Validation of assumptions on the endolymph motion inside the semicircular canals of the inner ear

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Figure on the title page: Velocity contours of the endolymph motion during a standard head maneuver (see section 1.3).
Abstract

In this work we investigate the validity of the following assumptions that were used in previous studies of the endolymph motion within the semicircular canals of the inner ear:

(a) quasi-steadiness and therefore exclusion of the time derivative,
(b) absence of advection,
(c) insignificance of Coriolis and centrifugal forces.

A parameter study is performed using the C++ based OpenFOAM® software library for numerical simulations on a finite volume grid. For validation purposes, the solutions are compared with results from the method of fundamental solutions to compute the flow in a quasi-steady Stokes flow regime. We conclude that the assumptions are valid for a standard head maneuver.
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1 Introduction

1.1 Anatomy of the inner ear

The human inner ear is embedded in the temporal bone and comprises the organs for hearing and balance, both sharing the same fluid space. This report focuses on the latter, namely the vestibular system. It consists of five sensors: three semicircular canals (posterior, lateral, superior) for the perception of angular motion, and two saccular organs (utricle, saccule) for the sensation of translational motion (figure 1).

![Figure 1: Picture of the human inner ear. The endolymph is drawn in purple, the perilymph in pink. (Source: http://www.graphicshunt.com/health/images/inner_ear-1732.htm)](image)

The canals contain a second fluid space filled with endolymph, which is separated from the surrounding fluid (perilymph) through a membrane. When turning the head, this membranous duct accelerates the endolymph and deflects the cupula, an elastic membrane inside the ampulla. The deflection is perceived by fibrous hair cells that change their firing rate accordingly. Since the viscous time scale is significantly smaller than the Cupula time constant (Obrist, 2012, p.18), we consider the endolymph dynamics alone and neglect the Cupula-endolymph interaction in a first approximation.

1.2 Geometry and mesh of the membranous labyrinth

Curthoys & Oman (1987) have carried out dimensional measurements of the membranous duct in the human lateral semicircular canal. Under the assumption of elliptic cross section profiles, they determined the ellipsis parameters for seven cross sections along the duct. Based on their study, the present geometry is derived by interpolating and smoothing the surface area between the seven sections, using the ANSYS® ICEM CFD™ software. In order to gain information about the grid resolution quality, all simulations are performed twice: first with a lower resolution of $N \approx 0.2 \cdot 10^6$ cells (figure 2a), then with a refined mesh of $N \approx 0.5 \cdot 10^6$ cells (figure 2b).

1.3 Head maneuver

A standard head maneuver (SHM) is applied to all transient simulations of this report. The SHM lasts three seconds and consists of a head rotation from a relative angle of $0^\circ$ to $120^\circ$. The rotation
Figure 2: Discretized computational domain of the membranous duct in the human lateral semicircular canal for different grid sizes of $N$ cells. The mesh is block-structured in an "o-grid" fashion using the ANSYS®|CEM CFD™ software. The underlying geometry was derived from measurements of Curthoys & Oman (1987).

axis is oriented perpendicular to the plane of the horizontal semicircular canal. A visualization of the motion pattern is given in figure 3. The underlying SHM equation is given in (1), with $t$ denoting time.

$$\alpha(t) = \begin{cases} 
0 & t < 0 \\
\frac{2\pi}{3 \cdot 2187}(-20t^7 + 210t^6 - 756t^5 + 945t^4) & 0 \leq t \leq 3s \\
\frac{2\pi}{3} & 3s < t 
\end{cases}$$  \hspace{1cm} (1)$$

Figure 3: Standard head maneuver: frame angle $\alpha(t)$ [—], angular frame velocity $\dot{\alpha}(t)$ [- -], and angular frame acceleration $\ddot{\alpha}(t)$ [··]. The graph is reproduced from Obrist et al. (2010).
1.4 Incompressible Navier–Stokes equations in a rotating reference frame

The incompressible Navier–Stokes equations (2) comprise the material derivative of the fluid velocity $w(\xi, t)$ in inertial frame coordinates $\xi$ at time $t$, as well as a diffusion term and a pressure gradient on the right-hand side. Vectors are denoted in bold letters, $\rho$ and $\nu$ depict fluid density and kinematic viscosity, respectively.

$$\frac{Dw(\xi, t)}{Dt} = \nabla \cdot (\nu \nabla w(\xi, t)) - \frac{1}{\rho} \nabla p(\xi, t)$$

The semicircular canals are activated through head rotation (frame angle $\alpha(t)$, angular frame velocity $\dot{\alpha}(t)$ and angular frame acceleration $\ddot{\alpha}(t)$) within the respective plane. A typical mathematical description of such a system (Gross et al., 2006, pp.2363-2369) requires the transformation of the inertial coordinates $\xi$ into coordinates $x$ of a moving reference frame, with the origin of $x$ lying on the axis of rotation. A transformation for a velocity $w(\xi, t)$ may then read:

$$w(\xi, t) = u(x, t) + \dot{\alpha}(t) \times x + v(\xi, t),$$

where $v(\xi, t)$ describes the translational motion of the reference frame. We can therefore identify $w$ to consist of relative, rotational and translational components. The material derivative of equation (3) reveals additional forces:

$$\frac{Dw(\xi, t)}{Dt} = \frac{Du(x, t)}{Dt} + \ddot{\alpha}(t) \times x + 2 \dot{\alpha}(t) \times u(x, t) + \dot{\alpha}(t) \times (\dot{\alpha}(t) \times x) + \frac{Dv(\xi, t)}{Dt}$$

Those forces are identified to be the frame, Coriolis and centrifugal acceleration, respectively:

$$a_{\text{frame}} = \ddot{\alpha} \times x$$

$$a_{\text{Coriolis}} = 2 \dot{\alpha} \times u$$

$$a_{\text{centrifugal}} = \dot{\alpha} \times (\dot{\alpha} \times x)$$

Using the definition of the material derivative,

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \cdot \nabla u,$$

and postulating zero transitional motion of the reference frame, along with a transformation of the diffusion term,

$$\nabla \cdot (\nu \nabla w(\xi, t)) = \nabla \cdot (\nu \nabla (u(x, t) + \dot{\alpha}(t) \times x))$$

$$= \nabla \cdot (\nu \nabla u(x, t)) + \nabla \cdot (\nu \nabla (\dot{\alpha}(t) \times x)) =$$

$$= \nabla \cdot (\nu \nabla u(x, t)) + \nabla \cdot (\nu (\nabla \dot{\alpha}(t) \times x + \dot{\alpha}(t) \times \nabla x)) =$$

$$= \nabla \cdot (\nu \nabla u(x, t)),$$

we finally substitute equation (4) into equation (2) and arrive at a momentum equation for the endolymph in reference frame coordinates:

$$\frac{\partial u(x, t)}{\partial t} + u(x, t) \cdot \nabla u(x, t) + \dot{\alpha}(t) \times x + 2 \dot{\alpha}(t) \times u(x, t) + \dot{\alpha}(t) \times (\dot{\alpha}(t) \times x) =$$

$$= \nabla \cdot (\nu \nabla u(x, t)) - \frac{1}{\rho} \nabla p(x, t)$$
1.5 Viscous time constant

During the acceleration of the head (according to the maneuver described in section 1.3) the moving walls of the semicircular canals displace the adjacent fluid layer along their path in the inertial system due to the no-slip condition. Since viscous stresses then act on the fluid body, a velocity profile is formed. This process acts with a delay in time, but since viscous stresses are dominant in low Reynolds number flows ($Re < 1$), we expect the latency to be of small value. The order of its value can be estimated a priori: equation (11) defines the viscous time constant $\tau_ν$ as a function of the canal radius $a$, dynamic viscosity $\nu$ and the first non-cupular eigenvalue $σ_1(φ)$ of the semicircular canal. The parameter $φ$ is a system constant, containing information about the cupula stiffness, the canal radius, the endolymph density and viscosity, and the curvature of the walls. Obrist (2008) has derived the value of $σ_1$ for various $φ$, varying insignificantly around $σ_1 \approx 5.78$ within an appropriate range for $φ$. The viscous time constant that is associated to the latency lies in the order of a few milliseconds, as calculated below.

$$\tau_ν = \frac{1}{σ_1(φ)} \frac{a^2}{ν} = \frac{1}{5.78} \frac{(160 \, \mu m)^2}{10^{-6} \, \text{m}^2/\text{s}} \approx 4.4 \, \text{ms}$$  (11)

The geometric parameter $a$ corresponds to typical human canal dimensions from the literature (Obrist, 2012, p.22), and the endolymph viscosity is assumed identical to the viscosity of water.

2 Results

Figure 4 shows the color-coded contours of the endolymph motion at $t = 0.83 \, \text{s}$ during the standard head maneuver (SHM, see section 1.3). There are cuts at different cross sections in the utricle, the ampulla and the semicircular duct, as well as in the horizontal plane of the canal. The simulation is performed with the OpenFOAM® software and is carried out by solving the continuity equation and momentum equation (10) on discretized finite volumes of the geometry described in section 1.2.

![Figure 4: Velocity magnitude of the endolymph in different cross sections at t = 0.83 s during the standard head maneuver (see section 1.3). Colors depict the range of the velocity magnitude. Glyph vectors reflect the velocity vectors.](image)

Because of the dominating size of the utricle, the viscous forces in the utricle cannot fully compensate for the radial differences in angular frame acceleration. This causes the formation of...
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a vortex, as it has already been observed in previous studies of our group, e.g. in Boselli et al. (2009), Boselli et al. (2012b), and Boselli (2012). Those studies neglect advection, the Coriolis force, the centrifugal force and the time derivative in the momentum equation. The results of this report, which are discussed in the following, will show that these simplifications are appropriate for the applied SHM.

2.1 Study of spatial forces and grid resolution

The discrepancies resulting from the absence of spatial forces (such as Coriolis, centrifugal, or advective forces) are expressed by the relative velocity error \( \epsilon_i(x) \), defined in equation (12).

\[
\epsilon_i(x) \equiv \frac{u^i(x) - u(x)}{U(x)}
\]

\[
u^i(x) = |\Delta^i|u(x,t)| = \frac{1}{3s} \int_{t=0}^{3s} |u^i(x,t) - |u(x,t)|| \cdot dt
\]

(13)

\[
U(x) = |u(x,t)| = \frac{1}{3s} \int_{t=0}^{3s} |u(x,t)| \cdot dt
\]

(14)

The index \( i \) specifies the investigated force, which can be either advection (\( i = A \)), or Coriolis and centrifugal forces (\( i = F \)). \( \Delta^i|u| \) is the difference in velocity magnitudes for simulations with and without the respective force of interest, where \( u \) corresponds to the simulation that considers all forces, and \( u^i \) is acquired by the respective assumption \( i \). Table 1 presents the maximum observed errors for different \( i \) and position \( x \).

<table>
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<tr>
<th>Force Type</th>
<th>Position</th>
<th>Error</th>
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<tr>
<td>Advection</td>
<td>( x_D )</td>
<td>( 1 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>( x_U )</td>
<td>( 5 \times 10^{-5} )</td>
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According to the results from table 1, the refinement of the mesh yields the greatest discrepancies observed (with an error of \( \epsilon_M(x_U) \approx 1\% \)), although still insignificant. The velocity errors caused by the absence of advection, \( \epsilon_A \), or of Coriolis and centrifugal forces, \( \epsilon_F \), are several orders of magnitude smaller than \( \epsilon_M \). When simulating a standard head maneuver on a semicircular canal, the spatial forces can therefore be safely neglected.

2.2 Study of transient forces

The dynamic process of the endolymph motion as a consequence of head rotation operates with some delay in time. This latency \( \Delta t \) can be observed in a transient simulation of the head maneuver (SHM, see section 1.3). Figure 5 shows time records of the velocity magnitude, taken at two positions on the reference frame: one located on the centerline within the membranous duct, the other one inside the utricle (for details see appendix B). Additionally, the head acceleration magnitude is plotted (dashed line). All quantities are normalized by their maximum amplitude. Between the input (frame acceleration) and the output quantity (fluid velocity) one can observe
Figure 5: The angular frame acceleration (dashed curve) is followed by short latencies $\Delta t$ in the velocity response of the viscous fluid within the duct and utricle (red and blue curves, respectively). The relative amplitude is given by $a_{rel}(t) = |a(t)| / \max(|a(t)|)$, where $|a|$ denotes the magnitude of the corresponding signal (angular frame acceleration, fluid velocity).

A phase lag. This latency reaches a value of $\Delta t_D \approx 5.8\text{ms}$ in the duct and $\Delta t_U \approx 37.6\text{ms}$ in the utricle (see close-up in figure 5B at around $t = 1.5\text{s}$). As for the duct, an a priori estimate of the viscous time constant $\tau_\nu$ was already introduced in section 1.5. A comparison between the estimated $\tau_\nu$ and the simulated $\Delta t_D$ shows that both values are of the same order, in contrast to the duration $T$ of the head maneuver:

$$
\begin{align*}
\tau_\nu &= 4.4\text{ ms} \\
\Delta t_D &= 5.8\text{ ms} \\
T &= 3000.0\text{ ms}
\end{align*} \quad \Rightarrow \tau_\nu \approx \Delta t_D \ll T
$$

Any head maneuver that implies the change of an input parameter over time generally requires the existence of a time derivative within the equation set. Otherwise the transient nature of the maneuver is not reflected in the simulation. However, if the latency of the system response in relation to the relevant time scale of the maneuver is some orders of magnitude smaller, one may make the assumption of quasi-steadiness.

Figure 6 compares steady with transient simulations at a time $t = 0.83\text{s}$ during the standard head maneuver and at cross-sections in the utricle, the ampulla and the duct. The present results have been carried out with a finite volume method (FVM). Additionally the results of F. Boselli are displayed; these were obtained on the same geometry (see appendix A), using a method of fundamental solutions (MFS) approach. The greatest discrepancies for the FVM simulations appear in the utricle and weigh approximately 1.7% of the cross-sectional average of the velocity magnitude.
Figure 6: Transient effects of the velocity magnitude along three cross-sectional cuts in duct, ampulla and utricle at time $t = 0.83\,s$. The present results, performed with a FVM approach using OpenFOAM® software, allow a direct comparison of the effects of the transient term which was switched on for the red, point-dashed curve. The results from Boselli et al. (2012a) were obtained with a MFS approach on the same geometry. All sets commonly neglect Coriolis force, centrifugal force, cupular forces and the advection term within the Navier-Stokes equations (10). The greatest discrepancies in velocity occurred in the utricle, yielding a value of $\delta_{\text{max}} \approx 1.7\%$.

3 Concluding remarks

The present simulations confirm the validity of all assumptions that were made by our colleague F. Boselli in previous studies of the endolymph motion. We find that the material derivative (including transient and advective mechanisms), $D\mathbf{u}/Dt$, as well as both the Coriolis and centrifugal accelerations can be safely neglected in the momentum equation (10). Discarding the advection term causes a maximum relative velocity error of $\epsilon^A \approx 5 \cdot 10^{-5}$, and the force terms yield a maximum error of $\epsilon^F \approx 3 \cdot 10^{-4}$. The latency is shown to be three orders of magnitude smaller than the relevant time scales, which justifies quasi-steady simulations for the standard head maneuver.

4 Acknowledgements

This work was supported by a grant from the Swiss National Science Foundation (SNF, no. 205321-138298).
A Geometry

The geometry of the horizontal semicircular canal was derived from a point cloud on the surface of the membranous labyrinth following geometric measurements by Curthoys & Oman (1987).

B Probe and sample locations

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References


