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Asymptotic Freedom at the Berezinskii-Kosterlitz-Thouless Transition without Fine-Tuning Using a Qubit Regularization

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We propose a two-dimensional hard-core loop-gas model as a way to regularize the asymptotically free massive continuum quantum field theory that emerges at the Berezinskii-Kosterlitz-Thouless transition. Without fine-tuning, our model can reproduce the universal step-scaling function of the classical lattice XY model in the massive phase as we approach the phase transition. This is achieved by lowering the fugacity of Fock-vacuum sites in the loop-gas configuration space to zero in the thermodynamic limit. Some of the universal quantities at the Berezinskii-Kosterlitz-Thouless transition show smaller finite size effects in our model as compared to the traditional XY model. Our model is a prime example of qubit regularization of an asymptotically free massive quantum field theory in Euclidean space-time and helps understand how asymptotic freedom can arise as a relevant perturbation at a decoupled fixed point without fine-tuning.

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The success of the standard model of particle physics shows that at a fundamental level, nature is well described by a continuum quantum field theory (QFT). Understanding QFTs nonperturbatively continues to be an exciting area of research, since defining them in a mathematically unambiguous way can be challenging. Most definitions require some form of short-distance [ultraviolet (UV)] regularization, which ultimately needs to be removed. Wilson has argued that continuum QFTs arise near fixed points of renormalization group flows [1]. This has led to the concept of universality, which says that different regularization schemes can lead to the same QFT. Following Wilson, traditional continuum quantum field theories are usually regulated nonperturbatively on a space-time lattice by replacing the continuum quantum fields by lattice quantum fields and constructing a lattice Hamiltonian with a quantum critical point where the long-distance lattice physics can be argued to be the desired continuum QFT. However, universality suggests that there is much freedom in choosing the microscopic lattice model to study a particular QFT of interest.

Motivated by this freedom and to study continuum quantum field theories in real time using a quantum computer, the idea of qubit regularization has gained popularity recently [2–10]. Unlike traditional lattice regularization, qubit regularization explores lattice models with a strictly finite local Hilbert space to reproduce the continuum QFT of interest. Euclidean qubit regularization can be viewed as constructing a Euclidean lattice field theory with a discrete and finite local configuration space, that reproduces the continuum Euclidean QFT of interest at a critical point. If the target continuum theory is relativistic, it would be natural to explore Euclidean qubit regularized models that are also symmetric under space-time rotations. However, this is not necessary, since such symmetries can emerge at the appropriate critical point. Lattice models with a finite dimensional Hilbert space that can reproduce continuum QFTs of interest were introduced several years ago through the D-theory formalism [11,12] and have been proposed for quantum simulations [13,14]. In contrast to qubit regularization, the D-theory approach allows the local Hilbert space to grow through an additional dimension when necessary. In this sense, qubit regularization can be viewed as the D-theory approach for those QFTs where a strictly finite Hilbert space is sufficient to reproduce the desired QFT.

Examples of using qubit regularization to reproduce continuum QFTs in the infrared (IR) are well known. Quantum spin models with a finite local Hilbert space are known to reproduce the physics of classical spin models with an infinite local Hilbert space near Wilson-Fisher fixed points [15]. They can also reproduce QFTs with topological terms like the Wess-Zumino-Witten theories [16]. Gauge fields have been proposed to emerge dynamically at some quantum critical points of simple quantum spin systems [17]. From the perspective of Euclidean qubit

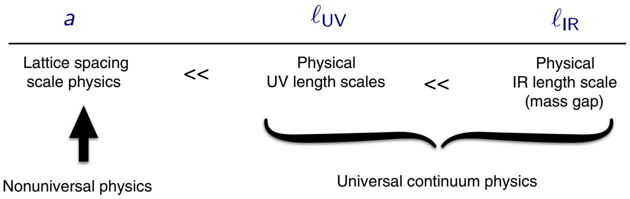

FIG. 1. Length scales in a lattice field theory that reproduces asymptotically free quantum field theories.

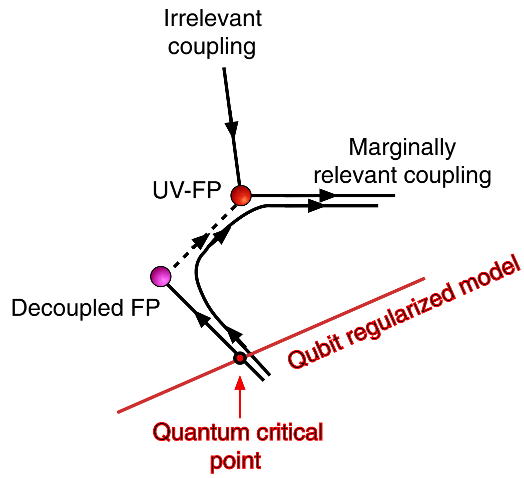

FIG. 2. Illustration of the renormalization group flow of a generic qubit regularized model that reproduces the physics of the asymptotically free QFTs. At the decoupled quantum critical point, qubit models describe the physics of a critical system containing two decoupled theories. However, when a small nonzero coupling is introduced between the theories, the long-distance physics flows toward the desired universal physics of the UV fixed point theory.

regularization, recently it was shown that Wilson-Fisher fixed points with $O(N)$ symmetries can be recovered using simple qubit regularized space-time loop models with $N + 1$ degrees of freedom per lattice site [18,19]. Similar loop models have also been shown to produce other interesting critical behavior [20–22]. Loop models are extensions of dimer models, which are also known to describe interesting critical phenomena in the IR [23,24]. All this evidence shows that Euclidean qubit regularization is a natural way to recover continuum QFTs that emerge via IR fixed points of lattice models.

A nontrivial question is whether we can also recover the physics of ultraviolet fixed points (UV-FPs), using qubit regularization. In particular, can we recover massive continuum QFTs that are free in the UV but contain a marginally relevant coupling? Examples of such asymptotically free (AF) theories include two-dimensional spin models and four-dimensional non-Abelian gauge theories. In the D-theory approach, there is strong evidence that the physics at the UV scale can indeed be recovered exponentially quickly as one increases the extent of the additional dimension [25–29]. Can the Gaussian nature of the UV theory emerge from just a few discrete and finite local lattice degrees of freedom, while the same theory then goes on to reproduce the massive physics in the IR? For this we will need a special type of quantum criticality where three length scales, as sketched in Fig. 1, emerge. There is a short lattice length scale a, where the nonuniversal physics depends on the details of the qubit regularization, followed by an intermediate length scale $\ell_{\rm UV} \gg a$, where the continuum UV physics sets in and the required Gaussian theory emerges. Finally, at long length scales $\ell_{\rm IR} \gg \ell_{\rm UV}$, the nonperturbative massive continuum quantum field theory emerges due to the presence of a marginally relevant coupling in the UV theory. The qubit regularized theory thus reproduces the universal continuum QFT in the whole region $\ell_{\rm UV}$ to $\ell_{\rm IR}$. The special quantum critical point must be such that $\ell_{\rm UV}/a \to \infty$.

Recently, a quantum critical point with these features was discovered in an attempt to find a qubit regularization of the asymptotically free massive nonlinear O(3) sigma model in two space-time dimensions in the Hamiltonian formulation [30]. Using finite size scaling techniques, it was shown that the qubit regularized model recovers all the three scales. In this Letter, we report the discovery of yet another example of a quantum critical point with similar features. In the current case, it is a Euclidean qubit regularization of the asymptotically free massive continuum quantum field theory that arises as one approaches the Berezinskii-Kosterlitz-Thouless (BKT) transition from the massive phase [31,32]. In both these examples, the qubit regularized model is constructed using two decoupled theories and the AF-QFT emerges as a relevant perturbation at a decoupled quantum critical point. The coupling between the theories plays the role of the perturbation that creates the three scales. A generic renormalization group flow diagram of such qubit regularized theories is illustrated in Fig. 2. An interesting feature of this discovery is that there is no need for fine-tuning to observe some of the universal features of the BKT transition that have been unattainable in practice with other traditional regularizations [33].

The BKT transition is one of the most widely studied classical phase transitions, since it plays an important role in understanding the finite temperature superfluid phase transition of two-dimensional systems [34]. One simple lattice model that captures the universal behavior of the physics close to the phase transition is the classical two-dimensional XY model on a square lattice given by the classical action,

$$S = -\beta \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j), \qquad (1)$$

where the lattice field $0 \leq \theta_i < 2\pi$ is an angle associated to every space-time lattice site i and $\langle ij \rangle$ refers to the nearest

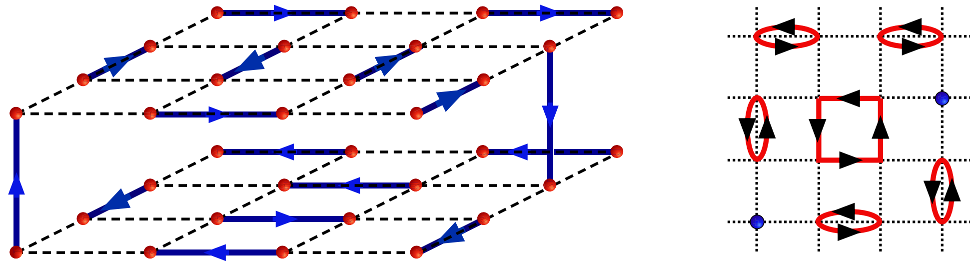

FIG. 3. The left figure shows an illustration of a dimer configuration that contributes to the partition function of the model arising from Eq. (3). Interlayer dimers (or instantons) have weight λ while the intralayer dimers have weight 1. By giving the dimers an orientation as illustrated, each dimer configuration can also be viewed as a configuration of self-avoiding oriented loops. The configuration on the right is such a mapping of the configuration on the left. The instantons are mapped into Fock-vacuum sites, shown as blue circles. The dimer model shows that the loop model is critical when $\lambda = 0$.

neighbor bonds with sites i and j. We refer to this as the ***bXY*** model. The lattice field naturally lives in an infinite dimensional Hilbert space of the corresponding one dimensional quantum model. Using high precision Monte Carlo calculations, the BKT transition has been determined to occur at the fine-tuned coupling of $\beta_c \approx 1.1199(1)$ [35,36]. The BKT phase transition has also been studied using the Villain model [37], which is better suited to analytic computations, as well as topological actions [38]. While the above approaches to the BKT transition require fine-tuning, the massive phase near the transition can be reached without fine-tuning through fermionic models [39]. It was recently shown how the two-flavor Schwinger model at $\theta = \pi$ reproduces the exponentially small mass gap expected near the BKT transition [40]. The model we consider in this Letter is similar, but without explicit gauge fields and constructed via hard-core bosons.

As one approaches the BKT transition from the massive phase, the long-distance physics of the Eq. (1) is known to be captured by the sine-Gordon model whose Euclidean action is given by [39],

$$S = \int dx d\tau \left[\frac{1}{2t}(\partial_\mu \theta_1)^2 + \frac{t}{8\pi^2}(\partial_\mu \theta_2)^2 - \frac{At}{4\pi^2}\cos\theta_2\right], \quad (2)$$

where $t \geq \pi/2$. The field $\theta_1(x,\tau)$ captures the spin-wave physics while the vortex dynamics is captured by the field $\theta_2(x,\tau)$. The BKT transition in this field theory language occurs at $t = \pi/2$, where the $\cos\theta_2$ term becomes marginal as one approaches the critical point and the physics is governed by a free Gaussian theory. In this sense, at length scales much larger than the lattice spacing, the physics of the lattice XY model is the same as an asymptotically free massive Euclidean continuum QFT, when β is tuned to β_c from smaller values.

Qubit regularizations of the classical XY model have been explored recently using various quantum spin formulations [41]. Lattice models based on the spin-1 Hilbert space are known to contain rich phase diagrams [42], and quantum field theories that arise at some of the critical points can be different from those that arise at the BKT transition. Also, the presence of a marginally relevant operator at the BKT transition can make the analysis difficult, especially if the location of the critical point is not known. In these cases, it becomes a fitting parameter in the analysis, increasing the difficulty. Since in our model the location of the critical point is known, our model can be analyzed more easily.

The model we consider in this Letter can be compactly written in terms of the Grassmann integral

$$Z = \int [d\bar{\psi} d\psi][d\bar{\chi} d\chi] \exp\left(\lambda \sum_i \bar{\psi}_i \psi_i \bar{\chi}_i \chi_i\right) \times \exp\left(\sum_{\langle ij \rangle} (\bar{\psi}_i \psi_i \bar{\psi}_j \psi_j + \bar{\chi}_i \chi_i \bar{\chi}_j \chi_j)\right), \quad (3)$$

where on each site i of the square lattice we define four Grassmann variables $\bar{\psi}_i, \psi_i, \bar{\chi}_i$, and χ_i and assume periodic lattices with L sites in each direction. We refer to Eq. (3) as the ***fXY*** model. Using the fermion bag approach [43], we can integrate the Grassmann variables and write the partition function as a sum over dimer configurations whose weight is given by λ^{N_I}, where N_I is the number of instantons (or Fock-vacuum sites). An illustration of such a configuration is given in Fig. 3. The interlayer dimers resemble t'Hooft vertices in the fermionic theory [44–46]. Thus, λ plays the role of the fugacity of instantons. It is easy to verify that the action of our model is invariant under $\bar{\psi}_j \psi_j \to e^{i\sigma_j\theta} \bar{\psi}_j \psi_j$ and $\bar{\chi}_j \chi_j \to e^{-i\sigma_j\theta} \bar{\chi}_j \chi_j$, where $\sigma_j = \pm$ tracks the parity of the site j. The critical behavior of this U(1) symmetry is connected to the BKT transition.

The configurations in Fig. 3 can also be viewed as configurations of oriented self-avoiding loops on a square lattice with Fock-vacuum sites if the dimers are given orientation as explained in the caption of the figure. The model we consider in this Letter is a variant of the qubit regularized XY model introduced in Euclidean space recently [4]. The loop model can be viewed as a certain limiting case of the classical lattice XY model Eq. (1)

written in the world-line representation [47], where the bosons are assumed to be hard-core. The main difference between our model in this Letter and the one introduced in [4] is that closed loops on a single bond are now allowed. Such loops seemed unnatural in the Hamiltonian framework that motivated the previous study, but seem to have profoundly different features in two dimensions because it is possible to view the loop configurations as a configuration of close-packed oriented dimers and argue for a critical point in our model at $\lambda = 0$ and a massive phase for $\lambda > 0$. The previous model does not have this property [48].

Using worm algorithms (see Ref. [49]) we study our model for various values of L and λ. At $\lambda = 0$, one gets two decoupled layers of close-packed dimer models, which is known to be critical [50–53]. The effect of $\lambda \neq 0$ was studied several years ago, and it was recognized that there is a massive phase for sufficiently large values of λ [54,55]. However, the scaling of quantities as $\lambda \to 0$ was not carefully explored. Recently, the subject was reconsidered [56], and the emergence of a long crossover phenomenon was discovered for small λ as a function of L. However, the universal properties of this crossover being related to the UV physics at the BKT transition was not appreciated. In this Letter, we demonstrate that the observed crossover phenomena captures the asymptotic freedom of Eq. (2). We do this by comparing the universal behavior of Eq. (3) with the traditional XY model Eq. (1) near the massive phase of the BKT transition [35,57,58].

To compare universal behaviors of Eq. (1) and Eq. (3) we compute the second moment finite size correlation length $\xi(L)$ defined as $\xi(L) = \sqrt{(\chi/F) - 1}/[2\sin(\pi/L)]$ (see Ref. [59]), where $\chi = G(0)$ and $F = G(2\pi/L)$ are defined through the two point correlation function

$$G(p) = \sum_j e^{ipx} \langle \mathcal{O}^+_{(x,\tau)} \mathcal{O}^-_{(0,0)} \rangle. \quad (4)$$

In the above relation, j is the space-time lattice site with coordinates (x, τ) and $\mathcal{O}^+_j$, $\mathcal{O}^-_j$ are appropriate lattice fields in the two models. In the XY model $\mathcal{O}^+_j = e^{i\theta_j}$, $\mathcal{O}^-_j = e^{-i\theta_j}$, while in the dimer model $\mathcal{O}^+_j = \mathcal{O}^-_j = \bar{\psi}_j \psi_j$. We demonstrate that the step-scaling functions [i.e., the dependence of $\xi(2L)/\xi(L)$ on $\xi(L)/L$] of the two lattice models show excellent agreement with each other in the scaling regime $\ell_{\rm UV} \gg a$, in Fig. 4.

Another interesting universal result at the BKT transition is the value of the helicity modulus, which can be defined using the relation, $\Upsilon = \langle Q_w^2 \rangle$, where Q_w is the spatial winding number of bosonic worldlines. In the XY model Eq. (1), it is usually defined using a susceptibility of a twist parameter in the boundary conditions [35]. In our model, we can easily compute the winding charge Q_w in each loop configuration illustrated in Fig. 3. The universal result in the massive phase as we approach the BKT transition is that $\Upsilon \approx 2/\pi$ in the UV up to exponentially small

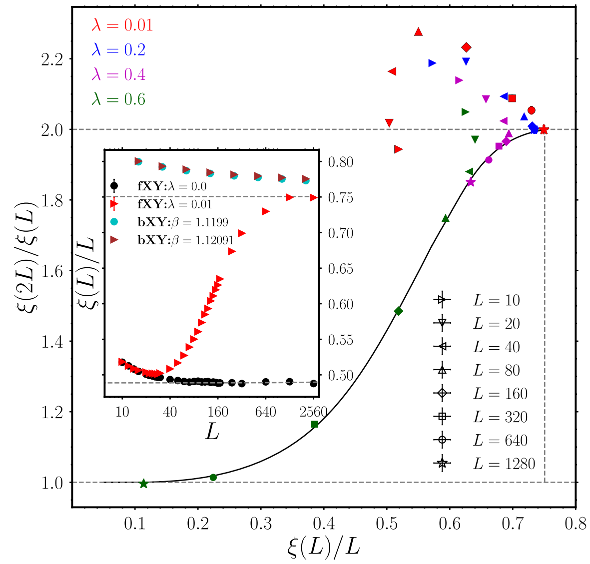

FIG. 4. The figure shows the universal step-scaling function [i.e., $\xi(2L)/\xi(L)$ vs $\xi(L)/L$] obtained from the XY model Eq. (1) (solid line) [60] and compares it with data from the model Eq. (3) at $\lambda = 0.01$ (red), 0.2 (blue), 0.4 (purple), and 0.6 (green), for various lattice sizes shown with different symbols. For small values of L, our data deviate from the solid line. We define $\ell_{\rm UV}$ as the minimum value of L when the data begin to fall on the solid line. From the figure we estimate $\ell_{\rm UV} \approx 80$ for $\lambda = 0.6$ and $\ell_{\rm UV} \approx 160$ for $\lambda = 0.4$. For very small λ we expect the $\xi(L)/L$ to approach the universal UV prediction of $\xi(L)/L = 0.750\,691\,2\ldots$ (see Ref. [35]), when $L \sim \ell_{\rm UV}$ before beginning to follow the solid line. We see this at $\lambda = 0.2$ and 0.01. Since at these couplings $\ell_{\rm UV} > 1280$, we predict that the data at these couplings will also eventually follow the solid line, but only for $L \gg \ell_{\rm UV}$, which we cannot access. To show this feature, in the inset we plot $\xi(L)/L$ as a function of L at $\lambda = 0.01$. Note that the data approaches $\xi(L)/L = 0.750\,691\,2\ldots$ when $L \sim \ell_{\rm UV}$ as expected. Based on our prediction above, this is only a plateau and that for $L \gg \ell_{\rm UV}$ (which we cannot access) $\xi(L)/L$ will eventually approach zero. The inset also shows that the large L behavior of $\lambda = 0$ is very different and stabilizes at $\xi(L)/L = 0.4889(6)$. In the inset we also show the data from [35] in the traditional XY model [Eq. (1)] at two values of β close to the transition. These data are still far from the universal value due to logarithmic corrections as explained in [35].

corrections [35], although in the IR $\Upsilon = 0$. While it is difficult to obtain the UV value in lattice calculations using the traditional model Eq. (1), in our model, we can see it emerge nicely at $\lambda = 0.01$. We demonstrate this in Fig. 5. Again, as expected, the value of Υ when $\lambda = 0$ is very different, since it is a theory of free bosons but at a different coupling. Using the different value of the coupling gives $\Upsilon \approx 0.606$ [60]. Our results provide strong evidence that the AF-QFT at the BKT transition emerges from our dimer model when we take the limit $L \to \infty$ followed by $\lambda \to 0$. The opposite limit leads to the critical theory of the decoupled dimer model.

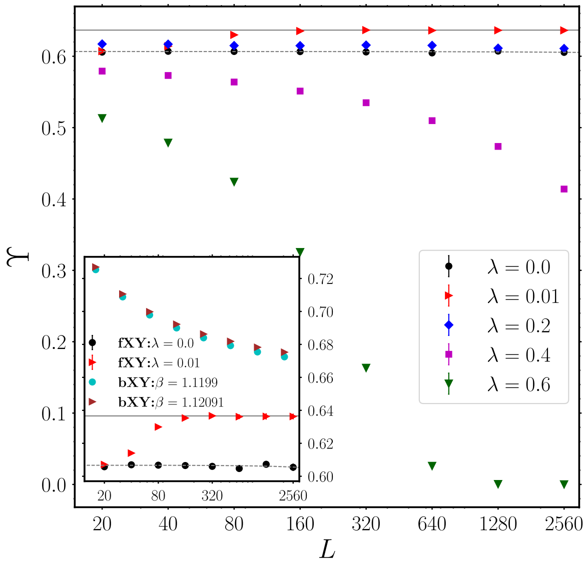

FIG. 5. The figure shows the helicity modulus Υ as a function of L for $\lambda = 0.6$, 0.4, 0.2, 0.01, and 0.0. We expect $\Upsilon \to 0$ for $L \gg \ell_{\mathrm{UV}}$ when $\lambda \neq 0$. This is clearly seen at $\lambda = 0.6$. At $\lambda = 0.4$, since ℓ_{UV} is larger, we only see the initial part of the decrease toward zero. At $\lambda = 0.2$, ℓ_{UV} is even larger, and so we only observe the flat part expected in the UV. At the UV fixed point we expect $\Upsilon \approx 2/\pi$. When $\lambda = 0.01$ we do observe the data converging well to this universal value (top solid line). Since $\lambda = 0.01$ is not really the critical point, this line, too, will eventually turn around at very large values of L and go to zero. On the other hand when $\lambda = 0$ we find that $\Upsilon \approx 0.606$ in the large L limit [60]. We demonstrate the difference between $\lambda = 0$ and $\lambda = 0.01$ in the inset, where we also show data from [35] for the helicity modulus in the traditional XY model Eq. (1) at two values of β close to the transition. Note that the values from the traditional model are far from $2/\pi$, a well-known difficulty due to large logarithmic corrections. In contrast, our qubit model is able to recover the UV physics more easily.

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[1] K. G. Wilson, The renormalization group and critical phenomena, Rev. Mod. Phys. **55**, 583 (1983).
[2] N. Klco and M. J. Savage, Digitization of scalar fields for quantum computing, Phys. Rev. A **99**, 052335 (2019).
[3] A. Alexandru, P. F. Bedaque, H. Lamm, and S. Lawrence (NuQS Collaboration), σ models on quantum computers, Phys. Rev. Lett. **123**, 090501 (2019).
[4] H. Singh and S. Chandrasekharan, Qubit regularization of the $O(3)$ sigma model, Phys. Rev. D **100**, 054505 (2019).
[5] M. C. Bañuls *et al.*, Simulating lattice gauge theories within quantum technologies, Eur. Phys. J. D **74**, 165 (2020).
[6] T. V. Zache, M. Van Damme, J. C. Halimeh, P. Hauke, and D. Banerjee, Toward the continuum limit of a $1+1$D quantum link Schwinger model, Phys. Rev. D **106**, L091502 (2022).
[7] A. Ciavarella, N. Klco, and M. J. Savage, Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis, Phys. Rev. D **103**, 094501 (2021).
[8] A. Mariani, S. Pradhan, and E. Ercolessi, Hamiltonians and gauge-invariant Hilbert space for lattice Yang-Mills-like theories with finite gauge group, Phys. Rev. D **107**, 114513 (2023).
[9] T. V. Zache, D. Gonzalez-Cuadra, and P. Zoller, Quantum and classical spin network algorithms for q-deformed Kogut-Susskind gauge theories, Phys. Rev. Lett. **131**, 171902 (2023).
[10] T. Hayata and Y. Hidaka, String-net formulation of Hamiltonian lattice Yang-Mills theories and quantum many-body scars in a non-Abelian gauge theory, J. High Energy Phys. 09 (2023) 126.
[11] U. J. Wiese, Quantum spins and quantum links: The D theory approach to field theory, Nucl. Phys. B, Proc. Suppl. **73**, 146 (1999).
[12] R. Brower, S. Chandrasekharan, S. Riederer, and U. J. Wiese, D theory: Field quantization by dimensional reduction of discrete variables, Nucl. Phys. **B693**, 149 (2004).
[13] D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U. J. Wiese, and P. Zoller, Atomic quantum simulation of $U(N)$ and SU(N) non-Abelian lattice gauge theories, Phys. Rev. Lett. **110**, 125303 (2013).
[14] U.-J. Wiese, From quantum link models to D-theory: A resource efficient framework for the quantum simulation and computation of gauge theories, Phil. Trans. R. Soc. A **380**, 20210068 (2021).
[15] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, Cambridge, England, 2011).
[16] I. Affleck and F. D. M. Haldane, Critical theory of quantum spin chains, Phys. Rev. B **36**, 5291 (1987).
[17] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Quantum criticality beyond the Landau-Ginzburg-Wilson paradigm, Phys. Rev. B **70**, 144407 (2004).
[18] D. Banerjee, S. Chandrasekharan, D. Orlando, and S. Reffert, Conformal dimensions in the large charge sectors

at the O(4) Wilson-Fisher fixed point, Phys. Rev. Lett. **123**, 051603 (2019).
[19] H. Singh, Qubit regularized O(N) nonlinear sigma models, Phys. Rev. D **105**, 114509 (2022).
[20] A. Nahum, J. T. Chalker, P. Serna, M. Ortuno, and A. M. Somoza, 3D loop models and the CP^{n-1} sigma model, Phys. Rev. Lett. **107**, 110601 (2011).
[21] A. Nahum, P. Serna, A. M. Somoza, and M. Ortuño, Loop models with crossings, Phys. Rev. B **87**, 184204 (2013).
[22] A. Nahum, J. T. Chalker, P. Serna, M. Ortuño, and A. M. Somoza, Deconfined quantum criticality, scaling violations, and classical loop models, Phys. Rev. X **5**, 041048 (2015).
[23] F. Alet, Y. Ikhlef, J. L. Jacobsen, G. Misguich, and V. Pasquier, Classical dimers with aligning interactions on the square lattice, Phys. Rev. E **74**, 041124 (2006).
[24] S. Kundu and K. Damle, Flux fractionalization transition in two-dimensional dimer-loop models, arXiv:2305.07012.
[25] W. Bietenholz, A. Gfeller, and U. J. Wiese, Dimensional reduction of fermions in brane worlds of the Gross-Neveu model, J. High Energy Phys. 10 (2003) 018.
[26] B. B. Beard, M. Pepe, S. Riederer, and U. J. Wiese, Study of CP(N-1) theta-vacua by cluster-simulation of SU(N) quantum spin ladders, Phys. Rev. Lett. **94**, 010603 (2005).
[27] C. Laflamme, W. Evans, M. Dalmonte, U. Gerber, H. Mejía-Díaz, W. Bietenholz, U. J. Wiese, and P. Zoller, $\mathbb{C}$P(N − 1) quantum field theories with alkaline-earth atoms in optical lattices, Ann. Phys. (Amsterdam) **370**, 117 (2016).
[28] S. Caspar and H. Singh, From asymptotic freedom to θ vacua: Qubit embeddings of the O(3) nonlinear σ model, Phys. Rev. Lett. **129**, 022003 (2022).
[29] J. Zhou, H. Singh, T. Bhattacharya, S. Chandrasekharan, and R. Gupta, Spacetime symmetric qubit regularization of the asymptotically free two-dimensional O(4) model, Phys. Rev. D **105**, 054510 (2022).
[30] T. Bhattacharya, A. J. Buser, S. Chandrasekharan, R. Gupta, and H. Singh, Qubit regularization of asymptotic freedom, Phys. Rev. Lett. **126**, 172001 (2021).
[31] V. L. Berezinsky, Destruction of long range order in one-dimensional and two-dimensional systems having a continuous symmetry group. I. Classical systems, Sov. Phys. JETP **32**, 493 (1971).
[32] J. M. Kosterlitz and D. J. Thouless, Ordering, metastability and phase transitions in two-dimensional systems, J. Phys. C **6**, 1181 (1973).
[33] R. Kenna and A. C. Irving, The Kosterlitz-Thouless universality class, Nucl. Phys. **B485**, 583 (1997).
[34] D. M. Stamper-Kurn and M. Ueda, Spinor bose gases: Symmetries, magnetism, and quantum dynamics, Rev. Mod. Phys. **85**, 1191 (2013).
[35] M. Hasenbusch, The two dimensional XY model at the transition temperature: A high precision Monte Carlo study, J. Phys. A **38**, 5869 (2005).
[36] M. Hasenbusch, The three-dimensional XY universality class: A high precision Monte Carlo estimate of the universal amplitude ratio A_+/A_-, J. Stat. Mech. (2006) P08019.
[37] J. V. José, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Renormalization, vortices, and symmetry-breaking perturbations in the two-dimensional planar model, Phys. Rev. B **16**, 1217 (1977).
[38] W. Bietenholz, M. Bögli, F. Niedermayer, M. Pepe, F. G. Rejon-Barrera, and U. J. Wiese, Topological lattice actions for the 2d XY model, J. High Energy Phys. 03 (2013) 141.
[39] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Oxford University Press, New York, 2002).
[40] R. Dempsey, I. R. Klebanov, S. S. Pufu, B. T. Søgaard, and B. Zan, Phase diagram of the two-flavor Schwinger model at zero temperature, arXiv:2305.04437.
[41] J. Zhang, Y. Meurice, and S.-W. Tsai, Truncation effects in the charge representation of the O(2) model, Phys. Rev. B **103**, 245137 (2021).
[42] W. Chen, K. Hida, and B. C. Sanctuary, Ground-state phase diagram of $s = 1$ XXZ chains with uniaxial single-ion-type anisotropy, Phys. Rev. B **67**, 104401 (2003).
[43] S. Chandrasekharan, Fermion bag approach to lattice field theories, Phys. Rev. D **82**, 025007 (2010).
[44] V. Ayyar and S. Chandrasekharan, Massive fermions without fermion bilinear condensates, Phys. Rev. D **91**, 065035 (2015).
[45] V. Ayyar and S. Chandrasekharan, Origin of fermion masses without spontaneous symmetry breaking, Phys. Rev. D **93**, 081701(R) (2016).
[46] S. Maiti, D. Banerjee, S. Chandrasekharan, and M. K. Marinkovic, Three-dimensional Gross-Neveu model with two flavors of staggered fermions, Proc. Sci., LATTICE2021 (**2022**) 510 [arXiv:2111.15134].
[47] D. Banerjee and S. Chandrasekharan, Finite size effects in the presence of a chemical potential: A study in the classical non-linear O(2) sigma-model, Phys. Rev. D **81**, 125007 (2010).
[48] The model introduced in [4], which did not contain two site loops between neighboring sites, was studied in $1 + 1$ dimensions by Hersh Singh, but the results were not published since the BKT transition required fine-tuning as usual.
[49] D. H. Adams and S. Chandrasekharan, Chiral limit of strongly coupled lattice gauge theories, Nucl. Phys. **B662**, 220 (2003).
[50] P. W. Kasteleyn, Dimer statistics and phase transitions, J. Math. Phys. (N.Y.) **4**, 287 (1963).
[51] M. E. Fisher, Statistical mechanics of dimers on a plane lattice, Phys. Rev. **124**, 1664 (1961).
[52] M. E. Fisher and J. Stephenson, Statistical mechanics of dimers on a plane lattice. II. Dimer correlations and monomers, Phys. Rev. **132**, 1411 (1963).
[53] N. Allegra, Exact solution of the 2d dimer model: Corner free energy, correlation functions and combinatorics, Nucl. Phys. **B894**, 685 (2015).
[54] S. Chandrasekharan and C. G. Strouthos, Kosterlitz-Thouless universality in dimer models, Phys. Rev. D **68**, 091502(R) (2003).
[55] N. Wilkins and S. Powell, Interacting double dimer model on the square lattice, Phys. Rev. B **102**, 174431 (2020).
[56] N. Desai, S. Pujari, and K. Damle, Bilayer Coulomb phase of two-dimensional dimer models: Absence of power-law columnar order, Phys. Rev. E **103**, 042136 (2021).
[57] J. Balog, Kosterlitz-Thouless theory and lattice artifacts, J. Phys. A **34**, 5237 (2001).

[58] J. Balog, M. Niedermaier, F. Niedermayer, A. Patrascioiu, E. Seiler, and P. Weisz, Does the XY model have an integrable continuum limit?, Nucl. Phys. **B618**, 315 (2001).

[59] S. Caracciolo, R. G. Edwards, A. Pelissetto, and A. D. Sokal, Asymptotic scaling in the two-dimensional O(3) sigma model at correlation length 10^5, Phys. Rev. Lett. **75**, 1891 (1995).

[60] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.132.041601 for we present a comprehensive overview of our algorithm, the observables, and the parameters used in their measurements. The first section delves into the analytic calculation of helicity modulus, covering both $\lambda = 0$ and $\lambda \to 0$ limits. The second section provides insights into the worm algorithm employed for the complete analysis. To validate the algorithm, the third section entails a comparison between the exact diagonalization (ED) and worm algorithm results on a 2^3 lattice. In the fourth section, critical exponents for the fermionic XY model are discussed, while the fifth section details the computation of the step scaling function for both the traditional (bosonic) and the fermionic XY model. The sixth section addresses the extrapolation of the infinite volume correlation length. The final section presents all our Monte Carlo data.