length of 1 m (B-1 series) or 1.2 m (B-2 series)

<table>
<thead>
<tr>
<th>Spec.</th>
<th>$l; b; h$</th>
<th>$\rho = A_s/\rho$</th>
<th>$f_{cm} = f_{cm}$</th>
<th>$E_{cm}$</th>
<th>$E_s$</th>
<th>$t_0$</th>
<th>$\varepsilon_e$</th>
<th>$\sigma_{II}$</th>
<th>$w_{max}$</th>
<th>$s_{cm}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>%</td>
<td>GPa</td>
<td>GPa</td>
<td>MPa</td>
<td>d</td>
<td>%</td>
<td>MPa</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>B-11</td>
<td>2500; 450; 150 608; -</td>
<td>0.45</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-12</td>
<td>2500; 450; 150 608; -</td>
<td>0.60</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-13</td>
<td>6010; -</td>
<td>0.70</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-14</td>
<td>6010; -</td>
<td>0.93</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-17</td>
<td>6010; -</td>
<td>0.47</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-21</td>
<td>6010; -</td>
<td>0.60</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-22</td>
<td>6010; -</td>
<td>0.70</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-23</td>
<td>6010; -</td>
<td>0.93</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
<td>6.4</td>
<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
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<tr>
<td>B-24</td>
<td>6010; -</td>
<td>0.47</td>
<td>32.9</td>
<td>2.43</td>
<td>32.0</td>
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<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
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<td>2.43</td>
<td>32.0</td>
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<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
<tr>
<td>B-26</td>
<td>6010; -</td>
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<td>n/a</td>
<td>617</td>
<td>36</td>
<td>Fig. 4.2</td>
</tr>
</tbody>
</table>

a) calculated from $f_{cm}$ assuming $E_s = 205$ GPa; b) determined from measured axial force $N/A_s$; c) in Jaccoud et al. (1984).
## Bending Tests

### Table A.4: Statically determinate bending members.

<table>
<thead>
<tr>
<th>Spec.</th>
<th>l; b; h; d</th>
<th>( \rho^a ); ( \rho'^a )</th>
<th>( f_{cm} ); ( f_{cm} )</th>
<th>( E_{cm} ); ( n )</th>
<th>( E_s ); ( f_s )</th>
<th>( t_0 ); ( t_0 )</th>
<th>( \epsilon_{sh} ); ( \phi )</th>
<th>( LS )</th>
<th>( M )</th>
<th>( \chi_m ); ( t_0 )</th>
<th>( w_{max} )</th>
<th>( s_{max} )</th>
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</thead>
<tbody>
<tr>
<td>unit</td>
<td>m</td>
<td>%</td>
<td>MPa</td>
<td>GPa</td>
<td>-</td>
<td>GPa</td>
<td>d</td>
<td>10^{-3}</td>
<td>kNm</td>
<td>mra</td>
<td>d/m</td>
<td>mm</td>
</tr>
<tr>
<td>Kenel &amp; Marti (2002)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>B1</td>
<td>3.6; 1.0; 0.2; 0.167</td>
<td>0.38</td>
<td>34</td>
<td>208</td>
<td>27</td>
<td>n/a</td>
<td>n/a</td>
<td>1</td>
<td>21.4</td>
<td>2.2</td>
<td>0.05</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>8010; n/a; 08/300</td>
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<td>0.30</td>
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<td>6</td>
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<td>0.40</td>
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<td>n/a</td>
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<td>16.9</td>
<td>1.9</td>
<td>0.20</td>
<td>n/a</td>
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<td></td>
<td>4010; n/a; 08/300</td>
<td>3.1&quot;</td>
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<td>16.3</td>
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<td>7.5</td>
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Table A. 5: Statically indeterminate beams.

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a) the reinforcement ratios refer to the effective depth ($\rho = A_*/bd$); b) flexural tensile strength; c) estimated; d) transverse reinforcement; e) splitting tensile strength; f) in Alvarez and Marti (1996); g) in Bakoss et al. (1982b; 1982a).
### Table A. 6: Slabs.

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<td>6.2</td>
<td>2.0</td>
<td>52.1</td>
<td>2.5</td>
</tr>
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</table>

a) assumed; b) flexural tensile strength; c) values from Gilbert & Guo (2005); d) shrinkage specimen 600/600 mm with same thickness as slabs e) average of 4 bays; f) assuming $d_{avg} = 0.8 \ h$; g) 28-day; h) splitting strength; i) at striking; k) 600 d; j) measured on specimens $\Omega 102$ mm, $l = 254$ mm; k) average of 4 panels; l) average of two panels.
Curriculum vitae

Clare Burns
Born on the 25 July 1977
Citizen of Effingen, AG

Education

1984 – 1997     Primary School in Effingen, Grammar School in Frick and Aarau, Matura Type C
1997 – 2002     Civil Engineering Studies at ETH in Zurich, graduated Dipl. Bauingenieur ETH

Work Experience

2000 – 2001     Trainee (part time), Engineering Office Wolfscher und Partner AG, Zurich
2002 – 2005     Site Engineer, Building Companies SERPREC Ltda. und Quiroga Ltda., Cochabamba, Bolivia
2005 – 2011     Assistant and Research Associate of Prof. Dr. P. Marti at the Institute of Structural Engineering, ETH Zurich