


sectorgap: An R Package for Consistent Economic Trend Cycle Decomposition

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Abstract

Determining potential output and the output gap—two inherently unobservable variables—is a major challenge for macroeconomists. This paper presents the R package **sectorgap**, which features a flexible modeling and estimation framework for a multivariate Bayesian state space model identifying economic output fluctuations consistent with subsectors of the economy. The proposed model is able to capture various correlations between output and a set of aggregate as well as subsector indicators. Estimation of the latent states and parameters is achieved using a simple Gibbs sampling procedure and various plotting options facilitate the assessment of the results. An illustrative example with Swiss data outline data preparation, model definition, estimation, and evaluation using **sectorgap**.

Keywords: R, state space models, time series, simulation smoother, Gibbs sampling, business cycle, output gap, potential output.

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1. Introduction

Determining the cyclical position of the economy—unobservable by nature—constitutes an essential problem in macroeconomics. The business cycle, also known as the output gap, is the difference between observed economic output and its potential, i.e., the level of sustainable and non-inflationary output (Hall and Taylor 1991).

Numerous modeling and estimation techniques have been developed, among them univariate and multivariate time series filters (e.g. Hodrick and Prescott 1997; Baxter and King 1999; Hamilton 2018; Quast and Wolters 2022), production-function based methods (e.g. Havik, Mc Morrow, Orlandi, Planas, Raciborski, Röger, Rossi, Thum-Thysen, and Vandermeulen 2014), and semi-structural unobserved component models that incorporate economic theory (Kuttner 1994; Gerlach and Smets 1999; Blagrove, Garcia-Saltos, Laxton, and Zhang 2015; Jarociński and Lenza 2018; Hasenzagl, Pellegrino, Reichlin, and Ricco 2022).

sectorgap enables the estimation of the output gap according to a new bottom-up approach which incorporates the cycle and trend dynamics of subsectors of the economy (Rathke and Streicher 2023). Similar to previous unobserved component models, the sectorgap (SG) model is able to incorporate correlations between output and the labor market and inflation developments. In addition, by incorporating subsectors of economic output and ensuring consistency between aggregate and subsector trends, the SG model provides a comprehensive overview of the emergence of economic fluctuations and secular trends.

In essence, **sectorgap** facilitates the estimation of a large Bayesian state space model. The model definition is kept general to ensure applicability in various settings. Unobserved states and parameters are obtained using a Gibbs sampling approach. Finally, **sectorgap** provides useful plotting options to analyze the results.

Section 2 defines the bottom-up sector gap methodology and elaborates on the aggregation constraints. The Bayesian estimation approach is detailed in Section 3. Section 4 gives a brief overview of the general functionality of **sectorgap** and Section 5 illustrates its usage by applying it to Swiss data. Section 6 gives a short overview to related software and Section 7 concludes.

2. A bottom-up output gap model

Our model is based on the methodology described by Rathke and Streicher (2023), but allows for a more general structure here.

Potential output and the output gap are latent processes and are thus modeled via an unobserved component model. The observed variable, aggregate output y_t —or gross domestic product (GDP)—splits up into a trend τ_{yt} and a cycle c_{yt} , the latter of which defines the so-called output gap g_{yt} . It is common to complement output by a set of aggregate variables $a_t := (a_{1t}, \dots, a_{kt})'$. Examples of such variables are price and labor market indicators.¹ In addition, we allow for subsector output $\check{y}_t = (\check{y}_{1t}, \dots, \check{y}_{nt})$ and subsector indicators $\check{a}_t := (\check{a}_{1t}, \dots, \check{a}_{mt})'$.

Similar to output, we decompose each of these additional variables into a trend and a cycle. The individual cycles of the aggregate variables a_t and the subsectors \check{y}_t are assumed to be

¹Okun's law (Okun 1963) connects employment and unemployment to the output gap and a Phillips curve relationship can be used to capture correlations to inflation.

linearly dependent on the output gap, while the cycles of the subsector indicators in \check{a}_t are correlated with their corresponding output subsectors \check{y}_t . Summing up, we obtain

$$\begin{pmatrix} y_t \\ a_t \\ \check{y}_t \\ \check{a}_t \end{pmatrix} = \begin{pmatrix} \tau_{yt} \\ \tau_{at} \\ \tau_{\check{y}t} \\ \tau_{\check{a}t} \end{pmatrix} + \begin{pmatrix} g_{yt} \\ g_{at} \\ g_{\check{y}t} \\ g_{\check{a}t} \end{pmatrix} = \begin{pmatrix} \tau_{yt} \\ \tau_{at} \\ \tau_{\check{y}t} \\ \tau_{\check{a}t} \end{pmatrix} + \begin{pmatrix} 1 & & & \\ \Psi_a(L) & I_k & & \\ \Psi_{\check{y}}(L) & & I_n & \\ & & \Psi_{\check{a}}(L) & I_m \end{pmatrix} \begin{pmatrix} c_{yt} \\ c_{at} \\ c_{\check{y}t} \\ c_{\check{a}t} \end{pmatrix}, \quad (1)$$

where $\Psi_j(x) = \psi_{j0}x^0 + \dots + \psi_{jq_j}x^{q_j}$, $j \in \{a, \check{y}, \check{a}\}$ are diagonal matrix lag polynomials of possibly different orders $q_a, q_{\check{y}}, q_{\check{a}}$.

For the trends we allow several different processes. Dropping the group indices for notational convenience, we have

$$\tau_t = \tau_{t-1} + \varepsilon_{\tau t}, \quad (2a)$$

$$\tau_t = \tau_{t-1} + \mu_{t-1} + \varepsilon_{\tau t}, \quad \mu_t = \mu_{t-1} + \varepsilon_{\mu t}, \quad (2b)$$

$$\tau_t = \tau_{t-1} + \mu_{t-1}, \quad \mu_t = \mu_{t-1} + \varepsilon_{\mu t}, \quad (2c)$$

$$\tau_t = \tau_{t-1} + \mu_{t-1} + \varepsilon_{\tau t}, \quad \mu_t = \gamma + \phi_{\mu}\mu_{t-1} + \varepsilon_{\mu t}. \quad (2d)$$

The innovations $\varepsilon_{\tau t}$ and $\varepsilon_{\mu t}$, with $\varepsilon_{\tau t} \perp \varepsilon_{\mu t}$ are normally distributed and may each be correlated across their respective group, but are assumed independent between groups. Equation (2a) describes a standard random walk and Equation (2b) a random walk with random drift, also called a local linear trend. Equation (2a) drops the innovation to the level of the local linear trend and finally, Equation (2d) assumes a stationary autoregressive drift of order one.

The output gap and all idiosyncratic cycles are modeled as stationary autoregressive processes, i.e.,

$$\Phi(L)c_t = \varepsilon_{ct}, \quad \varepsilon_{ct} \sim \mathcal{N}(0, \sigma_c^2), \quad (3)$$

where we again drop the group indices for convenience. Since Equation (1) already imposes a correlation between the cycles, we assume that the innovations ε_{ct} are independent. The diagonal matrix lag polynomials $\Phi_j(x) = 1 - \Phi_{j1}x - \dots - \Phi_{jp_j}x^{p_j}$, $j \in \{y, a, \check{y}, \check{a}\}$ are of order $p_y, p_a, p_{\check{y}}$, and $p_{\check{a}}$.

sectorgap also features a second group of subsectors $\check{\check{y}}_t$ which we exclude here to simplify notation. Expanding the model for a second subgroup is trivial. In addition, we allow for a group of variables with characteristics that do not fit into any of the above groups. For instance, an additional measure of inflation that shares the same trend as one of the variables in a_t . To keep notation simple, we refrain from including this group at this point and comment on it in Section 2.3.

Identification can be achieved by placing restrictions on the innovation correlations between trends, drifts, and cycles. The Clark (1987) model is a subversion of the model in Equation (1) – (3) and for identification Clark (1987) assumes an autoregressive cycle of at least order two and additionally drops possible correlation between trend and drift innovations, leaving within trend and within drift correlations possible.² Similar restrictions carry over to our model, and thus we enforce $\varepsilon_{\tau t} \perp \varepsilon_{\mu t}$ and recommend an autoregressive order larger two.

²The correlation between permanent and transitory shocks is likely negligible (Clark 1987; Morley, Nelson, and Zivot 2003; Morley 2007; Oh, Zivot *et al.* 2006)

2.1. Aggregation and constraints

In most cases, subsector trends are subject to known restrictions. The derivations in this section build on the assumption that data in levels enters the model using a log transformation $g(x) = \log x$. As a result, the estimated cycles can be readily interpreted since, e.g.,

$$\frac{Y_t}{T_{Yt}} - 1 \approx \log Y_t - \log T_{Yt} = y_t - \tau_{yt} = g_{yt}, \quad (4)$$

where T_{Yt} denotes the untransformed trend of aggregate output.³ By definition, aggregate real output $Y_t = \exp y_t$ is a chain-linked volume index, i.e., it can be represented as the weighted sum of real subsector output

$$Y_t = w_t^{P'} \check{Y}_t, \quad (5)$$

where $\check{Y}_t = (\check{Y}_{1t}, \dots, \check{Y}_{nt})$, $\check{Y}_{it} = \exp \check{y}_{it}$ and $w_t^P = (w_{1t}^P, \dots, w_{nt}^P)'$, $w_{it}^P = \check{P}_{it-1}/P_{t-1}$ contains relative previous period prices for the subsectors. Naturally, these restrictions can be carried over to the unobservable trends, i.e.,

$$\exp \tau_{yt} = w_t^{P'} \exp \tau_{\check{y}_t}, \quad (6)$$

where $\tau_{\check{y}_t} = (\tau_{\check{y}_{1t}}, \dots, \tau_{\check{y}_{nt}})$. Similarly, without loss of generality, let now $k = 1$ and let $A_t = \exp a_t$ represent an aggregate indicator in levels, for instance aggregate employment. For the corresponding subsector indicators $\check{A}_t = (\check{A}_1, \dots, \check{A}_m)$, $\check{A}_j = \exp \check{a}_j$, we have

$$A_t = 1_m' \check{A}_t$$

where 1_m is a $m \times 1$ vector of ones, which implies

$$\exp \tau_{at} = 1_m' \exp \tau_{\check{a}_t}. \quad (7)$$

We use an iterative procedure to approximate the non-linear constraints in Equations (6) and (7), which we elaborate on in Section 2.2.

In addition, we can deduce linear constraints for parts of the trend growth rates τ and drifts μ . Let now Y_t^{nom} , Y_{it}^{nom} , $i = 1 \dots, n$ denote the respective nominal output series. From Equation (5) and since $Y_{it}^{nom} = 1/100 P_{it} Y_{it}$, we have

$$\begin{aligned} \Delta y_t &\approx \frac{Y_t}{Y_{t-1}} - 1 = \frac{w_t^{P'} \check{Y}_t}{Y_{t-1}} - 1 \\ &= \frac{w_t^{P'}}{Y_{t-1}} \text{diag}(\check{Y}_{t-1}) \text{diag}(\check{Y}_{t-1})^{-1} \check{Y}_t - 1 \\ &= \frac{\check{P}'_{t-1}}{P_{t-1} Y_{t-1}} \text{diag}(\check{Y}_{t-1}) \text{diag}(\check{Y}_{t-1})^{-1} \check{Y}_t - 1 \\ &= w_t^{Y^{nom}'} \text{diag}(\check{Y}_{t-1})^{-1} \check{Y}_t - 1 \\ &\approx w_t^{Y^{nom}'} \Delta \check{y}_t, \end{aligned}$$

³In fact, the applied transformation is usually $g(x) = 100 \log x$, as this results in cycle estimates in percent. For ease of readability, we drop the multiplier.

where $w_t^{Y^{nom}} = (w_{1t}^{Y^{nom}}, \dots, w_{nt}^{Y^{nom}})'$, $w_{it}^{Y^{nom}} = \check{Y}_{it-1}^{nom}/Y_{t-1}^{nom}$ are the nominal output weights at $t - 1$ for which naturally, $w_t^{Y^{nom}'} \mathbf{1}_n = 1$ for all $t \in \mathbb{Z}$. Hence, the growth rate of output is a linear combination of the growth rates of subsector output.⁴ If we assume that relative previous period prices w_{it}^P are the same for output and its trend, this linear constraint carries over to trend growth rates and drifts

$$\Delta\tau_{yt} = w_t^{Y^{nom}'} \Delta\tau_{\check{y}t}, \quad (8)$$

$$\mu_{yt} = w_t^{Y^{nom}'} \mu_{\check{y}t}, \quad (9)$$

where $\mu_{\check{y}t} = (\mu_{\check{y}1t}, \dots, \mu_{\check{y}nt})'$. Both linear constraints can be imposed by adding an identity series to the observation equation (Doran 1992). For trend specifications without short-term changes to trend growth ($\varepsilon_{\tau_{yt}} = 0, \varepsilon_{\tau_{\check{y}t}} = 0$, see Equation (2c)), consistency between aggregate and subsector trends can be attained by enforcing Equation (9) only. On the other hand, if the model allows for shocks to trend growth, Equation (8) can be enforced through the trend shocks $\varepsilon_{\tau_{yt}}, \varepsilon_{\tau_{\check{y}t}}$ and via Equation (9).⁵

Analogously, for the subsector (employment) indicators, we obtain

$$\Delta\tau_{at} = w_t^A \Delta\tau_{\check{a}t}, \quad (10)$$

$$\mu_{at} = w_t^A \mu_{\check{a}t}, \quad (11)$$

where $w_t^A = (w_{1t}^A, \dots, w_{mt}^A)'$, $w_{jt}^A = \check{A}_{jt-1}/A_{t-1}$ denotes the weight of the indicator in sector i at point in time $t - 1$.

2.2. State space representation

The model in Equation (1) – (3) with different trend and cycle specification can be cast into state space representation. We stack all observation variables and their cycles, trends, and drifts in corresponding order, i.e.,

$$\begin{aligned} \bar{y}_t &= (y_t, a_t', \check{y}_t', \check{a}_t')' \\ c_t &= (c_{yt}, c'_{at}, c'_{\check{y}t}, c'_{\check{a}t})' \\ \tau_t &= (\tau_{yt}, \tau'_{at}, \tau'_{\check{y}t}, \tau'_{\check{a}t})' \\ \mu_t &= (\mu_{yt}, \mu'_{at}, \mu'_{\check{y}t}, \mu'_{\check{a}t})' \end{aligned}$$

where $o = 1 + k + n + m$ is the number of observables and o_μ the number of drifts. The state vector can be defined by

$$\alpha_t = (c_t', \dots, c'_{t-v}, \tau_t', \mu_t')',$$

⁴For annual figures, this holds with equality if the data is compiled using the Annual Overlap method. For quarterly figures, small deviations are possible which we assume to be zero for the trend component we are interested in.

⁵Note that $\Delta\tau_{yt} - \mu_{yt-1} = \varepsilon_{\tau_{yt}}, \Delta\tau_{\check{y}t} - \mu_{\check{y}t-1} = \varepsilon_{\tau_{\check{y}t}}$ and thus Equation (8) is satisfied if $\varepsilon_{\tau_{yt}} = w_t^{nom'} \varepsilon_{\tau_{\check{y}t}}$ and if Equation (9) holds.

where $s = (v+1)o + o_\mu$ is the number of states with $v = \max\{p, q\}$ where $q = \max\{q_a, q_{\check{y}}, q_{\check{a}}\}$ is the maximum lag of all loadings and $p = \max\{p_y, p_a, p_{\check{y}}, p_{\check{a}}\}$ the maximum autoregressive order of all cycles. The measurement and state equation are given by

$$\begin{aligned} \bar{y}_t &= c_{\bar{y}t} + Z_t \alpha_t, \\ \alpha_t &= c_{\alpha t} + T_t \alpha_{t-1} + R_t \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, Q_t), \end{aligned} \quad (12)$$

where $\tilde{s} = o + o_\tau + o_\mu$ denotes the number of state equations and $o_\tau \leq o$ the number of trends that include an innovation term. For ease of readability, we split the system matrices into three blocks, one concerning the vector of contemporaneous and lagged cycles $(c'_t, \dots, c'_{t-v})'$, one for the trends τ_t and a final block for the drifts μ_t . The structure of the blocks is indicated by vertical and horizontal lines. The system matrices of the state space model in Equation (12) are given by

$$\begin{aligned} Z_t &= \left[Z_t^0 \quad \dots \quad Z_t^v \mid I_o \mid 0 \right], & T_t &= \left[\begin{array}{cccc|cc} \Phi_1 & \Phi_2 & \dots & \Phi_p & 0 & & \\ I_o & 0 & & & & & \\ & & \ddots & & & & \\ & & & I_o & 0 & 0 & \\ 0 & \dots & 0 & 0 & 0 & 0 & \\ \hline & & & & & I_o & T_t^\tau \\ \hline & & & & & & T_t^\mu \end{array} \right], \\ R_t &= \left[\begin{array}{ccc|ccc} I_o & 0 & 0 & & & \\ 0 & 0 & 0 & & & \\ \vdots & \vdots & \vdots & & & \\ 0 & 0 & 0 & & & \\ \hline 0 & I_o & 0 & & & \\ 0 & 0 & I_o & & & \end{array} \right], & Q_t &= \left[\begin{array}{c} \Sigma_c \\ \hline \Sigma_\tau \\ \hline \Sigma_\mu \end{array} \right]. \end{aligned}$$

The submatrices in Z_t are defined by

$$Z_t^j = \begin{cases} I_o + \tilde{Z}_t^0 & j = 0, \\ \tilde{Z}_t^j & j > 0, \end{cases} \quad \tilde{Z}_t^j = \begin{bmatrix} 0 & & & \\ \psi_a^j & 0 & & \\ \psi_{\check{y}}^j & & 0 & \\ & & \Psi_{\check{a}}^j & 0 \end{bmatrix}$$

with $\Psi_{\check{a}}^j = \text{diag}(\psi_{\check{a}}^j)$ for $j \in \{0, \dots, q\}$, where $\psi_a^j, \psi_{\check{y}}^j, \psi_{\check{a}}^j$ are $k \times 1$, $n \times 1$, and $m \times 1$ vectors, respectively. The auxiliary matrices $\tilde{Z}_t^j, j \in \{0, \dots, q\}$ contain the (lagged) loadings of all aggregate variables a_t and the subsectors \check{y}_t on aggregate output y_t , and those of the subsector indicators \check{a} on the associate subsectors \check{y}_t . In addition, the matrix Z_t^0 links each observable with its contemporaneous cycle. Note that $\psi_a^j = 0$ for $j > q_a$, $\psi_{\check{y}}^j = 0$ for $j > q_{\check{y}}$, and $\psi_{\check{a}}^j = 0$ for $j > q_{\check{a}}$.

The state matrix T_t contains the diagonal autoregressive coefficient matrices

$$\Phi_j = \text{diag}(\phi_y^{j'}, \phi_a^{j'}, \phi_{\check{y}}^{j'}, \phi_{\check{a}}^{j'}), j \in \{1, \dots, p\}$$

of the cycles. Note that, depending on the specified autoregressive order, some elements on the diagonal of Φ_j are zero. In addition, the matrix T_t contains a number of identities for the lagged cycles and it defines the trend processes. T_t^r is a matrix of size $o \times o_\mu$, which connects the trends to their respective drifts, i.e., the rows associated with trends that do not contain a drift are zero. T_t^μ is a $o_\mu \times o_\mu$ diagonal matrix that links the drifts to their lags. For local linear trend specifications (Equation (2b)), the respective diagonal value is one, while for AR(1) drift specifications (Equation (2d)), it contains the autoregressive parameter ϕ_μ .

The vector $c_{\alpha t}$ contains zeros for all states except AR(1) drifts, for which it holds the constant γ . The matrix R_t connects each state equation to its corresponding innovation term (or none). Finally, the variance-covariance matrix Q_t contains all cycle, trend, and drift variances Σ_c, Σ_τ , and Σ_μ , which are matrices of sizes $o \times o$, $o_\tau \times o_\tau$, and $o_\mu \times o_\mu$. By assumption, Σ_c is diagonal, while the trend and drift variances Σ_τ, Σ_μ may contain off-diagonal values allowing for correlation within each group.

Constraints

To impose constraints on the trends of sector output and employment as discussed in Section 2.1, the system in Equation (12) needs to be adjusted. For $j \in \mathbb{Z}$ linear constraints on the states of the form

$$W_t \alpha_t = \eta_t,$$

where W_t is a $j \times s$ matrix and η_t a $j \times 1$ vector, the aggregation constraints can be imposed by a simple extension of the observation equation (Doran 1992). Non-linear constraints

$$f_t(\alpha_t) = 0$$

can be approximated by an iterative procedure using a first-order Taylor expansion

$$0 = f_t(\alpha_t) \approx f_t(\alpha_t^{(\iota)}) + Df_t(\alpha_t^{(\iota)}) (\alpha_t - \alpha_t^{(\iota)})$$

(Doran 1992). We can augment the observation equation analogous to linear constraints by defining

$$\begin{aligned} W_t &= Df_t(\alpha_t^{(\iota)}), \\ \eta_t &= Df_t(\alpha_t^{(\iota)}) \alpha_t^{(\iota)} - f_t(\alpha_t^{(\iota)}), \end{aligned}$$

where Df_t is the Jacobian matrix of the vector function $f_t : \mathbb{R}^s \rightarrow \mathbb{R}^j$. The iterative nature can easily be incorporated into the Bayesian estimation framework, i.e., $\alpha_t^{(\iota)}$ denotes the ι -th draw of the state vector.

We first consider the simple case where shocks to trend growth are set to zero, i.e., it suffices to impose linear constraints on the drifts, i.e., $\tau_{yt} = w_t^{Y^{nom}} \tau_{\bar{y}t}$ and $\tau_{at} = w_t^A \tau_{\hat{a}t}$. To that end, Z_t and \bar{y}_t are augmented. We define

$$\begin{aligned} \hat{\bar{y}}_t^{o+2 \times 1} &= (\bar{y}_t', 0, 0)', \quad \hat{Z}_t^{o+2 \times s} = \left[\begin{array}{ccc|c} Z_t^0 & \cdots & Z_t^q & I_o \\ 0 & \cdots & 0 & 0 \end{array} \middle| \begin{array}{c} 0 \\ \hat{Z}_t^\mu \end{array} \right], \\ \hat{Z}_t^\mu &= \left[\begin{array}{cccccc} 1 & 0 & -w_{1t}^{Y^{nom}} & \cdots & -w_{nt}^{Y^{nom}} & 0 \\ 0 & 1 & 0 & \cdots & 0 & -w_{1t}^A \end{array} \middle| \begin{array}{c} \cdots \\ -w_{mt}^A \end{array} \right], \end{aligned} \quad (13)$$

where, without loss of generality, we again impose $k = 1$, i.e., there is only one aggregate indicator in a_t which corresponds to the subindicators \check{a}_t . Note that the weights $w_{it}^{Y^{nom}}$, $i = 1, \dots, n$ and w_{jt}^A , $j = 1, \dots, m$ as defined in Section 2.1 are time-dependent. If constraints involve a residual $\eta_t \neq 0$, this term can be included in the vector c_{yt} .⁶

If the model allows for shocks to the trends, non-linear constraints are implemented in a similar way. Subsector trends satisfy

$$0 = f_t(\alpha_t) = \exp\{\tau_{yt}\} - w_t^{P'} \exp\{\tau_{yt}\} - r_t^Y$$

where the residual r_t^Y is zero if the subsectors are exhaustive. Thus, we have

$$\delta Df_t(\alpha_t) = \left(\exp \tau_{yt}^{(\iota)}, -w_t^{P'} \exp \tau_{yt}^{(\iota)} \right)'$$

and

$$\begin{aligned} \delta' W_t^Y &= \left(\exp \tau_{yt}^{(\iota)}, -w_t^{P'} \exp \tau_{yt}^{(\iota)} \right)' = \left(\tilde{w}_t^Y, -\tilde{w}_{1t}^Y, \dots, -\tilde{w}_{nt}^Y \right)^{(\iota)'}, \\ \eta_t^Y &= \exp \tau_{yt}^{(\iota)} \left(\tau_{yt}^{(\iota)} - 1 \right) - w_t^{P'} \text{diag} \left(\exp \tau_{yt}^{(\iota)} \right) \left(\tau_{yt}^{(\iota)} - 1 \right) + r_t^Y, \end{aligned}$$

where δ is a $(n+1) \times s$ selection matrix such that $\delta' \alpha_t = \left(\tau_{yt}, \tau_{yt}^{(\iota)} \right)'$. The adjustments for the non-linear constraint for the subindicators in Equation (7) follow analogously, i.e.,

$$\begin{aligned} \delta' W_t^A &= \left(\exp \tau_{at}^{(\iota)}, -1'_m \exp \tau_{at}^{(\iota)} \right)' = \left(\tilde{w}_t^A, -\tilde{w}_{1t}^A, \dots, -\tilde{w}_{nt}^A \right)^{(\iota)'}, \\ \eta_t^A &= \exp \tau_{at}^{(\iota)} \left(\tau_{at}^{(\iota)} - 1 \right) - 1'_m \text{diag} \left(\exp \tau_{at}^{(\iota)} \right) \left(\tau_{at}^{(\iota)} - 1 \right) + r_t^A. \end{aligned}$$

Finally, the state-space model in Equation (12) can be adjusted as follows,

$$\begin{aligned} \hat{y}_t^{o+2 \times 1} &= (\bar{y}_t', 0, 0)', & \hat{Z}_t^{o+2 \times s} &= \left[\begin{array}{ccc|c|c} Z_t^0 & \dots & Z_t^q & I_o & 0 \\ 0 & \dots & 0 & \hat{Z}_t^{\tau^{(\iota)}} & 0 \end{array} \right], \\ \hat{c}_{yt}^{o+2 \times 1} &= \left(c_{yt}', -\eta_t^Y, -\eta_t^A \right)', & \hat{Z}_t^{\tau^{(\iota)}} &= \left[\begin{array}{cccc|ccc} \tilde{w}_t^Y & 0 & -\tilde{w}_{1t}^Y & \dots & -\tilde{w}_{nt}^Y & 0 & \dots & 0 \\ 0 & w_t^A & 0 & \dots & 0 & -\tilde{w}_{1t}^A & \dots & -\tilde{w}_{mt}^A \end{array} \right], \end{aligned} \quad (14)$$

Alternatively, we may want to impose linear constraints on the trend growth rates. Similar to the constraints on the drifts, the system matrices in Equation (12) need to be augmented. By adding the trend shocks $\varepsilon_{\tau t}$ to the state equation, the additional constraints can again be imposed via Z_t . Let now

$$\begin{aligned} \hat{\alpha}_t^{s+o \times 1} &= (\alpha_t' | \varepsilon_{\tau t}')', & \hat{T}_t^{s+o \times s+o} &= \left[\begin{array}{c|c} T_t & \\ \hline & 0 \end{array} \right], & \hat{R}_t^{s+o \times \bar{s}} &= \left[\begin{array}{c|cc} R_t & & \\ \hline 0 & I_o & 0 \end{array} \right], \\ \hat{y}_t^{o+2 \times 1} &= (\bar{y}_t' | 0)', & \hat{Z}_t^{o+2 \times s+o} &= \left[\begin{array}{ccc|c|c|c} Z_t^0 & \dots & Z_t^q & I_o & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 & \hat{Z}_t^{\tau} \end{array} \right] \end{aligned}$$

with $\hat{Z}_t^{\tau} = \hat{Z}_t^{\mu}$.

⁶For instance if the subsector data is non exhaustive, i.e., if $Y_t \neq w_t^{P'} \check{Y}_t$ or $A_t \neq 1'_m \check{A}_t$.

2.3. Model extensions

In this section, we introduce a group of observable variables b_t which does not fall into any of the previous groups $a_t, \check{y}_t, \check{a}_t$. For each variable in b_t , a cycle, trend, and loading specification can be made. Without loss of generality, let now b_t be a scalar which follows

$$b_t = \tau_{bt} + \psi_b' \alpha_t + c_{bt}, \quad (15)$$

where α_t is the state vector from Equation (12) in Section 2.2 and c_{bt} is an $\text{AR}(p_b)$ cycle. The $s \times 1$ vector ψ_b contains loadings for the state vector, i.e., b_t can load on cycles as well as trends in α_t . The trend τ_{bt} can be one of the above specifications or dropped.⁷

The state space representation in Equation (12) can be easily augmented for extensions of the type outlined in Equation (15).

3. Estimation

sectorgap provides simple tools to define the multivariate state space model in Equation (12) based on previously specified model settings. To that end, it uses the extensive modeling framework of the R package **KFAS** (Helske 2017). The unobserved states α_t and parameters

$$\begin{aligned} \Theta &= \{\Theta_c, \Theta_\tau, \Theta_\mu\}, & \Theta_{c_j} &= \{\phi_{j1}, \dots, \phi_{jp_j}, \psi_{j0}, \dots, \psi_{jq_j}, \sigma_{c_j}^2\}, j \in \{y, a, \check{y}, \check{a}\}, \\ & & \Theta_\tau &= \Sigma_\tau \\ & & \Theta_\mu &= \{\gamma, \phi_\mu, \Sigma_\mu\} \end{aligned}$$

are then estimated using a Bayesian approach. To be more precise, we use a Gibbs algorithm structured in multiple blocks. The first three steps draw the parameters conditional on the states, i.e., $\Theta|\alpha_t$. In a final step, the unobserved states are drawn conditional on the previously drawn parameters using the simulation smoother of Durbin and Koopman (2002, 2012) provided by the R package **KFAS** (Helske 2017). To compute each posterior we generate R draws and discard the specified burnin phase (e.g. the first 50%). If specified, thinning is applied, i.e. the Gibbs procedure discards draws to prevent possible autocorrelation between draws. See Appendix A for details on the algorithm.

3.1. Prior distribution

For the prior distributions of all loading and autoregressive parameters, we assume a normal distribution, while for the variances we assume inverse-gamma or inverse-wishart priors. **sectorgap** uses weakly informative distributions to initialize the priors, see Table 3.1. To facilitate economically meaningful trend cycle decompositions, we adopt a smoothing parameter $\lambda = \mathbb{E}[\sigma_c^2]/\mathbb{E}[\sigma_i^2]$, $i \in \{\tau, \mu\}$ defining the ratio between the variance of cycle and trend or drift innovations. For instance, a smoothing constant of $\lambda = 100$ implies an a priori signal-to-noise ratio of 1%.

⁷An example of an equation with a dropped trend is when b_t shares a trend with one or more of the variables in \check{y}_t .

| Name | Support | Density | Parameters | |
|----------------------------|---|-----------------|---------------------|---------------------------------|
| $(\psi_0, \dots, \psi_q)'$ | \mathbb{R}^q | Normal | $\mu_0 = 0,$ | $\Sigma_0 = 1000I_q$ |
| $(\phi_1, \dots, \phi_p)'$ | $\mathbb{R}^p \times I_{\phi \in S_\phi}$ | Normal | $\mu_0 = (0, 0)'$, | $\Sigma_0 = 1000I_p$ |
| $(\gamma, \phi_\mu)'$ | $\mathbb{R}^2 \times I_{\phi_\mu \in S_\phi}$ | Normal | $\mu_0 = (0, 0)'$, | $\sigma_0^2 = 1000I_2$ |
| σ_c^2 | $(0, \infty)$ | Inverse-gamma | $\nu_0 = 6,$ | $s_0 = 4$ |
| Σ_μ^2 | $(0, \infty)$ | Inverse-wishart | $\nu_0 = 6,$ | $S_0 = 4\lambda^{-1}I_o$ |
| Σ_τ^2 | $(0, \infty)$ | Inverse-wishart | $\nu_0 = 6,$ | $S_0 = 4\lambda^{-1}I_{o_\tau}$ |

Table 1: Prior distributions. $I_{\phi \in S_\phi}$ denotes the indicator function and S_ϕ the stationary region of an AR(p) process. All indices are suppressed for the sake of readability. The normal distribution is parameterized via mean and variance, the inverse-gamma distribution via degrees of freedom ν and location s with mean $s/\nu-2$, the inverse-wishart distribution via degrees of freedom ν and $\kappa \times \kappa$ scale matrix S , with mean $1/\nu-\kappa-1S$.

3.2. Posterior distribution

Given the model specification and the prior distributions, we now outline a procedure to obtain the posterior distribution of the parameters and the states.

Let now $\bar{y}^T := (\bar{y}_1, \dots, \bar{y}_T)$ and analogously, for the state vector, $\alpha^T := (\alpha_1, \dots, \alpha_T)$. Since the posterior distribution of (α^T, Θ) conditional on the data \bar{y}^T is not given in closed form, we use a Gibbs sampling procedure to obtain draws from it. We have that

$$p(\alpha^T, \Theta | \bar{y}^T) = p(\alpha^T | \Theta, \bar{y}^T) p(\Theta | \alpha^T, \bar{y}^T) \quad (16)$$

and the first term on the right hand side factorizes to

$$p(\alpha^T | \Theta, \bar{y}^T) = p(\alpha_T | \Theta, \bar{y}_T) \prod_{t=d+1}^{T-1} p(\alpha_t | \alpha_{t+1}, \Theta, \bar{y}^T) \prod_{t=1}^d p(\alpha_t | \alpha_{t+1}, \Theta, \bar{y}^T),$$

where d denotes the number of non-stationary states. Samples from the first two terms can be readily obtained by applying the simulation smoother conditional on the model parameters and the data (Durbin and Koopman 2002, 2012). For the third term, the exact diffuse initialization is applied (see e.g. Helske 2017; Durbin and Koopman 2002, 2012).

For the second term in Equation (16), we assume the block independence

$$\begin{aligned} p(\Theta | \alpha^T, \bar{y}^T) &= p(\Theta | c^T, \tau^T, \mu^T, \bar{y}^T) \\ &= p(\Theta_\tau | \tau^T, \mu^T) p(\Theta_\mu | \mu^T) \prod_{j \in \{y, a, \check{y}, \check{a}\}} p(\Theta_{c_j} | c_j^T, \bar{y}^T). \end{aligned} \quad (17)$$

The next sections describes how to draw from each of the conditionals in Equation (17).

Trends and drifts

We specify prior and posterior for a general innovation vector ε_t of size $\kappa \times 1$ and in turn relate these results to each of the trend specifications in Equation (2a)–(2d). For the unknown variance Σ of ε_t , we impose

$$\Sigma \sim \mathcal{IW}(S_0, \nu_0)$$

as prior distribution, where S_0 is a $\kappa \times \kappa$ positive definite scale matrix and ν_0 denotes the degrees of freedom.⁸ Using standard results, we obtain

$$p\left(\Sigma \mid \varepsilon^T\right) = \prod_{t=1}^T p\left(\varepsilon_t \mid \Sigma\right) p\left(\Sigma\right) \propto \mathcal{IW}\left(S_*, \nu_*\right)$$

with

$$\begin{aligned}\nu_* &= \nu_0 + T, \\ S_* &= S_0 + \varepsilon_t' \varepsilon_t.\end{aligned}$$

For a random walk specification (Equation (2a)), we have Σ_τ with $\varepsilon_{\tau t} = \Delta\tau_t$. For a local linear trend (Equation (2b)), we have Σ_τ with $\varepsilon_{\tau t} = \Delta\tau_t - \mu_t$ and Σ_μ with $\varepsilon_{\mu t} = \Delta\mu_t$. If the trend innovation drops (Equation (2c)), only Σ_μ is required. Finally, in case of an AR(1) drift (Equation (2d)), the posterior distribution of the drift variance follows as described in the next section.

Cycles and loadings

For the observation equations, we can apply the results of Chib (1993). We define the univariate linear model with autoregressive errors

$$\tilde{y}_t = x_t' \beta + \varepsilon_t, \quad \Phi(L) \varepsilon_t = u_t, \quad u_t \sim \mathcal{N}\left(0, \sigma^2\right), \quad (18)$$

where $\Phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p$, β is a $b \times 1$ vector of coefficients and x_t is a $b \times 1$ vector of covariates. Define

$$\tilde{y}_t^* = \Phi(L) \tilde{y}_t, \quad x_t^* = \Phi(L) x_t,$$

for $t = p + 1, \dots, T$ and $\tilde{y}^* = (\tilde{y}_{p+1}^*, \dots, \tilde{y}_T^*)'$, $\tilde{y} = (\tilde{y}_{p+1}, \dots, \tilde{y}_T)'$ and $x^* = (x_{p+1}^{*'} , \dots, x_T^{*'})'$ are of dimension $T - p \times 1$ and $T - p \times b$, respectively.

We assume the prior distribution of the involved parameters factorizes, i.e.,

$$\pi\left(\beta, \sigma^2, \phi\right) = \pi\left(\beta\right) \pi\left(\sigma^2\right) \pi\left(\phi\right)$$

with $\phi = (\phi_1, \dots, \phi_p)'$ and for the individual prior distributions,

$$\begin{aligned}\beta &\sim \mathcal{N}_b\left(\beta_0, A_0^{-1}\right), \\ \sigma^2 &\sim \mathcal{IG}\left(s_0, \nu_0\right), \\ \phi &\sim \mathcal{N}_p\left(\phi_0, \phi_0^{-1}\right) I_{\phi \in S_\phi}.\end{aligned}$$

Note that the inverse-gamma distribution is parametrized via location s_0 and degrees of freedom ν_0 , which is equivalent to a parametrization via shape $\nu_0/2$ and scale $s_0/2$. The posterior distribution of the coefficient vector is given by

$$\begin{aligned}\beta \mid y &\sim \mathcal{N}_b\left(\beta_*, A_*^{-1}\right), & A_* &= A_0 + \sigma^{-2} X^{*'} X^*, \\ \beta_* & & \beta_* &= A_*^{-1} \left(A_0 \beta_0 + \sigma^{-2} X^* y^*\right),\end{aligned}$$

⁸This implies that Σ has mean $S_0/\nu_0 - \kappa - 1$.

that of the variance by

$$\sigma^2|y, \beta, \phi \sim \mathcal{IG}(s_*, \nu_*), \quad \begin{aligned} s_* &= s_0 + (y^* - X^*\beta)'(y^* - X^*\beta), \\ \nu_* &= T - p + \nu_0 + k \end{aligned}$$

and the autoregressive coefficient by

$$\phi|y, \beta, \sigma^2 \sim \mathcal{N}_p(\phi_*, \Phi_*^{-1}) I_\phi, \quad \begin{aligned} \Phi_* &= \phi_0 + \sigma^{-2} E' E, \\ \phi_* &= \Phi_*^{-1} (\Phi_0 \phi_0 + \sigma^{-2} E' E), \end{aligned}$$

where $E = \{\varepsilon_t\}_t, \varepsilon_t = (\varepsilon_{t-1}, \dots, \varepsilon_{t-p})$ is a $T - p \times p$ matrix (Chib 1993).

It is straightforward to see that each observation equation is a subgroup of this model. For instance, for the sector cycle equations, we have $\check{y}_{it} - \tau_{\check{y}_{it}} = \psi_{\check{y}_i} g_t + c_{\check{y}_{it}}$ with $c_{\check{y}_{it}} = \phi_{\check{y}_{i1}} c_{\check{y}_{it-1}} + \dots + \phi_{\check{y}_{ip}} c_{\check{y}_{it-p}} + \varepsilon_{c_{it}}$, i.e., the above model is of dimension $b = 1$ and p . In the case of the output gap, the covariate and coefficient vectors x_t and β are dropped. The AR(1) drift $\mu_t = \gamma + \phi_\mu \mu_{t-1} + \varepsilon_{\mu t}$ from the previous section is also a special case of the linear model in Equation (18). More precisely, $b = 2$ and $p = 0$.

4. Functionality of `sectorgap`

The output gap can be estimated using `sectorgap` with a simple, multi-step procedure. The package contains example data for Switzerland which is used here to guide through the process.

The first step concerns data preparation and the definition of the state space model. Model settings can be initialized via the function `initialize_settings()`, returning a nested list with settings for each group of variables, which can subsequently be modified. Based on the model settings, the function `prepare_data()` performs transformations to the raw data and computes weights for the constraints. Finally, the model can be defined via the function `define_model()`, returning an object of class `'ss_model'` which can be printed via the S3 method `print()`.

In a second step, the model parameters and unobserved states can be estimated via Bayesian methods. Prior densities and starting values for the MCMC chain can be initialized using `initialize_prior()` and the returned object of class `'prior'` can subsequently be modified if necessary. The Gibbs sampling procedure is performed using the function `estimate_model()`, producing an object of class `'ss_fit'`. The estimation function returns several MCMC summary statistics and convergence criteria, computed by the function `compute_mcmc_results()`. Optionally, `compute_mcmc_results()` can be also be directly called by the user given an object of class `'ss_fit'`, which is also what the function itself returns. In case the MCMC chain has not yet converged, the function `estimate_model()` can be called again by supplying the returned object of class `'ss_fit'` and the number of additional repetitions. S3 methods for printing are provided for objects of class `'prior'` and `'ss_fit'`.

The third and final step presents the results in the form of time series, density, and trace plots, provided by the S3 method `plot()`. In addition, the function `transform_results()` consolidates the estimated time series in long format.

5. Computing the output gap with `sectorgap`

This section exemplifies the estimation of the output gap using `sectorgap`, providing details on each of the three steps mentioned in Section 4.

5.1. Data

In our example, we use quarterly Swiss economic data sourced in November 2023, i.e., the 2023 Q2 vintage. Table 2 describes the variables. The aggregate variable y_t is real gross domestic product and the group of aggregate variables a_t consists of full-time equivalent employment fte_t and the unemployment rate u_t . Group b_t , which is characterized by individual settings for each variable, contains inflation based on the consumer price index π_t . The first subgroup \check{y}_t is real value added in three exhaustive subsectors of the production side, namely A–goods-producing industries, B–service industries, and C–government and adjustments. The associated nominal series add up to nominal output and thus, constraints can be placed on the trends and drifts. Full-time equivalent employment in sectors A, B, and C are used to inform the dynamics of subsector value added. These three series thus define \check{a}_t . The second group consists of four sectors on the expenditure side, namely i–total consumption, ii–investment, iii–exports, iv–imports. In contrast to subsector value added, the expenditure side does not add up to aggregate output. However, the residual of the growth contributions is automatically computed and thus constraints can be implemented.

In terms of the model outlined in Section 2,

$$\begin{aligned}y_t &= \log output_t, \\a_t &= (\log fte_t, u_t)', \\b_t &= \pi_t, \\\check{y}_t &= \log (va_{At}, va_{Bt}, va_{Ct})', \\\check{a}_t &= \log (fte_{At}, fte_{Bt}, fte_{Ct})', \\\check{\check{y}}_t &= \log (exp_{it}, exp_{iit}, exp_{iiit}, exp_{iivt})'.\end{aligned}$$

For illustrative purposes, the package contains the series in Table 2 alongside their nominal counterparts if applicable, which can be retrieved by

```
R> data("data_ch")
```

A nested list called `data_ch` is loaded into the working environment. It contains two lists with time series. `ts1` contains all series described in Table 2 while `ts1_n` contains the corresponding nominal series for aggregate output, and the production and expenditure side, i.e., $y_t, \check{y}_t, \check{\check{y}}_t$. Settings can be initialied and the data prepared by

```
R> settings <- initialize_settings()
R> data <- prepatate_data(
+   settings = settings,
+   ts1 = data_ch$ts1,
+   ts1_n = data_ch$ts1_n,
+   ts_start = c(1990, 1),
+   ts_end = c(2023, 2)
+ )
```

| | Variables | Description | Unit |
|---------------|--------------------------|---|---------------------------|
| y_t | <i>output</i> | gross domestic product | at 2020 prices in million |
| a_t | <i>fte</i> | full-time equivalent employment | in thousand |
| | <i>u</i> | ILO unemployment rate | in percent |
| b_t | π | consumer price inflation | year on year in percent |
| \tilde{y}_t | <i>va_A</i> | value added in sector A: Goods-producing industries | at 2020 prices in million |
| | <i>va_B</i> | value added in sector B: Service industries | at 2020 prices in million |
| | <i>va_C</i> | value added in sector C: Government and adjustments | at 2020 prices in million |
| \tilde{a}_t | <i>fte_A</i> | full-time equivalent employment in sector A: Goods-producing industries | in thousand |
| | <i>fte_B</i> | full-time equivalent employment in sector B: Service industries | in thousand |
| | <i>fte_C</i> | full-time equivalent employment in government sector | in thousand |
| \tilde{y}_t | <i>exp_i</i> | expenditure side sector i: Total consumption | at 2020 prices in million |
| | <i>exp_{ii}</i> | expenditure side sector ii: Investment | at 2020 prices in million |
| | <i>exp_{iii}</i> | expenditure side sector iii: Exports | at 2020 prices in million |
| | <i>exp_{iv}</i> | expenditure side sector iv: Imports | at 2020 prices in million |

Table 2: List of variables used to compute the sector output gap for Switzerland. **sectorgap** contains these series for the 2023 Q2 vintage alongside their nominal counterparts, if applicable.

The function `prepare_data()` performs the specified transformations to the raw data and uses the nominal and real series to compute weights for the constraints. It returns a list with four multiple time series objects containing the transformed input data, the untransformed real, nominal, and price series, and lists with weights for the different (sub-) groups.

It is straightforward to obtain predictions, i.e., by setting `ts_end = c(2027, 4)` for predictions until the end of 2027.

5.2. Model definition

Bases on the previously defined settings and data, the state space model can be defined by

```
R> model <- define_ssmodel(
+   settings = settings,
+   data = data
+ )
```

which returns an object of class ‘`ss_model`’. An S3 printing method is available returning the dimensions of the state space model and the set of stationary as well as non-stationary states.

```
R> model
```

Call:

```
define_ssmodel(settings = settings, data = data)
```

State space model object of class `ss_model`

----- Dimensions:

```
Time points: 134
Time series: 20
```

```
Innovations: 41
States:      60
```

```
----- Stationary states:
```

```
[1] "const"           "cycle_output"       "cycle_output_L1"
[4] "cycle_output_L2" "cycle_vaA"          "cycle_vaA_L1"
[7] "cycle_vaA_L2"    "cycle_vaB"          "cycle_vaB_L1"
[10] "cycle_vaB_L2"   "cycle_vaC"          "cycle_vaC_L1"
[13] "cycle_vaC_L2"   "cycle_fteA"         "cycle_fteA_L1"
[16] "cycle_fteB"     "cycle_fteB_L1"     "cycle_fteC"
[19] "cycle_fteC_L1"  "cycle_exp1"         "cycle_exp1_L1"
[22] "cycle_exp2"     "cycle_exp2_L1"     "cycle_exp3"
[25] "cycle_exp3_L1"  "cycle_exp4"         "cycle_exp4_L1"
[28] "cycle_employment" "cycle_employment_L1" "cycle_urate"
[31] "cycle_urate_L1" "cycle_inflation"    "cycle_inflation_L1"
```

```
----- Non-stationary states:
```

```
[1] "trend_output"    "drift_output"      "trend_vaA"
[4] "drift_vaA"       "trend_vaB"         "drift_vaB"
[7] "trend_vaC"       "drift_vaC"         "trend_fteA"
[10] "drift_fteA"      "trend_fteB"        "drift_fteB"
[13] "trend_fteC"      "drift_fteC"        "trend_exp1"
[16] "drift_exp1"      "trend_exp2"        "drift_exp2"
[19] "trend_exp3"      "drift_exp3"        "trend_exp4"
[22] "drift_exp4"      "trend_employment"  "drift_employment"
[25] "trend_urate"     "drift_urate"       "trend_inflation"
```

5.3. Estimation

Priors can be initialized by

```
R> prior <- initialize_prior(
+   model = model,
+   settings = settings,
+   lambda_t = 100,
+   lambda_d = 100,
+   df = 6
+ )
```

The returned object of class ‘prior’ is essentially a data frame defining the prior distribution of each parameter—normal, inverse-gamma, or inverse-wishart—and the initial value for the MCMC chain for which the unconditional means are used. S3 printing methods return

```
R> head(prior)
```

Call:

```
initialize_prior(model = model, settings = settings, lambda_d = 100,  
lambda_t = 100, df = 6)
```

Data frame object of class prior

----- Normally distributed parameters:

| variable | parameter name | mean | variance | initial value |
|----------|--------------------------|------|----------|---------------|
| output | cycle_output_AR | 0 | 1000 | 0 |
| output | cycle_output_AR_L1 | 0 | 1000 | 0 |
| vaA | cycle_load_vaA_output_L0 | 0 | 1000 | 0 |

----- Inverse-gamma/wishart distributed parameters:

| variable | parameter name | scale | degrees of freedom | initial value |
|----------|------------------|-------|--------------------|---------------|
| output | var_cycle_output | 4.00 | 6 | 1.00 |
| output | var_drift_output | 0.04 | 6 | 0.01 |
| output | var_trend_output | 0.04 | 6 | 0.01 |

Finally, the model can be estimated by

```
R> fit <- estimate_ssmodel(  
+   model = model,  
+   settings = settings,  
+   prior = prior,  
+   R = 50000,  
+   burnin = 0.5,  
+   thin = 10,  
+   HPDIprob = 0.68  
+ )
```

which starts a Gibbs sampling procedure and computes a set of summary statistics and highest posterior density intervals (HPDI) based on the specified value `HPDIprob`. For the latter, `estimate_ssmodel()` calls the function `compute_mcmc_results()`, which can also be called post estimation in order to adapt the HPDI. Both functions return an object of class `'ss_fit'` for which S3 printing methods return information on the Bayesian settings and convergence of parameters and states based on Geweke's convergence diagnostic (Geweke 1992).

```
R> fit
```

Call:

```
estimate_ssmodel(model = model, settings = settings, prior = prior,  
R = 50000, burnin = 0.5, thin = 10, HPDIprob = 0.68)
```

Fitted state space model of class `ss_fit`

----- Bayesian settings:

```
Repetitions:      50000
Burnin period (%): 0.5
Skipped draws:    9/10
HPDI (%):         68
```

----- Convergence at 10% level:

```
88/94 parameters have converged
40/41 states have converged for all time points (Bonferroni corrected)
```

Non-converged parameters:

```
[1] "cycle_load_fteC_vaC_L0"      "cycle_load_inflation_output_L0"
[3] "cycle_load_inflation_output_L1" "cycle_load_urate_output_L1"
[5] "var_drift_exp3"              "var_drift_vaC"
```

Non-converged states:

```
state time points
drift_vaB          9
```

In case sufficient convergence has not been achieved, Gibbs sampling can be continued by recalling the function `estimate_ssmodel()` and supplying the fitted object `fit` and the number of additional draws `R`:

```
R> fit <- estimate_ssmodel(
+   fit = fit,
+   R = 10000
+ )
```

5.4. Results

sectorgap provides convenient and extensive plotting options for the posterior densities of parameters and the estimated unobserved states.

```
R> plot(
+   fit,
+   plot_type = "density",
+   file_path = "doc/fig",
+   n_col = 4
+ )
```

Figures 1 and 2 exemplify the output for inverse-gamma and normally distributed parameters, respectively. Prior densities are shown in dashed grey while posterior densities are solid black. Figure 1 compares the prior and posterior densities for all drift innovation variances. For the

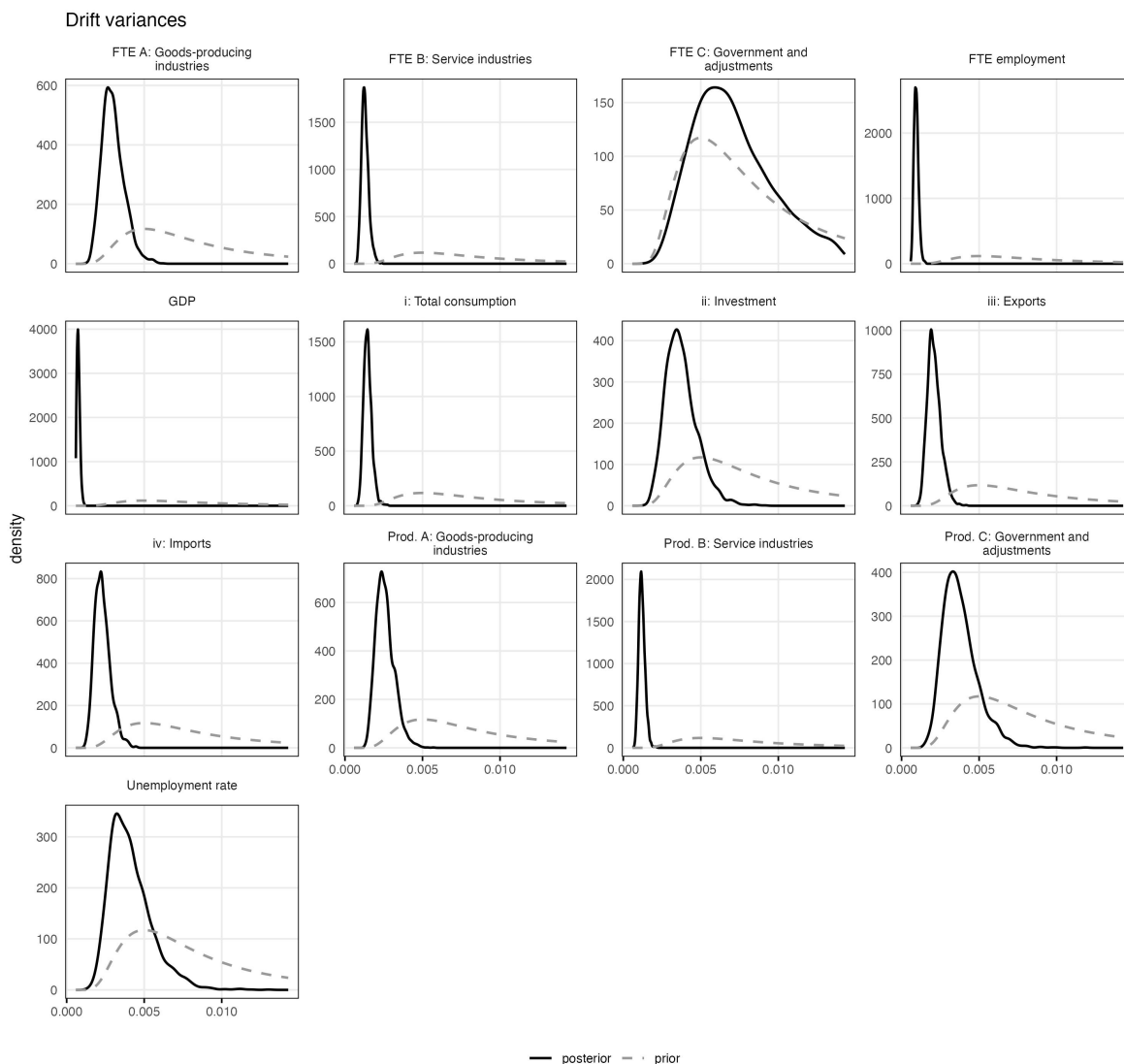


Figure 1: Posterior densities of drift innovation variances.

majority of variances, the posterior is well peaked and clearly distinguishable from the prior density, indicating that the data was informative in updating these parameters.

Similarly, Figure 2 shows the results for the normally distributed parameters that load on the aggregate output gap. The prior distributions were flat while the posterior distributions are well peaked. As expected, the employment loadings are positive, while the ones for unemployment are negative and for both indicators, the data suggests labor market frictions of at least two quarters. The loadings of the production and expenditure side are all positive, indicating a positive correlation to the business cycle. Goods-producing industries and exports and imports have a comparably strong connection to the output gap.

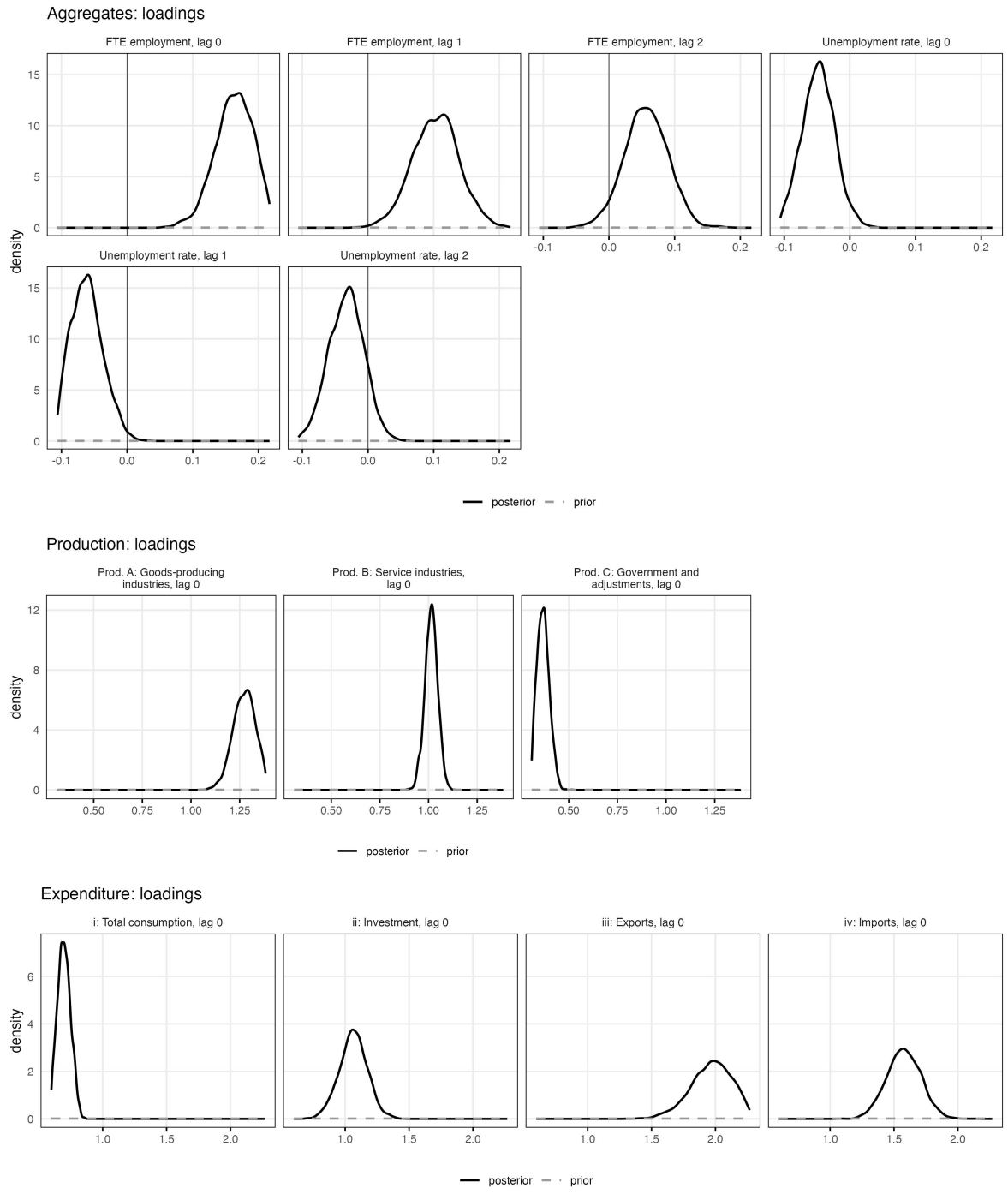


Figure 2: Posterior densities of selected loadings on the aggregate output gap.

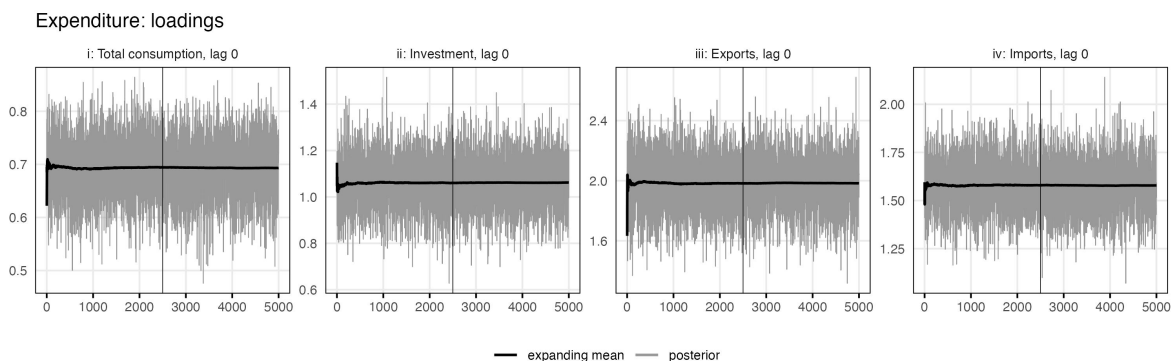


Figure 3: Trace plot for output loadings of the expenditure side.

In order to visually assess the convergence of the Gibbs sampler, trace plots can be obtained via

```
plot(
+   fit,
+   plot_type = "trace",
+   file_path = "doc/fig",
+   include_burnin = TRUE,
+   n_col = 4,
+   height = 2
+ )
```

The resulting plot for the output loading parameters of the expenditure side sectors can be found in Figure 3. The x-axis represents the number of posterior draws and the vertical line shows the burnin cut off. The thick black line is the expanding mean of the posterior draws. Visual inspection suggests that for the considered parameters, the Gibbs sampler indeed converged, which has been confirmed by a Geweke test in the previous section.

Complementary to the Swiss data set, **sectorgap** contains data on recessions in Switzerland to enhance visualizations. An extensive set of time series plots can be obtained via

```
data("recessions_ch")
R> plot(
+   fit,
+   plot_type = "timeseries",
+   data = data,
+   highlighted_area = recessions_ch,
+   n_col = 3,
+   file_path = "doc/fig"
+ )
```

One of the key results of the above analysis are the gap decompositions in Figure 4. Regarding output, large parts of the negative gap during the Nineties and after the Dotcom bubble can be attributed to sector B: Service industries. However, the aftermath of the financial crisis of



Figure 4: Output gap decomposition for the production and expenditure side.

2007–2008 and the scrapping of the floor to the euro by the Swiss National Bank (SNB) in January 2015 is mainly visible in sector A: Goods-producing industries. On the expenditure side, export and imports explain comparably large parts of the aggregate gap.

Figure 5 shows the corresponding decompositions of the output drift, revealing that the share of sector B: Service industries has been declining recently. This development is in part counteracted by the increasing share of sector A: Goods-producing industries. For the expenditure side, the contributions by iii–exports and iv–import appear mostly stable, while those for i–total consumption and ii–investment have been slightly decreasing during recent years. Note that the residual for the expenditure side is clearly visible since the expenditure side components are non-exhaustive. At the same time, the residual for the production side is negligible and is purely a side effect of the Bayesian estimation, as constraints were in place.

The individual series that load on the output gap can also be decomposed into idiosyncratic and shared contributions, the latter of which is attributed to the business cycle. Figure 6 exemplifies those decompositions. Dark shaded regions signal shared contributions and light shaded regions idiosyncratic ones. The impact of the business cycle on the employment and unemployment gaps is clearly visible. Similar results can be observed for the value added subsectors A: goods-producing industries and B: service industries and for the expenditure side components iii–exports and iv–imports.

The latest call to the function `plot()` with `plot_type = "timeseries"` also produces separate plots for each estimated state, i.e., cycles, trends, and drifts, alongside their HPDI. Figure 7 shows these plots for the production side. The resulting trends and drifts are fairly smooth, allowing for an economically meaningful interpretation.

Finally, `sectorgap` provides a function to summarize the resulting time series in long format,

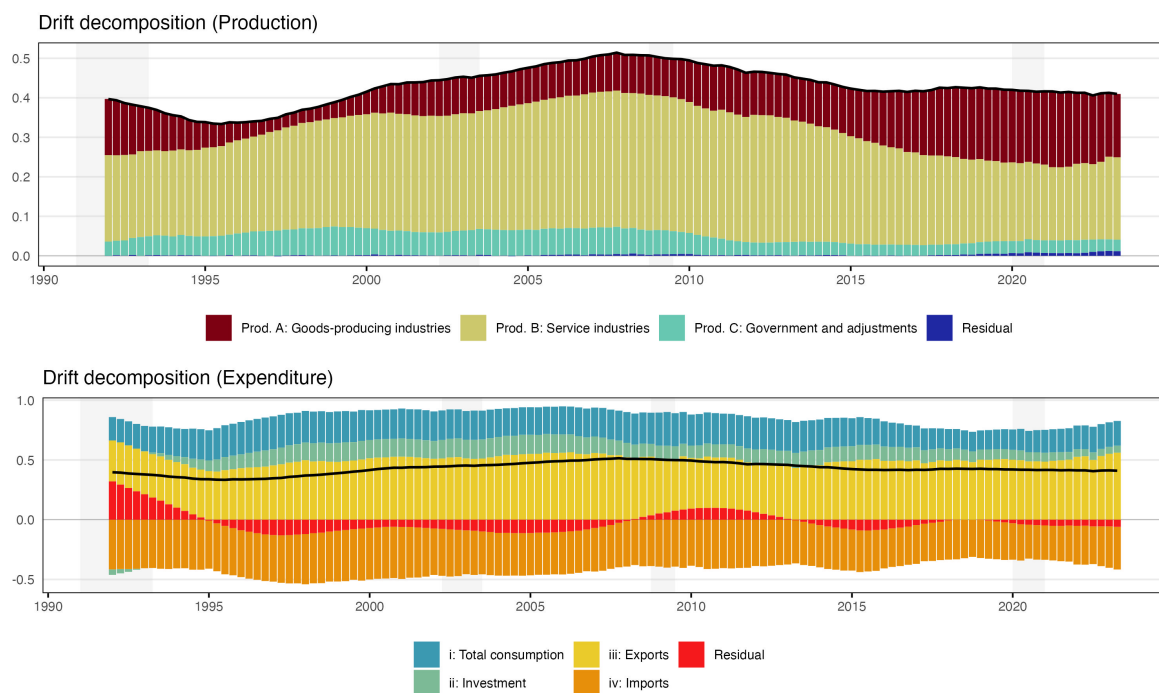


Figure 5: Drift decompositions for the production and expenditure side.

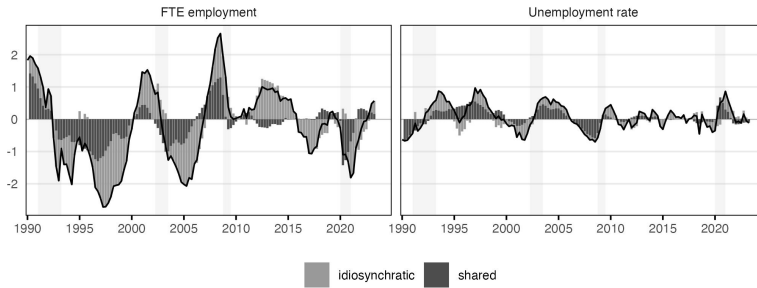
including mean and median estimates, HPDI bounds, group labels, variable labels:

```
data("recessions_ch")
R> df <- transform_results(
+   fit = fit,
+   data = data,
+   estimate = "median"
+ )
R> head(df)
```

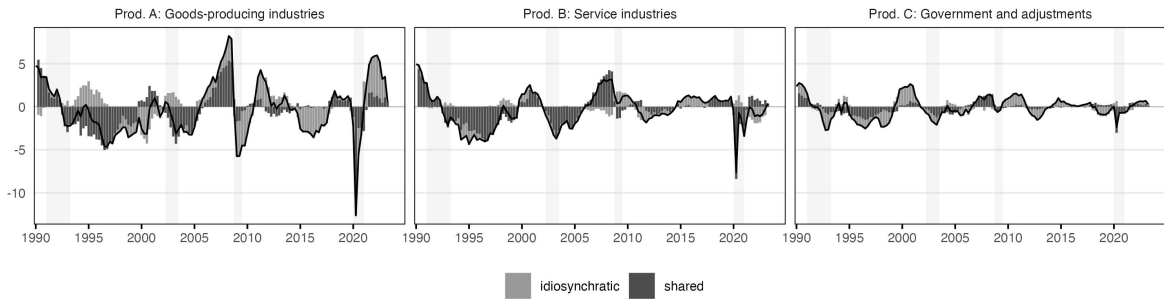
| | date | series | value | type | obs_name | obs | lb |
|---|------|--------------|--------------|-------|----------|----------|--------------|
| 1 | 1990 | trend_output | 1.073830e+05 | trend | output | NA | 1.057008e+05 |
| 2 | 1990 | drift_output | 3.614392e-01 | drift | output | NA | 2.765256e-01 |
| 3 | 1990 | trend_vaA | 2.971960e+04 | trend | vaA | 31169.46 | 2.880804e+04 |
| 4 | 1990 | drift_vaA | 4.585089e-01 | drift | vaA | NA | 2.659045e-01 |
| 5 | 1990 | trend_vaB | 6.246305e+04 | trend | vaB | 65628.77 | 6.129841e+04 |
| 6 | 1990 | drift_vaB | 3.778030e-01 | drift | vaB | NA | 2.562194e-01 |

| | ub | contr | group | group_label | series_label |
|---|--------------|-------|--------|--|-------------------------------------|
| 1 | 1.092838e+05 | NA | agg | Aggregate output | GDP |
| 2 | 4.778077e-01 | NA | agg | Aggregate output | GDP |
| 3 | 3.051303e+04 | NA | group1 | Production Prod. A: Goods-producing industries | Prod. A: Goods-producing industries |
| 4 | 6.390992e-01 | NA | group1 | Production Prod. A: Goods-producing industries | Prod. A: Goods-producing industries |
| 5 | 6.371592e+04 | NA | group1 | Production Prod. B: Service industries | Prod. B: Service industries |
| 6 | 5.075749e-01 | NA | group1 | Production Prod. B: Service industries | Prod. B: Service industries |

Aggregates gap decomposition



Production gap decomposition



Expenditure gap decomposition

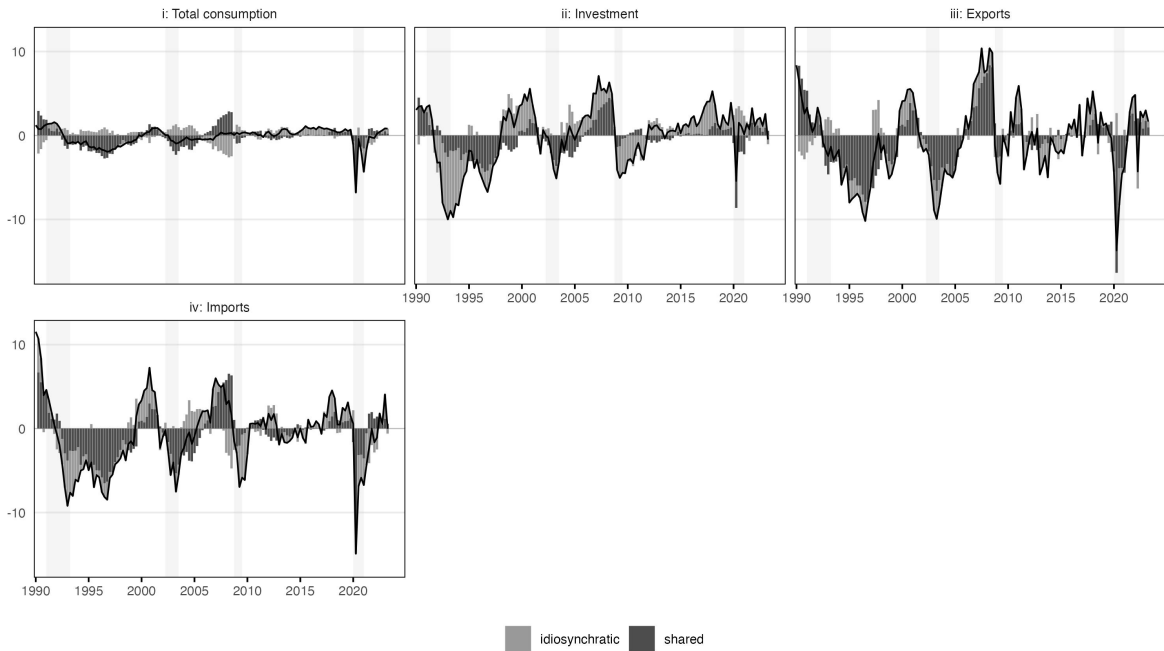


Figure 6: Sector gap decompositions for all groups that load on the output gap.

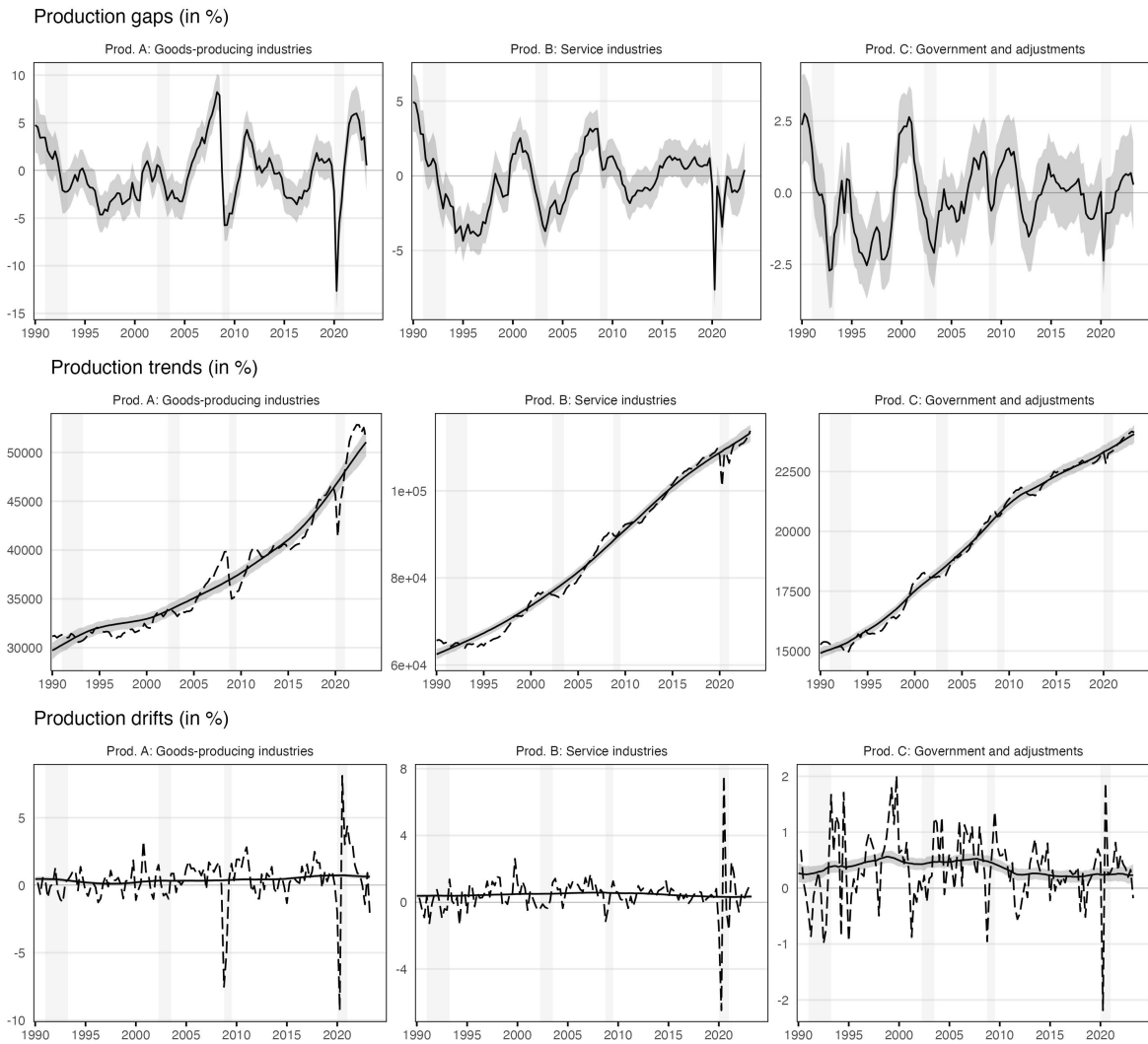


Figure 7: Production side gaps, trends, and drifts. The shaded area represents 68% HPDI. All estimated series are solid while the data is dashed.

6. Other software

There are no other software solutions for the estimation of a consistent bottom-up output gap as defined in Section 5.2. However, given the unobservability of the true business cycle, numerous other modeling frameworks exist. One such framework, for which a package is available, is the Cobb-Douglas production function methodology proposed by the European Commission (EC) (Havik *et al.* 2014; Blondeau, Planas, and Rossi 2021). The R package **RGAP** available on CRAN provides tools to estimate the output gap according to the EC approach (Streicher 2022).⁹ In addition, **RGAP** enables the estimation of the output gap according to Kuttner (1994), which is essentially a version of our model without subsectors. When it comes to models free from economic theory such as univariate and multivariate time series filters (Hodrick and Prescott 1997; Baxter and King 1999), there are a multitude of package options. An example is **mFilter**, which offers time series filters for trend and cycle extraction (Balcilar 2019).

7. Concluding remarks

In this paper, we introduce the R package **sectorgap** which enables the estimation of economic trends and cycles. By including subsectors of economic output, the proposed multivariate state space model allows for a comprehensive decomposition of the business cycle and potential output growth into its sectoral contributions. The resulting estimates are extremely useful for fiscal and monetary policy makers. They help to identify the specific causes of economic fluctuations and facilitate targeted political measures to stabilize the economic cycle.

sectorgap provides a transparent, reliable, and easy to use modeling and estimation framework and is suitable both for beginners and advanced R users. The model definition is kept as general as possible to allow for a wide range of possible applications. Bayesian estimation is performed using a simple Gibbs sampling procedure incorporating the simulation smoother implemented in **KFAS**. Finally, **sectorgap** provides tools to assess posterior densities and the convergence of the Gibbs sampler and offers an extensive set of time series plots of the resulting trends, cycles, and decompositions thereof. After individual data preparation, the output gap can be estimated with just a few lines of code.

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⁹The EC itself provides an Excel user interface that enables the estimation with Python.

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A. Estimation algorithm

The algorithm is structured in multiple Gibbs sampling blocks: The first three blocks sample parts of the parameter set $\Theta^{(\iota)}$ conditional on the states $\alpha^{(\iota-1)}$. The final block applies simulation smoothing to sample $\alpha^{(\iota)}$ conditional on the previously drawn parameters $\Theta^{(\iota)}$ (Durbin and Koopman 2002, 2012).

Initialization: We use the prior means to initialize all parameters $\Theta^{(0)}$ and apply the simulation smoother based on those parameters to initialize the states $\alpha^{(0)}$.

Recursion: For $\iota = 1, \dots, I$:

1. Trends and drifts: Draw all trend and drift variances

$$\begin{aligned} \Sigma_{\tau}^{(\iota)} &| \tau^{(\iota-1)}, \mu^{(\iota-1)} \\ \Sigma_{\mu}^{(\iota)} &| \mu^{(\iota-1)} \end{aligned}$$

For each AR(1) drift equation, additionally draw

$$(\gamma, \phi_{\mu})^{(\iota)} | \mu^{(\iota-1)}, \sigma^{2(\iota)}$$

2. Output gap equation: Draw autoregressive coefficients and cycle variance, i.e.,

$$\begin{aligned} \phi^{(\iota)} &| \tau^{(\iota-1)}, \sigma^{2(\iota-1)} \\ \sigma^{2(\iota)} &| \tau^{(\iota-1)}, \phi^{(\iota)} \end{aligned}$$

sequentially in this order as detailed in Section 3.2.2. If the characteristic polynomial $\Phi^{(\iota)}(x)$ has roots inside the unit circle, redraw $\phi^{(\iota)}$.

3. Observation equations including cycles, excluding trends: For each equation, draw autoregressive coefficients, loading coefficients, and cycle variances, i.e.,

$$\begin{aligned} \phi^{(\iota)} &| \tau^{(\iota-1)}, c^{(\iota-1)}, \sigma^{2(\iota-1)} \\ \psi^{(\iota)} &| \tau^{(\iota-1)}, c^{(\iota-1)}, \sigma^{2(\iota-1)}, \phi^{(\iota)} \\ \sigma^{2(\iota)} &| \tau^{(\iota-1)}, c^{(\iota-1)}, \phi^{(\iota)}, \psi^{(\iota)} \end{aligned}$$

sequentially in this order as detailed in Section 3.2.2. If the characteristic polynomial $\Phi^{(\iota)}(x)$ has roots inside the unit circle, redraw $\phi^{(\iota)}$.

4. States: Apply the simulation smoothing recursion (Durbin and Koopman 2002) to sample the unobserved states conditional on the previously drawn parameters

$$\alpha^{(\iota)} | \Theta^{(\iota)}.$$

Discard a specified burnin phase and finally, apply thinning, i.e., for the posterior distribution, select only the draws $\Theta^{(\iota)}$ and $\alpha^{(\iota)}$ with $\iota = \tilde{I}, \tilde{I} + \xi, \dots, I - \xi, I$, where $\xi \in \mathbb{N}$.

B. Decompositions

The decompositions of output trend growth $\Delta\tau_{yt}$, the output drift μ_{yt} and trend growth and drift of the aggregate indicator $\Delta\tau_{at}, \mu_{at}$ into its sector contributions directly follows by the aggregation constraints in Equations (8) – (11) detailed in Section 2.1.¹⁰

Similarly, we can deduce a decomposition of the output gap g_{yt} into a weighted average of subsector output gaps $g_{\check{y}t} = (g_{\check{y}_{1t}}, \dots, g_{\check{y}_{nt}})'$. We again assume that relative previous period prices w_t^P for real potential output $T_{Yt} = \exp \tau_{yt}$ are the same as for real output Y_t , i.e.,

$$T_{Yt} = w_t^{P'} T_{\check{Y}t}, \quad (19)$$

where $T_{\check{Y}t} = (T_{\check{Y}_{1t}}, \dots, T_{\check{Y}_{nt}})'$ with $T_{\check{Y}_{it}} = \exp \tau_{\check{y}_{it}}$. Then,

$$\begin{aligned} g_{yt} &= \frac{Y_t}{T_{Yt}} - 1 \\ &= \frac{w_t^{P'}}{T_{Yt}} \text{diag}(T_{\check{Y}t}) \text{diag}(T_{\check{Y}t})^{-1} \check{Y}_t - 1 \\ &= w_t^{PT'} \text{diag}(T_{\check{Y}t})^{-1} \check{Y}_t - 1 \\ &= w_t^{PT'} \check{g}_t \end{aligned}$$

where $w_t^{PT} = (w_{1t}^{PT}, \dots, w_{nt}^{PT})$ and $w_{it}^{PT} = \check{P}_{it-1} T_{\check{Y}_{it}} / P_{t-1} T_{Yt}$ with $1'_n w_t^{PT} = 1$ for all t by Equation (19). The decomposition of the employment gap g_{at} follows analogously by replacing w_t^P with 1_m .

¹⁰Without loss of generality, a_t with $k = 1$ denotes the aggregate indicator corresponding to the subindicators in \check{a}_t .

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