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THE ONE-STEP SOLUTION
OF THE ADVANCE TUNNEL HEAD PROBLEM

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Abstract. A numerical technique has been developed which uses a single computational step to calculate the stress, deformation and hydraulic head fields around an advancing tunnel heading. This is possible by reformulating and solving the underlying transient equations of poro-elastoplasticity in a reference frame which is fixed to the moving face. This leads to the equations being enhanced by additional terms with the advance rate as a parameter (as well as to additional matrices in the finite element formulation), while the spatial co-ordinate in the direction of tunneling undertakes the role of the eliminated time-coordinate in the integration of the path- or time-dependent constitutive equations. The proposed computational algorithm has considerable advantages in terms of computer- and post-processing time, thereby rendering possible the comprehensive analysis of a wide class of tunneling problems. The numerical examples presented in the paper concern the deformations and the ground pressures developing around an advancing shielded TBM in squeezing ground and the evolution of surface settlement during tunnel excavation in a consolidating ground.
1 INTRODUCTION

In problems characterized by constant conditions in the tunneling direction, the stress- and deformation fields are steady with respect to the tunnel heading, i.e. they “advance” together with the face in the direction of excavation. For this large class of problems, the step-by-step solution method, which models each excavation round by removal of ground elements at the tunnel face and activation of support elements behind the face [1, 2, 3], is computationally inefficient as it approaches the steady state asymptotically after simulating several excavation steps. Such an approach is costly particularly in the case of time-dependent ground behaviour.

The time-dependency of ground behaviour can be traced back to consolidation, creep and, in some rocks, chemical processes as well. It manifests itself in a variety of ways depending upon both the type of ground and the construction method, and may have important implications for the construction process or the life of a tunnel. Creep is associated with the rheological properties of the ground and becomes evident if the ground is overstressed - particularly as the state of failure is approached. It is, therefore, of paramount importance for weak rock under high stress (squeezing conditions). The time-dependency of low-permeability soft ground is mainly due to transient seepage flow processes that are triggered by the tunnel excavation and develop slowly over the course of time: The long-term deformations of the ground include, in general, changes to its pore volume and water content. The latter needs more or less time depending on the seepage flow velocity and thus on the permeability of the ground. In a low-permeability ground, the water content remains constant in the short term. Tunnel excavation generates excess pore pressures, however. As these are higher in the vicinity of the tunnel than further away, seepage flow starts to develop. So the excess pore pressures dissipate over the course of time, thereby altering the effective stresses and leading to additional time-dependent deformations (consolidation). The permeability of the ground is decisive for the rate of the pore pressure dissipation and thus for the time-development of the ground deformations or, if the latter are constrained by a lining, of the ground pressure.

In geological conditions with pronounced time-dependent ground behaviour during construction (e.g., shallow tunnels through clay deposits or deep tunnels through weak rock), the advance rate greatly influences the development of ground pressure and deformation as the deformations due to creep or consolidation are superimposed in the region around the working face upon those occurring due to the three-dimensional redistribution of stress caused by the excavation. Regarding consolidation processes, the higher the advance rate and the lower the permeability, the less will the pore pressures dissipate in the vicinity of the face and, consequently, the smaller the deformations will be. If, on the other hand, the permeability is high and the advance rate low, drained conditions, which are less favourable, will also prevail in the vicinity of the working face. Depending on the relation of advance rate to permeability, the ground response will be “undrained”, “drained” or somewhere in-between. The ratio of advance rate \( v \) to permeability \( k \) governs the stability, the deformations and the ground pressure acting upon a lining or shield in the vicinity of the face. Similar considerations apply for creep [2]: the lower the advance rate, the bigger the deformations will be. In the borderline case of a very high advance rate, only small, elastic deformations develop in the vicinity of the face. Consequently, a high advance rate reduces deformations close to the face, whether the reason for the time-dependency is creep or consolidation.

Consolidation and creep processes in tunneling are in general superimposed on the spatial stress redistribution that takes place in the vicinity of the advancing face. Since high deformation- and pore water pressure- gradients prevail in the vicinity of the tunnel face and the latter moves during the step-by-step simulation, however, either the finite element mesh has to be fine everywhere along the tunnel axis or adaptive re-meshing must be carried-out for each ex-
cavation step. Such an analysis is, therefore, extremely time-consuming even in the case of linear material behavior.

The method proposed in this paper makes possible the solution of the advancing tunnel heading problem in a single computational step, i.e. without an integration in the time-domain. The main idea on how to do this can be traced back to past works on steady crack propagation in elastoplastic media [4] and it is that the time-coordinate can be eliminated from the equations governing the steady state by carrying out appropriate transformations. Since the stress, pore pressure and deformation fields are apparently time-independent for an observer moving with the tunnel face, the obvious way to do that is by re-formulating the equations in a frame of reference that is fixed to the advancing heading (co-ordinate $x_f^*$ in Fig. 1). A similar approach was followed by [5, 6] in order to solve the elasto-viscoplastic tunneling problem and by [7, 8] for the analysis of the transient seepage flow during continuous tunnel excavation. The proposed method concerns the coupled problem of tunnel excavation through a porous, elastic or elastoplastic ground and represents a synthesis of these works. Section 2 outlines the numerical method, while Section 3 presents applications concerning the squeezing pressure developing upon an advancing shield and the settlement above a shallow tunnel.

![Diagram of the problem setup](image)

Figure 1: Layout of a problem having constant conditions in the horizontal direction with respect to geology, initial stress, depth of cover and water level (the co-ordinate $x_f$ is spatially fixed, while the co-ordinate $x_f^*$ is fixed to the advancing tunnel heading)
2 COMPUTATIONAL METHOD

2.1 Governing equations

Hydraulic-mechanical coupled processes are governed by the balance equations

\[ \frac{\partial \sigma \cdot n}{\partial x_i} + b_j = 0 \text{ (equilibrium)} \] (1)

and

\[ \frac{\partial u_k}{\partial x_k} + n c_p \frac{\partial n}{\partial x_k} + \frac{\partial q_k}{\partial x_k} = 0 \text{ (conservation of mass)} \] (2)

where \( \sigma(x_k, t) \), \( b_j \), \( \n c_p(x_k, t) \), \( n \), \( c_p \), \( p(x_k, t) \) and \( q_k(x_k, t) \) denote the stress tensor field, the body force vector, the volumetric strain field, the porosity, the compressibility of pore water, the pore pressure field and the flux vector field, respectively; the kinematic equations

\[ \epsilon_k = 0.5 \left( \frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k} \right) \] (3)

where \( \epsilon_k(x_k, t) \) and \( u_k(x_k, t) \) denote the strain and displacement fields, respectively; and the constitutive equations of seepage flow and stress response which, assuming Darcy's law and elasto-plastic behaviour, read as follows:

\[ q_k = \frac{\partial}{\partial x_k} \left( \frac{\partial n}{\partial x_k} \right), \] (4)

\[ \sigma_k = D_{ijkl} \frac{\partial \epsilon_j}{\partial x_k}, \] (5)

where \( k \), \( \epsilon_k(x_k, t) \) and \( \epsilon_k(x_k, t) \) denote the permeability, the hydraulic head field and the effective stress field, respectively \( (\epsilon_k' = \epsilon_k + \epsilon_k p) \), while the stiffness tensor \( D_{ijkl} \) depends, in general, on the effective stress state and on the direction of the strain increment [9].

2.2 Transformed equations

For the transformation of the equations of poroplasticity in a frame of reference that moves with the advancing tunnel face, we note that the position vector \( x_k^* \) of a point in this frame of reference is given by

\[ x_k^* = x_k - \sum_i \epsilon_k x_i (t) \text{ with } \frac{dx_k^*}{dt} = \dot{v}, \] (6)

where \( x_k \) and \( \dot{v}(t) \) denote the position vector of the point and the location of the face in the spatially fixed coordinate system, respectively, while \( \dot{v} \) is the advance rate and \( \sum_i \epsilon_k \) is Kronecker's delta (Fig. 1). Subsequently, let \( A(x_k, t) \) denote an arbitrary field function (e.g. a component of the stress tensor) and \( A^*(x_k^*) \) the respective function in the face-fixed coordinate system. Since the conditions considered are steady with respect to the moving tunnel face, the transformed function \( A^* \) does not depend on time \( t \). Based upon Eq. (6), the partial derivatives of \( A(x_k, t) = A^*(x_k^*) \) can be expressed in terms of \( A^* \):

\[ \frac{\partial A}{\partial x_k} = \frac{\partial A^*}{\partial x_k^*}, \quad \frac{\partial A}{\partial t} \bigg|_{x_k} = \frac{\partial v}{\partial x_i} A^* = \frac{\partial v}{\partial x_i} A \bigg|_{x_k}. \] (7)
On account of these transformation rules, eqs. (1), (3) and(4), that do not contain time-derivatives, remain formally the same, while the mass balance equation and the constitutive stress-strain relationship in the face-fixed co-ordinate system become:

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\mathbf{u} p) = 0 ,
\]

\[
\frac{\partial \varepsilon_{ij}}{\partial x_i} = D_{ijkl} \frac{\partial u_k}{\partial x_j}.
\]

For a geometrical illustration of the general relationship (7) and a physical interpretation of the transformed equations (8) and (9), refer to [10]. According to Eq. (7), at a steady state prevailing with respect to the moving face, the objective rate of any field variable \( A \) is related to its spatial derivative in the direction of tunnel excavation. Note that the mass balance equation (8) is enhanced by additional terms with the ratio \( \nu/k \) of advance rate to permeability as a parameter, while the co-ordinate \( x_i^* \) in the direction of advance undertakes the role of the eliminated time-coordinate in the incremental stress-strain equations (9). During the iterative solution of the non-linear finite element equations, the elasto-plastic incremental equation (9) has to be integrated repeatedly in order to calculate the residual forces. Due to the path-dependency of the elasto-plastic behavior, the integration at each particular point \( x_i^* \) should take into account its complete stress- and deformation-history. The latter is given by the stress and deformation states of the points of the interval \((x_i^*, x_{i+1}^*)\). Taking into account that the initial stress \( \sigma_{ij}^0 \) prevails far ahead of the face, the integration of Eq. (9) starts with the boundary condition \( \sigma_{ij}^0 (x^*_i) = \sigma_{ij}^0 \) and proceeds along the \( x_i^* \) - axis in an opposite direction to that of the tunnel advance (from right to left in Fig. 1). The calculation of the internal forces resulting from the tunnel support (e.g., from a shield or a lining, the latter possibly having non-linear or time-dependent mechanical properties [10]) is carried out in exactly the same way, the only difference being in the starting condition of the integration along the \( x_i^* \) - axis, as the deformations and the stresses of the support are zero at the installation point \( x_i^* \) (Fig. 1).

### 2.3 Matrix formulation

Since the equilibrium equations (1) remain formally unaffected by the transformations, their spatial discretisation leads to the standard matrix equation of nodal equilibrium:

\[
[K]\{u\} + [L]\{q\} = \{f\}
\]

where \( \omega \), \([K]\), \([L]\), \(\{u\}\), \(\{q\}\) and \(\{f\}\) denote the unit weight of water, the stiffness matrix, the coupling matrix, and the vectors of nodal displacements, hydraulic potentials and external forces, respectively [9]. On the other hand, applying the weighted residual Galerkin method to the transformed mass balance equation (8) leads to

\[
([H]\{V\})\{q\} = \{Q\},
\]

where \([H]\) and \(\{Q\}\) denote the common permeability matrix and nodal source vector, respectively, while \([V]\) and \([W]\) are additional matrices that result from the first, advance-rate-dependent term of Eq. (7). The coefficients of the respective element matrices are given by the following equations:

\[
W_{\text{pp}} = \int_{\Omega_p} \frac{\partial^2 N_p}{\partial x_i^2} N_p \, d\Omega , \quad V_{\text{pp}} = \int_{\Omega_p} \frac{\partial n_c}{\partial x_i} \frac{\partial^2 N_p}{\partial x_i^2} N_p \, d\Omega ,
\]
where $W_{p,k}$ and $V_{p,n}$ denote the contribution of the displacement $u_k$ and of the hydraulic potential of node $n$, respectively, to the mass balance of node $p$; and $N_n$ (with/without overscore) is the (potential/ displacement) shape function of node $n$. The calculation of the 2$^\text{nd}$ order derivatives of the shape functions is carried out by applying the chain rule twice [10].

2.4 Numerical implementation

Apart from the trivial case where $v/k \neq 0$, Eqs. (10) and (11) are coupled and yield the steady state displacement and hydraulic head fields straightforwardly in one step, i.e. without a time-iteration. The difficulties associated with the accuracy and stability of a marching scheme in the time-domain do not arise in this method.

Eq. (11) has been implemented easily in an existing finite-element code (the HYDMEC code of ETH Zurich [22]). It should be noted that the system of equations (10) and (11) is non-symmetric even in the case of a pure seepage flow analysis [8]. Symmetry is lost additionally due to the tunnel support, since the internal forces provided by the support elements depend not only on the displacements of their nodes but also on the nodal displacement at the installation point [6, 10]. This source of non-symmetry can be dealt with numerically by using the symmetric global stiffness matrix and solving the problem iteratively. The iterative approach fails however in the case of a coupled problem due to the additional matrix terms of Eq. (11) resulting from the transformation of the diffusion equation. Consequently, a non-symmetric solver must be used. For very large systems of equations, arising from the discretisation of true three-dimensional problems, iterative solvers may be advantageous. First trials with the flexible version of the generalized minimum residual method in combination with pre-conditioning by incomplete factorization have been successful (the SPARSKIT package of the University of Minnesota [11]). For the numerical examples of Section 3.2 of this paper, that involve about 93,000 nodes and 255,000 degrees of freedom, the PARDISO direct solver of the University of Basel [12] has been employed to solve the system of equations (10) and (11). The parallel solution of the linear equation system takes about 330 CPU seconds on a 3.0GHz, 8-core Intel Xeon-based Mac Pro.

Figure 2: Boundary effect in the case of elasto-plastic material behavior
In order to accelerate the integration of the elasto-plastic incremental equation (9), it is advantageous to choose a structured finite element mesh in the direction of tunnel axis (in the plane of the tunnel cross-section, however, the mesh may be unstructured). Jumping in this way from one Gauss sampling point to the next, the integration of Eq. 9 can be performed stepwise [6]. Attention must be paid to the integration sequence and to appropriate sorting of the Gauss sampling points.

An interesting "boundary effect" can be observed in the case of elasto-plastic behavior. Consider for example the axisymmetric model of an unsupported cylindrical tunnel. Fig. 2 shows the typical development of radial displacement along the tunnel. Note that the displacement increases unexpectedly in the vicinity of the left boundary (point B). Due to the boundary conditions the shear stress $\tau_{22}$ at point B is equal to zero. Consequently, the "elastic" shear strain $\gamma_{EL}^{22}$, depends linearly on the shear stress $\tau_{22}$, and is also equal to zero. The total shear strain $\gamma_{22}$ is, however, positive due to the partially irreversible shearing $\gamma_{22}^{PL}$ that occurred in the vicinity of the tunnel face (note distortion at point A in Fig. 2). From $\gamma_{22} = 0.5 (\bar{\tau}_{22} + \bar{\tau}_{12}) = \gamma_{22}^{PL} > 0$ and $\bar{\tau}_{12} = 0$ (boundary condition) it follows that $\bar{\tau}_{12} = 2 \gamma_{22}^{PL} > 0$, i.e. the axial gradient of the radial displacement at point B depends on the amount of accumulated plastic shear strain. So the observed anomaly results from the prescribed boundary constraints which are incompatible with the deformational state of the ground. According to numerical experiments, the effects of this incompatibility are spatially limited to the vicinity of the boundary by choosing a finer discretisation of the two or three element rows next to the boundary (see Fig. 3 and 6a).

3 APPLICATIONS

3.1 Squeezing pressure acting upon a shield

Squeezing, the phenomenon of large time-dependent deformations triggered by tunnel excavation, occurs mostly in weak rocks with a high deformability and a low strength, often in combination with a high overburden [13]. Squeezing normally develops slowly, although cases have also been known where rapid deformations occurred very close to the working face [14-16]. Laboratory tests, field observations and theoretical considerations show that consolidation processes are, besides creep, also highly important for weak rocks prone to squeezing [17]. It is hardly possible however to distinguish these two mechanisms from another phenomenologically. Here, the generation and subsequent dissipation of excess pore pressures will be considered as the dominant mechanism. In this case, the permeability of the ground governs the rate of pore pressure dissipation and, therefore, the intensity of squeezing. More precisely, where permeability is higher by a factor of ten, rock pressure and rock deformation will develop ten times faster. Permeability is a major source of prediction uncertainty: squeezing rocks, such as shales, mudstones or altered metamorphic rocks, exhibit very low permeabilities. The water inflows in tunnel portions with squeezing conditions are mostly negligible. The permeability coefficient $k$ of a practically impermeable rock (i.e., in the range of $k < 10^{-9}$ m/sec) cannot be predicted reliably even in the specimen scale. Additional difficulties exist in heterogeneous rock masses with alternating layers of weak and hard rock, as the latter are often fractured and therefore increase the overall permeability.

Squeezing ground may slow down or even obstruct TBM advance. As tunnel alignments cannot always avoid difficult geological zones with sufficient reliability, planning a TBM drive in potentially squeezing ground involves the assessment of a series of hazards concerning the machine (blockage of the cutter head, jamming of the shield) or in the back-up area (inadmissible convergences of the bored profile, damage to the support). This analysis has to
take into consideration the characteristics of the different TBM types (thrusting system, type of support, length of shield etc.), as TBM performance is the result of a complex interaction between machine, tunnel support and ground [18].

Subsequently, the role of pore water pressure dissipation will be illustrated by means of numerical results concerning a 500 m deep, Ø 10 m tunnel which crosses weak ground at a depth of 100 m beneath the water table. The numerical investigations are based on an axial symmetric model (Fig. 3). The ground is modelled as a saturated porous medium according to the principle of effective stresses. Seepage flow is taken into account using Darcy’s law. The mechanical behaviour is modelled as an isotropic, linear elastic, perfectly plastic material obeying the Mohr-Coulomb yield criterion and a non-associated flow rule. The material constants (Table 1) are typical for tectonically overstressed rocks from the Gotthard Base Tunnel [19]. For simplicity, both the lining and the shield are assumed to be rigid and only the simplified case of a constant gap size $\delta R$ between shield extrados and bored cross section is considered.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnel radius</td>
<td>$R$ 5 m</td>
</tr>
<tr>
<td>Radial gap size</td>
<td>$\delta R$ 0.05 or 0.15 m</td>
</tr>
<tr>
<td>Advance rate</td>
<td>$v$ variable</td>
</tr>
<tr>
<td>Shield length</td>
<td>10 m</td>
</tr>
<tr>
<td>Coefficient of skin friction</td>
<td>$\mu$ 0.25</td>
</tr>
<tr>
<td>Initial stress</td>
<td>$\sigma_0$ 12.5 MPa</td>
</tr>
<tr>
<td>Initial hydraulic head</td>
<td>$U_0$ 100 m</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E$ 1000 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$ 0.25</td>
</tr>
<tr>
<td>Angle of internal friction</td>
<td>$\phi$ 25°</td>
</tr>
<tr>
<td>Cohesion</td>
<td>$c$ 500 kPa</td>
</tr>
<tr>
<td>Dilatancy angle</td>
<td>$\delta$ 5°</td>
</tr>
<tr>
<td>Permeability coefficient</td>
<td>$k$ variable</td>
</tr>
</tbody>
</table>

Table 1: Parameter values of the deep tunnel example.

Figure 3: Computational domain, boundary conditions and spatial discretisation for a deep tunnel
Figure 4: Contour lines of the pore water pressure and distribution of convergence $u$ along an unsupported tunnel for different values of the ratio of advance rate $v$ to permeability $k$ (parameters: see Table 1).
At the far-field boundary the radial stress is kept constant and equal to the initial value \( \sigma_0 \). The respective hydraulic potential \( \phi \) is fixed to 78 m. This value is slightly lower than the initial value of \( \sigma_0 = 100 \) m. It takes into account the deviation from axial symmetry and it was estimated based upon a preliminary two-dimensional seepage flow analysis. At the excavation boundary the initial stress is reduced to zero. In order to avoid water inflow from the excavation boundary, a mixed seepage flow boundary condition is prescribed (no flow condition in the case of negative pore water pressures, atmospheric boundary pressure in the case of positive pore water pressures).

For simplicity, the hypothetical case of an unsupported tunnel will be considered first. Figure 4 shows, for different \( v/k \) -ratios, the contour lines of pore water pressure and the radial displacement \( u \) of the tunnel wall (after subtracting the deformations taking place ahead of the face). With increasing permeability (or, equivalently, decreasing advance rate), dissipation of the excess pore water pressures accelerates, i.e. it takes place more and more closely to the tunnel heading. The higher the permeability of the ground (or the slower the tunnel excavation), the quicker the consolidation process develops and the larger are the deformations occurring at a fixed distance from the face.

When tunneling with a shielded TBM, some limited convergence can occur, due to the gap between the shield and the surrounding ground. If the convergences develop quickly, the ground closes the gap near to the tunnel face, thereby starting to develop pressure upon the shield. If the permeability is low and the advance rate high, the gap remains open for a longer period and, consequently, the pressure acting upon the shield is lower or even zero. The thrust force needed to overcome shield friction during continuous excavation in given ground conditions depends on the length of the shield, on the amount of overboring, on the skin friction coefficient and on the advance rate. A reduction in the advance rate leads as a rule to an increase in the ground pressure, and thus to further deceleration or even standstill of the machine.

![Diagram](image)

Figure 5: The influence of ground permeability \( k \), of advance rate \( v \) and of amount of overboring \( \Delta R \) on the thrust required in shield tunneling through squeezing ground (parameters: see Table 1).
Figure 5 shows, for the same example, the effect of permeability $k$ on the thrust needed to overcome friction in the presence of a 10 m long shield. The diagram takes into account additionally the thrust needed for the boring process (15 MN). The curves apply to advance rates $v$ of 1 or 10 m/cd and an overboring $\alpha$ of 5 or 15 cm. Accordingly, the thrust requirements can be reduced considerably by a major overboring – this comes, however, at the cost of possible steering difficulties and reduced production rates. If the permeability $k$ is higher than about $10^{-7} - 10^{-8}$ m/s, the thrust requirements reach, in the present example, the limits of feasibility. In the case of a practically impermeable ground ($k < 10^{-10} - 10^{-11}$ m/s) the thrust needed to overcome friction is considerably lower even at moderate advance rates ($v = 1 - 10$ m/cd). In the permeability range of $k = 10^{-10} - 10^{-8}$ m/s, the results are highly sensitive to small variations in permeability, thus indicating a major source of prediction uncertainty as mentioned above.

3.2 Settlement above a shallow tunnel

Field measurements show clearly that settlements induced by tunneling through low-permeability clay deposits may increase for several months after excavation [21]. Groundwater recharge from the surface usually suffices in low-permeability ground for maintaining the water table even if the tunnel lining is unsealed. Due to the drainage action of the tunnel, however, the pore pressures will decrease over the course of time. Since this happens within a zone which is considerably more extended than the influence-zone of the actual excavation, the settlement trough widens over the course of time.

As mentioned above, the higher the excavation rate, the lower will be the deformations of a low-permeability ground in the vicinity of the face. The importance of rapid excavation becomes evident when considering the fact that in closed shield tunneling the deformations of the ground ahead of the face and around the shield (gap closure) represent by far the main sources of volume loss and surface settlement in shield tunneling.

Next, the opposed effects of consolidation and advance rate will be discussed based upon numerical results concerning a cylindrical shallow tunnel. The numerical model (Fig. 6a) consists of 27707 isoparametric, 20-node brick elements with quadratic and linear shape functions for the displacement field and the hydraulic head field, respectively. The tunnel support consists of a 0.20 m thick lining. Lining installation follows the advancing heading at a distance of 1 m (Fig. 6b). Both the ground and the lining have been modeled as linearly elastic materials with the constants given in Table 2. The mechanical boundary conditions are similar to the ones of Figure 3. Concerning the hydraulic boundary conditions, the tunnel heading and the unsupported span of 1 m are modeled as seepage faces under atmospheric pressure (the appropriate boundary condition is $\n = -z$, where $z$ denotes the geodetic height). We assume, furthermore, that the groundwater recharge from the surface (e.g. through rainfalls or an adjacent river or lake) suffices to maintain the elevation of the water table at a constant level ($\n = n_0$, where $n_0$ denotes the initial hydraulic head, i.e. the elevation of the natural, undisturbed water table). The drainage action of the tunnel results then in decreasing pore water pressures in the ground surrounding the tunnel. The decrease in the pore pressures may be temporary or permanent depending on the permeability of the lining. Concerning the latter, two borderline cases will be considered: (a) a highly permeable lining (the condition $\n = -z$ applies to the entire tunnel boundary); (b) an impervious lining (the no-flow boundary condition applies to the supported section of the tunnel boundary).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnel radius</td>
<td>$R$</td>
</tr>
<tr>
<td>Depth of cover (from tunnel axis)</td>
<td>$H$</td>
</tr>
<tr>
<td>Elevation of water table (from tunnel axis)</td>
<td>$H_w$</td>
</tr>
<tr>
<td>Advance rate</td>
<td>$v$</td>
</tr>
<tr>
<td>Coefficient of horizontal initial stress</td>
<td>$K$</td>
</tr>
<tr>
<td>Young's Modulus (ground)</td>
<td>$E$</td>
</tr>
<tr>
<td>Poisson's ratio (ground)</td>
<td>$\nu$</td>
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<tr>
<td>Permeability coefficient</td>
<td>$k$</td>
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<tr>
<td>Total unit weight (ground)</td>
<td>$\gamma$</td>
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<tr>
<td>Unit weight of water</td>
<td>$\gamma_w$</td>
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<tr>
<td>Compressibility of water</td>
<td>$c_w$</td>
</tr>
<tr>
<td>Porosity</td>
<td>$n$</td>
</tr>
<tr>
<td>Distance between lining and face</td>
<td>$e$</td>
</tr>
<tr>
<td>Lining thickness</td>
<td>$d$</td>
</tr>
<tr>
<td>Young's Modulus (lining)</td>
<td>$E_L$</td>
</tr>
<tr>
<td>Poisson's Number (lining)</td>
<td>$\nu_L$</td>
</tr>
</tbody>
</table>

Table 2: Parameter values for the shallow tunnel example.

Figure 6: Computational domain and spatial discretisation for a shallow tunnel
The basic difference between these two cases can be seen in Figure 7 which shows the contour lines of vertical displacement for $v/k > 0$ (i.e., either during excavation in a high permeability ground or at the steady state achieved during a long excavation standstill). In the first case (a highly permeable lining), the displacements increase monotonously and the transversal settlement trough deepens and widens with the distance from the tunnel face (Fig. 7a). In the second case (an impervious lining), a crater-like depression of the surface can be observed above the tunnel face (Fig. 7b). The soil surface experiences firstly a settlement as the tunnel face approaches ("Phase 1") and subsequently a heave ("Phase 2"). The phase 1 settlement is caused by the spatial stress redistribution and by the decreasing pore pressures (consolidation) in the vicinity of the heading. After the installation of the impervious lining, the pore pressure increase gradually and reach their natural values again at a certain distance behind the face. Consequently, the effective stresses decrease (unloading, swelling). The simplified assumption of linear elastic behavior (the same stiffness for loading and unloading) overestimates the swelling strains with the consequence that the phase 2 heave compensates for a considerable portion of the phase 1 settlement. With a more realistic material model (taking into account the stiffer ground response to unloading), this effect would be less pronounced and the final settlement would be governed by the deformations around the tunnel heading.

Figures 8 and 9 show the effect of advance rate $v$ and of permeability $k$ on the longitudinal settlement trough for the case of a highly permeable lining and for the case of an impervious lining, respectively. A more complete picture of the surface settlement is given by Fig. 10. According to Fig. 8, the conditions in the vicinity of the heading are practically undrained for $v/k > 10^6$, and practically drained for $v/k < 10^2$. Depending on the ratio $v/k$, the ground response lies somewhere between the undrained response and the drained response. The higher this ratio is, the further away from the advancing heading will the deformations reach a steady state. Note furthermore that the slope of the longitudinal settlement trough indicates the objective settlement rate since $\partial u/\partial t = -v \cdot \partial u/\partial x_1^*$ (cf. Eq. 7). Figure 9, which applies to the case of an impervious lining, shows that the crater-like depression of the surface occurs only at high permeability values or at low advance rates. High $v/k$-ratios reduce the time available to the consolidation process in the vicinity of the advancing face. This too illustrates the importance of rapid excavation for settlement limitation.
Figure 8: Longitudinal settlement trough for different values of the ratio of advance rate $v$ to permeability $k$ (highly permeable tunnel lining; parameters: see Table 2).

Figure 9: Longitudinal settlement trough for different values of the ratio of advance rate $v$ to permeability $k$ (impervious tunnel lining; parameters: see Table 2).
v/k = 15,600
(e.g., v = 2 m/d, $k = 1.5 \cdot 10^{-9}$ m/s)

v/k = 977
(e.g., v = 2 m/d, $k = 2.4 \cdot 10^{-8}$ m/s)

v/k ≠ 0

Figure 10: Contour lines of surface settlement for different values of the ratio of advance rate $v$ to permeability $k$ (excavation direction from right towards left; parameters: see Table 2).
4 CONCLUSIONS

The stress, deformation and hydraulic head fields around an advancing tunnel heading can be calculated in only one computational step by re-formulating and solving the underlying continuum-mechanical equations in a frame of reference which is fixed to the tunnel face. This approach is applicable even in the presence of consolidation processes or path-dependent mechanical behaviour. The proposed computational algorithm has considerable advantages in terms of computer- and post-processing time, thereby rendering possible the comprehensive analysis of a wide class of tunneling problems.

REFERENCES


