Doctoral Thesis

Search for Supersymmetry in Hadronic Final States Using MT2 in pp Collisions at sqrt(s) = 7 TeV and Discovery Potential of Top Partners in Composite Higgs Models

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Search for Supersymmetry in Hadronic Final States Using $M_{T^2}$ in pp Collisions at $\sqrt{s} = 7$ TeV
and
Discovery Potential of Top Partners in Composite Higgs Models

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presented by

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ABSTRACT

The standard model of particle physics has been extraordinarily successful in describing the vast majority of collider data of the past decades. However, a number of outstanding problems remain. In an attempt to overcome the deficiencies of the standard model, a variety of new physics models have been developed, among which composite Higgs and supersymmetric models are promising candidates.

In this thesis, a search for supersymmetry is presented using a data sample of $4.73 \text{ fb}^{-1}$ of proton-proton collisions collected at $\sqrt{s} = 7 \text{ TeV}$ with the CMS detector at the Large Hadron Collider. Fully hadronic final states are selected based on the variable $M_{T^2}$, an extension of the transverse mass in events with two invisible particles. The salient properties of the $M_{T^2}$ kinematic variable are presented and its ability to separate potential signal events from the standard model backgrounds is discussed. Two complementary searches are performed. The first targets the region of parameter space with medium to high squark and gluino masses, whereas the second is optimized to be sensitive to events with a light gluino and heavy squarks. In both analyses, the backgrounds arising from standard model processes are estimated using data control regions as well as simulation. No significant excess of events over the standard model expectations is observed. Exclusion limits are derived for the parameter space of the constrained minimal supersymmetric extension of the standard model, as well as on a variety of simplified model spectra.

Composite Higgs models are attractive, non-supersymmetric extensions of the standard model, in which a naturally light Higgs boson emerges from the spontaneous breaking of the global symmetry of a new strongly interacting sector. In these models, one or more sets of heavy top partners are typically introduced with masses well within the LHC reach. The collider signatures that these new quarks can produce are analyzed in detail. The final states with two (same-sign) or three leptons are found to be the most promising discovery channels. Exotic quarks of charge $5/3$ are a distinctive feature of these models. A new method is presented to reconstruct the mass of such exotic quarks from their leptonic decay, without relying on jets in the final state.
ZUSAMMENFASSUNG

Der Grossteil der experimentellen Daten von Beschleunigerexperimenten wird durch das Standardmodell der Teilchenphysik mit beeindruckender Präzision beschrieben. Es bleiben jedoch einige hartnäckige Probleme, welche zur Entwicklung von Modellen jenseits des Standardmodells geführt haben, wie beispielsweise die supersymmetrische Erweiterung des Standardmodells oder Modelle in welchen das Higgs-Boson als zusammengesetztes Teilchen beschrieben wird (Englisch: composite Higgs model).


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Chapter 1

INTRODUCTION

“By convention sweet, by convention bitter and by convention color;
but in reality atoms and void.”
(Democritus, circa 400 BC)

Leucippus and Democritus, who were ancient Greek philosophers, introduced the idea that matter consists of indestructible atoms surrounded by empty space. This idea, though based on grounds very different from today’s understanding of nature, remarkably resembles the modern description of the atomic structure of matter. Only at the dawn of the 20th century it was realized that atoms themselves have structure and consist of smaller entities: a nucleus surrounded by electrons. Experimental results in the 1930s established that even the atom’s nucleus is not fundamental, but instead comprised of neutrons and protons. Today, we know that neutrons and protons consist of three quarks, bound together by gluon fields.

More than a century of theoretical work and experimental results has led us to the formulation of an elegant theory of the elementary particles and their interactions. This theory, called the standard model of particle physics (SM), is widely regarded as one of the central achievements of human research in the 20th century. In the SM, the electromagnetic, weak and strong interactions are all formulated within the same mathematical framework of a relativistic quantum field theory.

The success of the SM in reproducing a huge amount of experimental data is impressive. Thanks to the Tevatron at Fermilab and the Large Hadron Collider (LHC) at CERN, the experimental frontier has advanced into the TeV range, however, no unambiguous hints of physics beyond the SM have yet been found. The new boson recently observed at the LHC with a mass of about 125 GeV has — at current precision — properties consistent with a SM Higgs boson. Still, there are strong arguments to suggest that the SM will have to be extended to describe physics at higher energies. Certainly, a new framework will be required at the Planck scale, $\Lambda_{\text{Planck}} \approx 10^{19}$ GeV, where quantum gravitational effects become important. The disturbing energy difference between the scale of electroweak symmetry breaking and the Planck scale, often referred to as the “hierarchy problem”, is another shortcoming of the SM which cannot be solved without introducing new
particles. Moreover, a variety of astrophysical and cosmological observations point to the existence of “dark matter”, for which the SM fails to provide an explanation.

In the last decades, several new models have been developed in order to address the shortcomings of the SM. Two such models, whose collider signatures are now extensively searched for at the LHC, are discussed in this thesis. These are supersymmetric and composite Higgs models.

Supersymmetric extensions of the SM have been a central topic in theoretical and experimental research since the early 1980s. Such models have attracted a lot of attention because they provide a weakly-interacting dark matter candidate and facilitate the gauge coupling unification. Moreover, the hierarchy problem is elegantly solved by exploiting the cancellation between bosonic and fermionic degrees of freedom. In this thesis, we present a search for supersymmetry in pp collisions collected with the Compact Muon Solenoid (CMS) detector at the LHC at a center-of-mass energy of 7 TeV. The analysis makes extensive use of the kinematic variable $M_{T^2}$ to select new physics candidate events.

A different mechanism to stabilize the electroweak scale is provided by composite Higgs models, which were originally introduced in the 1980s, but realistic and calculable models were developed only recently. These models feature a Higgs boson arising as a composite state of a new strongly interacting sector. Typically, heavy top partners are also introduced with masses within the reach of the LHC. In this thesis, we discuss the collider phenomenology of such heavy top partners and investigate their discovery potential at the LHC.

This thesis is organized as follows: In Chapter 2 a brief review of the standard model of particle physics is presented and some of its shortcomings are discussed. We introduce the minimal supersymmetric standard model and the composite Higgs model as attractive extensions to the SM and highlight their distinctive collider signatures. Chapter 3 is dedicated to the discussion of the LHC discovery potential of top partners in composite Higgs models. The CMS detector at the LHC is described in Chapter 4, and Chapter 5 contains a summary of the reconstruction of physics objects used within CMS. In Chapter 6, we introduce the kinematic variable $M_{T^2}$ and discuss some of its virtues. Finally in Chapter 7, a search for supersymmetry in pp collisions with the CMS detector is presented. A summary of this thesis is given in Chapter 8.
In the standard model (SM) of particle physics, the electromagnetic and weak interactions are unified in an electroweak interaction with an associated $SU(2) \times U(1)$ gauge symmetry. However, only the electromagnetic interaction is observed as a long-range force. The rest of the electroweak symmetry can only be observed in high-energy, short-interaction processes, as it is mediated by massive gauge bosons. Since gauge symmetries forbid a mass term for the gauge bosons, the electroweak symmetry must be broken at low energies. The SM provides an economical formulation of this electroweak symmetry breaking by introducing only one new degree of freedom: the Higgs boson.

The recent discovery of a new boson with a mass of about 125 GeV [1, 2] and properties which are — at this writing — compatible with the ones of the SM Higgs boson, marks a milestone in the history of particle physics. Yet, a fundamental piece is still missing; namely, the SM does not explain the dynamical origin of the breaking of the electroweak symmetry.

This chapter is organized as follows. The SM is introduced in Section 2.1, where first the electroweak symmetry and its breaking is reviewed. We then discuss some of the shortcomings of the SM and argue for the need of physics beyond the SM. In Sections 2.2 and 2.3, we discuss two such new physics models; the minimal supersymmetric standard model and a realistic composite Higgs model. This chapter roughly follows the line of argumentation of Refs. [3–7] and is further based on Refs. [8–12].

2.1 The Standard Model of Particle Physics

2.1.1 The Electroweak Symmetry and its Breaking

The SM is based on a $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, meaning that its Lagrangian is invariant under local transformations of the $G_{\text{SM}}$ group. The $SU(3)_C$ factor corresponds to the symmetry group of Quantum ChromoDynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons. The unified electromagnetic and weak interactions are incorporated in the
SM by interpreting $SU(2)_L \times U(1)_Y$ as the group of gauge transformations under which the Lagrangian is invariant.

The kinetic Lagrangian of the SM can be written as

$$L_{\text{kin}} = -\frac{1}{4} G^{a\mu}_\nu G^{a\mu\nu} - \frac{1}{4} W^{i\mu}_\nu W^{i\mu\nu} - \frac{1}{4} B^{\mu\nu} B^{\mu\nu} + \bar{\psi}_L i D_\mu \psi_L + \bar{\psi}_R i D_\mu \psi_R,$$

(2.1)

where $\psi_{L,R}$ runs over all SM fermions and the subscripts “L” and “R” denote left-handed and right-handed chiralities. The three families of fermions and their quantum numbers are summarized in Table 2.1. We have used the covariant derivative, defined as

$$D_\mu = \partial_\mu - i[g_s t^a G^a_\mu + g T^i_i W^i_\mu + g' Y B_\mu],$$

(2.2)

and introduced the short-hand notation $D_\mu = \gamma_\mu D^\mu$, where $\gamma_\mu$ denote the gamma-matrices satisfying the Clifford algebra. The generators of the $SU(3)_C$ and $SU(2)_L$ groups are denoted by $t^a$ and $T^i$, respectively. For a color triplet and a weak doublet they are $t^a = \lambda^a/2$ and $T^i = \sigma^i/2$, with $\lambda$ and $\sigma$ the Gell-Mann and Pauli matrices, respectively. $Y$ is called “weak hypercharge” and corresponds to the generator of the $U(1)_Y$ group. Furthermore, $f^{abc}$ and $\epsilon^{ijk}$ are the corresponding structure constants. We have used

$$G^{a\mu}_\nu = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g s f^{abc} G^b_\mu G^c_\nu,$$

(2.3a)

$$W^{i\mu}_\nu = \partial_\mu W^{i\nu}_\mu - \partial_\nu W^{i\mu}_\mu + g \epsilon^{ijk} W^{j}_\mu W^{k}_\nu,$$

(2.3b)

$$B^{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

(2.3c)

where $G^a_\mu (a = 1, \ldots, 8)$ are the eight gluon fields and $g_s$ is the QCD coupling constant, commonly expressed in terms of the fine-structure constant of the strong interaction $\alpha_s = \frac{g_s^2}{4\pi}$. $W^i_\mu (i = 1, \ldots, 3)$ and $B_\mu$ denote the three $SU(2)_L$ and the $U(1)_Y$ gauge fields, with the corresponding coupling constants $g$ and $g'$.}

**Table 2.1:** The three families of SM fermions and their quantum numbers. The set of numbers $(C, L)_Y$ designates the representations of the SM subgroups $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$.

<table>
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<tr>
<td>$q_L$</td>
<td>$(u^i_L)$, $(c^1_L)$, $(t^3_L)$</td>
</tr>
<tr>
<td>$u_R$</td>
<td>$(u^1_R)$, $(c^3_R)$, $(t^1_R)$</td>
</tr>
<tr>
<td>$d_R$</td>
<td>$(d^3_R)$, $(u^2_R)$, $(c^2_R)$</td>
</tr>
<tr>
<td>$s_R$</td>
<td>$(s^3_R)$, $(d^1_R)$, $(u^2_R)$</td>
</tr>
<tr>
<td>$b_R$</td>
<td>$(b^1_R)$, $(s^3_R)$, $(d^2_R)$</td>
</tr>
<tr>
<td>$l_L$</td>
<td>$(\ell^1_L)$, $(\ell^2_L)$, $(\ell^3_L)$</td>
</tr>
<tr>
<td>$e_R$</td>
<td>$(\ell^1_R)$, $(\ell^2_R)$, $(\ell^3_R)$</td>
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</table>
| $
u$  | $(\nu^1_L)$, $(\nu^2_L)$, $(\nu^3_L)$ |

We can write local gauge transformations as $U(x) = \exp[i T^a(x)]$, where $T^a$ runs over all 12 generators of $G_{SM}$ and $\alpha^a(x)$ are real. With this notation, and $A_\mu$ denoting the 12 corresponding gauge bosons, gauge transformations act as

$$\psi(x) \rightarrow U(x) \psi(x) \approx (1 + i T^a(x)) \psi(x),$$

(2.4a)

$$A_\mu(x) = A^a_\mu(x) T^a \rightarrow A_\mu + i[\alpha, A_\mu] + \frac{1}{g} \partial_\mu \alpha,$$

(2.4b)

and we find that $L_{\text{kin}}$ in Eqn. (2.1) has indeed a local $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry.

Note that the electroweak part of $L_{\text{kin}}$ is chiral, i.e. the left-handed and right-handed fermions carry different quantum numbers under the electroweak group (see Table 2.1). Fermion mass terms, which mix
left-handed and right-handed components, are not gauge invariant and therefore forbidden. Moreover, the
gauge invariance also forbids mass terms for the gauge bosons. Experimentally, however, we observe that
some of the gauge bosons have a non-vanishing mass, suggesting that the gauge symmetry must be bro-
ken. The existence of both massless photons and gluons indicate that $G_{SM}$ must be broken down to QCD,
$SU(3)_C$, and electromagnetism, $U(1)_{em}$. Since the unbroken $SU(3)_C \times U(1)_{em}$ symmetry is vector-like,
meaning that the left-handed and right-handed components of all fermions transform under the same represen-
tation, Dirac mass terms for fermions are allowed after electroweak symmetry breaking (EWSB).

We will now leave aside the details of how this symmetry breaking is achieved in the SM and instead
first study the Lagrangian after EWSB. This Lagrangian, omitting the QCD part, can be written as

$$
\mathcal{L} = -\frac{1}{2} W^{\pm}_{\mu} W^{-\mu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + m_W^2 W^{\mu} W^{-\mu} + m_Z^2 Z_{\mu} Z^{\mu} + \sum \left[ \bar{\psi}_L D \psi_L + \bar{\psi}_R D \psi_R \right] - \left[ \bar{u}_L V^i_{ij} m_u u_R^i + \bar{d}_L m_d d_R^i + \bar{e}_L m_e e_R^i + \text{h.c.} \right],
$$

(2.5)

where we have performed the following redefinition of fields,

$$
W^{\pm}_{\mu} = \frac{W^1_{\mu} \pm W^2_{\mu}}{\sqrt{2}}, \quad \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W^3_{\mu} \\ B_{\mu} \end{pmatrix}.
$$

(2.6)

The massive vector fields of the charged and neutral weak interactions are denoted by $W^{\pm}$ and $Z_{\mu}$, respec-
tively, and $A_{\mu}$ is the massless photon field. We have used $c_W$ and $s_W$ to denote the cosine and sine of the
weak angle, which satisfy

$$
e = g s_W = g' c_W.
$$

(2.7)

In Eqn. (2.5), $V^i_{ij}$ is the unitary $3 \times 3$ CKM matrix. The Lagrangian (2.5) successfully reproduces the
experimental data up the electroweak scale. However, as we will see in the following, it is inconsistent if
extrapolated to higher energies.

**Unitarity Violation**

**Figure 2.1:** Diagrams contributing to longitudinal $W^+_L W^-_L \rightarrow W^+_L W^-_L$ scattering.
We can use the Lagrangian (2.5) to calculate the cross section for longitudinal gauge boson scattering, which proceeds through the diagrams shown in Figure 2.1. It is found that the amplitude

$$A(W^+_L W^-_L \rightarrow W^+_L W^-_L)$$

grows as $\sim E^2$ for energies $E \gg m_W$, leading to a violation of unitarity at $\Lambda_{\text{unit}} \lesssim 1$ TeV. Consequently, at these energy scales, the theory breaks down and has to be replaced by something else. Perturbative unitarity thus requires an EWSB sector that unitarizes longitudinal gauge boson scattering.

### 2.1.2 The Electroweak Symmetry Breaking Sector

The minimal solution to the problems we described above consists in the introduction of a complex scalar field

$$\phi \equiv \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right), \quad (2.8)$$

transforming as $(1, 2)_2$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$. The full SM Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \quad (2.9)$$

with

$$\mathcal{L}_{\text{Higgs}} = |D_\mu \phi|^2 - V(\phi), \quad V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{4} (\phi^\dagger \phi)^2, \quad (2.10a)$$

$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i \lambda_{ij} u^i_R \lambda_{ij}^d d^i_R - \bar{q}_i \lambda_{ij} e^i_R \lambda_{ij}^e e^i_R + \text{h.c.}. \quad (2.10b)$$

where we have introduced Yukawa interaction of the fermions with the scalar field. For $\mu^2, \lambda > 0$, the potential $V$ is minimized by all non-vanishing field configurations with $\phi^\dagger \phi = 2\mu^2/\lambda$. By choosing the vacuum configuration

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \frac{2\mu}{\sqrt{\lambda}} \quad (2.11)$$

we spontaneously break the electroweak symmetry, meaning that the Lagrangian is invariant under the symmetry, but the vacuum is not (i.e. the generator of the broken symmetry does not preserve the ground state). A central aspect of spontaneous symmetry breaking is Goldstone’s theorem [13–15]. It states that for every spontaneously broken continuous global symmetry, there is a massless scalar boson with the quantum numbers of the broken generator. Note that these scalar fields, called Goldstone bosons, do not need to be fundamental scalars and can instead be composite states, made from fermions or other species.

Goldstone’s theorem, however, does not hold in gauge theories in which a local symmetry is spontaneously broken. Instead, the Higgs mechanism [16–18] operates; after breaking the local $G_{\text{SM}}$ symmetry to $SU(3)_C \times U(1)_{\text{em}}$, no massless scalar fields arise in the particle spectrum. Instead, the degrees of freedom carried by the Goldstone bosons manifest themselves as the longitudinal spin component of the broken gauge bosons, which have in turn acquired a mass. In fact, by expanding the potential $V$ around the vacuum configuration and rewriting Eqn. (2.9) in the unitary gauge (which locally removes the would-be Goldstone bosons) we reveal the particle content of the SM: a massive scalar Higgs boson $(h)$, massive $W^\pm$ and $Z$ gauge bosons, massless gluons and photons, massive quarks and charged leptons as well as massless
neutrinos. The masses of the bosons are

\[ m_W = \frac{1}{2} g v, \quad m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} v, \quad m_h = \mu \sqrt{2}. \quad (2.12) \]

Using Eqn. (2.7), we obtain the important relation

\[ \rho \equiv \frac{m_W^2}{m_Z c_W} = 1. \quad (2.13) \]

According to an empirical theorem, a computation resulting in 0 or 1 implies either a mistake in the calculation or a symmetry that explains the result. In the present case, there is indeed such a symmetry, called custodial symmetry, which is described in the following. More details can be found in Ref. [19].

### Custodial symmetry

The Higgs doublet introduced in Eqn. (2.8) contains four real degrees of freedom and the Higgs potential is actually invariant under any rotation of these four components, hence under a \( SO(4) \cong SU(2)_L \times SU(2)_R \) symmetry, where \( SU(2)_L \) is the gauge symmetry of the SM. This \( SO(4) \) symmetry can be made explicit by writing \( \mathcal{H} = (i \sigma^2 \phi^*, \phi) \), on which the \( SU(2)_L \times SU(2)_R \) acts as

\[ \mathcal{H} \rightarrow U_L \mathcal{H} U_R^\dagger, \quad (2.14) \]

with \( U_L \in SU(2)_L, \; U_R \in SU(2)_R \). Writing the Higgs potential as \( V = \frac{\lambda}{4} \left( \phi^\dagger \phi - v^2 \right)^2 \), we find

\[ V(\mathcal{H}) = \frac{\lambda}{4} \left( \text{Tr} \; \mathcal{H}^\dagger \mathcal{H} - v^2 \right)^2, \quad (2.15) \]

which is invariant under Eqn. (2.14). The fact that the Higgs acquires a non-vanishing vacuum expectation value (VEV)

\[ \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{i.e.} \; \langle \mathcal{H} \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (2.16) \]

spontaneously breaks the \( SU(2)_L \times SU(2)_R \) down to the vector subgroup \( SU(2)_D \) (with \( U_L = U_R \)), since for \( U_L = U_R \) the vacuum is invariant under the transformation (2.14). Under the unbroken \( SU(2)_D \) symmetry, the three gauge bosons \( (W^1_\mu, W^2_\mu, W^3_\mu) \) transform as a triplet, implying the same mass \( M_W \) for all \( W^i \). The mass term of the \( W^3 \), however, can also be obtained from the mass matrix in the basis of \( (\gamma, Z) \)

\[
\begin{pmatrix}
Z_\mu & \gamma_\mu
\end{pmatrix}
\begin{pmatrix}
M_Z^2 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
Z^\mu \\
\gamma^\mu
\end{pmatrix}
= \begin{pmatrix}
W^3_\mu & B_\mu
\end{pmatrix}
\begin{pmatrix}
c_W^2 M_Z^2 & c_W s_W M_Z^2 \\
-c_W s_W M_Z^2 & s_W^2 M_Z^2
\end{pmatrix}
\begin{pmatrix}
W^3_\mu \\
B_\mu
\end{pmatrix},
\]

where we used Eqn. (2.6). Entry \((1, 1)\) in the above matrix on the right hand side now implies

\[ M_W = c_W M_Z, \quad (2.18) \]

and thus \( \rho = 1 \).

It is important to note that the custodial symmetry is only approximate, as it is explicitly broken by the \( U(1)_Y \) interaction and by the difference in the up-type and down-type Yukawa couplings. This implies that at loop level, there will be extra corrections causing deviations from \( \rho = 1 \).
In addition to the custodial symmetry, the SM has an accidental $U(1)\times U(1)\times U(1)$ symmetry, leading to a conservation of the number of baryons and the number of leptons in each generation. Moreover, flavor changing neutral currents are heavily suppressed in the SM.

### Unitarity Restoration in the Standard Model

As the scalar kinetic term in Eqn. (2.10a) contains couplings of the Higgs field and the gauge bosons, the Higgs contributes to longitudinal gauge boson scattering. Taking into account the additional diagrams illustrated in Fig. 2.2, we find that the amplitude $A(W^+_LW^-_L \rightarrow W^+_LW^-_L)$ no longer grows as $\sim E^2$. Instead, the contribution of the order $\sim E^2$ exactly cancels between the Higgs and gauge terms. The Higgs boson of the SM thus exactly unitarizes longitudinal gauge boson scattering.

![Additional diagrams involving the Higgs boson contributing to longitudinal $W^+_LW^-_L \rightarrow W^+_LW^-_L$ scattering.](image)

#### 2.1.3 The Hierarchy Problem

Since the Higgs vacuum expectation value $v$ in Eqn. (2.11) is fixed by the Fermi coupling constant $G_F$ and found to be $v = (\sqrt{2}G_F)^{-1/2} \approx 174$ GeV, the Higgs mass must be of the order of $\sim 100$ GeV. The infamous hierarchy problem [5, 6, 20–23] arises from the fact that a fundamental scalar field is not natural to be light. This can be illustrated by calculating the correction to the Higgs mass from a fermion loop as illustrated in Fig. 2.3. If the fermion with mass $m_f$ couples to the Higgs with a term $-\lambda_f h\bar{f}f$ in the Lagrangian, we obtain a loop correction

$$\Delta m_h^2 = \frac{\lambda_f^2}{8\pi^2} \left[ -\Lambda^2 + 6m_f^2 \ln(\Lambda/m_f) + \ldots \right],$$

(2.19)

where $\Lambda$ is a cutoff used to regulate the momentum integral.

If the SM remains valid all the way up to the Planck scale $\Lambda_P = 10^{19}$ GeV, where we expect quantum gravity effects to become important, the loop corrections induced to the Higgs mass are of the order of $\sim 10^{18}$ GeV. The bare parameters in the Higgs potential need to exactly cancel these huge effects up to one part in $\sim 10^{10}$ to have a Higgs mass around 100 GeV. For the loop corrections from Eqn. (2.19) to be of the same order as the Higgs mass, new physics effects should appear already around the TeV scale.
Unwilling to accept an extremely fine-tuned realization of EWSB, we conclude that the SM should be seen as a parametrization rather than a dynamical explanation of EWSB. A light scalar Higgs seems very unnatural – unless protected by a new symmetry.

2.1.4 Precision Tests and Electroweak Constraints on New Physics

As illustrated above, the SM is theoretically not fully satisfactory and we expect new physics to appear at around the TeV scale to stabilize the electroweak symmetry breaking scale. However, the SM has been probed to a great degree of accuracy and found to be in excellent agreement with experimental data. This means that new physics effects on the quantities that we have measured so far must be tiny. In the following, we will illustrate the basic idea behind electroweak precision tests (EWPT) and describe how they can be used to constrain new physics models. This section is based on Refs. [5, 24–30], where more details can be found.

The gauge sector of the electroweak Lagrangian (2.9) has two independent parameters: $g$ and $g'$. Including the Higgs sector, we find $\lambda$ and $v$ as two more free parameters. Including the fermion sector, we have a large number of Yukawa couplings. However, only the top Yukawa coupling is reasonably large and we neglect the others. In summary, the SM has five relevant, independent parameters

$$g, g', v, y_t, \lambda.$$  \hspace{1cm} (2.20)

Any SM observable can thus be expressed in terms of these parameters. It is useful to exchange these parameters with observables that have been experimentally measured with great precision. A convenient choice is

$$G_F, M_Z, \alpha_{em}, m_t, m_h,$$  \hspace{1cm} (2.21)

which are respectively the Fermi constant, the $Z$ boson mass, the fine structure constant $\alpha_{em} = \frac{e^2}{4\pi}$, the top quark mass and the Higgs boson mass. At tree level, any electroweak quantity can be expressed in simple terms involving only the electroweak input parameters $G_F, M_Z$ and $\alpha_{em}$. Radiative corrections, however, generally introduce a dependence on the non-electroweak parameters. By measuring many electroweak observables with high accuracy, we can deduce the value of other parameters outside the electroweak sector. These values are then compared with direct measurements of the same parameters to test the SM for consistency. To do so, a global fit to electroweak precision data is typically performed. The good agreement
between the measured values and the best-fit result is illustrated in Fig. 2.4b for some of the electroweak precision observables. Figure 2.4a shows the famous Higgs “blue band” plot, illustrating that, assuming the SM is the correct theory of nature, the electroweak precision data favors a light Higgs.

(a) $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ vs. $m_h$ from a global fit to electroweak precision data, assuming the SM is the correct theory of nature.

(b) Agreement between the measured values for some of the precision electroweak observables and their SM prediction.

**Figure 2.4:** Higgs “blue band plot” and global fit results for electroweak precision observables. Figures taken from Ref. [31].

Having seen how well the SM compares with electroweak precision data, we can now use these experimental data to constrain new physics models. To this end, there are two main approaches:

- **Model-dependent:** One can completely specify the new physics model under study, compute the contribution from new physics to precision observables and repeat the fits to derive constraints on the extended set of input parameters (including the new physics).

- **Model-independent:** Alternatively, one could write down the full set of higher-dimensional operators consistent with the symmetries of the SM in an effective Lagrangian approach. Electroweak precision data will constrain the coefficients of the different operators.

Since the model dependent approach is typically computationally very challenging and time consuming,
such analyses (see e.g. Refs. [32, 33]) were only carried out for a few specific models. In the following, we focus on the model independent approach.

Effective theories are based on the idea that details of the physics at very high energies are irrelevant for the description of an observation at the much lower energies at which we perform experiments. The unknown short-distance physics can be parametrized by an infinite expansion of operators,

$$\mathcal{L}_{\text{eff}} = \sum_d \sum_i \alpha_i^{(d)} O_i^{(d)},$$  \hspace{1cm} (2.22)

taking into account the relevant symmetries and degrees of freedom at low energy. In Eqn. (2.22), the sum runs over all operators with mass dimension\(^1\) \(d\), implying that in 4-dimensional space the coefficients \(\alpha^{(d)}\) have mass dimension \(4 - d\). Consequently, the coefficients of the operators with \(d > 4\) have negative mass dimension and are therefore suppressed by a cutoff scale \(\Lambda\),

$$\alpha^{(d)} = \frac{a^d}{\Lambda^{d-4}},$$  \hspace{1cm} (2.23)

where \(a^d\) is a dimensionless coefficient that one naturally expects to be of order \(\sim 1\). Due to the coefficients with negative mass dimension, the effective Lagrangian is non-renormizable, which is expected as this description breaks down at energies greater than the cutoff scale \(\Lambda\). Since operators of mass dimension \(d > 4\) are suppressed with increasing power of \(E/\Lambda\), we can neglect the operators with large dimensions and instead concentrate on a finite set parametrizing new physics effects beyond the SM at low energies.

Assuming that the dominant new physics effects reside in gauge boson self-energies (propagators), corrections to electroweak precision observables can be parametrized using the Peskin-Takeuchi \(S, U\) and \(T\) parameters [27, 34]. The \(S\) parameter is sensitive to the number of left- versus right-handed fermion weak doublets, whereas \(U\) is constrained by the W boson mass and width [35, 36]. The \(T\) parameter is sensitive to weak isospin violation and related to \(\rho\) from Eqn. (2.13) via

$$\rho \approx \alpha(M_Z)T + 1,$$  \hspace{1cm} (2.24)

as \(T\) is the only Peskin-Takeuchi parameter that violates custodial symmetry.

One can now add \(S, T\) and \(U\) to the floating parameters in the global fits to electroweak precision data. The results [24]

$$S = 0.00^{+0.11}_{-0.10}$$  \hspace{1cm} (2.25)

$$T = 0.02^{+0.11}_{-0.12}$$  \hspace{1cm} (2.26)

$$U = 0.08^{+0.11}_{-0.11},$$  \hspace{1cm} (2.27)

where the Higgs mass has been varied between 115.5 and 127 GeV, are graphically illustrated in Figure 2.5. We will see in Section 2.3 how these results can be used to constrain new physics models.

\(^1\)Since the actions \(S = \int d^D x \mathcal{L} \) is dimensionless, the mass dimension of the Lagrangian in \(D\)-dimensional space is \(D\).
2.1. The Standard Model of Particle Physics

Figure 2.5: 1 σ constraints on the Peskin-Takeuchi S and T parameters, where the red contour assumes a Higgs boson mass of $115.5 \text{ GeV} < m_h < 127 \text{ GeV}$. Figure taken from Ref. [24].

2.1.5 The Need for Physics Beyond the Standard Model

Despite the astonishing agreement of the SM with experimental observations, there is strong evidence indicating that the SM cannot be the \textit{ultima ratio} of particle physics. Some of these observations are listed below.

- Recent observations have revealed that neutrinos have non-zero masses [37–39]. While the accommodation of massive neutrinos seems a minimal extension of the SM, the nature of the neutrino’s mass is still unknown.

- A variety of astrophysical observations has lead to the conclusion that the baryonic matter as described by the SM can only account for about 4% of the total energy and matter in the universe. The SM does not incorporate Dark Matter, which is estimated to account for about 84% of the total mass in the universe and known to interact at most weakly with other particles.

- The SM is unable to explain the observed matter-antimatter asymmetry in the universe.

We recall that in the absence of a symmetry that impedes the Higgs mass’ sensitivity to quantum corrections, a light fundamental scalar field seems unnatural. The hierarchy problem, if taken seriously, provides strong evidence for the existence of new physics beyond the SM. In an attempt to stabilize and explain the electroweak scale, new physics models almost inevitably introduce new particles, which in turn cause conflicts with constraints from electroweak precision data. Such conflicts can be avoided by

- making new particles heavy or suppressing their couplings to the SM particles;
- making the new physics at least approximately custodially symmetric and thus preventing large tree-level corrections to the $\rho$ parameter;
• or generally introducing new symmetries that forbid new particles to contribute to SM processes at tree level.

In the following, we will review two popular new physics models that fulfill the above requirements: the Minimal Supersymmetric Standard Model and a realistic Composite Higgs model.

2.2 The Minimal Supersymmetric Standard Model

Supersymmetry (SUSY) – a symmetry relating fermionic and bosonic degrees of freedom – is an exciting idea which was first developed in the 1970s for various reasons [40–42], including purely esthetic ones. Only later it was realized that SUSY provides an elegant solution to the hierarchy problem.

As of this writing, none of the superpartners of the SM particles has been discovered. If SUSY was unbroken, the SUSY particles would have to have the same masses as their SM partners. Clearly, no sleptons and squarks with masses equal to the ones of the SM leptons and quarks exist, indicating that SUSY must be a broken symmetry.

In the following, we describe the supersymmetric solution to the hierarchy problem, introduce the MSSM and review some of its consequences. This Section is based on Refs. [5–7, 12].

2.2.1 Supersymmetric Solution to the Hierarchy Problem

If we consider a heavy complex scalar particle $S$ with mass $m_S$ that couples to the Higgs with a Lagrangian term $-\lambda_S |h|^2 |S|^2$, the Feynman diagram in Figure 2.6 gives a correction to the Higgs mass of the form

$$\Delta m_H^2 = \frac{\lambda_S^2}{16\pi^2} \left[ \Lambda^2 - 2m_S^2 \ln (\Lambda/m_S) + \ldots \right],$$

(2.28)

leading to a partial cancellation of the Higgs mass correction due to the fermion loop in Eqn. (2.19). In a supersymmetric extension of the SM, in which each SM fermion is accompanied by two complex scalars with $\lambda_S = |\lambda_f|^2$, then the $\Lambda^2$ divergencies of Eqns. (2.28) and (2.19) would exactly cancel, leaving only a logarithmic correction behind. It can be shown that unbroken supersymmetry guarantees the cancellation of the quadratic divergencies in scalar squared masses to all orders of perturbation theory, providing a elegant solution to the hierarchy problem.
2.2.2 MSSM Gauge Eigenstates

In supersymmetric theories, each SM particle is organized in a supermultiplet and must have a superpartner with spin differing by 1/2 unit. The spin-0 partners of the quarks and leptons are called squarks and sleptons, indicating their scalar nature. Moreover, the fermionic superpartners of the gauge bosons are called gauginos. Each supermultiplet contains the same number of fermionic and bosonic degrees of freedom and the members of a supermultiplet must fall in the same representation of the SM gauge group. Thus, they have the same electric charges, weak isospin and color degrees of freedom. Since the known fundamental particles have quantum numbers and masses that prevent them from being supersymmetric partners of any other SM particle, supersymmetric theories necessarily need to extend the particle content of the theory. Because of the structure of SUSY, it turns out that one chiral supermultiplet to accommodate the Higgs boson is not enough. A Higgs chiral multiplet with $Y = 1/2$ can only give mass to charge +2/3 up-type quarks, such that a second multiplet with $Y = -1/2$ is needed to account for massive charge -1/3 down-type quarks and charged leptons. The Minimal Supersymmetric Standard Model (MSSM) contains the minimal number of particles to realize a supersymmetric model. The chiral and gauge multiplets of the MSSM are listed in Table 2.2.

Table 2.2: Chiral and gauge supermultiplets in the MSSM. The set of numbers $(C, L)_Y$ indicates the representations of the SM subgroups $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$.

<table>
<thead>
<tr>
<th>Names</th>
<th>spin-0</th>
<th>spin-1/2</th>
<th>spin-1</th>
<th>$(C, L)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>squarks, quarks</td>
<td>$(\bar{u}_L, \bar{d}_L)$</td>
<td>$(u_L, d_L)$</td>
<td>-</td>
<td>$(3, 2)_{+1/6}$</td>
</tr>
<tr>
<td>$(\times 3$ families)</td>
<td>$\bar{u}_R$</td>
<td>$(u_R)^c$</td>
<td>-</td>
<td>$(3, 1)_{-2/3}$</td>
</tr>
<tr>
<td>sleptons, leptons</td>
<td>$(\tilde{\nu}_L, \tilde{e}_L)$</td>
<td>$(\nu_L, e_L)$</td>
<td>-</td>
<td>$(1, 2)_{-1/2}$</td>
</tr>
<tr>
<td>$(\times 3$ families)</td>
<td>$\tilde{d}_R$</td>
<td>$(d_R)^c$</td>
<td>-</td>
<td>$(1, 1)_{+1}$</td>
</tr>
<tr>
<td>Higgs, higgsinos</td>
<td>$(H_u^0, H_u^0)$</td>
<td>$(\tilde{H}_u^+, \tilde{H}_u^0)$</td>
<td>-</td>
<td>$(1, 2)_{+1/2}$</td>
</tr>
<tr>
<td></td>
<td>$(H_d^0, H_d^-)$</td>
<td>$(\tilde{H}_d^0, \tilde{H}_d^-)$</td>
<td>-</td>
<td>$(1, 2)_{-1/2}$</td>
</tr>
<tr>
<td>wino, W boson</td>
<td>-</td>
<td>$(\tilde{W}^+, \tilde{W}^0)$</td>
<td>$(W^+, W^0)$</td>
<td>$(1, 3)_0$</td>
</tr>
<tr>
<td>bino, $B^0$ boson</td>
<td>-</td>
<td>$\tilde{B}^0$</td>
<td>$B^0$</td>
<td>$(1, 1)_0$</td>
</tr>
<tr>
<td>gluino, gluon</td>
<td>-</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>$(8, 1)_0$</td>
</tr>
</tbody>
</table>

The general MSSM Lagrangian involving chiral supermultiplets can be derived from the superpotential, which is an analytic function of the scalar components of the chiral supermultiplets. It can be written as

$$W_{MSSM} = \bar{u}_L y_u QH_u - \bar{d}_L y_d QH_d - \tilde{e}_L y_e LH_d + \mu H_u H_d,$$

(2.29)

where $Q, \bar{u}, \bar{d}, L$ and $e$ are the $SU(2)_L$ quark doublet, up-type quark singlet, down-type quark singlet, lepton doublet and lepton singlet superfields from Table 2.2, respectively, and $H_u$ and $H_d$ denote the two Higgs superfields.
2.2.3 Gauge Coupling Unification

It can be shown that the inverse gauge couplings of the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ run linearly with $\ln Q$ at one loop order in the renormalization group equation (RGE)

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(Q_0) + \frac{b_i}{2\pi} \ln Q/Q_0 \quad i = 1, 2, 3,$$

where $Q$ is the ‘running’ energy scale, $Q_0$ is a reference energy scale at which the couplings are well measured (usually taken to be $m_Z$). The constants $b_i$ depend on the gauge group and on the multiplets to which the gauge bosons couple. It is found that $\alpha_1^{-1}$ increases, but $\alpha_2^{-1}$ and $\alpha_3^{-1}$ decrease with $Q$. Gauge coupling unification is the hypothesis that at some high energy scale $Q_{GUT}$, the three couplings would be equal. In the SM, such a unification is excluded with high significance. In the MSSM, however, they do meet convincingly at a scale of $O(10^{16}\text{ GeV})$, provided that the superpartner masses are in the range of 100 GeV to 10 TeV. This result is non-trivial, as a lower scale would generate too fast proton decay and a higher scale (above the Planck scale) would require to include gravitational interactions. It is important to note that the apparent unification of gauge couplings is an unforced consequence simply of the matter content of the MSSM, and thus taken as a strong motivation for grand unified theories. An illustration of the running of the inverse gauge coupling is given in Figure 2.7.

![Figure 2.7: RGE running of gauge couplings in the SM and the MSSM. Figure taken from Ref. [24] and adopted.](image)

2.2.4 R-Parity

The MSSM superpotential in Eqn. (2.29) does not contain any lepton number ($L$) or baryon number ($B$) violating terms, even though such terms can be gauge invariant and renormalizable. This choice is experimentally motivated by the non-observance of proton decays, such as $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$. While
the lepton number and baryon number conservations are accidental symmetries of the SM, a fundamental symmetry is needed to make MSSM compatible with experimental constraints. This symmetry, which plays an important role in avoiding conflicts with electroweak precision tests (see Section 2.1.5), is called $R$-parity and defined by

$$R = (-1)^{3(B-L)+2s},$$

(2.31)

where $s$ is the spin of the particle. With this definition, all SM particles and the Higgs boson have even $R$-parity, while the squarks, sleptons, gauginos and higgsinos have odd $R$-parity. If $R$-parity is indeed a conserved quantum number in nature, the phenomenological consequences are:

- The lightest sparticle is stable, and if electrically neutral it could be an attractive candidate for non-baryonic dark matter.
- The decay products of all other SUSY particles must contain an odd number of LSPs.
- In accelerator experiments, SUSY particles can only be produced in pairs.

Other phenomenological consequences of $R$-parity conservation especially important to collider experiments will be discussed in Section 2.2.10.

### 2.2.5 SUSY Breaking

There is no theory consensus on how to ‘best’ spontaneously break supersymmetry, even though the mechanism by which SUSY breaking can be realized in nature has been the focus of an extensive theoretical work over several decades. The spontaneous SUSY breaking cannot be realized in the MSSM itself, since none of the MSSM fields can develop a non-zero vacuum expectation value without spoiling the gauge invariance. A common theory viewpoint, however, seems to be that spontaneous SUSY breaking could occur in a sector that is weakly coupled to the chiral supermultiplets of the MSSM.

Rather than studying models of spontaneous supersymmetry breaking, we can look for a parametrization of the SUSY-breaking terms at low energies, using an effective Lagrangian approach. It is important to notice that broken SUSY does no longer guarantee the cancellation of the quadratic divergencies in scalar squared masses. Demanding that broken SUSY is still to provide a solution to the hierarchy problem, we restrict ourselves to soft SUSY-breaking terms, i.e., terms that do not spoil the cancellation of the quadratic divergencies. It can be shown [6] that in case of soft SUSY breaking, the remaining correction to the Higgs squared mass must be of the form

$$\Delta m_H^2 = m_{\text{soft}}^2 \left[ \frac{\lambda}{16\pi^2} \ln(\Lambda/m_{\text{soft}}) + \ldots \right],$$

(2.32)

where $m_{\text{soft}}$ is a mass scale of the lightest few superpartners. To cure the hierarchy problem successfully, we conclude that the lightest SUSY particles cannot be too large; using $\Lambda = M_{\text{Planck}}$, one finds that the superpartners of the SM particles should indeed show up at the TeV scale.
The most general soft SUSY breaking Lagrangian that is compatible with gauge invariance and $R$-parity conservation in the MSSM reads

$$L_{\text{soft}}^{\text{MSSM}} = - \frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} \right) + \text{h.c.}$$ (2.33a)

$$- \left( \tilde{u}_a \tilde{Q}_u + \tilde{d}_a \tilde{Q}_d - \tilde{e}_a \tilde{L}_d \right) + \text{h.c.}$$ (2.33b)

$$- m_Q^2 |\tilde{Q}|^2 - m_U^2 |\tilde{U}|^2 - m_d^2 |\tilde{d}|^2 - m_e^2 |\tilde{e}|^2$$ (2.33c)

$$- m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - (b H_u H_d + \text{h.c.}),$$ (2.33d)

where the “$m$-terms” correspond to the sfermion masses, the “$M$-terms” to the gaugino masses, and the “$a$-terms” represent the trilinear scalar couplings and are matrices in generation space.

A careful count of the new parameters introduced in Eqn. (2.33) reveals that there are 105 new masses, mixing angles and phases that cannot be rotated away. Consequently, and opposed to the SUSY invariant MSSM-Lagrangian with only one new parameter ($\mu$) w.r.t. the SM, SUSY breaking introduces a tremendous arbitrariness in the theory. In a sense, however, this impression is misleading, since extensive regions of parameter space are excluded experimentally. This is due to the fact that generic values of many of the new parameters lead to flavor changing neutral currents or new sources of CP-violation at levels that are excluded by experiment.

There are various frameworks of SUSY breaking that guarantee a suppression of parameters leading to such contradictions with experimental data. In these models, the sector in which SUSY is spontaneously broken shares an interaction with the “visible sector” of chiral multiplets in the MSSM, and this interaction mediates SUSY breaking. There have been two main competing frameworks of spontaneous SUSY breaking.

The first one is the Planck-scale mediated supersymmetry breaking scenario, in which the mediating interaction is of gravitational strength. The gravitino, which is the massive, spin-3/2 goldstone boson associated to the spontaneous breaking of local supersymmetry in supergravity [43, 44], has typically a mass comparable to the MSSM sparticle masses. Since it only interacts gravitationally, it plays no role in collider physics. A particular model, called minimal Supergravity, will be investigated further in Section 2.2.9.

A second possibility is that electroweak and QCD gauge interactions are mediating the supersymmetry breaking. This scenario is called gauge mediated supersymmetry breaking. In these models, the gravitino is expected to be much lighter than the other MSSM sparticles, and thus most certainly the LSP.

### 2.2.6 The Higgs Sector and Electroweak Symmetry Breaking in the MSSM

The complete potential for the scalar fields in the MSSM reads

$$V = (|\mu|^2 + m_{H_u}^2)(|H_u|^2 + |H_0^0|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d|^2 + |H_0^0|^2)$$ (2.34a)

$$+ \left[ b(H_u^+ H_d^- - H_0^0 H_0^0) + \text{h.c.} \right] + \frac{g^2 + g'^2}{8} (|H_u^+|^2 + |H_d^-|^2 - |H_0^0|^2 - |H_0^-|^2)^2$$ (2.34b)

$$+ \frac{g^2}{2} |H_u^+ H_d^-|^2 + |H_d^+ H_u^-|^2.$$ (2.34c)
We can now investigate whether, and if so under what condition, this potential can have a minimum which breaks the \( SU(2)_L \times U(1)_Y \) down to \( U(1)_{em} \). Ignoring terms in \( H_u^+ \) and \( H_d^- \) — they do not acquire a VEV as electromagnetism is not broken — and after a bit of algebra, the conditions for \( H_0^u \) and \( H_0^d \) to obtain non-zero vacuum expectation values read:

\[
2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 > 2b > 0 \quad \text{and} \quad \left( |\mu|^2 + m_{H_u}^2 \right) \left( |\mu|^2 + m_{H_d}^2 \right) < b^2. \tag{2.35}
\]

We see that the above condition is not satisfied if \( m_{H_d}^2 = m_{H_u}^2 = 0 \), implying that electroweak symmetry breaking does not occur in exact supersymmetric models (with no SUSY breaking). More generally, the Eqn. (2.35) does not hold in case of \( m_{H_u}^2 = m_{H_d}^2 \), a condition that is often taken to hold at a high scale \( \sim 10^{16} \text{GeV} \). However, in the RG evolution the parameter \( m_{H_u}^2 \) is typically pushed to negative or small values \( m_{H_u}^2 \ll m_{H_d}^2 \) at the electroweak scale, as illustrated in Figure 2.8. This means that in the MSSM, electroweak symmetry breaking is actually driven by quantum corrections. This mechanism, usually called \textit{radiative electroweak symmetry breaking}, is the dynamical origin of electroweak symmetry breaking in the MSSM.

The Higgs sector in the MSSM accounts for eight degrees of freedom. After the neutral scalar components of \( H_u \) and \( H_d \) obtain non-vanishing vacuum expectation values

\[
\langle H_{u,d}^0 \rangle = v_{u,d}, \quad \tan \beta \equiv v_u/v_d, \tag{2.36}
\]

the \( SU(2)_L \times U(1)_Y \) symmetry is broken down to \( U(1)_{em} \) and three Goldstone bosons are eaten to give mass to the SM \( W^\pm \) and \( Z \) bosons. The relation between these VEVs and the \( Z \) boson mass can be written as

\[
v_u^2 + v_d^2 = v^2 = 2 \frac{m_Z^2}{g^2 + g'^2}. \tag{2.37}
\]

The remaining five real physical scalars form the five Higgs bosons of the MSSM: \( h^0, H^0, A^0 \) and \( H^\pm \).

The masses of these Higgs bosons are obtained, as in the SM, by expanding the potential around the minimum. A crucial point is that even though the masses \( m_{A^0}, m_{H^0} \) and \( m_{H^\pm} \) are unconstrained, the mass of the lightest Higgs \( m_{h^0} \) is bounded from above. At leading order one finds

\[
m_{h^0} \leq m_Z |\cos 2\beta| \leq m_Z. \tag{2.38}
\]

It is interesting to note that the smallness of the Higgs mass in the MSSM is a consequence of the fact that — unlike in the SM — the quartic term in Eqn. (2.34b) is fixed by the gauge couplings.

The above bound is not compatible with the results from Higgs boson searches at LEP and the LHC. Luckily, since SUSY is broken, the cancellation of the top-quark and top-squark loops to the Higgs mass is not exact. Neglecting effects from top-squark mixing as well as 2-loop effects, the bound is modified to

\[
m_{h^0} \leq m_Z + \frac{3m_t^2}{2\pi^2 v^4} \ln \left( m_S/m_t \right), \tag{2.39}
\]

where \( m_S^2 = \frac{1}{2}(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2) \) (see Section 2.2.7). In this simple approximation, the Higgs mass can be raised above the LEP exclusion of 114 GeV by choosing \( m_S \approx 500 \text{GeV} \). Raising it further to the celebrated
Chapter 2. Physics at the Large Hadron Collider

125 GeV would request \( m_S \approx 900 \) GeV. A rigorous study \cite{45}, taking into account 2-loop effects and using \( m_S = 2 \) TeV, has concluded on an upper bound on the lightest Higgs mass of \( m_{h^0} \lesssim 135 \) GeV. Clearly, if the LHC experiments had excluded a light Higgs and had not found a new boson at 125 GeV, the MSSM would have been in serious troubles. More remarks on the consequences of a 125 GeV Higgs are made in Section 2.2.8.

2.2.7 MSSM Mass Eigenstates

Another interesting feature of the MSSM is that after electroweak symmetry breaking and taking into account SUSY breaking effects, gauge eigenstates with identical quantum numbers would mix to form mass eigenstates. The gauge eigenstates of fermion superpartners of the third generation are particularly prone to mixing since their Yukawa couplings are typically large. It is common to make the approximation that only the third generation sfermions show significant mixing. Moreover, the neutral electroweak gauginos and the neutral higgsinos mix to form four mass eigenstates called neutralinos \( (\tilde{\chi}_i^0, i = 1, 2, 3, 4) \). The charged higgsinos and the winos mix to form two mass eigenstates called charginos \( (\tilde{\chi}_i^\pm, i = 1, 2) \). The mass eigenstates of the MSSM are listed in Table 2.3.

<table>
<thead>
<tr>
<th>Names</th>
<th>gauge eigenstate</th>
<th>mass eigenstate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higgs bosons</td>
<td>( H_u^0 ) ( H_d^0 ) ( H_u^\pm ) ( H_d^- )</td>
<td>( h^0 ) ( H^0 ) ( A^0 ) ( H^\pm )</td>
</tr>
<tr>
<td>squarks</td>
<td>( \tilde{u}_L ) ( \tilde{d}_R ) ( \tilde{d}_L ) ( \tilde{d}_R )</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td>( \tilde{s}_L ) ( \tilde{c}_R ) ( \tilde{c}_L ) ( \tilde{c}_R )</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td>( \tilde{t}_L ) ( \tilde{b}_R ) ( \tilde{b}_L ) ( \tilde{b}_R )</td>
<td>( \tilde{t}_1 ) ( \tilde{b}_2 )</td>
</tr>
<tr>
<td>sleptons</td>
<td>( \tilde{e}_L ) ( \tilde{e}_R ) ( \tilde{\nu}_e )</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td>( \tilde{\mu}_L ) ( \tilde{\mu}<em>R ) ( \tilde{\nu}</em>\mu )</td>
<td>(same)</td>
</tr>
<tr>
<td></td>
<td>( \tilde{\tau}_L ) ( \tilde{\tau}<em>R ) ( \tilde{\nu}</em>\tau )</td>
<td>(same)</td>
</tr>
<tr>
<td>neutralinos</td>
<td>( \tilde{B}^0 ) ( \tilde{W}^0 ) ( \tilde{H}_u^0 ) ( \tilde{H}_d^0 )</td>
<td>( \tilde{\chi}_1^0 ) ( \tilde{\chi}_2^0 ) ( \tilde{\chi}_3^0 ) ( \tilde{\chi}_4^0 )</td>
</tr>
<tr>
<td>charginos</td>
<td>( \tilde{W}^\pm ) ( \tilde{H}_u^+ ) ( \tilde{H}_d^- )</td>
<td>( \tilde{\chi}_1^\pm ) ( \tilde{\chi}_2^\pm )</td>
</tr>
<tr>
<td>gluino</td>
<td>( \tilde{g} )</td>
<td>(same)</td>
</tr>
</tbody>
</table>

2.2.8 Fine-Tuning in the MSSM

While softly broken SUSY successfully cures the hierarchy problem of the SM, the exclusion limits on the Higgs mass from the LEP and LHC experiments again lead to some degree of fine-tuning. This problem, which is often referred to as the little hierarchy problem \cite{7, 46, 47}, is summarized in the foregoing discussion.
Using Eqn. (2.35) and in the approximation of large $\tan \beta$, one can express the minimization condition on the Higgs potential as

$$\frac{1}{2} m_Z^2 = -|\mu|^2 - m_{\tilde{H}_u}^2.$$  

(2.40)

In order to satisfy this condition “naturally”, we may demand that the terms on the right hand side of Eqn. (2.40) are no more than an order of magnitude larger than the one on the left-hand side, as to avoid the need for miraculous cancellations. The RG evolution of the parameter $m_{\tilde{H}_u}$, however, receives contributions from the top squarks. When running down from a high scale $\Lambda_{GUT} \sim 10^{16}$ GeV to the electroweak scale (see e.g. Figure 2.7), it can be shown that $m_{\tilde{H}_u}$ receives a contribution of the form

$$\delta m_{\tilde{H}_u}^2 \approx -\frac{3y_t}{4\pi^2} m_S^2 \ln \left( \frac{\Lambda_{GUT}}{m_S} \right),$$

(2.41)

which amounts to roughly $2m_S^2$ when setting $y_t = 1$. If we now impose our naturalness condition and request that $m_{\tilde{H}_u}^2 \lesssim 10 \cdot \frac{1}{2} m_Z^2$, we conclude that

$$m_S^2 < 10 \cdot m_Z^2 / (\sim 4),$$

(2.42)

which implies

$$m_S \lesssim 140 \text{ GeV}.$$  

(2.43)

This is, however, a substantially lower value than what we have obtained in Section 2.2.6 to raise the Higgs mass above the LEP exclusion ($m_S \approx 500$ GeV). If we set $m_S \approx 500$ GeV (the smallest value consistent with the LEP exclusion) the factor 10 in Eqn. (2.42) would have to be changed to $\sim 120$, indicating that (within the validity of our simple calculation) the MSSM is already fine tuned to the % level.

In agreement with our simple calculation, it is generally true that recent results from the LHC Higgs searches, most notably the discovery of the new boson with a mass of 125 GeV, had cast the natural MSSM in a rather negative light. This situation tends to be aggravated with the negative results from the LHC sparticle searches, which have imposed stringent bounds on squarks of the first two generations and on the gluino. Such considerations have driven forward an extensive discussion [46, 48–50] among the theory community on the naturalness of SUSY models. These studies roughly agree on the conclusion that the MSSM has to be tuned to the % level or worse to obtain $m_{h^0} = 125$ GeV and the most natural SUSY spectrum includes (a.) a light stop $\lesssim 500$ GeV and (b.) a gluino mass below $\sim 1.5$ TeV.

There are, however, various supersymmetric models that go beyond the MSSM and address some of these fine-tuning problems. Compressed SUSY [51, 52], for instance, which feature a running gluino mass parameter that is substantially smaller than the wino mass parameter at the scale of apparent gauge coupling unification, are known to provide a solution to the little hierarchy problem. Similarly, the next-to-minimal model (NMSSM) [53] is known to be able to accommodate a 125 GeV Higgs with only a $\sim 10\%$ fine-tuning.

2.2.9 The Constrained MSSM

In this section, we review some aspects of the **Constrained Minimal Supersymmetric Standard Model** (CMSSM) [54], which is often also called **minimal Supergravity** (mSUGRA).
Figure 2.8: Running of the soft masses from the GUT scale to the electroweak scale. The three gaugino masses $M_1, M_2, M_3$ are labeled $\tilde{B}, \tilde{W}, \tilde{g}$ respectively. Figure taken from Ref. [54].

In these models, special boundary conditions are imposed on the soft SUSY breaking parameters in Eqn. (2.33) at the GUT scale, which reduce the vast parameter space of the softly broken MSSM to a much more predictive subspace. These boundary conditions, which are further chosen as to evade terms leading to dangerous flavor-changing neutral currents and CP violation, read

\begin{align}
M_3 &= M_2 = M_1 = m_{1/2}, \\
m_Q^2 &= m_{\tilde{u}}^2 = m_{\tilde{d}}^2 = m_L^2 = m_{\tilde{e}}^2 = m_0^2, \\
m_{\tilde{H}_u}^2 &= m_{\tilde{H}_d}^2 = m_0^2, \\
a_u &= A_0 y_u, \quad a_d = A_0 y_d, \quad a_e = A_0 y_e.
\end{align}

(2.44a, 2.44b, 2.44c, 2.44d)

Here, the $y$ matrices are the ones appearing in Eqn. (2.29) and $1$ denotes the unit matrix in family space. With this choice of boundary conditions, the 105 unconstrained new parameters of the softly broken MSSM are reduced to a small number of five parameters: the ratio of the Higgs VEVs at the electroweak scale $\tan \beta$ as well as the sign of the higgsino mass parameter $\text{sign}(\mu)$, the trilinear coupling $A_0$, the common scalar mass parameter $m_0$ and the common gaugino mass parameters $m_{1/2}$ at the GUT scale. The RG evolution from the GUT scale down to the electroweak scale allows to predict the entire MSSM spectrum from these parameters. An illustration of the RG evolution of these parameters is given in Figure 2.8. Typically, one finds the following relation to hold for the gaugino masses at the TeV scale:

\[ M_3 : M_2 : M_1 \approx 6 : 2 : 1. \]

(2.45)
We thus expect in mSUGRA the gluino to be a few times heavier than the states associated with the electroweak sector. It is also interesting to observe that the squarks of the first two generations cannot be lighter than about 0.8 times the mass of the gluino, because the gluino mass feeds into the squark masses through RG evolution. The lighter stop $\tilde{t}_1$ and lighter sbottom $\tilde{b}_1$ are likely to be the lightest squarks. Moreover, models with low $m_0 \lesssim m_{1/2}$ are characterized by a squark mass spectrum with $m_{\tilde{q}} \sim m_{\tilde{g}}$, whereas in the case of large values of $m_0$ we typically have $m_{\tilde{q}} \gg m_{\tilde{g}}$. The mass spectra of two mSUGRA benchmark points are shown in Figure 2.9.

2.2.10 SUSY Signals at Hadron Colliders

In this section, we outline general features of MSSM signatures relevant at hadron colliders. Here, we assume that $R$-parity is conserved and the lightest neutralino $\tilde{\chi}_0^0$ is the LSP. Consequently, sparticles can only be produced in pairs and cascade down to the LSP.
Sparticle Production at Hadron Colliders

At hadron colliders, such as the Tevatron and the LHC, sparticles can be produced in pairs from reactions of QCD strength

\[ gg \to \tilde{g} \tilde{g}, \tilde{q}_i \tilde{q}_j^*, \]  

\[ gq \to \tilde{g} \tilde{q}_i, \]  

\[ q\bar{q} \to \tilde{g} \tilde{g}, \tilde{q}_i \tilde{q}_j^*, \]  

\[ q\bar{q} \to \tilde{q}_i \tilde{q}_j^*, \]  

and from parton collisions of electroweak strength

\[ q\bar{q} \to \tilde{\chi}^+_{i} \tilde{\chi}^-_{j}, \tilde{\chi}^0_{i} \tilde{\chi}^0_{j}, \]  

\[ ud \to \tilde{\chi}^+_{i} \tilde{\chi}^0_{j}, \]  

\[ d\bar{u} \to \tilde{\chi}^0_{i} \tilde{\chi}^-_{j}, \]  

\[ qu \to \tilde{\ell}_{i}^+ \tilde{\ell}_{j}^-, \tilde{\nu}_{\ell} \tilde{\nu}_{\ell}^*, \]  

\[ ud \to \tilde{\ell}_{i}^+ \tilde{\nu}_{\ell}, \]  

\[ d\bar{u} \to \tilde{\ell}_{i}^- \tilde{\nu}_{\ell}^*, \]  

At the LHC, with a proton-proton initial state, the largest SUSY production cross-sections are typically found for colored sparticles via gluon-gluon and gluon-quark fusion. The strongly produced SUSY signal is in turn dominated by gluinos and the first two generation squarks, as illustrated in Figure 2.10. In the following, we will concentrate on SUSY signatures arising from the production of gluinos and first and second generation squarks.

Figure 2.10: LHC cross sections for sparticle production at \( \sqrt{s} = 7\,\text{TeV} \), calculated at NLO by PROSPINO [56]. Figure taken from Ref. [57].
Squark Decays

If the decay $\tilde{q} \rightarrow q \tilde{g}$ is kinematically allowed, it is expected to dominate since the squark-quark-gluino vertex has QCD strength. If instead the gluino is heavier, the squarks can decay into quark plus neutralino $\tilde{q} \rightarrow q \tilde{\chi}_i^0$ or quark plus chargino $\tilde{q} \rightarrow q \tilde{\chi}_i^\pm$, where the direct decay to the LSP $\tilde{q} \rightarrow q \tilde{\chi}_0^0$ is usually favored. The charginos and neutralinos produced in squark decays may undergo the two-body decays

$$\tilde{\chi}_i^0 \rightarrow Z \tilde{\chi}_j^0, \ W \tilde{\chi}_j^0, \ h \tilde{\chi}_j^0, \ \ell \tilde{\nu}, \ \nu \tilde{\nu} \tag{2.48}$$
and

$$\tilde{\chi}_i^\pm \rightarrow W^\pm \tilde{\chi}_j^0, \ Z \tilde{\chi}_1^\pm, \ h \tilde{\chi}_1^\pm, \ \ell \tilde{\nu}, \ \nu \tilde{\ell} \tag{2.49}$$
respectively, or a variety of three-body decays.

Gluino Decays

A gluino can only decay through a squark. If kinematically accessible, the two-body decay $\tilde{g} \rightarrow q \tilde{q}$ will dominate. As we have seen in Section 2.2.9, the third generation sbottom and stop quarks can easily be lighter than the other squarks from the first and second generation. In such a case, the decays $\tilde{g} \rightarrow t \tilde{t}_1$ and $\tilde{g} \rightarrow b \tilde{b}_1$ are likely to be the dominant ones. If instead the gluino is lighter than all squarks, as for instance in the LM9 benchmark point illustrated in Figure 2.9b, the gluino is forced to undergo three-body decays through off-shell squarks $\tilde{g} \rightarrow qq \tilde{\chi}_1^0$ and $\tilde{g} \rightarrow q \tilde{\chi}_1^\pm$. The decay $\tilde{g} \rightarrow q q \tilde{\chi}_0^0$ is likely to be the dominant channel.

Generic Signature of Strongly Produced Sparticles

Assuming $R$-parity conserving SUSY with a neutralino LSP, the produced sparticles must necessarily lead to two unobserved LSPs in the final state, which escape detection. The LSPs carry away at least $2m_{\tilde{\chi}_0^0}$ of missing energy. Since the true incoming momenta along the beam directions of the colliding partons are unknown at hadron colliders, only the component of the missing momentum transverse to the beam direction is observable. This missing transverse momentum is a key ingredient in SUSY searches at the LHC.

Furthermore, final states arising from strongly produced SUSY signals contain at least two quark partons, missing energy and potentially leptons. In case of a pair-produced gluino with subsequent three-body decay, at least four quark partons are expected in the final state. If the third generation squarks are lighter than the others, gluino decays through off-shell top and bottom squarks can contribute significantly, leading to the exciting signatures

$$pp \rightarrow \tilde{g} \tilde{g} \rightarrow tttt \tilde{\chi}_1^0 \tilde{\chi}_1^0, \quad \text{and} \quad pp \rightarrow \tilde{g} \tilde{g} \rightarrow bbb \tilde{\chi}_1^0 \tilde{\chi}_1^0. \tag{2.50}$$

We conclude that the final states arising from squark or gluino decays are characterized by a missing transverse energy signature, a large hadronic activity and potentially leptons. Since the third generation squarks are typically relatively light, SUSY signatures often contain multiple b quarks.
2.3 A Realistic Composite Higgs Model

2.3.1 Brief Overview

Composite Higgs models are based on the idea that a light Higgs could emerge as a pseudo-Goldstone boson from a new strongly interacting theory. This theory possesses a global symmetry that is spontaneously broken, giving rise to (at least) four Goldstone bosons that can be arranged into a complex $SU(2)_L$ doublet, which is in turn identified with the Higgs doublet. Upon gauging the electroweak symmetry group — by which we mean that the corresponding global symmetry is actually a local symmetry — the Higgs acquires a mass. This mass is typically small, as only generated at loop-level. Since the Higgs is a bound state and its mass not sensitive to loop corrections above the compositeness scale (of a few TeV), the hierarchy problem is elegantly solved.

Before immersing ourselves deeper in composite Higgs models, we stop to review a well known example of symmetry breaking in which nature has chosen a realization that does not suffer from any hierarchy problem: the chiral symmetry breaking in low energy QCD from which the pions emerge as light scalar fields. This discussion will help us understand the idea behind technicolor models and finally lead us to composite Higgs models. These sections are largely based on Refs. [4, 5, 27, 58–60].

2.3.2 Interlude: Chiral Symmetry Breaking in Low Energy QCD

The fermionic sector of the QCD Lagrangian with two massless flavors can be written as

$$\mathcal{L} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R,$$  \hspace{1cm} (2.51)

where we have used the QCD covariant derivative $D_\mu$ and defined the doublet $q_{L,R} = (u_{L,R}, d_{L,R})^T$. Given that the up- and down-quark masses are small, $m_u = 1.5 \sim 3.3$ MeV, $m_d = 3.5 \sim 6.0$ MeV [24], it is reasonable to treat these quarks as massless in a first approximation. The Lagrangian (2.51) has a global $U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ chiral symmetry. At low energies, QCD becomes strongly coupled, leading to quark-antiquark bound states. The vacuum state has a non-zero vacuum expectation value\(^\text{2}\) for the operator

$$\langle 0 | \bar{q} q | 0 \rangle = \langle 0 | \bar{q}_L q_R + \bar{q}_R q_L | 0 \rangle \neq 0,$$  \hspace{1cm} (2.52)

which is not invariant under the chiral symmetry $U_L \neq U_R$. The $SU(2)_L \times SU(2)_R$ symmetry is thus spontaneously broken to its vectorial subgroup $SU(2)_V$, called isospin, and the three Goldstone bosons associated to this symmetry breaking are the QCD pions $\pi^\pm, \pi^0$. These pions are composite states of the strongly interacting theory: they are bound states of the form $\bar{q} q'$.

As mentioned above, the up-type and down-type quarks are not actually massless, meaning that the chiral symmetry is only approximate to begin with. This results in the fact that the pions appear as light, but

\(^{2}\text{Strictly speaking, this is an assumption since the true QCD vacuum cannot be computed.}\)
massive particles and are in this sense called *pseudo-Goldstone bosons*. Experimentally, their masses were found to be $\sim 140$ MeV.

It is instructive to note that pion-pion scattering is affected by a similar unitarity problem as encountered in $WW$ scattering: the scattering amplitude grows with energy as $E^2$. Here, however, we know from experimental observations that there is no light scalar field playing the role of the Higgs in the SM. Instead, a whole tower of hadronic resonances is responsible for the restoration of unitarity; these resonances are exchanged at high energies in pion-pion scattering, where the most important contribution comes from the lightest vector meson ($\rho$).

Let us consider what happens if we turn on electroweak interactions. The electroweak gauge group $SU(2)_L \times U(1)_Y$ turns out to be a subgroup of the global symmetry group of the Lagrangian (2.51). By making the $SU(2)_L \times U(1)_Y$ local, i.e. by *gauging* the symmetry, we couple the gauge bosons to the quarks. The QCD vacuum breaks the symmetry spontaneously. $U(1)_{em}$ is the remaining *local* symmetry, and the $W^\pm$ and Z gauge boson acquire a mass by eating the corresponding would-be Goldstone bosons: the pions. It can be shown [4, 5] that this way, the gauge bosons would obtain masses of the form

$$m_W^2 = m_Z^2 e^2_W = g^2 f_\pi^2 / 4, \quad m_\gamma^2 = 0,$$

where $f_\pi = 92$ MeV denotes the pion decay constant. Clearly, $m_W(\text{QCD}) = g f_\pi / \sqrt{2} \approx 46$ MeV is far from the measured value of $m_W = 80.4$ GeV, indicating that the electroweak scale cannot be explained in this way. Moreover, the pions are actually observed as physical particles, and thus not eaten to account for the longitudinal polarization of the gauge bosons.

Still, the above conclusions suggest that the actual dynamics of the electroweak symmetry breaking could be just a scaled up version of QCD [20, 21, 23], with

$$f_\pi = 93 \text{ MeV} \to F_\pi = \sqrt{2} v = 246 \text{ GeV},$$

which is exactly the essence of *technicolor* models. Such models are attractive as they provide a dynamic realization of electroweak symmetry breaking and do not suffer from a hierarchy problem: there are no fundamental scalars in the theory. The lack of such a scalar field playing the role of the SM Higgs, however, implies the need for a different mechanism to account for massive leptons and quarks. This is accomplished in *extended technicolor* models. Such models, however, are typically quite challenged by experimental constraints on flavor-changing neutral current and precision electroweak measurements. Moreover, a light Higgs with a mass of $\sim 125$ GeV, as recently observed at the LHC, cannot easily be accommodated in technicolor models [61]. A detailed review of technicolor models can for instance be found in Refs. [24, 62, 63].

### 2.3.3 A Realistic Composite Higgs Model

In this section, we discuss an interpolation between technicolor models and the SM Higgs model that goes under the name of Composite Higgs model: a light Higgs emerges as a composite state from a new strongly
interacting sector. As first pointed out by Kaplan, Georgi and Dimopoulos [64–68], a composite Higgs can be naturally light if it is a pseudo-Goldstone boson of a new strong sector. Before discussing a specific model in detail, we will summarize some of the basic ideas of composite Higgs models.

**General considerations**

Composite Higgs models are generally based on the following ideas:

- We assume there is a new strongly interacting sector with a global symmetry group $G$. This symmetry is spontaneously broken at a scale $f > v = 174 \text{ GeV}$ to a subgroup $H$, where the SM electroweak gauge group $SU(2)_L \times U(1)_Y$ is a subgroup of $H$.

- The global symmetry breaking implies $n = \dim(G) - \dim(H)$ Goldstone bosons, out of which three are eaten to give mass to the $W^\pm$ and $Z$ gauge bosons. $G/H$ thus contains at least one $SU(2)_L$ doublet with hypercharge $\pm 1/2$, which we identify with the Higgs doublet.

- If $H$ was completely unbroken, the Higgs would be an exact, massless Goldstone boson. The Higgs potential vanishes at tree level, since at tree level $H$ is unbroken.

- The couplings of the SM gauge fields to the strong sector, however, *explicitly* break the global symmetry $G$, since they are invariant under SM electroweak gauge group but not under $G$. Due to this explicit symmetry breaking, the Higgs becomes a massive pseudo-Goldstone boson; loops of gauge bosons and SM fermions will generate a potential for the Higgs, which in turn breaks the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry.

- The Higgs is naturally light, since it is a pseudo-Goldstone boson and its mass is generated through radiative corrections at loop level. The hierarchy problem is solved as these virtual corrections saturate at the compositeness scale $\Lambda$.

- One might expect additional composite heavy bosons (the equivalent of the heavy mesons in QCD) to exist. Since composite Higgs models provide an effective description of the low-energy physics of the new strong sector, these states are integrated out and their mass is the natural cutoff $\Lambda \simeq 2\pi f$ of the theory. Its value is typically of the order of a few TeV: too large values of $\Lambda$ introduce substantial fine-tuning, while too low values cause large contributions to electroweak precision observables.

The minimal symmetry breaking pattern that fulfills the above requirements is $SU(3)$ broken down to $SU(2)_L \times U(1)_Y$, in which case we obtain exactly the $8 - 4 = 4$ Goldstone bosons we need to form the Higgs doublet. Such a model, however, would not be custodially symmetric and large corrections to the $\rho$ parameter would be expected. We thus focus our discussion on the minimal symmetry breaking pattern $SO(5)/SO(4)$ which preserves custodial symmetry. This particular model was discussed in Ref. [58], on which the following section is based. Non-minimal models, such as $SO(6)/SO(4)$, have more than four Goldstone bosons (and thus introduce additional scalars in the theory) and are for instance discussed in Ref. [69].
2.3. A Realistic Composite Higgs Model

A Realistic Model: The Higgs Sector

We study the low-energy phenomenology of a realistic composite Higgs model. In order not to depend on any specific high energy physics relevant at the scale of the strong dynamics that is not accessible to current experiments, we adopt an effective Lagrangian approach \cite{70, 71}. In this effective description, only a light composite Higgs boson as well as composite top partners lie below the cutoff of the theory.

The strongly interacting sector is described by a non-linear sigma model \cite{9}, with a UV cutoff given by

\[ \Lambda = \frac{4\pi f}{\sqrt{N_G}} \]

where \( N_G = 4 \) is the number of Goldstone bosons from the \( SO(5)/SO(4) \) symmetry breaking. The four Goldstone bosons are arranged into a fundamental \((4)\) of \( SO(4) \), or equivalently as a bidoublet of \( SU(2)_L \times SU(2)_R \). The Goldstones can be described by a scalar field

\[ \phi = \phi_0 e^{-iT^a h^a \sqrt{2}/f}, \]

subject to the constraint

\[ \phi^2 = f^2 \]

through which the \( SO(5) \to SO(4) \) global symmetry breaking is realized. In Eqn. (2.55), \( T^a \) denote the four broken generators and \( h^a \) the corresponding Goldstone bosons and \( \phi_0 = (0, 0, 0, f) \) is the vacuum state that preserves the \( SO(4) \equiv SU(2)_L \times SU(2)_R \) symmetry. Expanding the exponential, we get

\[ \phi = \sin \left( \frac{h/f}{h} \right) \left( h^1, h^2, h^3, h^4, h \cot(h/f) \right) = \left( \vec{\phi}, \phi_5 \right), \]

where we have defined \( h = \sqrt{\sum_a (h^a)^2} \). Gauging the \( SU(2)_L \) and the \( T^3_R \) generator of \( SU(2)_R \) explicitly breaks that \( SO(5) \) symmetry and induces a potential (and thus a mass) for the Higgs boson. It can be shown \cite{3, 5} that the usual relation

\[ m_W^2 = \frac{g^2 v^2}{2} \]

is satisfied, provided that we request

\[ s_\alpha \equiv \sin \left( \langle h \rangle / f \right) = \frac{\sqrt{2} v}{f}, \]

where \( \langle h \rangle \) is the vacuum expectation value of the composite Higgs, with its relation to \( v \) given in the above equation. With Eqn. (2.59), we can compute the couplings of the physical Higgs \( h \) to the gauge bosons by expanding the Higgs around its VEV, \( h^a = (0, 0, \langle h \rangle + h, 0)^T \). We find that the couplings of the Higgs to the gauge bosons \( (V = W, Z) \) are reduced with respect to the SM values by

\[ g_{VVh} = g_{VVh}^{SM} \sqrt{1 - s_\alpha^2}, \quad g_{VWh} = g_{VWh}^{SM} (1 - 2 s_\alpha^2). \]

While the particular dependence on \( s_\alpha \) depends on the group structure, the reduction of Higgs to gauge boson couplings is generic to composite Higgs models.

As a direct consequence of the reduced Higgs to gauge boson couplings, the Higgs production cross section via vector boson fusion will be reduced with respect to the SM prediction. For a symmetry breaking scale \( f = 500 \text{ GeV} \), the reduction would be at the order of 25\% \cite{70}. It is also interesting to note the effect
of Eqn. (2.60) on the restoration of unitarity in longitudinal gauge boson scattering. In the limit \( s_\alpha \to 0 \) the SM Higgs is recovered, in which the Higgs fully unitarizes the theory and the resonances of the strong sector become infinitely heavy and decouple. On the other hand, \( s_\alpha \to 1 \) corresponds to the technicolor limit in which the unitarity is restored by a tower of hadronic resonances.

**Constraints from Electroweak Precision Tests**

The reduced Higgs to gauge boson couplings also affect the electroweak precision \( S \) and \( T \) parameters. In fact, the leading contribution to \( S \) and \( T \) from diagrams involving the Higgs read in the SM

\[
\Delta S, \Delta T = a_{S,T} \log m_h + b_{S,T}, \tag{2.61}
\]

with \( a_{S,T} \) and \( b_{S,T} \) two constants. In the composite Higgs model, we obtain a correction to these parameters of the form

\[
\Delta S = \frac{1}{12\pi} \ln \left( \frac{m_h (\Lambda/m_h)^{s_\alpha} v^2}{m_{h,\text{ref}}^2} \right)^2 \Delta T = -\frac{3}{16\pi c_W^2} \ln \left( \frac{m_h (\Lambda/m_h)^{s_\alpha}}{m_{h,\text{ref}}^2} \right)^2 \tag{2.62}
\]

where \( \Lambda \) is the UV cutoff of the effective theory and \( m_{h,ref} \) denotes the reference Higgs mass used in the electroweak fit [70]. Moreover, we expect contributions to the \( S \) parameters from UV physics, but the \( T \) parameter to be protected by the \( SO(4) \) custodial symmetry in the strong sector. The contribution to \( S \) has been estimated [70] as

\[
\Delta S_\Lambda \sim \frac{4 s_W^2 g^2 v^2}{\alpha_\text{em}} \log \frac{3 \text{TeV}}{\Lambda} \approx 0.16 \left( \frac{3 \text{TeV}}{\Lambda} \right)^2. \tag{2.63}
\]

In Figure 2.11 we show the contributions from Eqn. (2.62) and Eqn. (2.63) for different values of \( s_\alpha \) on the \( S \) and \( T \) parameters. These effects result in a negative shift to the \( T \) parameter and a positive shift to the \( S \) parameter. We conclude that the corrections to \( S \) and \( T \) from Higgs compositeness and UV physics result in a sizeable contradiction with experimental constraints. Yet, one can expect other composite states to be as well below the cutoff of the theory. The next paragraph, in which we discuss how composite fermions are introduced in the model, is taken from work previously published in Ref. [59].

**A Realistic Model: The Fermionic Sector**

We consider vector-like resonances of composite fermions transforming in the fundamental representation of \( SO(5) \). We denote them by \( \Psi^i \), with the index \( i \) running over the multiplets included below the cutoff. The corresponding mass Lagrangian is [58, 72]

\[
-\mathcal{L}_{\text{SO}(5)} = m_\Psi^2 \bar{\Psi}^i \Psi^i + \frac{y_{ij}}{f} (\bar{\Psi}^i \phi) (\phi^j \Psi^j), \tag{2.64}
\]

where \( y_{ij} \) is a Hermitian matrix and \( \phi \) is the Higgs field defined in Eqn. (2.57). Under the electroweak gauge group, \( \Psi \) decomposes as \( \Psi = (Q, X, T) \), where \( Q \) and \( X \) are \( SU(2)_L \) doublets with hypercharge +1/6 and +7/6 respectively, and \( T \) is a \( SU(2)_L \) singlet with hypercharge 2/3. The \( X \) doublet introduces
Figure 2.11: 68%, 95% and 99% C.L limits on the $S$ and $T$ parameters for a fit to electroweak observables, using a reference Higgs mass $m_{h,\text{ref}} = 120$ GeV and fixing the anomalous $b_L$ to $Z$ coupling ($\delta g_{b_L}$) to $-2.5 \times 10^{-4}$. The effects of Higgs compositeness and UV physics are shown in blue and purple dots for $s_\alpha = 0.25, 0.33$ and 0.5. This Figure is taken from Ref. [58].

another quark of electromagnetic charge $2/3$, which can mix with the top, and a quark with charge $5/3$. Such quarks are one of the distinguishing features of the model. The SM quarks $q_L$ and $t_R$ have the same quantum numbers as $Q$ and $T$, respectively. The most generic interaction between the top sector and the new quarks is therefore of the form

$$-L^{\text{int}} = m^i_L \bar{q}_L Q^i_u + m^i_R \bar{T}^i_L t^i_R + \text{h.c.}. \quad (2.65)$$

Combining Eqns. (2.64) and (2.65) we obtain the mass matrices

$$-L^{2/3} = \begin{pmatrix} t_L \\ Q^u_L \\ X^u_L \\ T_L \end{pmatrix} \begin{pmatrix} 0 & m^T_L & 0 & 0 \\ 0 & m_\Psi + \frac{s_\alpha^2}{2} f y & \frac{s_\alpha^2}{2} f y & c_\alpha v y \\ 0 & \frac{s_\alpha^2}{2} f y & m_\Psi + \frac{s_\alpha^2}{2} f y & c_\alpha v y \\ m_R & c_\alpha v y & c_\alpha v y & m_\Psi + c_\alpha^2 f y \end{pmatrix} \begin{pmatrix} t_R \\ Q^u_R \\ X^u_R \\ T_R \end{pmatrix} + \text{h.c.} \quad (2.66)$$

for the quarks of charge $2/3$ and

$$-L^{-1/3} = \begin{pmatrix} b_L \\ Q^d_L \end{pmatrix} \begin{pmatrix} -\lambda_b v & m^T_L \\ 0 & m_\Psi \end{pmatrix} \begin{pmatrix} b_R \\ Q^d_R \end{pmatrix} + \text{h.c.} \quad (2.67)$$

for the quarks of charge $-1/3$. Here, we have introduced the short-hand $c_\alpha = \sqrt{1 - 2v^2/f^2}$. The indices $u$ and $d$ denote respectively the charge $2/3$ and $-1/3$ components of the doublet indicated. In the case of more fermionic resonances, the mass matrices are to be understood as in block form. Note that in Eqn. (2.67) we introduced an explicit $SO(5)$ breaking term

$$\mathcal{L}^b = \lambda_b \bar{q}_L \varphi b_R \quad (2.68)$$
to give a mass to the bottom quark. We could also generate a mass for the bottom quark in an $SO(5)$

preserving fashion. For example, we could couple the bottom quark to some new multiplets of $SO(5)$, as

we did for the top quark. This would come at the expense of introducing extra particles. Since the mass of

the bottom quark is small, we do not expect large effects from bottom compositeness. We opt therefore for

a minimal description, in which the bottom mass is generated with the current particle content of the model.

The couplings of the fermions to the Higgs boson are obtained expanding the second term in Eqn. (2.64)

around the VEV of $\phi$. For example, the couplings of the charge $2/3$ quarks to the Higgs boson are given by

$$\begin{bmatrix}
 t_L \\
 Q_L^u \\
 X_L^u \\
 T_L
\end{bmatrix}
\begin{bmatrix}
 0 & 0 & 0 & 0 \\
 s_\alpha c_\alpha & s_\alpha c_\alpha & \frac{1-2s_\alpha^2}{\sqrt{2}} & \frac{1-2s_\alpha^2}{\sqrt{2}} \\
 0 & s_\alpha c_\alpha & s_\alpha c_\alpha & \frac{1-2s_\alpha^2}{\sqrt{2}} \\
 0 & 1-2s_\alpha^2 & 1-2s_\alpha^2 & -2s_\alpha c_\alpha
\end{bmatrix}
\begin{bmatrix}
 t_R \\
 Q_R^u \\
 X_R^u \\
 T_R
\end{bmatrix}.$$  

(2.69)

The interaction Lagrangian of Eqn. (2.65) leads to a mixing between the fundamental fields $q_L$, $t_R$ and

the composite states $Q$ and $T$. The large top-quark mass can in fact be explained by a large mixing of the top

with composite states. The light quarks and fermions, on the other hand, are largely elementary, explaining

why the bounds on the compositeness of these particles can easily be evaded in this theory.

As previously explained and illustrated in Fig. 2.11, the Higgs sector typically yields large corrections
to the $S$ and $T$ parameters. It was shown in Refs. [58, 70, 72–74] that the vector-like composite fermions
we just introduced can help in restoring the agreement of the model with electroweak precision tests. It

turns out, however, that composite Higgs models with only one set of fermionic resonances below the cutoff

of the effective theory are very constrained. The charge $5/3$ quark is the lightest new particle predicted,

with a mass $m_{5/3} \lesssim 500$ GeV. Above it and rather close in mass ($\Delta m \lesssim 100$ GeV) is a charge $2/3$ quark.

The other quarks are typically much heavier. The most relevant collider signatures therefore come from the

production and decay of the charge $5/3$ quark. These signatures have been studied in Ref. [75].

The scenario dramatically changes if we include a second set of composite fermions below the cutoff [58, 76]. Constraints from electroweak precision tests become less stringent, and many different mass patterns

are allowed in the region accessible with early LHC data. In Chapter 3, we discuss the collider signatures

and discovery potential of this model.

### 2.4 On the Physics of Proton-Proton Collisions

As we already briefly discussed in Section 2.2.3, the size of the coupling of the strong interaction changes

with the size of the momentum transfer of the interaction. At leading order, this can be expressed as

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda_{QCD}^2)},$$  

(2.70)

where the constant $\beta_0$ is positive for color $SU(3)$. This equation implies that at low energies, the theory be-
comes strongly coupled and non-perturbative effects are important. The energy scale at which this happens
2.4. On the Physics of Proton-Proton Collisions

is $\Lambda_{\text{QCD}}$, which is experimentally measured to be of the order of 200 MeV. At large momentum transfers, however, the strong coupling is small; the quarks and gluons are asymptotically free and perturbative calculations can be used.

In this thesis, we are primarily interested in hard proton-proton collisions, i.e. collisions with a large momentum transfer. As pictorially illustrated in Figure 2.12, non-perturbative effects, however, play an important role even in such processes.

![Figure 2.12: Illustration of a proton-proton collision. Figure taken from Ref. [77] and adapted.](image)

A hard proton-proton collision, as illustrated above, involves different processes characterized by different energy scales. First, the interaction between two incoming partons, which involves a large momentum transfer, can be calculated in perturbative QCD. Second, the partons emerging from the hard interaction themselves emit radiation. Finally, the final state partons hadronize. This, however, is a non-perturbative process taking place at a scale comparable to $\Lambda_{\text{QCD}}$. Moreover, the spectator partons, which do not interact in the hard interaction, constitute the underlying event. The description of the non-perturbative processes relies on phenomenological models, which are tuned to experimental data.

Thanks to the factorization theorem [78], the cross section for the production of a final state $X$ from the scattering of two initial protons can be separated in a process-dependent, calculable partonic cross section, and a process independent non-perturbative part describing the hadrons involved. It can be expressed as

$$\sigma_{pp \rightarrow X} = \sum_{1,2} \int_0^1 dx_1 dx_2 f_1(x_1, \mu_F) f_2(x_2, \mu_F) \hat{\sigma}(x_1, x_2, s, \mu_F).$$

Here, $x_i$ denotes the fraction of the proton momentum carried by the parton taking part in the hard scatter and $\hat{\sigma}(x_1, x_2, s, \mu_F)$ is the partonic cross section for the process considered, with $s$ being the center-of-mass
energy squared. The functions \( f_i(x_i, \mu_F) \) are the parton distribution functions (PDF) describing the long distance behavior of the parton inside the proton. The sum runs over all pairs of partons that contribute to the process. In a leading-order picture, the PDF \( f_i(x_i, \mu_F) \) represents the probability to find parton \( i \) with momentum fraction \( x_i \) inside the incoming proton. The factorization scale \( \mu_F \) sets an arbitrary separation between soft and hard physics, and usually, \( \mu_F = Q \) is chosen. When calculated at all orders in perturbation theory, the dependence of the hadronic cross section on \( \mu_F \) has to vanish. At low orders, however, the dependence can be significant. The evolution of the PDFs from one scale to another is described by the celebrated DGLAP equations [79–81]. The MSTW PDFs at next-to-leading order are shown in Figure 2.13 for two different energy scales.

**MSTW 2008 NLO PDFs (68% C.L.)**

![Graph showing MSTW 2008 NLO PDFs at different energy scales.](image)

**Figure 2.13:** MSTW 2008 NLO PDFs at \( Q^2 = 10 \text{ GeV}^2 \) and \( Q^2 = 10^4 \text{ GeV}^2 \). Figures taken from Ref. [82].
We have seen in Chapter 2 that composite Higgs models provide an attractive mechanism to explain the lightness of the Higgs boson. In these models the Higgs boson arises as a composite state of some new, strongly interacting sector. The Higgs boson is not necessarily the only composite state of the new sector to be relatively light. We discussed that the mixing of the top quark with composite quarks could explain the large top mass. These composite quarks can give significant contributions to the electroweak precision observables, thus modifying the region of parameter space that is allowed for these models. In this chapter, we study the LHC discovery potential of the top partners of the two-multiplet composite Higgs model introduced in Section 2.3. This chapter is taken from work previously published in Ref. [59], where we limit ourselves to signatures accessible with early LHC data and thus focus on top partners lighter than 500 GeV.

3.1 Overview

The first glimpse of new physics at the LHC could well be due to new heavy quarks, which are a rather common feature of beyond the standard model (BSM) scenarios. The discovery potential of such heavy quarks has been studied in the context of little and littlest Higgs models [83, 84], warped extra dimensions [85, 86], fourth generation quarks [87, 88] and generic vector-like quarks in isospin singlets or doublets and with different hypercharge [89]. If new quarks are observed, we will need a way to understand which model they point at. For this reason, we focus on collider signatures that can be considered distinctive of the composite Higgs model under study. In particular, we look for configurations in which either two charge 5/3 quarks or a full, almost degenerate 4 of $SO(4)$ lie within the reach of the 2010 runs at the LHC. For these distinctive signatures, we discuss the phenomenology and study the discovery potential on the basis of 200 pb$^{-1}$ of collision data at $\sqrt{s} = 7$ TeV. We study the event yield with respect to the SM expectation in various
multi-lepton channels for different points in the parameter space. We outline a new method to reconstruct the mass of a charge 5/3 top partner exploiting its leptonic decay channel. We show that with only about 50 signal events in the same-sign dilepton final state, this method can be used to judge if the signal is mainly due to one charge 5/3 quark or rather produced by the contributions from multiple top partners.

3.2 Collider Signatures

3.2.1 Parameter-Space Scan

We scan over the parameter space of the two-multiplet composite Higgs model of Ref. [58] (see Section 2.3) in order to find regions compatible with electroweak precision data. From Eqns. (2.64)-(2.68) we see that there are 6 variables parametrizing the fermionic sector with one multiplet below the cutoff, and 11 for the case of two multiplets. In both cases, $s_\alpha$ is fixed through Eqn. (2.59), as we have $v = 174$ GeV and $f = 500$ GeV. We fix other two parameters in such a way to obtain the measured top and bottom masses

$$m_t = 172.4 \text{ GeV} \quad \text{and} \quad m_b = 4.2 \text{ GeV}.$$  \hspace{1cm} (3.1)

This is more easily done if we factor out of the mass Lagrangian (2.66) (or Eqn. (2.67) for the bottom quark) one of the parameters, say $M_1 (\lambda_b)$. Then we diagonalize the remaining part of the mass matrix and fix $M_1 (\lambda_b)$ so that the mass of the lightest quark is $m_t (m_b)$. We are left with eight free parameters in the case of the two-multiplet model. We require that the resulting quarks contribute to electroweak precision observables in such a way to make the model compatible with observations. We use the same fit as in Ref. [58] to assess the agreement between a point in parameter space and experimental constraints. Furthermore, we exploit the value of $\chi^2$ that parametrizes this comparison in order to drive our Vegas-based analysis [90]. The procedure is the following. We use Vegas to randomly sample on the eight-dimensional parameter space. For each point sampled, the value returned to Vegas as an ‘integrand’ is $1/\chi^2$. By construction, Vegas will focus its sampling on the points that lead to a higher value of the integrand $1/\chi^2$, i.e., to a better agreement with electroweak precision data; we retain points that are compatible at 99% C.L..

We further refine our search asking for signals which are characteristic of the two-multiplet model. As already mentioned, in the case of only one multiplet below the cutoff, the mass spectrum of the new resonances is typically rather spread out. The charge 5/3 quark has a mass of some few hundred GeV, while the charge -1/3 quark is very close to the cutoff. A signature of a two-multiplet model would then be a charge 5/3 quark in a 4 of $SO(4)$, i.e. very close in mass to two charge 2/3 and one charge -1/3 quarks. We require the mass difference among these particles to be $\lesssim 60$ GeV, so that decays through off-shell gauge bosons are strongly suppressed. Another typical signature of the model is the presence of both the charge 5/3 quarks. We take these two signatures as neat indications of this particular model and focus on their discovery potential with early LHC data. With 200 pb$^{-1}$ of collision data at 7 TeV, a significant number of
quarks with masses below \( \sim 500 \) GeV should be produced\(^1\). We will set this value as an upper bound in our search for the two distinctive patterns that we just discussed. As lower bounds, we set

\[
m_{5/3} > 365 \text{ GeV}, \quad m_{2/3}, m_{-1/3} > 260 \text{ GeV},
\]

where we use the most recent results from Tevatron searches on the exclusion of a charge 5/3 top partner [91]. For this quark, the only decay channel is \( tW^+ \), as in the reference. We do not use instead the most stringent exclusion bounds on the charge -1/3 and 2/3 quarks, [91, 92] and [93], as they assume the new quarks to decay entirely through W bosons or Z bosons. This is not the case in our model.

In summary, we define for the two-multiplet model two distinctive signatures in view of early LHC data:

- **XX signature**: both charge 5/3 quarks have a mass below 500 GeV,
- **4 of SO(4) signature**: the charge 5/3, two charge 2/3 and one charge -1/3 quarks have masses below 500 GeV and are degenerate within 60 GeV.

### 3.2.2 Phenomenology of the Two-Multiplet Model

We outline some of the basic features of the phenomenology that we expect from the two-multiplet model. This phenomenology is largely determined by the mass hierarchy of the 10 new quarks. The mass eigenstates (ordered according to increasing mass) of the new top-like quarks are named \( t_1, t_2, t_3, t_4, t_5 \) and \( t_6 \), whereas the charge 5/3 quarks and the bottom-like quarks will be denoted as \( x_1, x_2 \) and \( b_1, b_2 \), respectively. The Higgs boson mass is assumed to be 120 GeV.

The process \( gg \rightarrow q\bar{q} \) plays the dominant role in the production of heavy top partners at the LHC. Therefore, we analyze the decay chains that start from pair produced quarks. In Figures 3.1b and 3.1a we show two Feynman diagrams for a possible decay chain of pair-produced \( t_1 \) and \( x_1 \) top partners.

In all points of the parameter space that satisfy the selection criteria of Section 3.2.1, the two lightest new quarks are \( x_1 \) and \( t_1 \). The signatures from this model that could be observed early at the LHC will be therefore dominated by the decay modes of these two quarks. We also find that their mass difference is always too small for the heavier of the two to decay into the lighter. Consequently, only the following channels are accessible for the decay of the two lightest new quarks

\[
t_1 \rightarrow tZ, \quad t_1 \rightarrow t\bar{h}, \quad t_1 \rightarrow bW^+,
\]

\[
x_1 \rightarrow tW^+.
\]

The lightest bottom-like quark \( b_1 \), which we always find to be heavier than \( t_1 \) and \( x_1 \), decays predominantly via

\[
b_1 \rightarrow tW^-.
\]

\(^1\) For reasons that we will explain later, we focus on decay channels which produce at least two charged leptons in the final state. We estimate a leading order cross section of \( \sim 207.8 \) fb for pair production of a quark with a mass of 500 GeV. Taking into account the branching ratio for the W and Z bosons to decay leptonically, we cannot expect to observe more than a handful of events in the considered channels for an integrated luminosity of 200 pb\(^{-1}\).
3.2. Collider Signatures

(a) Feynman diagram for $x_1 \bar{x}_1$ pair-production.  
(b) Feynman diagram for $t_1 \bar{t}_1$ pair-production.

Figure 3.1: Feynman diagrams with example decay chains of pair-produced charge 5/3 and charge 2/3 top partners.

The other possible decays, $b_1 \rightarrow bZ$ and $b_1 \rightarrow bh$, are strongly suppressed because of the small off-diagonal couplings. Such small mixing is a consequence of the fact that the bottom quark is mainly elementary. We find that the decay $b_1 \rightarrow t_1 W^-$ is not kinematically accessible.

In case the $x_1$, $t_1$, $t_2$ and $b_1$ form a $4$ of $SO(4)$, with a maximal mass difference among the quarks of $\lesssim 60$ GeV, none of the new quarks can decay into another one; the decays through the $W$, $Z$ and $h$ bosons are not kinematically allowed. Consequently, all these four new quarks can only decay to the SM top and bottom quarks.

The phenomenology can be much richer in case of a $XX$ signature, in which both charge 5/3 quarks are below 500 GeV and no restriction on the maximal mass difference among the new quarks is imposed. However, the exclusion limits from the CDF experiment in combination with the upper bound of 500 GeV for early detection imposes strong restrictions on the cascade decays that are kinematically allowed. Often, the mass differences of these quarks are such that they only decay via the channels given in Eqns. (3.4) and (3.5). The two lightest quarks are $x_1$ and $t_1$, where either of the two can be the lighter one. Going up in mass, we find $b_1$ and $t_2$, or $t_2$ and $b_1$. The next heavier quark is either $x_2$ followed by $t_3$, or vice versa. The most common hierarchy is

$$m_{x_1} < m_{t_1} < m_{b_1} < m_{t_2} < m_{x_2}. \quad (3.6)$$

A rarer mass pattern is

$$m_{t_1} < m_{x_1} < m_{t_2} < m_{b_1} < m_{t_3} < m_{x_2} < m_{t_4}. \quad (3.7)$$

The quarks that do not appear in these relations have masses above 500 GeV. An example for a cascade decay accessible for various points is

$$t_2 \rightarrow x_1 W^- \rightarrow t W^+ W^- \rightarrow b W^+ W^+ W^-. \quad (3.8)$$

Both this cascade decay and the dominant decay modes from Eqns. (3.4) and (3.5) suggest that the model
can easily yield multi-lepton final states plus many jets. The SM is expected to produce only few events with such a signature.

### 3.3 Model Implementation in MadGraph and Categorization of Points

#### 3.3.1 Implementation

We implement the model in MadGraph 4 [94–96]. MadGraph 4 is a matrix-element based tree-level event generator that is capable of generating amplitudes and events for any given model describing high energy physics interactions. For such an event generator to be able to cope with a new physics model, the couplings and interactions of the new particles as defined in Section 2.3.3, in addition to the (modified) standard model interactions, have to be translated into a specific form. In MadGraph 4 these couplings are defined according to the convention from Helas [97]. The implementation of these couplings is done by means of the “usermod v1” framework. The decay widths and branching ratios of all unstable particles are calculated with BRIDGE [98].

We implement the model taking into account not only the couplings of the newly introduced particles, but also the changes in the standard model couplings arising from Higgs compositeness and from the mixing of the SM quarks with the new states.

#### 3.3.2 Benchmark Points in the Composite Higgs Model Parameter Space

Since the scan over the parameter space was optimized to search for points that satisfy the selection criteria of Section 3.2.1, the points returned are not necessarily very different from each other. For this reason, we arrange the points in groups that are expected to exhibit a similar phenomenology and focus on the representatives of these groups for a detailed study. We assign two points to the same group if all branching ratios of the new quarks with a mass below 500 GeV are of similar magnitude. When a group contains more than one point, we use the mass of the lightest new quark $m_{q, \text{low}}$ to select two representatives: the point with the lowest value of $m_{q, \text{low}}$ and the one with largest value of $m_{q, \text{low}}$. In the following, these two representatives will be referred to as low benchmark point (lBP) and high benchmark point (hBP) of a group. For the discussion of the discovery potential, we will restrict ourselves to the 30 benchmark points obtained in this way.

### 3.4 Event Generation and Detector Simulation

#### 3.4.1 Event Generation

For each benchmark point, we produce $10^5$ signal events with MadGraph 4. In particular, we generate events for pair production of all new quarks that have a mass below 500 GeV. The outcome of the MadGraph 4 event generation is a Les Houches event file [99], which we process with Pythia 6 [100] for the
showering and hadronization of the partonic events and for the simulation of the underlying event. Table 3.1 lists the mass of the lightest particle $m_{q,\text{low}}$ and the total leading order cross section for pair production of all considered quarks for each benchmark point.

**Table 3.1**: The 30 benchmark points of the two multiplet model with the mass of the lightest quark $m_{q,\text{low}}$ and the total cross section for pair production of all quarks below 500 GeV. The cross sections are calculated at leading order for $\sqrt{s} = 7$ TeV.

<table>
<thead>
<tr>
<th>BP</th>
<th>signature</th>
<th>$m_{q,\text{low}}$</th>
<th>cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[GeV]</td>
<td>[pb]</td>
</tr>
<tr>
<td>lBP 1</td>
<td>XX</td>
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</tr>
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</tr>
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</tr>
<tr>
<td>BP 17</td>
<td>XX, 4</td>
<td>x₁: 375.3</td>
<td>4.40</td>
</tr>
<tr>
<td>BP 18</td>
<td>XX, 4</td>
<td>x₁: 414.2</td>
<td>2.53</td>
</tr>
<tr>
<td>BP 19</td>
<td>XX</td>
<td>x₁: 438.3</td>
<td>1.66</td>
</tr>
<tr>
<td>BP 20</td>
<td>XX, 4</td>
<td>x₁: 414.2</td>
<td>2.53</td>
</tr>
</tbody>
</table>

As already mentioned in Section 3.2.2, we focus on final state signatures with at least two charged leptons and multiple jets. Consequently, every SM process that can lead to such final states represents a possible background. Table 3.2 lists the leading order cross section and the number of generated events for all relevant background processes. Note that single top production was neglected for this study. Its contribution is expected to be within the uncertainty of the pair production cross section. In order to estimate correctly the momentum spectrum of the jets in the transverse plane of the detector, we generate all partonic multiplicities needed for the SM backgrounds in MADGRAPH 4 and use PYTHIA 6 for the parton shower. The overlap between the phase-space description of the matrix-element calculator and the parton shower is removed using the MLM parton-jet matching prescription [101]. For the signal samples, the jets produced by the parton shower in the decay of very heavy particles are known to be satisfactory [102]. The underlying event is simulated with PYTHIA 6. For all samples, we use the CTEQ6 [103] parton distribution functions.

We would like to point out that the samples for the background processes were generated within the SM. We did not take into account the changes of the SM couplings introduced in the composite Higgs
Table 3.2: Background processes with the corresponding cross sections and the number of generated events. The divector boson sample VV+jets includes all processes with two W or Z bosons, except for the case of two W bosons with the same charge. In the first three samples, the vector bosons are forced to decay leptonically. The (*) indicates that we used the MADGRAPH 4 Les Houches events made available from the LCG Monte-Carlo Data Base [104] instead of generating the events ourselves.

<table>
<thead>
<tr>
<th>process</th>
<th>cross section [pb]</th>
<th>number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z + ≤ 3 j</td>
<td>2400</td>
<td>1302405*</td>
</tr>
<tr>
<td>W + ≤ 3 j</td>
<td>24170</td>
<td>12270142*</td>
</tr>
<tr>
<td>VV + ≤ 1 j</td>
<td>4.8</td>
<td>113764*</td>
</tr>
<tr>
<td>W±W± +2 j</td>
<td>0.2119</td>
<td>47070</td>
</tr>
<tr>
<td>W+W−W±</td>
<td>0.04105</td>
<td>49999</td>
</tr>
<tr>
<td>tância τ + ≤ 3 j</td>
<td>95</td>
<td>1395630*</td>
</tr>
<tr>
<td>tância τ+W± + ≤ 1 j</td>
<td>0.1687</td>
<td>66266</td>
</tr>
<tr>
<td>tância τ+Z</td>
<td>0.1038</td>
<td>49999</td>
</tr>
</tbody>
</table>

model. These modifications differ for each point in the parameter space of the model, but we expect the resulting effects on the SM backgrounds to be small. Also note that we only consider pair production of the new quarks for the signal samples. We neglect the contributions of other processes (such as single quark production) to the signal yield in multi-lepton final states. These additional contributions to the signal would enhance the excess over the SM expectation.

3.4.2 Detector Simulation

We use DELPHES [105] for the simulation of the response of a typical LHC detector. DELPHES is a recently developed simulation framework for a generic collider experiment. As CMS is one of the two general purpose detectors at the LHC, we use the CMS detector card for the DELPHES detector simulation. We reconstruct the jets with the anti-kt jet-clustering algorithm [106] and use a cone radius \( \Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} = 0.5 \). \( \phi \) denotes the azimuthal angle and the pseudorapidity \( \eta \) is defined as \( \eta = -\ln \tan \frac{\theta}{2} \), where \( \theta \) is the angle between the beam pipe and the trajectory of the particle. To adapt the performance of DELPHES to our needs, we make the following modifications.

- In DELPHES, the possibility of a jet being reconstructed as an electron is not taken into account. This, however, is expected to be a relevant source of fake electrons. In Ref. [107], the probability for a jet to be reconstructed as an isolated, identified electron is estimated to be at a level of \( 6 \cdot 10^{-6} \). We use this result and add jets to the isolated electron collection with the stated global probability.

- We set the global tracking efficiency to 100% for tracks with a transverse momentum of at least 0.9 GeV, but remove electrons from the electron collection with a probability of 10%.
3.4.3 Lepton and Jet Identification

We outline a robust and simple event selection that is suitable for early data from the LHC.

For the electrons and the muons, we demand a transverse momentum $p_T > 20$ GeV and a pseudorapidity $|\eta| < 2.4$. The first criterion ensures a robust identification of electrons and muons, both offline and on trigger level, whereas the second is made to restrict the leptons to the volume of the tracker. For this study, we are interested in prompt leptons coming from vector boson decays. To discriminate against leptons coming from semileptonic hadron decays, we apply a relative isolation criterion. In particular, we sum the $p_T$ of the tracks in a cone with $\Delta R < 0.3$ around the electron or muon and require this value to be smaller than 5% of the lepton momentum.

To obtain a robust jet selection, we demand the $p_T$ of a jet be larger than 50 GeV and require $|\eta| < 3$. The conservative choice of $p_T > 50$ GeV is made to minimize the contribution of fake jets. The second criterion marks the end of the electromagnetic and hadronic calorimeters. As electrons may be reconstructed as possible jet candidates, we reject those jets that are matched within $\Delta R < 0.2$ to an isolated electron. The jet collection can be further cleaned from such electrons by requiring that the jets should have an electromagnetic fraction (electromagnetic over hadronic energy deposits) of less than 0.98.

A more stringent lepton identification and isolation criterion would enhance the purity of the selection but cause the efficiency to decrease. The goal is to achieve a pure selection of prompt leptons without losing too much efficiency. For the $t\bar{t}$ sample from Table 3.2, we find an efficiency of 83% and a purity of 97% for the electrons. For the muons, we obtain an efficiency of 91% and a purity of 99%.

3.5 Discovery Potential at the LHC

3.5.1 Identification of Promising Channels

After applying the lepton and jet selection defined in Section 3.4.3, we investigate the number of events for a given jet multiplicity and lepton configuration for each background and signal process. The lepton configurations go from dilepton events — same-sign (SS) or opposite-sign (OS) — to events with up to five charged leptons in the final state. Each configuration, which is characterized by a certain lepton combination and jet multiplicity, is interpreted as a specific signal region with an associated selection efficiency. Since these efficiencies are based on a finite statistics of simulated events, we observe certain configurations with non-vanishing signal but zero background events. To avoid this issue, we calculate for all configurations lower and upper limits for the selection efficiency for the signals and backgrounds respectively, based on a 95% confidence level. This can be interpreted as a worst case scenario for the discovery of the model. The expected number of signal and background events are obtained by multiplying these efficiencies by the integrated luminosity of $200 \text{ pb}^{-1}$ times the corresponding cross section (as listed in Tables 3.1 and 3.2).

In Figures 3.2a and 3.2b we plot the jet multiplicity versus the lepton configuration for the total SM background and for the signal for BP 10, respectively. We denote by SS the configurations in which all
(a) Upper limit for the total number of background events.

(b) Lower limit for the expected number of signal events for BP 10.

(c) The number of expected signal events divided by the total number of background events.

**Figure 3.2:** The upper and lower limit for the total number of background events and the total number of signal events for BP 10, respectively, based on a 95% confidence level. The ratio plot obtained by dividing bin-by-bin the expected signal yield by the number of background events is shown in Figure 3.2c. The y-axis shows the jet multiplicity, whereas the lepton configuration is given on the x-axis.
the leptons have the same charge. Configurations in which at least one lepton has a different charge are denoted by OS. In the OS dilepton case, we also distinguish between the opposite-flavor (OF) and same-flavor (SF) configurations. For the bins in Figure 3.2a for which zero background events were found, we calculate an upper limit of 2.13 background events with a confidence of 95%. This number is dominated by the contributions from W+jets and Z+jets due to their large cross sections and the limited number of simulated events. In Figure 3.2c, we plot the number of signal events for BP 10 divided by the total number of background events. Using the signal-to-background ratio as a figure of merit, we conclude that the final states with SS dileptons and OS trileptons are the most promising channels for a possible discovery with 200 pb$^{-1}$ of collision data. This observation holds for all 30 benchmark points. The decrease in the plotted S/B ratio for large jet multiplicities is not expected in collision data, but due to the finite number of simulated events and the calculation of an upper limit on the number of background events. In the light of the above discussion, we will focus on the four channels: SS dilepton with 3 or 4 jets and OS trilepton with 2 or 3 jets.

3.5.2 Inclusive Discovery Potential

In order to quantify the discovery reach in the four channels defined above, we calculate the probability for the expected signal-plus-background observation to be caused by a fluctuation in the background distribution. We use $2 \log X$ as a test statistic, where $X$ is the ratio of the likelihood function for the signal-plus-background hypothesis $H_1$ to the likelihood function for the background hypothesis $H_0$ [108, 109].

The likelihood ratio $X_i$ for the channel $i$ can be defined as

$$X_i = \frac{L_{H_1,i}}{L_{H_0,i}},$$

where

$$L_{H_1,i} = \frac{e^{-(s_i+b_i)}(s_i+b_i)^{d_i}}{d_i!}, \quad L_{H_0,i} = \frac{e^{-b_i}(b_i)^{d_i}}{d_i!}.$$  (3.10)

Here, $s_i$ and $b_i$ denote the number of signal and background events, respectively, and $d_i$ is the number of observed candidates. Since the statistic $2 \log X$ for the outcome of multiple channels is the sum of the test statistics of the channels separately, we use $q = 2 \sum_{i=1}^{4} \log X_i$ for the combined four channels. We define the confidence level as

$$\text{CL}_{b} = P_b(q < q_{\text{obs}}|H_0),$$  (3.11)

where the probability sum assumes the presence of the background only. Note that the background confidence $1 - \text{CL}_{b}$ expresses the compatibility of the observation with the background hypothesis, since $\text{CL}_{b}$ is the probability that the background processes would give fewer than or equal to the number of events observed. For this reason, we use $\text{CL}_{b}$ to quantify the discovery potential. The background confidence $1 - \text{CL}_{b}$ can be compared with the widely used notion of standard deviations ($\sigma$) by using the convention from Ref. [24]. A 3$\sigma$ and 5$\sigma$ excess beyond the background expectation then corresponds to a one-sided background confidence level of $1 - \text{CL}_{b} = 1.35 \cdot 10^{-3}$ and $1 - \text{CL}_{b} = 2.87 \cdot 10^{-7}$, respectively.

The distributions of the test statistic for $H_0$ and $H_1$, often referred to as the test statistic probability density function (tPDF), are obtained by throwing Poisson numbers around $s_i + b_i$ and $b_i$ as a replacement
for $d_i$. The confidence level $\text{CL}_{b}$, and its uncertainty is calculated as follows. In the presence of data, $\text{CL}_{b}$ is given by the integral of the tPDF of the background hypothesis from $-\infty$ to the measured value of the test statistic $q_{\text{obs}}$. For this study, we replace this value by the mean of the tPDF for the signal-plus-background hypothesis to substitute collision data. The uncertainty on $\text{CL}_{b}$ is then obtained by changing the integration limit to the mean plus/minus one standard deviation of the signal-plus-background tPDF. To claim a $5\sigma$ excess over the background expectation, we have to be sensitive to $\text{CL}_{b}$ at the order of $10^{-7}$. For this reason, we generate $10^9$ pseudo-experiments for each of the tPDFs for $H_0$ and $H_1$.

Table 3.3 lists the number of expected events from the SM backgrounds in the four channels considered. The corresponding numbers for the 30 signal benchmark points are given in Table 3.4. As indicated in Table 3.4, we expect a signal evidence of at least 3 $\sigma$ for 23 benchmark points. For 10 points among these 23, the central $\text{CL}_{b}$ value corresponds to an excess over the SM expectation of at least 5 $\sigma$.

Table 3.3: The upper limit on the number of expected events for 200 pb$^{-1}$ of data for each of the background processes at a confidence level of 95%. Systematic uncertainties on cross sections are not taken into account.

<table>
<thead>
<tr>
<th>process</th>
<th>2l SS + 3j</th>
<th>2l SS + 4j</th>
<th>3l OS + 2j</th>
<th>3l OS + 3j</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z + \text{jets}$</td>
<td>1.7</td>
<td>1.0</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>$W + \text{jets}$</td>
<td>1.8</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>$V V + \text{jets}$</td>
<td>0.075</td>
<td>0.023</td>
<td>0.49</td>
<td>0.12</td>
</tr>
<tr>
<td>$W^\pm W^\pm jj$</td>
<td>0.099</td>
<td>0.019</td>
<td>0.0024</td>
<td>0.0024</td>
</tr>
<tr>
<td>$W^+ W^- W^\pm$</td>
<td>0.0035</td>
<td>0.00044</td>
<td>0.0012</td>
<td>0.00044</td>
</tr>
<tr>
<td>$t \bar{t} + \text{jets}$</td>
<td>2.1</td>
<td>0.83</td>
<td>0.52</td>
<td>0.25</td>
</tr>
<tr>
<td>$t \bar{t} W^\pm j$</td>
<td>0.19</td>
<td>0.075</td>
<td>0.052</td>
<td>0.016</td>
</tr>
<tr>
<td>$t \bar{t} Z^\pm$</td>
<td>0.036</td>
<td>0.011</td>
<td>0.085</td>
<td>0.063</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>5.90</strong></td>
<td><strong>3.02</strong></td>
<td><strong>3.86</strong></td>
<td><strong>3.16</strong></td>
</tr>
</tbody>
</table>

3.5.3 Discovery Potential of Two Benchmark Points

We now focus on the discovery potential of two promising benchmark points. One is BP 10, which has both $x_1$ and $x_2$ below 500 GeV; the other is BP 18. Both benchmark points exhibit a 4 of $SO(4)$ and have large cross sections (10.78 pb and 5.52 pb respectively), yielding a relevant excess over the SM background. We use a simple cut-based analysis and outline some features of their specific phenomenology.
Table 3.4: Number of expected events in each of the four channels for 200 pb\(^{-1}\) of integrated luminosity.
The background confidence level \(1 - \text{CL}_{b}\) with its uncertainty is also given. The \(1 - \text{CL}_{b}\) values marked with (*) correspond to benchmark points for which more than \(10^{9}\) pseudoexperiments would be needed for the tail of the tPDF of the background hypothesis to leak out of the integrated region.

<table>
<thead>
<tr>
<th>BP</th>
<th>2l SS + 3j</th>
<th>2l SS + 4j</th>
<th>3l OS + 2j</th>
<th>3l OS + 3j</th>
<th>(1 - \text{CL}_{b})</th>
</tr>
</thead>
<tbody>
<tr>
<td>lBP 1</td>
<td>7.22</td>
<td>5.82</td>
<td>4.45</td>
<td>4.12</td>
<td>(1.3 \times 10^{-6} \pm 2.2 \times 10^{-4} )</td>
</tr>
<tr>
<td>hBP 1</td>
<td>3.76</td>
<td>3.40</td>
<td>2.35</td>
<td>2.53</td>
<td>(2.2 \times 10^{-3} \pm 4.1 \times 10^{-2} )</td>
</tr>
<tr>
<td>lBP 2</td>
<td>6.70</td>
<td>5.59</td>
<td>4.13</td>
<td>3.42</td>
<td>(5.9 \times 10^{-6} \pm 2.1 \times 10^{-3} )</td>
</tr>
<tr>
<td>hBP 2</td>
<td>4.64</td>
<td>4.24</td>
<td>2.68</td>
<td>2.71</td>
<td>(4.8 \times 10^{-4} \pm 1.5 \times 10^{-2} )</td>
</tr>
<tr>
<td>lBP 3</td>
<td>12.70</td>
<td>9.67</td>
<td>8.49</td>
<td>7.27</td>
<td>* (1 \times 10^{-9} )</td>
</tr>
<tr>
<td>hBP 3</td>
<td>6.61</td>
<td>5.87</td>
<td>4.22</td>
<td>4.18</td>
<td>(2.4 \times 10^{-6} \pm 3.4 \times 10^{-4} )</td>
</tr>
<tr>
<td>lBP 4</td>
<td>8.10</td>
<td>7.01</td>
<td>4.65</td>
<td>4.38</td>
<td>(7.9 \times 10^{-8} \pm 5.2 \times 10^{-5} )</td>
</tr>
<tr>
<td>hBP 4</td>
<td>2.94</td>
<td>2.85</td>
<td>1.70</td>
<td>1.78</td>
<td>(1.1 \times 10^{-2} \pm 1.2 \times 10^{-1} )</td>
</tr>
<tr>
<td>BP 5</td>
<td>6.97</td>
<td>5.65</td>
<td>3.93</td>
<td>3.80</td>
<td>(3.7 \times 10^{-6} \pm 4.9 \times 10^{-4} )</td>
</tr>
<tr>
<td>BP 6</td>
<td>2.91</td>
<td>2.59</td>
<td>1.73</td>
<td>1.78</td>
<td>(1.4 \times 10^{-2} \pm 1.2 \times 10^{-1} )</td>
</tr>
<tr>
<td>BP 7</td>
<td>3.06</td>
<td>3.03</td>
<td>1.73</td>
<td>1.81</td>
<td>(9.1 \times 10^{-3} \pm 1.0 \times 10^{-1} )</td>
</tr>
<tr>
<td>BP 8</td>
<td>2.98</td>
<td>2.81</td>
<td>1.50</td>
<td>1.52</td>
<td>(1.4 \times 10^{-2} \pm 1.3 \times 10^{-1} )</td>
</tr>
<tr>
<td>BP 9</td>
<td>7.19</td>
<td>5.66</td>
<td>3.81</td>
<td>3.68</td>
<td>(3.8 \times 10^{-6} \pm 4.9 \times 10^{-4} )</td>
</tr>
<tr>
<td>BP 10</td>
<td>14.62</td>
<td>10.87</td>
<td>9.14</td>
<td>7.59</td>
<td>* (1 \times 10^{-9} )</td>
</tr>
<tr>
<td>lBP 11</td>
<td>10.37</td>
<td>8.63</td>
<td>5.56</td>
<td>5.40</td>
<td>* (1 \times 10^{-9} \pm 2.7 \times 10^{-7} )</td>
</tr>
<tr>
<td>hBP 11</td>
<td>5.70</td>
<td>5.08</td>
<td>3.32</td>
<td>3.63</td>
<td>(3.4 \times 10^{-5} \pm 2.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>lBP 12</td>
<td>7.87</td>
<td>5.87</td>
<td>4.20</td>
<td>3.53</td>
<td>(1.5 \times 10^{-6} \pm 2.3 \times 10^{-4} )</td>
</tr>
<tr>
<td>hBP 12</td>
<td>8.42</td>
<td>7.30</td>
<td>4.86</td>
<td>4.72</td>
<td>(3.7 \times 10^{-8} \pm 1.3 \times 10^{-5} )</td>
</tr>
<tr>
<td>BP 13</td>
<td>12.98</td>
<td>10.16</td>
<td>8.74</td>
<td>7.38</td>
<td>* (1 \times 10^{-9} )</td>
</tr>
<tr>
<td>BP 14</td>
<td>10.44</td>
<td>8.83</td>
<td>6.59</td>
<td>6.74</td>
<td>* (1 \times 10^{-9} \pm 2.2 \times 10^{-8} )</td>
</tr>
<tr>
<td>lBP 15</td>
<td>7.99</td>
<td>6.11</td>
<td>4.12</td>
<td>3.83</td>
<td>(8.4 \times 10^{-7} \pm 1.5 \times 10^{-4} )</td>
</tr>
<tr>
<td>hBP 15</td>
<td>6.89</td>
<td>5.71</td>
<td>4.24</td>
<td>3.99</td>
<td>(2.5 \times 10^{-6} \pm 3.6 \times 10^{-4} )</td>
</tr>
<tr>
<td>lBP 16</td>
<td>8.57</td>
<td>6.87</td>
<td>5.74</td>
<td>4.86</td>
<td>(2.3 \times 10^{-8} \pm 2.0 \times 10^{-6} )</td>
</tr>
<tr>
<td>hBP 16</td>
<td>2.32</td>
<td>2.21</td>
<td>1.42</td>
<td>1.34</td>
<td>(3.2 \times 10^{-2} \pm 3.0 \times 10^{-1} )</td>
</tr>
<tr>
<td>lBP 17</td>
<td>9.15</td>
<td>6.72</td>
<td>5.44</td>
<td>4.83</td>
<td>(2.0 \times 10^{-8} \pm 8.2 \times 10^{-6} )</td>
</tr>
<tr>
<td>hBP 17</td>
<td>2.43</td>
<td>2.17</td>
<td>1.35</td>
<td>1.40</td>
<td>(3.2 \times 10^{-2} \pm 2.0 \times 10^{-1} )</td>
</tr>
<tr>
<td>lBP 18</td>
<td>10.03</td>
<td>7.48</td>
<td>6.53</td>
<td>5.09</td>
<td>* (1 \times 10^{-9} \pm 6.8 \times 10^{-7} )</td>
</tr>
<tr>
<td>hBP 18</td>
<td>4.42</td>
<td>3.77</td>
<td>2.67</td>
<td>2.44</td>
<td>(9.7 \times 10^{-4} \pm 2.5 \times 10^{-2} )</td>
</tr>
<tr>
<td>BP 19</td>
<td>7.31</td>
<td>5.42</td>
<td>4.09</td>
<td>3.83</td>
<td>(3.2 \times 10^{-6} \pm 4.2 \times 10^{-4} )</td>
</tr>
<tr>
<td>BP 20</td>
<td>6.49</td>
<td>5.06</td>
<td>3.79</td>
<td>3.75</td>
<td>(1.3 \times 10^{-5} \pm 1.1 \times 10^{-3} )</td>
</tr>
</tbody>
</table>
Phenomenology of the Two Benchmark Points

As we can see from Table 3.1, the lightest new quark for BP 10 is the top-like $t_1$ with a mass of 316.6 GeV. The full mass hierarchy for the new quarks with a mass below 500 GeV reads

$$m_{t_1} (316.6) < m_{b_1} (365.2) < m_{t_2} (374.4) < m_{b_1} (377.3) \quad (3.12)$$
$$< m_{t_3} (377.9) < m_{b_2} (395.3) < m_{t_4} (473.3), \quad (3.13)$$

where the masses are given in GeV. For this point, the mass difference between the $t_4$ quark and the other quarks is such as to allow $t_4$ to decay into most of them. The full list of branching ratios for all the above listed quarks can be seen in Table 3.5.

Table 3.5: The branching ratios for the seven quarks with a mass below 500 GeV for BP 10. Note that the branching ratios may not add up to 1 as possible three-body decays might contribute.

<table>
<thead>
<tr>
<th>BP 10</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($t_1 \rightarrow bW^+$):</td>
<td>2.74 · 10^{-1}</td>
<td>BR($t_2 \rightarrow bW^+$):</td>
<td>5.47 · 10^{-5}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($t_2 \rightarrow tZ$):</td>
<td>3.80 · 10^{-1}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($t_2 \rightarrow th$):</td>
<td>6.20 · 10^{-1}</td>
</tr>
<tr>
<td></td>
<td>BR($t_4 \rightarrow bW^+$):</td>
<td>6.89 · 10^{-2}</td>
<td>BR($t_4 \rightarrow tZ$):</td>
</tr>
<tr>
<td></td>
<td>1.17 · 10^{-2}</td>
<td>BR($t_4 \rightarrow t_1 Z$):</td>
<td>7.03 · 10^{-2}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($t_4 \rightarrow b_1 W^+$):</td>
<td>3.01 · 10^{-2}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($t_4 \rightarrow t_2 Z$):</td>
<td>1.38 · 10^{-3}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($b_1 \rightarrow tW^-$):</td>
<td>9.96 · 10^{-1}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($b_1 \rightarrow bZ$):</td>
<td>3.78 · 10^{-5}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($b_1 \rightarrow th$):</td>
<td>2.29 · 10^{-5}</td>
</tr>
<tr>
<td>BR($x_1 \rightarrow tW^+$):</td>
<td>1.00</td>
<td>BR($x_2 \rightarrow tW^+$):</td>
<td>9.97 · 10^{-1}</td>
</tr>
</tbody>
</table>

For lBP 18, the mass hierarchy is

$$m_{b_1} (365.2) < m_{t_1} (367.6) < m_{b_1} (373.9) < m_{t_2} (403.1). \quad (3.14)$$

Given that the maximal mass difference among these quarks is about 40 GeV, their decay modes are described by Eqns. (3.4) and (3.5). In Table 3.6 we list the branching ratios corresponding to these decay modes.

Table 3.6: The branching ratios for the four quarks with a mass below 500 GeV for lBP 18.

<table>
<thead>
<tr>
<th>lBP 18</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($t_1 \rightarrow bW^+$):</td>
<td>1.56 · 10^{-2}</td>
<td>BR($t_2 \rightarrow bW^+$):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($t_2 \rightarrow tZ$):</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BR($t_2 \rightarrow th$):</td>
</tr>
<tr>
<td></td>
<td>BR($x_1 \rightarrow tW^+$):</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Cut-Based Analysis

We outline a simple, cut-based analysis for the two benchmark points to illustrate a complementary way to investigate the discovery potential of the model. For this analysis, we use the lepton and jet selections defined in Section 3.4.3. Given the results from Tables 3.3 and 3.4, we require as preselection criteria at least two same-sign (isolated) leptons (e, µ) with $p_T > 20\text{ GeV}$ and $|\eta| < 2.4$.

(a) Jet multiplicity for the two benchmark points and the sum of the SM backgrounds.

(b) $H_T$ distribution for the two benchmark points and the sum of the SM backgrounds.

(c) The $p_T$ of the leading jet after preselection for BP 10 (red), lBP 18 (blue) and the total SM background (grey).

Figure 3.3: The jet multiplicity and $H_T$ distributions as well as the distribution of the leading jet $p_T$ for the signal benchmark points BP 10 and lBP 18 and for the SM background after applying the preselection criteria. The distributions are scaled to $200\text{ pb}^{-1}$. 
Figure 3.3a shows the jet multiplicity distributions for BP 10, lBP 18 and the SM background after preselection for an integrated luminosity of 200 pb$^{-1}$. Based on these distributions we require the presence of at least two jets with $p_T > 50$ GeV. As a next step, we make use of the variable $H_T$, which is defined as the scalar sum of the transverse momentum of the selected jets and leptons. In Figure 3.3b we show the overlaid $H_T$ distributions for BP 10 (red), lBP 18 (blue) and the SM background (grey). The distributions were obtained only after imposing the preselection criteria. Clearly, this variable can be used as a powerful criterion to suppress the background contribution. For this reason, we impose a $H_T > 300$ GeV requirement on the selected events. From Figure 3.3c, we see that the leading jet tends to be harder for the two signal benchmark points with respect to the SM backgrounds. We thus impose a $p_T > 90$ GeV requirement on the leading jet. In summary, we impose the following selection criteria:

1. at least 2 jets with $p_T > 50$ GeV;
2. $H_T > 300$ GeV;
3. $p_T$ of the leading jet $> 90$ GeV.

**Table 3.7:** The efficiency of the preselection criteria, the total selection efficiency and number of expected events for each background and the two signal samples for an integrated luminosity of 200 pb$^{-1}$. The stated uncertainty on the number of expected events corresponds to the 68.3% confidence interval of this number. The total background sums up to 6.7 events.

<table>
<thead>
<tr>
<th>sample</th>
<th>preselection eff.</th>
<th>total selection eff.</th>
<th>expected # of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z + jets</td>
<td>$7.22 \times 10^{-5}$</td>
<td>$1.54 \times 10^{-6}$</td>
<td>$0.74^{+0.74}_{-0.37}$</td>
</tr>
<tr>
<td>W + jets</td>
<td>$2.97 \times 10^{-5}$</td>
<td>$1.63 \times 10^{-7}$</td>
<td>$0.79^{+0.79}_{-0.39}$</td>
</tr>
<tr>
<td>VV + jets</td>
<td>$2.19 \times 10^{-2}$</td>
<td>$6.15 \times 10^{-4}$</td>
<td>$0.59^{+0.07}_{-0.07}$</td>
</tr>
<tr>
<td>W$^\pm$W$^\pm$jj</td>
<td>$2.32 \times 10^{-2}$</td>
<td>$9.92 \times 10^{-3}$</td>
<td>$0.42^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>W$^+$W$^-$W$^\pm$</td>
<td>$2.24 \times 10^{-2}$</td>
<td>$1.26 \times 10^{-3}$</td>
<td>$0.010^{+0.001}_{-0.001}$</td>
</tr>
<tr>
<td>tt + jets</td>
<td>$8.67 \times 10^{-4}$</td>
<td>$1.89 \times 10^{-4}$</td>
<td>$3.6^{+0.2}_{-0.2}$</td>
</tr>
<tr>
<td>ttW$^\pm$ j</td>
<td>$2.44 \times 10^{-2}$</td>
<td>$1.11 \times 10^{-2}$</td>
<td>$0.37^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>ttZ$^\pm$</td>
<td>$1.67 \times 10^{-2}$</td>
<td>$8.24 \times 10^{-3}$</td>
<td>$0.17^{+0.01}_{-0.01}$</td>
</tr>
</tbody>
</table>

| BP 10          | $3.79 \times 10^{-2}$ | $2.87 \times 10^{-2}$ | $61.8^{+1.1}_{-1.1}$ |
| IBP 18         | $4.97 \times 10^{-2}$ | $3.75 \times 10^{-2}$ | $41.4^{+0.7}_{-0.7}$ |

In Table 3.7 we list the efficiencies of the preselection criteria and the total selection efficiency for the two signal samples and for each of the background processes. The efficiencies of the individual selection requirements have been studied individually. We also list the number of expected events after the full selection requirements have been applied. We find that for an integrated luminosity of 200 pb$^{-1}$ we can expect 62 and 41 events for BP 10 and IBP 18, respectively, but only a total of 6.7 events arising from the SM backgrounds. To estimate how much integrated luminosity we need to obtain a 5 $\sigma$ excess over
the SM expectation, we adapt the integrated luminosity in the calculation of the background confidence level $1 - \text{CL}_{b}$ until we reach a $5\sigma$ probability of $2.87 \cdot 10^{-7}$. For IBP 18 we find a $5\sigma$ significance for an integrated luminosity of $46^{+25}_{-22}\text{pb}^{-1}$ with an expected number of $9.52^{+5.17}_{-4.55}$ signal and $1.54^{+0.84}_{-0.74}$ background events. For BP 10 we find that a $5\sigma$ excess is expected for an integrated luminosity of $24^{+16}_{-12}\text{pb}^{-1}$, which corresponds to $7.42^{+4.94}_{-3.71}$ signal and $0.80^{+0.55}_{-0.40}$ background events. The central values and uncertainties are obtained in the same way as in Section 3.5.2. Systematic errors are not taken into account. These results show that a discovery of this model may already be feasible at the LHC with only a few dozen inverse picobarns of understood collision data.

3.6 Reconstructing the Mass of a Charge 5/3 Top Partner

Among the top partners, the charge 5/3 $x_1$ gives the largest contribution to the excess over the SM expectation in the SS dilepton channel. This is due to its low mass and the fact that it always decays to $tW^+$, which leads to

$$x_1 \rightarrow tW^+ \rightarrow bW^+W^+ \rightarrow b\ell^+\ell^+\nu\nu\ell$$

(3.15)

in the leptonic decay mode. For the $t_1$ quark, which is the only new quark that could be lighter than $x_1$, only few of its decay modes (see e.g. Eqn. (3.4)) produce SS dileptons in the final state. The accurate mass reconstruction of a charge 5/3 quark would be a big step towards the interpretation of the discovery. In the literature, different methods have been proposed for the reconstruction of its mass. These methods usually focus on pair production, so that they can exploit same-sign dileptons from the decay of one of the charge 5/3 quarks to select and identify the event. The mass is reconstructed using the fully hadronic decay mode of the two $W$ bosons coming from the other charge 5/3 quark [75, 89, 110]. In Ref. [111] an alternative method is presented. The mass of a charge 5/3 top partner is reconstructed in SS dilepton events via its transverse mass. This transverse mass is computed from the momenta of the two SS leptons, the missing transverse energy (from the two neutrinos) and the b jet belonging to the semileptonically (and not to the second, hadronically) decaying top quark.

In the following, we outline a new method to reconstruct the mass of a charge 5/3 quark $x_1$. We exploit the same channel as Ref. [111], but we only rely on the two SS leptons and use the shape of their invariant mass distribution to reconstruct $x_1$. This avoids b-tagging inefficiencies and the problem of assigning the correct b jet to the corresponding $x_1$ decay. We also consider the situation in which an excess of about 50 SS dilepton events (as expected for 200 pb$^{-1}$ of collision data) is caused by the presence of multiple top partners. In this case, we show how the method can be used to discriminate the signal against a hypothesized presence of $x_1$ only.
3.6.1 Mass Determination with 200 pb$^{-1}$ of Collision Data

The Method

In the decay of a pair-produced $x_1 \bar{x}_1$, the SS dileptons come from the same decay leg and the positively (negatively) charged leptons can be assigned to the decay of the $x_1$ quark ($\bar{x}_1$ quark). The invariant mass distribution of the SS dileptons contains information about the $x_1$ quark mass. In fact, the endpoint of this invariant mass distribution $m_{\ell\ell}^{\text{max}}$ is sufficient to determine $m_{x_1}$, since $m_{\ell\ell}^{\text{max}}$ can be expressed in terms of the masses of the particles involved in the decay (3.15). The mass of $x_1$ is the only unknown parameter in this relation. An accurate measurement of this endpoint, however, is not possible with only 200 pb$^{-1}$ of collision data. We can use, instead, the shape of the invariant mass distribution to determine $m_{x_1}$.

In Ref. [112], an analytic expression for the shape of the invariant mass distribution $M_{\ell c}$ for the supersymmetric decay $\tilde{g} \rightarrow t \tilde{t}_1$, $t_1 \rightarrow c \tilde{\chi}_1^0$ is presented$^2$. As the kinematic configuration of this decay is identical to Eqn. (3.15), we can use their results to model the shape of the invariant mass distribution of the SS dileptons from leptonic $x_1$ ($\bar{x}_1$) quark decays. This shape function, however, does not take into account the possibility of a leptonically decaying tau-lepton originating from a W boson decay. Also, an inclusive electron and muon spectrum without imposing kinematic selection criteria was assumed. These two assumptions are not satisfied in our realistic analysis. A fit of the full invariant mass distribution does therefore not lead to an accurate estimation of $m_{x_1}$. However, we find the shape of the tail of the distribution to be almost invariant under the effect of the selection criteria and the tau contribution$^3$. A fit of the tail of the distribution is thus a powerful means to extract the mass of $x_1$.

In Figure 3.4 we show the invariant mass distribution of the SS dileptons from a pair-produced $x_1$ quark with a mass of 365 GeV. This is the $x_1$ quark mass for BP 10 and IBP 18. We apply the same selection as in Section 3.5.3. Both the signal and the SM background (see Table 3.7) are normalized to an integrated luminosity of 200 pb$^{-1}$. With a leading order cross section of 1.64 pb for the signal, we estimate $15.3 \pm 0.2$ SS dilepton events due to the $x_1 \bar{x}_1$ decay. When fitting the tail of the total distribution from the signal plus the SM backgrounds with the shape function (starting from the peak of the distribution), we obtain a fitted mass $m_{\text{fit}}$ of $370.0 \pm 32.3$ GeV. By rescaling the generated distribution with the signal cross section, we underestimate the statistical fluctuations in the number of events per bin. The statistical uncertainty of about 32 GeV on the fitted mass, however, correctly represents the precision expected with about 15 signal events. We conclude that fitting the tail of the invariant mass distribution of the signal plus the SM background leads to a fairly accurate estimate of the $x_1$ quark mass$^4$.

The above method assumes the total production cross section of the charge 5/3 top partners to be dominated by pair-production. Neglecting the contribution of single quark production allows us to estimate the production cross section as a function of the quark mass. This neglected contribution affects neither the shape nor the endpoint, but changes the absolute normalization of the invariant mass distribution of the SS

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$^2$Spin effects were neglected in the calculation of the shape of the invariant mass distribution.
$^3$The systematic error introduced is $< 3\%$.
$^4$When only fitting the signal without the SM background, we find $m_{\text{fit}} = 372.4 \pm 30.3$ GeV.
Reconstructing the Mass of a Charge 5/3 Top Partner

Figure 3.4: Invariant mass of the SS dileptons from leptonic decays of a $x_1$ with mass 365 GeV. The signal is stacked on top of the SM. The peak in the SM distribution at an invariant mass of about 520 GeV is caused by a single $Z \to \ell^+\ell^-$ event that passed the selection cuts.

dileptons. The cross section for single quark production is typically small for relatively light top partners, but influenced by model-dependent electroweak couplings. For BP 10 and IBP 18 we find that the ratio of the leading order cross section for single $x_1$ production over $x_1\bar{x}_1$ pair production is about 5.8% and 2.3%, respectively. For these points, the errors introduced are smaller than the uncertainty of the next-to-leading order pair production cross section, which is approximately 20% for top partners with a mass of about 500 GeV [113].

Applying the Method to Two Benchmark Points

For the 4 of $SO(4)$ and the $XX$ signatures, the various top partners in addition to $x_1$ contribute to the excess of SS dilepton events and alter the invariant mass distribution. In the special case of BP 10 and IBP 18, there is a bottom-like $b_1$ with a mass of about 10 GeV above the $x_1$ mass. Since it predominantly decays to $W^-t$, it plays an important role for the additional production of SS dileptons. In order to obtain SS (rather than OS) dileptons from $b_1\bar{b}_1$ decays, one lepton has to come from the $b_1$ quark and the other from the $\bar{b}_1$ quark. Therefore, the invariant mass distribution of the SS dileptons from $b_1\bar{b}_1$ decays does not show an endpoint, but rather a tail that extends far into the high invariant mass region. This is in contrast to the SS dileptons from $x_1\bar{x}_1$ decays. In case of BP 10, two charge 5/3 quarks below 500 GeV contribute to the excess of SS dilepton events. Since the $x_2$ quark is more massive than the $x_1$, the invariant mass distribution due to its leptonic decay is broader and has a larger endpoint with respect to the $x_1$ contribution. The main effects of these additional top partners (including the charge 2/3 quarks) on the invariant mass distribution of the SS dileptons are an increased number of signal events and a large tail that hides the endpoint due to the light $x_1$. These effects can be used to determine whether or not the expected SS dilepton invariant mass distributions for BP 10 and IBP 18 can be explained by the hypothesized presence of a charge 5/3 top partner only.
In Figures 3.5a and 3.5b we show the invariant mass distribution of the SS dileptons for BP 10 and IBP 18, respectively, as expected to be observed with $200\text{ pb}^{-1}$ of collision data. The SM background is added to the signal distribution. As explained above, a fit of the tail of the distribution leads to a fairly accurate estimate of the $x_1$ mass, if the observation is caused by only one charge $5/3$ quark (plus the SM contribution). For BP 10, we obtain $m_{\text{fit}} = 395.5 \pm 24.6\text{ GeV}$ and for IBP 18, we find $m_{\text{fit}} = 388.6 \pm 29.7\text{ GeV}$. This shows that a fit of the total distribution, including the contributions from the various top partners and the SM backgrounds, leads to a systematic overestimate of the mass, which nevertheless remains within about $1\sigma$ of the true $x_1$ quark mass.

![Figure 3.5: The invariant mass distributions of the SS dileptons for BP 10 plus, IBP 18 and the SM background (red). The overlaid histogram (blue) represents the $m_{\ell\ell}$ distribution of a pair-produced $x_1$ quark with a mass as estimated from the fit. The (blue) dashed distributions correspond to the $1\sigma$ variation of the $x_1$ expectation due to the uncertainty on the fitted mass.](image)

(a) Fit to the $m_{\ell\ell}$ distribution for BP 10.  
(b) Fit to the $m_{\ell\ell}$ distribution for IBP 18.

As a next step, we calculate the cross section and simulate the expected signal for a pair-produced $x_1$ with the fitted masses. This signal plus the SM expectation gives the expected invariant mass distribution of the SS dileptons for a given mass hypothesis. For BP 10 and IBP 18 (Figures 3.5a and 3.5b), we see that neither of the two signal distributions can be explained assuming the presence of only one charge $5/3$ quark. In particular, we expect 62.5 and 6.7 SS dileptons from BP 10 and the SM background, respectively, whereas the contribution from a pair-produced $x_1$ quark with the fitted mass could only account for $10.6^{+3.3}_{-2.9}$ SS dileptons. The stated errors are due to the statistical uncertainty on the fitted mass only. For IBP 18, 42.4 SS dileptons are expected from the top partners with $200\text{ pb}^{-1}$. Assuming the presence of $x_1$ only, the tail of the distribution suggests an $x_1$ mass that can account for $11.2^{+5.4}_{-3.4}$ SS dileptons. For BP 10 and IBP 18 we are left with respectively 52 and 31 unexplained SS dileptons. The uncertainty on these numbers is dominated by the Poisson uncertainty of the 62.5 and 42.4 expected events.

The possibility of the signal to be mainly caused by a very light $t_1$ quark can be excluded in the following
3.7 Conclusions

We studied the collider phenomenology of a realistic composite Higgs model with two multiplets of new quarks. We showed that the phenomenology is very rich, and in particular we can obtain some distinctive signatures for our model. These are the cases when a full $4$ of $SO(4)$ or two charge $5/3$ top partners lie within the reach of the LHC. We scanned the parameter space of the model focussing on points that are consistent with EWPT observables and give these signatures. For these signatures we described the possible mass hierarchies and outlined the basic features of their phenomenology. We find that the trilepton and same-sign dilepton final states are the most promising ones for a discovery of the model.

We studied in detail the phenomenology of two benchmark points with a large production cross section. Both exhibit a $4$ signature and one has two charge $5/3$ quarks with a mass below $500\,\text{GeV}$. We presented a robust cut-based search strategy for an excess in final states with at least two same-sign leptons. After making a basic kinematic selection, only little background from the SM was found in this channel. We find
that for both benchmark points a few tens of inverse picobarns of understood collision data would suffice to observe a $5\sigma$ significance.

Since the SM contamination in the same-sign dilepton final state is small, this channel is not only well suited for observing an excess over the SM expectation, but also for reconstructing the masses of the new particles. Among the top partners, the light charge $5/3$ quark contributes the most to the excess of SS dilepton events. We described a new method to reconstruct the mass of such a quark via its leptonic decay. This method only relies on the reconstruction of the two same-sign leptons and exploits the shape of their invariant mass distribution. For both distinctive signatures of the model, the light top partners besides the charge $5/3$ quark also contribute to the excess of same-sign dilepton events. In this case, we showed how the mass reconstruction method could be used to judge if the excess of same-sign dilepton events is compatible with the presence of a charge $5/3$ quark only, or if it hints at the existence of additional top partners. For this, we used the fact that the cross section for pair production of top partners can be predicted as a function of mass. Already with an integrated luminosity of $200\,\text{pb}^{-1}$ and a corresponding statistics of about 50 signal events, we found an evident disagreement between the single $x_1$ hypothesis and the expected observation. Such a disagreement can be seen as an indication for the presence of top partners in addition to a charge $5/3$ quark.
Chapter 4

EXPERIMENTAL ENVIRONMENT

4.1 The Large Hadron Collider

The Large Hadron Collider (LHC) [114–117] is a two-ring hadron accelerator and collider located close to Geneva at the Franco-Swiss border at the European Organization for Nuclear Research (CERN). The picture of the geographical situation in the Geneva area is shown in Figure 4.1. The LHC was constructed in the 26.7 km long tunnel that was used from 1989 to 2000 for the Large Electron Positron Collider (LEP) [118] and went live to produce proton-proton collisions in November 2009.

![Figure 4.1: The geographical situation in the Geneva area with the LHC and its experiments. Figure taken from Ref. [119].](image)

The design goals of the LHC were defined to maximize the discovery reach for new physics and unravel
4.1. The Large Hadron Collider

The mechanism responsible for the breaking of the electroweak symmetry. 1232 superconducting dipole magnets were installed, which are operated at a temperature of 1.9 K and capable of providing a dipole magnetic field of 8.3 T to accelerate the two counter-rotating proton beams to a maximum beam energy of 7 TeV. Prior to being injected in the LHC, the proton-proton beams are prepared in a series of accelerators that successively increase their energy. The full system of the CERN accelerator complex is illustrated in Figure 4.2. In the linear accelerator LINAC 2 the protons are accelerated to 50 MeV and then fed into the Proton Synchrotron Booster (PSB), which further increases their energy to 1.4 GeV. The energy of the proton beams is then successively increased in the Proton Synchrotron (PS) and the Super Proton Synchrotron (SPS), where they reach an energy of 450 GeV before they are injected to the LHC. Most of these facilities were already used for LEP, but upgraded to meet the needs of the LHC.

\[ N_{\text{process}} = L\sigma_{\text{process}}, \]

Figure 4.2: Schematic illustration of the CERN accelerator complex. Figure taken from Ref. [120].

There are four main experiments situated in caverns within the LHC ring: ATLAS [121], CMS [122], LHCb [123] and ALICE [124]. ATLAS and CMS are two general purpose detectors, the latter will be described in detail in Section 4.2. LHCb is a b-physics experiment and ALICE is dedicated to the physics of heavy-ion collisions.

4.1.1 Status of the LHC Operations

The number of events of a certain process produced per second in the LHC proton-proton collisions is given by

\[ N_{\text{process}} = L\sigma_{\text{process}}, \]
where $L$ denotes the machine luminosity and $\sigma_{\text{process}}$ is the cross section of the process under study. The machine luminosity depends only on beam parameters and can be written as follows for two identical beams:

$$L = \frac{N^2 b n v}{AC}. \quad (4.2)$$

Here, $n$ is the number of colliding bunches per beam in a ring of circumference $C$ each containing $N_b$ particles. The beam velocity is denoted by $v$ and $A$ is the cross-sectional area of the two beams at the interaction point. Table 4.1 lists the most relevant parameters of the 2011 and 2012 beam conditions during proton-proton operations.

### Table 4.1: Design v.s. actual LHC parameters for 2011 and 2012 run periods. The integrated luminosity is defined as $\int L \, dt$, for which the quoted numbers represent the delivered values to each CMS and ATLAS.

<table>
<thead>
<tr>
<th>parameter</th>
<th>design</th>
<th>2011 running</th>
<th>2012 running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of mass energy $E_{\text{cm}}$ [TeV]</td>
<td>14</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Peak luminosity $L$ [cm$^{-2}$s$^{-1}$]</td>
<td>$10^{34}$</td>
<td>$3.6 \cdot 10^{33}$</td>
<td>$7.7 \cdot 10^{33}$</td>
</tr>
<tr>
<td>Integrated luminosity for CMS and ATLAS [fb$^{-1}$]</td>
<td>–</td>
<td>5.7</td>
<td>23.3</td>
</tr>
<tr>
<td>Bunch spacing [ns]</td>
<td>25</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Number of bunches $n$ per beam</td>
<td>2808</td>
<td>1380</td>
<td>1380</td>
</tr>
<tr>
<td>Number of protons per bunch $N_b$</td>
<td>$1.2 \cdot 10^{11}$</td>
<td>$1.5 \cdot 10^{11}$</td>
<td>$1.5 \cdot 10^{11}$</td>
</tr>
</tbody>
</table>

The LHC will continue to run with the 2012 beam conditions throughout 2012. After the technical stop during the Christmas shutdown, heavy ion collisions are scheduled until February 2013. Then, the LHC will be shutdown until the end of 2014 (Long Shutdown 1), during which period repair and upgrade work will take place to prepare for proton-proton collisions at a center of mass energy of 13 to 14 TeV in 2015.

### 4.2 The Compact Muon Solenoid

The CMS experiment is a general purpose detector designed to meet the goals of the broad LHC physics program, with an emphasis on elucidating the nature of electroweak symmetry breaking for which the Higgs boson is presumed to be responsible. During the design period, the main detector requirements were defined as follows [107]:

- a good and redundant muons system, providing excellent muon identification and momentum resolution;
- good charged particle momentum resolution and reconstruction efficiency in the inner tracker;
- excellent electromagnetic energy resolution;
- a hadron calorimeter with a hermetic geometric coverage allowing for good $E_{\text{miss}}$ resolution.
These requirements were met using a compact geometry, where the full silicon-based tracking system as well as the electromagnetic and (most of) the hadronic calorimeters are located within a strong superconducting solenoid magnet. The solenoid has a length of 13 m, a diameter of 6 m and provides an axial magnetic field of 3.8 T. A schematic view of the CMS detector is shown in Figure 4.3. The overall dimensions of the CMS detector are a total weight of 12500 tons, a length of 21.6 m and a diameter of 14.6 m. In the following, the various detector components are reviewed. A detailed description of the CMS detector can be found elsewhere [107, 122].

**Figure 4.3:** A schematic view of the CMS apparatus [107].

### Coordinate Conventions

The right-handed coordinate system adopted by CMS is centered at the nominal collision point inside the detector, the $x$-axis pointing radially inward toward the center of the LHC and the $z$-axis pointing along the beam direction toward the Jura mountains. The pseudorapidity is defined as $\eta = -\ln \tan(\theta/2)$, where $\theta$ is the polar angle measured from the $z$-axis. A one-quarter longitudinal view of the CMS detector is shown in Figure 4.4, with lines of constant $\eta$ superposed.

#### 4.2.1 Inner Tracking Detectors

The innermost subdetector is the central tracking system, which is designed to accomplish a precise vertex reconstruction and to measure the trajectories of charged particles with $p_T \gtrsim 100$ MeV. It consists of a
Figure 4.4: One quarter cross-sectional view of the CMS apparatus showing the $\eta$ coverage of the subdetectors. The dimensions are indicated in units of millimeters. Figure taken from Ref. [107].

A silicon pixel detector and a surrounding silicon microstrip detector, with overall dimensions of 5.8 m in length and 2.5 m in diameter. Figure 4.5 shows a cross-sectional view of the CMS tracking detector.

Figure 4.5: Cross-sectional view of the CMS inner tracking detectors [122].

Pixel Tracker

The pixel detector consists of three concentric cylindrical barrel layers with two endcap disks on each side of them. The three barrel layers are located at radii of 4.4 cm, 7.3 cm and 10.2 cm, whereas the two endcap disks are placed at $|z|=34.5$ cm and 46.5 cm and extend from 6 to 15 cm in radius. The pixel tracker comprises 66 million pixels, each with a size of $100 \times 150 \mu m^2$, and covers an area of about 1 m$^2$. A spatial resolution of
about 10 \( \mu m \) in \( r-\phi \) and about 20 \( \mu m \) in \( z \)-direction is achieved.

**Strip Tracker**

The silicon strip detector, occupying the radial region between 20 cm and 116 cm, is composed of three subsystems. The Tracker Inner Barrel and Disks (TIB/TID) consist of four barrel layers and three endcap disks, extending to 55 cm in radius. The Tracker Outer Barrel (TOB) is composed of six barrel layers and has an outer radius of 116 cm. Beyond the \( |z| < 118 \) cm coverage of the TOB, the nine Tracker EndCaps (TEC) disks extend the range to \( 124 < |z| < 282 \) cm. This layout comprises a total of 9.3 million silicon strips, covering an active area of 198 m\(^2\), and ensures at least \( \approx 9 \) hits in the strip tracker for the full \( \eta < 2.4 \) range.

**Tracker Performance**

The transverse momentum resolution for high momentum tracks with \( p_T \approx 100 \) GeV is \( \lesssim 2\% \) for \( |\eta| < 1.6 \). The resolution is degraded to about \( \delta p_T/p_T \approx 7\% \) for \( |\eta| = 2.4 \). The material budget in the tracker increases from \( \approx 0.4 \) radiation lengths (\( X_0 \)) at \( \eta = 0 \) to about \( X/X_0 \approx 1.8 \) at \( |\eta| = 1.6 \).

**4.2.2 Electromagnetic Calorimeter**

The design of the Electromagnetic Calorimeter (ECAL) was driven by the need for an excellent diphoton mass resolution, which is crucial for the observation of the Higgs boson in its decay to two photons. The ECAL, surrounding the tracker, is a hermetic and homogeneous calorimeter, comprising about 76000 lead tungstate (\( \text{PbWO}_4 \)) crystals. These crystals are characterized by a relatively low light yield of about 30 \( \gamma/\text{MeV} \) [107], but a short radiation length \( X_0 = 0.89 \) cm and a high density of about 8.3 g/cm\(^3\), thus allowing for the compact design of the calorimeter. A schematic view of the ECAL is shown in Figure 4.6.

The ECAL barrel (EB) covers \( |\eta| < 1.479 \) in pseudorapidity and has an inner radius of 129 cm. It comprises 36 identical “supermodules”, each covering half the barrel length and 20° in \( \phi \). The 61200 barrel crystals are mounted in a quasi-projective geometry so that the axes are tilted at 3° with respect to the vector from the center of CMS. The crystals cover approximately 1° in \( \Delta \eta \) and \( \Delta \phi \) and have a front face cross-section of \( 22 \times 22 \) mm\(^2\). The crystal length is 230 mm, corresponding to 25.8 \( X_0 \). The crystal scintillation light is collected using two silicon avalanche photodiodes (APDs) per crystal. About 80% of the scintillation light is emitted within 25 ns.

The two ECAL endcaps (EE) cover a pseudorapidity range of 1.479 < \( |\eta| < 3.0 \) and comprise each 7324 crystals. Each endcap is structured as two “Dees”, as can be seen in Figure 4.6. All endcap crystals are identical, have a front face cross-section of \( 28.6 \times 28.6 \) mm\(^2\) and a length of 220 mm (24.7 \( X_0 \)). A vacuum phototriode is glued on the back of each crystal to collect the scintillation light.

A preshower detector is situated between the inner tracking system and the ECAL endcaps, covering 1.653 < \( |\eta| < 2.6 \) in pseudorapidity. The active elements of the preshower device are two planes of silicon
Figure 4.6: A schematic view of the CMS Electromagnetic Calorimeter [122].

strip detectors. These lie behind disks of lead absorber, causing most photons to shower before the second silicon plane is reached. The preshower is installed mainly to improve the $\pi^0 - \gamma$ discrimination.

ECAL Performance

After installation of the ECAL and some losses due to infant mortality, stable fractions of 99.30% and 98.94% of crystals were found to be working in the EB and EE, respectively [125]. The energy resolution of the ECAL has been measured to be

$$\frac{\sigma(E)}{E} = \frac{2.8\%}{\sqrt{E(\text{GeV})}} \oplus \frac{12\%}{E(\text{GeV})} \oplus 0.3\%,$$

(4.3)

where the different terms denote the stochastic, noise and constant terms, respectively. The stochastic contribution arises from event-by-event fluctuations and photostatistics. The constant term accounts, for instance, for non-uniformity of the light collection and intercalibration errors, whereas the noise contribution arises from the electronics and digitization.

4.2.3 Hadronic Calorimeter

The Hadronic Calorimetry (HCAL) is crucial for the reconstruction of hadron jets and for the measurement of the missing transverse energy. The design of the HCAL is driven by the choice to install most of the CMS calorimetry inside the magnet coil. The four components of the HCAL, i.e., the Hadron Barrel (HB), the Hadron Endcap (HE), the Hadron Outer (HO) and the Hadron Forward (HF), are illustrated in Figure 4.7.
The HB is constructed as a scintillator/brass sandwich detector. The absorber material consist of a 4 cm thick steel front plate, 15 brass plates with a thickness of about 5 cm and a 7.5 cm thick steel back plate. The absorber plates are interleaved with plastic scintillator plates. The HB covers the pseudorapidity range $0 < |\eta| < 1.4$ with a segmentation of $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$. The effective thickness of the HB increases with pseudorapidity, with a minimum (maximum) of 5.82 (10.6) interaction lengths $\lambda_I$ at $\eta = 0$ ($|\eta| = 1.3$). The scintillation light is collected using hybrid photodiodes.

The HO is installed in the central region $|\eta| < 1.26$ outside the solenoid and consists of 10 mm thick scintillator and iron. It acts as a tail catcher, increasing the effective thickness of the HCAL to over 10 interaction lengths.

The HE covers $1.3 < |\eta| < 3.0$ in pseudorapidity with a segmentation of $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$ for $|\eta| < 1.6$ and $\Delta\eta \times \Delta\phi = 0.17 \times 0.17$ for $|\eta| > 1.6$. In the HE, the brass absorber plates with a thickness of 79 mm are interleaved with 9 mm thick scintillator plates. Including the electromagnetic crystals, a total of about 10 interaction lengths are reached in this part of the calorimetry.

The very forward region with $3.0 < |\eta| < 5.0$ is covered with a steel/quartz fiber calorimeter called HF. Its front face it located at a distance of 11.2 m from the interaction point. The Cherenkov light emitted in the quartz fibers is amplified with photomultiplier tubes. The granularity in $\eta$ and $\phi$ ranges from 0.1 to 0.3 and from $10^{\circ}$ to $20^{\circ}$, respectively.

**HCAL Performance**

About 99.3% of the HB, HE and HF channels, and 95.5% of the HO channels are reported operational [126]. The hadronic energy resolution of the HCAL, combined with the ECAL and for energies between 30 GeV
and 1 TeV, can be parameterized as [126]

$$\sigma_E / E = 85% / \sqrt{E (\text{GeV})} \oplus 7.4\%,$$

(4.4)

where the first term is stochastic in nature.

### 4.2.4 Muon System

The muon system, which is interleaved with the layers of the flux return plates, is the outermost detector of CMS. A precise and robust muon system was a central requirement from the earliest stages of the CMS design, as implied by the experiment’s name. The CMS muon system uses three types of gaseous particle detectors: drift tube (DT) chambers in the barrel (|\eta| < 1.2), cathode strip chambers (CSC) in the endcap (1.2 < |\eta| < 2.4), complemented by resistive plate chambers (RPC) covering the barrel and a large portion of the endcaps. The CMS muon system is shown in Figure 4.8.

![Figure 4.8: Schematic view of the CMS muon system [107].](image)

The barrel muon system comprises four layers of DTs, which are interspersed among the layers of the iron yoke plates, at radii of approximately 4.0, 4.9, 5.9 and 7.0 m from the beam axis. The DTs are made from aluminum cathodes and stainless steel anode-wires, filled with an argon carbon-dioxide mixture. The first three stations provide a measurement in \(r\), \(\phi\) and \(z\) coordinates. The fourth layer only provides a measurement of the muon coordinate in the \(r-\phi\) bending plane.

The two endcap muon systems each comprise 234 CSCs, which are proportional drift chambers with
gold-plated anode wires perpendicular to the cathode strips. The CSCs are arranged in three layers of concentric rings around the beam pipe.

In addition to the DTs and CSCs, a complementary muon system consisting of RPCs was installed in the barrel and endcap regions, providing a fast and highly-segmented trigger for the DTs and CSCs. The RPCs are double-gap chambers, filled with freon and isobutane gas.

**Performance of the Muon System**

About 96% and 98% of the channels in the CSCs and DTs are reported alive [127, 128]. When combining the measurement from the inner tracker and the muon systems, a muon momentum resolution of $\frac{\Delta p_T}{p_T} \approx 1\%$ (4%) is reached for $|\eta| < 0.8$ and $p_T = 10$ GeV (1 TeV). In the endcap region, the resolutions are degraded to $\sim 2\%$ and $\sim 10\%$ for the same transverse momenta.

### 4.2.5 Data Acquisition and Trigger System

With a total pp cross section of $\sim 100$ mb, about $10^9$ proton-proton interactions per second are anticipated at nominal LHC operation. With a typical data size of $\sim 1$ MB per event, such an event rate would result in an unmanageable $\sim 1000$ TB of data to be stored per second. The prime task of the trigger system is thus to reduce this huge rate by about a factor of $10^6$, down to a few hundreds of potentially interesting events per second that can be coped with by the data acquisition (DAQ) system.

The CMS trigger system consists of two main components, namely, the hardware trigger system called Level 1 (L1) and a software based High Level Trigger (HLT).

The L1 trigger employs custom hardware processors to analyze every bunch crossing and needs to come to a positive or negative decision within a maximal latency of 3.2 $\mu$s. The L1 decision is based on information from the muon system and the calorimetry only, read out with reduced granularity. The L1 output rate is limited to a maximum of 100 kHz, but typically of the order of 30 kHz.

The HLT processor farm further reduces the event rate by a factor of about 100. It employs tracking information in addition to the calorimetry and the muon systems. This software-based trigger decision beyond L1 allows for a high level of flexibility and great sophistication.
This analysis is based on a particle-flow (PF) event reconstruction [129], which aims at reconstructing and identifying individually each particle produced in the collision, namely charged hadrons, photons, neutral hadrons, electrons, and muons. This is achieved using an optimal combination of the information available from all subdetectors. Thanks to the strong magnetic field, the large silicon tracker and the high granularity ECAL, the CMS detector seems almost ideally suited for a PF event reconstruction, which leads to an improved performance for jets, taus and $E_{T}^{\text{miss}}$. In the following, some key elements for the reconstruction of PF jets, electrons, muons and taus are discussed.

The building blocks of the PF event reconstruction are charged-particle tracks, muon tracks and calorimeter clusters, which must be found with high efficiency and low fake rate. For this, a dedicated iterative tracking algorithm was developed [130]. First, very tight criteria are used to reconstruct tracks with a low fake rate. The hits assigned to these tracks are removed and the algorithm is reapplied with loosened selection criteria. With a total of five iterations, this algorithm allows to efficiently reconstruct charged particles’ tracks with as little as three hits down to a $p_T$ of 150 MeV. Next, topological clusters are formed from adjacent calorimeter cells separately in the ECAL and HCAL subdetector but the HF, starting from seeds above a given energy threshold. A geometrical linking algorithm is used to finally reconstruct each particle and get rid of potential double counting. For instance, a charged-particle track is linked to clusters in the ECAL and HCAL by extrapolating the last hit in the tracker to the calorimeters, taking into account typical electromagnetic and hadronic shower shapes.

5.1 Electron Reconstruction and Identification

We have seen in Section 4.2.1 that the material budget of the CMS inner tracker accounts for up to two radiation lengths at $|\eta| = 1.6$. Consequently, electrons may lose a significant fraction of their energy via the emission of bremsstrahlung photons while travelling through the material. Because of the strong axial
magnetic field, the energy deposits in the ECAL due to the electron and its Bremsstrahlung photons can be widely separated in azimuthal direction. For this reason, a special seeding and Bremsstrahlung recovery techniques have been adopted to reconstruct electrons [131], as briefly discussed in the following.

Two seeding methods are exploited. The ECAL-seeded reconstruction is efficient for isolated, high $p_T$ electrons. It starts from ECAL clusters above a given threshold, which are used to seed the reconstruction of the electron track. For this, a specific fit is carried out, using a Gaussian-Sum Filter, able to cope with sudden changes in curvature radius due to emitted Bremsstrahlung photons [132]. The track-seeded electron reconstruction is based on tracks passing tight selection criteria compatible with the electron hypothesis. This electron track is then extrapolated to the ECAL, taking into account the ECAL deposits which are tangent to the electron track from each tracker layer in order to recover Bremsstrahlung photons. The electron momentum is reconstructed by combining the information from the tracker and the ECAL. Finally, an electron identification is performed to discriminate from charged hadrons, using a multivariate boosted decision tree method based on multiple observables from calorimetry and tracking information.

**Electron Selection**

We select particle-flow electrons with transverse momenta $p_T > 10\text{ GeV}$ and $|\eta| < 2.4$. The tracks of the electron candidates are requested to point to the event primary vertex. Since we are primarily interested in prompt electrons arising from leptonic $W$ and $Z$-boson decays, we further require the electron candidates to be isolated. In particular, we require that the transverse momentum sum of charged hadrons, photons, and neutral hadrons surrounding the lepton within a cone of radius $\sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.4$, divided by the lepton transverse momentum value itself, be less than 0.2. The electron candidates are further requested to pass selection criteria designed to reduce the background from photon conversions, as described in Ref. [133].

**5.2 Muon Reconstruction and Identification**

Particle-flow muons are based on the standard CMS muon reconstruction [134, 135]. There are two main reconstruction approaches, each based on tracker tracks and tracks reconstructed in the muon system (standalone-muon tracks).

- **Global muon reconstruction (outside-in):** starting from a standalone muon, a matching track in the inner tracker is sought by comparing the parameters of the two tracks. A global-muon track is fitted combining hits from the muon system and the inner tracker.

- **Tracker muon reconstruction (inside-out):** high quality tracks reconstructed with the inner tracking detector are extrapolated to the muon system, taking into account the expected energy loss and multiple Coulomb scattering in the detector material. At least one muon segment must match the extrapolated track.
Since the tracker muon reconstruction requires only a single hit in the muon system and the global muon reconstruction is based on multiple muon segments, the tracker muon reconstruction is more efficient for muons with low momenta \( p \lesssim 5 \text{ GeV} \).

The particle-flow muon identification is optimized for high efficiency and low fake-rate not only for isolated muons, but especially for muons in jets. These non-isolated muons in jets are important for the PF event reconstruction, as missed or fake muons can bias the jet and \( E_T^{\text{miss}} \) measurements. This is mainly because non-identified muons are interpreted as charged hadrons and thus falsely expected to deposit energy in the calorimetry. In the PF muon identification, isolated muons are only required to be global muons. In addition to these isolated muons, PF \textit{tight} and PF \textit{loose} muons are obtained from the non-isolated muons based on an identification tuned to identify muons in jets. The efficiency of the PF muon reconstruction has been studied in Ref. [135] and found to be \( > 99\% \) both for data and simulation for muons with \( p_T > 5 \text{ GeV} \).

\section*{Muon Selection}

We select particle-flow muons with a transverse momentum \( p_T > 10 \text{ GeV} \) and \( |\eta| < 2.4 \). Since we are interested in muons from \( W \) and \( Z \) boson decays, we further require the muon candidate to pass the \textit{tight} muon identification from Ref. [134]. This requirement significantly reduces the background from heavy-flavor decays-in-flight. Furthermore, we require the muons to be isolated, using a particle-flow based isolation criterion identical to the one used for the PF electrons (see Section 5.1).

\section*{5.3 Tau Reconstruction and Identification}

About two-thirds of the tau leptons decay hadronically, typically into one or three charged hadrons and a number of \( \pi^0 \)’s. The \( \pi^0 \) mesons decay immediately into two photons. Within CMS, the tau reconstruction and identification makes extensive use of the particle-flow technique to reconstruct the individual tau decay products, namely charged hadrons and photons. There are two main reconstruction algorithms used. These are the “Tau Neural Classifier” and the “Hadron plus Strips (HPS)” algorithms described in detail in Refs. [136, 137]. For the work presented here, we use HPS taus. This algorithm employs criteria on the invariant mass and the multiplicity of charged hadrons and neutral pions in a narrow cone and requires that there be no other reconstructed particles above a certain \( p_T \) threshold within the jet [136]. Depending on the working point, the reconstruction efficiency for taus with \( p_T > 30 \text{ GeV} \) ranges from 20\% to 50\% with a corresponding fake rate of about 0.1\% and 2\% [137].

\section*{5.4 Photon Reconstruction and Identification}

The particle-flow photon reconstruction is not fully efficient, mainly due to electron-photon disambiguation and photon isolation requirements. For this reason, we use photons based on the standard CMS photon reconstruction documented in Refs. [138–141].
While a photon incident perpendicularly on the ECAL surface deposits most of its energy in a matrix of $5 \times 5$ crystals, the material in front of the detector causes many photons to convert in electron-positron pairs, whose trajectories are separated in azimuth due to the magnetic field. In order to recollect the full photon energy, a specific clustering algorithm [107] is used which builds “super-clusters” of crystals to account for several deposits at constant $\eta$ but spread in $\phi$. The direction of the photon momentum is estimated through an extrapolation of the super-cluster position to the event’s primary vertex.

The dominant background to prompt photon production stems from jets fragmenting mainly into boosted neutral mesons, which decay into two photons. The minimum opening angle in the laboratory frame between the two photons in a $\pi^0 \rightarrow \gamma \gamma$ decay is given by $\sim \frac{2m_{\pi}}{E_{\pi}}$, showing that the $\pi^0 - \gamma$ discrimination becomes more and more challenging with increasing energy. The situation is even more challenging in the ECAL endcap, due to the coarser granularity and the fact that the opening angle decreases with energy $E$ and not $E_T = E \cos(\theta)$.

On a statistical basis, however, a good $\pi^0 - \gamma$ discrimination can be achieved by exploiting the difference between the electromagnetic shower profiles of prompt photons and of photon pairs from neutral-meson decays. A standard choice of shower shape variable is given by a modified second moment of the electromagnetic energy cluster about its mean $\eta$ position, which expresses the extent in $\eta$ of the cluster. It is explicitly defined and discussed in more detail in Section 7.3.2.1. The signal can be further discriminated from the background using the fact that neutral mesons are produced in jets and thus not expected to be isolated. The ratio between the energy deposited in the ECAL and the HCAL can be used to test the compatibility of the shower with the photon hypothesis. The background from electrons is reduced by requiring the absence of hits in the first two layers of the pixel detector, consistent with an electron track matching the reconstructed photon position and energy.

**Photon Selection**

We use a photon identification and isolation identical to the one from Ref. [139]. We further require the photon candidate to have $p_T > 20$ GeV and $|\eta| < 2.4$, excluding the transition region between the ECAL barrel and endcap ($1.44 < |\eta| < 1.57$).

**5.5 Jet Reconstruction and Identification**

Because of QCD confinement, the quarks and gluons (partons) produced in a QCD hard scattering process are not observed as free particles. Instead, they undergo parton showering and hadronization leading to narrow cones of hadrons and other particles finally observed in the experiment [11]. The association of the particles originating from an initial parton is done using a jet finding algorithm. The standard algorithm used in CMS for jet-finding is the anti-$k_T$ jet clustering [106]. This algorithm is collinear and infrared safe, i.e., collinear splittings and soft emissions do not alter the jets.

---

1See Table 2 in Ref. [139]: “photon conversion method”
In the inclusive anti-\textit{k}_T algorithm, the clustering is performed as follows. First, the distances \(d_i\) and \(d_{ij}\) are calculated for each object \(i\) and pair of objects \(i\) and \(j\), respectively. These distances are defined as

\[
\begin{align*}
  d_i & = p_{T,i}^{-2} \\
  d_{ij} & = \min(p_{T,i}^{-2}, p_{T,j}^{-2}) \frac{\Delta^2_{ij}}{R^2},
\end{align*}
\]

where \(\Delta^2_{ij} = (\phi_i - \phi_j)^2 + (y_i - y_j)^2\) and \(y_i\) and \(\phi_i\) denote the rapidity \(y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)\) and the azimuthal angle of the object \(i\). In CMS, the distance parameter \(R\) is typically set to 0.5, which is also the choice for the work presented here. The clustering proceeds by identifying the smallest of all \(d_i\) and \(d_{ij}\). If it is a \(d_{ij}\), the objects \(i\) and \(j\) are recombined. If it is a \(d_i\), object \(i\) is called a jet and removed from the list of objects to cluster. Then, the distances are recalculated and the procedure iteratively repeated until no objects are left.

In the standard particle-flow event reconstruction, all reconstructed particle-flow particles are clustered in PF-jets [142, 143]. Here, we make a slightly different choice and form PF jets from all PF particles except from the selected electrons and muons (as defined above). With this procedure, no lepton-jet cross-cleaning needs to be performed at analysis level to avoid potential double counting.

### 5.5.1 Jet Energy Correction

The reconstructed jets are corrected to relate, on average, the measured energy to the one of the true particle jet, arising from all stable particles originating from the fragmenting parton and the underlying event. The jet energy correction corrects for the extra energy due to noise and pile-up, non-uniformity in \(\eta\) and \(p_T\), as well as for differences between data and simulation [144]. The uncorrected (raw) four-momentum \(p_{\mu}^{\text{raw}}\) of each jet is rescaled as

\[
p_{\mu}^{\text{corr}} = C(p_T, \eta) \cdot p_{\mu}^{\text{raw}},
\]

where

\[
C(p_T, \eta) = C_{\text{offset}}(p_T^{\text{raw}}) \cdot C_{\text{sim}}(p_T, \eta) \cdot C_{\text{rel}}(\eta) \cdot C_{\text{abs}}(p_T^{\text{raw}}).
\]

Here, \(p_T^{\text{raw}}\) and \(p_T^{\text{corr}}\) denote respectively the transverse momentum of the jet after applying the offset and all previous corrections.

The offset correction subtracts the energy from pile-up and noise, i.e. energy not associated to the hard scattering. The correction is based on a FastJet pileup subtraction procedure [145, 146]. In particular, an average \(p_T\) density \(\rho\) per unit area is estimated for each event using the \(k_T\) jet clustering algorithm [147–149] with a distance parameter \(R = 0.6\). \(\rho\) is defined on an event-by-event basis as the median of the distribution of \(p_{Tj}/A_j\), where \(j\) runs over all jets and \(A_j\) denotes the jet area, and is insensitive to the presence of hard jets. The jet area is calculated as follows: A large number of infinitesimally soft particles are added to the event and clustered with the true jet components. The jet area is defined as the region of \(\phi - y\) space it occupies. With these definitions, the offset correction is given by

\[
C_{\text{offset}}(p_T^{\text{raw}}, \eta, A_j, \rho) = 1 - \left( \frac{\rho - \langle \rho_{\text{UE}} \rangle}{p_T^{\text{raw}}} \right) \cdot \beta(\eta) \cdot A_j,
\]

where

\[
\beta(\eta) = \frac{\eta}{\ln(\rho)}.
\]
where $\langle \rho_{\text{UE}} \rangle$ is the $p_T$-density due to the UE, which needs to be subtracted from $\rho$. The non-uniformity of the detector response is small for PF jets, but nevertheless accounted for using the multiplicative factor $\beta(\eta)$. The calibration $C_{\text{sim}}$ corrects the energy of the reconstructed jet to the one of the generated particle jet in simulation. The correction for the relative jet energy scale corrects the response of a jet at any $\eta$ to the jet energy response in the central region $|\eta| < 1.3$. This correction factor is derived using a dijet $p_T$-balance technique [144]. Furthermore, the correction $C_{\text{abs}}$ is used to bring the absolute energy scale of central jets back to unity. To derive this correction factor, the missing transverse energy projection fraction (MPF) method [144, 150] is used in $\gamma$+jets and $Z$+jets events. The method relies on the idea that $\gamma$+jets and $Z$+jets events do not have genuine $E_T^{\text{miss}}$ and the bosons are well measured by the CMS detector. Figures 5.1a and 5.1b show respectively the total jet energy correction factor as a function of jet $p_T$ and its uncertainty as a function of $\eta$.

![Jet Energy Correction Factor](image1)

(a) Total jet energy correction factor as a function of $\eta$ for jets with $p_T > 50$ GeV

![Jet Energy Uncertainty](image2)

(b) Total jet energy correction uncertainty as a function of jet $p_T$ for central jets.

**Figure 5.1:** Jet energy correction factor and uncertainty for different jet types, including PF-jets. Figures taken from Ref. [144].

### 5.5.2 Jet Selection

We select particle-flow jets with $p_T > 20$ GeV and $|\eta| < 2.4$. The rather restrictive $\eta$ selection makes sure that selected jets are within the coverage of the inner tracking detector and not only measured with calorimetry. We further request jet candidates to pass PF jet identification criteria [142] developed to reject fake jets arising from spurious energy deposits, e.g. from noise in readout electronics or calorimeters. Using simulation, this jet identification was shown to be $> 99\%$ efficient for genuine jets, while rejecting the vast majority of fake jets.
5.5.3 Tagging of b-Quark Jets

Jets that arise from b-quark (and c-quark) hadronization and decay can be discriminated from light-flavored (uds quark and gluon) jets due to the presence of tracks not compatible with the primary vertex or displaced secondary vertices. This discrimination is typically based on the long proper lifetime of about 480 µs and the large mass of about 5.2 GeV for bottom mesons. This analysis makes use of the Simple Secondary Vertex (SSV) discriminator, which is briefly explained in the following. More information on various b-quark taggers used in CMS can be found in Refs. [151–153].

The SSV algorithm attempts to directly identify secondary vertices within a jet arising from b-hadron decays. The SSV discriminator is a function of the significance of the distance between the reconstructed secondary and the primary vertex (flight distance). There are two variants considered with a different minimal number of tracks assigned to the secondary vertex. The high efficiency version (SSVHE) requires at least two tracks assigned to the secondary vertex, whereas the high purity version (SSVHP) is based on at least three tracks. In Figure 5.2 we show the efficiencies and mistag rates for various b-tag discriminators. We use a SSVHP working point that yields a typical jet-tagging efficiency of 42% for b jets in our search region while the mistagging efficiency for light-flavored (uds quark and gluon) jets is of the order of 0.1% and for c jets, 6.3%.

(a) Tagging efficiency of b quarks v.s. mistag rate for light jets for various b-tag discriminators. (b) Tagging efficiency of b quarks v.s. c-quark jets for various b-tag discriminators.

**Figure 5.2:** Tagging efficiency for b-quarks v.s. mistag efficiency for various b-tagging algorithms used in CMS. Figures taken from Ref. [153].
As previously mentioned, the particle flow event reconstruction produces a complete list of all identified and reconstructed particles in the event. Since the information of all subdetectors is used, the PF missing transverse momentum $E_{T}^{\text{miss}}$ is simply computed as the negative vector sum of the transverse momenta of all particles reconstructed by the PF algorithm [143, 154]. Its absolute value is denoted by $E_{T}^{\text{miss}}$.

Various cleaning procedures and event filters are used to clean the reconstruction from anomalous sources of $E_{T}^{\text{miss}}$. Such sources include, for instance, beam induced background (mainly due to beam halo muons), anomalous signals in the ECAL and HCAL calorimeters and non-functional detector regions [154]. The mitigation of such sources of anomalous $E_{T}^{\text{miss}}$ measurements is further discussed in Section 7.1.2.

Figure 5.3: $E_{T}^{\text{miss}}$ resolution v.s. the scalar sum of the transverse energies of all PF particles $\sum E_{T}$ for three different algorithms to reconstruct $E_{T}^{\text{miss}}$. Figure taken from Ref. [154].

In Figure 5.3 we show the $E_{T}^{\text{miss}}$ resolution for three different algorithms used in CMS to reconstruct $E_{T}^{\text{miss}}$. It is measured as the $\sigma$ of a Gaussian fit to the $x$ and $y$ components of $E_{T}^{\text{miss}}$ in purely hadronic multijet events with no genuine $E_{T}^{\text{miss}}$. We see that the PF $E_{T}^{\text{miss}}$ resolution is superior to the other two algorithms.
In this chapter, the definition of the “stransverse mass” variable $M_{T2}$ is reviewed and its application as a discovery variable in searches for supersymmetry is discussed. An approximate formula for $M_{T2}$ is derived and some of its main properties and limitations are outlined. The event configurations and kinematics of the SM backgrounds at large $M_{T2}$ values are characterized in detail. In the context of a search for SUSY in the fully hadronic channel, it is crucial to understand the kinematics of these events, since the tail of the $M_{T2}$ distribution is known to be sensitive to a potential SUSY signal. This work was first documented in a CMS internal Analysis Note [155], from which this chapter is partly extracted.

6.1 Definition of $M_{T2}$

The “stransverse mass” variable $M_{T2}$ [156, 157] is the natural extension of the transverse mass $M_{T}$ to the case where two colored supersymmetric particles (“sparticles”) are pair-produced and both decay through a cascade of jets and possibly leptons to the lightest supersymmetric particle (LSP). The LSP is not visible in the detector and leads to a missing transverse momentum signature.

The variable $M_{T2}$ was introduced [156] to measure the mass of primary pair-produced particles in a situation where both ultimately decay into undetected particles (e.g., LSPs) leaving the event kinematics underconstrained. It assumes that the two produced sparticles give rise to identical types of decay chains with two visible systems defined by their masses $m_{\text{vis}}(i)$, transverse momenta $\vec{p}_{T\text{vis}}^{(i)}$, and transverse energies $E_{T\text{vis}}^{(i)} = (|\vec{p}_{T\text{vis}}^{(i)}|^2 + (m_{\text{vis}}^{(i)})^2)^{1/2}$. They are accompanied by the unknown LSP transverse momenta $\vec{p}_{T\tilde{\chi}}^{(i)}$. In analogy with the transverse mass used for the W boson mass determination [158], we can define two transverse masses ($i = 1, 2$):

$$ (M_{T2}^{(i)})^2 = (m_{\text{vis}}^{(i)})^2 + m_{\tilde{\chi}}^2 + 2 \left( E_{T\text{vis}}^{(i)} \vec{E}_{T\tilde{\chi}}^{(i)} - \vec{p}_{T\text{vis}}^{(i)} \cdot \vec{p}_{T\tilde{\chi}}^{(i)} \right), \quad (6.1) $$

These have the property (as in W-boson decays) that, for the true LSP mass $m_{\tilde{\chi}}$, their distributions cannot exceed the mass of the parent particle and thus present an endpoint at the value of the parent mass. The
momenta $\vec{p}_T^{\chi(i)}$ of the invisible particles are not experimentally accessible individually. Only their sum, the missing transverse momentum $\vec{p}_T^\text{miss}$, is known. Therefore, in the context of SUSY, a generalization of the transverse mass is needed and the proposed variable is $M_{T2}$. It is defined as

$$M_{T2}(m_{\tilde{\chi}}) = \min_{\vec{p}_T^{\chi(1)}+\vec{p}_T^{\chi(2)}=\vec{p}_T^\text{miss}} \left[ \max \left( M_T^{(1)}, M_T^{(2)} \right) \right],$$

where the LSP mass $m_{\tilde{\chi}}$ remains a free parameter. This formula can be understood as follows. As neither $M_T^{(1)}$ nor $M_T^{(2)}$ can exceed the parent mass if the true momenta are used, the larger of the two can be chosen. To make sure that $M_{T2}$ does not exceed the parent mass, a minimization is performed on trial LSP momenta fulfilling the $\vec{p}_T^\text{miss}$ constraint. The distribution of $M_{T2}$ for the correct value of $m_{\tilde{\chi}}$ has an endpoint at the value of the primary particle mass. If, however, $m_{\tilde{\chi}}$ is lower (higher) than the correct mass value, the endpoint will be below (above) the parent mass. Hereafter, we always set $m_{\tilde{\chi}} = 0$.

An analytic expression for $M_{T2}$ has been computed [159] assuming that initial-state radiation (ISR) can be neglected. In this case, two types of solutions are considered, namely the “balanced” and “unbalanced” one, as illustrated in Figure 6.1. The dependence of each of the transverse masses on the unseen particle

![Figure 6.1: Pictorial illustration of the balanced and unbalanced solutions for $M_{T2}$. Figures taken from Ref. [159].](image)

momentum follows roughly a parabolic curve. It may happen that the global minimum of one of the transverse masses is larger than the corresponding transverse mass of the other one. This situation satisfies the requirements of Eqn. (6.2) and is called unbalanced solution which is simply given by the minimum of the transverse mass, i.e.

$$M_{T2} = m_{\tilde{\chi}} + m_{\text{vis}(i)}. \tag{6.3}$$

From differentiating $M_{T2}^2$ by $\vec{p}_T^{\chi(i)}$ one finds that the stationary point for $M_T$ corresponds to the situation when the visible system and the LSP have equal velocities in the transverse plane

$$\frac{p_T^{\chi(i)}}{E_T^{\chi(i)}} = \frac{p_T^{\text{vis}(i)}}{E_T^{\text{vis}(i)}}. \tag{6.4}$$
In case none of the two transverse masses lead to an unbalanced solution, the solution will be the crossing point of the two transverse mass curves, which is called a balanced solution:

\[(M_{T2})^2 = m_{\tilde{\chi}}^2 + A_T + \sqrt{\left(1 + \frac{4m_{\tilde{\chi}}^2}{2A_T - (m_{\text{vis}(1)})^2 - (m_{\text{vis}(2)})^2}\right)\left(A_T^2 - (m_{\text{vis}(1)}m_{\text{vis}(2)})^2\right)}\]  (6.5a)

\[A_T = E_{T_{\text{vis}(1)}}E_{T_{\text{vis}(2)}} + \vec{p}_{T_{\text{vis}(1)}}\vec{p}_{T_{\text{vis}(2)}}.\]  (6.5b)

In practice, the determination of \(M_{T2}\) may be complicated by the presence of ISR or, equivalently, transverse momentum arising from decays that occur upstream in the decay chain. In this case, it may also be interesting to compute \(M_{T2}\) for subsystems [160], i.e. considering only part of each decay chain and treating the particles from upstream decays as “upstream transverse momentum” (UTM). If there is ISR or UTM, no analytic expression for \(M_{T2}\) is known but it can be computed numerically, using, e.g., the results of Ref. [161].

In this work, we use \(M_{T2}\) as a variable to distinguish potential new physics events from the SM backgrounds. The use of \(M_{T2}\) as a discovery variable was first proposed in Ref. [162], but here we follow a different approach. Several choices for the visible system used as input to \(M_{T2}\) can be considered: dijet events (as in Ref. [162]), the two jets with largest \(p_T\) in multijet events, or two systems of pseudojets defined by grouping jets together. In this study, we use the last method.

### 6.2 Hemispheres as Input to \(M_{T2}\)

A technique to group jets in multijet events into two visible systems, called pseudojets, is the “event hemispheres” method described in more detail in Ref. [163] (see Section 13.4). The hemisphere reconstruction works as follows: first, two initial axes (seeds) are chosen. Here, we take them as the directions of the two massless jets that yield the largest dijet invariant mass. Next, the other jets are associated to one of these axes according to a certain criterion (hemisphere association method). Here, we used the minimal “Lund distance” [100], generalized to the case of a massive particle \(i\) and a massless one \(k\) as

\[d^2_{ik} = \frac{1}{2}(E_iE_k - \vec{p}_i \cdot \vec{p}_k) \cdot \frac{4E_iE_k}{(E_i + E_k)^2} = 2(E_i - p_i \cos \theta_{ik}) \frac{E_iE_k^2}{(E_i + E_k)^2},\]  (6.6)

meaning that jet \(k\) is associated to the hemisphere with mass \(m_i\) rather than \(m_j\) if

\[(E_i - p_i \cos \theta_{ik}) \frac{E_i}{(E_i + E_k)^2} \leq (E_j - p_j \cos \theta_{jk}) \frac{E_j}{(E_j + E_k)^2}.\]  (6.7)

After all jets are associated to one or the other axis, the axes are recalculated as the sum of the momenta of all jets connected to a hemisphere and the association is iterated using these new axes until no jets switch from one group to the other.

This method was shown [163] to be about 85% efficient for the correct association of quark jets from squark decay in several CMSSM benchmark points at \(\sqrt{s} = 14\) TeV. It is, however, only 70-80% efficient for quark jets from gluinos, as they tend to be softer in the tested benchmark points. The efficiencies are
expected to be somewhat worse at 7 TeV, due to the lower energy. Moreover, the hemisphere reconstruction can also be affected by the presence of ISR jets.

### 6.3 Approximate Formula for $M_{T2}$

To illustrate the behavior of $M_{T2}$, we consider the simple example of $M_{T2}$ without ISR or UTM. It can be seen from Eqn. (6.5) that the angular and $p_T$ dependence of $M_{T2}$ is encoded in a variable $A_T$:

$$A_T = E_T^{\text{vis}(1)} E_T^{\text{vis}(2)} + \vec{p}_{T}^{\text{vis}(1)} \cdot \vec{p}_{T}^{\text{vis}(2)},$$

and $M_{T2}$ increases as $A_T$ increases. Therefore, the minimum value of $M_{T2}$ is reached in configurations where the visible systems are back-to-back. The maximum value is reached when they are parallel to each other and have large $p_T$.

We now assume that the visible systems’ mass is smaller than its transverse momentum, $m_{\text{vis}(i)} \ll |\vec{p}_{T}^{\text{vis}(i)}|$. To simplify the notation, we use the shorthand $m_i \equiv m_{\text{vis}(i)}$ and $p_i \equiv p_{T}^{\text{vis}(i)}$. Then, $A_T$ can be written as

$$A_T = p_1 p_2 \left[ \sqrt{1 + \frac{m_2^2}{p_1^2}} \right] \left[ \sqrt{1 + \frac{m_2^2}{p_2^2}} + \cos \phi_{12} \right] \simeq p_1 p_2 \left[ \frac{1}{2} \left( \frac{m_1^2}{p_1^2} + \frac{1}{2} m_2^2 + 1 + \cos \phi_{12} \right) \right] \tag{6.9}$$

$$= \frac{1}{2} p_1 p_2 \left[ \frac{m_1^2}{p_1^2} + \frac{m_2^2}{p_2^2} + 2(1 + \cos \phi_{12}) \right], \tag{6.10}$$

where $\phi_{12}$ is the angle between the two visible systems in the transverse plane. With this we obtain

$$A_T^2 - m_1^2 m_2^2 = \frac{1}{4} p_1^2 p_2^2 \left[ \left( \frac{m_1^2}{p_1^2} - \frac{m_2^2}{p_2^2} \right)^2 + 4 \left( \frac{m_1^2}{p_1^2} + \frac{m_2^2}{p_2^2} \right) (1 + \cos \phi_{12}) + 4(1 + \cos \phi_{12})^2 \right] \tag{6.11}$$

and thus for $M_{T2}$ we get

$$M_{T2}^2 = A_T + \sqrt{A_T^2 - m_1^2 m_2^2} \tag{6.12}$$

$$= \frac{1}{2} p_1 p_2 \left[ \frac{m_1^2}{p_1^2} + \frac{m_2^2}{p_2^2} + 2(1 + \cos \phi_{12}) \right] + \sqrt{ \left( \frac{m_1^2}{p_1^2} - \frac{m_2^2}{p_2^2} \right)^2 + 4 \left( \frac{m_1^2}{p_1^2} + \frac{m_2^2}{p_2^2} \right) (1 + \cos \phi_{12}) + 4(1 + \cos \phi_{12})^2 \right]. \tag{6.13}$$

Let us now assume that we are in a back-to-back configuration, such that $(1 + \cos \phi_{12}) = 0$. In this case, we obtain

$$M_{T2}^2 = \frac{1}{2} p_1 p_2 \left[ \frac{m_1^2}{p_1^2} + \frac{m_2^2}{p_2^2} + \sqrt{ \left( \frac{m_1^2}{p_1^2} - \frac{m_2^2}{p_2^2} \right)^2 } \right] \tag{6.15}$$

$$= \frac{1}{2} p_1 p_2 \left[ \frac{m_1^2}{p_1^2} + \frac{m_2^2}{p_2^2} + \frac{m_1^2}{p_1^2} - \frac{m_2^2}{p_2^2} \right]. \tag{6.16}$$

We conclude that large $M_{T2}$ values can still be obtained even if the visible systems are back-to-back, provided that the terms $\frac{m_{\text{vis}(i)}}{|\vec{p}_{T}^{\text{vis}(i)}|}$ are non-negligible. This is typically the case for QCD multijet events where at
least one of the visible systems consists of multiple jets and the invariant mass may be large. Choosing to define pseudojets as massless may therefore be a good approach towards suppressing QCD multijet events in the $M_{T2}$ tail.

If we now assign zero masses to the visible systems, and use again our original notation, Eqn. (6.14) implies that

$$(M_{T2})^2 = 2p_T^{\text{vis}(1)}p_T^{\text{vis}(2)}(1 + \cos \phi_{12}).$$

(6.17)

Note that this formula is exact in the case of massless visible systems, no ISR and $m_\tilde{\chi} = 0$. It can be seen that Eq. (6.17) corresponds to the transverse mass of two systems $(M_T)^2 = 2p_{T\text{sys}}^{\text{vis}(1)}p_{T\text{sys}}^{\text{vis}(2)}(1 - \cos \phi_{12})$, with $p_T^{\text{vis}} = -p_T^{\text{sys}}$ for one of the systems.

**Correlation between $M_{T2}$ and $E_T^{\text{miss}}$**

We follow the common practice to abuse notation with $E_T^{\text{miss}} \equiv \vec{p}_T^{\text{miss}}$. In typical SUSY events with large $E_T^{\text{miss}}$, the contribution from the $\phi_{12}$ term will tend to be large. Indeed, using $0 = E_T^{\text{miss}} + p_T^{\text{vis}(1)} + p_T^{\text{vis}(2)}$, we can write

$$(E_T^{\text{miss}})^2 = (|p_T^{\text{vis}(1)}| - |p_T^{\text{vis}(2)}|)^2 + 2|p_T^{\text{vis}(1)}||p_T^{\text{vis}(2)}|(1 + \cos \phi_{12}),$$

(6.18)

showing that in the case of symmetric events with $|p_T^{\text{vis}(1)}| \simeq |p_T^{\text{vis}(2)}|$, the expression (6.17) for $M_{T2}$ is nothing but $E_T^{\text{miss}}$ itself. For asymmetric events, we always have $M_{T2} < E_T^{\text{miss}}$. In Figure 6.2 we show the correlation between $M_{T2}$ and $E_T^{\text{miss}}$ for the LM6 CMSSM benchmark point and various SM backgrounds. The LM6 sample shows an accumulation of events along the diagonal $M_{T2} \approx E_T^{\text{miss}}$.

### 6.4 Kinematics of the SM Backgrounds in the $M_{T2}$ Tail

We have seen in the previous section that SUSY events with large expected $E_T^{\text{miss}}$ and jet acoplanarity will be concentrated in the large $M_{T2}$ region. In contrast, QCD dijet events, in which the two jets are back-to-back, populate the region of small $M_{T2}$ regardless of the value of $E_T^{\text{miss}}$ or jet $p_T$. In the present study, we choose the visible systems to be massless and set $m_\tilde{\chi} = 0$. Then, back-to-back dijet events will have $M_{T2} = 0$, as explained above. Hence, $M_{T2}$ has a built-in protection against jet mismeasurements in QCD dijet events, even if accompanied by large $E_T^{\text{miss}}$. However, QCD multijet events with large $E_T^{\text{miss}}$ may give rise to acoplanar pseudojets, leading to larger $M_{T2}$ values. For this reason, further protections against $E_T^{\text{miss}}$ from mismeasurements need to be introduced, as will be described in Section 7.1.2. Other SM backgrounds, such as $t\bar{t}$, single top-quark, and W+jets events with leptonic decays, or Z+jets events where the Z boson decays to neutrinos, contain true $E_T^{\text{miss}}$ and can also lead to acoplanar pseudojets.

In this section, we aim at shedding light on the kinematics of the QCD multijet, W+jets, Z+jets and $t\bar{t}$ events that populate the tail of the $M_{T2}$ distribution.
6.4. Kinematics of the SM Backgrounds in the $M_{T2}$ Tail

(a) Correlation between $M_{T2}$ and $E_T^{\text{miss}}$ for the LM6 SUSY signal.

(b) Correlation between $M_{T2}$ and $E_T^{\text{miss}}$ for QCD multijet events.

(c) Correlation between $M_{T2}$ and $E_T^{\text{miss}}$ for $Z(\nu\bar{\nu})+$jets events. The z-axis is in log-scale.

(d) Correlation between $M_{T2}$ and $E_T^{\text{miss}}$ for $t\bar{t}$ events. The z-axis is in log-scale.

Figure 6.2: Correlation between $M_{T2}$ and $E_T^{\text{miss}}$ for the LM6 SUSY signal and various SM backgrounds. The events along the diagonal are characterized by $|\vec{p}_T^{\text{vis}(1)}| \simeq |\vec{p}_T^{\text{vis}(2)}|$. The normalization of the z-axis is arbitrary.
6.4.1 $M_{T2}$ Kinematics of Electroweak Processes

$W(\ell\nu)$+jets and $Z(\nu\bar{\nu})$+jets events are the dominant SM background in the $M_{T2}$ tail for a hadronic events selection when no b-tagged jet is explicitly demanded, as will be shown in Chapter 7. While the contribution from leptonic $W$ boson decays is reduced by vetoing events with identified electrons and muons, the background from events where the lepton veto fails is still significant. These events contain electrons or muons which are outside the $p_T$ and $\eta$ acceptance or, to a lesser extent, fail the identification or isolation criteria (as will be explained in more detail in Section 7.3.3). The following considerations hold for $W$+jets and $Z$+jets events where the boson $p_T$ corresponds to $E_T^{\text{miss}}$.

We consider events with two visible systems (such as two pseudojets) and denote the $p_T$ of the $Z$ and $W$ bosons by $p_{TV}$. The transverse momenta of the two systems are denoted $|\vec{p}_{\text{vis}}^{(1)}| \equiv p_{T1}$ and $|\vec{p}_{\text{vis}}^{(2)}| \equiv p_{T2}$, with $p_{T1} \geq p_{T2}$. The azimuthal angle between them is called $\Delta \phi$. From momentum conservation we have:

$$p_{TV}^2 = p_{T1}^2 + p_{T2}^2 + 2p_{T1}p_{T2}\cos \Delta \phi \quad (6.19)$$

It is interesting to express the $p_{TV}$ as a function of the scalar sum of the pseudojet momenta $H_T$:

$$p_{TV}^2 = (p_{T1} + p_{T2})^2 - 2p_{T1}p_{T2} + 2p_{T1}p_{T2}\cos \Delta \phi \quad (6.20)$$

$$= H_T^2 - 2p_{T1}p_{T2}(1 - \cos \Delta \phi) \quad (6.21)$$

For $Z(\nu\bar{\nu})$+jets events and $W$+jets where the full $W$ boson $p_T$ is unobserved, the $p_{TV}$ represents $E_T^{\text{miss}}$. Moreover, defining $r = p_{T2}/p_{T1}$, we have

$$(E_T^{\text{miss}})^2 = H_T^2 - 2rp_{T1}^2(1 - \cos \Delta \phi) \quad (6.22)$$

$$= H_T^2 \left[1 - 2\frac{r}{(1 + r)^2}(1 - \cos \Delta \phi)\right] \quad (6.23)$$

and hence

$$\frac{E_T^{\text{miss}}}{H_T} = \sqrt{1 - 2\frac{r}{(1 + r)^2}(1 - \cos \Delta \phi)} \quad (6.24)$$

This shows that the $E_T^{\text{miss}}/H_T$ depends only on the ratio of the two pseudojet transverse momenta and on the angle between them, i.e. on shape variables. This also demonstrates that once the shape is fixed, the scale of $E_T^{\text{miss}}$ is set by the variable $H_T$. The dependence of $E_T^{\text{miss}}/H_T$ as a function of $\Delta \phi$ is illustrated in Figure 6.3a for several values of the ratio $r = p_{T2}/p_{T1}$. It is seen that the largest $E_T^{\text{miss}}$ for any momentum ratio is obtained at $\Delta \phi = 0$. For symmetric systems with $p_{T2}/p_{T1} = 1$, $E_T^{\text{miss}}$ goes to zero when $\Delta \phi = \pi$. For totally asymmetric pseudojets ($p_{T2}/p_{T1} = 0$), $E_T^{\text{miss}}$ remains maximal ($E_T^{\text{miss}} = H_T = p_{T1}$) independent of $\Delta \phi$.

From Eqn. (6.17) we further obtain

$$M_{T2}^2 = 2p_{T1}p_{T2}(1 + \cos \Delta \phi) = 2\frac{r}{(1 + r)^2}H_T^2(1 + \cos \Delta \phi), \quad (6.25)$$

and thus

$$\frac{M_{T2}}{H_T} = \sqrt{2\frac{r}{(1 + r)^2}(1 + \cos \Delta \phi)} \quad (6.26)$$
6.4. Kinematics of the SM Backgrounds in the $M_{T2}$ Tail

The dependence of $M_{T2}/H_T$ as a function of $\Delta \phi$ is shown in Figure 6.3b. In this case, only symmetric jet configurations reach the maximum $M_{T2}$ for $\Delta \phi = 0$. Asymmetric configurations reach a lower value at $\Delta \phi = 0$, although the difference is small for $p_{T2}/p_{T1} \geq 0.5$, and all drop to zero for increasing $\Delta \phi$. The totally asymmetric configuration has $M_{T2} = 0$ for all values of $\Delta \phi$.

From the comparison of Figures 6.3a and 6.3b (and the discussion in Section 6.3), we conclude that

- in all configurations we have $M_{T2} \leq E_T^{\text{miss}}$ and $M_{T2} \approx E_T^{\text{miss}}$ holds for symmetric events $|p_{T1}^{\text{vis}(1)}| \approx |p_{T1}^{\text{vis}(2)}|$ independently of the value of $\Delta \phi$;
- the events in the region of the largest $M_{T2}$ values correspond to nearly collinear and symmetric ($p_{T2}/p_{T1} \geq 0.5$) configurations with $M_{T2} \approx E_T^{\text{miss}} \approx H_T$ and large $H_T$;
- for very asymmetric jet configurations, $E_T^{\text{miss}}$ can remain large up to large values of $\Delta \phi$, but $M_{T2}$ tends to be small.

6.4.2 $M_{T2}$ Kinematics of QCD Multijet Events

Well-measured QCD dijet and multijet events, ignoring leptonic heavy-flavor decays, are characterized by back-to-back pseudojets with equal momenta. In the dijet case, the visible systems remain in a back-to-back configuration even if the momenta of the jets are mismeasured, and thus a small value for $M_{T2}$ is obtained. In QCD multijet events, however, mismeasurements of at least one jet can lead to pseudojets away from a back-to-back configuration. In all cases of mismeasurements, artificial $E_T^{\text{miss}}$ is generated, which satisfies
Chapter 6. $M_{T2}$ as a Discovery Variable

the conservation equation

\[
(E_{T}^{\text{miss}})^2 = p_{T1}^2 + p_{T2}^2 + 2p_{T1}p_{T2} \cos \Delta \phi, \tag{6.27}
\]

where the same convention for the variable names is used as above. Introducing again the ratio $r = p_{T2}/p_{T1}$, we obtain

\[
(E_{T}^{\text{miss}})^2 = p_{T1}^2 (1 + r^2 + 2r \cos \Delta \phi) \tag{6.28}
\]

and thus

\[
p_{T1}^2 = \frac{1}{1 + r^2 + 2r \cos \Delta \phi} (E_{T}^{\text{miss}})^2. \tag{6.29}
\]

Introducing this expression in Eqn. (6.17), we find

\[
M_{T2}^2 = 2p_{T1}^2 r (1 + \cos \Delta \phi) = \frac{2r(1 + \cos \Delta \phi)}{1 + r^2 + 2r \cos \Delta \phi} (E_{T}^{\text{miss}})^2, \tag{6.30}
\]

and finally

\[
\frac{M_{T2}}{E_{T}^{\text{miss}}} = \sqrt{\frac{2r(1 + \cos \Delta \phi)}{1 + r^2 + 2r \cos \Delta \phi}}. \tag{6.31}
\]

The dependence of $M_{T2}/E_{T}^{\text{miss}}$ as a function of $\Delta \phi$ is shown in Figure 6.4. Note that the fraction of pseudojet momentum lost due to mismeasurements is $(1 - r)$. If the event is well measured ($r = 1$), we have $M_{T2} = E_{T}^{\text{miss}} = 0$. In all other cases, $M_{T2} < E_{T}^{\text{miss}}$ and the events are shifted to lower $M_{T2}$ compared to the $E_{T}^{\text{miss}}$, showing again that $M_{T2}$ is to some extent protected against mismeasurements in QCD events. This effect is enhanced with decreasing values for $r$ and increasing $\Delta \phi$.

![Figure 6.4: $M_{T2}/E_{T}^{\text{miss}}$ as a function of $\Delta \phi$ for several values of the momentum ratio.](image)

As mentioned above, well-measured QCD multijet events have $\Delta \phi \approx \pi$. Jet mismeasurements, however, typically introduce a small kink resulting in $\Delta \phi$ between $\sim 2.5$ and $\pi$. This is exactly the region where
$M_{T2}$ tends to be much smaller than $E_T^{\text{miss}}$ (see Figure 6.4), showing that it is more advantageous to reject mismeasured QCD multijet events based on a minimal $M_{T2}$ rather than $E_T^{\text{miss}}$ requirement.

### 6.4.3 $M_{T2}$ Kinematics of $t\bar{t}$ Events

Top-quark production is an evident background in new physics searches, especially if the presence of a b-tagged jet is required. In order for a $t\bar{t}$ event to have large $M_{T2}$, large $E_T^{\text{miss}}$ needs to be present. The dominant contribution from the $t\bar{t}$ background to the hadronic SUSY search from Section 7.4 indeed stems from semileptonic decays with a high $p_T$ neutrino. The contribution from the fully leptonic channel is almost negligible, because of its small branching ratio and the need for two leptons to be “lost” for the event to pass the lepton veto.

The fully-leptonic channel, however, is an ideal $M_{T2}$ playground, with the two visible systems each consisting of a b quark and a charged lepton (see e.g. Ref. [164]). This is illustrated in Figure 6.5a. The only difficulty comes from the combinatorial problem of assigning the correct b quark to the corresponding charged lepton. The primary particle of the two symmetric decay chains is the top quark, and when using a zero test mass $m_{\tilde{\chi}}$ (the neutrino is nearly massless), the endpoint of the $M_{T2}$ distribution is indeed given by the top-quark mass.

The situation is different for the semileptonic case, as illustrated in Figure 6.5b. One visible system consists of a b quark only, whereas the other system contains the full hadronic top-quark decay. In such a situation in which the decay chains are asymmetric and only one side contributes to $E_T^{\text{miss}}$, it is not obvious that $M_{T2}$ would still be bounded from above by the mass of the common parent particle. However, we know from simulation that in this particular case $M_{T2} < m_t$ still holds. This can be understood quantitatively as follows.

First, we compute the transverse mass of the hadronically decaying top quark $M_T^{(1)}$. We denote the 4-
momentum of the visible system corresponding to the reconstructed hadronic top-quark as \((E_T^{(t)}, \vec{p}_T^{(t)})\). We further denote by \(\vec{p}_T^{\chi(1)}\) and \(\vec{p}_T^{\chi(2)}\) the two trial LSP momenta fulfilling the \(p_T^{\text{miss}}\) constraint, \(\vec{p}_T^{\chi(1)} + \vec{p}_T^{\chi(2)} = \vec{p}_T^{\text{miss}}\). We then have

\[
(M_T^{(1)})^2 = (E_T^{(t)} + |\vec{p}_T^{\chi(1)}|)^2 - (\vec{p}_T^{(t)} + \vec{p}_T^{\chi(1)})^2
\]

\[
= m_t^2 + 2|p_T^{\chi(1)}|(E_T^{(t)} - |p_T^{(t)}| |\cos \Delta \phi_t|),
\]

where we used \(E_T^{\chi(1)} = |\vec{p}_T^{\chi(1)}|\) since we assume a vanishing test mass \(m_\chi = 0\). By \(\Delta \phi_t\) we denote the azimuthal angle between the top quark and \(\vec{p}_T^{\chi(1)}\). We conclude that \(M_T^{(1)} > m_t\) holds for any trial LSP momentum \(\vec{p}_T^{\chi(1)}\).

For the transverse mass of the second system, a similar expression is obtained

\[
(M_T^{(2)})^2 = (E_T^{(b)} + |\vec{p}_T^{\chi(2)}|)^2 - (\vec{p}_T^{(b)} + \vec{p}_T^{\chi(2)})^2
\]

\[
= 2|p_T^{\chi(2)}||\vec{p}_T^{(b)}|(1 - \cos \Delta \phi_b),
\]

where \(\phi_b\) is the angle between the b quark and \(\vec{p}_T^{\chi(2)}\) and the small b-quark mass neglected.

We can now compare Eqn. (6.32) with Eqn. (6.34). If the leptonically decaying W boson \(p_T\) (which corresponds to \(E_T^{\text{miss}}\)) is small, the two trial LSP transverse momenta must balance \((\vec{p}_T^{\chi(1)} \approx \vec{p}_T^{\chi(2)}\)). In this case, we expect \(M_T^{(1)} > M_T^{(2)}\). In the other extreme in which \(\vec{p}_T^{(b)}\) is small, \(M_T^{(1)} > M_T^{(2)}\) is again expected, due to the large \(m_t^2\) term in Eqn. (6.32). These vague arguments suggest that \(M_T^{(1)} > M_T^{(2)}\) may hold for all trial LSP momenta fulfilling the \(E_T^{\text{miss}}\) constraint, implying that the solution for \(M_T^{(2)}\) would be unbalanced (see Section 6.1). In this case, we would have \(M_{T2} = m_t + m_\chi = m_t\). Since in the actual \(M_{T2}\) calculation we use massless visible systems, the value for \(M_{T2}\) could only decrease.

In practice, however, the situation is more complicated. ISR jets or UTM as well as jets assigned to the wrong visible system need to be taken into account. These effects give rise to a tail in the \(M_{T2}\) distribution, exceeding the top-quark mass, such that top-quark production is still an important background in a search for an excess due to new physics in the tail of the \(M_{T2}\) distribution.

### 6.5 On the Effect of Upstream Transverse Momentum on \(M_{T2}\)

The discussion in Sections 6.3, 6.4.1 and 6.4.2 were based on the simple expression (6.14), repeated here for convenience,

\[
(M_{T2})^2 = 2p_T^{\text{vis}(1)} p_T^{\text{vis}(2)} (1 + \cos \phi_{12}),
\]

valid in the case of massless visible systems, \(m_\chi = 0\) and no upstream transverse momentum \(p_T^{\text{upstream}}\). We used this simple expression to gain a qualitative understanding for the kinematic configurations of the main SM background at large \(M_{T2}\) values. In the search for SUSY using \(M_{T2}\) with the CMS detector to be discussed in Chapter 7, massless visible systems and a vanishing test mass \(m_\chi\) will be used. However, the \(p_T^{\text{upstream}}\) vector will generally be different from zero. The question thus arises to what level the above
6.5. On the Effect of Upstream Transverse Momentum on $M_{T2}$

conclusions are still valid in the realistic scenario with UTM. Hereafter, the $M_{T2}$ value obtained from Eqn. (6.36) will be referred to as $M_{T2 \text{approx}}$.

Figure 6.6 shows the difference between the exact $M_{T2}$ calculation and $M_{T2 \text{approx}}$ as a function of $p_T^{\text{upstream}}$ for the LM6 SUSY signal. In both calculations, we use $m_\chi = 0$ and massless pseudojets obtained from dividing each event in two hemispheres as described in Section 6.2. All selected jets, electrons and muons within $|\eta| < 2.4$ are used as input to the clustering algorithm and thus assigned to either of the two hemispheres. Since the $E_T^{\text{miss}}$ is calculated from all particle-flow candidates, as described in Section 5.6, UTM arises (mainly) from forward objects with $\eta > 2.4$ and can be expressed as

$$p_T^{\text{upstream}} = -E_T^{\text{miss}} - p_T^{\text{vis}(1)} - p_T^{\text{vis}(2)}. \tag{6.37}$$

It is seen from Figure 6.6 that the difference between $M_{T2}$ and $M_{T2 \text{approx}}$ broadens with increasing $p_T^{\text{upstream}}$, but never becomes larger than $p_T^{\text{upstream}}$ itself. This observation is found to hold for all samples of simulated events tested, including all SM backgrounds and a variety of SUSY signals. We elaborate on this effect in the following.

We recall that no closed, analytic expression exists for $M_{T2}$ in the general case with UTM. However, we have shown in Ref. [155] that $M_{T2}^2$ depends\(^1\) on the two terms $X$ and $Y$ defined as

$$X = 1/2(p_T^{\text{vis}(1)} + p_T^{\text{vis}(2)}) \cdot p_T^{\text{upstream}}, \tag{6.38}$$

$$Y = -(p_T^{\text{miss}} \cdot p_T^{\text{upstream}}) / |p_T^{\text{miss}}|, \tag{6.39}$$

which explicitly depend on $p_T^{\text{upstream}}$. While $p_T^{\text{upstream}}$ in known to enter in $M_{T2}$ also through different terms, it is $X$ and $Y$ which determine the configuration of the event in the transverse plane. Figure 6.7a shows the dependence of the difference between $M_{T2}$ and $M_{T2 \text{approx}}$ on $X$. It is seen that the distribution

\(^1\)While $M_{T2}$ depends explicitly on $X$, the dependence on $Y$ is found to be complicated involving various dimensionful coefficients. The exact dependence is not needed for the following discussion.
Chapter 6. $M_{T2}$ as a Discovery Variable

(a) The correlation between $(M_{T2} - M_{T2\,\text{approx}})$ and $X$.

(b) The correlation between $(M_{T2} - M_{T2\,\text{approx}})$ and $Y$.

Figure 6.7: $(M_{T2} - M_{T2\,\text{approx}})$ as a function of $X = 1/2(\vec{p}_T^{\text{vis}(1)} + \vec{p}_T^{\text{vis}(2)}) \cdot \vec{p}_T^{\text{upstream}}$ and $Y = -(\vec{p}_T^{\text{miss}} \cdot \vec{p}_T^{\text{upstream}})/|\vec{p}_T^{\text{miss}}|$ for the LM6 SUSY signal.

of $(M_{T2} - M_{T2\,\text{approx}})$ has a clear trend, showing that the difference is maximally affected positively (negatively) when the transverse momentum of the total visible system $\vec{p}_T^{\text{vis}} = \vec{p}_T^{\text{vis}(1)} + \vec{p}_T^{\text{vis}(2)}$ is parallel (anti-parallel) to the UTM vector $\vec{p}_T^{\text{upstream}}$. The correlation between $(M_{T2} - M_{T2\,\text{approx}})$ and $Y$ is shown in Figure 6.7b. There, it is seen that the difference is maximally affected positively (negatively) when the $\vec{p}_T^{\text{miss}}$ is anti-parallel (parallel) to $\vec{p}_T^{\text{upstream}}$. We thus conclude that

- the largest positive difference between $M_{T2}$ and $M_{T2\,\text{approx}}$ is obtained when $\vec{p}_T^{\text{upstream}}$ is parallel to $\vec{p}_T^{\text{vis}}$ and antiparallel to $\vec{p}_T^{\text{miss}}$;
- the largest negative difference between $M_{T2}$ and $M_{T2\,\text{approx}}$ is found when $\vec{p}_T^{\text{upstream}}$ is anti-parallel to $\vec{p}_T^{\text{vis}}$ and parallel to $\vec{p}_T^{\text{miss}}$.

These two event configurations are pictorially illustrated in Figures 6.8a and 6.8b. It is easily seen from the figures that for the maximally positive difference we have $|\vec{p}_T^{\text{miss}}| \geq |\vec{p}_T^{\text{vis}(1)} + \vec{p}_T^{\text{vis}(2)}|$, whereas for the maximally negative difference $|\vec{p}_T^{\text{miss}}|, |\vec{p}_T^{\text{upstream}}| \leq |\vec{p}_T^{\text{vis}(1)} + \vec{p}_T^{\text{vis}(2)}|$ is found.

While the two configurations illustrated in Figures 6.8a and 6.8b produce the largest difference between $M_{T2\,\text{approx}}$ and $M_{T2}$, the value of the difference is not know analytically. However, as noted above, it is known from simulation that $|M_{T2} - M_{T2\,\text{approx}}|$ is always smaller than $\vec{p}_T^{\text{upstream}}$. For reasons to be explained later, the restriction $\vec{p}_T^{\text{upstream}} < 70$ GeV will be used as an event selection criterion for the SUSY search using $M_{T2}$ with CMS described in Chapter 7. This criterion will thus implicitly make sure that the actual value for $M_{T2}$ deviates from the intuitive expression $M_{T2\,\text{approx}}$ (Eqn. (6.36)) by no more than 70 GeV.
On the Effect of Upstream Transverse Momentum on $M_{T2}$

Figure 6.8: The two event configurations responsible for the maximum positive and negative difference between $M_{T2}$ and $M_{T2\text{ approx}}$.

A Special Event Configuration

It is worth mentioning that there exists a special event configuration leading to vanishing $M_{T2}$, if calculated for massless visible systems and $m_{\tilde{\chi}} = 0$, as noticed in Ref. [165]. This is the case if the $\vec{p}_T^{\text{miss}}$ vector happens to lie in between the two visible systems $\vec{p}_T^{\text{vis}(1)}$ and $\vec{p}_T^{\text{vis}(2)}$, as illustrated in Figure 6.9. Is is seen from the figure that for such a configuration, $\vec{p}_T^{\text{miss}}$ can be decomposed in two trial LSP momenta aligned with $\vec{p}_T^{\text{vis}(1)}$ and $\vec{p}_T^{\text{vis}(2)}$ (indicated with the green, dashed arrows), in which case $M_{T2}^{(1)} = M_{T2}^{(2)} = 0$ results. Due to the minimization performed on the trial LSP momenta from Eqn. (6.2), $M_{T2} = 0$ is found.
Figure 6.9: Event configuration in which $\vec{p}_T^{\text{miss}}$ lies in between $\vec{p}_T^{\text{vis}(1)}$ and $\vec{p}_T^{\text{vis}(2)}$, leading to $M_{T2} = 0$. 
6.5. On the Effect of Upstream Transverse Momentum on $M_{T2}$
In this chapter we present a search for supersymmetry or similar new physics in hadronic final states in pp collisions collected with the CMS detector at the LHC at a center-of-mass energy of 7 TeV. The results are based on the data sample collected in 2011, corresponding to about 4.73 fb$^{-1}$ of integrated luminosity.

The search makes use of the “stransverse mass” variable $M_{T2}$ to select new physics candidate events. Although $M_{T2}$ was originally introduced to derive the masses of sparticles involved in the cascade decay, we use it here as a discovery variable since it is sensitive to the presence of SUSY-like new physics. As described in Section 6, the distribution of $M_{T2}$ reflects the produced particle masses, which are much lighter for the SM background processes than for the SUSY processes. Hence, new physics is expected to appear as an excess in the tail of $M_{T2}$.

The analysis is based on two complementary approaches. A first approach, the “$M_{T2}$ analysis”, targets events resulting from heavy sparticle production, characterized by large $E_T^{\text{miss}}$, at least three jets, and large $M_{T2}$. The SM backgrounds in the signal region consist of $W(\ell\nu)+$jets, $Z(\nu\nu)+$jets, $t\bar{t}$, and single-top events (the last two will be referred to collectively as top-quark background), which are estimated from data-control regions and simulation. This analysis loses sensitivity if the squarks are heavy and the gluinos light, in which case the production is dominated by gluino-gluino processes. The gluinos give rise to three-body decays with relatively small $E_T^{\text{miss}}$. Since the gluino decay is mediated by virtual squark exchange and the stop and sbottom are expected to be lighter than the first- and second-generation squarks, these events can be rich in b quarks. To increase the sensitivity to such processes, a second approach, the “$M_{T2}b$ analysis”, is developed, in which the threshold on $M_{T2}$ defining the signal region is lowered. To suppress the QCD multijet background, we demand at least one b-tagged jet and place a stricter requirement on the jet multiplicity. The $M_{T2}b$ analysis provides a larger signal-to-background ratio in the region of heavy squarks and light gluinos and hence improves our sensitivity to this scenario.
This analysis extends previous results of searches in fully hadronic final states from the CMS [166–169] and ATLAS [170–173] Collaborations. It is organized as follows: we present in Section 7.1 the data samples used and the event selection and proceed in Section 7.2 to outline the search strategy. This strategy is applied to the $M_{T2}$ analysis in Section 7.3 and to the $M_{T2}b$ analysis in Section 7.4. In these sections the background estimation methods are also discussed. Finally, we interpret the results in Section 7.5 in the context of the CMSSM as well as for a variety of simplified models. Note that this chapter is largely taken from work previously published in Ref. [174].

7.1 Event Selection

7.1.1 Data Samples and Simulation

Triggers

The data used in this analysis were collected by triggers based on the scalar sum of transverse momenta $H_T$ of energy-corrected calorimeter jets. Due to a continuous increase in the instantaneous luminosity of the LHC, the trigger evolved with time from the requirement $H_T > 440$ GeV to $H_T > 750$ GeV. In this analysis, only triggers with a threshold of 650 GeV or less have been used, corresponding to a total integrated luminosity of 4.73 fb$^{-1}$, slightly lower than the full 2011 dataset. This decision was taken as to avoid a significant increase in the offline $H_T$ requirement for a marginal gain in integrated luminosity only.

The turn-on curves of two $H_T$ trigger paths with different thresholds are shown in Figure 7.1. The trigger efficiency is calculated as the number of events passing both the trigger in question and a reference trigger divided by the total number of events passing the reference trigger. The reference trigger is a prescaled $H_T$ trigger with a lower threshold. The efficiencies are measured as a function of different offline $H_T$ computations: $H_T$ reconstructed from particle flow jets (black), and from calorimeter jets (blue). The turn-on is less steep for particle flow jets since calorimeter jets are used at HLT level. We conclude from the turn-on curves that the trigger paths used in this analysis are nearly 100% efficient for an offline $H_T > 750$ GeV requirement imposed on particle flow jets.

Simulated Events

The analysis is designed using simulated event samples created with the PYTHIA 6.4.22 [100] and MADGRAPH 5v1.1 [175] Monte Carlo event generators. These events are subsequently processed with a detailed simulation of the CMS detector response based on GEANT4 [176]. The events are reconstructed and analyzed in the same way as the data. The SUSY signal particle spectrum is calculated using SOFTSUSY [55] and for the decays SDECAY [177] is used. We use two CMS SUSY benchmark signal samples referred to as LM6 and LM9 [163] (as defined in Section 2.2.9) to illustrate possible CMSSM [54] yields. All samples are generated using the CTEQ6 [103] parton distribution functions (PDFs). For SM background simulated samples we use the most accurate calculation of the cross sections currently available, usually with next-to-
(a) Turn-on curve for a $H_T$ trigger path with a threshold of 600 GeV.
(b) Zoomed turn-on curve for a $H_T$ trigger path with a threshold of 600 GeV.

(c) Turn-on curve for a $H_T$ trigger path with a threshold of 650 GeV.
(d) Zoomed turn-on curve for a $H_T$ trigger path with a threshold of 650 GeV.

Figure 7.1: Turn-on curves for $H_T$ based trigger paths with different thresholds.
leading-order (NLO) accuracy. For the CMS SUSY benchmark signal samples we use NLO cross sections of 0.403 pb and 10.6 pb for LM6 and LM9, respectively, obtained by weighting the leading order cross sections from {\textsc{pythia}} with sub-process dependent K-factors calculated with {\textsc{prospino}} [56].

### 7.1.2 Signal Selection

In the following, we describe our cleaning and selection procedure common to the $M_{T2}$ and $M_{T2b}$ analyses. Since $E_T^{\text{miss}}$ is an important ingredient to $M_{T2}$, it is necessary to identify and filter out events that suffer from instrumental effects, such as calorimeter noise, and to protect against jet mismeasurements which are a potential source of artificial $E_T^{\text{miss}}$ in QCD multijet events. Secondly, physics backgrounds with true missing transverse energy, such as leptonic W-boson decays, (semi)leptonic decays of top quarks, or invisible Z boson decays need to be addressed.

#### Preselection

Events that contain a high number of poorly reconstructed tracks are likely to be introduced by beam induced hits in the pixel detector. To remove this background, each event with at least 10 tracks is required to have a fraction of high purity tracks of at least 20%. Furthermore, each event is required to contain at least one well reconstructed primary vertex [178].

Supersymmetric processes typically result in large hadronic activity (dominantly colored sparticles are produced) and large $E_T^{\text{miss}}$ (due to the LSPs). The combination of $H_T$ and $E_T^{\text{miss}}$ is thus a reasonable starting point to search for new physics. The criterion on $H_T$ is predetermined by the used triggers, as outlined in Section 7.1.1. At analysis level, we calculate $H_T$ from fully corrected particle-flow jets with $p_T > 50$ GeV and $|\eta| < 2.4$. We require $H_T > 750$ GeV to make the triggers nearly 100% efficient. At least three jets are further required, where a $p_T$ threshold of 40 GeV is used for jet counting. We demand the two leading jets to have $p_T > 100$ GeV. The value of $E_T^{\text{miss}}$ is required to exceed 30 GeV, to assure a meaningful direction for the computation of $M_{T2}$.

#### Cleaning of Instrumental Effects

Fully hadronic analyses sensitive to an excess in $E_T^{\text{miss}}$-like distributions are especially sensitive to badly measured events and noise. This is because anomalous detector noise can bias the $E_T^{\text{miss}}$ calculation, artificially promoting uninteresting events to the signal region. To reject events containing beam background or anomalous calorimeter noise, we apply a set of dedicated event filters, designed to isolate certain known sources of noise. These include filters against beam halo particles, various sources of HCAL noise, tracking failure, and biases in $E_T^{\text{miss}}$ due to ECAL dead cells. In Figure 7.2 we show the $M_{T2}$ distribution in data from events flagged as bad by any of the applied noise filters. It can be seen that there is a large accumulation of noise-tagged events in the tail of the $M_{T2}$ distribution. It was found that most of the tagged events with $M_{T2} \approx 700$ GeV show a distinct signature: an energetic deposit in the HCAL at constant $\phi$ over large range of $\eta$. This anomalous signature gives rise to at least two acoplanar jets with nearly identical $\phi$. Such events
Figure 7.2: Observed $M_{T2}$ distribution overlaid to the events flagged as noisy by a set of standard event filters.

Figure 7.3: $M_{T2}$ distribution in QCD multijet events for different intervals of $p_{T}^{\text{upstream}} = |\vec{E}_{T}^{\text{miss}} - \vec{H}_{T}^{\text{miss}}|$. The distributions are normalized to unit area.

are characterized by $H_{T} = E_{T}^{\text{miss}} \approx M_{T2}$ (as can be seen from Eqn. (6.18) and from Figure 6.3). Due to the $H_{T}$ preselection requirement of 750 GeV, these events naturally show up at $M_{T2} \approx 750$ GeV. After filtering out noisy events, we carefully inspect the tail of the $M_{T2}$ and $E_{T}^{\text{miss}}$ distributions by hand for signs of anomalous instrumental effects and noise.

Events containing jet candidates with $p_{T} > 50$ GeV that fail the jet identification criteria (described in Section 5.5.2) are rejected. To veto events where a significant fraction of the momentum imbalance arises from forward or soft jets, a maximum difference of 70 GeV is imposed on the modulus of the difference between the $\vec{E}_{T}^{\text{miss}}$ and $\vec{H}_{T}^{\text{miss}}$ vectors, where $\vec{H}_{T}^{\text{miss}}$ is the negative vector sum of all selected jets (with a maximal $|\eta| = 2.4$). Events rejected by this criterion are potentially badly measured and thus vetoed. In Figure 7.3 we show the $M_{T2}$ distribution for QCD multijet events for different intervals of $|\vec{E}_{T}^{\text{miss}} - \vec{H}_{T}^{\text{miss}}|$. We observe that large values of $|\vec{E}_{T}^{\text{miss}} - \vec{H}_{T}^{\text{miss}}|$ distort the shape of the $M_{T2}$ distribution, promoting QCD multijets events to larger $M_{T2}$ values. We recall from the discussion in Section 6.5 that the $|\vec{E}_{T}^{\text{miss}} - \vec{H}_{T}^{\text{miss}}| < 70$ GeV requirement is equivalent to restricting the upstream transverse momentum $p_{T}^{\text{upstream}}$ in the $M_{T2}$ calculation to 70 GeV. This restriction ensures that the simplified expression $M_{T2}^{2} \approx 2p_{T}^{(1)}p_{T}^{(2)}(1 + \cos \phi_{12})$ deviates from the exact $M_{T2}$ calculation by no more than 70 GeV.

Selection Criteria to Reduce the QCD Multijet Background

QCD multijet events with large $E_{T}^{\text{miss}}$ arise from jet mismeasurements or leptonic heavy flavor decays. The weak decay of a $b$ or $c$ quark can give rise to an unobserved neutrino, carrying away a fraction of the quark’s momentum in a direction typically close to the one of the parent quark. Consequently, the
the $E_T^{\text{miss}}$ vector is expected to be aligned in the transverse plane with the jet. A similar signature results from a mismeasurement of the $p_T$ of a single jet. For this reason, the QCD multijet background with large $E_T^{\text{miss}}$ can be reduced by requiring a minimal azimuthal difference $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}}) > 0.3$ between the directions of $E_T^{\text{miss}}$ and the jets. The correlation between $M_{T2}$ and $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}})$ for QCD multijet events is shown in Figure 7.4a. The efficiency of the $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}}) > 0.3$ criteria largely depends on

![Figure 7.4: Correlation between $M_{T2}$ and $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}})$ for simulated QCD multijet events.](image)

(a) Correlation between $M_{T2}$ and $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}})$ for simulated QCD multijet events.

![Figure 7.4: $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}})$ for QCD multijet events for three different bins of $E_T^{\text{miss}}$. The distributions are normalized to unit area.](image)

(b) $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}})$ for QCD multijet events for three different bins of $E_T^{\text{miss}}$. The distributions are normalized to unit area.

the jet $p_T$ threshold as well as on the number of jets used for the calculation of the variable. Using a low jet $p_T$ threshold, for instance, the $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}}) > 0.3$ requirement may also protect against events with a severely undermeasured jet whose $p_T$ would otherwise fall below the threshold. However, a low $p_T$ threshold also increases the chances for a healthy event with genuine $E_T^{\text{miss}}$ to have accidentally a jet aligned with $E_T^{\text{miss}}$. This is especially important to consider in a search for a signal with large jet multiplicity. To address this issue, we defined two working points (WP) for the $\Delta \phi_{\text{min}}(\text{jets}, E_T^{\text{miss}})$ variable:

- WP1: all jets with $p_T > 20$ GeV,
- WP2: leading 4 jets with $p_T > 20$ GeV.

The WP1 has been found optimal for the $M_{T2}$ analysis, whereas the WP2 was designed for the $M_{T2b}$ analysis, which targets a signal with large jet multiplicity.

Selection Criteria to Reduce Electroweak and Top-Quark Backgrounds

The leptonic $W$-boson decay accompanied by hard jets is a physics process associated with a naturally large $E_T^{\text{miss}}$, and potentially also large $M_{T2}$. In order to curb the contribution from such events, we veto events
containing at least one electron or muon as defined in Sections 5.1 and 5.2.

**Pileup Dependence of the Event Selection**

We check the dependence of the used kinematic variables on the number of pileup interactions. Ideally, the pileup dependence of jets and variables depending on jet kinematics is small, after having applied the pileup subtraction technique as explained in Section 5.5.1. In Figure 7.5 we show the $H_T$ distribution as well as the jet multiplicity as a function of the number of reconstructed primary vertices in $Z(\nu\bar{\nu})$+jets simulated events. It can be seen that the pileup dependence of these distributions is negligible.

![Graphs showing pileup dependence of $H_T$ and jet multiplicity in $Z(\nu\bar{\nu})$+jets simulated events.](image)

(a) Pileup dependence of $H_T$ in $Z(\nu\bar{\nu})$+jets simulated events. The last bin contains the overflow. (b) Pileup dependence of the jet multiplicity in $Z(\nu\bar{\nu})$+jets simulated events.

**Figure 7.5:** Pileup dependence of the jet multiplicity and $H_T$ distribution in $Z(\nu\bar{\nu})$+jets simulated events. The inclusive distribution (black) is split in subsamples with differing intervals of number of pileup interactions. The ratio plot shows the subsamples divided by the inclusive distribution. A $p_T$ threshold of 40 GeV is used for jet counting, as specified in the preselection requirements.

Applying the pileup subtraction to jets, however, does not cure the pileup dependence of all objects and variables relevant to this analysis. For instance, the lepton isolation efficiency decreases with increasing number of pileup interactions. This is, however, not a problem, as long as the pileup profile in simulation matches the one in data and thus no bias is introduced when exploiting the simulation to estimate backgrounds. For this, we reweight the number of pileup interactions generated in the simulated events to the distribution expected in data using the instantaneous luminosity profile for the relevant data-taking period. While this reweighting is performed on all samples of simulated events before applying any selection re-
Figure 7.6: Distribution of the number of primary vertices, for an electroweak control region and reweighting the pileup distribution in simulation.

requirements, it is vital to check the distribution of the number of vertices within the relevant event selection. This is shown in Figure 7.6 for an electroweak enhanced control region: $M_{T2} > 150$ GeV is required and the lepton veto is omitted. This distribution can be seen as a sanity-check, indicating that the pileup dependence of the kinematic distributions and variables used for the event selection is well modelled.

The $M_{T2}$ and $M_{T2}b$ Analyses

The $M_{T2}$ variable is computed after applying the selection criteria outlined in this section. We separately consider fully hadronic channels with $\geq 3$ jets and a tight $M_{T2}$ requirement (the $M_{T2}$ analysis), which is mostly sensitive to signal regions with large squark and gluino masses, and channels with $\geq 4$ jets, at least one tagged b jet, and a relaxed $M_{T2}$ requirement (the $M_{T2}b$ analysis), which increases sensitivity to regions with small gluino and large squark masses. The event selections of the two analyses are pictorially illustrated in Figure 7.7.

7.2 Search Strategy

We do not expect a significant number of QCD multijet events to appear in the signal regions at large $M_{T2}$. Nonetheless, we estimate an upper limit on the remaining QCD multijet background in the signal regions from data control samples. The main backgrounds, consisting of W+jets, Z+jets, and top-quark production, are evaluated from data control samples and simulation. A common strategy is applied to both the $M_{T2}$ and $M_{T2}b$ analyses:
• Two regions are defined in $H_T$, a low $H_T$ region $750 \leq H_T < 950\, \text{GeV}$ and a high $H_T$ region $H_T \geq 950\, \text{GeV}$. In each region, several adjacent bins in $M_{T2}$ are defined: five bins for the $M_{T2}$ analysis and four for the $M_{T2b}$ analysis. The lowest bin in $M_{T2}$ is chosen such that the expected QCD multijet background remains a small fraction of the total background. For the $M_{T2}$ analysis the lowest bin starts at $M_{T2} = 150\, \text{GeV}$ and for $M_{T2b}$ at $M_{T2} = 125\, \text{GeV}$.

• A dedicated method for each background is designed to estimate its contribution in the signal region from data control samples and simulation. The number of events and their relative systematic uncertainties are computed by means of these methods in each $H_T$, $M_{T2}$ bin. The methods are designed such that the resulting estimates are largely uncorrelated statistically.

• The predicted number of events for all background components and their uncertainties are combined, resulting in an estimate of the total background yield and its uncertainty in each bin.

• The estimated number of background events for each bin is compared to the number of observed events, and the potential contribution from a SUSY signal is quantified by a statistical method described in Section 7.5.

7.3 $M_{T2}$ Analysis

Figure 7.8 shows the measured $M_{T2}$ distribution in comparison to simulation. For $M_{T2} < 80\, \text{GeV}$ the distribution is completely dominated by QCD multijet events. For medium $M_{T2}$ values, the distribution is dominated by W+jets and Z($\nu\bar{\nu}$)+jets events with some contribution from top-quark events, while in the tail of $M_{T2}$ the contribution from top-quark production becomes negligible and Z($\nu\bar{\nu}$)+jets together with W+jets events dominate. We observe good agreement between data and simulation in the core as well as in
Figure 7.8: The $M_{T2}$ distribution with all selection requirements applied and $H_T \geq 750$ GeV. The different predictions for the SM backgrounds from simulation are stacked on top of each other. The LM6 signal distribution is not stacked. All distributions from simulation are normalized to the integrated luminosity of the data.
Table 7.1: Observed number of events and expected SM background yields from simulation in $M_{T2}$ bins for the low and high $H_T$ regions. These numbers are for guidance only and are not used in the final background prediction.

<table>
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<th>$W$+jets</th>
<th>Top</th>
<th>$Z(\nu\nu)$+jets</th>
<th>Total SM</th>
<th>Data</th>
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<td>9.22e+02</td>
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<td>9.40</td>
<td>20.3</td>
<td>50.3</td>
<td>69</td>
</tr>
<tr>
<td>$M_{T2}[275, 375]$</td>
<td>0.0</td>
<td>9.74</td>
<td>2.74</td>
<td>11.6</td>
<td>24.1</td>
<td>19</td>
</tr>
<tr>
<td>$M_{T2}[375, 500]$</td>
<td>0.0</td>
<td>3.63</td>
<td>0.69</td>
<td>6.07</td>
<td>10.4</td>
<td>8</td>
</tr>
<tr>
<td>$M_{T2}[500, \infty]$</td>
<td>0.0</td>
<td>1.54</td>
<td>0.20</td>
<td>3.55</td>
<td>5.29</td>
<td>6</td>
</tr>
<tr>
<td>$H_T \geq 950$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{T2}[0, \infty]$</td>
<td>1.22e+05</td>
<td>4.39e+02</td>
<td>6.32e+02</td>
<td>1.42e+02</td>
<td>1.23e+05</td>
<td>1.19e+05</td>
</tr>
<tr>
<td>$M_{T2}[150, 200]$</td>
<td>9.84</td>
<td>19.8</td>
<td>11.7</td>
<td>12.9</td>
<td>54.2</td>
<td>70</td>
</tr>
<tr>
<td>$M_{T2}[200, 275]$</td>
<td>0.47</td>
<td>13.7</td>
<td>5.25</td>
<td>10.5</td>
<td>30.0</td>
<td>23</td>
</tr>
<tr>
<td>$M_{T2}[275, 375]$</td>
<td>0.04</td>
<td>6.43</td>
<td>1.83</td>
<td>6.42</td>
<td>14.7</td>
<td>9</td>
</tr>
<tr>
<td>$M_{T2}[375, 500]$</td>
<td>0.0</td>
<td>1.63</td>
<td>0.40</td>
<td>2.54</td>
<td>4.57</td>
<td>8</td>
</tr>
<tr>
<td>$M_{T2}[500, \infty]$</td>
<td>0.0</td>
<td>1.10</td>
<td>0.16</td>
<td>2.16</td>
<td>3.42</td>
<td>4</td>
</tr>
</tbody>
</table>
the tail of the distribution. The white histogram (black dotted line) corresponds to the LM6 signal. It can be noted that in the presence of signal, an excess in the tail of $M_{T2}$ is expected.

The corresponding event yields for data and SM simulated samples, after the full selection and for the various bins in $M_{T2}$, are given in Table 7.1 for the low and the high $H_T$ regions. Contributions from other backgrounds, such as $\gamma$+jets, $Z(\ell\ell)$+jets and diboson production, are found to be negligible. It is seen that for all but one $M_{T2}$ bin, the observed number of events agrees within the uncertainties with the SM background expectation from simulation. In the low $H_T$ region, the $M_{T2}$ bin [200, 275] GeV exhibits an excess in data compared to background. We investigated whether the origin could be instrumental in nature, but did not find evidence for it. It could be of statistical origin. The excess has a marginal impact on the final observed limit. Note that the background numbers in Table 7.1 are for guidance only. Dedicated methods to estimate the different background contributions from control regions in the data and simulation are described in the following.

7.3.1 Prediction of the QCD Multijet Background

The simulation predicts that the QCD multijet background is negligible in the tail of the $M_{T2}$ distribution. Nevertheless, a dedicated method using a data control region was designed to verify that this is indeed the case.

We base this estimation on $M_{T2}$ and $\Delta \phi_{\text{min}} \equiv \Delta \phi_{\text{min}}(\text{jets}, E^\text{miss}_T)$, which is the difference in azimuth between $E^\text{miss}_T$ and the closest jet. The background in the signal region, defined by $\Delta \phi_{\text{min}} \geq 0.3$ and large $M_{T2}$, is predicted from a control region with $\Delta \phi_{\text{min}} \leq 0.2$. The two variables are strongly correlated, as illustrated in Figure 7.4a. However, a factorization method can still be applied if the functional form is known for the ratio of the number of events $r(M_{T2}) = N(\Delta \phi_{\text{min}} \geq 0.3)/N(\Delta \phi_{\text{min}} \leq 0.2)$ as a function of $M_{T2}$. The ratio $r(M_{T2})$ is displayed in Fig. 7.9 for simulated QCD multijet events. We find that for $M_{T2} > 50$ GeV the ratio falls off exponentially\(^1\). Therefore, a parameterization of the form

$$r(M_{T2}) = \frac{N(\Delta \phi_{\text{min}} \geq 0.3)}{N(\Delta \phi_{\text{min}} \leq 0.2)} = \exp(a - b M_{T2}) + c \quad (7.1)$$

has been used for $M_{T2} > 50$ GeV. The function is assumed to reach a constant value at large $M_{T2}$ due to extreme tails of the jet energy resolution response. In the following, we will describe how the parameters $a$ and $b$ can be extracted from data in a QCD dominated region. The parameter $c$ cannot be directly measured, but needs to be carefully estimated.

First, we study the method in simulation. The red curve in Figure 7.9 corresponds to a fit of the function in Eqn. (7.1) to QCD multijet events starting from 50 GeV. The parameters $a$, $b$, and $c$ are extracted from fitting the full $M_{T2}$ spectrum, starting at 50 GeV. The results for the fit parameters for both the high $H_T$ and low $H_T$ regions are shown in Table 7.2. The fitted parameter value for $c$ is compatible with a negligible QCD multijet contribution at large $M_{T2}$. In a second step, the fit is limited to the region $50 < M_{T2} < 80$ GeV, where contributions from background processes other than that from QCD multijets is small. As shown in

\(^1\)The exact value at which the exponential fall off starts depends on the imposed $E^\text{miss}_T$ preselection cut, which is here 30 GeV.
Figure 7.9: The ratio $r(M_{T2})$ as a function of $M_{T2}$ for QCD multijet simulated events. The red curve corresponds to an exponential plus constant fit in the region $M_{T2} > 50$ GeV. The green curve corresponds to an exponential fit in the region $50 < M_{T2} < 80$ GeV. The blue curve corresponds to an exponential plus a constant where the parameters of the exponential are taken from the previous fit, and the constant term is taken as the value of the exponential at $M_{T2} = 250$ GeV.

Table 7.2: Fit results for QCD multijet simulated events for both bins in $H_T$. The fit parameters are obtained twice from applying the fit to the inclusive $M_{T2} > 50$ GeV region and to the QCD multijet dominated region with $50 < M_{T2} < 80$ GeV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$750 &lt; H_T &lt; 950$ GeV</th>
<th>$H_T &gt; 950$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{T2} &gt; 50$ GeV</td>
<td>$M_{T2} &gt; 50$ GeV</td>
</tr>
<tr>
<td></td>
<td>$50 &lt; M_{T2} &lt; 80$ GeV</td>
<td>$50 &lt; M_{T2} &lt; 80$ GeV</td>
</tr>
<tr>
<td>$a$</td>
<td>$2.5 \pm 0.3$</td>
<td>$1.56 \pm 0.10$</td>
</tr>
<tr>
<td>$b$ [GeV$^{-1}$]</td>
<td>$(4.2 \pm 0.5) \times 10^{-2}$</td>
<td>$(2.39 \pm 0.13) \times 10^{-2}$</td>
</tr>
<tr>
<td>$c$</td>
<td>$(1.5 \pm 1.5) \times 10^{-2}$</td>
<td>$(3.0 \pm 1.4) \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Table 7.2, similar fit results for the parameters \(a\) and \(b\) are obtained. We further checked the robustness of the prediction by systematically varying the fit boundaries.

For the final results, we repeat the fit to data in the region \(50 \leq M_{T2} \leq 80\) GeV, after subtracting the small W+jets, Z+jets and top-quark background contributions using simulation. This is shown in Figure 7.10. The fitted parameter values for \(a\) and \(b\) are given in Table 7.3.

Table 7.3: Results for the parameters \(a\) and \(b\) when the fit is applied to data in the region \(50 \leq M_{T2} \leq 80\) GeV. Before the fit is performed, the small contribution from electroweak and top-quark backgrounds are subtracted with simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(750 &lt; H_T &lt; 950) GeV</th>
<th>(H_T &gt; 950) GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>1.61 ± 0.12</td>
<td>1.51 ± 0.16</td>
</tr>
<tr>
<td>(b) [GeV(^{-1})]</td>
<td>((2.5 ± 0.2) \times 10^{-2})</td>
<td>((2.2 ± 0.3) \times 10^{-2})</td>
</tr>
</tbody>
</table>

The constant term \(c\) cannot be extracted from data. Fixing \(c\) to the value obtained from simulation could potentially be dangerous, as the extreme tail of the jet energy resolution response might not be accurately modelled. For this reason, we conservatively fix the constant \(c\) to the value of the exponential at \(M_{T2} = 250\) GeV, where agreement with data can still be verified (see Figure 7.10).
Chapter 7. Search for Supersymmetry in Hadronic Final States Using $M_{T2}$

The results of the method are summarized in Table 7.4. In the lower $M_{T2}$ bins, where the exponential term dominates, the method reliably predicts the QCD multijet background. For higher $M_{T2}$ bins, where the constant term dominates, the method overestimates the number of QCD multijet events relative to the simulation, nonetheless confirming that the QCD multijet contribution is negligible. It is important to note that the conservative overestimate of the QCD multijet background in the very tail does not lead to aggressive exclusion limits on potential signals, since the final QCD multijet background prediction is still negligible compared to the total background in the tail.

Table 7.4: Prediction of the QCD multijet background from a fit to data with the parameter $c$ conservatively fixed to the value of the exponential at $M_{T2} = 250$ GeV. The results are compared to the estimate from simulation.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$750 &lt; H_T &lt; 950$ GeV</th>
<th>$H_T &gt; 950$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>simulation</td>
<td>data-prediction</td>
<td>simulation</td>
</tr>
<tr>
<td>[150,200)</td>
<td>3.08</td>
<td>9.84</td>
</tr>
<tr>
<td>[200,275)</td>
<td>0.0</td>
<td>0.47</td>
</tr>
<tr>
<td>[275,375)</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>[375,500)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>[500,∞)</td>
<td>0.0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

In order to check how potential signal contamination affects the QCD multijet background estimation, we add LM6 signal events (as defined in Section 2.2.9) to the data in the lower $\Delta \phi_{\text{min}}$ distribution. This results in a QCD multijet background estimate of 0.29 ± 0.2 events for $M_{T2} > 400$ GeV. The actual LM6 event yield in the same signal region amounts to 59 events, indicating that the overestimation of the QCD multijet background has a negligible effect.

The extreme case of total loss of a jet, leading to population of the high $M_{T2}$ tail, is studied in data using a sample of high $p_T$ mono-jet events obtained with a dedicated event selection. The total number of events is found to be compatible within the uncertainties with the number expected from the electroweak processes, confirming that the QCD multijet contribution is negligible and hence that the constant $c$ is small.

7.3.2 Prediction of the $Z(\nu \bar{\nu})$+jets Background

The estimate of the $Z(\nu \bar{\nu})$+jets background is obtained independently from two distinct data samples, one containing $\gamma$+jets events and the other $W(\mu \nu)$+jets events. In both cases the invisible decay of the $Z$ boson is mimicked by removing, respectively, the photon and the muon from the event, and adding vectorially the corresponding $\vec{p}_T$ to $\vec{E}_{T}^{\text{miss}}$. Since the $Z(\nu \bar{\nu})$+jets background estimates from the $\gamma$+jets and $W(\mu \nu)$+jets are statistically uncorrelated, we take the weighted average of the two predictions as the final estimate.
7.3.2.1 Method using $\gamma$+jets events

As pointed out in Ref. [179], the Z+jets and $\gamma$+jets processes differ because of the different electroweak vertex factors and the non-zero Z boson mass. For vector boson transverse momenta much larger than $m_Z$, we expect the ratio of photon over Z production to be determined by the ratio of the respective couplings to quarks and thus to flatten out. Since the event kinematics (including the hadronic activity) are very similar at high boson $p_T$, $\gamma$+jets events are ideally suited to predict the $Z(\nu\bar{\nu})$+jets irreducible background in the signal region. In Figure 7.11 we show differential production cross sections for Z+jets and $\gamma$+jets as a function of the boson $p_T$ as obtained from MadGraph 5v1.1 simulation. Above 300 GeV, the $Z(\nu\bar{\nu})/\gamma$ ratio is essentially flat. This correspondence between $\gamma$ and Z production at high $p_T$ is the foundation of $Z(\nu\bar{\nu})$+jets background predictions obtained from $\gamma$+jets events.

![Graphs showing differential cross sections for $Z(\nu\bar{\nu})$ and $\gamma$ as a function of vector boson transverse momentum.](image)

**Figure 7.11:** MadGraph 5v1.1 $Z(\nu\bar{\nu})$ and $\gamma$ production cross sections as a function of the vector boson transverse momentum.

**Relating the $Z(\nu\bar{\nu})$+jets to the $\gamma$+jets Event Yields**

To model the $Z(\nu\bar{\nu})$+jets background, we select data events that pass all analysis selection criteria and contain an identified and isolated photon as defined in Section 5.4. To mimic the $Z(\nu\bar{\nu})$ event kinematics, we add vectorially the reconstructed photon $\vec{p}_T$ to $E_T^{\text{miss}}$. Furthermore, to reduce the contamination from a potential signal to this event selection, we require $E_T^{\text{miss}}$ before adding the photon to it be less than 100 GeV. The $Z(\nu\bar{\nu})$+jets background is obtained as

$$N_{Z(\nu\bar{\nu})}^{\text{pred}}(M_{T2}) = R_{Z(\nu\bar{\nu})/\gamma}^{\text{sim}} \cdot P \cdot N_{\gamma}^{\text{data}}(M_{T2}).$$

(7.2)
Here, $N^{\text{data}}_{\gamma}(M_{T2})$ denotes the observed number of events in an interval of $M_{T2}$. To account for a small contribution of non-prompt photons, this number is corrected by the purity $P$ of the photon selection. The ratio $R^\text{sim}(Z(\nu\bar{\nu})/\gamma)$ corrects for photon acceptance and reconstruction efficiency as well as residual kinematic differences between $\gamma$+jets and $Z(\nu\bar{\nu})$+jets, and is obtained from simulation.

**Prompt Photons in Simulation**

Prompt photons arise from direct photon production as well as through parton-to-photon fragmentation. While the fragmentation photon over direct photon ratio is calculated\(^2\) to be smaller than 7% for isolated photons with $p_T > 100$ GeV, the two contributions are accounted for in the simulation. At the detector level, the purity of the prompt photon selection is diluted by the background arising from neutral meson decays (mainly $\pi^0$) produced in QCD jets. To also appropriately take into account these fake photons, we combine simulated samples of QCD multijet events with prompt $\gamma$+jets events when comparing data to simulation. However, the simulated samples for QCD multijet and prompt $\gamma$+jets events generated with MADGRAPH 5v1.1 are not mutually exclusive in the photon production. For instance, a QCD 2-to-2 process with a final-state radiation (FSR) photon added by the parton shower would be double counted since already included in prompt $\gamma$+jets simulation through the $\gamma + jj$ final state. Moreover, the MADGRAPH 5v1.1 $\gamma$+jets simulation includes phase space cuts as to avoid collinear and soft divergencies. The minimal distance of the photon to any parton, for instance, is requested to be $\Delta R > 0.3$. In order not to double count the contributions from QCD multijet and $\gamma$+jets simulation to prompt photon production, the two samples are combined according to the following prescription: For QCD multijet events, we form parton jets from the status 2 quarks and gluons (i.e. after parton showering) and compute the minimal $\Delta R$ of the prompt photons (coming from a status 3 parton) w.r.t. these parton jets. If the minimal $\Delta R$ exceeds 0.3, the event is vetoed.

**Measuring the Purity of the Photon Selection.**

While the isolation requirements of the photon selection remove the bulk of the neutral-meson background, a substantial contribution remains. This contribution is mainly due to fluctuations in the fragmentation of partons, where neutral mesons carry most of the energy and are isolated. To measure the contribution of this non-prompt photon background, we use a method similar to the one used in Ref. [180], exploiting the difference between the electromagnetic shower profiles of prompt photons and of photon pairs from neutral-meson decays. We use a modified second moment of the electromagnetic energy cluster about its mean $\eta$ position, denoted by $\sigma_{i\eta i\eta}$, which expresses the extent in $\eta$ of the cluster. It is defined by

$$
\sigma_{i\eta i\eta}^2 = \frac{\sum_{i}^{5\times5} w_i (\eta_i - \bar{\eta}_{5\times5})^2}{\sum_{i}^{5\times5} w_i}, \quad w_i = \max(0, 4.7 + \ln \frac{E_i}{E_{5\times5}}),
$$

where $E_i$ and $\eta_i$ are the energy and pseudorapidity of the $i^{\text{th}}$ crystal within the $5 \times 5$ electromagnetic cluster and $E_{5\times5}$ and $\bar{\eta}_{5\times5}$ are the energy and $\eta$ of the entire $5 \times 5$ cluster. Clusters belonging to isolated prompt

\(^2\)The calculation is documented in a CMS internal publication: CMS-AN-10-272.
photons tend to have very narrow and symmetric $\sigma^2_{\eta_i\eta_i}$ distributions, whereas photons produced in hadron decays show $\sigma^2_{\eta_i\eta_i}$ dominated by a long tail towards higher values.

We measure the purity of our photon selection as follows. A sample of events with identified and isolated photons with $p_T > 20$ GeV is selected, where all selection requirements are applied but no restriction on $M_{T2}$ is imposed. The event sample is dominated by low $p_T$ photons, where the shower shape provides high discrimination power between prompt photons and photons from hadron decays. Within this event selection, we perform a binned, extended maximum likelihood (ML) fit to $\sigma_{\eta_i\eta_i}$ of the selected photons. The templates for the signal (prompt photons) and the background are obtained from simulation. The contamination from electroweak and top backgrounds is found to be negligible. To increase the statistical discrimination of signal and background, the fit is performed separately for barrel and endcap photons. The likelihood can be written as

$$L = -\ln L = -(N_S + N_B) + \sum_{i=1}^{n} N_i \ln(N_S S_i + N_B B_i),$$

(7.4)

where $N_S$ and $N_B$ denote the number of signal and background events, $N_i$ is the observed number of events in bin $i$ and $n$ denotes the number of bins. $S_i$ and $B_i$ represent the signal and background templates, respectively. In Figure 7.12 we show the ML fits for ECAL barrel and endcap photons. From the fit, we obtain

**Figure 7.12:** Extended maximum likelihood fit to $\sigma_{\eta_i\eta_i}$ for the ECAL barrel and endcaps. The black bullets represent the data. The green dashed lines correspond to the best fit contribution of the prompt photon template, whereas the pink dashed lines represent the background best fit contributions. The blue histogram is the sum of the two templates.

the absolute contribution of signal and background to the inclusive events selection, which is dominated by low $p_T > 20$ GeV photons. The $M_{T2}$ distribution of the selected events is shown in Figure 7.13, where the background and prompt photon yields are fixed to the ML fit results. With these fixed normalizations, the extrapolation to the high $p_T$ tail is obtained from simulation. For the inclusive event selection we obtain a
purity of 85%, whereas for the subsample with $p_T > 120$ GeV the purity is estimated to be 98%, each with a relative uncertainty of 5% due to the statistical uncertainty of the ML fit.

**Validation of the $Z/\gamma$ Ratio Using Opposite-Sign Dileptons.**

The $Z$ boson $p_T$ spectrum is accessible in data using opposite-sign (OS) dileptons consistent with the $Z$-boson mass hypothesis. Due to the small branching ratio into electrons and muons of 6.7%, we expect limited statistics at high $p_T$. However, given that the $Z/\gamma$ ratio is expected to flatten out at high transverse momenta, valuable information can be obtained even with limited statistics. Figure 7.14 shows the $Z$-boson $p_T$ distribution in data and simulation as reconstructed in OS dilepton events using both electrons and muons. The plot also shows the inclusive $\gamma p_T$ distribution, which is compared to simulation. Generally, a good agreement between data and simulation is observed. Also, and most importantly, the $Z(\ell\ell)$ to $\gamma$ ratio is very well reproduced in simulation. Performing a linear fit on the ratio in Figure 7.14b from 300 to 600 GeV results in a fit parameter of $0.077 \pm 0.003$ ($\chi^2/\text{Ndof} = 23.3/22$) and $0.082 \pm 0.001$ ($\chi^2/\text{Ndof} = 35.1/22$) in data and simulation respectively. The fitted constants thus agree within a few percent, building confidence in the modelling of the $Z/\gamma p_T$ ratio.

**$Z(\nu\bar{\nu})/\gamma$ Ratio as a Function of $M_{T2}$.**

The ratio $R_{\text{sim}}^\text{ev}(Z(\nu\nu)/\gamma)$ from Eqn. (7.2) is calculated in simulation within our event selection to relate the photon event yield to the $Z(\nu\bar{\nu})$ background estimate. In the following, we probe the $Z(\nu\bar{\nu})/\gamma$ ratio at high vector boson transverse momenta with all selection requirements applied, but without imposing any explicit $H_T$ requirement. For this, we exploit a single photon trigger to select events containing a photon with

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**Figure 7.13:** $M_{T2}$ distribution of the event sample with a reconstructed photon. The photon momentum was added to $E_T^{\text{miss}}$ to mimic the $Z(\nu\bar{\nu})$ kinematics.
Figure 7.14: $Z(\ell\ell)$ boson and $\gamma p_T$ spectrum as well as the $Z(\ell\ell)$ to $\gamma$ ratio in data and simulation. We note that the $Z(\ell\ell)$ to $\gamma$ ratio is very well modelled up to high transverse momenta.

$p_T > 135\, \text{GeV}$ and add the photon momentum to $E_T^{\text{miss}}$. We further impose a $E_T^{\text{miss}} > 300\, \text{GeV}$ requirement to select events in a regime where the kinematic differences between $\gamma$+jets and $Z(\nu\bar{\nu})$+jets are expected to be small. The data are then compared to simulated $\gamma$+jets and $Z(\nu\bar{\nu})$+jets events. This comparison is shown in Figure 7.15. The peak at approximately $300\, \text{GeV}$ in $M_{T_2}$ is due to the $E_T^{\text{miss}} > 300\, \text{GeV}$ bias in the event selection. We note that for this selection of high $p_T$ vector bosons, the $Z(\nu\bar{\nu})/\gamma$ ratio as a function of $M_{T_2}$ is compatible with a flat line, indicating that the kinematic difference between the two samples is small indeed.

In a second step, we show in Figure 7.16 the $Z(\nu\bar{\nu})/\gamma$ ratio for the event selection used to carry out the $Z(\nu\bar{\nu})$+jets background estimate, for $750 \leq H_T < 950\, \text{GeV}$ and $H_T \geq 950\, \text{GeV}$. The binning corresponds to the one of the $M_{T_2}$ signal regions. Comparing Figure 7.16 with Figure 7.15b, we see that without the bias from $E_T^{\text{miss}} > 300\, \text{GeV}$, the ratio increases with $M_{T_2}$ for small values of $M_{T_2}$, but remains consistent with a flat line for $M_{T_2} \gtrsim 300\, \text{GeV}$. For $H_T > 950\, \text{GeV}$, the statistical uncertainty in the simulation is large and the central value is derived from a small number of events only. Based on our discussion in the previous paragraph, we are confident that a more stable central value for $R_{\text{sim}}(Z(\nu\bar{\nu})/\gamma)$ at $M_{T_2} \gtrsim 300\, \text{GeV}$ can be obtained by averaging the three last bins. Though residual kinematic differences between $\gamma$+jets and $Z(\nu\bar{\nu})$+jets may remain at high values of $M_{T_2}$ (which may cause a deviation from an entirely flat ratio), we expect such a bias to be small, and most importantly a flat ratio to be more stable than the ratio obtained if it were computed with limited statistics for each bin in the $M_{T_2}$ tail separately.
(a) $M_{T2}$ distribution for $\gamma$+jets in data and simulation and $Z(\nu\bar{\nu}$)+jets in simulation with $E_T^{\text{miss}} > 300$ GeV. The last bin contains the overflow.

Figure 7.15: $M_{T2}$ distribution for $\gamma$+jets in data and simulation and $Z(\nu\bar{\nu}$)+jets in simulation with $E_T^{\text{miss}} > 300$ GeV. For this selection of high $p_T$ vector bosons the kinematic difference between the two samples is expected to be small. The flat ratio plot supports this hypothesis.

(b) $Z(\nu\bar{\nu})/\gamma$ ratio for $E_T^{\text{miss}} > 300$ GeV in simulation.

Figure 7.16: $Z(\nu\bar{\nu})/\gamma$ ratio with all selection criteria applied, for $750 \leq H_T < 950$ GeV and $H_T \geq 950$ GeV in black and magenta respectively. The last bin is inclusive and contains the overflow.
Summary of the Method and Results

A sample of events with identified and isolated photons with $p_T > 20$ GeV is selected, where all selection requirements except that on $M_{T2}$ are imposed. This sample contains both prompt photons and photons from $\pi^0$ decays in QCD multijet events. The two components are separated by performing a maximum likelihood fit of templates from simulated events to the shower shapes. The event sample is dominated by low $p_T$ photons, where the shower shape provides high discrimination power between prompt photons and $\pi^0$s. The extrapolation of their contributions as a function of $M_{T2}$ is obtained from simulation.

The $Z(\nu\bar{\nu})$+jets background is estimated for each bin in $M_{T2}$ from the number of prompt photon events multiplied by the $M_{T2}$-dependent ratio of $Z(\nu\bar{\nu})$+jets to $\gamma$+jets events obtained from simulation. This ratio increases as a function of the photon $p_T$ (which drives the $M_{T2}$ value) and reaches a constant value above 300 GeV. A robust estimate of the $Z(\nu\bar{\nu})/\gamma$ ratio is the key to a reliable $Z(\nu\bar{\nu})$+jets background estimate.

The theoretical uncertainty of the $Z(\nu\bar{\nu})/\gamma$ ratio was studied in detail by the BlackHat group in Refs. [181, 182] in a phase-space region similar to the one relevant to this analysis. In Ref. [182], the $Z$+3-jet to $\gamma$+3-jet production cross sections and kinematic distributions were calculated at next-to-leading order (NLO) in $\alpha_s$ and the results were compared to those obtained using a parton shower matched to leading-order matrix-element. The two calculations were found to agree within 10%. The uncertainty of the ratio due to the parton distribution functions was found to be no more than 5%.

We validated the modeling of this ratio in data using $Z(\ell\ell)$+jets and $\gamma$+jets events. Moreover, cross-checks are carried out to study the stability of the $Z(\nu\bar{\nu})/\gamma$ ratio in simulation with varying $H_T$ requirements and vector boson transverse momenta. Based on these cross-checks, and the theoretical calculations mentioned above, we assign a conservative systematic uncertainty of 20% to the $Z(\nu\bar{\nu})/\gamma$ ratio for $M_{T2} \leq 275$ GeV. For the last three bins in Figure 7.16 (corresponding to the three highest $M_{T2}$ signal regions), we increase the uncertainty to 30%, which is justified by the limited number of leptonic $Z$ events in the high $p_T$ tail.

The total uncertainty of the $Z(\nu\bar{\nu})$+jets background prediction is composed of the following contributions:

- the statistical uncertainties from the number of $\gamma$+jets events,
- a normalization uncertainty in the shower shape fit of 5%,
- a systematic uncertainty of 20% (30%) for $M_{T2} < 275$ ($M_{T2} > 275$) GeV on the $Z(\nu\bar{\nu})$ to $\gamma$ ratio added in quadrature to the uncertainty arising from limited statistics in the simulation.

In Tables 7.5 and 7.6, the estimates of the $Z(\nu\bar{\nu})$+jets background are given in bins of $M_{T2}$ and $H_T$. We note that the predicted background is in good agreement with the estimate from $Z(\nu\bar{\nu})$+jets simulation.
Table 7.5: $Z(\nu\bar{\nu})$+jets background estimate for the $M_{T2}$ signal regions with $750 \text{ GeV} < H_T < 950 \text{ GeV}$.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N^\text{data}_\gamma$</th>
<th>background</th>
<th>$R^\text{sim}(Z(\nu\bar{\nu})/\gamma)$</th>
<th>data pred.</th>
<th>$Z(\nu\bar{\nu})$ simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 – 200</td>
<td>88 ± 9.4</td>
<td>0.88 ± 0.56</td>
<td>0.33 ± 0.07</td>
<td>28.7 ± 3.1 ± 6.4</td>
<td>27.9</td>
</tr>
<tr>
<td>200 – 275</td>
<td>68 ± 8.2</td>
<td>1.04 ± 0.56</td>
<td>0.42 ± 0.10</td>
<td>28.0 ± 3.5 ± 6.6</td>
<td>20.3</td>
</tr>
<tr>
<td>275 – 375</td>
<td>38 ± 6.2</td>
<td>-</td>
<td>0.46 ± 0.15</td>
<td>17.4 ± 2.8 ± 5.7</td>
<td>11.6</td>
</tr>
<tr>
<td>375 – 500</td>
<td>9 ± 3</td>
<td>-</td>
<td>0.46 ± 0.15</td>
<td>4.1 ± 1.4 ± 1.3</td>
<td>6.1</td>
</tr>
<tr>
<td>500 – $\infty$</td>
<td>4 ± 2</td>
<td>-</td>
<td>0.46 ± 0.15</td>
<td>1.8 ± 0.9 ± 0.6</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Table 7.6: $Z(\nu\bar{\nu})$+jets background estimate for the $M_{T2}$ signal regions with $H_T > 950 \text{ GeV}$.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N^\text{data}_\gamma$</th>
<th>background</th>
<th>$R^\text{sim}(Z(\nu\bar{\nu})/\gamma)$</th>
<th>data pred.</th>
<th>$Z(\nu\bar{\nu})$ simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 – 200</td>
<td>37 ± 6.1</td>
<td>0.07 ± 0.07</td>
<td>0.38 ± 0.09</td>
<td>13.9 ± 2.3 ± 3.4</td>
<td>12.9</td>
</tr>
<tr>
<td>200 – 275</td>
<td>24 ± 4.9</td>
<td>0.79 ± 0.49</td>
<td>0.44 ± 0.11</td>
<td>10.1 ± 2.1 ± 2.7</td>
<td>10.4</td>
</tr>
<tr>
<td>275 – 375</td>
<td>10 ± 3.2</td>
<td>-</td>
<td>0.63 ± 0.22</td>
<td>6.3 ± 2.0 ± 2.2</td>
<td>6.42</td>
</tr>
<tr>
<td>375 – 500</td>
<td>4 ± 2</td>
<td>-</td>
<td>0.63 ± 0.22</td>
<td>2.5 ± 1.3 ± 0.9</td>
<td>2.54</td>
</tr>
<tr>
<td>500 – $\infty$</td>
<td>4 ± 2</td>
<td>-</td>
<td>0.63 ± 0.22</td>
<td>2.5 ± 1.3 ± 0.9</td>
<td>2.17</td>
</tr>
</tbody>
</table>
7.3.2.2 Method using W+jets events

For the $Z(\nu\bar{\nu})+\text{jets}$ background estimate from $W(\mu\nu)+\text{jets}$ events, corrections are needed for lepton acceptance, lepton reconstruction efficiency, and the ratio between the production cross sections for $W$ and $Z$ bosons (including differences between the shapes of the distributions on which selection criteria are applied). The estimate can be described as

$$N_{Z(\nu\bar{\nu})(M_{T2})} = R_{\text{sim}}(Z/W) \cdot \frac{1}{\epsilon_{\text{acc}}\epsilon_{\text{reco/iso}}} \cdot N_{\text{data}}(M_{T2}),$$  \hspace{1cm} (7.5)

where

- $\epsilon_{\text{acc}}$ is the muon acceptance efficiency,
- $\epsilon_{\text{reco/iso}}$ is the muon reconstruction and isolation efficiency,
- $R_{\text{sim}}$ accounts for the differences in the cross sections and kinematic distributions between $Z(\nu\bar{\nu})+\text{jets}$ and $W(\mu\nu)+\text{jets}$ events.

While the muon acceptance efficiency is measured in simulation, the reconstruction and isolation efficiencies are taken from studies of $Z(\mu\mu)+\text{jets}$ events in data using a tag-and-probe method. These efficiencies are measured as a function of muon $p_T$, $\eta$ and $\Delta R$ of the muon to the closest jet. This parametrization was found ideal when applied to leptons reconstructed in events with a large hadronic activity. The isolation and reconstruction efficiency slightly depends on the $H_T$ and $M_{T2}$ requirement and ranges from $0.91 \pm 0.08$ to $0.98 \pm 0.15$ for the different signal regions. We explain in the following paragraph that the estimate is derived from a b-quark depleted control region, so that $R_{\text{sim}}$ also corrects for the different b-quark content.

W Enriched Event Selection

A sample of $W(\mu\nu)+\text{jets}$ events is obtained by applying all selection criteria described in Section 7.1, but requiring the presence of exactly one muon with $p_T > 20\text{ GeV}$ and $|\eta| < 2.4$ passing all quality and selection criteria. The contribution of top-quark production to this event selection is reduced by vetoing events with a b-tagged jet with $p_T > 20\text{ GeV}$ and $|\eta| < 2.4$. For this veto, a loose b-tag based on the SSVHE algorithm (see Section 5.5.3) is used. To reduce the potential signal contamination, we further require the transverse mass $M_T$ of the reconstructed $W$ boson to be $< 100\text{ GeV}$. In Figure 7.17 we show the $M_{T2}$ and muon $p_T$ distributions with the above mentioned selection criteria applied. We see that the kinematic distributions are well reproduced in simulation. Furthermore, we note that the dominant background in this W enriched event selection comes from leptonically decaying top-quarks, even though the b-tag veto is imposed. The backgrounds from top quarks and other processes need to be subtracted from the data in order not to overestimate the true $W(\mu\nu)+\text{jets}$ event yield. We explain in the following how the contribution from top-quark production to this event selection is estimated from a data control region. The remaining backgrounds are subtracted using simulation.
Chapter 7. Search for Supersymmetry in Hadronic Final States Using $M_{T2}$

Figure 7.17: Kinematic distributions for the W enriched control sample used to derive the $Z(\nu\nu)+$jets background estimate from $W(\mu\nu)+$jets events. The background labelled as “Other” consists of $W(e\nu)+$jets, $\gamma+$jets and $Z(\ell\ell)+$jets processes. The grey shaded area represents the statistical uncertainty in the simulation.

Estimating the Top-Quark Background

First, we convert the b-tag veto into a tight b-tag requirement to obtain a top-quark enriched event selection. We use a high-purity b-tag based on the SSV algorithm. In Figure 7.18 we show the same kinematic distributions as in Figure 7.17 but for the top-quark enriched event selection. The data are systematically about 20% low with respect to the expectation from simulation. This discrepancy has a negligible effect on the final background estimate and is well covered by the systematic uncertainties. The purity of the top-quark enhanced region is estimated to be $> 80\%$. With this event selection, we estimate the top-quark background in the W boson enriched event selection by correcting the identified top-quark decays for the b-tagging efficiency as

$$N_{top\,W\,enriched} = \left( N_{top\,enriched\,data} - N_{top\,enriched\,bkg} \right) \frac{\epsilon(b - veto)}{\epsilon(1\,b - tag)}. \quad (7.6)$$

Here, $\epsilon(b - veto)$ and $\epsilon(1\,b - tag)$ denote respectively the efficiencies for vetoing and selecting b-tagged events using the two b-tagging working points. These efficiencies are measured in simulation but corrected by data-to-simulation scale factors [152]. In Tables 7.7 and 7.8 we present the results of the estimated top-quark background to the W boson enhanced event selection. The quoted uncertainties on $N_{bkg\,W\,enriched}$ are statistical only. The dominant systematic uncertainties in the estimate of $N_{top\,W\,enriched}$ are due to the b-tagging efficiencies and the statistics in the simulation.
7.3. $M_{T2}$ Analysis

(a) Muon $p_T$ distribution for the top-quark enriched event selection

(b) $M_{T2}$ distribution for the top-quark enriched event selection, where the muon momentum is added to $\vec{E}_{miss}^T$.

Figure 7.18: Kinematic distributions for the top-quark enriched control sample. The grey shaded area represents the statistical uncertainty in the simulation.

Table 7.7: Estimation of the top-quark contamination in the $W$ boson enriched event selection for $750 < H_T < 950\text{GeV}$. The estimate from data $N^W_{\text{top enriched}}$ is to be compared to the estimate from simulation given in the last column. The top-quark background is evaluated by applying b tagging to the data to identify top-quark decays and then correcting for the b-tagging efficiency, as indicated in Eqn. (7.6).

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N^{\text{top}}_{\text{data}}$</th>
<th>$N^{\text{top}}_{\text{bkg}}$</th>
<th>$N^W_{\text{top enriched}}$</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 – 200</td>
<td>7</td>
<td>1.67 ± 1.67</td>
<td>8.14 ±3.08 (stat) ±1.74 (syst)</td>
<td>15.65 ± 0.55</td>
</tr>
<tr>
<td>200 – 275</td>
<td>4</td>
<td>1.19 ± 1.19</td>
<td>4.33 ±2.17 (stat) ±1.22 (syst)</td>
<td>10.18 ± 0.42</td>
</tr>
<tr>
<td>275 – 375</td>
<td>2</td>
<td>0.99 ± 0.99</td>
<td>1.49 ±1.06 (stat) ±1.00 (syst)</td>
<td>5.31 ± 0.31</td>
</tr>
<tr>
<td>375 – 500</td>
<td>0</td>
<td>0.47 ± 0.47</td>
<td>-</td>
<td>1.80 ± 0.21</td>
</tr>
<tr>
<td>500 – ∞</td>
<td>0</td>
<td>0.22 ± 0.22</td>
<td>-</td>
<td>0.90 ± 0.20</td>
</tr>
</tbody>
</table>
Table 7.8: Estimation of the top-quark contamination in the W boson enriched event selection for $H_T > 950$ GeV.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{\text{data}}^{\text{top}}$ enriched</th>
<th>$N_{\text{bkg}}^{\text{top}}$ enriched</th>
<th>$N_{\text{W}}^{\text{H}}$ enriched</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 − 200</td>
<td>5</td>
<td>1.14 $\pm$ 1.14</td>
<td>6.31 $\pm$ 2.82 (stat) $\pm$ 1.27 (syst)</td>
<td>7.06 $\pm$ 0.36</td>
</tr>
<tr>
<td>200 − 275</td>
<td>8</td>
<td>0.60 $\pm$ 0.60</td>
<td>12.15 $\pm$ 4.29 (stat) $\pm$ 1.41 (syst)</td>
<td>5.03 $\pm$ 0.32</td>
</tr>
<tr>
<td>275 − 375</td>
<td>2</td>
<td>0.64 $\pm$ 0.64</td>
<td>2.48 $\pm$ 1.75 (stat) $\pm$ 0.74 (syst)</td>
<td>2.30 $\pm$ 0.23</td>
</tr>
<tr>
<td>375 − 500</td>
<td>0</td>
<td>0.38 $\pm$ 0.38</td>
<td>-</td>
<td>0.80 $\pm$ 0.11</td>
</tr>
<tr>
<td>500 $\sim \infty$</td>
<td>0</td>
<td>0.21 $\pm$ 0.21</td>
<td>-</td>
<td>0.40 $\pm$ 0.12</td>
</tr>
</tbody>
</table>

Results

In Tables 7.9 and 7.10 we show the results of the $Z(\nu\bar{\nu})$+jets background estimate from $W(\mu\nu)$+jets events. $N_{\text{data}}^{\text{W}}$ denotes the number of observed events in the W boson enriched control region, $N_{\text{W}}^{\text{data}}$ and $N_{\text{W}}^{\text{top}}$ are the estimated contributions of W+jets and top-quark events to this control region, respectively. The contribution from processes other than W+jets and top-quark production is estimated using simulation and denoted by $N_{\text{other}}^{\text{sim}}$. Finally, the estimate of the $Z(\nu\bar{\nu})$+jets background in the hadronic signal region is given in the second to last column. The dominant systematic uncertainties on the $N_{\text{data}}^{\text{pred}}$ background estimate stem from the lepton selection and reconstruction efficiencies, the b-tagging efficiency, the acceptance from simulation, and the Z-to-W ratio $R^{\text{sim}}(Z/W)$. The uncertainty on $R^{\text{sim}}(Z/W)$ is due to the parton distribution functions, b-tagging scale factors and statistics of the simulation.

Comparing these results with the $Z(\nu\bar{\nu})$+jets background estimates using $\gamma$+jets events from Section 7.3.2.1, we see that the two estimate are generally in very good agreement. Discrepancies of about $2\sigma$ are found between the two estimates in the bins with $150 < M_{T2} < 200$ GeV and $200 < M_{T2} < 275$ GeV for $H_T > 950$ GeV. The discrepancies are likely to be due to fluctuations in the data in the W boson enriched control sample.

7.3.3 Prediction of the $W(\ell\nu)$+jets and Top-Quark Backgrounds

The backgrounds in the $M_{T2}$ signal regions from fully-hadronic decays of $t\bar{t}$, single-top and W+jets events is very small due to the lack of genuine $E_T^{\text{miss}}$. The backgrounds due to $W(\ell\nu)$+jets and to semi-leptonic decays of top quarks, however, are significant and have the following sources in common:

- lepton decays of the W boson, where the lepton is unobserved because it falls outside the $p_T$ or $\eta$ acceptance;
- to a lesser extent, lepton decays of the W boson, where the lepton is within the acceptance but fails to satisfy the reconstruction, identification, or isolation criteria;
Table 7.9: Results of the $Z(\nu\bar{\nu})$+jets background estimate from $W(\mu\nu)$+jets events for $750 < H_T < 950$ GeV. For the bins in which $N^{\text{top}}$ is marked with (*), the estimate cannot be obtained from data and simulation is used instead.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{W}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{W}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{\text{top}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{\text{top}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{\text{other}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$R_{\text{other}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{Z(\nu\bar{\nu})}^{\text{pred}}$ data $\pm_{\text{sys}}$</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 − 200</td>
<td>20 15.48 $\pm_{3.50}$ $\pm_{3.11}$</td>
<td>1.42 $\pm_{0.54}$ $\pm_{0.33}$</td>
<td>3.10 $\pm_{3.10}$</td>
<td>1.13 $\pm_{0.10}$</td>
<td>20.46 $\pm_{4.63}$ $\pm_{4.64}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 − 275</td>
<td>16 12.80 $\pm_{3.22}$ $\pm_{2.39}$</td>
<td>0.72 $\pm_{0.36}$ $\pm_{0.22}$</td>
<td>2.48 $\pm_{2.48}$</td>
<td>1.20 $\pm_{0.13}$</td>
<td>17.81 $\pm_{4.48}$ $\pm_{4.12}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>275 − 375</td>
<td>12 9.71 $\pm_{2.04}$ $\pm_{1.38}$</td>
<td>0.26 $\pm_{0.18}$ $\pm_{0.18}$</td>
<td>2.03 $\pm_{2.03}$</td>
<td>1.06 $\pm_{0.14}$</td>
<td>11.87 $\pm_{3.43}$ $\pm_{3.10}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>375 − 500</td>
<td>4 2.89 $\pm_{1.44}$ $\pm_{0.86}$</td>
<td>0.28 $\pm_{0.00}$ $\pm_{0.00}$</td>
<td>0.84 $\pm_{0.84}$</td>
<td>1.25 $\pm_{0.25}$</td>
<td>4.22 $\pm_{1.11}$ $\pm_{1.60}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500 − $\infty$</td>
<td>2 1.42 $\pm_{1.00}$ $\pm_{0.53}$</td>
<td>0.09 $\pm_{0.00}$ $\pm_{0.00}$</td>
<td>0.49 $\pm_{0.49}$</td>
<td>1.12 $\pm_{0.27}$</td>
<td>1.71 $\pm_{1.21}$ $\pm_{0.81}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.10: Results of the $Z(\nu\bar{\nu})$+jets background estimate from $W(\mu\nu)$+jets events for $H_T > 950$ GeV. For the bins in which $N^{\text{top}}$ is marked with (*), the estimate cannot be obtained from data and simulation is used instead.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{W}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{W}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{\text{top}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{\text{top}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{\text{other}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$R_{\text{other}}^{\text{data}}$ data $\pm_{\text{sys}}$</th>
<th>$N_{Z(\nu\bar{\nu})}^{\text{pred}}$ data $\pm_{\text{sys}}$</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 − 200</td>
<td>20 17.15 $\pm_{3.87}$ $\pm_{1.67}$</td>
<td>1.20 $\pm_{0.54}$ $\pm_{0.29}$</td>
<td>1.65 $\pm_{1.65}$</td>
<td>1.26 $\pm_{0.16}$</td>
<td>25.95 $\pm_{5.86}$ $\pm_{4.67}$</td>
<td>12.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 − 275</td>
<td>5 0.95 $\pm_{1.14}$ $\pm_{1.20}$</td>
<td>2.98 $\pm_{1.05}$ $\pm_{0.56}$</td>
<td>1.06 $\pm_{1.06}$</td>
<td>1.26 $\pm_{0.19}$</td>
<td>1.42 $\pm_{1.60}$ $\pm_{1.80}$</td>
<td>10.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>275 − 375</td>
<td>5 3.44 $\pm_{1.58}$ $\pm_{1.08}$</td>
<td>0.49 $\pm_{0.35}$ $\pm_{0.19}$</td>
<td>1.06 $\pm_{1.06}$</td>
<td>1.33 $\pm_{0.25}$</td>
<td>5.19 $\pm_{2.38}$ $\pm_{1.99}$</td>
<td>6.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>375 − 500</td>
<td>5 4.04 $\pm_{1.81}$ $\pm_{0.78}$</td>
<td>0.19 $\pm_{0.00}$ $\pm_{0.11}$</td>
<td>0.77 $\pm_{0.77}$</td>
<td>1.15 $\pm_{0.31}$</td>
<td>5.03 $\pm_{2.25}$ $\pm_{1.88}$</td>
<td>2.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500 − $\infty$</td>
<td>- - - - - - - - - - - - - - - -</td>
<td>- - - - - - - - - -</td>
<td>- - - - - - - - - -</td>
<td>- - - - - - - - - -</td>
<td>- - - - - - - - - -</td>
<td>2.17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• $W(\tau\nu)$ decays, where the $\tau$ decays hadronically.

We refer to leptons that fall into either of the first two categories as “lost leptons”.

### 7.3.3.1 Background from Lost Leptons

The number of events with lost leptons is estimated from a data control sample where a single lepton (e or $\mu$) is found. This one-lepton control sample should be similar in kinematics to the lost lepton background in the $M_{T2}$ signal regions. This way, we make sure that the simulation driven translation between the two phase spaces does not introduce unforeseen biases from extrapolations in kinematic variables and thus additional systematic uncertainties. However, with all selection criteria from Section 7.1 applied but the lepton veto inverted, the statistics in the one-lepton data control sample becomes scarce in the tail of the $M_{T2}$ distribution. Consequently, no reliable prediction of the lost lepton background can be obtained in the very $M_{T2}$ tail.

To overcome this dilemma, we choose to stay as close as possible in kinematics to the hadronic signal region, but to increase the statistics in the leptonic data control sample by omitting selection criteria which do not alter the shape of the $M_{T2}$ distribution. The $\Delta\phi_{\text{min}}(\vec{E}_{T}\text{miss, jets}) < 0.3$ requirement is ideal for this purpose: the efficiency of this selection criteria is found to be well modelled in simulation and to be flat as a function of $M_{T2}$ starting from $M_{T2} \gtrsim 100$ GeV. Furthermore, this selection criteria is designed to reduce the QCD multijet contamination, which is very small in the leptonic control region. In Figures 7.19a and 7.19b, we show the $M_{T2}$ distribution for the leptonic control regions with one muon and one electron, respectively. We observe that the data tends to be slightly low with respect to the simulation, indicating that the $M_{T2}$ shape might not be perfectly well modelled and gross kinematic extrapolations using simulation should be avoided. The lepton $p_T$ distributions are shown in Figures 7.19d and 7.19c, where an acceptable agreement between data and simulation is observed.

The signal contamination in the $M_{T2}$ tail for the leptonic event selection could potentially be large, as many SUSY models naturally produce leptons in combinations with large hadronic activity and significant $E_T^{\text{miss}}$. To reduce the potential signal contamination, we request the transverse mass $M_T$ of the lepton plus $E_T^{\text{miss}}$ system to be $< 100$ GeV. This requirement protects against leptons not originating from a W decay while maintaining a large efficiency for leptonic W boson and top-quark events.

Starting from the $M_{T2}$ distributions in Figures 7.19a and 7.19b, the background due to lost electrons and muons in the hadronic signal regions, denoted by $N^{\text{lost lepton}}_{e,\mu}$, is estimated as

$$N^{\text{lost lepton}}_{e,\mu} = (N^{\text{data}}_{e,\mu} - N^{\text{bkg}}_{e,\mu}) \frac{1 - \varepsilon_{e,\mu}}{\varepsilon_{e,\mu}} f_{\Delta\phi},$$

where $N^{\text{data}}_{e,\mu}$ and $N^{\text{bkg}}_{e,\mu}$ denote respectively the observed number of events and the background from processes other than $W(\ell\nu)+\text{jets}$ and semileptonic $t\bar{t}$ events in the leptonic control regions. The dominant contribution to $N^{\text{bkg}}_{e,\mu}$ comes from fully leptonic $t\bar{t}$ events. The factor $f_{\Delta\phi}$ corrects for the efficiency of the $\Delta\phi_{\text{min}}(\vec{E}_{T}\text{miss, jets}) < 0.3$ requirement, which is not applied to the leptonic event selection. Furthermore, $\varepsilon_{e,\mu}$ denotes the probability for a true $W(e\nu)$ ($W(\mu\nu)$) event within our kinematic selection to have the
(a) $M_{T2}$ distribution for the lepton control region with one identified muon.

(b) $M_{T2}$ distribution for the lepton control region with one identified electron.

(c) Muon $p_T$ distribution for the Lost Lepton control region with one identified muon.

(d) Electron $p_T$ distribution for the Lost Lepton control region with one identified electron. A $M_{T2} > 50$ GeV requirement is imposed to reduce the QCD multijet contamination to this event selection.

Figure 7.19: Kinematic distributions for the lepton control region with one identified electron or muon. All selection criteria from Section 7.1 are applied but the lepton veto is inverted and the $\Delta \phi_{\text{min}} (E_{T}^{\text{miss}}, \text{jets}) < 0.3$ requirement is omitted. The last bin contains the overflow.
electron (muon) reconstructed and to pass the identification and isolation criteria. The probabilities $\varepsilon_{e,\mu}$ are measured in simulation and defined to absorb the correction for the $M_T < 100 \text{ GeV}$ requirement in the leptonic events selection.

In Tables 7.11 and 7.12, we present the results for the lost lepton background estimate in the electron and muon channels, respectively, bin by bin for each signal region in $M_{T2}$ and $H_T$. For the bins with a zero event yield in data, we take the simulation as background estimate with an uncertainty of 100%. The large statistical uncertainties on the background predictions are due to the very small number of leptonic events in the $M_{T2}$ tail. The final systematic uncertainty consists of the statistical uncertainty of the simulation to estimate $\varepsilon_{e,\mu}, f_{\Delta \phi}$, and an additional 5% uncertainty added in quadrature to cover potential differences between data and simulation in the lepton identification and reconstruction efficiencies. These efficiencies were studied using a tag & probe technique and found to differ by no more than 5% in data and simulation. Moreover, we assign a conservative 100% uncertainty on the background $N_{bkg}^\mu$, which is estimated using simulation.

Table 7.11: Results for the lost lepton background estimates in the electron channel for the two bins in $H_T$.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{data}^e$</th>
<th>$N_{bkg}^e$</th>
<th>$\frac{1-\varepsilon_{e,\mu}}{\varepsilon_{e,\mu}} f_{\Delta \phi}$ simulation</th>
<th>$N_{lost \ lepton}^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 – 200</td>
<td>22</td>
<td>2.64</td>
<td>0.84 ± 0.17</td>
<td>18.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.4 ± 3.96 (stat) ± 4.01 (sys)</td>
</tr>
<tr>
<td>200 – 275</td>
<td>9</td>
<td>1.60</td>
<td>0.70 ± 0.18</td>
<td>9.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.20 ± 2.11 (stat) ± 1.73 (sys)</td>
</tr>
<tr>
<td>275 – 375</td>
<td>3</td>
<td>0.26</td>
<td>0.50 ± 0.21</td>
<td>3.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.38 ± 0.87 (stat) ± 0.59 (sys)</td>
</tr>
<tr>
<td>375 – 500</td>
<td>1</td>
<td>0.21</td>
<td>0.49 ± 0.32</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.39 ± 0.49 (stat) ± 0.27 (sys)</td>
</tr>
<tr>
<td>500 – $\infty$</td>
<td>0</td>
<td>0.02</td>
<td>0.53 ± 0.45</td>
<td>0.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_T &gt; 950 \text{ GeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 – 200</td>
</tr>
<tr>
<td>200 – 275</td>
</tr>
<tr>
<td>275 – 375</td>
</tr>
<tr>
<td>375 – 500</td>
</tr>
<tr>
<td>500 – $\infty$</td>
</tr>
</tbody>
</table>

Comparing the last two columns in Tables 7.11 and 7.12, we see that the background estimates agree well within the uncertainty with the simulation. However, we also note that the estimate from data is slightly lower than expected from simulation. This is due to the aforementioned observation that the simulation seems to overestimate the leptonic events yield in the $M_{T2}$ tail (see Figures 7.19a and 7.19b).

7.3.3.2 Background from Hadronically Decaying Tau Leptons

For the background contribution from hadronically decaying tau leptons, a method similar to the one described above is used. Note, however, that events with a reconstructed hadronic tau lepton are not vetoed in
Table 7.12: Results for the lost lepton background estimates in the muon channel for the two bins in $H_T$.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{\mu}^{\text{data}}$</th>
<th>$N_{\mu}^{\text{bkg}}$</th>
<th>$\frac{1}{\varepsilon_{\mu}} \int d\phi$</th>
<th>$N_{\mu}^{\text{lost lepton}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$750 &lt; H_T &lt; 950$ GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150 – 200</td>
<td>18</td>
<td>1.17</td>
<td>0.78 ± 0.15</td>
<td>17.9</td>
</tr>
<tr>
<td>200 – 275</td>
<td>13</td>
<td>1.05</td>
<td>0.56 ± 0.15</td>
<td>7.86</td>
</tr>
<tr>
<td>275 – 375</td>
<td>5</td>
<td>0.21</td>
<td>0.58 ± 0.21</td>
<td>4.06</td>
</tr>
<tr>
<td>375 – 500</td>
<td>2</td>
<td>0.23</td>
<td>0.39 ± 0.26</td>
<td>1.05</td>
</tr>
<tr>
<td>500 – ∞</td>
<td>1</td>
<td>0.02</td>
<td>0.50 ± 0.50</td>
<td>0.42</td>
</tr>
</tbody>
</table>

| $H_T > 950$ GeV |
|----------------|--------------------------|-------------------------|---------------------------------|-----------------------------|
| 150 – 200 | 11 | 1.03 | 0.79 ± 0.20 | 8.89 | 7.89 ± 2.62 (stat) ± 2.12 (sys) |
| 200 – 275 | 8 | 0.40 | 0.62 ± 0.19 | 5.35 | 4.74 ± 1.76 (stat) ± 1.49 (sys) |
| 275 – 375 | 2 | 0.14 | 0.61 ± 0.26 | 2.53 | 1.14 ± 0.86 (stat) ± 0.49 (sys) |
| 375 – 500 | 1 | 0.00 | 0.42 ± 0.34 | 0.70 | 0.42 ± 0.42 (stat) ± 0.34 (sys) |
| 500 – ∞ | 0 | 0.00 | 0.48 ± 0.43 | 0.50 | – |

the hadronic signal region. We thus need to control the full background of events containing a hadronic tau lepton, arising from $W(\tau \nu)$ decays in W+jets, $t\bar{t}$ and single-top events, regardless whether or not the tau lepton is reconstructed and identified as such.

From the events with a reconstructed hadronic tau lepton, the number of produced hadronic tau leptons within the same kinematic selection can be estimated as

$$N_{\text{pred \, had \, \tau}} = N_{\tau}^{\text{data}} - N_{\tau}^{\text{bkg}} \varepsilon_{\tau},$$

(7.8)

where $N_{\tau}^{\text{data}}$ denotes the number of events with one reconstructed tau, $N_{\tau}^{\text{bkg}}$ is the number of events with a fake tau and $\varepsilon_{\tau}$ denotes the probability for a hadronically decaying $W(\tau \nu)$ event to have the tau lepton reconstructed and identified. The probability $\varepsilon_{\tau}$ is measured in simulation and found to be approximately 20% for the kinematic regime relevant for this analysis. $N_{\tau}^{\text{bkg}}$ arises dominantly from QCD multijet events where a jet is falsely identified as a hadronic tau lepton. The fake rate for this to happen is measured in data in the $M_{T2} < 70$ GeV region (which is entirely QCD multijet dominated) and parametrized in bins of $p_T$ and $\eta$. In average, the fake rate is approximately 0.1%.

In Figure 7.20 we show the $M_{T2}$ distribution for events with one identified hadronically decaying tau lepton. For this event selection, only the $H_T > 750$ GeV requirement is imposed and the presence of at least three selected jets (including the tau lepton) is requested. All other kinematic selection criteria are omitted in order to gain statistics in the tail.

An important difficulty in this method arises from the small tau efficiency. The number of events with a reconstructed tau is already limited in the $M_{T2}$ tail even for the very relaxed selection criteria used in Figure 7.20. With the full selection requirements imposed, the tau leptons are depleted in the very $M_{T2}$ tail.
For this reason, we predict the inclusive hadronic tau background from the events with reconstructed tau leptons only for the relaxed selection criteria from Figure 7.20. The results of this exercise are presented in Table 7.13.

Table 7.13: Results of the hadronic tau lepton background estimate for the relaxed event selection with only the $H_T$ and jet multiplicity requirements applied. The middle column contains the background estimate and can be compared to the estimate from simulation in the last column.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{\text{data}}$</th>
<th>$750 &lt; H_T &lt; 950$ GeV</th>
<th>$H_T &gt; 950$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{\text{had } \tau}$</td>
<td>simulation</td>
<td></td>
</tr>
<tr>
<td>150 – 200</td>
<td>21 65.1 ± 16.9 (stat) ± 5.89 (sys)</td>
<td>62.4 ± 2.8</td>
<td></td>
</tr>
<tr>
<td>200 – 275</td>
<td>10 28.4 ± 11.3 (stat) ± 2.57 (sys)</td>
<td>34.5 ± 2.1</td>
<td></td>
</tr>
<tr>
<td>275 – 375</td>
<td>6 20.8 ± 8.92 (stat) ± 2.55 (sys)</td>
<td>16.3 ± 1.4</td>
<td></td>
</tr>
<tr>
<td>375 – 500</td>
<td>2 6.8 ± 5.15 (stat) ± 0.84 (sys)</td>
<td>8.6 ± 1.2</td>
<td></td>
</tr>
<tr>
<td>500 – ∞</td>
<td>0 -</td>
<td>2.3 ± 0.4</td>
<td></td>
</tr>
<tr>
<td>150 – 200</td>
<td>17 42.8 ± 13.1 (stat) ± 4.99 (sys)</td>
<td>34.8 ± 2.2</td>
<td></td>
</tr>
<tr>
<td>200 – 275</td>
<td>9 30.8 ± 10.9 (stat) ± 3.59 (sys)</td>
<td>23.1 ± 1.7</td>
<td></td>
</tr>
<tr>
<td>275 – 375</td>
<td>5 15.9 ± 7.32 (stat) ± 2.34 (sys)</td>
<td>10.4 ± 1.1</td>
<td></td>
</tr>
<tr>
<td>375 – 500</td>
<td>4 12.8 ± 6.54 (stat) ± 1.88 (sys)</td>
<td>4.2 ± 0.7</td>
<td></td>
</tr>
<tr>
<td>500 – ∞</td>
<td>1 3.10 ± 3.27 (stat) ± 0.46 (sys)</td>
<td>2.5 ± 0.4</td>
<td></td>
</tr>
</tbody>
</table>
The predicted number of hadronically decaying tau background events agrees with the true number from simulation. The systematic uncertainty on the background prediction is due to the tau lepton efficiency and fake rate as well as the statistical uncertainty on \( N_{\text{tr}}^{\text{bkg}} \) from Eqn. (7.8). We take the success of this validation test in the relaxed event selection as an indication that the hadronic tau background is reasonably well modelled. As mentioned above, a repetition of this background prediction with all selection requirements imposed is not feasible due to the low number of reconstructed tau leptons in the \( M_{T2} \) tail. Instead, we use the numbers of events from the simulation for the background estimate, with relative uncertainties taken bin-by-bin as the ones from the lost lepton background prediction. This choice of systematic uncertainty is motivated by the similar nature of these backgrounds.

### 7.3.4 Results

The results of the background estimation methods for each background contribution are summarized in Table 7.14 and shown in Fig. 7.21.

Table 7.14: Estimated event yields for each background contribution in the various \( M_{T2} \) and \( H_T \) bins. The predictions from control regions in data are compared to the expected event yields from simulation. Statistical and systematic uncertainties are added in quadrature.

<table>
<thead>
<tr>
<th>( H_T \leq 950 )</th>
<th>( Z \rightarrow \nu \bar{\nu} ) sim. data pred.</th>
<th>Lost lepton sim. data pred.</th>
<th>( \tau \rightarrow \text{had} ) sim. data pred.</th>
<th>QCD multijet sim. data pred.</th>
<th>Total bkg. data pred.</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{T2}[150, 200] )</td>
<td>27.9 ± 4.9</td>
<td>36.0 ± 7.1</td>
<td>22.5 ± 5.4</td>
<td>3.1 ± 3.5</td>
<td>83.3 ± 10.7</td>
<td>88</td>
</tr>
<tr>
<td>( M_{T2}[200, 275] )</td>
<td>20.3 ± 4.8</td>
<td>17.2 ± 3.9</td>
<td>12.7 ± 4.2</td>
<td>0.0 ± 0.5</td>
<td>47.4 ± 7.5</td>
<td>69</td>
</tr>
<tr>
<td>( M_{T2}[275, 375] )</td>
<td>11.6 ± 3.8</td>
<td>7.1 ± 1.9</td>
<td>5.4 ± 2.5</td>
<td>0.0 ± 0.07</td>
<td>23.4 ± 4.9</td>
<td>19</td>
</tr>
<tr>
<td>( M_{T2}[375, 500] )</td>
<td>6.1 ± 1.6</td>
<td>2.2 ± 0.9</td>
<td>2.2 ± 1.8</td>
<td>0.0 ± 0.08</td>
<td>7.4 ± 2.6</td>
<td>8</td>
</tr>
<tr>
<td>( M_{T2}[500, \infty] )</td>
<td>3.5 ± 0.9</td>
<td>1.1 ± 1.0</td>
<td>0.6 ± 0.5</td>
<td>0.0 ± 0.00</td>
<td>3.6 ± 1.4</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( H_T \geq 950 )</th>
<th>( Z \rightarrow \nu \bar{\nu} ) sim. data pred.</th>
<th>Lost lepton sim. data pred.</th>
<th>( \tau \rightarrow \text{had} ) sim. data pred.</th>
<th>QCD multijet sim. data pred.</th>
<th>Total bkg. data pred.</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{T2}[150, 200] )</td>
<td>12.9 ± 3.6</td>
<td>18.7 ± 5.3</td>
<td>12.7 ± 4.1</td>
<td>9.8 ± 5.5</td>
<td>56.6 ± 9.4</td>
<td>70</td>
</tr>
<tr>
<td>( M_{T2}[200, 275] )</td>
<td>10.5 ± 2.0</td>
<td>11.7 ± 3.7</td>
<td>7.1 ± 2.6</td>
<td>0.47 ± 0.7</td>
<td>23.2 ± 5.0</td>
<td>23</td>
</tr>
<tr>
<td>( M_{T2}[275, 375] )</td>
<td>6.4 ± 2.2</td>
<td>5.0 ± 1.7</td>
<td>3.3 ± 1.9</td>
<td>0.04 ± 0.07</td>
<td>12.1 ± 3.3</td>
<td>9</td>
</tr>
<tr>
<td>( M_{T2}[375, 500] )</td>
<td>2.5 ± 1.4</td>
<td>1.1 ± 0.6</td>
<td>0.9 ± 0.9</td>
<td>0.0 ± 0.04</td>
<td>4.6 ± 1.8</td>
<td>8</td>
</tr>
<tr>
<td>( M_{T2}[500, \infty] )</td>
<td>2.2 ± 1.5</td>
<td>0.6 ± 0.6</td>
<td>0.6 ± 0.6</td>
<td>0.0 ± 0.04</td>
<td>3.8 ± 1.7</td>
<td>4</td>
</tr>
</tbody>
</table>
Figure 7.21: $M_{T2}$ distribution from the background estimates compared to data for the $M_{T2}$ analysis. The figure on the left corresponds to the $750 \leq H_T < 950$ GeV region, while that on the right corresponds to $H_T \geq 950$ GeV. The predictions from simulated events for the LM6 signal model (not stacked) are also shown. The hatched band shows the total uncertainty on the SM background estimate.
7.4 $M_{T2b}$ Analysis

The selection criteria developed for the $M_{T2}$ analysis are not optimal for events with heavy squarks and light gluinos, such as are predicted by the SUSY benchmark model LM9. To improve sensitivity to these types of events, we perform the $M_{T2b}$ analysis based on loosened kinematic selection criteria and the requirement of a tagged b jet.

As outlined in Section 7.1.2, we require for the $M_{T2b}$ analysis that there be at least four jets with $p_T > 40\text{ GeV}$, and the leading jet to have $p_T > 150\text{ GeV}$. The $\Delta\phi_{\min}(\text{jets}, E_T^{\text{miss}}) > 0.3$ requirement is applied to the four leading jets only. We further require that at least one of the jets in the event be tagged as a b-quark jet, where we use the SSVHP algorithm described in Section 5.5.3. Furthermore, we loosen the restriction on $M_{T2}$ for the signal regions to $M_{T2} > 125\text{ GeV}$.

![Figure 7.22](image_url)

**Figure 7.22:** $M_{T2}$ for events with the $M_{T2b}$ selection criteria applied and with $H_T \geq 750\text{ GeV}$. The different predictions from simulation for the SM backgrounds are stacked on top of each other. The LM9 signal distribution is not stacked. All distributions from simulation are normalized to the integrated luminosity of the data. The last bin contains the overflow.

Figure 7.22 shows the $M_{T2}$ distribution for events that satisfy the $M_{T2b}$ selection criteria with $H_T \geq 750\text{ GeV}$. As for the $M_{T2}$ analysis (Fig. 7.8), the QCD multijet background dominates for $M_{T2} < 80\text{ GeV}$ but is strongly suppressed for $M_{T2} \geq 125\text{ GeV}$. In the signal region, top-quark events dominate the elec-
troweak contribution. The white histogram (black dotted line) corresponds to the LM9 signal. The corresponding event yields for data and SM simulation for the low and high $H_T$ regions are summarized in Table 7.15. Note that the background numbers in Table 7.15 are for guidance only. Dedicated methods to estimate the different background contributions from control regions in the data and simulation are described in the following.

Table 7.15: Observed number of events and expected SM background event yields from simulation in the various $M_{T2}$ bins for the $M_{T2}b$ event selection.

<table>
<thead>
<tr>
<th></th>
<th>QCD multijet</th>
<th>$W+$jets</th>
<th>Top</th>
<th>$Z(\nu\nu)+$jets</th>
<th>Total SM</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$750 \leq H_T &lt; 950$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{T2}[0,\infty]$</td>
<td>2.83e+04</td>
<td>4.53e+02</td>
<td>1.15e+03</td>
<td>1.41e+02</td>
<td>2.97e+04</td>
<td>2.99e+04</td>
</tr>
<tr>
<td>$M_{T2}[125,150]$</td>
<td>5.16</td>
<td>1.86</td>
<td>20.3</td>
<td>0.95</td>
<td>28.3</td>
<td>22</td>
</tr>
<tr>
<td>$M_{T2}[150,200]$</td>
<td>0.16</td>
<td>1.94</td>
<td>17.9</td>
<td>2.00</td>
<td>22.1</td>
<td>16</td>
</tr>
<tr>
<td>$M_{T2}[200,300]$</td>
<td>0.0</td>
<td>1.84</td>
<td>9.43</td>
<td>1.25</td>
<td>12.6</td>
<td>16</td>
</tr>
<tr>
<td>$M_{T2}[300,\infty]$</td>
<td>0.0</td>
<td>0.57</td>
<td>2.55</td>
<td>0.53</td>
<td>3.65</td>
<td>2</td>
</tr>
<tr>
<td>$H_T \geq 950$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{T2}[0,\infty]$</td>
<td>1.19e+04</td>
<td>2.18e+01</td>
<td>5.46e+02</td>
<td>6.51e+00</td>
<td>1.25e+04</td>
<td>1.23e+04</td>
</tr>
<tr>
<td>$M_{T2}[125,150]$</td>
<td>1.25</td>
<td>0.76</td>
<td>9.95</td>
<td>0.64</td>
<td>12.7</td>
<td>10</td>
</tr>
<tr>
<td>$M_{T2}[150,180]$</td>
<td>0.57</td>
<td>0.79</td>
<td>7.15</td>
<td>0.43</td>
<td>8.96</td>
<td>10</td>
</tr>
<tr>
<td>$M_{T2}[180,260]$</td>
<td>0.67</td>
<td>1.09</td>
<td>6.62</td>
<td>0.68</td>
<td>9.06</td>
<td>9</td>
</tr>
<tr>
<td>$M_{T2}[260,\infty]$</td>
<td>0.04</td>
<td>0.76</td>
<td>3.09</td>
<td>0.65</td>
<td>4.55</td>
<td>3</td>
</tr>
</tbody>
</table>

7.4.1 Prediction of the QCD Multijet Background

The QCD multijet contribution is estimated following the same approach as for the $M_{T2}$ analysis. We find that the function in Eq. (7.1) fitted to data in the region $50 < M_{T2} < 80$ GeV provides a good description of the QCD multijet background, also for events containing b-tagged jets. The fit to data is shown in Figures 7.23a and 7.23b for $750 < H_T < 950$ GeV and $H_T > 950$ GeV, respectively. The parameters $a$ and $b$, measured by applying the fit to the QCD multijet dominated region with $50 \leq M_{T2} \leq 80$ GeV, are listed in Table 7.16.

As in the case of the $M_{T2}$ analysis, we conservatively fix the constant $c$ to the value of the exponential at $M_{T2} = 250$ GeV, since it cannot be directly measured in data. The results of the QCD multijet background estimate for the various $M_{T2}$ bins for the low and high $H_T$ regions are shown in Table 7.17.
(a) The ratio \( r(M_{T2}) \) for \( 750 < H_T < 950 \) GeV.

(b) The ratio \( r(M_{T2}) \) for \( H_T > 950 \) GeV.

**Figure 7.23:** The ratio \( r(M_{T2}) \) in data with a fit to data superposed for the \( M_{T2}b \) event selection. The background from electroweak processes and top-quark production are subtracted using simulation. The green curve corresponds to an exponential fit in the region \( 50 < M_{T2} < 80 \) GeV. The blue curve corresponds to an exponential plus a constant where the parameters of the exponential are taken from the previous fit, and the constant term is taken as the value of the exponential at \( M_{T2} = 250 \) GeV.

**Table 7.16:** Results for the parameters \( a \) and \( b \) when the fit is applied to data in the region \( 50 \leq M_{T2} \leq 80 \) GeV. Before the fit is performed, the contributions from electroweak and top backgrounds are subtracted with simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( 750 &lt; H_T &lt; 950 ) GeV</th>
<th>( H_T &gt; 950 ) GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( 2.5 \pm 0.3 )</td>
<td>( 2.2 \pm 0.5 )</td>
</tr>
<tr>
<td>( b ) (GeV(^{-1}))</td>
<td>( (3.1 \pm 0.5) \times 10^{-2} )</td>
<td>( (2.4 \pm 0.8) \times 10^{-2} )</td>
</tr>
</tbody>
</table>

**Table 7.17:** Summary of the QCD multijet background estimate for the signal regions of the \( M_{T2}b \) analysis.

<table>
<thead>
<tr>
<th>( M_{T2} ) (GeV)</th>
<th>( 750 &lt; H_T &lt; 950 ) GeV</th>
<th>( H_T &gt; 950 ) GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>simulation</td>
<td>data-prediction</td>
</tr>
<tr>
<td>([125,150))</td>
<td>5.16</td>
<td>4.1 ( \pm 2.1 )</td>
</tr>
<tr>
<td>([150,200))</td>
<td>0.16</td>
<td>0.90 ( \pm 0.51 )</td>
</tr>
<tr>
<td>([200,300))</td>
<td>0.0</td>
<td>0.04 ( \pm 0.03 )</td>
</tr>
<tr>
<td>([300,\infty))</td>
<td>0.0</td>
<td>0.0 ( \pm 0.0 )</td>
</tr>
</tbody>
</table>
7.4.2 Prediction of the W(\ell\nu)+jets and Top-Quark Backgrounds

Events arising from top-quark production are the dominant background contribution in the signal region. The top-quark contribution is evaluated, together with the one from W(\ell\nu)+jets, in the same way as for the $M_{T2}$ analysis, using single-electron and single-muon events, as well as simulation for taus decaying to hadrons.

Background from Lost Leptons

The lepton control sample consists of events passing the selection criteria of the $M_{T2b}$ analysis, where the lepton veto is omitted and instead the presence of exactly one electron or one muon is requested. As in the case of the $M_{T2}$ analysis, we further impose a $M_T < 100$ GeV requirement to reduce the potential signal contamination and omit the $\Delta\phi_{\text{min}}(\text{jets}, E_{T\text{miss}}) < 0.3$ criteria.

In Figure 7.24 we show the distribution of four kinematic variables for this leptonic event selection. We see that top-quark production is the dominant process. Note that the event yield is systematically overestimated in the simulation: while the shape of the distributions is generally well reproduced, there seems to be an offset in the normalization. This is consistent with what was observed in the b-tagged event selection in Section 7.3.3.1.

Table 7.18: Results for the lost lepton background estimates in the electron channel for the two bins in $H_T$. The naming convention of the variables is the same as in Section 7.3.3.1 (see e.g. Eqn. (7.7)).

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_e^{\text{data}}$</th>
<th>$N_e^{\text{bkg}}$</th>
<th>$\frac{1 - e_e}{e_e} f_{\Delta\phi}$</th>
<th>$N_e^{\text{lost lepton}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$750 &lt; H_T &lt; 950$ GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125 − 150</td>
<td>2</td>
<td>0.92</td>
<td>1.24 ± 0.30</td>
<td>6.15</td>
</tr>
<tr>
<td>150 − 200</td>
<td>5</td>
<td>0.67</td>
<td>1.18 ± 0.27</td>
<td>5.85</td>
</tr>
<tr>
<td>200 − 300</td>
<td>1</td>
<td>0.48</td>
<td>1.25 ± 0.28</td>
<td>3.34</td>
</tr>
<tr>
<td>300 − ∞</td>
<td>0</td>
<td>0.20</td>
<td>0.40 ± 0.48</td>
<td>0.55</td>
</tr>
</tbody>
</table>

| $H_T > 950$ GeV |
|-----------------|------------------|------------------|--------------------------|---------------------|
| 125 − 150       | 3                | 0.31             | 1.14 ± 0.45              | 2.96                |
| 150 − 180       | 4                | 0.20             | 1.29 ± 0.41              | 2.52                |
| 180 − 260       | 1                | 0.38             | 1.37 ± 0.45              | 2.10                |
| 260 − ∞         | 1                | 0.13             | 1.04 ± 0.48              | 1.16                |

In Tables 7.18 and 7.19 we present the lost lepton background estimate for the $M_{T2b}$ signal regions for the electron and muon channels, respectively. For the bins with a zero event yield in data, we take the simulation as background estimate with an uncertainty of 100%. Due to the very limited number of events in the lepton control region in the $M_{T2}$ tail, the background estimate suffers from a large statistical uncertainty. The systematic uncertainty is evaluated the same way as in Section 7.3.3.1.
Figure 7.24: Control plots of the lepton control sample for the $M_{T2b}$ analysis.
Table 7.19: Results for the lost lepton background estimates in the muon channel for the two bins in $H_T$. The naming convention of the variables is the same as in Section 7.3.3.1 (see e.g. Eqn. (7.7)).

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N^{\text{data}}_\mu$</th>
<th>$N^{\text{bkg}}_\mu$</th>
<th>$\frac{1-\varepsilon_\mu}{\varepsilon_\mu} f_{\Delta\phi}$</th>
<th>$N^{\text{simulation}}_\mu$</th>
<th>$N^{\text{lost lepton}}_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$750 &lt; H_T &lt; 950$ GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125 – 150</td>
<td>3</td>
<td>0.54</td>
<td>$1.27 \pm 0.27$</td>
<td>6.68</td>
<td>$3.12 \pm 2.20$ (stat) $\pm 0.96$ (sys)</td>
</tr>
<tr>
<td>150 – 200</td>
<td>3</td>
<td>0.57</td>
<td>$1.03 \pm 0.23$</td>
<td>5.49</td>
<td>$2.51 \pm 1.79$ (stat) $\pm 0.81$ (sys)</td>
</tr>
<tr>
<td>200 – 300</td>
<td>1</td>
<td>0.36</td>
<td>$0.94 \pm 0.33$</td>
<td>2.80</td>
<td>$0.61 \pm 0.94$ (stat) $\pm 0.40$ (sys)</td>
</tr>
<tr>
<td>300 – $\infty$</td>
<td>0</td>
<td>0.17</td>
<td>$0.87 \pm 0.83$</td>
<td>0.74</td>
<td>–</td>
</tr>
<tr>
<td>$H_T &gt; 950$ GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>125 – 150</td>
<td>2</td>
<td>0.18</td>
<td>$1.55 \pm 0.35$</td>
<td>3.20</td>
<td>$2.83 \pm 2.20$ (stat) $\pm 0.69$ (sys)</td>
</tr>
<tr>
<td>150 – 180</td>
<td>2</td>
<td>0.23</td>
<td>$0.88 \pm 0.38$</td>
<td>2.09</td>
<td>$1.55 \pm 1.24$ (stat) $\pm 0.71$ (sys)</td>
</tr>
<tr>
<td>180 – 260</td>
<td>3</td>
<td>0.25</td>
<td>$0.91 \pm 0.27$</td>
<td>2.14</td>
<td>$2.49 \pm 1.57$ (stat) $\pm 0.77$ (sys)</td>
</tr>
<tr>
<td>260 – $\infty$</td>
<td>0</td>
<td>0.06</td>
<td>$0.86 \pm 0.51$</td>
<td>1.07</td>
<td>–</td>
</tr>
</tbody>
</table>

Background from Hadronically Decaying Tau Leptons

As in Section 7.3.3.2, we base the estimate of the background from tau leptons decaying to hadrons on simulation and assign to this background a conservative systematic uncertainty. Figure 7.25 shows the $M_{T2}$ distribution of the events passing the $H_T > 750$ GeV requirement and containing a b-tagged jet as well as a reconstructed hadronic tau decay. For this relaxed event selection, where the modelling can still be verified with data, a reasonable agreement between the simulation and the data is found.

Figure 7.25: $M_{T2}$ distribution for the events with a reconstructed hadronic tau decay. Only the $H_T > 750$ GeV requirement is imposed and a b-tagged jet is requested.
7.4.3 Prediction of the $Z(\nu\nu)+\text{jets}$ Background

The background from $Z(\nu\nu)+\text{jets}$ events is expected to be very small compared to the background from top-quark events. We estimate the background from $Z(\nu\nu)+\text{jets}$ events with the method based on $W+\text{jets}$ events discussed for the $M_{T2}$ analysis. As the selection of $W(\mu\nu)+\text{jets}$ events includes a $b$-tag veto to suppress the top-quark background, a ratio of efficiencies for $W(\mu\nu)+\text{jets}$ events with a $b$ tag to $W(\mu\nu)+\text{jets}$ events without a $b$ tag is taken into account. This ratio is obtained from simulation. Figure 7.26 shows the $M_{T2}$ distributions for the $W$-boson enriched selection (with $b$ tag veto) and for the top quark enriched selection, where the muon momentum is added to $\vec{E}_T^{\text{miss}}$. A good agreement between data and simulation is observed for both distributions. The results for the $Z(\nu\nu)+\text{jets}$ background estimate in the various $M_{T2b}$ signal regions are shown respectively in Tables 7.20 and 7.21 for $750 < H_T < 950 \text{ GeV}$ and $H_T > 950 \text{ GeV}$. The systematic uncertainties are evaluated as in Section 7.3.2.2.

7.4.4 Results

The results of the estimates for the various backgrounds are summarized in Table 7.22 and shown in Fig. 7.27.

![Graphs showing $M_{T2}$ distributions](image-url)
Table 7.20: Results of the Z(ντ)+jets background estimate for the $M_{T2}b$ analysis from W(μν)+jets events for $750 < H_T < 950$ GeV.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{\text{data}}$</th>
<th>$N_{W}^\text{data} \pm \text{sys}$</th>
<th>$N_{\text{top}}^\text{data} \pm \text{sys}$</th>
<th>$N_{\text{other}}^\text{data}$</th>
<th>$R^\text{sim}$</th>
<th>$N_{Z(\nu\bar{\nu})}^\text{pred} \pm \text{sys}$</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>125 − 150</td>
<td>8</td>
<td>$4.50 \pm 1.67$</td>
<td>$1.39 \pm 0.52$</td>
<td>$2.11 \pm 2.11$</td>
<td>$0.10 \pm 0.03$</td>
<td>$0.52 \pm 0.19$</td>
<td>0.9</td>
</tr>
<tr>
<td>150 − 200</td>
<td>12</td>
<td>$8.61 \pm 2.59$</td>
<td>$2.14 \pm 0.71$</td>
<td>$1.25 \pm 1.25$</td>
<td>$0.07 \pm 0.02$</td>
<td>$0.66 \pm 0.20$</td>
<td>1.9</td>
</tr>
<tr>
<td>200 − 300</td>
<td>13</td>
<td>$10.83 \pm 3.03$</td>
<td>$0.72 \pm 0.36$</td>
<td>$1.45 \pm 1.45$</td>
<td>$0.08 \pm 0.02$</td>
<td>$1.01 \pm 0.28$</td>
<td>1.2</td>
</tr>
<tr>
<td>300 − ∞</td>
<td>8</td>
<td>$6.71 \pm 2.37$</td>
<td>$0.08 \pm 0.14$</td>
<td>$1.21 \pm 1.21$</td>
<td>$0.09 \pm 0.03$</td>
<td>$0.63 \pm 0.26$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 7.21: Results of the Z(ντ)+jets background estimate for the $M_{T2}b$ analysis from W(μν)+jets events for $H_T > 950$ GeV.

<table>
<thead>
<tr>
<th>$M_{T2}$ [GeV]</th>
<th>$N_{\text{data}}$</th>
<th>$N_{W}^\text{data} \pm \text{sys}$</th>
<th>$N_{\text{top}}^\text{data} \pm \text{sys}$</th>
<th>$N_{\text{other}}^\text{data}$</th>
<th>$R^\text{sim}$</th>
<th>$N_{Z(\nu\bar{\nu})}^\text{pred} \pm \text{sys}$</th>
<th>simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>125 − 150</td>
<td>9</td>
<td>$7.83 \pm 2.62$</td>
<td>$0.18 \pm 0.18$</td>
<td>$1.00 \pm 1.00$</td>
<td>$0.05 \pm 0.03$</td>
<td>$0.42 \pm 0.14$</td>
<td>0.6</td>
</tr>
<tr>
<td>150 − 180</td>
<td>10</td>
<td>$7.27 \pm 2.40$</td>
<td>$1.81 \pm 0.68$</td>
<td>$0.92 \pm 0.92$</td>
<td>$0.10 \pm 0.03$</td>
<td>$0.88 \pm 0.29$</td>
<td>0.4</td>
</tr>
<tr>
<td>180 − 260</td>
<td>4</td>
<td>$0.93 \pm 0.99$</td>
<td>$2.60 \pm 0.87$</td>
<td>$0.46 \pm 0.46$</td>
<td>$0.06 \pm 0.03$</td>
<td>$0.07 \pm 0.07$</td>
<td>0.6</td>
</tr>
<tr>
<td>260 − ∞</td>
<td>5</td>
<td>$3.62 \pm 1.62$</td>
<td>$0.84 \pm 0.00$</td>
<td>$0.54 \pm 0.54$</td>
<td>$0.16 \pm 0.06$</td>
<td>$0.65 \pm 0.29$</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Table 7.22: Estimated event yields for each background contribution in the various $M_{T2}$ and $H_T$ bins. The predictions from control regions in data are compared to the expected event yields from simulation. Statistical and systematic uncertainties are added in quadrature. The total background prediction is compared to data in the last two columns.
7.5. Statistical Interpretation of the Results and Exclusion Limits

No significant deviation from the SM background prediction is observed and upper limits are set on a potential signal. The statistical approach used to derive limits follows closely the methodology of Ref. [183]. A brief description of the steps relevant to this analysis follows.

First, a likelihood function is constructed as the product of Poisson probabilities for each \((H_T, M_{T2})\) search bin as a function of the signal and background yields

\[
L = \prod_{i=1}^{N_{\text{bins}}} \frac{n_i!}{\mu_i^i n_i!} e^{-\mu_i}, \quad \mu_i = \lambda \cdot s_i + \sum_{j=1}^{N_{\text{bkg}}} b_{ij}.
\]

(7.9)

Here, \(n_i\) denotes the number of observed events in bin \(i\), \(i = 1 \ldots N_{\text{bins}}\). The prediction for the background source \(j\) and the signal in bin \(i\) are denoted \(b_{ij}\) and \(s_i\), respectively. The signal strength modifier \(\lambda\) is introduced to test signal cross section values \(\sigma = \lambda \sigma_{\text{sig}}\).

The predictions for both signal and background yields are subject to various uncertainties that are handled by introducing nuisance parameters \(\theta\). The nuisance parameters affect the results, but are not themselves of interest to the analysis. The signal and background expectations, therefore, become dependent on \(N_{\text{syst}}\) nuisance parameters \(\theta_m\) where \(m = 1 \ldots N_{\text{syst}}\): \(s(\theta_m)\) and \(b(\theta_m)\). The systematic uncertainties are taken to be either (positively or negatively) 100% correlated or fully uncorrelated, whichever is believed to be more appropriate or conservative. The likelihood function now becomes

\[
L(\text{data} | \lambda, \theta) = \text{Poisson}(\lambda \cdot s(\theta) + b(\theta)) \cdot p(\theta),
\]

(7.10)
where \( p(\theta) \) is the probability density function (pdf) associated to the given systematic uncertainty. Different choices of the pdf can be used to describe uncertainties on parameters. We take log-normal distributions as a suitable choice. The contribution to a bin’s prediction from a given systematic source \( \theta_m \) with log-normal uncertainty \( \kappa_{jm} \) in the background estimate of process \( j \), for instance, is given by

\[
b_{ij}(\theta) = b_{ij}^0 \cdot (\kappa_{jm})^{\theta_m},
\]

where \( b_{ij}^0 \) is the central prediction for bin \( i \), and \( \theta \) is a normal random variable with mean zero and unit variance.

In order to compare the compatibility of the data with the background-only and the signal-plus-background hypotheses, we construct the test statistic \( q_\lambda \) based on the profile likelihood ratio

\[
q_\lambda = -2 \ln \frac{L(\text{data}|\lambda, \hat{\theta}_\lambda)}{L(\text{data}|\hat{\lambda}, \hat{\theta})}, \quad \text{with } 0 \leq \hat{\lambda} \leq \lambda,
\]

where “data” can be the actual experimental observation or pseudo-data. Both the denominator and the numerator are maximized. In the numerator, the signal parameter strength \( \lambda \) remains fixed and the likelihood is maximized only for the nuisance parameters, whose values at the maximum are denoted \( \hat{\theta}_\lambda \). In the denominator, the likelihood is maximized for both \( \lambda \) and \( \theta \). \( \hat{\lambda} \) and \( \hat{\theta}_\lambda \) denote the values at which \( L \) reaches its global maximum in the denominator. The lower constraint \( 0 \leq \hat{\lambda} \) is imposed because the signal strength cannot be negative, while the upper constraint \( \hat{\lambda} < \lambda \) guarantees a one-sided confidence interval. The value of the test statistic for the actual observation is denoted \( q_{\lambda}^{\text{obs}} \). Note that this choice of test statistic is different from what was used in Section 3.5.2 and corresponds to the one agreed upon among the ATLAS and CMS experiments [183].

To set limits, a modified frequentist CLs approach is employed [109, 184]. We define the confidence levels (CL) for the background-only hypothesis (\( H_0 \)) and the signal-plus-background hypothesis (\( H_1 \)) as

\[
\text{CL}_{s+b} = P(q_\lambda \geq q_{\lambda}^{\text{obs}}|H_1),
\]

\[
\text{CL}_b = P(q_\lambda \geq q_{\lambda}^{\text{obs}}|H_0).
\]

Correspondingly, the \( p \)-value \( p_b = 1 - \text{CL}_b \) represents the probability for the background to produce a distribution of events as signal-like or more signal-like as the one observed. The \( p \)-value that expresses the compatibility of the data with the signal-plus-background hypothesis is \( p_\lambda = \text{CL}_{s+b} \). The CLs quantity is then defined as the ratio

\[
\text{CL}_{s} = \frac{\text{CL}_{s+b}}{\text{CL}_b}.
\]

In the modified frequentist approach, the value of CLs is required to be less than or equal to \( \alpha \) in order to establish a \((1 - \alpha)\) CL exclusion. To quote the upper limit on \( \lambda \) for a given signal at 95% CL, we adjust \( \lambda \) until we reach \( \text{CL}_{s} = 0.05 \).

To calculate \( \text{CL}_b \) and \( \text{CL}_{s+b} \), we need to construct the pdfs of the test statistic for the background-only and the signal-plus-background hypotheses. This is achieved by generating pseudo-data by randomly tossing Poisson numbers around the expected yields under the assumption of \( H_0 \) and \( H_1 \), after varying
Figure 7.28: A hypothesis testing example. The blue histogram corresponds to the distribution of the test statistic for the signal-plus-background hypothesis, whereas the red histogram represents the one of the background-only hypothesis. The distributions are not normalized to unit area. The value of the test statistic in data is represented by the thick, black vertical line. The red hashed area corresponds to $1 - \text{CL}_b$ and the blue hashed area is $\text{CL}_{s+b}$. The three thin, vertical lines indicate the expected value of the test statistic and the edges of the $1\sigma$ band, computed as the median of the background-only distribution and the quantiles at 16% and 84% probability.
the nuisance parameters according to their pdfs (see e.g. Eqn. (7.11)). Once these pdfs are generated, the observed CL$_s$ limit is calculated from these distributions and the actual observation of the test-statistic $q^\text{obs}_\lambda$. The expected CL$_s$ limit is calculated by replacing $q^\text{obs}_\lambda$ by the median of the test statistic pdf for the background-only hypothesis. Figure 7.28 shows an example of the hypothesis testing for the $M_{T2}$ analysis using a CMSSM signal point with $m_0 = 600\,\text{GeV}$ and $m_{1/2} = 520\,\text{GeV}$. We see that the observed CL$_s$ value is 0.08 whereas the median expected value is 0.03, and consequently, the observed limit does not exclude this particular signal point at 95% CL even though the expected limit would.

### 7.5.1 Exclusion Limits in the CMSSM Plane

Exclusion limits at 95% CL are determined in the CMSSM $(m_0, m_{1/2})$ plane [185]. The signal cross section is calculated at NLO and next-to-leading-log (NLL) accuracy [56, 186, 187]. At each point in the scan four CL$_s$ values are computed for $\lambda = 1$: the observed, the median expected, and the one standard deviation ($\pm 1\sigma$) expected bands. If the observed CL$_s$ value is smaller than 0.05, the point is excluded at 95% CL, resulting in the exclusion limits shown in Figure 7.29. The results from both the $M_{T2}$ and $M_{T2b}$ selections are shown in Figure 7.29a. In Figure 7.29b, the results are combined into a single limit exclusion curve based on the best expected limit at each point of the plane.

The dominant sources of systematic uncertainties on the signal model are found to be the jet energy scale and (for the $M_{T2b}$ analysis) the b-tagging efficiency. These two uncertainties are evaluated at each point of the CMSSM plane, typically ranging from 5 to 25% for the former and from 2 to 6% for the latter. Additionally, a 2.2% uncertainty is associated with the luminosity determination [188]. All these uncertainties are included in the statistical interpretation as nuisance parameters on the signal model.

Observed exclusion limits are also determined when the signal cross section is varied by changing the renormalization and factorization scales by a factor of 2 and using the PDF4LHC recommendation [189] for the parton distribution function uncertainty. The exclusion contours obtained from this method are shown by the dashed curves of Figure 7.29 and referred to as theory uncertainties.

The effect of signal contamination in the leptonic control region could be significant, yielding a potential background overprediction of about 1-15%. To account for this effect, the signal yields are corrected by subtracting the expected increase in the background estimate that would occur if the given signal were present in the data.

The results in Figure 7.29a establish that the $M_{T2}$ analysis is powerful in the region of large squark and gluino masses, corresponding to small $m_0$ and large $m_{1/2}$, while the $M_{T2b}$ analysis increases sensitivity to large squark and small gluino masses, corresponding to large $m_0$ and small $m_{1/2}$. Conservatively, using the minus one standard deviation ($-1\sigma$) theory uncertainty values of the observed limit, we derive absolute lower limits on the squark and gluino masses for the chosen CMSSM parameter set. We find lower limits of $m(\tilde{q}) > 1110\,\text{GeV}$ and $m(\tilde{g}) > 800\,\text{GeV}$, as well as $m(\tilde{q}) = m(\tilde{g}) > 1180\,\text{GeV}$ assuming equal squark and gluino masses.
Figure 7.29: Exclusion limit in the CMSSM \((m_0, m_{1/2})\) plane for the \(M_{T2}\) and \(M_{T2b}\) analyses with \(\tan\beta = 10\). The dark gray area marks the region in which the LSP is a \(\tau\) slepton and not a weakly interacting neutralino. The region where no EWSB is taking place (as \(\mu^2\) is negative) is marked in light gray color. A narrow adjacent strip is not considered in the limit setting since in this region the iterative procedure used to solve the MSSM RGEs does not converge \cite{185}. The regions excluded due to constraints on the slepton and chargino masses from LEP2 searches are also indicated.
7.5.2 Exclusion Limits for Simplified Model Spectra

In this section we interpret the results in terms of simplified model spectra (SMS) [190], which allow a presentation of the exclusion potential in the context of a larger variety of fundamental models, not necessarily in a supersymmetric framework. In SMS, a limited set of new particles and decay chains are used to produce different topological signatures. The new particles are produced in pairs and decay either directly or indirectly via intermediate particles to the LSP and SM particles. We studied the following topologies:

- gluino pair production, with \( \tilde{g} \rightarrow q\bar{q}\tilde{\chi}^0 \);
- gluino pair production, with \( \tilde{g} \rightarrow b\bar{b}\tilde{\chi}^0 \);
- gluino pair production, with \( \tilde{g} \rightarrow t\bar{t}\tilde{\chi}^0 \);
- gluino pair production, with \( \tilde{g} \rightarrow q\bar{q}Z\tilde{\chi}^0 \).

![Diagrams](image)

**Figure 7.30:** Diagrams of the studied SMS topologies: gluino pair production with \( \tilde{g} \rightarrow q\bar{q}\tilde{\chi}^0 \) in Figure 7.30a, \( \tilde{g} \rightarrow b\bar{b}\tilde{\chi}^0 \) in Figure 7.30b, \( \tilde{g} \rightarrow t\bar{t}\tilde{\chi}^0 \) in Figure 7.30c and \( \tilde{g} \rightarrow q\bar{q}Z\tilde{\chi}^0 \) in Figure 7.30d. Figures taken from Ref. [191].

Diagrams illustrating these processes are shown in Figure 7.30. The last of these models is used to demonstrate the sensitivity of the analysis in a high jet multiplicity topology, since the hadronic decay of the Z boson can lead to (maximally) 8 jets in the final state.

In Figure 7.31 the 95% CL excluded cross sections are reported as a function of the relevant masses for gluino pair production with \( \tilde{g} \rightarrow q\bar{q}\tilde{\chi}^0 \) using the \( M_{T2} \) analysis, and for \( \tilde{g} \rightarrow b\bar{b}\tilde{\chi}^0 \), \( \tilde{g} \rightarrow t\bar{t}\tilde{\chi}^0 \) and \( \tilde{g} \rightarrow q\bar{q}Z\tilde{\chi}^0 \).
7.5. Statistical Interpretation of the Results and Exclusion Limits

Figure 7.31: Exclusion limits for simplified model spectra for the \( M_{T2} \) and \( M_{T2b} \) analyses. The signal production cross sections are calculated at NLO and NLL accuracy [56, 186, 187].
Chapter 7. Search for Supersymmetry in Hadronic Final States Using $M_{T^2}$

$q\bar{q}Z\tilde{\chi}^0$ using the $M_{T^2}b$ analysis. Systematic uncertainties on jet energy scale and on b-tagging efficiencies are taken into account as nuisance parameters on the signal model. To minimize the effect of ISR modeling uncertainties, the region near the diagonal is excluded in the limit setting. Observed, median expected, and one standard deviation ($\pm 1\sigma$ experimental) expected limit curves are derived for the nominal signal cross section. Also shown are the $\pm 1\sigma$ variations in the observed limit when the signal cross section is varied by its theoretical uncertainties.

7.6 Conclusions

Summary of the $M_{T^2}$ & $M_{T^2}b$ Analyses

We have conducted a search for supersymmetry or similar new physics in hadronic final states using the $M_{T^2}$ variable calculated from massless pseudojets. $M_{T^2}$ is strongly correlated with $E_T^{\text{miss}}$ for SUSY processes, yet provides a natural suppression of QCD multijet background. The data set for this analysis corresponds to 4.73 fb$^{-1}$ of integrated luminosity in $\sqrt{s} = 7$ TeV pp collisions collected with the CMS detector during the 2011 LHC run. All candidate events are selected using hadronic triggers. Two complementary analyses are performed. The $M_{T^2}$ analysis targets decays of moderately heavy squarks and gluinos, which naturally feature a sizeable $E_T^{\text{miss}}$. This analysis is based on events containing three or more jets and no isolated leptons. We show that the tail of the $M_{T^2}$ distribution, obtained after this selection, is sensitive to a potential SUSY signal. A second approach, the $M_{T^2}b$ analysis, is designed to increase the sensitivity to events with heavy squarks and light gluinos, in which the $E_T^{\text{miss}}$ tends to be smaller. Therefore, the restriction on $M_{T^2}$ is relaxed. The effect of the loosened $M_{T^2}$ is compensated by requiring at least one b-tagged jet and a larger jet multiplicity, to suppress the QCD multijet background. For both analyses, the standard model backgrounds, arising from QCD multijet, electroweak, and top-quark production processes, are obtained from data control samples and simulation. No excess beyond the standard model expectations is found. Exclusion limits are established in the CMSSM parameter space, as well as for some simplified model spectra. Conservatively, using the minus one standard deviation ($-1\sigma$) theory uncertainty values, absolute mass limits in the CMSSM scenario for $\tan\beta = 10$ are found to be $m(\tilde{q}) > 1110$ GeV and $m(\tilde{g}) > 800$ GeV, and $m(\tilde{q}) = m(\tilde{g}) > 1180$ GeV assuming equal squark and gluino masses.

The $M_{T^2}$ & $M_{T^2}b$ Analyses in the Context of the CMS Searches for Supersymmetry

The plethora of potential SUSY signatures at the LHC range from signals resulting in fully hadronic to multilepton final states as well as from energetic signals accompanied by large $E_T^{\text{miss}}$ to signals nearly indistinguishable from the SM backgrounds. For this reason, the strategy deployed by CMS is based on multiple topological searches, which are sensitive to a large number of SUSY models, in combination with a few model-dependent searches targeting specific signatures. As already discussed in Section 2.2.10, the fully hadronic final state is well suited for a generic SUSY search, mainly due to the large production cross section of colored sparticles.
At the time of 7 TeV proton-proton collisions at the LHC there were five main CMS analyses searching for supersymmetry in hadronic final states, namely, the $\alpha_T$ analysis [168, 169], the Razor analysis [167], the “RA2 jets & $H_T^{\text{miss}}$“ analysis [166], the “RA2b b-tagged jets & $E_T^{\text{miss}}$“ analysis [192] and the $M_{T2}/M_{T2b}$ analysis [174]. The sensitivity of all these analyses but the “RA2b” analysis is illustrated in Figure 7.32a, where the results were interpreted in the framework of the CMSSM and the observed exclusion limits are indicated for each analysis. It is seen that the different hadronic analyses have similar sensitivities to a potential signal in a CMSSM-like scenario. In the region of large $m_{1/2}$, where signal events are typically accompanied by large $E_T^{\text{miss}}$, stringent exclusion limits are obtained with the “RA2 jets & $H_T^{\text{miss}}$“ and the “razor” analyses. However, in the region of large $m_0$, where the gluinos are much lighter than the first and second generation squarks and the $E_T^{\text{miss}}$ spectrum tends to be rather soft, the $M_{T2}/M_{T2b}$ analysis yields the most stringent exclusion. An alternative comparison of the sensitivity of the various analyses is shown in Figure 7.32b, where the exclusion limits are presented for the SMS topology of a pair-produced gluino with $\tilde{g} \rightarrow b\tilde{b}\tilde{\chi}^0$, as illustrated in Figure 7.30b.

![CMS Preliminary] L_{\text{int}} = 4.98 fb
\sqrt{s} = 7 \text{ TeV}

(a) Observed limits from several 2011 CMS SUSY searches plotted in the CMSSM $(m_0, m_{1/2})$ plane. The exclusion limits obtained with the hadronic searches are labelled “Jets+MHT”, “Razor”, “$\alpha_T$” and “$M_{T2}$”. This plot is taken from Ref. [193].

(b) Limits on the gluino and LSP masses for the SMS topology of a pair-produced gluino with $\tilde{g} \rightarrow b\tilde{b}\tilde{\chi}^0$ obtained with the CMS searches for supersymmetry in hadronic final states. This figure is taken from Ref. [194] and adapted.

Figure 7.32: Comparison of the 95% CL exclusion limits of several 2011 CMS SUSY searches in the CMSSM and for a specific SMS topology.

In an attempt to understand the overlaps and correlations among the different CMS analyses, lists of events in the signal sensitive regions were compared for data and simulated signal samples. It was found [195] that the overlap among the five mentioned hadronic analyses was surprisingly low, at the level of 1 to $\sim 40\%$ for the dominant backgrounds and less than $\sim 50\%$ for the signal samples. We interpret this result as strong indication that the different analyses add important complementarity to the CMS SUSY search effort. This complementarity mainly arises from the different optimizations of analyses to certain SUSY signatures, resulting, for instance, in a different composition of the SM backgrounds in the signal

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sensitive region. Such redundancy in analyses will be crucial to establish trustworthy evidence for new physics once an excess beyond the SM prediction has been observed.
Chapter 8

SUMMARY AND CONCLUSION

The Large Hadron Collider (LHC) represents the forefront of experimental high energy physics research, promising to shed light on some of the most persisting riddles of particle physics. With the discovery of a new boson with a mass of about 125 GeV at the LHC, a new era of particle physics has been heralded. The future LHC collision data to be recorded at $\sqrt{s} = 14$ TeV will allow us to probe the properties of this new particle in detail and test if it is indeed the elusive SM Higgs boson. Any measured deviation from the SM expectation would be a sign of new physics at the LHC.

We have reasons to believe that such deviations will indeed be found: Though the SM has so far been experimentally confirmed in many of its aspects, a number of outstanding problems remain. For instance, the SM Higgs boson is not naturally light, as radiative corrections drive its mass close to the Planck scale. This “hierarchy problem” cannot be solved in a satisfactory way without introducing new particles. Moreover, the SM fails to explain the nature of Dark Matter, whose existence is inferred from numerous astrophysical and cosmological observations. These arguments suggest that the SM is only a low energy description of a more complete theory.

In the past decades, a tremendous effort has been undertaken to develop new physics models which are consistent with experimental observations and solve some of the SM’s shortcomings. The most popular example is supersymmetry, that solves the hierarchy problem by exploiting the cancellation between the contributions given by fermions and by bosons to the Higgs boson self-energy. Another solution is provided by composite Higgs models. In these models, the Higgs boson is naturally light, since it emerges as a Goldstone boson from a spontaneously broken new global symmetry.

Top Partners in Composite Higgs Models

We argued that in composite Higgs models, the Higgs boson is not necessarily the only composite state of the new sector to be relatively light. In particular, the mixing of the top quark with composite quarks can explain the large top-quark mass. These composite quarks can give significant contributions to the
electroweak precision observables, thus modifying the region of parameter space that is allowed for these models. For this study we focused on a non-minimal realization of these models where two multiplets of top partners in the fundamental representation of $SO(5)$ were introduced.

We discussed the rich collider phenomenology of the introduced top partners and highlighted two distinctive signatures of the studied model, characterized by a full 4 of $SO(4)$ or two charge 5/3 top partners with masses within the LHC reach. Two benchmark points that show such a signature were investigated in detail. By means of a simple analysis we showed that in final states with at least two same-sign leptons a large excess over the SM expectation can be obtained already with a few tens of inverse picobarns of collision data.

Furthermore, a novel method to reconstruct the mass of a charge 5/3 quark was presented. The method relies on the fact that the mass of the charge 5/3 quark is encoded in the invariant mass distribution of the same-sign dileptons arising in its fully leptonic decay. It was shown that the method could be further used to judge whether or not an observed excess in the same-sign dilepton final state is compatible with the presence of a single charge 5/3 quark only, or if it hints at the existence of additional top partners.

Search for Supersymmetry in Hadronic Final States using $M_{T2}$ with the CMS Detector

A brief review of the minimal supersymmetric standard model (MSSM) was presented and the collider signatures of models with $R$-parity conservation were discussed. These signatures are typically characterized by significant missing transverse energy $E_T^{\text{miss}}$ and large hadronic activity arising from the cascade decays of the strongly produced squarks and gluinos to the lightest supersymmetric particle.

The kinematic variable $M_{T2}$ is an extension of the transverse mass $M_T$ in events with two invisible particles. The salient properties of this variable and its ability to separate a potential SUSY signal from the SM backgrounds were discussed in detail. The research presented here pioneered the use of massless pseudojets, defined by grouping jets together, as visible systems for the calculation of $M_{T2}$, in which case the tail of the $M_{T2}$ distribution is highly sensitive to a variety of SUSY signals. Unlike a potential signal, the SM backgrounds populate the regions of small to intermediate values of $M_{T2}$. Most notably, this observation also holds for the large background due to mismeasured QCD multijet events in fully hadronic final states.

A search for supersymmetry in hadronic final states with the CMS experiment was presented, where the $M_{T2}$ variable was used to select new physics candidate events. The 4.73 inverse femtobarns of data used in this analysis were collected in 7 TeV proton-proton collisions at the LHC. Two complementary analyses were performed. The $M_{T2}$ analysis is based on events with three or more jets and targets decays of moderately heavy squarks and gluinos, which naturally feature a sizable $E_T^{\text{miss}}$. A second search, called $M_{T2b}$ analysis, was developed to increase the sensitivity to SUSY models with heavy squarks and light gluinos. For this analysis, the restriction on $M_{T2}$ was relaxed, since the decay of a light gluino is mediated by a virtual squark exchange and consequently only little $E_T^{\text{miss}}$ is produced. Moreover, tagging of $b$ quarks was exploited as an additional handle on the signal-to-background ratio.

For both analyses, the standard model backgrounds arising from QCD multijet, electroweak and top-
quark production processes are obtained from data control samples and simulation. The contribution from each background component was estimated separately using a dedicated method. These elaborate methods complement the already existing techniques used within the CMS collaboration.

The agreement of the data with the SM expectation in the signal sensitive region was quantified by means of a statistical analysis. No excess beyond the standard model expectations was found. Exclusion limits were established in the parameter space of the constrained MSSM (CMSSM) with $\tan\beta = 10$. In this scenario, this search excludes at 95% C.L. the existence of gluinos with mass $m(\tilde{g}) > 800\text{ GeV}$ and of squarks with mass $m(\tilde{q}) > 1110\text{ GeV}$. In the case of equal squark and gluino masses, an exclusion of $m(\tilde{q}) = m(\tilde{g}) > 1180\text{ GeV}$ was obtained.

We also established exclusion limits in a variety of simplified model spectra, which allow a reinterpretation of the results away from the specific mass hierarchies and branching ratios predicted by the CMSSM. The process $p p \rightarrow \tilde{g}\tilde{g}, \tilde{g} \rightarrow b\overline{b}\tilde{\chi}_1^0$, for instance, is excluded at 95% C.L. for gluinos lighter than 1050 GeV and a LSP mass $m_{\tilde{\chi}_1^0} < 400\text{ GeV}$, using cross sections calculated at next-to-leading order in $\alpha_s$.

**Outlook**

At the beginning of the 7 TeV data taking, the long awaited new physics could have been “right around the corner”. However, the analysis of a large dataset of 7 TeV and 8 TeV proton-proton collisions has not revealed any new physics, but instead excluded a significant fraction of the parameter space of many new physics models. Yet, there is still a lot of room for natural new physics to appear at the TeV scale. In supersymmetric models for instance, compressed mass spectra are largely unconstrained and the sensitivity to direct production of third generation squarks is still very limited. The awaited 13 to 14 TeV proton-proton collisions will not only allow us to probe the properties of the Higgs boson, but also lead to a large increase in sensitivity to the already mentioned challenging supersymmetric signatures and to numerous other new physics models. The coming years offer great promise to find new physics at the LHC.
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[108] A. L. Read, *Modified frequentist analysis of search results (The CL(s) method)*.


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