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Abstract

We embed a banking model, depicting the duality of private money creation and credit extension, into a two-sector neoclassical model with financial frictions. Banks rely on central-bank reserve loans that are collateralized according to the central bank's collateral framework. We derive optimal static and dynamic haircut rules, which balance the efficient allocation of capital across sectors and bank-default costs. We offer a simple formula for haircuts that relies on four fundamental factors: liquidity demand, output elasticity of capital, production capacities in the bond-financed and loan-financed sectors, and capital-ownership shares. We calibrate the model to the US and find ranges for haircuts between 5% to 20% when we consider numerical scenarios for capital-ownership shares and productivity risk. Varying haircuts have also distributional effects: bondholders and workers may suffer from tight collateral requirements (large haircuts), while bankers benefit despite reduced leverage.

Keywords: Central bank, haircut rule, monetary policy, bank money creation, monetary system

JEL Classification: E42, E50, E51, E52, E58, G21

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1 Introduction

Central banks provide collateralized credit to banks, applying criteria for eligible collateral and so-called “haircuts” on this collateral. A haircut is the difference between the market value of an asset and what can be borrowed against it, that is, the collateral capacity. Throughout the history of central banks, such collateral frameworks have been part of central bank operations, as they belong to the organization and execution of liquidity provision to banks. They have also played a central role in the US’ and Europe’s financial history (Bordo, 2008; Kindleberger, 2015). Collateral frameworks were less in the focus during the Great Moderation, in which monetary policy was primarily defined in terms of a short-term interest rate, and where the control of the interest rates required comparatively small-scale operations. The situation, however, has changed considerably in the aftermath of the financial crisis of 2007/2008. Collateral frameworks played a pivotal role in stabilizing financial markets and institutions during the financial crisis and during the subsequent debt crisis in Europe (Bindseil et al., 2017). Moreover, after the crisis, central banks have employed a broader set of instruments to conduct monetary policy—large-scale unsterilized or sterilized asset purchases and policies aiming at credit markets directly, for example.

Collateral frameworks are typically broad as to the scope of eligibility criteria and they are quite complex. The Eurosystem collateral framework, in particular, has triggered a great deal of controversy with regard to its consequences on financial risk, capital misallocation, and moral hazard at banks (Nyborg, 2017; Sinn, 2014; Bindseil et al., 2017). Since we want to analyze the impact of collateral-framework design on the private sector, we are interested in the haircuts applied to individually-deposited loans, which are an important class of collateral assets. In Table 1, we present the haircut ranges of selected loan classes that the FED currently applies in the US.[1] Within a haircut range, the haircuts typically increase with the credit spread and the time remaining to maturity.

[1]The shown haircuts are effective since March 14, 2022. The FED provides them on its website `https://www.frbdiscountwindow.org/pages/collateral/collateral_valuation` (accessed on January 9, 2023. For some asset classes, the applied haircuts can exceed the upper boundary of the respective range.

We develop an analytical framework to examine how haircuts on eligible collateral should be optimally set, and how fundamental factors affect such haircuts. The basis of our model is a monetary version of a two-sector stochastic neoclassical model in which some firms, which are endowed with a risky production technology and are subject to moral hazard, can obtain financing only from banks, while other firms have frictionless access to the bond market. We adopt a money-creation approach to banking which depicts the dual role of banks as loan providers and money creators. This approach was studied, for example, by Jakab and Kumhof (2015) and Faure and Gersbach (2021).

Individually deposited loans	Fixed rate loans	Floating rate loans
Agricultural loans		
Minimal risk rated	5% - 10%	5% -19%
Normal risk rated	5% - 27%	5% -38%
Commercial and industrial loans & leases		
Minimal risk rated	5% - 15%	5% -20%
Normal risk rated	5% - 41%	5% -48%
Commercial real estate loans		
Minimal risk rated	5% - 44%	5% -51%
Normal risk rated	5% - 70%	5% - 74%
Construction loans		
Minimal risk rated	5% - 75%	5% -81%
Normal risk rated	7% - 82%	7% -84%

Table 1: FED haircut ranges for selected loan classes.

The monetary architecture is a hierarchical system in which commercial banks provide loans and thus create bank money (bank deposits). All trades by private agents are settled with bank deposits, which are by far the most important part of the monetary aggregates.[2] When the depositors of a bank move money to other banks in the payment process, interbank liabilities arise, which, in the monetary architecture, are settled with central bank money (reserves). The bank can acquire these reserves by taking out fully collateralized loans at the central bank or at other commercial banks. The central bank affects the collateral requirements in the interbank market through its own collateral

[2]For simplicity, we abstract from banknotes as means of payment.

framework and thereby governs the liquidity provision to banks.

We distinguish two instruments of monetary policy. First, the central bank sets interest rates on reserve loans and reserve deposits. Second, the central bank designs the collateral requirements banks have to meet to draw liquidity in the form of central-bank reserves. The central bank defines which assets are eligible to be collateralized, and sets haircuts on the collateral value of these assets. On purpose, we abstract from price rigidities, so that the central bank's interest-rate policy is neutral and we can isolate the allocative impact of haircuts. By tightening the collateral requirements and thus providing less liquidity to commercial banks, the central bank makes commercial banks decrease their leverage ratio. As a consequence, the loan-financed sector obtains fewer funds, whereas more funds are channeled to the bond-financed sector. The following trade-off is at the core of the central bank's haircut policy: loose capital requirements, on the one hand, allow high lending activity and an efficient allocation of funds across sectors. On the other hand, extensive loan issuance is accompanied by bank-solvency risk and the associated costs that banks impose on society through default. The central bank balances these two forces.

The *Taylor Rule* is the best-known analytical rule for optimal monetary policy in terms of interest-rate monitoring. It ties the policy interest rate to the divergence of actual inflation from target inflation and to the output gap. In our model, interest-rate policy is neutral, but the design of the collateral framework is not; haircuts affect bank leverage and thus drive the capital allocation between bond-financed and loan-financed firms. We develop a static haircut rule that explains how the central bank optimally sets the haircuts. In a stylized and simplified form, this haircut rule at time t reads

$$\psi_t = 1 - \gamma_t\big(1 - \alpha(1 + \tilde{a}_t^{\lambda})\rho_t\big). \tag{1}$$

It relates the optimal haircut ψ_t with four macroeconomic fundamentals: the commercial banks' liquidity demand (captured by γ_t), the output elasticity of capital α, the relation between the production capacities in the loan-financed and the bond-financed sector (captured by $\tilde{a}_t^{\lambda}$), where $\tilde{a}_t^{\lambda}$ also accounts for the default costs associated with loan-financing, and eventually the relation ρ_t of capital ownership between banks and households. We also establish a dynamic haircut rule that describes the interperiod transition of haircuts, taking into account the capital accumulation of households, who buy bonds and hold deposits, and bank owners.

In a quantitative analysis, we calibrate the model parameters to the US economy. We show that the calibrated model permits two steady states under optimal collateral policy. In the first, the central bank applies a loose collateral policy that allows banks to take on solvency risk, whereas in the second steady state, the central bank implements a greater haircut that rules out bank-solvency risk. We find that the economy attains a higher output level when the central bank permits bank default: the associated efficiency gains outweigh the arising bank-default costs.

We analyze how, away from the steady states, the optimal haircut changes when the bank owners' share of aggregate capital in the economy increases. We find that the higher the share of bank owners' capital, the higher the optimal haircut. For shares from 5% to 25%, the associated haircuts increase from 5% to 20%. In particular, we find an upward jump in the optimal haircut, considered as a function in the bank owners' capital share. There is a threshold of the bank owners' capital share, at which, by applying a great haircut that rules out solvency risk, the central bank attains the same output level as by applying a small haircut. When switching from the tight regime to the loose regime, the efficiency gains exactly offset the additional default costs. The central bank is thus indifferent between a tight collateral haircut policy and a loose policy.

For given capital endowments of households and bank owners, we find that households and workers in the loan-financed sector benefit from increasing (not necessarily optimal) haircut levels in terms of consumption, while bank owners and workers in the bond-financed sector suffer. A loose collateral policy allows banks to grant more loans to firms who are prone to moral hazard, so that these firms rent more capital. Since short-term labor supply is modeled to be immobile across sectors, capital becomes relatively abundant in the loan-financed sector and relatively scarce in the bond-financed sector. This affects the marginal factor products and the resulting factor prices in both sectors.

The greater marginal productivity of capital in the bond-financed sector translates into a higher interest rate on bonds, from which households benefit, in particular, and it drives down the wages in this sector. The lower marginal productivity of capital in the loan-financed sector pushes down interest rates on loans. Moreover, since bonds and deposits are perfect substitutes as stores of value and thus pay the same interest rate, the higher bond interest rate transmits to a higher deposit interest rate. The lower returns on loan issuance and the larger deposit-funding costs drive down the return on bank equity.

The increase of bank lending, however, has two opposing effects on the workers' consumption in the loan-financed sector. On the one hand, the abundance of capital in this sector drives up the wages. On the other hand, workers have to pay taxes to compensate for the government's expenses for bank resolution and deposit insurance, which increase with the banks' exposure to default. Since a firm's loan repayment depends on the idiosyncratic productivity shock it experiences, and since banks are unable to fully diversify their loan portfolios, banks are exposed to risk as well. Hence, more loan issuance translates into larger expected bank-resolution costs, which are financed through taxation. Remarkably, the benefits from increasing wages outweigh the additional tax costs incurred by the workers.

Finally, we calibrate our model to a stylized economy that is identical to the US economy, apart from the loan-to-bond-capital ratio, which we take from the Euro area. We conduct the same analysis as for the US economy and find qualitatively similar results. The only striking difference is a lower sensitivity of bank leverage to changes in the haircut level. Since the share of loan-financed firms is remarkably larger in the Euro area than in the US, already a small change of the bank leverage in response to a change of the haircut has a significant impact on capital accumulation in the two sectors. By applying the same haircut change, the central bank thus exerts a stronger impact on real outcomes in the mainly loan-financed economy than in the mainly bond-financed economy.

The remainder of this paper is organized as follows. In Section 2, we relate our work to the existing literature. We develop the model in Section 3 and the equilibrium is characterized in Section 4. We present optimal static haircut rules in Section 5 and discuss dynamic haircut rules in Section 6. In Section 7, we calibrate our model, study the response of optimal haircuts to changes in the environment, and identify the beneficiaries from loose collateral requirements. We conclude in Section 8. For convenience, all proofs and long derivations are relegated to the appendix.

2 Literature

We present an optimal monetary policy rule for the central bank's haircut, which it applies to collateral assets pledged by banks. This haircut rule complements the already existing interest-rate rules by Taylor (1993) and Woodford (2001), for instance. Proceeding from the model of Kiyotaki and Moore (1997), who explain how collateralized loan supply generates credit cycles, a growing literature on central banks' collateral frameworks has developed. Recently, Böser and Gersbach (2021) examined how the collateral framework impacts the banks' monitoring decisions because banks increase the collateral value of their outstanding loans if they monitor their customers more closely. Bindseil (2004) and Bindseil et al. (2017) provided qualitative work on the collateral framework of the Eurozone and explained its development in the last two decades. Nyborg (2017) also examined the European Central Bank's (ECB) collateral monetary policy and developed normative guidelines for the ECB's haircut schedule.

The need for banks and loan-financing is due to financial frictions, as micro-founded by Holmström and Tirole (1997) and Diamond (1984), for instance. Banks have a dual role in our model, as they issue loans and create money in the form of bank deposits. The practice of money and loan creation by commercial banks has a long history and was subject of many analyses and debates (e.g., Macleod, 1866; Wicksell, 1907; Hahn, 1920; Keynes, 1931; Schumpeter, 1954; Gurley and Shaw, 1960; Tobin, 1963; McLeay et al., 2014; Donaldson et al., 2018). The money banks create is a claim on fiat money created by the central bank. Different modeling approaches are applied to capture this setting (Skeie, 2008; Jakab and Kumhof, 2019; Wang, 2019; Bolton et al., 2020; Faure and Gersbach, 2021; Piazzesi et al., 2022; Wang, 2021; Li and Li, 2021; Parlour et al., 2022).[3]

We elaborate a macroeconomic model of banking based on the two-sector neoclassical model with financial frictions developed by Gersbach et al. (2022). While they abstract from a monetary layer in the economy, we introduce book money (deposits) as means of payment, like Faure and Gersbach (2021), and establish that the haircut rule does affect the real economy, while the interest rate applied by the central bank does not.

3 Model

3.1 Macroeconomic Environment

To analyze the impact of optimal haircut rules on aggregate production, we embed a model of equity- and deposit-financed banking into a two-sector neoclassical growth model. Time $t \in \{0, 1, 2, \dots\}$ is discrete and infinite. The model features six agents: households, investors, workers, (bond-financed and loan-financed) firms, the central bank, and commercial banks (henceforth simply "banks"). There are two goods: a capital good and an output good. Households and investors are endowed with the capital good. Firms use the capital good and employ workers to produce the output good. Transactions in the capital-good and output-good market are settled instantaneously with bank deposits, which are insured through governmental guarantees.[4] Households and workers either hold their proceeds from the capital-good market as deposits or invest them in corporate bonds. Investors use their proceeds to buy bank equity. While workers are hand-to-mouth consumers and spend their entire labor income on consumption, households and investors can accumulate capital over time.

Firms are penniless and production takes place under technologies generating constant returns to scale. We distinguish two sectors, based on the firms' types of financing: in Sector B (*bond financed*), firms have access to the bond market and finance their expenses

[3] A parallel literature has examined the properties of monetary systems when banks issue banknotes instead of deposits (e.g., Gersbach, 1998; de O. Cavalcanti and Wallace, 1999).

[4] Deposits are the main medium of exchange in our economy. With deposit insurance, they are generally superior to cash as medium of exchange and store of value. We abstract from cash in our analysis.

for production inputs by issuing bonds. Firms in Sector L (*loan financed*) are plagued by moral hazard and thus do not have access to the bond market.[5] Hence, they rely on loans granted by banks, which use a costless monitoring technology to alleviate this moral hazard. Banks finance the issued loans with equity and deposits. We impose a one-to-one matching of banks and firms in Sector L to fully subject the banks to the idiosyncratic shocks in this sector.

On purpose, we abstract from price rigidities to isolate the impact of haircut rules. As a consequence, monetary policy only affects the real economy via the haircut applied to collateralized loans, whereas interest-rate policies for central-bank reserves do not have any impact on the real economy. For interest-rate policies, the classical dichotomy between the real side and the monetary side of the economy holds.

At the start of each period t, a macroeconomic shock z_t realizes, which follows a first-order Markov chain with transition probabilities $P(z_{t+1} = z'|z_t = z) \equiv \sigma_{z'|z} \in (0, 1)$ for $z', z \in \mathcal{Z}$, where $\mathcal{Z}$ is the countable set of possible macroeconomic shocks. After the macroeconomic shock has realized, firms in Sector L face an additional individual shock $s_t \in \{\underline{s}, \overline{s}\}$ in period t, distributed according to $\mathbb{P}(s_t = \overline{s}|z_t = z) \equiv \eta_z \in (0, 1)$. The expression $\mathbb{E}_t[X_t]$ denotes the expectation of a random variable X_t conditional on the information set available at the start of period t, including the macroeconomic shock z_t but not including any idiosyncratic shocks s_t.

3.2 Money Creation

Our model embeds the current monetary architecture into a neoclassical growth model. Specifically, there are two forms of money: public money and private money. The central bank creates public money when it grants loans to banks which, in return, pledge assets as collateral. This publicly-created money is a claim on fiat money that banks hold against the central bank. The publicly-created money is called "central-bank reserves" or simply "reserves". Only banks have access to this kind of money. It is destroyed when banks pay back their loans to the central bank, and when the pledged assets are redeemed.

[5]There are several ways to micro-found moral hazard (Diamond, 1984; Holmström and Tirole, 1997).

Banks create private money by granting loans to firms. This money is a claim on fiat money against the respective bank. We call it "deposits". Hence, banks exert a dual role by granting loans and creating private money. Deposits are destroyed when investors buy bank equity, or when firms repay loans.

We now outline how public and private money are linked in our economy. When deposits are used for a monetary transaction between two private agents in the economy, the deposits flow from the buyer's bank to the seller's bank. According to the institutional framework, a liability of the buyer's bank to the seller's bank arises in the background in terms of central-bank reserves in the same nominal amount as the deposit transfer. Thus, the exchange rate between public and private money is set at 1. To receive the reserves needed to settle the transaction with the seller's bank, the buyer's bank can either try to borrow reserves on the interbank market or it can approach the central bank directly. In either case, it must fully collateralize the new reserve loan.

3.3 Timeline of One Period

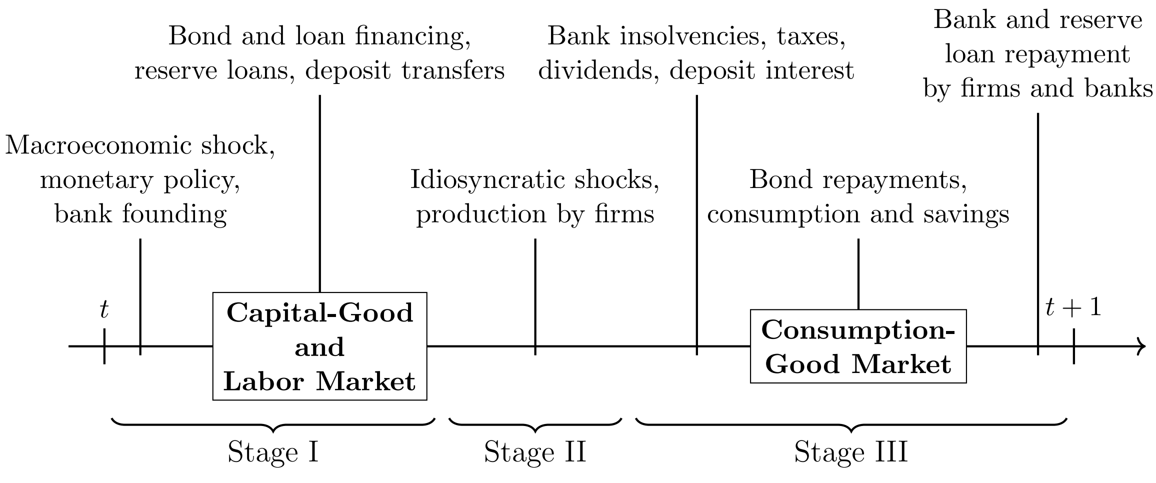

Figure 1: Timeline of a typical period t.

Since transactions are settled instantaneously, the timing of events is of great importance for our analysis. Figure 1 gives an overview of the sequence of events in a typical period t. We subdivide these events in three stages. Stage I comprises monetary policy and the loan, bond, and factor markets. At Stage II, firms in Sector L incur their idiosyncratic shocks and production takes place. Finally, at Stage III, banks may default, the output good is traded, and bonds and loans are repaid. We next describe each stage in detail and introduce the notation explaining the interactions of agents in the economy.

Stage I. At the start of period t, the macroeconomic shock z_t realizes. Then the central bank sets the interest rate for central-bank reserves and deposits, and designs the collateral framework in its lending facility by setting a haircut for banks' collateral. Moreover, the loan, bond, and factor markets clear. We depict the loan, bond, and factor markets in Figure 2, and we distinguish four substages within Stage I.

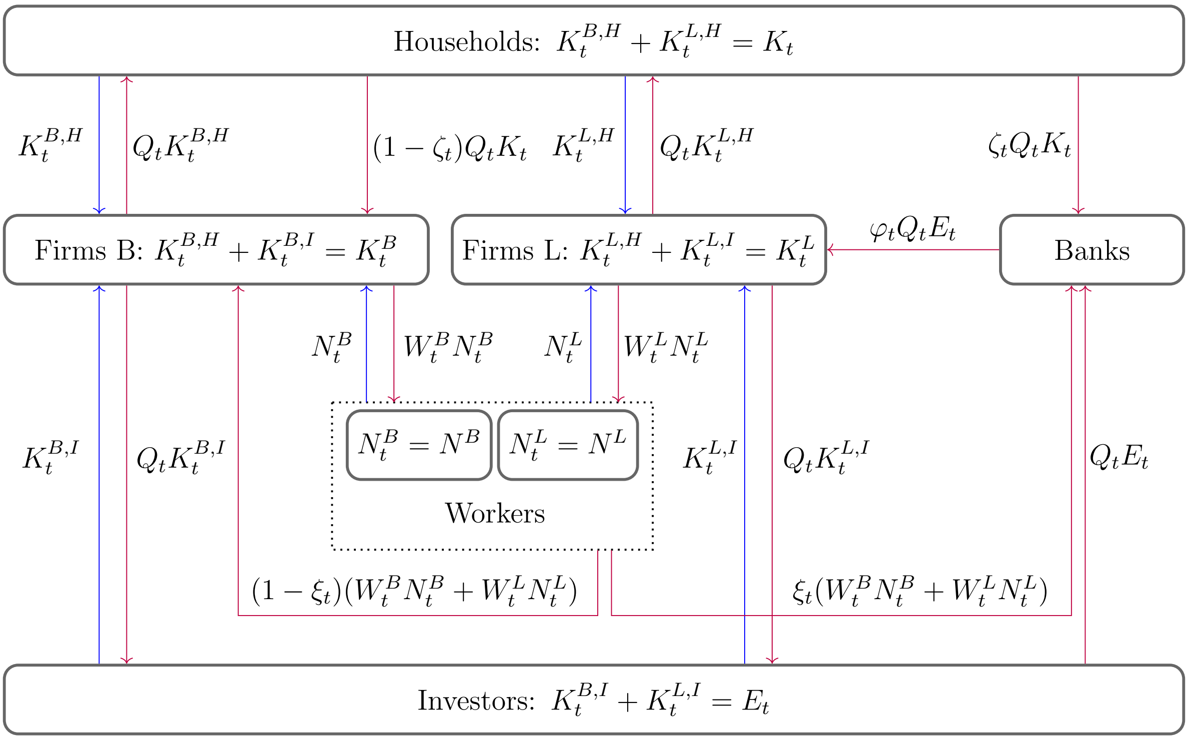

Figure 2: Transactions on the loan, bond, and factor markets at Stage I.

Stage I.A. Banks are founded and investors commit towards equity-financing banks. Investors promise to buy equity contracts at Stage I.D with deposits $Q_t E_t$ that they are going to receive as proceeds in the capital-good market at Stages I.B and I.C. Thereby, Q_t is the rental rate of capital and E_t investors' capital.

Stage I.B. Banks grant loans L_t to firms in Sector L and thereby create deposits on the liability side of their balance sheet. Since a part of deposits is going to flow out of each bank, due to transactions on the bond and factor markets, each bank takes out a loan at the central bank's lending facility to cover the reserve transactions in the background. The central bank accepts the banks' claims on loan repayment as collateral. Firms in Sector L use all their deposits to rent capital good K_t^L from investors ($K_t^{L,I}$) and households ($K_t^{L,H}$) at rental rate Q_t, and employ workers N_t^L at wage W_t^L.

Stage I.C. Households and workers keep shares ζ_t and ξ_t, respectively, of their proceeds from the capital-good market in the bank account and spend the remaining deposits on corporate bonds. These bonds are designed to mature after one period and represent repayment agreements in terms of deposits. Similarly to firms in Sector L, firms in Sector B spend their deposits acquired in the bond market on renting capital good K_t^B from investors ($K_t^{B,I}$) and households ($K_t^{B,H}$) at rental rate Q_t, and employing workers N_t^B at wage W_t^B.

In potentially multiple steps, households lend capital good and workers provide labor to firms in Sector B until the factor markets clear. Multiple steps may be needed, since households receive deposits from lending capital, which may be insufficient to buy the total amount of bonds they desire at once. In such cases, the current amount of deposits is used to buy bonds, providing the firms with further deposits to rent more capital. These additional deposits then are used by households to buy a further amount of bonds. We refer to this process in the future as "market clearing by deposit circulation".

Stage I.D. Investors stick to their promises at Stage I.A and buy the pledged equity contracts from banks with deposits $Q_t E_t$. This means that the respective deposits are destroyed. The total volume of loans $L_t = \varphi_t Q_t E_t$, granted to firms in Sector L, thus can be expressed as the product of loan-to-equity ratio $\varphi_t = L_t / Q_t E_t$ and bank equity $Q_t E_t$.

Stage II. In Sector L, the idiosyncratic shocks s_t realize and firms in both sectors engage in producing the output good.

Stage III. The output good is sold and interest rates on deposits and dividends on bank equity are paid; repayment obligations from bonds and loans are met. Figure 3 illustrates output sales and interest payments. We distinguish three substages.

Stage III.A. Banks have limited liability, so that the return on bank equity $\tilde{r}_t^E$ cannot be negative. A bank can face two scenarios: either it defaults or it does not. Therefore, within Stage III.A, we distinguish Case I (bank default) from Case II (no bank default).

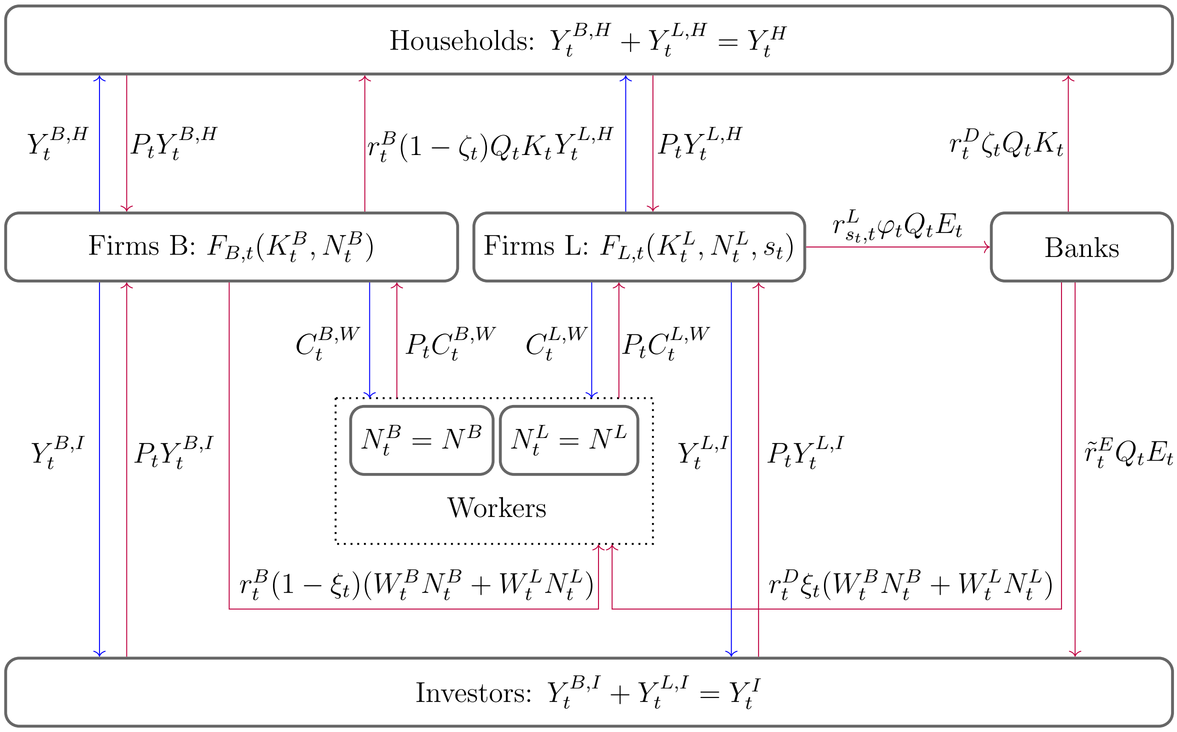

Figure 3: Transactions in the output market at Stage III.

Case I. When a bank is exposed to solvency risk, and the matched firm is hit by a negative idiosyncratic shock $\underline{s}$, this bank cannot meet its financial obligations to depositors (households and workers) and defaults. Then, the gross rate of return on bank equity $r_{\underline{s},t}^E$ is zero. Since deposits are insured, the government compensates the depositors for whatever the bank is unable to pay back. Thus, households and workers receive $r_t^D\zeta_tQ_tK_t$ and $r_t^D\xi_t(W_t^BN_t^B + W_t^LN_t^L)$ in total from the bank and the government. Moreover, the government makes up for the bank-resolution costs $\lambda F_{L,t}(K_t^L, N_t^L, \underline{s})$, $\lambda \in (0,1)$, that we model to scale with the output in Sector L. The government finances its expenses by taxes on workers' labor income.[6]

Case II. If the bank has not taken on any solvency risk, or the matched firm is hit by a positive idiosyncratic shock $\overline{s}$, banks pay the gross interest rate r_t^D on deposits and declare gross dividends $r_{s_t,t}^E$ on equity. Investors reconvert their equity holdings into deposits to participate in the output-good market.

Stage III.B. Firms in Sector L (B) sell $C_t^{L,W}$ ($C_t^{B,W}$) units of the output good to workers, $Y_t^{L,I}$ ($Y_t^{B,I}$) units to investors, and $Y_t^{L,H}$ ($Y_t^{B,H}$) units to households at price P_t,

[6]Workers are the only agents who pay taxes in our economy. This increases the tractability of the model, since workers, unlike households and investors, have a static utility maximization problem.

so that $C_t^W = C_t^{L,W} + C_t^{B,W}$, $Y_t^I = Y_t^{L,I} + Y_t^{B,I}$, and $Y_t^H = Y_t^{L,H} + Y_t^{B,H}$. Bonds mature and the output market clears by deposit circulation: while households and investors spend all their proceeds from the capital-good market on the output-good market, workers do not have their full labor income at command for output-good purchases, since they have to pay taxes. Households and investors either consume or invest the purchased units of output good. Workers, not having any savings technology, consume all purchased units of the output good.

Stage III.C. Firms in Sector L repay their bank loans, which have been issued at Stage I.B, at the state-contingent gross interest rate $r_{s_t,t}^L$. Thereby, all deposits in the economy are destroyed. Moreover, outstanding loans on the interbank market are settled, accounts at the central bank are cleared, and commercial banks are resolved at zero cost. Hence, at the end of period t, privately-created money (deposits) and publicly-created money (reserves) are destroyed.

3.4 Households

There is a unit mass of identical households. Each household is initially endowed with capital good $k_0 > 0$, so that the aggregate amount K_0 is identical with the individual endowment k_0. Households are perfectly competitive and take capital-good rental rates and interest rates on bank deposits and corporate bonds as given. We can thus focus on a representative household. Since households do not have any labor income, disposable income in period t is linear homogeneous with respect to the capital-good stock K_t.

The representative household chooses a sequence of consumption $\{C_t^H\}_{t=0}^{\infty}$, bank deposits $\{D_t\}_{t=0}^{\infty}$, corporate bonds $\{B_t\}_{t=0}^{\infty}$, and savings $\{I_t^H\}_{t=0}^{\infty}$ to maximize its lifetime utility, subject to a sequential budget constraint. At the start of each period t, after the macroeconomic shock z_t has realized, the household lends its capital-good stock K_t to firms at rental rate Q_t. It holds a share $\zeta Q_t K_t$, $\zeta_t \in [0, 1]$, of revenues $Q_t K_t$ as bank deposits and invests the remaining share $(1 - \zeta_t) Q_t K_t$ into bonds issued by firms.[7] At

[7]We could also assume that firms with access to the frictionless capital market are equity-financed instead of bond-financed. Due to the capital structure irrelevance principle of Modigliani and Miller (1958), this alternative approach to firm-financing would not affect our results.

the end of each period, the representative household is given the lent capital back, which depreciates at rate $\delta \in (0,1)$. The household buys Y_t^H units of the output good with the revenues from the capital-good market and the accrued interest payments. Thereof, it saves I_t^H units and consumes C_t^H units. Since there is no aggregate risk in the economy, the price P_t of the output good is deterministic and can be normalized as $P_t = 1$.

Given the initial endowment $K_0 > 0$ of capital good and the initial macroeconomic shock $z_0 \in \mathcal{Z}$, the household's utility maximization problem is given by

$$\max_{\{\zeta_t, C_t^H, I_t^H\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_H^t \ln(C_t^H) \right],$$

$$\text{subject to} \quad C_t^H + I_t^H \leq Y_t^H,$$

$$Y_t^H \leq [r_t^D \zeta_t + r_t^B (1 - \zeta_t)] Q_t K_t,$$

$$K_{t+1} = (1 - \delta) K_t + I_t^H,$$

where $\beta_H \in (0,1)$ denotes the households' time discount factor. As the utility function is strictly increasing in consumption, all constraints must be binding at the optimum. The optimization problem simplifies to

$$\max_{\{\zeta_t, K_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_H^t \ln(C_t^H) \right], \tag{2}$$

$$\text{subject to} \quad C_t^H = [r_t^D \zeta_t + r_t^B (1 - \zeta_t)] Q_t K_t + (1 - \delta) K_t - K_{t+1}. \tag{3}$$

When bank deposits and corporate bonds pay the same interest rate, we derive a linear capital accumulation rule, stated in Lemma 1.

Lemma 1 (Households' Capital Accumulation) *Let $r_t^D = r_t^B$. Then the households optimally accumulate capital according to the rule*

$$K_{t+1} = \beta_H (1 + r_t^D Q_t - \delta) K_t, \quad t \in \{0, 1, \dots\},$$

and consume $C_t^H = (1-\beta_H)(1 + r_t^D Q_t - \delta)K_t$.

Proof. See Appendix D.1. ■

3.5 Investors

There is a unit mass of identical investors. Each investor is initially endowed with capital good $e_0 > 0$, so that the aggregate amount E_0 is identical with the individual endowment e_0. Investors are perfectly competitive and take capital-good rental rates and returns on equity as given. We can thus focus on a representative investor.

S/he chooses a sequence of consumption $\{C_t^I\}_{t=0}^{\infty}$ and savings $\{I_t^I\}_{t=0}^{\infty}$ to maximize his/her lifetime utility subject to a sequential budget constraint. At the start of each period t, after the macroeconomic shock z_t has realized, s/he lends his/her capital endowment E_t at rental rate Q_t to firms. S/he uses the proceeds $Q_t E_t$ from the capital-good market to equity-finance banks. The equity-return factor $r_{s_t,t}^E$ of an individual bank depends on the idiosyncratic shock s_t of the firm that the specific bank finances. This will be detailed in Section 3.9. Each investor fully diversifies the risk which banks are exposed to. The representative investor thus earns the deterministic gross return $\tilde{r}_t^E Q_t E_t$ with $\tilde{r}_t^E = \mathbb{E}_t[r_{s_t,t}^E]$. At the end of each period, the representative investor receives the lent capital, which depreciates at rate $\delta \in (0,1)$, and buys Y_t^I units of the output good with the compounded revenues from the capital-good market. Thereof, s/he saves I_t^I units and consumes C_t^I units.

Given the initial endowment of capital good $E_0 > 0$ and the initial macroeconomic shock $z_0 \in \mathcal{Z}$, the representative investor's optimization problem is given by

$$\max_{\{C_t^I, I_t^I\}_{t=0}^{\infty}} \; \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_I^t \ln(C_t^I) \right],$$

$$\text{subject to} \quad C_t^I + I_t^I \leq Y_t^I,$$

$$Y_t^I \leq \tilde{r}_t^E Q_t E_t,$$

$$E_{t+1} = (1-\delta)E_t + I_t^I,$$

where $\beta_I \in (0,1)$ denotes the investors' time discount factor. As the utility function is strictly increasing in consumption, all constraints must be binding at the optimum. The optimization problem simplifies to

$$\max_{\{E_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_I^t \ln(C_t^I) \right], \tag{4}$$

$$\text{subject to} \quad C_t^I = \tilde{r}_t^E Q_t E_t + (1-\delta)E_t - E_{t+1}. \tag{5}$$

Similarly to households, investors accumulate capital following a linear accumulation rule, given in Lemma 2.

Lemma 2 (Investors' Capital Accumulation) *Investors optimally accumulate capital according to the rule*

$$E_{t+1} = \beta_I(1 + \tilde{r}_t^E Q_t - \delta)E_t, \quad t \in \{0, 1, \dots\},$$

and consume $C_t^I = (1-\beta_I)(1 + \tilde{r}_t^E Q_t - \delta)E_t$.

Proof. See Appendix D.2. ■

3.6 Workers

There is a mass $N_t > 0$ of workers. Each worker is endowed with one unit of labor that s/he inelastically supplies either to firms in Sector B at nominal wage W_t^B or to firms in Sector L at nominal wage W_t^L. Workers are immobile across sectors and we assume that N_t^B workers are employed in Sector B and N_t^L workers in Sector L, fulfilling $N_t^B + N_t^L = N_t$. The empirically observed polarization of the labor market, spatial frictions, and the intransferability of skills provide a rationale to this assumption of sector-specific immobile labor supply.[8] Since the capital good is elastically supplied in contrast to labor, the accumulation of funds in one sector can entail persistent wage inequalities between sectors.

[8] Autor and Dorn (2013) document job polarization and the associated wage inequality across sectors in the US between 1980 and 2005; Dustmann et al. (2009) did the same for Germany. For a comprehensive overview of the intransferability of skills, see Acemoglu and Autor (2011).

Workers are hand-to-mouth consumers. They do not have any capital accumulation technology and thus do not save. This assumption is made for tractability, but it also has an empirical counterpart, as this proportion of agents can be quite large. However, within a period, workers hold a share $\xi_t \in [0, 1]$ of their wage payments as deposits and invest the remaining share in bonds. Workers' consumption preferences are represented by the period utility function $u(C_t^W)$ with u, $u' > 0$, where C_t^W denotes their consumption. Because of perfect competition in the labor market, we focus on a representative worker who faces the period utility maximization problem

$$\max_{\xi_t \in [0,1]} u(C_t^W), \tag{6}$$

$$\text{subject to} \quad C_t^W = [r_t^D \xi_t + r_t^B (1 - \xi_t)](W_t^L N_t^L + W_t^B N_t^B) - \tau_t, \tag{7}$$

with τ_t denoting real lump-sum taxes.

3.7 Firms

Firms are profit-maximizing, one-period-lived, and protected by limited liability. They differ with respect to their access to financial markets: while firms in Sector B have access to a frictionless financial market, firms in Sector L are exposed to moral hazard with the consequence that the revenues they can credibly promise to pay back to external financiers are low. In particular, the pledgeable revenues are lower than the repayment obligations prevailing in the bond market. This limited pledgeability of revenues can be due to a lack of transparency and reputation towards potential creditors or might be micro-founded differently.[9] However, banks can alleviate moral hazard by monitoring the firms in Sector L at zero costs, so that these firms can rely on bank loans. Banks enforce contractual obligations of their creditors. In each sector, there is a unit mass of firms. Since firms are perfectly competitive, we consider a representative firm in each sector.

[9]See the seminal contributions of Holmström and Tirole (1997) and Kiyotaki and Moore (1997).

3.7.1 Production Technologies

In both sectors, firms operate under constant returns to scale, using a capital good and labor as the production factors. The production technologies have positive and decreasing marginal products in both factor inputs so that the Inada Conditions hold. We assume that the production functions is of the Cobb-Douglas type

$$F_{B,t}(N_t^B, K_t^B) = \tilde{A}_t^B (N_t^B)^{1-\alpha}(K_t^B)^{\alpha} \qquad \text{and} \qquad F_{L,t}(N_t^L, K_t^L, s_t) = \tilde{A}_{s_t,t}^L (N_t^L)^{1-\alpha}(K_t^L)^{\alpha},$$

where, in sector $i \in \{\text{B, L}\}$, $\tilde{A}_t^i$ indexes the sector-specific total factor productivity and K_t^i and N_t^i denote capital-good and labor input. In Sector B, the total factor productivity $\tilde{A}_t^B = (1+g_B)^t A_{z_t}^B$ is subject to technological sector-specific growth, $g_B > 0$, and the macroeconomic shock z_t. In Sector L, total factor productivity $\tilde{A}_{s_t,t}^L = (1+g_L)^t A_{z_t,s_t}^L$ is subject to technological sector-specific growth, $g_L > 0$, the macroeconomic shock z_t, and an idiosyncratic shock $s_t \in \{\underline{s}, \overline{s}\}$ which realizes after the factor allocation has taken place and before the firms produce the output good.

3.7.2 Profit Maximization Problems

The representative firm in sector $i \in \{\text{B, L}\}$ faces expenses $Q_t K_t^i + W_t^i N_t^i$ to purchase the input factors K_t^i and N_t^i. Since firm B has access to a frictionless financial market, it issues $Q_t K_t^B + W_t^B N_t^B$ bonds that pay gross interest rate r_t^B. Firm L relies on bank loans and requests a loan $Q_t K_t^L + W_t^L N_t^L$ at gross interest rate $r_{s_t,t}^L$ which depends on the idiosyncratic shock s_t.

Given factor prices W_t^B and Q_t, the optimization problem of firm B reads

$$\max_{N_t^B, K_t^B \geq 0} \; F_{B,t}(N_t^B, K_t^B) - r_t^B (Q_t K_t^B + W_t^B N_t^B). \tag{8}$$

Taking first-order conditions with respect to N_t^B and K_t^B, firm B's factor demand is determined by

$$(1-\alpha)\tilde{A}_t^B (N_t^B)^{-\alpha}(K_t^B)^{\alpha} = r_t^B W_t^B \qquad \text{and} \qquad \alpha \tilde{A}_t^B (N_t^B)^{1-\alpha}(K_t^B)^{\alpha-1} = r_t^B Q_t. \tag{9}$$

Firm L determines its factor demand, depending on the expectation of the repayment factor $r^L_{s_t,t}$, and thus of the idiosyncratic shock s_t. As discussed in detail in Section 3.9, banks enforce financial contracts state-contingently. Hence, the loan interest rates $r^L_{s_t,t}$ depend on the idiosyncratic shock $s_t \in \{\underline{s}, \overline{s}\}$. Firm L solves the optimization problem

$$\max_{N^L_t, K^L_t \geq 0} \; \mathbb{E}_t[F_{L,t}(N^L_t, K^L_t, s_t) - r^L_{s_t,t}(Q_t K^L_t + W^L_t N^L_t)]. \tag{10}$$

Taking first-order conditions with respect to N^L_t and K^L_t, firm L's factor demand is given by

$$(1-\alpha)\mathbb{E}_t[\tilde{A}^L_{s_t,t}](N^L_t)^{-\alpha}(K^L_t)^{\alpha} = \mathbb{E}_t[r^L_{s_t,t}]W^L_t \;\; \text{and} \;\; \alpha\mathbb{E}_t[\tilde{A}^L_{s_t,t}](N^L_t)^{1-\alpha}(K^L_t)^{\alpha-1} = \mathbb{E}_t[r^L_{s_t,t}]Q_t. \tag{11}$$

Due to perfect competition among firms, the financial contract induces zero profit for the representative firm L in every state, so that

$$r^L_{s_t,t}(W^L_t N^L_t + Q_t K^L_t) = \tilde{A}^L_{s_t,t}(N^L_t)^{1-\alpha}(K^L_t)^{\alpha} \equiv F_{L,t}(N^L_t, K^L_t, s_t), \quad s_t \in \{\underline{s}, \overline{s}\}.$$

Multiplying firm L's first-order condition with respect to labor with N^L_t, multiplying firm L's first-order condition with respect to capital with K^L_t, and summing up the resulting equations, we obtain

$$\mathbb{E}_t[\tilde{A}^L_{s_t,t}](N^L_t)^{1-\alpha}(K^L_t)^{\alpha} = \mathbb{E}_t[r^L_{s_t,t}](W^L_t N^L_t + Q_t K^L_t).$$

Using this equation, the state-contingent zero-profit conditions for firms in Sector L can be rearranged into

$$r^L_{s_t,t} = \frac{\tilde{A}^L_{s_t,t}(N^L_t)^{1-\alpha}(K^L_t)^{\alpha}}{Q_t K^L_t + W^L_t N^L_t} = \frac{\tilde{A}^L_{s_t,t}(N^L_t)^{1-\alpha}(K^L_t)^{\alpha}}{\mathbb{E}_t[\tilde{A}^L_{s_t,t}](N^L_t)^{1-\alpha}(K^L_t)^{\alpha}}\mathbb{E}_t[r^L_{s_t,t}] = \frac{\tilde{A}^L_{s_t,t}}{\mathbb{E}_t[\tilde{A}^L_{s_t,t}]}\mathbb{E}_t[r^L_{s_t,t}]$$

with $s_t \in \{\underline{s}, \overline{s}\}$. Using this equation and the first-order condition for capital in Sector L, we obtain

$$
\begin{aligned}
r^L_{s_t,t} Q_t &= \frac{\tilde{A}^L_{s_t,t}}{\mathbb{E}_t[\tilde{A}^L_{s_t,t}]} Q_t \mathbb{E}_t[r^L_{s_t,t}] = \frac{\tilde{A}^L_{s_t,t}}{\mathbb{E}_t[\tilde{A}^L_{s_t,t}]} \alpha \mathbb{E}_t[\tilde{A}^L_{s_t,t}] (N^L_t)^{1-\alpha} (K^L_t)^{\alpha-1} \\
&= \alpha \tilde{A}^L_{s_t,t} (N^L_t)^{1-\alpha} (K^L_t)^{\alpha-1} \qquad (12)
\end{aligned}
$$

for $s_t \in \{\underline{s}, \overline{s}\}$. Analogously, we derive

$$
r^L_{s_t,t} W^L_t = (1-\alpha) \tilde{A}^L_{s_t,t} (N^L_t)^{-\alpha} (K^L_t)^{\alpha}, \quad s_t \in \{\underline{s}, \overline{s}\}.
$$

From equating firm B's and firm L's first-order conditions with respect to the capital good in Equations (9) and (12), respectively, we obtain

$$
\begin{aligned}
& \frac{\tilde{A}^B_t (N^B_t)^{1-\alpha} (K^B_t)^{\alpha-1}}{r^B_t} = \frac{Q_t}{\alpha} = \frac{\mathbb{E}_t[\tilde{A}^L_{s_t,t}] (N^L_t)^{1-\alpha} (K^L_t)^{\alpha-1}}{\mathbb{E}_t[r^L_{s_t,t}]} \\
\Leftrightarrow \quad & \frac{K^B_t}{K^L_t} = \frac{N^B_t}{N^L_t} \left(\frac{\mathbb{E}_t[r^L_{s_t,t}]}{r^B_t} \frac{\tilde{A}^B_t}{\mathbb{E}_t[\tilde{A}^L_{s_t,t}]} \right)^{\frac{1}{1-\alpha}}, \qquad (13)
\end{aligned}
$$

which is the ratio of capital good employed in Sector B and Sector L.

3.8 Central Bank

The central bank provides liquidity to banks in the form of reserves which banks use to settle interbank liabilities. For this task, the central bank has two instruments for monetary policy: (a) it sets the gross interest rate on reserve loans and deposits, denoted by $r_{CB,t}$, and (b) it designs a collateral framework applied in the central bank's lending facility.[10] Banks borrow reserves from the central bank via collateralized loans to which the central bank applies a haircut $\psi_t \in [0, 1)$, following the haircut rules specified in Section 5. The only pledgeable assets in our economy are bank loans provided to firms in Sector L.

In each period t, the central bank chooses the monetary policy after the realization

[10] We abstract from a positive interest-rate spread between borrowing and depositing reserves, as such a spread could not play a welfare-improving role in our economy.

of the macroeconomic shock z_t and before the idiosyncratic shock s_t in Sector L realizes. Thereby, the central bank maximizes the periodwise utility of investors, households, and workers. The investors' portfolio of bank-equity holdings is fully diversified. The end-of-period incomes of households and workers are deterministic, as households and workers hold their proceeds from the factor markets, either as deposits or bonds. Hence, the income of neither of these agents is stochastic and the central bank thus does not have to account for the agents' risk aversion. We assume that there are no restrictions on redistributing income across agents.[11] Hence, the central bank sets $r_{CB,t}$ and ψ_t to maximize expected aggregate production, corrected for potential bank-default costs.

We derive in Section 4 that in equilibrium, the central bank's interest-rate policy is neutral: it does not impact the equilibrium outcome of the real economy. Thus, the central bank's optimization problem reduces to setting the haircut ψ_t to maximize the potentially default-cost-adjusted expected aggregate production.

3.9 Banks

There is a unit mass of identical, one-period-lived banks. Banks are protected by limited liability and are perfectly competitive. Hence, we can focus on a representative bank. The representative bank operates with equity-financing by the representative investor who commits to investing his/her entire proceeds from capital-good sales $Q_t E_t$. Banks maximize the expected gross return $\mathbb{E}_t[r^E_{s_t,t}]$ on equity. Moreover, banks offer deposit contracts where deposits D_t pay gross interest rate r^D_t. If a bank defaults, all its deposits are protected by a deposit insurance scheme, financed by lump-sum taxation on workers' income.[12] Banks provide loans to firms in Sector L and we denote the state-dependent lending interest factor by $r^L_{s_t,t}$. Banks can alleviate the moral hazard problem of firms in Sector L, since banks are able to costlessly monitor these firms and enforce their contractual obligations.

We assume a one-to-one matching between banks and firms in Sector L. Hence, each

[11] One could go further by forgoing this simplifying assumption and considering instead a social welfare function that assigns Pareto weights to each group of agents. Then the central bank would maximize this social welfare function by setting $r_{CB,t}$ and ψ_t accordingly in each period t.

[12] Bank equity is not insured however. This precludes bank equity from being a medium of exchange.

bank holds a non-diversified loan portfolio and is fully exposed to the idiosyncratic risk of the financed firm. The decision about loan supply L_t to the matched firm pins down the leverage ratio $\varphi_t \equiv L_t/Q_tE_t$ of the representative bank and its deposit-financing $D_t = L_t - Q_tE_t$. We assume that households and workers distribute their deposits across all banks equally.

A share $\gamma_t D_t$, $\gamma_t \in (0,1]$, of deposits D_t temporarily moves to other banks in period t. Arising interbank liabilities are settled with reserves on gross basis, i.e., they are settled without netting the deposit inflows. Hence, the liquidity demand of the representative bank amounts to $\gamma_t D_t = \gamma_t(L_t - Q_tE_t)$ and to satisfy interbank liabilities, the bank's reserve deposits D_t^{CB} and reserve borrowings L_t^{CB} have to fulfill $D_t^{CB} \geq \gamma_t D_t$ and $L_t^{CB} \geq \gamma_t D_t$. As the interest rates on reserve loans and reserve deposits are equal, there are no arbitrage opportunities at the central bank's lending and deposit facilities. Without loss of generality, we assume that $L_t^{CB} = \gamma_t D_t$. Since we focus on a representative bank, deposit inflows and deposit outflows must be equal, leading to $D_t^{CB} = L_t^{CB} = \gamma_t D_t$ and the bank's balance-sheet identity $L_t + D_t^{CB} = D_t + L_t^{CB} + E_t$.

Reserves can be borrowed from the central bank via collateralized loans. Firm loans are the only eligible assets for collateralization in our economy and are subject to a haircut $\psi_t \in [0,1)$. The expected loan repayment $\mathbb{E}_t[r^L_{s_t,t}]L_t$ is the bank's *collateral value*. Hence, the bank has the *collateral capacity* $(1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]L_t$, which must cover its liquidity demand. The bank faces the liquidity constraint

$$(1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]L_t \geq \gamma_t(L_t - Q_tE_t)r_{CB,t} \qquad \Leftrightarrow \qquad (1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]\varphi_t \geq \gamma_t(\varphi_t - 1)r_{CB,t}.$$

If $\gamma_t r_{CB,t} > (1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]$, this condition reads

$$\varphi_t \leq \frac{\gamma_t r_{CB,t}}{\gamma_t r_{CB,t} - (1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]} \equiv \varphi_t^L(\psi_t). \tag{14}$$

Otherwise, if $\gamma_t r_{CB,t} \leq (1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]$, there is no liquidity constraint and we set $\varphi_t^L(\psi_t) \equiv$

$+\infty$. The individual bank realizes equity return $r^E_{s_t,t} Q_t E_t$ with equity-return factor

$$r^E_{s_t,t} = \max\{(r^L_{s_t,t} - r^D_t)\varphi_t + r^D_t, 0\}. \tag{15}$$

Recall that the equity-return factor is bounded from below by zero, since the bank is protected by limited liability. The bank solves

$$\max_{\varphi_t \in [1, \varphi^L_t(\psi_t)]} \mathbb{E}_t[\max\{(r^L_{s_t,t} - r^D_t)\varphi_t + r^D_t, 0\}] Q_t E_t. \tag{16}$$

We assume that an individual bank chooses the greatest leverage if the bank is indifferent between several leverage levels. If the bank's matched firm in Sector L incurs a negative shock $s_t = \underline{s}$, a bank defaults if and only if

$$(r^L_{\underline{s},t} - r_{CB,t})\varphi_t + r_{CB,t} < 0.$$

If $r_{CB,t} > r^L_{\underline{s},t}$, this condition reads

$$\varphi_t > \frac{r_{CB,t}}{r_{CB,t} - r^L_{\underline{s},t}} \equiv \varphi^S_t, \tag{17}$$

where φ^S_t is the critical leverage ratio above which the bank incurs solvency risk. Otherwise, if $r_{CB,t} \leq r^L_{\underline{s},t}$, there is no solvency risk for all $\varphi_t \geq 1$, and we set $\varphi^S_t \equiv +\infty$. When φ_t exceeds φ^S_t, the loan repayment after a negative idiosyncratic shock does not cover the bank's liabilities towards households and workers. Then, the bank is resolved and the government assumes the outstanding liabilities and reimburses households through its deposit insurance scheme. Besides these expenses for deposit insurance, the government faces bank-resolution costs $\lambda F_{L,t}(N^L_t, K^L_t, \underline{s})$ with $\lambda \in (0,1)$.[13] Lemma 3 gives the bank's optimal leverage choice.

Lemma 3 (Optimal Choice of the Bank) *Without the possibility of solvency risk,*

[13] There are several ways to model costs of default (see e.g. Malherbe, 2020). We opt for a simple formula in which default costs are proportional to the value of assets that have to be recovered in case of default.

i.e., if $\varphi_t^L(\psi_t) \leq \varphi_t^S$, the bank's optimal choice of leverage is characterized by

$$\varphi_t = \varphi_t^L(\psi_t) \quad (\varphi_t = 1) \quad \textit{iff} \quad \mathbb{E}_t[r_{s_t,t}^L] \geq (<) r_t^D.$$

With the possibility of solvency risk, i.e., if $\varphi_t^L(\psi_t) > \varphi_t^S$, the bank's optimal choice of leverage is characterized by

$$\varphi_t = \varphi_t^L(\psi_t) \quad (\varphi_t = 1) \quad \textit{iff either (neither)} \quad \begin{cases} \mathbb{E}_t[r_{s_t,t}^L] \geq r_t^D \textit{ or (nor)} \\ r_{t,\bar{s}}^L > r_t^D \textit{ and } \varphi_t^L(\psi_t) \geq \frac{\mathbb{E}_t[r_{s_t,t}^L]/\eta_{z_t} - r_t^D}{r_{t,\bar{s}}^L - r_t^D} \end{cases}.$$

Proof. See Appendix D.3. ■

The rationale behind Lemma 3 is the following. If all liquidity-preserving leverage ratios guarantee bank solvency, i.e., if $\varphi_t^L(\psi_t) \leq \varphi_t^S$, limited liability is irrelevant for the bank's optimization problem—the limitation of downside risk does not bite for any liquidity-preserving leverage. The bank's optimization problem thus reduces to the comparison of the interest on deposits r_t^D and the expected interest on loans $\mathbb{E}_t[r_{s_t,t}^L]$. If r_t^D weakly exceeds $\mathbb{E}_t[r_{s_t,t}^L]$, the bank leverages up to its liquidity constraint $\varphi_t^L(\psi_t)$, as every additional unit of deposit-financed granted loan yields a non-negative expected profit $r_t^D - \mathbb{E}_t[r_{s_t,t}^L]$. Otherwise, the bank sticks to pure equity-financing, i.e., $\varphi_t = 1$.

If there are levels of liquidity-preserving leverage that induce solvency risk, i.e., if $\varphi_t^L(\psi_t) > \varphi_t^S$, limited liability matters in principle. However, if $r_t^D \geq \mathbb{E}_t[r_{s_t,t}^L]$, the bank implements $\varphi_t = \varphi_t^L(\psi_t)$ for the same reasons as in the case without any solvency risk as above. If $r_t^D < \mathbb{E}_t[r_{s_t,t}^L]$, the bank can nevertheless realize expected profits exceeding the expected profit associated with pure equity-financing ($\varphi_t = 1$) under certain circumstances. This is due to the limited liability of banks, that is, the bank's downside risk is bounded from below by $r_{\underline{s},t}^E = 0$ for all $\varphi_t > \varphi_t^S$. For the bank to realize an expected profit that exceeds the expected profit $\mathbb{E}_t[r_{s_t,t}^L]Q_tE_t$ from pure equity-financing, two conditions must be fulfilled. First, in case of a positive idiosyncratic shock $s = \bar{s}$ to the matched firm, the bank must be able to realize excess return $r_{\bar{s},t}^L - r_t^D > 0$ for every unit of loans. Second, the liquidity constraint has to be sufficiently loose, i.e., $\varphi_t^L(\psi_t) \geq \frac{\mathbb{E}_t[r_{s_t,t}^L]/\eta_{z_t} - r_t^D}{r_{t,\bar{s}}^L - r_t^D}$.

When these two conditions hold, the bank can leverage sufficiently high, so that the profit in the positive state, in combination with limited liability in the negative state, yields an expected profit that exceeds $\mathbb{E}_t[r^L_{s_t,t}]Q_t E_t$.

If, however, $r^D_t < \mathbb{E}_t[r^L_{s_t,t}]$ and either $r^L_{\bar{s},t} \leq r^D_t$ or $\varphi^L_t(\psi_t) < \frac{\mathbb{E}_t[r^L_{s_t,t}]/\eta z_t - r^D_t}{r^L_{t,\bar{s}} - r^D_t}$, the bank cannot realize an expected profit exceeding the expected profit associated with minimum lending ($\varphi_t = 1$), even when the bank takes into account its potential benefits from limited liability.

Next, we turn from the representative bank's optimization problem to the interbank market where banks can borrow (deposit) liquidity from (at) other banks. We assume that banks cannot differentiate between deposits held by firms, households, workers, and other banks. For simplicity, we assume that loan and deposit rates on the interbank market are identical. Hence, there prevails a uniform gross interest rate r^D_t on the (inter)bank deposit and loan market. Analogously to central-bank loans, interbank loans must be collateralized through bank loans issued to firms. These bank loans are subject to a haircut $\tilde{\psi} \in [0, 1)$. Lemma 4 characterizes the relation between liquidity pricing at the central bank's lending facility and on the interbank market.

Lemma 4 (Interbank Market) $r^D_t(1 - \psi_t) = r_{CB,t}(1 - \tilde{\psi}_t)$.

Proof. See Appendix D.4. ■

For simplicity, we assume that collateral standards on the interbank market mimic those of the central bank, i.e., $\tilde{\psi}_t = \psi_t$. Hence, it holds that $r^D_t = r_{CB,t}$. To sum up, the loan and deposit rate in the interbank market are equal to the central-bank interest rate.

4 Sequential-Markets Equilibrium

Subsequently, we define a sequential-markets equilibrium.

Definition 1 (Sequential-Markets Equilibrium) *For any given sequence of monetary policies* $\{\psi_t, r_{CB,t}\}_{t=0}^{\infty}$*, a competitive sequential-markets equilibrium is a sequence of capital allocations* $\{K^B_t, K^L_t\}_{t=0}^{\infty}$*, factor prices* $\{Q_t, W^L_t, W^B_t\}_{t=0}^{\infty}$*, gross interest rates*

$\{r_t^D, r_t^B, r_{\underline{s},t}^L, r_{\overline{s},t}^L, \tilde{r}_t^E\}_{t=0}^{\infty}$, *deposit-holding shares* $\{\zeta_t, \xi_t\}_{t=0}^{\infty}$, *bank leverage* $\{\varphi_t\}_{t=0}^{\infty}$, *consumption choices* $\{C_t^H, C_t^I, C_t^W\}_{t=0}^{\infty}$, *and wealth allocations* $\{K_{t+1}, E_{t+1}\}_{t=0}^{\infty}$, *such that*

(i) $\{\zeta_t, K_{t+1}\}_{t=0}^{\infty}$ *solves the representative household's maximization problem in* (2) *s.t.* (3)*;*

(ii) $\{E_{t+1}\}_{t=0}^{\infty}$ *solves the representative investor's maximization problem in* (4) *s.t.* (5)*;*

(iii) $\{\xi_t\}_{t=0}^{\infty}$ *solves the representative worker's maximization problem in* (6) *s.t.* (7)*;*

(iv) $\{K_t^B, K_t^L, N_t^B, N_t^L\}_{t=0}^{\infty}$ *solves the representative firms' maximization problems in* (8) *and* (10)*;*

(v) $\{\varphi_t\}_{t=0}^{\infty}$ *solves the representative bank's maximization problem in* (16)*;*

(vi) the labor market clears;

(vii) the capital-good market clears: $K_t^B + K_t^L = K_t + E_t$*;*

(viii) the loan market clears: $\varphi_t Q_t E_t = Q_t K_t^L + W_t^L N_t^L$*;*

(ix) the deposit market clears: $\zeta_t Q_t K_t + \xi_t Q_t E_t = (\varphi_t - 1) Q_t E_t$*;*

(x) the bond market clears: $(1-\zeta_t) Q_t K_t = Q_t K_t^B + W_t^B N_t^B$*;*

(xi) the output-good market clears: $Y_t^I + C_t^W + Y_t^H = F_{B,t}(K_t^B, N_t^B) + F_{L,t}(K_t^L, N_t^L, s_t)$.

The loan demand is equal to the loan-financed firm's expenses $Q_t K_t^L + W_t^L N_t^L$ for the capital good and labor, and the loan supply is equal to $\varphi_t Q_t E_t$. The loan market clearing condition thus is

$$\varphi_t Q_t E_t = Q_t K_t^L + W_t^L N_t^L \qquad \Leftrightarrow \qquad \varphi_t E_t = K_t^L + \frac{W_t^L}{Q_t} N_t^L.$$

From the loan-financed firm's first-order conditions with respect to capital good and labor in (11), we obtain

$$\frac{(1-\alpha)(N_t^L)^{-\alpha}(K_t^L)^{\alpha}}{W_t^L} = \frac{\mathbb{E}_t[r_{s_t,t}^L]}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} = \frac{\alpha(N_t^L)^{1-\alpha}(K_t^L)^{\alpha-1}}{Q_t} \qquad \Leftrightarrow \qquad \frac{W_t^L}{Q_t} = \frac{1-\alpha}{\alpha}\frac{K_t^L}{N_t^L}. \tag{18}$$

Analogously, the bond-financed firm's first-order conditions with respect to capital good and labor in (9) yield

$$\frac{(1-\alpha)(N_t^B)^{-\alpha}(K_t^B)^{\alpha}}{W_t^L} = \frac{r_t^B}{\tilde{A}_t^B} = \frac{\alpha(N_t^B)^{1-\alpha}(K_t^B)^{\alpha-1}}{Q_t} \quad \Leftrightarrow \quad \frac{W_t^B}{Q_t} = \frac{1-\alpha}{\alpha}\frac{K_t^B}{N_t^B}. \tag{19}$$

Accordingly, it holds that

$$\varphi_t E_t = K_t^L + \frac{1-\alpha}{\alpha} K_t^L \qquad \Leftrightarrow \qquad K_t^L = \alpha\varphi_t E_t.$$

It follows from capital-good market clearing that capital used by bond-financed firms is $K_t^B = K_t + E_t - K_t^L = (\varphi_t^M - \alpha\varphi_t)E_t$ with $\varphi_t^M \equiv (K_t + E_t)/E_t$. These results are summarized in

Lemma 5 (Capital Allocation) *In equilibrium, it holds that $K_t^L = \alpha\varphi_t E_t$ and $K_t^B = (\varphi_t^M - \alpha\varphi_t)E_t$.*

Lemma 6 characterizes the bank leverage in equilibrium.

Lemma 6 (Bank Leverage) *In equilibrium, the bank leverages up to its liquidity constraint $\varphi_t = \varphi_t^L(\psi_t) < \infty$ and it holds that $\varphi_t \in \left(1, \frac{1}{\alpha}\varphi_t^M\right)$.*

Proof. See Appendix D.5. ■

To establish that the interest rates $r_t^B = r_t^D$ on bonds and deposits are equal in equilibrium, we introduce some useful notation. For what follows, we write $\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L] \equiv \mathbb{E}_t[\tilde{A}_{s_t,t}^L] - (1-\eta_{z_t})\lambda\tilde{A}_{t,\underline{s}}^L$. Moreover, we define

$$\tilde{a}_t \equiv \frac{N_t^B}{N_t^L}\left(\frac{\tilde{A}_t^B}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]}\right)^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad \tilde{a}_t^\lambda \equiv \frac{N_t^B}{N_t^L}\left(\frac{\tilde{A}_t^B}{\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]}\right)^{\frac{1}{1-\alpha}}. \tag{20}$$

The coefficient $\tilde{a}_t^\lambda$ ($\tilde{a}_t$) describes the relation between the production capacities in Sectors B and L in terms of the labor supply and the expected total factor productivity up to a power factor $\frac{1}{1-\alpha}$ if there is (no) solvency risk. To obtain the desired equality $r_t^B = r_t^D$, we make

Assumption 1 *$\tilde{a}_t^\lambda \rho_t < 1$ with $\rho_t \equiv E_t/K_t$.*

Assumption 1 ensures that the marginal product of capital in Sector L relative to Sector B would be excessively high if firms in Sector L were only granted loans Q_tE_t, since these firms' loan demand would then be excessively high as well. Hence, banks grant loans $\varphi_t Q_t E_t$ with $\varphi_t > 1$ to make the loan market clear. After the sale of equity contracts, there are still deposits $(\varphi_t - 1)Q_tE_t > 0$ in the economy, so that workers and households must have positive deposit holdings. This is essential for proving

Lemma 7 (Bond and Deposit Rates) *Let Assumption 1 hold. Then, in equilibrium, bonds pay the same gross interest rate $r_t^B = r_t^D = r_{CB,t}$ as bank deposits and central-bank reserves.*

Proof. See Appendix D.6. ■

With the optimal capital allocations K_t^L and K_t^B from Lemma 5, with the optimal bank leverage $\varphi_t^L(\psi_t)$ from Lemma 6, and with Equations (9) and (12), the marginal product of capital in Sector L, which is state-contingent, and the marginal product of capital in Sector B read

$$r_{\underline{s},t}^L Q_t = \alpha \tilde{A}_{\underline{s},t}^L (N_t^L)^{1-\alpha} (\alpha \varphi_t^L(\psi_t) E_t)^{\alpha-1} \tag{21}$$

$$r_{\overline{s},t}^L Q_t = \alpha \tilde{A}_{\overline{s},t}^L (N_t^L)^{1-\alpha} (\alpha \varphi_t^L(\psi_t) E_t)^{\alpha-1} \tag{22}$$

$$r_t^B Q_t = \tilde{A}_t^B (N_t^B)^{1-\alpha} ((\varphi_t^M - \alpha \varphi_t^L(\psi_t)) E_t)^{\alpha-1}. \tag{23}$$

With the equality $r_t^B = r_{CB,t}$ from Lemma 7, the banks' liquidity constraint reads

$$\varphi_t^L(\psi_t) = \frac{\gamma_t r_t^B Q_t}{\gamma_t r_t^B Q_t - (1-\psi_t)\left[(1-\eta_{z_t}) r_{\underline{s},t}^L Q_t + \eta_{z_t} r_{\overline{s},t}^L Q_t\right]}. \tag{24}$$

Plugging this expression for $\varphi_t^L(\psi_t)$ into Equations (21) to (23), these three equations uniquely determine the equilibrium levels of $r_{\underline{s},t}^L Q_t$, $r_{\overline{s},t}^L Q_t$, and $r_t^B Q_t$, and thus also determine the bank leverage $\varphi_t^L(\psi_t)$ and all real equilibrium variables in the economy. A change of $r_{CB,t}$ affects the levels of gross interest rates $r_{\underline{s},t}^L$, $r_{\overline{s},t}^L$, and r_t^B, but it is set off by a change of Q_t, so that $r_{\underline{s},t}^L Q_t$, $r_{\overline{s},t}^L Q_t$, and $r_t^B Q_t$ remain unchanged and so does the real economy as a whole. However, a change of the haircut ψ_t induces a change in real variables, as can be seen in the expression for $\varphi_t^L(\psi_t)$ in Equation (24). Hence, the central bank's interest-rate policy is neutral, whereas its haircut policy is not.

Finally, we must ensure that the government can levy enough taxes from workers to cover its expenses for deposit insurance and bank resolution, so that the workers' consumption $C_t^W \geq 0$ is non-negative. Lemma 8 provides sufficient conditions that this is ensured.[14]

Lemma 8 (Workers' Consumption) *Let Assumption 1 hold and let $\alpha < \eta_{z_t}$. Then, in equilibrium, the taxes the government levies to cover its expenses for deposit insurance and bank resolution still guarantee a non-negative consumption $C_t^W \geq 0$ for workers.*

Proof. See Appendix D.7. ■

5 Monetary Policy

Since the central bank's interest-rate policy is neutral, the central bank steers the economy through the collateral framework for reserve loans. After the macroeconomic shock z_t has realized, but before loans are granted and money is created, the central bank sets the haircut $\psi_t \in [0, 1)$ on collateral assets to maximize the economy's expected aggregate output

$$\begin{aligned}\mathbb{E}_t[Y_t] &= F_{B,t}(N_t^B, K_t^B) + \mathbb{E}_t[F_{L,t}(N_t^L, K_t^L, s_t)(1 - \lambda \mathbb{1}\{\varphi_t > \varphi_t^S \wedge s_t = \underline{s}\})] \\ &= \tilde{A}_t^B (N_t^B)^{1-\alpha}(K_t^B)^{\alpha} + \mathbb{E}_t[\tilde{A}_{s_t,t}^L(1 - \lambda \mathbb{1}\{\varphi_t > \varphi_t^S \wedge s_t = \underline{s}\})](N_t^L)^{1-\alpha}(K_t^L)^{\alpha}. \quad (25)\end{aligned}$$

We account for the potential bank-resolution costs when the banks' leverage is sufficiently high, i.e., when $\varphi_t > \varphi_t^S$, and when the matched loan-financed firm incurs a negative productivity shock $s_t = \underline{s}$. In the remainder of the paper, we write $\mathbb{E}_t^{\lambda}[Y_t]$ instead of

[14]Since α denotes the output elasticity of capital, $\alpha \in [0.3, 0.4]$ is a common choice. Recall that η_{z_t} denotes the probability of a successful realization of the investment project, conditional on the macroeconomic shock z_t, so that $(1-\eta_{z_t})$ denotes the probability of bank default. Thus, a value $\eta_{z_t} \geq 0.5$ seems plausible. Together with Assumption 1, $\alpha \in [0.3, 0.4]$ and $\eta_{z_t} \geq 0.5$ are sufficient conditions for Lemma 8 to hold.

$\mathbb{E}_t[Y_t]$ when we want to emphasize that solvency risk is present in the economy, so that the expected output is default-cost-adjusted.

The central bank anticipates that, as seen in Lemma 6, banks leverage up to their liquidity constraint $\varphi_t^L(\psi_t)$ that, according to Lemma 5, determines capital allocations $K_t^L = \alpha\varphi_t^L(\psi_t)$ and $K_t^B = (\varphi_t^M - \alpha\varphi_t^L(\psi_t))E_t$. The capital allocations, in turn, pin down the economy's expected output in Equation (25). By decreasing the haircut ψ_t, the central bank increases the banks' liquidity constraint $\varphi_t^L(\psi_t)$, as captured in

Lemma 9 (Liquidity Constraint) *In equilibrium, $\varphi_t^L(\psi_t)$ is uniquely determined by ψ_t and decreases with ψ_t: $\dfrac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} < 0$.*

Proof. See Appendix D.8. ■

Dependent on the macroeconomic fundamentals, the central bank can face three different situations. First, the first-best allocation of funds is attainable: there is a haircut $\psi_t^E \in [0, 1)$ that induces a leverage ratio $\varphi_t^E = \varphi_t^L(\psi_t^E)$ so that equal financing conditions $\mathbb{E}_t[r_{s_t,t}^L] = r_t^B$ across sectors prevail and no default costs arise. We call the implementation of haircut ψ_t Regime E (*efficient*).

If the first-best allocation is not attainable since imposing $\mathbb{E}_t[r_{s_t,t}^L] = r_t^B$ would induce solvency risk, the central bank has two possibilities. First, the central bank can rule out solvency risk by setting a larger haircut denoted by ψ_t^S such that $\varphi_t^L(\psi_t^S) = \varphi_t^S$, as defined in Equation (17). Lemma 10 delivers the explicit formula of ψ_t^S below. By setting ψ_t^S, the central bank accepts an over-accumulation of funds in Sector B, that is, the marginal product of capital in Sector B falls short of the expected marginal product of capital in Sector L. We call this Regime S (*solvency risk ruled out*).

Lemma 10 (Solvency Risk) *Let ψ_t^S be the haircut for which $\varphi_t^L(\psi_t^S) = \varphi_t^S$, as defined in Equation* (17)*. Then,*

$$\psi_t^S = 1 - \frac{\gamma_t A_{z_t,\underline{s}}^L}{\mathbb{E}_t[A_{z_t,s_t}^L]}.$$

In equilibrium, there is solvency risk if and only if the central bank sets a haircut $\psi_t < \psi_t^S$.

Proof. See Appendix D.9. ■

The central bank can also permit bank default by setting a haircut ψ_t^D that induces a more balanced level of returns across sectors. This monetary policy is called Regime D (*default*). Deciding between Regimes S and D, the central bank balances the overaccumulation of capital in Sector B on the one side and bank-resolution costs on the other side. If the first-best solution is not attainable, the central bank implements Regime D (S) if and only if $\mathbb{E}_t^\lambda[Y_t(\psi_t^D)] \geq (<) \ \mathbb{E}_t[Y_t(\psi_t^S)]$. Proposition 1 characterizes the central bank's optimal static haircut rules in the three regimes.

Proposition 1 (Static Haircut Rules) *Let Assumption 1 hold. The central bank optimally implements the haircut*

$$\psi_t^E = 1 - \gamma_t \left(1 - \alpha\rho_t \frac{1+\tilde{a}_t}{1+\rho_t}\right), \tag{26}$$

and thus Regime E, if and only if Regime E does not induce solvency risk, i.e., if

$$\varphi_t^L(\psi_t^E) \leq \varphi_t^S \qquad \Leftrightarrow \qquad \frac{A^L_{z_t,\underline{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]} \geq 1 - \alpha\rho_t \frac{1+\tilde{a}_t}{1+\rho_t}.$$

If Regime E induced solvency risk, the central bank would either implement Regime S and thus haircut

$$\psi_t^S = 1 - \frac{\gamma_t A^L_{z_t,\underline{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]}, \tag{27}$$

which rules out bank-solvency risk, or it would implement Regime D and thus haircut

$$\psi_t^D = 1 - \gamma_t \left(1 - \alpha\rho_t \frac{1+\tilde{a}_t^\lambda}{1+\rho_t}\right) \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]}, \tag{28}$$

which entails bank-solvency risk. The central bank sets haircut ψ_t^D if and only if both $\mathbb{E}_t^\lambda[Y_t(\psi_t^D)] \geq \mathbb{E}_t[Y_t(\psi_t^S)]$ and $\psi_t^D \leq \psi_t^S$. Otherwise, it sets ψ_t^S.

Proof. See Appendix D.10. ■

The haircut rule in Equation (28) under Regime D—arguably the most plausible regime since we observe bank default in reality—can be approximated by

$$\psi_t^D = 1 - \gamma_t\big(1 - \alpha(1+\tilde{a}_t^\lambda)\rho_t\big), \tag{29}$$

which rationalizes the stylized haircut rule in Equation (1). The calibration results in Section 7.1 legitimate this approximation.[15] Given the liquidity demand γ_t and the default-cost adjusted relation $\tilde{a}_t^\lambda$ of the production capacities between sectors, the stylized haircut rule in Equation (29) describes a linear relation between the haircut ψ_t^D and the investors-to-households-capital ratio ρ_t. The larger ρ_t, the larger is the haircut ψ_t^D, since a large capital endowment of investors and thus an extensive equity-financing of banks requires a strong restriction of banks to leverage to induce the desired allocation of funds across sectors.

As a corollary of Proposition 1, we establish the equilibrium levels of bank leverage in the three regimes in

Corollary 1 (Equilibrium Bank Leverage) *Let Assumption 1 hold and let the central bank apply the optimal monetary policy characterized in Proposition 1. If the central bank applies Regime E, the equilibrium bank leverage is*

$$\varphi_t^E = \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t}\right).$$

If the central bank applies Regime D, the equilibrium bank leverage is

$$\varphi_t^D = \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t^\lambda}\right).$$

The equilibrium bank leverage in Regime S is is uniquely pinned down by equation

$$\varphi_t^S = \frac{\tilde{A}_t^B(N_t^B)^{1-\alpha}(\varphi_t^M - \alpha\varphi_t^S)^{\alpha-1}}{\tilde{A}_t^B(N_t^B)^{1-\alpha}(\varphi_t^M - \alpha\varphi_t^S)^{\alpha-1} - \tilde{A}_{\underline{s},t}^L(N_t^L)^{1-\alpha}(\alpha\varphi_t^S)^{\alpha-1}}.$$

[15] The calibration of the model with US pre-crisis data from 2004Q3 to 2007Q2, which is shown in Table 2, yields $\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]/\mathbb{E}_t[A_{z_t,s_t}^L] = 0.999$, which justifies to omit this ratio. Moreover, in the steady-state equilibrium under Regime D that the calibration supports, it holds that $\rho_t = 0.1245$, which, in turn, is shown in Table 3. Typically, the level of ρ_t thus is sufficiently low to make the approximation $\rho_t \approx 1/(1+\rho_t)$ suitable.

In all regimes, the bank leverage decreases with the investors-to-households-capital ratio:

$$\frac{\mathrm{d}\varphi_t^E}{\mathrm{d}\rho_t}, \frac{\mathrm{d}\varphi_t^D}{\mathrm{d}\rho_t}, \frac{\mathrm{d}\varphi_t^S}{\mathrm{d}\rho_t} < 0.$$

Proof. See Appendix D.11. ■ The central bank's monetary policy pins down the equilibrium allocation of funds between the bond-financed sector and the loan-financed sector within a period. Through this allocation, also the return factors $\tilde{r}_t^E Q_t$ and $r_t^D Q_t$ on investors' and households' capital are pinned down. As seen in Lemmata 1 and 2, by affecting these return factors, the central bank controls the capital accumulation of investors and households.

6 Comparative Statics and Dynamic Haircut Rules

In this section, we explore the comparative-statics properties of the haircut rules and we characterize dynamic haircut rules that indicate how haircuts change across time.

6.1 Avoidance of Bank Default

Recall from Proposition 1 that, if the first-best allocation of funds is not attainable and the central bank rules out bank default, it sets the haircut

$$\psi_t^S = 1 - \frac{\gamma_t A^L_{z_t,\underline{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]}.$$

We summarize the main properties of ψ_t^S in

Proposition 2 (Optimal Haircut in Regime S) *Haircut ψ_t^S decreases with the liquidity demand from banks, captured by γ_t, and increases (decreases) with the productivity of loan-financed firms in the high (low) productivity state, denoted by $A^L_{z_t,\overline{s}}$ ($A^L_{z_t,\underline{s}}$). Moreover, haircut ψ_t^S is independent of the productivity in the bond-financed sector, denoted by $A^B_{z_t}$.*

Proof. See Appendix D.12. ■

Setting haircut ψ_t^S, the central bank equalizes $\varphi_t^L(\psi_t^S)$ and φ_t^S and thereby rules out solvency risk. If the bank's liquidity demand, captured by γ_t, increases, the bank is more restricted in its lending activity because the rise of interbank liabilities must be covered with collateral. The maximal leverage ratio $\varphi_t^L(\psi_t^S)$ that still guarantees liquidity thus decreases. To keep the level of leverage up to φ_t^S, the central bank extends the bank's collateral capacity $(1-\psi_t^S)\mathbb{E}_t[r_{s_t,t}^L]\varphi_t^S Q_t E_t$ by reducing haircut ψ_t^S.

An increase of the positive idiosyncratic productivity shock $A_{z_t,\overline{s}}^L$ directly translates into an increase of the interest factor $r_{\overline{s},t}^L$ via the first-order condition of firms in Sector L with respect to capital. Therefore, the collateral value $\mathbb{E}_t[r_{s_t,t}^L]\varphi_t^S Q_t E_t$ rises as well. To keep collateral requirements tight, the central bank increases ψ_t^S and attenuates the increase in value of the collateral capacity $(1-\psi_t^S)\mathbb{E}_t[r_{s_t,t}^L]\varphi_t^S Q_t E_t$. As a result, the bank's liquidity constraint binds again at $\varphi_t^S = \varphi_t^L(\psi_t^S)$.

An increase of the positive idiosyncratic productivity shock $A_{z_t,\underline{s}}^L$ introduces two countervailing forces. On the one hand, there is an upward pressure on ψ_t^S for analogous reasons as for a rise of $A_{z_t,\overline{s}}^L$: a rise of $A_{z_t,\underline{s}}^L$ causes a rise of $r_{\underline{s},t}^L$ and of the collateral value, which makes the central bank tighten its collateral requirements. On the other hand, the rise of $r_{\underline{s},t}^L$ attenuates solvency risk, since a bank must take on a higher leverage, so that, given a negative shock to the matched firm, the bank's equity return is equal to zero (see Equation (15)). The solvency-preserving leverage ratio φ_t^S thus increases in $r_{\underline{s},t}^L$ (see Equation (17)). In response, the central bank relaxes its collateral requirements by reducing haircut ψ_t^S, and allows banks to expand their lending. Proposition 2 shows that the latter force is dominant.

We call haircut rules that link the haircut in period $t+1$ to the haircut in period t "dynamic haircut rules". For cases in which the central bank rules out solvency risk in every period, the dynamic haircut rule is characterized in

Proposition 3 (Dynamic Haircut Rule in Regime S) *If the first-best allocation of funds is not attainable and, in each period, the central bank optimally aims at avoiding* *bank default, the haircut follows the dynamic rule*

$$\psi_{t+1}^S = 1 - \xi_{t+1}^S + \psi_t^S \xi_{t+1}^S$$

for all $t \geq 0$*, with* $\xi_{t+1}^S \equiv \xi_{t+1}^\gamma \, \xi_{t+1}^d \, \xi_{t+1}^e$ *and*

$$\xi_{t+1}^\gamma \equiv \frac{\gamma_{t+1}}{\gamma_t}, \qquad \xi_{t+1}^d \equiv \frac{\tilde{A}_{z_{t+1},\underline{s}}^L}{\tilde{A}_{z_t,\underline{s}}^L}, \qquad \xi_{t+1}^e \equiv \frac{\mathbb{E}_t[A_{z_t,s_t}^L]}{\mathbb{E}_{t+1}[\tilde{A}_{z_{t+1},s_{t+1}}^L]}.$$

Proof. See Appendix D.13. ■

6.2 Acceptance of Bank Default

We next turn to the case when the central bank optimally accepts bank default. From Proposition 1, we know that, if the first-best allocation of funds is not attainable, the central bank accepts bank default by setting haircut

$$\psi_t^D = 1 - \gamma_t \left(1 - \alpha\rho_t \frac{1 + \tilde{a}_t^\lambda}{1 + \rho_t}\right) \frac{\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]}{\mathbb{E}_t[A_{z_t,s_t}^L]},$$

if and only if $\mathbb{E}_t^\lambda[Y_t(\psi_t^D)] \geq \mathbb{E}_t[Y_t(\psi_t^S)]$ and $\psi_t^D \leq \psi_t^S$. For the following analysis, we require that the latter two assumptions hold for all periods $t \geq 0$. The comparative statics of ψ_t^D are summarized in

Proposition 4 (Optimal Haircut in Regime D) *Suppose that the central bank selects Regime D in all periods. Haircut* ψ_t^D *increases with*

- $\rho_t = E_t/K_t$ *(and thus increases with* E_t *and decreases with* K_t*);*
- *default-cost parameter* λ*;*
- *productivity* $A_{z_t}^B$ *of the bond-financed sector;*
- *growth rate* g_B*;*

and decreases with

- *liquidity-demand parameter* γ_t*;*

- *productivity $A^L_{z_t,\bar{s}}$ of the loan-financed sector in the high productivity state;*

- *growth rate g_L.*

The effect on ψ^D_t of an increase of productivity $A^L_{z_t,\underline{s}}$ of the loan-financed sector in the high productivity state is ambiguous. If $\tilde{A}^B_t \geq \mathbb{E}^\lambda_t[\tilde{A}^L_{s_t,t}]$, $\frac{\mathrm{d}\psi^D_t}{\mathrm{d}\alpha}$ is positive.

Proof. See Appendix D.14. ■

For a given leverage ratio φ^D_t, an increase of $\rho_t = E_t/K_t$ induces a shift of capital from the bond-financed to the loan-financed sector. To attenuate this shift, the central bank increases haircut ψ^D_t, thereby reduces the bank's collateral capacity, and finally achieves a decrease in $\varphi^D_t = \varphi^L_t(\psi^D_t)$.

For a given leverage φ^D_t, an increase of default-cost parameter λ causes a decrease in $\mathbb{E}^\lambda_t[Y_t(\psi^D_t)]$. On the one hand, solvency risk becomes more expensive. On the other hand, the gains from taking on solvency risk remain unchanged, namely the high level of output if a positive idiosyncratic shock of the matched firm in Sector L realizes. Hence, a lower level of leverage $\varphi^D_t = \varphi^L_t(\psi^D_t)$ becomes optimal. which the central bank attains by implementing a greater haircut ψ^D_t.

Productivity gains in Sector B, i.e., an increase of $A^B_{z_t}$ or g_B, prompt the central bank to foster a reallocation of resources towards Sector B. The central bank lifts haircut ψ^D_t and, by this means, reduces the banks' collateral capacity. As a result, the banks reduce their lending to firms in Sector L—captured by a decrease of $\varphi^D_t = \varphi^L_t(\psi^D_t)$—, the deposit holdings in the economy simultaneously decrease, and firms in Sector B obtain more bond-financing.

When the banks' liquidity demand, reflected by γ_t, rises, the optimal leverage ratio φ^D_t—maximizing the expected default-cost-adjusted output—stays unaffected. Hence, the central bank eases the collateral framework by reducing haircut ψ^D_t to keep $\varphi^D_t = \varphi^L_t(\psi^D_t)$ unchanged.

An increase of productivity in Sector L in the good state, i.e., an increase of $A^L_{z_t,\bar{s}}$ or g_L, has an analogous effect on ψ^D_t as a rise of $A^B_{z_t}$ or g_B. The central bank fosters a reallocation of resources towards Sector L by slackening the collateral requirements, such that the banks expand their lending to firms in Sector L. That is, the central bank decreases ψ_t^D such that $\varphi_t^D = \varphi_t^L(\psi_t^D)$ increases.

However, an increase of productivity $A^L_{z_t,\underline{s}}$ in Sector L in the bad state has an ambiguous impact on ψ_t^D. On the one hand, the central bank has an incentive to loosen the collateral requirements, such that banks increase their lending towards firms in Sector L who, for their part, realize productivity gains. On the other hand, an increase of $A^L_{z_t,\underline{s}}$ means an increase of default costs if a negative shock $\underline{s}$ occurs, since the default costs scale with $A^L_{z_t,\underline{s}}$, so that the central bank may restrict bank lending by tightening the collateral requirements. It is *a priori* unclear whether balancing the productivity gains in Sector L and the additional default costs prompt the central bank to increase or decrease ψ_t^D. This depends on the model's parametrization and the current macroeconomic state.

For cases in which the central bank accepts solvency risk in every period, the dynamic haircut rule is characterized in

Proposition 5 (Dynamic Haircut Rule in Regime D) *If, in each period, the central bank optimally aims at accepting bank default, the haircut follows the dynamic rule*

$$\psi_{t+1}^D = 1 - \xi_{t+1}^D + \psi_t^D \xi_{t+1}^D,$$

where $\xi_{t+1}^D \equiv \xi_{t+1}^\gamma\ \xi_{t+1}^\rho\ \xi_{t+1}^d\ \xi_{t+1}^e$ *and*

$$\begin{aligned} \xi_{t+1}^\gamma &\equiv \frac{\gamma_{t+1}}{\gamma_t}, \qquad \xi_{t+1}^\rho \equiv \frac{1+\rho_{t+1}-\alpha\rho_{t+1}(1+\tilde{a}_{t+1}^\lambda)}{1+\rho_t-\alpha\rho_t(1+\tilde{a}_t^\lambda)}\,\frac{1+\rho_t}{1+\rho_{t+1}}, \\ \xi_{t+1}^d &\equiv \frac{\mathbb{E}^\lambda_{z_{t+1}}[\tilde{A}^L_{z_{t+1},s_{t+1}}]}{\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}, \qquad \xi_{t+1}^e \equiv \frac{\mathbb{E}_t[A^L_{z_t,s_t}]}{\mathbb{E}_{z_{t+1}}[\tilde{A}^L_{z_{t+1},s_{t+1}}]}. \end{aligned}$$

Proof. See Appendix D.15. ■

7 Quantitative Analysis

This section provides a quantitative assessment of the model, based on a calibration of the model to the US economy. To this end, we first provide empirical references and calibrate the model, so that the central bank accepts bank default. Given the calibrated parameters, we identify a second steady state in which the central bank rules out solvency risk. We then discuss how investors' and households' capital levels differ between the two steady states with and without bank default. Second, we conduct three simulation studies: we vary the productivity risk in Sector L and the investors-to-households-capital ratio to examine the effect on bank leverage and output, and to analyze the transition of policy regimes. Moreover, we study how the economy reacts to non-optimal changes of the haircut. We identify which agents benefit, and which agents suffer from increasing haircuts. Finally, we calibrate the model to a stylized, mainly loan-financed economy and elaborate on the differences between the resulting calibration and the calibration of the mainly bond-financed US economy.

7.1 Calibration

7.1.1 Calibration Targets and Parametrization

We calibrate the model to the US economy. Since banks take on solvency risk, we calibrate the model, such that the central bank permits bank default and implements Regime D according to the optimal policy characterized in Proposition 1. We match, similarly to Gersbach et al. (2022), the parameters in the model to microeconomic and macroeconomic pre-crisis data from 2004Q3 to 2007Q2. The calibration targets for the real economy are taken from Fernald (2014), the Penn World Table (PWT), and the Federal Reserve Economic Data (FRED). Financial data is derived from the Reports of Condition and Income (Call Reports) that each deposit-insured commercial bank is required to file to the Federal Financial Institutions Examination Council (FFIEC). Drechsler et al. (2017b) compile these data and provide a comprehensive overview of how the checking, savings, and time deposit rates in the US have evolved over time. In the calibration, the term "deposit rate" refers to "savings deposit rate". To obtain the deposit rate target $\overline{r}^D$, we compute the arithmetic average of the gross savings deposit rates from 2004Q3 to

2007Q2.[16] Drechsler et al. (2017a) provide these data with weekly frequency. Moreover, we use De Fiore and Uhlig (2011) who examine the capitalization of the bond-financed and loan-financed sectors in the US and the Euro area. The haircut target $\overline{\psi} = 0.1100$ is the Federal Reserve's haircut on commercial real estate loans with normal risk, fixed interest rate, zero coupon payments, and one-year maturity.[17]

Our model features twelve parameters to calibrate: the production technology parameters α, $\tilde{A}^B$, $\tilde{A}^L_{\underline{s}}$, and $\tilde{A}^L_{\overline{s}}$, the probability η of an idiosyncratic shock to a firm in the loan-financed sector, the capital depreciation rate δ, the labor endowment N and the loan-to-bond-labor ratio N^L/N^B, the time discount factors β_I and β_H, default-cost parameter λ, and liquidity-demand parameter γ. Besides these model parameters, investors' and households' capital endowments E and K are to be pinned down. When calibrating a steady state, we do not use time subscripts and we abstract from the macroeconomic shock. For this reason, we do not consider A^B and g_B separately but focus on $\tilde{A}^B = (1 + g_B)A^B$. A separate analysis would only make sense in a dynamic context. Analogously, we consider only $\tilde{A}^L_{\underline{s}}$ and $\tilde{A}^L_{\overline{s}}$ but not $A^L_{\underline{s}}$, $A^L_{\overline{s}}$, and g_L.

On the other side, we impose eight empirically motivated calibration targets. We make two normalizations—$\tilde{A}^B = 1.0000$ and $N = 1.0000$—, since these normalizations only affect the units of account for the measurement of labor, capital good, and productivity levels, but do not impact equilibrium properties and optimal monetary policy. We let the default probability be $1 - \eta = 0.02$. Motivated by intergenerational mobility of labor across sectors, we impose the equilibrium wage equality $W^B = W^L$ in steady state. The model entails three additional constraints: there are two steady-state conditions arising from the capital accumulation rules of households and investors in Lemmata 1 and 2, respectively. Moreover, the default-cost adjusted expected output $\mathbb{E}^\lambda[Y(\psi^D)]$ under a monetary policy that accepts bank default must weakly exceed the expected output $\mathbb{E}[Y(\psi^S)]$ if the central bank rules out solvency risk. For the sake of simplicity, we

[16] In principle, we could proceed in the same way, using checking deposit rates or time deposit rates. The choice of $\overline{r}^D$ is only constrained by the relation $\overline{r}^D \leq \overline{r}^E$, which our model requires to hold in equilibrium.

[17] The haircut is effective since March 14, 2022, and it can be found on the FED's website `https://www.frbdiscountwindow.org/pages/collateral/collateral_valuation` (accessed on January 9, 2022).

set $\mathbb{E}^\lambda[Y(\psi^D)] = \mathbb{E}[Y(\psi^S)]$. The number of constraints imposed on the calibration then matches the number of parameters to be pinned down. Calibration targets, normalizations, and the remaining constraints are summarized in Table 2, in which we also present the calibrated parameters that we derive in Appendix B.1.

	Variable	Description	Source	Value
Calibration targets	$\overline{\sigma}$	aggregate saving rate	FRED	0.1814
	$\overline{(K+E)/\mathbb{E}^\lambda[Y]}$	capital-to-output ratio	PWT	12.1202
	$\overline{r^D WN/\mathbb{E}^\lambda[Y]}$	labor share of income	Fernald (2014)	0.6358
	$\overline{\varphi}$	bank leverage	Call Report	9.9212
	$\overline{r}^E$	gross return on bank equity	Call Report	1.0320
	$\overline{r}^D$	gross return on bank deposits	Drechsler et al. (2017a)	1.0146
	$\overline{K^L/K^B}$	loan-to-bond-capital ratio	De Fiore and Uhlig (2011)	0.6667
	$\overline{\psi}$	haircut	Federal Reserve	0.1100
Constraints	$\overline{r}^E Q - \delta - \frac{1-\beta_I}{\beta_I}$	investors' steady state		0.0000
	$\overline{r}^D Q - \delta - \frac{1-\beta_H}{\beta_H}$	households' steady state		0.0000
	W^L/W^B	long-run wage equality		1.0000
	$\frac{\mathbb{E}^\lambda[Y(\psi^D)]}{\mathbb{E}[Y(\psi^S)]}$	optimality of bank default tolerance		1.0000
	η	probability of a pos. shock in Sector L	**free choice**	0.9800
Model parameters	α	output elasticity of capital		0.3642
	$\tilde{A}^B$	productivity in Sector B	normalization	1.0000
	$\tilde{A}^L_{\underline{s}}$	productivity in Sector L after neg. shock		0.8426
	$\tilde{A}^L_{\overline{s}}$	productivity in Sector L after pos. shock		1.0038
	δ	capital depreciation rate		0.0150
	N	labor endowment	normalization	1.0000
	N^L/N^B	loan-to-bond-labor ratio		0.6667
	β_H	time preference of households		0.9851
	β_I	time preference of investors		0.9846
	λ	default costs		0.0354
	γ	liquidity demand		0.9904

Table 2: Calibration.

7.1.2 Duality of Steady States

The levels of investors' and households' capital E and K for which the calibration targets are met, in particular, the optimality of Regime D, are shown in the first row of Table 3. However, these capital levels are not the only ones that sustain a steady state, given the parametrization in Table 2. The capital levels in the second row of Table 3 sustain another steady state in which the central bank optimally rules out bank default and thus applies Regime S. However, in this steady state, the calibration targets are not met. We show in Appendix B.2 that this additional steady state exists, and we show that it is the only steady state apart from the one matching the calibration targets.

	Capital levels						Output and leverage	
	ρ	E	K	$E+K$	K^L	K^B	$\mathbb{E}^\lambda[Y](\mathbb{E}[Y])$	φ
Regime D	0.1245	5.6018	45.0002	50.6020	20.2408	30.3612	4.1750	9.9212
Regime S	0.2058	8.6242	41.9120	50.5362	20.1750	30.3612	4.1740	6.4233

Table 3: Steady-state levels of capital, production, and leverage.

If we compare the steady-state capital allocations under Regimes D and S in Table 3, the lower investors-to-households-capital ratio ρ under Regime D is particularly striking. Intuitively, a low value of ρ requires a high leverage ratio φ and thus a small haircut ψ to attenuate the difference between the marginal returns of capital across sectors. For ρ sufficiently low, the central bank even allow banks to take on solvency risk.

	Interest rates					Factor prices		
	r^D	$r^L_{\underline{s}}$	$r^L_{\overline{s}}$	$\mathbb{E}[r^L_s]$	$\tilde{r}^E$	Q	W^L	W^B
Regime D	1.0146	0.8549	1.0185	1.0152	1.0320	0.0296	2.6163	2.6163
Regime S	1.0146	0.8566	1.0206	1.0173	1.0320	0.0296	2.6078	2.6163

Table 4: Steady-state interest rates and factor prices.

Apart from the distribution of capital between investors and households, we observe that the total amount of capital $E+K$ is larger under Regime D than under Regime S. Moreover, the default-cost-adjusted expected output $\mathbb{E}^\lambda[Y(\psi^D)]$ in Regime D is larger than the expected output $\mathbb{E}[Y(\psi^S)]$ in Regime S, which makes the central bank implement

	Consumption and taxes			
	C^I	C^H	C^W	τ
Regime D	0.0874	0.6787	2.6516	0.0029
Regime S	0.1345	0.6321	2.6510	0.0000

Table 5: Steady-state consumption and taxes.

Regime D. This reflects the plausible scenario in which the central bank does not rule out solvency risk because the output gains from large activities in the loan-financed sector outweigh expected default costs due to high leverage ratios.

We see that workers in Sector L particularly benefit from Regime D, compared to Regime S, as W^L is greater in Regime D. Since the haircut in Regime D is smaller and thus bank leverage is greater, more capital is channeled into Sector L, while the labor endowment N^L in Sector L is fixed. Hence, the marginal product of labor in Sector L increases and so do wages.

We see that the gross interest rates $r^D = r^B$ on bonds and deposits, which are equal in equilibrium, the return on bank equity $\tilde{r}^E$, and the rental rate of capital Q are the same in Regimes D and S. Therefore, the real returns on E and K are the same in both regimes as well. Nevertheless, the capital stock $E+K$ in the whole economy is distributed differently. Although $K+E$ is larger in Regime D, banks have a smaller capital level E in Regime D than in Regime S, whereas households have a larger capital stock K in Regime D than in Regime S. Since consumption of investors and households is given by

$$C^I = (1-\beta_I)(1+\tilde{r}^E Q - \delta)E \qquad \text{and} \qquad C^H = (1-\beta_H)(1+r^D Q - \delta)K,$$

respectively, and $\tilde{r}^E Q$ and $r^D Q$ do not considerably change, investors' consumption C^I is greater under Regime S, while households' consumption C^H is greater under Regime D, as can be seen in Table 5. Workers' consumption is given by

$$C^W = r^D(W^L N^L + W^B N^B) - \tau,$$

where τ denotes the taxes workers have to pay. Under Regime S, the bank does not take on solvency risk, such that no taxes are levied ($\tau = 0$), while under Regime D, taxes $\tau > 0$ cover the government's expenses for deposit insurance and bank-resolution costs. However, the higher level of W^L under Regime D compensates for the positive taxes, which results in a higher level of C^W under Regime D than under Regime S (see Table 5). Summarizing, both households and workers enjoy a higher level of consumption under Regime D than under Regime S, whereas the opposite holds for investors. Hence, the costs which banks impose on the society through default and which are carried by the government, and not by the banks themselves, are set off by the gains of a more efficient allocation of funds under Regime D and a higher level $E + K$ of total capital in the economy. This is a particularly interesting observation, as one might expect that the bank owners particularly gain in Regime D, since banks impose the externalities on the society.

7.2 Simulations

In the following three subsections, we discuss some simulation examples of our model. Based on the calibrated parameters in Table 2, we examine how different levels of productivity risk (Section 7.2.1) and the investors-to-households-capital ratio (Section 7.2.2) impact the optimal haircut policy and the resulting equilibrium outcomes. For each macroeconomic scenario, we compute the one-period equilibria under the optimal monetary policy and illustrate the resulting bank leverage, the haircut, and the expected output under Regimes D, E, and S. We discuss the intuition behind these intraperiod equilibria and analyze the transitions between the regimes. In Section 7.2.3, we simulate equilibria in the economy when the central bank does *not* apply the optimal haircut policy, and we show how the equilibrium outcomes change with the haircut level. We display aggregate production, bank leverage, factor products, prices, interest rates, taxes, and consumption levels, to scrutinize which agents benefit and which agents suffer from great haircut levels. Since we do not analyze any interperiod dynamics such as capital accumulation or total-factor-productivity (TFP) growth, we do not index the model variables by time t.

7.2.1 Productivity Risk in Sector L

Figure 4 shows the outcomes of the one-period equilibria for different values of the negative productivity shock $\tilde{A}^L_{\underline{s}} \in [0.7500, 1.0000]$ in the loan-financed sector. We take the parameters in Table 6 as the basis of the computations displayed in Figure 4. Recall that these parameters resulted from the steady-state calibration in Section 7.1, which Tables 2 and 3 display. We have chosen E such that $E + K = 1$ for $K = E/\rho$. However, since the whole economy scales with E, the choice of E does not have any impact on the descriptive analysis of the computations.

	Factor endowments				Production parameters					
	$E+K$	ρ	N^B	N^L	$\tilde{A}^B$	$\tilde{A}^L_{\underline{s}}$	η	α	λ	γ
Value	1.0000	0.1245	0.6000	0.4000	1.0000	1.0038	0.9800	0.3642	0.0354	0.9904

Table 6: Parameters for the computations in Figure 4.

We observe in the upper-left panel of Figure 4 that for high values of $\tilde{A}^L_{\underline{s}} \geq 0.9010$, the central bank exercises Regime E (*orange*) and sets haircut ψ^E accordingly. Thereby, the central bank achieves the first-best allocation of funds. Taking this as the baseline situation, we analyze the collateral policy when the negative productivity shock $\tilde{A}^L_{\underline{s}}$ in Sector L becomes smaller.

When $\tilde{A}^L_{\underline{s}}$ becomes smaller, i.e., for $\tilde{A}^L_{\underline{s}} \in (0.8426, 0.9010)$, banks would be exposed to default if the central bank tried to enforce equal financing conditions across sectors by implementing Regime E. Hence, the first-best allocation of funds is not feasible. To prevent banks from taking on inefficiently high leverage, the central bank tightens the collateral framework by incrementing ψ, as displayed in the upper-left panel. The central bank implements Regime S (*green*) until the lower threshold $\tilde{A}^L_{\underline{s}} = 0.8426$ is reached, where $\mathbb{E}[Y(\psi^S)]$ and $\mathbb{E}^\lambda[Y(\psi^D)]$ coincide.

At this point, the optimal haircut ψ jumps down and, accordingly, the optimal leverage $\varphi = \varphi^L(\psi)$ jumps up (center-left panel), as the central bank applies Regime D (*blue*) for $\tilde{A}^L_{\underline{s}} \leq 0.8426$. That is, the central bank accepts solvency risk but corrects the capital allocation for the costs arising from expected bank default. However, the transition from

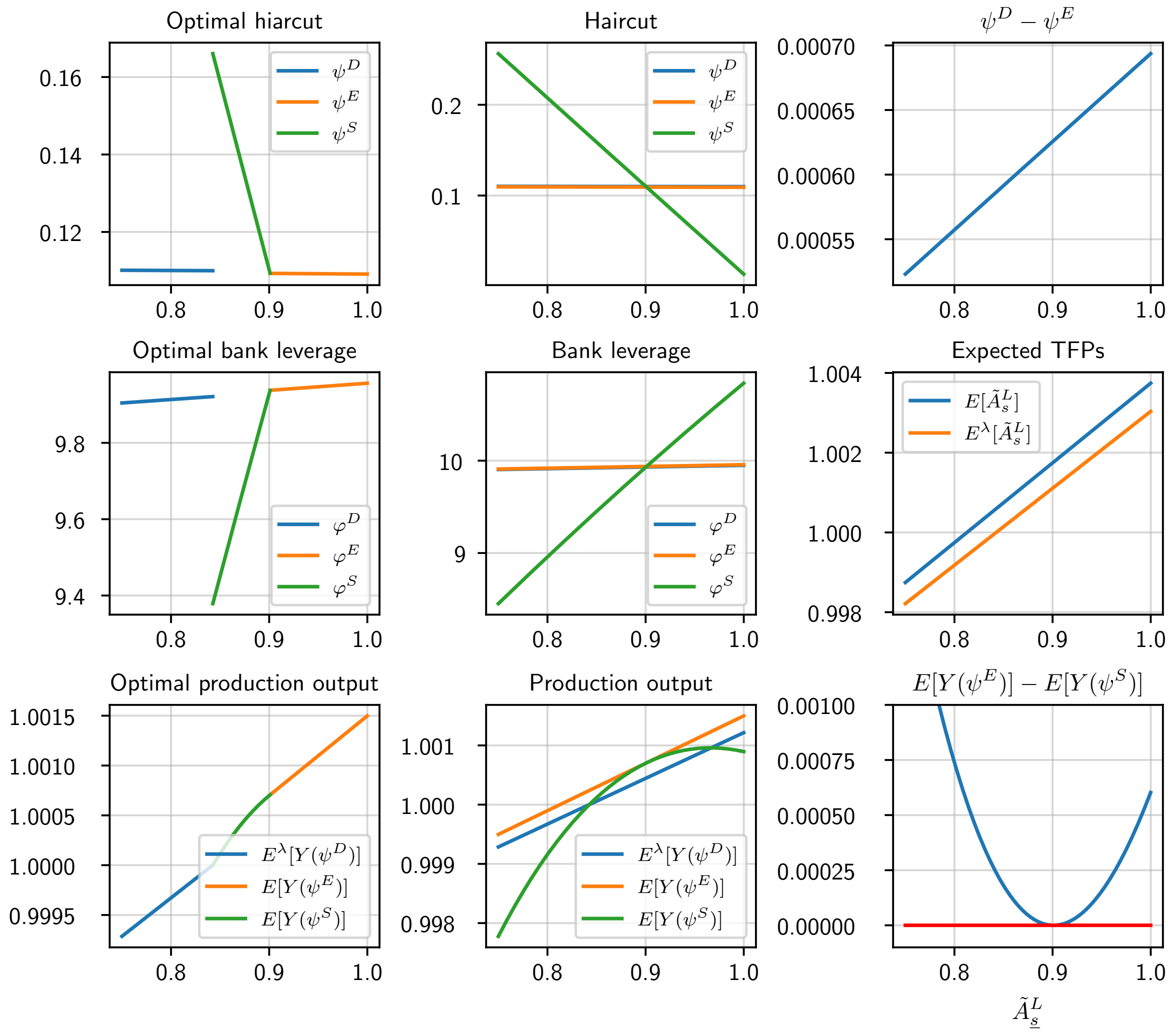

Figure 4: Computations for $\tilde{A}^L_{\underline{s}} \in [0.7500, 1.0000]$.

Regime S to Regime D is continuous in terms of the (default-cost-adjusted) expected output $\mathbb{E}[Y(\psi^S)]$ ($\mathbb{E}^\lambda[Y(\psi^D)]$), as one can see in the lower-left panel.

In the transition from Regime S to Regime D, two forces are at work. First, as $\tilde{A}^L_{\underline{s}}$ decreases, the expected default-cost-adjusted productivity $\mathbb{E}^\lambda[\tilde{A}^L_s]$ in Sector L decreases. This urges the central bank to foster a reallocation of funds from Sector L to Sector B. Thus, the central bank has an incentive to tighten the collateral framework by increasing ψ, and thus to push down bank leverage $\varphi^L(\psi)$. Second, *ceteris paribus*, also the bank-default costs $\lambda\tilde{A}^L_{\underline{s}}(N^L)^{1-\alpha}(K^L)^\alpha$ fall, since they scale with $\tilde{A}^L_{\underline{s}}$. This makes it more desirable for the economy as a whole that banks take on high leverage $\varphi^L(\psi)$, and thus calls for a small haircut ψ. We see in the upper-left and middle-left panels that ψ jumps down and $\varphi^L(\psi)$ jumps up when $\tilde{A}^L_{\underline{s}}$ falls below $\tilde{A}^L_{\underline{s}} = 0.8426$. For small values of $\tilde{A}^L_{\underline{s}}$, the central bank thus prefers a more efficient allocation of funds across sectors over ruling out solvency risk and bank-resolution costs.

7.2.2 Investors-to-Households-Capital Ratio.

In Figure 5, we provide the key endogenous variables of the one-period equilibria for different values of the investors-to-households-capital ratio $\rho \equiv E/K$ for the range $[0.0500, 0.5000]$. As the basis of the computations displayed in Figure 5, we take the parameters in Table 7, which resulted from the steady-state calibration in Section 7.1, displayed in Table 2. We adjust E and K in each computation, such that $E + K = 1$ and $E/K = \rho$. Thus, aggregate capital is the same in each computation.

	Factor endowment			Production parameters						
	N^B	N^L	$E+K$	$\tilde{A}^B$	$\tilde{A}^L_{\underline{s}}$	$\tilde{A}^L_{\bar{s}}$	η	α	λ	γ
Value	0.6000	0.4000	1.0000	1.0000	0.8426	1.0038	0.9800	0.3642	0.0354	0.9904

Table 7: Parameters for the computations in Figure 5.

For small investors-to-households-capital ratios $\rho \leq 0.1245$, the first-best allocation of funds is not feasible, since equal financing conditions across sectors would expose banks to solvency risk. This is visible in Figure 5, as the ψ^E-line (*orange*) lies under the ψ^S-line (*green*) in the upper-center panel. Bank equity is even so small that a leverage ratio that is so high that banks are exposed to solvency risk is necessary to achieve the second-best allocation of funds. Note that, for all $\rho \leq 0.1245$, the default-cost-adjusted expected output $\mathbb{E}^{\lambda}[Y(\psi^D)]$ is constant. This pertains, since capital

$$K^L = \alpha\varphi^D E = \frac{E + K}{1 + \tilde{a}^{\lambda}}$$

employed in Sector L (recall Lemma 5) under Regime D does not depend on ρ but scales with total capital endowment $E + K$, which is fixed across the computations. For analogous reasoning, K^B does not depend on ρ either. Hence, for different $\rho \leq 0.1245$, neither the absolute default costs nor the expected output change, as the central bank adjusts the haircut (and thus steers bank leverage). The allocation of capital across sectors stays the same, which can be seen in the bottom-left panel.

In the range $\rho \in (0.1245, 0.2100)$, bank equity is too small to permit the first-best allocation of funds. However, it is sufficiently high, so that the gains from shifting funds to Sector L, and thereby letting banks take on solvency risk, do not outweigh the expected default costs. The central bank thus implements Regime S, which is illustrated in the bottom-center panel by $\mathbb{E}^{\lambda}[Y(\psi^D)] - \mathbb{E}[Y(\psi^S)]$ being negative for $\rho \in (0.1245, 0.2100)$.

When bank equity is comparatively high, i.e., $\rho \in [0.2100, 0.5000]$, the central bank can implement the first-best allocation of funds by applying haircut ψ^E. In fact, at $\rho = 0.2100$, $\psi^E = \psi^S$ (upper-center panel) and $\varphi^L(\psi^E) = \varphi^L(\psi^S)$ (middle-center panel) such that the respective expectations of output $\mathbb{E}[Y(\psi^E)] = \mathbb{E}[Y(\psi^S)]$ (bottom-right panel) coincide as well. Then the associated leverage $\varphi = \varphi^L(\psi^E)$ is small enough, so that it does not expose banks to solvency risk. Yet it is large enough to ensure equal financing conditions in both sectors. On that account, it guarantees the necessary loan supply to firms in Sector L.

7.2.3 Haircut

In this section, we examine how the economy reacts to non-optimal monetary policy regimes. We thus analyze how the equilibrium outcomes change when the central bank

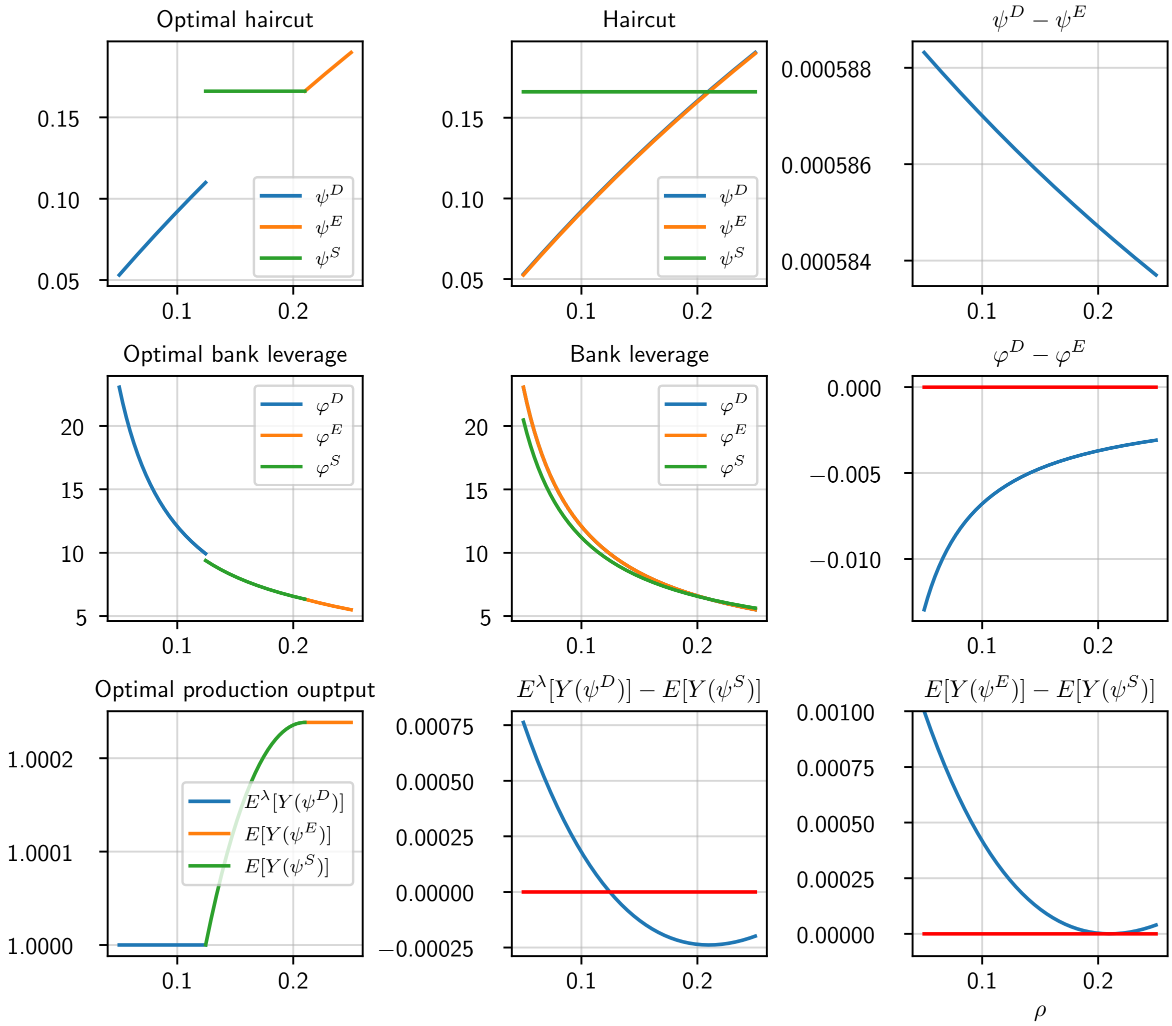

Figure 5: Computations for $\rho \in [0.0500, 0.5000]$.

changes the haircut. We take the model parametrization from the calibration in Table 2 and the steady-state levels of households' and investors' capital under Regime D in Table 3 as given. For an increasing haircut ψ, we study the development of aggregate production, bank leverage, factor prices, interest rates, consumption, and costs arising from bank default and deposit insurance. Figure 6 displays the equilibrium variables for haircut values ψ in $[0.0100, 0.2500]$ and illustrates the change from haircut regimes that induce bank-solvency risk to regimes that ensure bank solvency. Figure 7 displays the full range of eligible haircuts $[0.0100, 0.9999]$ and gives an overall impression how equilibrium outcomes change with the haircut. In every period, the rental rate of capital is normalized as $Q = 0.0296$, which is the value that Q takes on in the steady-state equilibrium under Regime D, such that the calibration targets are met. This allows o compare different haircut policies.

As the haircut ψ increases, the liquidity constraint $\varphi^L(\psi)$ becomes smaller, since the bank's collateral capacity decreases with the haircut (left panel, second row in Figure 7). Since in equilibrium, the bank leverages up to its liquidity constraint, the central bank steers the capital accumulation between the sectors by setting the haircut. When the central bank raises (lowers) the haircut and thereby induces a lower (higher) bank leverage, it channels capital to the bond-financed (loan-financed) sector.

The default-cost-adjusted expected output $\mathbb{E}^\lambda[Y(\psi)]$ attains its maximum at $\psi^D = \bar{\psi} = 0.1100$, since the model parameters are calibrated such that collateral Regime D maximizes the (default-cost-adjusted) expected output $\mathbb{E}[Y(\psi)]$ ($\mathbb{E}^\lambda[Y(\psi)]$) (right panel, first row in Figure 6). With the haircut ψ increasing in the domain $[\psi^D, \psi^S] = [0.1100, 0.1661]$, the default-cost-adjusted expected output $\mathbb{E}^\lambda[Y(\psi)]$ continuously decreases. At $\psi = \psi^S$, the haircut is sufficiently large, so that the arising liquidity constraint $\varphi^L(\psi^S)$ does not induce any solvency risk and the economy attains the expected output level $\mathbb{E}[Y(\psi^S)]$.[18] For haircuts ψ smaller but close to ψ^S, the default-cost-adjusted expected output $\mathbb{E}^\lambda[Y(\psi)]$ falls short of the expected output $\mathbb{E}[Y(\psi^S)]$ induced by ψ^S. On the one side, the absolute spread between the expected gross interest rates $\mathbb{E}[r^L_s]$ and r^B on loans and bonds is

[18]Recall that the model is calibrated such that $\mathbb{E}[Y(\psi^S)] = \mathbb{E}^\lambda[Y(\psi^D)]$ to ensure the (weak) optimality of Regime D.

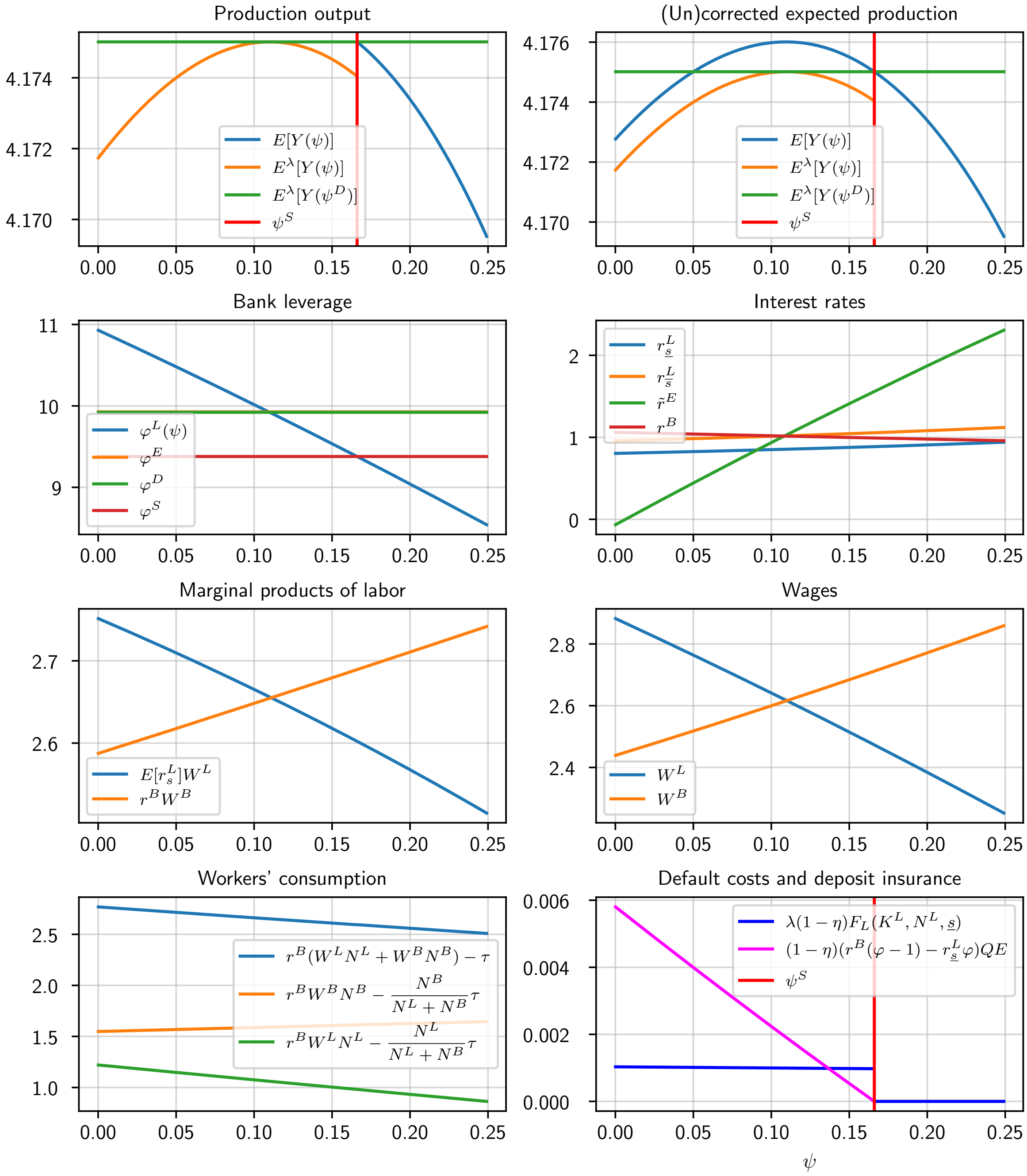

Figure 6: US: Computations for $\psi \in [0.0100, 0.2500]$.

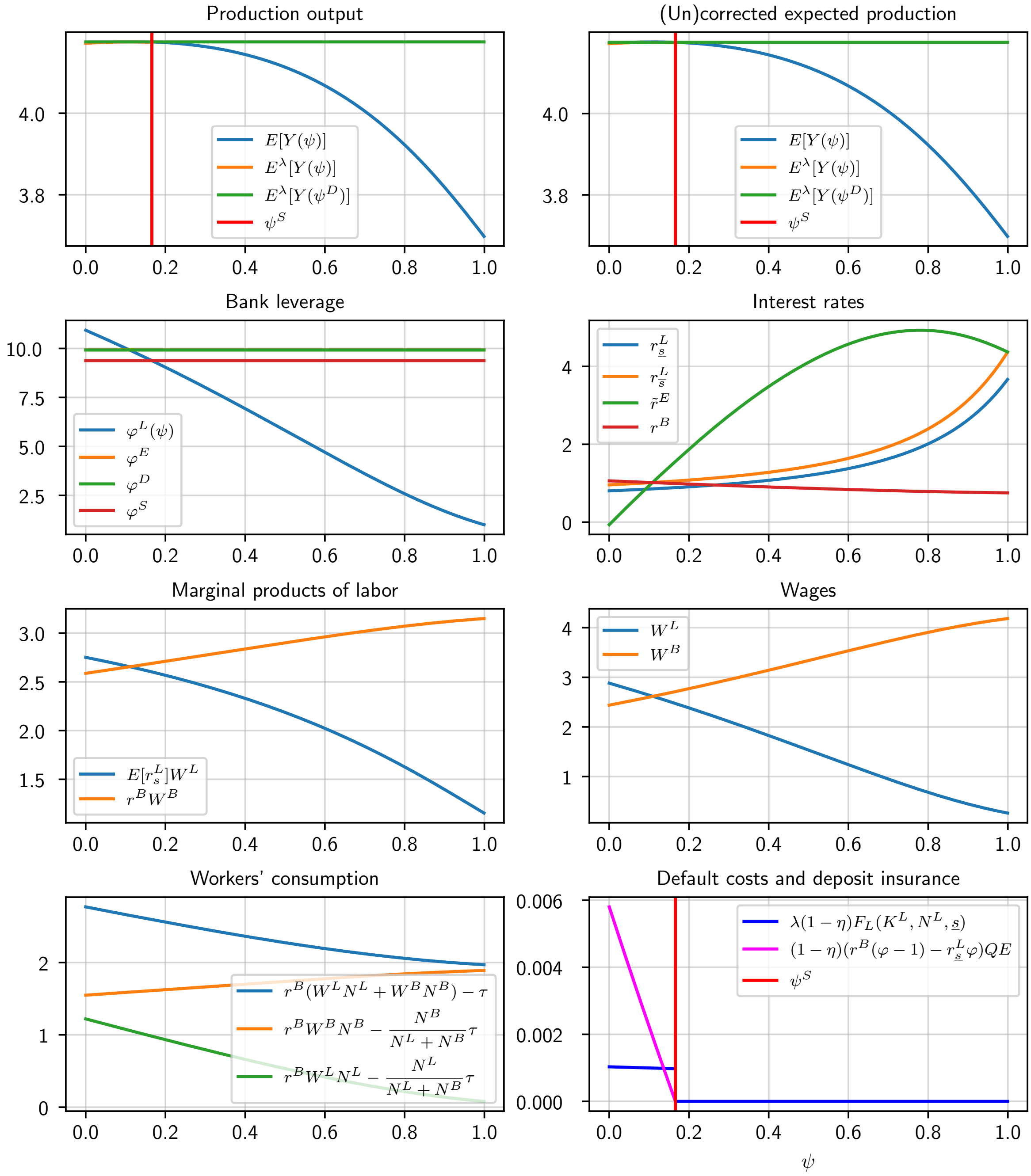

Figure 7: US: Computations for $\psi \in [0.0100, 0.9999]$.

smaller for ψ than for ψ^S, and the capital across sectors is thus allocated more efficiently. On the other side, ψ is small enough, so that default costs and expenses for deposit insurance arise (right panel, fourth row in Figure 6). Eventually, these default-related costs are larger than the efficiency gains compared to Regime S.

The change of capital allocation that is induced by different haircuts, is also reflected in the curves describing the factor prices and marginal products of labor (right panel, second row and both panels, third row in Figure 7). When the central bank tightens the collateral framework by increasing the haircut ψ, the central bank induces an additional accumulation of capital in the bond-financed sector, so that the marginal product of capital decreases in Sector B and increases in Sector L. Since the rental rate of capital Q is normalized across all haircut levels, these changes of the marginal products of capital in Sectors B and L translate into a falling gross interest rate on bonds and deposits $r^B = r^D$ and increasing state-contingent gross interest rates on loans $r^L_{\underline{s}}$, $r^L_{\bar{s}}$, and $\mathbb{E}[r^L_s]$ (right panel, second row in Figure 7). Vice versa, when the central bank increments the haircut ψ, the marginal product of labor in Sector B (L) increases (decreases), which is captured by an increase (decrease) of $r^B W^B$ ($\mathbb{E}[r^L_s]W^L$) (left panel, third row in Figure 7). Since r^B falls when ψ increases, W^B increases with ψ. Moreover, we observe that W^L overcompensates the increase of $\mathbb{E}[r^L_s]$ by falling faster than $\mathbb{E}[r^L_s]$ grows, such that the product $\mathbb{E}[r^L_s]W^L$ decreases (right panel, third row in Figure 7).

The surge of the expected return $\tilde{r}^E$ on bank equity for an increasing haircut $\psi \in (0, 0.7811]$ is particularly salient (right panel, second row in Figure 7). One could expect a decrease of $\tilde{r}^E$ on the whole domain of the haircut due to a decrease of bank leverage $\varphi = \varphi^L(\psi)$, but this force is overcompensated for two reasons. First, the decrease of leverage causes a higher expected marginal product of capital in Sector L, which translates into a higher expected gross interested rate $\mathbb{E}[r^L_s]$ on loans. Second, since more capital is channeled to Sector B when ψ increases, the marginal product of capital in Sector B falls. For Q fixed, this decreases the gross interest rate r^B on bonds. In equilibrium bonds and deposits pay the same gross interest $r^B = r^D$, so that the value of the banks' liabilities to deposit-holders decreases. This reduction in deposit-funding costs puts upward-pressure on $\tilde{r}^E$.

After examining how a change of the haircut affects factor prices and interest rates, we can analyze how the welfare of each group of agents changes with the haircut. Households and investors have linear capital accumulation and consumption rules according to Lemmata 1 and 2, so that households' and investors' welfare increase if and only if $r^B Q$ and $\tilde{r}^E Q$ increase, respectively. Hence, households benefit if the central bank loosens the collateral framework by lowering the haircut, whereas investors benefit if the central bank tightens the collateral framework up to a certain level $\tilde{r}^E = 0.7811$ where $\tilde{r}^E$ reaches its top (right panel, second row in Figure 7). Workers in Sector B benefit from an increase of ψ. Their wage W^B grows faster than the return $r^B = r^D$ on their deposit- and bond-holdings falls, so that their income $r^B W^B$ after interest payments increases with the haircut ψ (left panel, third row in Figure 7). Since bank-resolution costs (*blue*) and expenses for deposit insurance (*magenta*) decrease with the haircut and so do taxes (right panel, fourth row in Figure 7), the increase of income $r^B W^B$ translates into a higher consumption level of workers in Sector B (left panel, fourth row in Figure 7). The consumption of workers in Sector L decreases with ψ. In particular, they prefer a small haircut ψ that even allows banks to take on solvency risk and thus to generate taxes that finance the government's expenses for bank-resolution costs and deposit insurance. A small haircut ψ translates into a high level of bank leverage $\varphi^L(\psi)$ and a high level of capital $K^L = \alpha\varphi^L(\psi)E$ in Sector L, which entails productivity gains of labor N^L and a greater income $r^B W^L$. This high income compensates for the taxes $\frac{N^L}{N^L+N^B}\tau$ workers in Sector L have to pay. We put on record that investors and workers in Sector B benefit from a moderate increase of $\psi \in [0, 0.7811]$, whereas households and workers in Sector L suffer from it.

7.3 Mainly Loan-Financed Euro Economy

In the calibration presented in Section 7.1, we considered the mainly bond-financed US economy. However, the economic activities in other developed countries are mainly loan-financed. The Euro area, for instance, differs from the US economy in that regard.

De Fiore and Uhlig (2011) show that the loan-to-bond-capital ratio in the Euro area is $K^L/K^B = 5.4800$, which is more than eight times higher than in the US ($K^L/K^B = 0.6667$).

Analogously to the calibration to the mainly bond-financed US economy, we calibrate the model parameters to a mainly loan-financed economy. We want to isolate the effect of a change of the loan-to-bond-capital ratio on the calibrated parameters and the steady-state levels of capital. We adopt all calibration targets from the US economy and only change the calibration target of the loan-to-bond-capital ratio from $\overline{K^L/K^B} = 0.6667$ to $\overline{K^L/K^B} = 5.4800$. That is, we calibrate our model to a stylized economy that resembles the US in all respects, apart from the loan-to-bond-capital ratio, which is taken from the Euro area.

In Appendix C, we present the calibrated parameters, the equilibrium factor prices and interest rates, and the steady-state levels of capital, production, and consumption for the loan-financed economy. Table 12 shows the calibrated model parameters for this stylized economy. As for a mainly bond-financed economy, the obtained parametrization permits a steady state for both Regimes D and S. The two steady-state levels of capital, equilibrium (default-cost-adjusted) outputs and bank leverages are displayed in Table 13. Table 14 shows the equilibrium gross interest rates and factor prices in the respective steady states. In Table 15, we see the steady-state consumption levels of investors, households, and workers, and the taxes that workers have to pay in the steady state associated with Regime D.

Comparing the calibrated parameters for the mainly bond-financed and the mainly loan-financed economy (see Tables 2 and 12, respectively), we identify three striking differences, which are illustrated in Table 8. In the loan-financed economy, (a) the loan-to-bond-labor ratio N^L/N^B, which equals the calibrated loan-to-bond-capital ratio $\overline{K^L/K^B}$, is more than eight times higher than in the bond-financed economy. Moreover, (b) the negative TFP shock $\tilde{A}^L_{\underline{s}}$ and (c) the default-cost parameter λ are smaller in the mainly loan-financed economy, so that default costs are lower in case of bank insolvency. Since the loan-to-bond-labor ratio is higher, the attenuation of marginal products of capital across

	Parameters		
Economy	N^L/N^B	$\tilde{A}^L_{\underline{s}}$	λ
Bond-financed	0.6667	0.8426	0.0354
Loan-financed	5.4800	0.8278	0.0181

Table 8: Differences in the calibration between the mainly bond-financed and the mainly loan-financed economy.

sectors requires a larger share of capital in Sector L. Additionally, the lower default costs incentivize the central bank to induce this higher share of capital employed in Sector L. Hence, the differences (a), (b), and (c) in the calibration of the loan-financed economy support a greater capital accumulation in Sector L.

When we compare the steady-state variables under Regime D between the loan-financed and the bond-financed economy, we observe similarities and differences. The most striking ones are displayed in Table 9.

	Steady state levels under Regime D				Loan return	Consumption and taxes			
Economy	ρ	E	$E+K$	$\mathbb{E}^\lambda[Y^D]$	$\mathbb{E}[r^L_s]$	C^I	C^H	C^W	τ
Bond-financed	0.1245	5.6018	50.6020	4.1750	1.0152	0.0874	0.6787	2.6516	0.0029
Loan-financed	0.3056	11.8432	50.6020	4.1750	1.0149	0.1847	0.5846	2.6484	0.0061

Table 9: Similarities and differences of the steady-state variables under Regime D between the mainly bond-financed and the mainly loan-financed economy.

Although the calibrated parameters change between the two economies, we observe that in both economies, the aggregate capital levels $E+K$ under Regime D are the same and so are the default-cost-adjusted expected output levels $\mathbb{E}^\lambda[Y^D]$.[19] However, to attain the higher concentration of capital in Sector L for the same bank leverage $\overline{\varphi}$, the investors-to-households-capital ratio ρ has to be considerably higher in the mainly loan-financed economy, that is, the investors have to hold a higher share of the aggregate capital in the economy. Strikingly, the banks' expected return $\mathbb{E}[r^L_s]$ on loans is higher in the loan-financed economy. The lower level of productivity $\tilde{A}^L_{\underline{s}}$ after a negative shock in Sector L,

[19]We derive from our considerations in Section 7.1 that the steady-state level of aggregate capital $E+K$ does not depend on the loan-to-bond-capital ratio K^L/K^B. Aggregate capital is mainly driven by the capital-to-output ratio. From the aggregate-capital-to-investors-capital ratio $\varphi^M = \left(1+\frac{K^B}{K^L}\right)\alpha\overline{\varphi}$,

which we have already discussed in the context of default costs, leads to a lower repayment factor $r^L_{\underline{s}}$. Due to the different steady-state levels of investors' and households' capital, we also observe differing levels of steady-state consumption and taxes. Since investors (households) hold a higher (lower) level of capital in the loan-financed economy, compared to the bond-financed economy, their consumption level C^I (C^H) is also higher (lower). The higher tax level in the mainly loan-financed economy reflects the higher default costs due to the higher accumulation of capital in Sector L. Although the default-cost-driving parameters λ and $\tilde{A}^L_{\underline{s}}$ are smaller, the higher level of capital employed in Sector L induces higher government expenses for bank resolution and deposit insurance, which have to be covered by workers' taxes. This higher tax level exactly explains the difference in the workers' consumption level C^W between the loan-financed and the bond-financed economy.

In Section 7.2, we executed three simulation studies in the calibrated model of the mainly bond-financed economy. We observe that analogous simulation studies of changes in the productivity risk and the investors-to-households-capital ratio for the mainly loan-financed economy are qualitatively very similar. Therefore, we only present an analysis of the equilibrium outcomes for different levels of the haircut in analogy to Subsection 7.2.3. We analyze how the equilibrium outcomes in the mainly loan-financed economy react to non-optimal changes of the central bank's haircut policy. We take the model parameters from the calibration of the loan-financed economy (see in Table 12 in Appendix C) and the resulting steady-state capital levels under Regime D (see Table 13 in Appendix C) as given, and vary only the haircut ψ. The dynamics of the endogenous model variables are illustrated in Figure 8, which displays the local changes for the hair-

the labor endowment $N^L = \frac{N}{1+\overline{K^B}/\overline{K^L}}$, and Equation (36), we derive that

$$
\begin{aligned}
E + K = \varphi^M E &= \left[\frac{\overline{K+E}}{\mathbb{E}^\lambda[Y]} \frac{\tilde{A}^B}{\varphi^M} \left(1 + \frac{\overline{K^B}}{\overline{K^L}}\right) \right]^{\frac{1}{1-\alpha}} N^L (\alpha\overline{\varphi})^{\frac{\alpha}{1-\alpha}} \varphi^M \\
&= \left[\frac{\overline{K+E}}{\mathbb{E}^\lambda[Y]} \tilde{A}^B \left(1 + \frac{\overline{K^B}}{\overline{K^L}}\right) \right]^{\frac{1}{1-\alpha}} \frac{N}{1 + \frac{\overline{K^B}}{\overline{K^L}}} (\alpha\overline{\varphi})^{\frac{\alpha}{1-\alpha}} \left[\left(1 + \frac{\overline{K^B}}{\overline{K^L}}\right) \alpha\overline{\varphi} \right]^{-\frac{\alpha}{1-\alpha}} \\
&= \left[\frac{\overline{K+E}}{\mathbb{E}^\lambda[Y]} \tilde{A}^B \right]^{\frac{1}{1-\alpha}} N.
\end{aligned}
$$

cut range $[0.0100, 0.2500]$, and in Figure 9, which gives an overview of the whole haircut range $[0.0100, 0.9999]$.

The most striking difference compared to the mainly bond-financed economy is that the bank leverage $\varphi^L(\psi)$ in the loan-financed economy is less sensitive to changes of the haircut for moderate values $\psi \leq 0.5$ (left panel, second row in Figures 7 and 9). Since the investors' steady-state capital level in the loan-financed economy is significantly higher, a small change of the bank-leverage ratio in the mainly loan-financed economy induces the same change of the capital allocation between sectors L and B as a strong change of the bank-leverage ratio in the mainly bond-financed economy. If the haircut ψ changes, $\varphi^L(\psi)$ is less sensitive in the loan-financed economy for $\psi \leq 0.5$. For increasing $\psi > 0.5$, the sensitivity of $\varphi^L(\psi)$ in the mainly loan-financed economy increases, since the overaccumulation of capital in the loan-financed sector is attenuated.

8 Conclusion

The collateral policy of central banks is a pillar of monetary policy. In this paper, we embed a model of financial intermediation that stresses the dual role of banks in private money creation and credit extension into a two-sector stochastic neoclassical model. In analogy to the Taylor rule for interest-rate policies, we derive analytical haircut rules that specify the central bank's optimal monetary policy on collateral rather than on interest rates.

For a welfare analysis, we conduct a calibration of the model to the US economy. We find that welfare is higher when the central bank permits high bank lending activity than when it does not, since the gains from a more efficient allocation of funds between the loan-financed and the bond-financed sector outweigh the bank-default costs arising from extensive bank lending. Moreover, we find that—maybe counter-intuitively—banks are worse off when the central bank applies loose collateral requirements than when it applies tight ones. The reason is that, since the banks can leverage more extensively, their larger business volume does not compensate for the lower interest rates on loans,

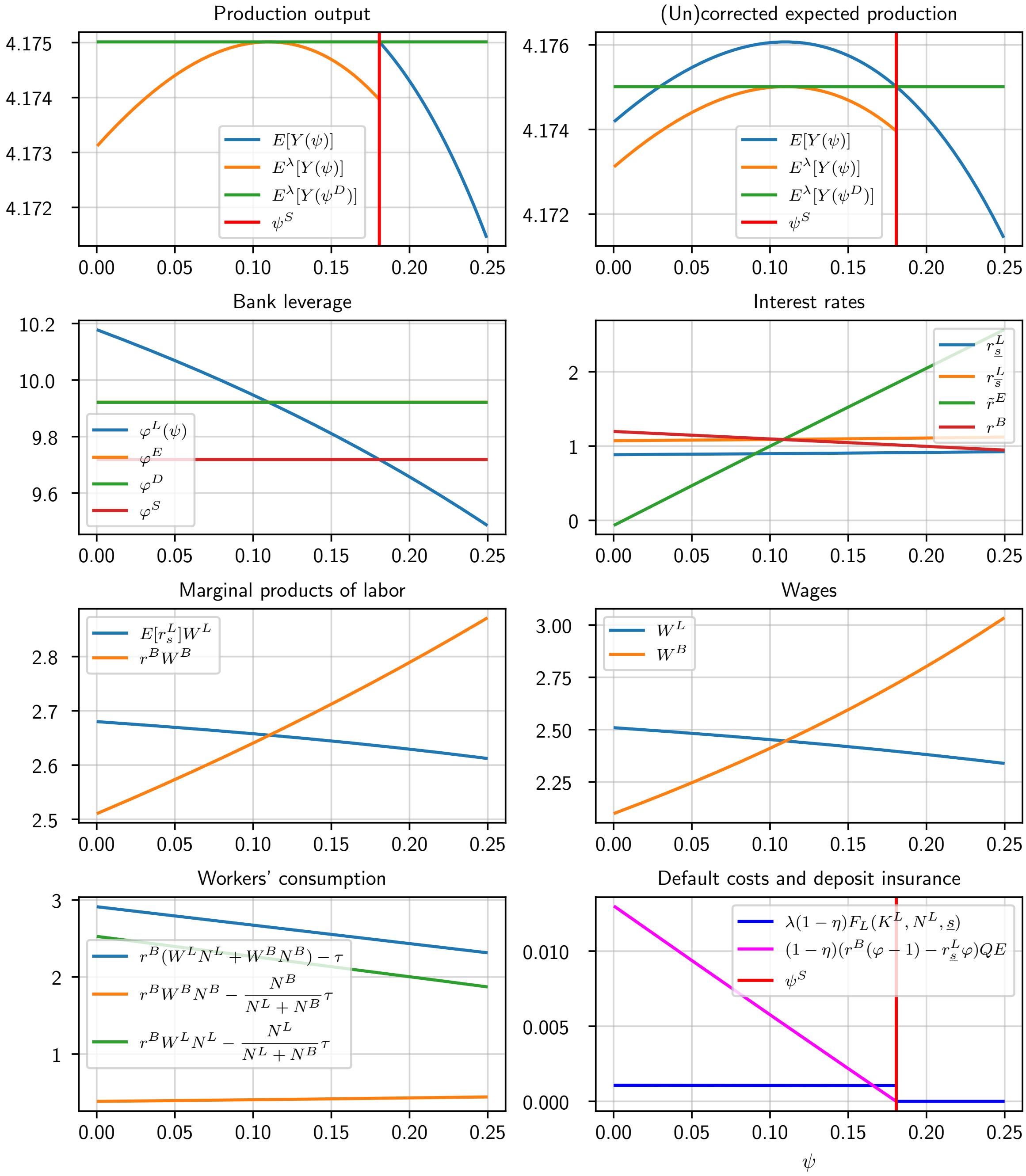

Figure 8: EU: Computations for $\psi_t \in [0.0100, 0.2500]$.

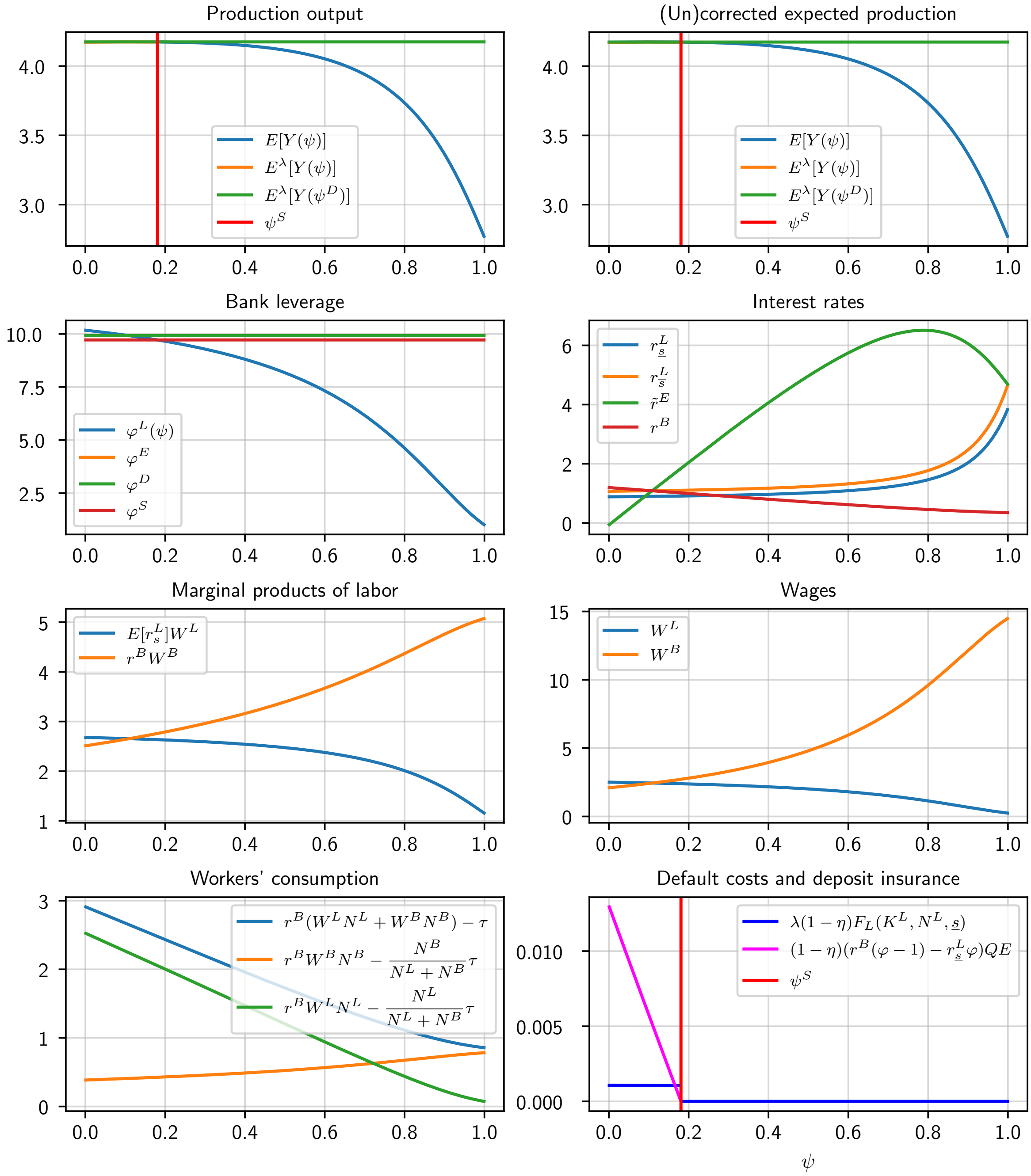

Figure 9: EU: Computations for $\psi_t \in [0.0100, 0.9999]$.

which are caused by the overaccumulation of capital in the loan-financed sector and the resulting depressed productivity of capital. Vice versa, the workers in the loan-financed sector benefit from small haircuts, since the higher labor productivity in this sector and the resulting higher wage level outweigh the taxes the workers have to pay to cover the government's expenses for bank resolution and deposit insurance. Bondholders benefit from loose collateral requirements as well, due to the high productivity of capital in the bond-financed sector, which is caused by the scarcity of capital in this sector.

We also calibrate the model to a stylized mainly loan-financed economy that resembles the US economy apart from the loan-to-bond-capital ratio, which we take from the economy of the Euro area. We find qualitatively very similar patterns in equilibrium outcomes and in the transition of monetary policy in different macroeconomic environments. The only striking difference between the two economies is that in the US economy, bank leverage is more responsive to haircut changes.

The simple formulae and properties of haircut rules may be a useful starting point for a more elaborate analysis. For instance, introducing monitoring decisions by banks may add a further factor that controls the optimal leverage of banks and thus the optimal collateral policy of the central bank. Introducing different time scales for production and maturity transformation is another promising direction of future research. Finally, generalizations of production and utility, the use of heterogeneous households, and a combination of this approach with price rigidities are long-term objectives that might allow to develop a comprehensive body of knowledge on optimal haircut rules.

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A Lists of Notation

	Symbol	Meaning
Parameters	ρ_t	investors-to-households-capital ratio at time t
	α	output elasticity of capital
	λ	bank-resolution cost in case of default as share of output of matched firm
	γ_t	liquidity demand of a bank as a share of issued deposits at time t
	β_I	investors' time discount factor
	β_H	households' time discount factor
	N_t^B	labor endowment employed in Sector B at time t
	N_t^L	labor endowment employed in Sector L at time t
	g_B	growth rate of total factor productivity in Sector B
	g_L	growth rate of total factor productivity in Sector L
	z_t	macroeconomic shock to firms in Sectors B and L
	s_t	idiosyncratic shock to firm in Sector L
	η_z	probability of idiosyncratic shock $\overline{s}$ given macro shock z.
	A_z^B	TFP in Sector B after macro shock z without TFP growth
	$A_{z,s}^L$	TFP in Sector L after macro shock z and idiosyncratic shock s without TFP growth
	$\tilde{A}_t^B$	TFP in Sector B at time t
	$\tilde{A}_{s_t,t}^L$	TFP in Sector L at time t after shock s_t
	$\tilde{a}_t$	relation between the production capacities in Sectors B and L without default risk
	$\tilde{a}_t^\lambda$	relation between the production capacities in Sectors B and L with default risk

Table 10: List of notation for model parameters.

	Symbol	Meaning
Capital endowments	E_t	investors' capital
	K_t	households' capital
	K_t^L	capital good purchased by firms in Sector L
	$K_t^{L,H}$	capital good sold by households to firms in Sector L
	$K_t^{L,I}$	capital good sold by investors to firms in Sector L
	K_t^B	capital good purchased by firms in Sector B
	$K_t^{B,H}$	capital good sold by households to firms in Sector B
	$K_t^{B,I}$	capital good sold by investors to firms in Sector B
Interest rates	$r_{CB,t}$	gross interest rate on central bank deposits and loans
	r_t^D	gross interest rate on bank deposits
	r_t^B	gross interest rate on bonds
	$r_{s_t,t}^L$	gross interest rate on loan to a firm hit by shock s_t
	$r_{s_t,t}^E$	gross dividend on bank equity
	$\tilde{r}_t^E$	average gross dividend on bank equity
	ψ_t	collateral haircut applied by central bank
	$\tilde{\psi}_t$	collateral haircut applied in interbank market
	$\tilde{r}_t^E$	average gross dividend on equity funding to a bank
	L_t	loan volume of the representative bank
	D_t	deposits' volume of the representative bank
	ζ_t	share of households' proceeds from the factor market held as deposits
	ξ_t	share of workers' labor income held as deposits
	φ_t	bank-leverage ratio
	φ_t^S	solvency constraint on bank-leverage ratio
	$\varphi_t^L(\psi_t)$	liquidity constraint on bank leverage given collateral haircut ψ_t
	Q_t	rental rate of capital
	W_t^B	wage to workers employed in bond-financed sector
	W_t^L	wage to workers employed in bond-financed sector
Output	Y_t	total output
	Y_t^H	output purchased by households
	$Y_t^{B,H}$	output purchased by households from firms in Sector B
	$Y_t^{L,H}$	output purchased by households from firms in Sector L
	$Y_t^{B,I}$	output purchased by investors from firms in Sector B
	$Y_t^{L,I}$	output purchased by investors from firms in Sector L

Table 11: List of notation for endogenous variables.

B Calibration Solution

B.1 Calibration to a Regime Permiting Bank Default

Step by step, we identify the parameters of the model such that the calibration targets and constraints in Table 2 are met. In particular, we use the equilibrium properties outlined in Section 4. To exploit the interest rate equality $r^B = \bar{r}^D$ and to use the optimal monetary policy in Proposition 1, we presume that Assumption 1, i.e., $\tilde{a}^\lambda \rho < 1$, holds in steady state. We then verify ex post that $\tilde{a}^\lambda \rho < 1$ indeed holds true, given the parametrization in Table 2 and the steady-state capital distributions in Table 3. Moreover, we will see that $\alpha < \eta$, so that with Lemma 8, workers' labor income suffices to cover government expenses for deposit insurance and bank-default costs.

Sector-specific labor endowments. First, we determine the sector-specific labor endowments N^L and N^B. Recall from Equations (18) and (19) that equating the first-order conditions with respect to capital and labor in both production sectors yields equations

$$\frac{W^L}{Q} = \frac{1-\alpha}{\alpha}\frac{K^L}{N^L} \qquad \text{and} \qquad \frac{W^B}{Q} = \frac{1-\alpha}{\alpha}\frac{K^B}{N^B}. \tag{30}$$

By solving these equations for Q, equating them, and then exploiting the long-term wage equality $W \equiv W^B = W^L$, we obtain

$$\frac{1-\alpha}{\alpha}\frac{K^L}{W^L N^L} = \frac{1-\alpha}{\alpha}\frac{K^B}{W^B N^B} \qquad \Leftrightarrow \qquad \frac{N^L}{N^B} = \overline{\frac{K^L}{K^B}}.$$

Since total labor supply is normalized to $N = 1$, we deduce from $N^L + N^B = N$ and the equation for loan-to-bond-labor share N^L/N^B from above that

$$N^L = \frac{\overline{\frac{K^L}{K^B}}}{1+\overline{\frac{K^L}{K^B}}} N = 0.4 \qquad \text{and} \qquad N^B = \frac{1}{1+\overline{\frac{K^L}{K^B}}} N = 0.6. \tag{31}$$

Output elasticity of capital. Next, we determine the output elasticity of capital α. We first show that in steady state under Regime D, it must hold that $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$, and we use the calibration target $\overline{r^D WN/\mathbb{E}^\lambda[Y]}$ for the labor share of income to derive an explicit solution for α. Recall from Lemma 5 that the equilibrium capital allocations are $K^L = \alpha\overline{\varphi}E$ and $K^B = (\varphi^M - \alpha\overline{\varphi})E$, so that

$$\overline{\frac{K^B}{K^L}} = \frac{\varphi^M - \alpha\overline{\varphi}}{\alpha\overline{\varphi}} \qquad \Leftrightarrow \qquad \varphi^M = \left(1 + \overline{\frac{K^B}{K^L}}\right)\alpha\overline{\varphi} \equiv \varphi^M(\alpha). \tag{32}$$

According to Corollary 1, the optimal leverage ratio φ^D under Regime D takes the form

$$\varphi^D = \frac{1}{\alpha}\left(\frac{\varphi^M}{1+\tilde{a}^\lambda}\right) \qquad \text{with} \qquad \tilde{a}^\lambda = \frac{N^B}{N^L}\left(\frac{\tilde{A}^B}{\mathbb{E}^\lambda[\tilde{A}^L_s]}\right)^{\frac{1}{1-\alpha}}. \tag{33}$$

Then, plugging the expression for $\varphi^M = \varphi^M(\alpha)$ in Equation (32) into the expression for φ^D in Equation (33), we obtain

$$\varphi^D = \left(1 + \overline{\frac{K^B}{K^L}}\right)\left(\frac{\overline{\varphi}}{1+\tilde{a}^\lambda}\right) = \overline{\varphi} \qquad \Leftrightarrow \qquad 1 + \overline{\frac{K^B}{K^L}} = 1 + \overline{\frac{K^B}{K^L}}\left(\frac{\tilde{A}^B}{\mathbb{E}^\lambda[\tilde{A}^L_s]}\right)^{\frac{1}{1-\alpha}},$$

where we used that $N^B/N^L = \overline{K^B/K^L}$. We infer that $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$.

Under Regime D, the central bank accepts bank default. Hence, expected default costs must be taken into account when the expected output is considered. Given the optimal capital accumulations in Lemma 5 and with $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$, the default-cost-adjusted expected output when the central bank applies ψ^D, is given by

$$\begin{aligned}\mathbb{E}^\lambda[Y(\psi^D)] &= \tilde{A}^B(N^B)^{1-\alpha}(K^B)^\alpha + \mathbb{E}^\lambda[\tilde{A}^L_s](N^L)^{1-\alpha}(K^L)^\alpha \\ &= \tilde{A}^B\left[(N^B)^{1-\alpha}(\varphi^M - \alpha\overline{\varphi})^\alpha + (N^L)^{1-\alpha}(\alpha\overline{\varphi})^\alpha\right]E^\alpha. \end{aligned} \tag{34}$$

Now, we derive an explicit expression for the output elasticity of capital α via the labor share of income $\overline{r^D WN/\mathbb{E}^\lambda[Y]}$ that is given as calibration target. With the relative factor prices from Equation (30), we can express total wage payments as

$$WN = W^L N^L + W^B N^B = \frac{1-\alpha}{\alpha}Q(K^L + K^B) = \frac{1-\alpha}{\alpha}Q\varphi^M E.$$

Note that wages are paid at the start of the period, whereas the output good is traded at the end. Hence, comparing the value of total output with labor income, we have to account for the interest payments on wage income, given by r^B and r^D. Since, under Assumption 1, interest payments on bonds and deposits are equal in equilibrium according to Lemma 7, the labor share of income reads

$$\begin{aligned}
\frac{\overline{r^D W N}}{\mathbb{E}^\lambda[Y]} &= \frac{1-\alpha}{\alpha} \frac{r^B Q \varphi^M E}{\mathbb{E}^\lambda[Y]} \\
&= \frac{1-\alpha}{\alpha} \frac{\alpha \tilde{A}^B (N^B)^{1-\alpha} (\varphi^M - \alpha\overline{\varphi})^{\alpha-1} E^{\alpha-1} \varphi^M E}{\tilde{A}^B \left[(N^B)^{1-\alpha} (\varphi^M - \alpha\overline{\varphi})^\alpha + (N^L)^{1-\alpha} (\alpha\overline{\varphi})^\alpha \right] E^\alpha} \\
&= (1-\alpha) \frac{(\varphi^M - \alpha\overline{\varphi})^{\alpha-1} \varphi^M}{(\varphi^M - \alpha\overline{\varphi})^\alpha + \left(\frac{N^L}{N^B}\right)^{1-\alpha} (\alpha\overline{\varphi})^\alpha} \\
&= (1-\alpha) \frac{\left(\overline{\frac{K^B}{K^L}} \alpha\overline{\varphi}\right)^{\alpha-1} \left(1 + \overline{\frac{K^B}{K^L}}\right) \alpha\overline{\varphi}}{\left(\overline{\frac{K^B}{K^L}} \alpha\overline{\varphi}\right)^\alpha + \left(\overline{\frac{K^B}{K^L}}\right)^{\alpha-1} (\alpha\overline{\varphi})^\alpha} \\
&= (1-\alpha) \frac{1 + \overline{\frac{K^B}{K^L}}}{\overline{\frac{K^B}{K^L}} + 1} \\
&= 1-\alpha,
\end{aligned}$$

where we have used $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$, the identity $N^L/N^B = \overline{K^L/K^B}$ of loan-to-bond-labor ratio and loan-to-bond-capital ratio, and $\varphi^M = \left(1 + \overline{\frac{K^B}{K^L}}\right) \alpha\overline{\varphi}$ from Equation (32). We obtain

$$\alpha = 1 - \frac{\overline{r^D W N}}{\mathbb{E}^\lambda[Y]} = 0.3642. \tag{35}$$

Note that, as in the standard real business-cycle model with one production sector and without any monetary dimension, the labor share of income is equal to the power with which the factor labor enters the Cobb-Douglas production function.

Investors' and households' capital endowments under Regime D. We compute the investors' and households' capital endowments under Regime D by using the fact that the capital-to-output ratio is at hand as a calibration target. With the equality $\varphi^M E = E + K$, the labor endowments, given in Equation (31), and the expression of

$\mathbb{E}^\lambda[Y]$ in Equation (34), the capital-to-output ratio reads

$$
\begin{aligned}
&& \frac{\varphi^M E}{\mathbb{E}^\lambda[Y]} &= \overline{\frac{K+E}{\mathbb{E}^\lambda[Y]}} \\
\Leftrightarrow && \varphi^M E &= \overline{\frac{K+E}{\mathbb{E}^\lambda[Y]}}\mathbb{E}^\lambda[Y] \\
\Leftrightarrow && \varphi^M E &= \overline{\frac{K+E}{\mathbb{E}^\lambda[Y]}}\tilde{A}^B\left[(N^B)^{1-\alpha}(\varphi^M-\alpha\overline{\varphi})^\alpha+(N^L)^{1-\alpha}(\alpha\overline{\varphi})^\alpha\right]E^\alpha \\
\Leftrightarrow && E &= \left[\overline{\frac{K+E}{\mathbb{E}^\lambda[Y]}}\frac{\tilde{A}^B}{\varphi^M}(\alpha\overline{\varphi})^\alpha(N^L)^{1-\alpha}\left(1+\overline{\frac{K^B}{K^L}}\right)\right]^{\frac{1}{1-\alpha}} \\
\Leftrightarrow && E &= \left[\overline{\frac{K+E}{\mathbb{E}^\lambda[Y]}}\frac{\tilde{A}^B}{\varphi^M}\left(1+\overline{\frac{K^B}{K^L}}\right)\right]^{\frac{1}{1-\alpha}}N^L(\alpha\overline{\varphi})^{\frac{\alpha}{1-\alpha}}. \qquad (36)
\end{aligned}
$$

Thus, we obtain E explicitly since N^L and α are already determined in Equations (31) and (35), respectively, and so is φ^M in Equation (32). Through $\varphi^M E = E+K$, we obtain households' capital K as well.

Intertemporal variables. Next, we determine the capital depreciation rate δ and the time discount factors β_I and β_H, which drive the capital accumulation in the economy. To be the steady-state capital stocks, E and K must be the fixed points of the investors' and households' laws of capital motion

$$K_{t+1} = (1-\delta)K_t + I_t^H \qquad \text{and} \qquad E_{t+1} = (1-\delta)E_t + I_t^I$$

in Lemmata 1 and 2, respectively. Since in steady state, it holds that $K = K_{t+1} = K_t$ and $E = E_{t+1} = E_t$, households and investors save $I^H = \delta K$ and $I^I = \delta E$, respectively. The savings rate $\overline{\sigma}$ thus fulfills $\overline{\sigma}\mathbb{E}^\lambda[Y] = \delta K + \delta E$. As the savings rate $\overline{\sigma}$ and the capital-to-output ratio $\overline{(E+K)/\mathbb{E}^\lambda[Y]}$ are calibration targets, we can compute δ from

$$\overline{\sigma} = \delta\overline{\frac{E+K}{\mathbb{E}^\lambda[Y]}} \qquad \Leftrightarrow \qquad \delta = \overline{\sigma}\left(\overline{\frac{E+K}{\mathbb{E}^\lambda[Y]}}\right)^{-1} = 0.0150.$$

We have already computed the households' and investors' capital levels K and E, and the output elasticity of capital α is known. Hence, we can derive the capital levels $K^L = \alpha\overline{\varphi}E$

and $K^B = (\varphi^M - \alpha\overline{\varphi})E$ employed in sectors L and B, respectively, from Lemma 5. This allows to determine the rental rate of capital

$$Q = \alpha \tilde{A}^B (N^B/K^B)^{1-\alpha}/\overline{r}^D$$

through the first-order condition of firms in Sector B with respect to capital. Moreover, the linear accumulation rules of households and investors in Lemmata 1 and 2 can be rearranged to

$$\beta_H = \frac{1}{1+\overline{r}^D Q - \delta} \qquad \text{and} \qquad \beta_I = \frac{1}{1+\overline{r}^E Q - \delta}.$$

Hence, we explicitly obtain β_I and β_H.

Total factor productivity in Sector L. We have already noted that the default-cost-adjusted expected TFP $\mathbb{E}^\lambda[\tilde{A}^L_s]$ in Sector L and the TFP $\tilde{A}^B$ in Sector B are equal in the calibrated steady state, i.e., $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$. Based on this equality, we investigate the relation of the TFPs $\tilde{A}^L_{\underline{s}}$ and $\tilde{A}^L_{\overline{s}}$ in Sector L after a positive and negative idiosyncratic shock, respectively, to the normalized total factor productivity level $\tilde{A}^B = 1$ in Sector B. Perfect competition on the loan market and the firms' first-order conditions with respect to capital yield

$$\begin{aligned} r^L_{\overline{s}} Q &= \alpha \tilde{A}^L_{\overline{s}} (N^L)^{1-\alpha} (K^L)^{\alpha-1} \quad \text{and} \\ r^B Q &= \alpha \tilde{A}^B (N^B)^{1-\alpha} (K^B)^{\alpha-1}. \end{aligned}$$

With $N^L/N^B = \overline{K^L/K^B}$ and $r^B = \overline{r}^D$, we obtain

$$r^L_{\overline{s}} = \frac{\tilde{A}^L_{\overline{s}}}{\tilde{A}^B} \overline{r}^D. \tag{37}$$

Regime D induces solvency risk, so that, due to limited liability of banks and with Equation (15), the return of investors on equity is expressed by

$$\overline{r}^E = \eta[(r^L_{\overline{s}} - \overline{r}^D)\overline{\varphi} + \overline{r}^D].$$

Using Equation (37) yields

$$\overline{r}^E = \eta \left[\left(\frac{\tilde{A}^L_{\overline{s}}}{\tilde{A}^B} \overline{r}^D - \overline{r}^D \right) \overline{\varphi} + \overline{r}^D \right] = \eta \left[\frac{\tilde{A}^L_{\overline{s}} - \tilde{A}^B}{\tilde{A}^B} \overline{\varphi} + 1 \right] \overline{r}^D.$$

Thus, we can express the TFP $\tilde{A}^L_{\overline{s}}$ in Sector L after a positive idiosyncratic shock as

$$\tilde{A}^L_{\overline{s}} = \left[\left(\frac{1}{\eta} \frac{\overline{r}^E}{\overline{r}^D} - 1 \right) \frac{1}{\overline{\varphi}} + 1 \right] \tilde{A}^B.$$

$\tilde{A}^L_{\overline{s}}$ can be explicitly determined, since all variables on the right-hand side are known. With $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$, we have

$$(1-\eta)(1-\lambda)\tilde{A}^L_{\underline{s}} + \eta \tilde{A}^L_{\overline{s}} = \mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B \qquad \Rightarrow \qquad \tilde{A}^L_{\underline{s}} = \frac{\tilde{A}^B - \eta \tilde{A}^L_{\overline{s}}}{(1-\eta)(1-\lambda)} \equiv \tilde{A}^L_{\underline{s}}(\lambda).$$

Hence, the TFP $\tilde{A}^L_{\underline{s}}$ in Sector L after a negative idiosyncratic shock is uniquely determined by λ.

Default costs. To determine the default-cost parameter λ, we examine the relation of liquidity-demand parameter γ to λ. Proposition 1 gives us

$$\psi^D = 1 - \gamma \left(1 - \alpha\rho \frac{1 + \tilde{a}^\lambda}{1+\rho} \right) \frac{\mathbb{E}^\lambda[\tilde{A}^L_s]}{\mathbb{E}[\tilde{A}^L_s]}.$$

From $\psi^D = \overline{\psi}$ and with $\tilde{a}^\lambda = \overline{\frac{K^B}{K^L}}$ and $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$, we can deduce

$$\gamma = (1 - \overline{\psi}) \left(1 - \alpha\rho \frac{1 + \overline{K^B/K^L}}{1+\rho} \right)^{-1} \frac{\mathbb{E}[\tilde{A}^L_s]}{\tilde{A}^B} \equiv \gamma(\lambda), \tag{38}$$

where we used $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$ again.[20] Since all variables on the right-hand side are known, except from $\mathbb{E}[\tilde{A}^L_s] = (1-\eta)\tilde{A}^L_{\underline{s}}(\lambda) + \eta\tilde{A}^L_{\overline{s}}$, which depends on the still unknown variable λ, we write $\gamma = \gamma(\lambda)$.

The default-cost parameter λ is pinned down by the constraint that Regime D indeed induces a default-cost-adjusted expected output that is weakly higher than under Regime S. As mentioned above, we thus set $\mathbb{E}^\lambda[Y(\psi^D)] = \mathbb{E}[Y(\psi^S)]$. Note that we can explicitly compute the default-cost-adjusted expected output under Regime D with the help of Lemma 5,

$$\mathbb{E}^\lambda[Y(\psi^D)] = \tilde{A}^B \left((\varphi^M - \alpha\overline{\varphi})^\alpha (N^B)^{1-\alpha} + (\alpha\overline{\varphi})^\alpha (N^L)^{1-\alpha} \right) E^\alpha,$$

independently of λ. In particular, we use equality $\mathbb{E}^\lambda[\tilde{A}^L_s] = \tilde{A}^B$. Next, we express $\mathbb{E}[Y(\psi^S)]$ by using the haircut ψ^S and the leverage ratio φ^S, which both depend on λ. According to Lemma 10, the haircut ψ^S is given by

$$\psi^S = 1 - \frac{\gamma\tilde{A}^L_{\underline{s}}}{\mathbb{E}[\tilde{A}^L_s]} = 1 - \frac{\gamma(\lambda)\tilde{A}^L_{\underline{s}}(\lambda)}{\tilde{A}^B + \lambda(1-\eta)\tilde{A}^L_{\underline{s}}(\lambda)}.$$

Then, since the representative bank leverages up to its liquidity constraint in equilibrium according to Lemma 6, i.e., $\varphi = \varphi^L(\psi)$, we implicitly obtain φ^S from Corollary 1, as

$$\varphi^S = \frac{\gamma(\lambda)\tilde{A}^B(N^B)^{1-\alpha}(\varphi^M - \alpha\varphi^S)^{\alpha-1}}{\gamma(\lambda)\tilde{A}^B(N^B)^{1-\alpha}(\varphi^M - \alpha\varphi^S)^{\alpha-1} - (1-\psi^S(\lambda))\mathbb{E}[\tilde{A}^L_s](N^L)^{1-\alpha}(\alpha\varphi^S)^{\alpha-1}} \equiv \varphi^S(\lambda).$$

We exploited that reserves, deposits, and bonds pay the same gross interest rate $r^D_{CB} = \overline{r}^D = r^B$ in equilibrium according to Lemma 7. Having determined $\varphi^S = \varphi^S(\lambda)$, we can

[20] Since $\gamma \in (0,1)$, Equation (38) imposes a constraint on our choice of $\overline{\psi}$:

$$\gamma < 1 \qquad \Leftrightarrow \qquad \overline{\psi} > 1 - \left(1 - \alpha\rho\frac{1+\tilde{a}^\lambda}{1+\rho}\right)\frac{\mathbb{E}^\lambda[\tilde{A}^L_s]}{\mathbb{E}[\tilde{A}^L_s]} \equiv \overline{\psi}_{\text{bound}}.$$

To calibrate a steady-state equilibrium such that the central bank applies Regime D, the target haircut $\overline{\psi}$ cannot be arbitrarily low; it must hold that $\overline{\psi} > \overline{\psi}_{\text{bound}}$.

solve

$$
\begin{aligned}
&\mathbb{E}^{\lambda}[Y(\varphi^D)] = \mathbb{E}[Y^S(\varphi^S)] \\
\Leftrightarrow\ &\mathbb{E}^{\lambda}[Y(\varphi^D)] = \tilde{A}^B(\varphi^M - \alpha\varphi^S(\lambda))^{\alpha}(N^B)^{1-\alpha} + (\tilde{A}^B + \lambda(1-\eta)\tilde{A}^L_{\underline{s}}(\lambda))(\alpha\varphi(\lambda))^{\alpha}(N^L)^{1-\alpha}
\end{aligned}
$$

for λ. With λ, all variables in the model are determined.

B.2 A Second Steady State Ruling out Bank Default

We show that there is a steady state, given the parametrization in Table 2, in which the central bank optimally rules out bank default, and we show that this is the only steady state apart from the one that meets the calibration targets in Table 2.

Suppose that there is a steady state such that the central bank applies Regime E. Since under Regime E, equal financing conditions $\mathbb{E}[r^L] = r^B = r^D$ across sectors prevail and no default costs occur, Equation (15) yields

$$
\tilde{r}^E = \varphi\mathbb{E}[r^L] - (\varphi - 1)r^D = r^D.
$$

Dividing the linear capital accumulation rules of investors and households in steady state yields

$$
\frac{\beta_I}{\beta_H} = \frac{1 + r^D Q - \delta}{1 + \tilde{r}^E Q - \delta} = 1.
$$

We have obtained a contradiction since $\beta_I < \beta_H$ in the calibration in Table 2.

Next, we show that, given the calibrated parameters in Table 2, there is a steady state in which the central bank applies Regime S. The bank leverage φ^S is uniquely pinned down by φ^M according to Corollary 1. To pin down φ^M, we use the linear capital accumulation rules of investors and households. Recall that, in steady state, it holds that

$$
\frac{1}{\beta_H} = 1 + r^D Q - \delta \qquad \text{and} \qquad \frac{1}{\beta_I} = 1 + \tilde{r}^E Q - \delta. \tag{39}
$$

When the central bank rules out solvency risk, bank default does not occur and Equation

(15) reads

$$\tilde{r}^E = \varphi^S \mathbb{E}[r^L] - (\varphi^S - 1) r^D.$$

Recalling that the interest rate equality $r^D = r^B$ holds in equilibrium according to Lemma 7, and exploiting the first-order condition with respect to capital in Sector B, we divide the Equations in (39) by one another and obtain

$$\begin{aligned}
\frac{\frac{1}{\beta_I} + \delta - 1}{\frac{1}{\beta_H} + \delta - 1} &= \frac{\varphi^S \alpha \mathbb{E}[\tilde{A}^L_s](N^L)^{1-\alpha}(\alpha\varphi^S)^{\alpha-1} - (\varphi^S - 1)\alpha \tilde{A}^B (N^B)^{1-\alpha}(\varphi^M - \alpha\varphi^S)^{\alpha-1}}{\alpha \tilde{A}^B (N^B)^{1-\alpha}(\varphi^M - \alpha\varphi^S)^{\alpha-1}} \\
&= \frac{\varphi^S \mathbb{E}[\tilde{A}^L_s](N^L)^{1-\alpha}(\alpha\varphi^S)^{\alpha-1}}{\tilde{A}^B (N^B)^{1-\alpha}(\varphi^M - \alpha\varphi^S)^{\alpha-1}} + 1 - \varphi^S \\
&= \frac{\mathbb{E}[\tilde{A}^L_s]}{\tilde{A}^B} \left(\frac{\varphi^M - \alpha\varphi^S}{\alpha\varphi^S} \frac{N^L}{N^B} \right)^{1-\alpha} \varphi^S + 1 - \varphi^S. \qquad (40)
\end{aligned}$$

Since φ^S is uniquely pinned down by φ^M, the right-hand side of the above equation can thus be read as a function in φ^M. Numerically, we can show that Equation (40) has a single solution at $\varphi^M = 5.8598$, which results in an investors-to-households-capital ratio $\rho = 0.2058$. From the first-order condition of firms in Sector B with respect to capital, equal interest rates on bonds and deposits, and with households' steady-state condition in Equation (39), we obtain

$$\begin{aligned}
& \frac{1}{\beta_H} + \delta - 1 = \alpha \tilde{A}^B (N^B)^{1-\alpha} \left(\varphi^M - \alpha \varphi^S(\varphi^M) \right)^{\alpha-1} E^{\alpha-1} \\
\Leftrightarrow \qquad & E = \left[\alpha \tilde{A}^B \left(\frac{1}{\beta_H} + \delta - 1 \right)^{-1} \right]^{\frac{1}{1-\alpha}} \frac{N^B}{\varphi^M - \alpha\varphi^S(\varphi^M)}. \qquad (41)
\end{aligned}$$

From investors' capital level, we obtain households' capital level $K = E/\rho$ as well. Finally, we have to verify that the expected output

$$\mathbb{E}[Y(\psi^S)] = \tilde{A}^B (N^B)^{1-\alpha}[K + (1 - \alpha\varphi^S)E]^\alpha + \mathbb{E}[\tilde{A}^L_s](N^L)^{1-\alpha}(\alpha\varphi^S E)^\alpha$$

under Regime S exceeds the default-cost-adjusted expected output

$$\mathbb{E}^\lambda[Y(\psi^D)] = \tilde{A}^B (N^B)^{1-\alpha}[K + (1 - \alpha\varphi^D)E]^\alpha + \mathbb{E}^\lambda[\tilde{A}^L_s](N^L)^{1-\alpha}(\alpha\varphi^D E)^\alpha$$

induced by Regime D and the associated equilibrium bank leverage

$$\varphi^D = \frac{1}{\alpha}\left(\frac{\varphi^M}{1+\tilde{a}^\lambda}\right).$$

The numerical computations show that the inequality $\mathbb{E}[Y(\psi^S)] > \mathbb{E}^\lambda[Y(\psi^D)]$ indeed holds true, given the investors' capital level E in Equation (41) and the households' capital level K corresponding with E and φ^M that we obtained from Equation (40). The numerical results for the capital levels E and K are displayed in Table 3.

C Calibration for a Mainly Loan-Financed Economy

	Variable	Description	Source	Value
Calibration targets	$\overline{\sigma}$	aggregate saving rate	FRED	0.1814
	$\overline{(K+E)/\mathbb{E}^{\lambda}[Y]}$	capital-to-output ratio	PWT	12.1202
	$\overline{r^D WN/\mathbb{E}^{\lambda}[Y]}$	labor share of income	Fernald (2014)	0.6358
	$\overline{\varphi}$	bank leverage	Call Report	9.9212
	$\overline{r}^E$	gross return on bank equity	Call Report	1.0320
	$\overline{r}^D$	gross return on bank deposits	Drechsler et al. (2017a)	1.0146
	$\overline{K^L/K^B}$	loan-to-bond-capital ratio	De Fiore and Uhlig (2011)	**5.4800**
	$\overline{\psi}$	haircut	Federal Reserve	0.1100
Constraints	$\overline{r}^E Q - \delta - \frac{1-\beta_I}{\beta_I}$	investors' steady state		0.0000
	$\overline{r}^D Q - \delta - \frac{1-\beta_H}{\beta_H}$	households' steady state		0.0000
	W^L/W^B	long-run wage equality		1.0000
	$\frac{\mathbb{E}^{\lambda}[Y(\psi^D)]}{\mathbb{E}[Y(\psi^S)]}$	optimality of bank default tolerance		1.0000
	η	probability of a pos. shock in Sector L	**free choice**	0.9800
Model parameters	α	output elasticity of capital		0.3642
	$\tilde{A}^B$	productivity in Sector B	normalization	1.0000
	$\tilde{A}^L_{\underline{s}}$	productivity in Sector L after neg. shock		0.8278
	$\tilde{A}^L_{\overline{s}}$	productivity in Sector L after pos. shock		1.0038
	δ	capital depreciation rate		0.0150
	N	labor endowment	normalization	1.0000
	N^L/N^B	loan-to-bond-labor ratio		5.4800
	β_H	time preference of households		0.9851
	β_I	time preference of investors		0.9846
	λ	default costs		0.0181
	γ	liquidity demand		0.9901

Table 12: Model calibration for a loan-financed economy.

	Capital levels						Output and leverage	
	ρ	E	K	$E+K$	K^L	K^B	$\mathbb{E}^{\lambda}[Y(\psi^D)]$	φ
Regime D	0.3056	11.8432	38.7588	50.6020	42.7930	7.8090	4.1750	9.9212
Regime S	0.6520	19.9011	30.5254	50.4265	42.6176	7.8090	4.1708	5.8799

Table 13: Steady-state levels of capital, production, and leverage in a mainly loan-financed economy.

	Interest rates					Factor prices		
	r^D	$r^L_{\underline{s}}$	$r^L_{\bar{s}}$	$\mathbb{E}[r^L_s]$	$\tilde{r}^E$	Q	W^L	W^B
Regime D	1.0146	0.8398	1.0185	1.0149	1.0320	0.0296	2.6163	2.6163
Regime S	1.0146	0.8420	1.0211	1.0176	1.0320	0.0296	2.6056	2.6163

Table 14: Steady-state interest rates and factor prices in a mainly loan-financed economy.

	Consumption and taxes			
	C^I	C^H	C^W	τ
Regime D	0.1847	0.5846	2.6484	0.0061
Regime S	0.3104	0.4604	2.6453	0.0000

Table 15: Steady-state consumption and taxes in a mainly loan-financed economy.

D Proofs

D.1 Proof of Lemma 1

If $r_t^B = r_t^D$, the portfolio allocation $\{\zeta_t\}_{t=0}^{\infty}$ between bank deposits and corporate bonds is irrelevant for the optimization. The first-order condition with respect to K_{t+1}, with $t \geq 0$, gives the intertemporal Euler Equation (EE)

$$\frac{1}{C_t^H} = \beta_H \mathbb{E}_t \left[\frac{1 + r_{t+1}^D Q_{t+1} - \delta}{C_{t+1}^H} \right],$$

where consumption is given by $C_t^H = (1 + r_t^D Q_t - \delta)K_t - K_{t+1}$. By *guess and verify*, we can show that the linear capital accumulation rule $K_{t+1} = \beta_H(1 + r_t^D Q_t - \delta)K_t$ solves EE

$$\frac{1}{(1-\beta_H)(1 + r_t^D Q_t - \delta)K_t} = \beta_H \mathbb{E}_t \left[\frac{1 + r_{t+1}^D Q_{t+1} - \delta}{(1-\beta_H)(1 + r_{t+1}^D Q_{t+1} - \delta)K_{t+1}} \right]$$

$$\Leftrightarrow \qquad \frac{1}{(1 + r_t^D Q_t - \delta)K_t} = \beta_H \mathbb{E}_t \left[\frac{1}{\beta_H (1 + r_t^D Q_t - \delta)K_t} \right].$$

With this linear accumulation rule, consumption is then given by $C_t^H = (1-\beta_H)(1 + r_t^D Q_t - \delta)K_t$. The linear rules also satisfy the transversality condition (TVC)

$$\lim_{t\to+\infty} \beta_H^t \frac{K_{t+1}}{C_t^H} = \lim_{t\to+\infty} \beta_H^t \frac{\beta_H(1 + r_t^D Q_t - \delta)K_t}{(1-\beta_H)(1 + r_t^D Q_t - \delta)K_t} = \lim_{t\to+\infty} \frac{\beta_H^{t+1}}{1-\beta_H} = 0.$$

Jointly satisfying EE and TVC suffices to give a solution to the optimization problem. For a proof, see Theorem 4.15 in Stokey et al. (1989).

D.2 Proof of Lemma 2

For all $t \geq 0$, the first-order condition with respect to E_{t+1} is given by the intertemporal Euler Equation (EE)

$$\frac{1}{C_t^I} = \beta_I \mathbb{E}_t \left[\frac{1 + \tilde{r}_{t+1}^E Q_{t+1} - \delta}{C_{t+1}^I} \right],$$

where consumption is given by $C_t^I = (1 + \tilde{r}_t^E Q_t - \delta) E_t - E_{t+1}$. By *guess and verify*, we can show that the linear capital accumulation rule $E_{t+1} = \beta_I (1 + \tilde{r}_t^E Q_t - \delta) E_t$ solves EE

$$\begin{aligned} \frac{1}{(1-\beta_I)(1 + \tilde{r}_t^E Q_t - \delta) E_t} &= \beta_I \mathbb{E}_t \left[\frac{1 + \tilde{r}_{t+1}^E Q_{t+1} - \delta}{(1-\beta_I)(1 + \tilde{r}_{t+1}^E Q_{t+1} - \delta) E_{t+1}} \right] \\ \Leftrightarrow \qquad \frac{1}{(1 + \tilde{r}_t^E Q_t - \delta) E_t} &= \beta_I \mathbb{E}_t \left[\frac{1}{\beta_I (1 + \tilde{r}_t^E Q_t - \delta) E_t} \right]. \end{aligned}$$

Consumption is then given by $C_t^I = (1-\beta_I)(1 + \tilde{r}_t^E Q_t - \delta) E_t$. The linear rules also satisfy the transversality condition (TVC)

$$\lim_{t \to +\infty} \beta_I^t \frac{E_{t+1}}{C_t^I} = \lim_{t \to +\infty} \beta_I^t \frac{\beta_I (1 + \tilde{r}_t^E Q_t - \delta) E_t}{(1-\beta_I)(1 + \tilde{r}_t^E Q_t - \delta) E_t} = \lim_{t \to +\infty} \frac{\beta_I^{t+1}}{1 - \beta_I} = 0.$$

Jointly satisfying EE and TVC suffices to give a solution to the optimization problem. For a proof, see Theorem 4.15 in Stokey et al. (1989).

D.3 Proof of Lemma 3

First, we focus on the situation where the bank cannot face any solvency risk, i.e., $\varphi_t^L(\psi_t) \leq \varphi_t^S$. In this case, the protection from losses by limited liability is irrelevant for all liquidity-preserving leverage ratios $\varphi_t \leq \varphi_t^L(\psi_t)$, so that the expected return on bank equity is given by

$$\mathbb{E}_t[r^E_{s_t,t}] = \mathbb{E}_t[(r^L_{s_t,t} - r^D_t)\varphi_t + r^D_t] = [(\mathbb{E}_t[r^L_{s_t,t}] - r^D_t)\varphi_t + r^D_t].$$

The expected profit is maximized for the leverage $\varphi_t = \varphi^L_t(\psi_t)$ if $\mathbb{E}_t[r^L_{s_t,t}] > r^D_t$, and for $\varphi_t = 1$ if $\mathbb{E}_t[r^L_{s_t,t}] < r^D_t$. If the expected loan return equals the interest factor on deposits, i.e., if $\mathbb{E}_t[r^L_{s_t,t}] = r^D_t$, the bank is indifferent between any leverage, i.e., $\varphi_t \in [1, \varphi^L_t(\psi_t)]$. We have assumed that in any situation where the bank is indifferent, it chooses the largest leverage, i.e., $\varphi_t = \varphi^L_t(\psi_t)$. Accordingly, without the possibility of solvency risk, the bank chooses $\varphi_t = \varphi^L_t(\psi_t)$ ($\varphi_t = 1$) if and only if $\mathbb{E}_t[r^L_{s_t,t}] \geq (<)\ r^D_t$.

Second, we focus on the situation where the bank can face solvency risk, i.e., $\varphi^L_t(\psi_t) > \varphi^S_t$. Solvency risk can only exist if the loan repayment of the matched firm in the bad state $\underline{s}$ does not cover the deposit claims of households and workers. This implies $r^D_t > r^L_{\underline{s},t}$. Taking limited liability into account, the expected return on bank equity satisfies

$$\begin{aligned}\mathbb{E}_t[r^E_{s_t,t}(\varphi_t)] &= \mathbb{E}_t[\max\{(r^L_{s_t,t} - r^D_t)\varphi_t + r^D_t, 0\}]t \\ &= \{\eta_{z_t}[(r^L_{t,\overline{s}} - r^D_t)\varphi_t + r^D_t] + \mathbb{1}\{\varphi_t \leq \varphi^S_t\}(1-\eta_{z_t})[(r^L_{\underline{s},t} - r^D_t)\varphi_t + r^D_t]\}.\end{aligned}$$

We distinguish three cases:

1. Financing loans with deposits is profitable even without limited liability ($\mathbb{E}_t[r^L_{s_t,t}] \geq r^D_t$).

2. Financing loans with deposits is costly ($\mathbb{E}_t[r^L_{s_t,t}] < r^D_t$), but there are excess returns in case of a positive shock ($r^L_{t,\overline{s}} > r^D_t$) and the liquidity constraint is loose, i.e.,

$$\varphi^L_t(\psi_t) \geq \frac{\mathbb{E}_t[r^L_{s_t,t}]/\eta_{z_t} - r^D_t}{r^L_{t,\overline{s}} - r^D_t}. \tag{42}$$

3. Financing loans with deposits is costly ($\mathbb{E}_t[r^L_{s_t,t}] < r^D_t$) and either there are no excess returns in case of a positive shock ($r^L_{t,\overline{s}} \leq r^D_t$) or the liquidity constraint is tight,

i.e., Inequality (42) does not hold.

Case 1. If $\mathbb{E}_t[r^L_{s_t,t}] \geq r^D_t$, the bank can realize a non-negative profit from any additional unit of loans extended to its matched firm. This would hold true even if the bank was not protected by limited liability. Hence, the bank maximizes its profit by taking on the largest possible leverage that preserves liquidity, i.e., $\varphi_t = \varphi^L_t(\psi_t)$.

Case 2. Without the benefits from limited liability, financing loans with deposits would be costly, i.e., $\mathbb{E}_t[r^L_{s_t,t}] < r^D_t$. Hence, among all leverage ratios $\varphi_t \in [1, \varphi^S_t]$ that do not induce any solvency risk, $\varphi_t = 1$ would dominate.

Nevertheless, for any leverage level there are excess returns—not *expected* excess returns—from loan-financing if the financed firm incurs a positive productivity shock ($s_t = \overline{s}$), i.e., $r^L_{t,\overline{s}} > r^D_t$. Due to limited liability, the bank can realize an expected equity return that weakly exceeds the expected return $\mathbb{E}_t[r^L_{s_t,t}]$ the bank would realize if solely equity-financed ($\varphi_t = 1$), and if implementing a sufficiently high leverage ratio $\varphi_t > \varphi^S_t$. This leverage ratio φ_t induces an expected equity return weakly exceeding $\mathbb{E}_t[r^L_{s_t,t}]$ if and only if

$$\eta_{z_t}[(r^L_{t,\overline{s}} - r^D_t)\varphi_t + r^D_t] \geq \mathbb{E}_t[r^L_{s_t,t}] \qquad \Leftrightarrow \qquad \varphi_t \geq \frac{\mathbb{E}_t[r^L_{s_t,t}]/\eta_{z_t} - r^D_t}{r^L_{t,\overline{s}} - r^D_t}. \tag{43}$$

The liquidity constraint $\varphi^L_t(\psi_t)$ fulfills Inequality (43), since Inequality (42) holds, so that such a haircut φ_t is eligible. It is clear that every leverage ratio $\varphi_t < \varphi^L_t(\psi_t)$ for which Inequality (43) holds, is strictly dominated by $\varphi^L_t(\psi_t)$.

Case 3. If the expected return $\mathbb{E}_t[r^L_{s_t,t}]$ on loans falls short of the interest factor of deposits r^D_t, and if either there are no excess returns in case of a positive shock ($r^L_{t,\overline{s}} \leq r^D_t$) or if the liquidity constraint is tight, that is, if Inequality (42) does not hold, the bank cannot realize an expected equity return exceeding $\mathbb{E}_t[r^L_{s_t,t}]$, even though the bank has limited liability. Recall that $\mathbb{E}_t[r^L_{s_t,t}]$ is the equity return that the bank realizes if it is solely equity-financed. Therefore, the expected profit is maximized for the smallest possible leverage, i.e., $\varphi_t = 1$.

D.4 Proof of Lemma 4

From Lemma 3, we know that the bank is either financing loans with deposits and is liquidity-constrained, i.e., $\varphi_t = \varphi_t^L(\psi_t)$, or it finances loans solely with equity and does not require any liquidity, i.e., $\varphi_t = 1$.

We focus on the former situation, where banks issue deposits and leverage as much as possible, without endangering liquidity. Note that the liquidity demand of the bank is given by $L_t^{CB} = \gamma_t(L_t - Q_t E_t)$. When borrowing liquidity from the central bank, the bank faces the liquidity constraint

$$(1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]L_t \geq L_t^{CB} r_{CB,t}.$$

The repayment of the borrowed liquidity is determined by the interest factor $r_{CB,t}$. In turn, when borrowing liquidity on the interbank market, the bank faces the liquidity constraint

$$(1-\tilde{\psi}_t)\mathbb{E}_t[r^L_{s_t,t}]L_t \geq L_t^{CB} r_t^D,$$

where the repayment of interbank loans is determined by the interest factor r_t^D. As the bank is liquidity-constrained, it borrows only from the central bank (other banks) if

$$\frac{(1-\psi)\mathbb{E}_t[r^L_{s_t,t}]L_t}{r_{CB,t}} > (<) \frac{(1-\tilde{\psi}_t)\mathbb{E}_t[r^L_{s_t,t}]L_t}{r_t^D}.$$

The interbank market can thus only be active if the liquidity supply from other banks is weakly exceeding the liquidity supply from the central bank, i.e.,

$$\frac{(1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]L_t}{r_{CB,t}} \leq \frac{(1-\tilde{\psi}_t)\mathbb{E}_t[r^L_{s_t,t}]L_t}{r_t^D} \qquad \Leftrightarrow \qquad r_t^D(1-\psi_t) \leq r_{CB,t}(1-\tilde{\psi}_t). \quad (44)$$

Like reserves, interbank deposits can be used to settle interbank liabilities. The bank which granted an interbank loan must thus ensure that if the interbank deposits, which have been created when the interbank loan was granted, are transferred to other banks, the liquidity to settle the resulting interbank liability is available, in the form of reserves. If interbank deposits are transferred, the bank can use the pledged bank loans and re-hypothecate them, i.e., use them as collateral, at the central bank to borrow reserves. The maximum amount of liquidity that can be obtained by the bank, using the collateral $\mathbb{E}_t[r^L_{s_t,t}]L_t$ associated with interbank loans, is given by $(1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]L_t/r_{CB,t}$. Hence, the interbank loans provided by the bank must satisfy

$$\frac{(1-\tilde{\psi}_t)\mathbb{E}_t[r^L_{s_t,t}]L_t}{r^D_t} \leq \frac{(1-\psi_t)\mathbb{E}_t[r^L_{s_t,t}]L_t}{r_{CB,t}} \qquad \Leftrightarrow \qquad r^D_t(1-\psi_t) \geq r_{CB,t}(1-\tilde{\psi}_t). \quad (45)$$

From Equations (44) and (45), it follows that $r^D_t(1-\psi_t) = r_{CB,t}(1-\tilde{\psi}_t)$.

In any situation where the bank is financing loans solely with equity ($\varphi_t = 1$), it issues no deposits, so that the interest factor on deposits is irrelevant for the real allocations. We thus assume without loss of generality that it also holds that $r^D_t(1-\psi_t) = r_{CB,t}(1-\tilde{\psi}_t)$ when the bank chooses the smallest possible leverage, i.e., $\varphi_t = 1$.

D.5 Proof of Lemma 6

First, we show that $\varphi_t < \varphi^M_t/\alpha$. The labor immobility across sectors and the Inada Conditions imply that firms in Sector B issue a positive amount of bonds for all levels of r^B_t. Hence, Sector B is active in equilibrium and the value of input factors employed in Sector L, $Q_tK^L_t + W^L_tN^L_t$, must be strictly smaller than the total value of input factors. As the loan market clears, i.e., $\varphi_tQ_tE_t = Q_tK^L_t + W^L_tN^L_t$, this is equivalent to

$$\begin{aligned}
& \varphi_tQ_tE_t < Q_t(K_t+E_t) + W^L_tN^L_t + W^B_tN^B_t \\
\Leftrightarrow \qquad & \varphi_tE_t < \varphi^M_tE_t + \frac{W^L_t}{Q_t}N^L_t + \frac{W^B_t}{Q_t}N^B_t \\
\Leftrightarrow \qquad & \varphi_tE_t < \varphi^M_tE_t + \frac{1-\alpha}{\alpha}K^L_t + \frac{1-\alpha}{\alpha}K^B_t \\
\Leftrightarrow \qquad & \varphi_tE_t < \varphi^M_tE_t + \frac{1-\alpha}{\alpha}(K_t+E_t) \\
\Leftrightarrow \qquad & \varphi_t < \frac{1}{\alpha}\varphi^M_t.
\end{aligned}$$

We used the factor price shares in Equations (18) and (19) and capital-good market clearing $K_t + E_t = K_t^L + K_t^B$.

Now, suppose that there is an equilibrium such that $\varphi_t = 1$. Then all deposits $Q_t E_t$ in the economy are destroyed at Stage I.D, as lined out in Section 3.3. Since firms in Sector L receive loans $L_t = Q_t E_t$, firms in Sector L are active and workers employed in Sector L receive wage payments $W_t^L N_t^L > 0$ as deposits at Stage I.B. At Stage I.C, these deposits end up at the bank account of either the workers or the households. In any case, this contradicts the complete destruction of deposits at Stage I.D and we infer that $\varphi_t > 1$ in equilibrium. From Lemma 3, it follows that $\varphi_t = \varphi_t^L(\psi_t)$.

D.6 Proof of Lemma 7

Since we assume that collateral standards on the interbank market mimic those of the central bank, i.e., $\tilde{\psi}_t = \psi_t$, we can infer from Lemma 4 that $r_t^D = r_{CB,t}$. Hence, we have to establish the equality $r_t^B = r_t^D$ of gross interest rates that bonds and deposits pay.

Suppose that $r_t^B < r_t^D$. Households and workers then hold all their proceeds from the factor markets as deposits and firms in Sector B do not receive any financing. The labor immobility across sectors and the Inada Conditions imply that firms in Sector B issue a positive amount of bonds for all levels of r_t^B. Hence, the bond market does not clear.

Conversely, suppose that $r_t^B > r_t^D$. Then households and workers spend all their proceeds from the factor markets on bonds. As a result, no agent in the economy holds deposits after the bond market has cleared. As deposit supply must meet deposit demand, banks are fully equity-financed, i.e., $\varphi_t = 1$. In equilibrium, the loan market clears and we obtain

$$Q_t K_t^L + W_t^L N_t^L = Q_t E_t, \quad \Rightarrow \quad K_t^L < E_t.$$

From capital-good market clearing $K_t^L + K_t^B = E_t + K_t$, it follows that $K_t^B > K_t$. The first-order conditions with respect to capital in both sectors read

$$r_t^B Q_t = \alpha \tilde{A}_t^B (N_t^B)^{1-\alpha} (K_t^B)^{\alpha-1} \qquad \text{and}$$
$$\mathbb{E}_t[r_{s_t,t}^L] Q_t = \alpha \mathbb{E}_t[\tilde{A}_{s_t,t}^L] (N_t^L)^{1-\alpha} (K_t^L)^{\alpha-1}.$$

We obtain

$$\frac{r_t^B}{\mathbb{E}_t[r_{s_t,t}^L]} = \left(\frac{N_t^B}{N_t^L}\right)^{1-\alpha} \frac{\tilde{A}_t^B}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} \left(\frac{K_t^B}{K_t^L}\right)^{\alpha-1} = (\tilde{a}_t)^{1-\alpha} \left(\frac{K_t^L}{K_t^B}\right)^{1-\alpha} < \tilde{a}_t^{1-\alpha} \rho_t^{1-\alpha},$$

where we used $\rho_t = E_t/K_t$. Since $\tilde{a}_t \rho_t < \tilde{a}_t^{\lambda} \rho_t < 1$ because of Assumption 1, we infer that $\mathbb{E}_t[r_{s_t,t}^L] > r_t^B > r_t^D$. As banks are fully equity-financed, they do not face any solvency risk. According to Lemma 3, $\mathbb{E}_t[r_{s_t,t}^L] > r_t^D$. This, together with the absence of solvency risk, implies that the representative bank leverages up to its liquidity constraint $\varphi_t^L(\psi_t)$, defined in (14). Since it always holds that $\psi_t < 1$, it also holds $\varphi_t = \varphi_t^L(\psi_t) > 1$. This is a contradiction.

D.7 Proof of Lemma 8

When the representative bank is not exposed to any solvency risk, no bank default occurs in the economy, the government faces neither expenses for deposit insurance nor for bank-resolution costs, and the government thus does not levy any taxes.

Assume that the representative bank takes on solvency risk and thus a share $(1-\eta_{z_t})$ of banks defaults. Then the gross return on loans these banks receive $(r_{\underline{s},t}^L \varphi_t Q_t E_t)$ does not cover all deposit liabilities. However, the government compensates for the residual claims against the defaulting banks and pays for bank-resolution costs $\lambda \tilde{A}_{\underline{s},t} (N_t^L)^{1-\alpha} (K_t^L)^{\alpha}$. The government finances these expenses by levying lump-sum taxes on workers' income.

Hence, workers' income must be sufficiently high to cover these expenses, i.e.,

$$
\begin{aligned}
&(1-\eta_{z_t})\left([r_t^D(\varphi_t-1)-r_{\underline{s},t}^L\varphi_t]Q_tE_t+\lambda\tilde{A}_{\underline{s},t}(N_t^L)^{1-\alpha}(K_t^L)^\alpha\right)<r_t^D\left[W_t^LN_t^L+W_t^BN_t^B\right]\\
\Leftrightarrow\ &(1-\eta_{z_t})\left(\lambda\tilde{A}_{\underline{s},t}(N_t^L)^{1-\alpha}(K_t^L)^\alpha-r_{\underline{s},t}^L\varphi_tQ_tE_t\right)<\\
&\qquad r_t^D\left(\left[W_t^LN_t^L+W_t^BN_t^B\right]-(1-\eta_{z_t})(\varphi_t-1)Q_tE_t\right).
\end{aligned}
\tag{46}
$$

Since Assumption 1 holds, we used the equality $r_t^B=r_t^D$ of interest rates on bonds and deposits, established in Lemma 7, such that the workers' wage after interest payments equals $r_t^D\left[W_t^LN_t^L+W_t^BN_t^B\right]$.

We prove that the left-hand side of Inequality (46) is negative, while the right-hand side is positive. Lemma 5 yields that $K_t^L=\alpha\varphi_tE_t$. With the expression for $r_{\underline{s},t}^L$ in Equation (12), we deduce that

$$
\begin{aligned}
&\lambda\tilde{A}_{\underline{s},t}(N_t^L)^{1-\alpha}(K_t^L)^\alpha-r_{\underline{s},t}^L\varphi_tQ_tE_t\\
&\quad=\lambda\tilde{A}_{\underline{s},t}(N_t^L)^{1-\alpha}(\alpha\varphi_tE_t)^\alpha-\alpha\tilde{A}_{\underline{s},t}(N_t^L)^{1-\alpha}(\alpha\varphi_tE_t)^{\alpha-1}\varphi_tE_t\\
&\quad=(\lambda-1)\tilde{A}_{\underline{s},t}(N_t^L)^{1-\alpha}(\alpha\varphi_tE_t)^\alpha<0.
\end{aligned}
$$

Recall that, according to Lemma 6, it holds that $1<\varphi_t<\frac{\varphi_t^M}{\alpha}$. Using the interest rate equality $r_t^B=r_t^D$ and Equations (18) and (19) with $\eta_{z_t}>\alpha$, it holds that

$$
\begin{aligned}
&r_t^D\left(\left[W_t^LN_t^L+W_t^BN_t^B\right]-(1-\eta_{z_t})(\varphi_t-1)Q_tE_t\right)\\
&\quad=r_t^D\left(\frac{1-\alpha}{\alpha}\varphi_t^M-(1-\eta_{z_t})(\varphi_t-1)\right)Q_tE_t\\
&\quad=r_t^D\left(1-\eta_{z_t}+(\eta_{z_t}-\alpha)\varphi_t+\frac{1-\alpha}{\alpha}\left(\varphi_t^M-\alpha\varphi_t\right)\right)Q_tE_t\\
&\quad>r_t^D(1-\eta_{z_t}+\underbrace{(\eta_{z_t}-\alpha)\varphi_t}_{>\eta_{z_t}-\alpha})Q_tE_t\\
&\quad>r_t^D(1-\alpha)Q_tE_t>0.
\end{aligned}
$$

Inequality (46) thus holds true.

D.8 Proof of Lemma 9

As shown in Section 4, the haircut ψ_t pins down the marginal products of capital in Equations (21) and (23) and, through these equations, also the liquidity constraint $\varphi_t^L(\psi_t)$, as expressed in Equation (24). Using the interest rate identity $r_{CB,t} = r_t^D = r_t^B$ from Lemma 7, we have

$$\varphi_t^L(\psi_t) = \frac{\gamma_t r_t^B Q_t}{\gamma_t r_t^B Q_t - (1-\psi_t)\mathbb{E}[r_{s_t,t}^L]Q_t}. \tag{47}$$

Recall that the firms' first-order conditions with respect to the capital good yield

$$\mathbb{E}_t[r_{s_t,t}^L]Q_t = \alpha \mathbb{E}_t[\tilde{A}_{s_t,t}^L](N_t^L)^{1-\alpha}(K_t^L)^{\alpha-1} \qquad \text{and} \qquad r_t^B Q_t = \alpha \tilde{A}_t^B (N_t^B)^{1-\alpha}(K_t^B)^{\alpha-1}.$$

With the capital allocations $K_t^L = \alpha\varphi_t E_t$ and $K_t^B = (\varphi_t^M - \alpha\varphi_t)E_t$ from Lemma 5, it holds that

$$\mathbb{E}_t[r_{s_t,t}^L]Q_t = \alpha \mathbb{E}_t[\tilde{A}_{s_t,t}^L](N_t^L)^{1-\alpha}(\alpha\varphi_t E_t)^{\alpha-1} \;\; \text{and} \;\; r_t^B Q_t = \alpha \tilde{A}_t^B (N_t^B)^{1-\alpha}((\varphi_t^M - \alpha\varphi_t)E_t)^{\alpha-1}.$$

We derive the simplified first-order derivatives

$$\frac{\mathrm{d}\mathbb{E}_t[r_{s_t,t}^L]Q_t}{\mathrm{d}\varphi_t} = \frac{(\alpha-1)\mathbb{E}_t[r_{s_t,t}^L]Q_t}{\varphi_t} \qquad \text{and} \qquad \frac{\mathrm{d}r_t^B Q_t}{\mathrm{d}\varphi_t} = \frac{\alpha(1-\alpha)r_t^B Q_t}{\varphi_t^M - \alpha\varphi_t}.$$

Differentiating Equation (47) with respect to ψ_t on both sides, we obtain

$$
\begin{aligned}
&\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} \\
&= \frac{1}{M_t^2}\left[M_t\gamma_t\frac{\mathrm{d}r_t^B Q_t}{\mathrm{d}\psi_t} - \gamma_t r_t^B Q_t \frac{\mathrm{d}M_t}{\mathrm{d}\psi_t}\right] \\
&= \frac{1}{M_t^2}\Bigg[(\gamma_t r_t^B Q_t - (1-\psi_t)\mathbb{E}_t[r_{s_t,t}^L]Q_t)\frac{\gamma_t\alpha(1-\alpha)r_t^B Q_t}{\varphi_t^M - \alpha\varphi^L(\psi_t)}\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} \\
&\qquad - \gamma_t r_t^B Q_t\Bigg(\frac{\gamma_t\alpha(1-\alpha)r_t^B Q_t}{\varphi_t^M - \alpha\varphi^L(\psi_t)}\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} \\
&\qquad\qquad - \frac{(1-\psi_t)(\alpha-1)\mathbb{E}_t[r_{s_t,t}^L]Q_t}{\varphi_t^L(\psi_t)}\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} + \mathbb{E}_t[r_{s_t,t}^L]Q_t\Bigg)\Bigg] \\
&= -\frac{1}{M_t^2}\Bigg[\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t}\left(\frac{\alpha}{\varphi_t^M - \alpha\varphi_t^L(\psi_t)} + \frac{1}{\varphi_t^L(\psi_t)}\right) \\
&\qquad \times \gamma_t(1-\alpha)(1-\psi_t)r_t^B Q_t\mathbb{E}_t[r_{s_t,t}^L]Q_t + \gamma_t r_t^B Q_t\mathbb{E}_t[r_{s_t,t}^L]Q_t\Bigg],
\end{aligned}
$$

where $M_t = \gamma_t r_t^B Q_t - (1-\psi_t)\mathbb{E}[r_{s_t,t}^L]Q_t > 0$. From that, we derive

$$
\begin{aligned}
\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} = -&\left[1 + \frac{\gamma_t(1-\alpha)(1-\psi_t)r_t^B Q_t\mathbb{E}_t[r_{s_t,t}^L]Q_t}{M_t^2}\left(\frac{\alpha}{\varphi_t^M - \alpha\varphi_t^L(\psi_t)} + \frac{1}{\varphi_t^L(\psi_t)}\right)\right]^{-1} \\
&\times \frac{\gamma_t r_t^B Q_t\mathbb{E}_t[r_{s_t,t}^L]Q_t}{M_t^2} < 0.
\end{aligned}
$$

D.9 Proof of Lemma 10

According to Lemma 6, in equilibrium, the bank leverages up to $\varphi_t = \varphi_t^L(\psi_t) > 1$. Exploiting the definitions of $\varphi_t^L(\psi_t)$ and φ_t^S in Equations (14) and (17), respectively, the bank faces solvency risk if and only if

$$\begin{aligned}
\varphi_t^S < \varphi_t^L(\psi_t) \qquad &\Leftrightarrow \qquad \frac{r_{CB,t}}{r_{CB,t} - r_{\underline{s},t}^L} < \frac{\gamma_t r_{CB,t}}{\gamma_t r_{CB,t} - (1-\psi_t)\mathbb{E}_t[r_{s_t,t}^L]} \\
&\Leftrightarrow \qquad \gamma_t r_{CB,t} - (1-\psi_t)\mathbb{E}_t[r_{s_t,t}^L] < \gamma_t (r_{CB,t} - r_{\underline{s},t}^L) \\
&\Leftrightarrow \qquad \psi_t < 1 - \frac{\gamma_t r_{\underline{s},t}^L}{\mathbb{E}_t[r_{s_t,t}^L]} \\
&\Leftrightarrow \qquad \psi_t < 1 - \frac{\gamma_t A_{z_t,\underline{s}}^L}{\mathbb{E}_t[A_{z_t,s_t}^L]} \equiv \psi_t^S.
\end{aligned}$$

In the last step, we used that the first-order condition with respect to capital in Sector L is $\mathbb{E}_t[r_{s_t,t}^L]Q_t = \mathbb{E}_t[\tilde{A}_{s_t,t}^L](N_t^L)^{1-\alpha}(K_t^L)^{\alpha-1}$, and that perfect competition among firms in Sector L implies that

$$r_{s_t,t}^L Q_t = \tilde{A}_{s_t,t}^L (N_t^L)^{1-\alpha}(K_t^L)^{\alpha-1}, \quad s_t \in \{\underline{s}, \overline{s}\}.$$

Finally, we used that $\tilde{A}_{s_t,t}^L = (1+g_L)^t A_{z_t,s_t}^L$. It is clear that $\psi_t^S \in [0,1)$, so that ψ_t^S is a valid haircut.

D.10 Proof of Proposition 1

We prove Proposition 1 by considering three subcases. In the first case, we treat the case when the first-best allocation is attainable without bank-solvency risk. In the second case, the first-best allocation of funds induces solvency risk, but the central bank rules out this risk with sufficiently tight collateral requirements. In the last case, the first-best allocation involves solvency risk and the central bank permits this solvency risk by implementing loose collateral requirements, that is, a small haircut.

First-best available. The first-best allocation of funds across sectors is derived by imposing equal borrowing conditions ($r_t^B = \mathbb{E}_t[r_{s_t,t}^L]$) for loan-financed and bond-financed firms. Using the capital allocations $K_t^L = \alpha\varphi_t E_t$ and $K_t^B = (\varphi_t^M - \alpha\varphi_t)E_t$ from Lemma

5, and using $\varphi_t = \varphi_t^L(\psi_t)$ from Lemma 6, the capital ratio in Equation (13) reads as

$$\frac{\varphi_t^M - \alpha\varphi_t^L(\psi_t)}{\alpha\varphi_t^L(\psi_t)} = \frac{N_t^B}{N_t^L}\left(\frac{\tilde{A}_t^B}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]}\right)^{\frac{1}{1-\alpha}} \equiv \tilde{a}_t \quad \Leftrightarrow \quad \varphi_t^L(\psi_t) = \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t}\right) \equiv \varphi_t^E. \quad (48)$$

According to Lemma 6, the representative bank leverages up to its liquidity constraint, i.e., $\varphi_t^E = \varphi_t^L(\psi_t^E)$. With the interest rate identity $r_{CB,t} = r_t^B$ from Lemma 7 and with the efficiency condition $r_t^B = \mathbb{E}_t[r_{s_t,t}^L]$, we obtain

$$\begin{aligned}
\varphi_t^E = \varphi_t^L(\psi_t^E) \qquad &\Leftrightarrow \qquad \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t}\right) = \frac{\gamma_t r_{CB,t}}{\gamma_t r_{CB,t} - (1-\psi_t^E)\mathbb{E}_t[r_{s_t,t}^L]} \\
&\Leftrightarrow \qquad \psi_t^E = 1 - \gamma_t\left(1 - \alpha\frac{1+\tilde{a}_t}{\varphi_t^M}\right) \\
&\Leftrightarrow \qquad \psi_t^E = 1 - \gamma_t\left(1 - \alpha\rho_t\frac{1+\tilde{a}_t}{1+\rho_t}\right).
\end{aligned}$$

Assumption 1 ensures that $\psi_t^E \in [0,1)$, so that ψ_t^E is a valid haircut. Setting ψ_t^E is welfare-maximizing and therefore optimal if ψ_t^E does not induce any solvency risk, i.e., $\varphi_t^L(\psi_t^E) \leq \varphi_t^S$. With Lemma 10, this inequality translates into

$$\psi_t^E \geq \psi_t^S \;\Leftrightarrow\; 1 - \gamma_t\left(1 - \alpha\rho_t\frac{1+\tilde{a}_t}{1+\rho_t}\right) \geq 1 - \frac{\gamma_t A_{z_t,\underline{s}}^L}{\mathbb{E}_t[A_{z_t,s_t}^L]} \;\Leftrightarrow\; \frac{A_{z_t,\underline{s}}^L}{\mathbb{E}_t[A_{z_t,s_t}^L]} \geq 1 - \alpha\rho_t\frac{1+\tilde{a}_t}{1+\rho_t}.$$

First-best unavailable: Ruled-out solvency risk. If $\psi_t^E < \psi_t^S$, the condition $r_t^B = \mathbb{E}_t[r_{s_t,t}^L]$ does not guarantee an efficient outcome anymore, since there are additional losses due to bank insolvency in case of a negative idiosyncratic shock. Note that in this case, the central bank can always select a sufficiently large haircut ψ_t^S that restricts bank lending enough to rule out bank default, i.e., $\varphi_t^L(\psi_t^S) = \varphi_t^S$. With the definition of φ_t^S in Equation (17), we have $\varphi_t^S > 1$. Moreover, by Lemma 9, $\psi_t^S > \psi_t^E$ implies that $\varphi_t^S = \varphi_t^L(\psi_t^S) < \varphi_t^L(\psi_t^E) = \varphi_t^E$. Since $\varphi_t^E < \frac{1}{\alpha}\varphi_t^M$, we conclude that φ_t^S lies in the valid range $\left(1, \frac{1}{\alpha}\varphi_t^M\right)$ defined in Lemma 6. Every haircut $\psi_t > \psi_t^S$ would rule out solvency risk as well according to Lemma 10. However, the associated leverage ratio $\varphi_t^L(\psi_t) < \varphi_t^S$ would induce an even stronger overaccumulation of capital in the bond-financed sector, which is not desirable. Therefore, the central bank always prefers haircut ψ_t^S to any haircut $\psi_t > \psi_t^S$ and it thus sets ψ_t^S if the first-best allocation is not attainable and if it wants to rule out solvency risk.

First-best unavailable: Accepted solvency risk. The central bank may also set a haircut that induces bank-solvency risk but corrects the capital allocation for the costs due to bank default. In the presence of solvency risk, i.e., if $\varphi_t^L(\psi_t) > \varphi_t^S$, or, equivalently, if $\psi_t < \psi_t^S$, the derivative of the default-cost-adjusted expected production $\mathbb{E}_t^\lambda[Y_t]$ with respect to ψ_t reads as

$$\begin{aligned}\frac{\mathrm{d}\mathbb{E}_t^\lambda[Y_t]}{\mathrm{d}\psi_t} &= E_t^\alpha \frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t}\Big[\alpha\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L](N_t^L)^{1-\alpha}(\alpha\varphi_t^L(\psi_t))^{\alpha-1} \\ &\qquad\qquad - \alpha\tilde{A}_t^B(N_t^B)^{1-\alpha}(\varphi_t^M - \alpha\varphi_t^L(\psi_t))^{\alpha-1}\Big] \\ &= \alpha E_t^\alpha \mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L](N_t^L)^{1-\alpha}\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t}\left[(\alpha\varphi_t^L(\psi_t))^{\alpha-1} - (\tilde{a}_t^\lambda)^{1-\alpha}(\varphi_t^M - \alpha\varphi_t^L(\psi_t))^{\alpha-1}\right].\end{aligned} \tag{49}$$

We see that

$$\varphi_t^L(\psi_t) = \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t^\lambda}\right) \equiv \varphi_t^D \tag{50}$$

is the unique root of $\frac{\mathrm{d}\mathbb{E}_t^\lambda[Y_t]}{\mathrm{d}\psi_t}$ in $\varphi_t^L(\psi_t)$, since φ_t^D is the only root of the term in square brackets in (49) and $\frac{\mathrm{d}\varphi_t^L(\psi_t)}{\mathrm{d}\psi_t} < 0$ according to Lemma 9. Remark that φ_t^D equates the marginal product of capital in Sector B and the default-cost-adjusted expected marginal product of capital in Sector L. However, it does not necessarily hold that $\varphi_t^S \equiv \varphi_t^L(\psi_t^S) < \varphi_t^D$, i.e., the leverage φ_t^D indeed induces solvency risk according to Lemma 10. For the moment, φ_t^D is only the root of the analytical expression in (49).

The expression for φ_t^D in Equation (50) and the general definition of $\varphi_t^L(\psi_t)$ in (14) yield the optimal haircut

$$\psi_t = 1 - \frac{\gamma_t r_{CB,t}}{\mathbb{E}_t[r_{s_t,t}^L]}\frac{\tilde{b}_t^\lambda - 1}{\tilde{b}_t^\lambda} \equiv \psi_t^D, \qquad \text{where} \qquad \tilde{b}_t^\lambda \equiv \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t^\lambda}\right).$$

Using the interest rate identity $r_{CB,t} = r_t^D = r_t^B$ from Lemma 7, and the first-order conditions with respect to capital in both sectors, that is,

$$r_t^B Q_t = \alpha \tilde{A}_t^B (N_t^B)^{1-\alpha} (K_t^B)^{\alpha-1} \qquad \text{and} \qquad \mathbb{E}_t[r_{s_t,t}^L] Q_t = \alpha \mathbb{E}_t[\tilde{A}_{s_t,t}^L] (N_t^L)^{1-\alpha} (K_t^L)^{\alpha-1},$$

we obtain

$$\begin{aligned}
\psi_t^D &= 1 - \frac{\gamma_t \tilde{A}_t^B}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} \frac{\tilde{b}_t^\lambda - 1}{\tilde{b}_t^\lambda} \frac{(N_t^B)^{1-\alpha}}{(N_t^L)^{1-\alpha}} \frac{(K_t^B)^{\alpha-1}}{(K_t^L)^{\alpha-1}} \\
&= 1 - \gamma_t \left(1 - \alpha \rho_t \frac{1 + \tilde{a}_t^\lambda}{1 + \rho_t}\right) \frac{\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]}{\mathbb{E}_t[A_{z_t,s_t}^L]},
\end{aligned}$$

where we used

$$\frac{K_t^B}{K_t^L} = \frac{\varphi_t^M - \alpha \varphi_t^L(\psi_t)}{\alpha \varphi_t^L(\psi_t)} = \frac{N_t^B}{N_t^L} \left(\frac{\tilde{A}_t^B}{\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]}\right)^{\frac{1}{1-\alpha}}$$

for $\varphi_t^L(\psi_t) = \frac{1}{\alpha}\left(\frac{\varphi_t^M}{1+\tilde{a}_t^\lambda}\right) \equiv \varphi_t^D$. Assumption 1 ensures that $\psi_t^D \in [0, 1)$, so that ψ_t^D is a valid haircut.

If $\psi_t^D \leq \psi_t^S$, we have $\varphi_t^D = \varphi_t^L(\psi_t^D) \geq \varphi_t^L(\psi_t^S) = \varphi_t^S$ and the bank indeed faces solvency risk if taking on leverage $\varphi_t^L(\psi_t^D) = \varphi_t^D$ (see Lemma 10). Hence, the haircut $\psi_t \in [0, \psi_t^S]$ that maximizes $\mathbb{E}_t^\lambda[Y_t]$ is ψ_t^D. Then the central bank sets the haircut ψ_t^D (ψ_t^S) if and only if $\mathbb{E}_t^\lambda[Y_t(\psi_t^D)] \geq \mathbb{E}_t[Y_t(\psi_t^S)]$. If, however, $\psi_t^D > \psi_t^S$, then there is no solvency risk at $\psi_t = \psi_t^D$. Since $\frac{\mathrm{d}\mathbb{E}_t^\lambda[Y_t]}{\mathrm{d}\psi_t}\big|_{\psi_t = \psi_t^D} = 0$ and $\frac{\mathrm{d}\mathbb{E}_t^\lambda[Y_t]}{\mathrm{d}\psi_t} > 0$ for $\psi_t \in [0, \psi_t^S]$, $\mathbb{E}_t^\lambda[Y_t]$ increases on $[0, \psi_t^S]$ and therefore attains its maximum at ψ_t^S for $\psi_t \in [0, \psi_t^S]$. Hence, in equilibrium, the central bank rules out solvency risk by setting $\psi_t = \psi_t^S$ if $\psi_t^D > \psi_t^S$.

D.11 Proof of Corollary 1

The expressions for the optimal leverage levels φ_t^E and φ_t^D in Regimes E and D follow from Equations (48) and (50) in the proof of Proposition 1. It is also clear that

$$\frac{\mathrm{d}\varphi_t^E}{\mathrm{d}\varphi_t^M} > 0 \qquad \text{and} \qquad \frac{\mathrm{d}\varphi_t^D}{\mathrm{d}\varphi_t^M} > 0.$$

To establish that φ_t^S is uniquely pinned down by φ_t^M, recall from Lemma 6 that the bank leverages up to its liquidity constraint $\varphi_t^L(\psi_t)$ in equilibrium. According to Lemma 9, the liquidity constraint is uniquely pinned down by the haircut ψ_t, so that

$$\varphi_t^L(\psi_t^S) = \varphi^S = \frac{r^B}{r^B - r^L_{\underline{s}}},$$

as defined Equation (17), is uniquely pinned down for a given parametrization, which determines ψ^S, and given levels of capital endowment E and K. From Equations (21) to (23), it is clear that the relation of E and K, captured by $\varphi^M \equiv (E+K)/E$, determines φ^S, rather than the absolute levels of E and K.

With Lemma 5, the first-order conditions of firms in Sectors B and L with respect to capital, presented in Equations (9) and (12), read

$$r^B Q = \alpha \tilde{A}^B (N^B)^{1-\alpha}(\varphi^M - \alpha\varphi^S)^{\alpha-1} \qquad \text{and} \qquad r^L_s Q = \alpha \tilde{A}^L_s (N^L)^{1-\alpha}(\alpha\varphi^S)^{\alpha-1}$$

for $s \in \{\underline{s}, \overline{s}\}$. Note that

$$\frac{\mathrm{d}r^B Q}{\mathrm{d}\varphi^M} = (\alpha-1)\frac{r^B Q}{\varphi^M - \alpha\varphi^S}\left(1 - \alpha\frac{\mathrm{d}\varphi^S}{\mathrm{d}\varphi^M}\right) \qquad \text{and} \qquad \frac{\mathrm{d}r^L_{\underline{s}} Q}{\mathrm{d}\varphi^M} = (\alpha-1)\frac{r^L_{\underline{s}} Q}{\varphi^S}\frac{\mathrm{d}\varphi^S}{\mathrm{d}\varphi^M},$$

such that

$$\begin{aligned}
\frac{\mathrm{d}\varphi^S}{\mathrm{d}\varphi^M} &= \frac{\frac{\mathrm{d}r^B Q}{\mathrm{d}\varphi^M}(r^B Q - r^L_{\underline{s}} Q) - r^B Q\left(\frac{\mathrm{d}r^B Q}{\mathrm{d}\varphi^M} - \frac{\mathrm{d}r^L_{\underline{s}} Q}{\mathrm{d}\varphi^M}\right)}{(r^B Q - r^L_{\underline{s}} Q)^2} \\
&= \frac{1}{(r^B Q - r^L_{\underline{s}} Q)^2}\left(\frac{\mathrm{d}r^L_{\underline{s}} Q}{\mathrm{d}\varphi^M} r^B Q - \frac{\mathrm{d}r^B Q}{\mathrm{d}\varphi^M} r^L_{\underline{s}} Q\right) \\
&= \frac{(1-\alpha) r^B Q r^L_{\underline{s}} Q}{(r^B Q - r^L_{\underline{s}} Q)^2}\left[\frac{1}{\varphi^M - \alpha\varphi^S} - \left(\frac{1}{\varphi^S} + \frac{\alpha}{\varphi^M - \alpha\varphi^S}\right)\frac{\mathrm{d}\varphi^S}{\mathrm{d}\varphi^M}\right].
\end{aligned}$$

We infer that

$$\frac{\mathrm{d}\varphi^S}{\mathrm{d}\varphi^M} = \left[1 + \frac{(1-\alpha) r^B Q r^L_{\underline{s}} Q}{(r^B Q - r^L_{\underline{s}} Q)^2}\left(\frac{1}{\varphi^S} + \frac{\alpha}{\varphi^M - \alpha\varphi^S}\right)\right]^{-1} \frac{(1-\alpha) r^B Q r^L_{\underline{s}} Q}{(r^B Q - r^L_{\underline{s}} Q)^2}\frac{1}{\varphi^M - \alpha\varphi^S} > 0.$$

From $\varphi_t^M \equiv 1 + 1/\rho_t$, we immediately obtain that

$$\frac{\mathrm{d}\varphi_t^E}{\mathrm{d}\rho_t}, \frac{\mathrm{d}\varphi_t^D}{\mathrm{d}\rho_t}, \frac{\mathrm{d}\varphi_t^S}{\mathrm{d}\rho_t} < 0.$$

D.12 Proof of Proposition 2

The derivative of ψ_t^S with respect to γ_t is given by

$$\frac{\mathrm{d}\psi_t^S}{\mathrm{d}\gamma_t} = -\frac{A^L_{z_t,\underline{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]} < 0.$$

Haircut ψ_t^S decreases with the productivity of loan-financed firms in the low productivity state, denoted by $A^L_{z_t,\underline{s}}$, as

$$\frac{\mathrm{d}\psi_t^S}{\mathrm{d}A^L_{z_t,\underline{s}}} = -\gamma_t \frac{\mathbb{E}_t[A^L_{z_t,s_t}] - (1-\eta_{z_t})A^L_{z_t,\underline{s}}}{(\mathbb{E}_t[A^L_{z_t,s_t}])^2} = -\frac{\gamma_t \eta_{z_t} A^L_{z_t,\bar{s}}}{(\mathbb{E}_t[A^L_{z_t,s_t}])^2} < 0,$$

and ψ_t^S increases with the productivity in the high productivity state, denoted by $A^L_{z_t,\bar{s}}$, as

$$\frac{\mathrm{d}\psi_t^S}{\mathrm{d}A^L_{z_t,\bar{s}}} = \frac{\gamma_t \eta_{z_t} A^L_{z_t,\underline{s}}}{(\mathbb{E}_t[A^L_{z_t,s_t}])^2} > 0.$$

Finally, note that $\frac{\mathrm{d}\psi_t^S}{\mathrm{d}A^B_{z_t}} = 0$, that is, ψ_t^S is independent of the productivity in Sector B.

D.13 Proof of Proposition 3

From Proposition 1, we know that the central bank sets haircut ψ_t^S in each period t in which it rules out solvency risk. Using in the definitions of ξ^γ_{t+1}, ξ^d_{t+1}, and ξ^e_{t+1}, we obtain

$$\begin{aligned}
1 - \xi^S_{t+1} + \psi_t^S \xi^S_{t+1} &= 1 + (\psi_t^S - 1)\xi^\gamma_{t+1}\ \xi^d_{t+1}\ \xi^e_{t+1} \\
&= 1 + \left(1 - \frac{\gamma_t A^L_{z_t,\underline{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]} - 1\right) \frac{\gamma_{t+1}\tilde{A}^L_{z_{t+1},\underline{s}}\mathbb{E}_t[A^L_{z_t,s_t}]}{\gamma_t \tilde{A}^L_{z_t,\underline{s}}\mathbb{E}_{t+1}[\tilde{A}^L_{z_{t+1},s_{t+1}}]} \\
&= 1 - \frac{\gamma_{t+1}\tilde{A}^L_{z_{t+1},\underline{s}}}{\mathbb{E}_{t+1}[\tilde{A}^L_{z_{t+1},s_{t+1}}]},
\end{aligned}$$

where we have used the expression for ψ_t^S in Proposition 1. This proves $\psi_{t+1}^S = 1 - \xi_{t+1}^S + \psi_t^S \xi_{t+1}^S$.

D.14 Proof of Proposition 4

The derivative of ψ_t^D with respect to ρ_t is given by

$$\frac{\mathrm{d}\psi_t^D}{\mathrm{d}\rho_t} = \gamma_t \alpha (1 + \tilde{a}_t^\lambda) \frac{\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]}{\mathbb{E}_t[A_{z_t,s_t}^L]} \frac{1}{(1+\rho_t)^2} > 0.$$

Recall that $\mathbb{E}_t^\lambda[A_{z_t,s_t}^L] = \eta_{z_t} A_{z_t,\bar{s}}^L + (1-\lambda)(1-\eta_{z_t}) A_{z_t,\underline{s}}^L$. Remark that with the definition of $\tilde{a}_t^\lambda$ in Equation (20), we obtain

$$\frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}\lambda} = \frac{\tilde{a}_t^\lambda}{(\alpha-1)\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]} \frac{\mathrm{d}\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]}{\mathrm{d}\lambda} = \frac{\tilde{a}_t^\lambda (1-\eta_{z_t}) \tilde{A}_{t,\underline{s}}^L}{(1-\alpha)\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]} > 0.$$

With this expression, we derive

$$\begin{aligned}
\frac{\mathrm{d}\psi_t^D}{\mathrm{d}\lambda} &= \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} \frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}\lambda} - \gamma_t \left(1 - \frac{\gamma_t \alpha \rho_t}{1+\rho_t}\right) \frac{1}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} \frac{\mathrm{d}\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]}{\mathrm{d}\lambda} \\
&= \tilde{a}_t^\lambda \frac{\gamma_t \alpha \rho_t (1-\eta_{z_t})}{(1+\rho_t)(1-\alpha)} \frac{\tilde{A}_{t,\underline{s}}^L}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} + \gamma_t \left(1 - \frac{\gamma_t \alpha \rho_t}{1+\rho_t}\right) \frac{(1-\eta_{z_t})\tilde{A}_{t,\underline{s}}^L}{\mathbb{E}_t[\tilde{A}_{s_t,t}^L]} > 0.
\end{aligned}$$

Haircut ψ_t^D increases with $A_{z_t}^B$, as

$$\frac{\mathrm{d}\psi_t^D}{\mathrm{d}A_{z_t}^B} = \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]}{\mathbb{E}_t[A_{z_t,s_t}^L]} \frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}A_{z_t}^B} = \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]}{\mathbb{E}_t[A_{z_t,s_t}^L]} \frac{\tilde{a}_t^\lambda}{(1-\alpha)A_{z_t}^B} > 0,$$

Note that by definition,

$$\tilde{a}_t^\lambda = \frac{N_t^B}{N_t^L} \left(\frac{\tilde{A}_t^B}{\mathbb{E}_t^\lambda[\tilde{A}_{s_t,t}^L]}\right)^{\frac{1}{1-\alpha}} = \frac{N_t^B}{N_t^L} \left(\frac{1+g_B}{1+g_L}\right)^{\frac{t}{1-\alpha}} \left(\frac{A_{z_t}^B}{\mathbb{E}_t^\lambda[A_{z_t,s_t}^L]}\right)^{\frac{1}{1-\alpha}}.$$

Since we assume labor supply to be fixed across sectors, it holds that

$$\frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}g_B} = \frac{t\tilde{a}_t^\lambda}{(1-\alpha)(1+g_B)} > 0.$$

Hence, haircut ψ_t^D increases with growth rate g_B, as

$$\frac{\mathrm{d}\psi_t^D}{\mathrm{d}g_B} = \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} \frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}g_B} = \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} \frac{t\tilde{a}_t^\lambda}{(1-\alpha)(1+g_B)} > 0.$$

We have argued in the proof of Proposition 1 that $\psi_t^D \in (0,1)$. We infer that

$$1 - \alpha\rho_t \frac{1+\tilde{a}_t^\lambda}{1+\rho_t} > 0,$$

which determines the sign of the derivative of ψ_t^D with respect to γ_t, that is,

$$\frac{\mathrm{d}\psi_t^D}{\mathrm{d}\gamma_t} = -\left(1 - \alpha\rho_t \frac{1+\tilde{a}_t^\lambda}{1+\rho_t}\right) \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} < 0.$$

Note that

$$\frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}A^L_{z_t,\bar{s}}} = \frac{-\tilde{a}_t^\lambda}{(1-\alpha)\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]} \frac{\mathrm{d}\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathrm{d}A^L_{z_t,\bar{s}}} = -\frac{\eta_{z_t}\tilde{a}_t^\lambda}{(1-\alpha)\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]} < 0.$$

Haircut ψ_t^D decreases with the productivity $A^L_{z_t,\bar{s}}$ of the loan-financed sector in case of a positive shock, since

$$\begin{aligned}
\frac{\mathrm{d}\psi_t^D}{\mathrm{d}A^L_{z_t,\bar{s}}} &= -\gamma_t \left[1 - \alpha\rho_t \frac{1+\tilde{a}_t^\lambda}{1+\rho_t}\right] \frac{\mathrm{d}}{\mathrm{d}A^L_{z_t,\bar{s}}} \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} + \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} \frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}A^L_{z_t,\bar{s}}} \\
&= -\gamma_t \left[1 - \alpha\rho_t \frac{1+\tilde{a}_t^\lambda}{1+\rho_t}\right] \frac{\lambda\eta_{z_t}(1-\eta_{z_t})A^L_{z_t,\underline{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]^2} - \frac{\gamma_t\alpha\rho_t\eta_{z_t}\tilde{a}_t^\lambda}{(1+\rho_t)(1-\alpha)\mathbb{E}_t[A^L_{z_t,s_t}]} < 0.
\end{aligned}$$

Haircut ψ_t^D decreases with g_L, as

$$\frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}g_L} = -\frac{t\tilde{a}_t^\lambda}{(1-\alpha)(1+g_L)},$$

and thus

$$\frac{\mathrm{d}\psi_t^D}{\mathrm{d}g_L} = \frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} \frac{\mathrm{d}\tilde{a}_t^\lambda}{\mathrm{d}g_L} = -\frac{\gamma_t \alpha \rho_t}{1+\rho_t} \frac{\mathbb{E}_t^\lambda[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} \frac{t\tilde{a}_t^\lambda}{(1-\alpha)(1+g_L)} < 0.$$

Identifying the sign of $\mathrm{d}\psi_t^D / \mathrm{d}A^L_{z_t,\underline{s}}$, we first observe that

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}A^L_{z_t,\underline{s}}}\frac{\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} \\
&\quad = \frac{(1-\lambda)(1-\eta_{z_t})\mathbb{E}_t[A^L_{z_t,s_t}] - (1-\eta_{z_t})\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]^2} \\
&\quad = (1-\eta_{z_t})\frac{(1-\lambda)\left(\eta A^L_{z_t,\bar{s}} + (1-\eta_{z_t})A^L_{z_t,\underline{s}}\right) - \left(\eta A^L_{z_t,\bar{s}} + (1-\lambda)(1-\eta_{z_t})A^L_{z_t,\underline{s}}\right)}{\mathbb{E}_t[A^L_{z_t,s_t}]^2} \\
&\quad = -\frac{\lambda(1-\eta_{z_t})\eta_{z_t}\tilde{A}^L_{t,\bar{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]^2},
\end{aligned}$$

so that

$$\frac{\mathrm{d}\tilde{a}^\lambda_t}{\mathrm{d}A^L_{z_t,\underline{s}}} = \frac{-\tilde{a}^\lambda_t}{(1-\alpha)\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}\frac{\mathrm{d}\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}{\mathrm{d}A^L_{z_t,\underline{s}}} = -\frac{(1-\lambda)(1-\eta_{z_t})\tilde{a}^\lambda_t}{(1-\alpha)\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}.$$

Hence,

$$\begin{aligned}
\frac{\mathrm{d}\psi^D_t}{\mathrm{d}A^L_{z_t,\underline{s}}} &= -\frac{\gamma_t}{1+\rho_t}\left[1-\alpha\rho_t\frac{1+\tilde{a}^\lambda_t}{1+\rho_t}\right]\frac{\mathrm{d}}{\mathrm{d}A^L_{z_t,\underline{s}}}\frac{\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]} + \frac{\gamma_t\alpha\rho_t}{1+\rho_t}\frac{\mathbb{E}^\lambda_t[A^L_{z_t,s_t}]}{\mathbb{E}_t[A^L_{z_t,s_t}]}\frac{\mathrm{d}\tilde{a}^\lambda_t}{\mathrm{d}A^L_{z_t,\underline{s}}} \\
&= \frac{\gamma_t}{1+\rho_t}\left[1-\alpha\rho_t\frac{1+\tilde{a}^\lambda_t}{1+\rho_t}\right]\frac{\lambda(1-\eta_{z_t})\eta_{z_t}\tilde{A}^L_{t,\bar{s}}}{\mathbb{E}_t[A^L_{z_t,s_t}]^2} - \frac{\gamma_t\alpha\rho_t(1-\lambda)(1-\eta_{z_t})\tilde{a}^\lambda_t}{(1+\rho_t)(1-\alpha)\mathbb{E}_t[A^L_{z_t,s_t}]}.
\end{aligned}$$

The sign of $\mathrm{d}\psi_t^D / \mathrm{d}A^L_{z_t,\underline{s}}$ is ambiguous. It depends on the parametrization of the model and it may differ for different macroeconomic shocks z_t and (endogenous) investors-to-households-capital ratios ρ_t.

To determine how ψ_t^D changes in α, we first observe that

$$\frac{\mathrm{d}\tilde{a}^\lambda_t}{\mathrm{d}\alpha} = \frac{N^B_t}{N^L_t}\ln\left(\frac{\tilde{A}^B_t}{\mathbb{E}^\lambda_t[\tilde{A}^L_{s_t,t}]}\right)\left(\frac{\tilde{A}^B_t}{\mathbb{E}^\lambda_t[\tilde{A}^L_{s_t,t}]}\right)^{\frac{1}{1-\alpha}}\frac{\mathrm{d}}{\mathrm{d}\alpha}\left[\frac{1}{1-\alpha}\right] = \ln\left(\frac{\tilde{A}^B_t}{\mathbb{E}^\lambda_t[\tilde{A}^L_{s_t,t}]}\right)\frac{\tilde{a}^\lambda_t}{(1-\alpha)^2}.$$

We obtain that, if $\tilde{A}^B_t \geq \mathbb{E}^\lambda_t[\tilde{A}^L_{s_t,t}]$, it holds that

$$\frac{\mathrm{d}\psi^D_t}{\mathrm{d}\alpha} = \frac{\gamma_t\rho_t}{1+\rho_t}\frac{\mathbb{E}^\lambda_t[\tilde{A}_{s_t,t}]}{\mathbb{E}_t[\tilde{A}_{s_t,t}]}\left(1+\tilde{a}^\lambda_t + \alpha\ln\left(\frac{\tilde{A}^B_t}{\mathbb{E}^\lambda_t[\tilde{A}^L_{s_t,t}]}\right)\frac{\tilde{a}^\lambda_t}{(1-\alpha)^2}\right) > 0.$$

D.15 Proof of Proposition 5

Note that

$$
\begin{aligned}
\frac{1-\psi_{t+1}^D}{1-\psi_t^D} &= \frac{\gamma_{t+1}}{\gamma_t}\frac{1+\rho_{t+1}-\alpha\rho_{t+1}(1+\tilde{a}_{t+1}^{\lambda})}{1+\rho_{t+1}}\frac{1+\rho_t}{1+\rho_t-\alpha\rho_t(1+\tilde{a}_t^{\lambda})}\frac{\mathbb{E}_{z_{t+1}}^{\lambda}[\tilde{A}_{z_{t+1},s_{t+1}}^L]}{\mathbb{E}_{z_{t+1}}[\tilde{A}_{z_{t+1},s_{t+1}}^L]}\frac{\mathbb{E}_t[A_{z_t,s_t}^L]}{\mathbb{E}_t^{\lambda}[A_{z_t,s_t}^L]} \\
&= \underbrace{\frac{\gamma_{t+1}}{\gamma_t}}_{\equiv\xi_{t+1}^{\gamma}}\underbrace{\frac{1+\rho_{t+1}-\alpha\rho_{t+1}(1+\tilde{a}_{t+1}^{\lambda})}{1+\rho_t-\alpha\rho_t(1+\tilde{a}_t^{\lambda})}\frac{1+\rho_t}{1+\rho_{t+1}}}_{\equiv\xi_{t+1}^{\rho}}\underbrace{\frac{\mathbb{E}_{z_{t+1}}^{\lambda}[\tilde{A}_{z_{t+1},s_{t+1}}^L]}{\mathbb{E}_t^{\lambda}[A_{z_t,s_t}^L]}}_{\equiv\xi_{t+1}^{d}}\underbrace{\frac{\mathbb{E}_t[A_{z_t,s_t}^L]}{\mathbb{E}_{z_{t+1}}[\tilde{A}_{z_{t+1},s_{t+1}}^L]}}_{\equiv\xi_{t+1}^{e}}.
\end{aligned}
$$

By defining $\xi_{t+1} \equiv \xi_{t+1}^{\gamma}\xi_{t+1}^{\rho}\xi_{t+1}^{d}\xi_{t+1}^{e}$, we see that haircuts vary over time according to $1-\psi_{t+1}^D = \xi_{t+1}(1-\psi_t^D)$ or, equivalently, $\psi_{t+1}^D = 1-\xi_{t+1}+\psi_t^D\xi_{t+1}$.