Elastic tube shape analysis and the corresponding velocity profile measurements were carried out under steady flow of both Newtonian and non-Newtonian aqueous solutions. The various degree of tube deformations were achieved by imposed compressive transmural pressures. It was seen that the tube shape changed from a circular shape to a two lobed shape, and the corresponding velocity profile changed from a parabolic to a bi-modal for the investigated fluids under different applied external pressures. The variation in the elastic tube shapes and the corresponding flow velocity profiles along the tube length were also observed. The experimental results depict that the tube cross sectional area decreased about an order of magnitude from the undeformed one, where the corresponding maximum flow velocity in the tube center increased by about a factor of two. Similarity tube laws represented well the experimental variation in tube cross sectional area with the transmural pressure. The tube shape and velocity profile were also quantified along the tube length, the velocity profiles were found to be bi-modal in the two lobe-shape domain and turned into uni-modal characteristics in the elliptical region. In addition, the steady flow of a non-Newtonian fluid through a collapsed elastic tube was simulated and a good agreement with the experiment was obtained with additional insight by considering the local quantities for shear rates and viscosities.

The detail understanding of steady and unsteady (pressure-controlled) flow characteristics of non-Newtonian fluids in collapsed elastic tube followed the interest to investigate experimentally the flow behavior of a non-Newtonian fluid during peristaltic propulsion through an elastic tube. The final goal of the present study was to understand the flow structure of non-Newtonian fluids under peristaltic squeezing of an elastic tube (in vitro modeled small intestine). The detailed knowledge gained about non-Newtonian peristaltic flow can be useful to explore and observe the influence of the peristaltic flow on the mass transport across the elastic membrane tube.
Steady and Unsteady Flow Characteristics of non-Newtonian Fluids in Deformed Elastic Tubes

A dissertation submitted to the ETH Zurich

for the degree of Doctor of Sciences

presented by
Samsun Nahar
MSc Polymer Science, a joint master program of FU Berlin, HU Berlin, TU Berlin and UP
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Prof. (Emeritus) Dr. Yasushi Takeda, co-examiner
Dr. Stéphane Fischer, co-examiner

2012
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Zurich, September 2012

Samsun Nahar
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<th>Meaning</th>
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<td>-</td>
<td>fitting parameters</td>
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<td>(m^2)</td>
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</tr>
<tr>
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<td>(m^2)</td>
<td>cross sectional area of undeformed tube</td>
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<tr>
<td>(A_m)</td>
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<td>total cross sectional area of the membrane</td>
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<tr>
<td>(c)</td>
<td>(m \ s^{-1})</td>
<td>sound velocity in the medium</td>
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<td>(d)</td>
<td>(m)</td>
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<tr>
<td>(d_{50})</td>
<td>(\mu m)</td>
<td>pore size of the membrane</td>
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<tr>
<td>(d_{\text{diameter}})</td>
<td>(m)</td>
<td>cone geometry diameter</td>
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<td>Unit</td>
<td>Meaning</td>
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<td>------</td>
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<td>velocity vector component measured by UVP</td>
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<td>$v_x$</td>
<td>m s$^{-1}$</td>
<td>local velocity in each channel</td>
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<td>m$^3$</td>
<td>solution volume in the respective cell</td>
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<td>$V_{mol}$</td>
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<td>$X$</td>
<td>m</td>
<td>length along the tube</td>
</tr>
<tr>
<td>$Z$</td>
<td>kg/(m$^2$s)</td>
<td>acoustic impedance</td>
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**Greek Letters**

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<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$\alpha$</td>
<td>rad</td>
<td>angle between $v_{UVP}$ and $v_{real}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>rad</td>
<td>angle between $v_{real}$ and $v_x$</td>
</tr>
<tr>
<td>$\dot{\gamma}$</td>
<td>s$^{-1}$</td>
<td>average shear rate in the deformed tube</td>
</tr>
<tr>
<td>$\dot{\gamma}_0$</td>
<td>s$^{-1}$</td>
<td>average shear rate in the undeformed tube</td>
</tr>
<tr>
<td>$\dot{\gamma}_{avg}$</td>
<td>s$^{-1}$</td>
<td>average shear rate in the deformed tube</td>
</tr>
<tr>
<td>$\dot{\gamma}_{wall}$</td>
<td>s$^{-1}$</td>
<td>average shear rate near the tube wall</td>
</tr>
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<td>$\dot{\Gamma}$</td>
<td>-</td>
<td>shear rate ratio ($\dot{\gamma}/\dot{\gamma}_0$)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-</td>
<td>velocity ratio ($v/v_0$)</td>
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<tr>
<td>$\Delta c$</td>
<td>mol/m$^3$</td>
<td>concentration difference in a time interval</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>mol/m$^3$</td>
<td>concentration difference between two cells</td>
</tr>
<tr>
<td>$\Delta L$</td>
<td>m</td>
<td>adjusted distance between two conductivity sensors</td>
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<tr>
<td>$\Delta p, \Delta P$</td>
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<tr>
<td>$\Delta r$</td>
<td>m</td>
<td>increase in tube radius</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>s</td>
<td>time interval</td>
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<tr>
<td>$\Delta x$</td>
<td>m</td>
<td>target measuring line shifted along horizontal direction</td>
</tr>
<tr>
<td>$\Delta X$</td>
<td>m</td>
<td>characteristic length, e.g. membrane thickness</td>
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### Notation

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<td>$\epsilon$</td>
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<td>$\eta$</td>
<td>Pa s</td>
<td>shear viscosity of the fluid</td>
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<tr>
<td>$\eta_0$</td>
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<td>zero shear viscosity</td>
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<td>$\eta_{avg}$</td>
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<td>average viscosity in the deformed tube</td>
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<tr>
<td>$\eta_{wall}$</td>
<td>Pa s</td>
<td>average viscosity near the tube wall</td>
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<tr>
<td>$\eta_m$</td>
<td>Pa s</td>
<td>average minimum viscosity when $A \to 0$</td>
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<td>rad</td>
<td>transducer incident angle</td>
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<tr>
<td>$\theta'$</td>
<td>rad</td>
<td>inclined slope of the tube wall w.r.t. the horizontal axis</td>
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<td>$\theta_{new}$</td>
<td>rad</td>
<td>transducer incident angle in contraction type of geometry</td>
</tr>
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<td>$\lambda$</td>
<td>s</td>
<td>time constant</td>
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<tr>
<td>$\Lambda$</td>
<td>-</td>
<td>viscosity ratio ($\eta/\eta_0$)</td>
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<tr>
<td>$\Lambda_0$</td>
<td>-</td>
<td>minimum viscosity ratio ($\eta_m/\eta_0$)</td>
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<tr>
<td>$\mu$</td>
<td>Pa s</td>
<td>viscosity of the NaCl aqueous solution</td>
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<tr>
<td>$\phi$</td>
<td>-</td>
<td>dimensionless association factor for the solvent</td>
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<tr>
<td>$\pi$</td>
<td>-</td>
<td>Pi-number</td>
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<tr>
<td>$\Pi$</td>
<td>-</td>
<td>dimensionless transmural pressure ratio</td>
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<tr>
<td>$\rho$</td>
<td>kg m$^{-3}$</td>
<td>density</td>
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<tr>
<td>$\tau$</td>
<td>Pa</td>
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<tr>
<td>$\omega$</td>
<td>-</td>
<td>cross sectional area ratio ($A/A_0$)</td>
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### Indices

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<td><strong>subscripts</strong></td>
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<td>avg</td>
<td>average</td>
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<td>tube inlet</td>
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<tr>
<td>N</td>
<td>normal</td>
</tr>
<tr>
<td>o</td>
<td>tube outlet</td>
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<tr>
<td>tm(down)</td>
<td>downstream transmural</td>
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Symbol Definition

\( tm(up) \) upstream transmural

**Dimensionless Numbers**

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<td>Reynolds number</td>
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<td>Schmidt number</td>
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<td>( Sh )</td>
<td>Sherwood number</td>
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**Abbreviations**

<table>
<thead>
<tr>
<th>Abbrev.</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>CMC</td>
<td>Carboxymethyl Cellulose</td>
</tr>
<tr>
<td>CT</td>
<td>Computer Tomography</td>
</tr>
<tr>
<td>ETH</td>
<td>Swiss Federal Institute of Technology</td>
</tr>
<tr>
<td>FPE</td>
<td>Food Process Engineering</td>
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<tr>
<td>FSO</td>
<td>Full Scale Output</td>
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<tr>
<td>GIT</td>
<td>Gastro-Intestinal Tract</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
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<tr>
<td>ID</td>
<td>Internal Diameter</td>
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<tr>
<td>LDA</td>
<td>Laser Doppler Anemometry</td>
</tr>
<tr>
<td>LMVT</td>
<td>Laboratory of Food Process Engineering (Lebensmittelverfahrenstechnik)</td>
</tr>
<tr>
<td>MII</td>
<td>Multiple Intraluminal Impedance</td>
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<tr>
<td>NaCl</td>
<td>Sodium Chloride</td>
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<tr>
<td>PA</td>
<td>Polyamide</td>
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<tr>
<td>PAA</td>
<td>Polyacrylamide</td>
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<tr>
<td>PD</td>
<td>Pressure Difference</td>
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<tr>
<td>PEG</td>
<td>Polyethylene Glycol</td>
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<tr>
<td>PG</td>
<td>Plexiglass</td>
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<tr>
<td>PVC</td>
<td>Polyvinyl chloride</td>
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## Notation

<table>
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<tr>
<th>Abbrev.</th>
<th>Definition</th>
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<tr>
<td>QNLR</td>
<td>Flow Controlled Non-Linear Resistance</td>
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<tr>
<td>UVP</td>
<td>Ultrasound Velocity Profiling</td>
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Abstract

The study of fluid flow characteristics in collapsible elastic tubes are useful to understand biofluid mechanics encountered in the human body. The pertaining mechanism of physiology in the human body is very complex and is not fully understood. Furthermore, there is little information available on the experimental flow characteristics of non-Newtonian fluids through elastic tubes. In addition, no scientifically useful publication exists on peristaltic rhythmic squeezing of elastic tubes for transport of non-Newtonian fluids. Therefore, this thesis highlights the thorough investigation of the flow behavior of both Newtonian and non-Newtonian fluids through an elastic tube. Steady and periodic (pressure controlled) flow behaviors were investigated under the influence of different transmural pressures. In addition, flow behavior of a non-Newtonian fluid during peristaltic propulsion (deformation controlled) through an elastic tube was also studied. These enable the understanding of the interaction between wall motion, fluid flow and intestinal transmembrane mass transfer. The detailed knowledge will be useful as crucial contribution to the mechanistic understanding of bio-accessibility and also bio-availability.

Therefore, in a first part of the work, steady and unsteady laminar flow characteristics of a Newtonian and two non-Newtonian (low and high shear thinning) chyme model fluid systems through a collapsible elastic tube were investigated under compressive transmural (internal minus external) pressures $P_{tm}$ using a Starling Resistor. It was observed that for a given steady volume flow rate $\dot{Q}$, the tube was buckled from an elliptical shape to a line or area contacted two lobe-shape as the critical external pressure $P_e$ was increased. The downstream $P_{tm}$ was found to get more negative than that at the upstream as the outlet pressure decreased due to stronger tube collapse resulting in a reduced cross sectional area. The collapsed elastic tube shape was determined by computer tomography (CT) image analysis. Near the tube outlet at different $P_e$, the velocity profiles were monitored using a pulsed ultrasound Doppler based velocity profiling (UVP) technique. The experimental results depict that the tube cross sectional area decreased about an order of magnitude from the undeformed one when the compressive pressure difference $P_{tm}$ reached a value of about -30 mbar at the downstream end. The corresponding maximum flow velocity in the tube center increased by about a factor of two. Similarity laws derived represented well the experimental variation in tube cross...
sectional area with the transmural pressure. The tube shape and velocity profile were also quantified along the tube length, the velocity profiles were found to be bi-modal in the two lobe-shape domain and turned into uni-modal characteristics in the elliptical region. In addition, the steady flow of a non-Newtonian fluid through a collapsed elastic tube was simulated and compared with the corresponding experimental data. A good agreement between simulation and experiment was obtained with additional insight by considering the local quantities for shear rates and viscosities. Moreover, the periodic flow characteristics in a pressure controlled unsteady flow were found to be governed by the corresponding ramp-up/ ramp-down of volume flow rate $\dot{Q}$ and applied external chamber pressure $P_e$, the latter simulating the periodic intestinal muscle contraction. It was observed that, the collapsed tube walls came closer and the pressure drop at a given $\dot{Q}$ increased as $P_e$ increased. In contrast, when $\dot{Q}$ was increased, the internal pressure slowly increased and the tube recovered to its original shape at higher flow rates decreasing the pressure drop. The evolution of flow velocity profiles during the periodic ramp flow was also measured. Maximum velocity in the open elastic tube cross section changed by an order of magnitude during a periodic flow cycle.

On the other hand, the peristalsis fluid dynamics is the key responsible for mixing and mass transport of nutrients from the food matrix through the gastrointestinal tract (GIT) membranes. Therefore, in a second part of the work, an in vitro small intestinal flow characteristics of the non-Newtonian chyme model fluid were investigated by transient ‘2-wave’-squeezing of an elastic tube under different speeds of peristalsis. Such peristaltic flow is the essential physiological transport mechanism in the GIT. The peristalsis involves both contraction and expansion type of flow (crest and trough of a wavelength). We met the challenge of implementing the UVP technique for appropriate peristaltic propulsion of a shear thinning fluid through an elastic tube by a special experimental setup. The experiment was carried out under different speeds of peristalsis and the velocity profiles were monitored both in the crest and trough zone while positioning the ultrasound transducer at different adjusted positions in the moving frame. The higher wave speed of peristalsis resulted in higher magnitude of back flow velocity (negative) both in the wave crest and trough regions with positive value being near the tube wall. Consequently, the higher value of back flow is expected to be responsible for the improved mixing and convection leading to higher mass transport through the intestinal wall. In addition, the approximated wall shear rates also found to be increased with increasing the peristaltic motion, which correspondingly is expected to enhance the mass transfer by reducing the viscosity of a shear thinning fluid. The pressure in the tube crest was found to be higher than that at the trough. The pressure difference between center of crest and trough was low since the mean fluid velocity was low and gap in the trough was relatively large.

In addition, a new experimental approach of UVP using multiple measurement line
based velocity profiles is proposed to investigate the real and axial velocity vectors under peristaltic motion, and a comparison with the single measurement line based velocity profiles is also demonstrated. It was found that the new experimental technique was well applicable to accurately estimate the velocity profiles both in the wave crest and trough. The detailed knowledge gained about non-Newtonian peristaltic flow is aimed to use in future for mass transport investigations across the elastic membrane tube wall.
Zusammenfassung


Daher wird in einem ersten Teil der Arbeit das stationäre und instationäre laminare Fließverhalten einer Newtonschen und zweier nicht-Newtonsscher (niedrige und hohe Scherentzung) speisebreiartiger Modellflüssigkeiten durch einen zusammendrückbaren elastischen Schlauch untersucht. Dies erfolgt mittels Starling Resistor unter Wirkung eines variabel einstellbaren transmuralen Druckgefälles $P_{tm}$. Dabei wird beobachtet, dass für einen gegebenen stationären Volumenstrom $\dot{Q}$, der Schlauch deformiert wird, wenn eine kritische transmurale Druckdifferenz $P_e - P_i$ überschritten wird. Dabei entwickelt sich der ursprüngliche kreisförmige Schlauchquerschnitt über eine eliptische Form bis hin zu einer zweiteiligen Bogenform mit dazwischen anliegenden Schlauchinnenflächen. Es wurde ferner gefunden, dass bei abnehmendem Austrittsdruck $P_{tm}$ am Schlauchaustritt negativer


1 Introduction

The systematic work was performed for the understanding of steady flow behavior in a deformed elastic tube (under different compressive transmural pressures), where UVP method was used for the flow field investigation and CT method was applied for the corresponding tube shape analysis. Those gained knowledge was then translated to study the peristaltic flow mechanism (a deformation controlled unsteady flow) while squeezing of an elastic tube (in-vitro small intestine) under various speeds of peristalsis. All the chapters written in this thesis can be structured as follows:

Chapter 2 describes the main aim and background of the present study, which also includes the summary of relevant literature delivered within last few decades.

Chapter 3 summarizes all the materials and applied methods used in the present study.

Chapter 4 focuses on the steady flow behavior investigation of both Newtonian and non-Newtonian fluids in a collapsible elastic tube. The different degree of tube deformation was attained by applying various external chamber pressure in a Starling Resistor set up. The shape of the deformed tube was analyzed by means of CT-method, while the corresponding velocity flow profiles were measured using UVP technique. The flow characteristics in the collapsed elastic tube were also expressed as a function of pressure drop in the tube, volume flow rate, upstream and downstream transmural pressures. Which also describes the influence of tube deformation on the fluid rheological properties. Finally, a comparison of the experimental data with the simulation result is also presented.

Chapter 5 represents a model developed for the prediction of velocity profiles of a shear thinning fluid flowing through a circular elastic tube, where the expansion in tube cross sectional area at higher flow rates is also illustrated. The model equations are well explained by the experimental results.
Chapter 6 describes the study of pressure-controlled unsteady flow behavior in a collapsible elastic tube during flow of a non-Newtonian fluid.

Chapter 7 presents an experimental approach for velocity profile measurement by UVP method when the flow streamlines in the deformed tube are non-parallel.

Chapter 8 illustrates the peristaltic flow behavior of a shear thinning fluid while squeezing of an elastic tube (in-vitro modeled small intestine). The new experimental approach (as described in chapter 7) was also implemented for the velocity vectors investigation both in the wave crest and trough under peristaltic motion.

Chapter 9 demonstrates an experimental approach to characterize a model membrane (representative of intestinal wall). The detailed knowledge gained here is aimed to couple with the non-Newtonian peristaltic flow information in future for mass transport investigations across the elastic membrane tube wall.

Chapter 10 summarizes an overall conclusion based on the summary of above mentioned individual chapters with some intended future work or outlook.
2 Background

2.1 Motivation

The investigation of fluid flow characteristics in elastic inflatable and collapsible tubes is important to biofluid mechanics encountered in the human body and other applications; for instance, transport of food and liquids in human throat (pharynx), tube (esophagus) connecting the throat and stomach, and intestines; blood flow through the veins, capillaries and arteries; air flow in the pulmonary airways. The knowledge on the mechanisms of pharyngeal, esophageal and intestinal transport of food and liquids is very useful for the treatment of patients with malfunctioning of these transport processes. The physiology of these applications in human body is very complex and is not fully understood.

Therefore, the presented research work was motivated to investigate in model experiments the flow behavior of Newtonian and non-Newtonian fluids in a collapsible elastic tube. The parameters were classified into the categories as: (i) the characteristics of the investigated fluids such as the composition, physico-chemical properties and the microstructure, (ii) the solid mechanical properties of the elastic tube (elastic modulus, Poisson ratio, strain and bending stresses), which can influence the tube deformation and wall motion, (iii) the applied external pressure to achieve various degree of tube deformation and the interaction between the tube wall motion and flow, (iv) three types of model flow experiments such as steady laminar, unsteady periodic (pressure controlled) and peristaltic (deformation controlled) unsteady flow.

The detail investigation comprised the study of fluid flow (steady laminar & unsteady periodic) characteristics in a collapsible elastic tube under the influence of expansive and compressive transmural (internal minus external) pressures using Starling Resistor. The experiments included the measurement of the volume flux as a function of pressure drop in the tube for different upstream and downstream transmural pressures, or the variations in the upstream and downstream transmural pressures with the pressure drop in the tube for different constant volume flow rates. In addition, the Similarity ‘tube law’ represented well the variation in tube cross sectional area under different transmural pressures.
The study also included the simultaneous measurement of tube deformation and the corresponding flow field. The collapsed tube shapes were analyzed by the computer tomography method using several images taken at different angles around the pressure chamber. Whereas, the pulsed ultrasound Doppler velocimetry technique was applied to monitor the velocity profiles of the investigated fluids flowing through the elastic tube of different degree of deformation (under applied $P_e$).

Furthermore, the flow behavior due to peristaltic squeezing of an elastic tube (\textit{in vitro} modeled small intestine) resulting in transport of non-Newtonian fluid was also studied. This enabled the understanding of the interaction between wall motion and flow behavior. The velocity field under peristaltic propulsion of a shear thinning fluid through an elastic tube was measured successfully by UVP technique in a special experimental setup. As the single measuring line based velocity profiles by UVP method is well applicable in the case of parallel flow streamlines, whereas the peristaltic flow is supposed to include both contraction and expansion type of flow (crest and trough of a wavelength) representing non-parallel streamlines. Therefore, an experimental technique of multiple line based velocity profiles using UVP was proposed to quantify the real and axial velocity vectors under imposed peristaltic motion.

In addition, a membrane (representative of intestinal wall) was characterized using a model fluid in terms of diffusion and mass transport phenomenon using a ‘mass transfer test device’. The distinct knowledge gained on peristaltic motion and the mass transfer can be coupled for the role of peristalsis on mass transport investigations using a membrane elastic tube.

### 2.2 Literature review

Fluid flow in elastic tubes is a large displacement fluid-structure interaction problem encountered in biofluid mechanics (Meng et al., 2005), peristaltic pumping (Shapiro et al., 1969) and other applications. Several works on the study of flows in collapsible tubes and channels have been well documented based on the intended biological applications (Grothberg and Jensen, 2004; Kamn and Pedley, 1989; Pedley, 1980; Shapiro, 1977a,b). Biofluid mechanics is important to the flow of fluids through vessels in the human body, as numerous fluid conveying vessels are elastic and subject to buckle non-axisymmetrically when the transmural pressure falls below a critical value (Heil, 1997). The interactions between the internal flow and wall deformation of these flexible elastic vessels determine the biological function or dysfunction. The examples of such vessels are the veins above the level of the heart, the airways during forced expiration, the pulmonary
capillaries and the blood vessels in the heart muscle during systole (Conrad, 1969; Pedley, 1980). Holt (1941) investigated how the collapse of veins might affect peripheral venous pressure. He set up a model where water flowed through a rigid pipe to a collapsible segment of thin-walled rubber tubing and out through a more rigid pipe. The exact flow and wave propagation in distended tubes was well understood (Lighthill, 1975), whereas the flow structure in collapsed vessels has been less investigated.

The problem of flow in collapsible tubes was extensively studied experimentally by many authors e.g. Bertram et al. (1990); Conrad (1969); Elad et al. (1992); Gavriely et al. (1989). The Starling Resistor is a classical bench-top experimental set up which was widely used (Groberg and Jensen, 2004; Hazel and Heil, 2003; Heil, 1997; Holt, 1941; Katz et al., 1969; Knowlton and Starling, 1912; Lyon et al., 1980; Shapiro, 1977b) to investigate flow through elastic tubes relevant to many applications. This involves a pressure chamber that encloses a finite-length elastic tube mounted between two rigid tubes and a fluid is pumped through the tube at a steady volume flow rate.

The tube's large deformation during the buckling was found to lead to a strong interaction between the fluid and solid mechanics which was described by nonlinear shell theory (Heil, 1997). Steady flow through a collapsible tube was found to be a multiple-valued function of the pressure drop across it, named as flow-controlled nonlinear resistance, QNLR (Conrad, 1969), where a systemic experimental pressure flow curves for both steady and unsteady flow conditions were presented. The significant system parameter for changes in tube cross-section was transmural pressure, which in turn affected the flow geometry. The typical fluid-structure interaction problem involving the flow passing a collapsible tube was studied both experimentally and theoretically, and represented with a relationship between transmural pressure and cross-sectional area and the factors which influenced it (Bertram, 1986, 1987; Elad et al., 1987; Flaherty et al., 1972; Jensen and Pedley, 1989; Katz et al., 1969; Scroggs et al., 2004; Shapiro, 1977b; Zhu and Wang, 2003).

On the other hand, Lyon et al. (1980) proposed the hypothesis for the pressure-flow relationships by the waterfall model studied in a Starling Resistor, described only for flows with lower Reynolds numbers. Whereas, the minimum Reynolds number for self-excited oscillation was precisely determined experimentally by Bertram and Tscherry (2006). There are few theoretical investigation of both the flow and the wall mechanics in three dimensional collapsible tubes (Heil, 1997, 1998; Heil and Pedley, 1996; Marzo et al., 2005; Roser and Peskin, 2001). In addition, the wall deformation and fluid flow was modeled using geometrically nonlinear shell theory and lubrication theory respectively (Heil and Pedley, 1996; Rosmery et al., 1977; Unhale et al., 2005; Whittaker et al., 2010). If the transmural pressure acting on
the tube is sufficiently negative then the tube buckles non-axisymmetrically and the subsequent large deformations lead to a strong interaction between the fluid and solid mechanics (Hazel and Heil, 2003).

The extensive experimental and theoretical contributions made by several authors mentioned above are of great value for the scientific community as well in the many biomedical and biomechanical applications. These enable the better understanding of the laminar and turbulent flows of Newtonian fluids through collapsible tubes and the solid mechanics of the tube. In contrast, there is little literature (Dodson et al., 1974; Nahar et al., 2012) on the experimental flow characteristics of non-Newtonian fluids through elastic tubes under the influence of different transmural pressures involving the interaction of the deformed tube wall with the fluids. In addition, there is also little information available on the unsteady-periodic flow or peristaltic-squeezing of elastic tubes for transport of such fluids. This investigation is important as the determination of time scales involved in the transport of non-Newtonian fluids is relevant to the transport of food in the human gastrointestinal tract and other applications.

The investigation methods for the shape of the deformed elastic tube and the corresponding flow field are also not yet well established. The local tube cross sectional area was measured by an electrical impedance technique (Kececioglu et al., 1981) or ultrasound imaging (Bertram and Ribreau, 1989) or remote sensing technique (Elad et al., 1989). The cross section was assumed to remain the same throughout the tube as the same amount of liquid was flowing with same mean velocity through each cross section at any given time (Holt, 1959). In contrast, different cross-section of vessel or rubber tube showing changes in shape during oscillations of pressure was also observed (Brooks and Luckhardt, 1916). Kresch and Noordergraaf (1972) proposed a mathematical analysis for the cross-sectional shape of a flexible tube as its internal pressure varies. Quantitative results were presented in terms of the physical parameters of the tube, such as wall thickness and Young’s modulus. In the present study, the different degree of deformed tube shapes were aimed to quantify by means of computer tomography method where a grid line pattern was constructed on the tube surface and several images were taken at different angles around the tube radius.

On the other hand, Laser Doppler Velocimetry was applied for the laminar and turbulent flow measurement (Bertram and Godbole, 1997), also for the local flow measurements in collapsible tubes under different phases in a period of oscillation (Ohba et al., 1997). In addition, the ultrasound Doppler velocity profiling (UVP) technique was developed (Takeda, 1986, 1987, 1995, 1999) to quantify the velocity profiles of flowing fluids in pipes or tubes or channels, which has not been yet tested for the flow in elastic tubes. UVP method was also combined with the pressure difference (PD) in a pipe section for the flow measurement of various opaque
2.2 Literature review

model and industrial suspension (Ouriev et al., 2000; Ouriev and Windhab, 2003; Ouriev et al., 2004; Wiklund and Stading, 2008). The UVP-PD technique also allowed the detailed in-line flow behavior investigation of the cocoa butter shear crystallization process (Birkhofer et al., 2008), and in-line rheometry for Newtonian and non-Newtonian fluids (Wiklund et al., 2007). A good overview of the measurement principle of in-line rheometry can be found in Birkhofer (2011). In all the cases as mentioned above, the UVP measurement accuracy was dependent on the transducer installation such as direct contact (using a flow adapter) of the ultrasound transducer with the fluid in pipes (Choi et al., 2002; Ouriev and Windhab, 2003) or positioning the transducer at the pipe wall. In the latter case, the measurement can be inside the near field and strongly affected by an inhomogeneous pressure distribution (Hoeks et al., 1991) and a variation in the Doppler angle (Bascom et al., 1986).

Peristaltic motion is the physical mechanism of the fluid flow driven by periodic progressive waves of contraction and expansion advancing axially along the distensible tube length (Abd elmaboud and Mekheimer, 2011; Elshehawey and Sobh, 1992). This mechanism is responsible for the transport of biological fluids in several physiological processes such as passage of urine from the kidneys to the bladder, the movement of chyme in the gastrointestinal tract, transport of food bolus through the esophagus, transport of blood in small blood vessels (Hariharana et al., 2008). Many studies on the peristaltic flow characteristics exist in the literature were based on the assumption of low Reynolds number and infinitely long wavelength in a two-dimensional channel or axisymmetric porous tube (Boehme and Friedrich, 1983; Burns and Parkes, 1967; Rao et al., 2004). In contrast, the distribution of velocity, pressure, wall shear stress for different peristaltic flow conditions characterizing flow at moderately higher Reynolds number were investigated by computational model (Xiao and Damodaran, 2002). Khir and Parker (2002) investigated wave speed in an elastic tube generating a semi sinusoidal wave along the walls. It was reported that wall motions cause a 30-35 % increase in absorption that was also confirmed by the corresponding analytical model. The movements induced little net flow but enhanced the mixing, which modified the concentration gradients of any substance in the chyme that was being absorbed (Macagno et al., 1982). A simple way of the fluid-mechanical phenomena inherent in peristalsis was studied by Shapiro et al. (1969), and obtained a coupling between longitudinal convective transport and transverse diffusive transport, led to a net longitudinal diffusion.

Moreover, several theoretical investigations have been carried out for Newtonian fluids, although it is known that most physiological fluids behave as non-Newtonian fluids. The rheological property of the fluid, wave shape and porous nature of the wall play an important role in peristaltic transport which is useful in understanding transport of chyme in small intestine (Rao et al., 2004). In
Background

In this regard, peristaltic motion of an Oldroyd fluid (considering as chyme) in an axisymmetric tube with a sinusoidal wave (Elshehawey and Sobh, 2001) or the flow of a power-law fluid through a cylindrical tube in the presence of a peripheral layer of another power-law fluid with different viscosity (Misra and Pandey, 2001) or the peristaltic wave propagation during swallowing of food bolus with different viscosities through the esophagus (Tripathi, 2011) were studied. Furthermore, the flow properties of a Johnson-Segalman fluid (representative of chyme in small intestine, Hayat et al. (2012)) in a tube with a sinusoidal peristaltic motion traveling down its wall was theoretically investigated (Wang et al., 2007), whereas Amornsamankul et al. (2006) developed a numerical technique to simulate the three dimensional pulsatile blood flow in stenotic arteries and to study the effect of the non-Newtonian viscosity of blood on the flow behavior. It was shown that the non-Newtonian behavior of blood has significant effects on the velocity profile of the blood flow and the magnitude of the wall shear stresses. Peristaltic flow behavior was also studied both in moving and fixed reference frames by Brown and Hung (1977). They found that, near the leading and trailing ends of the bolus, the backward velocity in the moving frame increased from the wall to the axis, while at the midsection of the bolus it decreased from the wall to the core.

There have been several techniques applied for the characterization of peristaltic flow behavior, such as flow visualization facilitated by fine air bubbles (Brown and Hung, 1977) or Multiple Intraluminal Impedance (MII) novel technique to detect the flow in a viscous organ by measuring changes in intraluminal impedance related to the movement of the bolus (Imam et al., 2007). In the case of laminar oscillating flow in elastic tubes, an additional velocity component was expected to occur in the radial direction which was successfully measured by LDA (Liepsch et al., 1985), where the pulsating flow was generated by superimposing an oscillating flow (sinusoidal) on the stationary flow. The transitional regime of a sinusoidal pulsatile flow in a straight, rigid pipe was investigated using particle image velocimetry, where the influence of different pulsatile conditions on the critical Reynolds number was demonstrated (Trip et al., 2012). Instantaneous pulsatile (considering fluid motion being composed of an oscillatory and a steady flow component) velocity profiles were measured in a cylindrical tube by UVP (Hughes and How, 1994). In addition, UVP method was also used for the time dependent velocity profiles measurement in a purely oscillating pipe flows (Yamanaka et al., 2002). The combined effect of assumptions relating to refraction, the speeds of sound in tissue and blood on the accuracy of Doppler ultrasound blood velocity measurements was investigated both theoretically and experimentally by Christopher et al. (1995). Among all those available methods mentioned above, UVP was considered to apply the flow behavior investigation in the present study under peristaltic motion while squeezing of an elastic tube.
3 Materials and Methods

3.1 Introduction

The study of flow behavior in a collapsible elastic tube includes the investigation of the fluid-structure interaction involving the flow passing through it (Zhu and Wang, 2003). There are very few published literature (Dodson et al., 1974; Nahar et al., 2012) on the experimental flow characteristics of non-Newtonian fluids in collapsed elastic tubes under the influence of compressive external pressures. Therefore, aqueous solutions of one Newtonian and two non-Newtonian shear thinning fluids were used in the present study. The solutions were characterized by rheological measurements. The collapsed elastic tube shapes (under the influence of transmural pressures) were determined using computer tomography based image analysis and the corresponding fluid flow velocity profiles were monitored by UVP technique.

3.2 Materials

3.2.1 PEG aqueous solution as Newtonian fluid

Polyethylene Glycol (PEG, $M_W \approx 3.5 \times 10^4$ g/mol; Clariant, Switzerland) aqueous solution (19.67 % w/w) was used as a Newtonian fluid (section 3.3) to investigate the steady flow behavior through collapsible elastic tubes. The solution was prepared using a rotor-stator device (Polytron PT6000, Kinematica AG) at constant temperature, $T = 22 \, ^\circ \text{C}$.

3.2.2 CMC and PAA aqueous solutions as non-Newtonian fluids

Carboxymethyl-cellulose (CMC; Blanose CMC 7MF, IMCD Switzerland AG) at 1.5 % w/w (with 0.1 M NaCl; $M_W = 2.5 \times 10^5$ g/mol) and Polyacrylamide (PAA;
Materials and Methods

SNF FLOERGER, France) at 0.01 % w/w ($M_W \approx 14 \times 10^6$ g/mol) aqueous solutions were used as non-Newtonian shear thinning fluids. CMC and PAA aqueous solutions were found to be less and highly shear thinning respectively with about same value of zero shear viscosity (section 3.3). CMC aqueous solution was used in the steady, unsteady periodic and peristaltic flow experiments in a collapsible elastic tube. In addition, polyamide particles ($\rho = 1030$ kg/m$^3$ and diameter of 20 µm; Dantec Dynamics, Skovlunde, Denmark) at 0.3 % w/w were added to the CMC aqueous solution during peristaltic flow experiment (velocity magnitude was very low) for better resolution of UVP measurement. The viscosity flow curve was not altered after adding the polyamide particle in the CMC aqueous solution (since small mass fraction of the added particles, which led to negligible particle-particle interaction). Whereas, PEG aqueous solution was only used in the steady flow experiment for comparison of the fluid-structure interaction of Newtonian and non-Newtonian (less and highly shear thinning) fluids. All those aqueous solutions were again prepared using rotor-stator device at $T = 22$ °C.

3.2.3 Elastic tube

A silicone elastic tube (Lindemann GmbH, Germany) with 20 mm inner diameter and 1 mm thickness was used in the model flow (steady, unsteady-periodic and unsteady-peristaltic) experiments. The elastic modulus of the tube was measured by stress-strain curve using Zwick device (Zwick Roell Z010, Zwick GmbH, Germany). The experimental setup and the measured stress-strain curve is shown in Figure 3.1. It is seen that the elastic modulus (stress to strain ratio) of the tube decreased with increase in strain (right side of Figure 3.1). Since, tube deformation in the present study was expected to be low, therefore, elastic modulus of the tube ($E = 4.7$ MPa) was obtained by linear fit of the stress-strain curve at the lower deformation region.

Figure 3.1: Experimental setup for measurement of elastic tube modulus (left), and the corresponding measured stress-strain curve (right).
3.2.4 NaCl aqueous solution and membrane for batch diffusion

The experiments for diffusion and mass transport kinetics were studied using aqueous solution of sodium chloride (NaCl, with purity of 99.5 %; Sigma-Aldrich, Switzerland) as a model solute to characterize a sinter ceramic membrane (ceramic plate filtration membrane for ultrafiltration, Fraunhofer IKTS, Germany). The circular membrane disk diameter was 120 mm with thickness of 2 mm and the fraction of open porosity of approx. 40%. The membrane disk consisted of two layers with pore sizes, $d_{50}$ of 2.5 µm and 10 nm for support ($\alpha$-$\text{Al}_2\text{O}_3$) and membrane ($\gamma$-$\text{Al}_2\text{O}_3$) respectively.

3.3 Methods

3.3.1 Rheology of fluids

The rheological properties such as shear rate dependent viscosities and frequency dependent dynamic storage and loss moduli were measured using an Anton Paar Physica (MCR 300) rheometer with concentric cylinder geometry (CC27, shear gap width = 1.13 mm) to confirm the concentration for similar zero shear viscosity and the corresponding inelastic behavior of the investigated fluids. The rheological measurements were carried out at $T = 22 \, ^\circ\text{C}$, which was also maintained in the flow loop used during the experiment.

Shear rate dependent viscosity

The measured shear rate dependent viscosities of the investigated fluids (PEG, CMC and PAA) at different concentrations are shown in Figure 3.2. PEG, CMC and PAA aqueous solutions at 19.67, 1.5 and 0.01 % w/w respectively showed about similar value of shear viscosity ($\eta_0 \approx 0.143 \, \text{Pa.s}$) at the shear rate of 0.1 1/s. Both non-Newtonian (1.5 % CMC and 0.01 % PAA) aqueous solutions represented the shear thinning behavior, while one being highly shear thinning (0.01 % PAA) than the other (1.5 % CMC) at the same applied shear rate ranges. In addition, the approximated average shear rate in the deformed elastic tube during steady flow of 0.01 % PAA upon applied external pressure was found to be higher than the experimental shear rate during rheological measurements. Therefore, the corresponding viscosity of PAA aqueous solution at the higher shear rate regime was estimated by Power law fitting of the experimental data (Figure 3.2 d). No significant variation in the solution properties and temperature were observed while
Materials and Methods

Figure 3.2: Measured shear rate dependent viscosities of aqueous fluids at different concentrations of PEG (a), CMC (b), PAA (c), and combined viscosity curves representing similar value of shear viscosity at shear rate of 0.01 1/s (d).

passing through the flow loop used in the experiment (the duration was very short ≈ 30min to 1h). The shear rate dependent viscosity of the investigated solutions were measured again after performing the flow experiment and no change in the flow curve was attained.

Inelastic behavior of the shear thinning fluids

The two shear thinning aqueous solutions (1.5 % CMC and 0.01 % PAA) were further investigated to measure the viscous modulus, $G''$ and elastic modulus, $G'$ under linear viscoelastic conditions of the oscillatory shear. $G''$ is seen to be an order of magnitude higher than $G'$ for 1.5 % CMC (Figure 3.3 a). Whereas 0.01 % PAA (Figure 3.3 b) shows slightly higher value of $G''$ than $G'$, indicating their inelastic shear thinning behavior. In contrast, the aqueous solution of 2.5
% CMC depicts dominating elastic properties with increasing frequency (Figure 3.3 a), which is a typical viscoelastic behavior. The inelastic shear thinning fluids were chosen in the present study to avoid the complex behavior under imposed flow conditions.

Figure 3.3: Frequency dependent elastic ($G'$) and loss ($G''$) moduli using oscillatory shear measurements with a constant deformation of 5 % at 22 °C for CMC (a) and PAA (b) aqueous solutions.

3.3.2 Pulsed Doppler Method-Ultrasound Velocity Profiling

Basic principle

Ultrasound Velocity Profiling (UVP) technique uses the Doppler effect of a sound wave scattered by a moving particle, which is subjected to a frequency shift and is proportional to the velocity of the particle. UVP is considered as both a method and device for measuring an instantaneous velocity profile in liquid flow along the ultrasonic beam axis (Takeda, 1986). The basic principle of UVP method is described in detail elsewhere (Birkhofer, 2007; Met-Flow, 2002; Takeda, 1986; Wiklund et al., 2007), which is schematically represented in Figure 3.4. Here an ultrasonic transducer emits short pulses of frequency that travels along the measurement axis into the flowing fluid with the speed of sound. When the ultrasonic pulse hits a reflective surface (e.g. a small moving particle suspended in the flowing liquid), part of the ultrasound energy scatters and echoes back. The transducer is switched to receiving mode directly after transmitting a pulse, and the time interval between two successive pulses is available for echo reception since the pulses are separated by a fixed pulse reception frequency. If the scattering particle is moving with a non-zero velocity component into the acoustic beam axis of the
transducer, Doppler shift of received echoed frequency takes place. The Doppler shifted frequency is determined after demodulation using either time-domain or frequency-domain based signal processing (Wiklund et al., 2007). The local velocity $v_x$ in each channel in the flow direction is determined by (Figure 3.4) (Garbini et al., 1982a):

$$v_x = \frac{c f_D}{2 f_e \sin \theta} = \frac{v_{UV P}}{\sin \theta}$$  \hspace{1cm} (3.1)

where $f_e$ is the ultrasound emission frequency, $f_D$ is the Doppler shifted frequency, $c$ is the sound velocity in the medium and $\theta$ is the incidence angle (angle between transducer and the elastic tube is defined as the Doppler angle).

![Figure 3.4: Schematic representation of the basic principle of UVP technique.](image)

The velocity profiles of investigated solutions flowing in an uncollapsed and collapsed elastic tube under imposed flow (steady, unsteady-periodic and unsteady-peristaltic) were measured using a UVP-DUO (Met-Flow SA, Lausanne, Switzerland) instrument. The general description of the instrument can be found in Met-Flow (2002). A MATLAB (MathWorks, Natick, MA, USA) based application with a Graphical User Interface (GUI) was developed by Birkhofer (2007) (where the communication with the UVP-DUO was made with an active X Library from Met-Flow SA) and was adopted and used in the experiment. A similar application also using the combination of MATLAB and the UVPAX ActiveX library from Met-Flow was developed and used in the experiments by Wiklund et al. (2007). A holder containing a single ultrasound transducer was immersed in the water filled Perspex cylindrical pressure chamber or in an open tank. The holder
was designed to contain multiple transducers as well. The ultrasound transducer was positioned at a Doppler angle of 70° with respect to the horizontal axis. The velocity profiles can be measured using a single transducer at different radial and axial locations of the elastic tube. The transducer was placed in direct contact with the elastic tube submerged in water in the pressure chamber without causing additional tube deformation.

### Ultrasound transducers

Ultrasound transducers with 4 MHz (5 mm active diameter and 8 mm housing diameter; TX-type, Imasonic, France) and 8 MHz (2 mm active diameter and 5 mm housing diameter) emission frequencies were used which were a transmitter as well as a receiver. The beam shape of various transducers has been extensively studied by Messer and Aidun (2009). They stated that for a given transducer diameter, the beam width decreases with increasing emitting frequency. The ultrasound transducers used in this study were well applicable for its better sensitivity and penetration depths compared to the other emission frequency transducers. A mirror image of the velocity profile was observed beyond the back wall distance of the tube which was the imaginary profile caused by ultrasound reflection at the back wall (confirms the depth associated to the path).

### Sound velocity and acoustic impedance

Sound velocity is the mass and the stiffness of the material through which the sound passes (Povey, 1997), which depends on several factors such as density, temperature and other properties of the medium. The well known “pulse-echo time of flight” technique was used for the measurement of sound velocities in various materials involved in this study. Here two ultrasound transducers were fixed in a double wall cylindrical stainless steel adapter (Figure 3.5, top), to monitor the transmitted pulse (Figure 3.5, bottom) where one was directly connected to a digital oscilloscope (TDS 2024, four channels, 200 MHz, 2 GS/s; Tektronix, Inc.) and second one to the corresponding UVP-DUO-connector. This oscilloscope was also connected to the UVP-DUO via serial port (RS-232). The distance between two transducers was adjusted by the sound velocity in water (1483 m/s), then the velocity in the sample was calculated by the time of flight of the pulse and the transducer distance.

In addition, the acoustic impedance, \( Z \) [kg m\(^{-2}\) s\(^{-1}\), or Ray] is also a material property and expressed as the product of the density, \( \rho \) [kg/m\(^3\)] and sound velo-
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Figure 3.5: A double wall stainless steel adapter (top) and schematic representation of the “pulse-echo time of flight” technique used for the sound velocity measurement (bottom).

Acoustic impedance, $Z \ [\text{m/s}]$ of that material (Met-Flow, 2002):

$$Z = \rho c \quad (3.2)$$

The measured sound velocity and the corresponding acoustic impedance values of the investigated materials in this study are given in Table 3.1. The densities of the fluids were measured using a density meter (DMA 38, Anton Paar).

**Refraction of ultrasound and critical Doppler angle**

Reflection and refraction of ultrasound occurs while propagating through the boundary of two layers (having different acoustic impedances), where some of the wave reflects and some transmits to the second medium. Ultrasound waves also refract at different angles while passing through the interface depending on the variation in material characteristics. The relationship of the incident and refractive angles with the sound velocities of different layers can be expressed by Snell’s law. The propagation of an ultrasound wave through different layers involved in the present study is schematically represented in Figure 3.6. It can be stated that the medium with higher sound velocity (into which the wave propagates) will lead to a larger refraction angle. In addition, the angle of incidence (known as the critical angle) can reach to a certain critical value when the propagating sound wave moves along the interface between two media.
3.3 Methods

Table 3.1: Sound velocity and acoustic impedance values of the investigated materials.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Density [kg/m³]</th>
<th>Sound velocity [m/s]</th>
<th>Acoustic impedance [MRay]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>998</td>
<td>1483</td>
<td>1.48</td>
</tr>
<tr>
<td>Elastic tube</td>
<td>1125</td>
<td>992</td>
<td>1.11</td>
</tr>
<tr>
<td>19.67 % PEG</td>
<td>1038</td>
<td>1586</td>
<td>1.64</td>
</tr>
<tr>
<td>0.01 % PAA</td>
<td>992</td>
<td>1500</td>
<td>1.48</td>
</tr>
<tr>
<td>1.5 % CMC</td>
<td>1011</td>
<td>1516</td>
<td>1.53</td>
</tr>
<tr>
<td>1.5 % CMC (+ 0.3 % PA)</td>
<td>1007</td>
<td>1499</td>
<td>1.51</td>
</tr>
</tbody>
</table>

In the present study, the adjusted angle of incidence (20° or 45°) was much below the critical angle, which was calculated by Snell’s law as follows:

\[
\frac{\sin \theta_3}{\sin \theta_2} = \frac{c_3}{c_2}
\]  \hspace{1cm} (3.3)

and,

\[
\frac{\sin \theta_2}{\sin \theta_1} = \frac{c_2}{c_1}
\]  \hspace{1cm} (3.4)

Then, combining Equations 3.3 and 3.4 give

\[
\frac{\sin \theta_1}{\sin \theta_3} = \frac{c_1}{c_3}
\]  \hspace{1cm} (3.5)

and

\[
\theta_1 = \sin^{-1} \left( \frac{c_1}{c_3} \sin \theta_3 \right)
\]  \hspace{1cm} (3.6)

If \( \theta_3 = 90° \), then \( \theta_1 = 81.62° \) (when \( c_3 = 1499 \) m/s and \( c_1 = 1483 \) m/s as indicated in Figure 3.6). Therefore, the critical Doppler angle must be larger than 8.38° (90° - \( \theta_1 \)) so that the whole of the incident wave is not reflected.
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Figure 3.6: The propagation of an incident sound wave through different layers.

Adjusted UVP parameters

The set of parameters adjusted and common for all profiles under imposed flow (steady, unsteady-periodic and unsteady-peristaltic) shown later are listed in Table 3.2.

Table 3.2: Set of UVP parameters adjusted and common in the flow experiments.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Steady flow</th>
<th>Unsteady-periodic flow</th>
<th>Unsteady-peristaltic flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel distance [mm]</td>
<td>0.38</td>
<td>0.38</td>
<td>0.56</td>
</tr>
<tr>
<td>Number of cycles [-]</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Pulse repetition frequency [Hz]</td>
<td>1007</td>
<td>1007</td>
<td>991</td>
</tr>
<tr>
<td>Emission frequency [MHz]</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Measurable depth [mm]</td>
<td>750</td>
<td>750</td>
<td>756</td>
</tr>
<tr>
<td>Number of pulse repetitions [-]</td>
<td>256</td>
<td>256</td>
<td>128</td>
</tr>
<tr>
<td>Velocity resolution [mm/s]</td>
<td>0.75</td>
<td>0.75</td>
<td>1.06</td>
</tr>
<tr>
<td>Time resolution (single profile) [ms]</td>
<td>254</td>
<td>254</td>
<td>130</td>
</tr>
<tr>
<td>Spatial resolution [mm]</td>
<td>0.38</td>
<td>0.38</td>
<td>0.37</td>
</tr>
<tr>
<td>Divergence of the ultrasound beam [°]</td>
<td>2.2</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>Voltage [V]</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Gain start [-]</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Gain end [-]</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>
3.3 Methods

3.3.3 Tube shape analysis by CT-method

The experimental investigation of tube geometry is limited due to lack of non-invasive technique, since any contact with a measuring device can alter the dynamic equilibrium of tube wall (Elad et al., 1992). There are several techniques applied to measure the cross sectional areas of collapsed tube such as electrical impedance method (Bertram, 1987) or ultrasound imaging (Bertram and Ribreau, 1989), which may have limited resolution. Elad et al. (1992, 1989) developed a remote sensing technique based on stereography method to measure the biomedical surfaces and presented the three dimensional measurements of collapsible tube.

In the present work, the shapes of the collapsed and uncollapsed elastic tube were measured optically using images taken at different angles. The computer tomography (CT) method was applied based on the backward projection of plural images for which several grid lines of equidistant (4 mm) spacing were drawn on the tube surface. A Canon PowerShot G2 camera with the resolution of 2272 ×1704 was mounted on an aluminum arm (surrounding the PG pressure chamber) at a distance of 434.5 mm from the tube surface, which can be rotated radially around the tube to obtain images at different angles (Figure 4.1). The maximum possible images obtained from the section corresponded to $170^\circ$ rather than the desired $180^\circ$ due to less number of grid lines captured by the camera especially when the elastic tube was more collapsed. This is an approximate approach to investigate the elastic tube shapes and geometries as the resolution varies depending on the degree of tube deformation under different applied external chamber pressures, $P_e$.

![Figure 3.7: Tube shape analysis by Computer Tomography (CT) method (right) by identifying the grid lines drawn on the tube surface (left and middle).](image)

For the correction of the distortion from the camera optics, several open source
software applications for the camera calibration were evaluated. The Camera Calibration Toolbox from the Computer Vision research Group of the California Institute of Technology\(^1\) was found to be the most suitable one. The light diffraction at the two interfaces of water/acrylic glass and acrylic glass/air was calculated. Thus it was possible to incorporate the corresponding correction in the image pre-processing.

A MATLAB based software was used where the CT method was implemented for the 3D reconstruction of the elastic tube. For this technique a grid pattern was painted on the tube and pictures were taken by rotating the radially oriented camera around the tube. The angle between the pictures taken was 5° or 10°. Image processing (contrast maximization etc.) was applied to identify the grid lines drawn on the tube surface (left and middle of Figure 3.7). Then a rotation matrix (according to the angle of the camera position) was applied to the images, which were combined using addition or multiplication of the grayscale values. The projection beam lines for the grid lines on the tube cross at one point resulting in the position in 2D as shown in right side of Fig. 3.7. Another MATLAB based software celltracker was used to detect the crossing points in pixel values. Since the internal cross sectional area was of interest to relate with the flow through collapsible tubes, additional data analysis was performed. The reference pixel value was calculated on the basis of the known outer tube diameter. Then the corresponding value for the tube wall thickness was subtracted to obtain the internal cross sectional area of the tube. The change in the wall thickness was considered to be negligible for both collapsed and inflated tubes. By combining the 2D position at several locations (performing an iteration along the tube length with a certain step change), the 3D tube shape was reconstructed.

### 3.3.4 Characteristic pressure measurement

The experiment of fluid flow behavior in collapsed elastic tube was performed by mounting an elastic tube in a controllable pressure chamber. The internal pressure inside the tube was assumed to be uniform over the cross section and two pressure sensors (SensorTechnics, Type: CTEM9350GY7, pressure range: 0 to 350 mbar with accuracy of ±1 % FSO) were installed on two hard PVC connectors for both the inlet and outlet aluminum pipes to measure the inlet $P_i$ and outlet $P_o$ pressures of liquid, which eventually gave the pressure drop ($\Delta P = P_i - P_o$) in the tube. One additional pressure sensor of the same type was connected to the pressure chamber to measure the externally applied pressure $P_e$ to deform the elastic tube. Three pressure transducers were connected to a data acquisition

\(^1\)http://www.vision.caltech.edu/bouguetj/calib_doc/
3.4 Conclusions

Three distinct sets of the measurements were carried out such as characteristic pressures, velocity profiles by UVP and tube shapes by image analysis. In order to check the reproducibility of the results presented, the individual set of experiments as mentioned were performed three to four times. All sets of experiments were well reproducible and showed no significant deviations. The corresponding standard deviations for characteristic pressures, velocity profiles and tube shapes are 0.0048, 0.0038 and 0.0278 respectively. Therefore, the error bars are not included and the data from one fully consistent set of experiments are presented.

Figure 3.8: Schematic representation of the characteristic pressure definitions used in the experiment.
4 Steady flow in deformed elastic tube

4.1 Introduction

The scientific research on the flow behavior of non-Newtonian fluids in inflatable and collapsible elastic tubes is important for understanding of biofluid mechanics in human body such as blood flow in arteries and veins, and food flow in pharynx, esophagus and small intestine. In addition, peristaltic flow (involves the contraction and expansion type of flow, in other words flow in a deformed elastic tube) is the essential physiological transport mechanism encountered in the most tubular organs of the human digestive system. Flow characteristics of fluids in elastic tubes are governed by the fluid micro-structural properties, elastic tube solid mechanics, interaction between tube deformation and fluids (Heil, 1997), and flow inducing applied stresses (Grotberg and Jensen, 2004). Some authors (Hazel and Heil, 2003; Heil, 1997; Lyon et al., 1980) contributed experimentally and theoretically, which enables the understanding of complexity of the Newtonian fluid laminar and turbulent flows in collapsible elastic tubes involving the tube solid mechanics. In contrast, there are very few (Dodson et al., 1974; Nahar et al., 2012) published literature on the experimental flow characteristics, the velocity distributions and rheological properties of non-Newtonian fluids in collapsed elastic tubes. Therefore, the present study challenge included the simultaneous measurement of both the deformed tube shape and the corresponding velocity flow field under the influence of compressive external pressures. Since peristalsis can be considered as a flow in a dynamically deformed elastic tube, hence as a simple model the bench of Starling Resistor set up was used where an elastic tube was deformed by imposed external chamber pressure. In this study, we investigated the steady laminar flow characteristics and spatial rheological properties of both Newtonian (PEG) and non-Newtonian inelastic shear thinning (CMC and PAA) aqueous solutions in an elastic tube deformed with different shapes due to compressive transmural (internal minus external) pressures $P_{tm}$. These involved the measurements of shapes of the deformed or collapsed elastic tube using CT-merhod and the corresponding fluid flow velocity profiles by UVP technique. UVP has been applied to monitor experimentally the flow field in non-collapsible tubes (Birkhofer et al., 2008)
Steady flow in deformed elastic tube
during steady and unsteady laminar flow of non-Newtonian fluids and particulate suspensions. We met the challenge of implementing the UVP technique to measure the velocity profiles during steady laminar flow of a non-Newtonian shear thinning solution in a collapsible elastic tube (Nahar et al., 2012). In addition, an interaction between the elastic tube deformation and the fluid flowing through it was also found, which can be represented by the measured pressure drop to volume flow rate relationship as well as the upstream and downstream transmural pressures. The aim of the present work was also to determine the effect of the reduction in cross-sectional area, and change in shape of the collapsed elastic tube on the flow velocity profiles of different fluids (Newtonian and non-Newtonian), which correspondingly influenced the average or spatial shear rates and viscosities. The detailed knowledge gained about the variation in rheological properties of the shear thinning fluids in deformed elastic tube will eventually help for better understanding of the effect of tube-deformation during peristalsis on diffusion encountered in small intestine during digestion.

4.2 Experiments: Starling Resistor

Several authors e.g. (Lyon et al., 1980) used the Starling Resistor setup for Newtonian fluid flow investigations in elastic tubes relevant to many applications. Figure 4.1 (top: actual, bottom: schematic) shows the present experimental setup consisting of a 300 mm inner diameter, 5.66 mm thick and 620 mm long cylindrical plexiglass (PG) pressure chamber fixed on each side a metal flange with an aluminum pipe. A silicone elastic tube (320 mm in length, 20 mm inner diameter, 1 mm thickness), was mounted between the two aluminum pipes with a slight axial extension to avoid the longitudinal bending especially when the tube inflates. The different states of the tube geometry were achieved by increasing the hydrostatic head connected with the water filled PG pressure chamber. The aluminum pipe on the right side was connected to a gear pump (MCP-Z, ISMATEC) and a PVC tank, which contained the investigated fluids for flow through the collapsible elastic tube. In addition, the schematic of collapsed elastic tube and position of ultrasound transducer for velocity profile measurement is also inserted in Figure 4.1 (bottom-right). All the consequent measurements were carried out at 22 °C in this work.
Figure 4.1: Experimental set up (top: photograph, and bottom: schematic) for flow behavior study of different fluids (Newtonian and non-Newtonian) through a collapsible elastic tube using Starling Resistor. (bottom-right: schematic of collapsed elastic tube and position of ultrasound transducer for velocity profile measurement).
4.3 Results and Discussion

The tube shape analysis and the corresponding velocity profile measurements were carried out under a steady flow of both Newtonian and non-Newtonian aqueous solutions. The various degree of tube deformations were achieved by imposed compressive transmural pressures. The velocity profiles were monitored by setting the ultrasound transducer as schematically represented in Figure 4.1 (bottom-right). It was seen that the tube shape changed from a circular shape to a two lobed shape, and the corresponding velocity profile changed from a parabolic to a bi-modal for both Newtonian and Newtonian fluids under different compressive $P_{tm(down)}$. The variation in the elastic tube shapes and the corresponding flow velocity profiles along the tube length were also observed. In addition, the influence of flowing fluid properties through the tube on the variation in deformation along the tube length under different applied external pressures was also observed. The steady flow behavior in collapsible elastic tube also included the investigation of characteristics pressure differences under different parameters such as external pressure ($P_e$) and fluid flow rate ($\dot{Q}$). The pressure drop ($\Delta P$) through the tube was found to be a function of $\dot{Q}$, the upstream ($P_{tm(up)} = P_i - P_e$) and downstream ($P_{tm(down)} = P_o - P_e$) transmural pressures. A relationship between the variation in tube cross sectional area under different $P_{tm}$ was established by the Similarity ‘tube law’ for different investigated fluids. Finally, influence of various degree of tube deformations on the rheological behavior, and the steady flow characteristics of the Newtonian and shear thinning fluids were also elucidated.

4.3.1 Tube shapes under various $P_{tm}$ at constant $\dot{Q}$

Experimentally reconstructed 3D tube shapes of both the uncollapsed (at $P_e = 18$ mbar or $P_{tm(down)} = 79$ mbar) and collapsed (at $P_e = 105$ mbar or $P_{tm(down)} = -18$ mbar) tubes are shown in Figure 4.2 while flowing of 1.5 % CMC aqueous solution through it (details are in section 3.3.3). The investigated length for tube shape analysis (both uncollapsed & collapsed) was chosen to be 190 mm from the rigid tube connection from the outlet. The 3D reconstruction of the tube shapes were performed by analyzing tube shapes at every 1 cm (undeformed) or 2 cm (deformed) along the tube length. The measured collapsed tube shape (Figure 4.2, bottom) was representing the region of strongest collapse near the downstream end as the fluid pressure decreased continuously in the stream-wise direction leading to a higher compression of the tube wall. Figure 4.3 represents the influence of flowing fluid properties on the variation in deformation along the tube length under different applied external pressures. The elastic tube was seen to be less collapsed for the Newtonian (19.67 % PEG) fluid, whereas it was more collapsed...
4.3 Results and Discussion

Figure 4.2: 3D reconstruction of the elastic tube: (top) uncollapsed tube shape at $P_e = 18$ mbar, considered as reference and (bottom) collapsed tube shape at $P_e = 105$ mbar.
Figure 4.3: Influence of flowing fluid properties through the tube on the variation in deformation along the tube length (front view) under different applied external pressures at \( \dot{Q} = 17 \text{ ml/s} \).

for inelastic shear thinning fluids (1.5 % CMC and 0.01 % PAA) exhibiting the same shear viscosity at shear rate of 0.1 s\(^{-1}\) and under same imposed external pressures. The reason is that, the internal pressure is minimized by increasing the fluid velocity during tube collapse by external pressure for the shear thinning fluids.

### 4.3.2 Variation in tube shape and velocity profile along the tube length at constant \( P_{tm} \) and \( \dot{Q} \)

The detailed illustration of the collapsed tube shapes and the corresponding flow velocity profiles at each section of 2 cm along the tube length for \( P_{tm(down)} = -18 \text{ mbar} \) \( (P_c = 105 \text{ mbar}) \) under a steady flow of 1.5 % CMC at \( \dot{Q} = 17 \text{ ml/s} \) are shown in Figure 4.4. The experimental results clearly showed that the elastic tube was deformed non-axisymmetrically since \( P_c \) exceeded the fluid pressure. The region of strongest tube deformation occurred near the tube outlet where the inside pressure was low. The monitored two lobed shape and corresponding bi-modal velocity profiles were obtained along the length of about 5 to 13 cm from the rigid tube connection at the outlet. The measured bi-modal velocity profiles exhibited a peak value in each of the two lobes. Since no line or area contact occurred in
these tube shapes, the pressure was uniform in each of the cross sections. The smaller lobe showed higher mean velocity and frictional force compared to those in the larger lobe. The analysis of these two lobes tube shapes showed that the gap between two walls varied from 2.7 (minimum) to 5.9 mm (maximum), and the corresponding velocity at the gap between two walls increased from 0.087 to 0.118 m/s. As the tube length increased from 13 to 19 cm, the gap between two walls further increased to 14.5 mm. Consequently, the tube shape changed to nearly elliptical as the pressure inside the tube increased and transmural pressure approached less negative value, the corresponding velocity profiles becoming unimodal. The velocity profiles measured in the two lobed shapes can be seen to have some noise effect at the center due to multiple echoes from the two tube walls which were closer to each other.

Figure 4.4: Illustration of the variation in the elastic tube shape and cross sectional area along the tube length at an external chamber pressure of about 105 mbar and the corresponding flow velocity profiles.
4.3.3 Effect of transmural pressures on tube shapes and velocity profiles at constant volume flow rate

The deformation of the tube was affected by the changes in the transmural pressures. The tube shape analysis and the velocity profile measurements were carried out near the tube outlet, since the maximum tube deformation appeared in this region. Figure 4.5 shows the variation in tube geometry and the corresponding velocity profiles at constant length \((X = 7 \text{ cm})\) from the rigid tube connection at the outlet and \(\dot{Q} = 17 \text{ ml/s}\) under various downstream transmural pressures \(P_{tm(down)}\). The inserted tube geometries were analyzed under different applied \(P_e\) while steady flow of 1.5% CMC aqueous solution through the tube. The tube geometries were also assumed to attain the same shapes with slight variation in the cross sectional area during steady flow of 19.67% PEG or 0.01% PAA solution in the tube. It can be seen that the tube shape changed from circular to two lobed shape, and the corresponding velocity profiles transformed from parabolic to bi-modal for both Newtonian and non-Newtonian fluids. The elastic tube exhibited a circular shape for positive \(P_{tm(down)}\) with a parabolic velocity profile, where the more shear thinning fluid (0.01% PAA) showed flattened profile at the tube center as expected. The \(P_{tm(down)}\) was found to be more positive for Newtonian fluid than that of the shear thinning fluids, since the internal pressure \(P_o\) was much higher than the applied external pressure \(P_e\). As long as the \(P_{tm(down)}\) became sufficiently negative, the elastic tube started to buckle from an elliptical to a two lobed shape. It was seen that a more negative \(P_{tm(down)}\) (-29.31 mbar) or higher \(P_e\) (125 mbar) was needed for Newtonian fluid compared to the shear thinning fluids (\(P_{tm(down)} \approx -18 \text{ mbar}\) and \(P_e = 105 \text{ mbar}\)) to obtain a two lobed shape of about same cross sectional area (Tables 4.3, 4.4 and 4.5). The reason is that the non-Newtonian fluid exhibits a significant pressure decrease in the reduced cross sectional area region due to much increase in fluid velocity compared to the Newtonian fluid (Bernoulli’s equation: \(p_1/\rho g + v_1^2/2g + h_l = p_2/\rho g + v_2^2/2g\), where \(v_1\) and \(v_2\) are the fluid flow velocities at the chosen points on a streamline, \(p_1\) and \(p_2\) are the pressures at the chosen points, \(\rho\) is the density of the fluid, \(g\) is the acceleration due to gravity and \(h_l\) is the pressure-head loss). The measured velocity profile changed from a parabolic to a bi-modal for two lobed shape. It was also observed that the average maximum velocity increased by a factor of two, and the tube cross sectional area decreased by a factor of six for a compressive \(P_{tm}\) of about -29 mbar (Newtonian fluid, PEG) and about -18 mbar (shear thinning fluids, CMC and PAA). Experimentally obtained average maximum velocity (during flow of 1.5% CMC) in each lobe for the bi-modal velocity profile at \(X = 7 \text{ cm}\) was nearly same which clearly represents the axisymmetric deformation of the elastic tube at that location. Whereas the elastic tube found to be deformed non-axisymmetrically during flow of PEG and PAA aqueous solutions as
4.3 Results and Discussion

Figure 4.5: Influence of various applied transmural pressures on the change in tube shapes and the corresponding velocity profiles in the tube during a steady volume flow rate ($\dot{Q} = 17$ ml/s) of PEG, CMC and PAA aqueous solutions.
the measured bi-modal velocity profiles showed different peak values in each lobe. The tube was supposed to deform axisymmetrically during flow of a Newtonian fluid (PEG) through it. The observed little discrepancy in the present experiment might be due to distortion of the tube center axis while mounting or non-uniform thickness along the tube length. On the other hand, a non-axisymmetric tube deformation was expected while flow of a highly shear thinning fluid (PAA) as the pressure inside the tube can certainly decrease and correspondingly increase in the fluid velocity when $P_{tm(down)}$ was approaching a negative value (higher applied $P_e$).

A reduction in the tube cross sectional area can lead to an increase in both the near-wall and average shear rates during a steady flow of both Newtonian and shear thinning fluids. As shown in Table 3.2, the spatial and time resolutions are reasonably good indicating precise measured velocities even in more collapsed tube. Consequently, the determined shear rate was also accurate. The average shear rate can be calculated using the obtained average velocity ($\dot{Q}/A$) in the tube where the cross-sectional area $A$ was estimated by the image analysis. Since the maximum shear rate was estimated near the tube wall, therefore it was calculated with the corresponding velocity gradient at that regime. The approximated average and near-wall shear rates in the deformed elastic tube with different cross sectional area during flow of Newtonian and non-Newtonian fluids are summarized in Tables 4.3, 4.4 and 4.5 respectively. The calculated maximum shear rates near the tube wall for PEG, CMC and PAA aqueous solutions are 10.5 s$^{-1}$, 10.7 s$^{-1}$ and 13.9 s$^{-1}$ respectively for circular tube shape. It is seen that CMC is showing similar shear rate value as PEG, since CMC is still in the Newtonian regime at that shear rate. Whereas PAA is representing much higher wall shear rate since it is strongly shear thinning at that shear rate. In addition, the approximated wall shear rates for two lobed shape for PEG, CMC and PAA aqueous solutions are 25.5 s$^{-1}$, 35.8 s$^{-1}$ and 57.4 s$^{-1}$ respectively.

4.3.4 Effect of external pressure on variation in pressure drop with volume flow rate

The flow behavior of fluids in elastic tubes depends on the micro-structural properties of the fluids, solid mechanics of the tube, and the interaction between the deformation of the tube and fluids. The applied external pressure $P_e$ should exceed a critical value to approach the different geometry of the tube shape (elliptical to line or area contacted two lobes). Figures 4.6a and 4.6b are representing the pressure drop in the tube ($\Delta P = P_i - P_o$) as a function of $P_e$ and $\dot{Q}$ respectively during steady flow of Newtonian fluid (19.67 % PEG). It is seen that $\Delta P$ at a given $\dot{Q}$ does not change until it reaches to a critical $P_e$ when the transition of
partial to complete tube collapse occurs with reduced tube cross sectional area. The applied critical $P_e$ region is about 105 to 120 mbar as seen in Figure 4.6a (dashed gray circle). The three distinct regions of applied critical $P_e$ for the tube deformation are also clearly observed. The value of critical $P_e$ depends on the balance of the fluid normal $F_N(P)$ and tube wall compression $F_{\text{tube}}$ forces with the applied external force $F_e(P_e)$, where the forces are almost at the equilibrium ($F_N + F_{\text{tube}} \simeq F_e(P_e)$). Figure 4.6b shows that $\Delta P$ in the tube increases linearly with increase in $\dot{Q}$ (schematic deformed tube shapes at $P_e = 120$ mbar for different $\dot{Q}$ and also at constant $\dot{Q} = 17$ ml/s for various applied $P_e$ are also given) below the critical $P_e$ region due to higher frictional force, as $\dot{Q}$ is a function of $Re$ rather than reduction in tube cross sectional area $A$. On the other hand, $\Delta P$ is higher for lower $\dot{Q}$ at higher $P_e$ since the tube cross sectional area is reduced above the critical $P_e$ region. Moreover, reduction in $A$ is much higher at lower $\dot{Q}$ than that of the higher $\dot{Q}$ ($F_{N,\text{higher}Q} \geq F_{N,\text{lower}Q}$) for a given $P_e$. Therefore, $\Delta P$ increases with increasing $\dot{Q}$ below critical $P_e$, and decreases above critical $P_e$ until a given $\dot{Q}$ when the elastic tube is reopen to its original shape.

The region of applied critical $P_e$ (about 97 to 105 mbar) is again clearly observed in Figures 4.7a and 4.8a (dashed gray circle), where the transition of partial to complete tube collapse occurs, during steady flow of 1.5 % CMC and 0.01 % PAA solutions respectively. The same feature is also observed for various values of $\dot{Q}$ and the transition region for tube collapse is also found to be common for the range of $\dot{Q}$ used in the present work. It is also seen that $\Delta P$ in the tube increases slightly linearly with increase in $P_e$ at a constant $\dot{Q}$ below the critical $P_e$ region (Figure 4.7a). In contrast, above the critical $P_e$ region, $\Delta P$ increases drastically involving a small non-linearity for the same $\dot{Q}$ since the tube walls come closer reducing $A$ at higher $P_e$ and increases the fluid friction on the tube wall. In addition, the slope in the upper critical $P_e$ region is found to be about 48 times higher than that at the lower critical $P_e$ region. Since $\Delta P$ in a pipe is a nonlinear function of the cross sectional area (according to Darcy-Weisbach equation: $\Delta P = fL\rho v_{\text{avg}}^2/2D$ (Green and Perry, 2008), where $f$ is the friction factor, $D = \sqrt{4A/\pi}$ is the diameter of tube of length $L$ and $v_{\text{avg}}$ is the average velocity of fluid of density $\rho$), the non-linearity present in the upper critical $P_e$ region can therefore be due to reduction in tube $A$. Figure 4.7b shows that $\Delta P (= P_i - P_o)$ increases with increase in $\dot{Q}$ of CMC aqueous solution up to critical $P_e$ due to increase of velocity and corresponding friction force in uncollapsed tube. When the applied $P_e$ is further increased from the critical value so that the transmural pressure difference becomes more negative, then the collapsed tube walls come closer reducing in $A$ and $\Delta P$ is increased at a given $\dot{Q}$. On the other hand, as $\dot{Q}$ is further increased, internal pressure is gradually increased and the tube recovered its original shape at higher $\dot{Q}$ thereby decreasing $\Delta P$ due to increase of $A$ and decreasing the corresponding average
Figure 4.6: Influence of applied external chamber pressure on the variation in pressure drop with steady volume flow of PEG solution in the elastic tube together with its schematic deformed shapes at $P_e = 120$ mbar for different flow rates and also at constant $Q = 17$ ml/s for various applied $P_e$. 

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Figure 4.7: Influence of applied external chamber pressure on the variation in pressure drop with steady volume flow of CMC solution in the elastic tube together with its schematic deformed shapes at $P_e = 120 \text{ mbar}$ for different flow rates and also at constant $\dot{Q} = 17 \text{ ml/s}$ for various applied $P_e$.  

"Below critical $P_e$ region" 

"Above critical $P_e$ region" 

Pressure drop through the tube, $\Delta P = P_i - P_o \text{ mbar}$ 

External chamber pressure, $P_e \text{ mbar}$ 

Flow rate, $Q \text{ ml/s}$ 

$\dot{Q}$ increase 

$P_{tm} = -37 \text{ mbar}$ 

$P_{tm} = -33 \text{ mbar}$ 

$P_{tm} = -29 \text{ mbar}$ 

$P_{tm} = -23 \text{ mbar}$
Steady flow in deformed elastic tube

Figure 4.8: Influence of applied external chamber pressure on the variation in pressure drop with steady volume flow of PAA solution in the elastic tube together with its schematic deformed shapes at $P_e = 120$ mbar for different flow rates and also at constant $Q = 17$ ml/s for various applied $P_e$. 
flow velocity in the tube. The situation is better explained by the observation of the tube geometry at the highest applied $P_e$ in this experiment (the change in the observed tube shape at the corresponding flow rate is schematically drawn in Figure 4.7b). It is seen that an area contacted collapsed tube shape is formed in the lower $\dot{Q}$ range with reduced $A$. As $\dot{Q}$ is slowly increased, the tube opens up and increasing its $A$ (due to deformability caused by tube elasticity), which results in a decrease of $\Delta P$ in the tube. On the other hand, for high shear thinning fluid (PAA) the critical $P_e$ is found to be more closer to each other for all $\dot{Q}$ compared to the other investigated fluids as in Figure 4.8a. The reason is that, $\dot{Q}$ does not have much influence on the increase in tube internal pressure rather than reduction in $A$, which can be observed above the critical $P_e$. In addition, the investigated range of $\dot{Q}$ is not to be high enough to increase the tube internal pressure due to increase in the fluid velocity. Therefore, the shape of the tube at a given $P_e$ does not change while increasing $\dot{Q}$, and correspondingly $\Delta P$ is increased for higher $\dot{Q}$ due to increase in frictional force inside the tube as shown in Figure 4.8b. It is also seen that $\Delta P$ is much higher for the deformed tube due to much higher reduction in $A$ compared to the other investigated fluids (PEG and CMC).

### 4.3.5 Variation in upstream and downstream transmural pressures with pressure drop at different volume flow rates

In the case of Newtonian fluid (high viscous) flow through a collapsible tube, it was demonstrated (Heil, 1997) that the downstream transmural pressures continue to linearly decrease with the pressure drop through the tube, which leads to move the point of strongest collapse further downstream whereas the upstream end of the tube becomes more strongly inflated.

Figures 4.9, 4.10 and 4.11 are representing the $\Delta P$ in the tube as a function of both upstream ($P_{tm(up)}$) and downstream $P_{tm(down)}$ transmural pressure differences for PEG, CMC and PAA aqueous solutions respectively. In our study of non-Newtonian fluids, the $P_{tm(down)}$ is found to be more negative than the upstream $P_{tm(down)}$ one as the outlet pressure decreases when the tube collapses more strongly. Heil (1997) observed similar behavior due to higher pressure drop even at low volume flow rate of highly viscous Newtonian fluid in the collapsed tube. It is also seen that the pressure inside the tube increases at the upstream end which leads to linear increase in the pressure drop through the tube for a constant $\dot{Q}$ while varying the $P_{tm(up)}$. The total pressure drop through the tube is dominated by the pressure drop in the most strongly collapsed region. The pre-buckling branches for different $\dot{Q}$ are observed (nearly vertical parts of the curves) where an increase in the transmural pressure reduces very slightly the pressure
Figure 4.9: Variation in the upstream (a) and downstream (b) transmural pressures with the pressure drop in the elastic tube at different constant volume flow rates of PEG.
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Figure 4.10: Variation in the upstream and downstream transmural pressures with the pressure drop in the elastic tube at different constant volume flow rates of CMC.

Figure 4.11: Variation in the upstream and downstream transmural pressures with the pressure drop in the elastic tube at different constant volume flow rates of PAA.
Steady flow in deformed elastic tube

drop through the tube. The curves of pre-buckling pressure drop through the tube are seen to be right shifted due to higher values of $\dot{Q}$. In addition, pressure drop in the tube increases in the region of tube collapse (for negative $P_{tm(down)}$) due to reduction in tube cross-sectional area $A$.

4.3.6 Effect of transmural pressure on tube cross-sectional area reduction

Similarity tube law:
The one-dimensional steady flow in compliant tubes is generally represented by the ‘tube law’, which is the relationship between the transmural pressure and cross-sectional area (Shapiro, 1977b). On the other hand, Elad et al. (1987) proposed the similarity tube law considering only the elastic, quasi-static behavior of the bronchi, which can describe the behavior of all geometries. The general nonlinear elastic behavior of the elastic tube under positive transmural pressure was well described by

$$P - P_e = K_p (\omega^{n_1} - 1)$$  \hspace{1cm} (4.1)

where $P$ is the pressure in the tube, $P_e$ is the external pressure, $\omega = A/A_0$ is the area ratio ($A$ and $A_0$ are the cross-sectional areas of the deformed and undeformed tube respectively), and $K_p$ and $n_1$ are the material coefficients. In contrast, the behavior in the collapsed region (negative transmural pressure) was described by a modified tube law (Shapiro, 1977b) as

$$P - P_e = K_p^2 (1 - \omega^{-n_2})$$  \hspace{1cm} (4.2)

As the simple fluid mechanical analysis demands a single description for the entire range of deformation, the similarity tube law over the entire range was empirically represented by

$$P_{tm} = K_p (\omega^{n_1} - \omega^{-n_2})$$  \hspace{1cm} (4.3)

where $P_{tm} = P - P_e$ is the transmural pressure difference with uniform pressure $P$ in the tube, and external pressure $P_e$, and $K_p$, $n_1$ and $n_2$ are the material coefficients.

Comparison with experimental results:
The similarity tube law of the measured data is represented (Figure 4.12) for both Newtonian (PEG) and non-Newtonian (CMC and PAA) aqueous solutions as a function of cross sectional area ratio $\omega$ and the dimensionless transmural
pressure ratio $\Pi = \frac{P_{tm(down)}}{K^b_p}$. The bending stiffness is defined as $K^b_p = E(h_0/r_0)^2/12(1 - \epsilon^2)$ (Elad et al., 1992), where $E$ is the elastic modulus of the tube wall, $\epsilon$ is the Poisson ratio, and $h_0$ and $r_0$ are the wall thickness and internal radius of the undeformed tube. The elastic modulus is measured by stress-strain curve using ZWICK device and it is assumed that $\epsilon = 0.5$ to reflect the near-incompressible behavior of rubber tubes and biological tissues (Heil, 1997). The comparable relevant material parameters of the elastic tube and the coefficients of the fitted Eq. 4.3 for different investigated fluids are summarized in Table 4.1 and Table 4.2 respectively.

### Table 4.1: Comparable elastic tube material parameters.

<table>
<thead>
<tr>
<th>Elastic tube parameters</th>
<th>Authors</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Present work</td>
</tr>
<tr>
<td>$h_0$ [mm]</td>
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</tr>
<tr>
<td>$r_0$ [mm]</td>
<td>10</td>
</tr>
<tr>
<td>$E$ [Pa]</td>
<td>$4.7 \times 10^6$</td>
</tr>
<tr>
<td>$K^b_p$ [Pa]</td>
<td>695.07</td>
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</table>

### Table 4.2: Coefficients of the fitted Eq. 4.3 for different investigated fluids.

<table>
<thead>
<tr>
<th>Parameters/ Fluids</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$K_p$ [Pa]</th>
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<tr>
<td>PEG</td>
<td>42</td>
<td>5.3</td>
<td>250</td>
</tr>
<tr>
<td>CMC</td>
<td>40</td>
<td>1</td>
<td>251</td>
</tr>
<tr>
<td>PAA</td>
<td>37</td>
<td>0.82</td>
<td>303</td>
</tr>
<tr>
<td>Water (Elad et al.)</td>
<td>44</td>
<td>1.3</td>
<td>135.93</td>
</tr>
</tbody>
</table>

Figure 4.12 shows that the similarity tube law fits well the experimental values except at the transition region of the thin elliptical shape to a point contacted two lobed tube shape for shear thinning fluids. As $P_e$ increased, $P_{tm(down)}$ decreased so that it began to be significantly negative in the transition region. The detection of the transition point was critical due to the interaction between the tube deformation and the fluid flow. This is because a small change in $P_e$ lead to the formation of a point contacted two lobed shape from a thin elliptical shape. In general, the elastic tube attains a circular shape at a positive transmural pressure whereas the cross sectional area of the collapsed tube reduces when the transmural pressure becomes negative (Elad et al., 1987). In this study, the tube shapes of full inflation to complete collapse were analyzed, which eventually led to a reduction in cross...
Figure 4.12: Experimental (symbols) variation in tube cross sectional area with compressive transmural pressure, which is well represented by the similarity tube law (full lines). Dashed gray line represents similarity tube law with fitted parameters of Elad et al. (1992).

A decrease of about an order of magnitude in tube cross-sectional area from the undeformed one was found for a compressive transmural pressure $P_{tm(down)}$ of -30 mbar ($\Pi \approx -5$) at the downstream for both CMC and PAA aqueous solutions. A major slope change in the transmural pressure-cross sectional area curve was observed when point contact and line contact in collapsed tube first occurred. The similarity tube law is well represented for the wide range of elastic tube material parameters and different fluids as shown in Figure 4.12 and Table 4.1. It is seen that the transition length for partial to rapid (first point contact) tube collapse is larger due to high elastic modulus and wall thickness (black and gray full lines for CMC and PAA respectively) compared to the work by Elad et al. (1992) (gray dash line). The first point contact was observed at about $\omega = 0.27$ (for water) by Elad et al. (1992) whereas in the present case it is at about $\omega = 0.11$ for moderate shear thinning fluid (CMC) and $\omega = 0.06$ for high shear thinning fluid (PAA). The tube cross sectional area decreased faster for high shear thinning fluid due to increase in the fluid velocity and correspondingly decrease in the tube internal pressure. Our tube wall thickness to radius ratio ($h_0/r_0$) is also about factor of 2 higher and the corresponding bending stiffness is about factor of 15 larger resulting the first point contact to appear at the more negative transmural pressure. Furthermore, in the case of idealized elastic tubes with circular cross sections, discontinuities in the pressure-area curve were found at $\omega = 0.15$ to
0.16 where opposite walls first come into contact as reported by Bertram (1987). The tube law is a semi-empirical approximation of the experimental data where the fitting coefficients vary depending on the fluid (Table 4.2) and elastic tube material properties. Consequently, a comparison of the present work is done with the existing literature data. The equation is also found to be applicable for other Newtonian fluid (PEG) when the fitting parameters are optimized based on its properties (dark-gray-dash-dot line in Figure 4.12). It is seen that the tube cross sectional area is not reduced significantly with the corresponding decrease in the $P_{tm(down)}$ for PEG due to increase in the tube internal pressure.

4.3.7 Effect of collapsed elastic tube geometry and velocity profile on fluid rheological properties

Figures 4.13, 4.14 and 4.15 show the qualitative summary of the flow characteristics of the investigated fluids (Newtonian and non-Newtonian) through deformed elastic tube. The variation in average flow velocity, the shear rate and the viscosity are represented as a function of tube cross sectional area, where all the quantities are defined as dimensionless. The reference average flow velocity ($v_0 = \dot{Q}/A_0$) in the undeformed tube used is about 0.05 m/s at $\dot{Q} = 17$ ml/s for PEG, CMC and PAA aqueous solutions. It can be seen that the velocity ratio ($\delta = v/v_0$) increases by an order of magnitude fitting with a power law ($\delta = 1.0005\omega^{-1}$) for a decrease in area ratio by less than factor of two during steady flow of PEG aqueous solution. The average shear rate $\dot{\gamma}_0$ for $\dot{Q} = 17$ ml/s in the undeformed tube is estimated by $\dot{\gamma}_0 = 4 v_0/r_0 = 21s^{-1}$ for Newtonian fluid. The equivalent radius $r_{eq} (= \sqrt[3]{4A}/\pi)$ is calculated from the experimentally obtained tube cross sectional area $A$ by tube shape analysis. It is found that the shear rate ratio ($\Gamma = \dot{\gamma}/\dot{\gamma}_0$) in the deformed tube increases only by a factor of two, which is again fitted well by a power law as $\Gamma = 1.0008\omega^{-1.5}$ and the corresponding average viscosity in the tube is found to be constant for PEG. All the calculated relevant flow parameters for PEG are summarized in Table 4.3. On the other hand, $\delta$ also increases by an order of magnitude for moderate shear thinning fluid (CMC) fitting with a power law ($\delta = 1.0109\omega^{-1.0037}$) where the area ratio decreases by more than an order of magnitude. Here, the average shear rate $\dot{\gamma}_0$ for $\dot{Q} = 17$ ml/s in the undeformed tube is estimated by $\dot{\gamma}_0 = \int_0^{r_{eq}} \dot{\gamma}rdr/\int_0^{r_{eq}} rdr = (2v_0/r_{eq})(3n+1)/(2n+1) = 14s^{-1}$ assuming a non-Newtonian shear thinning power-law liquid. The power law index $n$ is 0.8208 for CMC solution and the corresponding average viscosity in the tube is calculated using the Carreau equation. It is found that the shear rate ratio ($\Gamma$) in the deformed tube increases by a factor of 50, which is again fitted well by a power law $\Gamma = 1.0156\omega^{-1.5056}$. The corresponding viscosity ratio ($\Lambda = \eta/\eta_0$) decreases by a factor of two is fitted by $\Lambda = \Lambda_0 + k_1(1 - e^{-k_2\omega})$ where the fitted
Steady flow in deformed elastic tube

constants are \( \Lambda_0 = 0.1407, k_1 = 0.85 \) and \( k_2 = 6.243 \). \( \Lambda_0 = \eta_m/\eta_0 = 0.1407 \) is the minimum viscosity ratio, where \( \eta_m = 0.0193 \) Pa.s is the average minimum viscosity when the tube cross sectional area \( A \rightarrow 0 \), and \( \eta_0 = 0.137 \) Pa.s is the average viscosity in the undeformed tube. In addition, \( \delta \) is also seen to be increased by an order of magnitude for high shear thinning fluid (PAA) fitting with a power law \( (\delta = 1.005 \omega^{-1}) \) for a decrease in area ratio also by more than an order of magnitude. The average shear rate \( \dot{\gamma}_0 \) in the undeformed tube is \( 13 \) s\(^{-1} \) (assuming non-Newtonian shear thinning power-law liquid). The power law index \( n \) is 0.42 for high shear thinning PAA solution and the corresponding average viscosity in the tube is also obtained using the Carreau equation. The shear rate ratio \( (\dot{\Gamma}) \) in the deformed tube increases by more than a factor of 90, which is again fitted well by a power law \( \dot{\Gamma} = 1.0022 \omega^{-1.5} \). The corresponding viscosity ratio \( (\Lambda) \) decreases by a factor of fourteen is fitted by \( \Lambda = 0.9996 \omega^{0.87} \). Here the \( \eta_m = 0.00038 \) Pa.s (the average minimum viscosity when the tube cross sectional area \( A \rightarrow 0 \)) is approximated by a linear fit \( (\Lambda = \Lambda_0 + k' \omega, \text{with } \Lambda_0 = 0.0414 \text{ and } k' = 0.9625 \) ). The viscosities of CMC and PAA solutions are found to decrease while flowing at \( \dot{Q} = 17 \) ml/s through an elastic tube under different compressive \( P_{tm} \), the values of which are well comparable with the off-line rheological measurement (Table 4.4 for CMC and Table 4.5 for PAA). As \( P_e \) increases from 18 to 105 mbar, the average shear rate near the tube wall increases from 10.66 to 35.8 s\(^{-1} \) with the corresponding viscosity of 0.143 to 0.134 Pa.s (for CMC). Whereas the average wall shear rate also increases from 13.92 to 57.4 s\(^{-1} \) for PAA with decrease in the

![Figure 4.13: Non-Newtonian fluid flow characteristics of PEG solution during steady flow through deformed elastic tube.](image-url)
4.3 Results and Discussion

Figure 4.14: Non-Newtonian fluid flow characteristics of CMC solution during steady flow through deformed elastic tube.

Figure 4.15: Non-Newtonian fluid flow characteristics of PAA solution during steady flow through deformed elastic tube.
Steady flow in deformed elastic tube

corresponding viscosity from 0.0095 to 0.00414 Pa.s transforming the fluid to the shear thinning regime. The viscosity of PAA is drastically decreases due to its high shear thinning behavior than that of the CMC under same applied $P_e$. At low $P_e = 18$ mbar, the average shear rates in the tube and near the wall are found to be same value of about 11 s$^{-1}$ and 13 s$^{-1}$ for CMC and PAA respectively. At high $P_e = 105$ mbar, the average shear rate in the tube becomes almost equal to the average shear rate near tube wall thereby causing the fluid shear thinning over entire tube cross section for both shear thinning fluids. The decrease in the viscosity of a non-Newtonian fluid is useful to enhance the nutrient transport through the small intestine during digestion. The calculated Reynolds number $Re (= 2 \rho v_{avg} r_{eq}/\eta_{avg})$, as listed in Table 4.3, Table 4.4, Table 4.5) increases by a factor of seven for CMC in deformed tube ($Re = 55.8$ at $P_{tm} = -30$ mbar) compared to that in the undeformed tube ($Re = 7.7$), which confirms the flow region as laminar in the collapsible elastic tube. Whereas, for PAA the calculated $Re$ increases by a factor of sixty in deformed tube ($Re = 6729$ at $P_{tm} = -30$ mbar) compared to that in the undeformed tube ($Re = 105$), which indicates that the flow is transforming to the turbulent regime due to high increase in the velocity for shear thinning fluid.

4.4 Numerical simulation of steady flow in collapsed elastic tube

A direct comparison of the experimental data was made with the numerical simulation results (based on the open source CFD environment) during flow of a shear-thinning fluid (1.5 % CMC) in a collapsed elastic tube under applied $P_e = 105$ mbar. The simulation work was performed in a collaboration with Tanner et al. (Department of Mathematical Sciences, Michigan Technological University, U.S.A.) and was compared with the corresponding experimental results of the present work performed at ETH Zurich. The simulation details (mesh generation, mesh dependence, convergence) can be followed in (Tanner et al., 2012). The shear thinning behavior of the test fluid was represented well by the Bird-Carreau viscosity law (section 5.4.1). The computational domain of the deformed tube was reconstructed as shown in Figure 4.16 from the experimental tube geometry obtained by CT-method (Figure 4.2). The velocity profiles obtained by simulations were compared with the corresponding experimentally measured by UVP at various cross sections across the tube length. The comparative velocity magnitude distribution along the cross-section are illustrated in Figures 4.17 a, 4.18 a and 4.19 a with the velocity profiles (both Newtonian and Non-Newtonian fluids) in Figures 4.17 b, 4.18 b and 4.19 b for $X = 70$ mm, $X = 90$ mm and $X = 150$ mm,
4.4 Numerical simulation of steady flow in collapsed elastic tube

respectively. A good agreement between simulation and experiment was obtained. In particular, in the region of strongest collapse of the tube (downstream) at \( X = 70 \text{ mm} \) and \( X = 90 \text{ mm} \), where the bi-modal velocity profiles were quite well reproduced by the simulations. The velocity profile was found to be turned into an uni-modal with a corresponding elliptical tube shape (Figure 4.19) as long the axial position approaches to the tube upstream. In addition, a detailed shear rate field was computed from the calculated velocity field. Figure 4.20 a shows the distribution of shear rates in the cross-section at \( X = 50 \text{ mm} \), where the tube is the most deformed. The largest shear rates for the non-Newtonian fluid was obtained above 200 s\(^{-1}\) with a maximum value of 733 s\(^{-1}\). Moreover, it is seen that shear rates in the shear-thinning regime (at about \( 40 \text{ s}^{-1} \)) were attained significantly away from the tube wall with an average shear rate of about 103 s\(^{-1}\) in this cross-section. In order to better assess the non-Newtonian fluid flow behavior, the same flow has been simulated for a Newtonian fluid. The detailed comparison between Newtonian and non-Newtonian shear rate behavior along the cross-section \( X = 50 \text{ mm} \) is shown in Figure 4.20 b. A significant variation in the shear rates is observed at the position of strongest tube collapse (at the downstream), which is seem to be significant enough to transfer the fluid in its shear-thinning regime as reflected in the velocity profiles.
Figure 4.17: (a) The velocity magnitude distribution of the non-Newtonian fluid (1.5 % CMC) along the cross-section at $X = 70$ mm. (b) Comparison between the experimental and simulation velocity profiles at $X = 70$ mm. The cross-sectional coordinates are in mm and the velocity is in m/s.
4.4 Numerical simulation of steady flow in collapsed elastic tube

Figure 4.18: (a) The velocity magnitude distribution of the non-Newtonian fluid (1.5 % CMC) along the cross-section at $X = 90$ mm. (b) Comparison between the experimental and simulation velocity profiles at $X = 90$ mm. The cross-sectional coordinates are in mm and the velocity is in m/s.
Figure 4.19: (a) The velocity magnitude distribution of the non-Newtonian fluid (1.5% CMC) along the cross-section at $X = 150$ mm. (b) Comparison between the experimental and simulation velocity profiles at $X = 150$ mm. The cross-sectional coordinates are in mm and the velocity is in m/s.
4.4 Numerical simulation of steady flow in collapsed elastic tube

Figure 4.20: (a) The velocity magnitude distribution of the non-Newtonian fluid (1.5% CMC) along the cross-section at $X = 150$ mm. (b) Comparison between the experimental and simulation velocity profiles at $X = 150$ mm. The cross-sectional coordinates are in mm and the velocity is in m/s.
Table 4.3: Comparison of the experimental and theoretical values of rheological properties for PEG under different geometries of collapsed elastic tube and applied pressures $P_e$. $v_{avg}$ is the average velocity in the tube, and $r_{eq}$ and $Re$ are the equivalent radius and Reynolds number respectively. $\dot{\gamma}_{avg}$ and $\eta_{avg}$ are the average shear rate and viscosity in the tube respectively. $\dot{\gamma}_{wall}$ and $\eta_{wall}$ are the average shear rate and viscosity near the tube wall respectively.

<table>
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<tr>
<th>Points (Fig. 4.13)</th>
<th>Applied $P_e$ mbar</th>
<th>Measured $A$ mm$^2$</th>
<th>$v_{avg}$ m/s</th>
<th>$r_{eq}$ m</th>
<th>$\dot{\gamma}_{avg}$ s$^{-1}$</th>
<th>$\eta_{avg}$ Pa.s</th>
<th>Re</th>
<th>$\dot{\gamma}_{wall}$ s$^{-1}$</th>
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<td>342</td>
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<td>0.0104</td>
<td>19.03</td>
<td>0.143</td>
<td>7.25</td>
<td>10.52</td>
<td>0.143</td>
</tr>
<tr>
<td>b</td>
<td>90</td>
<td>329</td>
<td>0.051</td>
<td>0.0102</td>
<td>20.13</td>
<td>0.143</td>
<td>7.38</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>100</td>
<td>326</td>
<td>0.052</td>
<td>0.0100</td>
<td>20.42</td>
<td>0.143</td>
<td>7.42</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>105</td>
<td>266</td>
<td>0.063</td>
<td>0.0092</td>
<td>27.64</td>
<td>0.143</td>
<td>8.21</td>
<td>25.5</td>
<td>0.143</td>
</tr>
<tr>
<td>e</td>
<td>110</td>
<td>235</td>
<td>0.072</td>
<td>0.0086</td>
<td>33.35</td>
<td>0.143</td>
<td>8.74</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>f</td>
<td>120</td>
<td>207</td>
<td>0.082</td>
<td>0.0081</td>
<td>40.39</td>
<td>0.143</td>
<td>9.31</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
4.4 Numerical simulation of steady flow in collapsed elastic tube

Table 4.4: Comparison of the experimental and theoretical values of rheological properties for CMC under different geometries of collapsed elastic tube and applied pressures

<table>
<thead>
<tr>
<th>Points Applied</th>
<th>Measured P (mmHg)</th>
<th>Measured V (mm/s)</th>
<th>Re</th>
<th>$\dot{\gamma}_{avg}$</th>
<th>$\eta_{avg}$</th>
<th>$\dot{\gamma}_{wall}$</th>
<th>$\eta_{wall}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>18</td>
<td>342</td>
<td>0.050</td>
<td>0.0104</td>
<td>12.49</td>
<td>0.135</td>
<td>7.7</td>
</tr>
<tr>
<td>b</td>
<td>90</td>
<td>285</td>
<td>0.059</td>
<td>0.0095</td>
<td>16.45</td>
<td>0.134</td>
<td>8.5</td>
</tr>
<tr>
<td>c</td>
<td>100</td>
<td>114</td>
<td>0.149</td>
<td>0.0060</td>
<td>64.73</td>
<td>0.116</td>
<td>15.5</td>
</tr>
<tr>
<td>d</td>
<td>105</td>
<td>52</td>
<td>0.327</td>
<td>0.0040</td>
<td>211.11</td>
<td>0.089</td>
<td>29.6</td>
</tr>
<tr>
<td>e</td>
<td>110</td>
<td>32</td>
<td>0.538</td>
<td>0.0032</td>
<td>445.25</td>
<td>0.075</td>
<td>45.3</td>
</tr>
<tr>
<td>f</td>
<td>120</td>
<td>25</td>
<td>0.685</td>
<td>0.0028</td>
<td>639.39</td>
<td>0.069</td>
<td>55.8</td>
</tr>
</tbody>
</table>

$\dot{\gamma}_{avg}$ and $\eta_{avg}$ are the average shear rate and viscosity in the tube respectively. $\dot{\gamma}_{wall}$ and $\eta_{wall}$ are the average shear rate and viscosity near the tube wall respectively. $P_e$ and $V_{avg}$ are the equivalent pressure and Reynolds number respectively. $Re$, $\dot{\gamma}$, and $\eta$ are the equivalent Reynolds number and rheological properties in the tube, and geometries of collapsed elastic tube and applied pressures respectively. $P_e$ is the equivalent pressure in the tube, and geometries of collapsed elastic tube and applied pressures respectively.
Table 4.5: Comparison of the experimental and theoretical values of rheological properties for PAA under different geometries of collapsed elastic tube and applied pressures $P_e$. $v_{avg}$ is the average velocity in the tube, and $r_{eq}$ and $Re$ are the equivalent radius and Reynolds number respectively. $\dot{\gamma}_{avg}$ and $\eta_{avg}$ are the average shear rate and viscosity in the tube respectively. $\dot{\gamma}_{wall}$ and $\eta_{wall}$ are the average shear rate and viscosity near the tube wall respectively.

<table>
<thead>
<tr>
<th>Points (Fig. 4.15)</th>
<th>Applied $P_e$</th>
<th>Measured $A$</th>
<th>$v_{avg}$ m/s</th>
<th>$r_{eq}$ m</th>
<th>$\dot{\gamma}_{avg}$ s$^{-1}$</th>
<th>$\eta_{avg}$ Pa.s</th>
<th>$Re$</th>
<th>$\dot{\gamma}_{wall}$ s$^{-1}$</th>
<th>$\eta_{wall}$ Pa.s</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>18</td>
<td>342</td>
<td>0.049</td>
<td>0.0104</td>
<td>11.69</td>
<td>0.0098</td>
<td>106</td>
<td>13.92</td>
<td>0.0095</td>
</tr>
<tr>
<td>b</td>
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<td>19.06</td>
<td>0.0074</td>
<td>165</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
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<td>82</td>
<td>0.21</td>
<td>0.0051</td>
<td>99.81</td>
<td>0.0028</td>
<td>748</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>105</td>
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<td>0.34</td>
<td>0.0039</td>
<td>211.75</td>
<td>0.0018</td>
<td>1487</td>
<td>57.4</td>
<td>0.0041</td>
</tr>
<tr>
<td>e</td>
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<td>0.77</td>
<td>0.0026</td>
<td>718.45</td>
<td>0.0009</td>
<td>4540</td>
<td>-</td>
<td>-</td>
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<td>1.03</td>
<td>0.0022</td>
<td>1105.2</td>
<td>0.0007</td>
<td>6729</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
4.5 Conclusions

The steady flow characteristics of both Newtonian and non-Newtonian shear thinning fluids through a collapsible elastic tube were investigated at different compressive transmural pressures in a Starling Resistor setup. Since the outlet pressure in collapsed tube was lower due to reduced cross sectional area, the corresponding compressive downstream transmural pressure was more negative than that at the upstream. The tube cross sectional area decreased (at 9 cm from the downstream rigid tube connection) by only about factor of one for PEG and about factor of six for both CMC and PAA from the undeformed one under an applied $P_c$ of 105 mbar. The corresponding maximum velocity increased by a factor of two during steady flow of shear thinning fluids. Furthermore, the cross sectional area decreased by an order of magnitude from the undeformed one when $P_{tm(down)}$ is about -30 mbar for both shear thinning fluids. The novel UVP technique was successfully implemented to measure the velocity profiles in collapsible elastic tube during steady flow of both Newtonian and non-Newtonian fluids. The relationship between transmural pressure and tube cross sectional area was well represented by the similarity tube law. A critical external pressure caused collapse of elastic tube to form a two lobe-shaped cross-section over certain tube length near downstream, where measured UVP based velocity profiles were bi-modal. The bi-modal velocity profiles were found to be slowly turned into uni-modal for the change in two lobes shape to nearly elliptical as the tube length approached the upstream end. A detailed picture of flow characteristics of CMC solution was found by the simultaneous investigation of tube shapes and velocity profiles during steady laminar flow through elastic tube. The shear thinning behavior of both CMC and PAA solutions were clearly observed at a constant flow rate ($\dot{Q} = 17$ ml/s) as the tube cross-sectional area decreased due to increase in compressive transmural pressure. In addition, the viscosity of PAA was drastically decreased due to its high shear thinning behavior than that of the CMC under same applied $P_c$. Therefore, the high shear thinning fluids can be considered for efficient nutrient transport through the small intestinal wall during digestion. In addition, the steady flow of a non-Newtonian fluid through a collapsed elastic tube was simulated and compared with the corresponding experimental data. A good agreement between simulation and experiment was obtained with additional insight by considering the local quantities for shear rates and viscosities.
5 Prediction of velocity profiles in elastic tubes

5.1 Introduction

The rheology of particle suspensions, droplet emulsions and polymeric fluids during flow in tubes depends on their microstructure. The shear stress and shear rate distributions along the radius of the tube can be determined from the pressure drop and velocity profile during steady laminar flow at a given flow rate. The off-line and in-line rheological behaviors could be different when the microstructures of suspensions and emulsions depend on the corresponding individual flows, particle or droplet migration and other mechanisms. In contrast, the rheology of colloidal polymeric liquids can be almost the same (Goloshevsky et al., 2005) both in off-line and in-line measurements since the molecular architecture can be similar. The in-line rheological measurements are important in polymer and food industries for quality control in processing. The method of viscosity measurement is based on analysis of a velocity profile of the material flowing in a tube coupled with a simultaneous measurement of the pressure-drop driving the flow (Powell et al., 1994). The velocity profiles of the polymeric solution flowing in the tube have been studied based on the Nuclear magnetic resonance imaging (NMRI) (Goloshevsky et al., 2005; Li and McCarthy, 1995). Koeseli et al. (2006) considered this method to be expensive, complicated and undesirable when its magnetic field influences some processes. These and other authors (Wiklund et al., 2007) used UVP technique to measure the velocity profiles during the steady flow of xanthan aqueous solution in pipes. Birkhofer et al. (2008); Dogan et al. (2002); Wiklund and Stading (2008) investigated the in-line rheological behavior of food suspensions such as cocoa fat crystals, diced tomato, cellulose pulp, and mineral slurries using UVP. The present study investigated experimentally the velocity profiles using the in-line UVP technique during steady laminar flow of a shear thinning fluid in an elastic tube at different flow rates. These were compared successfully with those predicted by integrating the theoretical equation derived by equating the shear stress along the tube radius involving pressure drop to that of Carreau model using its parameters. If pressure driven flow is applied, the elastic tube is inflated depending on local pressure and elastic tube properties (e.g.
Young’s Modulus). The agreement between predicted and measured velocity profiles was found to be good. The predicted pressure drop was about the same as the experimental value at lower flow rates. In contrast, the measured pressure drop was lower than that predicted at higher flow rates due to inflation of tube, which was verified with image analysis of the increased cross-sectional area.

5.2 Experimental

The experimental study was performed using the Starling Resistor set-up as demonstrated in 4.1. The experiment included the simultaneous measurement of both the tube shape and the corresponding velocity flow field under steady flow of CMC 1.5 % aqueous solution at a range of volume flow rates. The shapes of the tube were analyzed by the CT-method and the corresponding velocity profiles were monitored using UVP technique. The details of the materials and methods are described in chapter 3.2.

5.3 Model

Consider the steady-state laminar flow of a non-Newtonian liquid through a circular tube of radius $r_t$ and length $l$ (see Figure 5.1). When $\Delta p$ is the pressure drop at a volume flow rate of $\dot{q}$, the shear stress $\tau$ at a radius $r$ is given by

$$\tau = \frac{\Delta p}{2l} r$$

(5.1)

The apparent viscosity $\eta$ is defined as

$$\eta = \frac{\tau}{\dot{\gamma}}$$

(5.2)
5.3 Model

where $\dot{\gamma}$ is the shear rate. For a shear thinning liquid for which the viscosity is constant at very low shear rates and decreases above a shear rate, the apparent viscosity $\eta$ according to Carreau model is given by

$$\eta = \frac{\eta_0}{[1 + (\lambda \dot{\gamma})^2]^{(1-m)/2}} \quad (5.3)$$

where $\eta_0$ is the zero shear rate viscosity and $\lambda$ is the time constant for the transition from constant viscosity to power law behavior occurring at a shear rate of $1/\lambda$. The exponent $m$ is smaller than 1, which corresponds to the power law region. This equation is originally based on molecular network theories allowing for the features of a pseudo-plastic liquid under simple shear. The values of $m$, $\eta_0$ and $\lambda$ can be determined from Eq. 5.3 using the experimental data of shear rate dependent viscosity $\eta$ measured with a rheometer. Thus the shear stress can be obtained as

$$\tau = \eta \dot{\gamma} = \frac{\eta_0 \dot{\gamma}}{[1 + (\lambda \dot{\gamma})^2]^{(1-m)/2}} \quad (5.4)$$

Equating this to the shear stress (Eq. 5.1) in a tube during steady-state laminar flow gives:

$$\tau = \frac{\Delta p}{2t} r = \frac{\eta_0 \dot{\gamma}}{[1 + (\lambda \dot{\gamma})^2]^{(1-m)/2}} \quad (5.5)$$

where the shear rate $\dot{\gamma}$ is equal to the radial velocity gradient:

$$\dot{\gamma} = -\frac{dv}{dr} \quad (5.6)$$

where $v$ is the liquid velocity at radius $r$. Thus Eq. 5.5 can be rewritten as

$$\Delta P R [1 + \dot{\Gamma}^2]^{(1-m)/2} - \dot{\Gamma} = 0 \quad (5.7)$$

where $\Delta P = \Delta p r_t \lambda/2 t \eta_0$ is the dimensionless pressure drop, $\dot{\Gamma} = \dot{\gamma} \lambda$ is the dimensionless shear rate and $R = r/r_t$ is the dimensionless radius. Since

$$\dot{\Gamma} = -\frac{dV}{dR} \quad (5.8)$$

Eq. 5.7 becomes

$$\Delta P R \left[1 + \left(-\frac{dV}{dR}\right)^2\right]^{(1-m)/2} + \frac{dV}{dR} = 0 \quad (5.9)$$

where $V = v\lambda/r_t$ is the dimensionless velocity. This can be integrated using finite difference method as follows. The dimensionless radius $R$ varies from 0 to 1 so that it can be divided into $N$ points equally separated by a distance of $\Delta R$. The
velocity \( V_N \) at the tube wall where \( R = R_N = 1 \) is zero. Thus the velocity gradient at any radial position \( R_{N-i} \) can be written as

\[
\dot{\Gamma} = -\left( \frac{dV}{dR} \right)_{N-i} = -\frac{(V_{N-i+1} - V_{N-i})}{\Delta R}
\]  

(5.10)

Substituting this in Eq. 5.9 gives

\[
\Delta P \left( \Delta R \right)^m R_{N-i+1} \left[ (\Delta R)^2 + (V_{N-i} - V_{N-i+1})^2 \right]^{(1-m)/2} + V_{N-i+1} - V_{N-i} = 0
\]  

(5.11)

which can be solved for each value of \( i \) from 1 to \( N \). When \( i = 1 \), Eq. 5.11 becomes

\[
\Delta P \left( \Delta R \right)^m \left[ (\Delta R)^2 + (V_{N-1})^2 \right]^{(1-m)/2} - V_{N-1} = 0
\]  

(5.12)

where \( V_N = 0 \) at \( R = R_N = 1 \) and \( \Delta R = 1/N \). This can be solved to obtain the value of \( V_{N-1} \) for a given value of \( \Delta P \). Using this value, Eq. 5.11 can be solved to obtain \( V_{N-2} \) when \( i = 2 \) and subsequently for \( i = 2, 3, 4, \ldots, N \). The volume flow rate \( \dot{q} \) is given by

\[
\dot{q} = 2\pi \int_0^r vr \, dr
\]  

(5.13)

which can be written as

\[
Q = 2\pi \int_0^1 VR \, dR
\]  

(5.14)

where \( \dot{Q} = \dot{q}\lambda/r_t^3 \) is the dimensionless volume flow rate. This can be rewritten as

\[
Q = 2\pi D R \sum_{i=1}^{N} (V_{N-i+1} R_{N-i+1})
\]  

(5.15)

The average velocity \( \bar{v} = q/\pi r_t^2 \) can be obtained from Eq. 5.13 as

\[
\bar{v} = 2 \int_0^1 VR \, dR
\]  

(5.16)

where \( \bar{V} = \bar{v}\lambda/r_t \) is the dimensionless average velocity, which can be rewritten as

\[
\bar{V} = 2\Delta R \sum_{i=1}^{N} (V_{N-i+1} R_{N-i+1})
\]  

(5.17)

The average shear rate is given by

\[
\bar{\gamma} = \frac{2}{r_t^2} \int_0^r \dot{\gamma} r \, dr
\]  

(5.18)

which can be written as

\[
\bar{\Gamma} = 2 \int_0^1 \dot{\Gamma} R \, dR
\]  

(5.19)

This can be rewritten in finite difference form as

\[
\bar{\Gamma} = 2\Delta R \sum_{i=1}^{N} (\dot{\Gamma}_{N-i+1} R_{N-i+1})
\]  

(5.20)
Thus the velocity profile along the radius of the tube of radius $r_t$ and finite length $l$ at a given pressure drop $\Delta p$ can be calculated with the finite difference Eq. 5.11 using the value of $m$, $\lambda$, and $\eta_0$ obtained by fitting the experimental rheological shear rate dependent viscosity data of the shear thinning solution to the Carreau constitutive Eq. 5.3. In addition, the volume flow rate, average velocity and average shear rate can also be calculated using Eqs. 5.15, 5.17 and 5.20.

5.4 Results and discussion

5.4.1 Rheology of CMC solution

The measured shear rate dependent viscosity of CMC aqueous solutions (1.5 %) shows a shear thinning behavior well represented by the Carreau model (Eq. 5.3) with the fitted parameters: $\eta_0 = 0.1452$ Pa.s, $\lambda = 0.02673$ s and $m = 0.7588$ as shown in 5.2.

Figure 5.2: Shear rate dependent viscosity of 1.5 % CMC aqueous solution at 22°C. Symbols: experimental, Full line: data fitted to the Carreau model (Eq. 5.3).
5.4.2 Velocity profiles measured using UVP

The velocity profiles were measured along the tube diameter (expressed as a function of channel number, the channel distance being 0.37 mm) during steady laminar flow of CMC solution in the undeformed silicone elastic tube of circular cross section. The velocity profiles were monitored by setting the ultrasound transducer at three different positions (bottom, front and top) of the elastic tube as schematically represented in figure 5.3. The experiment was carried out at three different volume flow rates as $\dot{q}_1 = 16.8$ ml/s, $\dot{q}_2 = 22$ ml/s and $\dot{q}_3 = 27.2$ ml/s Figure 5.3

![Figure 5.3: Velocity profile of 1.5 % CMC aqueous solution in the elastic tube at a volume flow rate of 16.8 ml/s as monitored from 3-different positions, the schematic of which is also inserted. Symbols: average profile.](image)

also shows the measured velocity profiles by averaging 100 profiles (represented by the symbols) for $\dot{q}_1 = 16.8$ ml/s. A slight difference among the velocity profiles is observed as monitored from different positions (bottom, front and top), because the elastic tube does not attain the perfect circular shape (the difference is better seen for the low flow rate value in Figure 5.4, and diminishes at the high flow rate, when the elastic tube becomes perfectly circular) and the positioning of the ultrasound transducer is also critical to adjust from different positions. It has been observed that the maximum velocity in the tube center increases with increasing the flow rate with a 1.5 times higher maximum velocity for $\dot{q}_3 > \dot{q}_2 > \dot{q}_1$. In addition, for a given volume flow rate, shear rate (velocity gradient) increases from the tube center to the tube wall, which also increases with higher flow rates.
5.4 Results and discussion

5.4.3 Velocity profile prediction

The velocity profiles for steady laminar flow of CMC solutions in the elastic tube were predicted by integration as described in Section 5.3 based on the pressure drop $\Delta p$ in the tube of length $l$. The $\Delta p$ increased with increasing the flow rates. The experimental $\Delta p$ for corresponding flow rates as $\dot{q}_3 > \dot{q}_2 > \dot{q}_1$ were $7.7 > 6.4 > 5.18$ mbar respectively. Figure 5.4 shows the measured and predicted velocity profiles

![Figure 5.4: 1.5 % CMC aqueous solution velocity distribution along the tube diameter at a volume flow rate of 16.8 ml/s. Symbols: Experimental. Dotted and full lines: Predicted using numerical solution with Carreau model.](image)

for $\dot{q}_1$ using the experimental $\Delta p$ where the center ($R = 0$) of the elastic tube is taken as reference for velocity profile data measured from 3-different positions. A good agreement between experiment (symbols) and prediction (dashed line) is found for lower flow rate $\dot{q}_1$. A slight deviation is observed from the predicted one due to ultrasound near field, deconvolution and reflection effects at the tube wall. As the flow rates are increased (i.e. $\dot{q}_2$ and $\dot{q}_3$), it is observed that the fitted $\Delta p$ give better prediction of the velocity profiles rather than using the experimentally measured $\Delta p$, which are shown in Figures 5.5 and 5.6 (dashed line: prediction with experimental $\Delta p$ and full line: prediction with fitted $\Delta p$). This discrepancy can be illustrated by the Figure 5.7, which shows that the measured and predicted variations in $\Delta p$ with $\dot{q}$ are close enough in the low $\dot{q}$-range. On the other-hand, the experimentally measured $\Delta p$ decreases compared to that of the predicted one at higher $\dot{q}$-range as well as with higher Reynold’s numbers due to slight increase
Figure 5.5: 1.5% CMC aqueous solution velocity distribution along the tube diameter at a volume flow rate of 22 ml/s. Symbols: Experimental. Dotted and full lines: Predicted using numerical solution with Carreau model.

Figure 5.6: 1.5% CMC aqueous solution velocity distribution along the tube diameter at a volume flow rate of 27.2 ml/s. Symbols: Experimental. Dotted and full lines: Predicted using numerical solution with Carreau model.
5.4 Results and discussion

Figure 5.7: Variation in pressure drop with volume flow rate of 1.5 % CMC aqueous solution in the tube. Symbols: Experimental. Full line: Predicted using numerical solution with Carreau model.

(about 5 %) in the tube cross sectional area for the deformability caused by tube elasticity (Y = 4.7 MPa). The average shear rate $\overline{\gamma} = \overline{\Gamma}/\lambda$ (Eq. 5.20) and average velocity $\overline{\nu} = q/\pi r_t^2$ in the tube are used for the calculation of Reynolds number $Re = \rho \overline{\nu} D/\eta$ at different $\dot{q}$ (listed in Table 5.1). The expansion in elastic tube cross sectional area was measured by tube shape image analysis using CT method and was also estimated by the Hagen-Poiseuille’s law (Eq. 5.21) as follows:

$$\dot{m} = \frac{\pi \Delta \rho d (r_t + \Delta r)^4 \rho}{8 \eta l}$$  \hspace{1cm} (5.21)

where $\dot{m}$ is the mass flow rate of aqueous solution and $\Delta r$ is the increase in radius of elastic tube.

The estimated and measured cross-sectional areas were found to be very close and the tube expansion with flow rates relative to the undeformed tube is represented in Figure 5.8 and summarized in Table 5.2. It is seen that both methods are well applicable for the detection of elastic tube expansion while increasing the flow rates. The detected change in the tube cross section was assumed to be constant over the length, which was also confirmed by analyzing the cross sectional area at different positions along the tube length.
Prediction of velocity profiles in elastic tubes

Figure 5.8: Comparison between the measured (tube shape image analysis) and estimated (Hagen-Poiseulle’s equation) expansion in tube cross sectional area with volume flow rate.

Table 5.1: The parameters used for Reynolds’ number $Re$ calculation for 1.5 % CMC aqueous solution with density $\rho = 1000 \text{ kg/m}^3$ flowing through the elastic tube of $D = 0.02 \text{ m}$ at different volume flow rates $\dot{q}$.

<table>
<thead>
<tr>
<th>$\dot{q}$, ml/s</th>
<th>$\dot{\gamma}$, 1/s</th>
<th>$\bar{v}$, m/s</th>
<th>$\eta$, Pa.s</th>
<th>$Re$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.8</td>
<td>12.72</td>
<td>0.5348</td>
<td>0.140</td>
<td>7.6</td>
</tr>
<tr>
<td>22</td>
<td>15.5</td>
<td>0.7003</td>
<td>0.139</td>
<td>10.0</td>
</tr>
<tr>
<td>27.2</td>
<td>21.5</td>
<td>0.8658</td>
<td>0.137</td>
<td>12.6</td>
</tr>
</tbody>
</table>

Table 5.2: Expansion in tube cross sectional area $A$ with volume flow rate $\dot{q}$ measured by tube shape image analysis and calculated by Hagen-Poiseulli’s equation.

<table>
<thead>
<tr>
<th>$\dot{q}$, ml/s</th>
<th>$A$, mm$^2$ calculated</th>
<th>$A$, mm$^2$ measured</th>
<th>% expansion in $A$ calculated</th>
<th>% expansion in $A$ measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.8</td>
<td>318.4</td>
<td>314</td>
<td>1.35</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>326.8</td>
<td>322</td>
<td>4.02</td>
<td>2.62</td>
</tr>
<tr>
<td>27.2</td>
<td>328.6</td>
<td>329</td>
<td>4.59</td>
<td>4.93</td>
</tr>
</tbody>
</table>
5.5 Conclusions

The shear thinning behavior of CMC 1.5% aqueous solution was well represented by the Carreau rheological constitutive equation with the parameters \((\eta_0, \lambda, m)\). The velocity profiles predicted by the numerical calculations of a model based on Carreau equation agreed well with the velocity profiles measured by UVP-technique for the steady laminar flow of shear thinning fluids through an elastic tube. The increase in the pressure drop with the volume flow rate was also successfully predicted. However, the measured pressure drop at higher flow rate was seen to be less than the predicted value as the cross-sectional area of the tube increased due to inflation. An increase of about 5% in tube cross sectional area relative to the undeformed tube was obtained for the flow rate of 27.2 ml/s, which was confirmed by both measurement (tube shape image analysis) and calculation (Hagen-Poiseulle’s law). Both methods were found to be well applicable for the detection of elastic tube expansion while increasing the flow rates.
6 Pressure-controlled unsteady flow in deformed elastic tube

6.1 Introduction

Study of unsteady flow in an elastic tube is of great interest for better understanding of unsteady flows involved in the human body, e.g., blood flow through arteries, peristaltic flow in stomach and intestine. Kamm and Shapiro (1979) have developed both theory and experiment for the understanding of unsteady flows in collapsible tubes when subjected to external pressure or body forces. The unsteady flow through a collapsible tube has been also studied while considering a sinusoidal upstream pressure within the range of the human heartbeat (Low and Chew, 1991). In addition, a numerical procedure for simulating unsteady, large amplitude flow containing shock-like quasi-discontinuities has been described by Kimmel et al. (1988), whereas Bertram and Pedley (1982) presented a simple, third-order lumped parameter model to describe unsteady flow in a short segment of a collapsible tube held between two rigid segments and contained in a pressurized chamber. In the present study, two sets of experiments were performed for the unsteady flow behavior investigation where the first set depicted the pressure controlled, and the second set included the deformation controlled unsteady flow. This chapter describes the experimental results of pressure controlled unsteady flow under different applied external pressures using a Starling Resistor. However, the acceleration and maximum velocity during blood flow in human body is quite high (Misra and Chakravarty, 1982). The aim of the present study is to understand the pressure controlled unsteady flow behavior in a deformed elastic tube under various compressive transmural pressures. In addition, this knowledge will be useful to understand peristaltic flow in an elastic tube. Furthermore the implementation of ultrasound Doppler velocity profile measurement method (UVP) to characterize the transient flow in elastic tubes is of additional interest and relevance.
6.2 Experiments of unsteady periodic flow

A Starling Resistor setup like used in the steady flow experiments (Figure 4.1, chapter 8) was also used for the pressure controlled unsteady flow experiments. An unsteady periodic flow was generated using a positive displacement pump (NEMO Progressing Cavity Pumps, Type: NM008BY, NETZSCH, Germany), and was operated in a well-controllable manner. The unsteady ramp-up (acceleration) and ramp-down (deceleration) was applied at different external chamber pressures, \( P_e \). A schematic representation of the pump-operation for an unsteady periodic flow is given in Figure 6.1 (a). Here also the shear thinning aqueous CMC solution (1.5% w/w) was used. The experiments included the measurement of pressure drop in the tube as a function of flow rates at different \( P_e \). In addition, the evolution of velocity profiles during periodic ramp flow in undeformed tube was monitored by UVP (using an ultrasound transducer of 4 MHz). About 126 profiles were measured during 1 cycle of periodic flow in elapsed time of 188 s. To increase the measurement accuracy of UVP, an average of 13 velocity profiles within a constant time interval (19 s) was taken. The calculation procedure for average velocity profiles as a function of time is demonstrated in Figure 6.1 (b).

Figure 6.1: (a) Schematic pump-operation for an unsteady periodic flow (holding time, \( t = 10 \) s), (b) Calculation procedure for average velocity profiles as a function of time evolution.
6.3 Results and Discussion

6.3.1 Effect of external pressure on pressure drop variation with volume flow rate

The periodic flow characteristics during ramp-up and ramp-down are governed by the volume flow rate and the applied external chamber pressure. In an unsteady flow (Figure 6.2) during the ramp-up (10 rpm/s), the flow characteristics of CMC solution were found to be similar to those received for steady laminar flow in the same range of $\dot{Q}$. This indicated that the pressure drop $\Delta P$ in the tube slowly increased as $\dot{Q}$ increased with time under a constant $P_e$ up to a critical value of $\dot{Q}_c$. As $\dot{Q}$ is further increased from that critical $\dot{Q}_c$, $\Delta P$ decreases due to the expansion of the collapsed elastic tube under the same applied external pressure $P_e$. In contrast, during the ramp-down (10 rpm/s) at higher applied external pressures (90 to 120 mbar), the pressure drop in the tube sharply decreased as $\dot{Q}$ decreased due to the instantaneous change of pressure measured at the inlet than that at the outlet. Moreover, as $\dot{Q}$ was reduced significantly, the tube walls came closer forming a line/area contact, thus reducing tube cross-sectional area. This in turn increased the drag force and hence the pressure drop. The change in tube shapes during a periodic flow cycle is also schematically drawn for $P_e = 120$ mbar (Figure 6.2), where a two lobed area contacted tube shape is formed. Figure 6.2 shows that the flow behavior during a periodic flow cycle is governed by the fluid flow rate into/out of the tube center due to an opening/closing of the tube. The fluid mechanistic parameters such as time scale, change in tube shape (due to elasticity) and interaction between fluid and tube deformation dominantly control the flow behavior of a periodic fluid flow cycle.

6.3.2 Temporal evolution of velocity profiles during unsteady periodic flow

The maximum velocity in the undeformed elastic tube center increased as time elapsed in a periodic flow cycle, where a higher average maximum velocity for the ramp-down than that for the ramp-up flow was observed. Figure 6.3 shows that the maximum velocity in the tube center increased by about an order of magnitude during 1 cycle of the periodic ramp flow in elapsed time of 188 s. The corresponding $\dot{Q}$ varied between 1.6 ml/s to 14.7 ml/s, which was lower than that in the steady flow experiments so that the expected unsteady flow regime was also laminar. In addition, the approximated shear rate variation during ramp-up was $3 \text{ s}^{-1}$ to $19 \text{ s}^{-1}$ and during ramp-down was $6 \text{ s}^{-1}$ to $20 \text{ s}^{-1}$, was found to be very close to each other.
Figure 6.2: Variation of pressure drop in the elastic tube as a function of flow rate during ramp flow of a CMC solution under different applied external chamber pressures. The change in tube shapes during a periodic flow cycle is also schematically drawn for $P_e = 120$ mbar.
6.4 Conclusions

The periodic flow characteristics during ramp-up and ramp-down were found to be governed by the fluid flow rate (into/out) due to change in tube shapes (due to elasticity) under various applied external chamber pressures. The velocity profiles evolution in undeformed tube during periodic ramp flow showed that the maximum velocity in the tube center increased by an order of magnitude as the time elapsed for 188 s. In addition, the approximated shear rate variation during ramp-up and ramp-down of a periodic flow was found to be very close to each other.
7 Ultrasound Doppler based flow profiling in non-parallel velocity fields

7.1 Introduction

The ultrasonic Doppler velocimetry (or ultrasonic velocity profiling, UVP) method is well recognized technique for flow profile estimation especially for opaque fluid system (Birkhofer, 2007; Birkhofer et al., 2008; Ouriev and Windhab, 2002; Takeda, 1986, 1987). The popularity comes from the applicability of UVP (Takeda, 1986; Wiklund et al., 2006) for at least one of the advantages (such as opaque fluid flow investigation, inexpensive, non-invasive to test fluid, easy to implement, and portable) over other methods such as Particle Image Velocimetry (PIV), Laser Doppler Anemometry (LDA), Electrical Impedance Tomography (EIT), X-ray radiography, Magnetic Resonance Imaging (MRI) and neutron radiography etc. UVP measures an instantaneous unidimensional velocity profile along a measurement line using the Doppler shift frequency information echoed by particles exist in the fluid (Takeda, 1986, 1987). Theoretically, UVP only gives information of a velocity projection of the true velocity vector along the measuring axis. In UVP-DUO (Met-Flow SA, Switzerland), a simple correction is used to get the corresponding real vector component by angle correction (Met-Flow, 2002) as shown in Figure 3.4. The correction is only valid when the stream lines are parallel to each other, and/or a flow is parallel to the tube wall in a pipe flow. This correction is again valid for the real velocity vector that creates an angle of 90-θ with the measuring line (Figure 3.4), and a velocity vector that is parallel to the wall. In many practical situations, non-circular tube or, non-parallel stream lines are involved. Flow mapping or flow profiling in complex geometries such as elbow, pump, valve, heat exchanger, orifice, contraction, expansion, nozzle, continuous stirring tank (CST) etc. is a research and industrial interest for different purposes. Conventional UVP-DUO technique can not be directly used for these complex flow circumstances. Recently, many researchers (Kotze et al., 2011; Nahar et al., 2012; Takeda and Kikura, 2002) are working with the complex geometries, where they
directly apply UVP-DUO in a single-line measurement for flow profiling. Moreover, many authors are using single-line based velocity profile by UVP to estimate in-line shear or extensional rate (Birkhofer, 2011; Choi et al., 2002; Hughes and How, 1993; Kotze et al., 2011; Nahar et al., 2012; Ouriev and Windhab, 2002; Wiklund et al., 2007). Therefore, estimation of accurate velocity profile by single-line based UVP is necessary before further processing of the obtained data. Although, inapplicability of single-line based UVP measurement is known to the researchers, they implied it as smaller error consideration in the deformed tube. However, in some cases the error can be significant when the impact of nonparallel flow streamlines is not considered. Some researchers have tried to improve the UVP measurement technique for flow mapping in a complex flow situation (Takeda, 1987; Takeda and Kikura, 2002). The approach was to implement multiple UVP-transducers to obtain information of true velocity vector for flow mapping (Murai et al., 2012).

In the present work, first we have explained inapplicability of conventional single-line UVP-measurement method for prediction of velocity profiles in non-parallel streamlines in a non-circular tube. In addition, a new experimental approach of multiple-line based velocity profile measurement has been proposed to solve this problem by determining the magnitude of real velocity vector ($v_{\text{real}}$) in the target measuring line, angle between the real velocity-vector and vertical axis or wall (in other words, direction of velocity-vector), and the velocity component ($v_x$) of the real velocity-vector in the flow direction. Although, present approach is an analytical solution to use UVP in a non-parallel streaming flow, an experimental result has been shown to visualize the implementation and the improvement. The experimental result shows the velocity vector profile (flow mapping), and the velocity in the flow direction in a deformed elastic tube.

### 7.2 Inapplicability of UVP for non-circular tube or non-parallel stream-lines

The basic principle of UVP technique has been schematically represented in Figure 3.4 (section 3.3.2). It is seen that the velocity profile of liquid in a circular tube can be estimated by single-line measurement using UVP technique. By UVP method, the projection ($v_{\text{UVP}}$) of the true velocity vector ($v_{\text{real}} \approx v_x$) on the measurement axis is monitored by Doppler-shifted echo-signal. Since the stream lines are parallel to the wall in a circular tube, the angle correction is applicable to get the true velocity vector using Eq. 3.1. Due to the parallel flow streamlines to the wall, the velocity profile in actual measuring line is same as the target measurement line. To understand the applicability of UVP method in a non-parallel flow
7.2 Inapplicability of UVP for non-circular tube or non-parallel stream-lines

condition, a symmetric contraction tube with infinitely thin wall (considering no wall effect) was taken as a model for non-parallel flow situation (Figure 7.1). In a contraction tube, the average velocity will increase by about 23.5 % due to a diameter decreased by only 10 % at a gives flow rate (Hint: \( Q = \bar{v}_1 \pi r_1^2 = \bar{v}_2 \pi r_2^2 \), where \( r_1 \) and \( r_2 \) are radii of a tube at inlet and outlet, \( \bar{v}_1 \) and \( \bar{v}_2 \) being the corresponding average velocities). Therefore, the assumption of the equivalent velocity profile along actual and target measuring line is not valid anymore for a contraction tube or any complex geometries, where the diameter changes over length. In Figure

![Figure 7.1: Schematic representation of a contraction flow geometry, where the velocity is increasing across the length due to decrease in diameter. The velocity profile of the actual measurement line by UVP is different than the target measuring line.](image)

7.1, it is seen that most of the measuring stream lines are either under or over predicted. The error can be minimized by reducing the Doppler angle which is again limited due to accuracy reason. On the other hand, the stream-lines are not parallel to the wall for the contraction flow condition. If the real velocity vector is tilted with the horizontal axis, and stays in the first-half of the the contraction tube from the transducer (Figure 7.2), then it is also seen that the angle correction (considering parallel flow with horizontal axis) is now giving wrong prediction (higher value than the real vector). In addition, if the tilted vector is taken from the second half of the tube, then the effect will be opposite. Therefore, asymmetric velocity profile can appear in a symmetric non-circular tube flow (e.g. hyperbolic nozzle, contraction jet). Therefore, the conventional angle correction as in UVP-DUO will not be valid for a non-parallel flow streams, where the flow streams are reasonably non-parallel to the wall and each other.
7.3 Proposed method for UVP based flow measurements

7.3.1 New measurement technique of UVP for non-circular tube

Former concept of flow mapping was to measure the velocity components of real velocity vector in the measuring lines, and then relate this information to the real vector as described in (Murai et al., 2012). The present approach is to split the real velocity vector in its two components, which can be measured by UVP. As it is known that a vector component can be only split into two vector components, which are perpendicular to each other (incident angles \( \theta \) or \( 90 - \theta \) should not be higher than critical incident angle). That means, the two measuring lines of UVP should be perpendicular to each other. The measured vector components give the two divided parts of the real vector as shown in Figure 7.3. Therefore, the magnitude of the real vector can be calculated accurately by the following equation (Eq. 7.1):

\[
\| \mathbf{v}_{\text{real}} \| = \sqrt{v_{UVP1}^2 + v_{UVP2}^2}
\]

where \( \mathbf{v}_{\text{real}} \) is the real velocity vector, and \( v_{UVP1} \) and \( v_{UVP2} \) are the vector components in two measuring lines, which are perpendicular to each other. In Figure 7.3, a new coordinate system is formed from the two perpendicular measuring lines.
7.3 Proposed method for UVP based flow measurements

Figure 7.3: The new approach to measure non-parallel velocity vector \((v_{\text{real}})\) by UVP method (\(\theta\) should not be higher than critical angle).

axes, where sign of the \(v_{UVP1}\) and \(v_{UVP2}\) are different depending on the direction of the real vectors (as shown in Figure 7.3). Hence, if the real vector is laid in a specific coordinate, then the corresponding split components will have the sign as mentioned in the Figure 7.3. Calculation procedure for real velocity vector in one coordinate (as in Figure 7.3) is described below. In Figure 7.3, the angle \(\alpha\) and \(\beta\) can be expressed by the following Eqs. 7.2 and 7.3:

\[
\alpha = \tan^{-1} \left( \frac{|v_{UVP2}|}{|v_{UVP1}|} \right) \tag{7.2}
\]

\[
\beta = \pi/2 - \alpha - \theta \tag{7.3}
\]

where, \(\alpha\) is the angle between \(v_{UVP1}\) and \(v_{\text{real}}\), and \(\beta\) is the angle between \(v_{\text{real}}\) and vector component of the real velocity vector along the x-axis \((v_x)\) or flow direction. Then, sum of \(\theta\) and \(\alpha\) gives the direction of the real velocity vector component with vertical axis, and the magnitude of a velocity component of the real velocity vector in the x-axis, \(v_x\) can be expressed by Eq. 7.4:

\[
v_x = v_{\text{real}} \cos \beta \tag{7.4}
\]

By obtaining all mentioned information at a given point, one can have full set of information of a velocity vector with direction relative to the vertical or horizontal axis at that point. By using multiple transducers (Figure 7.4), velocity
profiles can be estimated precisely for any complex geometries with non-parallel flow situation. In addition, the flow mapping of liquid flow in a complex geometry would also be possible with UVP-method using multiple transducers. For steady flow, two transducers would be enough to use the method in different points of a target measurement line by one after another. For more precise measurement in a complex geometry, multiple transducers with software code correction would be preferential for the implementation of the new methodology.

![Figure 7.4: The new approach for flow mapping of a parallel or a non-parallel velocity vector ($v_{\text{real}}$) by UVP method.]

### 7.3.2 Correction for wall diffraction

The ultrasonic waves are reflected (Messer and Aidun, 2009) and diffracted due to the tube wall. In the present approach, the diffraction can cause an error to predict the intersection point of two measuring lines, which can cause the inaccuracy of the measurement. Due to the diffraction of ultrasound through the wall, the wave diffracts and shifts horizontally (depending the thickness of wall) during second diffraction occurring from the inner solution of the tube, and again follow a line parallel to the incident wave line. The possible corrections for wall on the intersect-point of two measuring lines are described in the appendix. This correction is not important for very small value of $d$ and $d/D$ (where $d$ and $D$ are the thickness of wall, and diameter of the tube or the thickness of target measuring line respectively). In Figure 7.5, it is seen that the actual measuring point can be different than the target measuring point along the target measuring line. Therefore, a geometric solution is presented here which can be easily implemented by knowing the Doppler angle, tube wall thickness, sound velocities in the medium (aqueous solution) and tube wall. When $v_{\text{sound(solution)}} \geq v_{\text{sound(wall)}}$, then the target measurement point is shifted by $\Delta y$ in the vertical direction (shown in
7.3 Proposed method for UVP based flow measurements

Figure 7.5: Schematic presentation of the necessity of the wall correction due to diffraction of ultrasound.

Figure 7.5), which can be estimated by the following procedure.

$$\tan \alpha = \frac{x_1}{d} \quad (7.5)$$

$$\tan \theta = \frac{x_1 + \Delta x}{d} \quad (7.6)$$

From Eqs. 7.5 and 7.6, $\Delta x$ can be expressed by the following equation.

$$\Delta x = d(\tan \theta - \tan \alpha) \quad (7.7)$$

From Snell’s law,

$$\frac{\sin \theta}{\sin \alpha} = \frac{v_{\text{sound(solution)}}}{v_{\text{sound(wall)}}} \quad (7.8)$$

Therefore,

$$\alpha = \sin^{-1} \left( \frac{v_{\text{sound(wall)}}}{v_{\text{sound(solution)}}} \sin \theta \right) \quad (7.9)$$

$\Delta y$ can be expressed by the following equation.

$$\Delta y = \frac{\Delta x}{\tan \theta} \quad (7.10)$$

From Eqs. 7.7, 7.9 and 7.10, $\Delta y$ can be calculated by knowing the incident angle $\theta$, tube wall thickness ($d$), and the sound velocities in the solution and tube wall. $\theta$ will be different for contraction type of geometry, which can be corrected by $\theta_{\text{new}} = \theta + \theta'$, where $\theta'$ is the inclined slope of the wall with respect to the horizontal axis.
7.4 Exemplar experiment and result-discussion

The present approach is an analytical solution and measuring technique for true velocity component in a complex geometry, or complex flow situation. An experimental result is demonstrated for better understanding and visualization.

7.4.1 Experimental setup and material

Experiment was carried out in a vertically squeezed-elastic tube, which was circular before squeezing. The top and front views are shown in Figure 7.6 (left). Constant flow rate (144 ml/min) of 1.5 % carboxymethyl-cellulose (CMC; Blanose CMC 7MF, IMCD Switzerland AG) aqueous solution with the flow direction from left to right was maintained. The aqueous solution of CMC contains 0.3 % polyamide particle for better resolution, where the small amount of polyamide did not change the viscous behavior of the solution (Figure 3.2b). For mentioned operating flow rate, the average wall shear rate at the measuring target region \(\dot{\gamma}_{\text{wall}} = 32\dot{Q}/\pi D^3\), where \(\dot{\gamma}_{\text{wall}}, \dot{Q}, D^3\) are wall shear rate, fluid flow rate, and equivalent diameter at the target measuring line respectively) was lower than the shear needed to reach the shear-thinning region (shear rate of about 60 \(s^{-1}\) or higher) for CMC. The lower flow rate was also maintained to avoid any expansion of elastic tube. The inner diameter of the circular tube was 20 mm with wall thickness of 1 mm. The target measuring line was 29 mm away from the squeezed section as shown in Figure 7.6 (right). An ultrasound transducer of 8 MHz emission frequency with 2 mm active diameter and 5 mm housing diameter is used. The details of the UVP method applied here are described in section 3.3.2. The measured sound velocities in the elastic tube wall and 1.5 % CMC aqueous solution (with 0.3 % polyamide) were 992 m/s and 1499 m/s at 22 °C.
respectively. Accordingly the critical incident angle was calculated ($81^\circ$) and the incident angles ($45^\circ$) were chosen lower than that critical angle. The two measuring lines were adjusted so that they intersect at a point in the target measuring line (Figure 7.4 and Figure 7.6 (right)). The transducers were moved in a way that the crossing point always stays on the target measuring line (length of target measuring line was 18.5 mm). The adjacent crossing points were adjusted at 3.0 mm away from each other. The diffraction effect of wall on the measurement line was considered, though it was very small due to very thin wall (approx. $d/D = 0.05$, where $d$ is the tube wall thickness and $D$ is the inner tube diameter). Maximum six crossing-points were investigated along the target measuring line. The velocity profiles of circular tube and the velocity profile at 6 mm away from the squeezed section were also monitored at same flow rate to observe the effect of contraction on the shape of velocity profiles.

7.4.2 Velocity profile using multiple-line based measurement

Figure 7.7 shows the measured velocity profiles by single-line measurement using UVP in an elastic tube both at circular ($\geq 90$ mm away from the squeezed section) and squeezed sections (6 mm away from squeezed section and the measuring line L3 as shown in Figure 7.6 (right)). The parabolic and symmetric velocity profile is observed for the circular tube, whereas a non-symmetric velocity profile is monitored for the squeezed tube. The real and target measuring lines were different for squeezed-tube due to the contraction. The error from this conventional single-line measuring technique is not predictable due to two opposite influencing effects (as described in the section 7.2).

![Figure 7.7: Velocity profile by UVP method in a circular tube (left) and squeezed elastic tube (right) at the measuring line, L3, as shown in Figure 7.6 (right). Asymmetric profile of the squeezed tube is clearly shown in right figure.](image-url)
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Figure 7.8: (left) Non-parallel velocity vectors ($v_{\text{real}}$) by UVP method in a squeezed elastic tube and (right) the velocity component ($v_x$) in the flow direction in the same measuring line of the real vectors.

Figure 7.9: Velocity profile of the target measuring line by the conventional way from two measuring lines (L1 and L2) shown in Figure 7.6 (right), and the velocity component in flow direction ($v_x$) using new technique for getting approximate true velocity profile in a squeezed elastic tube.
7.5 Conclusions

The new measuring technique was used to obtain the velocity vector in a target measuring line, which was 29 mm away from the squeezed area, and the length of target measuring line was 18.5 mm as shown in Figure 7.6 (right). The real velocity vectors along the target measuring line, and the corresponding velocity components in the flow direction (here x-axis) are shown in Figure 7.8. It is clearly seen that the real velocity vectors are neither parallel to wall nor each other. In addition, the velocity components in the flow direction of the real velocity vectors are correctly measured, and approximated velocity profile is observed to be symmetric. The velocity profiles along the measuring lines L1 and L2 (Figure 7.6 (right), using conventional single-line measuring technique), and multiple-line measurement method are shown in Figure 7.9. The approximated velocity profile using multiline measurement technique is seen to be symmetric and the maximum velocity is also obtained at the tube center. On the other hand, the conventional single-line measurement method shows a non-symmetric velocity profile, and the velocity prediction at first-half and second-half region of the tube from the transducer is smaller and larger respectively compared to that if the real velocity profile. Therefore, the new approach of multiple-line measurement technique is well applicable for non-parallel flow condition, where conventional single-line measurement technique is not suitable or less accurate.

7.5 Conclusions

The multiple-line based approach for velocity profile measurement using UVP is an improved technique for parallel to non-parallel flow stream lines. It gives the magnitude of the real velocity vector as well as the direction. In addition, the velocity component of the real velocity vector in the horizontal or flow direction can be calculated. The study also includes the inapplicability of direct implementation by single-line based measurement by UVP method in non-parallel flow condition of a complex geometry. The experiments showed that the proposed method is an excellent technique for obtaining more precise result in a contraction flow, and it can be implemented to other complex geometries as well. Moreover, the measurement approach can also be implemented for other Doppler based velocity profiling method.
8 Deformation-controlled (peristaltic) unsteady flow in elastic tube

8.1 Introduction

The detail understanding of steady and unsteady (pressure-controlled) flow characteristics of non-Newtonian fluids in collapsed elastic tube followed the interest to investigate experimentally the flow behavior of a non-Newtonian shear thinning fluid during peristaltic propulsion through an elastic tube. Peristaltic flow is a primary physiological transport mechanism encountered in the most tubular organs of the human body (e.g. transport of food through the esophagus, and carrying and mixing motion of chyme in the small intestine). The experimental and theoretical features of peristaltic flow have been studied extensively for Newtonian fluids by several authors (Lew et al., 1971; Weinberg et al., 1971; Yin and Fung, 1971). In addition, an in-vivo study has been carried out to characterize and visualize the flux over time in small bowel segments (Gutzeit et al., 2010). For more realistic non-Newtonian fluid systems, systematic work is still missing. Furthermore, most of the physiological fluids and foods are known to be non-Newtonian and their rheological properties (shear and extensional viscoelasticities) play an important role in the peristaltic flow characteristics. The main study challenge here was to investigate the velocity fields under peristalsis (due to squeezing of an elastic tube), which were not measured previously due to lack of applicable methods. We met the challenge of implementing the UVP technique for monitoring the velocity fields during appropriate peristaltic propulsion of a shear thinning fluid through an elastic tube (Nahar et al., 2012). The final goal of the present study was to understand the flow structure of non-Newtonian fluids under peristaltic squeezing of an elastic tube (in vitro modeled small intestine). The detailed knowledge gained about non-Newtonian peristaltic flow can be useful to explore and observe the influence of the peristaltic flow on the mass transport (i.e. diffusion) across the elastic membrane tube wall (representative of small intestinal wall).
8.2 Experimental set up for peristaltic flow

A silicone elastic tube (1165 mm long) of the same type used in steady and unsteady flow experiments was mounted between two aluminum pipes and immersed in a water filled open tank (width 250 mm and length 1300 mm) as shown in Figure 8.1 (top). The corresponding schematic diagram of the experimental setup is also given in Figure 8.1 (bottom), where three distinct UVP measuring lines both horizontal (ML1L-H, ML1-H and ML1R-H) and vertical (ML1L-V, ML1-V and ML1R-V) positioning of ultrasound transducer are also inserted. Three pairs of rollers (outer diameter of 30 mm) were placed (three rollers on both top and bottom surface of the tube) to induce the peristaltic motion on the elastic tube. The distance between two adjacent rollers along tube length as well as the gap (4 mm) between top and bottom rollers were so adjusted that generates the peristaltic wave with wavelength of 82 mm. The additional experimental parameters were also adjusted in comparison with the *in vivo* small intestine as listed in Table 8.1. The speed of the rollers was controlled by an electric motor (MCD EPOS 60W, maxon motor), which generated the variation in the peristaltic motion while squeezing of the elastic tube. The elastic tube was filled with a non-Newtonian 1.5 % CMC aqueous solution with 0.3% Polyamide tracer particles (20µm), which was shear thinning (Figure 3.2b) and the two outlets were connected to the aqueous solution filled reservoir. Two optical fiber based pressure transducers with a control unit (Samba sensors, Sweden) were used for the measurement of local and total pressure difference in a wavelength at different speeds of peristalsis. The velocity profiles were monitored by UVP technique with an ultrasound transducer (5 mm outer diameter and 2 mm beam or active diameter) of 8 MHz emission frequency (supplied by private communication via Prof. Y. Takeda). The relevant and adjusted UVP parameters have been described in section 3.3. The tube shape in a crest at the measuring line ML1-H/ML1-V was constructed by CT-method as described also in section 3.3. Figure 8.2 is showing the front (a) and top (b) views, and the corresponding obtained tube shape (c) of the crest by CT-method.

8.3 Results and Discussion

8.3.1 Peristaltic flow of shear thinning fluid

Peristaltic flow investigation of a shear thinning fluid involved the measurements of axial velocity profiles using the UVP technique (based on the parallel flow streamlines) under various speeds of peristalsis. As peristalsis involves both contraction
8.3 Results and Discussion

Figure 8.1: Experimental setup for the peristaltic propulsion of shear thinning fluid in an elastic tube (top). Schematic representation showing peristaltic squeezing of an elastic tube with the positions of pressure sensors ($P_1$ & $P_2$) and ultrasound transducer (bottom). Three different measuring lines are adjusted for velocity profile measurement both horizontally (ML1L-H, ML1-H and ML1R-H) and vertically (ML1L-V, ML1-V and ML1R-V) under peristaltic motion.
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<table>
<thead>
<tr>
<th>Parameters</th>
<th>in vivo (Stoll et al., 2010)</th>
<th>in vitro (Egorov et al., 2002)</th>
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<td>1.08 ± 0.25</td>
</tr>
<tr>
<td>Thickness [mm]</td>
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<td>Elastic modulus [MPa]</td>
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<td>3.9 ± 0.71</td>
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<td>Average velocity [mm/s]</td>
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<tr>
<td>Elastic modulus [MPa]</td>
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<td>3.9 ± 0.71</td>
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<td>3.9 ± 0.71</td>
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<td>Elastic modulus [MPa]</td>
<td>1.08 ± 0.25</td>
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<td>Wavelength [mm]</td>
<td>80.1</td>
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Table 8.1: Comparison between physiological (in vivo) and adjusted experimental (in vitro) parameters for the study.
8.3 Results and Discussion

(a) (b) (c)

Figure 8.2: The front (a) and top (b) views, and the corresponding obtained tube shape by CT-method (c) at the measuring line ML1-H/ML1-V in a crest.

and expansion type flow (crest and trough of a wavelength), so the streamlines are assumed to be non-parallel to the tube wall. Therefore, a new measurement technique was proposed (chapter 7) to determine the magnitude of real velocity vector ($v_{\text{real}}$), angle between $v_{\text{real}}$ and vertical axis or wall (direction of velocity-vector), and the velocity component ($v_x$) of $v_{\text{real}}$ along the flow direction under peristaltic motion, and a comparison with the single measurement line based velocity profile has also been demonstrated. The calculation steps for determining the velocity vectors can be followed as described in chapter 7. In addition, the wall shear rates (velocity gradients near the tube wall) both in the wave crest and trough were also approximated from the measured velocity profiles. Furthermore, the average shear rate in the wave crest and the elongation rate in the wave trough were also calculated. The obtained shear rates and consequently viscosities of a shear thinning fluid are eventually helpful to estimate the improved mixing and mass transfer mechanism encountered in the intestinal wall. The pressure difference between crest and trough within a wavelength was also measured under peristaltic motion of a shear thinning fluid through an elastic tube.

8.3.1.1 Accuracy of UVP technique for peristaltic flow

The velocity profiles were monitored by UVP both in the crest and trough region of the ‘2-wave’-squeezing of an elastic tube. The ultrasound transducer was placed at three distinct measuring lines (positioning both horizontally and vertically at the crest, and only horizontally at the trough as shown in Figure 2) with a Doppler
angle of 70° in a moving frame. In the moving frame, the transducer was moving longitudinally at the same speed as the wave speed in both directions (left to right, \( L \rightarrow R \) or right to left, \( R \rightarrow L \)) depending on the study interest. The measured velocity profiles were found to be uniform and stable for the entire measurement (in both directions) as can be seen by the uniform color graph along the measuring line (Figure 8.3). The color graph uses color coding (here for velocity as depicted on the right corner of each window) to display the saved velocity profiles in the whole file. In addition, a larger tube width was observed as monitored from the front (horizontal) whereas smaller tube width was seen while monitoring from the top (vertical) as expected. All the measured velocity profiles shown later were transformed into the laboratory frame by adding or subtracting of the wave speed respect to the wave direction (as \( v_{\text{real}} = v_{\text{meas}} + v_{\text{wave}} \) for \( L \rightarrow R \), and \( v_{\text{real}} = v_{\text{meas}} - v_{\text{wave}} \) for \( R \rightarrow L \), where \( v_{\text{real}} \), \( v_{\text{meas}} \) and \( v_{\text{wave}} \) are the actual, measured and peristaltic wave velocities, respectively).

![Color graph of the measured velocity profiles](image)

Figure 8.3: Color graph of the measured (both horizontally and vertically) velocity profiles under different speeds of peristalsis (while peristaltic wave moving in both directions: left to right or right to left) which confirms the uniformity of the measurement along the measuring length.

### 8.3.1.2 Velocity profiles in the wave crest

The measured velocity profiles in a wave crest at the measuring line ML1-H (horizontal) are represented as a function of distance inside the tube in Figure 8.4 under different wave speeds (as 3, 5, 7 and 10 mm/s). The results show that higher wave speed of peristalsis develop in higher back flow (fluid velocity in the
opposite direction of the wave speed) at the crest center, which is of specific interest for improved mixing and mass transfer conditions of nutrients. A positive velocity was detected near the tube wall while moving the peristaltic wave from $L \rightarrow R$ (Figure 8.4a) or $R \rightarrow L$ (Figure 8.4b), since the moving rollers to impose the peristalsis were in direct contact with the tube wall. In contrast, the velocity magnitudes were gradually becoming larger and negative (representing a back flow) towards the tube center due to forward movement of the peristaltic wave from left to right (Figure 8.4a) and correspondingly the fluid was moving backward. The reason is that the pressure rises inside the leading wave (fluid ahead the roller) than that in the lagging wave (fluid behind the roller) during peristalsis which can lead to a mixing mechanism near the wave trough. The behavior is vice versa when the peristaltic wave was moving from $R \rightarrow L$ (Figure 8.4b), where the obtained positive velocity magnitudes are represented as negative values by correction with the relative speeds of the rollers. In both cases, when the wave speed of peristalsis go faster that is velocity near the tube wall was more positive and correspondingly the velocity magnitude at the tube center became more negative (back flow) due to higher pressure drop depending on the wave speed. Furthermore, the velocity magnitude at the crest center is seen to be higher while the peristaltic wave moving from $R \rightarrow L$ compared to than that from $L \rightarrow R$, since the tube geometry monitored by ultrasound was different (variation in the tube shape and cross sectional area) along the adjusted measuring line. The calculated shear rate magnitude near the wall by the corresponding velocity gradient along the measuring line ML1-H/ML1-V is also found to be increased from 0.15 to 0.71 s$^{-1}$ as the peristaltic motion varied from 3 to 10 mm/s, respectively. The average shear rate in the wave crest was calculated by $\gamma_{avg} = 4 v_{avg}/r_{eq}$, with average velocity magnitude in the crest $v_{avg} = 1$ mm/s, and equivalent radius of the crest $r_{eq} = \sqrt{A/\pi} = 7.6$ mm (as cross sectional area is 184 mm$^2$), considering the test fluid as Newtonian. The approximated magnitude of the average shear rate in the wave crest is about 0.52 s$^{-1}$. The corresponding viscosity values are seen to remain in the Newtonian regime as the test fluid (CMC 1.5 %) was less shear thinning and the imposed peristaltic motion was not high enough to transform it to the shear-thinning regime (shear rate about 50 s$^{-1}$). Therefore, the use of a highly shear thinning fluid e.g. polyacrylamide aqueous solution (as represented in Figure 3.2c) will be of interest. The accuracy of the measurement was also confirmed by symmetric and overlapping of the normalized (by velocity at crest center) velocity profiles at different peristaltic speeds as shown in Figures 8.4(c,d). The measured velocity fields are found to be influenced by the memory of the traveling waves and wave speeds. It is seen that the normalized velocity profiles during peristaltic wave moving from $L \rightarrow R$ are having little deviation for variation in speeds of peristalsis (Figure 8.4c), where the measured velocity profiles are affected by lagging of a wave respect to the direction of wave speed.
On the other hand, the wave moving from $R \rightarrow L$ is leading a wave that shows a more symmetric and overlapping normalized velocity profiles (Figure 8.4d) under various peristaltic speeds. In addition, velocity profiles were also measured at two other measuring lines ML1L-H and ML1R-H in the crest (Figure 8.1) which were 20 mm behind and ahead from ML1-H, respectively under different wave speeds of peristalsis and only moving from $L \rightarrow R$. The velocity profiles were monitored by horizontal positioning of the ultrasound transducer at the peristaltic speeds of 5 mm/s (Figure 8.5a) and 10 mm/s (Figure 8.5b) along three distinct measuring lines. The results depict that the maximum velocity magnitude (negative) inside the tube is much higher while measuring at ML1L-H compared to those measured at measuring lines ML1-H and ML1R-H (Figure 8.5) and a higher positive velocity value is observed at the tube wall. The reason is that the measuring line ML1L-H was close to and leading the wave crest with much higher contraction (smaller cross sectional area) in the first-half measuring region from the tube center resulting to a higher negative velocity values at that regime. The corresponding approximated shear rate magnitude near the wall is about $2.57 \, s^{-1}$. On the other hand, the second-half measuring region from the tube center is showing a less negative velocity values which confirms a region of bigger cross sectional area. In contrast, the measuring line ML1-H was aligned at the center of the wave crest with a uniform cross sectional area along the measuring line showing more symmetric velocity profile. Furthermore, the measuring line ML1R-H was fixed at the end of the wave crest with a small contraction in the second-half of the measuring line, representing a more negative velocity magnitude than that obtained in the first-half with more uniform tube shape. The velocity profiles measured at ML1R-H is showing slightly higher negative value at the end part of the measuring line since it starts in leading to the wave crest with little contraction (small cross sectional area). Hence, the resultant velocity magnitude differs depending on the corresponding tube deformation (tube cross sectional area), so the target and actual measuring lines for UVP measurement do not remain identical (as explained in chapter 7). Here, the calculated shear rate magnitude near the wall is about $1.7 \, s^{-1}$. The velocity profiles were again measured by vertical positioning of the ultrasound transducer at the peristaltic speeds of 5 mm/s (Figure 8.6a) and and 10 mm/s (Figure 8.6b) moving from $L \rightarrow R$ along three distinct measuring lines ML1L-V, ML1-V and ML1R-V. Here a larger velocity magnitude (negative) is observed in the tube center while measuring at ML1R-V than those measured along measuring lines ML1-V and ML1R-V (Figure 8.6) with a positive velocity value at the tube wall. In addition, the back flow is found to be higher (more negative velocity magnitude at the tube center) for faster speed of peristalsis. The reasons can be speculated as explained for the horizontal measurements. In this case, the approximated shear rate magnitudes near the wall are found to be same ($1.7 \, s^{-1}$) as measured along the three vertically adjusted measuring lines.
Figure 8.4: Experimentally measured velocity profiles (a, b) and the corresponding normalized velocity profiles (c, d) in the wave crest at the measuring line ML1-H by horizontal positioning of ultrasound transducer and wave traveling from left to right (a, c) or right to left (b, d), under different speed of peristalsis.
Deformation-controlled (peristaltic) unsteady flow in elastic tube

![Flow velocity vs Distance inside tube](image)

(a)  (b)

Figure 8.5: Measured velocity profiles in the wave crest at three optimized measuring lines ML1L-H, ML1-H, ML1R-H by horizontal positioning of ultrasound transducer at the peristaltic speeds of 5 mm/s (a) and 10 mm/s (b), moving from left to right.

![Flow velocity vs Distance inside tube](image)

(a)  (b)

Figure 8.6: Measured velocity profiles in the wave crest at three optimized measuring lines ML1L-V, ML1-V, ML1R-V by vertical positioning of ultrasound transducer at the peristaltic speeds of 5 mm/s (a) and 10 mm/s (b), moving from left to right.
8.3 Results and Discussion

8.3.1.3 Velocity profiles in the wave trough

The velocity profiles were measured at the first and second wave trough (of the ‘2-wave-squeezing’, Figure 8.7) with the same history of wave propulsion as the peristaltic wave moving from \( L \rightarrow R \) (Figure 8.8a for trough-1) and (Figure 8.8b for trough-2). The higher wave speed of peristalsis results in higher velocity magnitude (negative) inside the tube, which corresponds to higher back flow with a positive velocity value at the tube wall. The measured velocity profiles are representing a higher velocity magnitude in the first-half of the investigated region along the adjusted measuring lines in the wave troughs (trough-1 & trough-2), indicating more contraction of the squeezed tube at that region (so the target and actual measuring lines were seen to be different by UVP). The velocity magnitude measured in the second trough (trough-2, Figure 8.8b) is observed to be more negative than that of the first trough (trough-1, Figure 8.8a) as trough-2 is forwarded by the trough-1 in the wave direction. The estimated magnitude of shear rates near the wall of wave trough varied from 3 to 4 \( s^{-1} \) and the magnitude of approximated maximum elongation rate from crest to the trough is 0.44 \( s^{-1} \) under the peristaltic motion of 10 mm/s.

8.3.1.4 Velocity profiles in the wave trough at Doppler angle of 90°

The peristalsis involves both contraction and expansion type of flow (crest and trough of a wavelength). We met the challenge of implementing the UVP technique for appropriate peristaltic propulsion of a shear thinning fluid through an elastic tube (in vitro modeled small intestine) by a special experimental setup (Figure 8.1). It is known that there is only axial component present (no vertical component) in a parallel flow, therefore while monitoring the velocity profile at 90° results no Doppler effect of the scattering particle and no velocity profile is obtained (Lemmin and Rolland, 1997). On the other hand, in a contraction flow
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Figure 8.8: Measured velocity profiles in two wave troughs as trough-1 (a), and trough-2 (b) by horizontal positioning of ultrasound transducer, and wave traveling from left to right under different speed of peristalsis.

It is expected that the flow velocity is not only uniaxial rather having some radial or tangential components. Therefore, the velocity profiles were measured in the second wave trough at an incident angle of $0^\circ$ (Doppler angle is $90^\circ$) while peristaltic wave moving from $L \rightarrow R$ (Figure 8.9a) or right to left (Figure 8.9b). It can be seen that the measured velocity profiles include some arbitrary velocity magnitudes depending on the wave speeds and traveling direction, which confirms the presence of different velocity components in addition to the axial component at the contraction region of a peristaltic wave. The flow field can also be affected by the wave amplitude during the peristaltic motion, therefore the velocity vector investigation is of interest for better understanding of the flow behavior under peristalsis.

8.3.2 Velocity vectors under peristaltic motion

The experimentally measured velocity profiles at a Doppler angle of $90^\circ$ gave an indication for the existence of different velocity components (non-parallel flow) in a peristaltic motion. The flow profiling by means of UVP was carried out along the target measuring lines both in wave crest (ML1-H/ML1-V) and wave trough (trough-2) to investigate the non-parallel flow under peristalsis. The new measuring technique has been implemented based on the proposed approach and principle as described in chapter 7.
8.3 Results and Discussion

Figure 8.9: Measured velocity profiles in trough-2 by horizontal positioning of ultrasound transducer with a Doppler angle of 90°, and wave traveling from left to right (a), or right to left (b), under different speeds of peristalsis.

8.3.2.1 Multiple measurement line based velocity profiles in the wave crest

As shown in Figure 7.4, two ultrasound transducers were placed at a Doppler angle of 45° for each both in crest and trough. Then the peristaltic motion was imposed under a wave speed of 10 mm/s. The measuring lines along the two adjusted transducers intersected in one point, where obtained intersecting points were 10 and 11 for crest and trough, respectively. The first crossing point was set at the vicinity of the inner tube wall and the distance among successive crossing points were maintained at 3 mm. The two transducers measured the two split-up components of the real velocity vector under peristalsis then the magnitude of the real vector was obtained by Eq. 7.1. Figure 8.10 shows the obtained real velocity vectors in the wave crest (detail calculation steps have been described in section 7.3.1). It can be seen that the velocity is positive near the tube wall and gradually become negative along the center of the tube crest. The corresponding axial velocity components are shown in Figure . A similar flow profile was obtained as in Figure 8.4a by the new experimental approach of multiple measurement line based velocity flow profiling (Figure 8.11) in comparison with the single measurement line based velocity profile by UVP-technique (the detail reason has been given in section 8.3.1.2). Finally, a comparison between the single and multiple-line based velocity profiles by UVP technique is attained. Therefore, the two velocity profiles measured at Doppler angle of 45° and -45° (so that two measuring lines are perpendicular to each other, and passes though the center of the wave crest) by means
Figure 8.10: Non-parallel real velocity vectors ($v_{real}$) in the wave crest by new approach of UVP flow mapping.

Figure 8.11: Non-parallel velocity components ($v_x$) in the flow direction along the same measuring line of $v_{real}$ in the wave crest by new approach of UVP flow mapping.
of the single-line approach and the corresponding velocity profile using the new multiple-line method are shown in Figure 8.12. It is seen that the axial velocity components obtained using both methods (single-line and multiple-line) are well comparable, where the velocity profile by new multiple-line method is relatively symmetric (as expected) unlike over the single-line method. The reasoning is well described in chapter 7. In brief it can be stated that the measured velocity profile is over predicted in the first-half or under predicted in the second-half region of the tube from the transducer.

8.3.2.2 Multiple measurement line based velocity profiles in the wave trough

In a similar manner, the new multiple-line based measuring technique was also applied to investigate the velocity field in the wave trough. The obtained real velocity vectors and axial velocity components in the trough-2 are shown in Figures 8.13 and 8.14, respectively. The velocity components in a wave trough is found to be more asymmetric (non-uniform) and unpredictable due to two main possible reasons: (i) the adjusted gap between two tube walls in the wave trough was very small (4 mm) which included some reflection of the ultrasound from the tube walls and influenced the measurement accuracy. (ii) The peristaltic motion is
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Figure 8.13: Non-parallel real velocity vectors ($v_{\text{real}}$) in the wave trough by new approach of UVP flow mapping.

Figure 8.14: Non-parallel velocity components ($v_x$) in the flow direction along the same measuring line of $v_{\text{real}}$ in the wave trough by new approach of UVP flow mapping.
8.3 Results and Discussion

an unsteady flow which is pronounced in the wave trough and correspondingly a higher magnitude of the non-parallel flow components. As an unsteady state of the flow is present, therefore the use of two transducers along 11 adjusted measuring lines at different times might not result the actual velocity profiles for each trial. However, this problem can be resolved by using 22 transducers at once during the investigation.

Figure 4.9 shows a comparison between the velocity profiles in the trough measured by single-line based and proposed multiple-line based methods. It is seen that the single-line based UVP method is only estimating the velocity magnitudes in the first-half of the trough from the transducer as the measured velocity profiles along the two measuring lines (which are so adjusted that they pass through the center of the trough) are found to be uni-modal. On the other hand, the velocity profile is accurately estimated by the new multiple-line based method, which is seen to be bi-modal as expected respect to the corresponding shape of the trough (according to the previous experience of steady flow investigation).

8.3.2.3 Alternative approach for the estimation of shear rate profile

The measured velocity profiles along the horizontal measuring line ML1-H in the wave crest at the peristaltic wave speeds of 3 and 10 mm/s were fitted by the
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2nd order polynomial equations \( v = 0.0057x^2 - 0.156x - 0.1758 \) with \( R^2 = 0.88 \), where \( v \) is the velocity value and \( x \) is the corresponding distance inside the tube crest) and \( v = 0.0292x^2 - 0.8036x + 2.61 \) with \( R^2 = 0.98 \) respectively. Then the corresponding shear rate profiles were approximated by the velocity gradients \( (dv/dx = 0.0114x - 0.156) \) and \( (dv/dx = 0.0584x - 0.8036) \) at each point in the tube crest (Figure 8.16). Therefore, the wall shear rate magnitude varied from 0.156 to 0.8036 s\(^{-1}\) for the wave speed of 3 to 10 mm/s respectively. In the similar manner, the obtained wall shear rates along the horizontal measuring lines ML1L-H and ML1R-H were 2.4 and 1.4 s\(^{-1}\), while the peristaltic wave moving at the speed of 10 mm/s. In addition, the estimated wall shear rates along the vertically adjusted measuring lines ML1L-V, ML1-V and ML1R-V were again found to be same of about 2.55 s\(^{-1}\) for wave speed of 10 mm/s.

Figure 8.16: The approximated shear rate profile along the horizontal measuring line ML1-H in the wave crest, while peristaltic wave moving from left to right at the speeds of 3 and 10 mm/s.

On the other hand, the shear rate profile in the wave trough-2 is approximated from the obtained velocity profile by the new multiple-line based method for UVP (Figure 8.15) is shown in Figure 8.17. As the velocity profile in the trough-2 is seen to be bi-modal; therefore the shear rate profiles are separately approximated in each lobe by 2nd order polynomial equations \( v = 0.225x^2 - 4.11x - 1.55 \) with \( R^2 = 0.84 \), and \( v = 0.1798x^2 - 7.65x + 61.94 \) with \( R^2 = 0.92 \). Then the corresponding velocity gradients are \( (dv/dx = 0.45x - 4.11) \) and \( (dv/dx = 0.36x - 7.65) \), with the wall shear rate magnitudes of 4.11 and 7.65 s\(^{-1}\) for the wave speed of 10 mm/s. It is seen that the shear rate values in the wave trough is much higher than that in
the wave crest as expected, which in turn result the much decrease in the viscosity correspondingly higher back flow and efficient mixing.

### 8.3.3 Pressure difference under peristalsis

The local pressure in a wavelength was measured by placing pressure sensors \( P_1 \) at the crest center, \( P_2 \) at the tube trough as schematically shown in Figure 8.1 in direct contact with the fluid. The speeds of peristalsis were varied as 2, 4, 6, 8 and 10 mm/s. The two pressure sensors were also connected with moving part of the peristalsis (the sensors were always seen to be at the same position as the peristaltic wave travels). The distance between two sensors were about 25 mm. The pressure difference between tube crest and its corresponding two troughs was measured. The crest region showed a higher pressure compared to the trough region, since the magnitude of back flow velocity in the wave trough was much higher (negative) compared to that in the wave crest. Therefore, a higher velocity head corresponded to a lower pressure head as in the wave trough according to Bernoulli’s law. The pressure difference between crest and trough of a peristaltic wave was found to be increased with increasing the wave speed as expected (Figure 8.18). However, the pressure difference between center of crest and trough was found to be low about 0.2 to 0.7 mbar due to low mean fluid velocity and relatively large gap in the trough (required to locate the pressure sensors).
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8.4 Conclusions

The study of *in vitro* small intestinal flow (peristaltic squeezing of an elastic tube) of non-Newtonian fluids is important to biofluid mechanics encountered in the human body. The velocity profiles and pressure differences were investigated experimentally under peristaltic flow of a non-Newtonian shear thinning CMC aqueous solution in an elastic tube. The higher wave speed of peristalsis resulted in higher magnitude of back flow velocity (negative) both in the wave crest and trough regions with positive value being near the tube wall. Consequently, the higher value of back flow is expected to be responsible for the improved mixing and convection leading to higher mass transport through the intestinal wall. In addition, the approximated wall shear rates also found to be increased with increasing the peristaltic motion, which correspondingly is expected to enhance the mass transfer by reducing the viscosity of a shear thinning fluid. The pressure in the tube crest was found to be higher than that at the trough. The pressure difference between center of crest and trough was low since the mean fluid velocity was low and gap in the trough was relatively large. In addition, a new experimental approach of UVP flow profiling has been proposed to investigate the real and axial velocity vectors under peristaltic motion, and a comparison with the conventional technique was also demonstrated. It was found that the proposed method was well applicable to accurately estimate the velocity profiles both in the wave crest and trough. The detailed knowledge gained about non-Newtonian peristaltic flow is aimed to use in future for mass transport investigations across the elastic membrane tube wall.
9 Diffusion and mass transfer kinetics

9.1 Introduction

The ultimate goal of this work was to investigate the nutrient mass transfer through elastic membrane tube under peristaltic flow using an in-vitro small intestinal model. Therefore, in the first place, a ‘mass transfer test device’ was designed, constructed and diffusion and mass transfer of the model fluid system with nutrient for the selected membrane was experimentally investigated. The test device was useful to characterize the membrane under various control parameters and know the importance of convective mass transport over the diffusion. The findings will guide to anticipate the dominating role of peristalsis (convection of nutrients) during nutrient transport through the membrane of gastrointestinal tract (GIT).

9.2 Experimental set up for batch diffusion

The experimental setup for the batch diffusion measurement is schematically represented in Figure 9.1. Two diffusion cells were fixed vertically, which were separated/conducted by a sinter ceramic membrane. The diffusion cell-1 (bottom) was filled with 3% NaCl aqueous solution (600 ml) whereas the diffusion cell-2 (top) was filled with deionized water (550 ml) and the conductivity of the solutions were detected in-line using two conductivity sensors (SevenCompact Conduct. S230, Kit, Mettler-Toledo (Schweiz) GmbH) placed on the respective diffusion cells. Measurement range of Conductivity is 0.001 µS/cm - 1000 mS/cm with resolution Cond. (variable) of 0.001 - 1 and accuracy Cond. of 0.5%). Two distinct locations were adjusted for the conductivity sensors: at the vicinity of the membrane (distance between two sensors: 57 mm) and away from the membrane (distance between two sensors: 97 mm) as schematically shown in Figure 9.1. The driving force of the diffusion across the membrane was controlled by two rotors; rotor-1 (4-blade-impeller) and rotor-2 (cone geometry with an angle of 5° and 100
mm diameter) were placed respectively in diffusion cell-1 and diffusion cell-2. The speeds of the two rotors were adjusted by two motors (MCD EPOS 60W, maxon motor, Germany) as fixed with the respective rotating shafts. Two pressure sensors (Type: CTE9001GY0, SensorTechnics GmbH, Germany) were also located in the respective diffusion cells to monitor the variation in pressure. The experiment was performed at atmospheric pressure and pressure difference between two cells was found to be constant ($\approx 15.5$ mbar) over time at about $22 \pm 1^\circ C$ temperature.

Figure 9.1: Experimental setup for batch diffusion measurement. (a) Inner cross section, (b) outer front view and (c) adjusted positions of in-line conductivity measurement is enlarged with schematic representation of the concentration gradient along the length.
9.3 Results and Discussion

9.3.1 NaCl diffusion through membrane

The experiment was carried out on batch diffusion of the solute NaCl molecules from 3% NaCl aqueous solution in bottom cell-1 to the deionized water in top cell-2 through the membrane (experimental setup in Figure 9.1) with constant rotational speed N1 of 50 rpm in cell-1 while varying the rotational speeds N2 (30, 50, 100, 300 rpm) in cell-2. The conductivity of the solution was measured inline as positioning the conductivity sensors at two distinct locations (schematically shown in Figure 9.1). The measured conductivity was then converted to the molar concentration of NaCl using the calibration curve and represented as a function of time in Figure 9.2. It can be seen that the solute NaCl concentration in aqueous solution in bottom cell-1 decreases over time due to NaCl diffusion through the membrane into the deionized water in top cell-2, in which consequently the NaCl concentration increases. Therefore, the mass flow rate of the solute molecules per unit area (mass flux) from one cell to another is equal in magnitude but opposite in sign. A higher concentration gradient was observed between both cells for lower rotational speed (50 rpm) compared to the higher rotational speed (300 rpm) in cell-2 due to lower mass flux at lower lower rotational speed. In addition, the homogeneity in the solute concentration near the membrane reached faster (Figure 9.2 a) than away from it (Figure 9.2 b).

Figure 9.2: Variation of NaCl concentration in aqueous solution with time during molecular motion from high to low concentration cell through membrane at 5.7 cm (a) and 9.7 cm (b) distances between two conductivity sensors.
9.3.2 Mass flux and diffusivity of the membrane

Figure 9.3 shows the variation of NaCl concentration in aqueous solution over time with the rotational speeds in both cells fixed at N1/N2 = 50/300 rpm. The solution conductivity was measured by positioning the sensors at 2.85 cm (L = 5.7 cm) and 4.85 cm (L = 9.7 cm) from the membrane. It is seen that there is no significant difference in the measured NaCl concentration profiles over time detected in two different locations in cell-2 at higher rotational speed. Consequently, it is assumed that the solution in cell-2 is almost homogeneously mixed at 300 rpm or higher rotational speed. In that situation, the main resistance to mass transport from one cell to another is the diffusivity of the membrane, which is the main deterministic factor. Therefore, we assume that the concentration measured close to the membrane at the higher rpm is almost the concentration at the membrane interface. The mass flux $J \, [\text{mol/m}^2/\text{s}]$ can be expressed as a function of driving force acting only in the membrane as follows (Cussler, 2009):

$$J = D_m \frac{\Delta C}{\Delta X} \quad (9.1)$$

where $D_m \, [\text{m}^2/\text{s}]$ is the diffusion coefficient or diffusivity of the membrane, $\Delta C \, (= C_{cell1} - C_{cell2}) \, [\text{mol/m}^3]$ is the concentration difference between two cells at a certain time and $\Delta X \, [\text{m}]$ is the characteristics length which is here considered as thickness of the membrane. In addition, the mass flux can also be calculated mathematically as (Geankoplis, 1993)

$$J = \frac{\Delta c \, V}{\Delta t \, A_m} \quad (9.2)$$

with solute concentration difference $\Delta c$ in time interval $\Delta t$ and $V$ is the solution volume in the respective cell and $A_m$ is the total cross sectional area of the membrane. Since the mass flux through the membrane is equal to the flux, which enriches the concentration in cell-2. Hence by equating equations 9.1 and 9.2, the diffusivity of the membrane yields,

$$D_m = \frac{\Delta c \, V \, \Delta X}{\Delta t \, A_m \, \Delta C} \quad (9.3)$$

9.3.3 Overall mass transfer coefficient

In general, the mass transfer resistance from one cell to another is not only the function of diffusivity of the membrane but also a function of the diffusive and convective mass transport coefficients in the liquid phase. The mass flux $J \, [\text{mol/m}^2/\text{s}]$
Figure 9.3: NaCl concentration profile over time as measured by positioning the sensors at 2.85 cm (L = 5.7 cm) and 4.85 cm (L = 9.7 cm) from the membrane shows no significant difference at higher rotational speed (300 rpm) in cell-2.

can be expressed as a function of overall mass transfer coefficient, $K$ [m/s] (Geankoplis, 1993):

$$J = K \Delta C = K(C_{cell1}(t_i) - C_{cell2}(t_i))$$  \hspace{1cm} (9.4)

where $C_{cell1}(t_i)$ and $C_{cell2}(t_i)$ are the detected concentrations in the respective cells at time $t_i$. Consequently, the overall mass transfer coefficient $K$ of the solute is also defined as (Geankoplis, 1993):

$$\frac{1}{K} = \frac{1}{k_{cell1}} + \frac{\Delta X}{D_m} + \frac{1}{k_{cell2}}$$  \hspace{1cm} (9.5)

where $k_{cell1}$ and $k_{cell2}$ are the mass transfer coefficients of the solute in the diffusion cell-1 and cell-2 respectively. The equation 9.5 can be rearranged to calculate the mass transfer coefficient in the liquid phase as

$$k_{cell2} = \frac{2}{K - \frac{\Delta X}{D_m}}$$  \hspace{1cm} (9.6)

For simplicity, considering $k_{cell1} \approx k_{cell2}$ due to the same solution phase with approximately same volume in both cells.
9.3.4 Mass transfer coefficient and diffusivity in liquid

The mass transfer coefficient in the liquid phase includes both the convective and diffusive terms $k_{\text{conv.}}$ and $D_{ab}$ respectively, which can be stated as:

$$k_{\text{cell 2}} = k_{\text{conv.}} + \frac{D_{ab}}{\Delta L}$$  \hspace{1cm} (9.7)

or alternatively

$$k_{\text{conv.}} = k_{\text{cell 2}} - \frac{D_{ab}}{\Delta L}$$  \hspace{1cm} (9.8)

where $\Delta L$ is the adjusted distance between two conductivity sensors in a single cell. The diffusion in the liquid phase can be calculated by Wilke-Chang equation (Wilke and Chang, 1955):

$$D_{ab} = \frac{7.4 \times 10^{-8} (\phi M_{mol})^{1/2}}{\eta V_{mol}^{0.6}} T$$  \hspace{1cm} (9.9)

with dimensionless association factor for the solvent, $\phi = 2.6$ (for water) (Wilke and Chang, 1955), molecular mass of the solvent, $M_{mol} = 18.01528$ g/mol, viscosity of the solvent, $\eta = 1$ mPa.s, molar volume of the solute (NaCl), $V_{mol} = 35.93$ cm$^3$/mol and $T = 295$ K. All the calculated mass transport coefficients (overall, convective and in liquid phase) are shown in Figure 9.4 for different rotational speeds at constant temperature. The Figure 9.4 also shows the diffusion terms for membrane and in liquid phase. The diffusivity of the membrane seen to be lower in comparison with the convective mass transport coefficients. It is also seen that the convective mass transport is highly dominating compared to the diffusion term in the liquid phase, where the convective mass transport is higher for faster rotational speeds due to increase in the molecular convection. Therefore, the present study is an indication to elucidate the higher convective mass transport through the small intestinal wall during the associated peristaltic motion (source of convection). In addition, the mass transport coefficients in the liquid phase, $k_{\text{cell 2}}$ at different rotational speeds and the corresponding convective mass transport coefficients, $k_{\text{conv.}}$ are very close to each other, as the diffusivity in the liquid phase is very small compared to that of the convective terms. The overall mass transfer coefficient, $K$ is influenced by both the membrane diffusivity (dominating parameter) and mass transfer coefficients in the liquid phase. The overall mass transfer coefficients is also found to be increased with increasing the rotational speeds due to the higher concentration gradient between two interfaces of the membrane. The relationship between flow characteristics and mass transfer coefficients (convective and diffusive) is expressed in general by the dimensionless numbers for scaling as (Green and Perry, 2008):

$$Sh = a Re^b Sc^{1/3}$$  \hspace{1cm} (9.10)
where $a$ and $b$ are the fitting parameters which are optimized by the experimental values. In addition, Sherwood number (Sh); Reynolds number (Re) and Schmidt number (Sc) are defined as follows

$$Sh = \frac{\text{convective mass transfer}}{\text{diffusive mass transfer}} \approx \frac{k_{\text{cell}}}{D_{ab}/d_{\text{stirrer}}} \quad (9.11)$$

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} \approx \frac{\rho N d_{\text{stirrer}}^2}{\mu} \quad (9.12)$$

where characteristic length, $d_{\text{stirrer}} = 0.0675$ m, is the average of the stirrer (cone geometry) diameter, $d_{\text{diameter}} = 0.1$ m and average thickness of the cone, $d_{\text{thickness}} = 0.035$ m, $N$ is the rotational speed, $\rho$ and $\mu$ are the density and viscosity of the NaCl aqueous solution.

$$Sc = \frac{\text{viscous diffusion}}{\text{molecular (mass) diffusion}} \approx \frac{\mu/\rho}{D_{ab}} \quad (9.13)$$

Figure 9.5 represents the overall mass transport coefficient as a function of Reynolds number, Re or convection phenomena. It shows that the higher mass transfer coefficient is obtained at higher Re, and seem to reach a constant value in the turbulent region ($Re \geq 10,000$). Mass transport across the membrane is found to
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\[ \frac{1}{K_{\text{overall}}} = 1 \times 10^7 \Re^{-0.053} \]

\[ R^2 = 0.92833 \]

Figure 9.5: The overall mass transport coefficient, \( K_{\text{overall}} \) as a function of Reynolds number, \( \Re \).

be the most slowest (in other word, determining step) compared to the convection. The boundary layer thickness at the interfaces of membrane is lower at higher \( \Re \) than that of lower \( \Re \). Therefore, higher concentration gradient at higher \( \Re \) gives larger overall mass transport. In contrast, the investigated range of convection or \( \Re \) is in a narrow window. In addition, the studied range of \( \Re \) is much higher compared to the associated \( \Re \) in GIT by peristalsis. However, it is definite that the convection or peristalsis could be much more dominating compared to the diffusion for mass transport through the GI-membrane.

The fitting parameters \( a \) and \( b \) in Eq.9.10 are found to be 0.54 and 0.25 respectively by minimizing the error between experimental \( Sh \) and calculated \( Sh \) (using an optimization tool). The Figure 9.6 shows that the Sherwood number, \( Sh \) is a linear function of \( \Re^{1/4} \, Sc^{1/3} \). The observed range of \( \Re \) is from 2270 to 22,730 \((\Re \geq 10,000 \text{ for turbulent flow in stir tank})\) and the Schmidt number, \( Sc \) is about 575 (a constant value which is independent of the rotational speeds). As mentioned earlier that the investigated area of \( \Re \) is relatively narrow and higher compared to the condition in GIT, due to the experimental inconvenience and limited operating conditions in the set up. However, the Figure 9.6 is clearly representing that the convective mass transport coefficient can be estimated while knowing the flow condition (velocity by UVP), fluid properties (density, viscosity, etc.) and the diffusivity of a solute in the medium. Therefore, one can approximate the overall mass transport coefficient (Eq.9.5) of a solute through a membrane under peristalsis or induced flow condition.
9.4 Conclusions

In this study, the approach to completely characterize the mass mass transport phenomena involving a membrane has been demonstrated. The membrane characterization in terms of diffusivity of solute across it (intrinsic property of the membrane) was represented in the batch diffusion experiment. The obtained membrane property can be used to estimate the convective and overall mass transfer coefficients in a flow condition. A relationship between $Sh$ and $Re^{1/4} Sc^{1/3}$ was also shown, where the flow behavior could be monitored by UVP for known physical properties of the medium and solute (e.g. $\rho$, $\mu$, $D_{ab}$). Then one can presume the mass transport coefficient of solute in the medium. Finally, the overall mass transfer coefficient including membrane can be estimated since membrane diffusivity is a distinct property at constant temperature and pressure.

Figure 9.6: Relationship between convective and diffusive mass transfer with the inertial and viscous forces as well as the viscous and molecular diffusion in terms of Sherwood number, $Sh$; Reynolds number, $Re$ and Schmidt number, $Sc$. 

\[ Sh = 0.54 Re^{1/4} Sc^{1/3} \]
\[ R^2 = 0.93 \]
10 Conclusions

The flow behaviors of both Newtonian and non-Newtonian fluids under steady and periodic flow conditions through a collapsible elastic tube were investigated at different compressive transmural pressures in a Starling Resistor setup. The elastic tube was found to collapse from circular to a two lobe-shaped cross-section over certain tube length near downstream at a critical applied external pressure $P_e$, while steady flow was investigated with various fluids. The corresponding measured UVP based velocity profiles were bi-modal, which were seen to be gradually turned into uni-modal for the elliptical shaped cross-section as it approached the upstream end. The similarity tube law represented well the relationship between transmural pressure and tube cross-sectional area. Whereas, the periodic flow behaviors during ramp-up and ramp-down were found to be controlled by the rate of fluid flow (into/out) due to change in tube shapes (due to elasticity) under various external pressures. The shear thinning behavior of both CMC and PAA solutions were clearly observed for the steady flow as the tube cross-sectional area decreased for the more negative transmural pressures. In addition, the viscosity of PAA was observed to be decreased drastically for its high shear thinning behavior than that of the CMC under same value of applied $P_e$. Therefore, the high shear thinning fluids can be of interest for efficient nutrient transport through the small intestinal wall during digestion. Moreover, a good agreement between simulation and corresponding experimental data was obtained during a steady flow of the CMC shear thinning aqueous solution through a collapsed elastic tube. As the shear thinning behavior of CMC aqueous solution was well represented by the Carreau model, it was possible to predict the velocity profiles by the numerical calculations, which was again agreed well with the measured velocity profiles by UVP-technique. The linear increase in the pressure drop with the volume flow rate was also successfully predicted, although the measured pressure drop at higher flow rate was found to be less than the predicted value due to bigger tube cross-sectional area for its elasticity.

Moreover, the study of in vitro small intestinal flow (peristaltic squeezing of an elastic tube) of non-Newtonian fluids involves the fluid dynamics encountered in mixing and propulsion of food in small intestine, which is important to biofluid mechanics encountered in the human body. The velocity profiles and pressure differences were investigated experimentally under various peristaltic motion of a
shear thinning CMC aqueous solution in an elastic tube. The higher wave speed of peristalsis developed in higher magnitude of back flow velocity (negative) both in the wave crest and trough regions, which is expected to be responsible for the improved mixing and convection leading to higher mass transport through the intestinal wall. In addition, the approximated wall shear rates also found to be increased with increasing the peristaltic motion, which correspondingly is expected to enhance the mass transfer by reducing the viscosity of a shear thinning fluid. In addition, the pressure measured in the tube crest was found to be higher than that at the trough, due to higher magnitude of back flow velocity in the wave trough compared to that in the wave crest. It is known that an additional velocity component does occur in the radial direction during laminar oscillating flow in elastic tubes. So, the assumption of single measurement line based velocity profiles by UVP method (considering only the parallel streamlines) lack to give the whole picture of the flow behavior under peristaltic motion. Therefore, a new experimental approach of UVP using multiple measurement line based velocity profiles for parallel to non-parallel flow streamlines has been proposed. The new measuring technique was well applicable to quantify the magnitude of real velocity vector with its direction, and also the velocity components of the real velocity vector in the horizontal and vertical directions. This new proposed method was successfully implemented to accurately estimate the velocity profiles both in the wave crest and trough under peristaltic motion. The detailed study of non-Newtonian peristaltic flow is important to biofluid mechanics encountered in the human body. Moreover, the proposed measurement approach can be also implemented for other Doppler based velocity profiling method.

An additional study was also performed to completely characterize the mass transport phenomena involving a membrane in a batch diffusion experiment. The obtained membrane property was aimed to estimate the convective and overall mass transfer coefficients in a flow condition, where the flow behavior could be monitored by UVP for known physical properties of the medium and solute. The detailed knowledge gained about non-Newtonian peristaltic flow is aim to couple with the mass transport phenomena for the detail investigations of mass transport across the elastic membrane tube wall in the further research work.


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